



Fintech Valuation

Is Real Option Valuation a suitable approach to value US listed
FinTech companies?

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ABSTRACT

The aim of this study is to assess whether a real option valuation model used on the internet industry is suitable to value FinTech's. More specifically, try to answer how close or far the estimated firm value is from observed market value disclosed in the financial statement. We argue that commonly used valuation models are limited when it comes to valuation of FinTech businesses. Additionally, we explain that FinTechs are facing many similarities in comparison with Internet companies in the 2000's. We adopt and apply a real option model, originally developed to value Internet companies, on a U.S listed FinTech, Square Inc. Revenues are imitated and forecasted as a discrete approximation of a continuous time stochastic mean-reverting process. Further, the revenues are risk-adjusted, to avoid the need of estimation of an uncertain WACC, and therefore discounted at an appropriate risk-free rate. The model estimates FinTechs stock prices closer to the traded market price than a traditional NPV DCF-model, since traditional NPV DCFs neglect the value from the flexibility option. Stock prices of FinTechs might be rational if one put a high enough value on the growth of revenues. Notwithstanding that the model requires estimation of many parameters, it is suggested to look closer into how life-expectancy of FinTechs affect the valuation and find a better predictor or proxy.

Keywords: FinTech, Real Option, Valuation, Asset Pricing, Risk-Adjusted, Capital-Budgeting

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ABSTRATO

O objetivo deste estudo é avaliar se o modelo de avaliação para opções reais utilizado na indústria da internet é aplicável para a avaliação de FinTech's. Mais especificamente, procura mensurar o desvio da avaliação entre o valor intrínseco da empresa e o registado nas demonstrações financeiras das empresas da indústria de FinTech. Adicionalmente, é feita uma comparação com as empresas tecnológicas dos inícios da primeira década do século XXI. É adotado um modelo desenvolvido na avaliação da FinTech, Square Inc. As receitas são estimadas como uma aproximação discreta de um processo de reversão estocástica da média. Seguidamente, as receitas são ajustadas ao risco evitando assim estimar o WACC, descontando, por isso, à taxa sem risco. O modelo estima os preços das ações da FinTech mais próximos do preço de mercado negociado do que o modelo tradicional de VAL-DCF. Os preços das ações das FinTechs poderão ser racionais se colocarmos um valor alto o suficiente no crescimento das receitas. De qualquer forma, o modelo requer a estimativa de muitos parâmetros. Sugere-se examinar mais pormenorizadamente como a expectativa de vida de uma empresa ou projeto da FinTech afeta o valor e estabelecer um proxy mais adequado.

Palavras-Chave: FinTech, Opção Real, Avaliação, Preço de Ativos, Ajuste ao Risco, Orçamentação de Capital

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Before I started university, I used to work within constructions, and I was highly engaged in how accuracy and improvements could result in increased lifetime of the products the firm delivered to the market. That motivated me to start studying business, where my main interest was strategy and logistics at that time, as I found it hard to understand mathematics. After some years of study my interest in mathematics, statistics and finance evolved and I decided to go on with a major in finance. As you can read, I have always been curious about improvement, efficiency, analytics and accuracy, and these skills were highly needed throughout my dissertation. Actually, when I started to work on my decided thesis topic, I was thinking that this would be straight-forward work. I was wrong! Quickly I understood that my topic required a lot more work. I learned a lot about myself, time management and accuracy in my masters. To achieve this result, I am very thankful for the support from family, friends, colleagues and professors. I am very thankful for the deep insight and knowledge, and the support I received from my supervisor, João Freire de Andrade. I would also like to mention professors José Corrêa Guedes, who guided me with some statistical advice. In the end I have found my subject very interesting and I will continue to revise the model and research further in the future.

Thank you for reading my dissertation!

Best Regards
Marius Sæterbø

EXECUTIVE SUMMARY

The FinTech business and its disruptive innovations has made many investors questioning the rational value of FinTechs' earning potential. Traditional Net Present Value approaches and DCF models has shown to undervalue many FinTechs, given the assumption that market prices are the rational and correct prices for FinTech companies. The question was raised in the late 1990s and early 2000s with respect to Internet companies. Many investors thought the price was irrationally high, while others justified the value of Internet companies. Some businesses succeeded and gained enormous earnings in the following year after huge investments in R&D. We have the opinion that FinTech companies are facing a similar environment as Internet companies. Since traditional DCFs seems not to explain the value of the FinTech business we decided to study whether a real option approach better explain the stock price in the market.

We adopt a real option model published by Schwartz and Moon (2001), originally developed to value high-growth Internet companies and modifies the model slightly to value Square Inc, a US-listed FinTech within the payments vertical. Throughout our research we found that initial condition in the model and how we estimate them has a large impact on the valuation result. The model is especially sensitive to initial growth rate of revenues, and the revenues volatility, the market price of risk, and the projected period length of forecast.

Our study shows that a traditional DCF valuation results in an estimated share price tremendously lower than the closing price in the market at year end 2018, for Square Inc. Notwithstanding that the simulations seems to better explain the stock prices, even though it overvalues the stock.

Throughout the study we discovered that there might be some weaknesses and limitation with the framework of the model. We suggest to look closer into 1) how one determines the length of the forecast period, 2) how one can accurately estimate the market price of risk, 3) which assumptions are made with respect to the volatility of revenue growth rate, 4) which effect periodization vs expensing of R&D-expenses has on the value and 5) how the bankruptcy condition can be made more realistic. Adjustments are needed and we believe that further research will be interesting.

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LIST OF ABBREVIATIONS

Bankruptcy: when a firm is in condition where they are not able to meet their payment obligation and are accruing bankruptcy costs or are insolvent/liquidated.

Capital-Budgeting: A framework where you budget cash inflow and cash outflow from different activities to gain arrange calculation.

Cashflow: A stream of cash that's flow, either cash inflow or cash outflow.

Comparables: Multiples that describe different ratios of key number in financial statements or forecast, i.e. EV/EBITDA.

FinTech: A term that combines "Financial" and "Technology" to Financial Technology. See page 1.

Half-Life: The period of time it takes before a given value is halved.

Mean-Reverting process: A process where a stochastic time series converges back to the mean as time goes.

Monte Carlo Simulation: A way to model uncertainty and probability, where you assign random variables as input. From that you obtain a probability distribution of possible outcomes.

Real Option: A right, but not an obligation, to acquire or dispose a real asset if certain condition is met. That could be flexibility in order to expand or abandon a project, or the right to take control of natural recourses.

Risk-adjusted Cashflow: Cashflows are risk-adjusted when you can replicate a stream of uncertain cashflows into a certain future stream of cashflow.

Valuation: A process where you put a value on an asset.

GLOSSARY

APV = Adjusted Present Value

CAPM = Capital Asset Pricing Model

CF = Cashflow

COGS = Cost of Goods Sold

DCF = Discount Cash Flow

DDM = Dividend Discount model

EBITDA = Earnings Before Interest Tax Depreciation and Amortization

ESOP = Employee Stock Option Plan

EV = Enterprise Value

FCFE = Free Cashflow Equity

FCFF = Free Cashflow Firm

I/B/E/S = Institutional Brokers Estimate System

IPO = Initial Public Offering

IT = Information Technology

NPV = Net Present Value

OLS = Ordinary Least Squares

PP&E = Property, Plant & Equipment

NYSE = New York Stock Exchange

SQ = Square Inc.

R&D = Research & Development

VBA = Visual Basic for Applications

WACC = Weighted Average Cost of Capital

DISSERTATION

1 Introduction

1.1 Topic presentation

Valuation of financial institutions and services, specifically FinTechs and its niches, can be very complicated, difficult and more demanding than ever (Wilson, 2018), much due to an increase in speed of changes in innovations and disruptions. Over many decades, the industry has gone through large changes, within fields as operation management, customer service, regulations, innovation and digitalization. The two latter are said to make a difference when it comes to efficiency and profitability in financial institutions the last recent years.

Innovation can be defined as, developing new products or improving existing technologies, processes, designs and marketing to solve problems, increase efficiency, reach new customers and ultimately increase profits. Or, disruptive or step changes that transform the business in some significant way (Gartner IT Glossary).

Digitalization on the other hand are defined as, taking an analog process and changes it to a digital form without any different-in-kind changes to the process itself (Gartner IT Glossary).

The combination of innovation and digitalization within financial sector has its own name, FinTech, which refers to Financial Technology. Gomber, Koch, and Siering (2017) describes FinTech in the following way:

« [...] the connection of modern and, mainly, Internet-related technologies (e.g., cloud computing, mobile Internet) with established business activities of the financial services industry (e.g., money lending, transaction banking) »

A statement by KPMG might clarify even better what FinTech is (KPMG, 2017):

« Fintech is often, and in our view wrongly, understood to be separate from financial services. In fact, we believe that fintech is an evolution of financial services and that every business in the sector must engage with it if they are to survive. The City has reinvented itself many times – fintech is, put simply, the latest iteration of this evolution of how financial services will better meet the needs of its business, retail and institutional customers »

The companies that are focusing on FinTech are now posing threat to already existing incumbent banks and other financial institutions, as they are challenging the way financial services are offered. At the same time, FinTech are raising new opportunities, new products, new markets and process improvements. This raises questions when it comes to valuation of

FinTech companies since they are, acquiring targets for incumbent banks, seeking to perform an IPO, or of interest for investors that are trading FinTech stocks at stock exchanges.

1.2 Managerial and academic relevance

Nowadays there are numerous different valuation models designed for various kind of use, from valuing a single part of a project to a whole company. There are limitations, pros and cons in all of them.

<u>Valuation model</u>	<u>Pros</u>	<u>Cons</u>
Discount Cash Flow	<ul style="list-style-type: none"> Extremely detailed and include many assumptions about business. Determines the intrinsic value of a business. Includes expectation of future cashflows. Suitable in stable businesses 	<ul style="list-style-type: none"> Requires many assumptions Prone to error, overcomplexity and overconfidence. Terminal value represent a large amount of the value. Challenging to estimate future cost of capital such as WACC.
Comparables / Multiples	<ul style="list-style-type: none"> Few assumptions needed. Easy to estimate and understand. Capture current trends in the market. 	<ul style="list-style-type: none"> Disregard future states as they focus on one single static period. Anyway, there are some lead ratios. Requires an amount of comparable companies. Not reliable when comparable are thinly traded.
Asset in place	<ul style="list-style-type: none"> Easy to apply in cases of liquidation to value residual equity. Accurate in investments niches, such as real estate. 	<ul style="list-style-type: none"> Neglect the value of intangibles such as R&D expenses. Additional calculation needed. Disregard prospective earnings.

