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CENTRAL BANK DIGITAL CURRENCY IN THE INSIDE-OUTSIDE MONEY COMPETITION MECHANISM

Rocco Alfarano

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ABSTRACT

The present thesis studies how a widely-accessible and interest-bearing CBDC affects the inside-outside money competition mechanism and in turn monetary equilibria. Consumers perception of CBDC as an alternative to bank deposits and the structure of the banking sector are crucial factors to determine the CBDC role in such a mechanism. Given imperfect competition in the banking sector, CBDC is found to be an effective monetary policy tool capable of influencing households' money-holdings directly through the deposit rate channel. In particular, the higher the substitutability across the two forms of digital money, the higher the correlation between the interest rates they bear.

A presente tese estuda como uma CBDC acessível pelo público e com juros afeta o mecanismo de competição pela moeda e, por sua vez, os equilíbrios monetários. A percepção dos consumidores sobre a CBDC como alternativa aos depósitos bancários e a estrutura do setor bancário são fatores cruciais para determinar o papel da CBDC nesse mecanismo. Dada a concorrência imperfeita no setor bancário, constata-se que o CBDC é um instrumento eficaz de política monetária capaz de influenciar os agregados familiares diretamente através do canal de taxa de depósito. Em particular, quanto maior a substituibilidade entre as duas formas de moeda digital, maior a correlação entre as suas taxas de juros.

KEYWORDS: *CBDC, Inside money, Outside money, money competition, monetary policy.*

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INTRODUCTION

In order to lay a common ground for all readers to understand the theme discussed, this thesis introduction is organized in three subparagraphs. The first one introduces the reader to CBDC definition, taxonomy and design; in the second one a variety of arguments on CBDC desirability are proposed; in the third one, the key questions and the objectives of the thesis are provided.

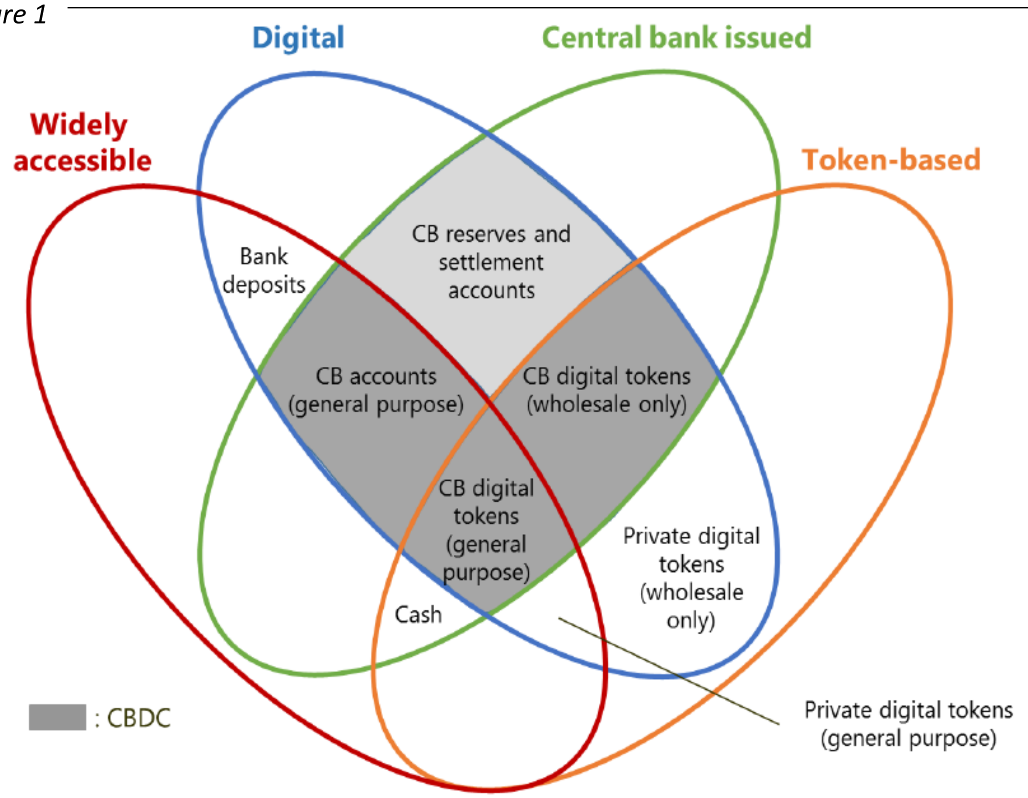
The thesis is articulated in 5 chapters. After the introduction, the literature review (chapter 2) and the models (chapter 3) are presented. At last, chapter 4 contains results and in chapter 5 conclusions are drawn.

1.1 CBDC DEFINITION, TAXONOMY AND DESIGN

Central bank digital currency (CBDC), also known as digital base money (DBM), is a new innovative type of central bank money, and is generically defined as (i) a central bank (CB) liability, (ii) assuming digital form, (iii) denominated in the legal unit of account, (iv) different than reserve balances or settlement accounts.

In order to understand better what CBDC is and how it relates to other types of money, a Venn-diagram is proposed in *Figure* 1. The diagram illustrates a taxonomy of money based on the combination of four key properties: (i) issuer (central bank or not), (ii) form (digital or

Figure 1



physical), (iii) accessibility (wide or restricted) and (iv) technology (account-based or token-based)[1]. Referring back to the definition, the first two properties are decisive for CBDC to be defined as such. The other two properties combined allow us to distinguish between CBDC families (dark grey areas in *Figure 1*): (i) public CB accounts, (ii) public CB token, (iii) and wholesale CB token.

CBDC can bear various design features. The main features concern: availability (24/7 or limited), anonymity, transfer mechanism (centralized or decentralized), interest-bearing, presence of limits or caps.

The combination of all these features, namely the chosen CBDC design, will determine the attractiveness of CBDC for the public, as well as the way it will interact with other elements into the economy. In particular, the interest-bearing feature can make the difference between a policy tool and a mere payment mean; anonymity and availability can result as attractive properties playing a role in the money competition mechanism; while limits and/or caps on quantities can be useful for the policy maker to control eventual undesirable implications or to steer usage in a certain direction.

This thesis is focused on widely accessible CBDC. Indeed accessibility for the general public seems to represent a key feature to make CBDC a relevant policy tool. For this reason, I will provide only a separate and short hint on wholesale CB token in *Box 1*.

Public CB accounts would be nothing else than deposits that can be held by the public at the CB. They would differ from bank deposits in the underlying riskiness, in the interest rate paid out, in the efficiency of payments and settlements.

It has to be noted that account-based CBDC would then require the CB to know a lot about its "customers", and this raises the question of whether the CB wants to get involved in this kind of client-firm relation with citizens.

Token-based CBDC would represent the legal tender in a fashion resembling a cryptocurrency. It would represent a solution where CB stays aside from providing a direct service to the public. Despite the fact this kind of money has never been adopted so far, distributed ledger technology (DLT) appears to be the best candidate to back the instrument

[1] Account- and token-based money are distinguished by identification requirement. For the former, the key question is whether a transaction has been made by the true owner of the account or by a thief; for the latter, the key question is whether the transferred money is valuable or counterfeit. As you can notice from the Venn-diagram, CB reserve is account-based money while Bitcoin (private digital token on the graph) is token-based money. An account-based CBDC would then take the same form of actual reserves while a CB token would resemble a cryptocurrency.

functioning. Although DLT maturity in the field of central banking is still uncertain for the purpose of implementation.

Box 1. Wholesale CB token

Digital CB money is already available in the form of reserves or settlements to monetary counterparties and some non-monetary counterparties. Therefore a wholesale CB token would not sound as a novelty if the set of potential recipients matched the today CB monetary interlocutors.

On the matters of monetary policy and financial stability, the effects of CBDC issuance are supposed to be strictly related to how vast the pool of users is and whether it is attractively remunerated. Indeed, as pointed out by the *Bank for International Settlement*, a well-remunerated wholesale CBDC accessible by institutional investors can influence their holdings of low-risk instruments (short-term government bills, repos, etc.) and then it would help establish a hard floor under money market rates. This contingency is benefic in terms of narrowing policy rates dispersion, which is identified as a primary indicator of the level of passthrough inefficiency. Furthermore, such wholesale CBDC would represent a positive tool to make the settlement systems more efficient and secure, and to help CB financial activity monitoring[2].

1 Passthrough Efficiency in the Fed's New Monetary Policy Setting, Darrell Duffie and Arvind Krishnamurthy. *Graduate School of Business, Stanford University. (Text available here:* https://www.hoover.org/sites/default/files/duffiekrishnamurthy.pdf)

2 Central Bank Digital Currency, Committee on Payments and Market Infrastructure, Markets Committee. *Bank for International Settlement. (Text available here:* https://www.bis.org/cpmi/publ/d174.htm)

Several concrete examples of CBDC or initiative on the matter have been carried out up to today. The ones definitely noteworthy are:

- *Implemented CBDC:*

  Dinero electrónico, implemented by the CB of Ecuador, it was a concrete example of widely accessible CBDC through CB accounts. Citizens could open an account online through the official app, and deposit/withdraw money at designated transaction centers. CB of Ecuador was, therefore, competing against the private sector in providing a service for retail transactions. The money was made available for usage in 2015 and it is now in a state of decommissioning, because of scarce usage.

E-dinar, implemented by Tunisia postal authority in 2015, and *E-CFA*, adopted in Senegal first (2017) and possibly in all the UEMOA countries in the future, seems to follow basically the same structure of the Ecuadorian CBDC, but very few clear information are available in this regard.

- *E-krona*, still in the process of experimentation by the Riskbank, represents a clear example of widely accessible CBDC. The Swedish CB will make use of such innovation as a reaction to the unceasing decline in the use of cash observed in the country. With the objective of offering a sort of "digital cash", the design features would be trivially oriented toward a combination of anonymity/integrity, unlimited availability, no interest-bearing, and decentralized transfer mechanism; but concretely Riskbank left other possibilities –like a centralized interest-bearing CBDC- open.
- *DNBcoin*, developed by the Dutch National Bank only for internal test purposes, is a prototype token-based CBDC, experimented in two versions, which gave important results and feedback on how technically good a DLT is in backing legal tender. The third prototype seems to look toward applications in the field of financial market infrastructures for the settlement of complex financial transactions[2]. On this same matter, Bank of Canada has run research under the project Gasper and Bank of Singapore under the project Ubin.

1.2 WIDELY ACCESSIBLE CBDC DESIRABILITY

Why should a CB issue CBDC? There are several points in favour of CBDC issuance, and there are as many counter-arguments. Whether CBDC is desirable for a country's economy it will depend on how those pros and cons balance for that specific case.

In general, we can recognise three broad areas of motivations for CBDC issuance: (i) monetary policy (MP) transmission and implementation; (ii) competitiveness and efficiency of payment systems; (iii) financial inclusion –to which *Box 2* is dedicated-.

Regarding MP transmission and implementation, wider digital access to the C may strengthen the pass-through of the policy rate trough the transmission channels to money and lending

[2] Ron Berndsen Speech at DNB: https://www.dnb.nl/binaries/Speech%20Ron%20Berndsen_tcm46-342846.pdf.

markets, especially if CBDC results attractively designed (mainly in terms of anonymity and interest-bearing). For example, if households considered retail interest-bearing CBDC to be a substitute to bank deposits, banks would have less scope for independently setting the interest rate on retail deposits. Indeed banks would find it harder not to increase deposit rates in tandem with the CBDC rate, which in turn would become a direct MP tool to depositors. One more point on this matter is that CBDC can possibly serve as an instrument to alleviate the zero lower bound if designed to tolerate negative interest rates. For this to be effective, it could be needed a simultaneous withdrawal of higher denomination banknotes or a general abating the demand for cash[3]. On this matter, Francisco Rivadeneyra[4]'s claim is noteworthy:

"When you have a CBDC which can bear a positive or negative interest rate you have much more power for implementing MP. So, in theory, it should be welfare improving. But how people would feel to be charged with negative rates is unknown".

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[3] "Monetary Policy and Digital Currencies: Much Ado about Nothing?", Christian Pfister, *Bank of France*.

[4] Francisco Rivadeneyra is Senior Researcher Advisor at Bank of Canada. The citation has been extracted from the panel debate "The impact of CBDC (Central Bank Digital Currency) and its current state" organized by Deconomy. The debate is available online on the official page of the association: https://www.youtube.com/watch?v=Mw541OU2eEg&t=7s .

Box 2 Financial Inclusion

Financial inclusion means the possibility of an economic agent to access money and therefore participate in the economic machine. As the world moves toward a less usage of cash, replacing it with technological solutions, a part of society –made of people uncomfortable, unable or unauthorized to use those technological solutions- may result marginalized.

CBDC is often indicated as an instrument capable of fostering financial inclusion, as it can be designed to allow almost everyone to access it.

Developed countries, which are well endowed in their means of payments, may not need a CBDC to foster furthermore financial inclusion or transfers efficiency. In countries like Sweden –where cash is disappearing- this theme is important instead, as many individuals who cannot access bank accounts could end up financially excluded. In an underdeveloped country it would instead represent a terrific innovation capable of including into the financial world many citizens today marginalized, and capable of making the payment system relevantly more efficient for the benefit of all the economic participants.

On this same regard, we also have to recognized the role of CBDC as a resiliency resort: while cash is likely to represent the ultimate resiliency ground against a total disruption of the payment system, a CBDC (reliant on a separate system) would represent a resiliency ground to guarantee remote payments in case of private-sector knock-out.

In other words, CBDC can foster financial inclusion –especially in underdeveloped countries- and guarantee financial inclusion in case of private-sector disruption.

1.3 THESIS OBJECTIVES

Core central banking is about monetary policy, financial stability, and payment services. Money is indeed the beating heart of central banks activity.

