Beta Analysis on the US Stock Market

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Asset Allocation

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Abstract

This study conducts extensive research in the NYSE and S&P500 markets, exploring various investment strategies based on Beta. Throughout this work, I present results for 156 portfolios, structuring them according to the systematic risk characteristics of each stock. I investigate Beta, a measure of systematic risk, to build my portfolios.

The "betting against beta" factor demonstrates considerable effectiveness applied across a broad spectrum of assets. This approach, which prioritizes investments in low-beta assets while reallocating resources from high-beta assets, consistently exhibits superior performance in risk-adjusted returns. Expanding on Frazzini & Pedersen (2014), I identify that those portfolios composed of low-beta assets provided robust returns and showed significant resilience to market volatility.

I closely analyzed the impact of variations in the rolling window on the performance of beta-based portfolios. This analysis revealed how adjustments in beta calculation methodologies can influence the outcome of the strategy. The existing literature used 5-years for correlations to calculate beta. I expand this by exanimating 3-years to 7-years rolling window for correlations. My findings show that the strategy does not suffer major disturbances which highlight the robustness of the strategy. Future research can examine the effect of using larger rolling windows for volatility calculation.

In summary, the results of this research strongly suggest the effectiveness of stock beta-based strategies, especially when applied in a context of broad asset diversification. These findings contribute to the advancement of academic knowledge and offer practical insights for investors in making informed and effective decisions.

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Title: Beta Analysis on the US Stock Market

Key Words: Beta, systematic risk, betting against beta, risk-adjusted returns, market volatility, rolling window analysis, investment strategies, USA stock market, empirical finance, factor investing.

Resumo

Conduzi uma pesquisa nos mercados da NYSE e S&P500, explorando várias estratégias de investimento com base no Beta. Apresento resultados para 156 carteiras estruturadas de acordo com as características de risco sistemático. Investiguei o Beta, medida de risco sistemático, para construir minhas carteiras com base nesse valor.

O fator "apostar contra o beta" demonstrou considerável eficácia quando implementado num amplo espectro de ativos. Essa abordagem, que prioriza investimentos em ativos de beta baixo enquanto realoca recursos de ativos de beta alto, exibiu consistentemente desempenho superior em retornos ajustados ao risco. Expandindo o trabalho de Frazzini & Pedersen (2014), identifiquei que carteiras compostas por ativos de baixo beta proporcionaram retornos robustos.

Analisei o impacto das variações na janela usada na estimação no desempenho das carteiras baseadas em beta. Essa análise revelou como ajustes nas metodologias de cálculo do beta podem influenciar o resultado da estratégia. A literatura existente utilizou 5 anos para estimar correlações e um ano para volatilidade para calcular o beta. Expando isso examinando uma janela móvel de 3 a 7 anos para correlações. A estratégia não sofre grandes perturbações, destacando a robustez da estratégia. Pesquisas futuras podem examinar o efeito do uso de janelas móveis maiores para o cálculo da volatilidade.

Em resumo, os resultados sugerem fortemente a eficácia de estratégias baseadas no beta de ações, especialmente quando aplicadas em um contexto de ampla diversificação de ativos. Essas descobertas contribuem para o avanço do conhecimento acadêmico e oferecem conhecimentos práticos para investidores na tomada de decisões informadas.

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Palavras-chave: Beta, Risco sistemático, apostar contra beta, Retornos ajustados ao risco, Volatilidade de mercado, Análise de janela deslizante, Estratégias de investimento, Mercado de ações dos EUA, Finanças empíricas, Investimento em fatores.

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Chapter 1

1. Introduction

The Betting Against Beta (BAB) strategy presented by Frazzini & Pedersen (2014) has sparked considerable debate and interest within the financial academic sphere. Its very existence poses a challenge to the Capital Asset Pricing Model (CAPM) that predicts a positive linear relationship between risk and return. Yet, empirical evidence seems to persistently demonstrate that assets with lower beta often outperform those with higher beta when adjustments for risk are considered. This apparent contradiction raises fundamental questions about the nature of risk and return in financial markets and suggests that there may be unexplored mechanisms at play that dictate investment performance.

The work of Black (1972) and Frazzini & Pedersen (2014) have postulated that the constraints imposed on investors' ability to borrow may lead to a preference bias towards certain high-beta stocks. As investors with constraints invest in more risky assets to get higher returns, an investor without these constraints may be able to exploit an arbitrage opportunity.

Chapter 2 highlights the existing literature that examines the role of ‘beta’ as a factor to consider when investing. This section displays the methodology employed in each case, the data used and the results that the different authors have achieved.

Chapter 3 thoroughly examines the data and methodology utilized to explore the advantages of investing using the measure of beta as a key indicator. This section displays how the different portfolios were built based on the existing literature.

I am particularly interested in how changing the period over which we assess market relationships, known as the correlation rolling window, influences the outcomes of the 'Betting Against Beta' strategy. This is important because I want to check the strategy's reaction to diverse ways of defining beta. Frazzini & Pedersen (2014) definition of beta has several specific features, for example, using different time frames to measure market correlations and market volatility. They believe that market volatility changes happen more quickly than correlations, so they suggest using one year’s worth of data for market volatility and five years for correlations.

In contrast, to Frazzini & Pedersen (2014), Kenneth French data library use a regression of monthly returns over 5 years. In this study, I look at what happens if we change the time frame for measuring these stock correlations using 3, 4, 5, 6 and 7 years as the rolling window. Furthermore, I also present a beta using a 5-year rolling window for volatilities and correlations. The point is to see if the 'Betting Against Beta' approach is still effective when we tweak how beta is calculated. This could give us new information on how flexible this strategy is and if we can rely on it when we measure beta in different ways. To do this I built 156 portfolios, as I have 6 different way of calculating beta, 2 markets studied (S&P 500 and NYSE), 10 portfolios formed on deciles of beta, portfolios formed on above or below the median beta, and the portfolio of the BAB strategy.

To expand on my motivation for conducting this exercise, recent studies in finance have highlighted the importance of defining variables in financial research. Menkveld et al. (2021), Walter et al. (2023), and Soebhag et al. (2023), emphasize that the findings in finance can depend on the choices made in the definition of the variables. The choice of time periods can significantly impact the results.

The main findings of this study are the following. Low beta stocks have better risk adjusted returns than high beta stocks under all the betas calculated in all the markets studied (S&P 500 and NYSE). Changing the rolling window does not seem to affect the BAB strategy. This tells us that the strategy is robust. The anomaly is less present on the S&P 500 than on the NYSE. The rolling window chosen by Frazzini & Pedersen (2014) was the best performing one for the S&P500 in terms of risk adjusted returns.

Chapter 2

2. Literature Review

In this chapter, I explore various academic papers debating the beta strategy in different assets. First, I start with the concept of beta, and then dive into the betting against beta strategy. The beta anomaly is an intriguing phenomenon in finance, it has been researched by several academics as it questions the traditional Capital Asset Pricing Model (CAPM). The presence of this anomaly has generated quite a debate on how we are able to capture it.

The Capital Asset Pricing Model (CAPM), proposed by Sharpe (1964) implies a linear relationship between systematic risk and expected returns. Beta is a measure of systematic risk of an asset, a beta of one would mean the asset has the same systematic risk as the market.

The CAPM equilibrium is given by the Security Market Line (SML):

$$E(r_i) = rf + \beta(E(r_m) - rf) \tag{1}$$

Where $E(r_i)$ is the expected return of security "i", rf is the risk-free rate, $E(r_m)$ the expected return of the market and β is the beta coefficient of security "i".

Nevertheless, empirical tests consistently show that low-beta stocks outperformed high-beta stocks on a risk-adjusted basis. Black et al (1972) argue that this happens as the security market line is too flat. To explain this, he points out that that a CAPM with restricted borrowing would be better at explaining this anomaly.

Miller and Scholes (1972) suggest that the alpha on an asset, which is the measure of the excess return of an investment compared to the return predicted by its risk, should depend, in a systematic way, on their beta. This means that high-beta assets tend to have negative alphas, and that low-beta assets tend to have positive alphas. In other words, the high-beta assets tend to be overpriced while the low-beta tend to be underpriced.

Frazzini & Pedersen (2014) used Black findings to develop the "Betting against Beta" strategy (BAB) exploiting the beta anomaly. This is a strategy where they go long (buy) in the low beta assets and go short (sells) in the high beta assets.

The authors have tested their strategy on US equities, 20 international equity markets, Treasury bonds, corporate bonds, and futures. Their data on US equities contains 23,538 stocks in total, starting from 1926 to 2012. For the local market index of the United States, they use CRSP value-weighted index and excess returns are calculated by subtracting the US Treasury bill rate.

To calculate the beta coefficients, they do not compute the standard five-year regression as the Kenneth French data library does. To calculate beta as a regression one would have as an independent variable the broad market returns and as a dependent variable as the asset returns, the slope coefficient of this regression would be the beta. Liu, Stambaugh, & Yuan, Y. (2018) on their paper 'Absolving beta of volatility' run a regression each month over a moving window of 60 month (5 years), this way of calculating goes in line with the way the Kenneth French data library does.

Instead, Frazzini & Pedersen (2014) decompose the slope of the regression, as the correlation between the asset and the market multiplied by the asset's volatility (standard deviation) divided by the market volatility. Mathematically this would be:

$$\hat{\beta}_i = \widehat{\rho_{im}} * \frac{\hat{\sigma}_i}{\hat{\sigma}_m} \quad (2)$$

Where $\hat{\rho}$ is the correlation between the asset and the market, $\hat{\sigma}_i$ is the asset standard deviation, and $\hat{\sigma}_m$is the market standard deviation.

This way they can compute the correlation calculation for 5-year rolling window using overlapping three-day log returns, and they can compute the volatility using 1-year rolling window using log returns.

The reason the correlation is calculated using a larger rolling window is to account for the fact that correlations tend to move slower than volatilities.

Regarding the three-day log return ($r_{i,t}^{3d} = \sum_{k=0}^{2} \ln\left(1 + r_{t+k}^{i}\right)$) this is to control for nonsynchronous trading, which happens when the prices of assets are not updated simultaneously, this has an effect on only correlations and not volatility, as volatility measures only one asset at the time.

Based on this beta, they proceed to form the portfolio the following way.

First, they shrink beta:

$$\hat{\beta}_i = w\hat{\beta}_i^{TS} + (1-w) * \hat{\beta}_i^{XS} \tag{3}$$

Where w = 0.6 and $\hat{\beta}_i^{XS}$ = 1 for all periods and across all assets. This does not affect how securities are sorted but it does affect the construction of the portfolio. By doing this the betas are shrunk towards 1.

Following this, as mentioned before, the assets are ranked in ascending order based on their beta values. Let z e the vector of beta rank and $\bar{z}$ is the average rank. In other words, if I have 10000 assets and I rank them 1 to 10000, then the average rank $\bar{z}$ would be 5000, the median, and the asset with the highest beta would have a z of 10000. Next, they weight the assets of the low beta and the high beta by:

$$W_H = k(z - \bar{z})^+ \tag{4}$$

$$W_L = k(z - \bar{z})^- \tag{5}$$

Where k is a constant calculated as $k = 2/1_n'|z-\bar{z}|$. The + and – at the end of the parenthesis indicate the positive and negative elements of the vector. This way, assets that have high betas will be overweighted on the W_H vector and low betas will be overweighted on the W_L vector. Note that under this construction each weight (high and low) vector will always add up to one.

Finally, to construct the BAB factor, both portfolios are rescaled to have a beta equal to 1 when the portfolio is formed. To achieve this, each portfolio is divided by their respective betas. The return calculation is the following:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L} * (r_{t+1}^L - r^f) - \frac{1}{\beta_t^H} * (r_{t+1}^H - r^f) \tag{6}$$

Where $r_{t+1}^L = r_{t+1}' W_L$, $r_{t+1}^H = r_{t+1}' W_H$, $\beta_t^L = \beta_t' W_L$ and $\beta_t^H = \beta_t' W_H$.

The objective of calculating the returns this way is to make the portfolio a zero-beta portfolio, as the long investment have a beta of one and the sort of investment also have a beta of 1 so it will net to zero in the long-short strategy.

Frazzini & Pedersen (2014) found that this factor produces significant positive risk-adjusted returns. The BAB yields positive Sharpe ratios across 18 of the 19 MSCI developed countries, showing a consistent trend across equity markets. On the US Equities, BAB had a Sharpe ratio of 0.78.

The theory behind these results is better explained by the CAPM with restricted borrowing. Where the equilibrium required return for a security (s) is given by:

$$E_t(r_{t+1}^s) = rf + \psi_t + \beta_t^s \lambda_t \tag{7}$$

The risk premium at moment t can be calculated as $\lambda_t = E(r) - rf - \psi_t$ and ψ_t is the average Lagrange multiplier which measures the tightness of funding constraints (a larger ψ_t indicates a tighter portfolio constrains). The alpha of the security decreases in the beta.

$$\alpha_t^s = \psi_t(1 - \beta_t^s) \tag{8}$$

Note that the Sharpe ratio for an efficient portfolio is the highest with a beta lower than one.

The expected returns of a self-financing BAB factor should be positive as:

$$E_t(r_{t+1}^{BAB}) = \frac{\beta_t^H - \beta_t^L}{\beta_t^L \beta_t^H} \psi_t \geq 0 \tag{9}$$

The expected return of the BAB factor depends on the ex-ante beta spread and the funding tightness.

The authors conclude that there is a way for investors, which are not restricted to borrowing, to invest in stocks with low betas and apply leverage, to get this abnormal return. Frazzini, Kabiller, & Pedersen (2018) on 'Buffett's alpha' tell us that investors like Warren Buffet tend to buy stocks that are "safe" (with low beta and low volatility).

Frazzini & Pedersen (2014) findings where critiqued by Novy-Marx & Velikov (2018) on their paper "Betting Against Betting Against Beta". Novy-Marx & Velikov (2018) argue that the performance of beta is mainly driven by the non-standard procedure used.

The first non-standard procedure mentioned before is that Frazzini & Pedersen (2014) used a rank-weighted portfolio instead of simple equal-weighted portfolios, which puts more weight on stocks with extreme betas. The problem with this is that it does not weigh stocks in proportion to their market capitalization. So, stocks which are the most illiquid tend to gain high weights, the abnormal excess returns of these stocks are compensating for the liquidity risk the investment holds.

The second non-standard procedure is to make BAB market neutral by leveraging. On the formula that calculates the BAB returns, note that it divides the returns on the High beta portfolio and the low beta portfolio by their corresponding betas. This way each portfolio should have a beta of 1 and should net to zero in the long/short strategy. Note that this makes BAB strategy to be not dollar-neutral in terms of only the risky securities and assumes it to be financed with the risk-free rates.

The borrowing cost used to finance the hedge assumed in the strategy should be:

$$(\beta_L^{-1} - \beta_H^{-1}) * r^f \tag{10}$$

The third non-standard procedure is that as they are using five years of overlapping three-day returns with volatilities estimated using one year of daily data to calculate "beta" which this procedure does not actually yield market betas.

The standard Beta should be calculated using a 5-year rolling window as:

$$\hat{\beta}_{i5} = Cov(r_{i5}, r_{m5})/var(r_{m5}) = \hat{\rho}_{i5} * \frac{\hat{\sigma}_{i5}}{\hat{\sigma}_{m5}} \tag{11}$$

Frazzini & Pedersen (2014) calculated the betas as:

$$\hat{\beta}_{FP} = \hat{\rho}_{i5} * \frac{\hat{\sigma}_{i1}}{\hat{\sigma}_{m1}} \tag{12}$$

To transform Frazzini & Pedersen (2014) "Betas" to standard Betas we should do the next computation:

$$\hat{\beta}_{FP} = \frac{\hat{\sigma}_{i1}/\hat{\sigma}_{i5}}{\hat{\sigma}_{m1}/\hat{\sigma}_{m5}} \hat{\beta}_{i5} \tag{13}$$

To add to the existing literature, in this study I examine this critique and explore different ways of calculating beta. This adds to the literature as different correlation windows were not studied before.

Novy-Marx & Velikov (2018) show that a great deal of the performance is driven by overweighing the market's smallest (market cap), least liquid stocks, and that the strategy is ignoring transaction costs and implementation issues. The transaction cost would reduce the profitability by almost 60%, however it still yields positive returns. The authors concluded that this performance does not support their underlying theory as the beta calculated is a non-standard estimation.

