Backward Partial Cross-ownership and Upstream R&D

Pedro Ruivo

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Supervisor: Professor Duarte Brito

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Abstract

In this article, we investigate the welfare effects of vertical backward ownership in vertical markets with upstream cost-reducing RD investments, emphasizing the role of input price discrimination. We first examine how backward partial cross- ownerships among firms influences welfare outcomes under uniform input pricing and input price discrimination. Under uniform input pricing, we replicate the in- variance result of Greenlee and Raskovich (2006) but demonstrate that this in- variance disappears when investment is incorporated. In this case, vertical partial cross-ownerships consistently enhance welfare. Under input price discrimination, the results are more nuanced: increasing cross ownership may either improve or reduce welfare. Finally, we take the ownership structure as given and study the welfare effects of input price discrimination. We find that transitioning from uniform input pricing to input price discrimination can have both positive and negative welfare implications, depending on specific parameter configurations.

Keywords: Partial Cross Ownership=PCO, Input Price Discrimination=IPD, Uniform Input Pricing=UIP, Research and Development=R&D

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Resumo

Este artigo investiga os efeitos sobre o bem-estar de participações verticais no âmbito de mercados verticais com investimentos em ID de redução de custos, colocando ênfase especial no papel da discriminação de preços dos inputs. Inicialmente, analisamos como as propriedades cruzadas parciais entre empresas influenciam o bem-estar sob uma política de preços uniforme assim como sob discriminação de preços. Sob preços uniformes, é replicado o resultado invariante de Greenlee and Raskovich (2006) e demonstrado que, quando o investimento é introduzido no modelo, este resultado deixa de se verificar e concluímos que participações cruzadas parciais verticais aumentam sempre o bem-estar. Sob discriminação de preços, os resultados tornam-se ambíguos pelo que um aumento no grau de participação cruzada não aumenta sempre o bem-estar, podendo até reduzir o mesmo. Finalmente, tomamos a estrutura de propriedade como dada e estudamos os efeitos da discriminação de preços no bem-estar. Descobrimos que passar de uma política de preços uniformes para uma política de discriminação de preços tanto pode influenciar o bem-estar positivamente como negativamente, dependendo de uma série de parâmetros.

Palavras-chave: Partial Cross Ownership=PCO, Input Price Discrimination=IPD, Uniform Input Pricing=UIP, Research and Development=R&D

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Contents

1 Introduction

The effects of partial cross ownership (PCO), both vertical and horizontal, have been explored extensively by academics. Among them, Farrel and Shapiro (1990) and Greenlee and Raskovich (2006) show that horizontal and vertical PCO may have welfare-increasing effects, respectively.

However, when considering cost-reduction investments, the body of research becomes more limited despite rising interest in recent years. For non-vertical industries, López and Vives (2019) considered cost-reducing R&D investments in a Cournot oligopoly with cross-ownerships, having found that the welfare and consumer surplus effects are ambiguous, depending on the scale of R&D spillovers. Also regarding the effects of horizontal PCO on innovation, Shelegia and Spiegel (2024) find that PCO among rival firms may harm or benefit consumers depending on a variety of parameters. For vertical industries, Hu et al (2022) considered a vertical market where the upstream firm engages in cost-reducing R&D, having found that total surplus may increase with horizontal PCO as long as the upstream R&D investment is inefficient. Jin et al (2024) added to this paper, having found that if a different definition of downstream producer surplus was used, PCO could have detrimental effects on consumer surplus and social welfare. Nevertheless, these articles analyzed horizontal ownership structures. An exception is Hunold and Shekhar (2022) who shows that non-controlling partial backward ownership can encourage costless innovation, increasing efficiency, industry profitability and consumer welfare.

This thesis aims to examine the effects of vertical backward ownership in a setting where upstream cost-reduction R&D investments are considered. Particular emphasis is placed on the role of input price discrimination (IPD), which is motivated by the possible existence of asymmetric ownership.

We model the downstream market as a homogenous product industry, which must purchase an input from an upstream supplier engaging in cost-reducing R&D. Both downstream firms are assumed to own a share of the upstream firm, receiving a percentage of its profits, but not acquiring any form of control rights. The interaction between the three firms is structured as a two-stage game: in the first stage, the upstream firm determines the input price and its investment level and may or may not be able to discriminate between its customers; in the second stage, the downstream firms compete à la Cournot. To analyze this setup, we considered two approaches: one assuming uniform input pricing

(UIP), and another allowing for IPD.

With UIP, we replicate the invariance result identified by Greenlee and Raskovich (2006) in scenarios that exclude R&D investments. Furthermore, we extend their findings by demonstrating that when investment levels are accounted for, vertical PCO has a positive impact on welfare. In contrast, the implications of IPD are less straightforward. Lestage (2021) analyzed the effects of IPD in a similar set up, but without considering the possibility of the input producer investing in order to reduce its marginal cost, and found that welfare is maximized when the ownership structure is symmetric, both under UIP and IPD. We show that, when investment is considered, this result is no longer the same.

