UNIVERSIDADE CATÓLICA PORTUGUESA

Pairs Trading

Cointegration-based methods

Applied to the Cryptocurrency Market

by

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Católica Porto Business School, Universidade Católica Portuguesa
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Resumo

O Pairs Trading tornou-se uma arbitragem estatística de grande relevância para a utilização de testes de cointegração no campo da econometria e da economia. Nesta dissertação, o mercado selecionado a considerar é o mercado das criptomoedas, pelo que se comparam as três abordagens mais conhecidas dos testes de cointegração: o teste Augmented Dickey-Fuller, o teste de Johansen e o teste de Phillips Peron. Os pares são testados quanto à cointegração através do período de 3 meses e de 6 meses, sendo posteriormente transacionados num período com a mesma duração. As criptomoedas incluídas no estudo são as 10 criptomoedas com a maior capitalização de mercado entre 1 de janeiro de 2022 e 1 de julho de 2023. O desempenho de cada carteira é comparado com o portefólio de referência. Embora todas as carteiras tenham apresentado um retorno superior ao portefólio de referência, independentemente de haver ou não custos de transação, fixados em 1%, não é possível concluir se o procedimento de formação e simulação de 6 meses obteve maior rentabilidade em comparação com o procedimento de 3 meses. Das três abordagens propostas, o teste Augmented Dickey-Fuller foi o que melhor conseguiu prever relações de cointegração entre os pares, com um retorno excedente médio de 268,68%.

Palavras-chave: Pairs trading; Cointegração; Criptomoeda

Abstract

Pairs Trading has become a well-known statistical arbitrage for the use of cointegration tests in the field of econometrics and economics. In this thesis, the selected market to consider is the cryptocurrency market, and therefore compare the three best-known approaches of cointegration tests: the Augmented Dickey-Fuller test, Johansen's test, and Phillips Peron's test. The pairs are tested for cointegration through the 3-month and a 6-month period, and later traded in a window of the same length. For the study, it is required to have 10 cryptocurrencies with the highest market capitalization between January 1st,2022 to July 1st, 2023. Each portfolio's performance is evaluated toward the appropriate buy-and-hold benchmark. Although all portfolios outperformed the buy-and-hold benchmark, with and without transaction costs set to 1%, it is not possible to conclude if the 6-month trading and testing procedure yielded a higher return rather to the 3-month procedure. From the three proposed approaches, the Augmented Dickey-Fuller test was best at predicting a cointegrated relationship, with an excess mean return of 268,68%.

Keywords: Pairs Trading; Cryptocurrencies; Cointegration

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Chapter 1

Introduction

The 1987 research made by the economists Clive Granger and Robert Engle published in the highly respected journal Econometrica and made them earn the 2003 Nobel Prize in Economics. The notion of cointegration, first presented by Granger in 1981, was the focus of the study. Granger's attempt to prove to his colleague David Hendry that pairs of integrated series could not constitute a stationary process produced an unexpected discovery, Henry was in fact right and expanded the characteristic to incorporate cointegration. Granger added cointegration to this realization in his development. Granger and Engle worked together to improve the paper after it was first rejected for reasons such as its lack of empirical applicability. This led to the ground-breaking 1987 publication "Cointegration and Error Correction: Representation Estimation and Testing." (Syczewska, 2011).

In the middle of the 1980s, Wall Street quant Nunzio Tartaglia, who worked at American Bank Morgan Stanley alongside with his co-workers, developed the statistical pairs trading with the goal of creating quantitative arbitrage plans with cutting-edge statistical methods.

The relevant aspect concerning pairs trading is to first identify a pair of stocks with similar historical price movements. Then, if there is substantial price divergence, a simultaneous short-long position is created with the prediction that the pair's divergence is just momentary and that, if it diverges, it will eventually converge. By saying this, this approach involves the sequential purchase of an undervalued stock and shorting an overvalued stock, therefore, the value of the long position will rise while the short position will decrease.

Although most of the papers on cointegration are done on stocks, as written by Maggiora and Skerman (2009), Khan (2011) and Wuthisatian (2014), this thesis is based on a more recent market and has yet to explore the cryptocurrency market (Maggiora & Skerman, 2009; Khan, 2011; Wuthisatian, 2014).

This market became acknowledged and more specifically, with the creation of the first cryptocurrency which was Bitcoin in 2008 during the world financial crisis. Since then, it has been considered to be an alternative to the usual or "normal" currencies. All the other cryptocurrencies that are not Bitcoin, are considered Altcoins, which depend on the Bitcoin price.

The main difference concerning the cryptocurrency market is the fact that it is independent of any government and central banks since both of these institutions were the main reason for the world financial crises in 2008, as stated by the creator of Bitcoin, Nakamoto, n.d.. This alternative has several advantages such as reduced transaction costs and speed, diversity, and easy to setup and it is pseudonymous since through the years it has become acknowledged that cryptocurrency leaves a digital footprint which can later be tracked (Nair, 2021). Nevertheless, there are some disadvantages, such as the higher risk of losing the investment since the value of the cryptocurrency differs according to its fluctuation and also since this market is still very unknown and there is still very little information for its investors. Another issue is the possibility of being used for illegal and criminal operations, such as the example of the cryptocurrency Terra which was later discovered to be a scam (Rice, 2019).

The cryptocurrency market has one similar aspect to the stock market, supply and demand determines prices rather than tangible products, meaning that the exchange between cryptocurrencies can fluctuate widely. For some economists, the market price is highly correlated with the cost of producing cryptocurrencies, which is required to use a large amount of electricity. Also, owing cryptocurrency, it is the same as having real and physical coins, which means one has value and can be traded as typical coins. Nowadays, with the evolution of technology and cryptocurrency, it is possible to make online purchases of goods and services and "store them" to increase value over time. The transference between cryptocurrencies is done from one digital wallet to another, and in relation to the wallet is considered to be a personal database which can be stored in a computer, smartphone, or any device that is a cloud and it connects with the user through the use of a digital code (Kristoufek, 2020).

Another aspect related to cryptocurrency is the blockchain which is a shared, immutable ledger that allows transactions to take place in a decentralized manner and the main characteristics are decentralization, persistence, anonymity, and auditability. Blockchain is ideal for delivering information, because not only does it provide immediate and completely transparent information, but it can also be accessed by authorized members (Tapscott & Tapscott, 2016).

The main goal of this dissertation is to examine the profit and risk of a proposed pairs trading strategy for the cryptocurrency market. The date used, as a total of 548 daily observations, which includes the daily closing prices of the 10 cryptocurrencies with the highest market capitalization between January 1st, 2022, and July 1st, 2023. The Augmented Dickey-Fuller (ADF) test will initially be applied to unit root test the cryptocurrencies. Following the ADF test, Johansen's

(JOE) and Phillips Perron's (PP) test will be used to pair and test for cointegration. The chosen pairs will then be combined into a portfolio, which will later be traded and therefore it will analyse its performance, make a comparison to a benchmark portfolio, and present the results.

From the results, it seems as though Engle-Granger's approach employing the Augment Dickey-Fuller test was the best predictor of cointegration relationships. Also, the results do not differ that much from with or without transaction costs.

The remaining part of this thesis is structured as follows. Chapter 2 presents existing literature on pairs trading, correlating with cointegration and later on cryptocurrency markets. Chapter 3 describes the three mains chosen cointegration approaches, which consist of three types of tests, Augmented Dickey Fuller Test, Johansen's Test and Phillips Perron's Test. Chapter 4 explains the methodology used in this dissertation and is followed by a discussion of the results in Chapter 5. To conclude, the last chapter mentions the main aspects that were concluded and expands on shortcomings and possible future research.

Chapter 2

Literature Review

2.1. Pairs Trading Strategy

Since the end of the 1990s, pairs trading alongside with statistical arbitrage and risk arbitrage have become one of the most common strategies to be used by hedge funds. (Gatev et al., 2006) Pairs trading consists of the simultaneous opening of long and short positions in two assets with a balance point between them. In this way, the earnings from a long position cover the losses from a short position and vice versa, meaning that the market risk is close to zero, as is the joint beta strategy. Therefore, the key elements that form the success of a trade consist of determining the balance point between two securities and the point in time that prices move sufficiently away from the balance point to take positions (Blázquez et al., 2018).