Asness et al. (2019) introduces some new factors, like Betting-Against-Correlation (BAC) and Scaled MAX (SMAX), which is related to measures of sentiment, to decompose the BAB strategy. They also use the Betting Against Volatility (BAV) factors. They tested their strategies on 58,415 stocks from 24 countries. Their date frame is between January 1926 and December 2015, 90 years of data. Their findings indicate that BAC produces strong performance in the US and International markets.

Asness et al. (2019) construct BAC portfolios in a similar way to BAB portfolios formed by Frazzini & Pedersen (2014). The difference is that they first rank the stocks by volatility each month and assign them to five quintiles and then form the BAC portfolios in each quintile, they do this to control volatility. The portfolio weights for the high and low correlation are given by:

$$W_H{}^q = k^q(z^q - \bar{z}^q)^+ \tag{14}$$

$$W_L{}^q = k^q(z^q - \bar{z}^q)^- \tag{15}$$

Where q is each quantile, z is the rank of the security, $\bar{z}$ is the average rank and k is a normalizing constant determined by $k = 2/1'_{n(q)}|z^q - \bar{z}^q|$.

The return of BAC of each quintile is:

$$r_{t+1}^{BAC(q)} = \frac{1}{\beta_t^{L,q}} * \left(r_{t+1}^{L,q} - r^f\right) - \frac{1}{\beta_t^{H,q}} * \left(r_{t+1}^{H,q} - r^f\right) \tag{16}$$

Where the returns of the low-correlation and high-correlation portfolios are $r_{t+1}^L = r'_{t+1}W_L$, $r_{t+1}^H = r'_{t+1}W_H$, and the corresponding betas are $\beta_t^L = \beta'_t W_L$ and $\beta_t^H = \beta'_t W_H$.

The BAC return is given by an equal-weighted return of the five quintiles.

$$r_{t+1}^{BAC} = \frac{1}{5}\sum_{q=1}^{5} r_{t+1}^{BAC(q)} \tag{17}$$

The BAV factor is calculated in a similar way to this portfolio only stocks are first sorted into quintiles based on correlation instead of volatility.

Asness et al. (2019) also consider the factor that goes long stocks with low MAX return (LMAX) or low idiosyncratic volatility (IVOL). MAX is the average of the five highest daily returns over the last month. For the low MAX return, they create a new factor to help differentiate the alternative hypothesis (Volatility), they create scaled MAX (SMAX) factor that goes long on stock with a low MAX return divided by the ex-ante volatility and short the stocks on stocks that have a high SMAX. They find that in the US LMAX, IVOL and SMAX all produce significant alphas with respect to the Fama-French five-factor model. SMAX produces a stronger performance than LMAX and IVOL.

Barroso & Maio (2018) study the risk dynamics of the BAB anomaly. They find that a risk-managed strategy achieves a Sharpe ratio of 1.28, with an information ratio of 0.94. Risk-scaling has economic gains especially in momentum and BAB factors. Then the authors decompose the risk of the strategy into specific and market risk and find that the market risk plays a minor role while the specific component shows predictability. To compute this risk-managed strategy they compute the realized variance from daily returns in the previous twenty-one sessions for each month and factor. The realized variance of the factor 'F' is:

$$RV_{F,t} = \sum_{j=0}^{20} r^2_{F,\, d_t - j}. \qquad (18)$$

And the realized volatility $\hat{\sigma}_{F,t} = \sqrt{RV}$. The portfolio weight is defined as: $W_t = \sigma_{target}/\hat{\sigma}_{F,t}$

The risk-scaled factors are then $F^*_{t+1} = F_{t+1}W_t$

Barroso & Maio (2018) found that the scaled BAB has a higher alpha with respect to the market and only twenty five percent is explained by equity risk factors.

AQR (Applied Quantitative Research) Capital Management, is a well-known US hedge fund founded by Cliff Asness in 1998. This fund employs various diversified strategies, including the Betting against Beta (BAB) strategy by Frazzini & Pedersen (2014). They explain to their investors that the security market line for U.S. stocks is too flat relative to the CAPM and is better explained by the CAPM with limited borrowing. These types of constraints often force investors to overweight in risky securities, which means that high-beta assets offer lower risk-adjusted returns than low-beta assets, which is opposing the CAPM's predictions.

Chapter 3

3. Data Description and Methodology

In this chapter I explain the process followed to get the data used in this study. Furthermore, I will describe the methodology used to create different portfolios using the beta factor to construct them and highlight the different ways in which this measure can be calculated to then see how it can affect the performance of the portfolios.

3.1. Data Description

The data utilized in my analysis was collected mainly from two sources, Refinitiv Eikon (DataStream) and Wharton Research Data Services (WRDS).

To enhance the depth of my analysis I have focused my study by examining only the American market stocks, this market was chosen for three main reasons.

The first reason is that the United States is one of the largest and most influential of the markets in the world. Second, the regulatory framework and reporting standards of the United States offer extensive and high-quality data. Finally, in Refinitiv, the only two countries where the dividend payment data is available, for the period used, is the USA and Canada.

To obtain the stocks returns I used Refinitiv Eikon as my source, I gathered the 43 years of daily data on the New York Stock Exchange (NYSE), and the Standard & Poor's 500 (S&P 500) from 01/01/1980 to 31/12/2023. I used this sample as I want to examine the last data available and to have at least over 40 years of data to gain statistical significance. As investors do not only take the stock price returns into account but also the dividend proceeds from shares, I used the Return Index as the data type. The return index shows the theoretical growth in value of a share assuming that dividends are re-invested to purchase additional units of the stock.

If Return Index is equal to 100 on the base date, then:

$$RI_t = RI_{t-1} * \frac{PI_t}{PI_{t-1}} * (1 + \frac{DY_t}{100} * \frac{1}{N}) \quad (19)$$

Where RI_t is the return index on day t, PI_t is the price index on day t, DY_t is the dividend yield percentage on day t of the price index and N is the number of working days in a year. Refinitiv assumes N to be equal to 260 days. Note that the calculation ignores tax and re-investment charges.

For the market I downloaded the CRSP Value Weighted Index (which was the same that was used by Frazzini & Pedersen (2014)) using WRDS as the source.

The Center for Research in Security Prices, LLC (CRSP) keeps and maintains the most complete collection of security prices and returns data for the USA Market. Which includes NYSE, AMEX, NASDAQ, and ARCA stock markets. The Value Weighted return (VWRETD) indices contain the daily, including all distributions, on a value-weighted market portfolio. This is excluding the American Depository Receipts (ADRs).

To get the risk-free interest rate of the United State Market, I downloaded the Fama/French 3 Factors [Daily] from the Kenneth French Data Library web page. This risk-free rate is the one-month Treasury bill rate from Ibbotson Associates, which is a well-known investment research and data firm. As the interest rate was presented in percentage, this data was divided by 100. Furthermore, to complete the missing dates, I used the last risk-free rate when there was missing data, meaning if a date did not have a risk free, the last risk-free interest rate available was used. The average monthly interest rate for the risk-free interest rate is approximately 0.24% for the period studied, annualized this would be 2.90%.

Table 1: Summary of the data used in this study.
This table provides the source, frequency, and period of the data.

Data	***Source***	***Frequency***	***Start Date***	***End Date***
S&P500	Refinitiv Eikon (DataStream)	Daily	1980	2023
NYSE	Refinitiv Eikon (DataStream)	Daily	1980	2023
CRSP Value Weighted	Wharton Research Data Services (WRDS)	Daily	1980	2023
Risk-Free Rate	Kenneth French Data Library	Daily	1980	2023

The CRSP Value Weighted for the period 1987 to 2023 produced these statistics:

Table 2: CRSP Value Weighted Statistics.
This table shows the excess returns, volatility, Sharpe ratio, kurtosis, and skewness of CRSP Value Weighted from 1987 to 2023. Excess returns and volatility are in percentage. Sharpe Ratio is annualized, the rest of the metrics are monthly.

CRSP VW	VW Monthly
Excess returns	0.85
Volatility	4.47
Sharpe Ratio	0.66
Kurtosis	2.58
Skewness	(0.87)

The reason I am showing 1987 to 2023 and not 1980 to 2023 is to be able to compare this to the portfolios I formed, which used 7 years of data to be formed.

3.2. Methodology

3.2.1. Methodology: Beta Calculation

I estimated betas using rolling regressions on daily data of the stocks excess returns on the market excess returns. The log returns and three-day log returns ($r_{i,t}^{3d} = \sum_{k=0}^{2} \ln{(1 + r_{t+k}^{i})}$) were computed using python. To get the excess returns the daily risk-free rate was subtracted to the returns, log returns and three-day log returns. To calculate beta, I divided the calculation as Frazzini & Pedersen (2014) did, see Equation (2).

First, I calculated the volatility of each stock by computing the standard deviation using a 1 year rolling window on the log returns. And did the same for the market.

Next, I divided the rolling standard deviation of each stock by the rolling standard deviation of the market. This variable was called the 'volatility ratio' which is the relative volatility of the stock against the market. If this variable is higher than one, then the stock has more systematic risk than the market.

Regarding the correlation aspect of beta, I used a three overlapping day log returns and calculated the correlation of each stock to the market over a 5-year rolling window. Finally, to compute beta, the volatility ratio was multiplied with the correlation.

To shrink beta, I use the same computation as Frazzini & Pedersen (2014), see Equation (3).

Next, I formed different betas using different rolling windows in the correlation calculation. To do this the entire process was repeated using 3-year to 7-year rolling windows for correlation. To add on, I also calculated beta as the standard basis which uses a 5-year rolling window for both the volatility and the correlation.

In total I calculated six different betas, shown in Table 3.

Table 3: Betas calculated.
This table shows the different betas calculated and the components used in it. I used daily data to get them as the existing literature does.

Beta	Variation ratio	Correlation
Beta 1-3	1-year rolling window on excess log returns	3-year rolling window on three-day excess log returns.
Beta 1-4	1-year rolling window on excess log returns	4-year rolling window on three-day excess log returns.
Beta 1-5	1-year rolling window on excess log returns	5-year rolling window on three-day excess log returns.
Beta 1-6	1-year rolling window on excess log returns	6-year rolling window on three-day excess log returns.
Beta 1-7	1-year rolling window on excess log returns	7-year rolling window on three-day excess log returns.
Beta 5-5	5-year rolling window on excess log returns	5-year rolling window on excess log returns.

As I calculated all these different betas, I realized that to compare them, there would be an issue with the data available in each beta. As Beta 1-7 requires more information (7-years) a stock that only has 6 years of data would not be considered in the portfolio formed with this beta, but it would be considered in the portfolio formed with the Beta 1-6. To solve this issue and be able to compare them I made the following computation. If Beta 1-7 has an error, this error would also appear in the other betas. This way I have the same pool of stocks for all the betas calculations. Before doing this, my results showed an increase in all the portfolios risk adjusted returns when I used shorter rolling windows. This was happening because shorter rolling windows would mean that investors could choose from more stocks.

3.2.2. Methodology: Portfolio Formation

Once betas were computed, I sorted them in ascending order and ranked them. Using this ranking I divided the stocks into 10 deciles, this way decile 1 would be a cluster of the stocks with the lowest betas and decile 10 would be a cluster of stocks with the highest betas. On these clusters I used an equal weighted approach to estimate each of the portfolios. This way, if I have 500 stocks each portfolio would be formed of 50 stocks, where each stock would have an equal weight of 2%.

Next, I created three portfolios where I did a similar approach as the portfolios just mentioned. I ranked the betas and calculated the median of this rank. Then I formed two equal-weighted portfolios where one is formed on the stocks with betas higher than the median and the other is formed on the betas lower than the median. I called the portfolio with the betas above the median as PH and the portfolio with the betas below the median as PL.

Then I ranked each stock (where z denotes the rank) and calculated the median of the rank ($\bar{z}$). I formed the weights as in Equation 4 and 5.

Finally, to construct the BAB factor, each portfolio was scaled to have a beta of one by dividing it by the vector of betas multiplied by the weights, this way, by the property of beta, beta ex-ante will equal to "0".

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L} * (r_{t+1}^L - r^f) - \frac{1}{\beta_t^H} * (r_{t+1}^H - r^f) \quad (20)$$

Where $r_{t+1}^L = r_{t+1}' W_L$, $r_{t+1}^H = r_{t+1}' W_H$, $\beta_t^L = \beta_t' W_L$ and $\beta_t^H = \beta_t' W_H$.

To build monthly portfolios, I used the last weight, and therefore the last beta of each month and transformed the returns to monthly returns:

$$r_m = \prod_0^t (1 + r_d) - 1 \quad (21)$$

Where r_m is the monthly return, r_d is the daily return and t is the number of days in the month.

Note that on the portfolio formation I am not taking any type of transaction cost. Also, I am not considering liquidity issues associated with buying nor selling stocks. Furthermore, I am assuming that investors can take debt at risk free rates.

Table 4: Portfolios Calculated

This table presents the thirteen portfolios formed in this study. Portfolios P1 to P10 are portfolios formed on the beta ranking percentiles. P1 is a pool of stocks with the lowest beta and P10 is a pool of stocks with the highest beta. PL is a pool of stocks with betas below the median while PH is the one formed with stocks above the median rank. BAB is 'betting against beta' portfolio. All the portfolios are monthly.

Portfolio Name	***Portfolio Description***
P1	Portfolio formed on the 10% lower Betas. Lower percentile (P1 ≤ 10%).
P2	Portfolio formed on 10% to 20% percentile. (10% < P2 ≤ 20%).
P3	Portfolio formed on 20% to 30% percentile. (20% < P3 ≤ 30%).
P4	Portfolio formed on 30% to 40% percentile. (30% < P4 ≤ 40%).
P5	Portfolio formed on 40% to 50% percentile. (40% < P5 ≤ 50%).
P6	Portfolio formed on 50% to 60% percentile. (50% < P6 ≤ 60%).
P7	Portfolio formed on 60% to 70% percentile. (60% < P7 ≤ 70%).
P8	Portfolio formed on 70% to 80% percentile. (70% < P8 ≤ 80%).
P9	Portfolio formed on 80% to 90% percentile. (80% < P9 ≤ 90%).
P10	Portfolio formed on the 10% higher Betas. Higher percentile. (90% < P10).
PL	Portfolio formed on the betas lower than the median. (PL≤ 50%).
PH	Portfolio formed on the betas higher than the median. (50% < PH).
BAB	Betting Against Beta factor portfolio.

3.2.3. Methodology: Portfolio Performance Evaluation

As the portfolios formed have each different rolling windows, I have made all the portfolios start on the same date to be able to compare them. As the data starts in 1980 and Beta 1-7 portfolios need 7 years of data to start operating, the 1st of January 1987 is the starting date of each investment. All portfolios end on 12/31/2023, this way I have observed the performance over 37 years (444 months).

To compare all the portfolios, I calculated the average monthly excess return, monthly volatility, annual Sharpe ratio, CAPM alpha, three factor alpha, excess kurtosis, and skewness.

To annualize the Sharpe ratio, I used the following formula:

$$SR = \frac{(Average\ Excess\ returns)*12}{StDev(Monthly\ Excess\ returns)*\sqrt{12}} \tag{22}$$

Note that according to Frazzini & Pedersen (2014) paper BAB factor returns are achieved by using excess returns, so they are themselves excess returns and therefore we should not subtract the risk-free rate to calculate the Sharpe ratio.

Chapter 4

On this chapter, I present the performance of the portfolios. The presentation examines the beforementioned key metrics on each portfolio and analyzes the effect of using different betas.

4. Results

As mentioned before, I have constructed six betas, and thirteen portfolios for each beta assessed on two markets (S&P500 and NYSE), a total of 156 portfolios. The way I analyze them is by first looking at the portfolios formed on deciles. Then the portfolios formed as below and above the median beta. And finally, examine the BAB factor.

4.1 Benchmark AQR

The data was retrieved from AQR, the data set is related to "Betting Against Beta" (Frazzini and Pedersen, 2014). It includes all available common stocks in the merged CRSP / XpressFeed data. Using the period from 1987 to 2023 AQR BAB performance was:

Table 5: AQR: US Equities Betting Against Beta Factor.
This table shows the average excess returns, alphas (with their t-stat below), volatility, Sharpe Ratio, excess kurtosis, and skewness from 1987 to 2023. All the data is monthly except Sharpe Ratio which is annualized. Excess returns, volatility and alphas are in monthly percentage.