Unsurprisingly, CBDC has become an increasingly interesting topic for research and discussion in the CB field as it could have the potential to be positively impactful on the matters of monetary policy, financial stability and payment services[5].

[5] Central Bank Digital Currency, Committee on Payments and Market Infrastructure, Markets Committee. *Bank for International Settlement. (Text available here: https://www.bis.org/cpmi/publ/d174.htm).*

On a broader level, CBDC raises questions about the role of central bank money, direct access to central bank liabilities and the structure of financial intermediation. While all these themes are going to be on the background of the writing, the present thesis will focus on the specific aspect of inside-outside money competition.

In particular, the objectives of such dissertation are: (i) to recreate an interesting theoretical framework for inside-outside money competition analysis at the light of a widely accessible interest-bearing CBDC presence into the economy; (ii) to study the role of such CBDC into the competition mechanisms through the specified theoretical framework; (iii) to formalize how CBDC can be useful for the scope of MP.

For this end, I will make use of a monetary equilibrium model where households are provided with government-issued outside-money in both cash and CBDC forms, and with privately-issued inside-money in the form of deposits. In particular, the agents interacting in such a frame will be: (i) households, (ii) government, and (iii) financial intermediaries. While each agent behaviour is obviously key to the resulting competition mechanism, the agents' behaviour is not obvious in its-self, even more so if we are concerned with something still experimental like CBDC is:

- How does the public perceive CBDC related to other types of money? Are CBDC and bank deposits treated as substitutes or complements? And if they are treated as substitutes, are they perfect or imperfect substitutes?
- How does the government make use of CBDC? Is it going to be a seigniorage-revenues maximization tool? Or is it going to foster welfare?
- How do financial intermediaries compete in the supply of inside-money? What is the cost structure they face?

The present thesis pretends to address the over-mentioned objectives by engaging in a discussion for the listed unresolved questions.

As a warning, it has to be said that all the issues concerning CBDC technological feasibility, implementability and CB credibility threat are omitted. On the other hand, the theme of CBDC desirability is central for such a dissertation, which pretends indeed to draw some pros toward that direction.

2 LITERATURE REVIEW

The main reference for this thesis is Marimon et al. (2003). The paper presents a theoretical framework for studying competition between inside and outside money. The researchers find that inside money competition may (i) have a disciplinary role on equilibrium inflation by imposing an upper bound constraint, in the case of a fully committed revenues maximizing CB; (ii) drive outside money out of circulation as inside money issuers become more and more efficient, in the case of non-fully committed revenue-maximizing CB; (iii) have no impact on the economy, in the case of a Ramsey government pursuing the Friedman rule.

Davoodalhosseini (2018) estimates that the gains from introducing an account-based interest-bearing CBDC in Canada as an increase in consumption by around 16%. He uses a model where CBDC is supposed to be more costly for agents to use than cash because of the absence of anonymity, and where CBDC have discriminating rates (household rate and business rate).

Chiu et al. (2019) build a model with imperfect competition in the banking sector which shows that CBDC, even in the case of low usage, would serve as an outside option for households, thus limiting banks' market power in the deposit market. So they estimate that CBDC can raise bank lending by around 7% and increase output by around 1%.

Michael Kumhof and Clare Noone (2018) offer three models where an interest-bearing CBDC is accessible in different sectors (wholesale, public, and indirect public) where they study the question of how CBDC could affect the size and composition of commercial bank balance sheets. They find that if the introduction of CBDC follows a set of core principles, bank funding is not necessarily reduced, credit and liquidity provision to the private sector need not to contract, and the risk of a system-wide run from bank deposits to CBDC is addressed.

John Barrdear and Michael Kumhof (2016) elaborate a monetary DSGE model to study the macroeconomic consequences of a widely accessible interest-bearing CBDC. They find that CBDC issuance of 30% of GDP, against government bonds, could permanently raise GDP by as much as 3%, due to reductions in real interest rates, distortionary taxes, and monetary transaction costs. In their view, countercyclical CBDC price or quantity rules, as a second monetary policy instrument, could substantially improve the CB ability to stabilize the business cycle.

3 THE MODEL

All the models that will be presented represent a more or less elaborated extension of Marimon et al. (2003). The economy is populated by a representative household, one or more financial intermediaries, and a government. While Marimon et al. (2003) study the interaction between cash and bank deposits in the specific context of money perfect substitutability and perfect competition in the banking sector, in the present thesis a widely-accessible CBDC is included into the competition framework and various scenarios –that allow making useful confrontations to learn how diverse environments would lead CBDC to play a different role into the money competition mechanism- are taken into account. The scenarios are going to be dependent on whether:

(i) Households treat CBDC and bank deposit as perfect or imperfect substitutes,
(ii) The banking sector is perfectly or imperfectly competitive,
(iii) The government is a Ramsey-government or a transfer-maximizer.

3.1 HOUSEHOLD

The household maximizes an objective function, *V*, which involve consumption of a money-good, c_t^1, consumption of a credit-good, c_t^2, and total labour, n_t, for an infinite number of periods starting from *t=0*,

$$V = \sum_{t=0}^{\infty} \beta^t [\, u(c_t^1) + \, u(c_t^2) - \alpha \, n_t].$$

The household is endowed with a unit of time which is spendable for leisure and total labour. Part of the total labour supplied serves as a mean to deposits issuance, c_t^f, and the rest is used for the production of consumption goods, respectively n_t^1 and n_t^2. Consumption goods production is obtained through a one-to-one relation to labour, so that equilibrium in the goods markets requires,

$$c_t^1 = n_t^1 \;\; and \;\; c_t^2 = n_t^2.$$

Therefore feasibility in the labour market implies that:

$$n_t = c_t^1 + c_t^2 + c_t^f.$$

Money-good and credit-good are intended as in Lucas and Stokey (1987). Therefore, payment instruments are needed to carry out the money-good consumption period after period. The payment instruments household is possibly supplied with are: (i) cash, M_t, namely physical currency; (ii) CBDC, $CBDC_t$, designed to be widely-accessible and interest-bearing; (iii) bank deposit, E_t, intended as a digital currency substitute privately issued by financial intermediaries. All those payment means are indifferently effective for purchases and they are always traded at par[6]. The first two payment instruments will be referred to as outside-money, as they represent CB liabilities, while bank deposits are inside-money. Note that cash is unable to pay interests because of its physical nature, while digital instruments bear gross interest rates respectively referred to as I^{CBDC} and I^{f}.

Note that the household's payment preferences are solely based on money return, namely the interest rate paid by money itself, and they do not account for other potentially relevant factors such as anonymity, riskiness and usability[7]. Money holdings held in a given period t will act as inputs for the payment function $\square_t$, which in turn will determine the effective monetary output usable for money-good consumption. Such payment function is homogeneous of degree one and pretends to parametrically reflect money substitutability grades:

(eq. 0) $$\square_t\,(M_t, CBDC_t, E_t) = \left[M_t^{1-\rho} + \left(CBDC_t^{1-\varphi} + E_t^{1-\varphi}\right)^{\frac{1-\rho}{1-\varphi}}\right]^{\frac{1}{1-\rho}}$$

Where the parameter ρ measures substitutability between cash and digital money, and φ is instead the substitutability between CBDC and bank deposits. They vary between zero (included) and one (excluded), where the zero-level corresponds to perfect substitutability. We will consider (i) the case where cash is perceived as imperfect substitute of digital money, while CBDC and deposits are treated as perfect substitutes, namely $\varphi = 0$ and $\rho \in (0,1)$; (ii) and a more general case where all types of money are treated as imperfect substitutes (to simplify the analysis the substitutability parameters are set equal, so that $\rho = \varphi,\ \rho \in (0,1)$).

Transactions timing follows Svensson (1985). The good market meets at the beginning of the period: the household purchases money-goods, consume money-goods and credit-goods. After that, the asset market meets: the credit-goods, real wages, n_t, and government transfers,

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[6] Cash, CBDC and deposits may be traded with a diverse exchange rate each. For the purposes of the thesis, this eventuality is phased out by assumption, so that the agents can convert every type of money into others one-to-one.

[7] People may perceive cash as costly to hold because of physical inefficiency. While digital money appears to allow much more consumption opportunity.

g_t, are paid and the household adjusts its portfolios of payment instruments and bonds, b_t^h. Bonds pay a gross interest rate $I_t \equiv \left(\frac{P_{t+1}}{P_t}\right) R_{t+1}$, where P_t and R_t are respectively the price level and the real interest rate at time t. As a consequence, in each period the households will have to carry a portfolio of payment instruments to the next period in order to accommodate the future money-good consumption.

We assume that the bond rate is always above money rates, which constitutes a classic monetary friction: the households want to limit the exposure to money-holding inefficiency, given by the cost-opportunity of missing a bond investment. The absence of such an assumption would include into the analysis the possibility of the zero lower bound, a topic closely related to CBDC –as we saw in the introduction-, which need special attention and focus that this paper will not dedicate.

The household problem is then formalized as follows:

$$\underset{\{c_t^1, c_t^2, n_t, M_{t+1}, E_{t+1}, CBDC_{t+1}, b_{t+1}^h,\}}{MAX} \quad V$$

$$M_{t+1} + CBDC_{t+1} + E_{t+1} + b_{t+1}^{\mathrm{h}} P_t \leq M_t + CBDC_t\, I_t^{CBDC} + I_t^f E_t + P_t b_t^h R - P_t(c_t^1 + c_t^2) + P_t n_t + P_t g_t + \pi_t$$

$$P_t\, c_t^1 \leq \square_t(M_t, CBDC_t, E_t)$$

$$\lim_{T\to\infty} \frac{b_{T+1}^h}{\prod_{s=0}^{T} I_s}$$

$$M_0, R_0 b_0^H, I_0^f E_0 \; are\ given$$

In order to simply the algebra, the marginal conditions are derived under the assumption that $\varphi = \rho$ and $\rho \in (0,1)$, namely cash is assumed as substitutable for deposits as CBDC. The resulting Lagrangian is:

$$L = \sum_{t=0}^{\infty} \beta^t [\, u(c_t^1) + u(c_t^2) - \alpha\, n_t] + \sum_{t=0}^{\infty} \lambda_t \big[M_t + CBDC_t\, I_t^{CBDC} + I_t^f E_t + P_t b_t^h R - P_t(c_t^1 + c_t^2) + P_t n_t + P_t g_t + \pi_t - M_{t+1} - CBDC_{t+1} - E_{t+1} - b_{t+1}^{\mathrm{h}} P_t\big] + \sum_{t=0}^{\infty} \mathrm{\tau_t} \left[\left(M_t^{1-\rho} + CBDC_t^{1-\rho} + E_t^{1-\rho}\right)^{\frac{1}{1-\rho}} - P_t\, c_t^1\right]$$

The problem's FOCs are:

(c_{t+1}^1) $\qquad \beta^{t+1}\, u'^{(c_{t+1}^1)} = P_{t+1}[\lambda_{t+1} + \tau_{t+1}]$

(c_t^2) $\qquad \beta^t\, u'^{(c_t^2)} = P_t \lambda_t$

$(n_t) \qquad \beta^t \alpha = P_t \lambda_t$

$(M_{t+1}) \qquad \lambda_t = \lambda_{t+1} + \tau_{t+1} \left(M_t^{1-\rho} + CBDC_t^{1-\rho} + E_t^{1-\rho}\right)^{\frac{\rho}{1-\rho}} M_{t+1}^{-\rho}$

$(CBDC_{t+1}) \qquad \lambda_t = \lambda_{t+1} I_{t+1}^{CBDC} + \tau_{t+1} \left(M_t^{1-\rho} + CBDC_t^{1-\rho} + E_t^{1-\rho}\right)^{\frac{\rho}{1-\rho}} CBDC_{t+1}^{-\rho}$

$(E_{t+1}) \qquad \lambda_t = \lambda_{t+1} I_{t+1}^{f} + \tau_{t+1} \left(M_t^{1-\rho} + CBDC_t^{1-\rho} + E_t^{1-\rho}\right)^{\frac{\rho}{1-\rho}} E_{t+1}^{-\rho}$

$(b_{t+1}) \qquad P_t \lambda_t = P_{t+1} \lambda_{t+1} R$ Where $I_{t+1} = \frac{P_{t+1}}{P_t} R$, so that $\lambda_t = \lambda_{t+1} I_{t+1}$

$(\lambda_{t+1}) \qquad M_{t+1} + CBDC_{t+1} + E_{t+1} + b_{t+1} P_t$
$= M_t + CBDC_t I_t^{CBDC} + I_t^f E_t + P_t b_t R - P_t(c_t^1 + c_t^2) + P_t n_t + P_t g_t + \pi_t$

$(\tau_{t+1}) \qquad P_{t+1} c_{t+1}^1 = \left(M_{t+1}^{1-\rho} + CBDC_{t+1}^{1-\rho} + E_{t+1}^{1-\rho}\right)^{\frac{1}{1-\rho}}$

Furthermore, consider the following set of variables:

$s_{t+1} \equiv I_{t+1} - I_{t+1}^{CBDC}$. This differential will be called "policy spread" as it results from government's policy. In the eyes of the household, it represents the cost-opportunity of holding CBDC. In the government's eyes it measures CBDC marginal seigniorage revenue, indeed such spread indicates how cheap CBDC issuance is with respect to debt.

$\theta_{t+1} \equiv I_{t+1} - I_{t+1}^{f}$. This differential will be called "intermediation spread" as it comes from financial intermediaries. In the eyes of households, it plays as the cost opportunity of holding bank deposits. For banks, it is instead the marginal revenue gained through deposit issuance, under the assumption that they invest all the deposit inflows in bonds.

Furthermore, assume $u(c^1) = \frac{(c^1)^{1-\sigma}}{1-\sigma}$ and $u(c^2) = \frac{(c^2)^{1-\sigma}}{1-\sigma}$.