Table 1: Types of valuation models (Corporate Finance Institute, 2019b)

Discount Cash Flow (DCF) models, such as the Dividend Discount Model (DDM), Free Cash Flow to Equity/Firm (FCFE/FCFF) model or Adjusted Present Value (APV) model, are much used on already existing and stable companies, with a certain expectation of future states and cash flows.

Multiples or comparable models has been found suitable for valuation as long the firm being evaluated, are in the same industry, are facing the similar business risk and has a similar capital structure. When it comes to valuation of FinTech projects or companies, the above mention models might pose many limitations.

Firstly, FinTechs are seen as risky investments, either they are in the business of optimization and digitalization of already existing processes, developing new products, or focusing on

emerging markets. As with other comparable start-ups, there is a high uncertainty about how future revenues of FinTechs will evolve. Especially, if these FinTech solutions are likely to be competitive in the future or if they are being disrupted before they even start to create value.

Secondly, since the investments needed for creating value in the future are risky, it is hard to access capital. Capital is needed to finance investments in hardware, software, knowledge, research and development (R&D), etc. In an early stage, when projects or firms are lacking cash inflows, they are facing high bankruptcy risk. Or in worst case, liquidation as they are not able to offer investors or creditors any collateral that has residual value if they might need additional financing. This from the perspective that the technology developed might be worthless if its disrupted. Furthermore, it can lead to expensive cost of capital, equity and/or debt, which in hand can result in biased valuation if not adjusted for.

Thirdly, investments in development can be thought of as flexible, since the developer has an option to abandon, expand or continue the business as it is, which the above-mentioned models do not cover on a regular basis.

The purpose of this thesis is to find and investigate an objective valuation of FinTech's, which firstly takes the high risk and uncertainty into account in terms of revenues, secondly exclude the potential bias in cost of capital, and thirdly are putting value on the flexibility option.

The above mentioned limitations can lead overvalued or undervalued Net Present Value (NPV) (Lee & Shin, 2018, p. 40). To solve this problem, a Real Option valuation approach will be examined. The main difference from the traditional DCF methodology is that in the real option framework cashflows are made risk-neutral, so we can discount the cashflows at risk-free rate instead of potential biased Weighted-Average-Cost-of-Capital (WACC). Anyway, it cannot be stated that real option valuation is better than another valuation methodology. It's about which simplification that has to be done and certain assumption that are done about the uncertainty of the future, and hence how sensitive the valuation is on the simplifications and assumptions.

A real option model from the research paper "Rational Pricing of Internet Companies" by Schwartz and Moon (2001) will be adopted and apply on the Square Inc, a U.S listed company listed on New York Stock Exchange (NYSE). Square Inc. is a FinTech company in the niche of FinTech payment solutions.

The model applied was developed to value stocks of internet companies. The abstract of Schwartz and Moon (2001) can be seen in Appendix 1. FinTech companies shares many

similarities with internet companies as mention in the article such as continues-time revenues, high revenue growth rates, high revenue volatility and real chance for bankruptcy. One could also say that FinTechs either are a sub-sector of internet companies or that FinTechs uses internet as a medium to perform their operating activities. We do not address this question in the dissertation.

This dissertation will be of academic relevance, as a first step, to access whether the model of Schwartz and Moon (2001) is suitable and estimates valuation of FinTechs close to, or far away from, the stock market. More specifically, we compare historical closing prices for a specific valuation period with the estimated price obtained in the model. With that said, an initial assumption is that the stock market is efficient, which mean that the stock price reflects all available information. Or, on the other hand one could say that the stock market either overvalues or undervalues the stock.

1.3 Problem statement

From the perspective of firm valuation, in relation to the challenges described above, the problem to be answered in this dissertation is as follows:

Is Real Option Valuation a suitable approach to value U.S listed FinTech companies?

1.4 Research Questions

The problem statement poses a variety of more specific questions. The following are to be answered throughout the dissertation.

- 1.4.1 *Are **assumptions and simplifications** made for **FinTechs** reasonable in relation to **Internet Companies**?*
- 1.4.2 *How close is the **estimated share price** to the **actual market price** on the date of valuation, in absolute and relative terms?*
- 1.4.3 *How many **percentages of times** are the **valuation estimates** within a **10 percent range** to the **actual market value**?*
- 1.4.4 *In which year is it more **likely** to face for **bankruptcy**?*
- 1.4.5 *Is the **Real Option Value** higher or lower than the **Net Present Value**, and by how much in absolute and relative terms?*

1.5 Thesis structure

As already introduced, a firm valuation of Square Inc. is to be done. First, a general introduction of the FinTech will be given in the literature review. Further, the research paper by Schwartz and Moon (2001) are to be reviewed, as internet companies shares many similarities with FinTech's and the model might be usable to value FinTech's. Next, a qualitative review of the valuation model by Schwartz and Moon (2001) is given.

The data collection process will primary be done through Thomson Reuters Eikon and Datastream. The model to be used is based on 21 parameters, that either can be observed directly from historical data or estimated. Some parameters are based on the analysts experience and own projections. Further, the methodology and mathematic equations to be used to estimate parameters, cashflows and firm value will be explained in the methodology chapter. Furthermore, the calculations and estimation needed are to be done in Excel, some parts with VBA and Monte Carlo Simulation.

Necessary figures and tables will be given throughout the paper. Any assumptions or simplification to be done will be linked to relevant research, in similar or related topics. If not accessible, a clear economic justification will be given. The valuation of Square Inc. will be analysed and discussed in the chapter of discussion and result analysis. This chapter also includes comparison of the firm value obtained in the simulation and the firm value observed in the market on the specific valuation date, more precisely from the financial statement at a quarterly end date. Suggestion for future research will be given and finally a conclusion will summarize the dissertation.

2 Literature Review

2.1 FinTech

The words “Financial” and “Technology” can be concatenated to “FinTech”. FinTech are mainly described as the connection of modern IT in combination with established business activities of the financial industries (Gomber et al., 2017). FinTech’s are also referred to as innovators and disruptors in the financial sector, which either focus on business model innovation or offer new solutions for the existing financial market (Gomber et al., 2017).

2.1.1 Drivers

The innovations FinTech’s brings to the financial industry is partly driven by favourable regulations, sharing economy and information technology (Lee & Shin, 2018). Other drivers are; changing role of IT, changing consumer behaviour, changing ecosystem and changing regulation (Alt & Puschmann, 2012, p. 204; 2016, p. 24). Traditionally banks are facing regulatory constraints, while non-depository institutions fall outside these regulations (Buchak, Matvos, Piskorski, & Seru, 2018). Technological advances driven by the internet revolution changed the face of the financial services industry and led to the development of electronic finance (e-finance) (Lee & Shin, 2018). E-finance in hand, that also are used in the fintech landscape, refers to all forms of financial services performed through electronic means, including the internet. In relation to information technology, Buchak et al. (2018) finds significant differences on which information fintech lenders use to set mortgage interest rates, compared to non-fintech lenders. FinTech’s uses big data in addition to standard pricing variables. Digitalization is also one of the most important driver of the FinTech evolution, since financial products are almost exclusively based on digital information (Puschmann, 2017). Not only are processes improved with respect to the digitalization, one can also observe a fundamental reorganization of the financial industries business models and value chain (Puschmann, 2017). On the other hand, Philippon (2015) shows that advances in financial technology have failed to reduce intermediation costs. In that spirit, Buchak et al. (2018) finds evidence that fintech lenders charges a higher interest rates (a premium of 14-16 basis point) than non-fintech lenders. Further, it appears that fintech’s offer more convenience rather than cost savings.

2.1.2 Business models, ecosystem and competition

Traditional financial services are being challenged by FinTech's. Lee and Shin (2018) identified five elements in the fintech ecosystem, shown in Figure 1.

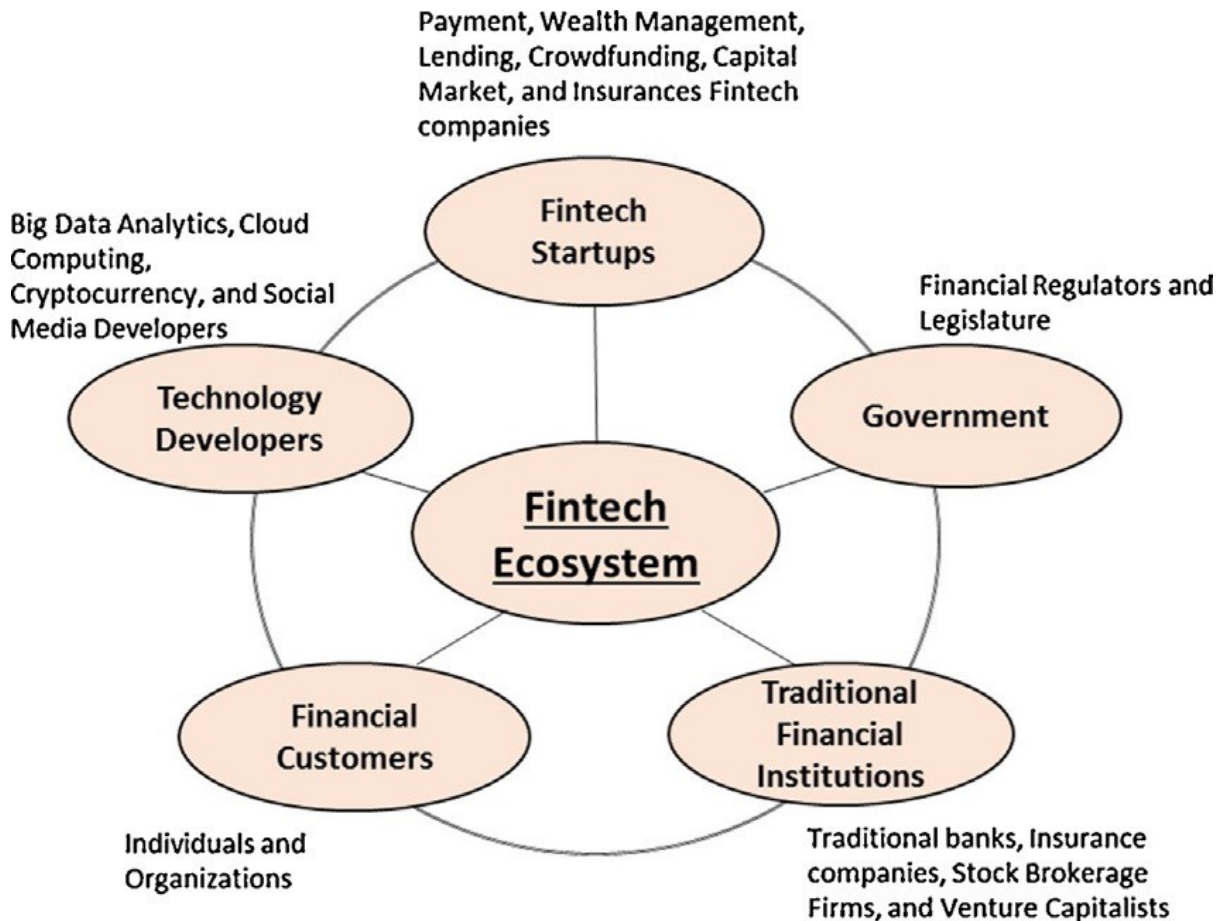


Figure 1: The five elements of the Fintech Ecosystem

FinTech companies concentrate on affordable and cost-efficient Internet-based business models in order to attack established financial services providers (Gomber et al., 2017). Their ability to unbundle services is one of the major drivers of growth in the fintech sector, as traditional financial institutions are disadvantaged in this situation (Lee & Shin, 2018). Lee and Shin (2018) identified six FinTech business models (also called verticals): payment, wealth management, crowdfunding, lending, capital market and insurance. These fintech's and their business models redefine how people store, save, borrow, invest, move, spend, and protect money (Accenture, 2016). FinTech startups differentiated themselves from traditional financial firms with personalized niche services, data-driven solutions, an innovative culture, and a nimble organization (Lee & Shin, 2018). A results of Buchak et al. (2018) suggest that both increased regulatory burdens and technological improvements have contributed to the decline of traditional banks' market share. Regulatory differences explains around 60% of the growth