In the mid-1980s, the first practice of pairs trading is attributed to the Wall Street quant Nunzio Tartaglia, who worked at the Morgan Stanley group. The main goal was to create quantitative arbitrage strategies using state-of-the-art statistical techniques and one of the techniques, that was developed for trading was concerning trading securities in pairs. The work concerned identifying pairs of securities whose prices tended to move together, meaning that whenever an anomaly in the relationship was happening, the pair would be traded with the idea that the anomaly would correct itself (Vidyamurthy, 2004).

Gatev et al. (2006) published papers on pairs trading, whereas the distance method was introduced, which uses the minimum distance between normalized historical prices to match stocks into pairs. The results show that when trading, the spread deviates two standard deviations from its mean, yielding relatively large annualized excess returns of up to 11% which normally exceed conservative transaction-cost estimates (Gatev et al., 2006).

The cointegration approach, which Vidyamurthy (2004), explains in detail, is one of the first parameterized techniques that is also applied in practice. The paired assets are chosen by Vidyamurthy (2004), following the cointegration connection between two financial instruments. This approach is based on the assumption that two cointegrated assets would track the same long-term trend and, in the event of a deviation, should revert to their mean. The benefit of using cointegration is that it allows one to check the intended mean-reversion of the pair and provides a statistical explanation for the pair's selection. Engle and Granger (1987) and Johansen (1991) tests are the two primary techniques for cointegration testing. Vidyamurthy (2004) offers a framework that might serve as the foundation for further study, even though it does not include actual results of the cointegration approach (Vidyamurthy, 2004).

Nevertheless, in the previous decade, the concept of cointegration was increasingly applied to financial econometrics, in connection with time series analysis and macroeconomics It is an extremely powerful technique, which allows dynamic modelling of non-stationary time series. The fundamental observation that justifies the application of the concept of cointegration in the analysis of stock prices is that a system involving non-stationary stock prices in levels can have a common stochastic trend (Caldeira & Moura, 2013).

2.1.1. Cointegration

Engle and Granger (1987) were pioneers in the development of the concept of cointegration, in which made them win a Noble Prize in Economics in 2003. The concept relies on even if a group of variables are non-stationary, there is still a possibility that in some cases a particular linear combination of this variable is stationary, therefore, the series moves in a lockset-like pattern (Vidyamurthy, 2004).

That means that a linear combination of X_t and Y_t can form an $I(0)$ and a stationary process.

A linear combination of X_t and Y_t is obtained by regressing one of the time series on the other

$$Y_t \ = \ \hat{\beta}_1 + \hat{\beta}_1 X_t + \ \hat{e}_t \tag{1}$$

By taking the residuals we get

$$\hat{e}_t \ = Y_t + -\hat{\beta}_1 + \hat{\beta}_2 X_t \tag{2}$$

If $\hat{e}_t \sim I\ (0)$ and stationary, then X_t and Y_t are cointegrated and $\hat{e}_t$ in the context of pairs trading will be the spread between assets in a pair (Asteriou & Hall, 2006).

2.1.2. Stationary

The basic principle of stationarity is that a stochastic process's behaviour is determined by probability rules that remain constant across time. Regardless of the process's duration, a stationary process's statistical features remain constant.

There are two different kinds of stationarity - strict stationarity, and weak stationarity also referred to as covariance stationarity. A process $\{Y_t\}$ is said to be strictly stationary if the joint distribution of $Y_{t1}, Y_{t2}, \dots, Y_{tn}$ is the same as $Y_{t1-k}, Y_{t2-k}, \dots, Y_{tn-k}$ for all t time periods and all k lags (Cryer & Chan, 2008).

For the purpose of this dissertation, the weak stationary process will be considered, so therefore it does not consider the joint distribution of the random variables.

For a weakly stationary process, it follows that $E(Y_t) = E(Y_{t-k})$ for every t and k. Hence, the mean function is constant over time (Cryer & Chan, 2008).

In addition,

$$Var\ (Y_t) = Var\ (Y_{t-k}), \tag{3}$$

for every t and k which makes the variance constant over time. The covariance is independent of time and only a function of the lag length (Cryer & Chan, 2008).

Therefore, the moments of a weakly stationary process are as follows:

Moment	Criteria	Formally
1st Mean	Mean is constant over time and independent of time	$\mu_t = \mu_{t-k}$
2st Mean	Variance is constant over time and independent of time	$\mu_{t,t} = \gamma_{0,0}$
3st Mean	Covariance is constant over time and independent of time	$\mu_{t,t-k} = \gamma_{0,k}$

Table 1 - Moments of a Stationary Process

Also, the shocks to stationary time series are temporary over time, which mean that the effects of the same shocks will dissipate throughout time. Then the time series will revert to its long-run time level (Asteriou & Hall, 2006).

2.1.3. Random walk

A stochastic process is a sequence of random variables and serves as a model for an observed time series. Also, an important stochastic process for modelling financial assets is the random walk, and the observed process $\{Y_t: t = 0, \pm 1, \pm 2, \dots\}$ is as follows

$$Y_t = 0 \tag{4}$$

$$Y_t = e_1 \tag{5}$$

$$Y_t = e_1 + e_2 \tag{6}$$

$$Y_t = Y_{t-1} + e_2 \tag{7}$$

And the first difference of a random walk because

$$\nabla Y_t = e_t \tag{8}$$

where e_t is a stationary process (Asteriou & Hall, 2006).

2.1.4. White noise

The most fundamental illustration of a probabilistic time series is the white noise. It is constructed by taking a value from a normal distribution at each time instance. In addition, the paraments for the normal distribution do not change over time and are considered to be fixed, so time series in this instance, is equal to repeatedly taking samples from a probability distribution. A white noise process is denoted as

$$Y_t = e_t \tag{9}$$

To sum up this aspect, when taking into consideration the white noise series, the variance of the value at each point in the series is the variance of the normal distribution used for drawing the white noise values (Cryer & Chan, 2008).

2.1.5. Unit Root

A process that regresses on itself is considered to be an autoregressive process. The assumption of an $AR(1)$ model is that the time series of Y_t is mostly determined through the value in the previous period. As such, events in time t are strongly influenced by those in time $t - 1$ and, and events in time $t + 1$ will, in turn, be heavily influenced by the series in the current time t (Asteriou & Hall, 2006).

Consider the following $AR(1)$ model

$$Y_t = \phi Y_{t-1} + e_t, \tag{10}$$

where the residuals are white noise, there are in general three cases:

Case 1: If $|\phi| < 1$ then the series is stationary.

Case 2: If $|\phi| = 1$ then the series is non-stationary, that is has a unit root.

Case 3: If $|\phi| > 1$ then the series will explode.

Also, another way to say this is that to test the order of integration of a series is the same as number of times the series needs to be differenced in order to become stationary which also equals, the number of unit roots.

2.1.6. Augmented Dickey-Fuller Test

The ADF test eliminates any autocorrelation by adding lag terms to the Y variable, which is a unit root test. Either the Schwartz Bayesian criterion (SBC) or the Akaike information criterion (AIC) determines the number of lags. The test has the following form

$$\Delta Y_t \ = a_0 \ + \ a_1 Y_{t-1} \ + \ a_2 t \ + \sum_{i=1}^{n} \beta_i \nabla Y_{t-1} \ + \ e_i, \tag{11}$$

where a_0 is the intercept, $\sum_{i=1}^{n} \beta_i \nabla Y_{t-1}$ is the sum of the differentiated lagged Y's together with their coefficients. The null of the test is $a_0 = 0$ and the alternative hypothesis $a_0 < 1$. Rejecting the null will indicate that Y_t does not exhibit a unit root and therefore is stationary. This is obtained by comparing the ADF test statistic with a critical value at a given significance level (Asteriou & Hall, 2006). The test statistic of the ADF test is given by

$$ADF_{obs} = \frac{\widehat{\alpha_1}}{\hat{\sigma}_{\widehat{\alpha_1}}}. \tag{12}$$

2.1.7. The log-normal process

The log-normal process, which assumes that an asset's price logarithm presents a random walk process, is the most commonly used model for financial asset modelling. This is known as a martingale in probability theory and suggests that the asset's price in the following period will be about equal to its price in the present period. This indicates that, given all previous values, the conditional expectation of a value at the future time point equals the current value. As stated in section regarding the random walk, a stationary process is produced by taking the initial difference of a random walk. This process may also be seen as the asset's return or the increase in the value of a random walk at a specific moment (Vidyamurthy, 2004).

