



UNIVERSIDADE CATÓLICA PORTUGUESA

Measuring the differences in productivities of Nations

A stochastic frontier approach

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Abstract

It is broadly accepted that differences in efficiency and productivity growth greatly contribute to the enormous differences in income across countries. Inefficiency levels were estimated for a panel of 40 countries, 34 of which are OECD-members and the remaining 6 are emergent economies, for the period of 2001-2011, using a stochastic frontier model based on the Battese and Coelli (1995) time-varying inefficiency model. Environmental variables were found to have an important role in explaining differences in technical efficiencies across countries. In particular, a high contribution of the agricultural sector and of natural resources rents to the economy, impediments to free trade such as tariffs, a bad business environment, a high number of patents, a high level of government debt and the financial crisis contribute negatively to technical efficiency. On the other hand, a good health status and good institutions help countries to be located closer to the frontier. Afterwards, productivity growth was decomposed using the Kumbhakar and Lovell (2000) primal frontier approach. The results showed that differences in TFP growth between developed and developing countries are the main drivers of the differences in the growth rates of GDP *per worker*, although differences in the factor accumulation also play an important role. Over the 2001-2011, we observed a general improvement in the technical efficiency of countries, which was outweighed by a downward shift in the stochastic production frontier.

Keywords: Technical efficiency, total factor productivity, productivity growth, stochastic frontier analysis.

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1. Introduction

Presently, it is widely accepted that differences in efficiency and productivity growth greatly contribute to the enormous differences in income across countries. Therefore, the purpose of this research is to evaluate the efficiency and productivity growth of nations and explore the causes behind those differences. Inefficiency levels will be estimated using a stochastic frontier model based on the Battese and Coelli (1995) time-varying inefficiency model and productivity growth will be decomposed using the Kumbhakar and Lovell (2000) primal frontier approach.

Jones and Romer (2010) selected the large differences in income across countries as one of the new stylized factors. According to the World Bank's World Development Indicators, in 2011, USA's output *per worker* (converted to 1990 constant international dollars using PPP rates) was approximately 7.5 times higher than output *per worker* in India. Intuitively, this means that a worker in America produces in one hour approximately as much output as a worker in India produces in an entire day.

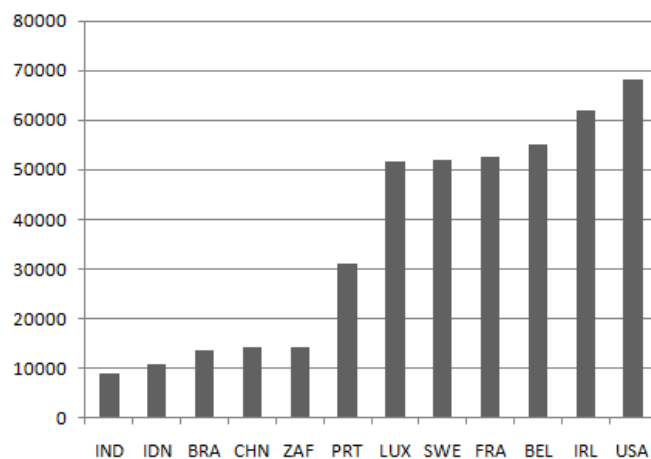


Figure 1: GDP per person employed (constant 1990 PPP \$) in 2011
(Source: World Development Indicators, World Bank, 2011)

But why are some countries far richer than others? Determining the causes of the discrepancies in the levels of production, and consequently in the standards of living, across countries, is a demanding and complex challenge that several authors have tried to address.

In order to understand the enormous disparities in economic performances between countries, one should consider first looking at the determinants of economic growth, that is, factors that explain the increase in a country's income *per capita* over a long period of time. One of the most famous attempts to explain those determinants was presented by Solow (1956, 1957), which established the roots of the neoclassic theory of economic growth. In his 1956's seminal article, Solow delivered his neoclassical model, which can be seen as an extension of the Harrod-Domar model. He demonstrated that, in order to sustain long-term economic growth, there must be continuous advances in technology so to contradict the effects of diminishing returns that would in due course cause economic growth to cease. The 1957 article established the accounting framework for explaining income growth. By assuming a Cobb-Douglas production function, Solow was able to decompose economic growth into contributions from factor accumulation (such as labor and capital) and total factor productivity (henceforth TFP). He found out that the growth rate of productivity, measured as a residual term, the Solow residual, has a predominant role in determining the GDP *per capita* growth rate.¹ Following the same line of thought, Kuznets (1971) concluded that the high rate of productivity growth accounts for most of the growth of product *per capita*. He reported that, even considering hidden costs and inputs, growth in productivity accounts for more than half of the growth in output *per capita*. Consequently, if the rate of change of productivity exerts such enormous influence on the growth rate of GDP *per capita*, as advocated by these two authors, according to

¹ Recall that the growth rate of TFP is given by: $\hat{A}(t) = g(t) - \alpha_K(t)g_K(t) - \alpha_L(t)g_L(t)$, where $g(t)$, $g_K(t)$ and $g_L(t)$ are the growth rates of output, capital and labor respectively and $\alpha_K(t)$ and $\alpha_L(t)$ are the capital share and the labor share.

the Solow model, it can be concluded that most of the economic growth is exogenously determined. Therefore, reliance on the exogenous technological progress as an essential variable to explain economic growth poses one of the biggest limitations of the initial neoclassical approach. This point of view was first expressed by Moses Abramovitz, who dubbed this term “a measure of our ignorance about the causes of economic growth” (Abramovitz, 1956).

Latter attempts to scrutinize the content of the Solow residual gave rise to a new set of theories named “endogenous growth theories”. By endogenizing a country’s technology, these theories advocate that factor accumulation is not sufficient to explain differences in income growth and try to explain the differences in the growth of the residual by analyzing the choices of the public and private sector.² As an example, in Romer’s (1990) model, growth is motivated by technological change that emerges from deliberate investment decisions made by profit-maximizing agents. According to its defenders, these theories provide policymakers with more relevant information regarding the determinants of long-run economic growth than the standard neoclassical framework.

However, in the recent past there have been a plethora of empirical studies that contradicted the idea that physical and human capital accumulation did not suffice to explain the differences in levels and growth rates of output. Mankiw, Romer and Weil (1992) concluded that the augmented Solow model (an extension of the original neoclassical Solow model that includes human as well as physical capital) provides a very good picture of the cross-country data. They predicted that the augmented Solow model accounts for about 80% of the cross-country variance in income in 1985. Alwyn Young (1994) documented the fundamental role played by factor accumulation (rather than the rise in

² The neoclassical framework postulated that a common (exogenously determined) technology was shared by every country due to the non-rivalry and non-exclusivity nature of the technological progress (note that the growth of the residual, that is the growth of productivity, essentially mirrors this technological progress). Consequently, technological progress could not explain differences in GDP *per capita* across countries and one had to look for differences in the factor accumulation.

productivity) in explaining the astonishing post-war growth of the East Asian countries. Klenow and Rodriguez-Clare (1997) called these set of studies the “neoclassical revival”, mainly because they advocate that differences in physical and human capital are the main contributors to the differences in the level and growth rate of GDP.

The lively debate that one has witnessed is based on the relative importance of factor accumulation or productivity in contributing for differences in economic performance. Simply put, $Output = F(Factors, Productivity)$, where factors include physical and human capital, and economists do not seem to agree on which variable (factors or productivity) contributes more to the differences in income levels and growth rates. According to Klenow and Rodriguez-Clare (1997), this debate is of great importance because the implications of each view (the factors or the productivity view) can differ substantially. For instance, technology-based models of productivity, by assuming scale effects due to the non-rival nature of technology creation and adoption, propose that international trade openness can have direct effects on income levels and growth rates. The neoclassical approach does not share this view, and assumes that productivity is common across countries. More recently, this crucial assumption was again questioned by several empirical studies – Knight, Loyaza and Villanueva (1993), Islam (1995) and Caselli, Esquivel and Lefort (1996), to name a few – which proved that the income-convergence predicted by the neoclassical framework is occurring but conditionally to the existence of differences in productivity between countries. In fact, by analyzing recent contributions to the economic growth literature, one can observe an increasing focus on the productivity growth as the main driver of long-term income growth and cross-country differences in income. Klenow and Rodriguez-Clare (1997) used the Mincer-regression to estimate the levels and growth rates of human capital, found out that differences in the level and growth rate of the TFP play a fundamental role in explaining the differences in

income levels and growth rate. Hall and Jones (1999) focused on levels instead of growth rates and calculated the TFP level as the Solow residual. They concluded that differences in physical and human capital can only partially explain differences in GDP *per worker* and that a great part of the variance in income *per capita* is due to a large fluctuation in the level of the Solow residual across countries. Easterly and Levine (2001) identified the TFP as the main contributor to the cross-country differences in the level and growth rate of income *per capita* and named it a stylized factor. In 2013, the Organization for Economic Co-operation and Development identified the productivity growth as a key factor to improve income *per capita* and hence standards of living.

The results in recent economic growth literature, by favoring the importance of productivity over factor accumulation in explaining the differences in income levels and growth, gave rise to the need of better comprehend TFP and its determinants in order to design policies most conducive to TFP growth, and consequently, long-run economic growth. Consequently, several authors have tried to address this issue. These studies emphasize the importance of institutions and government policies (Hall and Jones, 1999; Acemoglu et al., 2004; Afonso and St. Aubyn, 2013), human capital (Barro, 2001; Aiyar and Feyrer, 2002; Afonso and St. Aubyn, 2013), trade openness (Edwards, 1998; Baldwin and Gu, 2003; Dollar and Kraay, 2004), the roles of natural resources (Delíktas and Bacilar, 2005), among others, in boosting productivity growth.

By estimating the total factor productivity change (TFPC) for the 34-OECD countries plus the emergent economies of Brazil, China, India, Indonesia, Russian Federation and South Africa for the 2001-2011 period and evaluating the relative contribution of each factor to their efficiency, and consequently to their TFPC, this empirical study intends to be a valuable contribution to the ongoing debate on the determinants that foster productivity growth.

This work proceeds as follows: chapter 2 provides a literature review of the existing methods to estimate efficiency and productivity changes. Chapter 3

provides a brief description of the the Battese and Coelli (1995) time-varying inefficiency model as well as a description of the data and the estimated model. Chapter 4 provides the results and their discussion. Lastly, chapter 5 states the concluding remarks.

2. Literature Review

The ability of a nation to convert inputs, such as human and physical capital, into outputs, influences its capacity to generate long-run economic growth, and consequently, to improve the standards of living of its society. This ability, commonly referred to as productivity, can be analyzed for a number of other decision making units (DMU's) such as firms, plants, industries or regions, and its measurement has been made in different research fields.³

The general idea behind the calculation of productivity is based on the fact that it reflects output differences that cannot be explained by differences in the factor accumulation. To put it formally, one can think of a production function of the type

$$Y_{it} = A_{it}F(X_{it}) \quad (2.1)$$

where Y , the output of a decision making unit (firm/plant/industry/region/country) i in period t , results from the combination of $(1 \times N)$ vector of inputs X and the term A , which reflects the amount a given DMU is capable of producing from a certain quantity of inputs, given the technological level.⁴ Hence, A_{it} is the TFP for the DMU i in period t and can be calculated as the ratio of produced output to total inputs used:

$$TFP_{it} \equiv A_{it} = \frac{Y_{it}}{F(X_{it})} \quad (2.2)$$

³When productivity is mentioned, one is referring to the total factor productivity, also known as multifactor productivity, although there are several other measures of productivity. See *Measuring Productivity* OECD Manual (2001) for a brief sum up of the main productivity measures.

⁴Note that $F(\cdot)$ reflects the state of technology and is assumed to be common to all DMUs. This assumption prevails in the traditional neoclassical framework, where A_{t+1}/A_t (index i dropped) represents simultaneously productivity growth and technological progress. Further in the analysis, one will drop the assumption of common technology among units, which will dismantle the equality between productivity growth and technological progress, and one will decompose TFP growth into its several sources.

A wide range of methodologies for productivity estimation is available, and researchers have to choose the one that best suits their purposes. To facilitate this process, Del Gatto et al. (2011) proposed a way of grouping the different methods of productivity measurement, which one will follow. Although this approach does not consider all the existing methods, it comprises the main blocks of techniques for productivity estimation. They use three main criteria: (i) whether the method allows for a micro and/or macro analysis; (ii) whether it is a frontier or a non-frontier approach and (iii) whether it is a deterministic or econometric method.

	Deterministic Methodologies	Econometric Methodologies	
		Parametric	Semi-Parametric
Frontier	Data Envelopment Analysis (DEA) (Micro/Macro) Free Disposal Hull (FDH) (Micro/Macro)	Stochastic Frontier Analysis (SFA) (Micro/Macro)	
Non-Frontier	Growth Accounting (Macro) Index Numbers (Micro/Macro)	Growth Regressions (Macro)	Proxy-Variables (Micro)

Table 1: Methodologies used to measure productivity
Source: Del Gatto et al. (2011), *Measuring Productivity*

Del Gatto et al. (2011) start their analysis by dividing methodologies according to the type of data sets that they can be applied to. Methods applied to macro data sets measure productivity of aggregate units (industries, countries, regions), while techniques that use micro data sets aim to measure productivity of individual units (firms, plants). As Table 1 shows, this criteria is not mutually exclusive, as there are methods that can use both micro and macro data (DEA, FDH, SFA and Index Numbers). Then, the authors distinguish

between frontier and non-frontier models. Frontier models allow for the existence of inefficiency in the productive processes, while non-frontier models assume that the observed output always equals the potential level of production at each moment in time.⁵ Finally, they divide the surveyed methods between deterministic, where the measure of TFP is calculated, and econometric, where TFP is estimated.

An extensive comparison between non-frontier and frontier models goes beyond the scope of this analysis, which is to provide sufficient background to understand the frontier method used to calculate inefficiency and decompose TFP presented in the next chapters. Consequently, only a brief review on the non-frontier models will be presented and a more detailed discussion is provided in the Del Gatto et al. (2011) survey.

2.1 Non-Frontier Models

Non-frontier methodologies comprise methods that can be applied to (i) aggregate data sets only, such as the growth accounting and the growth regressions methods; (ii) individual data sets only, such as the proxy-variables models; (iii) both aggregate and individual data sets, such as the index numbers method.

The first attempts to measure productivity growth were performed using aggregate data of the US economy, via growth accounting methodology (Abramovitz, 1956; Solow, 1957). Under this method, TFP growth (also known as Solow residual or technological progress) was calculated as the difference between output growth and a weighted average of the inputs growth rates. The authors found out that the TFP growth was responsible for almost 90% of the

⁵ This assumption results from the fact that the latter methods assume that units share the same technology (embodied by the production function).

output growth. Several extensions of this methodology were proposed in order to overcome some limitations in the estimation of the residual. In particular, the level accounting methodology, although it shares the same objective of the previous methodology, it estimates TFP levels instead of growth rates. Hall and Jones (1999) support this extension, advocating that recent contributions to the economic growth literature focus in levels instead of growth rates and prove that cross-country differences in growth rates are merely transitory (Easterly et al., 1993; Parente and Prescott, 1994; Barro and Sala-i-Martin, 1995).

Growth regressions are an alternative methodology to estimate TFP. Although it is also an extension of the original Solow model, this approach has the advantage of estimating TFP from a structural equation that it identifies and not from a residual exercise. Furthermore, it has the advantage of not requiring the use of data on physical capital stocks, as it is very likely to contain significant measurement error problems.

More recently, studies started to focus on estimating firm-level productivity, essentially due to the development of a theoretical literature based on the assumption of heterogeneity among firms and the increasing availability of individual data. The proxy-variable method was developed to mitigate the problems related to estimating firm-level productivity, such as simultaneity, price dispersion and selectivity. It consists of identifying a proxy variable that is a function of the TFP (such as intermediate goods or investment) and calculating the inverse of that function, in order to express TFP as a function of the proxy variable itself.

Index numbers, the last non-frontier model mentioned in the survey, can be applied to both micro and macro data sets. Index numbers can be used to calculate changes in TFP directly from input and output prices and quantities, known as the TFP index numbers. The indexes most widely used to measure

productivity are the Törnqvist, the Laspeyres, the Paasche and the Fisher. Notice that index numbers can also be used together with frontier models (the Malmquist productivity index).

One characteristic that is common to all previously mentioned methods is the fact that they expect producers (either in aggregate or in individual terms) to be fully efficient, meaning that they operate on the production frontier, where observed output matches potential output. Yet, in the presence of inefficiency, both productivity measurement and productivity change will be affected, assuming that inefficiency varies over time. Consequently, measurements of TFP growth based on non-frontier methods will lead to biased results. Frontier models account for the presence of inefficiency and their main advantage over the non-frontier models is their capability of decomposing two main sources of productivity growth: technical efficiency change and technical (or technological) change. This characteristic provides useful information to the policymakers, since policies required to address these two sources of productivity growth are likely to be different. For instance, technological progress can be promoted using policies that induce innovation (such as public investment in R&D), while efficiency requires, for instance, policies oriented to education improvement. Frontier models will be presented in the following subchapter.

2.2 Frontier Models

Frontier models presuppose the existence of physical or economic representations of the production technology. Economic representations of the production technology include cost, revenue and profit frontiers and, contrarily to the production frontier, they require the use of information on both prices and quantities of inputs and outputs and an imposition of a behavioral

objective on producers. These frontiers result from the optimization problem that the producer successfully solves – at least in theory. In the case of a production frontier, it represents the maximum attainable output from a set of inputs, given the technology available, or alternatively, the minimum amount of inputs necessary to produce a given level of output, at the current technology level. The cost frontier gives the set of inputs that minimize the cost of producing a given level of output with the available technology and given the input prices, while the revenue frontier shows the set of outputs that maximize revenue from a given set of inputs, given the output prices and the available technology. The profit frontier results from the maximization of revenue and minimization of cost. While a production frontier represents the best set of inputs or outputs that can be achieved technically, the last three frontiers represent the best combinations that can be achieved in economic terms.

There is, however, empirical evidence that shows that producers do not always successfully solve their optimization problems. In fact, those frontiers are used as benchmarks to make relative economic performance evaluations, where distances to a particular frontier provide measures of efficiency. Technical efficiency is measured as the distance to a production frontier, while allocative efficiency is represented by the distance to an economic (cost, revenue or profit) frontier. The combination of both efficiencies constitutes the “overall efficiency” concept.⁶

The recognition that inefficiency exists and that it affects productivity led to the creation of methods capable of incorporating it into the measurement of the latter: the frontier methods. This characteristic and their capability of decomposing the sources of productivity change made them a very popular

⁶ One will forego the concepts of allocative and overall efficiency as well as economic frontiers in the overview of technical efficiency and productivity change-related concepts presented in Appendix A, since the empirical analysis performed in chapter 3 will not consider the prices of inputs or outputs. This brief overview intends to be a helpful insight in understanding the mechanisms behind the frontier model used in the following chapter.

instrument used in several empirical studies of TFP growth. TFP growth will now explicitly result from a decomposition of productivity change into technological change, which pushes the frontier of feasible production upward, and efficiency change, which corresponds to movements towards the production frontier. The Del Gatto et al. (2011) survey named three frontier methods that can be applied to both micro and macro data sets: data envelopment analysis (DEA), free disposal hull (FDH) and stochastic frontier analysis (SFA).

