UNIVERSIDADE CATÓLICA PORTUGUESA

MDCF Valuations Models

Multi-factor Discounted Cash Flow Valuations

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Católica Porto Business School

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Multi-factor Discounted Cash Flow Valuations

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by

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As I could not stop greeting, thanks to all the authors hereby cited, for their contributions to the literature and for their philanthropic initiatives in the economical/financial sphere. Such personalities leave me with the feeling of Isaac Newton, or earlier, Bernard of Chartres: *We are like dwarfs on the shoulders of giants, so that we can see more than they, and things at a greater distance, not by virtue of any sharpness of sight on our part, or any physical distinction, but because we are carried high and raised up by their giant size."*.

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Resumo

Nos modelos de avaliação assentes em *cash flows* descontados (DCF), os fluxos de caixa são descontados a uma taxa, que representa o custo de oportunidade do capital. Nos primeiros trabalhos dos anos 60, Markowitz (1952), Treynor (1961, 1962), Sharpe (1964), Lintner (1965) e Mossin (1966) renomearam o CAPM como o modelo transversal de apuramento do retorno esperado dos capitais próprios. Paralelamente, as contribuições de Fama & French (1992, 1993, 2015) e Carhart (1992, 1995, 1997), não geraram mudanças significativas na incorporação do risco do investimento nos modelos de avaliação DCF.

Ao testar os primeiros modelos de avaliação de Fisher (1930) e Kaplan & Ruback (1994), com os modelos de risco de capitais próprios, CAPM, FF3, FFC e FF5, nas empresas mais maturas (1987 a 2017) do índice SP100, num total de 1364 avaliações, foi estatisticamente determinado que os modelos de avaliação multifatoriais de fluxos de caixa descontados (MDCF) superam os retornos transversais do CAPM para fins de avaliação empresarial, com um T-test com valores p inferiores ao nível convencional de 5%.

Infelizmente para os utilizadores destes modelos, as metodologias transversais não são tão precisas quanto as metodologias de avaliação históricas (HDCF) e, como tal, qualquer modelo pode superar um modelo transversal. Além do mais, as avaliações HDCF permitem a ponderação de erros, fornecendo valores de referência e correções para preços justos.

Palavras-chave: DCF, *cash flows* descontados, modelos de avaliação, avaliação de empresas, modelos multifatoriais, CAPM, FF3, FFC, FF5, avaliações históricas, avaliações transversais, preço justo.

(Trabalho escrito com 14134 palavras)

Abstract

DCF valuation models discount cash flows at a rate that reflects the risk of the investment, the opportunity cost of capital. The early works of the 60s by Markowitz (1952), Treynor (1961, 1962), Sharpe (1964), Lintner (1965) and Mossin (1966) renowned the CAPM as the cross-sectional equity return model. In parallel, since the contributions of Fama & French (1992, 1993, 2015) and Carhart (1992, 1995, 1997), there were no significant changes in the risk assessment of such valuation models.

By testing the early valuation models of Fisher (1930) and Kaplan & Ruback (1994), with the CAPM, FF3, FFC, and FF5 equity models, throughout the most mature companies (1987 to 2017) of the SP100 index, in a total of 1364 valuations, it was statistically proved that the Multi-factor Discounted Cash Flows (MDCF) valuation models outperform the cross-section of returns of the CAPM for valuation purposes, with a T-test p-value lower than the 5% conventional level.

Unfortunately for practitioners, cross-sectional methodologies are not as accurate as the Historical Discounted Cash Flow (HDCF) valuation methodologies, and as such, any given model can outperform a cross-sectional model. Furthermore, HDCF valuations allow for the input of errors, providing reference values and fair price corrections.

Keywords: DCF, Discounted Cash Flows, Valuation models, Firm Valuation, multi-factor models, CAPM, FF3, FFC, FF5, Historical valuations, cross-sectional valuations, price fairness.

(Written work with 14134 words)

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Abbreviations and Acronyms

3MDCF	3 Multi-factor Discounted Cash Flow Model
4MDCF	4 Multi-factor Discounted Cash Flow Model
5MDCF	5 Multi-factor Discounted Cash Flow Model
AI	Artificial Intelligence
APV	Adjusted Present Value
B	Big
B/M	Book to Market ratio
BE/ME	Book equity to Market equity ratio
CAPM	Capital Asset Pricing Model
CAPV	Compressed Adjusted Present Value
CEO	Chief Executive Officer
CF	Cash Flows
CFF	Cash Flows from Financing activities
CFI	Cash Flows from Investment activities
CFO	Cash Flows from Operating activities
CMA	Conservative minus Aggressive
C.OIL	Crude Oil
DCF	Discounted Cash Flows
FCF	Free Cash Flow

FF3	Fama & French 3 Factor model
FF4	Carhart 4 Factor model
FF5	Fama & French 5 Factor Model
FFC	Fama French Carhart 4 Factor Model
Fig	Figure
H	High
H3MDCF	Historical 3 Multi-factor Discounted Cash Flow Model
H4MDCF	Historical Multi-factor Discounted Cash Flow Model
HDCF	Historical Discounted Cash Flows
HML	High minus Low
IBOR	Interbank Offered Rates
i.e.	Id Est (Latin for that is)
IFRS	International Financial Reporting Standards
INV	Investments
L	Low
LSQ	Least Squares
M&A	Merges and Acquisitions
MAPE	Mean Absolute Percentage Error
MDCF	Multi-factor Discounted Cash Flow Model
MMC	Maximized Monte Carlo
N	Neutral

OIS	Overnight Index Swap
OP	Operations
P.P.	Percentual Points
PR1YR	1 Year Persistence
RM	Market Return
RMW	Robust minus Weak
S	Small
S&P100	Standards & Poor's 100 Index
S&P500	Standards & Poor's 500 Index
SCL	Security Characteristic Line
SMB	Small minus Big
SML	Security Market Line
SP100	Standards & Poor's 100 Index
SP500	Standards & Poor's 500 Index
T-Bills	Treasuries Bills
US	United States
WACC	Weighted Average Cost of Capital

Introduction

This work aims to improve on the discounted cash flows valuation methods through the incorporation of the most contemporaneous and accepted literature, such as incorporating the Fama & French (1993) 3-factor model, the Carhart's (1997) 4-factor model and the Fama & French (2015) 5-factor model. Despite the capital asset pricing (CAPM) being the most world-wide accepted method, there are no studies testing whether the market risk factor determined by CAPM is, in fact, the most efficient determinant of the companies' return on equity, underlying the opportunity cost of capital in a firm's valuation process.

In addition, this study aims to understand if there are reasons to prefer an equity return model over another, and how different return models behave in firm valuation, through time and company. Also, given the more complex factoring of the three, four and five-factor models it is important to understand whether such complexities are worth to be borne in a valuation process.

Multifactor models have been gathering more and more consensus in explaining the cross-section of returns. Nevertheless, this study is not only focussed on the cross-sectional results, as different markets can have different valuation preferences. The notion that different types of investors can cluster in different types of firms is also considered. As such, this study also addresses an investment portfolio that accounts for every investor type most likely observed in a given firm (HDCF model) versus the cross-sectional model.

The firm valuation methodology stands today as an incomplete procedure at the literature level. There are newer studies that have not been incorporated into the main valuation procedures. Incorporating and testing such newness in the discounted cash flow methods will allow for a more effective and unveiling pricing procedure - understanding investor types and market drivers allows for a financial environment inference, as Fama & French (1995) also mentions.

Through a historical projected forecast, from 1987 to 2017, on the SP100, the best cross-sectional model will be asserted, with the consideration of the Mean Absolute Percentage Error (MAPE) measure. In this study 3 hypotheses are raised: a multi-factor discounted cash flows model (MDCF) has lower MAPE than a plain DCF model; an MDCF model's extra variable (complexity) is significant; and the Historical DCF (HDCF) models have lower MAPE than cross-sectional MDCF models – concluding that hypothesis 1 and 3 are statistically significant at the conventional levels, and the hypothesis 2 does not hold for the 5MDCF model against the 3MDCF and the 4MDCF models, in the SP100.

The limitations on such a study *a priori* refer to the industrial characteristics of the conducted valuations. There is no individual determination of any parameters for any of the firms, as adjustments or judgments advised by Mckinsey (2015). Also, the cross-sectional results obtained are limited to the SP100 performance and may differ from market to market. So, the lack of idiosyncratic measures can bias some of the results presented, and a more traditional, individual approach to any of the 1364 valuations performed can imply different valuation prices.

In the first chapter, the literature will be reviewed, considering the background of the Discounted Cash Flow models and the equity return models. Chapters II and III will elaborate on what type of data was selected and the needed methodology to value a firm. Chapter IV underlies this study's methodology toward building the 3MDCF, the 4MDCF and the 5MDCF valuation models. The results and implications will be confronted in Chapter V, with special examples of different outputs from the models and their literature context. Finally, the conclusions will be presented in Chapter VI.

Chapter I
Literature Review

1.1. Background

The practice of discounted cash flows hitherto applied for financial purposes has its origins dating from the Old Babylonian period, between 1800-1600 B.C., with written knowledge on compound interests (Neugebauer, 1951). The first applications of the present value methodology were on loans and life insurance with historical probabilities being assessed. The valuation of fixed assets took developments from actuarial science, engineering economy and political economy (Parker, 1968).

Medieval Christian Church had banned usury and, until the XVI century, interest tables were highly confidential and existed only in manuscripts. In spite, the mathematical address on compound interests was being written from the Leonardo of Pisa (1202) onwards in European books of mathematics. Steven (1582) published the book *Tables of Interests* and introduced the concept of present value for deciding between two loans. Further on, an unpublished book by James Dodson (1756) investigated the principles of assurance companies and how to calculate risk premiums, and Price (1843) introduced bond tables and the concept of internal rate of return.

Non-financial investments were disregarded until the XVIV century for their small size and toughness in forecasting cash-flows (Parker 1968). Nearly a century ago, railways shaped the scope of investment analysis. Wellington (1887), Hoskold (1877) and Pennel (1914) introduced forecasting techniques and the concepts of rate of return and present value to capital expenditures on railways and mines. Goldman (1920) defined the present value of capital as a condition of the initial cost (C), the interest rate (r), the life of the investment (n), re-investments (m), and operating costs (a) as:

$$V'_{mn} = \frac{C\,[1-(1+r)^{-mn}]}{1-(1+r)^{-n}} + \frac{a\,[1-(1+r)^{-mn}]}{r}. \quad (1)$$

It was not until Political Economy described some investment axioms that we found the thus-far known concept and formulation of discounted cash-flows. Marshall (1907) mentioned the possibility to value an investment at any given time, by compounding past capital accumulation and discounting future capital flows. Furthermore, the author mentions that if cash-flows are not certain, a risk allowance must be made so that the value compensates for a given risk. These concepts are very familiar today, nevertheless, the author's observations are denoted. One special outlook is the mention that if a given future accumulated amount, instead of being discounted, can be invested, such investment returns cannot be counted as an additional return for the evaluated investment. Bohm-Bawerk (1903) also brought in contribution the reasoning for cash-flows to be discounted at the rate of the same year they are being generated. Fisher (1930) pointed out the present value of a single investment with a rate of return over cost different than the internal rate of return, which finally led Boulding (1949) to present the formula to calculate the internal rate of return (i) of an enterprise as in equation (2), for any series of known revenue (x) in n periods.

$$\text{Enterprise Value} = \frac{x_1}{(1+i)} + \frac{x_2}{(1+i)^2} + \frac{x_3}{(1+i)^3} \dots + \frac{x_n}{(1+i)^n} \quad (2)$$

The discounted cash-flow methodology, in the 50s, was concerned with defining the discount rate and understanding how capital expenditure rationality was attained – the accounting rate of return and the payback period were the most applied methods that far (Parker, 1968). A parallel study from Modigliani & Miller (1958) described how uncertain and volatile cash streams can be approached and how industries or peers are to be characterized. Furthermore, the acknowledgement of capital structure, financial risk premiums and expected return per share were drawn – much more similar to contemporaneous methodologies constraints.