Similarly, a normal distribution may be defined as the set of improvements from a random walk that are generated by taking the first difference. Nevertheless, the predicted increment of a random walk is zero due to the random walk's martingale feature, which is inconvenient when attempting to predict asset values in order to make money. The predicted value two steps further in time is still zero, but with increased variance. Nonetheless, the researcher can forecast the increase to the next value in a stationary process due to the mean-reverting characteristic of stationary time series. Even so, financial assets are described as non-stationary random walks, where the expected value is equal to the current value. However, the researcher can identify linear combinations of assets whose time series are combined stationary and so predictable because of cointegration (Vidyamurthy, 2004).

2.2. Cryptocurrency Market

Cryptocurrency is a decentralized medium of exchange which uses cryptographic functions to conduct financial transactions. Cryptocurrencies leverage the Blockchain technology to gain decentralization, transparency, and immutability (Tapscott & Tapscott, 2016).

Cryptocurrencies are a digital method which relies on transaction verification independent of banks. It’s a peer-to-peer system that can allow anyone anywhere to send and receive payments. Cryptocurrency payments exist only as digital entries to an online database that describe individual transactions, as opposed to actual money that is carried about and traded in the real world. The transactions that are made while transferring Bitcoin money are entered into a public ledger. Digital wallets are where cryptocurrency is kept. The reason cryptocurrency got its moniker is because it verifies transactions via encryption. This means advanced coding is involved in storing and transmitting cryptocurrency data between wallets and public ledgers, whereas the main aim of encryption is to provide security and safety. Also, cryptocurrency exchanges are open 7 days a week, 365 days a year (Pacheco, 2022).

The emerge of cryptocurrency happened after the publication of the influential whitepaper by Nakamoto, which resulted in the creation of Bitcoin in 2009. At that time, this was described as "A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution". Until this day, the whereabout of the creator of Bitcoin is unknown, remaining the main mystery of the cryptocurrency market, since it was the first and the biggest cryptocurrency to ever been created. Later, in 2015, Ethereum was launched, which is a special blockchain with a special token called Ether. This cryptocurrency has a relevant feature which is the ability to create new tokens on the Ethereum Blockchain. After that, several cryptocurrencies have emerged and nowadays the cryptocurrency market is composed by 23,000 cryptocurrencies (CoinMarketCap, 2023; Yang, 2019).

Digital currency exchange (DCE) or cryptocurrency exchange is a business that permits customers to trade cryptocurrencies. Cryptocurrency exchanges can act as matching platforms by collecting a charge or as market makers, often utilizing the bid-ask spread as compensation for services. (Fang et al., 2022).

Also, it is required to mention the existing of blockchain, which is a digital ledger of economic transactions that can be used to record not just financial transactions, but any object with an intrinsic value (Tapscott & Tapscott, 2016). To simplify, a Blockchain is a collection of timestamped, immutable data records that are maintained by a group of independent computers. These data blocks are linked to one another in a chain and are all secured by cryptographic principles.

The process by which miners add blocks to the Blockchain involves retrieving transactions from the previous block, combining them with the hash of the previous block to get a new hash, and storing the resultant hash in the current block. Transactions are accepted by Blockchain miners, who then authenticate and disseminate them throughout the network. Each node has to enter the transaction into its database after the miner validates it (Fang et al., 2022).

Yang (2019), as well as Krafft et al. (2018), analysed market dynamics and behavioural anomalies respectively to understand effects of market behaviour in the cryptocurrency market. The second paper. discussed potential ultimate causes, potential behavioural mechanisms and potential moderating contextual factors to enumerate the possible influence of GUI and API on cryptocurrency markets. Then it was highlighted the potential social and economic impact of human-computer interaction in digital agency design. On the other hand, Yang (2019), applied behavioural theories of asset pricing anomalies in testing 20 market anomalies using cryptocurrency trading data. The results showed that anomaly research focused more on the role of speculators, which gave a new idea to research the momentum and reversal in the cryptocurrency market (Yang, 2019; Krafft et al., 2018).

Huang (2024), in the paper, had the main goal of comprehending the relationship between cryptocurrencies and the financial market. Current studies focus on correlation, and if two markets are highly associated, they can hedge risk. If there was little correlation, a diversified portfolio may be constructed. If, in times of market turbulence, the correlation between the two markets remains relatively low and steady, then the safe haven hypothesis is supported. Nonetheless, the majority of these studies are able to identify a meaningful connection but seldom concentrate on the cause of the association. Though it is also more likely to be the result of an external shock, there may be an interaction between the two markets. Therefore, opting to identify causality in the study approach as opposed to correlation raises the possibility of receiving market signals for risk avoidance and gaining a deeper understanding of the interconnections between these two markets. The results obtained consists of the fact the cryptocurrency market is independent of the financial market and that cryptocurrencies can be used in order to diversify risk and be an alternative investment that is not related to the traditional market (Huang, 2024).

Chapter 3

Cointegration-Based Methods

Lucas (1999) and Alexander (1999) were the initial researchers who demonstrated how the cointegration approach may be applied to asset allocation. The main characteristics, for instance, mean reverting tracking errors, enhanced weight stability and better use of the information comprised in the stock prices, allow a flexible design of various funded and self-financing trading strategies, from index and enhanced index tracking to long-short market neutral and alpha transfer techniques (Caldeira & Moura, 2013).

3.1. Engle-Granger's approach

Engle and Granger (1987) introduced an approach to test for cointegrated relationships between different time series. In order to comprehend the approach, it is necessary to consider two given time series X_t and Y_t, where X_t is $I(0)$ and Y_t is $I(1)$. Thereby, any linear combination of the series

$$\theta_1 X_t + \theta_2 X_t \tag{13}$$

will always be $I(1)$, that is non-stationary since the behaviour of the non-stationary $I(1)$ series will dominate the behaviour of the stationary series.

Nevertheless, if X_t and Y are both $I(1)$, then a linear combination of the series in the equation above 13 is most likely to be non-stationary$I(1)$ too. Although this is the most common case, there are some exceptions in which for instance, two non-stationary time series can be linearly combined to produce a stationary process and therefore the time series can be cointegrated.

Also, it is demanding to estimate the parameters of the long-term relationship and whether the time series are cointegrated, and therefore, to verify for cointegration and estimate the parameters of the relationship, Engle and Granger proposed a method, which is as follows:

First of all, it is required to test if the time series are integrated in the same order. For the purpose of this thesis, it is tested through the Augmented Dickey-Fuller test, so it can be concluded the number of unit roots. It is relevant that the time series cannot be stationary and must be integrated in the same order. Second, if the variables are integrated in the same order, the long-run relationship is estimated by regressing one variable on the other.

$$Y_t \ = a_0 \ + \ \beta_1 X_t + e_t \tag{14}$$

which can be written as

$$e_t \ = Y_t - a_0 \ - \ \beta_1 X_t \tag{15}$$

If e_t is stationary, $(I(0))$, then the variables are cointegrated. This can be tested through two tests, ADF or the PP test, only this time on the residual time series. If the test is rejected, it can be concluded that the variables have a cointegrated relationship. The procedure towards the JOE test will be outlined below in section 3.3.

One of the drawbacks of using this strategy is that it is not possible to identify the regressor. As an example, consider the time series X_t and Y_t, the approach does not explain which time series to regress on the other and why. One can either regress X_t on Y_t or vice versa. Asymptotic theory suggests that as the sample size approaches infinity, testing for cointegration on the residuals of two regressions becomes equivalent. However, in practical economic applications, large samples are rarely available. Consequently, it is common to observe a scenario where one regression indicates cointegration while the other does not, due to the limitations of having relatively small sample sizes in real-world economic studies (Asteriou & Hall, 2006).

Nevertheless, there are limitations such as that when a process is stationary, yet close to having a unit root, the power is considerably low and also, the null is rejected when the moving average root of the process is negative. Therefore, the Augment Dickey-Fuller test can be tested for more than one cointegrated relationship.

3.2. Phillips-Perron Test

The PP test was developed as a generalization of the ADF test, which makes a more reasonable assumption regarding the error terms.

The regression test takes the following $AR(1)$ form.

$$Y_t = a_0 + a_1 Y_{t-1} + e_t, \tag{16}$$

where the null is that $a_1 = 1$ and the alternative that $a_1 < 1$. Rejecting the null will indicate that Y_t does not have a unit root and is therefore stationary.

The PP test modifies the coefficient a_1 from the $AR(1)$ regression for the serial correlation in e_t, whereas the ADF test adds lag differentiation terms to accommodate higher-order correlations. Also, it is relevant to mention that this thesis does not cover the derivation of the PP test.