According to Del Gatto et al. (2011), one can distinguish FDH and DEA from SFA using the deterministic/econometric criteria: while in DEA and FDH, the estimation of frontier functions and the measurement of technical efficiency are performed in a deterministic environment, the latter assumes a stochastic context. Thus, while the first two methods involve mathematical programming, the last one requires econometric methods. It is the nature of these methods that provide their main advantages and limitations. On the one hand, the first set of methods, because of their deterministic nature, assumes that all deviations of observed output from potential output are due to technical inefficiency. Consequently, all observations lie on or below the production frontier. Any other possible source of these deviations, i.e. unobserved measurement errors, omitted variables and stochastic noise, is not considered, which may result in an upward bias of inefficiency measures. Additionally, DEA and FDH require large data sets in order to better approximate the “best-practice” frontier to the real production frontier. Although traditional literature on DEA presented, as a limitation of this methodology, the fact that it did not allow for the estimation of standard errors and tests of hypothesis, recent literature has shown that it is possible to define a statistical model that allows for the determination of statistical properties of the non-parametric frontier estimators (Simar and Wilson, 2000).

On the other hand, the deterministic frontier approaches do not require the imposition of a functional form for the technology set. The stochastic frontier approach, by separating the error term into an inefficiency term and a noise term, allows the distinction between inefficiency and other causes of the differences between the observed output and the potential one. This separation is only possible due to the imposition of a distributional form for the inefficiency term, which might affect the results considerably. Likewise, the imposition of a specific functional form for the production frontier constitutes another important limitation.

Empirical works have used extensively both DEA and SFA, but not the FDH model, created by Deprins et al. (1984). Although it constitutes a more flexible model than the DEA (as it only assumes free disposability while DEA assumes also convexity) the FDH model did not gain much acceptance among the efficiency measurement studies. For this reason, one will not consider this model in the following analysis of the most widely used frontier models.⁷

2.2.1 Data Envelopment Analysis (DEA)

The idea behind the DEA approach to measure TFP change consists in using linear programming methods to envelope the observed combinations of input and outputs in order to construct a non-parametric piecewise surface (also known as the “best-practice” frontier) so that all observed points lie on or below that frontier and then use it to identify the contribution of the different sources of the TFP change.⁸

⁷ The following analysis was adapted from Coelli (1995).

⁸ For the purpose of this analysis, in the DEA framework, one will consider the estimation of technical efficiency but not the estimation and decomposition of productivity change. The estimation and decomposition of productivity change will be mentioned for the SFA framework, as it is the framework used in the empirical analysis performed in chapter 3, and TFP change will be estimated and decomposed in chapter 4.

The piecewise-linear convex hull method to estimate frontiers proposed by Farrell (1957) did not receive much attention until Charnes, Cooper and Rhodes (1978) changed it into a mathematical programming problem, which they called data envelopment analysis (DEA). They proposed an input-oriented model with constant returns to scale (CRS). Subsequent papers presented extensions of the original model by assuming different assumptions, such as Banker, Charnes and Cooper (1984), who presented the variable returns to scale (VRS) model.

The Constant Returns to Scale (CRS) Model

Let's assume there is information regarding K inputs and M outputs for each of the N firms. The KxN input matrix, X, and the MxN output matrix, Y, represent the data for all the firms. One way of representing the CRS model is through the definition of a linear programming problem:

$$\begin{aligned}
 & \min_{\theta, \lambda} \theta \\
 \text{st. } & -y_i + Y\lambda \geq 0, \\
 & \theta x_i - X\lambda \geq 0, \\
 & \lambda \geq 0,
 \end{aligned} \tag{2.3}$$

where λ is a Nx1 vector of constants and θ is a scalar, representing the efficiency score for the i-th firm. θ varies between 0 and 1, where 1 indicates a point in the frontier meaning that the firm is technically efficient. This linear programming problem has to be solved N times because θ is obtained for each firm.

The Variable Returns to Scale (VRS) Model

The constant returns to scale assumption works well in a perfectly competitive scenario, where all firms operate at the optimal scale. However, imperfect competition, government regulations, constraints on finances,

externalities, among others, may cause firms to not operate in the optimal scale. In these cases, technical efficiency measured under the CRS framework is biased by scale efficiencies. An extension of the CRS was proposed by Banker, Charnes and Cooper (1984) to correct this shortcoming: the variable returns to scale (VRS) model. The VRS linear programming problem can be derived from the previous CRS programming problem by adding the convexity constraint, $N1'\lambda = 1$:

$$\begin{aligned}
& \min_{\theta, \lambda} \theta \\
& \text{st } -y_i + Y\lambda \geq 0, \\
& \theta x_i - X\lambda \geq 0, \\
& N1'\lambda = 1, \\
& \lambda \geq 0,
\end{aligned} \tag{2.4}$$

where $N1$ is an unitary vector ($N \times 1$). This approach provides a “tighter” frontier to the data points and thus the technical inefficiency scores that result from it are less than or equal to the ones obtained from the CRS model. Another advantage of this model over the CRS model is the fact that it ensures that an inefficient firm is compared with firms of a similar size. Figure 2 shows the differences between the frontiers obtained from a CRS model and a VRS model for the case of one input and one output. One can see the implications of each method on the input-efficiency measures. For the case of firm B, using the CRS-DEA model, its technical inefficiency is given by the distance BB_2 , while in the case of the VRS-DEA model, the technical inefficiency is given by the distance BB_1 , where $\overline{BB_1} < \overline{BB_2}$.

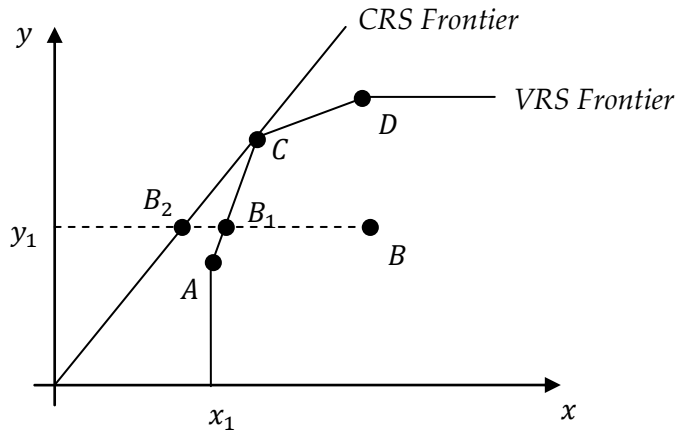


Figure 2: Production Frontiers obtained from CRS-DEA and VRS-DEA

Output-Oriented Models

The previous input-oriented models aimed to identify technical inefficiency as a proportional drop in the input quantity, holding output levels constant. This is the Farrell's input-oriented measure of technical inefficiency. It is also possible to compute technical inefficiency as a proportional increase in output production, holding input levels constant. The two measures of inefficiency provide the same results under the CRS assumption, but not under VRS. The choice of the orientation of the model will depend on the purpose of each firm: either to maximize output maintaining the input levels fixed or to minimize input usage for the current output level (for example, when a firm has orders to fill and thus the input quantities seem to matter the most). These models are very similar to their input-oriented counterparts. In particular, the output-oriented CRS model can be defined from the following linear programming problem:

$$\begin{aligned}
 & \max_{\phi, \lambda} \theta \\
 & \text{st } -y_i + Y\lambda \geq 0, \\
 & \phi x_i - X\lambda \geq 0, \\
 & \lambda \geq 0,
 \end{aligned} \tag{2.5}$$

where ϕ corresponds to the proportional increase in outputs that the i -th firm can achieve. To obtain the output-oriented VRS model, just add convexity constraint, $N1'\lambda = 1$.

One last note on the input and output-oriented models regards the fact that both estimate the same frontier and consequently the efficient firms are the same. It is only the efficiency measures of the inefficient firms that may change.

Other Extensions

Recent developments in the DEA approach include: the measurement of allocative efficiency assuming availability of price information and a behavioral objective such as cost minimization (Ferrier and Lovell, 1990), revenue maximization (Färe, Grosskopf and Lovell, 1985) or profit maximization (Färe and Grosskopf, 1997); non-discretionary variables (Banker and Morey, 1986; Afonso and St. Aubyn, 2013); environmental variables (Fried, Schmidt and Yaisawarng, 1999); incorporation of a stochastic element (Sengupta, 1990); panel data and Malmquist index approach to calculate technical change and technical efficiency change (Färe et al., 1994).

2.2.2 Stochastic Frontier Analysis (SFA)

The stochastic frontier analysis was pioneered by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977). These authors, independently, proposed the estimation of the following stochastic frontier:

$$\ln(y_i) = F(x_i; \beta) + v_i - u_i \quad , i = 1, 2, \dots, N \quad (2.6)$$

where y_i is the output produced by firm i , x_i is the vector of input quantities used by firm i , $F(\cdot)$ represents the production function, β a vector of unknown

parameters to be estimated, v_i is the symmetric noise component of the error term assumed to be independently and identically distributed as $N(0, \sigma_v^2)$ and u_i is the non-negative technical inefficiency component of the error term, independent of v_i and derived from a $N(0, \sigma_u^2)$ distribution truncated above at zero.⁹ The firm's observed level of production is bounded above by the sum of a parametric function of known inputs (and unknown parameters) and a random error term reflecting measurement error of the level of production or other external factors, such as climate, strikes, economic crisis that cause the position of the deterministic part of the frontier, $F(x_i; \beta)$, to vary across firms. The greater the difference between realized production, $F(x_i; \beta) + v_i - u_i$, and the corresponding stochastic frontier level of production, $F(x_i; \beta) + v_i$, the greater the level of technical inefficiency.

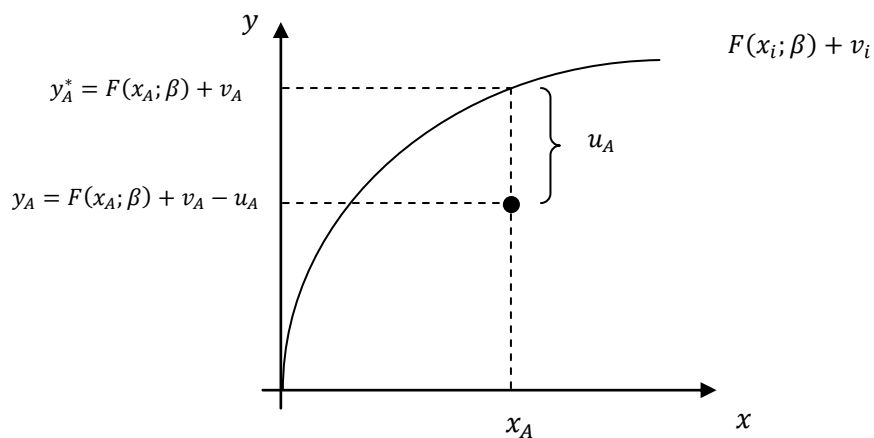


Figure 3: Stochastic Production Frontier

The output-oriented measure of technical efficiency is then given by the ratio of observed output to the corresponding stochastic frontier output:¹⁰

⁹ Note that when $\sigma_v^2 = 0$, this model becomes a deterministic frontier model.

¹⁰According to Aigner, Lovell and Schmidt (1977), this is the correct measure of technical efficiency, instead of $\frac{y_i}{\exp(F(x_i; \beta))}$, because the first ratio distinguishes technical inefficiency from other sources of disturbance that the firm cannot control.

$$\begin{aligned}
TE_i &= \frac{y_i}{\exp(F(x_i; \beta) + v_i)} = \frac{\exp(F(x_i; \beta) + v_i - u_i)}{\exp(F(x_i; \beta) + v_i)} = \frac{\exp(F(x_i; \beta)) * \exp(v_i) * \exp(-u_i)}{\exp(F(x_i; \beta)) * \exp(v_i)} \\
&= \exp(-u_i) \tag{2.7}
\end{aligned}$$

The technical efficiency measure, TE_i , varies between 0 and 1. In order to estimate it, one has to estimate first the parameters of the stochastic production frontier. The β 's in this model are estimated using the Maximum Likelihood (ML) method, under the assumption that the error terms follow appropriate distributions. In particular, Aigner, Lovell and Schmidt (1977) assumed that v_i is independently and identically distributed following a normal distribution and that u_i is distributed independently of v_i and derived from a half-normal or an exponential distribution. The other possible method to estimate the parameters is the COLS method. Although the COLS method is computationally less demanding than the ML method, the latter is asymptotically more efficient, making it a more popular estimation method.

This stochastic model allows the overcoming of two of the limitations of the traditional deterministic models: no consideration of noise and the impossibility of estimation standard errors and tests of hypothesis. However, this model has some other limitations, as mentioned before. In particular, the selection of a specific distributional form for the inefficiency term without a solid *a priori* justification constitutes an important limitation, which can be partially solved with the assumption of more general distributions, such as the truncated-normal or the two-parameter gamma. Another important limitation of this model is the imposition of a specific functional form for the production frontier. The most common production function used in empirical estimations of frontier models is the logarithmic transformation of the Cobb-Douglas, mostly due to its simplicity. However, this form imposes strong restrictions: returns to scale are equal for all firms and elasticities of substitution are equal to one. One way of alleviating the problem of assuming a functional form is through adopting

more flexible ones. In particular, the translog form, although it is not flawless, imposes no restrictions on the returns to scale and the elasticities of substitution.

Panel Data

Up to this moment, the estimation of the frontier was performed in the cross-sectional framework, i.e., the case of N firms observed at one moment in time. The case of multiple firms observed in several moments in time, i.e., the panel data case, provides a richer set of information, which is proved to have many advantages for the frontier estimation. In particular, panel data provides consistent estimators of firm efficiencies (given that T is large); increases the degrees of freedom; eliminates the need to make assumptions on the distribution of the u_i ; does not require independency between the inefficiencies and the regressors and allows for the estimation of both technical change and technical efficiency change over time.¹¹ Islam (1995) states that the main advantage of the panel techniques is related to the fact that it allows for differences in the aggregate production function across countries. Temple (1999) argues that the panel data approach allows to control omitted variables that are persistent over time, which otherwise could cause estimation problems.

Pitt and Lee (1981) presented an extension of the Aigner, Lovell and Schmidt (1977) half-normal model to account for panel data:

$$\ln(y_{it}) = f(x_{it}; \beta) + v_{it} - u_{it} \quad , i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (2.8)$$

which parameters were estimated using ML estimation. This model accommodates the situation where technical inefficiency varies across

¹¹ The importance of not requiring the inefficiencies to be independent of regressors can be seen, for example, in the case where inefficiencies can be correlated with the inputs producers select.

producers and over time for each producer. The authors also proposed a model where technical inefficiency varies across producers but is constant over time:

$$\ln(y_{it}) = f(x_{it}; \beta) + v_{it} - u_i \quad , i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (2.9)$$

Battese and Coelli (1988) provided an extension of this model that allowed the u_i to have a truncated normal distribution. These models provide consistent estimators of the u_i when T is large. However, the assumption of time-invariant technical inefficiency is reasonable only when the time period is short, since managers are expected to improve their performance based on previous experience.

Kumbhakar (1990) and Battese and Coelli (1992) proposed extensions of the Pitt and Lee (1981) time-varying inefficiency model that take the form:

$$u_{it} = f(t).u_i \quad (2.10)$$

where $f(.)$ is a function that determines the way technical inefficiency varies over time. For the Kumbhakar (1990) model, $f(t) = [1 + \exp(bt + ct^2)]^{-1}$ and u_i has an half-normal distribution, while for the Battese and Coelli (1992) model, $f(t) = \exp[-\eta(t - T)]$ and u_i is assumed to have truncated normal distribution. b , c and η are parameters to estimate. The inclusion of the time trend allows for the estimation of both technical change and technical inefficiency change over time. Cornwell et al. (1990) proposed an alternative specification of the time-varying inefficiency term: $u_{it} = \beta_{1i} + \beta_{2i}t + \beta_{3i}t^2$. Lee and Schmidt (1993) assumed $u_{it} = d_t.u_i$, where d_t are time dummies representing the time effects and u_i are either fixed or random firm-specific effects.

The previous presentation did not involve a comprehensive review of the panel data stochastic frontier models. A list of other panel data stochastic frontier models is provided in Appendix B.

Determinants of Inefficiency

The ability of a firm to convert inputs into outputs is influenced by the exogenous variables that characterize the environment in which the production takes place. These variables are neither the inputs nor the outputs of the production process, but nonetheless affect the performance of the firm.¹² Empirical studies that have explored the relationship between environmental variables and estimated technical efficiencies include Pitt and Lee (1981), Kalirajan (1981, 1989) and Kalirajan and Shand (1989). These papers adopt a two-stage approach, in which the first-stage estimates either the inefficiency effects or the technical efficiencies of the firms using an estimated stochastic frontier (excluding the exogenous variables) and the second-stage involves regressing them on the environmental variables. Through the second-stage, one can see that exogenous variables influence the output indirectly, through its effect on predicted inefficiency effects or the levels of technical efficiency. Exogenous variables do not influence the production frontier, but they influence the position of the producer in relation to the frontier. This approach has, however, some econometric problems. In the first place, it is assumed in the first-stage that the inefficiencies are identically distributed, yet this assumption is contradicted in the second-stage, where they are assumed to have a functional relationship with the exogenous variables. Secondly, the exclusion of the environmental variables in the first-stage leads to biased estimators of the parameters of the production frontier and biased technical inefficiency measures, in the case where exogenous variables are correlated with the regressors.¹³

To overcome these inconsistencies, Pitt and Lee (1981) investigated the sources of technical inefficiency by regressing the estimated firm intercepts on

¹² One will consider only non-stochastic variables, which are the ones that are observable at the time production decisions are taken (eg. degree of government regulation, age of the labor force). Stochastic variables can be interpreted as sources of production risk (eg. weather).

¹³ For a more detailed discussion, see Caudill et al. (1995) and Wang and Schmidt (2002).

firm specific characteristics or by including these variables into the production frontier and jointly estimate the parameters. More recently, Kumbhakar, Ghosh and McGuckin (1991), Reifschneider and Stevenson (1991) and Huang and Liu (1994) proposed stochastic frontier models in which inefficiencies are an explicit function of the firm specific factors and the parameters are estimated using a single-stage ML estimation. Kumbhakar, Ghosh and McGuckin (1991) assumed technical inefficiency effects as non-negative truncations of a normal distribution with the mean being a linear function of exogenous variables with unknown coefficients, and unknown variance. The authors considered also the allocative inefficiency effects which result from the failed attempt to profit maximization. In the empirical application of their model, the authors found out that the farmers' level of education and the size of their farming operations affect technical inefficiency significantly. Reifschneider and Stevenson (1991) presented a model where the technical inefficiency effects are a function of non-negative explanatory variables and a non-negative random variable, which has a half-normal, gamma or exponential distribution. The model was applied to the generation of electricity in the USA in three separate periods. They concluded that the inefficiency function is important to the estimation of the production frontier. Huang and Liu (1994) proposed a model where the non-negative technical inefficiency effects are a linear function of variables that reflect firm characteristics and a random error term, which is the truncation of a normal distribution with zero mean and whose point of truncation depends on the firm characteristics. The authors investigated the electronics industry in Taiwan in a cross-section framework. Battese and Coelli (1995) proposed an extension of the Huang and Liu (1992) model to account for panel data, where the technical inefficiencies are a function of firm-specific variables and time.¹⁴ The inclusion of the time variable allows for the estimation of technical efficiency change and technical change. The model was applied to paddy

¹⁴ A variant of this model will be used in the empirical analysis performed in chapter 3.

farmers in Aurepalle, India, for the period 1975-1985 and the authors found out that the farmers' age increases inefficiency and schooling has the opposite effect.