The 1960s, with the affirmation of computational technology and econometric liberation from mechanical calculations, allowed big data analysis and massive developments in studies concerned with stock return behaviour (Fama, 2014). At such time, Fama (1965, 1970) was debating whether or not asset prices reflect all available information. The efficient markets hypothesis is still the benchmark for many pricing theories, but at the time, there was no asset pricing model. One, until today, untested implication comes from assuming that a time t set of information used in the market to price an asset at t (Θtm) (see equation (3)) is a permanent factor through time (Fama, 2014).

$$E\,(R_{t+1} \mid \Theta_{tm}) = E(R) \qquad (3)$$

As such, an era began, where on the one hand, the rate of return on equity was beginning to be documented for a particular investor's interest point of view (stock returns), and on the other hand, discounted cash-flow models were being optimized for firm valuation and project valuation purposes.

A major implication for discounted cash-flow models was introduced by Modigliani & Miller's (1963) paper, where the after-tax leveraged return was said to have less variability, according to higher leverage or higher marginal corporate tax rate. Also, tax-shields were considered as value bearing cash-flows, where the value of a leveraged company (V_L) is equal to the value created by the tax-shields (t_{DL}) plus the value of the unlevered company (V_U):

$$V_L = V_U + t_{DL} \qquad (4)$$

The consideration of interest tax-shields led Myers (1966, 1974) to suggest the adjusted present value (APV), a firm valuation model based on two components. First, the free cash-flows discounted value method, and second, the present value of the interest tax-shields, discounted at the cost of debt - Miles & Ezzell (1980) suggested that the tax-shields should be discounted at the cost of debt for the first year and at the cost of capital for the following years, and Harris and pringle (1985) suggest discounting tax-shields at the unlevered cost of capital.

Despite Miller (1977) arguing that, when accounting for the investor's personal taxation, tax benefits are diluted, Kaplan & Ruback (1994) also came up with a model, proposed as the Compressed APV – as suggested by Myers. The Compressed APV discounts the capital cash-flows (free cash-flows plus the interest tax-shields) at the discount rate for an all-equity firm. The use of an unlevered discount rate allows for a simplified procedure since the WACC calculation demands yearly inputs for changes in capital structure.

Fernandez (2004) raised questions for when combining cash-flows and tax-shield flows since both have different levels of risk and are being combined. Following the methodology of capital cash-flow's valuation, Fernandez proposed discounting them at the $WACC_{BT}$:

$$WACC_{BT} = \frac{(EK_e + DK_d)}{(E+D)} \quad (5)$$

Where E stands for equity, D for debt, K_e stands for (6), K_d stands for the debt required return, K_u is the unlevered cost of equity and T is the corporate tax rate.

$$K_e = K_u + [(^D/_E) * (1\text{-}T) * (K_u - K_d)] \quad (6)$$

Scientific work towards discounted cash-flows is building upon the approximation to the transaction value of companies with many different characteristics. Such heterogeneity has been blocking a universal explanation of the cross-sectional results to define the better model. The joint hypothesis (Fama, 1970) reports that whether the market is inefficient, or whether the asset pricing model is insufficiently explanatory of reality, is inconclusive, still. Such impediments and a vast literature on stock returns (i.e., Return to Equity) set a stand on discounted cash-flow valuations.

1.2. The CAPM

Back into the 60s, the portfolio choice theory of Markowitz (1952) – with investor's rationality based on the average returns and standard deviation as the risk measure – led to the introduction of CAPM by the unpublished studies of Treynor (1961, 1962) and Sharpe (1964), Lintner (1965) and Mossin (1966). It was Sharpe (1964) that contributed with a risk measure based on a single index (the market index). Evaluating a security, or a portfolio's risk based on the market risk, is a much more straightforward methodology, but the risk measure evaluated is always systematic. The risk that a given company translates to a portfolio (the idiosyncratic risk) is not inferred by CAPM. Assuming investors have diversified portfolios, the return they expect is equal to the risk-free asset return, plus a market premium based on the market risk (beta), as follows:

$$R_e = R_f + Beta * Risk\ Premium \quad (7)$$

Black (1972) reports that there is no risk-free security. The introduction of the zero-beta portfolio as a risk-free measure allowed for a better explanation of the cross-section of stock returns. However, CAPM and the efficient market hypothesis became questioned in the late 80s, when Banz (1981) and Rosenberg et. al. (1985) reported that smaller firms were outputting higher risk-adjusted returns and the book-to-market ratio had as well a positive relation with investment returns.

Fama and French (1992, 1993) assessed all similar reports that handicap CAPM, extended the test for an integrated market, with bonds, and formulated the 3-factor model (FF3) extension on CAPM. The lower earnings from firms with higher Book equity to Market equity - BE/ME ratio - (undervalued in the market) are reported to persist for at least five years before, and five years after the measurement - and vice-versa. The size factor, also related to profitability, indicated smaller earnings on assets for smaller firms and proneness to long earnings depressions – smaller firms did not enjoy the economic boom of the late 1980s. The FF3 model considers the market risk premium (the excess market return), the size risk premium (the difference in returns between small and big firms with the same weighted average of BE/ME) and the book-to-market risk premium (the difference between the simple average of returns from the high and the low BE/ME groups with the same weighted average firm size).

When investment funds were analysed for their knowledge outperformance in stock picking strategies, Carhart (1997) found that the transaction costs dilute momentum strategies' gains in stock markets. However, the author's 4-factor model does better explain the cross-sectional variations described by Fama & French (1996). Carhart's 4-factor model combines the Fama & French 3-factor model with the Jegadeesh & Titman's (1993) one-year momentum anomaly so that the momentum risk premium is equal to the average returns of the 30^{th} higher group earners (for 11 months returns lagged 1 month) minus the lower 30^{th}. Nevertheless, investment costs, transaction costs and load fees are, still highly relevant features in investment fund performance.

The latest most accepted model comes from Fama & French's (2015) 5-factor model paper. Besides the investors' concerns with the market, the size and the book-to-market ratio, profitability and investment patterns are also risk premia that explain the cross-section of returns. When accounting for those factors, the book-to-market ratio becomes redundant and can be substituted for the momentum factor concerns of investors. The four-factor model without the B/M risk premium, the 5-factor model, and the five-factor model with momentum premium are all described as better models (Fama & French, 2015). At this time point, various hypotheses for portfolio dimension and selection are specified, and special concerns with such impacts on bias generation are documented.

Harvey et. al. (2016) compiled all the factors studied from 1967 to 2012. From the 316 factors that were studied thus far, only 50 remained as possible explanators of the cross-section of stock returns, which also must be weighted with McLean & Pontiff's (2015) observation, that the publishing of a study on a given stock market anomaly, makes that anomaly less anomalous and more prone to be valued by the market. As such literature ends with no definite disclosure, it comes to pair the discounted cash-flows methodologies of the XXI century.

The need for a cross-sectional model makes the scientific field believe that the CAPM is irrevocably suited for international firm valuation, which holds since the moment of its appearance. The further developments in DCF models, such as the incorporation of the effect of depreciation on the continuum value, introduced by Jennergren (2008), only focus on the FCF forecasting parameters. The author's riskiness reflection is not based on the investor risk behaviour, but rather on the growth of capital expenditures (c), as shown in equation (8):

$$\text{Continuum value} = \frac{FCF_{t+1}}{(WACC-c)} \quad (8)$$

Interdependency problems also exist on the other edge of the literature, with studies such as the ones from Döring, S. et al. (2021) Brochet, F. et al. (2022) and Naffa & Fain (2022), that reveal a focus on how the projects' investment horizons, the CEO's tenures and, most recently, market concerns of sustainable practices (ESG), impact stock returns but not firm valuation.

Empirically, the optimal association between Discounted Cash Flow models and investor risk models, that explains the required/expected equity return, has not yet been performed[2].

[2] Roll (1986) was concerned with miss-pricing in merges and acquisitions (M&A) - "*such bids are likely to represent positive errors in valuation*" – but never questioned if the valuation models were themselves miss-pricing assets, but rather focussed on the synergies' problematics.

Chapter II
Methodology - Data

2.1. Cash Flows

Stone (1972) introduced forecasting methodologies in corporate cash management systems[3], intentioned to smooth such variable. Forecasting ability was thereafter linked with cost-saving measures by the maximization of resource allocation. This led to many linear and non-linear model proposals within cash flow forecasting. Stone & Miller (1987) accounted for the different statistical properties of cash flow components and stated that major cash flows, such as in loans, that have properties known through time and amount, should be forecasted differently than non-major cash flows, which are all transactions that do not have such properties.

Cash Flow Management systems (for the purposes of firm valuation) allow valuators to understand the timing and certainty of firms' Cash Flows. By knowing how cash is being raised and used, valuators can perceive how agents are allocating resources and how sustainable firm policies are (Heider & Heyler, 2003)

Despite recognising such endeavours, for firm valuation purposes, the aim is to estimate the Free Cash Flows – the cash flows that are available to remunerate all investors (equity and non-equity), according to Mckinsey (2015). In such aspect, considering Stone & Miller's (1987) study, it is necessary to calculate the Cash Flows from Operating Activities[4], the Cash Flows from Investment

[3] See Appendix 1 for a deeper understanding of what affects cash flows.
[4] Cash Flows from Operating Activities are calculated based on the assumption of the Indirect Method of report.

Activities and the Cash Flows from Financing Activities, discussed further in the next section.

2.1.1. Cash Flows from Operating Activities (CFO)

These cash flows correspond to the cash movements within regular business operations, usually product-related corporate functioning, or more contemporaneously service-oriented operations. The Firm's operations value derives from its Net Income[5], Depreciation & Amortization, Deferred Taxes, Non-Cash Items and Changes in Working Capital – this and all the following data was retrieved from Thomson Reuters Database, from 1987 to 2021, regarding the SP100's firms with available information for the time-period.

Managing the Cash Flows from Operating Activities (for the purposes of firm valuation) allow valuators to know the firms' ability to maintain the current operations, the capacity to invest without having to bear financing costs, and to share profits and repay obligations, according to Reider & Heyler (2003).