of shadow banks (Buchak et al., 2018). While fintech is generally considered a threat to traditional financial firms, it also provides ample opportunities for FinTechs to gain a competitive advantage over competitors. In order to grow businesses and secure venture capital, fintech startups can choose to compete with the traditional financial institutions or to collaborate with them (Lee & Shin, 2018). The authors also state that many major financial firms have started to collaborate, coexist or compete with FinTech startups.

2.1.3 Investments and flexibility

Lee and Shin (2018) argue that the ability to assess the value of projects accurately will be critical in an increasingly competitive business environment, so the selection of promising fintech projects is challenging. Further, financial institutions may choose to invest in internal fintech projects in competition with fintech start-ups. Alternatively, financial institutions use collaborative investments with fintech start-ups without requiring internal innovation. A survey of Lee and Shin (2018) show that traditional financial institution invest in a variety of ways: (1) partnering with fintech's or technology companies, (2) outsourcing fintech services from fintech's, (3) providing venture capital to fintech's, (4) incubating/accelerating fintech start-ups, (5) acquiring/buying fintech's, and (6) developing internal fintech's. Further, financial institutions are going to take an immediate investment or a wait-and-see approach to the above-mentioned investment options based on the volatility and project duration of the specific fintech's.

2.1.4 Real Option Approach

Since many fintech projects are experimental and being developed in highly fluid economic and regulatory environments, real options may be an appropriate evaluation method (Lee & Shin, 2018). Lee and Shin (2018) states that there are characteristics, similar to financial options, that make real options an appropriate application for fintech projects. With that, real options are the right, but not the obligation, to take an action during a period of time or by an expiration date. Such real options are (1) option to defer, which gives management the option to wait/learn more to see if a project will be profitable; (2) option to expand, which gives management the option to invest more in a project that is profitable; (3) option to abandon, which gives management the option to abandon a project that is operating at a loss and sell or redeploy the assets; and (4) option to contract, which gives management the option to scale back a project that is operating at a loss. Values of real options for projects or equity can be calculated using the Black-Scholes model (Black & Scholes, 1973) or the binomial option

pricing model (Cox, Ross, & Rubinstein, 1979) if the estimates of the underlying asset's value and variance are obtained. For real options, using decision trees is recommended, as it allows the ability to set up the possibilities of the project according to what management believes it to be or data obtained from simulations (Lee & Shin, 2018). Furthermore, decision trees are more intuitive to decision makers, and solutions can be framed flexibly and realistically without confined assumptions of other real option pricing models.

2.2 “Rational Pricing of Internet Companies” by Schwartz and Moon

The high and much volatile valuations of fintech's, both start-ups and listed on stock exchanges, has been much of discussion the recent years. The same was said about internet companies, when they had their breakthrough in the 1990s and 2000.

In the following sections of the literature review, it is referred to Schwartz and Moon (2001) research paper “Rational Pricing of Internet Companies”. Citation will not be given throughout the review, besides of other supplementary literature that was required for their work.

2.2.1 Introduction

In the year 1990 to 2000 one of the most discussed topics in relation to firm valuation was the sky-high valuations of Internet companies. Entrepreneurs has earned millions or even billions in the emergence of internet firms. This, even though the actual companies had significant losses, that also were growing during time. Some money managers have said that internet stocks had been bid upward irrationally by day traders. Others have seen the value in which internet was transforming the way business was transacted. Due to this, Schwartz and Moon (2000) developed a valuation model (revisited in 2001) to value Internet companies, where they applied real option theory and capital budgeting techniques. Further, they conducted a study on the Amazon and eBay stock, to access whether the market was rationally pricing the stock.

2.3 Valuation framework

The following paragraphs will review the qualitative parts of the methodology and valuation framework by Schwartz and Moon (2000), while the quantitative part are referred to in the methodology chapter.

2.3.1 Model structure in “Rational Pricing of Internet Companies (2000)”

Schwartz and Moon built the model in continuous time, formed a discrete approximation, estimated parameters, solved the model by simulation and finally performed sensitivity analysis. The general set-up of the model is based on modern capital budgeting, which involve cash inflow, cash outflow, ratios, simulations and time value (discounts). As a first step, historical data from the firm’s financial statement and its industry is used to estimate initial parameters in the model. As a second step, cashflows was estimated quarterly. The terminal value was assumed to be multiple times Earnings Before Interest, Taxes, Depreciation and Amortization (EBITDA), which is frequently used by practitioners to set a continuation value. Third step, according to commonly used practice, is to discount expected net cashflow for each quarter to obtain the total value of the firm. As a fourth step, to take care of path dependencies in the cashflow estimations, a Monto Carlo simulation is used to estimated revenues and calculate the firm value. The fifth and final step is to obtain the equity value and hence the share price. With respect to the firm’s capital structure financing sources, such as debt, convertible bonds and employee stock options, has to be taken into consideration and subtracted from the total firm value to obtain the stock price.

2.3.2 Revenues

A stochastic mean reverting process is explaining the revenues of internet companies in the model. The drift of this mean reverting process is the expected growth rate of revenues and is assumed to converge to the long-term growth rate of the industry. Furthermore, the unexpected changes in the revenue process is estimated through the volatility of the revenue growth rate and is also assumed to converge deterministically to a normal level. The unexpected changes in revenue growth rate and its unexpected drift may be correlated and is taken care of in the model. The revenue process of the model is the most uncertain element of the valuation. That is, the uncertainty about changes in revenue and the expected growth rate of revenues. To deal with such uncertainty, a risk-adjusted process by Brennan and Schwartz (1982) are obtained under some simplifications.

2.3.3 Costs

Two components are determining the costs, namely cost of goods sold (GOGS) and other expenses. Other expenses are assumed to have a fixed part and a variable part. Both GOGS and the variable part of costs is proportional to the revenues. Schwartz and Moon (2001) suggest implementing a stochastic cost function which reflects uncertainty in the cost structure (such as e.g. capital expenditures, depreciation, competitors, market share and technological developments).

2.3.1 Taxes, profits, cash and loss carry-forward

The firm's profit in each period is given by revenues less costs and taxes. Corporate tax is only paid if there is no loss carry-forward. This means that the tax rate is zero, if there are any loss carry-forward. For simplicity, Schwartz and Moon (2000) neglected depreciation tax shield. If the profits are positive, the cash balance increases by the amount of profit and vice versa if there are losses. Further, the cash balance earns interest which is added to the revenues in the next period. If the cash balance reaches zero, the company is assumed bankrupt. This is a clear simplification, as it ignores the possibility for additional financing.

2.3.2 Total firm value

Schwartz and Moon assume that no dividend is paid until the firm is matured, at time T , and that the firm at this time is liquidated for valuation purposes. This implies that profits from all periods (every quarter) are summed up in the cash balance, at time T . Finally, cash balance is discounted under an assumed constant risk-neutral measure (the equivalent martingale measure), at the risk-free rate, to obtain the Net Present Value.

2.3.3 Equity valuation

To obtain the equity value, claims such as employees stock options, debt and convertible bonds including their respective principal and after-tax coupon have to be subtracted from the total firm value. For simplification matters, Schwartz and Moon assume that options are exercised, and convertible bonds are converted into shares, if the firm does not go bankrupt in its lifetime. With this assumption the number of shares outstanding has to be adjusted, before estimation of the stock price.

2.3.4 Parameters and Sensitivity

The model by Schwartz and Moon, as described above, requires 24 parameters. Some of these are initial values obtained directly from the firms' balance sheet and income statement. Others are obtained by estimation on historical data or the analysts own projections. The valuation is highly sensitive to initial condition and parameter specification, and they are found to significant effect on the value as explained earlier.

2.3.5 Monte Carlo Simulation

Monte Carlo Simulation is a statistical method applied in modelling the probability of different outcomes in a problem (Corporate Finance Institute, 2019a). The Monte Carlo method is commonly used in finance to perform simulation to forecast different future outcomes when there is uncertainty about the impact of risk (Investopedia, 2019).¹ The main ideas behind are the repeated random inputs of the random variable and the distribution of outcomes, namely the probability distribution (Corporate Finance Institute, 2019a). Monte Carlo simulations has been used to model project cashflows that faces uncertainty. A numerous amount of simulation results in a range of Net Present Values (NPV), which are averaged to form an expectation of the most likely outcome. It is also used to price option where random paths for an underlying asset is generated, each having associated payoff (Investopedia, 2019).

¹ Corporate Finance Institute and Investopedia are not academic publishers, but we cite them since their explanation gives a basic overview of the Monte Carlo method within finance.

3 Methodology

3.1 Data Collection

The data being used in valuation model consists of firm specific data, industry data and data from the financial market as a whole. More specifically, the data that is needed is secondary data collected from various sources. Historical financial statement data are mainly retrieved from Eikon and Datastream by Thomson Reuters. When needed, or for quality matters, Compustat – Capital IQ from Standard & Poor is also used in addition. Further, financial forecasts related to the firm being valued are collected from I/B/E/S by Thomson Reuters.² Most data are collected on an annual basis, but in some cases quarterly data will was used to capture the most recent changes and its volatility for some of the parameters to be estimated.

To simplify the model and to control for possible seasonal effects, forecasts in the valuation model are performed on a yearly basis. This due to the fact that it's hard to find evidence that there are any seasonal effects for FinTechs, both on a yearly and quarterly basis. This is line with what Schwartz and Moon (2001) suggested and implemented in their framework to value internet companies.