However, there are several restrictions. For example, the power is very low when a process is stationary but on the verge of having a unit root; also, the null hypothesis is rejected when the process's moving average root is negative. As a result, many cointegrated relationships may be tested using the Phillips-Perron test.

3.3. Johansen's approach

In order to comprehend the Johansen's test, it is essential to understand Vector autoregression (VAR). A vector autoregression is a matrix that contains two or more regressions, in each variable is regressed on n number of lags of the other variables and n number of lags of the variable itself. Each variable is also regressed on a constant. A VAR system can take the following equation.

$$Y_t \; = a \; + \; \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \ldots + \beta_n Y_{y-n} \; + \; e_t, \tag{17}$$

where Y_t is a vector, β_k act as an j by j matrix of the coefficients, where $k = 1,2,3, \ldots, n$, a represent a j by one matrix of the constants and e_t represent the error terms in the same matrix as a.

For instance, if a model has three or more variables, there is then a possibility that more than one cointegration relationship exists. For n number of variables that can be at most $(n - 1)$ cointegration. The JJ test can detect multiple cointegrated relationships through the use of a VAR system, and as comparing with the Engle and Granger's approach, whose can only detect one cointegrated relationship.

The derivation of Johansen's approach to detect cointegration for a vector of two-time series $X_t = [Y_t, Z_t]$, is as follows,

$$\begin{cases} Y_t & = \pi_{11}Y_{t-1} + \pi_{12}Z_{t-1} + e_{1t} \\ Z_t & = \pi_{21}Y_{t-1} + \pi_{22}Z_{t-1} + e_{2t} \end{cases}. \tag{18}$$

Now Y_t and Z_t are cointegrated, if

$$\begin{cases} \Delta Y_t & = \alpha(\beta_1 Y_{t-1} + \beta_2 Z_{t-1}) + e_{1t} \\ \Delta Z_t & = \alpha(\beta_1 Y_{t-1} + \beta_2 Z_{t-1}) + e_{2t} \end{cases}, \tag{19}$$

where $\beta_1 Y_{t-1} + \beta_2 Z_{t-1}$ is a stationary process.

This can also be represented using matrices.

$$\begin{pmatrix} \Delta Y_t \\ \Delta Z_t \end{pmatrix} = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \cdot \begin{pmatrix} Y_{t-1} \\ Z_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}. \tag{20}$$

Then Y_t and Z_t are cointegrated, if

$$\begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} = \begin{pmatrix} \alpha_1\beta_1 & \alpha_1\beta_2 \\ \alpha_2\beta_1 & \alpha_2\beta_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \cdot (\beta_1\ \beta_2), \tag{21}$$

where $\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix}$

Thereby Y_t and Z_t are cointegrated if the rank of Π is one. The rank of the matrix Π represents the maximum number of linearly independent rows of Π(Asteriou & Hall, 2006).

The rank of Π is estimated through two different ratio tests, in which both are based on eigenvalues, that is the number of characteristic roots.

The first method tests the null that $Rank(\Pi) = r$ against the alternative that $Rank(\Pi) = r + 1$. In other words, the null is that there are r cointegrated vectors and at most r cointegrated relationships. Meanwhile, the alternative suggests that there are $r + 1$ vectors. The method orders the eigenvalues in descending orders and tests if they are significantly different from zero. For example, consider n characteristic roots, $-\lambda_1 > \lambda_2 > \lambda_3 > \cdots > \lambda_n$. If there is no cointegration, then all roots will be equal to zero. Hence, $-T \ \ln(1 - \hat{\lambda}_{r+1})$ will also be zero. Nonetheless, if the rank is equal to one implying one cointegrated relationship, then $\lambda_1 > 0$ which leads to $-T \ \ln(1 - \hat{\lambda}_{r+1}) < 0$.

There are two methods to get the statistics used to test if the characteristic roots are different from zero. The first is as follow

$$\lambda_{max}(r, r + 1) = -T \ \ln(1 - \hat{\lambda}_{r+1}) \qquad (22)$$

The second method is conducted by the likelihood ratio test for the trace of Π.

The null, in this case, is that the number of cointegrated vectors is at most r. Where the test statistic is

$$\lambda_{trace} = -T \ \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_{r+1}) \qquad (23)$$

However, the Johansen's test relies on the assumption that the cointegration vector remains constant throughout the testing period which is a long assumption especially when the test period is long since long-run relationships of the underlying variable can vary. Furthermore, applying the VAR approach is of theoretical nature which may make it more difficult to understand the model.

Chapter 4

Methodology and Data

4.1. Data

The data required for this thesis has been imported from Coin Market Cap, which consists of data from the 10 cryptocurrencies with the highest market capitalization on January 1st, 2022. In terms of time series, it is from January 1st, 2022, to July 1st, 2023. Unlike stocks, cryptocurrencies are traded 24 hours daily, so there isn't any closing day to consider. The price used in the tests will be the 24-hour price change and are denoted in United States dollars ($).

For this thesis, the best 10 cryptocurrencies do not include cryptocurrencies that were created after January 1st, 2022. The selected cryptocurrencies and each market capitalization between the specific time series are mentioned in Table 3, below presented. A brief description of each cryptocurrency can be found in Appendix A.

	Name	Symbol	Market Cap (01/01/2022)	Market Cap (01/07/2023)
1	Bitcoin	BTC	902104M$	593974M$
2	Ethereum	ETH	448538M$	231370M$
3	Solana	SOL	55189M$	7479M$
4	Cardano	ADA	46109M$	10216M$
5	XRP	XRP	40380M$	24744M$
6	Polkadot	DOT	28229M$	6423M$
7	Dogecoin	DOGE	22957M$	9587M$
8	Shiba Inu	SHIB	18743M$	4499M$
9	Algorand	ALGO	11090M$	941M$
10	Litecoin	LTC	10447M$	7857M$

Table 2 - Cryptocurriencies by Market Capitalization

4.2. Selection of Cryptocurrencies

When selecting the cryptocurrencies for the thesis, given the date is January 1st, 2022, it was required to know information and details on each one to ensure that the data selected is reliable and relevant. Having said this, it was necessary to eliminate 10 cryptocurrencies, which are, Binance Coin, Tether, USD Coin, Terra, Avalanche, Polygon, Crypto.com, Binance USD, Wrapped Bitcoin and Uniswap.

Both cryptocurrencies Avalanche and Polygon had to be ruled out from the relevant data for the thesis, simply because the initial date is different from the rest of the other cryptocurrencies. As mentioned, the initial date is January 1st, 2022, whereas the initial information data for these cryptocurrencies is January 2nd, 2022.

Tether, USD Coin, Binance USD and Wrapped Bitcoin are considered to be stablecoins, meaning that the main goal of each one is to reduce the price volatility about either a unit of account or a store of value. In this case, these four coins are related to the US dollars. For this reason, its data would not be the most truthful since the market fluctuations would be concerning the market value of the US dollar (Richards, 2018).

Binance coin, Crypto.com and Uniswap are exchange coins, which function in a very similar way to stock exchanges. Its main purpose is to work within the specific exchange platform making it more accessible for security hacks or violations of the users. Furthermore, exchange coins might not be fully reliable, by not offering enough data, as well as not being sufficiently transparent when it comes down to managing them (Adams et al., 2021; Pacheco, 2022).

Terra was created by Terraform Labs in 2018, belonging to Do Kwon and Daniel Shin. At that time, this cryptocurrency was created to be an algorithmic stablecoin to the Terra network (the specifics of a stablecoin were already mentioned previously). In this case, the cryptocurrency was manipulated through an algorithm than rather a unit of account, as per usual the US Dollar. In May 2022, a valuable amount of the cryptocurrency was unstaked[1], causing the stablecoin to be depeg[2] and therefore, investors started to sell the cryptocurrency and due to this, it proved to be a scam and therefore caused the crash of the cryptocurrency (Briola et al. 2022).

4.3. Selection of pairs

In order to form pairs, it is required to test if there is cointegration and also, if each one of the time series has a unit root. The maximum value of cointegrated pairs is 45 when conducting these tests with 10 cryptocurrencies. The maximum number of pairs is given by

$$\frac{n(n-1)}{2} \tag{24}$$

[1] The term "unstaked" describes the procedure of taking assets out of a blockchain network that have previously been locked or staked.