Decomposition of TFP Change

So far, the discussion has focused on estimating technical inefficiencies from cross-sectional or panel data. In the presence of panel data, we can move one step further and estimate TFP change and decompose it into its various sources. This raises two questions: (i) how can productivity change be measured; and (ii) what are the sources of productivity change. Diewert (1992) provided an answer to question (i): given that productivity change occurs when an index of outputs and an index of inputs change at different rates, he proposed the use of index number techniques to construct a Fisher or Törqvist productivity index. However, these indexes require price and quantity information, as well as assumptions on the structure of technology and on producers' behavior. Productivity change can also be estimated using non-parametric techniques (such as DEA) or econometric approaches (such as SFA) to construct a Malmquist productivity index. This index does not require price information or assumptions on producers' behavior, although it still requires information on the structure of technology.

Although both types of techniques (index numbers and non-parametric/econometric techniques) provide answers to question (i), only non-parametric/econometric techniques provide an answer to question (ii) and only econometric techniques do it in a stochastic environment.

A stochastic frontier approach to the estimation and decomposition of the TFP change in a panel context is proposed by Kumbhakar and Lovell (2000). The primal (production frontier) approach, as the authors named it, consists in

a quantity-based method in which they propose the use of econometric techniques to estimate a production frontier against which the magnitude of productivity change is estimated and then decomposed into its various sources, namely a technical change component, a returns to scale component and a technical efficiency change component.

The general stochastic production frontier model is presented in equation (2.11), where y is the vector of output produced, $f(t, x, \beta)$ is the deterministic part of the stochastic production frontier, x is the vector of production factors used, β is the vector of the parameters defining the production technology, t is the time trend serving as a proxy for technical change, $v \sim \text{idd } N(0, \sigma_v^2)$ is the random error term and $u \sim \text{NT}(\mu, \sigma_u^2)$ is the output-oriented technical inefficiency.

$$y = f(t, x, \beta) \cdot \exp(v) \cdot \exp(u), \quad u \geq 0 \quad (2.11)$$

Given the previous assumptions on the distributions and independence of the two error components, v and u , the model can be estimated by ML. Once maximum likelihood estimates of the parameters in (2.11) are obtained, we can decompose TFP change for each producer into its various sources.

A measure of the rate of technical change is given by:

$$T\Delta = \frac{\partial \ln f(t, x, \beta)}{\partial t} \quad (2.12)$$

where $T\Delta > 0$ means an upward shift in the production frontier and technological progress, $T\Delta < 0$ represents a downward shift in the production frontier and technological regress and $T\Delta = 0$ means that the frontier remained unchanged. Technical efficiency change can be given by:

$$TE\Delta = -\frac{\partial u}{\partial t} \quad (2.13)$$

where $TE\Delta > 0$ means that the producer moved towards the production frontier, that is, became more efficient, $TE\Delta < 0$ represents a movement away of the frontier and $TE\Delta = 0$ means that the position of the producer in relation to the production frontier remained unchanged. For the scalar output case, TFP change is defined by a Divisia index of productivity change:

$$TFP = \dot{y} - \dot{X} = \dot{y} - \sum_n S_n \dot{x}_n \quad (2.14)$$

where a dot over a variable represents its rate of change, $S_n = \frac{w_n x_n}{E}$ is the observed expenditure share of the input x_n , $w = (w_1, \dots, w_N) > 0$ is an input price vector and $E = \sum_n w_n x_n$ is the total expenditure. In the absence of price information, $S_n = \frac{\varepsilon_n}{\varepsilon}$, where $\varepsilon_n = \frac{\partial \ln f(t, x, \beta)}{\partial \ln x_n}$ are the elasticities of output with respect to each of the inputs and $\varepsilon = \sum_n \varepsilon_n$ is the scale elasticity, which provides a measure of returns to scale characterizing the production frontier.

$$TFP = \dot{y} - \sum_n \left(\frac{\varepsilon_n}{\varepsilon} \right) \dot{x}_n \quad (2.15)$$

By differentiating the deterministic part of (2.11), we obtain:

$$\dot{y} = \frac{\partial \ln f(t, x, \beta)}{\partial t} + \sum_n \varepsilon_n \dot{x}_n - \frac{\partial u}{\partial t} \quad (2.16)$$

Combining (2.15) and (2.16), we have:

$$\begin{aligned} TFP &= \frac{\partial \ln f(t, x, \beta)}{\partial t} + \sum_n \varepsilon_n \dot{x}_n - \frac{\partial u}{\partial t} - \sum_n \left(\frac{\varepsilon_n}{\varepsilon} \right) \dot{x}_n \\ &= \frac{\partial \ln f(t, x, \beta)}{\partial t} + \left(\frac{\varepsilon - 1}{\varepsilon} \right) \sum_n \varepsilon_n \dot{x}_n - \frac{\partial u}{\partial t} \\ &= \frac{\partial \ln f(t, x, \beta)}{\partial t} + (\varepsilon - 1) \sum_n \left(\frac{\varepsilon_n}{\varepsilon} \right) \dot{x}_n - \frac{\partial u}{\partial t} \\ &= \Delta T + (\varepsilon - 1) \sum_n \left(\frac{\varepsilon_n}{\varepsilon} \right) \dot{x}_n + \Delta TE \end{aligned} \quad (2.17)$$

Equation (2.17) exhibits the decomposition of the productivity change into a technical change component (ΔT), a scale component $[(\varepsilon - 1) \sum_n \frac{\varepsilon_n}{\varepsilon} \dot{x}_n]$ and a technical efficiency component (ΔTE). If either production technology or technical efficiency does not change through time, it does not affect productivity change. The influence of scale economies on productivity change depends on the type of technology and the input usage. Assuming constant returns to scale, the change in the input usage will not alter productivity change. However, under non-constant returns to scale, input growth ($\sum_n \frac{\varepsilon_n}{\varepsilon} \dot{x}_n > 0$) and scale elasticity $\varepsilon > 1$ or input contraction ($\sum_n \frac{\varepsilon_n}{\varepsilon} \dot{x}_n < 0$) and scale elasticity $\varepsilon < 1$ lead to an increase in total factor productivity.

So far, we have decomposed TFP change from the general stochastic production frontier model in equation (2.11). To provide a good basis for the TFP change estimation and decomposition performed in chapter 3, one will consider the panel data case of I producers through T periods of time and a time-varying production frontier expressed in the following translog form:

$$\begin{aligned} \ln y_{it} = & \beta_0 + \sum_n \beta_n \ln x_{nit} + \beta_t t + \frac{1}{2} \sum_n \sum_k \beta_{nk} \ln x_{nit} \ln x_{kit} + \frac{1}{2} \beta_{tt} t^2 \\ & + \sum_n \beta_{nt} \ln x_{nit} t + v_{it} - u_{it} \end{aligned} \quad (2.18)$$

where $v \sim \text{idd } N(0, \sigma_v^2)$ is random error term and $u_{it} \geq 0$ is the technical efficiency error term. Additionally, we consider the technical inefficiency function suggested by Battese and Coelli (1995), although there are several other possible parameterizations of the $u^{15,16}$:

$$u_{it} = (z_{it} + t)\delta + w_{it} \quad (2.19)$$

¹⁵ We present some alternative parameterizations of the u_{it} previously in the present subchapter.

¹⁶ Although the original parameterization of the technical inefficiency is $u_{it} = z_{it}\delta + w_{it}$, we decided to add explicitly the time variable – as the authors did in their empirical application – in order to show how the decomposition of TFP change is performed.

where u_{it} 's are non-negative truncations of the $N((z_{it} + t)\delta, \sigma^2)$ -distribution and w_{it} is the random variable defined by the truncation of the normal distribution with zero mean and variance σ^2 .

One can see that the time variable t shows up in the production frontier and in the second error term. According to Kumbhakar and Lovell (2000), the main econometric problem is to separate the two roles played by t : as a proxy for the technical change in the deterministic part of the stochastic production frontier and as an indicator of technical efficiency change in the second error term, u . In specification (2.19) the separate time effects can be disentangled by linearly adding the variable t . Based on the previous assumptions regarding the distributions and independence of u_{it} and v_{it} , the probability density function of the composite error term, $v_{it} - u_{it}$, is derived and the log likelihood function for the model (2.18) is obtained. By maximizing the log likelihood function, one obtains estimates of the parameters of the model, which one can then use to predict technical inefficiencies. From equation (2.17), we can see that the estimation and decomposition of TFP change require the estimation of ΔT , ΔTE , $\varepsilon_n \forall n$ and ε , which can be obtained after the estimation of the parameters of the model:

$$\hat{T}\Delta = \frac{\partial \ln f(t, x_{nit}, \hat{\beta})}{\partial t} = \hat{\beta}_t + \hat{\beta}_{tt}t + \sum_n \hat{\beta}_{nt} \ln x_{nit} \quad (2.20)$$

$$\widehat{TE}\Delta = -\frac{\partial \hat{u}_{it}}{\partial t} = -\hat{\delta} \quad (2.21)$$

$$\hat{\varepsilon}_n = \frac{\partial \ln f(t, x_{nit}, \hat{\beta})}{\partial \ln x_n} = \hat{\beta}_n + \sum_k \hat{\beta}_{nk} \ln x_{kit} + \hat{\beta}_{nt}t, \quad n = 1, \dots, N \quad (2.22)$$

$$\hat{\varepsilon} = \sum_n \left(\hat{\beta}_n + \sum_k \hat{\beta}_{nk} \ln x_{kit} + \hat{\beta}_{nt}t \right) \quad (2.23)$$

TFP change can be obtained for each producer by substituting (2.20) – (2.23) into (2.17). Notice that, if no restrictions are made, all components of equation (4.7) are time specific and all components except ΔTE are producer specific. ΔT varies across time, unless $\beta_t = \beta_{tt} = \beta_{nt} = 0 \forall n$, and across producers, except if technological progress is neutral with respect to the inputs ($\beta_{nt} = 0 \forall n$). ΔTE varies across time, following the same path across producers, unless $\delta = 0$. $[(\varepsilon - 1) \sum_n \left(\frac{\varepsilon_n}{\varepsilon}\right) \dot{x}_{nit}]$ varies across time and producers except when technology is of the Cobb-Douglas type ($\sum_n \beta_n = 0$ and $\beta_{nk} = \beta_{nt} = 0 \forall n, k$) or $\dot{x}_{nit} = 0 \forall n, i, t$. We will test these parametric restrictions in chapter 3.

2.2.3 Empirical Applications of Frontier Models to Evaluate the Efficiency Productivity Growth of Nations

The frontier models presented in the previous discussion considered firms in a particular industry as the unit of analysis (Pitt and Lee, 1981; Kalirajan, 1981, 1982, 1989; Kalirajan and Flinn, 1983; Kalirajan and Shand, 1989; Kumbhakar, 1991; Reifschneider and Stevenson, 1991; Huang and Liu, 1992; Battese and Coelli, 1995). The objective of this subchapter is to name a few empirical studies that use the mentioned frontier approaches - DEA and SFA - to evaluate the efficiency and productivity growth, but where the unit of analysis is countries, instead of firms.

Färe et al. (1994) analyzed productivity growth in 17 OECD countries for the 1979-1988 period, using data envelopment analysis (DEA) techniques to construct the world technology frontier and to compute Malmquist productivity indexes. These indexes were then decomposed into efficiency change, reflecting how much closer a country gets to the world frontier (they called it “catching up”), and technical change, reflecting how much the world

frontier shifts (they named it “innovation”). The results showed that U.S. productivity growth is slightly higher than the average, totally motivated by the technical change component. Japan had the higher productivity growth, with almost half due to efficiency change.

Deliktas and Balcilar (2005) estimated the relative technical efficiencies and efficiency change, technical change and TFP change in the transition economies for the period 1991-2000, using the stochastic frontier approach. They estimated a translog stochastic production function, where the output (GDP) of a country is assumed to be a function of two inputs, capital and labor. By using the Battese and Coelli (1995) time-varying inefficiency model, the authors were able to measure technical efficiency and technical change in the transition economies, and at the same time model technical inefficiency effects as a function of country-specific socioeconomic factors, liberalization and democratization indices, and time period under the Soviet Regime.

Pires and Garcia (2012) estimated and decomposed TFP change using SFA. They estimated a world production frontier for 75 countries for the period 1950-2000. Then they decomposed TFP change for a smaller sample of countries using the “Bauer-Kumbhakar” approach. TFP change is decomposed not only into technical efficiency change, scale effects and technical change, but also into allocative efficiency change. They concluded that productivity accounts for almost all the differences of growth between developed and developing countries and that allocative efficiency is responsible for a large part of them. In fact, their main contribution in relation to other SFA studies is showing that allocative efficiency plays an important role in the economic growth of nations.

Afonso and Aubyn (2013) used the DEA approach to compute Malmquist productivity indexes as well as the SFA analysis for a set of OECD countries for the periods 1970, 1980, 1990 and 2000, choosing GDP *per* worker as output and

three inputs – human capital, public physical capital *per* worker and private physical capital *per* worker – and an environmental variable related to governance. Results showed that (i) private capital is important for growth; (ii) public and human capital contribute positively for growth but they may not be statistically significant; (iii) government effectiveness is important to explain inefficiency, where better governance influences a country's performance positively.

The estimation of relative inefficiencies and TFP growth, in chapter 3, will follow a pattern somewhat similar to the ones proposed by Deliktas and Balcilar (2005), Pires and Garcia (2012) and Afonso and Aubyn (2013). Similarly to Deliktas and Balcilar (2005), the stochastic frontier model used will be an adaptation of the Battese and Coelli (1995) time-varying inefficiency model. The adoption of this model allows for the inclusion of exogenous variables to explain differences in inefficiencies. The production technology used will be the translog production function, due to its flexibility. The decomposition of the TFP is based on a primal approach adapted from Kumbhakar and Lovell (2000). The main contribution of this analysis, in comparison to the one proposed by Deliktas and Balcilar (2002), is the enlargement of the sample of countries by including all OECD countries plus the emergent economies of Brazil, China, India, Indonesia, Russian Federation and South Africa, analyzed for a more recent period (2001-2011). Additionally, some of the determinants of productivity growth included in the analysis will be different and more likely to affect all countries in the world, although with different intensities. Consequently, this analysis is more easily projected/ extended to the rest of the world. This is due to the fact that the purpose of Deliktas and Balcilar (2002) was to explain relative efficiencies among transition economies and those economies are affected by variables that are less likely to affect other countries (for example, the time period under the Soviet Regime). The main contribution

of this analysis, in relation to the one proposed by Pires and Garcia (2012), is the fact that technical inefficiency effects are assumed to be a function of country-specific explanatory variables. Those variables help understand the reasons behind the differences in inefficiencies across countries. Our main contribution in relation to the empirical application of Afonso and Aubyn (2013) is the extension of the sample of countries to include all the OECD countries plus the emergent economies and the inclusion of a broader set of country-specific variables to explain cross-country differences in inefficiencies, such as the weight of agriculture in the economy, the role of trade, the business environment, innovation, governance, among others.

3. The Empirical Model

3.1 Stochastic Frontier Time-Varying Inefficiency Model

The idea of an inefficiency stochastic frontier production function discussed in chapter 2. is now applied to a macroeconomic scenario in which the 34-OECD countries plus the emergent economies of Brazil, China, India, Indonesia, Russian Federation and South Africa are the producers of output (e.g. real GDP at current PPPs *per worker*) using a set of inputs (e.g. human and physical capital *per worker*). Countries can then be thought of as operating on the frontier or below it. The distance between the observed output and the corresponding value on the frontier reflects the level of inefficiency of a country in a certain moment in time. Over time, a country can increase or decrease its level of inefficiency and “catch-up” to the frontier or even the frontier itself can move upwards or downwards, reflecting technical progress or regress, respectively. Additionally, a country can move along the frontier by changing the scale of operations and experiencing the effects of returns to scale. Thus, a country can experience output growth due to factor accumulation or productivity growth, where the last component results from the joint effects of technical efficiency change, technical change and scale change. The estimation and decomposition of productivity change will be performed under the stochastic frontier time-varying inefficiency model of Battese and Coelli (1995) and the Kumbhakar and Lovell (2000) primal approach discussed in subchapter 2.2.2. A brief presentation of the Battese and Coelli’ model will be provided before the presentation of the data and the estimated model.

Battese and Coelli (1995) proposed a stochastic frontier model for (balanced/unbalanced) panel data, in which the technical inefficiency effects are explicitly formulated as a function of a set of explanatory variables.

The authors started the presentation of the theoretical model by defining the stochastic frontier production function for panel data as:

$$Y_{it} = \exp(x_{it}\beta + V_{it} - U_{it}) \quad (3.1)$$

where

- Y_{it} represents the production of the i -th DMU ($i = 1, 2, \dots, N$) in the t -th period ($t = 1, 2, \dots, T$);
- x_{it} is a $(1 \times k)$ vector of values of known functions of inputs of the production function associated with the i -th DMU in the t -th period;
- β is a $(k \times 1)$ vector of unknown parameters to be estimated;
- V_{it} 's represent random errors assumed to be iid $N(0, \sigma_V^2)$, independently distributed of the U_{it} 's;
- U_{it} 's are non-negative random variables associated with technical inefficiency of production and are assumed to be independently distributed, obtained by truncation at zero of the $N(z_{it}\delta, \sigma^2)$ -distribution;
- z_{it} is a $(1 \times m)$ vector of DMU-specific variables which may vary over time;
- δ is a $(m \times 1)$ vector of unknown coefficients of the DMU-specific inefficiency variables.

The technical inefficiency effect, U_{it} , in the stochastic frontier production function (3.1) is defined as:

$$U_{it} = z_{it}\delta + W_{it} \quad (3.2)$$

where W_{it} is the random variable defined by the truncation of the normal distribution with zero mean and variance σ^2 , such that the truncation point is $-z_{it}\delta$, i.e., $W_{it} \geq -z_{it}\delta$. These assumptions are consistent with the fact that the U_{it} 's are non-negative truncations of the $N(z_{it}\delta, \sigma^2)$ -distribution. The inefficiency frontier model defined by equations (3.1) and (3.2) specifies the stochastic frontier production function (which can be Cobb-Douglas, translog or any other type) in respect to the observed levels of production and the U_{it} 's as a function of explanatory variables, the z_{it} 's, and a vector of coefficients, δ . The z_{it} 's include variables that explain the shortfall of the observed levels of production in relation to the corresponding stochastic frontier levels of production, $\exp(x_{it}\beta + V_{it})$. Note that, when the vector of coefficients δ is the null vector, the explanatory variables cannot explain inefficiency effects and the distribution would become the $N(0, \sigma_u^2)$ distribution truncated above at zero originally proposed by Aigner, Lovell and Schmidt (1977).