Combining equations (c_t^2) and (n_t) the marginal condition for credit-good consumption is derived:

$(eq. 1) \qquad c_t^2 = \left(\frac{1}{\alpha}\right)^{\frac{1}{\sigma}}$

Credit-good consumption depends on the marginal disutility of labour and on consumption preferences. The agent will continue consuming credit-goods as long as its marginal utility compensates for the disutility of labour. Obviously, credit-good consumption choice is regardless of the monetary sphere.

Combining (E_{t+1}), $(CBDC_{t+1})$, (M_{t+1}) and (b_{t+1}), marginal conditions for relative demand of payment instruments are obtained:

$$(eq. 2) \qquad \frac{CBDC_{t+1}}{E_{t+1}} = \left(\frac{\theta_{t+1}}{s_{t+1}}\right)^{\frac{1}{\rho}}$$

$$(eq. 3) \qquad \frac{CBDC_{t+1}}{M_{t+1}} = \left[\frac{(I_{t+1} - 1)}{s_{t+1}}\right]^{\frac{1}{\rho}}.$$

CBDC demand relative to deposits is given by the relative convenience of CBDC holding with respect to deposits, namely the spreads ratio, and by the grade of money substitutability. CBDC demand relative to cash follows the exact same principle, where $(I_{t+1} - 1)$ is interpreted as the cost opportunity of holding cash. Therefore, when the policy spread equals the intermediation spread, households will be demanding CBDC and deposits in the same extent; when the policy spread equals the cost-opportunity of holding cash, households will be demanding the same quantity of CBDC and cash.

Note that lower substitutability parameters make money types more substitutable. In other words, the elasticity of money demand increases. As a result, households become more susceptible to interest rate shifts, and in the extreme case of perfect substitutability –where $\rho = 0$- they will be willing to hold only the most remunerative mean of payment. Note that in the most general case where $\rho \neq \varphi$, namely cash is not as substitutable as CBDC with respect to deposits, the CBDC-to-cash ratio is not as simple as described in equation 3, but the ratio is complicated by a factor which grows with the substitutability parameters discrepancy.

Consider the following notation: $m_{t+1} \equiv \frac{M_{t+1}}{P_{t+1}}$, $cbdc_{t+1} \equiv \frac{CBDC_{t+1}}{P_{t+1}}$, $e_{t+1} \equiv \frac{E_{t+1}}{P_{t+1}}$.

Solving the whole system of FOCs, the money demands and money-good are obtained:

$$(eq. 4) \qquad m_{t+1}^{h} = \frac{\{\alpha[1 + H_{t+1}]\}^{-\frac{1}{\sigma}}}{H_{t+1}^{m}}$$

$$where \;\; H_{t+1}^{m} \equiv \left[1 + \left(\frac{I_{t+1} - 1}{s_{t+1}}\right)^{\frac{1-\rho}{\rho}} + \left(\frac{I_{t+1} - 1}{\theta_{t+1}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}$$

$$(eq. 5) \qquad cbdc_{t+1}^{h} = \frac{\{\alpha[1 + H_{t+1}]\}^{-\frac{1}{\sigma}}}{H_{t+1}^{cbdc}}$$

$$where\ H_{t+1}^{cbdc} \equiv \left[1 + \left(\frac{s_{t+1}}{I_{t+1} - 1}\right)^{\frac{1-\rho}{\rho}} + \left(\frac{s_{t+1}}{\theta_{t+1}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}$$

$$(eq.6) \qquad e_{t+1}^{h} = \frac{\{\alpha[1 + H_{t+1}]\}^{-\frac{1}{\sigma}}}{H_{t+1}^{e}}$$

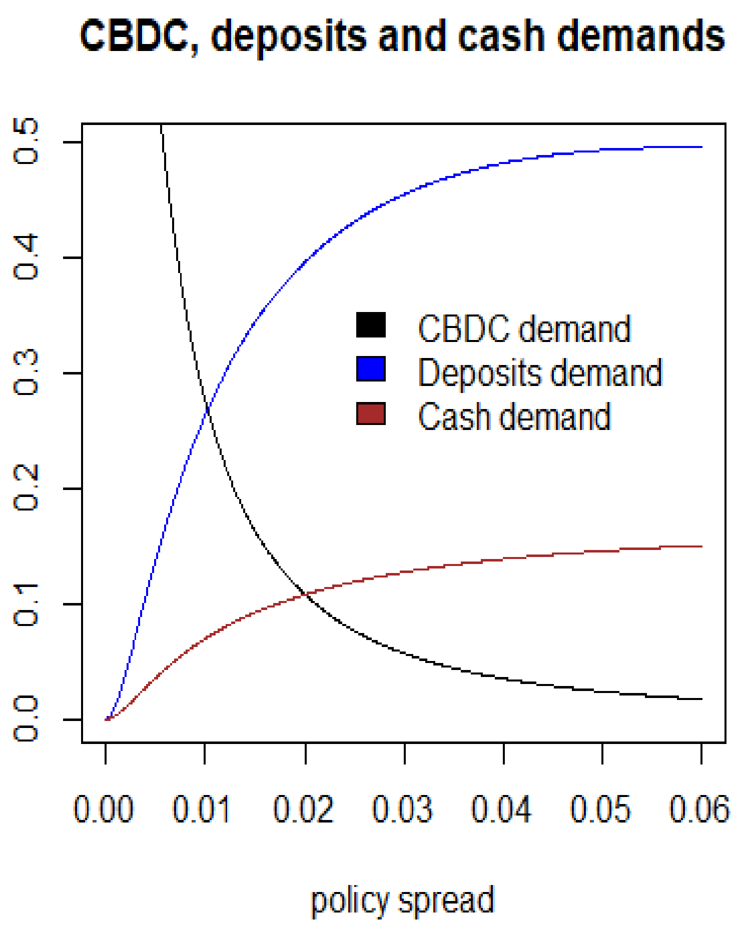
$$where\ H_{t+1}^{e} \equiv \left[1 + \left(\frac{\theta_{t+1}}{I_{t+1} - 1}\right)^{\frac{1-\rho}{\rho}} + \left(\frac{\theta_{t+1}}{s_{t+1}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}$$

$$(eq.7) \qquad c_{t+1}^{1} = \{\alpha[1 + H_{t+1}]\}^{-\frac{1}{\sigma}}$$

$$where\ H_{t+1} \equiv (s_{t+1})\left(H_{t+1}^{cbdc}\right)^{-\rho} = (\theta_{t+1})(H_{t+1}^{e})^{-\rho} = (I_{t+1} - 1)(H_{t+1}^{m})^{-\rho}.$$

Money demands are determined in equilibrium as functions of the labour disutility, consumption preferences, spreads combination and the level of money substitutability parameters. When the policy spread increases, coherently with the assumption of imperfect substitutability of money substitutes, CBDC demand gets deteriorated while deposits and cash demand improve. In particular, as illustrated in *Graph 1,* given the parameterization provided in *table 1,* deposit and cash demands are concave and increasing functions of the policy spread while CBDC demand is a decreasing and convex function of the policy spread. An intuition to understand why the former are concave while the latter is convex, lies in equation 7: note that as the policy spread increases, the optimal money-good consumption falls ceteris paribus. Given that the cash-in-advance constraint is binding, less money-good consumption means less money held in aggregate. As a result, when the policy spread increases CBDC is substituted by deposits and cash, but not in the exact same extent.

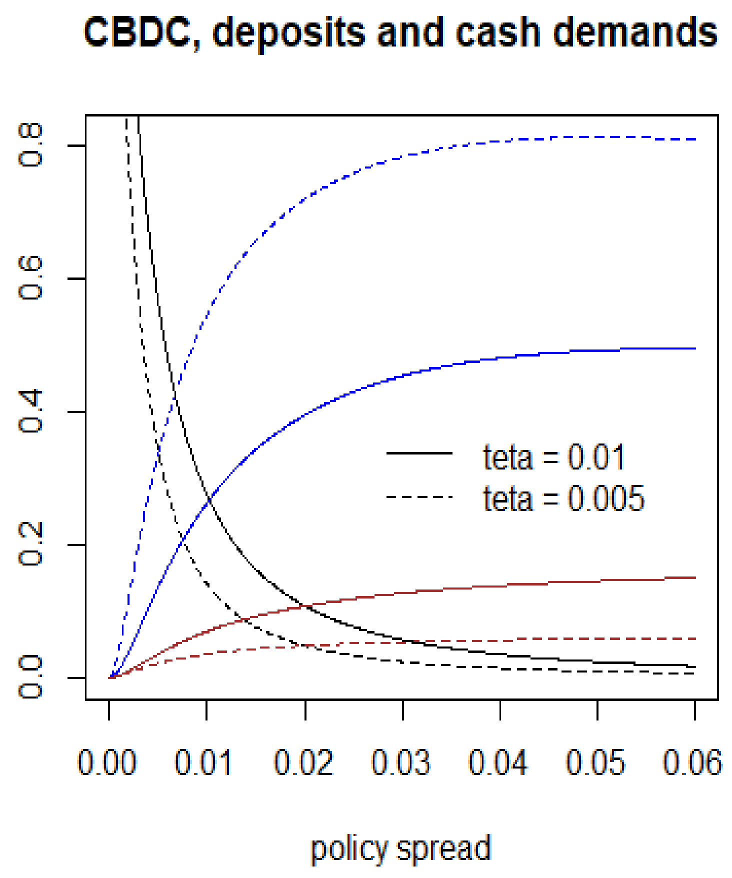
Graph 1



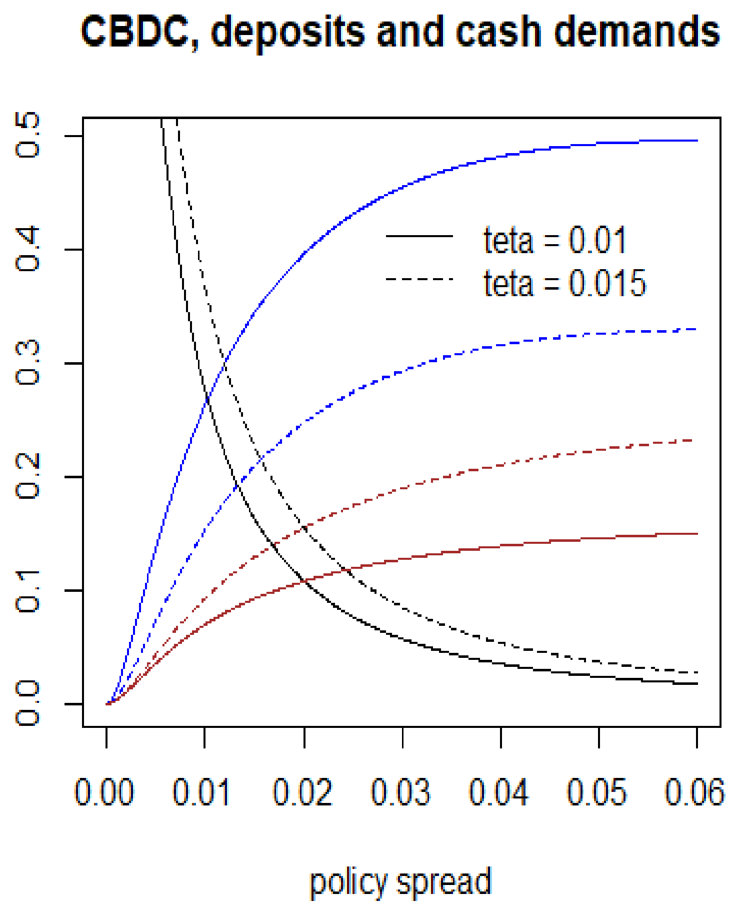
Accordingly, when the intermediation spread grows, deposit demand gets dimed while CBDC and cash become more demanded (ceteris paribus). This mechanics is illustrated in *Graph 3*, where the dashed curves represent the new demands after the intermediation spread increases from 1% to 1.5%.

Furthermore, the substitutability parameter is key in the determination of the household portfolio composition. As *Graph 4* and *Graph 5* show, given the parameterization in *table 1*, money demands become steeper and steeper as ρ approaches zero because of the enlargement of demand elasticity.