[2] A depeg is a situation where the value of a stablecoin fluctuates in relation to the asset it is tethered to.

where n is the number of cryptocurrencies.

4.3.1. Identifying pairs – ADF, PP, and JOE method

As previously mentioned on the subject of 3.1., it is not relevant to test stationary time series for cointegration. Having said this, first, all the cryptocurrencies must be tested on whether their individual time series have a unit root by the ADF test. Those that have a unit root can then be tested for cointegration with other cryptocurrencies according to the ADF, PP and JOE test and must be made at the 5% significance level.

When testing for a unit root, each cryptocurrency will be tested on whether each individual time series is integrated into order 1. For the purpose of this thesis, each cryptocurrency will only be tested if it is integrated into order 1 and it can only be tested through the ADF test even though time series can potentially be integrated into different orders.

Concerning the ADF and PP cointegration test, it is necessary to regress the time series on each other and determine if the residual time series is stationary. A stationary residual time series between two cryptocurrencies would mean rejecting the null hypothesis that the time series exhibits a unit root.

The regression test for the ADF test is found in equation 12 and the regression test for the PP test is found in equation 16.

For the JOE cointegration test, a bivariate vector of two cryptocurrencies is set up. Consequently, Y_t and Z_t are the time series of two cryptocurrencies that are individually integrated in order 1 (see equation 17).

After this, α and β are estimated using Johansen's reduced rank regression, which generates the rank of Π, derived from section 3.3. and represents the number of cointegrated relationships which in this thesis must be either one or zero. Also, the maximum eigenvalue will be the basis for testing the null hypothesis and the statistic test can be found in equation 22.

4.4. Methodology

4.4.1. Testing and Trading Windows

The methodology for trading pairs will follow the following scheme:

1. Test if each cryptocurrency exhibits a unit root over a 3- or 6-month testing window through the ADF test. In total, there will be 5 testing windows for the 3-month period and 2 testing windows for the 6-month period.
2. All cryptocurrencies that exhibit a unit root will be tested for cointegration through the ADF, PP or JOE test over the 3- or 6-month testing window. The cointegrated cryptocurrencies will form pairs and a portfolio of pairs will be formed.
3. All cointegrated cryptocurrencies pairs will be traded over a trading window that spans over the last day of the testing window to 3 or 6 months further in time, so therefore there will be in total 5 trading windows for the 3-month method and 2 for the 6-month method.

The testing and trading windows for the 3-month and 6-month procedures are illustrated in the figure below.

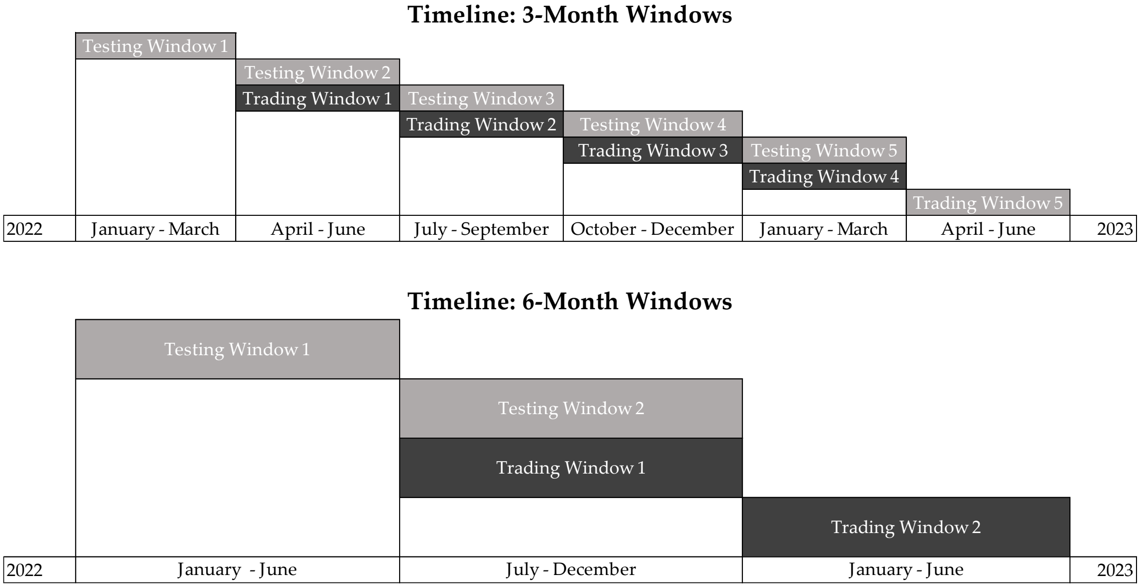

Figure 1 - Testing and Trading Window

For instance, if both Bitcoin and Ethereum have a unit root and are cointegrated during the first three months of the testing window, then use the pairs trading strategy on a pair of Bitcoin and Ethereum during the first three months of the trading window, which is more than three months later than the testing window's start date. Test if Bitcoin and Ethereum are a cointegrated pair in testing window 2 after trading window 1 has ended. Proceed to trade Bitcoin and Ethereum in trading window 2 if they are a pair in testing window 2. Therefore, there should not be a trade of the pair in testing window 2 if it is not a pair. Cointegration pairs are tested for cointegration in multiple windows since cointegrated relationships could potentially break over different time periods. This can be the result of regulations of certain cryptocurrencies or sharp shifts in the demand for various cryptocurrencies as a result of technological advancements. The following are the testing and trading periods for the three- and six-month procedures.

4.4.2. Z-score and Trading Strategy

The strategies for pairs trading are based on the capitalization of the oscillations around the mean of the spread, which usually requires the trader to trade an equal amount in asset Y of price Y_t and asset X of the price βX_t by

$$Y_t \quad = \beta X_t, \tag{25}$$

where β is the coefficient that makes the price of X and Y equal to when a position is opened. (Ting, 2018)

Recall that $\hat{e}_t$ represents the spread between two assets at a given time and can be obtained by regressing one asset on the other. In order to easily generate trading signals, it is computed the dimensionless z-score defined as

$$z_t \quad = \frac{\hat{e}_t - \mu_{spread}}{\sigma_{spread}}, \tag{26}$$

where z_t is the standard deviation away from 0 at time t, $\hat{e}_t$ is the value of the spread at a given time t, μ_{spread} is the mean of the spread and σ_{spread} is the standard deviation of the spread (Palomar, 2020).

Positions are usually taken when z_t drift from 0 and reaches a certain value, and these rules are mentioned as thresholds in this paper and are pre-specified trading strategies on when to open and close positions. Also, the spread is expected to be shorted when the standard deviation reaches a specific positive threshold, and the trader is expected to long the spread if the spread reaches a certain negative threshold.

Also, thresholds are to be set in order to maximize returns and minimize the amount of transactions. Therefore, it should be as far as possible from zero while still capitalizing on the mean-reverting property of the spread to reduce transaction costs and increase the possibility of high-quality trade (Ting, 2018).

The trading strategy taken into consideration for all the portfolios is the following one:

Trading Strategy

Short the spread if $z_t > 1.75$	Buy βX shares and short-sell Y shares.
Long the spread if $z_t < -1.75$	Short-sell βX and buy Y shares.
Stop-loss if $z_t > 4$	Exit both positions
Stop-loss if $z_t < -4$	Exit both positions
Close positions if the spread reverts back to its mean $z_t = 0$	Exit both positions

Table 3 - Trading Strategy
Note: z_t is the value of the normalized spread at a time t.

Also, it is relevant to mention, that a 1% transaction fee will be deducted from the cumulative return of each position when it's either open or closed for each portfolio that has transaction charges.

4.4.3. Buy and Hold Benchmark

The buy and hold index consists of the cumulative return of buying and holding each cryptocurrency in a trading without considering a pairs trading strategy. The buy-and-hold return of each portfolio, meaning Augmented Dickey-Fuller, Johansen's, or Phillips Perion is given by

$$Return\ Test_{Buy\ and\ Hold} = \frac{Return\ Cryptocurrency\ in\ window\ t}{Number\ of\ Cryptocurrencies\ in\ window\ t}, \tag{27}$$

where $window\ t$ is the amount of trading windows. The return of each test is then combined by

$$Return\ Test_{Buy\ and\ Hold} = \frac{Return\ ADF_{Buy\ and\ hold} + Return\ JOE_{Buy\ and\ hold} + Return\ PP_{Buy\ and\ hold}}{3}, \quad (28)$$

The buy and hold benchmark will be used to compare the return of a pairs trading strategy.