US BAB	AQR Monthly
Excess returns	0.67
CAPM α	0.82
	4.64
Three Factor α	0.73
	4.36
Volatility	3.72
Sharpe Ratio	0.63
Kurtosis	3.53
Skewness	-0.42

4.2. S&P 500

First, to analyze the data used for this market, I formed my own S&P 500 Value weighted, using the returns from the DataStream (RI) from each stock.

For computing the weights, I used MV/Total MV of one period before. Finally, to get the returns of the S&P 500 Value Weighted I multiplied these two.

$$R^{SP500\,VW}{}_t - rf = \sum \frac{MV^i{}_{t-1}}{Total\ MV_{t-1}} * \left(\frac{RI_t^i}{RI_{t-1}^i} - 1\right) - rf \qquad (23)$$

Table 6: S&P500: Value Weighted Excess Returns.
This table shows the average excess returns, volatility, Sharpe ratio, excess kurtosis, and skewness of S&P 500 Value Weighted that I created from 1987 to 2023. Excess returns are in percentage. Sharpe Ratio is annualized, the rest of the metrics are monthly.

S&P 500	VW Monthly
Excess returns	0.85
Volatility	4.36
Sharpe Ratio	0.68
Kurtosis	1.98
Skewness	-0.64

4.2.1. S&P 500: Deciles Portfolios

As presenting all the portfolios statistics of each of the portfolios would take too much time, I will provide a summary of the most important statistics of all the beta sorted percentile portfolios first and then dig into the other portfolios formed. I use the Three-factor Alpha Fama, E. F., & French, K. R. (1992). I chose this statistic because it is the same measure adopted in Liu, Stambaugh, and Yuan (2018).

Table 7: S&P500: Decile Portfolio Three Factor Alphas.
This table shows the three-factor alpha of each of the decile portfolios formed on each of the betas from 1987 to 2023. To calculate the three factor Alpha, I performed the regression of monthly excess returns where the explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios. Stocks are weighted by ranked betas and the portfolios are rebalanced every calendar month. The values are in monthly percentage. The 5% statistical significance is indicated in bold.

Three Factor Alphas	***P1***	***P2***	***P3***	***P4***	***P5***	***P6***	***P7***	***P8***	***P9***	***P10***
Beta 1-3	**0.36**	**0.18**	0.14	0.17	0.12	-0.04	-0.02	0.07	-0.07	-0.28
Beta 1-4	**0.31**	**0.23**	0.17	0.17	0.11	-0.06	0.05	0.08	-0.12	-0.31
Beta 1-5	**0.32**	**0.27**	0.14	0.13	0.20	-0.05	0.04	-0.01	-0.07	-0.33
Beta 1-6	**0.32**	**0.31**	0.15	0.18	0.07	0.08	0.00	-0.03	-0.11	-0.32
Beta 1-7	**0.33**	**0.25**	0.18	0.17	0.08	0.10	-0.02	-0.04	-0.14	-0.28
Beta 5-5	**0.39**	**0.29**	0.17	0.02	0.03	0.07	-0.01	-0.02	-0.14	-0.16

The first thing noticed from this table is how portfolios formed with lower beta ranked stocks tend to outperform the portfolios formed with higher beta ranked stocks in all the betas calculated. The effect of using different betas (different rolling windows for correlations) does not seem to make profound changes in the three-factor alphas. The highest alpha is achieved by the lowest beta portfolio, P1 using Beta 5-5.

4.2.2. S&P 500: High Beta and Low Beta Portfolios

Now I will analyze the portfolios formed on betas below the median beta and above the median beta. Table 8 holds the results for the portfolio of stocks which are below the median beta, PL, most of the results are quite similar, changes in the rolling window do not seem to affect the portfolio formation drastically.

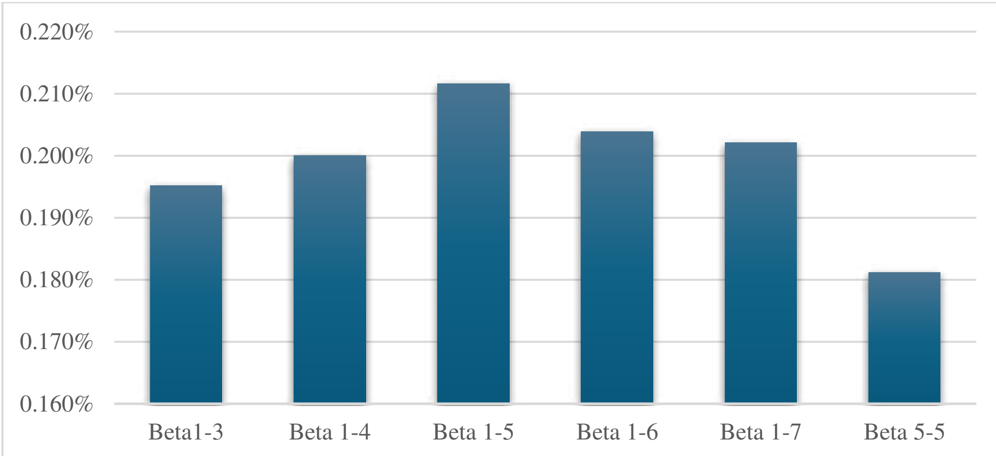
Table 8: S&P500: Portfolios statistics formed on Low Betas (PL).
This table shows the average excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, excess kurtosis and skewness of the PL Portfolio calculated using different betas formed on the S&P 500 from 1987 to 2023. Used three decimal places for the metrics. excess returns and the alphas are in percentage.

PL	Beta 1-3	Beta 1-4	Beta 1-5	Beta 1-6	Beta 1-7	Beta 5-5
Excess returns	0.926	0.926	0.938	0.928	0.926	0.933
CAPM α	**0.265**	**0.268**	**0.279**	**0.270**	**0.267**	**0.252**
	(2.93)	(3.01)	(3.16)	(3.07)	(3.07)	(2.73)
Three Factor α	**0.195**	**0.200**	**0.212**	**0.204**	**0.202**	**0.181**
	(2.51)	(2.61)	(2.77)	(2.67)	(2.66)	(2.30)
Volatility	3.976	3.935	3.93%	3.924	3.919	4.071
Sharpe Ratio	0.809	0.815	0.826	0.819	0.818	0.793
Kurtosis	4.360	4.265	4.114	4.099	4.107	3.920
Skewness	-1.046	-1.027	-1.002	-1.006	-1.000	-1.000

The results indicate a remarkable consistency in excess returns and volatility. The Portfolio PL exhibits a high Sharpe ratio and positive significant alphas for all the portfolios. Beta 1-3 demonstrates the highest excess kurtosis among the portfolios.

Beta 1-5 is the beta that was used by Frazzini & Pedersen (2014). Note that this was the PL that produced the highest results in terms of Sharpe Ratio and Alphas (See figure 1). Regarding skewness, all portfolio iterations exhibit negative skewness, which indicates a distribution with a longer left tail.

Figure 1: S&P500: Three Factor Alphas PL.
This figure shows the Three Factor Alphas calculated on PL portfolios computed using each Beta.



From Figure 1 we can observe that the highest alpha belong to the PL formed on Beta 1-5.

On the portfolios formed on High betas (PH) we can see the following results:

Table 9: S&P500: Portfolios statistics formed on High Betas (PH).
This table shows the average excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, excess kurtosis and skewness of the PH Portfolio calculated using different betas formed on the S&P 500 from 1987 to 2023. Used three decimal places for some of the metrics.

PH	Beta 1-3	Beta 1-4	Beta 1-5	Beta 1-6	Beta 1-7	Beta 5-5
Excess returns	1.174	1.175	1.163	1.173	1.175	1.168
CAPM α	-0.000	-0.003	-0.014	-0.006	-0.003	0.012
	(-0.00)	(-0.02)	(-0.11)	(-0.04)	(-0.02)	(0.10)
Three Factor α	-0.068	-0.073	-0.085	-0.077	-0.075	-0.054
	(-0.72)	(-0.77)	(-0.87)	(-0.79)	(-0.76)	(-0.60)
Volatility	6.718	6.743	6.753	6.766	6.781	6.589
Sharpe Ratio	0.606	0.604	0.597	0.601	0.600	0.614
Kurtosis	2.892	2.916	2.924	2.888	2.879	3.000
Skewness	-0.176	-0.182	-0.186	-0.186	-0.188	-0.197

Regardless of the way beta was calculated, PH presents mostly negative alphas, none of which are significant. In terms of risk adjusted return Beta 5-5 was the one that produced the best results.

Comparing PL to PH, PH has higher average excess returns but in terms of risk adjusted returns they are far worse. Excess Kurtosis and negative skewness are smaller for the PH portfolio.

4.2.3. S&P 500: Betting Against Beta (BAB)

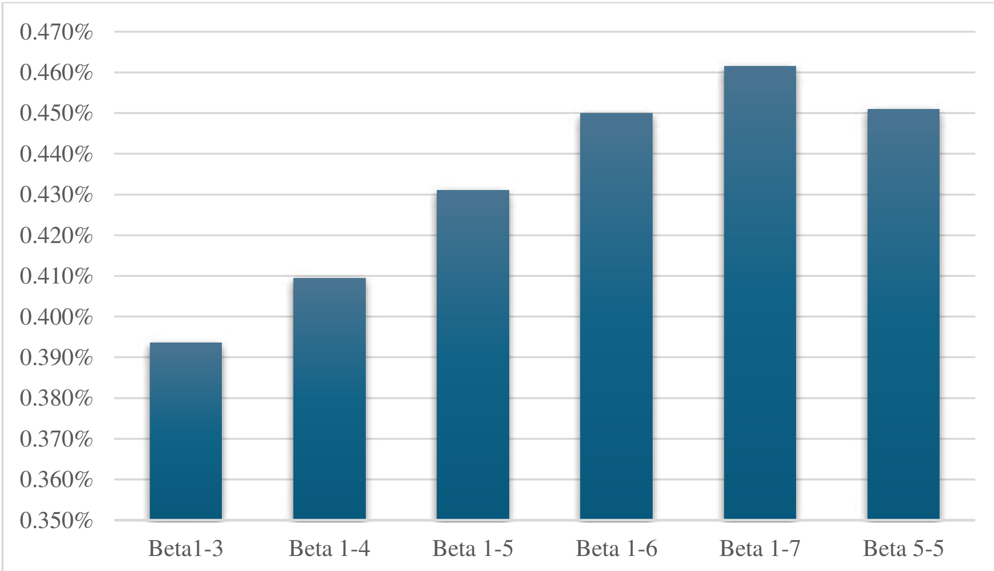
When applied the BAB strategy to the S&P 500 data the results were not as good as expected. See the statistics on table 10 of the betting against beta applied to the S&P 500.

Table 10: S&P500: BAB factor statistics.
This table shows the average excess Returns, the alphas with their respective t-stats, volatility, Sharpe ratio, excess kurtosis and skewness of the BAB Portfolio calculated using different betas formed on the S&P 500 from 1987 to 2023. Used 3 decimal places for some of the metrics. Beta ex-ante is the average beta on the portfolio formation.

BAB	Beta 1-3	Beta 1-4	Beta 1-5	Beta 1-6	Beta 1-7	Beta 5-5
Excess returns	0.215	0.216	0.229	0.237	0.245	0.267
CAPM α	**0.455**	**0.466**	**0.486**	**0.503**	**0.513**	**0.502**
	(2.50)	(2.57)	(2.68)	(2.74)	(2.76)	(2.86)
Three Factor α	**0.394**	**0.409**	**0.431**	**0.450**	**0.462**	**0.451**
	(2.29)	(2.39)	(2.53)	(2.61)	(2.64	(2.70)
Volatility	3.983	3.987	4.005	4.054	4.097	3.842
Sharpe Ratio	0.19	0.19	0.20	0.20	0.21	0.24
Kurtosis	4.44	4.37	4.54	4.60	4.63	3.51
Skewness	-0.79	-0.82	-0.86	-0.87	-0.89	-0.68
Beta Ex Ante	0.000	0.000	0.000	0.000	0.000	0.000

Compared to the AQR results the BAB strategy applied to the S&P 500 does not seem to yield a satisfactory performance. The alphas are positive and significant but compared to the BAB strategy of AQR they are relatively low. This goes in line with Green, J., Hand, J. R., & Zhang, X. F. (2017) findings, where the top 500 stocks that compose the S&P 500 are large by definition, and anomalies are often weak in these types of sets. Regarding the different betas used, there is no substantial difference between them. A small trend is observable in the next Figure.

Figure 2: S&P500: Three Factor Alphas BAB.
This figure shows the Three Factor Alphas calculated on BAB portfolios computed using each Beta.



There is a tendency to increase its performance when a larger rolling window is used.

The weights of the strategy are given by the next table:

Table 11: S&P500: BAB Factor Long and Short Weights.
This table presents the weights on each part of the strategy. The Long (Buy) is the average of the long side of the strategy which is $1/\beta_L$. The Short Sell is the average of the short side of the strategy which is $1/\beta_H$. The borrowing is $(\beta_L^{-1} - \beta_H^{-1})$.

BAB	Beta 1-3	Beta 1-4	Beta 1-5	Beta 1-6	Beta 1-7	Beta 5-5	Average
Long (Buy)	1.52	1.51	1.50	1.50	1.50	1.54	1.51
Short (Sell)	0.85	0.85	0.84	0.84	0.84	0.92	0.86
Financed at Risk Free Rate	0.67	0.66	0.66	0.66	0.66	0.62	0.65

In average the BAB factor is long on $1.51 on low beta stocks, which is financed by short selling $1.51 risk-free securities, and short sells $0.86 of high beta-stock, with 0.86$ invested in the risk-free rate.

4.3. NYSE

As done with the S&P500, I formed a NYSE Value weighted using the data from my analysis.

$$R^{NYSE\,VW}{}_t - rf = \sum \frac{MV^i{}_{t-1}}{Total\ MV_{t-1}} * \left(\frac{RI_t^i}{RI_{t-1}^i} - 1 \right) - rf \qquad (24)$$

Table 12: NYSE: Value Weighted Excess Returns.
This table shows the excess returns, volatility, Sharpe ratio, excess kurtosis, and skewness of NYSE Value Weighted from 1987 to 2023. Excess returns are in percentage. Sharpe Ratio is annualized, the rest of the metrics are monthly.

NYSE	VW Monthly
Excess returns	1.20
Volatility	0.05
Sharpe Ratio	0.92
Kurtosis	3.46
Skewness	0.76

4.3.1. NYSE: Decile Portfolios

Table 13 proves again how portfolios formed with lower beta ranked stocks tend to outperform the portfolios formed with higher beta ranked stocks in all the Betas calculated.

Contrasting the results of the table above with the ones of the S&P 500, NYSE lowest beta portfolio (P1) demonstrates a notably higher alpha, suggesting that the 'Betting Against Beta' strategy may find more fertile ground in the NYSE environment.

This could be attributed to the broader market composition of the NYSE, which may offer a more diverse set of investment opportunities and risks compared to the more selective S&P 500.

Table 13: NYSE: Decile Portfolio Three Factor Alphas.

This table shows the three-factor alpha of each of the decile portfolios formed on each of the betas from 1987 to 2023. To calculate the three factor Alpha, I performed the regression of monthly excess returns where the explanatory variables are the monthly returns from Fama and French (1993) mimicking portfolios. Stocks are weighted by ranked betas and the portfolios are rebalanced every calendar month. The values are in monthly percentage. The 5% statistical significance is indicated in bold.

Three Factor Alphas	*P1*	*P2*	*P3*	*P4*	*P5*	*P6*	*P7*	*P8*	*P9*	*P10*
Beta 1-3	**0.93**	**0.33**	**0.25**	0.08	0.16	0.12	0.05	-0.10	-0.02	0.02
Beta 1-4	**0.93**	**0.28**	**0.26**	0.10	0.12	0.17	0.00	0.02	-0.09	0.06
Beta 1-5	**0.87**	**0.36**	**0.32**	0.07	0.15	0.10	0.03	-0.08	-0.02	0.07
Beta 1-6	**0.84**	**0.33**	**0.31**	0.19	0.06	0.10	0.08	-0.06	-0.04	0.05
Beta 1-7	**0.83**	**0.24**	**0.31**	**0.23**	0.13	0.06	0.05	-0.05	0.01	0.04
Beta 5-5	**1.09**	**0.27**	**0.30**	**0.20**	0.13	0.14	0.03	-0.07	-0.06	-0.09

Interestingly, the variation in alphas due to different rolling windows for beta computation appears to have a minimal impact on portfolio performance. This could indicate that the chosen rolling window, whether it be three, four, five, six, or seven years, does not significantly alter the systemic risk captured by the beta measure, or the market has not expressed sufficient variation within this time frame to impact the outcomes discernibly.