The technical efficiency measure for the i -th DMU in the t -th period is defined as the ratio of observed output to the corresponding stochastic frontier output:

$$TE_{it} = \frac{Y_{it}}{\exp(x_{it}\beta + V_{it})} = \frac{\exp(x_{it}\beta + V_{it} - U_{it})}{\exp(x_{it}\beta + V_{it})} = \exp(-U_{it}) = \exp(-z_{it}\delta - W_{it}) \quad (3.3)$$

Since the technical inefficiency effects may vary over time and DMU's, the fact that a DMU is more inefficient relative to another DMU in period t does not necessarily imply that the same will happen in the next period. Consequently, the ranking of DMU's defined in terms of technical efficiency may change over time. Maximum Likelihood was the method used to estimate the parameters of the inefficiency model (3.1) – (3.2). The likelihood function and its partial derivatives with respect to the parameters of the model can be found in Appendix C.

3.2 Data and Sample

Data on the 34-OECD countries plus the emergent economies of Brazil, China, India, Indonesia, Russian Federation and South Africa for the 2001-2011 period was collected from the World Development Indicators database, published by the World Bank, and from the PENN World Table, version 8.0. The present study considers a balanced panel data with a total of 440 observations. Below, we provide the definitions of each series used in the econometric estimation performed in subchapter 3.3, as well as a general description of the considered countries regarding some of these series. Finally, we describe the procedures used to select the countries and the time period considered in the empirical application.

The output variable used is the real GDP per person engaged measured at current prices (2005 US\$), converted by the purchasing power parity (PPP) in order to include differences in relative prices of non-tradable goods. This variable is obtained by dividing output-side real GDP at current PPPs (CGDPO) from PWT 8.0 by the number of persons engaged (*emp*) in each county. Two inputs were used in the empirical application and collected from PWT 8.0: (i) the index of human capital per person (*hc*), constructed using data on the average years of schooling (calculated for population with 15 years of age and more) from Barro and Lee (2013) and rates of return for completing different sets of years of education (Psacharopoulos, 1994); (ii) capital stock per person engaged (*ck*), obtained by dividing the variable capital stock at current PPPs by the number of persons engaged. Notice that the index *hc* is calculated per person and not *per worker*. The creators of the index consider it a good indicator of the human capital *per worker* because it considers the average years of schooling for the working-age population (15-64 years). We opted to include output and input variables scaled by worker instead of absolute values, to make these variables more comparable across countries.

Besides the data collected from PWT 8.0 on output and inputs, there were also considered series reflecting countries' specific characteristics that may influence technical inefficiency and productivity change, which were collected from the World Development Indicators database from World Bank. Nine aggregated dimensions were included in the analysis: the economic structure of a country, trade, demography, institutions, innovation, urbanization, government finance and the global financial crisis.

The economic structure of a country is illustrated in the model by the following variables: (i) value added of agriculture, measured as a percentage of the GDP; (ii) value of the rents from total natural resources, which is composed by the sum of oil rents, natural gas rents, coal rents (hard and soft), mineral rents, and forest rents, calculated as a percentage of the GDP. Both variables are good proxies of the weight of the primary sector in the economy. Accounting for the contribution of total natural resources rents to economic output constitutes an important basis for the analysis of the sustainability of the economic growth, as these rents reflect the degree of liquidation of a country's capital stock. In fact, if countries use rents from nonrenewable resources, such as fossil fuels and minerals, and rents from deforestation to support current consumption, instead of investing it in new capital to replace the used one, they are compromising their future economic growth.

The trade dimension is represented by the variable tariff rate, which is the average of applied tariff rates weighted by the product import shares of each partner country. This variable considers the fact that not all countries set equal tariffs to all products, and may set high tariffs to protect favored domestic sectors. In fact, agriculture and labor-intensive manufactures are the sectors that face the greatest trade barriers and given the fact that poor people in developing countries tend to work in these sectors, one could promote economic growth in these countries by removing (or even decreasing) the

barriers to merchandise trade. This variable has a negative impact on the trade dimension: the greater the tariff rate, *ceteris paribus*, the less open the country is to trade.

Demography is present in the econometric estimation through the variable life expectancy at birth, measured in total years. This is an important indicator of the health status of a country and, according to the World Bank definition, it is one of the most frequently used to compare socioeconomic development across countries. A higher life expectancy at birth, *ceteris paribus*, places the country in a higher position in a socioeconomic development ranking.

According to Ulabasoglu and Doucouliagos (2008), institutions can be defined as the rules and organs that set the production climate. Therefore, the institutions' dimension is illustrated in our model by the variables time to resolve insolvency, which mirrors the quality of the business environment of a country, and regulatory quality, which reflects the governance of a country.¹⁷ The selection of both series encompassed the construction of two correlation matrixes. The first matrix included all variables belonging to the World Bank Doing Business Index as well as the real GDP per person engaged, while the second matrix included all variables of the Worldwide Governance Indicators (WGI) plus the output variable. Each of the correlation matrix showed us that it was better to include only one variable reflecting the business environment and one reflecting the governance of a country in order to avoid multicollinearity problems (as the correlations between the variables in each matrix were sufficiently high) and that the variables time to resolve insolvency and regulatory quality were good choices, given the strong correlation with GDP per person engaged. In relation to the first variable, the longer the necessary time to resolve insolvency, *ceteris paribus*, the worse is the business

¹⁷ Institutions can be distinguished between political and economic, although we will focus only on the latter.

environment, and consequently, the economic performance of a country. This comes from the fact that economic activity requires good and efficient regulations that are easy to understand and to implement. Countries with weak regulations tend to have a higher informal sector where firms grow at a slower pace, have less access to finance and employ fewer workers, and hence overall economic performance tends to be weaker. Regulatory Quality is a broad indicator of the governance dimension that reflects the ability of a government to effectively formulate and implement policies which allow and enhance development of the private sector. It includes the following areas, among others: the extent and effect of taxation, intensity of local competition, effectiveness of anti-trust policy, investment freedom and financial freedom.¹⁸ The greater is the regulatory quality of a country, *ceteris paribus*, the stronger tends to be the private sector and hence the stronger is the overall economic performance.

The choice of the variable resident patent applications to represent the innovation dimension followed the same reasoning: a correlation matrix of all indicators of the Global Innovation Index (GII) and the real GDP per person engaged was constructed and the analysis resulted in the selection of this variable. According to the World Bank definition, resident patent applications are “those for which the first-named applicant or assignee is a resident of the State or region concerned.”

Urban population measured as a percentage of the total population reflects the degree of development of a country. In principle, cities generate jobs and income, and deliver education, health care and other services more efficiently than rural areas. Countries with higher percentage of urbanization, *ceteris*

¹⁸ A complete list of the components of this indicator can be easily obtained in the WGI website: <http://info.worldbank.org/governance/wgi/index.aspx#home>

paribus, tend to be more developed since high percentage of urban population is normally associated with industry and services-based economies.

The government finance dimension is represented in the econometric estimation by the variable central government debt measured as a percentage of GDP. High levels of public debt are likely to have negative impact on economic growth, although some believe that this effect becomes relevant only after a certain threshold.

Finally, the effects of the global financial crisis on the economic performance of countries are represented by a dummy variable that assumes the value 1 for the 27-EU countries for the 2009-2011 period and 0 otherwise. This variable intends to differentiate the impact that the crisis had on the 27-EU countries in comparison to other regions, given that those effects were seriously aggravated after the financial crisis turned into a sovereign debt crisis in 2009, which put high constraints on public finances and fiscal sustainability in several euro area and EU countries.

One last note on the treatment of the data is related to the fact that the missing values of each series were substituted by the respective series' means.

The selected sample of countries includes two distinct groups of countries: the 34-OECD countries and 6 emergent economies. The first set of countries was responsible, in 2011, for approximately 75% of the world GDP, while the second set of emergent countries accounted for approximately 15% of the world GDP. In terms of size, approximately 20% of the world population lived in the OECD countries and about 45% lived in the considered emergent economies. The OECD countries' average GDP *per capita* was approximately 3 times higher than the world GDP *per capita*, while the GDP *per capita* of the six emergent countries was only 80% of the world GDP *per capita*. Measured in terms of persons employed, the GDP *per person* employed of the OECD countries was

approximately 2.5 times higher than the world GDP *per person* employed, while the GDP *per person* employed of the emergent economies was about 70% of the world GDP *per person* employed. Although the OECD countries showed a better economic performance when measured in levels, the inverse occurs when we are considering growth rates. All the emergent economies showed a substantively higher GDP and GDP *per capita* growth rate than the respective world growth rates, while for the OECD economies the reverse happened. On average, the emergent countries exhibited a GDP growth rate of approximately 5.4%, with special emphasis to China which grew at a 9.3% rate. On the other hand, the OECD countries exhibited a growth rate of approximately 1.7%, while the world economy grew at the rate of 2.8%.

Country	Population (in millions)	GDP constant 2005 US\$ (in millions)	GDP <i>per capita</i> , PPP (constant 2005 international \$)	GDP per person employed (constant 1990 PPP \$)
OECD members	1249,21	38830376,90	30831,42	47647,58
World	6965,94	53236185,77	10201,53	18552,25
China	1344,13	4194935,26	7417,89	14203
Brazil	196,94	1126722,92	10264,01	13592
India	1221,16	1325841,89	3277,01	8875
Indonesia	243,80	402426,05	4071,65	11002
Russian Federation	142,96	948263,63	14731,03	19012
South Africa	50,59	299675,67	9729,56	14203

Table 2: Comparison in levels of OECD and emergent countries for 2011
Source: World Bank

Country	Population growth (annual %)	GDP <i>per capita</i> growth (annual %)	GDP growth (annual %)
OECD members	0,67	1,05	1,73
World	1,17	1,64	2,83
China	0,48	8,78	9,30
Brazil	0,88	1,83	2,73
India	1,28	4,98	6,33
Indonesia	1,29	5,13	6,49
Russian Federation	0,40	3,87	4,29
South Africa	1,18	2,24	3,46

Table 3: Comparison in growth rates of OECD and emergent countries for 2011

Source: World Bank

In spite of the evident differences between the two sets of countries representing both developed and emerging economies, those economies play an extremely important role in the global economy. In fact, together, these countries account for approximately 90% of the world GDP and are home to approximately 65% of the world's population. Hence, the choice of the countries was based on the following reasoning: the sample had to include countries which size or strategic importance made them crucial players in the global market. One last note on this subject concerns the exclusion of poor countries. According to Pires and Garcia (2011), the decision of exclusion could lead to a biased analysis through selection, if it weren't for the adoption of a flexible production frontier that allows for output elasticities with respect to inputs to vary across countries and over time. In principle, we are protected from selection bias, given that we assume a translog production function in the empirical application.

Regarding the time frame considered in the analysis, we decided to include observations for the 2001-2011 period because of the entry into force of the euro in 2001. Among other things, we wanted to analyze the impacts of the global financial crisis and the inclusion of years previous to 2001 would reflect structural changes different from the ones we wanted to analyze. 2011 was the last year for which the desired data was available.

3.3 Estimation

The stochastic frontier production function considered in this empirical application is

$$\begin{aligned} \ln(y_{it}) = & \beta_0 + \beta_h \ln(h_{it}) + \beta_k \ln(k_{it}) + \beta_t t \\ & + \frac{1}{2} [\beta_{hh}(\ln(h_{it}))^2 + \beta_{kk}(\ln(k_{it}))^2 + \beta_{tt}t^2] + \beta_{hk} \ln(h_{it}) \ln(k_{it}) + v_{it} \\ & - u_{it} \end{aligned} \quad (3.4)$$

where the technical inefficiency is assumed to be a function of

$$\begin{aligned} u_{it} = & \delta_0 + \delta_1(\text{agriculture}_{it}) + \delta_2(\text{nat resources}_{it}) + \delta_3(\text{tariffs}_{it}) \\ & + \delta_4(\text{life expect}_{it}) + \delta_5(\text{time insolv}_{it}) \\ & + \delta_6(\text{regulatory}_{it}) + \delta_7(\text{patents}_{it}) + \delta_8(\text{urban}_{it}) + \delta_9(\text{gov debt}_{it}) \\ & + \delta_{10}(\text{dEUcrisis}_{it}) + \delta_{11}(t) \\ & + w_{it} \end{aligned} \quad (3.5)$$

where

- y is the output-side real GDP at current PPPs (in 10 thousands 2005US\$) per person engaged;
- h is the index of human capital per person, based on years of schooling and returns to education;
- k is the capital stock at current PPPs (in 10 thousands 2005US\$) per person engaged;
- t is the year of the observation;
- agriculture is the value added of agriculture measured as a percentage of GDP;
- nat resources is the total natural resources rents as a percentage of GDP;
- tariffs is the applied tariff rate, weighted mean, all products (%)
- time insolv is the number of years necessary to resolve an insolvency;
- regulatory is a composite index reflecting regulatory quality, which assumes values between -2.5 and 2.5, where higher values correspond to better outcomes;

- *patents* is the resident patent applications measured in thousands of units;
- *urban* is the urban population as a percentage of the total population;
- *gov debt* is the total central government debt as a percentage of GDP;
- *dEUcrisis* is a dummy variable that assumes the value 1 for the 27-EU countries for the years 2009 to 2011 and 0 otherwise;
- v_{it} , u_{it} and w_{it} are defined as in the theoretical model.

Translog is the selected functional form of the frontier production function in (3.4) given its desirable properties, in particular, the fact that it does not impose prior constraints on the elasticities of substitutions between inputs and the fact that output elasticities with respect to each of the inputs are country and time specific. The technical progress is assumed to be Hicks neutral given that $\beta_{ht} = \beta_{kt} = 0$, which implies that technological changes affects all inputs proportionally.¹⁹ Finally, we can see that the variable t is present in both equations (3.4) and (3.5), in order to include both technical change and time-varying inefficiency effects. Similarly to the original Battese and Coelli (1995) model, the t variable in equation (3.4) accounts for Hicks neutral technical progress, while the t variable in equation (3.5) specifies that the inefficiency effects may change linearly with time. The presence of this variable in both equations, along with the distributional assumptions on the inefficiency effects, constitute the trick that allows to disentangle the effects of technical change and inefficiency change.

Productivity change and its decomposition can be obtained in a similar way to the presented in subchapter 2.2.2:

¹⁹ The choice of the Hicks neutral technical progress as well as the translog functional form resulted from the realization of likelihood ratio tests on alternative specifications of the frontier production function to determine the best functional form and the best type of technical progress. These tests are present in the Appendix D.

$$T\Delta = \beta_t + \beta_{tt}t \quad (3.6)$$

$$TE\Delta = -\delta_{11} \quad (3.7)$$

$$\varepsilon_h = \beta_h + \beta_{hh}\ln h_{it} + \beta_{hk}\ln k_{it} \quad (3.8)$$

$$\varepsilon_k = \beta_k + \beta_{kk}\ln k_{it} + \beta_{hk}\ln h_{it} \quad (3.9)$$

$$\varepsilon = (\beta_h + \beta_k) + (\beta_{hh} + \beta_{hk})\ln h_{it} + (\beta_{kk} + \beta_{hk})\ln k_{it} \quad (3.10)$$

$$S\Delta = (\varepsilon - 1) \left[\left(\frac{\varepsilon_h}{\varepsilon} \right) \dot{x}_h + \left(\frac{\varepsilon_k}{\varepsilon} \right) \dot{x}_k \right] \quad (3.11)$$

$$T\dot{F}P = T\Delta + TE\Delta + S\Delta \quad (3.12)$$

Equation (3.6) shows us that technical progress only varies over time. This results from the fact that technical progress is assumed to be Hicks neutral. Technical efficiency change is constant across producers and time because of the linear specification of the variable t in equation (3.5). Output elasticities with respect to inputs (equations (3.8) and (3.9)) and the scale elasticity (equation (3.11)) vary over time and across producers, as expected. Consequently, TFP change in equation (3.12) is expected to vary over time and across producers.

4. Results

4.1. Empirical model specification and estimates

The maximum likelihood estimates of the parameters of the model (3.4) – (3.5) were obtained using the software Stata.²⁰

Several alternative specifications of the model (3.4) – (3.5) were tested using the likelihood ratio test statistics $LR = -2\{\log[Likelihood(H0)] - \log[Likelihood(H1)]\}$. All tests favored the adoption of a translog production function with Hicks-neutral technological progress with the presence of inefficiency effects, which are a linear function of the z -variables.²¹

Maximum likelihood estimates of the parameters of the selected model as well as the standard errors, t -statistics and 95% confidence intervals are presented in table 4. All parameters of the stochastic frontier production function are significant at 5%. Although β_h is negative, the elasticity of output *per worker* with respect to human capital *per worker*, evaluated at the sample mean, is 0.24, indicating that an increase of 1% in the human capital index *per worker* causes an average increase of 0.24% in GDP *per worker*. On the other hand, β_k is positive, as expected, and an increase of 1% in physical capital *per worker* leads to an average increase of 0.59% in GDP *per worker*. The Allen partial elasticity of substitution between human and physical capital *per worker*, measured at the sample mean, is approximately 0.1, suggesting that both inputs are substitutes and strongly inelastic, which means that those factors have limited substitution possibilities.²² Finally, the elasticity of scale,

²⁰ The estimation of the model with Stata required the incorporation of an external command created by Frederico et al. (2013) that estimates panel data stochastic frontier models. This command allows the estimation of a wider set of time-varying inefficiency models than the traditional Stata command `xtfrontier`, including, among others, the Battese and Coelli (1995) model.

²¹ The results of these tests are presented in Appendix D.

²² The calculations of the elasticity of substitution between the two inputs are presented in Appendix E.

measured at the sample mean, assumes the value of 0.83, which reflects decreasing returns to scale. This means that if a country increases all inputs (human and physical capital *per worker*) by 1%, GDP *per worker* will increase, on average, 0.83%. The coefficient of β_t is negative and more than offsets the positive effect coming from the positive sign and magnitude of β_{tt} , contributing to a negative technical change every year. This indicates technological regress over the period 2001-2011, with an estimated annual rate of -7.06%. The estimated variance parameter $\gamma = \frac{\sigma_u^2}{\sigma^2}$, where $\sigma^2 = \sigma_u^2 + \sigma_v^2$, is approximately 0.6, meaning that 60% of total composed error variance of the stochastic production function is explained by the variance of u . This reflects the importance of incorporating technical inefficiency in the model. The standard errors of u and v are statistically different from 0, indicating that inefficiency effects are stochastic and that the traditional production function does not fit the data well.²³

²³ According to Batese and Coelli (1995), if σ_u and σ_v were not statistically different from zero, the model would reduce to a traditional mean response function with the z-variables belonging to the production function.

