The Effectiveness of Adding Commodities to a Multi-Asset Portfolio

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Abstract:

Commodity investment is fundamentally motivated by a desire to improve the performance of portfolios composed of stocks and bonds. Throughout this paper we analyze the out-of-sample performance effects derived from including commodities in a stock-bond portfolio for seven distinct asset allocation models – equally and strategically weighted portfolios, risk-parity, reward-to-risk timing, as well as, minimum-variance, mean-variance, and Black-Litterman. We analyze seven commodity groups and consider two distinct investor profiles, while constructing portfolios in which commodities are picked in a consistent standard format, and portfolios in which commodities are dynamically picked and adjusted every month. Precious metals emerge from our static portfolio allocation analysis as the commodity group with the clearest and most significant portfolio benefits. Concurrently, dynamic portfolio allocation yields very appealing returns, performing well in terms of risk-return tradeoff measures, while delivering more consistent results across asset allocation models when compared to static allocation. Portfolio gains remain highly linked to the macroeconomic environment, demonstrating that investments in industrial metals generate considerable improvements for favorable subperiods of stability and growth, whereas precious metals are particularly beneficial in the face of unstable subperiods.

Keywords: portfolio development, asset allocation modelling, commodity investment, stock-bond portfolio, static portfolio allocation, dynamic portfolio allocation

A Eficácia de Adicionar *Commodities* a um Portfólio com Múltiplos Ativos

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Sumário:

O investimento em *commodities* é fundamentalmente motivado pelo desejo de melhorar o desempenho de portfólios compostos por ações e obrigações. Ao longo deste estudo, analisamos os efeitos do desempenho *out-of-sample* resultantes da inclusão de *commodities* num portfólio composto por ações e obrigações para sete modelos de alocação de ativos distintos –*equally-weighted, strategically-weighted, risk-parity, reward-to-risk timing,* bem como, *minimum-variance, mean-variance,* e *Black-Litterman* Analisamos sete classes *commodities* e consideramos dois perfis de investidores distintos, enquanto construímos portfólios em que as *commodities* são escolhidas num padrão consistente, e portfólios em que as *commodities* são escolhidas e ajustadas dinamicamente todos os meses. Os metais preciosos emergem da nossa análise de alocação estática como a classe de *commodities* com os benefícios mais significativos. A alocação dinâmica produz retornos muito atrativos, tendo um bom desempenho em termos de medidas de retorno ajustadas ao risco, enquanto proporciona resultados mais consistentes ao longo dos modelos de alocação de ativos, quando comparados com a alocação estática. Os ganhos dos portfólios permanecem altamente ligados a fatores macroeconómicos, demonstrando que os investimentos em metais industriais geram melhorias consideráveis para subperíodos favoráveis de estabilidade e crescimento, enquanto que os metais preciosos são particularmente benéficos em subperíodos instáveis.

Palavras-chave: construção de portfólios, modelagem da alocação de ativos, investimento em *commodities,* portfólio de ações e obrigações, alocação estática, alocação dinâmica

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1. Introduction

"Commodities tend to zig when the equity markets zag."

– Jim Rogers (2013)

Investors and traders have traditionally remained focused on investing in equity markets, despite commodity markets playing a significantly larger part in the world stage. This may no longer be the case, with the past decade seeing a dramatic increase in investor's interest in commodity markets. In fact, commodities have increased in popularity as an alternative asset class for both institutional and private investors. This new interest has materialized into swift investment growth over the last few years, particularly in the form of commodity futures and commodity index funds.

Over the long run, commodities have offered investors a positive rate of return, despite being a considerably volatile asset class – commodity prices do not usually go up in a straight line and it is very common for investors and traders to experience periods of negative returns. According to Worah (2011) as cited in Baker et al. (2018), commodities are extremely correlated with global growth and characterized as being supply constrained. Contrary to stock prices, which are based on the market's long-term appraisal of a firm's discounted cash flows, commodity prices are governed by demand-supply dynamics, typically integrating considerations of a more short-term nature. Commodities might be employed to achieve diversified goals – they are used by investors to diversify risk in a portfolio context due to apparently low correlations with traditional asset classes of stocks and bonds, thus gaining more attractive risk-adjusted returns, or simply to hedge against inflation. In our research, we explore the employment of commodities as a tool for enhancing the performance of multi-asset portfolios initially composed of solely stocks and bonds.

The years from 2000 through 2013 marked a period of strong growth in commodity markets, while also playing particularly important role in the demise of equities during the 2008-2009 bear market. ETF Securities (2011) states that commodities outperformed major asset classes in 2010, establishing their position as the best performing asset class from 2000 to 2011. The year of 2011 represents a peak in the evolution of commodity markets – with the total value of commodities under management reaching a record number of $412 bn in March 2011 [Carpenter (2011)]. According to Croft and Norrish (2013), commodity investments more than doubled between 2007 and 2013, from

$170 bn to roughly $410 bn[1]. The diversification benefits of commodities when this asset class is incorporated into a stock-bond portfolio is one possible reason for the commodity market boom of recent years. In fact, Geman (2005) and Daskalaki et al. (2011, 2014) state that commodity price movements are associated with several specific risk factors, such as geopolitical events, event risk, and weather. The way these factors affect commodities' value differs significantly from how they drive the value of equities and bonds. This serves to justify the small or even negative correlations of commodities' returns with the returns of assets that belong to traditional asset classes. Additionally, investing in commodities has emerged as an effective strategy throughout inflationary periods. Since commodities are real assets, they have an intrinsic value and are able to reflect price movements and can thus be used as a hedging instrument against both expected and unexpected inflation [Bodie and Rosansky (1980), Gorton and Rouwenhorst (2006), Erb and Harvey (2006)].

However, a substantial decline in overall commodity prices took place following their 2011 peak. A possible explanation for this negative trend suggests that the sizable downward adjustment in the economic growth pace of China, characterized for its manufacturing-intensive economy, and of several other emerging economies at the time, were to blame. Consequently, by January 2015 the primary commodities' price index had reached levels 40% lower than at their 2011 heights. This five-year long negative trend was finally reversed only in January 2016, albeit the last quarter of 2018 saw commodity prices falling once more, driven by an overall decline in energy commodities. More recently, in the second half of 2019, precious metals such as gold and silver started to break out. Investors have been looking for alternative currencies, since the reduction in US yields decreased the dollar's appeal as a holding asset. Still, the World Bank (2019) documented that prices of almost 60% of commodities fell during the third quarter of 2019. This unexpected decline can perhaps be explained by the sharp slowdown in manufacturing and goods trade, alongside a deteriorating macroeconomic environment in general, and in particular due to trade tensions between the US and China, weighing negatively on demand for commodities. Our intuition is that, although commodities may not currently be an attractive asset class as a stand-alone investment, they may enhance an investor's risk-return profile by providing diversification benefits and thus adding value to a stock-bond portfolio.

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[1] The rose in commodity investments between 2007 and 2013 was accompanied by an exceptional increase in commodity prices between 2003 and 2008. This period is well-known in the investing community as the third commodity boom since 1950 [Daskalaki et al. (2017)].

Taking the aforementioned as a starting point and following the approach developed by Bessler and Wolff (2015), we build portfolios that combine stocks and bonds with commodity groups and with individual commodities. We consider two distinct commodity groups, the diversified S&P GSCI index and the S&P GSCI light energy index, and five individual commodities, namely, energy, industrial metals, precious metals, livestock, and agriculture. We build portfolios in which commodities are picked in a consistent standard format (static portfolio allocation), and portfolios in which commodities are dynamically picked and adjusted every month. We end up constructing ten different types of portfolios – (i) a portfolio composed solely of equities and bonds; (ii) portfolios that are combinations of the traditional assets (stocks and bonds) with each commodity, making a total of seven portfolios; (iii) a portfolio that, each month, combines the two traditional assets with the commodity that presents the lowest correlation with the stock index in that month; and (iv) a portfolio combining, for each month, stocks, bonds and the pair of commodities showing the lowest correlation in that month.

Then, we analyze the out-of-sample portfolio benefits from including commodities in a stock-bond portfolio for seven different asset allocation strategies – two naïve asset allocation rules, reward-to-risk timing, risk-parity, minimum-variance, mean-variance, and Black-Litterman. Our study adds value to the existing literature by building portfolios in which commodities are dynamically selected and by computing and comparing several performance measures. Such an approach allows us to overcome the drawbacks of relying on the results from adding commodities to multi-asset portfolios only for Sharpe ratio, as is shown in Bessler and Wolff (2015), thereby and enabling us to obtain a more consistent overview of the real gain from optimal diversification. We further analyze the portfolio benefits of commodities for different investment styles, specifically for conservative and aggressive investment strategies, throughout different sub-periods. In this manner we are able to gather additional insight on whether the benefits of including commodities in a multi-asset portfolio depend on the type of commodity, on the employed asset allocation model, on the investor's level of risk-aversion, or even on different market environments.

We find it possible to considerably enhance portfolio performance measures by adding precious metals to a stock-bond portfolio, under static portfolio allocation methodology. Our study documents that such combination of assets yields very appealing returns and, most importantly, performs well in terms of risk-return tradeoff measures. We conclude that Black-Litterman, of the many asset allocation models considered for empirical analysis purposes, delivers portfolios with superior performance. Static portfolio allocation methodology proves that there are, in fact, some positive out-of-sample performance effects derived from the inclusion of commodities in a stock-bond portfolio. For instance, the maximum benefit an aggressive investor with a risk aversion coefficient equal to 2 achieves, from including commodities statically, corresponds to an increase in average return of 1.71% every year and in certainty-equivalent (annualized) of 1.32%. A conservative investor with a risk aversion coefficient equal to 10, in turn, is able to increase average return by 0.7% every year and boost certainty-equivalent return by 0.60%.

Our study documents that dynamic portfolio allocation yields very appealing returns, performing well in terms of risk-return tradeoff measures, while delivering more consistent results across asset allocation models when compared to static allocation. By picking the weights dynamically and adjusting them every month, an investor is able to increase its certainty-equivalent return up to 3.5% every year. Moreover, we find that the benefits of including commodities largely depend on macroeconomic factors. Investments in industrial metals generate considerable improvements for favorable subperiods of stability and growth, whereas precious metals are particularly beneficial in the face of unstable subperiods. For instance, in subperiods with low volatility, a conservative investor with a risk aversion coefficient of 10, improves Sharpe ratio (annualized) by 0.73 by investing in industrial metals, along with stocks and bonds. Contrarily, in bear markets, a conservative investor increases Sharpe measure by 0.39 every year by including precious metals in a stock-bond portfolio.

This dissertation is therefore structured as follows: Section 2 documents the existing literature on the diversification benefits from adding commodities to a multi-asset portfolio; Section 3 presents the data used in the analysis; Section 4 develops on the out-of-sample estimation procedures, the asset allocation models employed for portfolio's construction and the performance measures chosen for analysis purposes; Section 5 discusses empirical results for the entire dataset, while providing additional insight into our sub-periods; and Section 6 provides a summarizing conclusion for our study.

2. Literature Review

Our research contributes to the literature that studies diversification gains derived from the inclusion of commodities in a stock-bond portfolio. There exists a vast amount of literature documenting the diversification potential of commodities, thus demonstrating that commodities have established themselves as an important asset class and the portfolio effects of commodities are of interest to both academics and practitioners. Different authors have reported different outcomes however, which may be explained by the diverse research setups employed. Hence, there is currently not much unanimous empirical evidence on whether adding commodities to a multi-asset portfolio is a valuable investment strategy.

There are already a large number of papers recognizing that incorporating commodities in the opportunity set does improve the risk-return tradeoff of investors' portfolios. For instance, the works of Bodie and Rosansky (1980), Greer (1994), and Conover et al. (2010) on different data sets have provided a theoretical foundation for suggestions that by moving from a stock-only portfolio to a portfolio that combines stocks with commodities, investors are able to enhance their risk-return profile. Fortenbery and Hauser (1990), Satyanarayan and Varangis (1996), Abanomey and Mathur (1999), Jensen et al. (2000), and Idzorek (2007) conclude that the actual returns investment frontier shifts upwards when commodity futures are added to a multi-asset portfolio. Georgiev's (2001) and Gibson's (2004) works on the subject saw the construction of portfolios with distinct commodity allocations, resulting in the improvement of risk-return characteristics in the mean-variance space. You and Daigler (2012) instead document the diversification gains of commodity futures by employing two asset allocation models – mean-variance and Sharpe optimization. In their work, Scherer and He (2008) conduct a regression of commodity indices on a portfolio composed of traditional assets and through a *p*-value analysis of the regression estimates, conclude that commodities do indeed possess a potential for diversification. Nijman and Swinkels (2008) suggest improvements in institutional investors' asset-liability management are due when commodities are added to the investment opportunity set. Furthermore, Galvani and Plourde (2010) report that during the 1990 to 2008 period, the presence of four energy future contracts could potentially reduce the risk of a portfolio composed of energy stocks.

Additionally, the portfolio effects of commodities have been documented from the perspective of expected utility, alongside the investor's level of risk aversion. For example, Ankrim and Hensel (1993) suggest the presence of an improvement in portfolios' performances for the period between

1972 and 1990, by including commodities in the investment universe, independent of the level of risk tolerance. In stark contrast, Anson (1999) concluded that the utility an investor derives from investing in commodities is an increasing function of an investor's degree of risk aversion.

Some authors address the same question regarding the effectiveness of having commodities in a portfolio, but with the particularity of investigating commodities' diversification gains in certain sub-periods, in view of the changes in the monetary policy of the United States' Federal Reserve. For instance, Conover et al. (2010) report that the diversification benefits of commodity investments have existed not only during periods of restrictive monetary policy, but also during periods of expansive policies. Whereas Jensen et al. (2002) concluded that diversification gains arise only during restrictive periods of monetary policy.

Other studies go one step further and conduct spanning tests[2] in order to gather insight on whether the shift of the efficient frontier when commodities are included in a multi-asset portfolio is statistically significant or not. On the one hand, Galvani and Plourde (2010), and Daskalaki and Skiadopoulos (2011), conclude that commodities lead to a significant enhancement of the initial frontier, by rejecting the spanning tests' null hypothesis that traditional asset classes, such as stocks and bonds, span returns of commodity futures. In stark disagreement to this conclusion, Cao et al. (2010) reported that adding commodities to the investment opportunity set does not significantly shift the initial frontier. Furthermore, through the conducting of spanning tests for 25 individual commodities, Belousova and Dorfleitner (2012) came to the conclusion that the diversification gains from adding commodities to a portfolio depend on the type of commodity one takes into consideration.

There exists a large body of literature analyzing the contribution of commodities to shifts in the efficient frontier, visually or with spanning tests, within an in-sample setting. In-sample studies are only able to prove that commodities would have enhanced the efficient frontier during a certain period [t, t*], conditioned by the fact that asset returns were known in advance. Therefore, an in-sample

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[2] Spanning tests study whether one set of risky assets can improve the investment opportunity set of another set of risky assets. If one rejects the null hypothesis of a spanning test, it means that the augmented investment opportunity set of both test and benchmark assets provides a significant shift of the initial efficient frontier of the set of benchmark assets [Huberman and Kendal (1987), and Kan and Zhou (2012)].

approach carries the drawback of assuming perfect forecasts of expected asset returns, variances, and correlations. In-sample studies allow us to get a sense of the maximum potential benefit of including commodities in a multi-asset portfolio, assuming absence of estimation errors. According to Welch and Goyal (2007) however, forecasts often enclose considerable estimation errors. An alternative and more realistic approach would be to consider an out-of-sample analysis, where an investor has to calculate portfolio weights at time t for the following period $[t, t + 1]$ with the data accessible at time t, in order to avoid possible forecast estimation errors. In accordance with Broadie (1993), and You and Daigler (2012), one can state that out-of-sample optimized portfolios perform lower than the ex-post efficient frontier estimated within an in-sample setting.

Still, the literature in out-of-sample contribution of commodities remains limited and considerably ambiguous. For instance, Daskalaki and Skiadopoulos (2011) report an absence of portfolio benefits from including commodities in an out-of-sample setting in the case of first generation commodity indices[3] (Deutsche Bank Liquid Commodity Index, S&P Goldman Sachs Commodity Index, and the Dow-Jones-UBS Commodity Index) in which a passive equity index, the S&P 500, is picked. Cotter et al. (2017) implement conditional mean-variance portfolio allocation strategies, with the goal of exploiting predictability, and the fixed-weight portfolio strategy. The authors report in-sample portfolio performance improvements when commodities and currencies are considered, but do not find diversification gains within an out-of-sample setting. Conversely to such conclusions however, You and Daigler (2012) and Giamouridis et al. (2014) report an improvement in out-of-sample performance of optimized portfolios that include commodities. It is important to point out that these two studies only employ Markowitz (1952) sample-based mean-variance and non-mean-variance investment strategies. Nonetheless, the mean-variance model comprises several limitations documented in the literature, which may distort the portfolio effects of commodities in out-of-sample studies, such as estimation error maximization [Michaud (1989)], high transaction costs [Best and Grauer (1991)], and the presence of corner solutions [Broadie (1993)]. In addition, the mean-variance strategy does not necessarily present the most accurate depiction of the benefits from investing in commodities since it relies on two assumptions – either the distribution of asset

[3] According to Daskalaki and Skiadopoulos (2011), *"first generation indices mimic passive commodity futures portfolio strategies where only long positions in the constituent futures are allowed"*.

returns is normal, or investors' preferences are defined by a quadratic utility function [Markowitz (1952)]. However, neither of these assumptions is expected to hold throughout a realistic scenario. Regarding the first of these, Peiro (1999) demonstrated that stock indexes' returns are not normally distributed, especially for short-term horizons, while Gorton and Rouwenhorst (2006), and Kat and Oomen (2007), reached the same conclusion when considering commodity futures. As for the second assumption, according to Daskalaki and Skiadopoulos (2011), a quadratic utility function shows increasing absolute risk aversion with respect to wealth and negative marginal utility after a certain level of finite wealth, with neither of these findings being consistent with investors' status as rational actors. Barroso and Saxena (2020), in turn, demystify the fact that portfolio optimization usually struggles in out-of-sample scenarios. The authors prove that out-of-sample forecast errors are not totally random, but have predictable patterns instead, and hence, may be reduced using their own history.