This infers a level of resilience in the 'Betting Against Beta' approach, suggesting that it is not excessively sensitive to the period selection for beta computation.

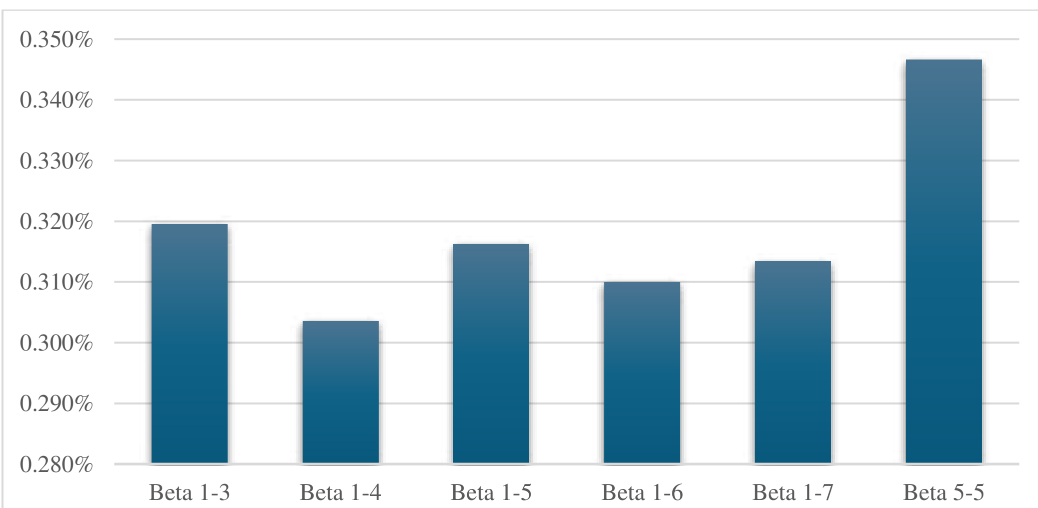
4.3.2. NYSE: High beta and Low beta Portfolios

Table 14: NYSE: Portfolios statistics formed on Low Betas (PL).
This table shows the average excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, excess kurtosis and skewness of the PL Portfolio calculated using different betas formed on the NYSE from 1987 to 2023. Used three decimal places for some of the metrics.

PL	Beta 1-3	Beta 1-4	Beta 1-5	Beta 1-6	Beta 1-7	Beta 5-5
Excess returns	1.021	1.002	1.015	1.008	1.010	1.059
CAPM α	**0.390**	**0.372**	**0.383**	**0.377**	**0.379**	**0.412**
	(3.91)	(3.79)	(3.91)	(3.82)	(3.86)	(4.11)
Three Factor α	**0.319**	**0.304**	**0.316**	**0.310**	**0.313**	**0.347**
	(4.16)	(4.00)	(4.15)	(4.02)	(4.07)	(4.51)
Volatility	3.922	3.900	3.909	3.912	3.907	4.000
Sharpe Ratio	0.902	0.890	0.900	0.892	0.895	0.917
Kurtosis	6.126	6.077	5.745	5.572	5.707	4.980
Skewness	-1.265	-1.295	-1.252	-1.224	-1.234	-1.114

Table 14 demonstrates the strong performance of low beta portfolios in the NYSE, as they consistently generated positive excess returns and achieved significant alphas. Despite using different beta measures, the portfolios did not change drastically.

Figure 3: NYSE: Three Factor Alphas PL.
This figure shows the Three Factor Alphas calculated on PL portfolios computed using each Beta.



From Figure 3 it is observable that the Beta 5-5 was the measure that yielded the best results for the PL portfolio on the NYSE. However, there is no clear trend on the changes of the rolling window.

Table 15: NYSE: Portfolios formed on High Betas (PH).
This table shows the average excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, excess kurtosis and skewness of the PH Portfolio calculated using different betas formed on the NYSE from 1987 to 2023. Used three decimal places for some of the metrics.

PH	Beta 1-3	Beta 1-4	Beta 1-5	Beta 1-6	Beta 1-7	Beta 5-5
Excess returns	1.204	1.223	1.209	1.217	1.215	1.166
CAPM α	0.126	0.144	0.133	0.139	0.137	0.104
	(0.92)	(1.04)	(0.96)	(1.00)	(0.98)	(0.77)
Three Factor α	0.016	0.032	0.020	0.026	0.022	-0.011
	(0.18)	(0.35)	(0.22)	(0.28)	(0.25)	(-0.13)
Volatility	6.365	6.389	6.378	6.385	6.386	6.270
Sharpe Ratio	0.655	0.663	0.657	0.660	0.659	0.644
Kurtosis	3.956	3.934	4.042	4.056	4.002	4.418
Skewness	-0.244	-0.240	-0.252	-0.258	-0.263	-0.304

High beta portfolios (PH), achieve positive excess returns, however the alphas here are not significant[1]. Comparing PL and PH within the NYSE shows how low beta portfolios deliver superior risk-adjusted returns.

The high beta portfolios, while yielding higher returns, do not exhibit the same level of alpha or efficiency as evidenced by their Sharpe ratios, highlighting the potential for low beta strategies to provide a more optimal balance between return and risk for discerning investors.

[1] Note that the alpha statistics shown are for equal-weighted returns, so the slight positive and insignificant alpha of high-beta stocks is not incompatible with the stylized fact that vale weighted portfolios tend to have negative alphas.

4.3.3. NYSE: BAB

Table 16: NYSE: BAB factor statistics.
This table shows the average excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, excess kurtosis and skewness of the BAB Portfolio calculated using different betas formed on the NYSE from 1987 to 2023. Used three decimal places for some of the metrics.

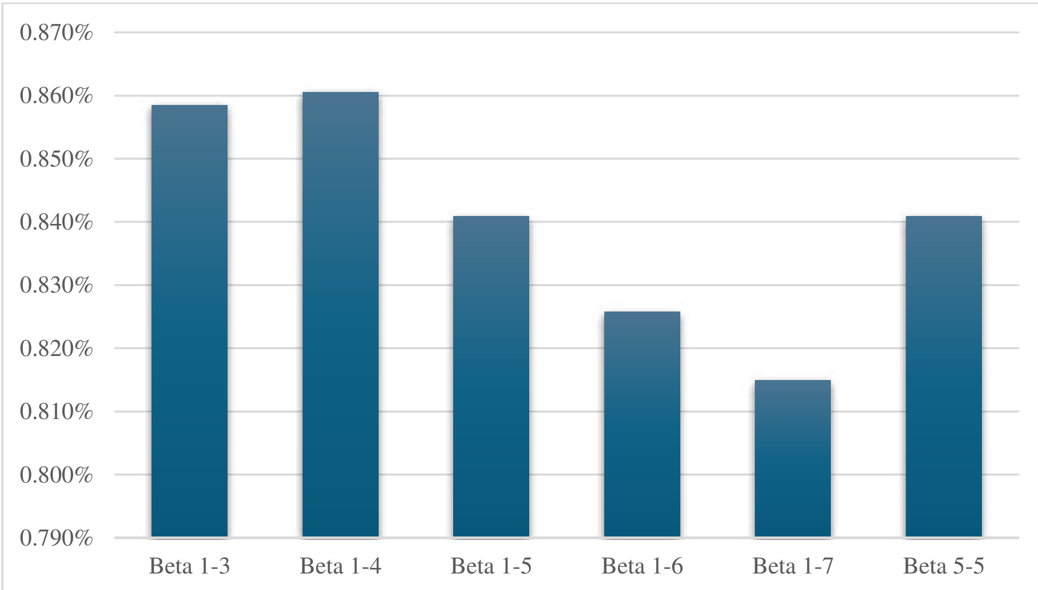
BAB	Beta 1-3	Beta 1-4	Beta 1-5	Beta 1-6	Beta 1-7	Beta 5-5
Excess returns	0.788	0.761	0.720	0.698	0.686	0.720
CAPM α	0.859	0.854	0.830	0.809	0.795	0.830
	(5.05)	(5.08)	(5.04)	(4.86)	(4.70)	(5.04)
Three Factor α	0.858	0.861	0.841	0.826	0.815	0.841
	(5.06)	(5.12)	(5.11)	(4.97)	(4.82)	(5.11)
Volatility	3.559	3.535	3.474	3.514	3.565	3.477
Sharpe Ratio	0.767	0.746	0.718	0.688	0.667	0.718
Kurtosis	2.875	2.137	1.769	1.799	1.942	1.769
Skewness	0.113	-0.011	-0.179	-0.176	-0.188	-0.179
Beta Ex Ante	0.00	0.00	0.00	0.00	0.00	0.00

Table 16 on the NYSE's Betting Against Beta (BAB) factor offers insightful revelations about the strategy's performance.

The BAB portfolio displays excess returns across all beta variants, with the highest returns noted in the Beta 1-3 and Beta 1-4 groups. Alphas are significantly positive. The BAB strategy's ability to produce substantial alphas confirms its potential for superior risk-adjusted performance.

The portfolio's Sharpe ratios, though positive, present a downward trend as we move from Beta 1-3 to Beta 1-7. The consistency in volatility across beta measures emphasizes the strategy's stable risk profile, while the kurtosis and skewness indicate a relatively normal distribution of returns with slight variations.

Figure 4: NYSE: Three Factor Alphas BAB.
This figure shows the Three Factor Alphas calculated on PL portfolios computed using each Beta.



Overall, the BAB strategy within the NYSE displays a noteworthy capability to generate excess returns while maintaining a stable risk profile. In this case the bigger the rolling window for the correlation, the less alpha is achieved, contrary to the S&P 500 results.

Table 17: NYSE: BAB Factor Long and Short Weights.
This table presents the weights on each part of the strategy. The Long (Buy) is the average of the long side of the strategy which is $1/\beta_L$. The Short Sell is the average of the short side of the strategy which is $1/\beta_H$. The borrowing is $(\beta_L^{-1} - \beta_H^{-1})$.

BAB	Beta 1-3	Beta 1-4	Beta 1-5	Beta 1-6	Beta 1-7	Beta 5-5	Average
Long (Buy)	1.73	1.72	1.73	1.73	1.73	1.73	1.73
Short (Sell)	0.97	0.98	0.99	0.99	0.99	0.99	0.98
Financed at Risk Free Rate	0.76	0.74	0.74	0.74	0.74	0.74	0.74

In average the BAB factor is long on $1.73 on low beta stocks, which is financed by short selling $1.73 risk-free securities, and short sells $0.98 of high beta-stocks, with $0.98 invested in the risk-free rate.

Chapter 5

5. Conclusion

Throughout the course of this dissertation, I have demonstrated that portfolios constructed with low beta stocks tend to outperform their high beta counterparts, providing superior risk-adjusted returns, a finding that resonates with the assertions of existing academic studies. Moreover, the low beta portfolios exhibited commendable performance in the context of the Sharpe ratio, when compared to high beta portfolios. This pattern of outperformance is consistent across the two markets that were the subject of this study.

In addition, this research has tried to shed a light of the conditions under which the Betting Against Beta (BAB) strategy is most effective. The evidence suggests that the BAB strategy performs the best in environments characterized by a rich diversity of stock choices, as seen in the New York Stock Exchange (NYSE). In contrast, the S&P500 which is composed of the United States' most actively traded stocks and represents a substantial portion of the US market capitalization, did not provide a good environment for the BAB strategy as this set has generally weaker anomalies.

Furthermore, the robustness of the BAB strategy was put to the test by varying the correlation windows used in beta calculations, stretching from three to seven years. The stability in the performance of the BAB strategy and the construction of percentile portfolios, regardless of the length of these correlation windows, underscores the solidity of the strategy. This suggests that the BAB strategy is not overly sensitive to changes in its underlying beta calculation methodology, a testament to its potential as a reliable investment approach.

In summary, my findings contribute a non-trivial amount of empirical evidence surrounding the BAB strategy, underlining its capability to generate significant alphas and risk-adjusted returns for investors. These insights offer a valuable resource for both academics and practitioners, emphasizing the practical benefits of low-beta investing. However, this study has limitations which can be evaluated in future research. The study only focuses on the US stock markets, more assets and markets can be used. Trading cost and higher cost of borrowing can be added to evaluate the robustness of the strategy. More windows used to estimate the volatility ratio component could be tested. More asset pricing models could be tested.

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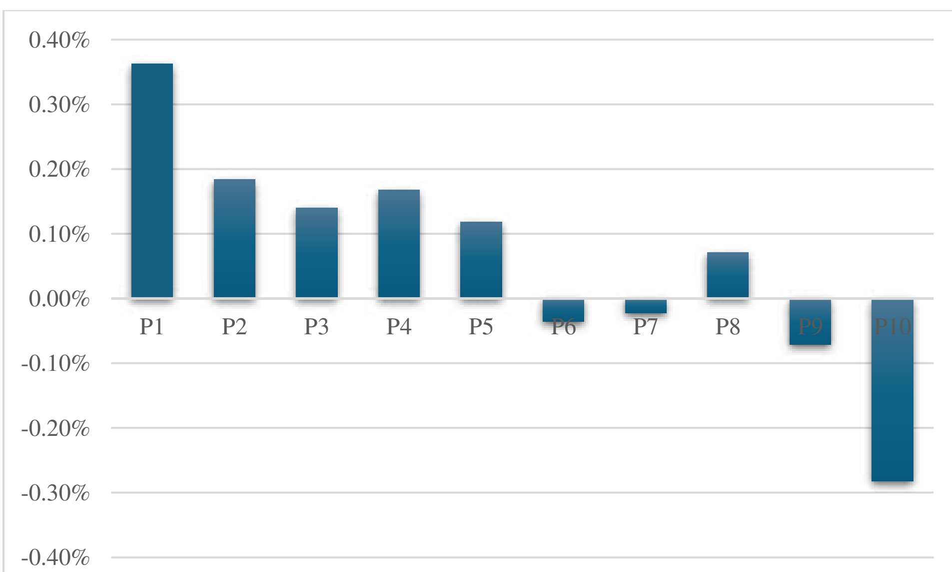
Appendix

Percentile Portfolios Results

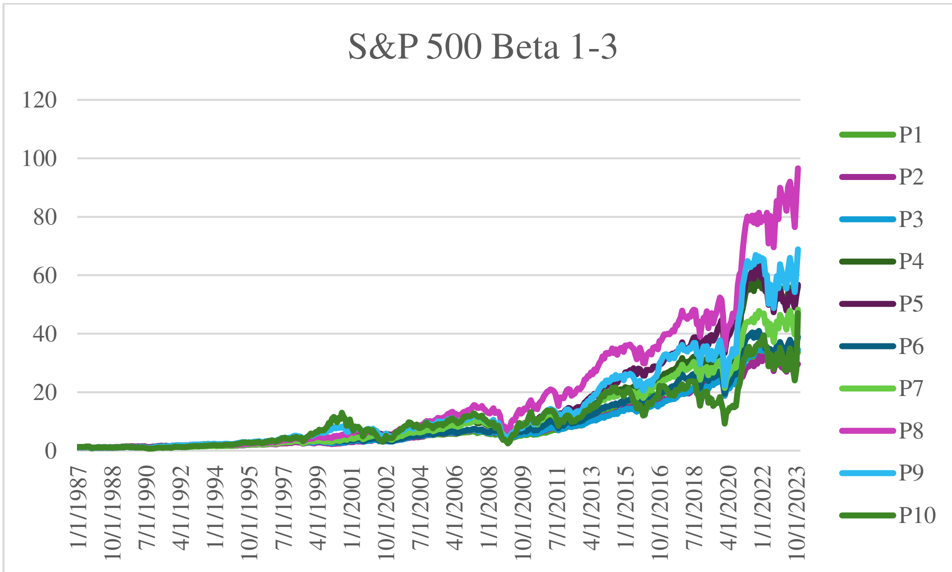
Appendix Table 1: S&P500: Beta portfolios statistics using Beta 1-3.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the S&P 500. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 1-3	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	0.86	0.83	0.89	1.02	1.03	0.97	1.04	1.22	1.22	1.41
CAPM α	**0.43**	**0.25**	**0.21**	**0.24**	0.19	0.05	0.06	0.16	-0.02	-0.25
	3.21	2.48	2.04	2.02	1.89	0.44	0.51	1.28	-0.12	-0.86
Three Factor α	**0.36**	**0.18**	0.14	0.17	0.12	-0.04	-0.02	0.07%	-0.07	-0.28
	2.86	1.99	1.50	1.61	1.31	-0.40	-0.24	0.70	-0.53	-1.11
Volatility	0.04	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.11
Sharpe Ratio	0.83	0.78	0.74	0.74	0.73	0.63	0.63	0.68	0.58	0.46
Kurtosis	5.22	3.80	3.97	4.84	2.66	2.76	2.84	2.66	3.86	5.35
Skewness	-0.69	-1.10	-0.93	-0.54	-0.49	-0.54	-0.49	-0.36	-0.08	0.82

Appendix Figure 1: S&P500: Three Factor Alphas Beta 1-3, Percentile Portfolios.
This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-3.