Number of observations = 440			Observations per country: Minimum = 11				
Number of Countries = 40			Maximum = 11				
Number of Years = 11			Average = 11				
Log Likelihood = 217.0422			Prob > $\chi^2 = 0.0000$				
			Wald $\chi^2 (7) = 628.00$				
ln y		Coefficients	Standard Error	z	P > z	95% Confidence Interval	
						Lower	Upper
Frontier							
<i>ln(h)</i>	β_h	-5.809128	1.147038	-5.06	0.000	-8.057282	-3.560974
<i>ln(k)</i>	β_k	0.7064641	0.1479122	4.78	0.000	0.4165614	0.9963668
<i>t</i>	β_t	-0.0981486	0.0253975	-3.86	0.000	-0.1479267	-0.0483704
<i>ln(h) * ln(h)</i>	β_{hh}	4.113806	1.469301	2.80	0.005	1.234029	6.993583
<i>ln(k) * ln(k)</i>	β_{kk}	-0.2920147	0.0573869	-5.09	0.000	-0.4044909	-0.1795384
<i>t²</i>	β_{tt}	0.0045607	0.0020993	2.17	0.030	0.0004461	0.0006754
<i>ln(h) * ln(k)</i>	β_{hk}	.6141446	0.2106014	2.81	0.005	0.1856937	1.042595
<i>Const.</i>	β_0	3.753954	0.51971394	7.22	0.000	2.735283	4.772624
μ							
<i>Agriculture</i>	δ_1	0.0307487	0.0054211	5.67	0.000	0.0201235	0.0413738
<i>Nat Resources</i>	δ_2	0.0048014	0.0016155	2.97	0.003	0.0016351	0.0079677
<i>Tariffs</i>	δ_3	0.014778	0.0035337	4.18	0.000	0.0078522	0.0217039
<i>Life expect</i>	δ_4	-0.0035429	0.0023082	-1.53	0.125	-0.0080669	0.000981
<i>Time insolv</i>	δ_5	0.0497739	0.0065565	7.59	0.000	0.0369234	0.0626244
<i>Regulatory</i>	δ_6	-0.1080597	0.0233531	-4.63	0.000	-0.153831	-0.0622885
<i>Patents</i>	δ_7	0.0007503	0.000132	5.68	0.000	0.0004915	0.0010091
<i>Urban</i>	δ_8	-0.0008899	0.0008697	-1.02	0.306	-0.0025944	0.0008146
<i>Gov debt</i>	δ_9	0.0008052	0.0003302	2.44	0.015	0.0001581	0.0014524
<i>dEUcrisis</i>	δ_{10}	0.1145139	0.0360738	3.17	0.002	0.0438106	0.1852172
<i>t</i>	δ_{11}	-0.0623471	0.0182371	-3.42	0.001	-0.0980911	-0.026603
<i>Const.</i>	δ_0	0.9465922	0.2487099	3.81	0.000	0.4591297	1.434055
	σ_u	0.1154147	0.0237713	4.98	0.000	0.0798969	0.1755017
	σ_v	0.0968468	0.026857	3.61	0.000	0.0562387	0.1667767
	$\lambda = \frac{\sigma_u}{\sigma_v}$	1.222701	0.0499933	24.6	0.000	1.124716	1.320686

Table 4: Stochastic Frontier Time-Varying Inefficiency Model

4.2. Inefficiency

With respect to the inefficiency model, given by equation (3.5), it is important to bear in mind that the set of country-specific variables considered in the analysis help explaining part, but not all, of the differences in inefficiencies, and hence, productivity growth across countries and over time. All parameters, except δ_4 and δ_8 , are statistically significant at 5%.

The weight of agriculture in the economy is assumed to have a positive effect in inefficiency ($\delta_1 > 0$), suggesting that the higher the contribution of the agricultural sector to the overall economy, *ceteris paribus*, the higher is the inefficiency of a country. *Ceteris paribus*, countries with lower weight of the primary sector are expected to be more efficient, and hence, more productive. This result goes along with the majority of the empirical studies on the TFP determinants. Chanda and Dalgaard (2003) argue that 30 to 50% of cross-country variation in TFP is due to the sectoral composition of GDP. In particular, the allocation of resources between non-agricultural and agricultural sectors affects productivity growth, with agricultural shares affecting negatively TFP. Jaumotte and Spatafora (2007) found out that the transition of economic activity from agricultural to non-agricultural sectors has two types of sectoral effects: (i) the sectoral reallocation effect, which reflects an increase in aggregate TFP (and hence, temporarily, productivity growth) due to the reallocation of resources from a low-productivity to a high-productivity sector; (ii) the sectoral composition effect, which reflects the fact that when a majority of a country's value added comes from high-productivity growth sectors, this will raise its TFP growth.

Resource abundance is expected to have a negative impact on the efficiency of a country, given that $\delta_2 > 0$. This result seems puzzling, since natural resources increase wealth and purchasing power over imports and,

consequently, are expected to boost investment and growth rates. Evidence has shown, however, that countries with high stocks of natural resources tend to be less productive and grow slower than resource-poor countries. Sachs and Warner (2001) named it a “curse of natural resources”. Several reasons were pointed out to explain this result. In particular, Sachs and Warner (1995) mentioned that laziness is associated with resource-rich countries because they have access to easy wealth which does not require major efforts to obtain and to multiply, eliminating the need to become more productive. Another explanation is related to the fact that abundance of natural resources leads to corruption, rent-seeking and poorer governance. The third reason is associated with the Dutch disease, which claims that natural resources promote de-industrialization. The authors found evidence that resource-rich countries had a higher ratio of output of the non-traded sector to output of the tradeable (non-resource) manufacturing sector and that the tradeables production was concentrated in natural resources rather than manufacturing, which is consistent with the Dutch disease hypothesis. Finally, Sachs and Warner (2001) found evidence that countries rich in natural resources tend to be high-price economies and, partly as a consequence, they tend to miss export-led growth opportunities and other kind of growth opportunities. Additionally, high prices tend to attract less efficient producers.

Our results show that tariffs affect negatively efficiency ($\delta_3 > 0$). Tariffs, as well as nontariff measures, are used by countries to control their imports, with the final objective of protecting domestic industries or to raise fiscal revenues. These measures constitute an obstacle to free trade. Several authors investigated the relationship between trade and productivity growth. According to Isaksson (2007), those studies focused on analyzing many factors of the relationship, in particular, the diffusion of technology from industrialized countries to LDCs, the scope of learning-by-exporting, the benefits of trade to productivity growth

evaluated by types of trade (imports versus exports), the types of traded goods that contain more technology. One effect of trade liberalization that goes along with our results is the fact that it causes competition to increase, which may encourage firms to reduce inefficiency by pushing them to use inputs more rationally, or, ultimately, force the least inefficient firms to leave the market. Consequently, average productivity of the sector, as well as of the economy, will increase. Many other reasons are behind the result that trade liberalization affects productivity positively, in particular: the improved access to imported intermediate inputs of higher quality and variety; access to bigger markets, which facilitates the exploitation of economies of scale and product specialization; access to better technology embodied in intermediate and final imported goods. One of the components of trade, imports, was found to have highly significant positive impacts on productivity. Mayer (2001) looked at imports as a mechanism to introduce foreign (and better) technology into domestic production, which will enhance productivity. Isaksson (2001), on the other hand, found that imports only contribute to the incorporation of better technology into domestic production and, consequently, to the improvement of productivity, if the importer country has the necessary absorptive capacity (for example, the required level of human capital). Acemoglu and Zilibotti (2001) favor this result, by arguing that even if developing countries had access to the same technology by importing it from developed countries, there would still exist differences in productivities due to the inadequacy of this technology to the characteristics of those countries (such as low absorptive capacity measured by the level of human capital).

Life expectancy at birth is an indicator of the health status of a country. A healthy worker is expected to be more productive, *ceteris paribus*, and hence contribute positively to economic growth. In addition to the direct impact of health on both energy to work and working hours, Bloom, Canning and Sevilla

(2004) reported three additional channels through which health may influence economic performance: (i) improved health and survival of the young children may create incentives to reduce fertility and, therefore, increase labor participation; (ii) higher life expectancy may lead to a higher investment in education, given the fact that the number of available years to recover the investment increases; (iii) individuals become more cautious and want to save more for retirement, which may result in a higher accumulation of physical capital. Although the variable life expectancy is significant only at 12.5%, our results corroborate the premise that health contributes positively to productivity and economic performance, given that higher life expectancy at birth influences efficiency positively ($\delta_4 < 0$). Many empirical studies focused on investigating the impact of health on TFP and economic growth. Cole and Neumayer (2003) used three health indicators – proportion of undernourished within a country, the incidence of malaria and other waterborne diseases and life expectancy – to investigate the impact of poor health on TFP for a sample of 52 developed and developing countries for the period of 1965-1996. They found out that poor health impacts negatively TFP, although the effect is larger for African countries. Bloom and Sachs (1999) concluded that 50% of the differences between African countries' growth rates and the rest of the world's growth rates can be explained by demographic and health variables. Barro and Sala-I-Martin (1995), Barro (1997), Gallup and Sachs (2000), among others, estimated a positive growth effect, with different magnitudes, of an increase in life expectancy. As an example, the latter authors estimated, for the period of 1960-1990, for 104 countries, that an increase of one year on the population's life expectancy would lead to an output increase of 4%. Although, in general, authors seem to agree on the importance of health on economic growth, the significance of this indicator varies across groups of countries, according to Knowles and Owen (1995). In fact, the authors found that health is a very significant determinant of TFP, except in the case of the OECD countries. The

fact that the variable life expectancy is statistically significant only at the 12.5% level of significance may be related to the fact that the majority of our sample is composed by OECD-countries, which are expected to have similar levels of health standards. Consequently, life expectancy becomes a non-differentiating factor among this group of countries.

The business environment and the governance of a country are defined by its institutions. A bad business environment, proxied in our model by a high value of the variable time to resolve insolvency, is expected to influence efficiency negatively ($\delta_5 > 0$). On the other hand, a country with better governance is expected to be more efficient ($\delta_6 < 0$), *ceteris paribus*. In fact, good institutions are expected to set a good environment that promotes private investment, productivity and economic growth. Hall and Jones (1998) reported that differences in institutions and government policies, which, for them, constitute the social infrastructure of a country, are responsible for the majority of the differences in physical and human capital accumulation, productivity and output *per worker* across countries. Acemoglu et al. (2004), after concluding that economic institutions are fundamental in explaining differences in economic growth of countries, they tried to explain the variety of economic institutions across countries. They concluded that economic institutions determine not only the economic performance of a country but also the distribution of income among the different groups in the society. Thus, this will generate winners and losers and the choice of the economic institutions is dependent on the political power of each of these groups. The distribution of the political power is then determined by political institutions (which allocate *de jure* political power) and the distribution of resources (which allocate *de facto* political power). Good economic institutions emerge “when political institutions allocate power to groups with interests in broad-based property rights enforcement, when they create effective constraints on power-holders, and when there are relatively few

rents to be captured by power-holders (Acemoglu et al., 2004).” Several other authors confirmed the importance of institutions on TFP and economic growth, which support our results (Easterly and Levine, 2001; Rodrik et al., 2002; Ulubasoglu and Doucouliagos, 2004; Afonso and St. Aubyn, 2013).

It is widely accepted that innovation contributes positively to TFP and economic performance, evaluated in levels and in growth rates (Guellec and van Pottelsberghe de la Potterie, 2001; Ulku, 2004; Abdih and Joutz, 2005). That being said, our results, which show that patents influence efficiency negatively ($\delta_7 > 0$), although with a very small magnitude, seem contradictory. Two questions immediately arise from this result: Do patents really provide an incentive to innovate? Are patents a good proxy of innovation? Hall (2007) answered the first question by presenting new reasons to why patents might in fact disincentive innovation, which contradicted the traditional view on patents. The traditional view supported the idea that patents encouraged innovation. In fact, the inventor, by patenting its invention, is excluding others from using his invention for a limited period, in exchange for revealing the description and implementation of the invention. This would, on one hand, incentive innovation, given that it prevents immediate imitation, and, on the other hand, the sharing of the invention’s secrets would help others to innovate more easily, based on the knowledge contained in the innovation. Instead, the idea that patents might discourage innovation is based on the fact that they tend to “increase the costs for subsequent innovators, especially when these innovators need to combine inventions from many sources (Hall, 2007)”. The second question has been addressed by a few authors (Griliches, 1991; Breschi and Lissoni, 2001; Thompson and Fox-Kean, 2005; Shearmur, 2012), who stated that using patents as a proxy of innovation may be problematic for a number of reasons: patents only register major product innovations, large firms may opt to patent for precaution (i.e. they may patent inventions that they don’t have the

intention to place in the market just to keep others from doing it), small firms may opt to not divulge the secrets of their inventions or, simply, may not have the necessary means to acquire a patent. Notice that small firms tend to be the type of firms that are most associated with innovation (Hall et al., 2009) and, therefore, using patents as its proxy may be truly misleading.

Urbanization shows up in our model as statistically insignificant, although with the expected sign ($\delta_8 < 0$). Nevertheless, empirical studies have shown that urbanization has a positive influence on TFP and on economic growth through agglomeration effects, which contribute to the reduction of transaction costs, to the creation of economies of scale (Kumar and Kober, 2012). Indirectly, urbanization can be seen as a determinant of TFP growth, given that it favors the concentration of innovative activities and innovation (Shearmur, 2012), which, as previously mentioned, is an important determinant of TFP growth. In fact, cities are conducive to the concentration of very diverse economic agents as well as a very diverse “ethnic, cultural and social fabric (Shearmur, 2012)”, which are pre-requirements for innovation.

Government debt is shown to have a negative impact on efficiency ($\delta_9 > 0$). Countries with lower government debt are expected to be closer to the frontier, *ceteris paribus*. Some authors have tried to analyze the relationship between government debt and TFP growth and, hence, economic growth (Pattillo et al., 2002; Schclarek, 2004; Checherita and Rother, 2010). In particular, Patillo et al. (2002) found that the quality, rather than the level, of investment is an important channel through which growth is negatively affected by the high levels of government debt. Specifically, the expectation of high future taxation needed to repay the debt may distort the investment decisions towards, for example, short-run projects with a lower positive impact on productivity growth.

The effect of the financial crisis, which showed, on a global scale, major impacts on macroeconomic variables especially after 2009, on the efficiency of the 27-EU countries is expected to be negative, given that $\delta_{10} > 0$. Balakrishnan et al. (2009), in particular, analyzed the impact of 88 banking crises, over the past 40 years for a large sample of countries, on medium-term output growth. The authors reported a persistent and substantive decrease of output after the banking crises. TFP was pointed out as the main responsible for the output losses observed in the short-run, although it recovered to pre-crisis levels in the medium-run. Nevertheless, the authors registered some positive effects of the crises on TFP, which magnitude is not sufficient to overcome the negative effects. The negative effects are related to the more precautionous attitude of the financial system in allocating funds, which may not be willing to lend resources to more productive and high-return but more risky projects. Additionally, less innovation associated with cuts in research and development may have negative impacts on productivity. Finally, lack of financing may affect high-productivity firms, which may be obliged to leave the market or to contract their operations. The positive effects of the crises on TFP are related to a cleansing effect, where less efficient firms are forced to adopt more efficient practices, or even forced to leave the market.

Finally, time is also an important factor to explain differences in inefficiencies across countries and for the period considered in the analysis, given that the coefficient of the time variable, δ_{11} , is highly significant. The negative estimate of the time trend coefficient shows a positive evolution of the technical efficiency among the 40 countries for the period of 2001-2011. This result goes along with the findings of Pires and Garcia (2012).

One last note on the determinants of inefficiency is related to the fact that no variables reflecting the labor quality of each country are included in the inefficient model (3.5) as environmental variables influencing inefficiency

directly. A labor force with a high degree of human capital (for example in the form of high levels of education) is expected to better acquire and use relevant knowledge. In fact, it is a crucial determinant of a country's ability to innovate (Romer, 1990) and, especially for developing and least developed countries, it influences their capacity to adopt foreign technology, mainly developed in OECD economies, given that "many technologies are designed to make optimal use of the skills of these richer countries' workforce (Acemoglu and Zilibotti, 2001)". So, no doubt that human capital is an essential variable, however, we opted to not include it in the inefficiency model, due to the fact that labor quality, measured as the index of human capital, is already incorporated in the stochastic frontier production function in (3.4). Consequently, human capital will influence directly the position of the stochastic frontier, rather than the position of the country in relation to the frontier.

4.3. Technical efficiency and returns to scale

A ranking of countries constructed for the technical efficiencies and returns to scale is presented in table 5.²⁴ The values of the technical efficiencies and returns to scale correspond to the respective means calculated for the overall period for each country. Regarding the technical efficiencies ranking, we can see that no country is situated on the frontier (given that $TE_i < 1, \forall i$). The top of the ranking is occupied by European countries, led by Ireland. The bottom of the ranking is occupied by the emergent economies, as expected, where the less inefficient country in the sample is Indonesia. South Africa stands out of this group of countries, occupying the 30th place in the ranking, surpassing Slovenia, Portugal, Japan, Hungary and Czech Republic. This position may be explained

²⁴More detailed information regarding efficiency levels per country and per year as well as output elasticities with respect to human and physical capital and average elasticities per country are presented in Appendixes F, G, H and I, respectively.

by the relative low contribution of agriculture to the economy in comparison to the important contribution of more productive sectors, such as a well developed manufacturing sector (agriculture value added corresponds to 3% of GDP in comparison to the 10% for the group of emergent economies). In addition, South Africa has a relatively good business environment, where the time necessary to resolve insolvency is approximately 2 years, in comparison to the 3 years and 3 months for the emergent economies. Finally, the average number of resident patents applications *per year* in South Africa is one of the lowest in the sample. A somewhat surprising position is the one occupied by the United States, which assumes the 18th position in the ranking. A deeper analysis of the data allows us to conclude that the middle position that the United States occupies may be due to (i) the relatively high contribution of the natural resources rents to the economy, which is somewhat close to the OECD average (they are responsible for approximately 1.4% of American GDP versus 2.1% for the OECD countries); (ii) the relatively high level of protectionism, where the average tariff is 1.7% versus 2.3% for the OECD countries and 1.85% for the overall sample; (iii) the “just slightly” higher life expectancy than the OECD average (77.73 versus 76.91 years); (iii) the extremely high number of resident patents applications in comparison to the OECD average (215.000 versus 25.500 patents, respectively); (iv) the relatively high level of government debt (52% versus 54% for the OECD countries). Therefore, we can see that the United States performance evaluated in terms of the previous indicators is close to the OECD average performance, which contradicts what we initially expected, that is, the United States occupying the top positions of the OECD ranking. Finally, Portugal shows up in the 32nd position out of 40, with a technical efficiency lower than the overall average. The main drivers of the low technical efficiency of Portugal seem to be the low levels of governance (an average of 1.07 versus 1.3 for the OECD countries) and the high levels of public debt (74.5% versus 54% for the OECD countries). According to the OECD 2013

publication “Portugal: Reforming the State to promote growth”, Portugal lacks an efficient public sector capable of promoting a business environment that is more favorable to investment, job creation and innovation, in order to ensure a stronger private sector development. Additionally, Portugal needs institutional reforms that encompass greater sustainability of fiscal management.