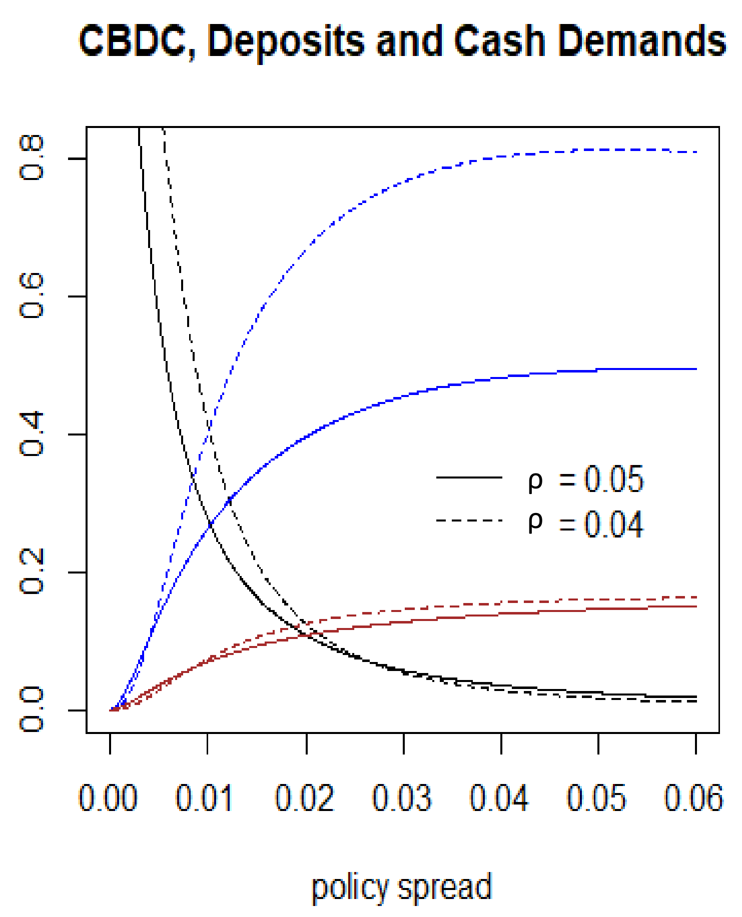
Graph 2



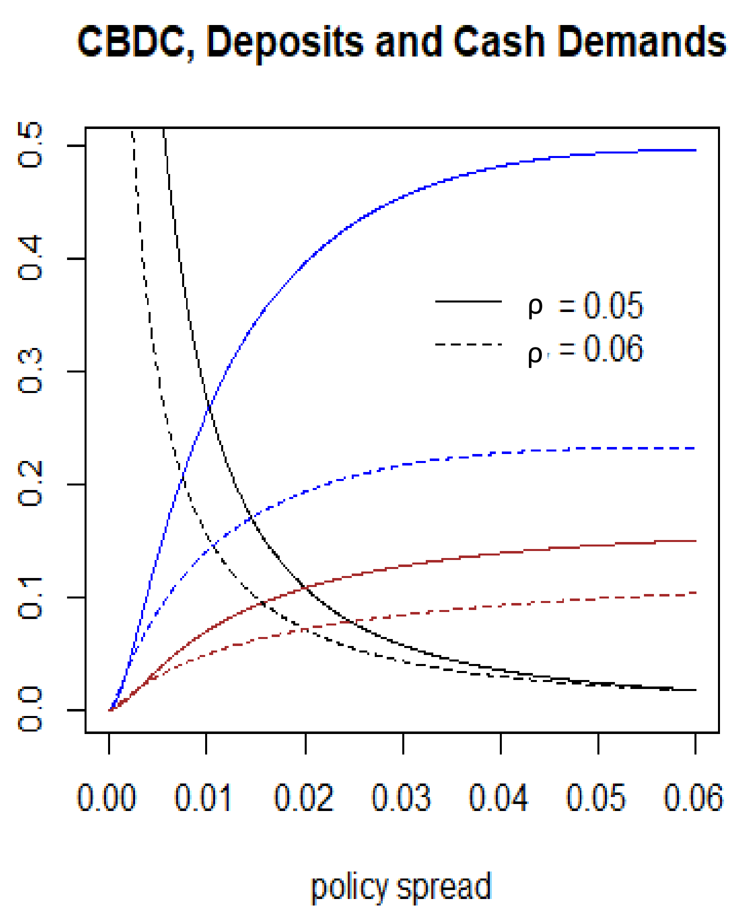
Graph 3



Graph 4

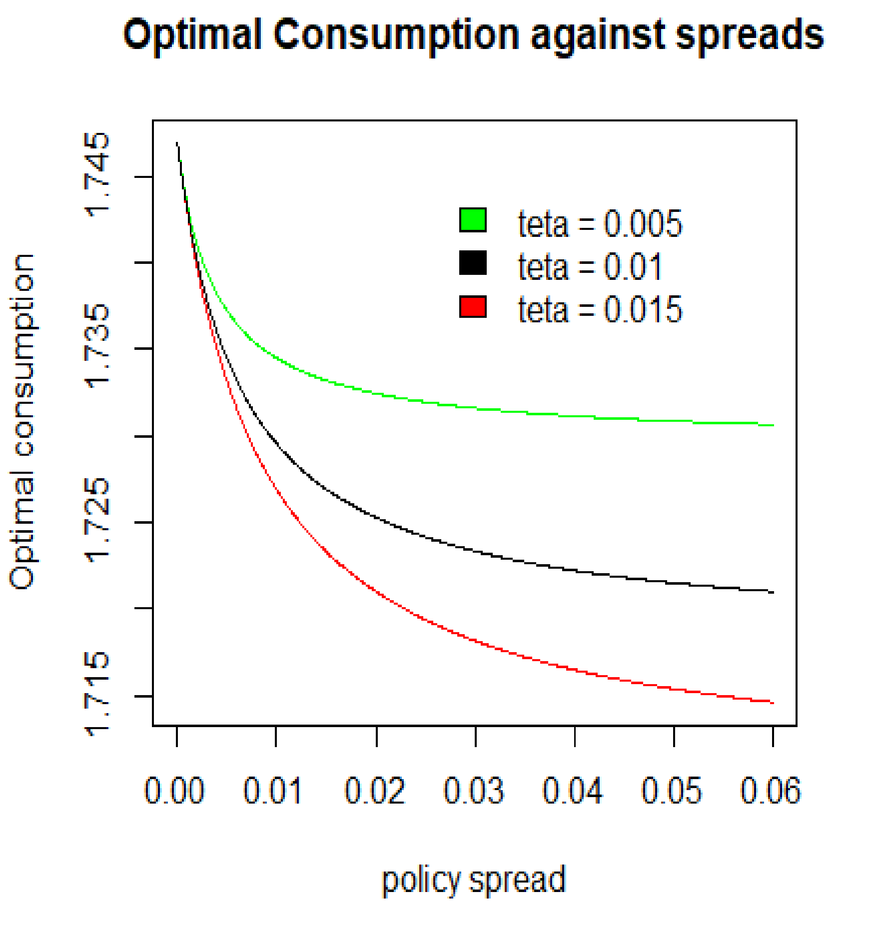


Graph 5



The optimal money-good consumption is determined in equilibrium as a function of the linear cost of labour, consumption preferences, spreads and money substitutability (contained in the algorithm H_{t+1}). Everything else constant, higher spreads mean less money-good consumption because holding money becomes more costly, namely relatively less convenient than a bond investment. This is clearly shown in *Graph 6.*

Graph 6



Perfect substitutability case

Assume $\varphi = 0$ and $\rho \in (0,1)$. Then we have:

$$\square_t(M_t, CBDC_t, E_t) = [M_t^{1-\rho} + (CBDC_t + E_t)^{1-\rho}]^{\frac{1}{1-\rho}}$$

The new FOCs are:

$(c_{t+1}^1)'$ $\quad \beta^{t+1}\, u^{'(c_{t+1}^1)} = P_{t+1}[\lambda_{t+1} + \tau_{t+1}]$

$(c_t^2)'$ $\quad \beta^t\, u^{'(c_t^2)} = P_t\lambda_t$

$(n_t)'$ $\quad \beta^t\, \alpha = P_t\lambda_t$

$(M_{t+1})'$ $\quad \lambda_t = \lambda_{t+1} + \tau_{t+1}\ [M_t^{1-\rho} + (CBDC_t + E_t)^{1-\rho}]^{\frac{\rho}{1-\rho}}\, M_{t+1}^{-\rho}$

$(CBDC_{t+1})'$ $\quad \lambda_t = \lambda_{t+1}\, I_{t+1}^{CBDC} + \tau_{t+1}\ [M_t^{1-\rho} + (CBDC_t + E_t)^{1-\rho}]^{\frac{\rho}{1-\rho}}\, (CBDC_{t+1} + E_{t+1})^{-\rho}$

$(E_{t+1})'$ $\quad \lambda_t = \lambda_{t+1} I_{t+1}^f + \tau_{t+1}\, [M_t^{1-\rho} + (CBDC_t + E_t)^{1-\rho}]^{\frac{\rho}{1-\rho}}\, (CBDC_{t+1} + E_{t+1})^{-\rho}$

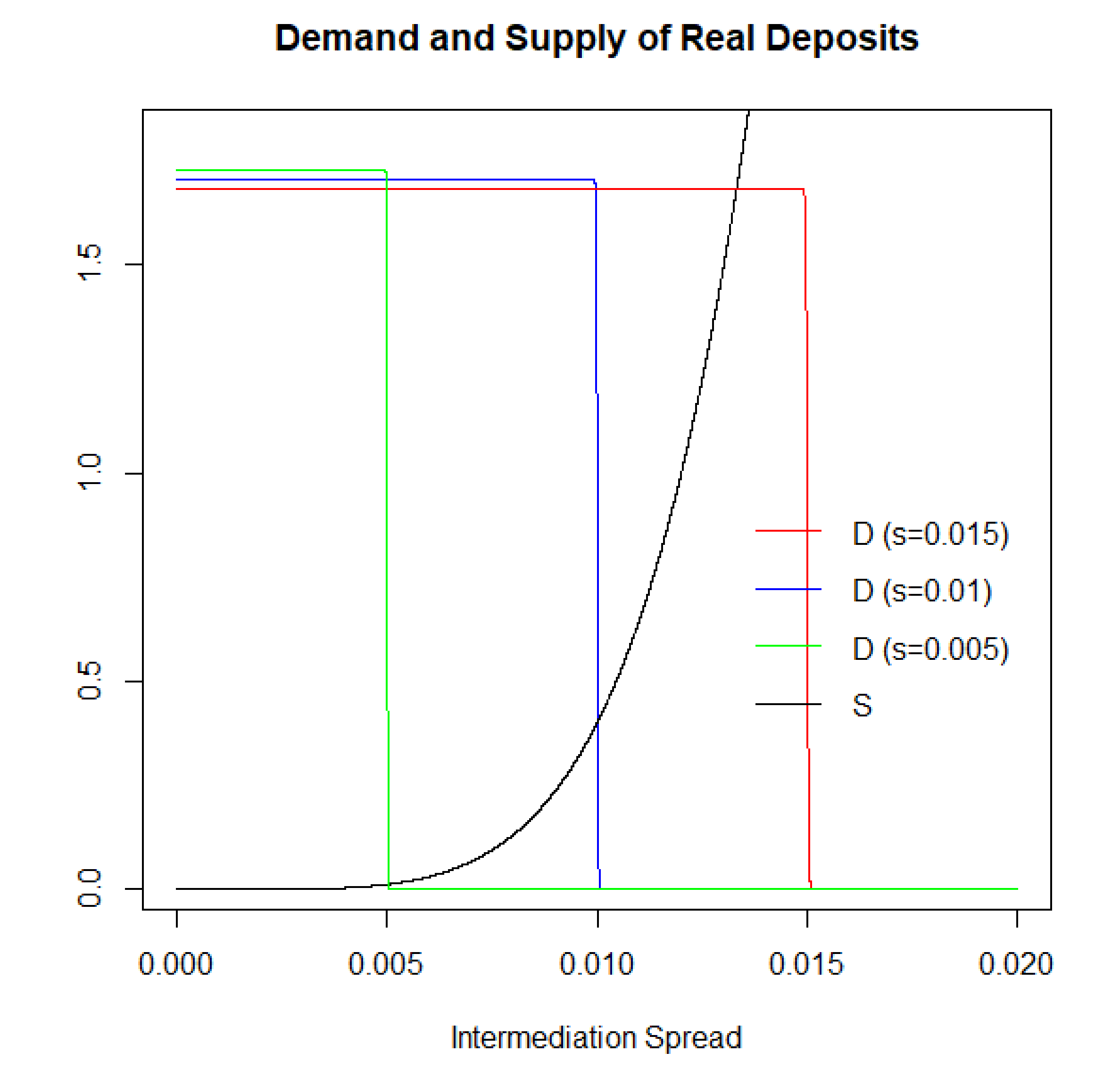
$(b_{t+1})'$ $\quad P_t\lambda_t = P_{t+1}\lambda_{t+1}R$ *Where* $I_{t+1} = \frac{P_{t+1}}{P_t} R$, *so that* $\lambda_t = \lambda_{t+1}\, I_{t+1}$

$(\lambda_{t+1})'$ $\quad M_{t+1} + CBDC_{t+1} + E_{t+1} + b_{t+1}P_t$
$= M_t + CBDC_t\, I_t^{CBDC} + I_t^f E_t + P_t b_t R - P_t(c_t^1 + c_t^2) + P_t n_t + P_t g_t + \pi_t$

$(\tau_{t+1})'$ $\quad P_{t+1}\, c_{t+1}^1 = [M_t^{1-\rho} + (CBDC_t + E_t)^{1-\rho}]^{\frac{1}{1-\rho}}$

Note that $(CBDC_{t+1})'$ and $(E_{t+1})'$ hold at the same time if and only if $I_{t+1}^{CBDC} = I_{t+1}^f$. This is due to the fact that –given perfectly substitutable digital substitutes - households are willing to hold only the most remunerative substitute; and in case of equal remuneration, they are actually indifferent on which digital payment means to use. In other words, the sole equilibrium where both digital instruments are valued requires remuneration rates equality. *Graph 7* provides an illustrative explanation by showing how deposit demand –as a function of the intermediation spread- shifts when the policy spread moves. So, when $\theta_{t+1} = s_{t+1}$ households hold a portfolio with cash and digital money, where the amount of digital money can be optimally any combination of CBDC and deposits, but as soon as the equality gets broken the optimum will move to a corner solution where only the less costly digital money is valued.

Graph 7



Assuming $s_{t+1} \geq \theta_{t+1}$ and CRRA preferences, the following equations hold in equilibrium:

(eq. 1) $$c_t^2 = \left(\frac{1}{\alpha}\right)^{1/\sigma}$$

(eq. 8) $$\frac{CBDC_{t+1}+E_{t+1}}{M_{t+1}} = \left[\frac{(\mathrm{I}_{t+1}-1)}{\mathrm{s}_{t+1}}\right]^{\frac{1}{\rho}}.$$

Equation 8 describes how the demand of digital money relative to cash is given by the holding costs ratio and by the substitutability level. Note that the right-hand side is identical to the right-hand side of equation 3. Remind also that the policy spread is nothing else than the difference between the bond rate and the CBDC rate. Therefore, the government strategy in setting the CBDC rate turns out to be key for money demands control: if the CBDC rate was set to be autonomous and fixed, an increase in the bond rate would result into an identical increase of the policy spread, leaving the CBDC-to-cash ratio unaltered; in the case of a CBDC rate anchored to the bond rate –meaning that the government is fixing the policy spread rather than the CBDC rate itself- a bond rate raise would determine a demand shift in favour of digital money, as cash would become relatively more costly to hold.