2.1.2. Cash Flows from Investing Activities (CFI)

The Cash Flows from Investing Activities reflect, in terms of firm value, the proportion that accounts for the generation/expense of cash in investment projects carried out by the business. Firm valuation accounts for the capital expenditures and the other investing cash flow items the firm holds, such as the acquisition of businesses.

Managing the Cash Flows from Investing Activities (for the purposes of firm valuation) allow valuators to know how cash applications are contributing to the future enhancement of the firm (Reider & Heyler, 2003).

[5] Net Income in Cash Flow Statements refer to income before extraordinary items.

2.1.3 Cash Flows from Financing Activities (CFF)

Such cash flows refer to how much cash has been usually outflowing (or sometimes inflowing) from the business financing functioning, allowing company value to reflect on the businesses' financing, dividend, and debt policies. Data-wise, it corresponds to its financing Cash Flow Items, Total Cash, Dividends Paid, Issuance/Retirement of Stock and Issuance/Retirement of Debt.

Managing the Cash Flows from Financing Activities (for the purpose of firm valuation) allow valuators to know what the cash flow claims from future capital providers are (Reider & Heyler, 2003).

2.1.4 Exchange rate effects

Globalization, virtualization, and its trend-like behaviour in the corporate sphere make exchange rate accountability much heavier on firms' total value. International Firms' cash-flows are not quoted by the same currency and, therefore, it is prevailingly demanded to calculate the net change in cash, capturing the cash-flows from exchange conversion of cash and cash equivalents. The Net Cash at the beginning of the period must be discounted from the Cash Interests and Taxes, and its final period Net Cash value.

2.1.5. Free Cash Flows

The Cash Flows available to remunerate all corporate investors are the sum between the Operating Activities CF and the Financing Activities CF, discounted by the business' Cash Flows from Investment Activities.

Richardson (2006), Harford (1999) and Lang et al. (1991), brought many contributions for reflecting on how we can imply whether or not a firm has good investment opportunities, if the firm is likely to have higher agency costs based on its access to cheap capital (implying the firm does not incur in Stewardship Theory), if the firm is overinvesting, and how the firm's corporate governance allows to mitigate overinvestment. These studies are implied by Free Cash Flow regressions and estimations.

2.2. Risk-Free rate

Black's (1972) & Brennan's (1971) studies refer to the theoretical framework for which risk-free is not considered as standard. In such cases, where there are borrowing restrictions or less risky assets than Treasury Bill rates, the risk-free rate should account for the excess return between the condition and the Zero Beta Portfolio.

The Zero Beta Portfolio can be estimated using the classical multivariable linear regression, according to Gibbons (1982), Shanken (1986) and MacKinlay (1987). Black et al. (1972) and Fama & MacBeth (1973) stated such procedure as an estimation by the time series regression of a Security. Finding Market Beta and using a cross-sectional regression on such beta to compute the risk-free - Zero Beta Portfolio.

The Zero Beta Portfolio can be proxied by the rates of Long-term Government Default-Free bonds, according to McKinsey (2015) – the United States of America and Western Europe (Germany) have extremely low betas. Furthermore, the corporate native currency should match the risk-free assets so that the inflation rate is consistent amongst the corporate cash flows and the discount rate.

2.2.1. Zero Beta Portfolio

The classical multivariate regression departs on the consideration of R_t, t = 1, ... , T, as being the return on a firm in period t, and R_{mt} as being the market return on period t and the equations associated with the regressions of R_t on R_m and a constant, on a portfolio model (where the individual-equation disturbances are heteroskedastic and contemporaneously cross-correlated). The Zero Beta Portfolio return equation is written by taking the Slopes (betas: β_i) and interceptions (alphas: a_i) and the equilibrium beta relations so that:

$$\alpha_i - \gamma(1 - \beta_i) = 0 \tag{9}$$

Where the scalar γ represents the Zero Beta Portfolio return[6] and i = 1, …, n companies.

The problem with estimating a Zero Beta Portfolio arises from Dimensionality, Portfolio Repacking and Identification[7]. Beaulieu et. al. (2013) refurbished the statistical test procedures so that 1) there is better testing efficiency and 2) there is a confidence set reliable for the Zero Beta Portfolio – where Maximized Monte Carlo (MMC)[8] test procedures are used with non-linear models with non-Gaussian errors.

2.2.2. Government Default-Free Bonds

The Government Default-Free Bonds can be used as a proxy for the Zero Beta Portfolio return. Mckinsey (2015) and Damodaran (2002), assert that this

[6] See Black (1972) and Brennan (1971) for defining the Risk-Free rate.

[7] See Marie et al. (2012) studies for defining Dimentionality, Portfolio repacking and Identification's statistical characteristics.

[8] See Dufour (2006)

methodology is robust as long as the maturity and currency of the Government Default-Free Bonds match the Cash Flows. As an example, a Cash Flow ($) generated in year 5 should be discounted using a 5-year Zero Coupon Bond ($).

The year of 2022 has been particularly known for its high inflation rates worldwide, according to The Financial Times. In such cases, where inflation is high or unstable, both the cash flows and the risk-free rate must be acknowledged by their real value rather than through nominal terms[9]. Damodaran (2002) refers that if capital can flow freely to the highest real returns' economies, then the inflation-indexed treasury is a worldwide proxy for real returns. Otherwise, the real growth rate (long term) of the domestic economy, equals the expected real return of such an economy.

2.2.3. Other Proxies

Some economies do not have default-free bonds to proxy for the Zero Beta Portfolio. In such cases, Damodaran (2002), suggests the usage of other reasonable estimations of the risk-free rate:

- By using the safest and largest firms' Long-Term Borrowing rate and subtracting 0,5% (AA Default Spread proxy).
- By using a dollar-denominated forward contract on the local currency and deriving the local borrowing rate with the interest rate parity and the treasury bond rate[10]
- By adjusting the local Government Bond by the estimated default spread on the Bond.

Other studies such as the one provided by Zhen et.al (2022), find that Gold, T-bills, Overnight Index Swaps (OIS) and Interbank Offered Rates (IBOR) can

[9] See McKinsey (2015) for how to forecast Cash Flows in unstable/high inflation rate periods.
[10] See Damodaran (2002) for the local borrowing rate formula.

also be good proxies for the Risk-Free rate, depending on which markets are addressed.

2.3. Market Return

According to Mckinsey (2015), Market Returns are expressed both in real and nominal value. The US' real Market Returns average 7%[11]. The nominal market return is calculated as the logarithmic difference between the yearly closing prices of the S&P500 for this study.

Merton (1980) indicates that the usage of the historical realized returns comes with some biases that one must account for. The fact that the expected return of the market must be higher than the riskless rate, and that the nominal rate depends on inflation, means that realized returns are biased and do not always correspond to the equilibrium conditions of the asset pricing model.

2.4. Beta

Beta is an estimated measure that refers to the risk a given investment has. When Beta equals 1 the investment risk is equal to the market risk. The Historical Beta is estimated by regressing the company's log returns (Ri) on the market's (S&P500) log returns (Rm), as follows in equation (10).

$$R_i = \alpha + \beta R_m \tag{10}$$

Where a is the interception of the regression and Beta is the slope. The analysis of the interception's value allows to infer whether the firm has

[11] This value can be used to estimate the real Market Risk Premium on high/ unstable inflation periods by subtracting the inflation-indexed US bond, McKinsey (2015).

over/underperformed its risk measure (Damodaran, 2002). Furthermore, the R squared (R^2) allows us to know how much risk is weighted to the market and how much risk is idiosyncratic, and the estimation's standard error measure can be used to compute confidence intervals for the "true" Beta values (Damodaran, 2002).

Both Damodaran (2002) and Mckinsey (2015), talk about three estimation characteristics that should be addressed. The measurement period can vary from Beta data providers. Black et.al (1972) and Alexander & Chervany (1980) find ideal measurements for 5-year periods; nevertheless, the estimation period must consider whether the risk characteristics of the firm are changing, such as capital structure or corporate strategy (Mckinsey, 2015). One other aspect of calculating Beta is the observations interval. Dimson and Scholes et.al. (1979, 1977), find that weekly or monthly observations reduce the biases of non-trading days. More contemporaneous studies, such as the ones of Chen & Reeves (2012) and Bollerslev & Zhang (2003), describe how filters on daily data and 5-minute returns can generate better accuracy than the latter. The last aspect regards the market benchmark against which the returns will be regressed. The CAPM assumes the market as all existing assets – stocks, bonds, private equity and human assets. Mckinsey (2015) finds that the SP500 is a sufficiently diversified portfolio that is a good, and most commonly used, proxy – The S&P500 correlation is 0.958 against MSCI World Index, meaning their returns change at a practically identical rate.

2.5. Total Debt

According to the International Financial Reporting Standards (IFRS), the total debt represents all interest-bearing debt outstanding of a firm. It is normally[12] accounted as the sum of: Notes Payable/ Short-Term Debt – the notes payable, the short-term borrowings, demand notes payable, revolving credit (short-term), interest-bearing short-term payables to related parties, short-term loans from stakeholders and any other short-term interest-bearing liabilities; Current Portion of Long-Term Debt/Capital Leases – represent the portion of Long-Term Debt or Capitalized Lease Obligations to be paid within one year; Long-Term Debt – (Long-term) bonds, debentures, bank borrowings, notes payable, mortgage loans, senior debt, bills of exchange, interest-bearing borrowings from stakeholders and any other interest-bearing liability; And Capital Lease Obligations – the portion of capitalized lease obligations to be repaid in over one year.

The computation of an enterprise Discounted Cash Flows valuation leads preliminarily to valuations based on the free cash flows, the capital available to remunerate all equity and debt holders. While Mckinsey (2015) uses a net debt approach, subtracting the value of cash & cash equivalents and other non-operating assets, such as non-consolidated assets, to make the value debt-free from any shareholders' stake, Damodaran (2002) says that the use of gross debt-to-value ratios and a later subtraction of total debt must bear identical outcomes. Furthermore, the use of net debt-to-value ratios requires, according to Damodaran, an ulterior computation of the net debt cost, which differs from the cost of debt (from total debt).

[12] Normally – on non-financial Firms.

2.6. Corporate Tax rate

Corporate Tax rates, according to Mckinsey (2015), must refer to the marginal tax rate. The marginal tax rate accounts for the realizable rate of its referred period. The estimation of the marginal tax rate can be done through simulation. Graham (1996) stated that investment-grade companies' rate can be proxied by the statutory tax rate, and that typical companies' rate is on average 5 p.p. bellow the statutory rate, since they do not always use in full tax shields -the data collected refers to the realizable tax rate - the sum of current and deferred tax rates (Domestic, Foreign, Local and Other).

Chapter III
Methodology – Assumptions

3.1. Return on Equity

3.1.1. Capital Asset Pricing Model

A consensus between Treynor's (1961, 1962), Sharpe's (1964), Lintner's (1965) and Mossin's (1966) papers is the notion that the return of an individual security is a result of its systematic risk. Systematic risk, as stands, can be measured through the single index model proposed by Sharpe (1964)[13]. As such, the Security Market Line (SML) estimates the return of a given security (or portfolio) in terms of the risk-free rate and the systematic risk of the security.