In relation to the returns to scale ranking, it provides some intuitive results and some that are not so intuitive. We expect top positions of the ranking, where countries are characterized by increasing returns to scale ($RTS > 1$), to be occupied by countries that have relatively low levels of physical and human capital *per worker*. There are, however, countries that don't fit the previous description. A closer analysis allowed us to conclude that there are other factors that may be behind increasing returns to scale, in particular: a very developed financial system (Switzerland); high degree of innovation (Finland); the presence of natural resources (Norway, Israel) and recent access to EU structural funds (Czech Republic). Relatively homogeneous European countries, such as Germany, Belgium, United Kingdom, France, Italy, Austria, Ireland and Sweden are concentrated in the middle of the table, with decreasing returns to scale. Those countries conciliate relatively high levels of physical and human capital *per worker* with high investment in innovation. The bottom positions are occupied by countries with very high levels of human and physical capital *per worker* (Australia, Canada, United States), with the clear exception of the Russian Federation. The last position occupied by the Russian Federation may be related to the extreme macroeconomic turbulence that characterized the 1990s for this country and the type of regime, characterized by very low levels of governance. For the particular case of Portugal, its mid-low position may result from the very low levels of human capital *per worker* and relatively high levels of physical capital *per worker* and the still low level of resources allocated to innovation.

Rank	Country	TE	Country	RTS
1	Ireland	0,89	South Africa	1,29
2	Norway	0,83	Czech Republic	1,24
3	Luxembourg	0,82	Indonesia	1,20
4	United Kingdom	0,82	Luxembourg	1,14
5	Sweden	0,78	Switzerland	1,12
6	Canada	0,76	Finland	1,09
7	Netherlands	0,75	Norway	1,09
8	Belgium	0,75	Spain	1,06
9	Switzerland	0,74	Israel	1,04
10	Austria	0,74	Mexico	1,00
11	Denmark	0,72	Brazil	0,98
12	Finland	0,72	Belgium	0,97
13	New Zealand	0,71	Germany	0,96
14	France	0,71	Japan	0,94
15	Australia	0,70	United Kingdom	0,94
16	Germany	0,70	Sweden	0,93
17	Israel	0,70	Austria	0,91
18	United States	0,70	Turkey	0,89
19	Italy	0,68	Denmark	0,89
20	Spain	0,68	New Zealand	0,87
21	Iceland	0,65	Italy	0,86
22	Poland	0,65	France	0,86
23	Estonia	0,63	Ireland	0,86
24	Greece	0,63	India	0,83
25	Mexico	0,61	Korea, Rep,	0,83
26	Chile	0,60	Estonia	0,81
27	Slovak Republic	0,60	Slovak Republic	0,78
28	Korea, Rep.	0,59	Netherlands	0,76
29	Turkey	0,59	Iceland	0,73
30	South Africa	0,59	Portugal	0,71
31	Slovenia	0,58	China	0,70
32	Portugal	0,58	Hungary	0,59
33	Japan	0,58	United States	0,59
34	Hungary	0,57	Slovenia	0,52
35	Czech Republic	0,53	Australia	0,50
36	Russian Federation	0,48	Chile	0,45
37	Brazil	0,39	Canada	0,43
38	China	0,37	Greece	0,34
39	India	0,30	Poland	0,31
40	Indonesia	0,29	Russian Federation	0,22
Overall Mean		0,64		0,83
OECD Mean		0,68		0,82
Non-OECD Mean		0,40		0,87

Table 5: Rankings in levels: Technical Efficiencies and Returns to Scale

4.4. Decomposition of economic growth

So far, we have analyzed the countries' behavior in terms of levels of technical efficiency and returns to scale. Table 6 provides a decomposition of the economic growth into factor accumulation (human and physical capital), change in TFP, where the latter is divided into changes in technical progress, technical efficiency and scale effects, and random shocks.²⁵ We can see that emergent economies grew at a higher annual rate than the OECD countries (approximately 4 times higher). This resulted from the fact that: (i) both human and physical accumulation was higher in developing nations; (ii) the change in TFP was less negative for those countries. Regarding the components of productivity, we see that technical and technical efficiency change affected both group of countries in the same manner. As previously mentioned, this resulted from the assumption of Hicks-neutral technological progress and the assumption that technical efficiency changes linearly with time, respectively. We observed an improvement of technical efficiency over time, for both groups, which was outweighed by the technological regress. In fact, the improvement of technical efficiency of countries can be explained, at least in part, by the downward shift of the production frontier. Although the technological regress seems counterintuitive, several other authors also reported it, even though they analyzed much less controversial periods than the one in this analysis (Rao and Coelli, 1998; Deliktas and Balçilar, 2005; Pires and Garcia, 2012). The change in scale effects is the only responsible for the differences in TFP change between groups. It affected negatively the TFP change of both groups of countries, although with a smaller magnitude for the non-OECD members. This resulted from the positive rate of accumulation of inputs combined with decreasing returns to scale verified in both groups. Finally, thanks to the Kumbhakar and Lovell (2000) TFP decomposition, we were able to separate the normal behavior

²⁵The sources of economic growth per country can be consulted in Appendix J.

of the economies from the random shocks, where the last were obtained as a residual term, subtracting the growth rate of inputs *per worker* adjusted by the respective expenditure share and the growth rate of TFP to the growth rate of GDP *per worker*.

Variable	Group of Countries	Compound Annual % Rate*
GDP <i>per worker</i> growth	OECD Members	1,20
	Non-OECD Members	4,69
	Difference**	-3,49
Human Capital <i>per worker</i> accumulation	OECD Members	0,06
	Non-OECD Members	0,19
	Difference	-0,13
Physical Capital <i>per worker</i> accumulation	OECD Members	3,48
	Non-OECD Members	3,52
	Difference	-0,04
Change in TFP	OECD Members	-1,64
	Non-OECD Members	-1,18
	Difference	-0,46
Technical Change	OECD Members	-7,07
	Non-OECD Members	-7,07
	Difference	0,00
Change in Technical Efficiency	OECD Members	6,23
	Non-OECD Members	6,23
	Difference	0,00
Change in Scale Effects	OECD Members	-0,80
	Non-OECD Members	-0,34
	Difference	-0,46
Random Shocks	OECD Members	-0,70
	Non-OECD Members	2,16
	Difference	-2,86

Table 6: Sources of economic growth 2001–2011 per groups of countries: compound annual % change

*The values in this table were calculated by taking a simple arithmetic mean over the countries that constitute each group of the compound annual rates of change of each variable.

** The difference is calculated in terms of percentage points.

5. Conclusion

In the beginning of the present research, we posed the question “Why are some countries far richer than others?”, which we tried to provide an answer to under the framework of the Battese and Coelli (1995) stochastic frontier time-varying inefficiency model as well as the Kumhakar and Lovell (2000) primal approach to the decomposition of the productivity growth, using a panel of 40 countries, 34 of which are OECD-members and the remaining 6 are emergent economies, for the period of 2001-2011.

The results showed that differences in TFP growth between developed and developing countries are the main drivers of the differences in the growth rates of GDP *per worker*, although differences in the factor accumulation also play an important role. However, given that the developing countries performed better in terms of growth rates of GDP *per worker*, factor accumulation and productivity in the period of analysis, it seems that they might be under a process of catching up. Or alternatively, the results may be inherent to this turbulent decade, which had particular severe effects on the developed countries. In particular, the separation of the normal behavior of the economy from the random shocks favors this conclusion, given that those shocks, probably coming from the demand side, affected the growth of the developed countries negatively.

The decomposition of productivity growth into its various sources, namely technical efficiency change, technical change and change in returns to scale, allowed us to conclude that the differences in TFP change across countries are due to differences in the change in returns to scale. This resulted from the type of stochastic frontier model that we adopted, in particular a translog production function with Hicks neutral technical progress and the inefficiency effects as a

linear function of time. Nevertheless, we concluded that environmental variables are important to explain differences in the technical efficiency levels. Specifically, a high contribution of the agricultural sector to the economy, a high value of natural resources rents, impediments to free trade such as tariffs, a bad business environment, a high number of patents, a high level of government debt and the financial crisis contribute negatively to technical efficiency. On the other hand, a good health status and good institutions help countries to be located closer to the frontier. Over the 2001-2011 period, we observed a general improvement in the technical efficiency of countries along with a downward shift in the stochastic production frontier.

Further improvement of this work may include the enlargement of the sample to the least developing countries as well as the enlargement of the period of analysis. Additionally, the assumption that technical efficiency varies linearly with time and in the same manner for all countries may be too restrictive. An alternative could be to adopt a formulation of the technical inefficiency term similar to the one proposed by Kumbhakar (1990) and Battese and Coelli (1992).

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Appendix

Appendix A: Technical Efficiency and Productivity Change

Before presenting and discussing the concepts of technical efficiency and productivity change further, one should make a clear distinction between efficiency and productivity. Most of the times, they are used interchangeably, although they don't mean the same thing.

Productivity of a firm corresponds to the ratio of the output(s) that it produces to the input(s) that it uses. This ratio is easy to calculate if the firm uses a single input to produce a single output. However, in the case of several inputs and outputs, they have to be aggregated so that productivity remains the ratio of two scalars. In the case of multiple inputs, one assumes productivity to be the total factor productivity (TFP), which is a productivity measure involving all factors of production, since partial productivity measures (such as labor productivity or capital productivity) often give wrong ideas of the overall productivity. Efficiency can be seen as the distance between the quantity of input and output observed and the quantity of input and output defined from a particular frontier, corresponding to the best possible frontier for that firm in its cluster (industry).²⁶ It can be divided into technical and allocative efficiency, and the combination of both provides a measure of the overall efficiency. One will focus only on the technical efficiency concept, which will be further developed and formal definitions of it will be provided.²⁷

To render the distinction between productivity and efficiency more intuitive, one can represent the case of a production process where a single input (x) is

²⁶If the unit is countries instead of firms, the cluster could be defined as the group of countries considered in the sample. This is the case of the empirical analysis performed in chapter 3.

²⁷ The following analysis on the technical efficiency measures will not contain a decomposition of the various sources of technical efficiency. For that, consult Färe et al. (1985,1994).

used to produce a single output (y). Figure A.1 is adapted from Coelli et al. (2005) and illustrates this difference and further introduces the concept of scale economies. Productivity of a firm is given by the slope of the ray that goes from the origin to the point of production of that firm, y/x . If a firm is operating at point A, it would be technically inefficient, as it is operating in a point below the production frontier. If this firm were to move to point B, productivity would increase (as the slope of the ray would be greater) and become technically efficient. However, this firm could further increase its productivity by moving to point C, which is the point of maximum possible productivity (as the ray at point C is tangent to production frontier). This movement is an example of exploiting scale economies. The fact that a firm may be producing at the frontier (that is, it is technically efficient) but may still improve its productivity by changing its scale of operations, reflects the importance of this concept.

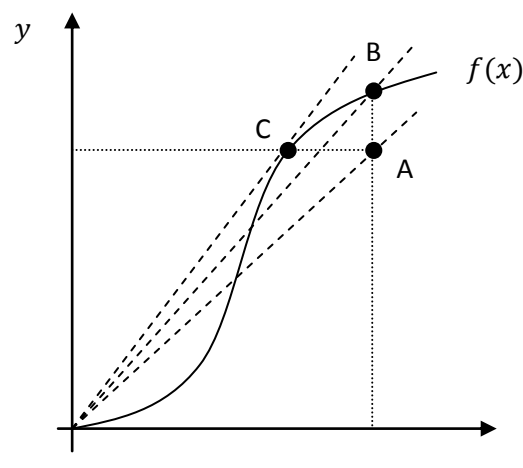


Figure A.1: Productivity, Technical Efficiency and Scale Economies

Efficiency and productivity, in any case, can be seen as two complementary concepts. The measures of efficiency are more precise than those of productivity, given that they require a comparison with the most efficient frontier, and because of that, they can complete those of productivity.

Technical Efficiency

One of the reasons that justify the measurement of technical efficiency is “*the recognition that a gap exists between the theoretical assumption of full technical efficiency and empirical reality.*” (Kalirajan and Shand, 1999). According to Kumbhakar and Lovell (2000), the main issue here lies in quantifying the distance from the input-output bundle of each producer to the production frontier. This distance can be provided by technical efficiency measures, which can be input or output-oriented. An input-oriented measure of technical efficiency answers the question “By how much can input quantities be proportionately reduced and still produce the same levels of output?”, while an output-oriented measure provides an answer for the question “By how much can you proportionately increase output quantities without changing the input quantities used?”. Full technical efficiency requires the producers to produce on the production frontier and, consequently, no input quantity can be reduced without reducing output and no output can be increased without increasing the input level.

Farrell (1957) introduced a new impetus to the efficiency measurement discussion by extending the work of Debreu (1951) and Koopmans (1951) to define a firm efficiency measure that accounts for multiple inputs. He defined technical efficiency of a firm as consisting of two components: technical efficiency, which reflects the ability of a producer to obtain maximal output from a given set of inputs, and allocative efficiency, which reflects the ability of a producer to use the inputs in optimal proportions, given the input prices. The combination of both efficiencies provides the “overall efficiency” concept.²⁸ Farrell illustrated his idea of an input-oriented technical efficiency measure by considering the case of one output, y , and two inputs, x_1 and x_2 , under the

²⁸ Farrell used the term *price efficiency* instead of *allocative efficiency*.

assumption of constant returns to scale.²⁹ Technical efficiency here is measured against the unit isoquant SS' in Figure A.2 of fully efficient producers.³⁰

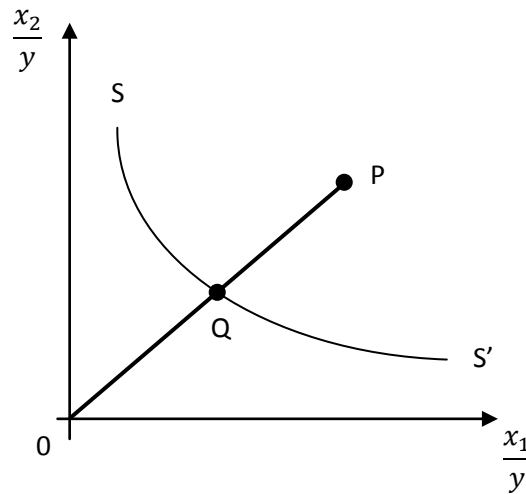


Figure A.2: An input-oriented measure of technical efficiency ($N=2$)

Assuming that a producer uses the quantity of inputs P to produce a unit of output, the technical inefficiency (TI) of that producer can be measured as $\overline{QP}/\overline{OP}$, which represents the percentage by which all inputs need to be reduced to achieve technically efficient production. The technical efficiency of that producer is given by $\overline{OQ}/\overline{OP}$, which corresponds to $1-TI$. TE varies between 0 and 1, where a value of 1 implies that the producer is fully technically efficient. Point Q is technically efficient because it lies on the isoquant SS' .

Coelli et al. (1999) presented an illustration for the output-oriented technical efficiency measure, where they consider the case of two outputs (y_1 and y_2) and 1 input (x), under constant returns to scale.³¹

²⁹ The assumption of constant returns to scale allows the technology to be represented by a unit isoquant. Farrell provides extensions of his method in order to accommodate the cases of multiple inputs, multiple outputs and non-constant returns to scale. Kumbhakar and Lovell (2000) provide a brief summary on these most complex cases.

³⁰ These efficiency measures assume that the production frontier of fully efficient producers is known. In reality, however, this production frontier is not known and has to be estimated from observations of a sample of producers. Estimation of these frontiers using DEA and SFA methods are discussed in sub-chapters 2.2.1 and 2.2.2.

³¹ This assumption allows the technology to be represented by a unit production possibility curve in a bi-dimensional space.

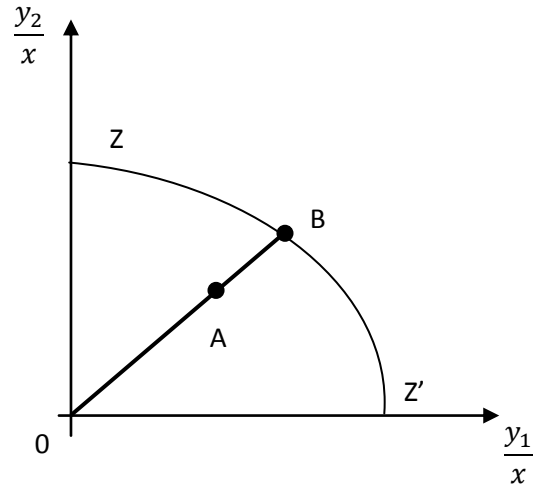


Figure A.3: An output-oriented measure of technical efficiency ($M=2, N=1$)

Here, point B is technical efficient, since it lies in the production possibilities frontier ZZ' and \overline{AB} represents technical inefficiency, which is the amount by which outputs could be increased without requiring an increase in the input quantity. Technical efficiency of that producer is given by $\overline{OA}/\overline{OB}$ and varies between 0 and 1.

For the case of a single-output and two inputs (which is the case one pursues in chapter 3), we can follow Kumbhakar and Lovell (2000)'s definitions of input- and output-oriented measure of technical efficiency, respectively:

$$TE_i(x, y) = \min\{\theta : y \leq f(\theta x)\} \quad (A.1)$$

$$TE_o(x, y) = [\max\{\phi : \phi y \leq f(x)\}]^{-1} \quad (A.2)$$

where $f(\cdot)$ is the production technology. Under the assumption of constant returns to scale, input- and output-oriented efficiency measures provide the same results, although they differ when increasing and decreasing returns to scale are assumed (Fare and Lovell, 1978).

So far, we have measured technical efficiency along a ray from the origin to the observed production point, where only proportional reductions on input quantities or proportional increases on output quantities were allowed. Those

are called radial efficiency measures. A non-radial efficiency measure, also known as directional efficiency measure, allows for non-proportional variations of inputs and outputs. Several authors proposed non-radial efficiency measures (Färe and Lovell, 1978; Zieschang, 1984; Chambers, Chung and Färe, 1998). Recent applications of these efficiency measures include: modeling plant capacity (Färe and Grosskopf, 2000; Färe et al., 2010), assigning prices to non-market outputs including undesirable outputs applied to the agricultural sector in particular (Hudgins and Primont, 2004). Although recent literature shows an increasing focus on these measures, one uses instead radial output-oriented measures of technical efficiency on the empirical application presented in chapter 3.

Productivity Change

In the discussion above, efficiency was assumed to remain constant over time and across producers. However, if efficiency varies, either across producers or through time, its variation constitutes a source of producer performance variation. One measure of performance is productivity change, and so, if efficiency changes through time, it makes a contribution to TFP change. Furthermore, in the case where price information is not available, productivity change can be decomposed into (i) technical change, which measures shifts of the frontier over time, (ii) technical efficiency change, which measures the movement of an economy towards (or away from) the production frontier, and (iii) scale change, which measures the contribution of returns to scale. Notice that only in the case where technical efficiency does not vary in time and under the assumption of constant returns to scale (and persistent allocative efficiency, in the case where price information is available), TFP change and technical change are synonymous, as was common to assume in early studies of productivity change.