To conclude our analysis, we glance over to the effects of IPD on welfare, taking the ownership structure as given. Lestage (2021) previously showed that IPD, which is encouraged by asymmetric vertical PCO, could increase or decrease welfare through a combination of two effects: the consumption reallocation effect and the output effect. In our analysis, the introduction of cost-reducing investments reveals a similar final result, with IPD impacting welfare positively or negatively, but through a new combination of effects: the investment effect and the output effect.

The remainder of this dissertation is organized as follows. Section 2 reviews the relevant literature. In Section 3 we present the model, with Sections 3.1 and 3.2 analyzing its implications under UIP and IPD, respectively. In Section 4 we study the welfare effects of IPD. Finally, Section 5 concludes. All proofs are in the Appendix.

2 Literature Review

This study mainly contributes to the literature on the effects of PCO. On this literature, there are two main branches of analysis: vertical and horizontal partial cross ownerships. Regarding the latter, its effects have been deeply examined, and the literature has set that partial horizontal ownership structures unambiguously soften competition (O'Brien and Salop (1999), Reynolds and Snapp (1986)). This softening occurs as horizontal PCO enables firms to internalize some portion of their rivals' profits, thereby reducing their incentives to compete aggressively. In specific settings, partial horizontal ownerships may even "lessen competition more than a monopoly" (Brito, Ribeiro, and

Vasconcelos, 2019).

However, when considering a horizontal ownership structure with cost-reducing R&D investments, the literature is fairly recent, with the results differing from the previous. López and Vives (2019) considers cost-reducing R&D investments with spillovers in a Cournot oligopoly with PCO, aiming to answer "How do R&D and output levels vary with the degree of internalization of rivals' profits?". As such, they show that, for demands not excessively convex, increases in PCO increase R&D and output for high enough spillovers, being possible that they improve consumer surplus and welfare, while they increase R&D but decrease output for intermediate levels of spillovers. Despite analyzing a different market structure, we find similar results regarding the investment level, with R&D investments increasing with increases in PCO in both our strands of analysis. With no spillovers and under Bertrand competition, Shelegia and Spiegel (2024) show that the less aggressive price competition that results from PCO may promote innovation. They identify two effects of opposite signs: the price effect that increases the benefits from cost-reducing investments and the cannibalization effect that, contrary to the case of spillovers, lowers the incentive to invest as this hurts the rival firm.

Developing on the effects of horizontal PCO when considering cost-reducing investments, Hu et al (2022) considered a vertical market with two downstream firms engaging in crossholding and an upstream firm engaging in cost-reducing R&D. Assuming that the downstream firms are symmetric, they find that downstream crossholdings weaken downstream competition, decreasing the investment of the upstream firm and increasing the input price, resulting in a negative effect on the downstream firms' profits. However, if the investments are inefficient, it is possible that total surplus increases with crossholdings despite consumer surplus always decreasing.

With regard to vertical PCO, the results tend to be more ambiguous. Vertical ownership interests can be viewed as tactics by which downstream firms attempt to capture share by lowering their own costs and "raising rivals' costs" (Salop and Scheffman (1983)). Regarding vertical PCO, these tend to lower the acquirer's effective marginal cost since the downstream holder has a claim on upstream profits, changing both the upstream and downstream decisions. Greenlee and Raskovich (2006), focusing on minority ownership interests which do not allow for any presence of control, analyze a model in which they incorporate the downstream firm decisions to acquire ownership in their respective supplier. They find that producer surplus and total surplus are invariant across all ownership profiles in various homogeneous Cournot settings with constant and symmetric costs. In Section 3.1, we add to this literature by introducing upstream cost-reducing R&D investments into the analysis and we show that when upstream investment is considered, this invariance result no longer holds, with consumer surplus and welfare increasing with the level of partial vertical ownership.

Regarding innovation in the presence of vertical PCO, Hunold and Shekhar (2022) addressed this in a very specific setting. In their model, downstream firms with no PCO prefer to (costlessly) help an inefficient supplier lower its costs in order to increase their outside option and obtain better conditions in the contract with their supplier. The authors discuss how partial ownership of the upstream efficient supplier by the downstream firms can change this outcome, leading also to a cost reduction of the efficient supplier, with higher industry profits and consumer benefits.

An important aspect of asymmetric non-controlling vertical shareholding is that it is likely to be a foundation for input price discrimination since the influence of each firm on the supplier's decisions will vary depending on their respective degree of shareholding, and this introduces a new question regarding the analysis of vertical PCO, which is the introduction of input price discrimination (IPD) into the discussion.