CAPM as an *ex-ante* model, which aims to estimate a future return, is given by equation (11). The *ex-post* model proposed by Jensen (1968) in equation (12), is an expression of the Security Characteristic Line (SCL), where the excess returns of the security are related to the excess returns of the single index. Even though historical valuations are being performed, the goal is to simulate a forecast and therefore the methodology followed is the one proposed in equations (11) and (11.1).

$$E(R_i) = R_f + \beta_{im} [E(R_m) - R_f] + \varepsilon_i \text{[14]} \qquad (11)$$

$$\beta_{im} = \frac{\sigma_i \cdot \rho_{(i,m)}}{\sigma_m} \qquad (11.1)$$

[13] Jones (2012) describes the common market return index as any dominant influent on returns, despite being a stock index or not.
[14] Bai & Ng (2002) define εi as the idiosyncratic component of E(Ri).

$$R_{it} - R_{ft} = \alpha_i + \beta_{im} (R_{mt} - R_{ft}) + u_{it} \quad (12)^{15}$$

Where R_i is the return of security i, R_f is the risk-free rate, β_{im} is the systematic risk of the security i in relation to the market m and R_m is the market return.

3.1.2. Fama & French's 3-Factor Model

Fama & French (1992,1993) found evidence that the companies' size and book-to-market ratio do explain much of the abnormal returns of CAPM. Fama & French (1995) elaborated on a portfolio arrangement for further economical comprehension. The size factor is divided in two groups: Small and Big (SMB), according to the 50th percentile of the market equity (the stock value times the shares outstanding). The book-to-market factor is divided into three groups: High, Medium and Low (HML), according to the first 30%, the last 30% and the in-between 40% (through the book common equity divided by the market equity at the end of the period t-1)[16]. The weighted average returns of SMB and HML are computed in equation (13) and the coefficients are obtained by a linear LSQ regression (the regression procedure is explained in Chapter II – Beta).

$$R_i - R_f = \alpha + \beta_1 * (R_m - R_f) + \beta_2 * SMB + \beta_3 * HML + \varepsilon_i \quad (13)$$

Where α is the "Jensen's alpha"[17], β_1 is the market risk coefficient, β_2 is the size coefficient, β_3 is the B/M ratio coefficient and ε_i is the idiosyncratic risk of security i - the Fama & French 3 factors from 1987 to 2021 were disclosed by the Kenneth French Data Library.

[15] See Jensen (1968).

[16] The different combinations of size and book-to-market ratio performances allow valuators to infer on future business conditions (Fama & French, 1995).

[17] See Jensen (1968).

3.1.3. Carhart's 4-Factor Model

Deriving from Carhart (1992, 1995, 1997), the momentum factor has been included in the most known Fama & French 3-factor model. Similarly to the motivation for the latter CAPM extension, Carhart's four-factor model is the result of anomalous cross-sectional returns. The momentum factor was originally developed as the one-year momentum in stock returns (PR1YR) – it is the result of the weighted average of the 30th percent highest returns minus de weighted average of the 30th percent lowest returns (both lagged one month) of the portfolio. The equation follows the CAPM structure as shown in (14).

$$R_i - R_f = \alpha + \beta_1 * (R_m - R_f) + \beta_2 * SMB + \beta_3 * HML + \beta_4 * PR1YR + \varepsilon_i \quad (14)$$

The analysis of the momentum factor can also be useful for an empirical understanding of the scenarios. Charhart (1997) described the expected performance of past winners, rankings, rank progression, and momentum loadings (that can be seen as trend-following investments for firm valuation purposes). Furthermore, the observable returns can be linked to the firm's sensitivity to the factors – such sensitivity is said to be explained by the amount of liabilities and market leverage a firm has[18].

The Carhart 4 factors from 1987 to 2021 were disclosed by the Kenneth French Data Library and afterward LSQ regressions were made to determine the factors' coefficients.

[18] See Subhrendu et al. (2015) for more detail on which components may boost or decay a firm's sensibility to size, book-to-market ratio and momentum factors.

3.1.4. Fama & French's 5-Factor Model

The Fama & French five-factor model (FF5) arose from the early studies of Miller and Modigliani (1961) - the market value of a firm, at the time, was described by the well-known Dividend Discount Model, and such rationality allowed for the theoretical understanding and application of the effects of earnings and book equity (investment) in stock returns, which led to the five-factor model arrangement (Fama & French, 2015).

The determination of the factors is more complex in the FF5 model than in the FF3 model, where a 2x3 portfolio sort considers 2 Size groups, 3 B/M (book to market), 3 OP (operating profitability) and 3 INV (investment) groups. Size is sorted by the first percentile 50 (small, S) and the remaining (big, B), B/M, OP and INV groups are sorted by the lowest 30% (L) the highest 30% (H) and the neutral group between (N). The weighted averages for the factors' determination are described in Table 1.

Table 1: Factors and their components, Fama & French (2015)

$$SMB_{B/M} = (SH + SN + SL)/3 - (BH + BN + BL)/3$$
$$SMB_{OP} = (SR + SN + SW)/3 - (BR + BN + BW)/3$$
$$SMB_{Inv} = (SC + SN + SA)/3 - (BC + BN + BA)/3$$
$$SMB = (SMB_{B/M} + SMB_{OP} + SMB_{Inv})/3$$
$$HML = (SH + BH)/2 - (SL + BL)/2 = [(SH - SL) + (BH - BL)]/2$$
$$RMW = (SR + BR)/2 - (SW + BW)/2 = [(SR - SW) + (BR - BW)]/2$$
$$CMA = (SC + BC)/2 - (SA + BA)/2 = [(SC - SA) + (BC - BA)]/2$$

Fama & French (2015) argued about miss accuracy issues caused by the lack of diversification of some sorted portfolios due to their group segmentation (e.g., 2x2 *vs* 2x3 portfolios), and how dropping the HML factor[19] helps to diversify

[19] See Fama & French (2015) about the redundancy of HML factor when combining Profitability and Investment patterns.

portfolios that would account for momentum as well. The "2x3" 5 factors from 1987 to 2021 were disclosed by the Kenneth French Data Library and LSQ regressions were made to determine the Beta coefficients in equation (15).

$$R_i - R_f = a + \beta_1 * (R_m - R_f) + \beta_2 * SMB + \beta_3 * HML + \\ + \beta_4 * RMW + \beta_5 * CMA + \varepsilon_i \quad (15)$$

3.2. Cost of Debt

The cost of debt is an estimated parameter that proxies the cost of obtaining external funds to finance projects. The investment grade firms have easier access to the bond market, and since they operate with lower interest rates than those of banking services, the firm's long-term bonds' yield to maturity is a good proxy for the amount of capital being repaid to debtholders (McKinsey, 2015). Smaller, unrated, and below-investment-grade companies[20] rely the most on the banking system and their cost of debt is better proxied by analysing recent borrowing activity or by integrating a synthetic rating methodology (Damodaran, 2002).

Overall, Damodaran (2002) estimates the cost of debt as the sum of a riskless rate of return, the default spread and the tax benefits (marginal tax shields)[21]. The default spread stood as the conversion rate (Table 2) for the companies' Credit Combined Implied Rating available in the Refinitiv database. By following rating agencies' - such as Moody's, Standard & Poor's, or Flitch's - ratings, similar estimations could be derived.

[20] Firms that do not have liquid bonds also fall within this category (Damodaran, 2002).
[21] The cost of debt is recorded as the after-tax cost of debt.

Table 2: Synthetic and Rating spread, Damodaran Database

>	≤ *to*	*Rating is*	*Spread is*
-100000	0.199999	D2/D	20.00%
0.2	0.649999	C2/C	17.50%
0.65	0.799999	Ca2/CC	15.78%
0.8	1.249999	Caa/CCC	11.57%
1.25	1.499999	B3/B-	7.37%
1.5	1.749999	B2/B	5.26%
1.75	1.999999	B1/B+	4.55%
2	2.2499999	Ba2/BB	3.13%
2.25	2.49999	Ba1/BB+	2.42%
2.5	2.999999	Baa2/BBB	2.00%
3	4.249999	A3/A-	1.62%
4.25	5.499999	A2/A	1.42%
5.5	6.499999	A1/A+	1.23%
6.5	8.499999	Aa2/AA	0.85%
8.50	100000	Aaa/AAA	0.69%

Chapter IV
Methodology – Modelling

4.1. Discounted Cash Flow Valuation Models

The value of an asset that generates future cash flows is measured by the present value of such cash flows (Damodaran, 2002). Fisher's (1930) equation serves as the standard discounting model procedure to value a firm, as presented in equation (16). The most common method to discount future cash flows is to account for the expected cash flows and discount them back at a rate that reflects their risk (i.e., the opportunity cost). Alternatively, one can discount cash flows that are the certain equivalents, or guaranteed cash flows at the risk-free rate, as in equation (17) (Damodaran, 2010).

$$\text{Value of asset} = \frac{E(CF_1)}{(1+r)} + \frac{E(CF_3)}{(1+\text{r})^2} + \frac{E(CF_3)}{(1+\text{r})^3} \ldots + \frac{E(CF_n)}{(1+\text{r})^n} \tag{16}$$

$$\text{Value of asset} = \frac{CE(CF_1)}{(1+rf)} + \frac{CE(CF_2)}{\left(1+r_f\right)^2} + \frac{CE(CF_3)}{\left(1+r_f\right)^3} \ldots + \frac{CE(CF_n)}{\left(1+r_f\right)^n} \tag{17}$$

Firms' longevity is far beyond computable reason and therefore, after a 5 to 10 years forecast[22], future cash flows are computed for their residual/terminal value, as in equation (18), which can be done through three different approaches, according to Damodaran (2002). A stable future growth rate can be solved through a perpetual growth model. A multiples' model can also be applied to

[22] The 5-year cash-flow forecast returned the most adjusted market returns for the methodology in this study. See Casanova (1998) for different business cycle definitions (between 1-10 years).

estimate the terminal value. Or, as in this study, it is assumed that the terminal value is the liquidation value[23].

$$\text{Value of asset} = \sum_{t=1}^{t=n} \frac{E(CF_t)}{(1+r)^t} + \frac{E(terminal\ value)^n}{(1+r)^n} \quad (18)$$

4.1.1. Free Cash Flow Model

The Free Cash Flow Model discounts the free cash-flows (cash-flows available to remunerate all equity and debt holders) at the respective riskiness rate, as in equation (19). The discount rate is calculated as the cost of capital, as in equation (20), where the return on equity (R_e) is calculated as the CAPM's return, D0 represents total debt at period 0, and the capital structure is the average of the next five years capital structure[24].

$$\text{Value of company} = \sum_{t=1}^{t=n} \frac{E(CF_t)}{(1+WACC_t)^t} + \frac{E(terminal\ value)^n}{(1+WACC_n)^n} - D_0 \quad (19)$$

$$WACC = R_e * \text{Equity capital structure} + R_d * \text{Debt capital structure} \quad (20)$$

4.1.2. Compressed APV

The Kaplan & Ruback (1994) model implies that the free cash-flows and the interest tax-shields are combined and discounted at the same time-period rate with regards to the period's managed capital structure, as follows in equation

[23] The liquidation value is assumed to be the average of the next 5 years' market price – See Damodaran (2002) to estimate the liquidation value.