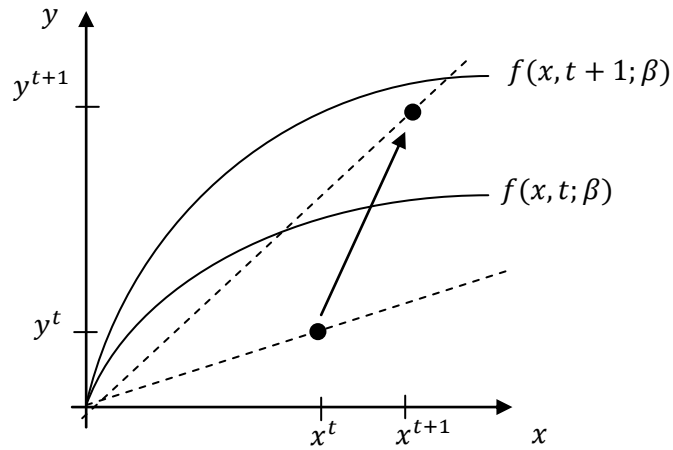


Figure A.4: Decomposition of productivity growth

Figure A.4, adapted from Kumbhakar and Lovell (2000), illustrates the decomposition of productivity growth for the case of a single input that is used to produce a single output and a producer that expands from (x^t, y^t) to (x^{t+1}, y^{t+1}) . Technical progress has occurred from period t to $t+1$, since $f(x, t + 1; \beta) > f(x, t; \beta)$. Although production in both periods is technically inefficient ($y^t < f(x^t, t; \beta)$ and $y^{t+1} < f(x^{t+1}, t + 1; \beta)$), technical efficiency has improved, since $\frac{y^t}{f(x^t, t; \beta)} < \frac{y^{t+1}}{f(x^{t+1}, t+1; \beta)}$. Productivity, measured as the slope of the ray that connects observed output-input bundle to the origin, increased from t to $t+1$, since $\left(\frac{y^t}{x^t}\right) < \left(\frac{y^{t+1}}{x^{t+1}}\right)$.

Appendix B: Panel Data Stochastic Frontier Models

<u>Model</u>	<u>Nature of u</u>	<u>Model specification</u>	<u>Estimation Method</u>	<u>Distribution</u>	<u>Heteroscedasticity in v</u>	<u>Heteroscedasticity in u</u>
True fixed-effects model (Greene, 2005)	Time-varying	$y_{it} = \alpha_i + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_{it}$ $v_{it} \sim N(0, \sigma_v^2)$	Maximum Likelihood Dummy Variable	Exponential, Truncated Normal, Half Normal	Yes	Yes
True random-effects model (Greene, 2005)	Time-varying	$y_{it} = \alpha + w_i + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_{it}$ $v_{it} \sim N(0, \sigma_v^2)$	Simulated Maximum Likelihood	Exponential, Truncated Normal, Half Normal	Yes	Yes
ML random-effects time-varying inefficiency effects model (Battese and Coelli, 1995)	Time-varying	$y_{it} = \alpha + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_{it}$ $u_{it} = z_{it}\delta + w_{it}$ $v_{it} \sim N(0, \sigma_v^2)$	Maximum Likelihood	Truncated Normal	Yes	Yes
Iterative Least Squares time-varying fixed-effects model (Lee and Schmidt, 1993)	Time-varying	$y_{it} = \alpha + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_{it}$ $u_{it} = d_t \cdot u_i, \text{ where } d_t \text{ are time dummies}$ $v_{it} \sim N(0, \sigma_v^2)$	Iterative Least Squares	-	No	No
ML random-effects time-varying efficiency decay model (Battese and Coelli, 1992)	Time-varying	$y_{it} = \alpha + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_{it}$ $u_{it} = \exp[-\eta(t - T)] \cdot u_i$ $v_{it} \sim N(0, \sigma_v^2)$	Maximum Likelihood	Truncated Normal	No	No
ML random-effects flexible time-varying efficiency model (Kumbhakar, 1990)	Time-varying	$y_{it} = \alpha + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_{it}$ $u_{it} = [1 + \exp(bt + ct^2)]^{-1} \cdot u_i$ $v_{it} \sim N(0, \sigma_v^2)$	Maximum Likelihood	Half Normal	No	No
Modified-LSDV time-varying fixed-effects model (Cornwell, Schmidt and Sickles, 1990)	Time-varying	$y_{it} = \alpha + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_{it}$ $u_{it} = \beta_{1i} + \beta_{2i}t + \beta_{3i}t^2$ $v_{it} \sim N(0, \sigma_v^2)$	Modified Within Group	-	No	No
ML random-effects model with time-invariant efficiency (Battese and Coelli, 1988)	Time-invariant	$y_{it} = \alpha + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_i$ $v_{it} \sim N(0, \sigma_v^2)$	Maximum Likelihood	Truncated Normal	No	No
ML random-effects model with time-invariant efficiency (Pitt and Lee, 1981)	Time-invariant	$y_{it} = \alpha + x'_{it}\beta + \varepsilon_{it}$ $\varepsilon_{it} = v_{it} - u_i$ $v_{it} \sim N(0, \sigma_v^2)$	Maximum Likelihood	Half Normal	No	No
Fixed-effects model (Schmidt and Sickles, 1984)	Time-invariant	$y_{it} = (\alpha - u_i) + x'_{it}\beta + v_{it}$ $v_{it} \sim N(0, \sigma_v^2)$	Within Group	-	No	No

Appendix C: Maximum Likelihood Function and Partial Derivatives

Assuming that there are T_i observations for the i -th DMU, where $1 \leq T_i \leq T$ and $Y_i \equiv (Y_{i1}, Y_{i2}, \dots, Y_{iT_i})'$ is the vector of the T_i production values, the logarithm of the likelihood function for the sample observations, $y \equiv (y'_1, y'_2, \dots, y'_T)$, is defined as³²:

$$L^*(\theta; y) = -\frac{1}{2} \left(\sum_{i=1}^N T_i \right) \{ \ln 2\pi + \ln \sigma_S^2 \} - \frac{1}{2} \sum_{i=1}^N \sum_{t=1}^{T_i} \left\{ \frac{(y_{it} - x_{it}\beta + z_{it}\delta)^2}{\sigma_S^2} \right\} - \frac{1}{2} \sum_{i=1}^N \sum_{t=1}^{T_i} \{ \ln \Phi(d_{it}) - \ln \Phi(d_{it}^*) \} \quad (C.1)$$

where

- $\theta = (\beta', \delta', \sigma_S^2, \gamma)$;
- $\sigma_S^2 = \sigma_V^2 + \sigma^2$;
- $\gamma = \frac{\sigma^2}{\sigma_S^2}$;
- $d_{it} = \frac{z_{it}\delta}{\sqrt{\gamma\sigma_S^2}}$;
- $d_{it}^* = \frac{\mu_{it}^*}{\sigma_*}$;
- $\mu_{it}^* = (1 - \gamma)z_{it}\delta - \gamma(y_{it} - x_{it}\beta)$;
- $\sigma_* = \sqrt{\gamma(1 - \gamma)\sigma_S^2}$;
- $\Phi(\cdot)$ is the distribution function for the standard normal random variable.

The partial derivatives of the log-likelihood function in (C.1) with respect to the parameters $\beta, \delta, \sigma_S^2$ and γ are defined as:

³² For simplicity of notation, the authors defined Y_{it} as the logarithm of the production for the i -th DMU in the t -th time period.

$$\frac{\partial L^*}{\partial \beta} = \sum_{i=1}^N \sum_{t=1}^{T_i} \left\{ \frac{(y_{it} - x_{it}\beta + z_{it}\delta)}{\sigma_S^2} + \frac{\phi(d_{it}^*) \gamma}{\Phi(d_{it}^*) \sigma_*} \right\} x'_{it} \quad (C.2)$$

$$\frac{\partial L^*}{\partial \delta} = - \sum_{i=1}^N \sum_{t=1}^{T_i} \left\{ \frac{y_{it} - x_{it}\beta + z_{it}\delta}{\sigma_S^2} + \left[\frac{\phi(d_{it})}{\Phi(d_{it})} \frac{1}{\sqrt{\gamma} \sigma_S^2} - \frac{\phi(d_{it}^*) (1-\gamma)}{\Phi(d_{it}^*) \sigma_*} \right] \right\} z'_{it} \quad (C.3)$$

$$\begin{aligned} \frac{\partial L^*}{\partial \sigma_S^2} = & -\frac{1}{2} \left(\frac{1}{\sigma_S^2} \right) \left\{ \left(\sum_{i=1}^N T_i \right) - \sum_{i=1}^N \sum_{t=1}^{T_i} \left[\frac{\phi(d_{it})}{\Phi(d_{it})} d_{it} - \frac{\phi(d_{it}^*)}{\Phi(d_{it}^*)} d_{it}^* \right] \right. \\ & \left. - \sum_{i=1}^N \sum_{t=1}^{T_i} \frac{y_{it} - x_{it}\beta + z_{it}\delta}{\sigma_S^2} \right\} \end{aligned} \quad (C.4)$$

$$\frac{\partial L^*}{\partial \gamma} = \sum_{i=1}^N \sum_{t=1}^{T_i} \left\{ \frac{\phi(d_{it})}{\Phi(d_{it})} \frac{d_{it}}{2\gamma} + \frac{\phi(d_{it}^*)}{\Phi(d_{it}^*)} \left[\frac{y_{it} - x_{it}\beta + z_{it}\delta}{\sigma_*} + \frac{d_{it}^* (1-2\gamma)}{2\gamma(1-\gamma)\sigma_*^2} \right] \right\} \quad (C.5)$$

where $\phi(\cdot)$ is the density function for the standard normal random variable.

The derivation of the likelihood function and its partial derivatives with respect to the parameters of the model can be found in Battese and Coelli (1993).

Appendix D: Likelihood Ratio Tests

The LR tests allow us to check whether the restricted model (where we set restrictions on the parameters of the model) is more or less appropriate than the unrestricted model, given our data. If the value of the LR test is greater than the tabulated value of the χ^2 -distribution with degrees of freedom equal to the difference in the number of degrees of freedom between the unrestricted and the restricted model for the 95% level of significance, the null hypothesis is rejected and we opt for the unrestricted model. We tested for (i) the translog functional form where the null hypothesis (H0) is the Cobb-Douglas functional form with $\beta_{tt} = \beta_{Kt} = \beta_{Lt} = \beta_{KL} = \beta_{KK} = \beta_{LL} = 0$; (ii) the existence of technological progress, with $\beta_t = \beta_{tt} = \beta_{Kt} = \beta_{Lt} = 0$ as the null hypothesis; (iii) the type of technological progress, where $\beta_{Kt} = 0$ is the H0 for Harrod neutral technological progress, $\beta_{Lt} = 0$ is the H0 for Solow neutral technological progress and $\beta_{Kt} = \beta_{Lt} = 0$ is the H0 for Hicks neutral technological progress; (iv) the technical inefficiency being a linear function of the country-specific variables, where H0 is $\delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = \delta_7 = \delta_8 = \delta_9 = \delta_{10} = \delta_{11} = 0$; (v) combinations of the previous restrictions. The results of these tests are presented in the next table. In addition, the presence of technical inefficiency in the model is tested by setting H0 equal to $\delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = \delta_7 = \delta_8 = \delta_9 = \delta_{10} = \delta_{11} = \sigma_u = \sigma_v = 0$. The presence of stochastic inefficiency effects is tested by restricting $\sigma_u = \sigma_v = 0$, which, by looking at the corresponding t-statistics, allows us to reject the null hypothesis.

<i>Model</i>	<i>Full translog</i>	<i>Harrod Neutral</i>	<i>Solow Neutral</i>	<i>Hicks Neutral</i>	<i>Translog without TP</i>	<i>Cobb-Douglas with TP</i>	<i>Translog without Inefficiency</i>
Harrod Neutral $\beta_{Kt} = 0$	0.04 $\chi^2(1) = 3.84^{**}$	—	—	—	—	—	—
Solow Neutral $\beta_{Lt} = 0$	1.88 $\chi^2(1) = 3.84^{**}$	NC	—	—	—	—	—
Hicks Neutral $\beta_{Kt} = \beta_{Lt} = 0$	2.71 $\chi^2(2) = 5.99^{**}$	2.68 $\chi^2(1) = 3.84^{**}$	0.83 $\chi^2(1) = 3.84^{**}$	—	—	—	—
Translog without TP $\beta_t = \beta_{tt} = \beta_{Kt} = \beta_{Lt} = 0$	11.6* $\chi^2(5) = 11.07^{**}$	11.56* $\chi^2(4) = 9.49^{**}$	9.72* $\chi^2(4) = 9.49^{**}$	8.89* $\chi^2(3) = 7.81^{**}$	—	—	—
Cobb-Douglas with TP $\beta_{tt} = \beta_{Kt} = \beta_{Lt} = \beta_{KL} = \beta_{KK} = \beta_{LL} = 0$	91.5* $\chi^2(6) = 12.59^{**}$	91.46* $\chi^2(5) = 11.07^{**}$	89.62* $\chi^2(5) = 11.07^{**}$	88.78* $\chi^2(4) = 9.49^{**}$	NC	—	—
Cobb-Douglas without TP $\beta_t = \beta_{tt} = \beta_{Kt} = \beta_{Lt} = \beta_{KL} = \beta_{KK} = \beta_{LL} = 0$	96.25* $\chi^2(8) = 15.51^{**}$	96.21* $\chi^2(7) = 14.07^{**}$	94.37* $\chi^2(7) = 14.07^{**}$	93.54* $\chi^2(6) = 12.59^{**}$	84.65* $\chi^2(3) = 7.81^{**}$	4.75 $\chi^2(2) = 5.99^{**}$	—
Translog with Inefficiency not being a linear function of z-variables $\delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = \delta_7 = \delta_8 = \delta_9 = \delta_{10} = \delta_{11} = 0$	231.79* $\chi^2(11) = 19.68^{**}$	NC	NC	NC	NC	—	—
Hicks Neutral with Inefficiency not being a linear function of z-variables $\beta_{Kt} = \beta_{Lt} = \delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = \delta_7 = \delta_8 = \delta_9 = \delta_{10} = \delta_{11} = 0$	231.73* $\chi^2(13) = 22.36^{**}$	NC	NC	229.02* $\chi^2(11) = 19.68^{**}$	NC	NC	0.24 $\chi^2(2) = 5.99^{**}$

* An asterisk on the value of the test statistic indicates that it exceeds the corresponding critical value of the test statistic at the 5% level of significance for the χ^2 -distribution and so the null hypothesis is rejected

** Two asterisks indicates the critical value of the test statistic at the 5% level of significance for the χ^2 -distribution

Appendix E: Elasticity of substitution

The Allen partial elasticity of substitution (AES) between human *per* worker h and physical capital *per* worker k is given by:

$$\sigma_{hk} = \frac{f_h x_h + f_k x_k}{x_h x_k} \frac{F_{hk}}{F} = \frac{f_h f_k (f_h x_h + f_k x_k)}{x_h x_k (2f_h f_k f_{hk} - f_{hh} f_k^2 - f_{kk} f_h^2)} \quad (E.1)$$

where F is the determinant of the bordered hessian matrix and F_{hk} is the cofactor of f_{hk} . The bordered hessian matrix for the three input case is:

$$\begin{bmatrix} 0 & f_h & f_k \\ f_h & f_{hh} & f_{hk} \\ f_k & f_{hk} & f_{kk} \end{bmatrix} \quad (E.2)$$

where f_h and f_k are the first partial derivatives of the production function $f(\cdot)$ with respect to inputs h and k , respectively, f_{hh} and f_{kk} are the second direct partial derivatives with respect to inputs h and k , respectively, and f_{hk} is the second cross partial derivative. f_h , f_k , f_{hh} , f_{kk} and f_{hk} are calculated as follows:

$$f_h = \frac{\partial \ln y}{\partial \ln h} \frac{y}{h} = \varepsilon_h \frac{y}{h} \quad (E.3)$$

$$f_k = \frac{\partial \ln y}{\partial \ln k} \frac{y}{k} = \varepsilon_k \frac{y}{k} \quad (E.4)$$

$$f_{hh} = \frac{y}{h^2} [\beta_{hh} + \varepsilon_h (\varepsilon_h - 1)] \quad (E.5)$$

$$f_{kk} = \frac{y}{k^2} [\beta_{kk} + \varepsilon_k (\varepsilon_k - 1)] \quad (E.6)$$

$$f_{hk} = \frac{y}{hxk} [\beta_{hk} + \varepsilon_h \varepsilon_k] \quad (E.7)$$

Under the AES framework, human and physical capital *per* worker are substitutes if, after an increase in the price of one of the factors, the quantity used of the other increases, that is, $\sigma_{hk} > 0$. The two inputs are complements if, when the price of one input raises, the quantity used of the other input

decreases, that is, $\sigma_{hk} < 0$. Given that σ_{hk} depends on the input levels, we calculated the elasticity of substitution at the sample mean. Therefore, the estimated bordered hessian matrix is:

$$\begin{bmatrix} 0 & 0.4322 & 0.18262 \\ 0.4322 & 2.3916 & 0.07903 \\ 0.18262 & 0.07903 & -0.0096 \end{bmatrix} \quad (E.8)$$

Substituting the values present in the matrix (E.8) into equation (E.1), we obtain an elasticity of substitution of 0.1, indicating that human and physical capital *per worker* are substitutes.