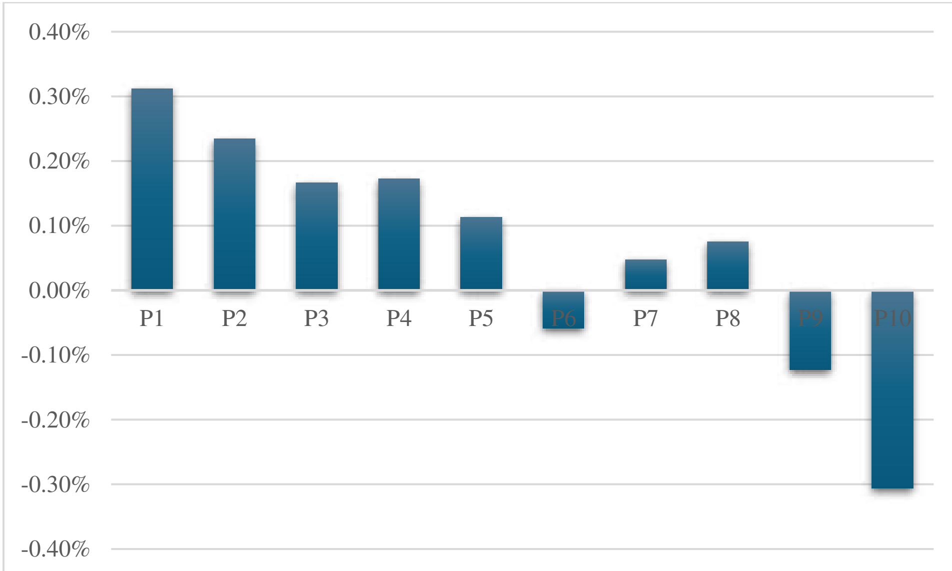
Appendix Figure 2: S&P500: Cumulative Excess Returns Beta 1-3, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-3.



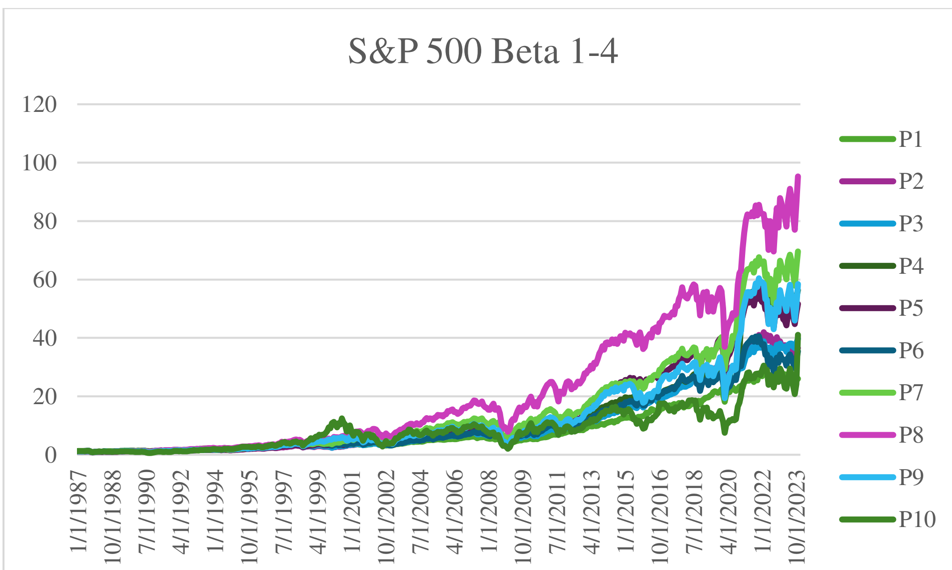
Appendix Table 2: S&P500: Beta portfolios statistics using Beta 1-4.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the S&P 500. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 1-4	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	0.79	0.88	0.92	1.02	1.01	0.95	1.12	1.22	1.19	1.39
CAPM α	**0.38**	**0.30**	**0.24**	**0.25**	0.18	0.03	0.13	0.16	-0.06	-0.27
	3.04	2.82	2.31	2.00	1.68	0.25	1.15	1.29	-0.42	-0.90
Three Factor α	**0.31**	**0.23**	0.17	0.17	0.11	-0.06	0.05	0.08	-0.12	-0.31
	2.67	2.40	1.81	1.54	1.20	-0.66	0.49	0.75	-0.93	-1.18
Volatility	0.03	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.11
Sharpe Ratio	0.81	0.81	0.76	0.73	0.71	0.61	0.68	0.69	0.56	0.45
Kurtosis	4.71	3.47	4.20	5.85	3.55	2.74	2.82	3.04	3.63	5.18
Skewness	-1.01	-0.86	-1.05	0.07	-0.59	-0.59	-0.39	-0.45	-0.08	0.80

Appendix Figure 3: S&P500: Three Factor Alphas Beta 1-4, Percentile Portfolios.
Figure 2 shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-4.



Appendix Figure 4: S&P500: Cumulative Excess Returns Beta 1-4, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-4.



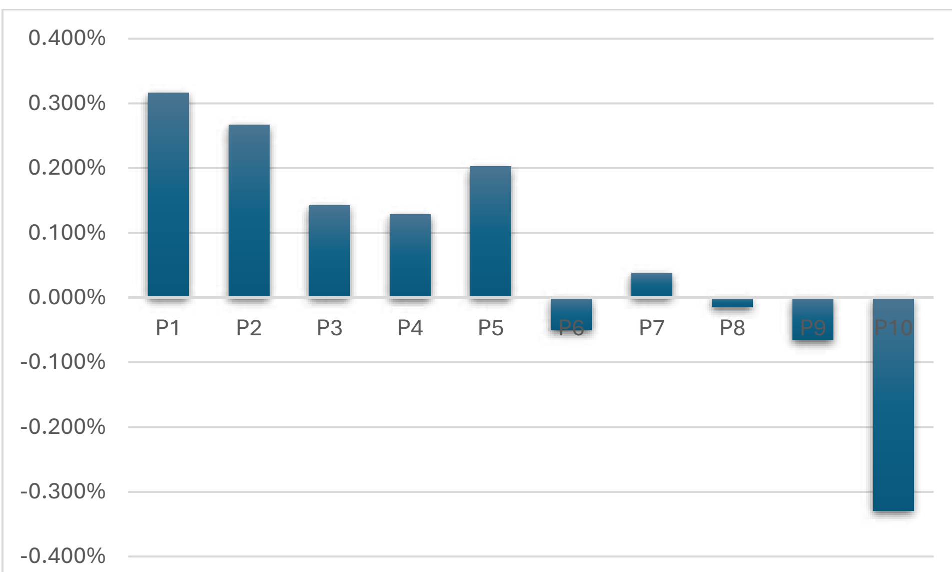
Appendix Table 3: S&P500: Beta portfolios statistics using Beta 1-5.

This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the S&P 500. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

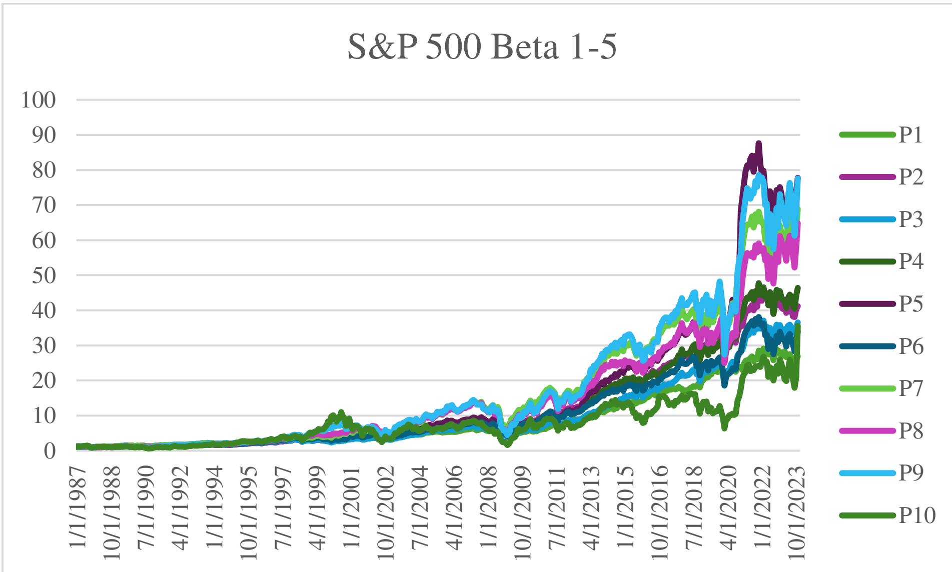
Portfolios Beta 1-5	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	0.80	0.91	0.90	0.97	1.12	0.94	1.12	1.13	1.26	1.36
CAPM α	**0.38**	**0.33**	**0.22**	**0.19**	**0.27**	0.03	0.13	0.07	0.00	-0.30
	3.10	3.25	1.99	2.13	2.14	0.27	1.08	0.55	0.00	-0.97
Three Factor α	**0.32**	**0.27**	0.14	0.13	0.20	-0.05	0.04	-0.01	-0.07	-0.33
	2.73	2.86	1.46	1.60	1.76	-0.55	0.39	-0.14	-0.50	-1.26
Volatility	0.03	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.11
Sharpe Ratio	0.82	0.85	0.73	0.74	0.75	0.61	0.67	0.63	0.59	0.44
Kurtosis	4.09	2.83	3.78	3.23	5.78	2.71	3.12	3.04	3.62	5.26
Skewness	-1.00	-0.80	-0.90	-0.74	-0.17	-0.59	-0.30	-0.44	-0.21	0.83

Appendix Figure 5: S&P500: Three Factor Alphas Beta 1-5, Percentile Portfolios.

Figure 3 shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-5.



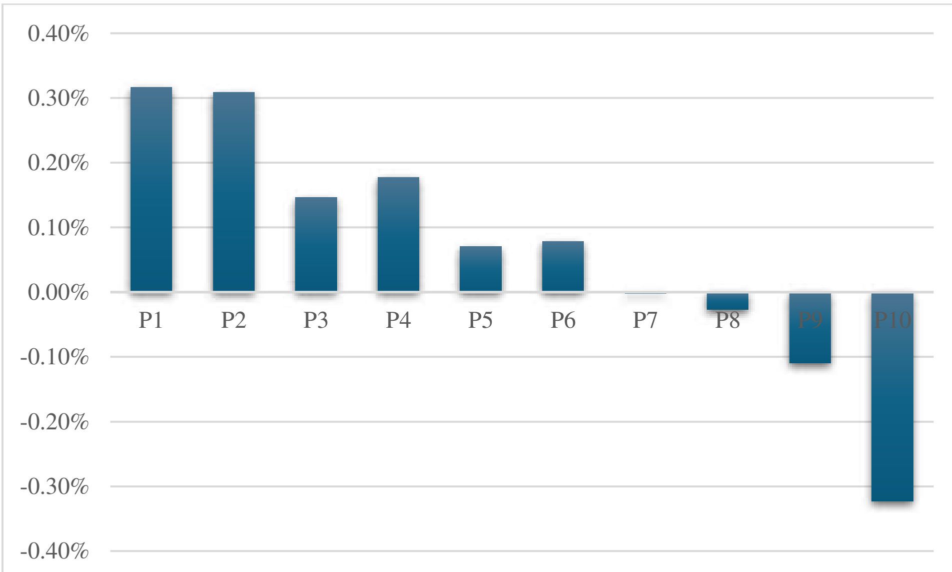
Appendix Figure 6: S&P500: Cumulative Excess Returns Beta 1-5, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-5.



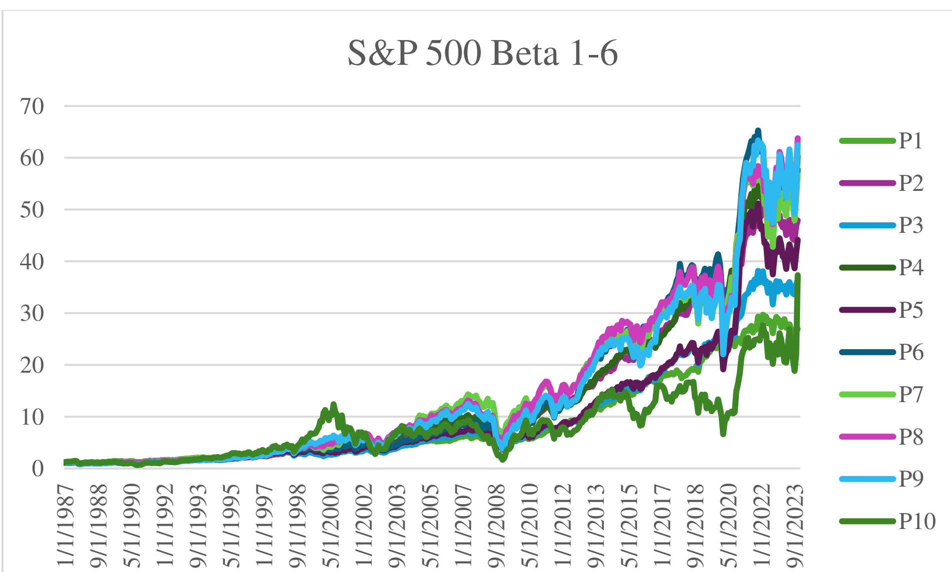
Appendix Table 4: S&P500: Beta portfolios statistics using Beta 1-6.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the S&P 500. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 1-6	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	0.80	0.94	0.90	1.02	0.98	1.07	1.08	1.14	1.20	1.38
CAPM α	**0.38**	**0.37**	**0.22**	**0.24**	0.14	0.16	0.09	0.06	-0.05	-0.28
	3.10	3.61	2.03	2.57	1.24	1.45	0.74	0.47	-0.32	-0.93
Three Factor α	**0.32**	**0.31**	0.15	**0.18**	0.07	0.08	-0.00	-0.03	-0.11	-0.32
	2.73	3.26	1.52	2.11	0.69	0.84	-0.02	-0.26	-0.85	-1.22
Volatility	0.03	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.11
Sharpe Ratio	0.82	0.88	0.74	0.78	0.68	0.70	0.65	0.63	0.57	0.44
Kurtosis	3.83	2.67	3.82	3.43	4.14	2.66	2.97	3.11	3.50	5.22
Skewness	-0.92	-0.84	-0.97	-0.73	-0.48	-0.52	-0.39	-0.42	-0.20	0.82

Appendix Figure 7: S&P500: Three Factor Alphas Beta 1-6, Percentile Portfolios.
This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-6.



Appendix Figure 8: S&P500: Cumulative Excess Returns Beta 1-6, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-6.