[24] Target capital structures account for the needed funding, flexibility and robustness. Target capital structure can be proxied by peer group comparison, credit-rating analysis and cash-flow analysis (McKinsey, 2015). Acquisitions and cash surpluses/deficits also explain deviations from target capital structures (Byoun (2008) and Uysal (2011)). Furthermore, according to Mckinsey (2015), these are the market-value capital structures.

(12). Cornell & Green (1991) proposed the methodology to calculate funding betas. In both papers, Dimson's (1979) Beta adjustments to Equity Beta (Be), Preferred stock Beta (Bp) and Debt Beta (Bd) in equation (21) are applied[25].

$$B_u = \frac{[Be * E + Bp * P + Bd * D]}{[E + P + D * (1-T)]} \tag{21}$$

So that $R_e = R_u$

$$R_u = \alpha + \beta_u * (R_m - R_f) \tag{21.1}$$

$$\text{Value of company} = \sum_{t=1}^{t=n} \frac{E(CF_t)}{(1+WACC_t)^t} + \frac{E(terminal\ value)^n}{(1+WACC_n)^n} - D_0 \tag{22}$$

4.2. Multi-factor Discounted Cash Flow Valuation Models

4.2.1. 3 Multi-factor Discounted Cash Flow Model

The 3 Multi-Factor discounted cash flow model assumes that investors need a risk premium higher (lower) given the Size and B/M ratio of a given firm (Fama & French (1992,1993)). This method is an extension of the Free Cash-Flow Model. It accounts for the free cash flows generated by the firm and discounts them at a WACC built with the return to equity of such type of investors, described in equation (13).

4.2.2. 4 Multifactor Discounted Cash Flow Model

The 4 Multi-Factor discounted cash-flow model is also an extension to both the Free cash-flow model and the 3 multi-factor model. Some investors require higher (lower) risk premiums for companies that have persistent negative

[25] The SP100 companies were proxied to be riskless investments for debt holders in this study – following Damodaran's (2002b) suggestion on equity holders as the only risk bearers.

(positive) returns (Carhart, 1997). So, the discounted WACC is built with consideration of the return of investors described in equation (14).

4.2.3. 5 Multifactor Discounted Cash Flow Model

The last multi-factor model in this study, the 5MDFC model, is an extension of both the FCF model and the 3MDCF model. For investors concerned with Size, B/M ratio, Profitability and Investment policy (Fama & French, 2015), the free cash-flows are discounted at a WACC built with the returns shown in equation (15).

Chapter V
Results & Implications

The most important aspect of this study is that it aims for an unbiased scenario. Throughout industries and economical events (the ".com" bubble in 2000, or the financial crisis of 2008), there is no adjustment to the methodology. All the results are a function of a change in the equity's required rate of return. The performance metric of the models is assessed by computing the Mean Absolute Percentage Errors (MAPE). Nevertheless, the standard deviations of the errors, the R^2, and the maximum likelihood estimations are also presented.

This study presents clear arguments against the cross-section of returns. Despite the fact that the multi-factor models perform better in the overall statements, any model can provide a better forecast in a given firm valuation.

Figure 1: Bristol Valuation – Forecasted Simulation

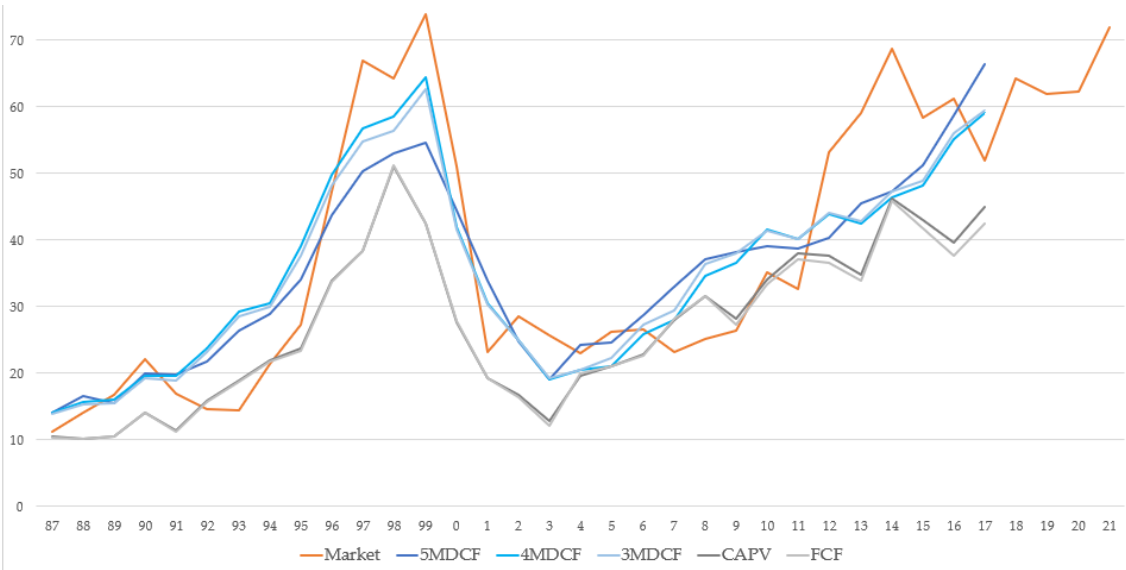


Table 3: Bristol Valuation – Forecasted Simulation data description

5MDCF	4MDCF	3MDCF	CAPV	FCF
Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
18,91%	18,06%	17,95%	20,52%	20,28%
0,17075	0,19831	0,19029	0,13383	0,13847

Figure 2: Bristol Valuation – Historical data

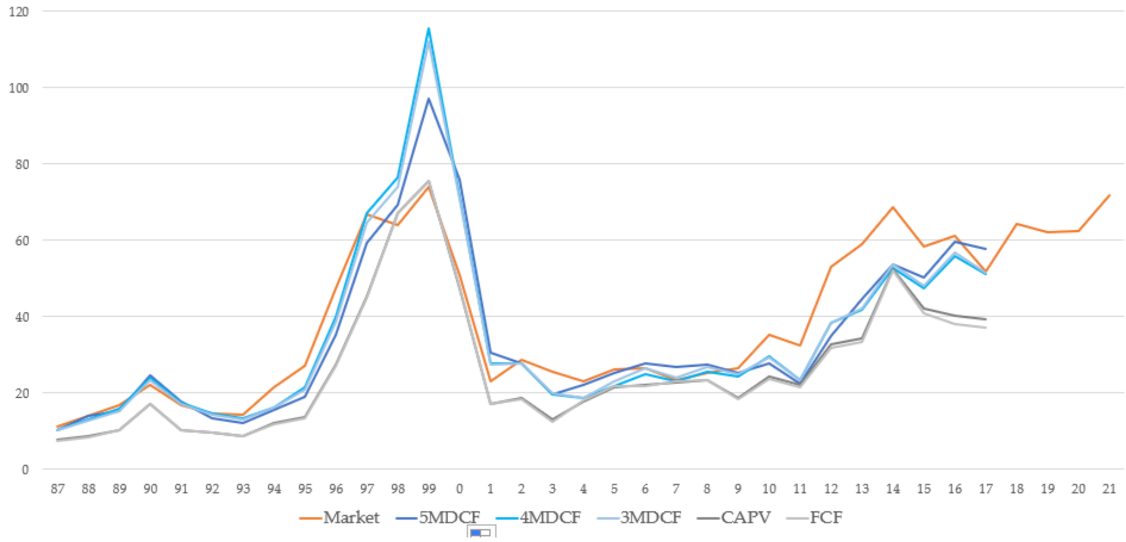


Table 4: Bristol Valuation – Historical data description

5MDCF	**4MDCF**	**3MDCF**	**CAPV**	**FCF**
Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
11,48%	8,53%	9,14%	22,49%	22,81%
0,12024	0,12830	0,12141	0,13447	0,13940

The firm valuation performed in Fig.1 is built upon the assumption that the investor is pricing the firm's residual value as a reflection of the future market's 5-year average worth, discounted at each of the discounting rates of every DCF model described in this study. In other words, it assumes that the investor's prevision is worth an average of the market pricing over the next five years – which deems a very good forecasting outlook. In Fig.2 (*Ceteris Paribus*) it is assumed that the investor gets the perpetual value exactly right. The assumption that the residual value is gotten exactly right by a given investor is not carried forward in this study. Nevertheless, it is possible to observe in Table 3 and Table 4 that the geometric average of the Mean Absolute Percentage Error (MAPE) is drastically lower when the investor gets its perpetual forecast exactly right, except for the 98-01 period where market expectancy better explains the data. But one particularly important observation is that, depending on which terminal value methodology is applied and the accuracy of such method, the most adequate riskiness rate changes. To value Bristol, an investor that gets a forecast within the future average, finds it better to use the 3-MDCF model. On the other hand, if the investor gets the forecasts exactly right, another model is best.

Fig. 3 and Fig. 4 are exactly the opposite example of such a case. Whether or not the consideration that future average information results in a better present valuation impacts Fama's (1970) efficient market hypothesis is a much more complex debate to have. However, it clearly seems to suggest that some behavioural or insider phenomena happen in the market. This study is a purely industrial valuation and, despite the fact that any growth/value company characterization, riskiness or macro event should be valued differently, it brings evidence that all available information (Fig.4) goes as long as market expectations (Fig.3) go in defining a company's price. If one then defines available information as a sufficient determinant of stock prices, one could consider the lack of some factor in defining the stock price, as shown in Fig.4.