Bessler and Wolf (2015), whose work inspired the current research, analyze for the 1983 to 2013 period, the in- and out-of-sample portfolio benefits arising from including commodities in a stock-bond portfolio for a wide range of investment strategies, overcoming the drawbacks of employing only the mean-variance model. They corroborate the theory that out-of-sample attainable benefits of including commodities in the investment universe are much lower when compared to in-sample analysis. The authors conclude that portfolio effects vary greatly between different types of commodities, suggesting that aggregate commodity indices, industrial metals, precious metals, and energy, improve the performance of a multi-asset portfolio for most of the investment strategies employed. For instance, when considering an aggressive investor, the authors report a Sharpe ratio of 0.81 for an out-of-sample portfolio composed of equities, bonds, and industrial metal commodities when utilizing the Black-Litterman model. A considerably higher value than that obtained from the stock-bond portfolio's Sharpe ratio of 0.58, despite the application of the same investment strategy. In contrast to these results, the authors find very small portfolio effects for agriculture and livestock commodities. They undertook to construct an equally weighted commodity index that excludes agriculture and livestock commodities, eventually concluding that it achieves superior performance for most asset allocation models. Moreover, Bessler and Wolf (2015) report that portfolio effects vary for sub-periods – for the period coinciding with the financial crisis, between 2008 and 2013, the authors report an absence of portfolio benefits for any commodity group. The exception to this case takes place solely when the risk-parity model is employed, since it does not consider the overall increasing correlations of commodities with traditional asset classes during the financial crisis [Silvennoinen and Thorp (2013), and Büyüksahin and Robe (2014)].

The work of Daskalaki et al.'s (2017) provide us with valuable insight into the current academic discourse on commodities, through the implementation of a non-parametric stochastic dominance efficiency (SDE[4]) approach in order to build optimal portfolios with and without commodities while addressing the research question of whether commodities should be included in investors' portfolios both in- and out-of-sample. The authors conclude that commodities do provide diversification benefits, with an effect becoming more evident when second and third generation commodity indices[5] are considered. Furthermore, and also worth mentioning, Cai et al. (2018) analyze co-movements and causality relationships between crude oil, precious metals, and agriculture products throughout a period ranging from 1986 through 2017. They report that commodity prices co-move in most of the frequencies employed and that causality relationships change over time for distinct time horizons, concluding that mixed commodities portfolios offer diversification benefits at the mid-term horizons.

Furthermore, several studies, namely those of Domanski and Health (2007), Mayer (2010), Cheng and Xiong (2013), and Adams and Glück (2015), reflect on the growth and financialization of commodity markets in recent years. In fact, the increasing interest in commodities itself has had positive effects, such as generating higher liquidity and boosting price efficiency. Nonetheless, the large inflow of investment capital into commodity markets over this past decade has raised concerns on whether and how financialization alters commodity prices. The works of Silvennoinen and Thorp (2013), and Büyüksahin and Robe (2014) for instance, sought to focus on the effect of financialization on the increasing correlations between commodities and traditional asset classes. Moreover, Tang and Xiong (2012), Gruber and Vigfusson (2012), and Basak and Pavlova (2013), identify increasing correlations within different commodity groups. Beckmann et al. (2014) further suggest that commodity prices may be influenced by monetary policy, which consequently generates higher correlations, constraining the portfolio effects of commodities. Moreover, an analysis of the consequences of financialization of commodity markets by Chen et al. (2019) conclude that there is

[4] The SDE approach overcomes the need to use a particular utility function to describe investors' preferences and does not make assumptions on the distribution of asset returns [Scaillet and Topaloglou (2010)].

[5] According to Daskalaki et al. (2017), *"second and third generation commodity indices mimic dynamic (long and long/short, respectively) commodity futures portfolio strategies, which exploit popular commodity trading signals such as momentum and switches from backwardated to contangoed markets and vice versa"*.

indeed an increasing co-movement between commodities' and stocks' returns, suggesting decreased diversification gains as a result of increased capital inflow in commodity markets.

Lastly, commodity investments have been subject to an ongoing and controversy filled discussion on whether commodities can be simultaneously ethical and profitable [Bessler and Wolff (2015)]. So far, the literature on this subject is far from reaching a consensus. Authors such as Cheng and Xiong (2013), and Pies et al. (2013), suggest the presence of incoherent empirical findings on the effects of commodity investments on prices and volatilities. The public is therefore perhaps justified for looking at investments in commodities with suspicion, with some critics blaming investors for increases in agriculture prices and volatility. For this reason, before offering products containing commodities, such as agriculture and livestock, asset management firms have undertaken to appraise the tradeoff between reputational costs and expected diversification benefits arising from commodity investments.

3. Data

3.1. Databases

Our data on stocks, bonds and aggregate and individual commodities was obtained from Thomson Reuters Datastream. All time-series are denominated in the US dollar. We extract monthly total return index data for stocks, bonds, and commodities and, for the relevant time period, specifically between January 1989 through December 2019. For the calculation of the optimized portfolios in the out-of-sample setting, we consider a rolling window of 36 months. As such, the evaluation period has a start date of three years later, in January 1992.

Stocks are represented by the S&P 500 index, while bonds are represented by the US benchmark 10-year government bond index. For the commodity investments, we consider the S&P Goldman Sachs commodity index family, which, according to Tang and Xiong (2012), has a leading role in the commodity market, given that its performance is tracked by several index funds. We picked two commodity indices – the diversified S&P GSCI index, and the S&P GSCI light energy index. The former one allows investors to track the average price evolution of five commodities – energy, industrial metals, precious metals, livestock, and agriculture. As of the 31st of December 2019, the dollar weights of the GSCI index stood at 62.64% for energy, 11.16% for industrial metals, 4.14% for precious metals, 6.65% for livestock, and 15.41% for agriculture. One can observe a strong overweight of the energy sector, an effect brought about by the S&P GSCI status as a production-based index. The S&P GSCI light energy index in turn, allows us to overcome this constraint given that the production weights in the energy sector are divided by four, thereby increasing the relative weights of non-energy S&P GSCI constituents, while providing a "lower exposure to energy and a more balanced weighting of different commodity groups" [Bessler and Wolff (2015)]. Moreover, our data set includes individual commodities, thereby enabling us to study the relative benefits of different commodity types. Hence, we extract data on the S&P GSCI energy, S&P GSCI industrial metals, S&P GSCI precious metals, S&P GSCI livestock, and S&P GSCI agriculture. Finally, for the risk-free rate we consider the yield of a three-month US T-Bill.

Our final sample is composed of 3,720 observations, corresponding to the monthly asset returns of the S&P 500 index, the US government bond index, the US T-Bill, the two S&P GSCI aggregate indices, and the five S&P GSCI individual indices, covering the 372 months from January 1989 through December 2019.

3.2. Descriptive statistics

Table 1 reports average statistics for the monthly asset returns of stock and bond indices, the risk-free rate, and seven commodity indices, for an evaluation period running from January 1992 to December 2019, comprising a total of 336 monthly observations. The annualized[6] means are 10.36% p.a. for the S&P 500 index, 5.52% p.a. for the government bond index, 2.58% p.a. for the S&P GSCI index, and 1.25% p.a. for the S&P GSCI light energy index. The aggregate commodity indices show average annualized returns as being considerably lower when compared to the stock index, a difference which can be explained by substantial price falls in commodities in recent years. Of particular relevance is the consideration that the annualized mean for the January 1992 to December 2013 sub-period for the S&P GSCI (S&P GSCI light energy) is 5.62% p.a. (3.43% p.a.), whereas the average annualized return for the more recent sub-period from January 2014 to December 2019 for the S&P GSCI (S&P GSCI light energy) returns -8.58% p.a. (-6.70% p.a.). For the individual commodity groups, average returns vary greatly, from -1.27% for agricultural products to 6.71% for precious metals. The two risk measures used, annualized[7] standard deviation and non-parametric value-at-risk, suggest that all commodity investments are substantially riskier than both bonds and equities. Energy, industrial metals, agriculture, and the S&P GSCI all exhibit the highest return volatilities and values-at-risk. For the period between 1992 to 2019, the average risk-free rate is 2.48% p.a., considerably higher than the average returns of S&P GSCI light energy, livestock, and agricultural products, providing us with negative Sharpe ratios for all three commodity groups (-0.08, -0.21 and -0.20, respectively). In addition, the Sharpe ratios for the remaining commodity groups are lower than those for equities and bonds, providing a foundation to the theory that commodity indices are not an attractive asset class as a stand-alone investment. Lastly, the Jarque-Bera tests return statistically significant results at the 5% level for all asset classes, rejecting the assumption of normality of returns distribution.

[6] We compute the annualized means by multiplying the monthly statistic by 12.
[7] We compute the annualized standard deviation by multiplying the monthly statistic by the square root of 12.

Table 1: Descriptive statistics of asset returns

Table 1 reports sample moments (mean, standard deviation, skewness, and kurtosis), Sharpe ratios, Value-at-Risk, and Jarque-Bera statistics for the stock and bond indices, the risk-free rate, and the seven commodity indices considered in our final sample. The evaluation period covers 336 months from January 1992 to December 2019. 'Mean' is defined as the annualized time-series mean of monthly returns; 'Std. Dev.' is the associated annualized volatility; 'Skewness' and 'Kurtosis' correspond to the third and fourth moment of the return distribution, respectively; 'Sharpe Ratio' is the annualized ratio of the average return earned in excess of the risk-free rate per unit of volatility of the respective asset classes; '$VaR_{95\%}$' denotes the non-parametric 95%-value-at-risk for the respective asset classes during the period from January 1992 to December 2019; 'JB (*p*-value)' corresponds to the *p*-value of the Jarque-Bera test for determining whether sample data have values of skewness and kurtosis that follow a normal distribution.

	Mean (%)	Std. Dev. (%)	Skewness	Kurtosis	Sharpe Ratio	VaR95% (%)	JB (*p*-value) (%)	Obs.
S&P 500	10.36	14.00	-0.71	4.46	0.56	12.66	0.00	336
US GOV	5.52	7.08	0.15	4.51	0.43	6.13	0.00	336
T-Bill	2.48	2.08	0.25	1.55	0.00	0.94	0.00	336
GSCI	2.58	20.50	-0.36	4.60	0.00	31.15	0.00	336
GSCI LE	1.25	14.42	-0.75	6.74	-0.08	22.46	0.00	336
Energy	4.79	29.48	0.04	4.05	0.08	43.70	0.05	336
Ind. Metals	5.15	19.23	-0.15	5.07	0.14	26.47	0.00	336
Prec. Metals	6.71	16.26	0.05	4.09	0.26	20.03	0.02	336
Livestock	-0.54	14.12	-0.32	3.26	-0.21	23.78	3.46	336
Agriculture	-1.27	18.96	0.20	4.34	-0.20	32.45	0.00	336

Although commodities may not represent an attractive asset class as stand-alone investments, they can provide diversification benefits when included in a multi-asset portfolio, thereby improving the investor's risk-return profile. To ensure such an effect, the correlations between commodities and the traditional asset classes, namely equities and bonds, must be low or preferentially negative. Table 2 reports the correlation coefficients of the asset returns over our January 1992 to December 2019 timeframe, enabling us to derive some knowledge on the potential diversification gains from adding commodities to a multi-asset portfolio. Regarding the correlation between different commodities with various stocks (column 1), we observe low but significantly positive correlations for the aggregate commodity indices, energy, and agriculture. The correlation between the S&P 500 index and industrial metals is statistically significant at the 1%-level, with the correlation coefficient being considerably higher (0.4) than for the other pairs of stocks and commodities. For precious metals and livestock, we do not find a significant correlation with stocks, rendering these types therefore suitable for complementing a stock-only portfolio. As regards the correlation between commodities and bonds (column 2), one can discern that bond returns are significantly negatively correlated with the aggregate commodity indices, energy, and industrial metals. Livestock and agriculture possess non-significant negative correlations with bond returns, while precious metals demonstrate significantly positive correlations. Similar to the findings of Bessler and Wolff (2015), the correlation matrix of asset returns shows that in overall terms, correlations between the traditional asset classes and commodity indices are low, and in many cases even negative, thus suggesting that commodities are an auspicious asset class for enhancing the risk-return tradeoff of portfolios originally composed of stocks and bonds.

Table 2: Correlation matrix of asset returns

The table below shows the correlation matrix for the stock and bond indices, the risk-free rate and the seven commodity indices for the evaluation period from January 1992 to December 2019. '*' and '**' indicate values significantly different from 0 at the 1% and 5% level, respectively.

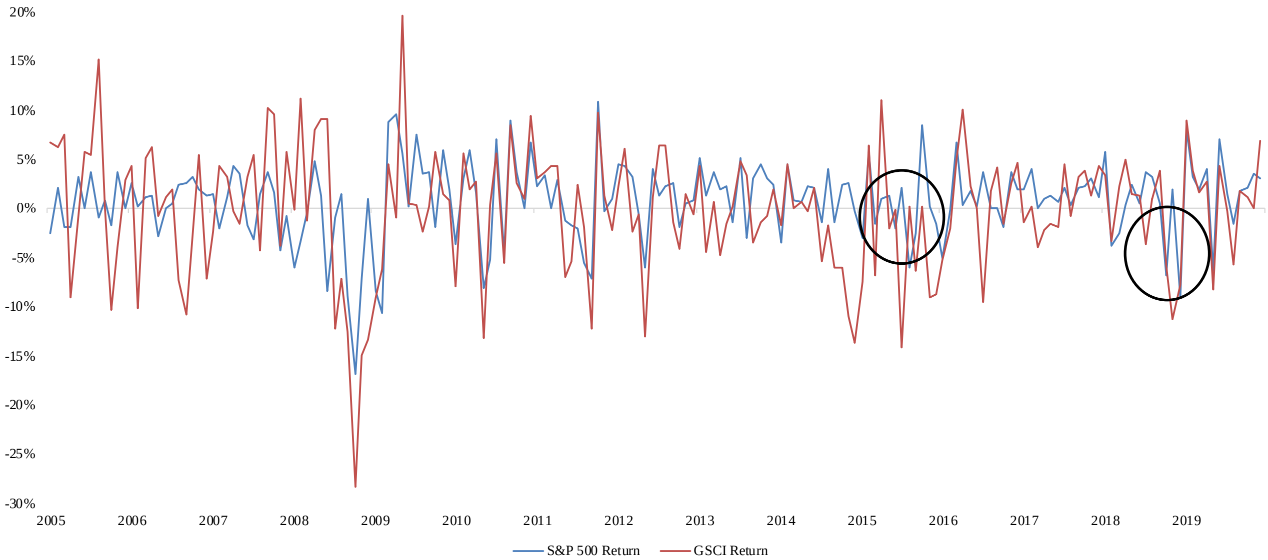
Correlation		Basis portfolio			Commodities						
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	S&P 500	1.00	-0.19*	0.03	0.28*	0.34*	0.23*	0.4*	0.04	0.06	0.23*
(2)	US GOV	-0.19*	1.00	0.04	-0.14**	-0.14**	-0.11**	-0.23*	0.18*	-0.07	-0.03
(3)	T-Bill	0.03	0.04	1.00	0.08	0.08	0.08	0.05	-0.03	0.04	0.04
(4)	GSCI	0.28*	-0.14**	0.08	1.00	0.92*	0.97*	0.44*	0.25*	0.16*	0.35*
(5)	GSCI LE	0.34*	-0.14**	0.08	0.92*	1.00	0.81*	0.58*	0.36*	0.23*	0.62*
(6)	Energy	0.23*	-0.11**	0.08	0.97*	0.81*	1.00	0.32*	0.17*	0.10	0.18*
(7)	Ind. Metals	0.4*	-0.23*	0.05	0.44*	0.58*	0.32*	1.00	0.32*	0.06	0.28*
(8)	Prec. Metals	0.04	0.18*	-0.03	0.25*	0.36*	0.17*	0.32*	1.00	-0.06	0.25*
(9)	Livestock	0.06	-0.07	0.04	0.16*	0.23*	0.10	0.06	-0.06	1.00	-0.01
(10)	Agriculture	0.23*	-0.03	0.04	0.35*	0.62*	0.18*	0.28*	0.25*	-0.01	1.00

Figure 1 describes the behavior over time of monthly returns of stocks, bonds and commodities for the period corresponding to the last 15 years of our final sample (2005-2019). Panel A depicts our comparison between stock and commodity returns, while Panel B shows the evolution of bond and commodity returns, with the aim of analyzing correlation through time and identifying periods of negative correlation between asset pairs. A visual analysis of Panel A and Panel B from Figure 1 allows us to identify four relevant periods in the world's economy where the correlation between commodities and a traditional asset class, such as stocks and/ or bonds, was noticeably negative. First, during the period between 2007 and 2010, roughly corresponding to that of the Subprime Mortgage Crisis, an overall large decline in stock markets was felt. Commodity markets also shrank during the years following the crisis, with commodity returns reaching considerable negative values, lower than even those shown by stocks. On the contrary, we can observe a large spike in bond returns during this same period, reaching an average monthly return of approximately 10% by the end of 2008. These

Figure 1: Average returns through time

Figure 1 portrays the evolution of the average monthly returns of stocks, bonds, and commodities throughout the last 15 years of our full sample period, directly corresponding to the period between January 2005 to December 2019. Stocks are represented by the S&P 500 index; bonds are represented by the US benchmark 10-year government index; while commodities are represented by the aggregate commodity index, S&P GSCI. Panel A itself compares stock and commodity returns, while Panel B depicts the evolution of bond and commodity returns. The circles in black show our identified periods of negative correlations between the pairs of assets.