Price discrimination in economic theory has long been a topic of debate, particularly regarding its welfare implications. In final good markets, third-degree price discrimination can potentially enhance welfare compared to uniform pricing, provided that it increases total output (Schmalensee 1981; Schwartz 1990; Varian 1985). However, the introduction of an intermediate market complicates this analysis, yielding less definitive results. The welfare effects of IPD have been extensively studied, often under the assumption of linear demand. A key finding in this literature is that IPD does not alter the total output produced compared to UIP, which often leads to a reduction in social welfare (DeGraba, 1990; Valletti, 2003). This reduction arises because IPD distorts the efficient allocation of the same output among downstream firms since lower input prices are charged to less efficient firms.

Despite this distortion, several studies highlight conditions under which IPD can enhance welfare. For instance, IPD may be beneficial when downstream firms operate in multiple product markets (Arya and Mittendorf, 2010), or when they engage in backward integration into input supply (Katz, 1987). Similarly, differences in firms' efficiency in transforming inputs into outputs can create welfare-enhancing opportunities for IPD (Yoshida, 2000). Welfare improvements are also observed when downstream firms differ in both cost and quality (Chen, 2017) or simply in terms of quality (Brito, Tselekounis, and Vasconcelos, 2019). Additionally, IPD may be welfare-improving when downstream firms hold unilateral (Hu, Mizuno, and Song, 2022) or bilateral (Tselekounis, 2023) passive partial ownership. However, these findings show the complexity of IPD's welfare effects since there are cases where IPD increases total output but reduces social welfare, and the opposite also occurs.

Most studies in this branch treat demand and cost structures as fixed, although some exceptions explore scenarios involving cost-reducing investments by downstream firms before input prices are set. For example, DeGraba (1990) demonstrates that IPD can stifle investment incentives and harm welfare when achieving lower marginal production costs requires higher fixed investment costs. On the other hand, when investment-contingent costs are considered, IPD can encourage investment and enhance welfare (Inderst and Valletti, 2009). Furthermore, if an upstream monopolist is able to commit to input prices before investment decisions are made, IPD can stimulate investment when it is made by technological laggards. This dynamic can improve welfare when the initial cost gap between firms is small enough for the laggard to overtake the leader (Lestage and Li, 2022). Similarly to our case, Pinopoulos (2020) considers the possibility of an upstream monopolist investing in order to reduce costs. His contribution to the extant literature assuming linear tariffs, quadratic investment cost and linear demand is the affirmation that banning IPD may decrease welfare. The type of pricing and the reason for discrimination are, however, different from the case we study. Indeed, Pinopoulos (2020) considers the case of observable and unobservable two-part tariffs whereas we use observable unit prices and the rationale behind input price discrimination is not the existence of asymmetric cross-ownership but, instead, the existence of different downstream costs.

Making the connection between these areas of research, Lestage (2021) studies how input price discrimination and vertical PCO interact. Initially, the author addresses whether it is optimal for an upstream monopolist to discriminate against its downstream affiliates and shows that a monopolistic input provider always discriminates against its downstream shareholders. Then, discussing the implications of the invariance principle mentioned above (Greenlee and Raskovich (2006)), they study the welfare effects of IPD motivated by vertical PCO, and find that under downstream quantity competition, there exists, for all given asymmetric ownership structures, a unique threshold such that discriminatory input pricing improves welfare if the degree of product substitutability is below this threshold, and reduces welfare otherwise. In the case of homogeneous products, the degree of product substitutability exceeds the above mentioned threshold and IPD reduces welfare. Incorporating upstream R&D investments, we build on this foundation and find that IPD may produce positive or negative effects on welfare depending on the magnitude of two effects: the output effect and a new investment effect. The consumption reallocation effect present in Lestage (2021) is not present in our model due to the fact that it assumes a downstream homogeneous-product industry.

3 The Model

We consider the following vertical market with two stages. An upstream firm, firm u, produces an input which is sold to two downstream firms, firm 1 and firm 2. The price of the input for downstream firm i is denoted by w_i and it is assumed to be uniform in the first case under analysis, while price discrimination is allowed in the second case. Firm u has constant marginal production costs, c, which can be reduced through investing. By investing, the upstream firm reduces its marginal costs by $x < c$, bearing investment cost $I(x)$. The investment cost is assumed to be quadratic and given by $I(x) = yx^2/2$, with y representing investment efficiency. A low y means that it is relatively cheap to lower the marginal cost, that is, that investment is relatively efficient. The quadratic cost assumption is standard in the literature and means that the investment needed to obtain one additional unit of cost savings increases with the extent of these savings.