4.4.4. Sharpe Ratio

William Forsyth Shape introduced the Sharpe Ratio as a method to measure mutual funds returns adjusted to risk exposure. According to Sharpe, the purpose of the ratio is to describe the difference between the risk-free rate and the expected return for each additional unit of volatility. In general, investors prefer a portfolio with high a Sharpe ratio (SR) over a portfolio with a low Sharpe ratio, ceteris paribus.

The Sharpe ratio is given by

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}, \quad (29)$$

where $E(r_p)$ is the expected return of the portfolio, r_f is the risk-free rate and σ_p is the standard deviation of the portfolio (Sharpe, 1994).

Chapter 5

Empirical Results

The purpose of this section is to evaluate the performance of the methodology of this study. This is achieved through the number of cointegrated cryptocurrencies whose individual time series exhibit a unit root and also have a cointegrated relationship for each testing period by the ADF, JOE and PP tests. The cointegrated pairs that exhibit a cointegration relationship will form an individual portfolio which will be compared with other portfolios that have the same testing and trading window procedure.

5.1. Cointegrated Pairs

The table below shows the number of cointegrated pairs for every testing method, 3-month and 6-month window.

Augmented Dickey-Fuller´s approach detected the most pairs for the 3-month method and for the 6-month method. Furthermore, 21 pairs were identified through Johansen's approach following the 3-month procedure and 4 were formed with the 6-month procedure. The PP test detected only 18 pairs, 12 in the

3-month window and 6 for the 6-month window procedure. ADF test detected 23 pairs with the 3-month procedure and 9 pairs to the 6-month procedure. In total, 56 pairs were identified with the 3-month window method and 19 pairs were identified through the 6-month method.

Number of cointegrated Pairs; 3-month window			
	ADF	PP	JOE
Trading Window 1	3	1	0
Trading Window 2	6	5	8
Trading Window 3	1	3	2
Trading Window 4	1	3	5
Trading Window 5	12	0	6
Total:	23	12	21
Number of cointegrated Pairs; 6-month window			
Trading Window 1	4	2	2
Trading Window 2	5	4	2
Total:	9	6	4

Table 4 - Number of Traded Cointegrated Pairs
Note: Number of pairs where each individual time series has a unit root and the residual series is stationary (p<0,05)

5.2. Performance Evaluation

The following table shows the performance results of each trading window, for the ADF test, PP test, JOE test, and Buy & Hold (benchmark portfolio) which exhibits the cumulative return, standard deviation, and Sharpe ratio without transaction costs and with transaction costs set at 1% for every position.

If a cryptocurrency generates more than one cointegration pair with another cryptocurrency in the same time period, it will only be counted once for the purpose of calculations for the buy-and-hold benchmark.

In order to present the results with the most closely resemble to reality, the results mentioned above were taken into account with transaction costs and without transaction costs which the values do not vary much.

Taking into consideration the values presented in the table, it is possible to confirm that, with the transaction cost the test that obtained the highest return was the ADF test, with the return of 330.54% and 201.15%, respectively.

The strategy with the lowest return was the buy and hold benchmark, which had a return of 6,58% for the 3-month window and 100,54% for the 6-month period.

Regarding the time periods, the strategy that has the highest return in the 3 months is ADF and with the lowest return is B&H. As for the 6 months, the ones that give the most return are ADF and JOE, 201,15% and 49,17%, respectively.

The highest cumulative return with transaction costs following the 6-month window method at a time point was found for the Phillips Peron portfolio, with the pair ADA/DOGE, accumulating a return of 57,66%, on trading window 1.

The lowest cumulative return with transaction costs following the 6-month window method at a time point was found for the Phillips Peron portfolio, with the pair DOGE/LTC, accumulating a return of -12,08%, on trading window 1.

It is possible to conclude that the time periods to be used differ from each strategy, not being able to find a relationship between the highest or lowest return. Having said this, for the ADF and JOE test, the highest return presented is for the 3-month period, while for the PP and B&H test it is for the 6-month period.

The values described specifically on cointegration pairs can be found in the appendix.

	Without transaction costs							
	ADF		PP		JOE		B&H	
Windows	3-Month	6-Month	3-Month	6-Month	3-Month	6-Month	3-Month	6-Month
Return	334,17%	203,18%	111,34%	119,20%	294,95%	49,67%	6,58%	100,54%
StDev	0,306	0,119	0,120	0,088	0,251	0,041	0,354	0,400
SR	187,526	152,218	90,929	55,198	201,556	51,622	0,186	2,512
	With transaction costs							
	ADF		PP		JOE		B&H	
Windows	3-Month	6-Month	3-Month	6-Month	3-Month	6-Month	3-Month	6-Month
Return	330,54%	201,15%	110,11%	117,77%	292,00%	49,17%	6,58%	100,54%
StDev	0,303	0,118	0,119	0,087	0,249	0,040	0,354	0,400
SR	185,210	150,696	89,822	54,214	199,540	51,106	0,186	2,512

Table 5 - Performance Results

Note: Performance statistics for pairs trading portfolios through the ADF test, Johanseen's test, Phillip's Peron's, and a buy and hold strategy; Risk-Free risk = 0

Chapter 6

Conclusion

For the purpose of this thesis, the pairs trading strategy was rigorously applied to the cryptocurrency market alongside the implementation of cointegration methodologies. The primary objective was to implement and compare the Augmented Dickey-Fuller test, the Johansen's test and the Phillips Peron's test. After conducting the tests, the efficacy of each one of these methodologies was meticulously evaluated. Afterwards, all the results were compared to the buy-and-hold benchmark portfolio, also taking into account the possibility of transaction costs.

Based on the findings, the pairs trading strategy did outperform the buy-and-hold benchmark portfolio, meaning that, in the time window, selling the pairs resulted in considerably higher returns. Consequently, adherence to pairs trading strategies, despite being possible to have higher returns, it is also dependent of the market direction.

It is feasible to conclude, through this analysis, that there is no correlation between the highest and lowest returns, indicating that the optimal time periods to be employed, vary depending on the chosen method, rather than time frames.

This study, clearly shows that, depending on the method applied, results oscillate.

If a look is taken upon the three different proposed approaches, considering the highest returns analysed, Engle-Granger's approach employing the Augmented Dickey-Fuller test was the best predictor of cointegration relationships. Despite showing higher results within the three-month window, rather than the six-month window, this is the approach that shows the best results, those being an excess mean return of 268,68%.

The transaction costs, set at 1%, did not have a meaningful impact on the return, given the low number of opened and closed positions, assuming a spread oscillating around 1.75 standard deviations between cryptocurrencies. This is due to the fact that cryptocurrency market functions, in its majority, only with openings. Therefore, closes should not be considered, since, in theory, these do not happen.

The research findings conclude that the pairs trading strategy based on cointegration relationships can successfully be applied to the cryptocurrency market and profits can be generated. By applying the cointegration methods, several pairs, were formed. 56 pairs were formed whilst the duration of the three-month window. During the six-month window 19 was the number of pairs formed. By adding these, it is possible to realise that a total of 75 pairs were created. When acessing these numbers, it is possible to confirm what the conjunctures above state, as well as, what this thesis analysed throughout its course.

6.1. Limitations and Future Research

This thesis acknowledges some limitations, such as the performance of a pairs trading portfolio is highly dependent on the trading rules of the pairs trading algorithm. For example, even if the spread demonstrates mean-reverting behaviour, the strategy may not effectively leverage these characteristics if the spread fails to oscillate around 1.75 standard deviations. As a result, an alternative statistical test for portfolio building combined with various algorithmic trading strategies may yield superior results.

After conducting the tests and withdrawing the conclusion of this dissertation is possible to comprehend that there is a possibility to conduct future research of this subject. For instance, the methodology and statistical cointegration test outlined in this paper could be tested on other asset classes, such as EFTs of stock indexes or traditional commodities.

Also, in this case, all the available capital has been invested in each trading window, regardless of the number of pairs traded within that window. This means that when a cointegrated pair is identified in a testing window, the entire capital is allocated to that specific pair. Therefore, a suggestion for further research is to allocate the capital in different approaches. For instance, allocate perhaps 3% to 10% of invested capital in one pair regardless of the number of pairs in a portfolio.