Figure 3: Johnson & Johnson Valuation - Forecasted Simulation

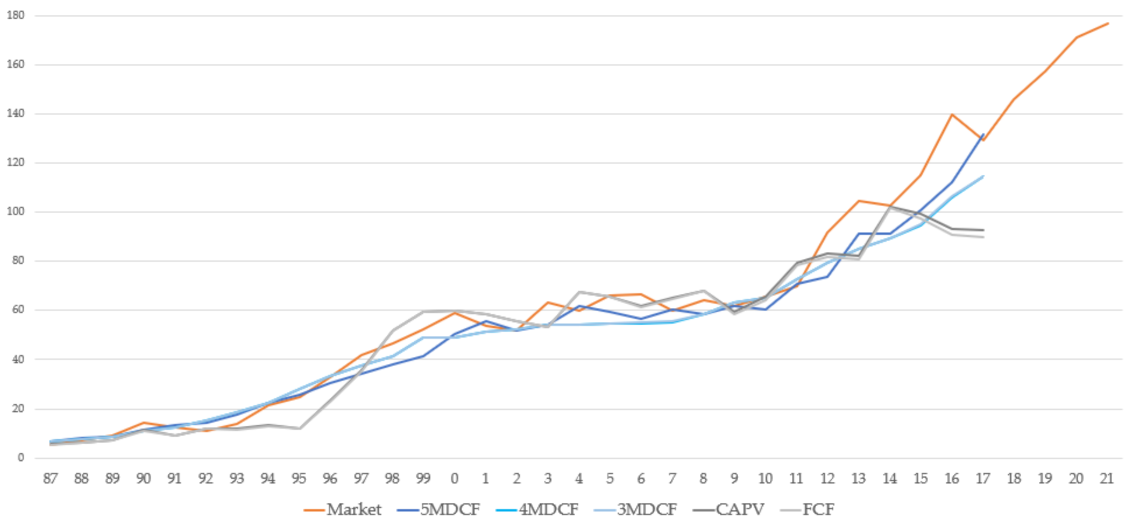


Table 5: Johnson & Johnson Valuation - Forecasted Simulation data description

5MDCF	**4MDCF**	**3MDCF**	**CAPV**	**FCF**
Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
7,22%	7,05%	7,78%	8,78%	10,17%
0,08439	0,09516	0,09443	0,11867	0,12417

Figure 4: Johnson & Johnson Valuation - Historical data

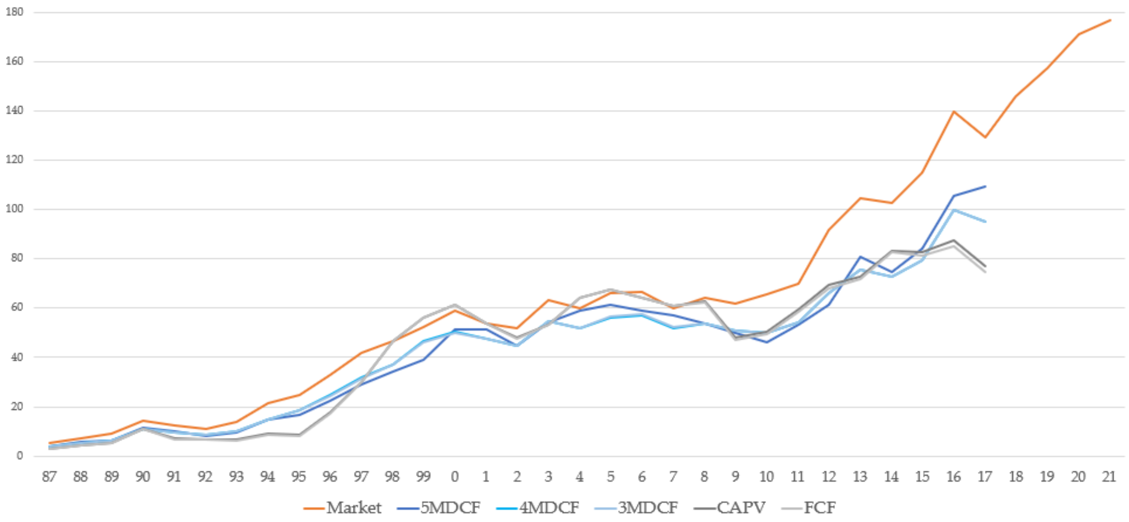


Table 6: Johnson & Johnson Valuation - Historical data description

5MDCF	4MDCF	3MDCF	CAPV	FCF
Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
18,46%	21,12%	21,11%	12,37%	13,43%
0,09123	0,06516	0,06578	0,18802	0,19376

The final example presented in this study (Fig.5 and Fig.6), helps to illustrate that neither the expected average nor the actual market pricing, or the current cross-sectional factors explain the totality of stock value. Chevron stock valuations generate huge MAP Errors, especially from 2008 onwards. Whether such macro event shaped investors' behaviour or made investors, in fact, more emotional, is actually an unexplainable observation. However, it is clear that, in such cases, neither the future average, nor the at-date values drove an aproximate valuation. From this follows the assumption that, in fact, another factor is needed to price this stock.

If one contests Fama's (1970) efficient market untestable assumption, that $E(Rt+1 \mid \Theta tm) = E(R)$, meaning, the information set is constant through time, Fig. 4 and Fig. 6 suggest that, at some point in time, the information set needed to value a stock might actually change (e.g., from 2008 onwards in Fig. 6), or again, a factor is missing. From 2008 forward, there is an evident spread between the models and the market price. The literature has been suggesting that the market is efficient and that deviations are due to over/under reactions and methodological differences (Fama, 1998). Nevertheless, Fig.6 shows strong deviations from market values and the MDCF models close much of the spread, indicating that factor selection is, in fact, a great contributor to market value.

Closing the topic, there seems to be no reason to expect cross-sectional optimization in firm valuation. While some firms derive more often value from expectations, others do from the information available. Equally, some investors price the market risk and some other price risks such as momentum, size, or others… There is a market fragmentation within the market, where each company derives value singularly. As Harvey et. al. (2016) stated, when a new factor is disclosed, it is more prone to explain abnormal returns, which may be a valid explanation for the gap case of Chevron valuation in Figs. 5 and 6 - a drawback to the existence of a constant information set in the market.

Figure 5: Chevron Valuation - Forecasted Simulation

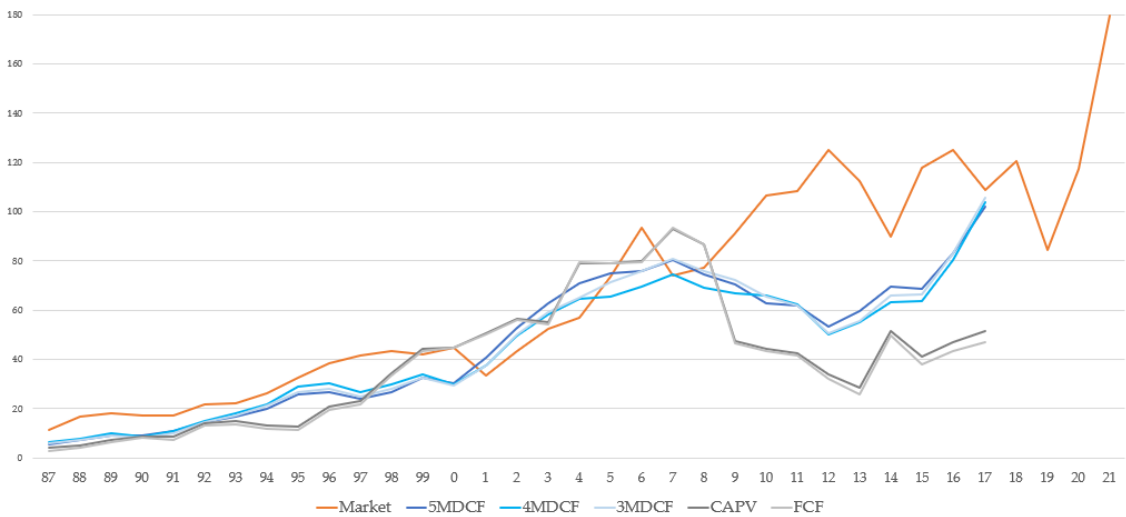


Table 7: Chevron Valuation - Forecasted Simulation data description

5MDCF	4MDCF	3MDCF	CAPV	FCF
Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
24,74%	22,14%	22,50%	28,67%	31,52%
0,15241	0,16032	0,16710	0,21882	0,23638

Figure 6: Chevron Valuation - Historical data

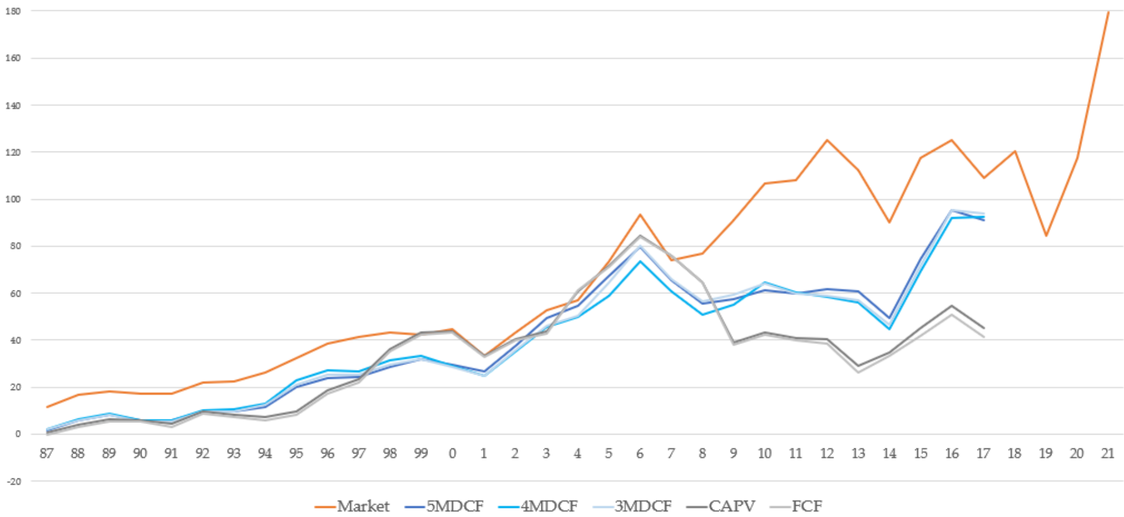


Table 8: Chevron Valuation - Historical data description

5MDCF	4MDCF	3MDCF	CAPV	FCF
Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
30,53%	34,11%	32,86%	26,10%	26,35%
0,20313	0,17876	0,19496	0,29088	0,30995

These abnormal types of stock market movements may also be due to stock overvaluation. Beneish & Nichols (2009) also approached the problem and stated that price collapses are ex-post evidence of previous overvaluation, as observable in 2012-2014. Nevertheless, they stated that this is an inconclusive phenomenon regarding the market efficiency hypothesis. Even if in 2014-15 the market valuation came much closer to a rational model valuation Beneish & Nichols' study referred to 12 months adjustments in overvaluation. As such, this observation seems not to be linked with the over-appreciation of a stock, a behavioural adjustment nor insider information. On the other hand, a later price conversion to the rational pricing models clearly suggests a bubble in the market or a missing factor, even if only for a given time-period.

Considering the lack of cross-sectional accuracy of any particular model, this study suggests performing Historical Discounted Cash Flow (HDCF) Valuation procedures, meaning that single firms have their own factors and that any given method will perform better with a given model. Even more, different time frames suggest having different market information sets – a question that remains unexplained by this study. The Historical DCF valuations average 29,87% Mean Absolute Percentage Error (MAPE), with a standard deviation of 0,30186. Overall, the HDCF valuation obtained the higher R2, suggesting that picking the best model, better suits the data – See Table 9 – log-likelihood also points that HDCF models better explain the SP100 stock value.