At the downstream level, we consider an homogeneous-product industry, with two firms competing in quantities and facing no additional costs other than the payment they make to their common input supplier, firm u. To produce one unit of final product, the downstream firms use one unit of input and therefore, the downstream firm's marginal cost is equal to the input price. As far as demand for the downstream product is concerned, the market demand is assumed to be linear and given by $p(Q) = a - bQ$, where $Q = q_1 + q_2$ denotes the aggregate quantity supplied by the downstream firms and p is the unit price.

With respect to model parameters, we assume that $a, b > 0$ and $a > c$.

As far as the other model parameters are concerned, we make the following assumptions:

Assumption 1a: $3by\left(2 - s_1 - s_2\right) - 2\frac{a}{c} > 0$.

Assumption 1b: $by\left(12\left(1 - s_2\right)\left(1 - s_1\right) - \left(s_1 - s_2\right)^2\right) - 2\left(2 - s_1 - s_2\right)\frac{a}{c} > 0$.

Assumptions 1a and 1b ensure, respectively for the uniform pricing and the input price discrimination cases, that the upstream firm's marginal cost after equilibrium investment remains positive. This also implies that second-order conditions for profit maximization by the upstream firm and other non-negativity constraints related to equilibrium quantities hold.

With respect to the ownership structure, we assume that there is partial backward vertical ownership, that is, both downstream firms own an exogenously given percentage of the supplier, being represented by $s_1 \geqslant 0$ for firm 1 and $s_2 \geqslant 0$ for firm 2. This ownership structure does not allow for any degree of control, constraining s_1, s_2 such that $1 - s_1 - s_2 > \frac{1}{2}$. This means that the downstream firms are not individually or collectively majority owners of the upstream firm whose decisions are made to maximize own profit. However, partial ownership entitles each downstream firm to a percentage of the upstream firm's profit. Hence, each of the downstream firms have their operating profits designated by $\pi_1 = [p(Q) - w_1]q_1$ and $\pi_2 = [p(Q) - w_2]q_2$, with the total profits of each downstream firm, V_1 and V_2, being given by $V_1 = \pi_1 + s_1\pi_u$ and $V_2 = \pi_2 + s_2\pi_u$. As far as the upstream firm is concerned , its total profit is $V_u = (1 - s_1 - s_2)\pi_u = (1 - s_1 - s_2)\left[w_1q_1 + w_2q_2 - (c - x)Q - I(x)\right]$, accounting for the downstream firms' participation in its capital.

Instead of the firms themselves holding stocks of their common supplier the same type of objective function would be possible in the presence of common ownership, that is, in the presence of individual shareholders that owned stock in more than one firm. Consider, for example that the three firms have each an independent shareholder than owns the majority of their stock and hence controls his or her firm. If the majority shareholder of firm i also held a minority stake s_i in firm u, then it would instruct the management of firm i to include a percentage s_i of firm u's profit in the manager's objective function and maximize $\pi_i + s_i\pi_u$.

Given this set up, consumer surplus and social welfare are respectively given by

$$\begin{aligned} CS &= \int_0^Q (a-bz)dz - (a-bQ)\,Q = \frac{1}{2}Q^2 b \\ W &= \int_0^Q (a-bz)dz - (c-x)Q - I(x). \end{aligned}$$

The timing of the game is as follows: In the first stage of this game, the upstream firm chooses both the investment level, x, and the input prices, w_1 and w_2. When investing, the upstream firm knows if it is allowed or not to price discriminate. In the second stage, the downstream firms engage in quantity competition and, having observed all previous decisions, choose their output levels simultaneously. The downstream price is assumed to adjust to clear the market. The game is solved using backward induction.

3.1 Uniform Input Price

In this section, we consider the scenario in which input price discrimination is banned and the upstream firm is forced to charge an uniform input price, w, to both downstream firms, despite their possible asymmetries with respect to their ownership of firm u. As such, the objective function of the two downstream firms and of the upstream monopolist are, respectively,

$$\begin{aligned} V_i &= \pi_i + s_i \pi_u = [p(Q) - w]q_i + s_i\,[w - (c-x))Q - I(x)] \\ V_u &= (1 - s_1 - s_2)\,[w - (c-x))Q - I(x)] \end{aligned}$$

The following lemma presents the equilibrium of this game.