All of the estimations and analysis has been done in Excel. Some statistical calculations are performed through VBA-programming. Monte Carlo simulations has been conducted to model risk, growth distribution and uncertainty, and finally to get rid of path dependencies which are present in each single simulation.

3.2 Parameter Specification

All parameters to be used in the model and its estimation procedure are presented in Table 2. Further explanation of the exact data used in the estimation is given in chapter 4. In the follow paragraphs we explain the discrete approximation of the valuation model. Please refer to Rational Pricing of Internet Companies by Schwartz and Moon (2001) for the continuous time model.

² Institutional Brokers Estimate System (I/B/E/S) is a database where selected stock analyst provides their forecast of US listed firms. I/B/E/S is owned by Thomson Reuters. More than 900 firms and analysts provide I/B/E/S with estimates, on a frequent basis, of 33 forecast items.

<u>Parameter</u>	<u>Notation</u>	<u>Estimation procedure</u>
Initial revenue	R_0	Obtained from current income statement
Initial loss carry-forward	L_0	Obtained from current balance sheet
Initial cash and cash equivalents	X_0	Obtained from current balance sheet
Initial property, plant and equipment	P_0	Obtained from current balance sheet
Initial expected growth rate in revenues	μ_0	Percentage change from past revenues and analyst projection of future growth
Initial volatility in revenues	σ_0	Standard deviation of percentage change in revenues over recent past
Initial volatility in expected growth rate of revenues	η_0	Inferred from stock volatility or implied volatility of options
Initial variable cost as fraction of revenues	γ_0	Estimated by regression of cost of EBITDA on revenues over recent past and/or forecast for the upcoming year
Initial volatility of variable cost	φ_0	Standard deviation of percentage change in cost of EBITDA
Fixed cost	F	Estimated by regression of cost of EBITDA on revenues over recent past and/or forecast for the upcoming year
Long-term growth rate in revenues	$\bar{\mu}$	Growth rate of stable firms in the industry/sector
Long-term volatility in growth rate of revenues	$\bar{\sigma}$	Standard deviation of percentage change in revenue for a stable firm in the industry/sector
Long-term variable cost fraction of revenues	$\bar{\gamma}$	Expected long-term fraction of variable cost.
Long-term volatility of variable cost	$\bar{\varphi}$	Expected long-term standard deviation of variable cost fraction
Correlation between change in revenues, change in expected growth rate and variable cost fraction	ρ	Estimated from income statement or analyst own projections
Market price of risk in the revenue factor	$\bar{\lambda}$	Obtained from the beta of the stock multiplied with the standard deviation of the market portfolio
Mean-reversion coefficient	κ	Estimated on assumption on the half-life of growth rate or volatility in relation to the length of forecast period and long-term growth or volatility
Depreciation and Amortization rate	DR	Estimated as the average of past depreciation and amortization divided by previous year property, plant and equipment
Capital Expenditures rate	CR	Estimated as the average capital expenditures divided by revenues in recent past
Risk-free interest rate	r_f	Appropriate U.S T-bill rate
Corporate tax rate	τ_c	Obtained from tax code
Horizon for forecast period	T	Arbitrary long-term horizon until the company has become stable in the industry
Time increment	Δt	Chosen with respect to data availability and forecast horizon
Terminal value multiple	M	EV/EBITDA multiple of analyst own projection

Table 2: Model parameters

3.3 Capital Budgeting

3.3.1 Revenue process

3.3.1.1 Revenues

The revenue dynamics of FinTech companies at any given time, $R_{(t+\Delta t)}$, is imitated by the following risk-adjusted stochastic equation:

$$R_{(t+\Delta t)} = R_{(t)} e^{\left(\left[\mu_{(t)} - \bar{\lambda} \sigma_{(t)} - \frac{\sigma_{(t)}^2}{2} \right] \Delta t + \sigma_{(t)} \sqrt{\Delta t} \varepsilon_1 \right)}$$

with; drift, $\mu_{(t)}$, that is the expected growth rate of revenues; market price of risk, $\bar{\lambda}$, in the revenues; volatility, $\sigma_{(t)}$, that is the unanticipated changes in revenue; and ε_1 , that is a standard normal random variable.

3.3.1.2 Growth rate of revenues

The expected growth rate of revenues is assumed to follow a mean-reverting process:

$$\mu_{(t+\Delta t)} = \mu_{(t)} e^{-\kappa \Delta t} + (1 - e^{-\kappa \Delta t}) \bar{\mu} + \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}} \eta_{(t)} \varepsilon_2$$

with long-term growth rate $\bar{\mu}$, i.e. the growth can initially be very high when they are achieving success and then converge stochastically to a sustainable rate of growth for the industry the firm belong to in the long run. The speed of convergence to the long-term growth rate is explained by the mean-reversion coefficient, κ . That is, any deviation in the growth rate, $\pi_{(t)}$, from its long-term growth rate, $\bar{\mu}$, is expected to be halved in the time period $\ln(2)/\kappa$. The unanticipated changes in the expected growth rate is given by $\eta_{(t)}$, where ε_2 is a standard normal random variable.

3.3.1.3 Volatility in the revenue process

The revenues have two sources of uncertainty, one in the revenue process and the other in the process for expected growth rate. The unanticipated changes in the revenues at any time, $\sigma_{(t)}$ is given by:

$$\sigma_{(t)} = \sigma_0 e^{-\kappa t} + (1 - e^{-\kappa t})\bar{\sigma}$$

and is assumed to converge deterministically to a more normal level, $\bar{\sigma}$, in the long-term, whereas the unanticipated changes in the expected growth rate, $\eta_{(t)}$, is given by:

$$\eta_{(t)} = \eta_0 e^{-\kappa t}$$

and is assumed to converge deterministically to zero.

3.3.2 *Costs*

3.3.2.1 Cost structure

The cost structure comprises both variable and fixed costs. Schwartz and Moon (2001) estimated costs through a linear regression with the intercept measuring the fixed costs and the slope, the variables costs. Thus, the cost function at any point in time, $C_{(t)}$, consists of two components, respectively a variable part and a fixed part:

$$C_{(t)} = \gamma_{(t)}R_{(t)} + F$$

where the variable costs fraction, $\gamma_{(t)}$, is proportional to the revenues.

3.3.2.2 Variable cost process

To capture uncertainty, such as cost level, future competitors, technological development and markets shares, a stochastic mean-reverting process is assumed for the variable costs:

$$\gamma_{(t+\Delta t)} = \gamma_{(t)}e^{-\kappa\Delta t} + (1 - e^{-\kappa\Delta t})\bar{\gamma} + \sqrt{\frac{1 - e^{-2\kappa\Delta t}}{2\kappa}}\varphi_{(t)}\varepsilon_3$$

where the proportional fraction of revenues, $\gamma_{(t)}$, is assumed to converge to the long-term average for the industry, $\bar{\gamma}$. The speed of convergence is given by the mean-reversion parameter, κ . Similar to the revenue process, any deviation in the variable cost from its long-term average is assumed to be halved in the period of $\ln(2)/\kappa$. Further, the stochastic part in the variable

costs are given by, ε_3 , a standard normal random variable. The unanticipated changes in the variable costs at any time, $\varphi_{(t)}$, is given by:

$$\varphi_{(t)} = \varphi_0 e^{-\kappa t} + (1 - e^{-\kappa t}) \bar{\varphi}$$

and is assumed to converge deterministically to a normal level, $\bar{\varphi}$, for the industry.

3.3.2.3 Fixed cost

The fixed cost in the cost structure, F , is given by the initial estimated fixed cost and is assumed to be constant throughout the valuation period.

3.3.3 Correlation between revenues and costs

The unanticipated changes in revenues, the unanticipated changes in its growth rate and the unanticipated change in variable cost might be correlated. Cholesky Decomposition is used to generate correlated standard normal random variables $\varepsilon_1, \varepsilon_2$ and ε_3 . See Appendix 2 for Cholesky Decomposition in VBA. For simplifying reason, and to prevent overcomplication of the mode, correlation can be set to 0 for all variables.³

3.3.4 Long-term convergence

Throughout time the firms cashflows are assumed to be less volatile since the company reverts to a more stable company within its industry. This is given by the mean reverting processes for the revenues and variable cost, with their respectively decreasing volatilities. That is, as time goes by the volatilities converge to:

$$\lim_{t \rightarrow \infty} \eta_{(t)} = 0$$

$$\lim_{t \rightarrow \infty} \sigma_{(t)} = \bar{\sigma}$$

$$\lim_{t \rightarrow \infty} \varphi_{(t)} = \bar{\varphi}$$

and hence revenue growth rate and the variable cost fraction converge to:

$$\lim_{t \rightarrow \infty} \mu_{(t)} = \bar{\mu}$$

$$\lim_{t \rightarrow \infty} \gamma_{(t)} = \bar{\gamma}$$

³ Schwartz and Moon (2001) conclude that correlation do not have any significant effect on the valuation. Same were the result on previous papers before 2001 when the model was revisited.

3.3.5 Depreciations

The depreciation at any time is assumed to be a fraction, DR, of the previous period Property, Plant and Equipment (PP&E):

$$Dep_{(t)} = DR * P_{(t-\Delta t)}$$

Depreciation is diluting PP&E as times go by, while capital expenditures is adding value, so the dynamics of PP&E is given by:

$$P_{(t)} = P_{(t-\Delta t)} - Dep_{(t)} + CapEx_{(t)}$$

where $CapEx_{(t)}$ are allowed to be known for an initial period, $CX_{(t)}$, and after that assumed to be a constant fraction, CR, of revenues. That is:

$$CapEx_{(t)} = \begin{cases} CX_{(t)}, & t \leq \bar{t} \\ CR * R_{(t)}, & t > \bar{t} \end{cases}$$

3.3.6 Profits and Loss Carry-Forward

3.3.6.1 Profit/Loss

Earning Before Tax (EBT) is given by the periods revenues less costs and depreciations, so profits after tax is given by:

$$Y_{(t)} = (R_{(t)} - C_{(t)} - Dep_{(t)})(1 - \tau_c)$$

where the corporate tax, τ_c , is assumed to be constant throughout the forecast period.

3.3.6.2 Loss Carry-Forward

The dynamics of loss carry-forward depends on the previous year loss carry-forward and the profits of the current year and are given by:

$$L_{(t)} = \begin{cases} L_{(t-\Delta t)} - Y_{(t)}, & L_{(t-\Delta t)} > 0 \\ \max\{-Y_{(t)}, 0\}, & L_{(t-\Delta t)} = 0 \end{cases}$$

that is, if the initial loss carry-forward, $L_{(t-\Delta t)}$;

- is positive and the profit is positive, accumulated loss carry-forward decreases
- is positive and the profit is negative, accumulated loss carry-forward increases
- is 0 and profit is negative, loss carry-forward increases by the amount negative profit

3.3.6.3 Corporate tax

Whether the company pays corporate tax or not at any time, t , is given by the following function:

$$\tau_c = \begin{cases} \tau_c, & L_{(t)} = 0 \text{ and } Y_{(t)} \geq 0 \\ 0, & L_{(t)} > 0 \text{ or } Y_{(t)} < 0 \end{cases}$$

where the tax rate is equal to τ_c if accumulated loss carry-forward is zero and net income is positive at the same time, and 0 if either accumulated loss carry-forward is positive or net income is negative.