Table 9: Cross-sectional data description

	5MDCF	4MDCF	3MDCF	CAPV	FCF	H-DCF
Average	31,53%	31,63%	31,90%	35,19%	37,86%	29,87%
Geomean	19,51%	19,21%	19,47%	22,46%	24,24%	17,29%
St. dev.	0,31474	0,30089	0,30941	0,37031	0,39080	0,30186
R2	79,86	79,22	79,23	73,05	69,70	80,54
likelihood	317,18	379,65	340,92	89,28	12,75	390,12

Considering the following hypothesis under a T-Statistic test, according to the sample's kurtosis, we have:

H0 : DCF model X average MAPE ≥ DCF model Y average MAPE

H1 : DCF model X average MAPE < DCF model Y average MAPE

Table 10: T-Statistic test

T-test vs	5MDCF	4MDCF	3MDCF	CAPV	FCF	HDCF
5MDCF	-	-0,08662	-0,30827	-2,78155	-4,66298	1,40393
4MDCF	0,08662	-	-0,22786	-2,75410	-4,66750	1,52498
3MDCF	0,30827	0,22786	-	-2,51940	-4,42114	1,73113
CAPV	2,78155	2,75410	2,51940	-	-1,83509	4,11105
FCF	4,66298	4,66750	4,42114	1,83509	-	5,97818
HDCF	-1,40393	-1,52498	-1,73113	-4,11105	-5,97818	-

Table 11: P-values for the T-Statistic test

P value vs	5MDCF	4MDCF	3MDCF	CAPV	FCF	HDCF
5MDCF	-	0,46757	0,38663	0,02487**	0,00478***	0,88349
4MDCF	0,53243	-	0,41546	0,02558**	0,00477***	0,89902
3MDCF	0,61337	0,58454	-	0,03270**	0,00575***	0,92076
CAPV	0,97513	0,97442	0,96730	-	0,07020*	0,99264
FCF	0,99522	0,99523	0,99425	0,92980	-	0,99803
HDCF	0,11651	0,10098*	0,07924*	0,00736***	0,00197***	-

In fact, the Historical DCF valuation is the one that gathers the best p-values for this test. There is strong statistical evidence, at the conventional levels of 10%*, 5%**and 1%*** (Table 11), to reject the hypothesis that the HDCF valuations' MAPE is higher or equal to the MAPE of any other model, which leads to the conclusion that the HDCF model is the most statistically adequate to evaluate the SP100 firms' price. Even if the p-value of the HDCF model *vs* the 5MDCF model is 0,11651, we can say that these HDCF models' mean errors are not equal to or greater than the errors of the 5MDCF model within an 88% confidence interval – the Z-test statistics reject, at the conventional levels, the FCF, CAPV, 3MDCF, 4MDCF and 5MDCF models' hypothesis *vs* the HDCF valuation methods (See Appendix 3).

If one considers the cross-sectional valuations, the 5MDCF, 4MDCF and 3MDCF models' tests gave enough statistical evidence, at conventional levels, to discard the null hypothesis. As such, the T-test provided enough statistical evidence to infer that the MDCF models have an average error inferior to the average error of the DCF models. If one considers the cross-sectional model that has the lowest p-values, then the 5MDCF model has the most statistical evidence to better fit the dataset of the SP100. Nevertheless, there is no reason to disregard the null hypothesis that the average MAPE of the 5MDCF model is equal or greater than the 4MDCF model, nor the 5MDCF average *vs* the 3MDCF, neither in the 4MDCF *vs* 3MDCF test.

The formal test for the significance of the extra variables in MDCF models, so that one can access whether or not the cross-section of the 5MDCF model is better than the 4MDCF and the 3MDCF and amongst each other, is the likelihood-ratio test, as follows bellow (Table 12).

H_0 : MDCF model $\beta_1 = \beta_2 = \ldots = \beta_n = 0 \equiv \beta_{set1}$

H_1 : MDCF model $\beta_1 \vee \beta_2 \vee \ldots \vee \beta_n \neq 0$

Table 12: Likelihood-ratio test for the MDCF models' extra variables

	4MDCF	**3MDCF**	**CAPV**	**FCF**
5MDCF	-124,97256	-47,4121185	456,8870584	610,7810709
p-value	-	-	(<0,001) ***	(<0,001) ***
4MDCF		77,5604431	581,85962	735,7536325
p-value		(<0,001) ***	(<0,001) ***	(<0,001) ***
3MDCF			504,2991769	658,1931894
p-value			(<0,001) ***	(<0,001) ***

The tests of all the models presented p-values lower than the conventional level of 1%, except for the ones of the 5MDCF model *vs* the 4MDCF and *vs* the 3MDCF, which leads to the conclusion that the validity of the FCF and the CAPV, in regard to their simplicity, is statistically rejected versus any Multi-factor DCF model. Nevertheless, the test does not reject the hypothesis that the 3MDCF model is a valid model *vs* the 5MDCF but, on the other hand, the 4MDCF model, with significant differences against all the basis models, had the best performance in the tests. Therefore, it is concluded that both the 3MDCF and the 5MDCF models are, complexity-wise, less fit for the SP100, since there is statistical reason to reject the 3MDCF *vs* the 4MDCF, and no reason to prefer the 5MDCF model.

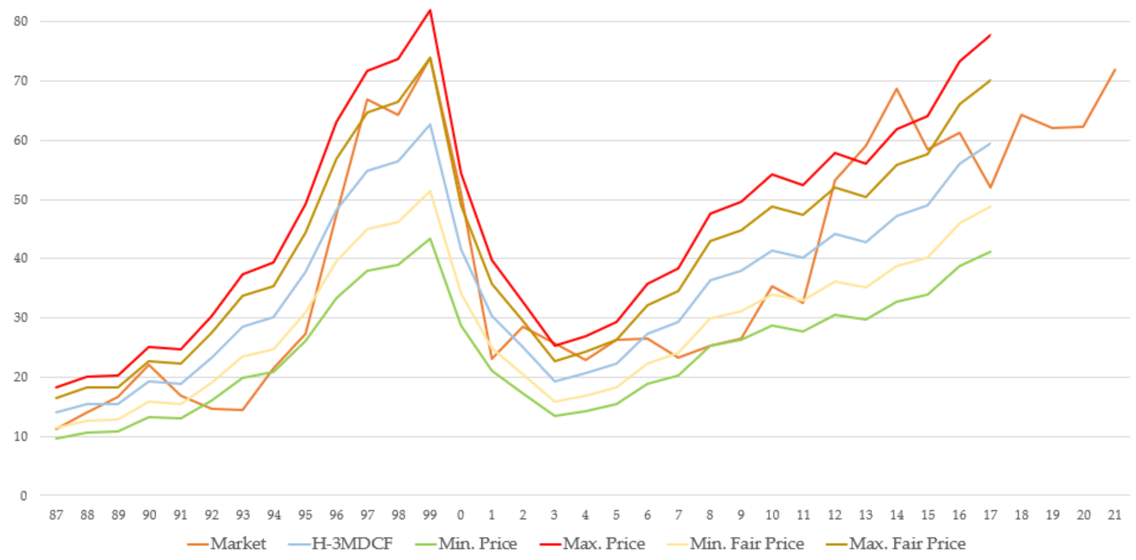
Table 13: Forecast simulation Bristol valuation with error measures (H-3MDCF)

2017	Reference	Min. Price	Min. Fair Price	Market price
(Share) Valuation	$ 59,45	$ 41,15	$ 48,78	$ 51,98
(Share) Error	±18,30			

Following the rule-based price fairness and its impacts (Maxwell, 2002), the fairness of a price, for the buyer side, is the result of the price perception and the willingness to buy. It is important to provide rational pricing information as it sets the buyers' attitude towards the seller and the buyers' purchasing appetite. Adopting an HDCF methodology helps to justify the reference price and a fair seller attitude. As such, the error measures (if not for a lack of factors) provide insights into the seller-buyer power structure and contractual fairness.

In addition, Maxwell's study indicates that when the seller has contractual power, the perception of the reference price and willingness to buy changes, which leads to unfair buys that may harbour resentment. Furthermore, information display augments price perception and willingness to buy.

Figure 7: Forecast Simulation of Bristol valuation with error measures (H-3MDCF)



As such, with Maxwell's (2002) model of fairness, one can set the minimum price expected as the economical price (the cheap price according to Huppertz et. al. (1978)) and the minimum fair price as the price that follows the social exchange rules that satisfy the minimal mutual expectation from the parties. Historical valuation presents itself as the best way to satisfy the rules of process described, and the error measures likewise satisfy the rules of outcome. Figure 7 illustrates an example of such a procedure, from 1987 to 2017, and Table 13 denotes the latter values.

The historical valuation not only establishes factors, but also withdraws the process and the outcome's rules that determine the buying behaviour of investors. The valuation in Fig.7 does not seem to need any other factor rather than market excess return, size and book-to-market ratio, but it lacks the comprehension of how the behavioural determinants, such as the assumed fair price assessment and exchange power effects, empirically drive versus constraint the market value.

Chapter VII
Conclusions

Valuating firms cross-sectionally clearly results in poorer outcomes than relying on historical models. HDCF models have statistically been proven to better fit the companies' data at the conventional confidence set. Nevertheless, the contributions of the cross-sectional studies, remain as important as ever. The cross-sectional models are of extreme importance to establish the building blocks of HDCF valuations.

It is not certain if any missing (cross-sectional) factor is lacking in the most accepted multi-factor models, but when asserting for individual company valuations, some present evident gaps, of which some are explained simply by the market's expectation of future value, while others are persistent through expectation forecasts and real value forecasts. A possible solution for behavioural deviations from the expected valuations can be furthermore explained by the incorporation of the error measures, thus assessing a fair-trading value expectancy.

Scientifical valuation books, such as Mckinsey's (2015), Damodaran's (2002), and the multi-factor equity return models here exploited, are the best practice routines so that one can be confident in affirming that a factor is missing in the market information set, or simply assume that the company is just highly overvalued (as in Figs. 5 and 6).

Converging with Fama & French's (1992, 1993, 2015) and Carhart's (1992, 1995, 1997) works, the FF3, FFC and FF5 models do better explain the cross-section of returns of the SP100, with the 4MDCF model outperforming in the overall, t-tests plus likelihood ratio tests. If one is intended to valuate a company by a cross-sectional model, the literature is accurate, as the simpler Capital Asset Pricing Model is more prone to abnormal valuations.

Despite having a definite conclusion, this study did not identify the exact reasons why there are observable gaps between the 5 different models and the market valuations[26]. Furthermore, as Merton (1980) argues, the realized market returns are not flawless proxies; the "true" values of the beta were not calculated according to Damodaran (2002), and the cost of debt was proxied with fixed spread rates.

The empirical results also support Fama's (1998) statement regarding the positive impacts methodological changes have in abnormal returns, in CAPM, and in every other model. Changing parameters implies a change in model efficacy. Nevertheless, if Fama's (1970) efficient market hypothesis holds, there must be a model that best delivers all the information available assessments and forecasts a reference value only subjected to behavioural deviations.

Further research on the market's information time-set continuity, further multi-factor equity model testing, and more trial with specific DCF model variations, such as the APV proposed by Myers (1966, 1974) and Jennergren's (2008) continuum value model, would present an extraordinary understanding of how the market values companies. Furthermore, extending the research towards the SP500, the Nasdaq, and abroad markets, could uncover the discrepancies and similarities between them, allowing for an understanding of the investor's risk behaviour across different market segments. Finally, when "to-fit" models become easily derived from the cross-sectional infrastructure, further developments could reflect on how price fairness, or other behavioural explanations, justify the market price waves.

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[26] According to Kilian & Park (2009) the crude oil shocks account for 22% of the long-run variation in U.S. real stock returns. A trial valuation with the H-4MDCF (+ the crude oil factor (WTI)) model for Chevron, lowered the average MAPE from 22,14% to 17,40%. A satisfying result with a T-test p-value of 0,15306. The result suggests that crude oil prices can be a determinant value factor for this company, or industry (See appendix 4).