Lemma 1: *Let Assumption 1a hold. Under uniform input price:*

i) the equilibrium input price and cost reduction from investment is given by

$$\begin{aligned} w(s_1, s_2) &= \frac{3by(c\,(1-s_1-s_2)+a) - 2a}{3by\,(2-s_1-s_2) - 2} \\ x(s_1, s_2) &= \frac{2(a-c)}{3by\,(2-s_1-s_2) - 2} \end{aligned}$$

ii) the equilibrium output is given by

$$
\begin{aligned}
q_1(s_1,s_2) &= \frac{(1-2s_2+s_1)}{3by(2-s_1-s_2)-2}y(a-c) \\
q_2(s_1,s_2) &= \frac{(1-2s_1+s_2)}{3by(2-s_1-s_2)-2}y(a-c) \\
Q(s_1,s_2) &= \frac{(2-s_1-s_2)}{3by(2-s_1-s_2)-2}y(a-c)
\end{aligned}
$$

iii) Operational profits are given by

$$
\begin{aligned}
\pi_u(s_1,s_2) &= \frac{y(a-c)^2}{3by(2-s_1-s_2)-2} \\
\pi_1(s_1,s_2) &= \frac{(1-2s_2+s_1)(1-2(s_1+s_2))}{(3by(2-s_1-s_2)-2)^2}by^2(a-c)^2 \\
\pi_2(s_1,s_2) &= \frac{(1-2s_1+s_2)(1-2(s_1+s_2))}{(3by(2-s_1-s_2)-2)^2}by^2(a-c)^2
\end{aligned}
$$

iv) Consumer surplus and welfare are given by

$$
\begin{aligned}
CS(s_1,s_2) &= \frac{1}{2}\left(\frac{(2-s_1-s_2)}{3by(2-s_1-s_2)-2}y(a-c)\right)^2 b \\
W(s_1,s_2) &= \frac{1}{2}y(a-c)^2\frac{5by(2-s_1-s_2)^2-4}{(3by(2-s_1-s_2)-2)^2}.
\end{aligned}
$$

Next, we provide the results of comparative statics. Differentiating CS and W with respect to s_1 yields

$$
\begin{aligned}
\frac{\partial CS}{\partial s_1} &= 2by^2(a-c)^2\frac{2-s_1-s_2}{(3by(2-s_1-s_2)-2)^3} \\
\frac{\partial W}{\partial s_1} &= 2by^2(a-c)^2\frac{4-5(s_1+s_2)}{(3by(2-s_1-s_2)-2)^3},
\end{aligned}
$$

from which we conclude that $\frac{\partial CS}{\partial s_1} > 0$ and $\frac{\partial W}{\partial s_1} > 0$ for all parameter values that verify assumption 1a, meaning that vertical PCO increases consumer surplus and welfare in a setting where the upstream firm does not discriminate between the downstream firms.

The proof of Lemma 1 also presents, for comparison purposes, the equilibrium in the absence of cost reducing investment, that is, the equilibrium if the upstream monopolist's marginal cost is merely equal to c. Consumer surplus and welfare are, in this case, given by

$$CS=\frac{1}{18}\frac{(a-c)^2}{b} \text{ and } W=\frac{5}{18}\frac{(a-c)^2}{b}$$

which, as expected, do not depend on the ownership structure. This invariance result corresponds to Greenlee and Raskovich (2006) Proposition 4 (ii) that states that in a Cournot n-firm oligopoly with constant and symmetric marginal downstream costs then "producer surplus and total surplus are invariant across all ownership profiles", which implies that consumer surplus is also invariant to the ownership structure.

We thus contribute to this literature by showing that when upstream investment is a relevant feature of an industry the "invariance result" no longer holds, and consumer surplus and welfare increase with the level of partial vertical ownership. We now explain why.

Differentiating the equilibrium outcomes $x(s_1,s_2)$ and $w(s_1,s_2)$ with respect to s_1 yields

$$\frac{\partial x(s_1,s_2)}{\partial s_1}=\frac{6by\,(a-c)}{(3by\,(2-s_1-s_2)-2)^2},\quad \frac{\partial w(s_1,s_2)}{\partial s_1}=3by\,(a-c)\,\frac{(3by-2)}{(3by\,(2-s_1-s_2)-2)^2}$$

Therefore, we conclude that $\frac{\partial x(s_1,s_2)}{\partial s_1}>0$, meaning that an increase in the degree of participation by any of the downstream firms in their input supplier always boosts the investment level. Optimal investment is obtained from maximizing the upstream firm's profit

$$\pi_u=(w-(c-x))Q(w,x)-y\frac{x^2}{2}$$

with

$$Q(w,x)=\frac{2a-2w+(s_1+s_2)\,(w-c+x)}{3b}.$$

Optimal investment in firm u´s perspective corresponds to the investment level that makes the marginal benefit of increasing x, $Q(w,x)+(w-(c-x))\frac{\partial Q(w,x)}{\partial x}$, equal to its marginal cost, yx.

Lowering the marginal cost of producing the input by 1€ will lead to an upstream profit increase of Q and, in the presence of partial ownership, also leads to an increase in input sales: $\frac{\partial Q(w,x)}{\partial x}>0$ if and only if $s_1+s_2>0$. To the extent that the downstream firms receive part of the upstream firm's profit, they consider part of the reduction in input costs as a reduction in their own costs, leading to higher output. There are thus two terms on the marginal benefit of investing: not only each unit becomes cheaper to produce but there will also be an increase in the number of units sold. Given w, the two effects are larger in the presence of vertical backward PCO: i) for any x, the equilibrium output is larger, because downstream firms pay part of the cost to themselves and ii) the equilibrium quantity increases with x because part of the cost reduction is appropriated by the downstream firms who, with lower costs, have an incentive to produce more. This means that PCO increases the marginal benefits of investing, which increases investment, leading an increase in consumer surplus and welfare.