3.3.7 Cash Balance

The dynamics of cash available to the firm at any time, $X_{(t)}$, evolves as follows:

$$X_{(t)} = X_{(t-\Delta t)}e^{r_f\Delta t} + Y_{(t)} + Dep_{(t)} - CapEx_{(t)}$$

where the cash balance available from the previous period earns an untaxed interest, which is achieved by continuously compounding at the risk-free rate, $e^{r_f\Delta t}$. This is done to make the valuation insensitive to future distribution of cash to investors. Additionally, the depreciations are added back since they are a non-cash cost and capital expenditures are deducted since they represent a cash outflow in order to invest and maintain non-current assets such as PP&E and technology.

3.3.8 Bankruptcy condition

If the cash balance at any time becomes negative the firm is assumed bankrupt. This is of course a simplifying assumption, and a deviation from the model by Schwartz and Moon (2001). It neglects the possibility of future financing. On the other hand, as in a real-world situation, many firms go bankrupt, even though the amount of cash is positive. Seeing these two contradictions together, it seems reasonable that the simplifying bankruptcy condition holds on average.

More advanced bankruptcy condition can be implemented, such as in the original model of Schwartz and Moon (2001), or a least-squares approach by Longstaff and Schwartz (2001) that can estimate the optimal amount of financing based on cross-sectional information from the simulations.

3.3.9 Terminal value

The firm is assumed to have continuation value if it does not reach any bankruptcy during the forecast period. Explicitly, the continuation value at time T is assumed to be a multiple M of the EBITDA in the last forecast year. Hence, the terminal value is given by:

$$TV_T = M(R_{(T)} - C_T) = M * EBITDA_{(T)}$$

where Schwartz and Moon (2001) uses and argues that a multiple of 10 is a conservative measure that is widely used by practitioners as proxy.

3.4 Valuation

3.4.1 Total Firm Value and Enterprise Value

According to standard theory, the present value of a firm is the sum of all expected future cashflows discounted at its cost of capital. In case of the model described above, all future cashflows are risk-adjusted and accumulated in the firm's cash balance until horizon T.

To avoid making the valuation result sensitive to any dividend pay-outs, a simplified assumption is that the firm pays no dividend during the forecast period. That is, all cash and profits are retained in the firm until the firm has reverted to a stable company within its industry at an arbitrary time, T.

The total value of the firm has two components, namely the cash balance and the terminal value. To determine the present value of the firm, these components are discounted under a risk neutral measure, the equivalent martingale measure, at risk-free rate:

$$V_0 = E_Q\{X_T + TV_T\}e^{-r_f T}$$

The value of the company at current time is therefore dependent on seven state variables, namely; revenues, expected growth rate, variable cost, loss carry-forward, cash balance, PP&E and time, and given by the following function:

$$V_0 = f(R, \mu, \gamma, L, X, P, t)$$

This implies that one single valuation will be path dependent. Monte Carlo Simulations are conducted to get rid of these path dependencies. The present value of the total firm value, V_0 , is therefore given by average of all the simulations.

3.4.1 Estimation of Stock Price

Long-term debt and current proportion of long-term debt must be deducted from the total firm value, as these are already existing financing sources, to arrive at an equity value of the firm:

$$Total\ Equity = V_o - Debt$$

And further, estimated share price is given by the equity value divided by the number of shares outstanding:

$$Share\ Price = \frac{Total\ Equity}{Number\ of\ Shares\ Outstanding}$$

where the amount of number of shares outstanding depends on whether employees stock options plans (ESOP) are exercised or not. For simplifying reasons, ESOP are assumed to be fully exercised if the estimated share price, S , is higher than the exercise price on the stock options, X . Hence the share price will be diluted as the number of shares outstanding increases by the amount exercised. Numbers of Shares Outstanding are therefore given by:

$$Number\ of\ Shares\ Outstanding = \begin{cases} Outstanding\ Shares, & S < X \\ ESOP + Outstanding\ Shares, & S \geq X \end{cases}$$

When this is done, we are able to estimate the modelled share price in the valuation. Analysis are to be performed once we are able to compare the modelled price and the price in the stock market.

4 Results' Analysis and Discussion

In this dissertation we applied the model explained in the methodology chapter on Square Inc. (SQ), a FinTech company listed on the New York Stock Exchange (NYSE). Square was founded in 2009 and made an Initial Public Offering (IPO) in November 2015. By the end of the fiscal year 2018 (31.12.2018), Square Inc. had an enterprise value of 23,05 billion US dollars and a market capitalization of 23,19 billion dollars.

4.1 Parameters Estimation

As in all valuations, this model is based on a large amount of assumptions and simplifications which differ from the real world. The approach depends critically on initial parameters such as the growth rates, volatility of growth rates, and the risk parameters that make the cashflows risk-adjusted. A comprehensive list of all estimated values can be seen in Table 3.

<u>Parameter</u>	<u>Notation</u>	<u>Value</u>
Initial revenue	R_0	3298
Initial loss carry-forward	L_0	886
Initial cash and cash equivalents	X_0	1124
Initial property, plant and equipment	P_0	142
Initial expected growth rate in revenues	μ_0	30,9%
Initial volatility in revenues	σ_0	0,130
Initial volatility in expected growth rate of revenues	η_0	0,171
Initial variable cost as fraction of revenues	γ_0	0,92
Initial volatility of variable cost	φ_0	7,4%
Fixed cost	F	147
Long-term growth rate in revenues	$\bar{\mu}$	8,3%
Long-term volatility in growth rate of revenues	$\bar{\sigma}$	0,045
Long-term variable cost fraction of revenues	$\bar{\gamma}$	0,92
Long-term volatility of variable cost	$\bar{\varphi}$	0,037
Correlation between change in revenues, change in expected growth rate and variable cost fraction	ρ	0
Market price of risk in the revenue factor	$\bar{\lambda}$	0,565
Mean-reversion coefficient	κ	0,198
Depreciation and Amortization rate	DR	40%
Capital Expenditures rate	CR	2%
Risk-free interest rate	r_f	2,69%
Corporate tax rate	τ_c	21,0%
Horizon for forecast period	T	15
Terminal value multiple	M	10
Time increment	Δt	1

Table 3: Estimated Parameters

Below we explain how we estimated the most crucial parameters in the model from historical and present data. I/B/E/S forecasts of revenues and capital expenditures are used in the model as a best proxy of future outcome. Since a higher quantity of analysts provides I/B/E/S with annually forecasts rather than quarterly forecasts, annually forecasts are used. Hence the time increments are one year in the valuation of Square Inc.

4.1.1 Revenues dynamics

As the starting value for initial revenue, we used the reported revenues of fiscal year 2018, that was 3298 million dollars. The initial growth rate of revenues from 2018 to 2019 is assumed to be 30,9% continuously compounded, estimated from analysts forecast for 2019. The long-term growth rate of revenues is assumed to be 8,3%, proxied by the historical median growth of revenues in the IT Services industry (GICS).

The initial volatility of revenues is estimated from data from the last 12 quarters and resulted in an annualized volatility of 0,130. In the long-term this volatility is assumed to decrease to a level of 0,045. As a proxy for initial volatility of the growth rate of revenues, we used the historical volatility of the market portfolio, that is taken to be 0,171 and thereafter decreases and converge to zero in the long-term.

Simulated revenues and expected revenues are risk-adjusted, while NPV revenues are not. With NPV revenues we mean the estimated revenues if we were to use a traditional DCF model, that is that the revenues are not risk-adjusted as in our real option approach. Each single simulation yields different revenues paths. If we run a high enough number of simulations, we would obtain simulated revenues which closely matches expected revenues. The result of revenue evolution can be seen in the Figure 2.

4.1.1 Cost dynamics

To arrive at an amount for the fixed cost and an initial variable cost fraction of revenues we run a regression with revenues as the independent variable and cost of EBITDA as the dependent variable. Using data from fiscal year 2017, 2018 and forecasts for 2019, the obtained result from the regression is 147 million dollars as fixed cost and 0,92 as a variable fraction of revenues. Figure 3 shows the resulting regression. It is important to keep in mind that when the business grows, the higher is the costs, but the fixed cost decreases as a proportion of revenues and hence the cost per dollar of revenue decreases (economy of scale).