Panel A: Comparisson between stock and commodity returns



Panel B: Comparisson between bond and commodity returns

values resulted in positive correlations between stock and commodity returns, while simultaneously leading to negative correlations between bond and commodity returns in the period immediately following the 2008 financial crisis. Second, stock markets experienced a significant drop during 2011 due to concerns with the United States' slowdown in economic growth, along with a simultaneous credit rating downgrade by Standard and Poor's from AAA to AA+ as a result of an event known in the investing community as "Black Monday", which took place on August 8th, 2011. A few months prior to 2011, a period of negative correlation between bond and commodity returns can be observed in Panel B. Third, the political and financial uncertainty during the 2015-2016 period, specifically due to the Chinese GDP growth slowdown, Greek debt default, the end of quantitative easing in the United States, and the Brexit referendum, marked a period of negative correlations between the two traditional asset classes and commodities. Finally, we can identify a period of negative correlations between stocks and commodities, and bonds and commodities, between 2018 and 2019, marked by the US-China trade war and Brexit developments.

4. Methodology

4.1. Out-of-sample estimation procedure

In our analysis, we implement out-of-sample asset allocation strategies, which consider data that investors would have had available when making portfolio investment decisions. We opted not to follow an in-sample approach – although we could have obtained insight into the maximum possible benefits of including commodities in a stock-bond portfolio, in-sample strategies cannot be implemented by investors in real time.

In order to then implement out-of-sample asset allocation strategies, we rely on a rolling sample approach to compute out-of-sample performances for the seven asset allocation models covering the evaluation period from January 1992 through December 2019. This approach follows on DeMiguel et al. (2009) and Bessler and Wolff (2015)'s published works. Given our $T = 336$ months dataset of asset returns, we pick an estimation window of length $M = 36$ months. For the first trading day of each month t, starting from $t = M + 1$, we compute portfolio weights by using the data of the previous M months. These weights are then used to assess portfolio performance during the following month $[t, t + 1]$. We then repeat this process by summing up the return of the next period in our dataset and ignoring the earliest return, until we reach the end of our dataset.

4.2. Asset allocation models

To analyze the diversification gains for different types of commodities when included in a stock-bond portfolio, we considered seven distinct asset allocation models within our analysis, following on the work of Bessler and Wolff (2015). Notice that all strategies were computed with monthly rebalancing. Table 3 presents an overview of the asset allocation models picked for our study. We employ two naïve asset allocation rules, one with equally-weighted portfolios (1/N), and another with strategically-weighted portfolios (st.w.), as well as two simple asset allocation strategies, reward-to-risk timing (RRT), risk-parity (RP). These approaches are tested along three portfolio optimization strategies, namely, minimum-variance (MinVar), mean-variance (MV), and Black-Litterman (BL).

The asset allocation strategies employed require different numbers of input parameters. For instance, the implementation of the naïve asset allocation rules do not require the estimation of any input parameters. As for the risk-based strategies, risk-parity (RP) does require the estimation of asset volatilities, while minimum-variance (MinVar) relies on the estimation of both volatilities and correlations. The reward-to-risk timing (RRT), mean-variance (MV), and Black-Litterman (BL) models are built on the tradeoff between risk and return, hence requiring the estimation of asset returns, volatilities and correlation matrix. According to Bessler and Wolff (2015), within an in-sample setting, MV dominates the remaining strategies, therefore we computed this investment strategy solely in the analysis of the in-sample benefits of adding commodities to a stock-bond portfolio. However, MV has been subject to some critique within the literature regarding its inferior performance in out-of-sample studies due to estimation errors in the input parameters, as stated by Garlappi et al. – *"The classical mean-variance approach to portfolio selection estimates the moments of asset returns via their sample counterparts and ignores the estimation error"* [Garlappi et. al (2007)]. For this reason, throughout our application of an out-of-sample approach, we complement the employment of MV with six additional investment strategies. Some authors favor risk-based strategies, such as minimum-variance and the risk-parity models, arguing that *"estimation errors in returns are higher than in the covariance matrix"* [Bessler and Wolff (2015) as cited in Chopra and Ziemba (1993)]. Lastly, there are other investment strategies which aim to overcome the MV strategy's estimation errors by adopting Bayesian estimation methods, as in Black and Litterman (1992), or through re-sampling techniques as in Michaud (1989).

Table 3: Summary of asset allocation models considered for the empirical analysis

The table below provides an overview of the seven asset allocation models chosen for our study. The last column of the table shows relevant abbreviations, in line with those defined in Bessler and Wolff (2015), which we will employ from now on when referring to different asset allocation strategies in future tables.

#	Model	Abbreviation
Naive asset allocation rules with rebalancing		
(1)	Equal weights	1/N
(2)	Strategic weights	st.w.
Simple asset allocation strategies		
(3)	Reward-to-risk timing	RRT
(4)	Risk-parity	RP
Portfolio optimization models		
(5)	Minimum-varince	MinVar
(6)	Sample-based mean-variance	MV
(7)	Black-Litterman	BL

Throughout our analysis, we distinguish between conservative and aggressive investor clienteles and analyze the benefits of commodities for each type of investor separately. We refrain from implementing all strategies for the two investor types. For instance, in the out-of-sample setting we only employ risk-parity and minimum variance strategies for the conservative investor type. Given that these strategies aim to attribute higher weights to assets which present lower volatility, such as bonds, they should therefore be preferable to conservative investors. The 1/N and reward-to-risk timing are implemented exclusively for the aggressive investor profile. These strategies allocate a considerable amount of wealth to stocks and commodities, thereby generating riskier portfolios when compared to risk-parity or minimum-variance, thus emerging as a more suitable option for aggressive investors.

4.2.1. Naïve asset allocation rules

We start by analyzing the diversification gains of adding commodities to a stock-bond portfolio by employing naïve asset allocation strategies, which rely neither on any theory or any data [Tu and Zhou (2011)]. This category of strategies has increased in popularity, especially with private investors [Benartzi and Thaler (2001)]. We build the portfolios with rebalancing in order to keep the weights for each asset constant over time.

A 1/N strategy is employed, distributing wealth equally to each of the N assets available at each rebalancing date. There exists a vast amount of literature comparing the performance of the 1/N rule with optimal portfolio choice theory. For instance, according to Jobson and Korkie (1980), *"naïve formation rules such as the equal weight rule can outperform the Markowitz rule"* due to estimation errors. Michaud (2008) further concludes that *"an equally-weighted portfolio may often be substantially closer to the true MV optimality than an optimized portfolio"*. With similar findings, DeMiguel et. al (2009) compare the 1/N rule with many extensions of the Markowitz rule and conclude that, in an out-of-sample setting, the benefit from optimal diversification is completely offset by the presence of estimation errors. Furthermore, we compute strategically-weighted portfolios (st.w.), in which each asset has a strategic weight, enabling us to define strategic weights customized to each type of investor clientele reflecting their level of risk aversion. Following Anson (1999), Erb and Harvey (2006), Conover et. al (2010), and Bessler and Wolff (2015), we set the strategic weights for commodities to 5% and 15% for conservative and aggressive investor types respectively. In line with Bessler and Wolff (2015)'s approach, the strategic weights for bonds are set at 80% (20%), and 15% (65%) for stocks, assigned to conservative (aggressive) investors. For the base case of a stock-bond portfolio, the strategic weight of commodities is zero, with more wealth being allocated to bonds when considering the conservative investor, whereas for the aggressive investor more wealth is allocated to stocks.

4.2.2. Risk-parity

The risk-parity strategy has become increasingly popular among investors since the 2008 financial crisis, "as frustrated investors have struggled to meet return targets by leveraging low-risk or low-beta assets" [Anderson et al. (2012)]. According to Maillard et al. (2010), institutional investors have been progressively looking at risk budgeting techniques, which means *"[analyzing] portfolios in terms of risk contributions rather than in terms of portfolio weights".* Furthermore, risk-parity has become a standard practice for hedge funds. This asset allocation model assigns the same volatility risk budget to each component of the portfolio, so that each asset class contributes equally to portfolio risk:

$$\boldsymbol{w_i} = \frac{\mathbf{1}/\widehat{\boldsymbol{\sigma}}_i^2}{\sum_{i=1}^{N}(\mathbf{1}/\widehat{\boldsymbol{\sigma}}_i^2)} \qquad \mathbf{(1)}$$

Where $\widehat{\boldsymbol{\sigma}}_i^2$ stands for the sample variance. There is little consensus within the literature on the performance of the risk-parity strategy. Anderson et al. (2012) document that risk-parity strategies often outperform a 60/40 equity/bond fixed allocation and a value-weighted portfolio. Chaves et al. (2011) on the other hand, report that risk-parity does not consistently outperform equal-weighting or 60/40 portfolios, but suggest that it does perform substantially better when compared to minimum-variance and mean-variance portfolios. The RP model benefits from what Baker et al. (2011) call "low-risk anomaly" – *"low-volatility portfolios [...] have offered an enviable combination of high average returns and small drawdowns".* This means that low-risk assets offer a higher premium per unit of volatility. For our analysis, we expect a larger allocation to bonds, due to their lower volatility, when comparing to stocks and commodities.

4.2.3. Reward-to-risk timing

Kirby and Ostdiek (2012) developed a new method of portfolio selection, the reward-to-risk timing strategy, which determines portfolio weights based on the historical reward-to-risk ratio. This approach resembles a momentum strategy, since it considers both risk and return, overweighting asset classes with high returns and low volatilities. The portfolio weights of the RRT model are given by:

$$\boldsymbol{w_i} = \frac{\widehat{\boldsymbol{\mu}}_i^+ / \widehat{\boldsymbol{\sigma}}_i^2}{\sum_{i=1}^{N} (\widehat{\boldsymbol{\mu}}_i^+ / \widehat{\boldsymbol{\sigma}}_i^2)} \quad \textbf{(2)}$$

In which $\widehat{\boldsymbol{\mu}}_i$ represents the sample mean return of asset i; $\widehat{\boldsymbol{\sigma}}_i^2$ stands for the sample variance; and $\widehat{\boldsymbol{\mu}}_i^+ = \boldsymbol{max}\ (\widehat{\boldsymbol{\mu}}_i, \mathbf{0})$, to prevent short sales. Kirby and Ostdiek (2012) document that the RRT strategy exploits sample information in a way that mitigates the impact of estimation risk, overcoming the main drawback associated with the mean-variance strategy. Moreover, the authors conclude that RRT outperforms MV in many different settings after transaction costs, due to RRT maintaining turnover at a low level by not requiring optimization while computing portfolio weights directly by following Eq. (2) instead. Following our rolling sample approach, the reward-to-risk measure is estimated over the past 36 months, thus the change in portfolio weights from one month to the next is smaller due to the 35 identical return observations which are considered in the calculation of the portfolio weights in two consecutive months. Finally, for our study we expect the RRT strategy to allocate more wealth to both stocks and commodities, given that they usually generate higher returns when compared to bonds.

4.2.4. Minimum-variance

The minimum-variance approach, inspired by the early works of Haugen and Baker (1991), has increased in popularity with the introduction of the large number of indexes[8] being currently adopted by many exchange-traded funds. Minimum-variance strategies also benefit from an increased appreciation for risk management since the financial crisis, beyond their inherent low-risk anomaly benefits, just as with the RP. The MinVar portfolio is the portfolio with the lowest variance for a given covariance matrix, and provides the solution to the following minimization problem:

$$\min_{\boldsymbol{w}} \boldsymbol{w}' \boldsymbol{\Sigma} \boldsymbol{w} \quad \textbf{(3)}$$

Where $\boldsymbol{w}$ represents the vector of portfolio weights and $\boldsymbol{\Sigma}$ the covariance matrix of asset returns. The composition of the minimum variance portfolio depends exclusively on the correlation of assets, which can be estimated much more precisely than expected returns, thereby reducing the estimation

[8] For example, the FTSE Global Minimum Variance Index Series.

risk. For the MinVar approach, we expect a slightly larger allocation to commodities, when comparing to the RP model, due to low or even negative correlations with traditional asset classes.

4.2.5. Mean-variance

Modern portfolio theory, first introduced by Markowitz (1952), forms the basis for many investors' portfolio allocation decisions. The author studies the role of return and risk as criteria in the evaluation of portfolios and documents that all investors, regardless of their level of risk aversion, are drawn to the same set of efficient portfolios – portfolios which, for a certain level of risk, maximize an investor's total return. Hence, in the MV model, the investor optimizes the tradeoff between the mean and variance of portfolio returns:

$$\max_{\boldsymbol{w}} \boldsymbol{U} = \boldsymbol{w}'\boldsymbol{\mu} - \frac{\boldsymbol{\gamma}}{\mathbf{2}}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \tag{4}$$

In which $\boldsymbol{U}$ is the investor's utility, $\boldsymbol{w}$ is the vector of portfolio weights, $\boldsymbol{\mu}$ is the vector of return estimates, $\boldsymbol{\Sigma}$ is the covariance matrix, and $\boldsymbol{\gamma}$ is the risk aversion coefficient. We set the risk aversion parameter equal to 10 and 2 for conversative and aggressive investors respectively, relying on the findings of Fletcher and Hillier (2005), and Daskalaki and Skiadopoulos (2011). Moreover, we consider an upper volatility bound as a constraint in the mean-variance optimization problem in order to obtain portfolios with similar volatility as strategically-weighted ones. Considering the historical volatility of strategically-weighted portfolios previous to the evaluation period, we posit that the conservative (aggressive) investor types accept a maximum annualized expected portfolio volatility of 7.5% (13.5%). By constraining the input parameters, we avoid extreme portfolio allocations and improve the out-of-sample performance [Jagannathan and Ma (2003)].

4.2.6. Black-Litterman

The BL model belongs to the category of quantitative asset allocation models, being widely used in the finance industry by global investors, such as pension funds and insurance companies. The model proposes a Bayesian methodology as a more effective combination of currently held managers' opinions and historical data [Satchell and Scowcroft (2000)].

The BL model aims at minimizing the estimation errors in return estimates. This is done by combining information from two distinct sources in order to create a final estimate of expected returns – 'implied' returns, and thus to assess what the current market tells investors about expected excess equilibrium returns, along with 'subjective' return estimates which reflect investment managers' views. The model offers two major advantages. Firstly, for each asset, investors can choose between producing return forecasts or staying neutral. Secondly, the reliability of each return estimate can be covered, enabling investors to discriminate between realistic estimates and simple guesses. Hence, the general idea behind the BL model is that investors should only depart from the reference portfolio (market or benchmark portfolio picked) if they can produce reliable return forecasts.

We implement a sample-based version of the BL model, following Bessler and Wolff (2015)'s and Bessler et al. (2017)'s approaches. To calculate a combined return estimate, which consists of a matrix-weighted average of implied returns and subjective returns in relation with the correlation structure [Lee (2000)], we follow the original Black and Litterman (1992) formula:

$$\hat{\boldsymbol{\mu}}_{\boldsymbol{BL}} = [(\boldsymbol{\tau}\Sigma)^{-\mathbf{1}} + \boldsymbol{P}'\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{P}]^{-\mathbf{1}}[(\boldsymbol{\tau}\Sigma)^{-\mathbf{1}}\boldsymbol{\Pi} + \boldsymbol{P}'\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{Q}] \quad \mathbf{(5)}$$

Where $\boldsymbol{\Pi}$ is the vector of implied asset returns, $\boldsymbol{\tau}$ is a parameter which measures the reliability of implied return estimates, $\boldsymbol{Q}$ is the vector of subjective returns, $\boldsymbol{\Omega}$ is a diagonal matrix, where the reliability of each subjective return appears on the diagonal, Σ is the covariance matrix, and $\boldsymbol{P}$ is the identity matrix.

In the original model, implied excess returns are derived using a reverse optimization technique, given that the market or benchmark portfolio weights result from a mean-variance optimization. In line with Bessler and Wolff (2015), we compute implied returns based on the strategic weights defined in section 4.2.1., applying the formula below:

$$\boldsymbol{\Pi} = \boldsymbol{\gamma}\Sigma\boldsymbol{\varphi} \quad \mathbf{(6)}$$

Where $\boldsymbol{\gamma}$ represents the risk-aversion coefficient, Σ is the covariance matrix and $\boldsymbol{\varphi}$ is the vector of portfolio weights of the reference portfolio (in our study, the strategic weights). However, we are aware that the reference portfolio picked to compute implied excess returns naturally influence the BL model's out-of-sample performance. The factor τ measures the uncertainty of implied returns. Black and Litterman (1992) state that this parameter usually ranges from 0.025 to 0.3, and for the purposes of our study τ is thus set to 0.3. Views are computed as the sample means of the respective asset returns. Finally, for a portfolio of *N* assets, Ω is a *N* x *N* diagonal matrix. The reliability of the subjective returns is reflected in the diagonal of matrix Ω and is calculated as the variance of the historical forecast errors e_i during the sample period, using the same rolling estimation window as for return estimates[9]. Following the approach in Satchell and Scowcroft (2000), the posterior covariance matrix is thus derived as:

$$\sum_{\boldsymbol{BL}} = \sum + [(\boldsymbol{\tau}\sum)^{\mathbf{-1}} + \boldsymbol{P'\Omega^{-1}P}]^{\mathbf{-1}} \quad \textbf{(7)}$$

After obtaining the estimation of the combined returns and the posterior covariance matrix, we follow the traditional risk-return optimization problem – maximizing investor's utility as shown in Eq. (7), using the same utility function, risk-aversion coefficient, and constraints as in the MV model.

4.3. Performance measures

Our goal is to study the performance of portfolios which include commodities while comparing them with our base case, a stock-bond portfolio, across a variety of asset allocation models, while distinguishing between conservative and aggressive investors. We compute six performance measures without considering the use of leverage[10], the presence of transactions costs, or short sales.