The equilibrium input price w may either increase or decrease with s_1. With no investment present, the input price would unambiguously increase with s_1 (the equilibrium expression for the input price with no investment can be found in the proof of lemma 1). With investment, however, a higher investment that results from an increase in s_1 leads to lower marginal costs for the input producer, which has the opposite effect on the price input, which, depending on the dominating effect may either increase or decrease with s_1. In any case, the relevant input price for each firm is $w(1-s_1)+s_1(c-x)$ which decreases in s_1 :

$$\begin{aligned} w(1-s_1)+s_1(c-x) &= \frac{2a-3aby-3bcy+3abys_1+3bcys_2}{-6by+3bys_1+3bys_2+2} \\ \frac{\partial\left(w(1-s_1)+s_1(c-x)\right)}{\partial s_1} &= \frac{-9b^2y^2(1-s_2)(a-c)}{\left(3by(2-s_1-s_2)-2\right)^2}<0 \end{aligned}$$

This means that total output and consumer surplus will increase with s_1.

3.2 Input Price Discrimination

We now allow for input price discrimination, meaning that the upstream firm can charge a different input price to each of the downstream firms, w_1 and w_2. Recall that downstream firms are not necessarily symmetric as far as their share in the upstream monopolist's profit is concerned. As such, the relevant formulations of the profits for the downstream firms are $\pi_1=(p(Q)-w_1)q_1$ and $\pi_2=(p(Q)-w_2)q_2$ and, for the upstream firm, is $\pi_u=w_1q_1+w_2q_2-(c-x)Q-I(x)$.

The following Lemma presents the equilibrium with input price discrimination.

Lemma 2: *Let Assumption 1b hold. Under input price discrimination:*

i) the equilibrium input price and cost reduction from investment is given by

$$
\begin{aligned}
w_1(s_1,s_2) &= \frac{by\left(a\left(6-s_1-5s_2\right)-c\left(11s_1+7s_2+s_1^2+s_2^2-14s_1s_2-6\right)\right)-2a\left(2-s_1-s_2\right)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)} \\
w_2(s_1,s_2) &= \frac{by\left(a\left(6-5s_1-s_2\right)-c\left(7s_1+11s_2+s_1^2+s_2^2-14s_1s_2-6\right)\right)-2a\left(2-s_1-s_2\right)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)} \\
x(s_1,s_2) &= \frac{2\left(2-s_1-s_2\right)\left(a-c\right)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)}
\end{aligned}
$$

ii) the equilibrium output is given by

$$
\begin{aligned}
q_1(s_1,s_2) &= \frac{y\left(1-s_1\right)\left(s_1-3s_2+2\right)\left(a-c\right)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)} \\
q_2(s_1,s_2) &= \frac{y\left(1-s_2\right)\left(s_2-3s_1+2\right)\left(a-c\right)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)} \\
Q(s_1,s_2) &= \frac{y\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)\left(a-c\right)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)}
\end{aligned}
$$

iii) Operational profits are given by

$$
\begin{aligned}
\pi_u(s_1,s_2) &= \frac{\left(2-s_1-s_2\right)y\left(a-c\right)^2}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)} \\
\pi_1(s_1,s_2) &= \frac{\left(1-s_1\right)\left(2-7s_1-3s_2+8s_1s_2\right)\left(2-3s_2+s_1\right)by^2\left(a-c\right)^2}{\left(by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)\right)^2} \\
\pi_2(s_1,s_2) &= \frac{\left(1-s_2\right)\left(2-3s_1-7s_2+8s_1s_2\right)\left(2-3s_1+s_2\right)by^2\left(a-c\right)^2}{\left(by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)\right)^2}
\end{aligned}
$$

iv) Consumer surplus and welfare are given by

$$
\begin{aligned}
CS(s_1,s_2) &= \frac{b}{2}\left(\frac{y\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)\left(a-c\right)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)}\right)^2 \\
W(s_1,s_2) &= y\left(a-c\right)^2\frac{\begin{array}{c}by\left(20\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)\\ -16\left(1-s_1-s_2\right)-4\left(s_1+s_2\right)^2\end{array}}{2\left(by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)\right)^2}.
\end{aligned}
$$

We now study the effects of cross-holdings on welfare. In contrast with the case of UIP, it is now possible that $\frac{\partial W}{\partial s_1}<0$. Indeed, contrary to the case of uniform pricing, we now show that increasing cross ownership may lead to a reduction in welfare when in the presence of input price discrimination. To illustrate this, we assume that $s_2 = 0$ in order to obtain shorter expressions and thus simplify the analysis.