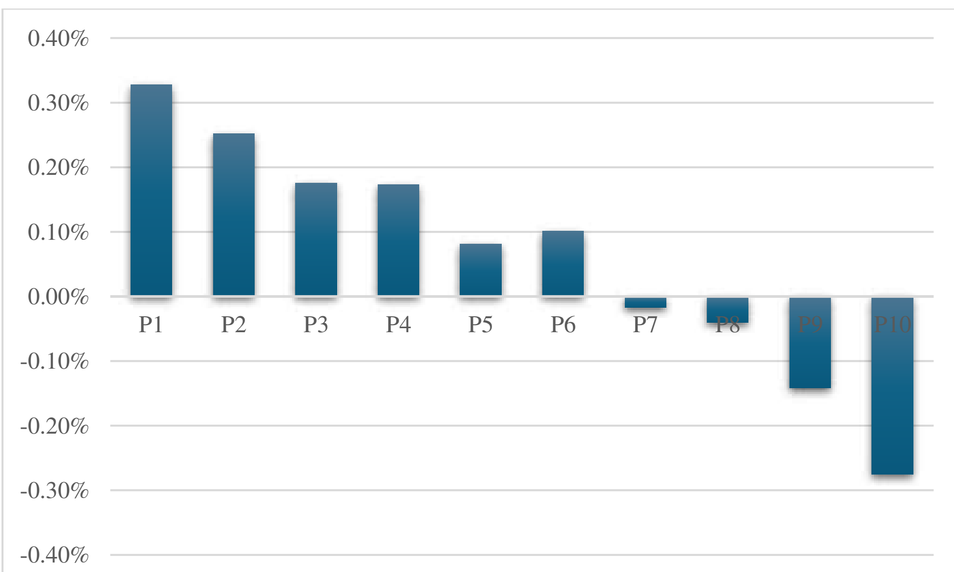
Appendix Table 5: S&P500: Beta portfolios statistics using Beta 1-7.

This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the S&P 500. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

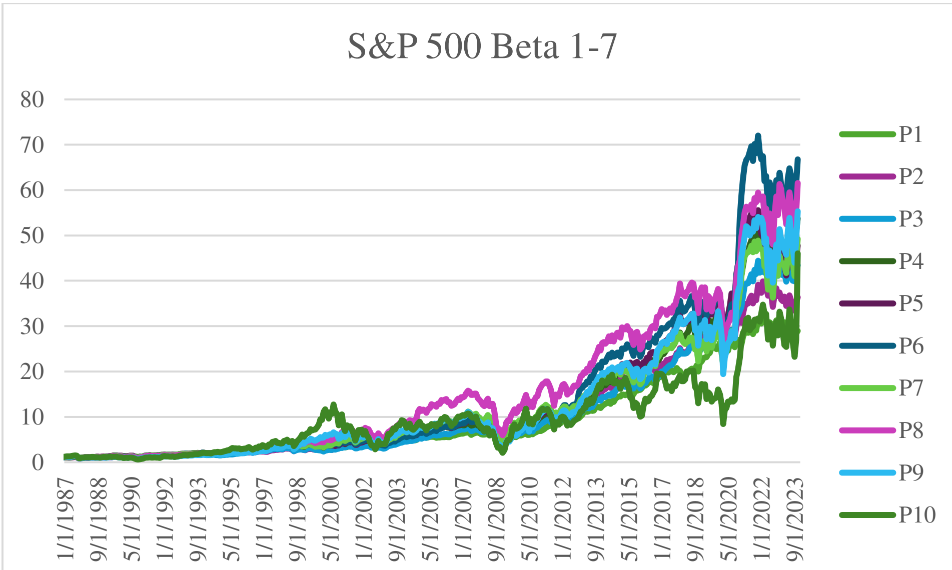
Portfolios Beta 1-7	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	0.82	0.88	0.94	1.00	0.99	1.09	1.04	1.13	1.18	1.43
CAPM α	**0.39**	**0.32**	**0.25**	**0.23**	0.15	0.19	0.07	0.04	-0.08	-0.24
	3.20	3.07	2.30	2.52	1.55	1.53	0.61	0.34	-0.49	-0.76
Three Factor α	**0.33**	**0.25**	**0.18**	**0.17**	0.08	0.10	-0.02	-0.04	-0.14	-0.28
	2.84	2.66	1.81	2.07	0.96	0.97	-0.19	-0.39	-1.09	-1.03
Volatility	0.03	0.04	0.04	0.04	0.05	0.05	0.06	0.06	0.07	0.11
Sharpe Ratio	0.84	0.83	0.76	0.77	0.70	0.70	0.64	0.62	0.56	0.46
Kurtosis	3.79	2.64	4.32	2.90	3.69	2.75	2.82	3.12	3.45	5.27
Skewness	-0.92	-0.83	-0.93	-0.71	-0.81	-0.33	-0.44	-0.40	-0.20	0.83

Appendix Figure 9: S&P500: Three Factor Alphas Beta 1-7, Percentile Portfolios.

This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-7.



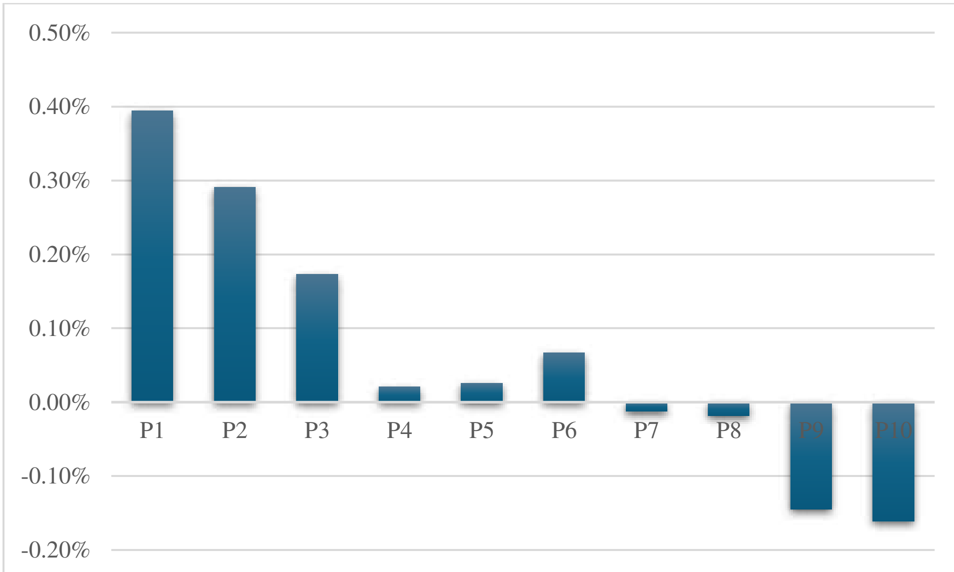
Appendix Figure 10: S&P500: Cumulative Excess Returns Beta 1-7, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-7.



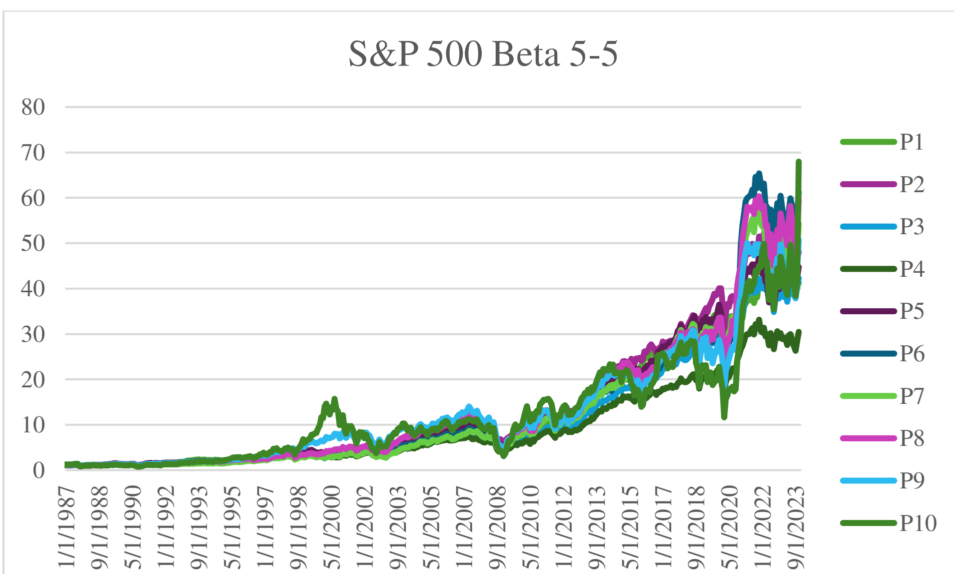
Appendix Table 6: S&P500: Beta portfolios statistics using Beta 5-5.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the S&P 500. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 5-5	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	0.90	0.95	0.93	0.88	0.99	1.08	1.07	1.11	1.13	1.45
CAPM α	**0.45**	**0.36**	**0.25**	0.09	0.12	0.15	0.08	0.06	-0.07	-0.15
	3.43	3.32	2.41	0.86	1.02	1.19	0.64	0.48	(0.50)	(0.57)
Three Factor α	**0.39**	**0.29**	0.17	0.02	0.03	0.07	-0.01	-0.02	-0.14	-0.16
	3.13	2.95	1.91	0.21	0.27	0.60	(0.13)	(0.19)	(1.23)	(0.69)
Volatility	0.04	0.04	0.04	0.05	0.05	0.06	0.06	0.06	0.07	0.10
Sharpe Ratio	0.86	0.86	0.77	0.65	0.66	0.67	0.64	0.63	0.56	0.50
Kurtosis	3.12	3.51	2.88	3.37	3.52	3.45	2.91	3.52	3.14	4.14
Skewness	(0.83)	(0.83)	(0.85)	(0.71)	(0.67)	(0.15)	(0.35)	(0.40)	(0.18)	0.55

Appendix Figure 11: S&P500: Three Factor Alphas Beta 5-5, Percentile Portfolios.
This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 5-5.



Appendix Figure 12: S&P500: Cumulative Excess Returns Beta 5-5, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 5-5.

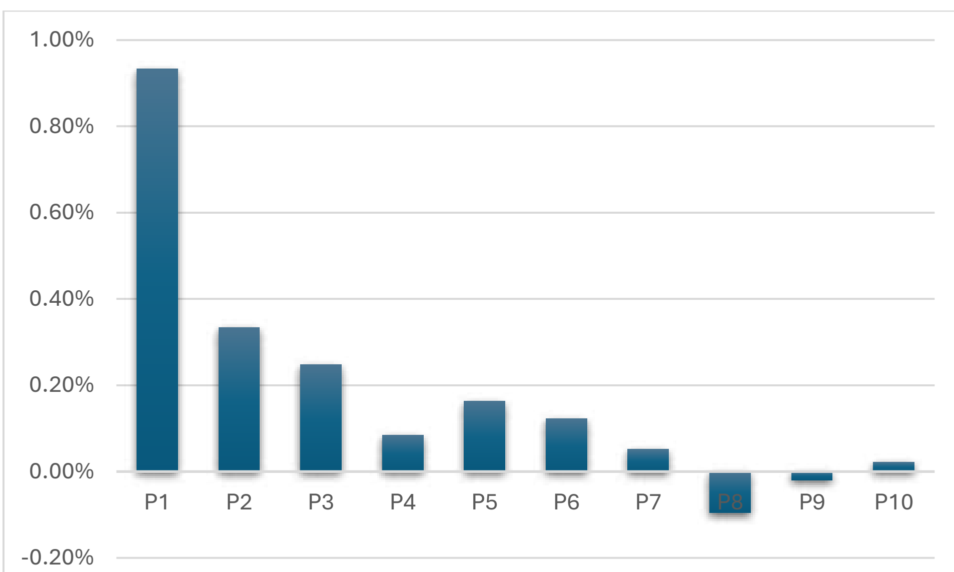


Appendix Table 7: NYSE: Beta portfolios statistics using Beta 1-3.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the NYSE. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

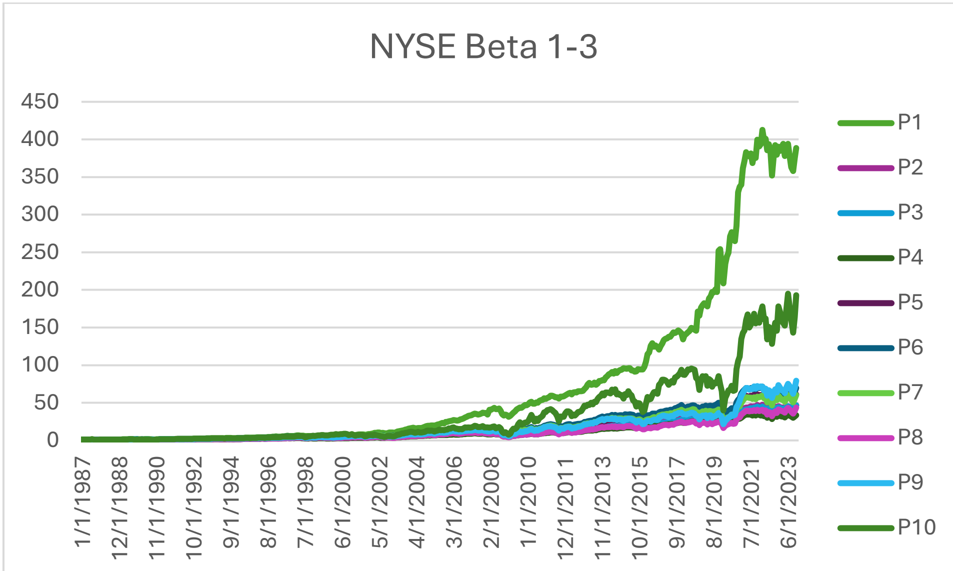
Portfolios Beta 1-3	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	1.42	0.93	0.95	0.90	1.05	1.10	1.08	1.04	1.21	1.58
CAPM α	**0.99**	**0.40**	**0.31**	0.16	0.25	0.22	0.15	0.02	0.10	0.14
	6.34	3.67	2.90	1.47	1.99	1.81	1.18	0.14	0.64	0.60
Three Factor α	**0.93**	**0.33**	**0.25**	0.08	0.16	0.12	0.05	-0.10	-0.02	0.02
	6.28	3.60	2.81	1.00	1.67	1.42	0.57	-0.93	-0.17	0.12
Volatility	3.96	3.61	4.03	4.49	4.97	5.28	5.59	6.13	6.74	8.96
Sharpe Ratio	1.25	0.90	0.82	0.69	0.73	0.72	0.67	0.59	0.62	0.61
Kurtosis	7.23	8.44	6.60	4.86	5.60	4.07	3.98	3.32	3.68	4.81
Skewness	0.49	-1.62	-1.14	-1.05	-0.41	-0.62	-0.62	-0.28	-0.21	0.43

On table 15 the portfolios on the first percentile, P1, have the higher alpha and Sharpe ratio but at the same time have the second highest excess kurtosis and exhibit positive skewness. The difference between P1 and the rest is more extreme than in the S&P 500.

Appendix Figure 13: NYSE: Three Factor Alphas Beta 1-3, Percentile Portfolios.
This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-3.



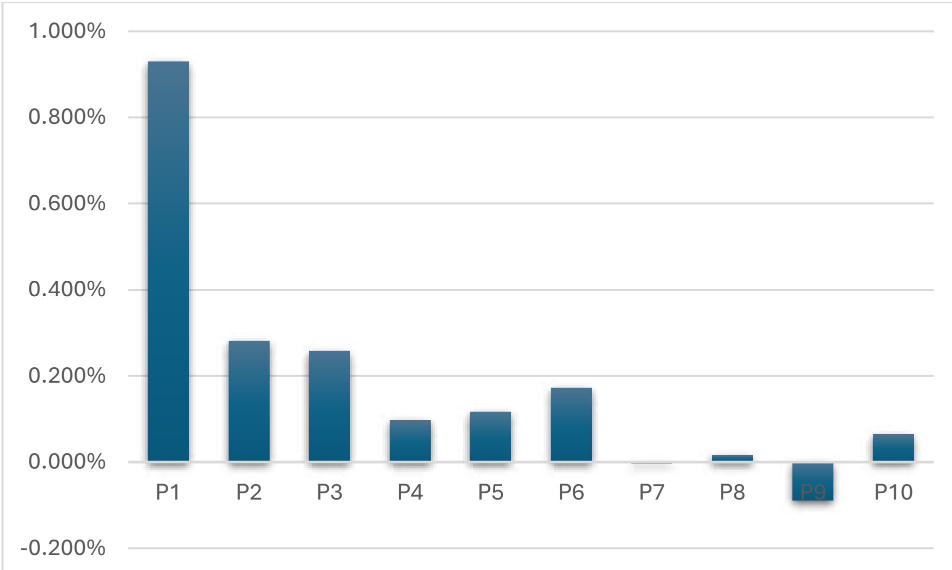
Appendix Figure 14: NYSE: Cumulative Excess Returns Beta 1-3, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-3.