We compute the portfolio's annualized expected return and volatility, as well as the annualized Sharpe ratio[11], defined as the sample mean of excess returns (over the risk-free asset), divided by their sample standard deviation. However, the Sharpe ratio implicitly assumes that investors are indifferent in discriminating between upside and downside risk, which may generate biased results when the distribution of returns is skewed – thus Sharpe measure generates portfolio rankings that *"are biased in that they implicitly regard the potential for large gain and large loss as* equally undesirable" [Keating and Shadwick (2002)]. In fact, for positively skewed return distributions, performance is achieved with less risk than the Sharpe measure proposes, whereas conversely, a standard deviation understates risk for negatively skewed return distributions. To

[9] The literature proposes many different ways to calculate matrix Ω. Meucci (2010) assumes an overall level of confidence in the views that is constant over time. However, in this approach, no time-varying information on the reliability of the subjective returns is included. Hence, the approach we employed does include time-varying information of the reliability of views, enabling us to obtain a superior out-of-sample performance.
[10] We do not present results for leveraged strategies, since *"the portfolio benefits of commodities do not change when using leverage in investment strategies"* - Bessler and Wolff (2015).
[11] Proposed by Sharpe (1994).

overcome this limitation we calculate a downside risk measure, the Sortino ratio proposed by Sortino and Price (1994), which estimates the average excess return relative to the downside deviation:

$$\boldsymbol{Sortino\ ratio} = \frac{\boldsymbol{R_p - r_f}}{\boldsymbol{\sigma_d}} \qquad \textbf{(8)}$$

Where $\boldsymbol{R_p}$ refers to the actual or expected portfolio return, $\boldsymbol{r_f}$ is the risk-free rate and $\boldsymbol{\sigma_d}$ represents the downside deviation, meaning the negative deviation of a portfolio return's from a threshold return, in our case presented as the average portfolio return. The Sortino ratio differs from the Sharpe measure in that it only considers the standard deviation of the downside risk, providing a better assessment of a portfolio's risk-adjusted performance, particularly for positively skewed return distributions, given that positive volatility can be considered a good outcome.

As an alternative measure, we compute the certainty equivalent return, which considers all the moments of the returns' distribution and is defined as the risk-free rate that an investor is willing to accept instead of adopting a particular risky portfolio strategy. We consider two alternative ways to obtain the certainty equivalent return – the first ($\boldsymbol{CE_1}$) follows a quadratic utility function, whereas the second ($\boldsymbol{CE_2}$) uses a power utility function. $\boldsymbol{CE_1}$ is computed through Eq. (9), following the work of DeMiguel et. al (2009).

$$\boldsymbol{CE_1 = \mu - \frac{\gamma}{2}\sigma^2} \qquad \textbf{(9)}$$

Where $\boldsymbol{\mu}$ and $\boldsymbol{\sigma^2}$ are the mean and variance of excess returns, while $\boldsymbol{\gamma}$ represents the risk-aversion coefficient, which equals 10 for the conservative investor, and 2 for an aggressive one.[12] $\boldsymbol{CE_2}$ was computed through the formula shown below, applying a power utility function with constant relative risk-aversion parameter ($\boldsymbol{\gamma}$), following Brandt (1999).

$$\boldsymbol{CE_2 = [(1-\gamma)\bar{U}]^{\frac{1}{(1-\gamma)}} - 1} \qquad \textbf{(10)}$$

In which $\boldsymbol{\bar{U}}$ represents the time-series average of the CRRA utility function for each monthly return.

[12] Following DeMiguel et al. (2009), Eq. (9) refers to the utility function of a mean-variance investor, which represents a good approximation of the CE of an investor with quadratic utility.

With the aim of evaluating the amount of trading necessary to implement each portfolio strategy, we further calculate the portfolio turnover of strategy i ($\boldsymbol{PT_i}$) as the average absolute change of the portfolio weights $\boldsymbol{w}$ through the $\boldsymbol{T}$ rebalancing points in time across $\boldsymbol{N}$ available assets, as demonstrated in the works of DeMiguel et. al (2009), and Bessler and Wolff (2015).

$$\boldsymbol{PT_i = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{N}\left(\left|w_{i,j,t+1} - w_{i,j,t+}\right|\right)} \tag{11}$$

Where $\boldsymbol{w_{i,j,t}}$ is the weight of asset j at time t in strategy i; $\boldsymbol{w_{i,j,t+}}$ represents the portfolio weight before rebalancing at $t + 1$; while $\boldsymbol{w_{i,j,t+1}}$ represents the desired portfolio weight after rebalancing at $t + 1$. The turnover quantity defined by Eq. (11) can be interpreted as the average percentage of wealth traded within each period. As documented by DeMiguel et. al (2009), we expect portfolio optimization models, specially BL and MV, to present higher turnover than simple asset allocation strategies, such as RP, or naïve asset allocation rules (st.w.).

5. Empirical results

5.1. Portfolio performance analysis

We analyze out-of-sample benefits of commodities by using two distinct portfolio allocation techniques – static portfolio allocation (Section 5.1.1), and dynamic portfolio allocation (Section 5.1.2). Throughout our employment of both allocation techniques, we distinguish between conservative and aggressive investors.

5.1.1. Static selection of commodities

We draw upon Bessler and Wolff's (2015) methodology as a starting point for our analysis. Thus, this section focuses on comparing our base portfolio, composed of equities and bonds, with seven portfolio types, in which commodities are picked in a consistent standard format. Each of the seven portfolios are combinations of stocks and bonds with a commodity, selected prior to analyzing portfolios' performances. This process is repeated across seven different asset allocation strategies.

Table 4 shows results when considering the conservative investor type within our full-time series, ranging from January 1992 through December 2019. The performance measures considered are all net of transaction costs. Note that improvements in comparison to the base portfolio are highlighted in bold.

The Sharpe ratio, Sortino ratio, along with certainty-equivalent returns, demonstrate that contrary to Bessler and Wolff's (2015) results, no commodity type consistently enhances the risk-return tradeoff of a stock-bond portfolio for all asset allocation strategies. Nevertheless, one can observe that precious metals provide substantial portfolio benefits, as can be seen with the BL, MV, RP, and st.w. strategies, whereas industrial metals work with BL, MinVar, RP and st.w. strategies. Precious metals offer the largest increase in certainty-equivalent returns with both BL (0.60% p.a.) and MV (0.24% p.a.) strategies, while industrial metals offer the largest increase for BL (0.44% p.a.) and RP (0.37% p.a.). Energy commodities slightly enhance the risk-return profile under the st.w. framework. All commodity types are able to reduce portfolio's volatility within the RP and st.w. frameworks – the light energy version of the S&P GSCI reduces portfolio volatility by 0.47% p.a.,

Table 4: Portfolio benefits of static selected commodities for conservative investor

Table 4 reports out-of-sample portfolio performance, both for stock-bonds portfolios, and portfolios complemented with commodities for different asset allocation strategies from January 1992 through December 2019. We present our results for the conservative investor profile, with a maximum volatility ceiling of 7.5% p.a. and a risk aversion coefficient equal to 10. The base portfolio is composed solely of US stocks and bonds, while our remaining portfolios are complemented with the indicated commodity group. Improvements relative to the stock-bond portfolio are marked in bold. 'Return' refers to the annualized time-series average of monthly returns. 'Volatility' denotes the corresponding annualized standard deviation. 'Sharpe' shows the annualized Sharpe ratio, while 'Sortino' stands for the annualized Sortino ratio for each portfolio. 'CE' denotes annualized certainty-equivalent return calculated using mean-variance utility function. 'Turnover'' represents the average turnover for each individual portfolio.

Asset Allocation strategy	Performance measure	Stock-bond	Stock-bond portfolio complemented with commodities						
			GSCI	GSCI LE	Energy	Industrial M.	Precious M.	Livestock	Agriculture
Conservative investor									
BL	Return (%)	8.16	7.34	7.22	7.67	**8.61**	**8.85**	6.14	6.71
	Volatility (%)	7.36	7.57	7.60	7.52	7.39	7.49	7.58	7.62
	Sharpe	0.77	0.64	0.62	0.69	**0.83**	**0.85**	0.48	0.55
	Sortino	1.08	0.91	0.85	0.98	**1.10**	**1.12**	0.59	0.66
	CE (%)	5.44	4.47	4.32	4.84	**5.88**	**6.04**	3.27	3.80
	Turnover (%)	8.58	10.15	11.16	9.51	**6.27**	10.28	16.12	**5.79**
MV	Return (%)	7.67	7.37	7.47	7.35	7.65	**8.12**	7.14	7.60
	Volatility (%)	7.12	7.33	7.56	7.20	7.38	7.42	7.23	7.58
	Sharpe	0.73	0.67	0.66	0.68	0.70	**0.76**	0.64	0.68
	Sortino	1.03	0.93	0.85	0.95	0.92	**1.08**	0.93	0.89
	CE (%)	5.13	4.69	4.61	4.75	4.93	**5.37**	4.52	4.73
	Turnover (%)	7.34	9.02	10.21	8.24	8.91	9.46	11.45	8.80
MinVar	Return (%)	6.90	6.04	5.68	6.27	6.70	6.37	5.46	5.81
	Volatility (%)	5.82	**5.75**	**5.52**	5.83	**5.41**	**5.67**	**5.59**	**5.75**
	Sharpe	0.76	0.62	0.58	0.65	**0.78**	0.69	0.53	0.58
	Sortino	1.05	0.84	0.79	0.88	**1.08**	0.98	0.74	0.83
	CE (%)	5.21	4.39	4.16	4.57	**5.24**	4.77	3.90	4.16
	Turnover (%)	3.36	4.31	4.44	3.92	4.39	4.54	4.52	4.38
RP	Return (%)	7.15	6.76	6.22	6.99	**7.31**	6.98	5.88	5.81
	Volatility (%)	6.01	**5.80**	**5.54**	**5.91**	**5.65**	**5.97**	**5.61**	**5.75**
	Sharpe	0.78	0.74	0.68	0.76	**0.85**	0.75	0.61	0.58
	Sortino	1.11	1.04	0.96	1.07	**1.20**	**1.13**	0.89	0.83
	CE (%)	5.35	5.08	4.69	5.25	**5.71**	5.20	4.31	4.16
	Turnover (%)	3.01	3.56	3.82	3.31	3.61	3.79	3.84	4.38
st.w.	Return (%)	6.38	6.22	6.16	6.34	6.35	**6.43**	6.06	6.03
	Volatility (%)	5.96	**5.69**	**5.67**	**5.78**	**5.64**	**5.85**	**5.65**	**5.76**
	Sharpe	0.65	**0.66**	0.65	**0.67**	**0.69**	**0.68**	0.63	0.62
	Sortino	0.99	0.97	0.95	0.98	**1.00**	**1.01**	0.95	0.93
	CE (%)	4.61	4.60	4.55	**4.67**	**4.76**	**4.72**	4.47	4.37
	Turnover (%)	1.83	1.98	1.91	2.08	1.96	1.93	1.91	1.96

with RP and industrial metals decreasing portfolio volatility by 0.33% p.a. under the st.w. framework. All commodities, with the notable exception of energy, reduce volatility when a MinVar strategy is applied. In contrast, only precious and industrial metals enhance portfolio returns, with precious metals improving portfolio return by 0.7% p.a. with BL and 0.45% p.a. with MV, while industrial metals increase portfolio return by an average 0.45% p.a. with BL. In the specific terms of portfolio turnover, agriculture and industrial metals both see a reduction of 2.79% and 2.31%, respectively.

A comparison of the results for different asset allocation strategies reveals that BL performs best in terms of risk-return tradeoff when precious and industrial metals are included in the base portfolio. In fact, BL's higher performance was already documented in the literature by Kirby and Ostdiek (2012), and Bessler and Wolff (2015). We observe further improvements in portfolio performance for MV when precious metals are included, in direct contradiction to Daskalaki and Skiadopoulos (2011)'s work, who did not find portfolio improvements of including commodities for investors applying an MV strategy.

Table 5 depicts results when considering an aggressive investor. Once again, there is no commodity class which consistently improves the risk-return tradeoff of a stock-bond portfolio for all asset allocation strategies. Similar to our results for a conservative investor, precious metals provide the largest enhancement of the risk-return profile. Certainty-equivalent returns increased by 1.32% p.a.[13] , 1.09% p.a., and 0.68%, when applying BL, MV, and RRT strategies, respectively. The Sharpe and Sortino ratios show some small improvements in portfolio performance when livestock is added to a stock-bond portfolio within an RRT model. All commodity types were shown to reduce portfolio volatility under the st.w. framework (livestock reduces a portfolio's volatility by 1.72% p.a.) all asset allocation strategies. Similar to our results for a conservative investor, precious metals provide the largest enhancement of the risk-return profile. Certainty-equivalent returns increased by

Table 5: Portfolio benefits of static selected commodities for aggressive investor

Table 5 reports out-of-sample portfolio performance for stock-bonds portfolios, and portfolios complemented with commodities for different asset allocation strategies from January 1992 through December 2019. We present results for

[13] This value means that, on average, every year, we could cover 1.32% extra fees from adding precious metals to a portfolio initially composed solely of equities and bonds.

an aggressive investor profile, accepting a maximum volatility ceiling of 13.5% p.a. and a risk aversion coefficient equal to 2. The base portfolio is composed solely of US stocks and bonds, while our remaining portfolios are complemented with the indicated commodity group. Improvements relative to the stock-bond portfolio are marked in bold. 'Return' refers to the annualized time-series average of monthly returns. 'Volatility' denotes the corresponding annualized standard deviation. 'Sharpe' shows the annualized Sharpe ratio, while 'Sortino' stands for the annualized Sortino ratio for each portfolio. 'CE' denotes annualized certainty-equivalent return calculated using mean-variance utility function. 'Turnover'' represents the average turnover for each individual portfolio.

Asset Allocation strategy	Performance measure	Stock-bond	Stock-bond portfolio complemented with commodities						
			GSCI	GSCI LE	Energy	Industrial M.	Precious M.	Livestock	Agriculture
Aggressive investor									
BL	Return (%)	10.11	9.08	9.20	9.66	**10.30**	**11.82**	**10.13**	**11.55**
	Volatility (%)	10.68	11.68	11.89	11.70	12.18	12.45	11.97	12.65
	Sharpe	0.71	0.57	0.57	0.61	0.64	**0.75**	0.64	**0.72**
	Sortino	0.98	0.80	0.74	0.87	0.89	**1.04**	0.88	0.90
	CE (%)	8.97	7.72	7.79	8.29	8.82	**10.29**	8.69	**9.95**
	Turnover (%)	9.97	14.82	14.81	14.15	**5.42**	12.89	19.14	**4.47**
MV	Return (%)	10.43	9.84	9.66	10.00	10.42	**11.89**	9.90	8.98
	Volatility (%)	10.72	11.86	12.18	11.72	12.19	12.22	10.90	11.74
	Sharpe	0.74	0.62	0.59	0.64	0.65	**0.77**	0.68	0.55
	Sortino	0.99	0.86	0.72	0.93	0.86	**1.03**	0.92	0.69
	CE (%)	9.28	8.43	8.18	8.63	8.93	**10.37**	8.71	7.60
	Turnover (%)	12.76	14.07	16.07	13.74	15.91	16.02	15.21	15.26
RRT	Return (%)	8.93	8.42	7.98	8.77	**9.16**	**9.63**	8.25	8.28
	Volatility (%)	6.40	6.42	**6.07**	6.63	6.49	6.54	**6.32**	**6.29**
	Sharpe	1.01	0.93	0.91	0.95	**1.03**	**1.09**	**0.91**	0.92
	Sortino	1.26	1.15	1.06	1.21	**1.28**	**1.31**	**1.10**	1.11
	CE (%)	8.53	8.01	7.61	8.33	**8.74**	**9.21**	7.85	7.89
	Turnover (%)	9.18	10.45	11.08	10.14	10.21	10.90	11.18	10.42
1/N	Return (%)	8.06	6.20	5.75	6.96	7.05	7.59	5.13	4.92
	Volatility (%)	7.21	9.13	7.65	11.64	**6.72**	7.70	**6.83**	8.70
	Sharpe	0.77	0.41	0.43	0.38	0.68	0.66	0.39	0.28
	Sortino	1.05	0.53	0.52	0.57	0.71	0.96	0.53	0.40
	CE (%)	7.54	5.37	5.17	5.61	6.60	7.00	4.66	4.16
	Turnover (%)	2.41	3.09	2.63	3.78	3.01	2.78	2.68	2.97
st.w.	Return (%)	9.50	8.31	8.10	8.65	8.69	8.93	7.82	7.73
	Volatility (%)	11.02	**10.19**	**9.86**	**10.79**	**10.36**	**9.43**	**9.30**	**9.38**
	Sharpe	0.64	0.57	0.57	0.57	0.60	**0.68**	0.57	0.56
	Sortino	0.83	0.72	0.72	0.74	0.78	**0.90**	0.74	0.69
	CE (%)	8.29	7.27	7.13	7.48	7.61	8.04	6.96	6.85
	Turnover (%)	2.90	3.08	**2.87**	3.39	3.05	2.94	2.90	3.03

1.32% p.a.[14] , 1.09% p.a., and 0.68%, when applying BL, MV, and RRT strategies, respectively. The Sharpe and Sortino ratios show some small improvements in portfolio performance when livestock is added to a stock-bond portfolio within an RRT model. All commodity types were shown to reduce portfolio volatility under the st.w. framework (livestock reduces a portfolio's volatility by 1.72% p.a.) yet again, with only precious and industrial metals enhancing portfolio returns – precious metals improve portfolio return by 1.71% with BL, while industrial metals increase returns by 0.22% p.a. when an RRT approach is applied. Regarding turnover, we see a reduction by the inclusion of agriculture and industrial metals of 5.50% and 4.55%, respectively.

Similarly to our results for the conservative investor, BL provides the largest gains in terms of portfolio returns, Sharpe and Sortino ratios, and certainty-equivalent returns. In line with Bessler and Wolff's (2015), and DeMiguel et al. (2009), we observe that MV does not consistently outperform naïve strategies across the different portfolios constructed, and that no commodity class improves the risk-return tradeoff of a stock-bond portfolio when applying an equally-weighted 1/N strategy.