Although, in this thesis, there were used 3 cointegration methods, there are other methods that could be taken into consideration such as the use of the Phillips-Ouliaris test or even the simple Dickey-Fuller test to find cointegrated pairs. Also, every open position in this study is closed by the end of a trading window which is not either ideal or realistic. So, for further research, a suggestion would be to hold positions until each is closed according to the pairs trading strategy.

Lastly, analysing a shorter time window is suggested, which can be an example to follow the methodology of this paper with a 1-month window. However, shorter windows also lead a higher transaction cost meaning that there will always be a trade-off between the length of windows and transaction costs.

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Appendix A

Description of each cryptocurrency

Bitcoin (BTC) – This cryptocurrency is the first decentralized, peer-to-peer digital currency and its original creator is known under the pseudonym, Satoshi Nakamoto. The main purpose of this coin was to allow “online payments to be sent directly from one party to another without going through a financial institution” according to Nakamoto, creating a system that is transparent, secure, and free from the control of any single authority. The innovation of Bitcoin lies in its use of blockchain technology, a distributed ledger that records all transactions across a network of computers, ensuring the integrity and chronological order of transactions. This approach solves the double-spending problem without the need for a trusted third party, making Bitcoin a groundbreaking financial innovation that has paved the way for the development of other cryptocurrencies and blockchain applications (CoinMarketCap, 2023).

Ethereum (ETH) – Founded by Vitalik Buterin and Gavin Wood in 2015, being the second most popular cryptocurrency. This cryptocurrency is quite similar to Bitcoin, however, it differs in that users can create applications that "run" on the blockchain like software "runs" on a computer, and also it can store and transfer personal data. These applications, known as decentralized applications (dApps), leverage Ethereum's smart contract functionality, enabling automated, programmable transactions without the need for intermediaries. This feature has facilitated the development of decentralized finance (DeFi) platforms, non-fungible tokens (NFTs), and other blockchain-based innovations, establishing

Ethereum as a foundational technology in the wider blockchain ecosystem (Rodeck & Curry, 2024).

Solana (SOL) – It was launched in 2020 by Anatoly Yakovenko, with the aim to provide a faster and more efficient alternative to Ethereum, hence it is often referred to as the "Ethereum killer". Contrary to the description of it being one of the oldest and slow, Solana is actually known for its high-speed blockchain, capable of processing thousands of transactions per second, thanks to its innovative Proof of History (PoH) consensus combined with the Proof of Stake (PoS) mechanism. This design significantly reduces the blockchain's energy consumption compared to traditional Proof of Work (PoW) systems, making it a more environmentally friendly option. Its architecture not only supports smart contracts and decentralized applications (dApps) like Ethereum but does so with lower transaction costs and higher scalability, addressing some of the key challenges faced by earlier blockchains (Beck & Adams, 2024).

Cardano (ADA) – Launched by Charles Hoskinson, a co-founder of Ethereum, Cardano aims to provide a more balanced and sustainable ecosystem for cryptocurrencies. It distinguishes itself with its research-driven approach, being developed through peer-reviewed research to ensure a secure, flexible, and scalable blockchain platform. The heart of its innovation lies in its unique proof-of-stake consensus mechanism, Ouroboros, which not only reduces energy consumption significantly compared to proof-of-work systems but also ensures high levels of security and scalability. This mechanism allows Cardano to be among the most environmentally sustainable platforms in the blockchain space. Besides its ecological benefits, Cardano also focuses on providing interoperability and regulatory compliance, with a vision to enable a more inclusive and transparent global financial system. Its layered architecture separates the settlement layer from the computational layer, allowing for the creation of smart contracts and dApps without compromising the network's performance (Coinbase, 2024).

XRP (Ripple) – Launched in 2012, XRP is the native cryptocurrency of the Ripple network, designed primarily for digital payment processing and remittances, aiming to facilitate instant, secure, and low-cost international transactions. Unlike Bitcoin, which was created as a decentralized digital currency, XRP focuses on serving as a bridge currency in financial services, enabling the exchange between different currencies, cryptocurrencies, and even commodities seamlessly. RippleNet, the underlying network, operates differently from traditional blockchain technology by utilizing a unique consensus protocol that significantly reduces transaction confirmation times and energy consumption. This makes it particularly attractive for banks and financial institutions, positioning XRP not just as an investment or a medium for exchange but also as a foundational technology for modernizing global payments. Ripple's vision extends beyond simple cryptocurrency transactions, aiming to revolutionize the financial sector by offering a decentralized infrastructure for financial operations on a global scale (Rodeck & Adams, 2023; Lacapra, 2023).

Polkadot (DOT) – Launched by Dr. Gavin Wood, an ex-cofounder of Ethereum, and developed by the Web3 Foundation, a Swiss Foundation aimed at facilitating a fully functional and user-friendly decentralized web, Polkadot represents a next-generation blockchain protocol. It distinguishes itself through its unique multi-chain framework, which enables different blockchains to interoperate seamlessly, allowing for the transfer of any type of data or asset across chains. This interoperability seeks to create a web where independent blockchains can share information and transactions in a trust-free environment, breaking down the silos that exist between different cryptocurrencies and enabling a more integrated and scalable network. Polkadot uses a novel proof-of-stake consensus mechanism that is designed to provide high security, scalability, and innovation. By supporting the development of custom blockchains (parachains) and connecting them with established networks, Polkadot aims to address the issues of scalability, governance, and interoperability that have hampered the adoption of blockchain technology. Through its advanced features, Polkadot not only facilitates the creation of new decentralized applications and services but also paves the way for a more interconnected and decentralized internet (Web3 Foundation, 2020).

Dogecoin (DOGE) – Created in 2013 by software engineers Billy Marcus and Jackson Palmer, Dogecoin was initially conceived as a lighthearted joke to poke fun at the wild speculation in cryptocurrencies at the time. It features the face of the Shiba Inu dog from the "Doge" meme as its logo and mascot, embodying the fun and friendly ethos the creators wanted to promote. Despite its origins as a "meme coin," Dogecoin quickly gained a massive following and evolved into a legitimate and widely used cryptocurrency. Its low transaction fees and fast transaction times have made it popular for tipping and microtransactions on social media and online forums. Furthermore, Dogecoin's vibrant and supportive community has been known to rally around charitable causes and fundraising efforts, showcasing the coin's potential for positive impact beyond mere speculation. Over the years, Dogecoin has grown in popularity and acceptance, moving beyond its initial status as a joke to become a symbol of the whimsical and inclusive spirit of the cryptocurrency community (Coinbase, 2024).

Shiba Inu (SHIB) – Launched in 2020 by an anonymous individual or group known as "Ryoshi," Shiba Inu was created as a direct competitor to Dogecoin, positioning itself within the meme coin category but with distinct characteristics. Unlike Dogecoin, SHIB operates on the Ethereum blockchain, leveraging the robust smart contract capabilities and decentralized finance (DeFi) ecosystem Ethereum provides. This allows for functionalities beyond simple transactions, including staking, liquidity provision, and participation in decentralized applications (dApps). Shiba Inu's supply is intentionally vast, with a total supply in the trillions, to encourage low per-unit prices and make it accessible to a broad audience. This strategic choice underscores its community-focused nature, aiming to create a decentralized spontaneous community building around it. The SHIB ecosystem also includes its own decentralized exchange, ShibaSwap, and has plans to expand its utility through various projects, including an NFT platform and additional tokens, contributing to its vision of being more than just a meme coin but a comprehensive ecosystem (Shiba Inu, 2023).

Algorand (ALGO) – Founded by Silvio Micali, a Turing Award-winning computer scientist, Algorand is designed to offer a high-speed, efficient, and secure blockchain network that addresses the scalability challenges prevalent in older cryptocurrencies like Bitcoin. Micali's vision was to create a platform that could facilitate instant transaction finality without the compromises on security and decentralization that plague many other networks. Utilizing a pure proof-of-stake (PPoS) consensus mechanism, Algorand enables a scalable blockchain solution that can support a wide range of applications, from financial services and traditional asset tokenization to decentralized finance (DeFi) and beyond. The PPoS approach not only speeds up transactions but also drastically reduces the energy consumption associated with proof-of-work (PoW) systems, making Algorand an environmentally friendly alternative. Additionally, Algorand's architecture is designed to be inclusive, allowing for wide participation in the network's security and governance. With its commitment to high throughput and low transaction fees, Algorand aims to empower developers and businesses to build on a blockchain that can handle the demands of global users without sacrificing performance or security (Gilad et al., 2017.).