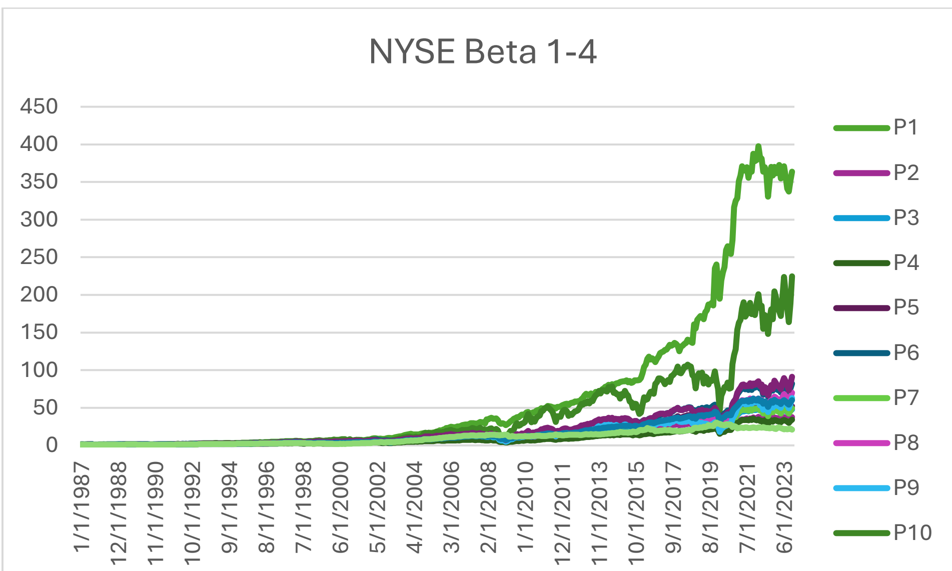
Appendix Table 8: NYSE: Beta portfolios statistics using Beta 1-4.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the NYSE. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 1-4	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	1.41	0.89	0.95	0.90	1.01	1.13	1.04	1.14	1.17	1.62
CAPM α	**0.98**	**0.35**	**0.32**	0.17	0.20	0.27	0.10	0.13	0.04	0.18
	6.33	3.19	3.13	1.50	1.81	2.14	0.74	0.98	0.23	0.77
Three Factor α	**0.93**	**0.28**	**0.26**	0.10	0.12	0.17	-0.00	0.02	-0.09	0.07
	6.27	2.97	3.11	1.12	1.41	1.90	-0.03	0.16	-0.78	0.34
Volatility	0.04	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.09
Sharpe Ratio	1.24	0.84	0.83	0.69	0.72	0.75	0.63	0.66	0.59	0.62
Kurtosis	6.37	8.22	5.24	5.91	4.03	3.43	4.17	3.48	3.65	4.74
Skewness	0.66	-1.55	-1.14	-1.15	-0.83	-0.57	-0.54	-0.41	-0.23	0.54

Appendix Figure 15: NYSE: Three Factor Alphas Beta 1-4, Percentile Portfolios.
This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-4.



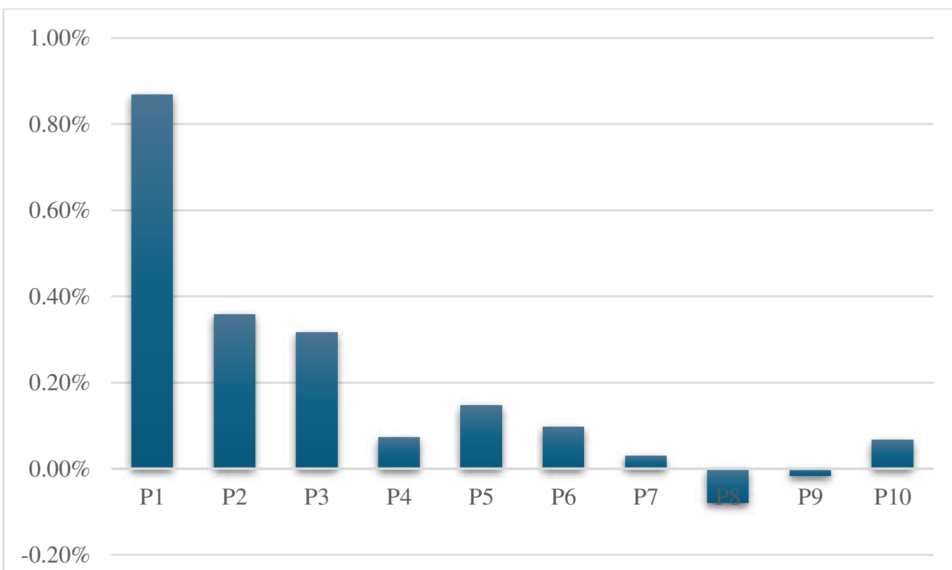
Appendix Figure 16: NYSE: Cumulative Excess Returns Beta 1-4, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-4.



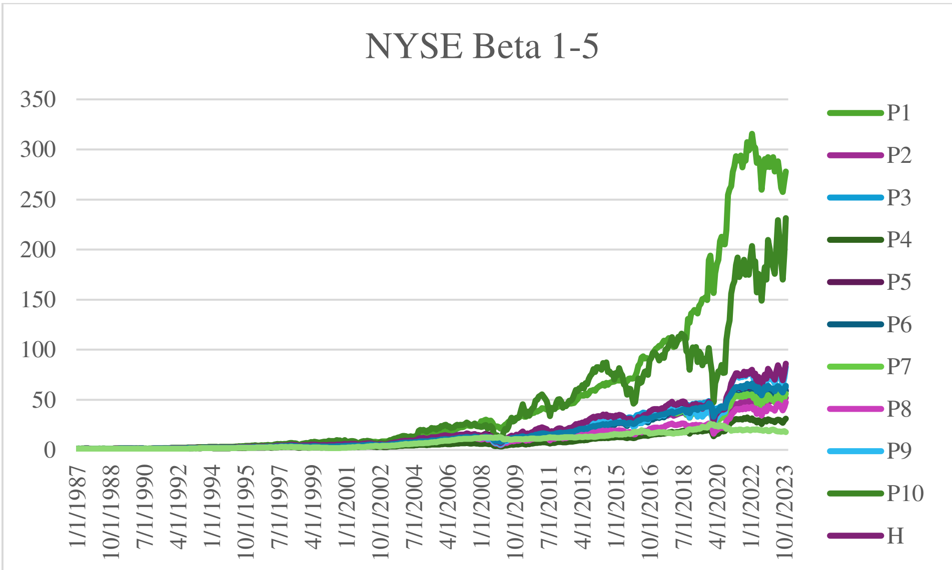
Appendix Table 9: NYSE: Beta portfolios statistics using Beta 1-5.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the NYSE. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 1-5	P1 (Low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (High beta)
Excess returns	1.34	0.96	1.01	0.88	1.05	1.05	1.07	1.06	1.22	1.64
CAPM α	**0.92**	**0.42**	**0.38**	0.15	**0.23**	0.19	0.13	0.04	0.11	0.19
	5.97	3.91	3.51	1.32	1.96	1.61	0.99	0.27	0.70	0.79
Three Factor α	**0.87**	**0.36**	**0.32**	0.07	0.15	0.10	0.03	-0.08	-0.02	0.07
	5.90	3.82	3.53	0.86	1.61	1.12	0.31	-0.81	-0.15	0.34
Volatility	0.04	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.09
Sharpe Ratio	1.19	0.92	0.87	0.68	0.73	0.70	0.65	0.60	0.63	0.62
Kurtosis	6.91	6.95	5.13	4.64	4.52	3.41	4.92	3.88	3.27	4.72
Skewness	0.60	-1.41	-0.90	-0.92	-0.92	-0.56	-0.54	-0.45	-0.17	0.50

Appendix Figure 17: NYSE: Three Factor Alphas Beta 1-5, Percentile Portfolios.
This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-5.



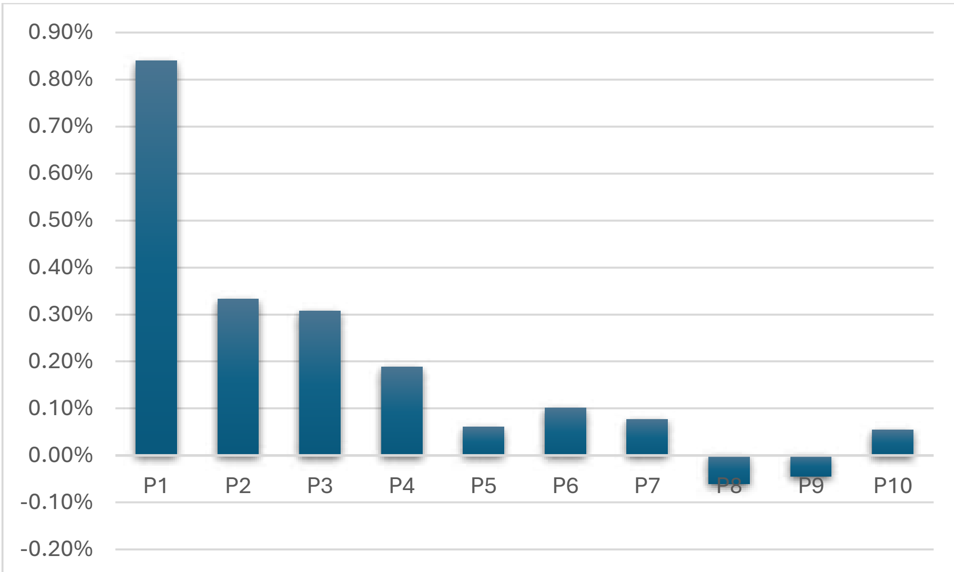
Appendix Figure 18: NYSE: Cumulative Excess Returns Beta 1-5, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-5.



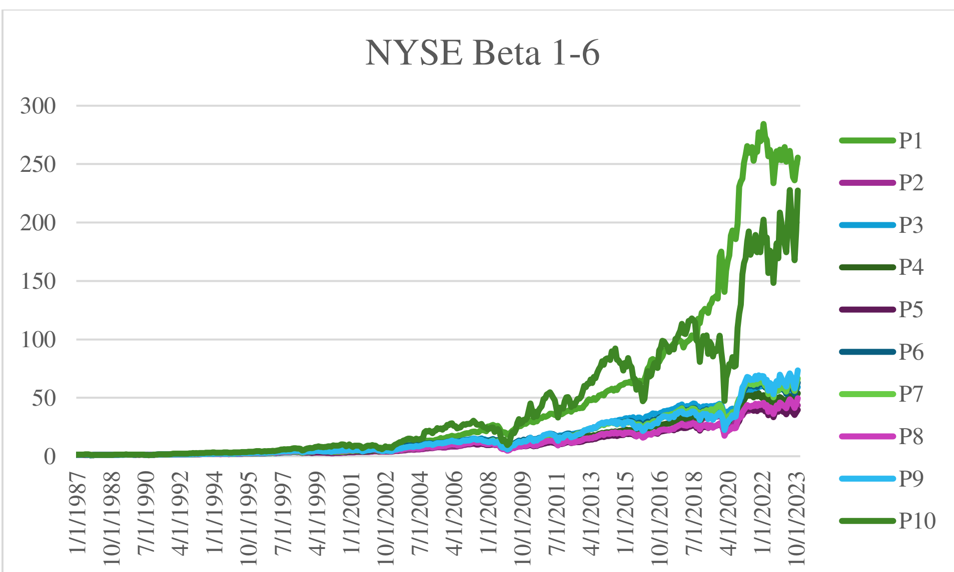
Appendix Table 10: NYSE: Beta portfolios statistics using Beta 1-6.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the NYSE. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 1-6	P1 (Low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (High beta)
Excess returns	1.33	0.92	1.00	1.00	0.95	1.08	1.11	1.06	1.20	1.64
CAPM α	**0.89**	**0.39**	**0.37**	**0.26**	0.15	0.20	0.18	0.05	0.08	0.19
	5.83	3.60	3.40	2.47	1.23	1.63	1.37	0.39	0.51	0.77
Three Factor α	**0.84**	**0.33**	**0.31**	**0.19**	0.06	0.10	0.08	-0.06	-0.04	0.05
	5.75	3.45	3.42	2.32	0.67	1.17	0.83	-0.62)	-0.41)	0.28
Volatility	0.04	0.04	0.04	0.04	0.05	0.05	0.06	0.06	0.07	0.09
Sharpe Ratio	1.17	0.89	0.86	0.77	0.67	0.71	0.68	0.61	0.62	0.62
Kurtosis	7.22	7.57	4.78	4.37	4.04	3.76	4.80	3.31	3.48	4.96
Skewness	0.62	-1.60	-0.88	-0.94	-0.82	-0.64	-0.54	-0.38	-0.27	0.51

Appendix Figure 19: NYSE: Three Factor Alphas Beta 1-6, Percentile Portfolios.
This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-6.



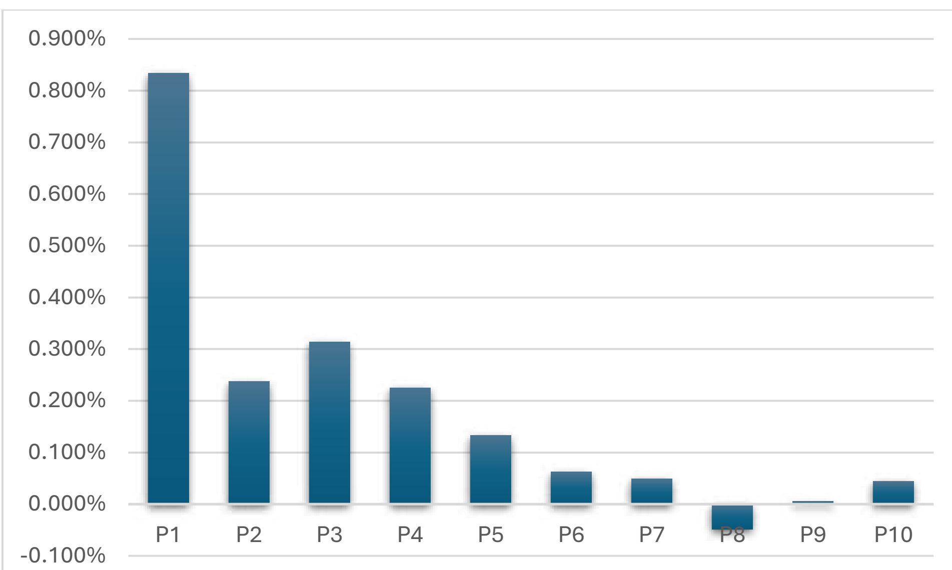
Appendix Figure 20: NYSE: Cumulative Excess Returns Beta 1-6, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-6.



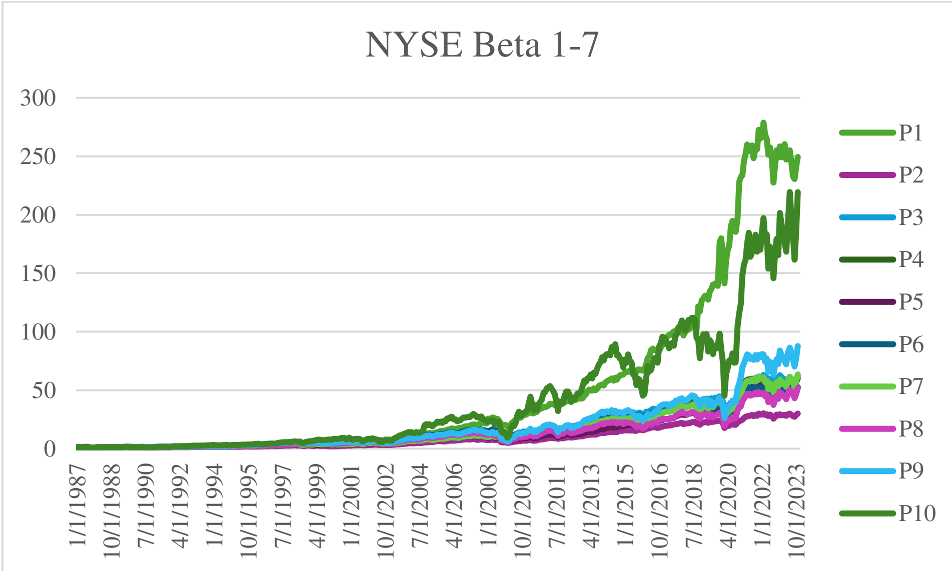
Appendix Table 11: NYSE: Beta portfolios statistics using Beta 1-7.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the NYSE. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 1-7	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	1.32	0.83	1.01	1.02	1.01	1.03	1.10	1.07	1.23	1.63
CAPM α	**0.87**	**0.30**	**0.37**	**0.29**	0.22	0.16	0.14	0.07	0.12	0.18
	5.71	2.83	3.50	2.73	1.86	1.32	1.16	0.53	0.80	0.74
Three Factor α	**0.83**	**0.24**	**0.31**	**0.23**	0.13	0.06	0.05	-0.05	0.01	0.04
	5.67	2.54	3.61	2.66	1.50	0.73	0.55	-0.47	0.05	0.22
Volatility	0.04	0.04	0.04	0.04	0.05	0.05	0.06	0.06	0.07	0.09
Sharpe Ratio	1.16	0.81	0.87	0.80	0.72	0.68	0.67	0.62	0.64	0.61
Kurtosis	7.57	6.79	5.45	3.55	4.55	3.50	4.41	3.49	3.45	5.09
Skewness	0.54	-1.52	-1.11	-0.62	-0.92	-0.67	-0.53	-0.32	-0.26	0.51

Appendix Figure 21: NYSE: Three Factor Alphas Beta 1-7, Percentile Portfolios.
Figure 20 shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 1-7.