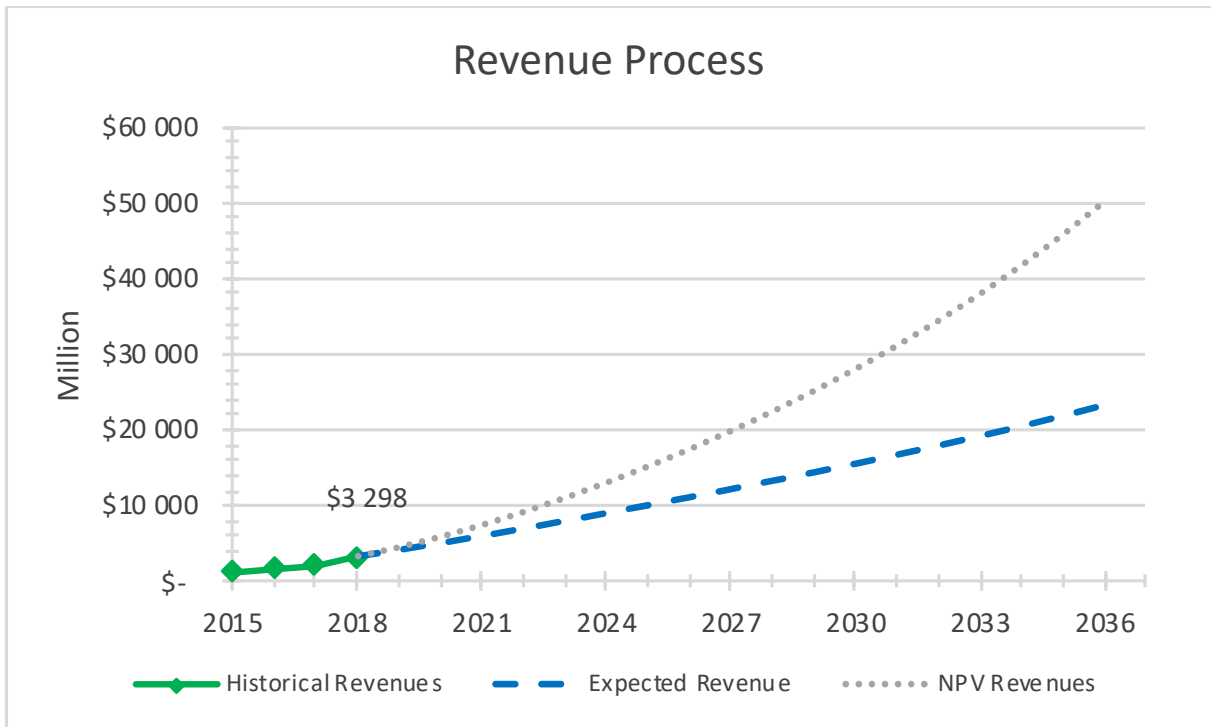


Figure 2: Revenue Process

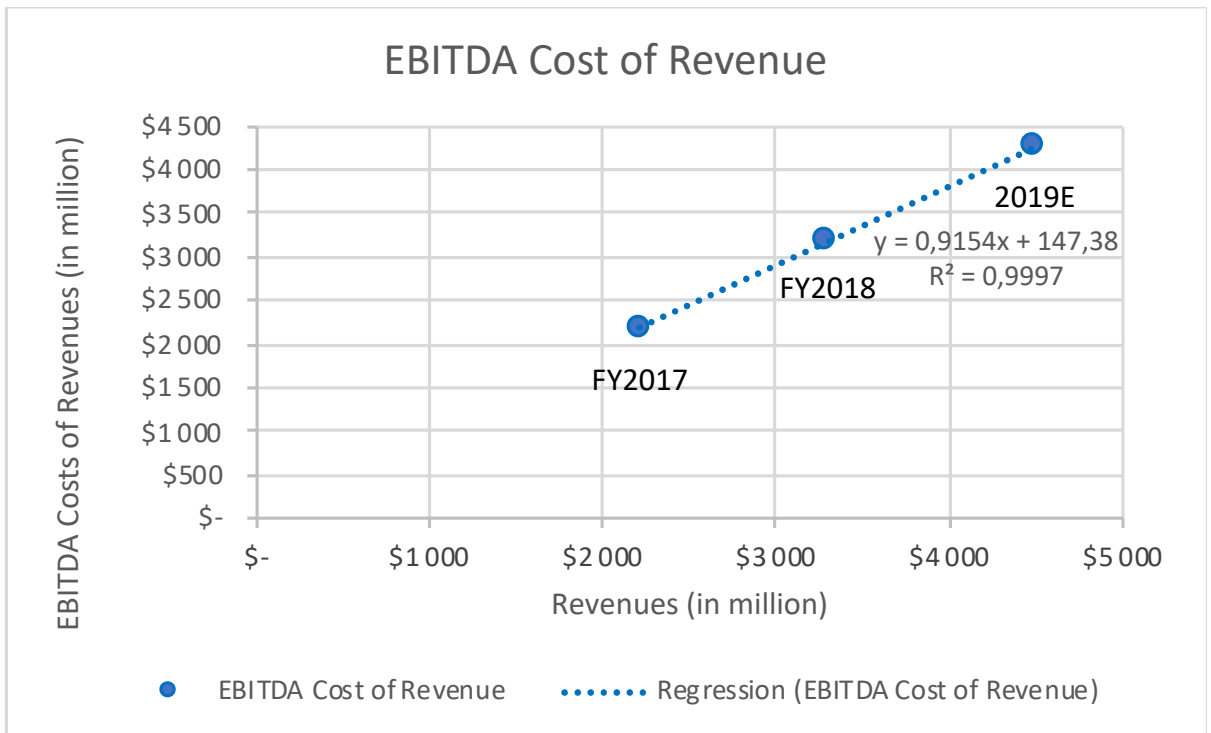


Figure 3: EBITDA Costs of Revenues

The initial volatility for the variable cost process is assumed to be 0,074, estimated from the historical cost of EBITDA data from 2015 and forecasts until 2020. In the long-term the volatility of variable cost is assumed to be halved and converge to 0,037.

4.1.2 Correlation and Half-Life of Deviations

In the valuation we assumed that there is no correlation between the Monte Carlo random variables in the revenue process, revenues growth process and finally the variable cost process. We examine the possible effect of correlation in the sensitivity analysis later on.

For simplifying reasons, we assumed that the speed of adjustment in the mean-reversion process for revenue, the growth rate of revenue and the variable cost process are the same. We infer the coefficient and its half-life from the initial and long-term growth rate of revenues.

This result in mean-reversion coefficient of 0,198, which refers to a half-life of 3,5 years. With this half-life, expected revenue growth closely matches analyst's prediction of revenue for fiscal year 2010 and 2021. Figure 4 shows how the half-life relates to growth rates and analysts forecast.

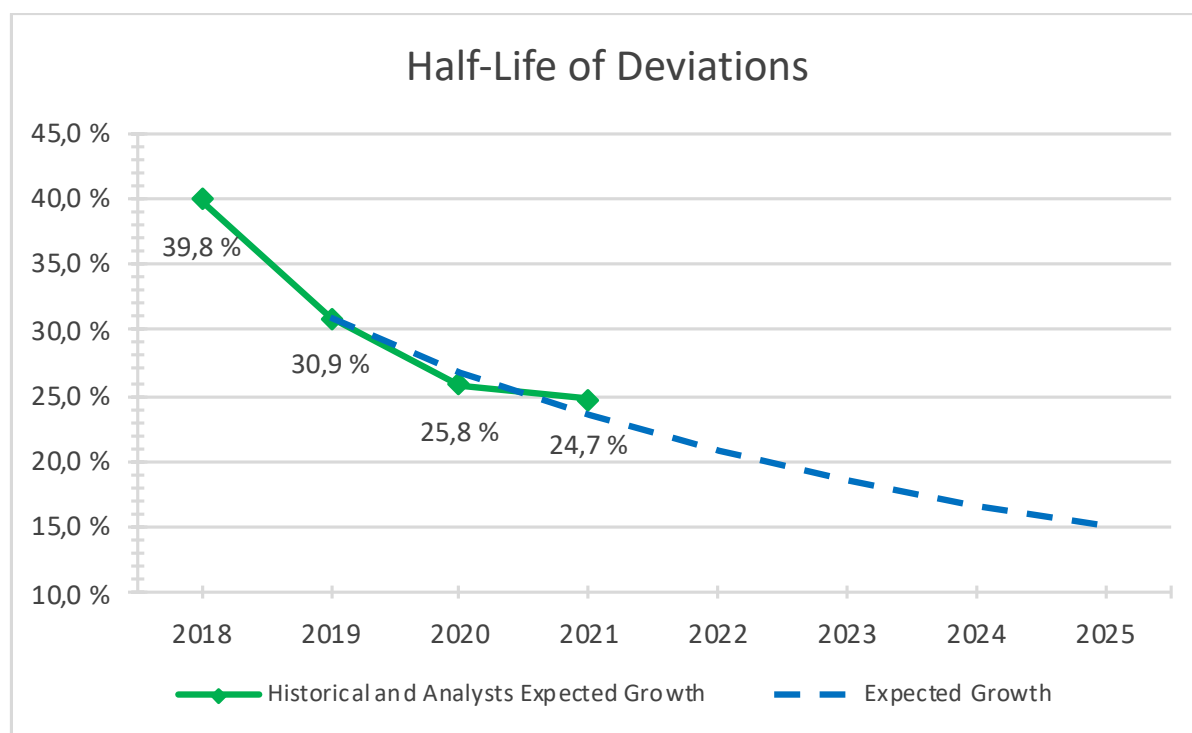


Figure 4: Half-Life of Deviations

4.1.3 Balance Sheet Data, Capital Expenditures and Depreciations

Accumulated losses at the end of the fiscal year 2018 amount to 886 million dollars. Cash and Short-Term Investments sums up to 1124 million dollars and is used as initial cash balance available to the firm for future financing purposes.

Reported Net property, plant and equipment (10-K includes capitalized software) at the end of fiscal year 2018 was 142 million dollars.⁴ Due to short lifetime of capitalized software, computers and data centre equipment, the historical amortization and depreciation expense over the last three years has been high. Hence the depreciation and amortization rate are taken to be 40% of prior year ending balance for net property, plant and equipment. We use analysts forecast of capital expenditures for year 2019 to 2022. After that, capital expenditures are taken to be 2% of the current year's revenues.

4.1.4 Risk and Market Parameters

The corporate tax rate in the US was 21% in 2018, and is assumed to be constant throughout the forecast period.⁵ As a proxy for the risk-free rate concerning the expected lifetime of the company we used the 10-year treasury rate, which was 2,69% at the valuation date (U.S. Department of the Treasury, 2019).

The 5-year monthly stock beta was 3,31 and the adjusted beta was 2,54. We choose to go further with the adjusted beta, because we think the 5-year monthly was abnormally high. Annual volatility of the market portfolio (S&P 500) is estimated to be 17,1% and is used to estimate the market price of risk to risk-adjust the revenue cashflows in the risk-neutral framework.⁶

4.1.5 Forecast Horizon and Terminal Value Multiple

Many FinTech firms are often acquired by other competitors or banks that drives innovation through FinTechs. Others are collaborating with other FinTechs in terms of bundled products, while others are start-ups. These facts make it hard to put a specific lifetime on FinTech because their products and solutions are on different stages in the life cycle. McKinsey found that the average life-span of companies listed on S&P 500 was 61 years in 1958 and in 2011 it was down to 18 years, and it's still decreasing (Desmet, Duncan, Scanlan, & Singer, 2015). According to Professor Richard Foster at Yale University the average lifetime was down to 15 years in 2018. We simplified and assumed that the lifetime of a FinTech is 15 years. We also argue and justify this choice with the fact that FinTechs often are acquired, bankrupt or disrupted by others.

⁴ This is Net Property, Plant and Equipment reported in Squares Inc. 10-K. Deviates from what Thomson Reuters Eikon reports. The number is checked by deducting accumulated depreciation from Gross Property, Plant and Equipment.

⁵ US corporate tax rate was reduced from 35% to 21% under The Tax Cuts and Job Act of December 2017, effective from January 2018.

⁶ The corresponding implied volatility on CBOE S&P 500 (VIX) at the valuation date was 25,42%.

As the Terminal value multiple, we chose a multiple of 10 times EBITDA, which is a commonly used EV/EBITDA-multiple applied by practitioners. The IT Services and Information Technology sectors has an average forward multiple at respectively 12,84 and 12,59. A multiple of 10 is therefore assumed to be a conservative measure, which we find suitable in a real option valuation since it is not that likely to put to high value on the terminal period. Especially, since we choose a forecast time of 15 years.