The results for static portfolio allocation derived from tables 4 and 5 can be summarized as follows: first, performance effects are not consistent across commodity types and asset allocation models; second, precious metals are the top class of commodities in terms of enhancing the risk-return profile of a stock-bond portfolio; third, BL appears to be the asset allocation model which delivers portfolios consisting of combined stocks, bonds, and commodities, with superior performance; and finally aggressive investors achieve a slightly higher maximization in terms of risk-return tradeoff by adding precious metals to a stock-bond portfolio under the BL framework , since certainty-equivalent improves 1.32% for the aggressive comparing to 0.60% for the conservative investor.

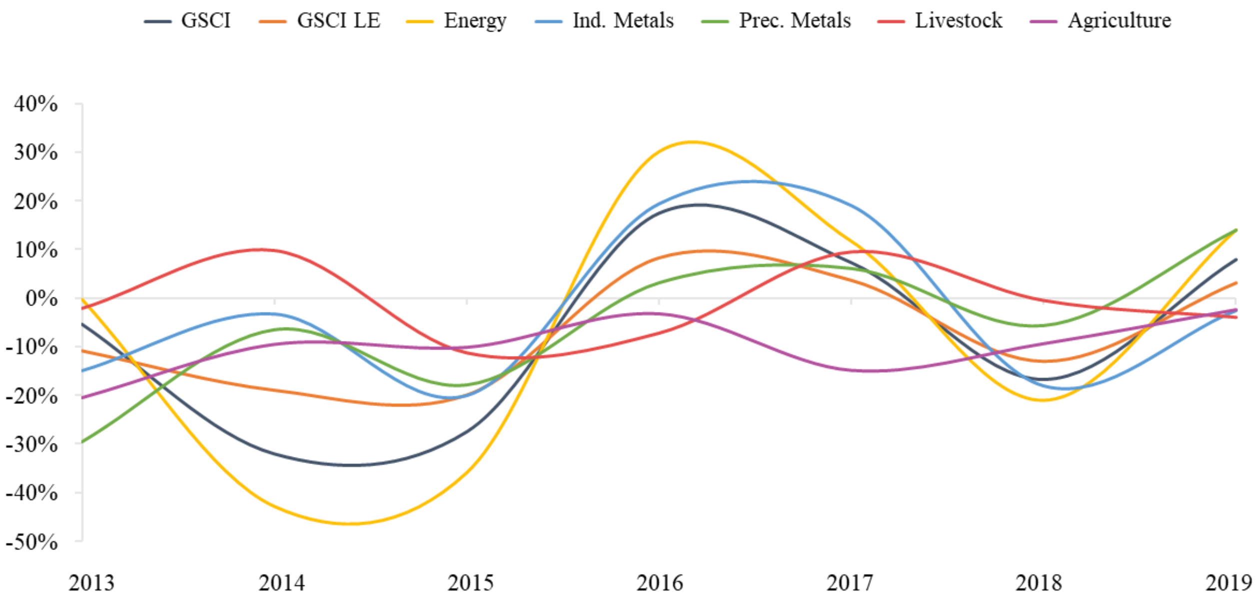
Static portfolio allocation analysis suggests that positive out-of-sample performance effects from adding commodities to a stock-bond portfolio are present but appear to be capped. A comparison with Bessler and Wolff's original work which inspired the present study shows that they found stronger and more consistent results across portfolio types and asset allocation strategies. Nevertheless, discrepancies identified between our results and Bessler and Wolff's conclusions may

[14] This value means that, on average, every year, we could cover 1.32% extra fees from adding precious metals to a portfolio initially composed solely of equities and bonds.

be explained by both sets of studies being conducted across different time frames and weights being chosen with respect to different data samples – our period of analysis ends in December 2019, whereas Bessler and Wolff's ends in December 2013. According to Worah (2011), and as further cited in Baker et al. (2018), commodities are quite significantly correlated with global growth, as such macroeconomic factors can be expected to play an important role in determining commodities' returns. There were many changes taking place within the macroeconomic environment between 2013 and 2019, negatively affecting commodity returns and consequently limiting positive commodities' portfolio effects. Figure 2 shows the evolution of commodities' returns for the period ranging between 2013 and 2019.

Figure 2: Commodities' returns (Dec 2013 – Dec 2019)

Figure 2 portrays the evolution of the annual returns of seven commodity types, two of them being aggregate commodity indices (GSCI, and GSCI Light Energy), while the remaining five are individual commodities (energy, industrial metals, precious metals, livestock, and agriculture), covering the time period between December 2013 through December 2019.



From Figure 2, one can observe that a substantial decline in overall commodity prices took place between 2013 and 2015. A possible explanation for this negative trend could be that the sizable downward adjustment in the pace of economic growth in China, characterized for its manufacturing-intensive economy, along with a similar trend in several other emerging economies at that time. Consequently, by January 2015 all commodity classes depicted in Figure 2, with the exception of livestock, showed negative returns, with energy presenting the lowest return at approximately -40%, a negative trend which was inverted during 2016. The following year, however, saw commodity prices fall once more, driven by an overall decline in the price of energy commodities throughout 2017, with all commodity classes eventually achieving negative returns by the end of 2018. The World Bank (2018) documented that metal prices dropped by approximately 10% during the third quarter of 2018 due to weak global demand, along with concern regarding the consequences of the US-China trade dispute on growth in China, which accounts for 50% of the worldwide demand for metals. Conversely, the closure of the world's largest supplier of aluminum alongside a production shortage in China, ended up constraining the supply side, inadvertently supporting the prices of several metals. Overall agricultural prices on the other hand, fell by nearly 7% in the third quarter of 2018. Robust supply of most oilseeds and grains, with the exception of wheat, trade tensions between US and China, along with depreciation in the currency value of emerging markets and developing economies', particularly of the Brazilian real, are some of the reasons for the weakness of agricultural prices in this period. Figure 2 shows that for all seven commodity types we are considering for analysis, a positive trend can be observed at the beginning of 2019 – as commodity-specific supply shocks boosted the prices of many commodities. However, the World Bank (2019) documented a market turnaround during the third quarter of 2019 (this cannot be visually confirmed by Figure 2, given our use of annual data for graph construction), in which 60% of the categories of commodities experienced price drops. This decline can be explained by the sharp slowdown in manufacturing and trade of various goods, alongside a deteriorating macroeconomic environment in general, and in particular due to significant trade tensions between the US and China, weighing negatively on the global demand for commodities.

Finally, taking a closer look at the behavior of precious metals in recent years, the second half of 2019 saw precious metals such as gold and silver start to break out, corroborating our empirical results regarding precious metals as the class of commodities delivering superior effects on portfolio performance. In fact, the World Bank's Precious Metals Index increased by 12.9% in the third quarter of 2019, demonstrating an ease of monetary policy by the US Federal Reserve – investors have been looking for alternative currencies since the reduction in interest rates, and consequently in US yields, decreased the dollar's appeal as a holding asset. The price of gold increased by 12.6% in the third quarter of 2019, motivated by four main macro drivers: (i) increased demand from the Central Bank, the growth of gold-back exchange-traded funds, and a rise in jewelry sales around the world and India in particular; (ii) a cut in interest rates by the US Federal Reserve; (iii) higher global policy uncertainty, benefitting gold prices, given its consideration by investors as a safe heaven; and (iv) fear of stagflation, a mix of growth stagnation and inflation, which motivated investors to add gold to their portfolios in order as a hedge investment. The price of silver also rose, by 14.3% in the third quarter of 2019, mainly due to jewelry demand motivated by the attractiveness of silver prices when compared to gold (an increase in the gold-to-silver price ratio), and to increasing solar panel demand, given that photovoltaic panels use up approximately 10% of the total global annual production of silver.

5.1.2. Dynamic selection of commodities

In this section, we go one step further in our study on the out-of-sample benefits of commodities by constructing two new portfolios in which, with the vital modification that commodities are picked in a dynamic manner. Our first portfolio set is composed of stocks, bonds, and the commodity which returns the lowest correlation[15] with the S&P 500. In practice, this means that we pick one commodity type, from among the seven distinct commodity classes we are considering for analytical purposes (GSCI, GSCI LE, energy, industrial metals, precious metals, livestock, and agriculture), with the lowest correlation to stocks. Every month, this pair of commodities and stocks is adjusted. The second of our dynamic portfolios is composed of stocks, bonds, and the pair of commodities which return the lowest correlation. In other words, every month we select the two commodity types, from the seven available, which have the lowest correlation between themselves, and we proceed to adjust the pair every single month. In line with the approach we employed for our static portfolio allocation method in section 5.1.1, we compare the two dynamic portfolios with the stock-bond portfolio across our seven asset allocation strategies (1/N, st.w., RP, RRT, MinVar, MV, and BL) for both the conservative and aggressive investor profiles. The main idea behind the construction of two dynamic portfolios is the leveraging of portfolio volatility. According to Markowitz (1952), modern portfolio theory states that adding assets to a diversified portfolio which presents low correlations, may decrease portfolio risk without compromising returns. In the same vein, adding diversification should increase risk-adjusted performance measures, such as Sharpe and Sortino ratios or certainty-equivalent returns, when compared to similar portfolios with

[15] Notice that, at any instant, the correlation is computed with past data only, so that the strategy is reproducible for investors.

lower level of diversification. Particularly if the decrease in the portfolio's volatility compensates for any possible decrease in portfolio's return.

Figure 3 presents portfolio performance for our conservative investor profile throughout the entirety of the time series, with three performance measures being carried out, all net of transaction costs, these being specifically portfolio return, volatility, and Sharpe ratio. From Figure 3, we can observe that a conservative investor can obtain considerable positive performance effects by picking commodities in a dynamic way. In all our asset allocation models, commodities' benefits are stronger for the portfolio in which one chooses the commodity with the lowest correlation with the S&P 500 (Dynamic A), when compared to the portfolio where one picks the pair of commodities with the lowest correlation (Dynamic B). Still, the Sharpe ratio increases for both Dynamic A and Dynamic B portfolios, in comparison to the base case scenario, the stock-bond portfolio, for all asset allocation models employed. When comparing our distinct asset allocation models, we conclude that BL performs best in terms of risk-return tradeoff –with a return increase for our Dynamic portfolio A of roughly 1.90% p.a. and a volatility reduction of 2.62% p.a., resulting in a Sharpe ratio 0.83 higher than the one of stock-bond portfolio; as for portfolio Dynamic B, return is increased by 1.77% p.a., while standard deviation is reduced by 2.33% p.a., leading to a Sharpe ratio of 1.48, approximately 0.71 higher than that of our base portfolio. After BL, MV is the model which delivers portfolios with the best risk-return profile. Under the MV framework, both our Dynamic A and Dynamic B portfolios experience increases in the Sharpe ratio, of 0.65 and 0.44 respectively. MinVar delivers portfolios with considerably low volatility – despite decreases in dynamic portfolios' returns, the decline in volatility is strong enough to compensate and even generate increasing Sharpe ratios. Nevertheless, MinVar emerges from our strategies as the provider of lower benefits in terms of risk-return profile, by adding commodities to a portfolio solely composed of equities and bonds.

In Figure 4, we present the remaining performance measures considered for our analysis – turnover, certainty-equivalent return, and the Sortino ratio – of the conservative investor under the BL framework, with our asset allocation model appearing to deliver the superior results. A complete analysis of portfolio performance and results for conservative investors within our remaining asset allocation strategies is included here as Appendix 1. The turnover measure reveals that both dynamic portfolios increase the amount of trading necessary to implement the BL strategy; in comparison to our stock-bond portfolio, Dynamic portfolio A increases turnover, by an average of 22.11% every month, whereas our Dynamic portfolio B increases it by 26.04% every month.

Figure 3: Portfolios' returns, volatility and Sharpe for the conservative investor

Figure 3 shows out-of-sample portfolio performance from January 1992 to December 2019 for stock-bond portfolios, portfolios which combine stocks, bonds, and the commodity with the lowest correlation with the S&P 500 ('Dynamic A'); and portfolios that combine stocks, bonds, and the pair of commodities with the lowest correlation between themselves ('Dynamic B'). Our base portfolio consists solely of US stocks and bonds. We present the results for our conservative investor profile, which accepts a maximum volatility of 7.5% p.a., with a risk aversion coefficient equal to 10. 'Return' refers to the annualized time-series average of monthly returns. 'Volatility' denotes the corresponding annualized standard deviation. 'Sharpe' shows the annualized Sharpe ratio. Panel A depicts the portfolios constructed under the BL framework, Panel B refers to the MV model, Panel C to MinVar, Panel D to RP, and Panel E to our st.w. model.

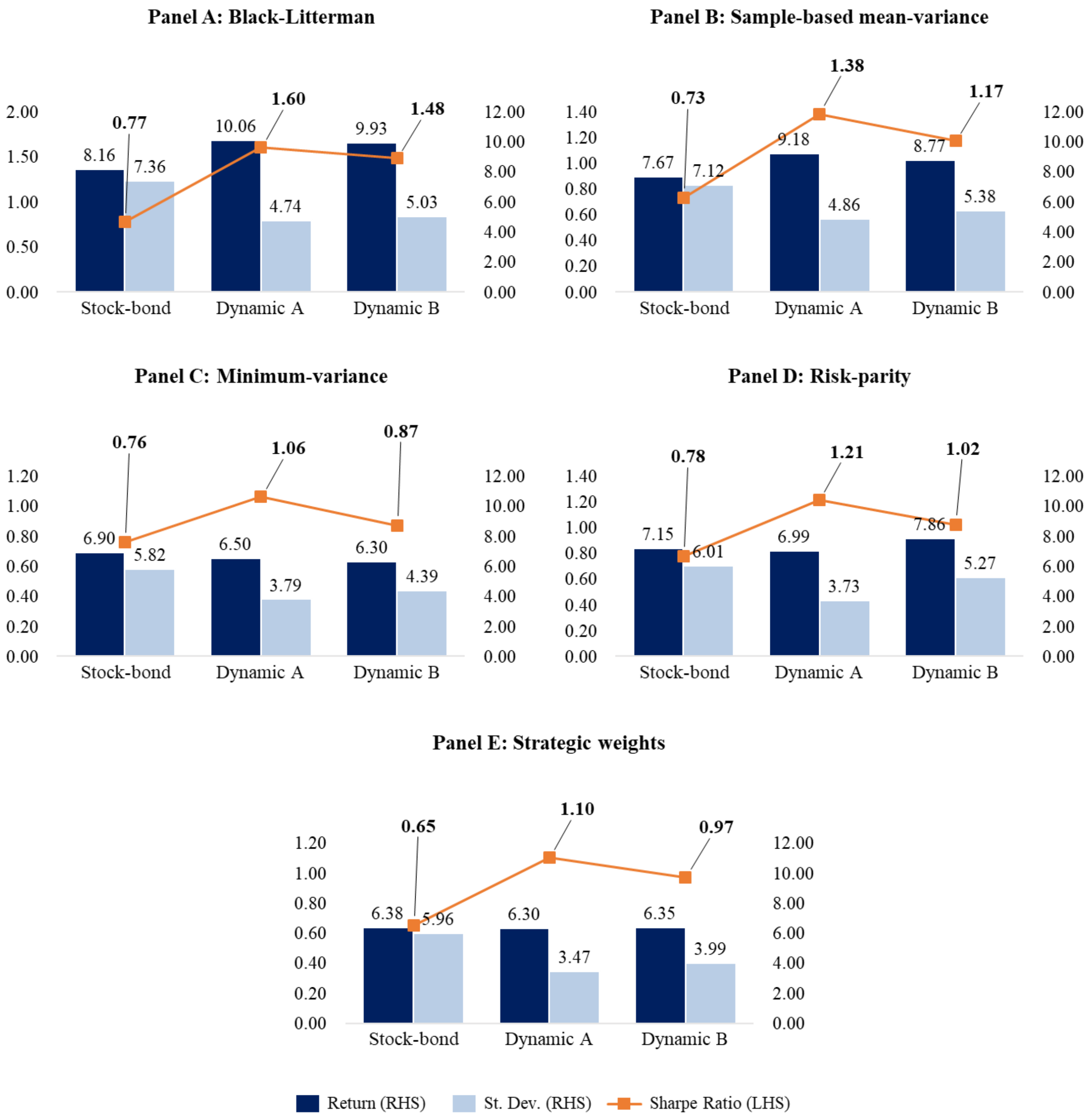
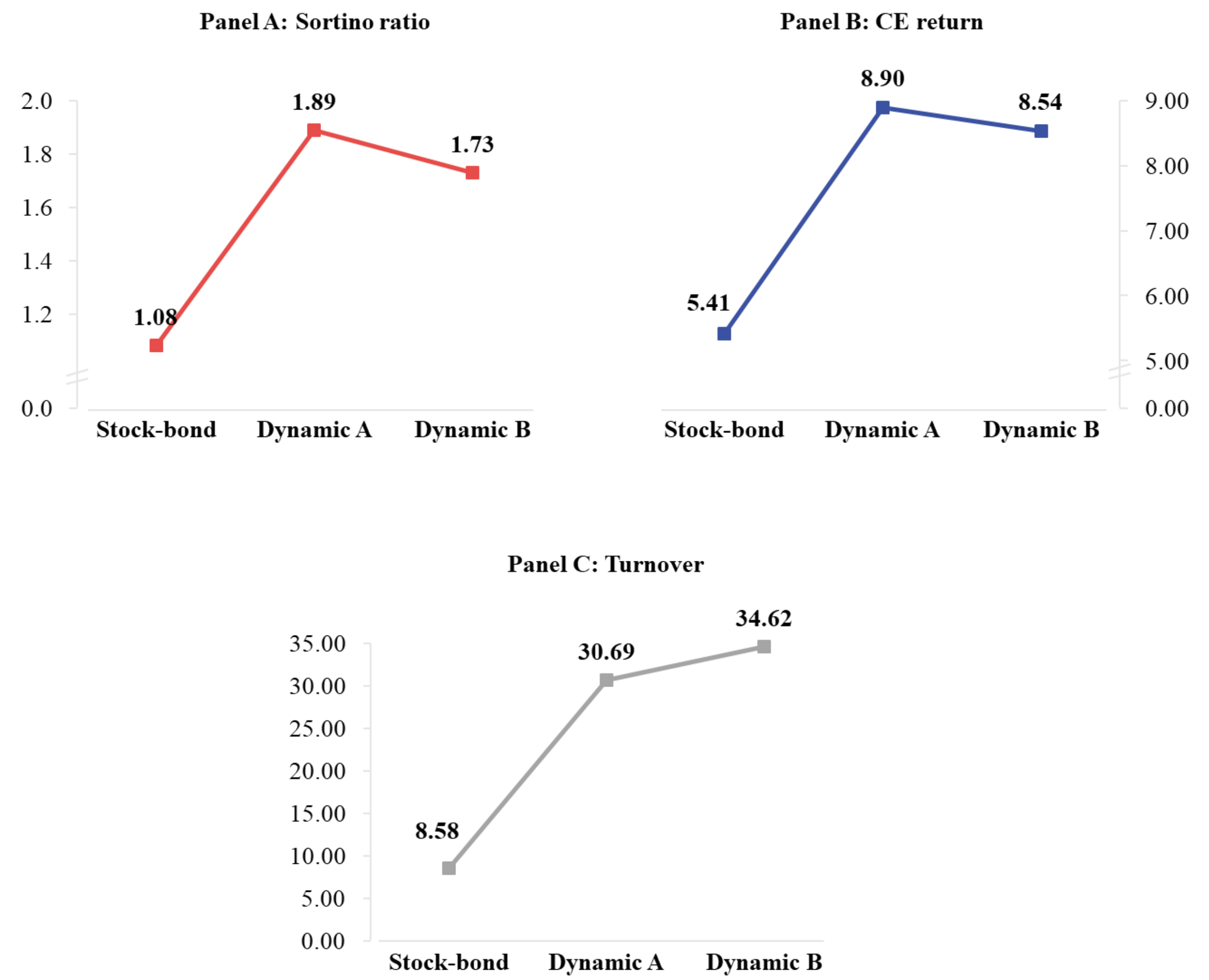