When $s_2 = 0$, assumption 1b can be simplified to $by\left(12\left(1-s_1\right)-s_1^2\right)-2\left(2-s_1\right)\frac{a}{c} > 0$ or $by > \frac{2(2-s_1)}{12-12s_1-s_1^2}\frac{a}{c}$. Note that this implies that $by > \frac{1}{3}$ and that $by > \frac{2}{3(2-s_1)}$ which will be useful later on.[1]

Figure 1 illustrates how welfare changes with s_1 in a specific numeric example, with $a = 10$, $b = 1$, $c = 6$ and $y = 20$.

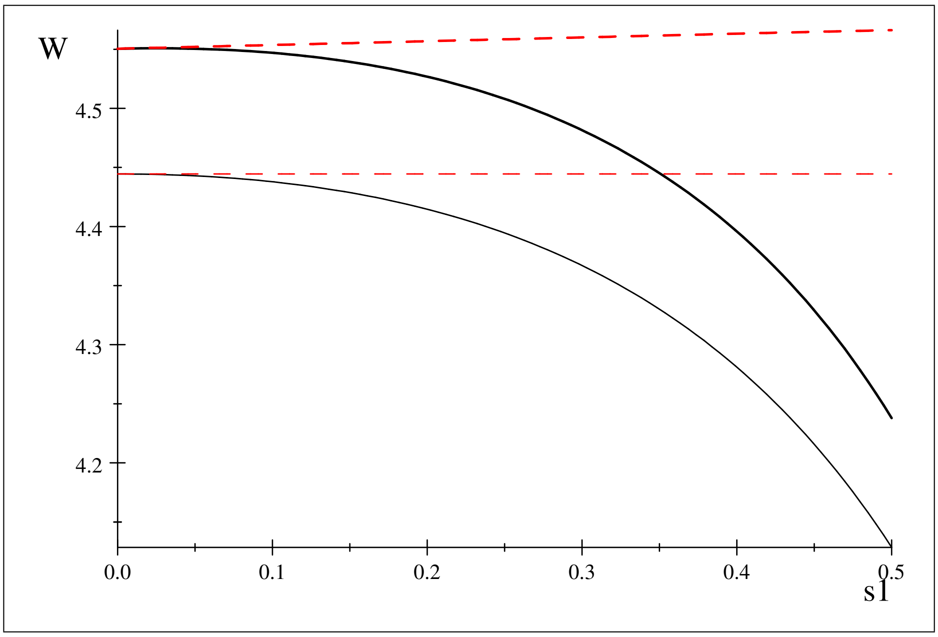

Figure 1: Welfare as a function of s_1 when $a = 10$, $b = 1$, $c = 6$, $y = 20$. The dashed curves corresponds to UIP and the solid curves to IPD. The thin curve corresponds to the case of no investment and the thick curves to the case with investment.

With IPD, as

$$\frac{\partial W}{\partial s_1} = -2by^2\left(a-c\right)^2 \frac{24s_1 + 60s_1^2 - 6s_1^3 - s_1^4 - 32 + 32bys_1\left(1-s_1\right)\left(2-s_1\right)}{\left(by\left(12-12s_1-s_1^2\right)-2\left(2-s_1\right)\right)^3},$$

[1] This follows from $\frac{2(2-s_1)}{12-12s_1-s_1^2}\frac{a}{c} > \frac{2(2-s_1)}{12-12s_1-s_1^2} = \frac{1}{3} + \frac{1}{3}s_1\frac{s_1+6}{12-12s_1-s_1^2}$.and $\frac{2(2-s_1)}{12-12s_1-s_1^2} = \frac{2}{3(2-s_1)} + \frac{8}{3}\frac{s_1^2}{(2-s_1)\left(12-12s_1-s_1^2\right)}$

the impact of s_1 on welfare may be negative if the expression above is negative, that is, if

$$24s_1 + 60s_1^2 - 6s_1^3 - s_1^4 - 32 + 32bys_1(1-s_1)(2-s_1) > 0$$

which is equivalent to

$$by > -\frac{24s_1 + 60s_1^2 - 6s_1^3 - s_1^4 - 32}{32s_1(1-s_1)(2-s_1)}.$$

This is always possible for some admissible parameter values as there is no upper bound on by. Thus, although social welfare is always increasing with s_1 at $s_1 = 0$, an increase in the degree of ownership may lead to lower welfare.[2]

This contrasts with the case under UIP in which welfare always increases with s_1.