4.2 Results

The market price of Square Inc stocks on December 31, 2018 was \$56,09, whereas the market capitalization and enterprise value summed up to, respectively 23,19 and 23,05 billion dollars. Below we summaries the resulting estimates we obtained in the Monte Carlo Simulation (Real Option Approach), Expected Value and Net Present Value, where the two last are static valuations since we only need to run one simulation to obtain the result.

4.2.1 Simulation

We ran 10.000 Monte Carlo simulations to get a range of firm values and to get rid of path dependencies. If we were to control for possible seasonal effect, the model would be even more complex, and the present value of the firm obtained from the simulations would have an even higher variance. Therefore, we have chosen to use apply the model with time increments of one year. This because most estimates that are needed as input in the model are accessible and more reliable on a yearly basis, since they are collected from larger group analysts. Further, by using yearly increments we also avoid the possible effect of seasonality in the revenue process. By taking seasonality into account, it would have been more likely to overcomplicate the model.

An estimated stock price of \$69,82, that is 24% above the market price, was obtained from the simulations.

The result obtained from Monte Carlo Simulations is the value of the firm seen form a real option perspective, since all the different states, in each single valuation, are brought together to form a unique distribution of future outcomes. Namely a risk-neutral distribution of the revenues process that can be seen as a probability distribution, typically used in traditional option pricing models and similar to Cox et al. (1979) or Black and Scholes (1973).

The firm went bankrupt in 29,5% of the cases in the simulations. This is under the assumption that the firm burns all cash available and do not have the access to any further financing in the future. A summary of bankruptcy occurrence in each year can be seen in Table 4.

<u>Year</u>	<u>Number of Bankruptcies</u>	<u>Probability of Bankruptcy</u>
1	0	0
2	452	4,5%
3	714	7,1%
4	486	4,9%
5	310	3,1%
6	210	2,1%
7	176	1,8%
8	139	1,4%
9	132	1,3%
10	97	1,0%
11	63	0,6%
12	63	0,6%
13	50	0,5%
14	57	0,6%
15	0	0%
<u>Total</u>	<u>2949</u>	<u>29,5%</u>

Table 4: Probability of Bankruptcy

4.2.2 Net Present Value

By running one single simulation without the stochastic processes in revenues, growth rate of revenues and variable costs (i.e. initial- and long-term volatilities are set to zero) we obtained the Net Present Value of the firm's stock, estimated to be \$7,10. This is tremendously lower than the simulated real option value and illustrates that traditional DCF-models underestimate firm value of high-growth firms and the value of flexibility in investments, namely the real option value. This is in line with Schwartz and Moon 2001 result in the valuation of eBay, and also the potential risk for under-pricing that (Lee & Shin, 2018, p. 40) mentioned as a limitation with traditional NPV- and DCF-models.

To obtain the NPV-result we discounted future cashflows at the Weighted-Average-Cost-of-Capital (WACC), estimated to be 10,9%. Cost of Debt was assumed to be 4,82%, calculated as the weighted average of the interest rate paid on the firms issued bonds. Cost of Equity was calculated by using the Capital Asset Pricing Model (CAPM) and is taken to be 16,7%, estimated with a beta of 2,54 (Adjusted Beta) and a risk premium in the market of 5,5%.

4.2.3 Expected Value

By running one simulation with all standard normal random variables set to zero, we can obtain the expected value of the firm and hence the stock price. Cashflows are now risk-neutral, while future states are thought of as certain. This results in an estimated stock price of \$34,51. This is 38% below the market price, but at the same time higher than the NPV. It illustrates again that traditional DCF-models underestimate the firm value of firms with high-growth potential and flexible investments.⁷

Table 5 summarizes the results obtained from the valuations.

	<u>Stock Price</u>	<u>Deviation from market</u>
Market Price	\$ 56,09	
Real Option Value	\$ 69,82	24%
Net Present Value	\$ 7,10	-87%
Expected Value	\$ 34,51	-38%

Table 5: Valuation Results

4.3 Sensitivity Analysis

Since the model consists of 24 different inputs, we evaluate how sensitive the estimated stock price is by performing, in most cases, a perturbation of 10% in each single parameter, while the other are kept constant as in the base case. Table 6 reports the sensitivity in the share price caused by changes in the given parameters.

<u>Parameter</u>	<u>Base value</u>	<u>Perturbed Value</u>	<u>Share Price</u>	<u>Probability of Bankruptcy</u>
Base Case			\$ 69,82	29,5 %
μ_0	30,9 %	34 %	\$ 78,44	30,1 %
σ_0	0,130	0,143	\$ 72,16	29,6 %
η_0	17,1 %	18,8 %	\$ 82,22	30,1 %
γ_0	0,92	0,95	\$ 63,36	37,5 %
φ_0	0,074	0,01	\$ 70,40	32,4 %
F	\$ 147	\$ 162	\$ 73,76	31,3 %
$\bar{\mu}$	8,3 %	9,2 %	\$ 72,54	30,7 %
$\bar{\sigma}$	0,045	0,050	\$ 71,49	30,0 %
$\bar{\gamma}$	0,92	0,95	\$ 71,29	29,6 %
$\bar{\varphi}$	0,037	0,041	\$ 69,46	30,6 %
$\bar{\lambda}$	0,565	0,622	\$ 71,63	29,5 %
κ	0,198	0,218	\$ 71,72	29,3 %
r_f	2,69 %	3,69 %	\$ 61,44	29,5 %
T	15	16	\$ 79,30	30,5 %
M	10	11	\$ 72,88	30,7 %

Table 6: Sensitivity Analysis

⁷ If we assume that the stock is priced correctly by investors, i.e. the stock is traded in a strong form efficient market.

4.4 Discussion and Future Research

4.4.1 Growth rates

From year 1 to year 5 the distribution of growth rate of revenues expands in range, and after it shrinks again until year 15. On one hand, it is intuitive that a successful start-up sees higher rates in its growth after some years, as it takes time to achieve market shares. On the other hand, an unsuccessful firm will see low or even extreme negative growth rates, which intuitively will result in bankruptcy when all cash available is used to cover cash costs in the start-up years. Therefore, the probability of very high or even very low growth outcomes are more likely in year 5. In the long-run, the distribution shrinks as the growth rate converge to the predetermined long-term rate. Figure 5 shows how the distribution of growth rates in revenues evolves.

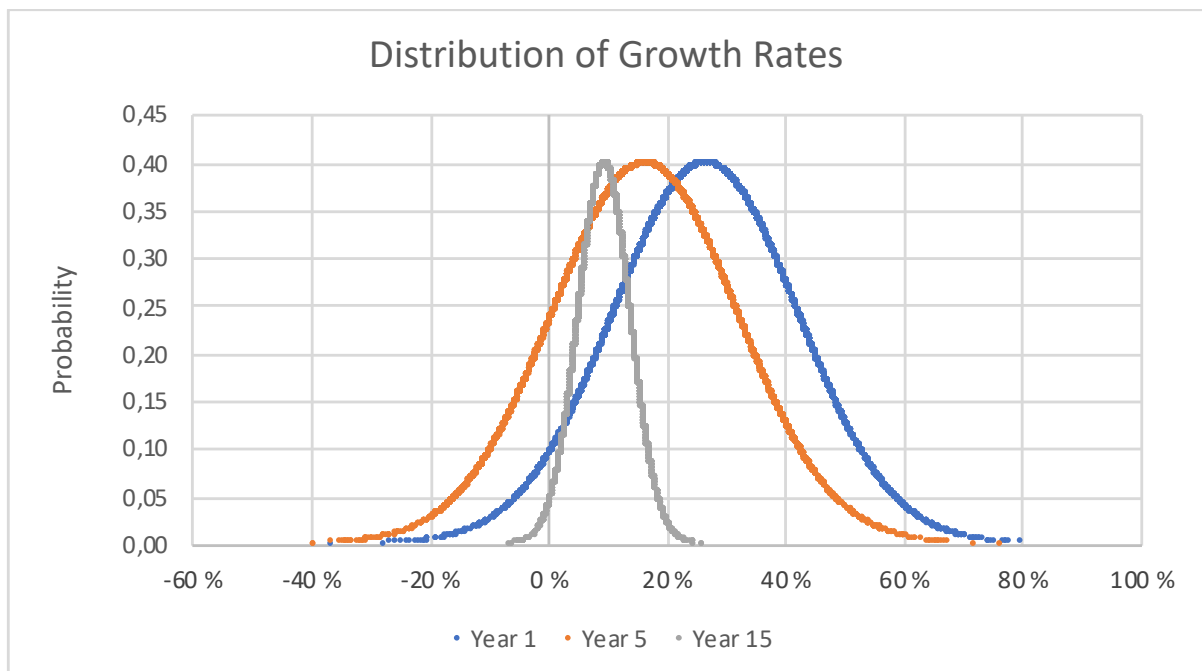


Figure 5: Distribution of Growth Rates

4.4.2 EDITDA cost of revenues

In the simulated result we allowed for stochastic variability in the assumed variable cost of EBITDA from year to year, to incorporate the variability one can see in firms cost over time. The fraction of variable cost converges to the predetermined long-term fraction set in the model and can be seen in Figure 6. The distribution spread first increases before it decreases over time, since we have estimated higher initial volatility than the long-term volatility.