Figure 4: Sortino ratios, CE and Turnover for a conservative investor under the BL model

Figure 4 shows out-of-sample portfolio performance from January 1992 through December 2019 for stock-bond portfolios, and the commodity with the lowest correlation with the S&P 500 ('Dynamic A'); and portfolios that combine stocks, bonds, and the pair of commodities with the lowest correlation ('Dynamic B'), under our BL framework. The base portfolio consists solely of US stocks and bonds. We present results for the conservative investor profile, accepting a maximum volatility of 7.5% p.a. and a risk aversion coefficient equal to 10. 'Sortino' stands for the annualized Sortino ratio for each portfolio. 'CE' denotes the annualized certainty-equivalent return calculated using a power utility function. 'Turnover" represents the average monthly turnover for each portfolio.



This result is not surprising, given that during dynamic portfolio allocation, we adjust the commodity we are picking every month, resulting in higher average turnover values. In order to compensate for the increase in turnover, the certainty-equivalent return and the Sortino ratio reveal that picking commodities in a dynamic way does enhance the risk-return profile of a stock-bond portfolio for a conservative investor under our BL framework. Dynamic portfolio A increases certainty-equivalent return by 3.50% p.a. and the Sortino ratio by 0.81 p.a., while Dynamic portfolio B boosts certainty-equivalent return by 3.12% and the Sortino ratio by 0.65. As for our remaining asset allocation models besides BL, Appendix 1 demonstrates that dynamic portfolio allocation consistently enhances both the certainty-equivalent and Sortino ratio for a conservative investor.

Figure 5 depicts our portfolios' performances for an aggressive investor throughout our full data sample. Three performance measures are analyzed, all net of transaction costs, these being portfolio return, volatility, and the Sharpe ratio. Figure 5 shows that an aggressive investor also finds very positive performance effects from picking commodities in a dynamic way. As we found for our conservative investor profile, commodities' benefits are stronger when included in portfolio A, than they are when included in Dynamic portfolio B. In line with static portfolio allocation, none of the dynamic portfolios containing commodity groups enhance the risk-return tradeoff (Sharpe measure) of a stock-bond portfolio when only our equally-weighted 1/N model is considered. For our remaining asset allocation strategies, the Sharpe ratio showed increases for both of our Dynamic portfolios, over the base case scenario. In the superior performing BL model within Dynamic portfolio A, returns increased by 2.50% p.a. and volatility decreased by 3.01% p.a., resulting in a Sharpe ratio increase of 0.61. As for Dynamic portfolio B, returns increased by 2.02% p.a., while the standard deviation decreased by 2.57% p.a., resulting in a Sharpe measure 0.48 higher than that of the stock-bond portfolio.

In Figure 6, we present the remaining performance measures considered for our analysis (turnover, certainty-equivalent return, and the Sortino ratio) of an aggressive investor under the BL framework, with the asset allocation model providing the highest performance. The results of our thorough analysis of the portfolio performance of an aggressive investor when applying our remaining asset allocation strategies, are included here as Appendix 2. Regarding the turnover measure, both dynamic portfolios increment the amount of trading necessary to implement the BL strategy – Dynamic portfolio A increases turnover by an average of 38.09% every month, while Dynamic portfolio B increases it by 41.00% every month. The certainty-equivalent return and Sortino ratio

Figure 5: Portfolios' returns, volatility, and the Sharpe ratio for an aggressive investor

Figure 5 shows out-of-sample portfolio performance from January 1992 through December 2019 for stock-bond portfolios, portfolios that combine stocks, bonds, and the commodity with the lowest correlation with the S&P 500 ('Dynamic A'), and portfolios that combine stocks, bonds, and the pair of commodities with the lowest correlation between themselves ('Dynamic B'). Our base portfolio consists only of US stocks and bonds. We present results for an aggressive investor profile, accepting a maximum volatility ceiling of 13.5% p.a. and a risk aversion coefficient equal to 2. 'Return' refers to the annualized time-series average of monthly returns. 'Volatility' denotes the corresponding annualized standard deviation. 'Sharpe' shows the annualized Sharpe ratio. Panel A depicts the portfolios constructed under the BL framework, Panel B refers to the MV model, Panel C to MinVar, Panel D to RP, and Panel E to the st.w. model.

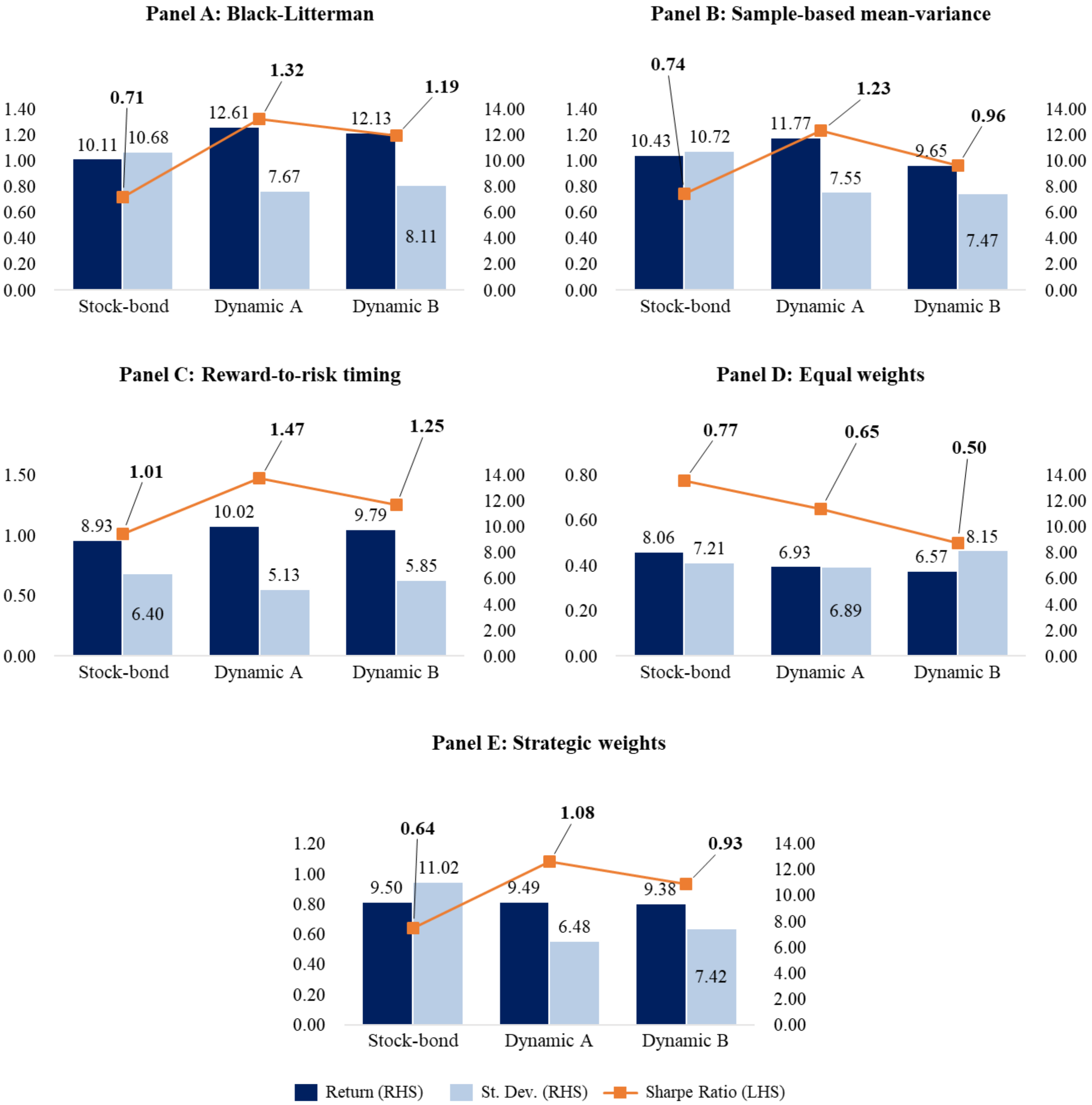
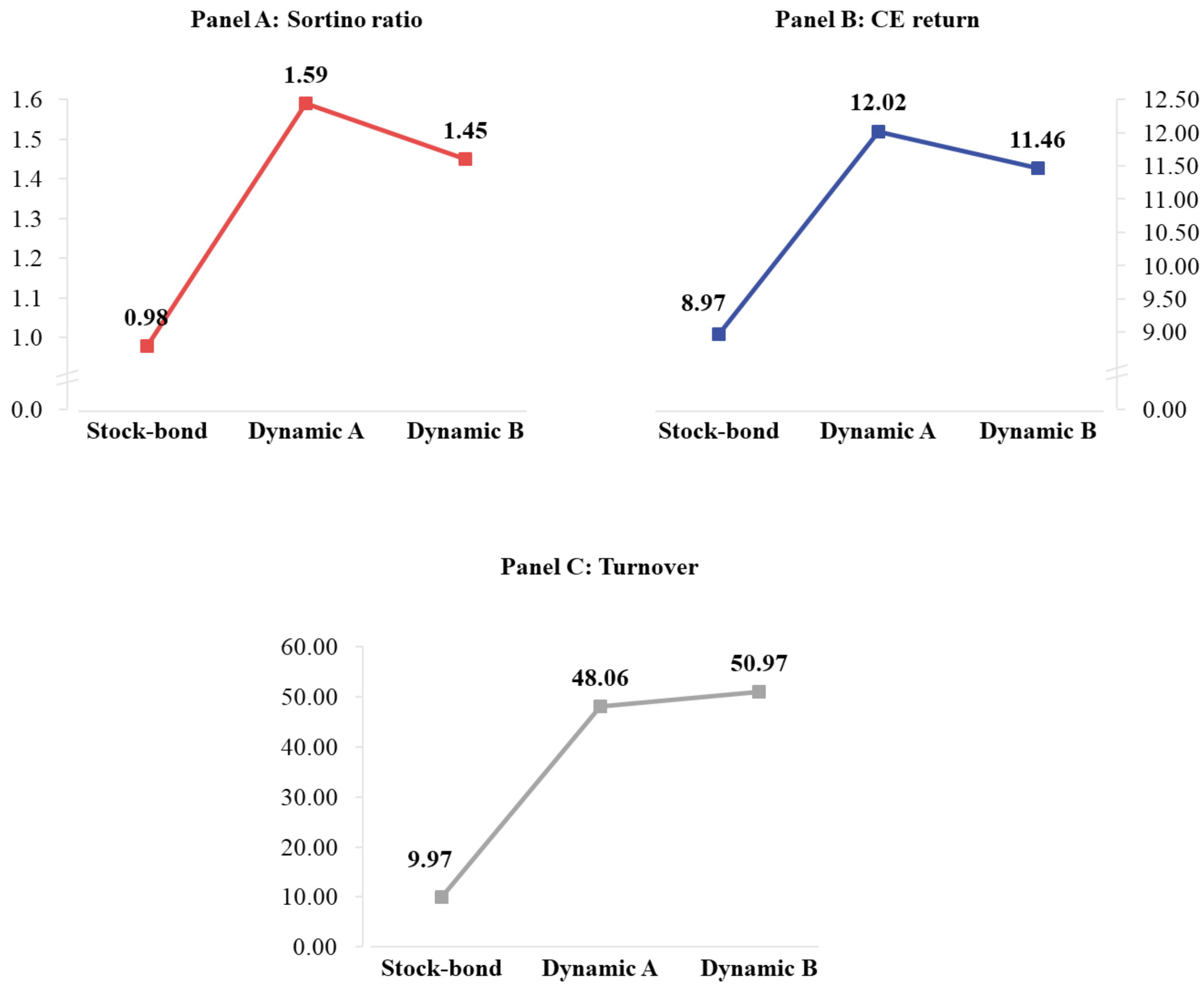


Figure 6: Turnover, CE, and the Sortino ratio for an aggressive investor under a BL model

Figure 6 shows out-of-sample portfolio performance from January 1992 through December 2019 for stock-bond portfolios that combine stocks, bonds, and the commodity with the lowest correlation with the S&P 500 ('Dynamic A'), along with portfolios which combine stocks, bonds, and the pair of commodities with the lowest correlation ('Dynamic B'), under the BL framework. The base portfolio consists only of US stocks and bonds. We present our results for the aggressive investor profile, accepting a maximum volatility of 13.5% p.a. with a risk aversion coefficient equal to 2. 'Sortino' stands for the annualized Sortino ratio for each portfolio.' 'CE' denotes the annualized certainty-equivalent return calculated using a power utility function. Turnover" represents the average monthly turnover for each portfolio.



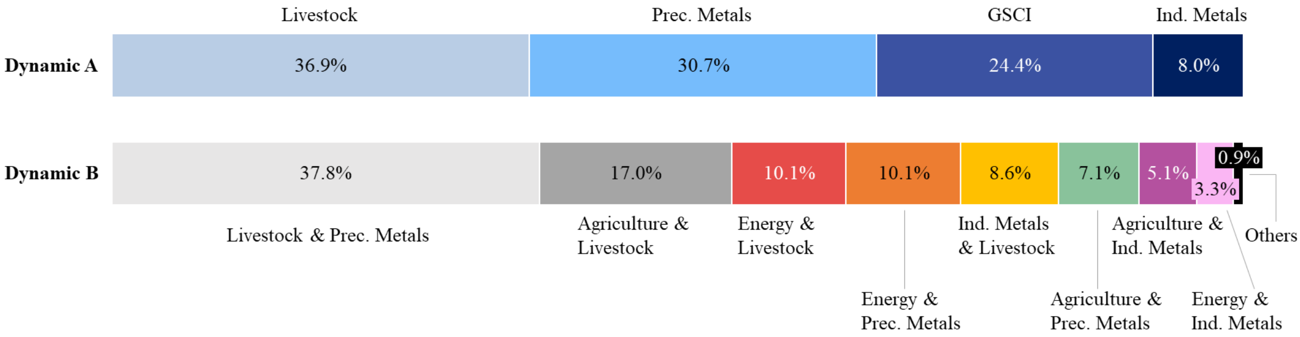
show improvements in the risk-return tradeoff of an aggressive investor for portfolios in which commodities are dynamically chosen and adjusted every month. For Dynamic portfolio A, the certainty-equivalent return and Sortino ratio are increased by 3.05% p.a. and 0.61 p.a., respectively. As for Dynamic portfolio B, certainty-equivalent return increases by 2.49% p.a., while the Sortino ratio increases by 0.47. Our remaining asset allocation models besides BL are expanded upon in Appendix 2, which displays that Dynamic portfolio A enhances certainty-equivalent return under the BL, MV, and RRT frameworks, whereas Dynamic portfolio B is only able to boost the certainty-equivalent return within a BL model.

An analysis of dynamic portfolio allocation suggests commodities do add value to a stock-bond portfolio – an investor is able to increase its certainty-equivalent return by up to 3.50% every year. In fact, when compared to static portfolio allocation, the portfolio effects of adding commodities to a stock-bond portfolio are substantially higher, and more consistent across the asset allocation models implemented in our study. In fact, by constructing portfolios in which weights are picked in such a way that correlation between the S&P 500 and one commodity class is minimized (Dynamic portfolio A) or in which weights are chosen so that correlation between a pair of commodities is the lowest possible (Dynamic portfolio B), we are able to achieve diversification benefits while improving risk-adjusted performance measures.