(eq. 9) $$m_{t+1}^{h} = \frac{\{\alpha[1+H_{t+1}]\}^{-\frac{1}{\sigma}}}{H_{t+1}^{m}}$$

Where $$H_{t+1}^{m} = \left[1 + \left(\frac{I_{t+1}-1}{s_{t+1}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}$$

(eq. 10) $$cbdc_{t+1}^{h} + e_{t+1}^{h} = \frac{\{\alpha[1+H_{t+1}]\}^{-\frac{1}{\sigma}}}{H_{t+1}^{d}}$$

Where $$H_{t+1}^{d} = \left[1 + \left(\frac{s_{t+1}}{I_{t+1}-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}$$

(eq. 11) $$c_{t+1}^{1} = \{\alpha[1+H_{t+1}]\}^{-\frac{1}{\sigma}}$$

Where $$H_{t+1} = \frac{s_{t+1}}{{H_{t+1}^{d}}^{\rho}} = \frac{I_{t+1}-1}{{H_{t+1}^{m}}^{\rho}}$$

3.2 FINANCIAL INTERMEDIARIES

One or more banks offer deposits, E_{t+1}^{f}, at the pre-mentioned gross interest rate. Assume them to be price takers and to honour their liabilities. Assume also that deposits contracts are enforceable through banking regulation. In this way, we rule out the possibility of tricky banks behaviour, namely over-issuing to deflate liabilities and advantageous defaults. The cash flow of the financial intermediary in period $t \geq 0$ is:

$$(\pi_t) \qquad E_{t+1}^{f} - E_{t}^{f}\, I_{t}^{f} - P_{t}\, b_{t+1}^{f} + P_{t-1}\, b_{t}^{f}\, I_{t} - P_{t} c_{t}^{f}$$

The financial intermediation technology consists of a real issuance cost, c_{t+1}^{f}, paid for the supply of deposits at redemption time. Such cost is a function of the outstanding deposits:

(eq. 11.5) $$c_{t}^{f} = k\left(\frac{E_{t}^{f}}{P_{t}}\right)^{1+\varepsilon}.$$

Furthermore, we assume the financial intermediary to be holding the total amount deposited as bonds, $P_t\ b^f_{t+1}$, which pay gross interest I_{t+1}. So the business strategy of banks is very simplified: bank offer deposits to households at a relatively low interest, in order to make a margin on bonds. The intermediation spread, θ_{t+1}, which is nothing else than the difference between the bond rate and the deposit rate, therefore represents banks' deposit issuance marginal revenue. Banks cash flow becomes,

$$(\pi_{t+1})\ \ E^f_{t+1}(\theta_{t+1}) - \ P_{t+1}c^f_{t+1}.$$

Perfect competition case

Consider $\varepsilon = 0$ and the banking sector populated by an infinite number of financial institutions, which engage in perfect competition. Bank will then offer deposits at the marginal issuance cost, implying null profits,

(eq. 11.51) $\quad (\pi_{t+1} = 0)\quad \theta_{t+1} = k$

Perfect competition is then fixing the intermediation spread.

One bank case

Consider $\varepsilon > 0$ and only one bank operating into the banking sector. The single bank's cash flow at time $t+1$ will be:

$$\pi_{t+1} = E_{t+1}\ \left[\theta_{t+1} - k\left(e^f_{t+1}\right)^{\varepsilon}\right].$$

The maximization behaviour leads to the following real deposit supply function:

(eq. 11.6) $\quad e^f_{t+1} = \left[\frac{\theta_{t+1}}{k\ (1+\varepsilon)}\right]^{\frac{1}{\varepsilon}}.$

As a result, the single bank will experience a consistent positive inflow,

(eq. 11.7) $\quad \pi_{t+1} = P_{t+1}\ \frac{\varepsilon}{k^{\varepsilon}}\left(\frac{\theta_{t+1}}{1+\varepsilon}\right)^{\frac{1+\varepsilon}{\varepsilon}}.$

If profits constituted an endogenous variable into the household problem, then the household could, in some cases, be encouraged to increase deposit holdings as a strategy to get more dividends. We want to exclude such behaviour, therefore profits will enter the households' dynamic budget constraint exogenously.

The main difference between the two proposed specifications of the financial intermediary problem concerns the intermediation spread equilibrium determination and dynamics. In the case of perfect competition there is indeed no dynamics: the only possible equilibrium level of the intermediation spread is determined by the zero-profit condition. Such inflexibility can be source of inflation boundaries in the case of perfect substitutability of cash and deposits, as Marimon et al. (2003) pointed out. Said that, this paper moves from Marimon et al. (2003) findings to explore new equilibria in multiple scenarios, with the unprecedented presence of a well-designed CBDC.

If we consider the perfect substitutability case, $\rho = \varphi = 0$, and $CBDC_t = 0\ \forall t$, we are essentially back to Marimon et al. (2003), where cash and deposits can be both valued in equilibrium only when the bond interest rate is such that the cost opportunity of holding cash binds the cost opportunity of deposits holding. As a result, the government turns out to be constrained (from above) by inside-money competition in setting the inflation level, otherwise the economy would switch into a cashless economy, whose eventuality results problematic from the government perspective both in terms of implied seigniorage uselessness and limited monetary policy efficacy and implementability.

3.3 GOVERNMENT

The government takes policy decisions concerning transfers, g_t, currency supply, M_t and $CBDC_t$, and bond supply, b_t, in every period. M_0 and $R_0 b_0$ are given. The choice variables behind such policy decisions will be g_t, I_t and I_t^{CBDC}.
For the sake of simplicity, seigniorage is assumed to be the only source of public revenues. When both types of outside-money are valued, the dynamic budget constraint of the government is:

$$M_{t+1} + CBDC_{t+1} + P_t b_{t+1} \leq M_t + CBDC_t\, I_t^{CBDC} + P_t R_t\, b_t + P_t g_t, \quad \forall t.$$

The classic no-Ponzi condition has to hold:

$$\lim_{T \to \infty} \frac{b_{T+1}}{\prod_{s=0}^{T} I_s} = 0.$$

The government dynamic budget constraint can be iterated over time and expressed as an intertemporal relation as follows:

$$\sum_{t=0}^{\infty} \beta^{t+1}\, m_{t+1}\,(I_{t+1}-1) + \beta^{t+1}\, cbdc_{t+1}\left(\mathrm{I}_{t+1} - \mathrm{I}_{t+1}^{\mathrm{CBDC}}\right) = \frac{M_0}{P_0} + b_0 R_0 + \sum_{t=0}^{\infty} \beta^t\, g_t.$$

The government is assumed to internalize other agents' behaviour. As a result, the intertemporal budget constraint becomes:

(eq. 11.8) $\sum_{t=0}^{\infty} \beta^{t+1}\, m_{t+1}^{h}\,(I_{t+1}-1) + \beta^{t+1}\, cbdc_{t+1}^{h}\left(\mathrm{I}_{t+1} - \mathrm{I}_{t+1}^{\mathrm{CBDC}}\right) = \frac{M_0}{P_0} + b_0 R_0 + \sum_{t=0}^{\infty} \beta^t\, g_t.$

Where m_{t+1}^{h} and $cbdc_{t+1}^{h}$ are real outside-money holdings demanded by households as a function of the interest rates and preferences.

Furthermore, the government is committed to an inflation target mandate. As a result, it anchors the bond interest rate to the real interest rate in order to guarantee a constant inflation rate over time at the targeted level. From $I_t \equiv \left(\frac{P_{t+1}}{P_t}\right) R_{t+1}$, given the inflation target Π, we get that:

$$I_t = (1+\Pi)\, R_{t+1}.$$

We explore two government behavioural specifications:

(i) A Ramsey government which maximizes social welfare,

(ii) A transfer-maximizer government which aims to get the most out of seigniorage.

The government is constrained in the use of the bond rate. The additional policy tool the government has got, namely the CBDC rate, will then serve as a mean to reach the government's goal.

Ramsey government

In the case of Ramsey government, the goal is to maximize social welfare. It means the economy will be directed toward a solution where monetary frictions are attenuated. Being constrained in the use of the bond rate, the best the government can do is to impose $I_{t+1}^{CBDC} = I_{t+1}$.

This intuition can be clearly caught from equation 7 and *Graph 6*: as the spreads approach the zero-level, real consumption, and in turn social welfare, becomes bigger and bigger. The CBDC rate would indeed follow the bond rate, making CBDC absolutely friction-free. Note that, in the special case where all payment means are perceived as perfect substitutes

($\rho = \varphi = 0$), this government behaviour would imply a scenario where it is not profitable for banks to issue any deposit (given $k > 0$) and cash would be valued only if the bond interest rate was set to zero. The economy would then settle at the Friedman rule. In the proposed model, this is not the case. Indeed, in any specification, the government is committed to the inflation target mandate so that the bond rate is firmly anchored to the real interest rate.

Transfer-maximizer government

In such a case, the government goal is to maximize transfers with commitment, according to its preferences $G(g_t)$. Such function G is assumed to be increasing and strictly concave. The government then chooses g_t and I_{t+1}^{CBDC} in order to solve the following problem:

$$MAX \sum_{t=0}^{\infty} \beta^{t}\, G(g_t)$$

$$\text{s.t. } \sum_{t=0}^{\infty} \beta^{t+1}\, m_{t+1}^{h}\,(I_{t+1} - 1) + \beta^{t+1}\, cbdc_{t+1}^{h}\, \mathrm{s}_{t+1} = \frac{M_0}{P_0} + b_0 R_0 + \sum_{t=0}^{\infty} \beta^{t}\, g_t$$

$$where\ \ I_{t+1} = \Pi\, R_{t+2}.$$

Given the constraint multiplier γ, the marginal conditions are:

$$(g_t) \qquad G'(g_t) = \gamma$$

$$(I_{t+1}^{CBDC}) \qquad \gamma\, \beta^{t+1}\left[(I_{t+1} - 1)\frac{\partial m_{t+1}^{h}}{\partial s_{t+1}} + cbdc_{t+1}^{h} + \mathrm{s}_{t+1}\,\frac{\partial cbdc_{t+1}^{h}}{\partial s_{t+1}^{CBDC}}\right] = 0$$

From equation (g_t) it is deduced that government transfers are a constant stream in the optimum.

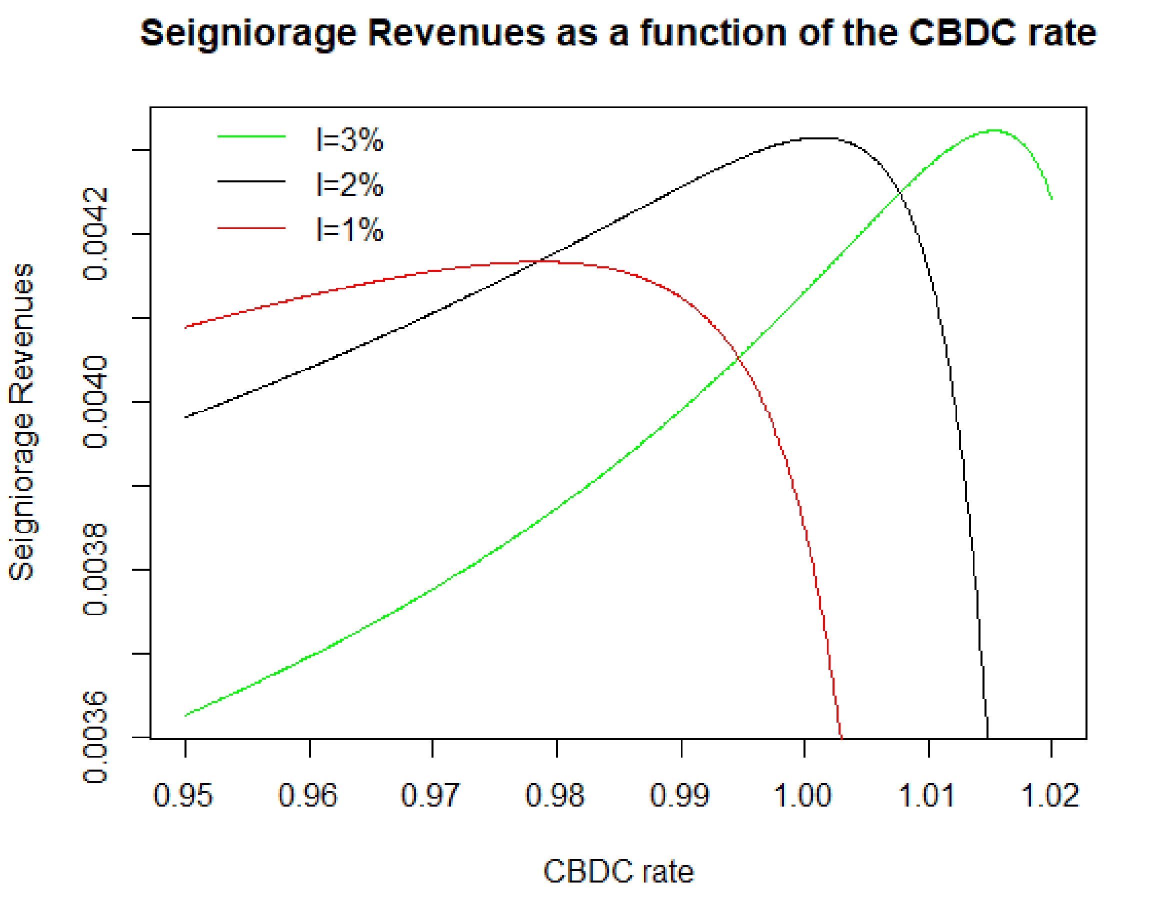
Note also that the government can finance its transfers only through total seigniorage revenues, indeed the government problem is nothing else than a total seigniorage maximization. In particular, both types of outside-money generate seigniorage revenues, where cash-seigniorage is given by the function $m_{t+1}^{h}\,(I_{t+1} - 1)$ and CBDC-seigniorage by $cbdc_{t+1}^{h}\, \mathrm{s}_{t+1}$, and the total seigniorage is the sum of the two. By assuming CRRA preferences, we get total seigniorage function concave and non-monotone with respect to the CBDC rate. This is illustrated in *Graph 8.*

From equation (I_{t+1}^{CBDC}) the optimal CBDC rate is derived:

$$(eq.12) \qquad s_{t+1} = \frac{\partial m^h_{t+1}/\partial s_{t+1}\ (I_{t+1}-1)}{-\,\partial cbdc^h_{t+1}/\partial s_{t+1}} - \frac{cbdc^h_{t+1}}{\partial cbdc^h_{t+1}/\partial s_{t+1}}$$

The CBDC rate choice can be seen as the result of two trade-off forces, the former concerns competition between cash and CBDC, the latter is CBDC related. When the CBDC rate falls, and in turn the policy spread goes up, households claim more cash to the detriment of CBDC, therefore cash-seigniorage is emphasized while CBDC-seigniorage gets dimed. This trade-off intuition is described by the first addend of equation 12. Furthermore, as the policy spread widens, CBDC supply gets dimed while the marginal CBDC-seigniorage revenue (which is the policy spread itself) increases. This trade-off intuition is captured by the second addend of equation 12. Ultimately, the government will move the policy spread to the point at which the negative effect on CBDC-seigniorage is just offset by the positive effect on CBDC-seigniorage, generating in fact the highest total seigniorage.