Appendix F: Efficiency levels by country and by group, 2001-2011

Country	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	Mean
Australia	0,46	0,52	0,59	0,62	0,67	0,70	0,75	0,78	0,82	0,88	0,91	0,70
Austria	0,50	0,57	0,62	0,67	0,72	0,77	0,81	0,84	0,83	0,88	0,90	0,74
Belgium	0,53	0,60	0,65	0,69	0,73	0,76	0,81	0,84	0,83	0,88	0,88	0,75
Brazil	0,30	0,30	0,27	0,29	0,31	0,38	0,41	0,45	0,48	0,52	0,55	0,39
Canada	0,53	0,58	0,63	0,69	0,74	0,77	0,82	0,85	0,87	0,92	0,93	0,76
Chile	0,41	0,44	0,46	0,50	0,54	0,62	0,66	0,70	0,71	0,76	0,81	0,60
China	0,27	0,29	0,31	0,32	0,35	0,37	0,40	0,40	0,43	0,44	0,44	0,37
Czech Republic	0,41	0,45	0,41	0,44	0,47	0,49	0,55	0,58	0,59	0,68	0,70	0,53
Denmark	0,50	0,57	0,59	0,64	0,68	0,73	0,81	0,84	0,83	0,88	0,91	0,72
Estonia	0,47	0,56	0,57	0,59	0,62	0,65	0,68	0,68	0,66	0,70	0,76	0,63
Finland	0,49	0,56	0,61	0,66	0,69	0,74	0,79	0,83	0,80	0,87	0,90	0,72
France	0,54	0,59	0,62	0,64	0,68	0,71	0,75	0,79	0,79	0,83	0,87	0,71
Germany	0,50	0,55	0,60	0,63	0,67	0,72	0,76	0,79	0,77	0,85	0,87	0,70
Greece	0,47	0,52	0,55	0,58	0,61	0,64	0,66	0,70	0,70	0,74	0,77	0,63
Hungary	0,42	0,49	0,51	0,54	0,58	0,60	0,60	0,61	0,61	0,65	0,69	0,57
Iceland	0,44	0,49	0,52	0,59	0,62	0,63	0,67	0,72	0,75	0,84	0,89	0,65
India	0,19	0,22	0,24	0,22	0,26	0,30	0,32	0,33	0,36	0,41	0,43	0,30
Indonesia	0,23	0,22	0,22	0,23	0,25	0,27	0,29	0,33	0,35	0,38	0,41	0,29
Ireland	0,66	0,78	0,88	0,91	0,94	0,93	0,93	0,94	0,93	0,94	0,96	0,89
Israel	0,50	0,54	0,56	0,60	0,65	0,70	0,75	0,78	0,82	0,88	0,91	0,70
Italy	0,51	0,54	0,58	0,61	0,64	0,68	0,72	0,77	0,76	0,82	0,84	0,68
Japan	0,40	0,43	0,49	0,53	0,54	0,57	0,60	0,64	0,66	0,73	0,76	0,58
Korea, Rep.	0,45	0,47	0,52	0,52	0,56	0,57	0,60	0,65	0,65	0,71	0,78	0,59
Luxembourg	0,60	0,68	0,72	0,74	0,81	0,86	0,92	0,90	0,90	0,94	0,95	0,82
Mexico	0,44	0,49	0,47	0,52	0,57	0,60	0,64	0,67	0,69	0,77	0,82	0,61
Netherlands	0,54	0,60	0,64	0,68	0,74	0,77	0,82	0,85	0,84	0,88	0,91	0,75
New Zealand	0,47	0,52	0,57	0,61	0,66	0,67	0,78	0,82	0,88	0,91	0,93	0,71
Norway	0,59	0,62	0,69	0,76	0,84	0,88	0,90	0,93	0,94	0,96	0,97	0,83
Poland	0,43	0,47	0,50	0,53	0,58	0,63	0,71	0,76	0,79	0,84	0,88	0,65
Portugal	0,43	0,46	0,48	0,51	0,56	0,59	0,63	0,66	0,66	0,69	0,72	0,58
Russian Federation	0,24	0,27	0,31	0,36	0,39	0,47	0,53	0,58	0,62	0,70	0,74	0,48
Slovak Republic	0,43	0,47	0,47	0,50	0,55	0,60	0,63	0,68	0,69	0,74	0,79	0,60
Slovenia	0,42	0,47	0,51	0,52	0,55	0,58	0,63	0,66	0,66	0,69	0,71	0,58
South Africa	0,41	0,45	0,49	0,51	0,56	0,59	0,62	0,64	0,68	0,73	0,77	0,59
Spain	0,50	0,55	0,58	0,61	0,65	0,68	0,70	0,75	0,76	0,80	0,84	0,68
Sweden	0,55	0,62	0,67	0,74	0,79	0,81	0,84	0,87	0,86	0,91	0,94	0,78
Switzerland	0,51	0,59	0,61	0,67	0,70	0,73	0,78	0,84	0,87	0,93	0,94	0,74
Turkey	0,40	0,44	0,45	0,50	0,56	0,58	0,61	0,65	0,70	0,76	0,81	0,59
United Kingdom	0,59	0,67	0,73	0,77	0,81	0,86	0,90	0,91	0,89	0,92	0,93	0,82
United States	0,49	0,54	0,58	0,62	0,66	0,70	0,73	0,77	0,82	0,87	0,91	0,70
Mean	0,46	0,51	0,54	0,57	0,61	0,65	0,69	0,72	0,73	0,78	0,81	0,64
OECD	0,49	0,54	0,58	0,62	0,66	0,69	0,73	0,77	0,77	0,82	0,86	0,68
Non-OECD	0,27	0,29	0,30	0,32	0,35	0,40	0,43	0,46	0,49	0,53	0,56	0,40

Appendix G: Output elasticities with respect to human capital,
2001-2011

Country	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Australia	-1,00	0,47	0,57	-1,80	0,72	0,69	0,36	0,53	-0,11	0,30	-2,15
Austria	-1,45	0,48	0,15	0,45	0,34	1,02	0,26	0,98	-0,13	0,59	0,40
Belgium	-0,01	0,51	0,27	1,11	0,72	0,50	-0,94	-1,09	1,28	0,99	0,79
Brazil	1,01	0,43	0,68	1,04	1,12	0,69	0,55	-1,91	1,13	-1,07	0,18
Canada	-1,02	0,60	0,51	-2,86	0,91	-0,24	0,67	-1,00	0,77	-1,13	0,60
Chile	-1,44	0,71	0,47	0,79	1,20	0,71	-2,59	-1,09	0,82	-1,47	0,40
China	-1,36	0,93	0,72	0,36	-2,11	0,72	0,33	0,48	0,29	0,95	0,52
Czech Republic	0,89	0,07	0,38	0,78	0,63	1,19	0,32	1,12	0,76	0,84	0,93
Denmark	-0,80	0,28	0,43	-0,32	0,40	1,17	0,33	0,80	1,10	0,72	-1,01
Estonia	0,95	0,08	0,33	-1,42	0,30	0,45	0,03	1,06	1,14	0,64	-1,22
Finland	0,56	0,70	0,70	-1,14	0,24	0,85	1,04	0,95	0,65	0,60	0,58
France	1,02	0,52	-2,47	-1,50	0,46	0,59	1,01	0,99	0,88	0,56	0,62
Germany	0,73	0,32	0,73	-2,37	0,27	0,73	0,75	0,80	0,80	0,38	0,38
Greece	0,84	0,36	0,86	-3,54	-0,20	-3,05	0,50	0,63	0,72	0,29	-1,73
Hungary	-2,68	1,07	0,80	0,52	0,73	-2,71	0,87	0,68	0,85	0,74	-0,24
Iceland	-3,37	0,31	0,79	0,44	0,67	0,82	0,60	0,65	0,32	0,62	0,00
India	0,96	0,29	0,63	-0,83	1,56	-1,15	0,87	1,22	-0,32	1,08	-1,66
Indonesia	0,34	0,40	1,05	1,02	1,55	0,67	0,35	0,77	0,77	0,25	-0,50
Ireland	0,83	-0,10	0,97	-1,05	1,54	0,63	1,07	0,12	-0,33	-2,32	0,89
Israel	0,39	-0,44	-0,58	0,79	1,52	1,09	0,87	-0,91	0,55	1,12	0,89
Italy	0,54	-0,64	-0,16	0,87	1,50	0,41	0,15	0,89	-0,38	-1,31	0,86
Japan	0,24	-0,05	-0,08	-1,20	1,47	0,43	-0,46	0,58	0,75	1,00	0,96
Korea, Rep.	0,86	-0,14	1,38	-1,29	1,43	-2,20	-0,37	0,46	0,76	0,96	0,54
Luxembourg	0,52	1,33	0,93	0,59	1,35	0,60	1,02	-0,86	0,97	0,57	-0,66
Mexico	0,61	0,59	0,88	-0,34	1,30	0,67	0,44	-0,88	0,77	0,33	0,67
Netherlands	0,43	0,60	-0,48	0,77	0,35	0,71	0,31	-0,91	-0,79	1,08	0,28
New Zealand	0,44	0,13	0,60	0,50	0,27	0,42	-1,65	1,13	0,48	1,15	0,26
Norway	0,72	0,04	0,83	0,70	1,39	-1,56	0,35	1,10	0,63	0,57	0,73
Poland	-0,02	-0,15	-3,46	0,59	1,33	-0,42	0,02	-0,09	0,72	0,28	-2,67
Portugal	-0,98	-0,85	0,65	0,82	0,36	0,18	-0,33	0,57	0,77	0,33	0,68
Russian Federation	0,97	-0,96	-2,76	0,73	0,21	-1,21	-0,69	1,37	0,46	-3,16	0,55
Slovak Republic	0,61	-0,95	0,52	0,74	0,09	0,72	-0,05	1,25	-2,25	0,67	0,28
Slovenia	-0,83	-1,19	-2,85	1,09	-0,04	0,63	-0,06	0,51	0,63	1,10	0,26
South Africa	0,88	0,85	0,55	0,60	-0,12	1,05	0,01	1,24	0,50	1,05	1,17
Spain	0,83	0,61	0,71	0,76	-0,16	-0,07	-0,01	-0,19	0,88	0,50	1,16
Sweden	0,81	0,50	0,64	0,82	-0,77	0,81	0,95	1,30	-3,27	0,89	1,13
Switzerland	0,61	0,46	0,57	0,38	-0,78	0,82	0,79	0,62	1,11	0,84	1,07
Turkey	0,59	0,64	1,04	0,69	-0,88	0,57	-2,95	1,02	1,00	0,92	1,03
United Kingdom	0,28	0,65	-0,13	-0,41	-0,57	0,51	0,54	0,88	0,03	0,82	1,01
United States	0,48	-2,79	-1,70	0,89	-1,00	0,41	0,43	0,60	0,56	0,63	1,00

Appendix H: Output elasticities with respect to physical capital,
2001-2011

Country	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Australia	0,84	0,74	0,45	1,04	0,58	0,37	0,41	0,42	0,71	0,52	0,85
Austria	0,95	0,74	0,54	0,70	0,48	0,54	0,43	0,62	0,70	0,42	0,76
Belgium	0,75	0,72	0,52	0,44	0,39	0,33	0,79	0,86	0,50	0,61	0,64
Brazil	0,60	0,74	0,38	0,45	0,50	0,50	0,41	1,06	0,55	0,89	0,89
Canada	0,71	0,48	0,45	0,94	0,37	0,78	0,29	0,72	0,77	0,90	0,55
Chile	0,72	0,45	0,47	0,61	0,42	0,59	0,91	0,70	0,48	0,65	0,47
China	0,72	0,33	0,36	0,47	0,83	0,45	0,78	0,42	0,51	0,55	0,44
Czech Republic	0,39	0,55	0,45	0,72	0,66	0,52	0,49	0,44	0,36	0,58	0,56
Denmark	0,82	0,52	0,36	0,79	0,48	0,52	0,51	0,59	0,53	0,62	0,88
Estonia	0,37	0,93	0,39	0,63	0,54	0,48	0,73	0,53	0,45	0,61	0,89
Finland	0,49	0,38	0,59	0,70	0,54	0,59	0,58	0,55	0,54	0,61	0,68
France	0,48	0,44	0,87	0,72	0,74	0,46	0,60	0,46	0,75	0,64	0,61
Germany	0,57	0,51	0,48	0,85	0,85	0,48	0,61	0,51	0,76	0,71	0,75
Greece	0,58	0,50	0,58	1,08	0,77	0,97	0,73	0,65	0,75	0,74	0,72
Hungary	0,93	0,66	0,35	0,49	0,35	0,92	0,34	0,61	0,46	0,36	0,41
Iceland	1,06	0,77	0,38	0,45	0,37	0,58	0,69	0,63	0,49	0,39	0,36
India	0,32	0,77	0,70	0,78	0,51	0,55	0,55	0,41	0,68	0,54	0,66
Indonesia	0,77	0,72	0,62	0,59	0,51	0,61	0,82	0,65	0,35	0,39	0,49
Ireland	0,38	0,39	0,70	0,71	0,51	0,64	0,63	0,90	0,70	0,85	0,76
Israel	0,46	0,47	0,50	0,43	0,51	0,46	0,70	0,81	0,58	0,50	0,75
Italy	0,74	0,52	0,40	0,40	0,52	0,68	0,54	0,62	0,68	0,91	0,76
Japan	0,52	0,68	0,71	0,71	0,52	0,68	0,79	0,35	0,75	0,61	0,43
Korea, Rep.	0,58	0,69	0,45	0,72	0,53	0,85	0,80	0,38	0,77	0,36	0,59
Luxembourg	0,46	0,47	0,56	0,56	0,56	0,66	0,59	0,84	0,43	0,35	0,71
Mexico	0,48	0,42	0,46	0,70	0,58	0,48	0,69	0,85	0,50	0,42	0,37
Netherlands	0,48	0,39	0,68	0,35	0,44	0,36	0,74	0,85	0,75	0,44	0,50
New Zealand	0,78	0,51	0,66	0,43	0,47	0,48	0,71	0,51	0,43	0,44	0,40
Norway	0,72	0,54	0,57	0,63	0,55	0,98	0,76	0,51	0,29	0,31	0,63
Poland	0,74	0,59	1,07	0,65	0,57	0,79	0,35	0,69	0,58	0,40	0,90
Portugal	0,71	0,48	0,67	0,41	0,44	0,53	0,44	0,42	0,57	0,48	0,46
Russian Federation	0,62	0,50	0,95	0,48	0,49	0,88	0,53	0,45	0,68	1,00	0,36
Slovak Republic	0,48	0,86	0,66	0,55	0,52	0,67	0,71	0,52	0,85	0,66	0,43
Slovenia	0,83	0,89	0,96	0,51	0,56	0,40	0,69	0,42	0,32	0,50	0,42
South Africa	0,57	0,58	0,66	0,39	0,58	0,70	0,68	0,53	0,69	0,53	0,53
Spain	0,59	0,67	0,37	0,76	0,58	0,74	0,69	0,71	0,49	0,53	0,52
Sweden	0,58	0,66	0,41	0,76	0,52	0,62	0,50	0,50	1,04	0,38	0,51
Switzerland	0,66	0,73	0,40	0,47	0,52	0,41	0,60	0,39	0,50	0,57	0,53
Turkey	0,62	0,31	0,61	0,53	0,52	0,48	0,96	0,56	0,55	0,48	0,54
United Kingdom	0,75	0,64	0,75	0,67	0,70	0,37	0,65	0,55	0,54	0,50	0,55
United States	0,74	0,93	1,01	0,57	0,78	0,39	0,51	0,41	0,42	0,64	0,55

Appendix I: Output elasticities with respect to human (ε_h) and physical capital (ε_k) and returns to scale (ε): average 2001-2011

Country	ε_h	ε_k	ε
Australia	-0,13	0,63	0,50
Austria	0,28	0,62	0,91
Belgium	0,38	0,60	0,97
Brazil	0,35	0,63	0,98
Canada	-0,20	0,63	0,43
Chile	-0,13	0,59	0,45
China	0,17	0,53	0,70
Czech Republic	0,72	0,52	1,24
Denmark	0,28	0,60	0,89
Estonia	0,21	0,59	0,81
Finland	0,52	0,57	1,09
France	0,24	0,62	0,86
Germany	0,32	0,64	0,96
Greece	-0,39	0,73	0,34
Hungary	0,06	0,54	0,59
Iceland	0,17	0,56	0,73
India	0,24	0,59	0,83
Indonesia	0,61	0,59	1,20
Ireland	0,20	0,65	0,86
Israel	0,48	0,56	1,04
Italy	0,25	0,61	0,86
Japan	0,33	0,61	0,94
Korea, Rep,	0,22	0,61	0,83
Luxembourg	0,58	0,56	1,14
Mexico	0,46	0,54	1,00
Netherlands	0,21	0,54	0,76
New Zealand	0,34	0,53	0,87
Norway	0,50	0,59	1,09
Poland	-0,35	0,67	0,31
Portugal	0,20	0,51	0,71
Russian Federation	-0,41	0,63	0,22
Slovak Republic	0,15	0,63	0,78
Slovenia	-0,07	0,59	0,52
South Africa	0,71	0,59	1,29
Spain	0,45	0,60	1,06
Sweden	0,35	0,59	0,93
Switzerland	0,59	0,53	1,12
Turkey	0,33	0,56	0,89
United Kingdom	0,33	0,61	0,94
United States	-0,05	0,63	0,59

Appendix J: Sources of economic growth 2001–2011: compound annual % change

Country	¹ Economic growth	² Human Capital accumulation	² Physical Capital accumulation	Productivity Change				³ Random Shocks
				TFP	Technical Progress	Technical Efficiency	Scale Effects	
Australia	0,01	-0,06	2,02	-1,81	-7,07	6,23	-0,98	-0,13
Austria	0,48	0,10	2,14	-1,05	-7,07	6,23	-0,21	-0,71
Belgium	-0,04	0,10	2,84	-0,92	-7,07	6,23	-0,09	-2,06
Brazil	0,55	0,34	1,41	-0,87	-7,07	6,23	-0,03	-0,34
Canada	-0,24	-0,27	2,05	-1,84	-7,07	6,23	-1,01	-0,18
Chile	2,59	-0,20	4,73	-3,32	-7,07	6,23	-2,48	1,38
China	7,68	0,21	9,31	-3,70	-7,07	6,23	-2,87	1,86
Czech Republic	1,21	0,05	1,61	-0,44	-7,07	6,23	0,39	-0,02
Denmark	1,10	0,06	3,42	-1,23	-7,07	6,23	-0,40	-1,14
Estonia	4,34	0,06	6,88	-2,17	-7,07	6,23	-1,33	-0,44
Finland	0,55	0,13	2,01	-0,64	-7,07	6,23	0,19	-0,95
France	0,70	0,17	4,70	-1,52	-7,07	6,23	-0,68	-2,66
Germany	1,13	0,34	2,14	-0,92	-7,07	6,23	-0,09	-0,42
Greece	0,57	-1,24	9,40	-6,21	-7,07	6,23	-5,38	-1,38
Hungary	2,00	0,02	6,49	-3,47	-7,07	6,23	-2,64	-1,04
Iceland	0,59	0,16	1,22	-1,21	-7,07	6,23	-0,37	0,41
India	6,29	0,27	5,96	-1,89	-7,07	6,23	-1,06	1,95
Indonesia	3,55	0,43	3,24	-0,10	-7,07	6,23	0,73	-0,02
Ireland	2,97	0,10	8,35	-2,05	-7,07	6,23	-1,22	-3,44
Israel	-1,86	0,06	-1,05	-0,88	-7,07	6,23	-0,04	0,02
Italy	-0,51	0,12	2,47	-1,19	-7,07	6,23	-0,36	-1,92
Japan	0,73	0,14	1,98	-0,95	-7,07	6,23	-0,12	-0,44
Korea, Rep.	2,35	0,14	5,26	-1,77	-7,07	6,23	-0,94	-1,29
Luxembourg	-0,87	0,14	1,41	-0,61	-7,07	6,23	0,22	-1,80
Mexico	1,18	0,44	2,55	-0,84	-7,07	6,23	0,00	-0,97
Netherlands	0,96	0,05	4,28	-1,89	-7,07	6,23	-1,05	-1,48
New Zealand	-0,18	0,12	0,43	-0,91	-7,07	6,23	-0,07	0,17
Norway	1,78	0,23	2,63	-0,58	-7,07	6,23	0,25	-0,50
Poland	2,77	-0,27	3,65	-3,15	-7,07	6,23	-2,32	2,54
Portugal	1,67	0,17	5,59	-2,51	-7,07	6,23	-1,67	-1,59
Russian Federation	7,77	-0,40	-0,39	-0,22	-7,07	6,23	0,62	8,78
Slovak Republic	3,39	0,02	3,99	-1,73	-7,07	6,23	-0,90	1,11
Slovenia	0,88	-0,02	4,97	-3,18	-7,07	6,23	-2,35	-0,88
South Africa	2,30	0,28	1,61	-0,28	-7,07	6,23	0,55	0,70
Spain	1,75	0,28	5,25	-0,51	-7,07	6,23	0,33	-3,27
Sweden	0,89	0,08	2,14	-0,98	-7,07	6,23	-0,14	-0,34
Switzerland	1,53	0,13	1,30	-0,67	-7,07	6,23	0,16	0,77
Turkey	4,57	0,40	4,72	-1,38	-7,07	6,23	-0,54	0,83
United Kingdom	0,46	0,14	3,84	-1,09	-7,07	6,23	-0,25	-2,44
United States	1,37	-0,02	2,96	-2,05	-7,07	6,23	-1,21	0,48

(1) Growth rate of GDP per worker engaged; (2) Growth rates of human and physical capital per worker adjusted by input expenditure share; (3) Obtained as a residual