Finally, in order to gather more insight on our dynamic portfolios' performances, we present here the commodity groups chosen for the construction of dynamic portfolios as Figure 7. In the case of Dynamic portfolio A, an investor will choose to invest in livestock for approximately 37% of the months in our timeframe, a figure which drops to 31% for precious metals. This result is in line with our expectations – table 2 showed that precious metals and livestock were the commodity groups returning the lowest correlations with the S&P 500 (0.04 for precious metals and 0.06 for livestock) for the period from January 1992 through December 2019. As for Dynamic portfolio B, the pairs of commodities more frequently chosen would be livestock and precious metals (approximately 38% of the months), which returned a negative correlation of 0.06 during the full sample period, and agriculture and livestock (17% of the months), with a negative correlation of 0.01. Energy and livestock, energy and precious metals, and industrial metals and livestock, are also pairs of commodities which show very low correlation and hence, are frequently picked throughout our period of analysis.

Figure 7: Commodity groups dynamically selected

Figure 7 shows the distribution of commodity groups across our two dynamic portfolios for the full sample period, ranging from January 1992 through December 2019, corresponding to a total of 336 months. For portfolio 'Dynamic A', a single commodity type was chosen each month from the seven commodity classes (GSCI, GSCI LE, energy, industrial metals, precious metals, livestock, and agriculture) considered for our analytical purposes, possessing the lowest correlation with the S&P 500 – in Figure 7, we display the preponderance of each commodity group, determined by the number of months we pick each commodity class over the 336 months. For portfolio 'Dynamic B', the pair of commodities with the lowest correlation between themselves was chosen for every month – in Figure 7, we show the weight of each pair of commodities, meaning the number of months each pair is chosen for over the 336 months (category 'Others' refers to GSCI & Livestock, GSCI LE & Livestock, and Ind. Metals & Prec. Metals pairs). Note that commodity classes and pairs of commodities which do not appear on Figure 7 are also excluded from the monthly selection process.



5.2. Analyzing commodity portfolio weights

Our analysis in section 5.1 demonstrates that commodities' portfolio benefits are largely dependent on the asset allocation model implemented. This is particularly applicable when considering a static portfolio allocation case. In order to gather more insights on the process by which commodities accrue such benefits in some models but not others, we set out to investigate the portfolio weights of commodities within distinct asset allocation strategies. Table 6 shows average portfolio weights of commodities for many asset allocation models, applied alongside different types of portfolios. We also present the relevant weights' standard deviations with the aim of showing how commodity weights change over time, and the maximum portfolio weight of commodities during our full-sample period. Commodity portfolio weights for naïve asset allocation rules are not further presented here, given that they are constant over time for all portfolios. It is thus considered that a conservative investor will always invest 5.00% in commodities under our st.w. framework[16].

An aggressive investor within our 1/N model will invest 33.33% in commodities for portfolios composed of stocks, bonds, and one commodity group, with 25.00% in each commodity type for Dynamic portfolio B (which is composed of 4 assets). For our st.w. model, an aggressive investor will allocate 15% of his total wealth to commodities[17]. From Table 6, one can observe that all asset allocation strategies presented allocate a significant portion of wealth (at the 1% level of statistical significance) to commodities.

The MV model shows the highest standard deviation of commodity portfolio weights for both of our investor profiles. This result is in line with the presence of corner solutions and substantial portfolio reallocations of the MV strategy, as documented in the literature by Broadie (1993), and Best and Grauer (1991). The maximum portfolio weight is 63.45% for a conservative investor and 100% for the aggressive profile, demonstrating the MV model's well-known corner solutions – note that this brief overweighting of commodities in portfolios appears in high contrast with the main goal of portfolio diversification. Our BL model emerged as the top performer asset allocation model for both static and dynamic portfolio allocations, allocating lower average weights to commodities, portfolio shares below that of our other competing models. For a conservative investor, average weight of commodities in the BL model ranges from 4.15% for our stock-bond-energy portfolio, and 13.86% for our stock-bond-livestock portfolio. When considering an aggressive investor, average allocation to commodities ranges from 8.84% for the stock-bond-agriculture portfolio, to 34.98% for dynamic portfolio B, composed of stocks, bonds, and a pair of commodities.

Table 6 further reports that aggressive investors allocate larger values of total wealth to commodities, a far from surprising result given that commodities are an asset class characterized for high levels of volatility, and hence, more suitable to the preferences of an aggressive investor. An important conclusion which can be derived from our analysis of commodity portfolio weights is that

[16] For Dynamic portfolio B, we assumed an equal split of the 5.00% weight between the two commodities under investment.
[17] In a similar fashion to our conservative investor scenario, when considering Dynamic portfolio B we assumed an equal split of the 15.00% weight between the two commodities under investment.

Table 6: Commodity portfolio weights

The table below reports the portfolio weights of commodities in both our out-of-sample static and dynamic optimized portfolios, for simple asset allocation strategies (RRT, and RP), along with portfolio optimization models (MinVar, MV, and BL). 'Mean' shows the average portfolio weight of commodities during the 1992 through 2019 period. "Std.Dev.' refers to the corresponding standard deviation of the commodity portfolio weight. 'Maximum' denotes the maximum portfolio weight allocated to each indicated commodity group or groups for the period ranging from 1992 through 2019. Panel A shows these results for a conservative investor, while Panel B does so for an aggressive investor. Note that for portfolio 'Dynamic B', our only portfolio including two commodities instead of just one, the weights presented refer to the total weigh invested in both commodities. '*' indicates values significantly larger than 0 at the 1% significant level.

Asset Allocation strategy	Commodity portfolio weight	Stock-bond portfolio complemented with commodities								
		GSCI	GSCI LE	Energy	Industrial M.	Precious M.	Livestock	Agriculture	Dynamic A	Dynamic B
Panel A: Conservative investor										
BL	Mean (%)	5.95*	7.61*	4.15*	11.04*	7.16*	13.86*	7.18*	7.56*	10.08*
	Std.Dev. (%)	4.20	6.41	2.75	5.35	6.00	5.37	5.49	6.43	9.05
	Maximum (%)	15.20	26.19	10.57	20.50	22.36	24.51	21.10	25.79	36.87
MV	Mean (%)	8.72*	14.74*	5.21*	12.01*	10.09*	20.19*	11.76*	12.61*	21.35*
	Std.Dev. (%)	8.82	14.93	5.22	13.67	11.37	23.34	15.22	11.56	23.76
	Maximum (%)	31.23	50.24	21.58	35.48	41.67	61.26	47.92	42.05	63.45
MinVar	Mean (%)	7.41*	9.95*	4.51*	10.45*	7.52*	17.17*	9.45*	10.32*	20.22*
	Std.Dev. (%)	5.19	9.63	1.98	3.93	6.76	6.61	6.38	6.37	8.41
	Maximum (%)	27.19	44.16	9.26	17.66	35.71	26.36	20.07	36.25	48.69
RP	Mean (%)	7.28*	14.06*	3.37*	7.66*	8.12*	8.25*	7.22*	10.25*	12.69*
	Std.Dev. (%)	3.94	7.38	1.43	3.32	6.14	5.73	3.06	8.56	10.92
	Maximum (%)	20.48	36.88	7.33	15.44	27.80	25.50	17.70	26.76	36.76
Panel B: Aggressive investor										
BL	Mean (%)	11.85*	10.89*	11.47*	13.42*	22.58*	16.31*	8.84*	26.87*	34.98*
	Std.Dev. (%)	10.30	13.26	7.91	8.05	15.96	15.09	11.77	17.95	19.93
	Maximum (%)	63.79	70.97	52.73	55.45	86.65	44.55	39.83	89.99	95.50
MV	Mean (%)	26.25*	21.92*	22.02*	26.32*	33.37*	20.91*	20.62*	35.22*	47.48*
	Std.Dev. (%)	32.80	37.54	26.33	35.03	41.57	32.47	32.34	44.89	55.67
	Maximum (%)	78.29	100.00	65.43	83.27	100.00	100.00	100.00	100.00	100.00
RRT	Mean (%)	9.75*	14.56*	7.51*	10.78*	14.81*	12.32*	8.15*	15.31*	20.44*
	Std.Dev. (%)	11.09	15.39	8.56	12.87	15.41	11.11	10.95	16.03	24.06
	Maximum (%)	61.30	75.84	51.65	59.53	70.33	56.41	59.46	82.60	94.34

the portion of shares allocated to a commodity group is not necessarily correlated with the performance of that portfolio. It appears instead that there is a considerable number of asset allocation models allocating substantial amounts of shares to a certain commodity type, without providing the relevant portfolio performance improvements. These results appear to confirm our reasoning and results thus far. Although aggressive investors do attribute higher weights to commodities, the benefits achieved from including commodities in a stock-bond portfolio are not necessarily larger than the benefits obtained when considering the conservative investor instead.

5.3. Contribution of commodities in different market environments

In this section we undertake a deeper analysis on the effects of adding commodities to a stock-bond portfolio in different market environments. To that purpose, we divided the full evaluation period into six distinct subperiods according to three different criteria – (i) the economy criteria, which leads to expansionary and recessionary subperiods, (ii) the market criteria, generating bull market and bear market subperiods, and lastly (iii) a volatility criteria, separating the full sample into low volatility and high volatility subperiods.

For our economy criteria, Bessler and Wolff's (2015) work on the classification of subperiods as expansionary and recessionary served as our methodological template. For the period overlapping our study with Bessler and Wolff's, specifically from January 1992 through December 2012, we followed their subperiod classification. We applied the same methodology for the rest of the years which compose our sample (from January 2013 through December 2019), combining monetary and stock market signals as documented in Jensen and Mercer (2003), and Bessler and Wolff (2015). On the one hand, monetary signals consist of the first change of short-term interest rates by a central bank which moves in the opposite direction of the previous trend. On the other hand, stock market signals are derived from a simple moving average, on the underlying assumption that the stock market trend is inverted should the S&P 500's 24-months moving average cross the actual index from above (expansionary state) or below (recessionary state). To classify a subperiod as expansionary or recessionary, evidence of both monetary and stock market signals must be found. Figure 8 shows monetary and stock market signals for the period ranging from January 2013 through December 2019. As becomes evident upon visual inspection, the period depicted corresponds to an expansionary state, since the federal funds target rate shows a positive trend while the S&P 500 index is above the 24-months moving average. A further detailed classification of periods in expansionary and recessionary states has been included in our paper as Appendix 3. Note that for further portfolio performance analysis purposes, all expansionary states were grouped together, as were the recessionary states, leading to two main subperiods. For the market criteria, we designated months as 'bull market' months when the return of the S&P 500 was above the 75th percentile. Conversely those below the 25th percentile were designated as 'bear market' months, enabling us to obtain two final subperiods (bull market and bear market) across the same timeframe. In a similar, fashion when addressing our volatility criteria we classified months with a VIX (1990) below the 25th percentile as being “low volatility”, while those above the 7th percentile were designated as 'high volatility' months We thus end up composing two further subperiods (low volatility and high volatility).

Figure 8: Monetary and stock market signals (January 2013 – December 2019)

Figure 8 depicts the evolution of the S&P’s 24-month moving average, alongside the evolution of the actual S&P 500, and the federal funds target rate, for the period ranging from January 2013 through December 2019.

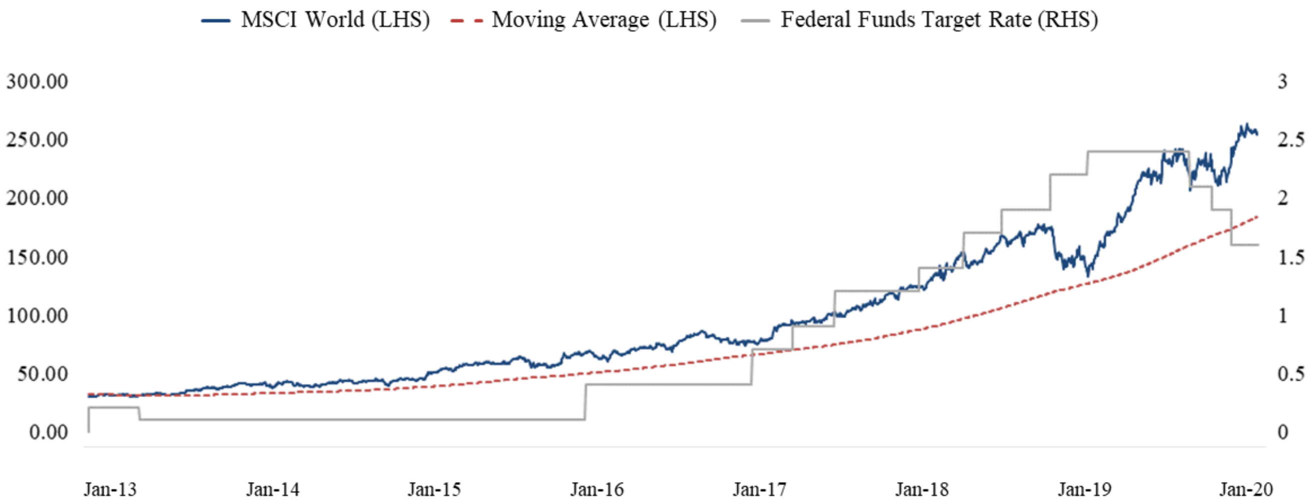


Table 7 shows the Sharpe ratios we obtained for optimized stock-bond portfolios and portfolios including commodities throughout our six subperiods. We selected only the subperiod results for the conservative investor profile for display, given that our results for an aggressive investor one were identical. From Table 7, one can further observe that the benefits of including commodities within a stock-bond portfolio depend on macroeconomic factors. In line with our previous results for the full sample period, dynamic portfolios show a strong and consistent performance across our different asset allocation models in subperiods. The clearest and significant portfolio benefits of dynamically selected commodities are in expansion, bull markets, and low volatility subperiods.

Industrial metals largely enhance the performance of our base portfolios for favorable subperiods – when the economy is in an expansionary stage, markets can be designated as bull, and volatility will be low. This result is intuitive and further supports Bessler and Wolff's results, given that the demand for industrial metals is usually pro-cyclical, reflecting the current state of the economy.

Table 7: Contribution of commodities across selected subperiods

This table shows portfolio performance (Sharpe measures) for distinct asset allocation models across six subperiods, covering the period ranging from January 1992 through December 2019 for a conservative investor, accepting a maximum volatility of 7.5% p.a., along with a risk aversion coefficient equal to 10. Improvements in relation to the stock-bond portfolio are highlighted in bold.

Period	Asset Allocation strategy	SR stock-bond portfolio	Sharpe ratio stock-bond portfolio complemented with commodities								
			GSCI	GSCI LE	Energy	Industrial M.	Precious M.	Livestock	Agriculture	Dynamic A	Dynamic B
Expansion	BL	0.90	0.78	**1.03**	0.83	**1.26**	**1.18**	0.79	**1.05**	**1.79**	**1.68**
	MV	0.87	0.87	0.80	0.85	**1.15**	**1.08**	0.72	0.87	**1.55**	**1.22**
	MinVar	0.97	0.78	0.72	0.85	**1.13**	0.80	0.65	0.74	**1.20**	**1.01**
	RP	0.99	0.91	0.83	0.95	**1.21**	0.86	0.75	0.85	**1.34**	**1.14**
	st.w.	0.76	**0.89**	**0.87**	**0.91**	**0.93**	**0.86**	0.73	0.72	**1.23**	**1.09**
Recession	BL	0.58	0.44	0.32	0.47	0.31	**0.73**	0.47	0.35	**0.94**	**1.21**
	MV	0.56	0.40	0.31	0.46	0.05	**0.67**	0.12	0.06	**0.78**	**0.95**
	MinVar	0.60	0.50	0.50	0.49	0.52	0.60	0.50	0.47	0.51	0.60
	RP	0.59	0.59	0.57	0.59	**0.71**	**0.77**	0.51	0.47	**0.69**	**0.78**
	st.w.	0.60	0.57	0.58	0.56	0.59	**0.75**	0.60	0.57	0.55	**0.69**
Bull market	BL	2.84	2.74	**3.36**	**3.15**	**5.21**	**2.98**	**3.46**	**3.81**	**5.97**	**5.44**
	MV	2.55	**2.57**	**2.71**	2.50	**2.81**	**2.90**	2.45	**2.61**	**2.94**	**2.47**
	MinVar	2.06	1.63	1.42	1.83	**2.10**	1.74	1.40	1.63	**2.39**	**2.11**
	RP	2.11	1.77	1.45	1.88	**2.25**	1.78	1.43	1.67	**2.65**	**2.28**
	st.w.	1.95	1.55	1.33	1.73	**2.01**	1.64	1.32	1.55	**2.36**	**2.09**
Bear market	BL	-1.72	**-1.70**	**-1.40**	**-1.44**	**-1.65**	**-1.34**	-2.77	-2.80	**-1.37**	**-1.21**
	MV	-1.45	-1.56	-1.53	**-1.44**	-1.45	**-1.40**	-1.70	-1.53	**-1.22**	**-1.13**
	MinVar	-0.94	-0.99	**-0.90**	-1.02	**-0.82**	**-0.79**	-0.95	**-0.80**	-0.95	**-0.64**
	RP	-0.96	-1.03	-0.97	-1.04	**-0.84**	**-0.77**	-0.96	**-0.83**	**-0.95**	**-0.69**
	st.w.	-0.95	-1.02	-0.96	-1.03	**-0.83**	**-0.79**	-0.95	**-0.82**	-1.00	**-0.68**
Low volatility	BL	3.47	3.09	**3.61**	**3.51**	**4.20**	**3.56**	3.25	**3.67**	**5.13**	**4.79**
	MV	3.70	3.57	3.67	3.34	**3.94**	**3.38**	3.37	3.36	**3.94**	**3.91**
	MinVar	2.41	2.16	2.23	2.17	**2.57**	**2.56**	2.06	2.33	**2.50**	**2.09**
	RP	2.45	2.22	2.27	2.20	2.57	2.63	2.09	2.37	**2.60**	**2.12**
	st.w.	2.13	1.91	1.99	1.92	2.29	2.26	1.82	2.05	**2.23**	1.86
High volatility	BL	0.33	0.15	0.11	0.20	-0.36	**0.41**	-0.60	-0.52	**0.51**	**0.46**
	MV	0.23	0.07	0.04	0.16	0.16	**0.57**	0.01	0.07	0.04	0.12
	MinVar	0.50	0.30	0.25	0.39	0.36	**0.69**	0.20	0.32	0.19	0.38
	RP	0.52	0.30	0.27	0.41	0.37	**0.75**	0.19	0.33	**0.55**	**0.81**
	st.w.	0.61	0.29	0.29	0.44	0.40	**0.79**	0.22	0.36	0.25	0.43

In fact, there is a large concentration of industrial metals, such as copper, steel, and aluminum in construction and production of capital goods (e.g. heavy machinery). The stock-holding behavior of manufacturers explains this procyclical demand. As economies accelerate, manufacturers tend to build up their stocks of raw materials in anticipation of higher levels of production, thereby increasing the demand for industrial metals. On the contrary, when an economy slows down, manufactures will usually reduce their stocks of raw materials, and thus reducing demand for industrial metals. We found that precious metals are particularly beneficial during the less favorable subperiods. This result is not particularly surprising, given that precious metals, and particularly gold, are traditionally viewed as safe investments during times of economic recession, financial uncertainty, high inflation, and depreciating currency rates. During periods of recession, stock prices will typically fall due to a reduction in most companies' profits, encouraging speculative purchasing of gold due to its relative value robustness as an asset. Gold also becomes attractive in times of negative real interest rates (meaning inflation is higher than nominal interest rates). When real interest rates return negative values, depositing one's savings into a bank becomes less attractive, while investing in gold becomes more attractive. Finally, gold has an intrinsic value, in contrast to fiat currencies, and one which cannot be depreciated.