Graph 8



No inflation target mandate

In the case of non-commitment to an inflation target mandate, both policy instruments would be employed.
In that case, as in Marimon et al. (2003), the Ramsey government would impose both rates at the zero-level, which implies an unconditioned Friedman rule.
A seigniorage maximizer government would instead look for the ideal combination of bond and CBDC rates to get the most in terms of total seigniorage. It is interesting to note that in such scenario the government appears to compete against itself as it is the provider of two competing payment means. In that case the FOCs would be:

$(g_t) \qquad G'(g_t) = \lambda$

$$(I_{t+1}) \qquad \lambda\, \beta^{t+1} \left[m^h_{t+1} + (I_{t+1} - 1)\ \frac{\partial m^h_{t+1}}{\partial I_{t+1}} + cbdc^h_{t+1} + \left(\mathrm{I}_{t+1} - \mathrm{I}^{\mathrm{CBDC}}_{t+1}\right) \frac{\partial cbdc^h_{t+1}}{\partial I_{t+1}} \right] = 0$$

$$\left(I^{CBDC}_{t+1}\right) \quad \lambda\, \beta^{t+1} \left[(I_{t+1} - 1)\ \frac{\partial m^h_{t+1}}{\partial I^{CBDC}_{t+1}} - cbdc^h_{t+1} + \left(\mathrm{I}_{t+1} - \mathrm{I}^{\mathrm{CBDC}}_{t+1}\right) \frac{\partial cbdc^h_{t+1}}{\partial I^{CBDC}_{t+1}} \right] = 0$$

3.4 CLEARING CONDITIONS

In equilibrium markets clear. As discussed in the household section, equilibrium in the goods and labour markets imply:

(eq. 13) $\qquad n_t = c^1_t + c^2_t + c^f_t.$

Equilibrium in the financial markets implies:

(eq. 14) $\qquad b_t = b^h_t + b^f_t,$

(eq. 15) $\qquad M_t = M^h_t,$

(eq.16) $\qquad E^f_t = E^h_t,$

(eq. 17) $\qquad CBDC_t = CBDC^h_t.$

4.0 EQUILIBRIA

In this chapter I will discuss the model equilibria in all the set up specifications for all the agents. I will then discuss what is the CBDC role into the specified economy through the money competition mechanism.

4.1 PERFECT DIGITAL SUBSTITUTES IN A PERFECTLY COMPETITIVE BANKING SECTOR.

Perfect competition in the banking sector entails a fixed intermediation spread θ_t^f, always inflexibly equal to the marginal cost of deposit issuance k. Government commitment to the inflation target implies that the bond rate, I_t, is anchored to exogenous variables such as the real interest and the inflation target level.

Perfect substitutability across digital instruments implies that only the most convenient digital substitute is actually held in equilibrium, while in case of even any combination of CBDC and deposits (summing up to the same required value) would be optimal as the agent is indifferent between the two.

Ramsey government

As discussed in section 3.3, a Ramsey government –in the attempt of maximizing social welfare- would mitigate monetary frictions by imposing the policy spread at zero.

(eq. 18) $s^{eq} = 0$

(eq. 19) $\theta^{f^{eq}} = k$

(eq. 20) $I^{eq} = (1 + \Pi)R$

Equations 18 and 19, $(CBDC_{t+1})'$, $(E_{t+1}^{'})$, and equation 16 imply that:

$$E^{eq} = 0$$

From equation 11.5 we get:

$$c^f = 0$$

For any $k > 0$, deposits are deviated out of circulation as their holding cost, k, is indeed higher than the one inherent in CBDC.

From equations 9, 10, 15 and 17 it derives that:

$$M^{eq} = 0$$

(eq. 21) $cbdc^{eq} = \alpha^{-\frac{1}{\sigma}}$

And from $(\tau_{t+1})'$, equations 1, 11 and 21 we know that:

(eq. 22) $c^{1^{eq}} = c^{2^{eq}} = cbdc^{eq}$

Cash is deviated out of circulation as well, and the stock of CBDC –under the binding cash-in-advance constraint- is then just equal to the money good consumption amount. Furthermore, given the absence of monetary frictions, the money-good consumption matches the credit-good consumption in equilibrium.
As a result, inside-money competition does not play any role in conditioning the Ramsey-government choice. Such attainment confirms Marimon et al. (2003) conclusion, according to which inside-money competition might generate boundaries on government manoeuvre from above, and such boundaries are not being met by a government pursuing the Friedman-rule.

Given $c^f = 0$, from equations 13, 21 and 22 we get:

$$n^{eq} = 2\,\alpha^{-\frac{1}{\sigma}}$$

From the government problem we know that the amount of transfers is zero, because of zero seigniorage revenues. The amount of bonds in circulation is derived from the household budget constraint which has to bind in equilibrium.
Finally, we assumed constant inflation –guaranteed by commitment to the inflation target- and constant real interest rate over time. As a result, in equilibrium the bond interest rate increases constantly at the price level pace, as well as all the nominal variables, such as the stock of CBDC.

Transfer-maximizer government

Given that the government internalizes other agents' behaviour, CBDC rate is going to be set at the light of the concomitant inside-money competition.
CRRA preferences assure that the total seigniorage is a concave function of the policy spread and that it has a unique maximum[8].

(Assumption 1) In the absence of inside-money competition, the policy spread which maximizes the total seigniorage function is assumed to be higher than k. Given that the function is monotone, the derivative is then positive for $s_{t+1} = \theta^f_{t+1} = k$.

[8] For the chosen parameterization, *Graph 7* proves the statement.

As a result, in order to catch the whole digital money demand, the government best choice is to set a policy spread slightly smaller than the competitive intermediation spread as a Bertrand competitor would do.

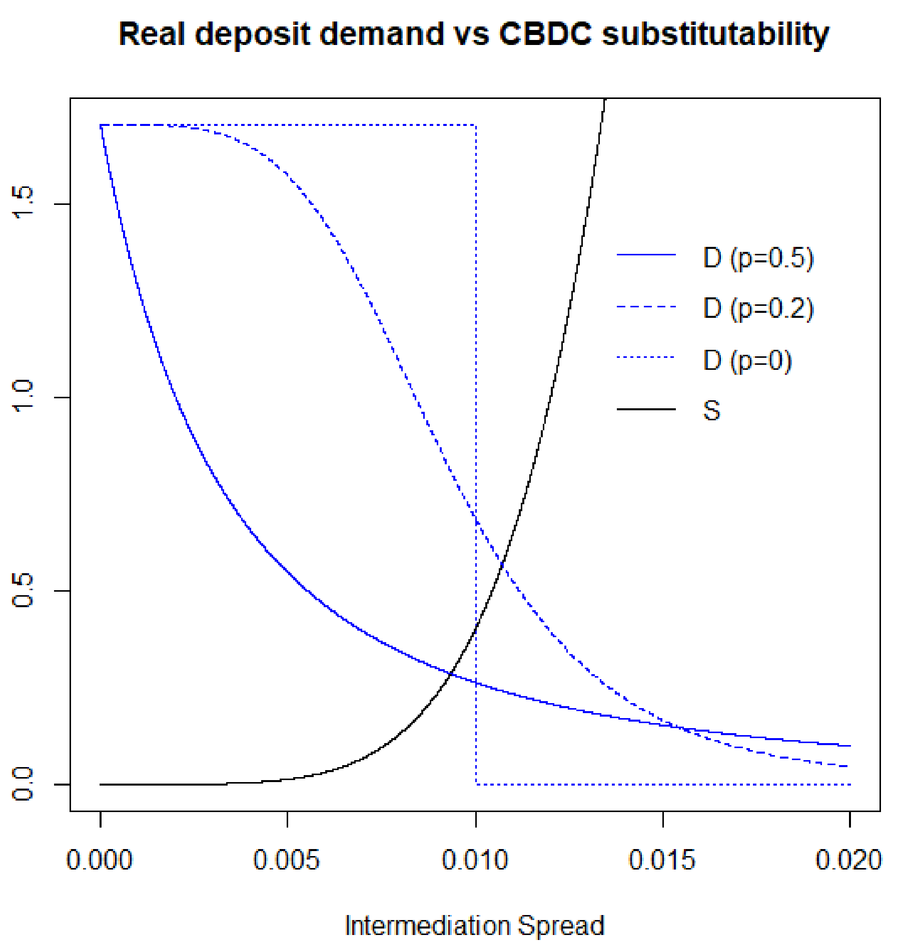
(eq. 21) $s^{eq} = k - \mu$ where μ is positive and arbitrarily small.

(eq. 19) $\theta^{f^{eq}} = k$

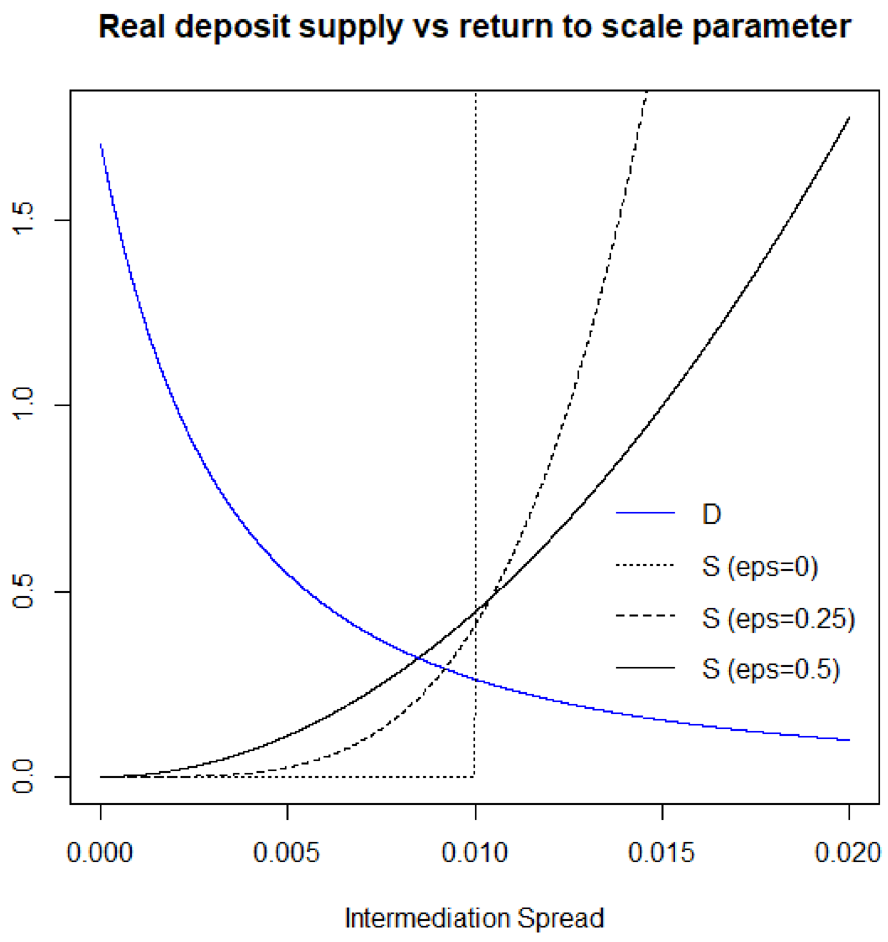
(eq. 20) $I^{eq} = (1 + \Pi)R$ where R and Π are constant.

It then comes obvious that inside-money competition –in such setting- generates boundaries to the government, which is forced to step the CBDC rate up to a point where CBDC is valued. Such mechanics is illustrated in *Graph 9* and *Graph 10,* where the deposit demand and supply are drawn in various contingencies. In particular, *Graph 10* shows deposit supply as a vertical line in the case of perfect competition ($\varepsilon = 0$), meaning that for any quantity of deposits the intermediaries are set to deliver always the same spread. *Graph 9* shows that in this case ($\rho = 0$) deposit demand is not smooth, meaning that the government is constrained to set a policy spread lower or equal to the intermediation spread because of the eventual zeroing of its CBDC-seigniorage revenues.