Overall, this study has shown that the MDCF models have better cross-sectional results compared to the plane DCF models, and that the HDCF methodology helps to improve forecast accuracy, depending on the investors' methodology, timeframe, and expectations.

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Appendix I
Cash Flow & Firm Valuation

Cash is King

Reider’s & Heyler’s (2003) book on Cash Flow Management is structured to manage the “lifeblood of the organization”. Businesses must make functional and economically efficient decisions. Cash-oriented performance pursuit brings up historical managerial concepts such as Fordism and Toyota’s Lean Manufacturing System (LMS) and all of the Management-field research into determining why, how, and when corporate value is obtained.

Marketing and Sales departments have many managerial instruments to track cash activity. Studies such as the ones of Day (2011), Kumar et al. (2016) and Puntoni et al. (2006) illustrate that methodologies are being implemented to maximize the cash efficiency of firms. Market anticipation, AI and networking as more adaptive business models, are dealing with increasing ramifications of consumer contact points and micro-segmented markets. In Firm Valuation terms, it demands valuators to know how corporate resources are being deployed and how much periodical cash is being generated, especially toward competitive advantage.

Corporate human capital also performs as a critical determinant of Cash Flows. The management policies, i.e., Authoritarian, are a pure reflection of the cash is king mentality. Well-being and performance have been particularly shaping the XXI century with the incorporation of ESG ethics, but cash, the king, is a survival factor in corporate nature.

Cash Flow Management & Firm Valuation

The corporate making of cash can come from a decrease in its assets, by liquidating client accounts, selling inventory, equipment… or cash can be generated by increasing liabilities with equity or debt issuance. On the other hand, corporate cash applications concern actions such as inventory purchases, credit extensions, repurchase of shares and dividend payments. Cash Flow Management for Firm Valuation focuses on the realisation of whether the firm has smooth Cash Flows; Are the corporate projects well implemented; Is the company proactive on investment opportunities; Are the cash outflows in the form of debt and equity sustainable.

Cash Flow Management is, preferably, a daily activity of the firm and a forecast includes 12 months of sales, cash receipts, cash disbursements and cash balances analysis. Public firms' valuation can take yearly intervals, or often in the USA, quarterly. The key understanding is what are the corporate's cash reserves and what is the corporation doing to deal with the cash excesses and shortfalls.

Appendix II
T-Test Statistic for Walmart company

5MDCF	4MDCF	3MDCF	CAPV	FCF
Avg / S.dev	Avg /S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
16,39%	19,41%	28,39%	13,39%	12,90%
0,13893	0,14978	0,38487	0,16316	0,17125

T-test vs	5MDCF	4MDCF	3MDCF	CAPV	FCF
5MDCF	-	-0,8602	-1,709749	0,81794	0,9245818
4MDCF	0,860217	-	-1,268473	1,58487	1,668617
3MDCF	1,709749	1,26847	-	2,09286	2,144751
CAPV	-0,817943	-1,5849	-1,58487	-	0,1209467
FCF	-0,924582	-1,6686	-2,144751	-0,12271	-

p value vs	5MDCF	4MDCF	3MDCF	CAPV	FCF
5MDCF	-	0,24016	0,11472	0,75033	0,7736
4MDCF	0,759839	-	0,166146	0,87307	0,8814
3MDCF	0,88528	0,83385	-	0,91428	0,9174
CAPV	0,249668	0,12693	0,126932	-	0,5426
FCF	0,226395	0,11857	0,082577 *	0,45678	-

Appendix III
Z-Test Statistic

H0 : DCF model X average MAPE ≥ DCF model Y average MAPE

H1 : DCF model X average MAPE < DCF model Y average MAPE

Z-test vs	**5MDCF**	**4MDCF**	**3MDCF**	**CAPV**	**FCF**	**H-DCF**
5MDCF	-	-0,11788	-0,42526	-4,22520	-7,31319	1,91363
4MDCF	0,12331	-	-0,32152	-4,29631	-7,52639	2,12499
3MDCF	0,43258	0,31267	-	-3,86540	-7,00658	2,37917
CAPV	3,59114	3,49095	3,22970	-	-2,62459	5,21760
FCF	5,88986	5,79492	5,54737	2,48699	-	7,43105
H-DCF	-1,99528	-2,11820	-2,43869	-6,40077	-9,62053	-

p value vs	**5MDCF**	**4MDCF**	**3MDCF**	**CAPV**	**FCF**	**H-DCF**
5MDCF	-	0,45308	0,33532	<0,001***	<0,001***	0,97217
4MDCF	0,54907	-	0,37391	<0,001***	<0,001***	0,98321
3MDCF	0,66734	0,62273	-	<0,001***	<0,001***	0,99132
CAPV	0,99984	0,99976	0,99938	-	0,00434***	0,99999
FCF	0,99999	0,99999	0,99999	0,99356	-	0,99999
H-DCF	0,02301**	0,01708**	0,00737***	<0,001***	<0,001***	-

Appendix IV
Chevron Valuation (with crude oil factor)

Figure 8: Chevron H-4MDCF + COIL - Forecasted Simulation

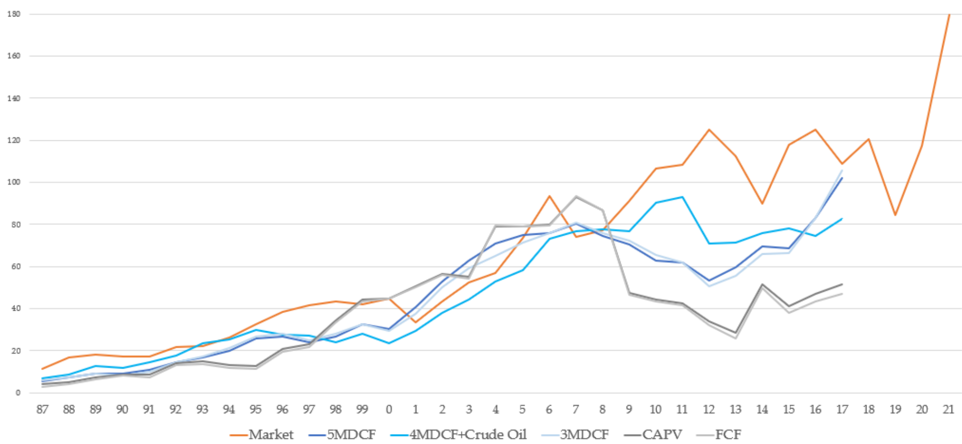


Table 14: Chevron Valuation* - Historical data description

5MDCF	H-4MDCF*	3MDCF	CAPV	FCF
Avg / S.dev	Avg /S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
24,74%	17,40%	22,50%	28,67%	22,14%
0,15241	0,14223	0,16710	0,21882	0,16032

*With Crude Oil Factor

Figure 9: Chevron H-4MDCF + COIL - Historical data

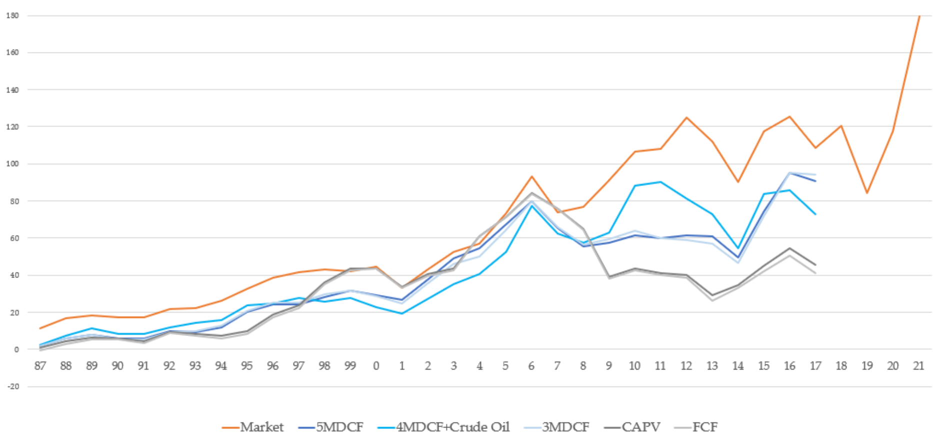
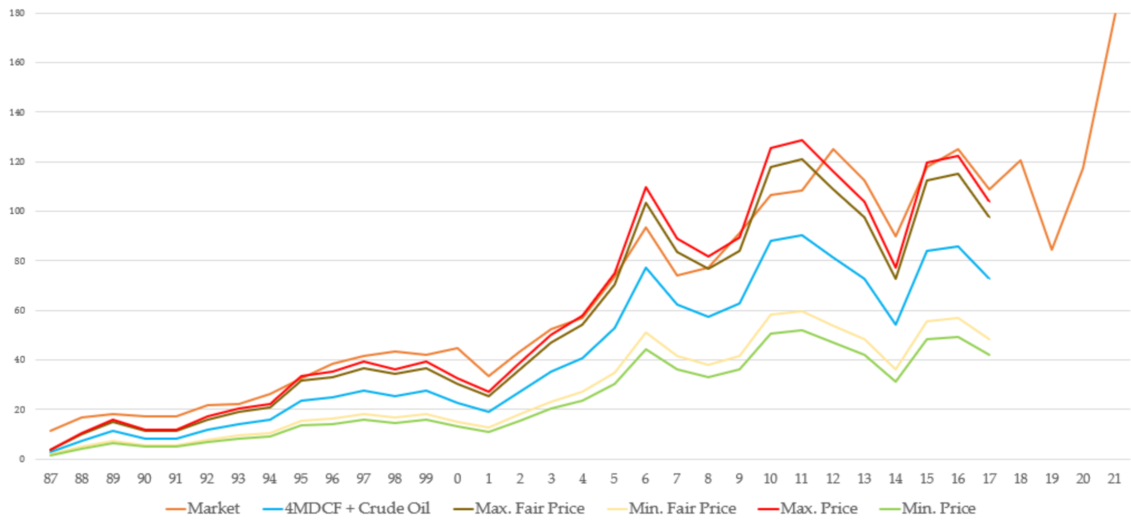


Table 15: Chevron Valuation* - Forecast Simulation description

5MDCF	H-4MDCF*	3MDCF	CAPV	FCF
Avg / S.dev	Avg /S.dev	Avg / S.dev	Avg / S.dev	Avg / S.dev
30,53%	33,80%	32,86%	26,10%	22,14%
0,20313	0,12802	0,19496	0,29088	0,16032

Figure 10: Historical Data valuation of Chevron with error measures (H-MDCF)



Since a price fairness error measure can explain the market deviations from the reference price of the H-4MDCF + C.OIL model, it is suggested that, for this company, the crude oil factor was the missing criteria. Nevertheless, the same may not be true for other companies. Furthermore, this is a valuation that assumes that the investor will get the terminal value exactly right, which can fall out of reasoning for a forecasting model.

Overall, we can see that Chevron's price is subject to some behavioural influence that sets its prices as high as the Max. Price expectancy, if this model is to be a valid criterion for this company.