Litecoin (LTC) – Developed in 2011 by Charlie Lee, a former Google engineer, Litecoin was introduced as a lighter alternative to Bitcoin, often dubbed as the silver to Bitcoin's gold. This digital currency was designed to address some of the limitations of Bitcoin, such as long transaction confirmation times and scalability issues. By incorporating a different hashing algorithm, Scrypt, Litecoin enables faster transaction processing times, with a goal of achieving a block confirmation time of 2.5 minutes, compared to Bitcoin's 10 minutes. This makes it a more practical choice for daily transactions and microtransactions. Additionally, Litecoin has a higher maximum supply limit of 84 million coins, four times that of Bitcoin's 21 million, which contributes to its accessibility and potential for wider adoption. Despite these differences, Litecoin maintains many of Bitcoin's core features, including its decentralized nature and a proof-of-work consensus mechanism, preserving the security and integrity of the network. Over the years, Litecoin has established itself as a reliable and efficient cryptocurrency, serving as a testing ground for technological innovations that could later be applied to Bitcoin, such as Segregated Witness (SegWit) and the Lightning Network, further enhancing its value within the digital currency ecosystem (Adams & Napoletano, 2022).

Appendix B

Testing and Trading Windows

Testing window (3-months)		
	Start date	*End date*
Testing window 1	01/01/2022	01/04/2022
Testing window 2	01/04/2022	01/07/2022
Testing window 3	01/07/2022	01/10/2022
Testing window 4	01/10/2022	01/01/2023
Testing window 5	01/01/2023	01/04/2023

Trading window (3-months)		
	Start date	*End date*
Trading window 1	01/01/2022	01/05/2022
Trading window 2	01/05/202	01/08/2022
Trading window 3	01/08/2022	01/11/2022
Trading window 4	01/11/2022	01/02/2023
Trading window 5	01/02/2023	01/05/2023

Testing window (6-months)		
	Start date	*End date*
Testing window 1	01/01/2022	01/07/2022
Testing window 2	01/07/2022	01/01/2023

Trading window (6-months)		
	Start date	*End date*
Trading window 1	01/07/2022	01/01/2023
Trading window 2	01/01/2023	01/07/2023

List of Pairs from the ADF test 3-month window

Date	Testing Window	Asset 1	Asset 2	ADF p-value
01/01/2022-01/04/2022	1	BTC	DOGE	0,0216
01/01/2022-01/04/2022	1	DOT	ALGO	0,0159
01/01/2022-01/04/2022	1	DOGE	ALGO	0,0241
01/04/2022-01/07/2022	2	SOL	DOGE	0,0137
01/04/2022-01/07/2022	2	SOL	SHIB	0,0180
01/04/2022-01/07/2022	2	XRP	DOGE	0,0268
01/04/2022-01/07/2022	2	DOGE	ALGO	0,0002
01/04/2022-01/07/2022	2	DOGE	LTC	0,0001
01/04/2022-01/07/2022	2	ALGO	LTC	0,0176
01/07/2022-01/10/2022	3	ADA	LTC	0,0040
01/10/2022-01/01/2023	4	ETH	SHIB	0,0189
01/01/2023-01/04/2023	5	BTC	XRP	0,0074
01/01/2023-01/04/2023	5	SOL	DOT	0,0173
01/01/2023-01/04/2023	5	SOL	DOGE	0,0008
01/01/2023-01/04/2023	5	SOL	SHIB	0,0327
01/01/2023-01/04/2023	5	SOL	LTC	0
01/01/2023-01/04/2023	5	ADA	DOGE	0,0192
01/01/2023-01/04/2023	5	ADA	LTC	0,0308
01/01/2023-01/04/2023	5	DOT	SHIB	0,0388
01/01/2023-01/04/2023	5	DOT	ALGO	0,0034
01/01/2023-01/04/2023	5	DOT	LTC	0,0299
01/01/2023-01/04/2023	5	SHIB	ALGO	0,0001
01/01/2023-01/04/2023	5	ALGO	LTC	0,0444

List of Pairs from the ADF test 6-month window

Date	Testing Window	Asset 1	Asset 2	ADF p-value
01/01/2022-01/07/2022	1	SOL	DOGE	0,0292
01/01/2022-01/07/2022	1	ADA	ALGO	0,0076
01/01/2022-01/07/2022	1	DOT	LTC	0,0127
01/01/2022-01/07/2022	1	DOGE	ALGO	0,0142
01/07/2022-01/01/2023	2	BTC	DOGE	0,0169
01/07/2022-01/01/2023	2	ETH	DOT	0,0146
01/07/2022-01/01/2023	2	ADA	SHIB	0,0451
01/07/2022-01/01/2023	2	DOT	DOGE	0,0283
01/07/2022-01/01/2023	2	DOT	SHIB	0,0274

List of Pairs from the PP test 3-month window

Date	Testing Window	Asset 1	Asset 2	PP p-value
01/01/2022-01/04/2022	1	XRP	SHIB	0,0435
01/04/2022-01/07/2022	2	ETH	DOGE	0,0024
01/04/2022-01/07/2022	2	XRP	DOGE	0,0094
01/04/2022-01/07/2022	2	DOGE	SHIB	0,0273
01/04/2022-01/07/2022	2	DOGE	ALGO	0,0004
01/04/2022-01/07/2022	2	DOGE	LTC	0,0001
01/07/2022-01/10/2022	3	BTC	SHIB	0,0331
01/07/2022-01/10/2022	3	ADA	LTC	0,0055
01/07/2022-01/10/2022	3	SHIB	LTC	0,032
01/01/2023-01/04/2023	4	DOT	SHIB	0,0366
01/01/2023-01/04/2023	4	DOT	ALGO	0,0008
01/01/2023-01/04/2023	4	SHIB	ALGO	0,0152

List of Pairs from the PP test 6-month window

Date	Testing Window	Asset 1	Asset 2	PP p-value
01/01/2022-01/07/2022	1	ADA	DOGE	0,0408
01/01/2022-01/07/2022	1	DOGE	LTC	0,0178
01/07/2022-01/01/2023	2	BTC	SHIB	0,0406
01/07/2022-01/01/2023	2	ETH	SHIB	0,0025
01/07/2022-01/01/2023	2	ADA	SHIB	0,0020
01/07/2022-01/01/2023	2	DOT	SHIB	0,0032

List of Pairs from the JOE test 3-month window

Date	Testing Window	Asset 1	Asset 2	JO p-value
01/04/2022-01/07/2022	2	SOL	DOGE	0.0004732898
01/04/2022-01/07/2022	2	SOL	SHIB	0.0236221084
01/04/2022-01/07/2022	2	ADA	XRP	0.0476047300
01/04/2022-01/07/2022	2	XRP	DOGE	0.0159374990
01/04/2022-01/07/2022	2	DOT	DOGE	0.0110542215
01/04/2022-01/07/2022	2	DOGE	ALGO	0.0003613086
01/04/2022-01/07/2022	2	DOGE	LTC	8.5488693272
01/04/2022-01/07/2022	2	ALGO	LTC	0.0234639315
01/07/2022-01/10/2022	3	ETH	DOT	0.0426633504
01/07/2022-01/10/2022	3	ETH	LTC	0.0140785276
01/10/2022-01/01/2023	4	BTC	SOL	0.0487887389
01/10/2022-01/01/2023	4	ADA	DOT	0.0396275987
01/10/2022-01/01/2023	4	ADA	ALGO	0.0054350827
01/10/2022-01/01/2023	4	DOT	ALGO	0.0206170554
01/10/2022-01/01/2023	4	SHIB	ALGO	0.0365814767
01/01/2023-01/04/2023	5	SOL	DOT	0.0134411432
01/01/2023-01/04/2023	5	SOL	SHIB	0.0225907441
01/01/2023-01/04/2023	5	SOL	LTC	0.0417479469
01/01/2023-01/04/2023	5	ADA	LTC	0.0164637701
01/01/2023-01/04/2023	5	DOT	SHIB	0.0059727677
01/01/2023-01/04/2023	5	SHIB	LTC	0.0347708682

List of Pairs from the JOE test 6-month window

Date	Testing Window	Asset 1	Asset 2	JO p-value
01/01/2022-01/07/2022	1	SOL	DOGE	0.004034008932443054
01/01/2022-01/07/2022	1	DOT	LTC	0.011181876350973285
01/07/2022-01/01/2023	2	ETH	DOT	0.02260688370920188
01/07/2022-01/01/2023	2	DOT	SHIB	0.028361512968243713