Graph 9



Graph 10



Equations 21, 19, $(CBDC_{t+1})'$, (E'_{t+1}), and 16 imply that:

$$E^{eq} = 0$$

From equation 11.5 we get:

$$c^f = 0$$

From the household problem and the clearing conditions, we have that:

(eq. 22) $$m^{eq} = \frac{\left\{\alpha\left[1+\frac{k}{\left[1+\left(\frac{k}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{I^{eq}-1}{k}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}}$$

(eq. 23) $$cbdc^{eq} = \frac{\left\{\alpha\left[1+\frac{k}{\left[1+\left(\frac{k}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{k}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}}$$

(eq. 24) $$c^{1^{eq}} = \left\{\alpha\left[1+\frac{k}{\left[1+\left(\frac{k}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}$$

(eq. 1) $$c^{2^{eq}} = \alpha^{-\frac{1}{\sigma}}$$

From the equations just reported, we see how the inefficiency of inside-money, k, together with the inflation target, is a key determinant of the money-consumption outcome. g^{eq} is determined through equation 11.8, as the government gives all the seigniorage revenues gained back to households in the form of transfers. Bond equilibrium quantity is determined from the household budget constraint.

As a result, inside-money competition appears to discipline government policy usage of CBDC against the threat of a takeover. Such attainment is again conformed to Marimon et al. (2003) findings, where the bond interest rate (and then inflation) is bounded in a context of perfect substitutability between cash and deposits. In the model just presented –in a context of perfect substitutability between CBDC and deposits- we find bond rate free of pursuing the inflation target and the CBDC rate constrained instead.

4.2 PERFECT DIGITAL SUBSTITUTES WITH A SIGLE INSIDE-MONEY PROVIDER.

Assume R and Π constant. Bond interest rate is then constant as in equation 20. As shown before, the unique bank's supply of deposits is an increasing function of the intermediation spread. The demand for deposits, in a context of perfect substitutability between digital instruments, is given by:

(eq. 25) $$e_t^h = \begin{cases} \frac{\{\alpha[1+H_{t+1}]\}^{-\frac{1}{\sigma}}}{H_{t+1}^d}, & for\ \theta_{t+1} < s_{t+1} \\ \delta, & for\ \theta_{t+1} < s_{t+1} \\ 0\ , & for\ \theta_{t+1} > s_{t+1} \end{cases}$$

Where δ can be any number between zero and $\frac{\{\alpha[1+H_{t+1}]\}^{-\frac{1}{\sigma}}}{H_{t+1}^d}$.

Deposit supply is described in equation 11.6.

Graph 7 shows how the deposit demand, as a function of the intermediation spread, changes according to the policy spread in this equilibrium scenario. In words, when the intermediation spread is lower than the policy spread, deposits are cheaper to hold than CBDC, so the demand for CBDC is zero while deposits are valued in equilibrium together with cash. On the other hand, when the intermediation spread is higher than the policy spread, deposits are not valued. When the cost opportunity of holding deposits coincides with the CBDC one, households are indifferent on which digital money to hold, so that any combination of the two (summing to $\frac{\{\alpha[1+H_{t+1}]\}^{-\frac{1}{\sigma}}}{H_{t+1}^d}$) is possible in the optimum.

Graph 7 allows us to catch the important intuition that the deposit market happens to meet where the intermediation spread is equal to the policy spread, except for the cases where the policy spread is too large. This eventuality is excludable because it would imply zero CBDC-seigniorage revenues, so that neither a Ramsey nor a transfer-maximizer government would choose any spread so large. So, from equation 25 and equation 11.6 we get that:

(eq. 26) $$\theta^{eq} = s^{eq}$$

As a result, $(CBDC_{t+1})'$ and $(E_{t+1})'$ are both satisfied in equilibrium so that both digital instruments are valued. From equations 9, 10, 11.6, 16 and 26 we deduce that:

(eq. 27) $$cbdc^{eq} = \frac{\left\{\alpha\left[1+\frac{s^{eq}}{\left[1+\left(\frac{s^{eq}}{I-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{s^{eq}}{I-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}} - \left[\frac{s^{eq}}{k\,(1+\varepsilon)}\right]^{\frac{1}{\varepsilon}}$$

(eq. 27.1) $$e^{eq} = \left[\frac{s^{eq}}{k\,(1+\varepsilon)}\right]^{\frac{1}{\varepsilon}}$$

(eq. 27.2) $$m^{eq} = \frac{\left\{\alpha\left[1+\frac{I-1}{\left[1+\left(\frac{I-1}{s^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{I-1}{s^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}$$

Equation 27 explains how the deposits market determines the holding shares of deposits and CBDC. Referring again to *Graph 7*, please notice that the total demand for digital money (first addend of equation 27) is given by the top-left horizontal line, which indeed happens to match deposit demand when $\theta_{t+1} < s_{t+1}$. Note that for higher equilibrium spreads, the bank would supply an higher amount of deposits (second addend of the equation 27) while the government would integrate for the remaining part of the total demand. In particular, according to the proposed parameterization, when $s = 0.5\%$ basically all the digital money is in CBDC form; when $s = 1\%$ households hold some deposits but CBDC is still the favourite digital money; when $s = 1.5\%$ CBDC is not valued and only deposits are used. As this result is highly dependent on the bank's behaviour, the cost structure of the bank (namely k and ε) turns out to be key.

Ramsey government

As discussed before, a Ramsey government –in the attempt of maximizing social welfare- would mitigate monetary friction by imposing a zero policy spread. For an intermediation spread equal to zero the unique bank is not willing to supply a positive amount of deposits, so that the deposit market is in equilibrium with null demand and supply of deposits. Equilibrium conditions match the ones described in section 4.1 in the subparagraph *Ramsey* *government.*

So, inside-money competition does not play any role in conditioning the Ramsey-government choice and such attainment goes in line with our previous finding in an environment with perfectly competitive banks.

Transfer-maximizer government

A transfer-maximizer government which internalizes agents' behaviour will undertake policy decisions at the light of such malleable deposit market. In a context of perfect competition between banks we concluded that CBDC policy manoeuvre space was bounded because of a low and inflexible intermediation spread imposition; now –on the contrary- the CBDC interest rate clearly plays a disciplinary role into the deposit market, whose equilibrium rate is anchored to the CBDC one.

As described in equation 10, when the spreads increase households are willing to hold less digital money (in total) than before.

As the spreads increase, the unique bank supplies more, and –given that the government supply CBDC on the remained demand for digital money- it implies that the share of deposits gets higher relative to CBDC. The opposite occurs when the policy spread falls. This mechanism is explained in equation 27 and *Graph 7*.

The government will account for this operating mechanism in imposing the policy spread maximizing seigniorage revenues. In math, we have that the government maximizes the problem presented in section 3.3, whose solution is described in equation 12, integrating equations 17 and 17.2 in it. So we have that the equilibrium policy spread is the solution of the following equation:

$$
s_{t+1}=\frac{\partial\dfrac{\left\{\alpha\left[1+\dfrac{I-1}{\left[1+\left(\frac{I-1}{s_{t+1}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{I-1}{s_{t+1}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\Bigg/\partial s_{t+1}\;(I_{t+1}-1)}{-\,\partial\dfrac{\left\{\alpha\left[1+\dfrac{s_{t+1}}{\left[1+\left(\frac{s_{t+1}}{I-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{s_{t+1}}{I-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}-\left[\frac{s_{t+1}}{k\,(1+\varepsilon)}\right]^{\frac{1}{\varepsilon}}\Bigg/\partial s_{t+1}}-
$$

$$
\frac{\dfrac{\left\{\alpha\left[1+\dfrac{s_{t+1}}{\left[1+\left(\frac{s_{t+1}}{I-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{s_{t+1}}{I-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}-\left[\frac{s_{t+1}}{k\,(1+\varepsilon)}\right]^{\frac{1}{\varepsilon}}.}{\partial\dfrac{\left\{\alpha\left[1+\dfrac{s_{t+1}}{\left[1+\left(\frac{s_{t+1}}{I-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{s_{t+1}}{I-1}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}-\left[\frac{s_{t+1}}{k\,(1+\varepsilon)}\right]^{\frac{1}{\varepsilon}}\Bigg/\partial s_{t+1}}
$$

⇒ (eq. 28) $s^{eq}=s(k,\varepsilon,\Pi,R,\rho,\sigma,\alpha)$

This attainment is in line with Chiu et al. (2019) finding, according to which CBDC -even in the case of low usage- would serve as an outside option for households, thus limiting banks' market power in the deposit market. Therefore CBDC is found to be an effective monetary policy tool capable of sharp effect on depositors by directly influencing the deposit rate. Note that –in this environment- also movements of the bond interest rate would be effective in conditioning the deposit rate, and it would be still valid even without a CBDC into the economy. This is in line with Woodford (2000), who argues that cash maintains its MP effectiveness even in the case of deposit takeover, so that CBDC is not really a necessity. In our model we look at a government with multiple objectives –inflation target and a Ramsey or seigniorage purpose- which are not obtainable at once with a single policy instrument. Equilibrium consumptions, from equations 1 and 24, are equal to:

$$c^{1^{eq}} = \left\{ \alpha \left[1 + \frac{s^{eq}}{\left[1 + \left(\frac{s^{eq}}{I-1} \right)^{\frac{1-\rho}{\rho}} \right]^{\frac{\rho}{1-\rho}}} \right] \right\}^{-\frac{1}{\sigma}} \text{ and } c^{2^{eq}} = \alpha^{-\frac{1}{\sigma}}$$

As you can see, the inefficiency of inside money does not influence directly the equilibrium level of money-good consumption –as we found out in the scenario with perfect competition in the banking sector through equation 24- but the influence is indirect through the government choice of the policy spread, which appear corrupted by inside-money competition but not as rigidly as we ascertained in the former case.

4.3 IMPERFECT DIGITAL SUBSTITUTES IN A PERFECTLY COMPETITIVE BANKING SECTOR.

In the case of imperfect substitutability of digital instruments, there is no threat of deposits takeover (i.e. a corner solution) as households are always willing to hold a positive amount of each payment means in their portfolios, except for the cases of zero spreads. A *Ramsey government* imposes a zero policy spread in every period, as discussed before. Equilibrium conditions match the ones in section 4.1.

A *transfer-maximizer government* imposes the policy spread that solves equation 12. As in equations 2 to 6, the household portfolio composition will result from the outstanding interest rates and payment preferences. The bigger the difference between money holding costs, the more unbalanced the portfolio composition will be; and the more money types are perceived as perfect substitutes the bigger such unbalancing will be. As in equation 2, for I_t^f higher than I_t^{CBDC}, the model predicts that the households payment choice (at time t) will result in a deposit-to-CBDC ratio higher than one; in particular, we know that the bigger the rates differential the bigger the ratio. As φ get closer to zero such rates differential becomes more and more relevant, to the point that –when $\varphi = 0$- even the tiniest positive rate differential would result in a deposit takeover.

So, from equations 11.51, 20, 12 and Assumption 1 we have that:

$$\theta^{eq} = k$$

$$I^{eq} = (1 + \Pi)R$$

$$s^{eq} = s(k, \varepsilon, \Pi, R, \rho, \sigma, \alpha) > k$$

From the household problem and clearing conditions we get:

(eq. 29) $$m^{eq} = \frac{\left\{\alpha\left[1+\frac{(I^{eq}-1)}{\left[1+\left(\frac{I^{eq}-1}{s^{eq}}\right)^{\frac{1-p}{p}}+\left(\frac{I^{eq}-1}{\theta^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{I^{eq}-1}{s^{eq}}\right)^{\frac{1-\rho}{\rho}}+\left(\frac{I^{eq}-1}{\theta^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}}$$

(eq. 30) $$cbdc^{eq} = \frac{\left\{\alpha\left[1+\frac{(s^{eq})}{\left[1+\left(\frac{s^{eq}}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}+\left(\frac{s^{eq}}{\theta^{eq}}\right)^{\frac{1-p}{p}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{s^{eq}}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}+\left(\frac{s^{eq}}{\theta^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}}$$

(eq. 31) $$e^{eq} = \frac{\left\{\alpha\left[1+\frac{(\theta^{eq})}{\left[1+\left(\frac{\theta^{eq}}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}+\left(\frac{\theta^{eq}}{s^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{\theta^{eq}}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}+\left(\frac{\theta^{eq}}{s^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}}$$

(eq. 32) $$c^{1^{eq}} = \left\{\alpha\left[1+\frac{(s^{eq})}{\left[1+\left(\frac{s^{eq}}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}+\left(\frac{s^{eq}}{\theta^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}$$

What is interesting here is that deposits demand is now a decreasing and well-behaved function of the intermediation spread, and depends on the interest rate combination and on preferences (both on payments and consumption), in contrast with the discontinue demand function we dealt with in the perfect substitutability case. The government accounts for the fixed intermediation spread in delivering the optimal policy spread, but it no longer represents a concrete boundary to monetary policy. In other words, we can say that inside money competition plays again a role in conditioning CBDC rate adoption, but the contingency of imperfect substitutability across digital instruments –which rules out the possibility of deposit takeover- alleviates what was previously identified as a boundary.

The idea that the CBDC rate is lower than the deposit rate, which comes from the assumption 1, is coherent with the intuition provided by the BIS (2018) according to which the CBDC rate would operate as a floor for market rates.

4.4 IMPERFECT DIGITAL SUBSTITUTES WITH A SIGLE INSIDE-MONEY PROVIDER.

A *Ramsey government,* as we have already learnt, would adopt a zero policy spread implying the equilibrium described in section 4.1.