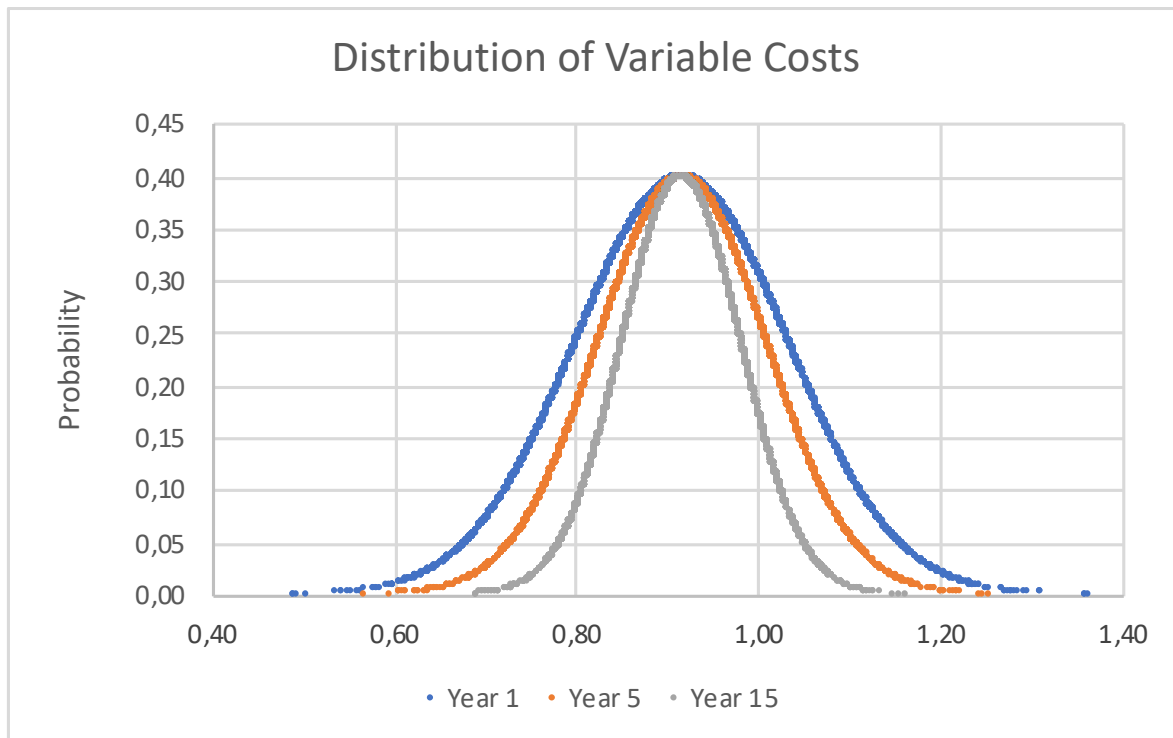


Figure 6: Distribution of Variable Cost fraction

However, if we assumed that there is no randomness in the variable cost fraction, i.e. no volatility in the cost function, and by increasing the variable cost fraction from 92% to 95%, the firm's profit-margin changes from 4,9% to break-even in year 1. Likewise, the profit would have been break-even if the fixed cost increases from \$147 million to \$299 million. With these changes, the firm would have gone bankrupt respectively in year 23 and 11 with today's level of financing. This is of course explained by the model, but this is not likely to be a true scenario in the real world.

4.4.3 Volatilities

The initial volatility in the growth rate of revenue, η_0 , is as earlier mentioned the most crucial parameter in the valuation. Higher initial volatility means a higher variance in the distribution of growth rates, which in line affects the real option component and has significant effect on the valuation. Increasing this parameter from 17,1 % to 18,8 % results in 16,7 % higher firm value, an increase from \$ 32 084 to \$ 37 319 million dollars.

The other volatility parameters do not have that much effect as the above mentioned. Increasing the other initial parameters increases the distributions of revenues and variable cost fraction. This in line, of course make an impact on the distributions in the next period. Likewise, a

decrease shrinks the distributions. Changing the long-term volatilities does not have a large effect on the valuation.

4.4.4 Correlations and Half-Life of Deviations

Our result from the analysis shows that both correlations and Half-Life of Deviation has an insignificant effect on the share price.

4.4.5 Investments

We used the same method as Schwartz and Moon (2001) when we forecasted capital expenditures and depreciations. Some capitalized software expenses are included in Property, Plant and Equipment stated by Square, since U.S GAAP accounting rules allow some certain types of capitalizing. However, we suggest that one look closer into how research and development expenses are forecasted. Due to accounting principles these costs are measured fully as expenses in the year they accrue. For valuation purposes, it would be reasonable to capitalize these costs as well and forecast impairments and amortizations, since it's expected that these investments generate cashflows over multiple years in the future. Damodaran (1999) argues that research and development expenses should be treated as tax-deductible expenses and that they can have a significant impact on certain metrics such as operating income, expected growth and capital. Increasing R&D over time will result in higher return on capital and vice versa after reclassifying. This in line will affect the valuation result. R&D incentives vary between countries. For example, there are less or almost zero incentive in the US compared to EU-countries according to a report published by EY (2018).

4.4.6 Life expectancy and forecast horizon

The life expectancy of FinTechs has a great impact on the firm value, and it is hard to determine the appropriate lifetime of these companies. In this valuation we assumed a lifetime of 15 years. This is today's average lifetime of Fortune 500 firms, and it has decreased tremendously during the last decades. We suggest that the lifetime could be set to a lower quartile and still be justified for businesses within tech and start-ups due to competition and innovation.

Another fact is that in many cases there is a high probability that the firm either goes bankrupt, keep its business as it is, are being acquired or merges with another firm. All cases may have a value: 1) If the firm continues as it is, they are having the flexibility to expand in future, which can be seen as a real option. 2) If the firm is being acquired, the acquiring company might see possible synergies and need of technologies or they actually see the target company as a treat

and want to stop the innovation. 3) Merges are giving potential synergies, similar to acquisitions. The value from either of the cases might vary a lot.

As terminal value indicator we decided to use, an exit multiple of EV/EBITDA, the same approach as Schwartz and Moon (2001) did in their valuation of Internet companies. Mercer Capital reported that FinTechs in the segments Payments, Solutions and Technology are seeing valuation multiple EV/EBITDA of respectively 13.1, 13.7 and 16.2 in 2018 (Bose, Davis, McLeod, & Wilson, 2018). If we were to use Gordons Growth Formula, we would have seen a growth rate that was higher than the risk-free interest rate. We therefore used a terminal value multiple of 10 times EV/EBITDA, which is seen as a conservative multiple and in line with Schwartz and Moon (2001).

For future research we suggest that one look closer into how lifetime of a FinTech affect the value of the firm and try to find other suitable proxies for both lifespan and/or terminal value.

5 Conclusion

This paper's aim has been to study whether a real option model used to value Internet Companies was suitable to value FinTech companies. The valuation was conducted by applying a model published by Schwartz and Moon (2001), developed to value high growth Internet Companies since their valuation was thought to be irrationally high in the 1990s and beginning of 2000. We stated that FinTech Companies either are facing many similarities with Internet Companies, or that they are a sub sector of the IT sector which Internet Companies belongs to. FinTechs are requiring heavy investments in the start phase before they get a solid market share and potentially start to create value. There is a high likelihood that they are being disrupted by other innovative business models or new products and solutions. Another fact is that their platforms require a lot of Research & Development. Their platform needs to be, and is, scalable when the transaction quantity increases, which mean more investments when the customer base increases. The investments made - often R&D-expenses before the products/solutions are launched - to build up or expand a FinTech start-up/company is in many ways' intangibles, so it is difficult for FinTech Companies to obtain funding, since there is an absence of collateral in the development phase. The investments are in a way flexible since the firm can choose whether to expand, abandon or continue the investment in another or same manner. Compared to expansion, this is the same flexibility as in call option, where you have the right but not the obligation to exercise the option. Many FinTechs requires a large customer base before they are being profitable, since marginal revenues are low and fixed cost are high, but at the same time scalable. To obtain a large customer base, marketing and marketing cost is a significant part that does also increase the total costs and pushes profit margins down. All these facts about FinTechs share many of the same characteristics as Internet Companies in their early years.

FinTech's creation of value is mainly driven by their competitive advantage in the B2B- and consumer market, attached to their revenue's streams. More specific, how much profit they are able to generate per dollar of revenue. The growth rate of revenues has a major impact on the valuation result. We see a wider spread in the distribution of growth rate, in our simulations, after some year, before they tighten again. Same is true for the fraction of variable costs. This is in line with what one could expect in the real world, when one look at a start-up or a company that experience very high growth throughout the lifecycle of a successful product.

Our result from the valuation showed that we obtained a price of \$ 69,82 dollar per share for Square Inc. The closing price in the stock market on December 31, 2018 was \$ 56,09 dollar per

share. All callable employee stock options in our valuation were assumed exercised and the dilution is included in the above-mentioned share price. The real option approach estimated a share price 24 % above the closing price. The estimated share price was in a 10% range of the market price in only 365 out of 6808 simulated share prices⁸, that is 5 % of the cases. In 61 % of the cases, the estimated price was lower than the market price, and in the remaining 39 % the estimated share price was higher. This implies a negatively (or left) skewed distribution of simulated firm value and share price.

From the performance of analysing the probability of bankruptcy, our simulation result shows a bankruptcy probability of 29,5 % throughout the forecast period. The probability for bankruptcy is highest in and from year 3 until 6, respectively 7,1 % and down to 2,1 %, whereas it decreases slowly to 0 % in the terminal period, in year 15. For future research we do suggest that one look closer in to how the bankruptcy condition affects the valuation result. Further, the valuation is highly sensitive to the initial volatility of the expected growth of revenue growth. This is the valuation most sensitive parameter and that might pose difficulties when estimating this parameter. We do also suggest that one look more specific into another way to choose the forecast period and terminal value, since these parameters have a large effect on the total sum of cashflows. Especially, the terminal period.

However, our valuations show that the estimated share price with the real option approach is closer to the actual closing price in the market, than the share price obtained in a traditional Net Present Value DCF. This gives us evidence about the fact that it is suitable to use a real option approach for valuing Square Inc. The main reason behind this is that an NPV DCF understates the value of flexibility. Hence, the flexibility is seen as the real option part in our valuation. Our result is valid for Square Inc., so we suggest looking closer into how the result will be if one applies the model to other FinTechs in the same and other verticals.

⁸ This is the amount of estimated share prices were the company does not go bankrupt. 6808 out of 10000 simulation.

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APPENDICES

1 Abstract

Rational Pricing of Internet Companies Revisited

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Abstract

In this article we expand and improve the Internet company valuation model of Schwartz and Moon (2000) in numerous ways. By using techniques from real options theory and modern capital budgeting, the earlier paper demonstrated that uncertainty about key variables plays a major role in the valuation of high growth Internet companies. Presently, we make the model more realistic by providing for stochastic costs and future financing, and also by including capital expenditures and depreciation in the analysis.

Perhaps more importantly, we offer insights into the practical implementation the model. An important challenge to implementing the original model was estimating the various parameters of the model. Here, we improve the procedure by setting the speed of adjustment parameters equal to one another, by tying the implied half-life of the revenue growth process to analyst forecasts, and by inferring the risk-adjustment parameter from the observed beta of the company's stock price. We illustrate these extensions in a valuation of the company eBay.

Keywords: internet valuation; real options; asset pricing

JEL Classifications: G12

2 Cholesky Decomposition

```
Function CHOL(matrix As Range)

Dim i As Integer, j As Integer, k As Integer, N As Integer
Dim a() As Double 'the original matrix
Dim element As Double
Dim L_Lower() As Double

N = matrix.Columns.Count

ReDim a(1 To N, 1 To N)
ReDim L_Lower(1 To N, 1 To N)

For i = 1 To N
    For j = 1 To N
        a(i, j) = matrix(i, j).Value
        L_Lower(i, j) = 0
    Next j
Next i

For i = 1 To N
    For j = 1 To N
        element = a(i, j)
        For k = 1 To i - 1
            element = element - L_Lower(i, k) * L_Lower(j, k)
        Next k
        If i = j Then
            L_Lower(i, i) = Sqr(element)
        ElseIf i < j Then
            L_Lower(j, i) = element / L_Lower(i, i)
        End If
    Next j
Next i

CHOL = Application.WorksheetFunction.Transpose(L_Lower)

End Function
```