5.4. Alternative optimization constraints

As a means of assessing the robustness of the results, a recalculation of our optimized portfolios was carried out, with the addition of short-term position allowances (Table 8). We can further observe that the performance improvements from including commodities within a stock-bond portfolio appear to increase when short-selling is allowed, and particularly so for an aggressive investor. This result is unsurprising, given that commodities are a highly volatile asset class – when an investor limits short-selling activity, he risks missing out on short term profit-generating opportunities[18]. In a similar manner to the results derived with short-selling constraints, dynamic portfolios generate our greatest improvement within the Sharpe measure. For static portfolio allocation, precious metals remain the commodity class offering the largest benefits.

[18] Results for 1/N, strategically-weighted, RP, and RRT strategies remain unchanged because they do not contemplate the possibility of short-selling in their setup.

5.5. Alternative rebalancing frequency

Portfolio rebalancing enables investors to maintain their desired target allocation. According to Albertazzi et. al (2016), periodically rebalancing portfolios allows investors to minimize the tendency for 'portfolio drift', potentially reducing exposure to risk relative to target asset allocation. In this section, we construct portfolios considering quarterly rebalancing rather than monthly rebalancing. Table 9 sets out our results for a conservative investor adopting such a quarterly rebalancing frequency. A visual inspection reveals that our results are once more in line with our previous work, with dynamic portfolios and precious metals in the static case performing better in terms of risk-return tradeoff. Of additional interest is the observation that risk-adjusted returns are not meaningfully different by our portfolio being rebalanced in a monthly or quarterly basis. This is because we are assuming the absence of transaction costs within our analysis. If transactions costs were to be included, it is likely that portfolio benefits would be larger for less frequent rebalancing, given that higher diversification advantages due to more rebalancing events would also require additional transaction costs. As such, after taking transactions costs into account we expect our quarterly rebalanced portfolios to deliver superior performances than our monthly ones.

Table 8: Contribution of commodities with short selling

This table shows portfolio performance (Sharpe measures) when short selling is allowed, covering the period ranging from January 1992 through December 2019. We restricted short selling for a maximum of -10% for each asset. We present the conservative investor, accepting a maximum volatility of 7.5% p.a., along with a risk aversion coefficient equal to 10, and the aggressive investor, who accepts a maximum volatility of 13.5% and has a risk aversion coefficient of 2. Improvements in relation to the stock-bond portfolio are presented in bold.

Asset Allocation strategy	SR stock-bond portfolio	Sharpe ratio stock-bond portfolio complemented with commodities								
		GSCI	GSCI LE	Energy	Industrial M.	Precious M.	Livestock	Agriculture	Dynamic A	Dynamic B
Conservative investor										
BL	0.79	0.66	0.63	0.69	**0.83**	**0.87**	0.48	0.56	**1.63**	**1.47**
MV	0.69	0.68	0.66	0.68	**0.70**	**0.79**	0.62	0.69	**1.40**	**1.20**
MinVar	0.76	0.62	0.57	0.65	0.75	0.67	0.50	0.58	**1.01**	**0.79**
RP	0.78	0.74	0.68	0.76	**0.85**	0.75	0.61	0.58	**1.21**	**1.02**
st.w.	0.66	0.66	0.65	**0.67**	**0.69**	**0.68**	0.63	0.62	**1.10**	**0.97**
Aggressive investor										
BL	0.69	0.58	0.59	0.64	0.67	**0.79**	0.64	**0.73**	**1.34**	**1.23**
MV	0.64	0.62	0.59	**0.65**	**0.67**	**0.81**	**0.68**	0.56	**1.33**	**1.05**
RRT	1.01	0.93	0.91	0.95	**1.03**	**1.09**	**1.31**	0.92	**1.47**	**1.25**
1/N	0.77	0.41	0.43	0.39	0.68	0.66	0.39	0.28	0.65	0.50
st.w.	0.64	0.57	0.57	0.57	0.60	**0.68**	0.57	0.56	**1.08**	**0.93**

Table 9: Contribution of commodities with quarterly rebalancing

This table shows Sharpe measures for quarterly rebalancing, covering the period ranging from January 1992 through December 2019. We present the conservative investor, accepting a maximum volatility of 7.5% p.a., along with a risk aversion coefficient equal to 10. Improvements in relation to the stock-bond portfolio are shown in bold.

Asset Allocation strategy	SR stock-bond portfolio	Sharpe ratio stock-bond portfolio complemented with commodities								
		GSCI	GSCI LE	Energy	Industrial M.	Precious M.	Livestock	Agriculture	Dynamic A	Dynamic B
BL	0.78	0.65	0.62	0.70	**0.84**	**0.87**	0.48	0.56	**1.57**	**1.46**
MV	0.72	0.67	0.66	0.68	0.70	**0.77**	0.62	0.67	**1.37**	**1.19**
MinVar	0.75	0.62	0.59	0.66	**0.77**	0.69	0.55	0.58	**1.06**	**0.87**
RP	0.78	0.74	0.68	0.75	**0.85**	0.75	0.63	0.58	**1.21**	**1.04**
st.w.	0.66	0.64	0.65	**0.67**	**0.68**	**0.69**	0.63	0.62	**1.08**	**0.99**

6. Conclusion, limitations, and avenues for future research

This research focuses on the construction of portfolios which include different commodity types, across distinct asset allocation models – equally and strategically-weighted portfolios, risk-parity, reward-to-risk timing, minimum variance, mean-variance, and Black-Litterman – while analyzing the out-of-sample performance effects of adding commodities to a stock-bond portfolio. We consider here investments into individual commodity groups, such as energy, industrial metals, and precious metals, but without disregarding investments on commodities largely criticized for ethical reasons, namely agriculture and livestock. Portfolio construction was carried out across two distinct methodologies, one in which commodities are picked in a consistent standard format (static portfolio allocation), and another in which commodities are dynamically picked and adjusted every month. In our first of the two distinct dynamic portfolio types we constructed, weights are picked, so that correlation between the S&P 500 and commodities is minimized. For our second dynamic portfolio, weights were chosen so that correlation between a pair of commodities is minimized. We further delved into the portfolio benefits of commodities for conservative and aggressive investor profiles across different market environments.

The data we obtained from empirical testing suggest that when static portfolio allocation is applied, the resulting performance effects are not consistent across commodity groups and asset allocation strategies. We further find that precious metals consistently emerge as the commodity group with clearly superior risk-return improvements when compared to a benchmark stock-bond portfolio. This result is supported by the fact that precious metals have experienced a resurgence in recent years, reflecting the easing of monetary policy by the US's Federal Reserve, with investors now considering alternative currencies given the cut in interest rates and its effects on US yields, decreasing the dollar's appeal as a holding asset. Our results also demonstrate that Black-Litterman delivers the superior performance. Our work on static portfolio allocation approach further indicates that there are some positive out-of-sample performance effects – the maximum benefit from static commodity inclusion corresponds to an increase in the certainty-equivalent of 1.32% per year. However, commodities' effects appeared to be constrained by static portfolio allocation, particularly given their significant correlation with global growth. The contemporary macroeconomic environment has undergone several and often dramatic changes within recent years, such as the trade tensions between US and China, which have weighed negatively on commodity returns. Of particular interest is our finding that dynamic portfolios deliver substantial benefits, with the average investor able to increase their CE return by up to 3.5% year on year. Our result in this case is supported within the literature by Markowitz (1952)'s work, who demonstrated that adding assets to a diversified portfolio with particularly low correlations, may decrease portfolio risk without compromising return, thereby enhancing risk-adjusted performance measures. When comparing our dynamic portfolio results with those of static alternatives, we find beneficial effects of up to 2.90% p.a., with significantly improved consistency across asset allocation strategies.

We further confirm that the benefits of including commodities depend greatly on macroeconomic factors. Industrial metals seem to largely improve performance throughout favorable subperiods, namely during expansionary economies, bull markets, and low volatility states, thereby establishing that demand for this commodity class is procyclical. Precious metals specifically stand out for being beneficial during less favorable subperiods, with gold in particular being perceived as a "safe investment" throughout periods of high instability.

The research we carried out returned a variety of interesting and relevant results, which nevertheless must be further developed and incremented upon before practical applications can be considered. Future iterations of our work must increase robustness tests further to eliminate any remaining and unforeseen biases which may remain. Specific areas of improvement within our methodology that could be further developed to enhance the quality of our study must also be addressed Starting with asset allocation models and asset selection, a wider selection of models and more asset class variety is likely to only further improve the performance and quality of our testing and results at virtually no cost beyond additional time and effort.. Our work sought to draw and build upon Bessler and Wolff (2015)'s study of commodity futures contracts and commodity spot prices. It could have been of further interest for that purpose to develop a sensitivity analysis of the strategic weights for the Black-Litterman model. Moreover, including a strategy involving volatility timing could have further improved our analysis of dynamic portfolio allocation. Finally, although an out-of-sample approach is applied within our work, the relevant analysis is based on historical average returns, variances, and covariances, which may sometimes deliver sub-par estimates for future returns. Future avenues of research and iterations upon our work will focus on developing a return prediction model with which to apply return forecasts for portfolio optimization. Doing so will undoubtedly further solidify the portfolio performance improvements achieved within our work, while creating a robust framework for future portfolio development through asset allocation modelling.

7. References

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8. Appendices

Appendix 1: Portfolio benefits of dynamic selected commodities for conservative investor

The table below reports out-of-sample portfolio performance for stock-bonds portfolios, portfolios that combine stocks, bonds, and the commodity with the lowest correlation with the S&P 500 ('Dynamic A'), and portfolios that combine stocks, bonds, and the pair of commodities with the lowest correlation ('Dynamic B') for different asset allocation strategies from January 1992 to December 2019. We present the results for the conservative investor profile, which accepts a maximum volatility of 7.5% p.a. and has a risk aversion coefficient equal to 10. The basis portfolio is composed solely of US stocks and bonds. Improvements relative to the stock-bond portfolio are marked in bold. 'Return' refers to the annualized time-series average of monthly returns. 'Volatility' denotes the corresponding annualized standard deviation. 'Sharpe' shows the annualized Sharpe ratio and 'Sortino' stands for the annualized Sortino ratio for each portfolio. 'CE #1' and 'CE #2' denote the annualized certainty-equivalent returns calculated using mean-variance and power utility functions, respectively. 'Turnover" represents the average monthly turnover for each portfolio.

Asset Allocation strategy	Performance measure	Stock-bond	Stock-bond portfolio complemented with commodities	
			Dynamic A	Dynamic B
Conservative investor				
BL	Return (%)	8.16	**10.06**	**9.93**
	Volatility (%)	7.36	**4.74**	**5.03**
	Sharpe	0.77	**1.60**	**1.46**
	Sortino	1.08	**1.89**	**1.73**
	CE #1 (%)	5.44	**8.94**	**8.56**
	CE #2 (%)	5.41	**8.90**	**8.54**
	Turnover (%)	8.58	30.69	34.62
MV	Return (%)	7.67	**9.18**	**8.77**
	Volatility (%)	7.12	**4.86**	**5.38**
	Sharpe	0.73	**1.38**	**1.17**
	Sortino	1.03	**1.63**	**1.46**
	CE #1 (%)	5.13	**8.00**	**7.32**
	CE #2 (%)	5.10	**7.95**	**7.27**
	Turnover (%)	7.34	22.41	28.53
MinVar	Return (%)	6.90	6.50	6.30
	Volatility (%)	5.82	**3.79**	**4.39**
	Sharpe	0.76	**1.06**	**0.87**
	Sortino	1.05	**1.37**	**1.17**
	CE #1 (%)	5.21	**5.78**	**5.34**
	CE #2 (%)	5.18	**5.74**	**5.32**
	Turnover (%)	3.36	13.56	13.68
RP	Return (%)	7.15	6.99	**7.86**
	Volatility (%)	6.01	**3.73**	**5.27**
	Sharpe	0.78	**1.21**	**1.02**
	Sortino	1.11	**1.57**	**1.53**
	CE #1 (%)	5.35	**6.29**	**6.47**
	CE #2 (%)	5.33	**6.27**	**6.43**
	Turnover (%)	3.01	11.52	13.53
st.w.	Return (%)	6.38	6.30	6.35
	Volatility (%)	5.96	**3.47**	**3.99**
	Sharpe	0.66	**1.10**	**0.97**
	Sortino	0.99	**1.43**	**1.97**
	CE #1 (%)	4.61	**5.70**	**5.55**
	CE #2 (%)	4.60	**5.70**	**5.53**
	Turnover (%)	1.83	5.94	6.24

Appendix 2: Portfolio benefits of dynamic selected commodities for aggressive investor

The table below reports out-of-sample portfolio performance for stock-bonds portfolios, portfolios that combine stocks, bonds, and the commodity with the lowest correlation with the S&P 500 ('Dynamic A'), and portfolios that combine stocks, bonds, and the pair of commodities with the lowest correlation ('Dynamic B') for different asset allocation strategies from January 1992 to December 2019. We present the results for the aggressive investor profile, which accepts a maximum volatility of 13.5% p.a. and has a risk aversion coefficient equal to 2. The basis portfolio is composed solely of US stocks and bonds. Improvements relative to the stock-bond portfolio are marked in bold. 'Return' refers to the annualized time-series average of monthly returns. 'Volatility' denotes the corresponding annualized standard deviation. 'Sharpe' shows the annualized Sharpe ratio and 'Sortino' stands for the annualized Sortino ratio for each portfolio. 'CE #1' and 'CE #2' denote the annualized certainty-equivalent returns calculated using mean-variance and power utility functions, respectively. 'Turnover" represents the average monthly turnover for each portfolio.

Asset Allocation strategy	Performance measure	Stock-bond	Stock-bond portfolio complemented with commodities	
			Dynamic A	Dynamic B
Aggressive investor				
BL	Return (%)	10.11	**12.61**	**12.13**
	Volatility (%)	10.68	**7.67**	**8.11**
	Sharpe	0.71	**1.32**	**1.19**
	Sortino	0.98	**1.59**	**1.45**
	CE #1 (%)	8.97	**12.02**	**11.47**
	CE #2 (%)	8.97	**12.02**	**11.46**
	Turnover (%)	9.97	48.06	50.97
MV	Return (%)	10.43	**11.77**	9.65
	Volatility (%)	10.72	**7.55**	**7.47**
	Sharpe	0.74	**1.23**	**0.96**
	Sortino	0.99	**1.48**	**1.22**
	CE #1 (%)	9.28	**11.20**	9.09
	CE #2 (%)	9.27	**11.18**	9.09
	Turnover (%)	12.76	25.95	29.73
RRT	Return (%)	8.93	**10.02**	**9.79**
	Volatility (%)	6.40	**5.13**	**5.85**
	Sharpe	1.01	**1.47**	**1.25**
	Sortino	1.26	**1.72**	**1.49**
	CE #1 (%)	8.53	**8.70**	8.08
	CE #2 (%)	8.53	**8.70**	8.08
	Turnover (%)	9.18	33.24	38.85
1/N	Return (%)	8.06	6.93	6.57
	Volatility (%)	7.21	**6.89**	8.15
	Sharpe	0.77	0.65	0.50
	Sortino	1.05	0.86	0.79
	CE #1 (%)	7.54	6.46	5.91
	CE #2 (%)	7.54	6.46	5.91
	Turnover (%)	2.41	8.11	8.67
st.w.	Return (%)	9.50	9.49	9.38
	Volatility (%)	11.02	**6.48**	**7.42**
	Sharpe	0.64	**1.08**	**0.93**
	Sortino	0.83	**1.28**	**1.16**
	CE #1 (%)	8.29	7.39	6.63
	CE #2 (%)	8.27	7.34	6.62
	Turnover (%)	2.90	9.24	13.35

Appendix 3: Definition of economy subperiods

The table below presents six subperiods of which three are categorized as recessionary, and three as expansionary. Bessler and Wolff's methodology was applied for the period that does not overlap between our study and Bessler and Wolff's (January 2013 through December 2019).

Period	Economy state
Jan 1992 - Fev 1994	Recession
Mar 1994 - Jan 2001	Expansion
Fev 2001 - Jun 2004	Recession
Jul 2004 - Mar 2008	Expansion
Apr 2008 - Dec 2012	Recession
Jan 2013 - Dec 2019	Expansion