Transfer-maximizer *government*

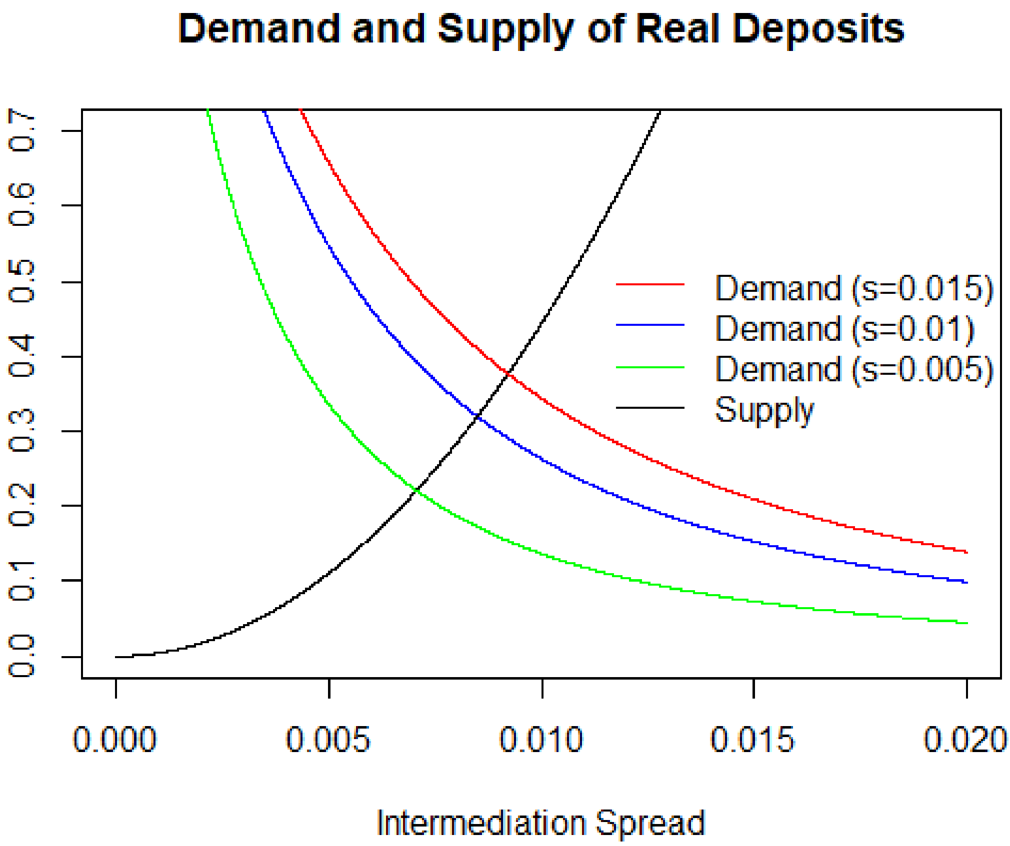
Assume constant inflation target Π and real interest rate R, so that equation 20 holds. A government oriented toward seigniorage, would account for inside-money dynamics in delivering the optimal policy spread. In order to catch this intuition, let us focus on the deposit market. In this setting, we finally have both deposit demand and supply well-behaved, so that we find that the equilibrium intermediation spread is an increasing function of the other spreads. Indeed, from equations 11.6, 6 and 16 we have that θ^{eq} is such that the following equality is satisfied:

$$\frac{\left\{\alpha\left[1+\frac{(\theta^{eq})}{\left[1+\left(\frac{\theta^{eq}}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}+\left(\frac{\theta^{eq}}{s^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}}\right]\right\}^{-\frac{1}{\sigma}}}{\left[1+\left(\frac{\theta^{eq}}{I^{eq}-1}\right)^{\frac{1-\rho}{\rho}}+\left(\frac{\theta^{eq}}{s^{eq}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{1}{1-\rho}}}=\left[\frac{\theta^{eq}}{k\,(1+\varepsilon)}\right]^{\frac{1}{\varepsilon}}$$

⇒ (eq. 33) $\theta^{eq}=\theta(s^{eq})$

Graph 11 illustrates such equilibrium dynamics:

Graph 11



From the government problem we obtain:

(eq. 34) $s^{eq} = s(\theta^{\text{eq}})$

Equation 33 describes how an higher policy spread means more demand for deposits and in turn an higher resulting intermediation spread. As we found out in the case of perfect substitutability, the single bank behaviour is disciplined by CBDC policy, anyway imperfect substitutability –through a smooth decreasing deposit demand- makes such disciplinary role less stringent, so that the intermediation spread is not perfectly anchored to the policy spread, but it is positively correlated to it. Again, this case scenario may provide an example of how CBDC can be used as an effective and direct MP tool through the deposit transmission channel.

Equation 34 depicts the fact that the government looks at the bank action in a competitive way. When the government increases the CBDC rate, the bank –in the attempt to maximize profits- moves the intermediation spread in the same direction, and the combination of the two spreads generates an effect in terms of total-seigniorage, for instance, cash-seigniorage would increase for sure. So the equilibrium, deriving from the intersection of equations 33 and 34, corresponds to a state of the world where the government set the CBDC rate such that total seigniorage is maximized and the bank imposes the intermediation spread that actually maximizes its profit.

Finally, the equilibrium level of the spreads differs from the one identified in section 4.3, where the intermediation spread was exogenously given by the inside-money inefficiency parameter k, and the government –once set the bond interest rate according to the inflation target- reacted to those exogenously determined values. In this section, the intermediation spread resulting from banking activity is endogenous and correlated to the policy spread, that makes the government policy decision more complex and most importantly it makes CBDC a proper policy tool. All the other equilibrium conditions match the ones reported in section 4.3.

4.4 CASE STUDY: E-krona

We can think at Sweden's money competition from a model perspective where the banking sector is perfectly competitive, and available payment means are treated as imperfect substitutes. Furthermore, we could account for the special Swedish household payment preferences which turns out not to be in favour of cash, probably because of the conjunction of a stronger sensitiveness to money convenience, efficiency, accessibility, and a weaker sensitiveness to anonymity. So, it appears fair to consider the cost of holding cash boosted by a specific parameter accounting for that. So, with reference to section 4.3, the monetary equilibrium applied to the Swedish case would appear as an equilibrium where households hold relatively few cash, where CBDC is not impactful on the deposit market and –under the assumption of a small k- CBDC would not really induce a relevant improvement in the digital payment market, which is in fact already perfectly competitive and efficient. As a conclusion, given the assumptions we set for this case, we may convene that CBDC would not play a distinctive role in the money-competition mechanism in Sweden, and that it would not represent a direct monetary policy tool through the deposit channel. The question is then why the Riskbank sees CBDC as a needed solution against cash progressive disappearance. The answer cannot be found in the model built up in these pages. Referring to Riskbank, there are various reasons why CBDC is needed:

- Firstly, cash scarce usage would eventually threat cash acceptance. Therefore, households would no longer have access to outside money, which has a lower credit and liquidity risk than inside money. In other words, CBDC would induce a more sound payment system.
- Secondly, CBDC can guarantee a usable alternative in the case of private-sector prolonged disruption.
- The lack of an outside money option can evolve in a context where monopoly situations can easily arise. As we discussed in paragraph 4.2 and 4.4, CBDC and cash –even in the case of short or null usage- put pressure on deposit interest rate as they represent the alternative payment option households can adopt in the case of banks' change of attitude toward the price charged.

  Whether this circumstance is realistic or not, it is a question that requires further investigation on the composition of the Swedish banking sector. Anyway, I categorized this case study into a context of perfectly competitive banking sector as –on top of my knowledge- payment services appear to be often totally free of charge.
- In a context of cash disappearance, CBDC would allow all those people who are excluded from having a bank account to continue having access to money. Such argument –presented in the introduction as the financial inclusion argument- is founded on the assumption that CBDC will be as accessible as cash is, which seems plausible even though the contingent technology barrier.
- In a context of cash disappearance, CBDC would allow citizens to hold risk free state-money in the case of financial unease. Otherwise, the fundamental trust in the Swedish monetary policy system risks declining.
  As we discussed in the introduction, CBDC brings about risks concerning CB credibility connected to the possibility of CBDC fiasco or to excessive contact with the general public. As a conclusion on the matter, Riskbank will have to face possible credibility challenges in any case; anyway, a properly designed CBDC appears the best solution for this specific issue.

5.0 CONCLUSIONS

CBDC may represent a relevant innovation for monetary policy, payment systems and financial integration for underdeveloped countries –where the payment systems are slow in developing by the hand of the private sector, and financial integration is a significant issue- while may not be perceived as such in the context of a developed economy. Our model –even though it does not integrate such level of detail- confirms such intuition on the grounds that CBDC is assumed to be costless in real terms by construction, while bank deposits are not. In a scenario where k is very high –namely the banking sector very inefficient in supplying deposits- the introduction of a CBDC would represent a potentially worthier tool in terms of social welfare enhancement. This first consideration refers to CBDC as an innovation into the payment market, which is supposed to be fostering social welfare independently of other monetary policy questions. It is then not surprising that countries like Ecuador and Tunisia have adopted a model of CBDC which does not bear interest: they were primarily interested in the opportunity of participating in the respective retail payment markets.

Furthermore, Canada and Singapore central banks' effort in studying DLT applications for interbank settlements lies on the same line. CBDC may bring about efficiency gains in the wholesale transfer and settlement systems –like reducing intermediation costs-, regardless of other monetary policy questions.

We have seen many specifications of the model, where households perceive CBDC and deposits as perfect or imperfect substitutes; the banking sector is inhabited by one price-taker bank or infinite perfectly-competitive banks; the government is a Ramsey or a transfers-oriented government.

In the case of Ramsey-government, the equilibria in all the specifications are characterized by a CBDC dominance over other instruments, which are in fact deviated out of circulation. Such occurrence enhances social welfare, as the cost opportunity of holding money –namely the real cost of consumption today- is drop down to zero and privately issued money –which bears real costs- is not valued.

So, the discussed models allow us to conclude that a CBDC in the hand of a Ramsey-government can serve as a second monetary policy tool for welfare enhancement, and can play a role in disciplining the deposit market toward lower intermediation rates. In concordance with Marimon et al. (2003), inside money competition does not impose any boundary to the Ramsey-government monetary policy activity.

A transfer-maximizer government, as discussed previously, may want to impose a relatively high policy spread in the view of maximizing its preferences for transfers. When CBDC and deposits are perceived as perfect substitutes the policy spread and the intermediation spread appear to be perfectly correlated in equilibrium: when the banking sector is perfectly competitive, the CBDC rate follows the exogenously determined deposit rate; when the banking sector is inhabited by a single price-taker bank, the deposit rate is anchored to the CBDC rate instead.

As we concluded in sections 4.1, perfect competition in the banking sector turns out to have a disciplinary role over CBDC rate decision, as the CB needs to avoid deposit takeover in order to gain seigniorage revenues to finance its transfers. This finding goes perfectly in line with Marimon et al. (2003), who concluded that inside-money competition happens to play a disciplinary role over the bond interest rate under similar assumptions.

As we discussed in section 4.2, in the case of single price-taker bank, even though the government's policy spread choice is influenced by the deposit supply, in equilibrium the intermediation spread is anchored to the policy spread. As a conclusion, we can say that in this contingency CBDC appears to be an effective monetary policy tool, capable of influencing –in a perfect way- the deposit market rate.

In conclusion, what appears clear from the analysis illustrated so far is that CBDC efficacy as a monetary policy tool depends highly on the structure and/or competitiveness of the banking sector in which it is supposed to operate. Where the banking sector is not competitive or it is inefficient, CBDC is powerful and potentially welfare-enhancing, while it may be superfluous where the banking sector is already well competitive and efficient.

When CBDC and deposits are perceived as imperfect substitutes, the policy spread and the intermediation spread appear to be positively (but not perfectly) correlated.

As discussed in section 4.3, inside-money competition puts pressure over the government CBDC rate choice without the threat of a deposit takeover, meaning that the exogenously determined intermediation spread does not represent in this case a stringent boundary to the policy spread. We conclude that imperfect substitutability alleviates inside-money competition boundary over monetary policy action.

As discussed in section 4.4, the CBDC rate works as an effective monetary policy tool as it directly influences the deposit rate. On this regard, it is fundamental to notice that money demand elasticity is key for CBDC to be relevant. Indeed, as money substitutability goes down, elasticity decreases and CBDC gets less impactful on the deposit market as the correlation between CBDC and deposit rates weakens.

In conclusion, CBDC relevance as an effective monetary policy tool appears definitely reliant on consumers' perception of the available payment means as well as on the structure and competitiveness of the banking sector. CBDC is found to be more beneficial as a monetary policy instrument in a context where CBDC and deposits are perceived as perfect substitutes and where the banking sector lacks competitiveness and efficiency.

5.1 LIMITATION AND FUTURE RESEARCH

The thesis limitations and the possibility of future improvements are clearly related to:

(i) the simplicity of the model used. In particular, the model ignores that households have preferences over anonymity, efficiency and other money design features, and that they are risk-averse; a more detailed specification of the banking activity (including lending and investment activities) would make the analysis more reliable, realistic and could also tell us more about other variables involved into the mechanics.

(ii) The absence of proper calibration. Calibrating the model would allows us to estimate the impact of CBDC policy manoeuvres on monetary aggregates demand and on the broader economy.

In general, the future economic research in the field of CBDC is probably going to be based on CBDCs case studies and lessons learnt from CBs in the attempt of experimentations and implementation.

Table 1

PARAMETER	DEFINITION	VALUE
α	*constant marginal disutility of labour.*	0.8
β	*Utility and profits discount factor.*	0.33
ε	*Bank return to scale. Marginal cost of deposits issuance is indeed: MC=*k**(*$1+\varepsilon$*)**e^{ε}	0
k	*Constant marginal real cost of deposit issuance when* $\varepsilon = 0$. *MC=*k**(*$1+\varepsilon$*)**e^{ε}	0.01
Π	*Inflation target.*	1.5%
R	*Real interest rate*	0.5%
ρ	*Imperfect substitutability between cash and digital money. It varies between zero and one, where zero means perfect substitutability.*	0.5
φ	*Imperfect substitutability between CBDC and bank deposits. It varies between zero and one, where zero means perfect substitutability.*	0.5
σ	*Household constant elasticity parameter of consumption.*	0.4

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