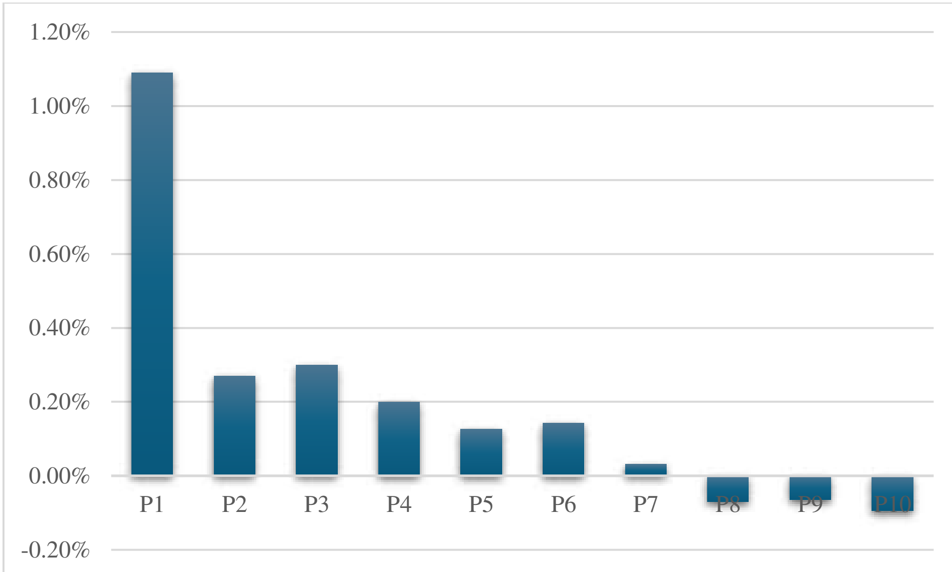
Appendix Figure 22: NYSE: Cumulative Excess Returns Beta 1-7, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 1-7.



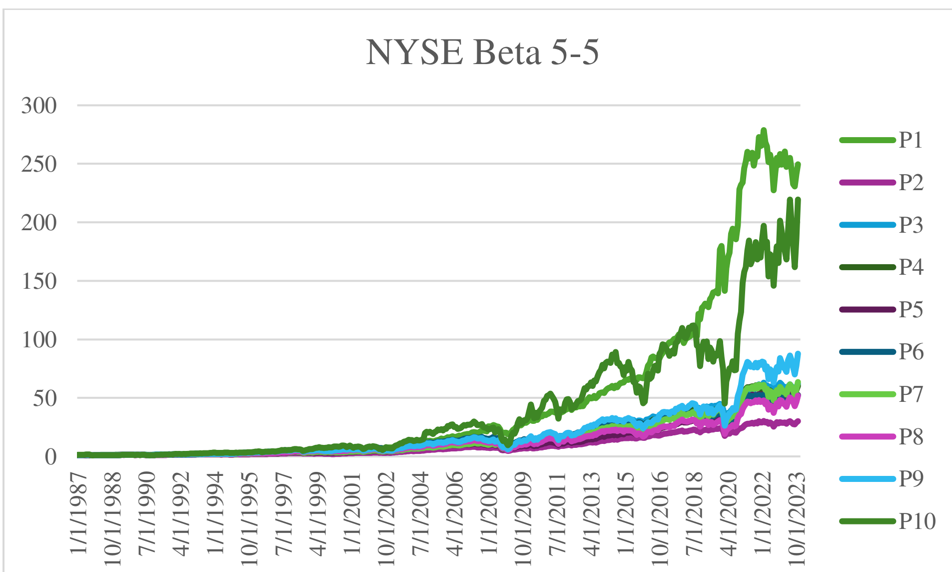
Appendix Table 12: NYSE: Beta portfolios statistics using Beta 5-5.
This table shows the excess returns, the alphas with their respective t-stats, volatility, Sharpe ratio, Kurtosis (Which is Excess Kurtosis) and Skewness of each portfolio formed on the NYSE. Excess returns and alphas are in percentage. All the metrics are monthly, except Sharpe Ratio which is annualized.

Portfolios Beta 5-5	**P1 (Low beta)**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**	**P8**	**P9**	**P10 (High beta)**
Excess returns	1.59	0.91	1.01	1.01	1.01	1.15	1.08	1.08	1.16	1.36
CAPM α	**1.12**	**0.33**	**0.36**	**0.27**	0.22	0.25	0.13	0.05	0.06	0.03
	6.98	2.80	3.30	2.55	1.82	1.72	0.98	0.37	0.39	0.13
Three Factor α	**1.09**	**0.27**	**0.30**	**0.20**	0.13	0.14	0.03	-0.07	-0.06	-0.09
	7.16	2.61	3.38	2.43	1.45	1.26	0.33	-0.70	-0.56	-0.61
Volatility	0.04	0.04	0.04	0.04	0.05	0.06	0.06	0.06	0.07	0.08
Sharpe Ratio	1.33	0.80	0.85	0.78	0.72	0.71	0.66	0.61	0.60	0.58
Kurtosis	6.78	6.95	3.28	4.80	3.84	5.65	4.60	4.27	4.36	4.46
Skewness	0.39	-1.43	-0.84	-0.95	-0.62	-0.33	-0.32	-0.44	-0.21	0.22

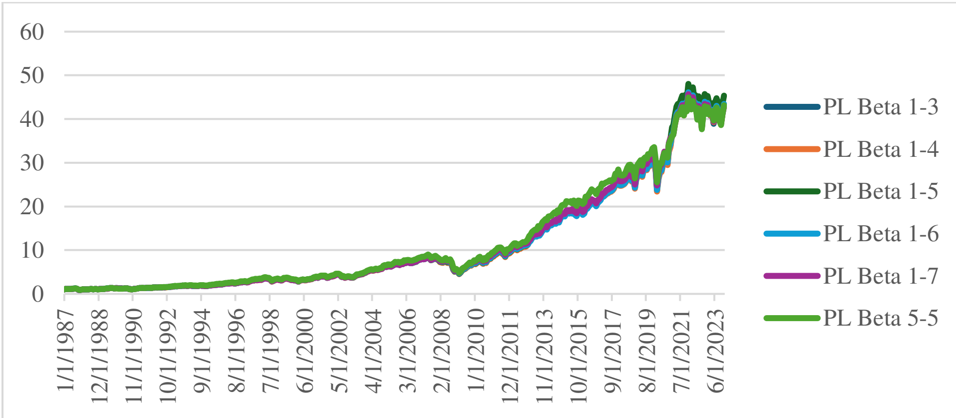
Appendix Figure 23: NYSE: Three Factor Alphas Beta 5-5, Percentile Portfolios.
This figure shows the Three Factor Alphas calculated on P1 to P10 portfolios computed using Beta 5-5.



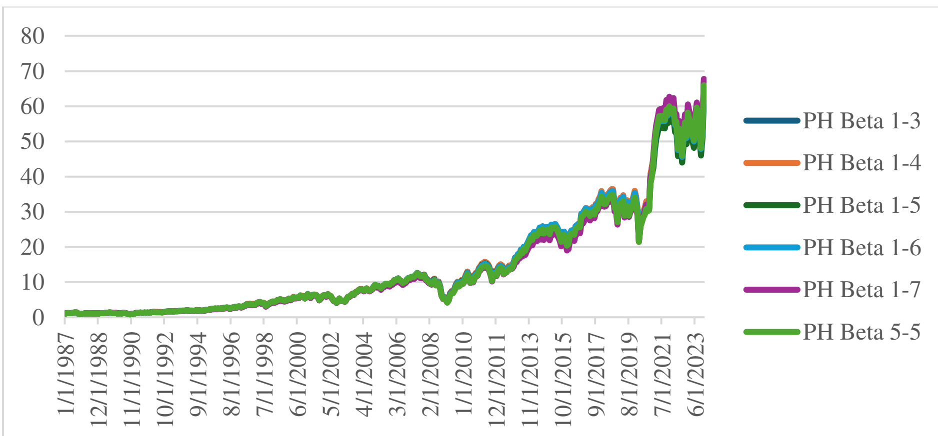
Appendix Figure 24: NYSE: Cumulative Excess Returns Beta 5-5, Percentile Portfolios.
This figure shows the Cumulative Excess Returns calculated on P1 to P10 portfolios computed using Beta 5-5.



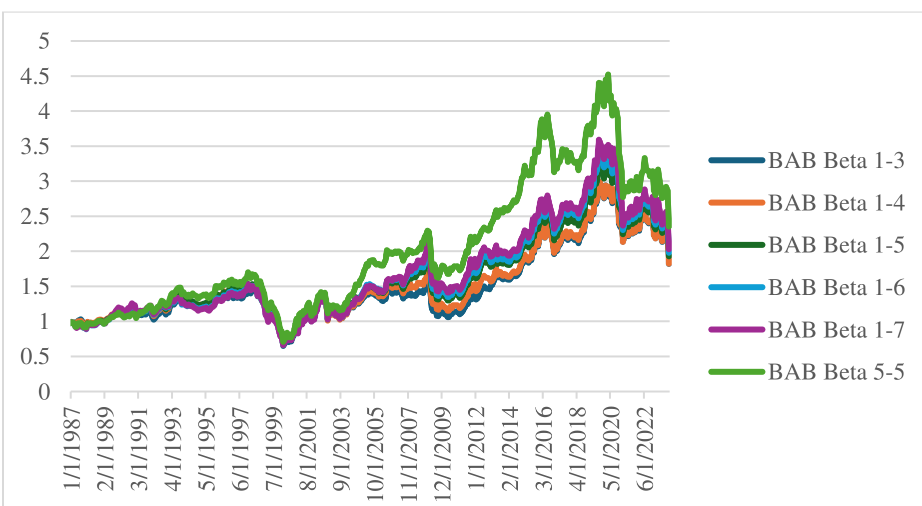
Appendix Figure 25: S&P500: Cumulative Excess Returns PL.
This figure shows the Cumulative Excess Returns calculated on PL portfolios.



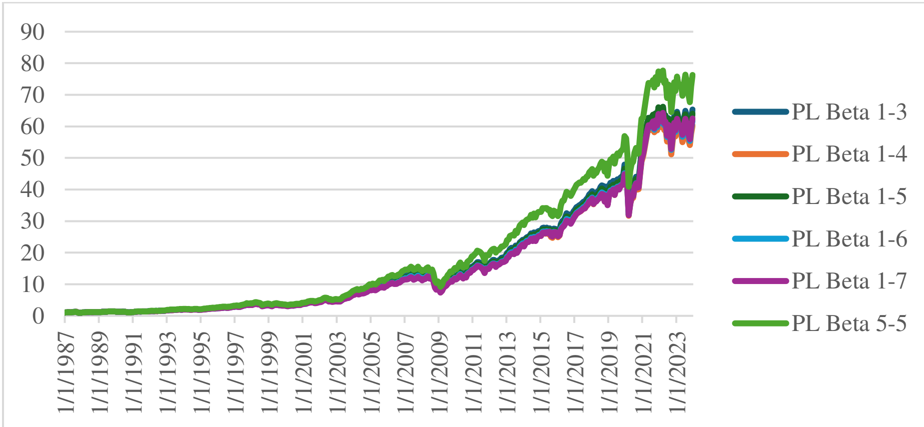
Appendix Figure 26: S&P500: Cumulative Excess Returns PH.
This figure shows the Cumulative Excess Returns calculated on PH portfolios.



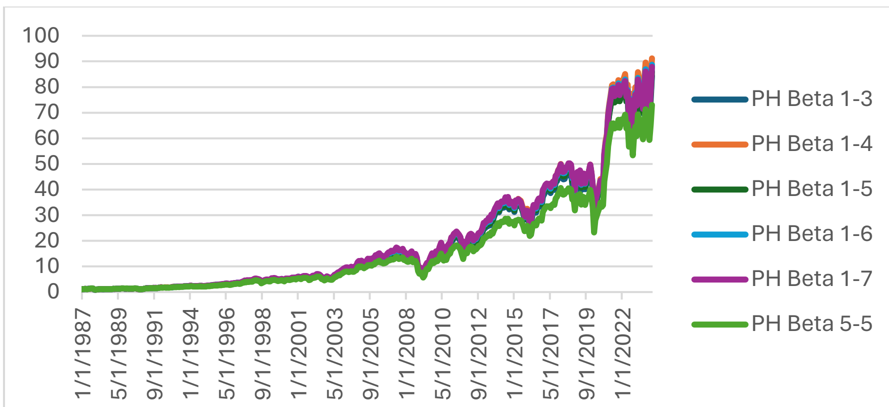
Appendix Figure 27: S&P500: Cumulative Excess Returns BAB.
This figure shows the Cumulative Excess Returns calculated on BAB portfolios.



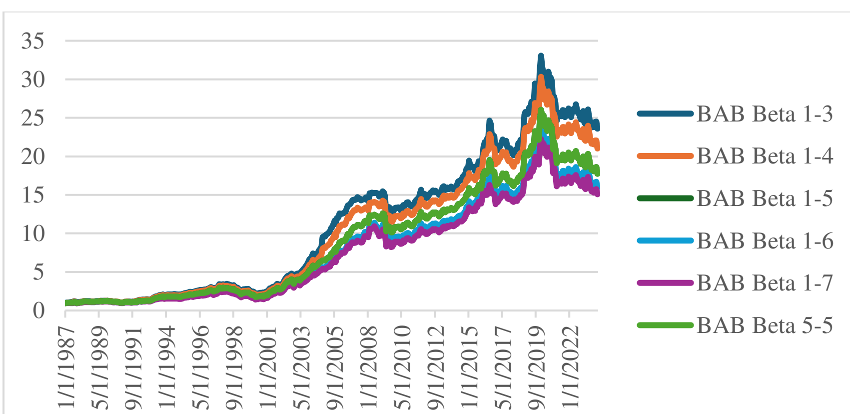
Appendix Figure 28: NYSE: Cumulative Excess Returns PL.
This figure shows the Cumulative Excess Returns calculated on PL portfolios.



Appendix Figure 29: NYSE: Cumulative Excess Returns PH.
This figure shows the Cumulative Excess Returns calculated on PH portfolios.



Appendix Figure 30: NYSE: Cumulative Excess Returns BAB.
This figure shows the Cumulative Excess Returns calculated on BAB portfolios.



CAPM Alpha

For the CAPM alpha of each portfolio I made the following regression and got the intercept using monthly data.

$$E(r_t^p - rf) = \propto + \beta_{MKT}(MKT)$$

Where $MKT = E(CRSP\ VW - rf)$.

Three-Factor Alpha Fama-French

For the three-factor alpha, I expanded the regression by adding SMB (small minus big) and HML (High minus Low).

$$E\left(r_t^p - rf\right) = \propto + \beta_{MKT}MKT + \beta_{SMB}SMB + \beta_{HML}HML$$

To test the statistical significance of the alphas calculated I added the t-statistic.

$$t - stat = \frac{\alpha}{s/\sqrt{n}}$$

Excess kurtosis:

$$Excess\ Kurtosis = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum\left(\frac{x_j - \bar{x}}{s}\right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)}$$

Skewness:

$$\text{Skewness} = \frac{\text{n}}{(n-1)(n-2)}\sum\left(\frac{x_j - \bar{x}}{s}\right)^3$$