To understand the difference, it is useful to decompose the change in welfare in two effects. Welfare depends on total output (because the product sold by the downstream firms is assumed to be homogeneous) and also on the investment level. Changing the ownership structure will affect both the equilibrium output and the investment level. So, the effect on welfare can be decomposed in

$$\begin{aligned}
\frac{\partial W}{\partial s_1} &= \frac{\partial W}{\partial Q}\frac{\partial Q}{\partial s_1} + \frac{\partial W}{\partial x}\frac{\partial x}{\partial s_1} \\
&= (a - bQ - (c - x))\frac{\partial Q}{\partial s_1} + (Q - yx)\frac{\partial x}{\partial s_1} \\
&= (p - c + x)\frac{\partial Q}{\partial s_1} + (Q - yx)\frac{\partial x}{\partial s_1}
\end{aligned}$$

Under uniform price, we have

$$\begin{aligned}
p - c + x &= \frac{2by(2-s_1)(a-c)}{3by(2-s_1)-2} > 0 \\
\frac{\partial Q}{\partial s_1} &= \frac{2y(a-c)}{(3by(2-s_1)-2)^2} > 0 \\
Q - xy &= -\frac{ys_1(a-c)}{3by(2-s_1)-2} < 0 \\
\frac{\partial x}{\partial s_1} &= \frac{6by(a-c)}{(3by(2-s_1)-2)^2} > 0
\end{aligned}$$

Increasing s_1 leads to an increase in output which is welfare increasing as, in equilibrium, the downstream price p exceeds the input marginal cost $c - x$. It also leads to higher

[2] As $\frac{\partial W}{\partial s_1}$ evaluated at $s_1 = 0$ is equal to $= by^2\frac{(a-c)^2}{(3by-1)^3} > 0$ (recall that $by > \frac{1}{3}$) welfare always starts by increasing with s_1.

cost reducing investment but this has the opposite effect. The increase in investment exceeds the costs savings: As the overall effect is positive under UIP, the first (positive) effect dominates the second (negative) one.

Under input price discrimination, the same effects are now given by

$$\begin{aligned}
p-c+x &= \frac{8(1-s_1)\,by\,(a-c)}{by\left(12-12s_1-s_1^2\right)-2(2-s_1)}>0 \\
\frac{\partial Q}{\partial s_1} &= \frac{2\left(4s_1-s_1^2+4-4bys_1(2-s_1)\right)y\,(a-c)}{\left(by\left(12-12s_1-s_1^2\right)-2(2-s_1)\right)^2}<>0 \\
Q-xy &= \frac{-s_1(s_1+2)\,y\,(a-c)}{by\left(12-12s_1-s_1^2\right)-2(2-s_1)}<0 \\
\frac{\partial x}{\partial s_1} &= \frac{2(s_1+2)(6-s_1)\,by\,(a-c)}{\left(by\left(12-12s_1-s_1^2\right)-2(2-s_1)\right)^2}>0
\end{aligned}$$

All the effects change quantitatively, but one effect, $\frac{\partial Q}{\partial s_1}$, may even change sign when one moves from uniform input pricing to input price discrimination. To understand this effect, recall that the downstream firms are Cournot competitors and that firm 1 partially pays the marginal cost to itself (to the extent that it receives part of the upstream firm's profit). Indeed, recall that firm 1 makes its output decision as if its marginal cost is given by $w_1(1-s_1)+s_1(c-x)$, whereas firm 2 makes the output decision with marginal cost w_2 (because we are here assuming $s_2=0$). Note that firm 1 receives part of the upstream firm's revenue but also shares a part of its costs. This makes firm 1's output less sensitive to the input price when compared to firm 2's. Its demand for the input is less elastic and firm 1 is charged a higher input price, a result that holds when $s_1>s_2$ and not necessarily when $s_2=0$:

$$\begin{aligned}
& w_1(s_1,s_2)-w_2(s_1,s_2) \\
=\ & \frac{4by\,(s_1-s_2)(a-c)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)}>0
\end{aligned}$$

However, in terms of effective (or perceived) costs, the ones relevant for the output decision, firm 1 will have lower costs than firm 2:

$$\begin{aligned}
& (1-s_1)w_1+s_1(c-x)-\left((1-s_2)w_2+s_2(c-x)\right) \\
=\ & \frac{-by\,(s_1-s_2)(2-s_1-s_2)(a-c)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)}<0
\end{aligned}$$

As is well known in the linear Cournot model, total output decreases with the simple average of the marginal costs, which in the case of the duopoly in question is $\frac{w_1(1-s_1)+s_1(c-x)+w_2(1-s_2)+s_2(c-x)}{2}$. The perceived marginal cost of firm 1 decreases with s_1 by less than the cost of firm 2 increases, as illustrated in Figure 2 for the same numeric example, leading to an effective average marginal cost that increases with s_1. As a result, total output decreases with s_1 under IPD, which may lead to lower welfare.

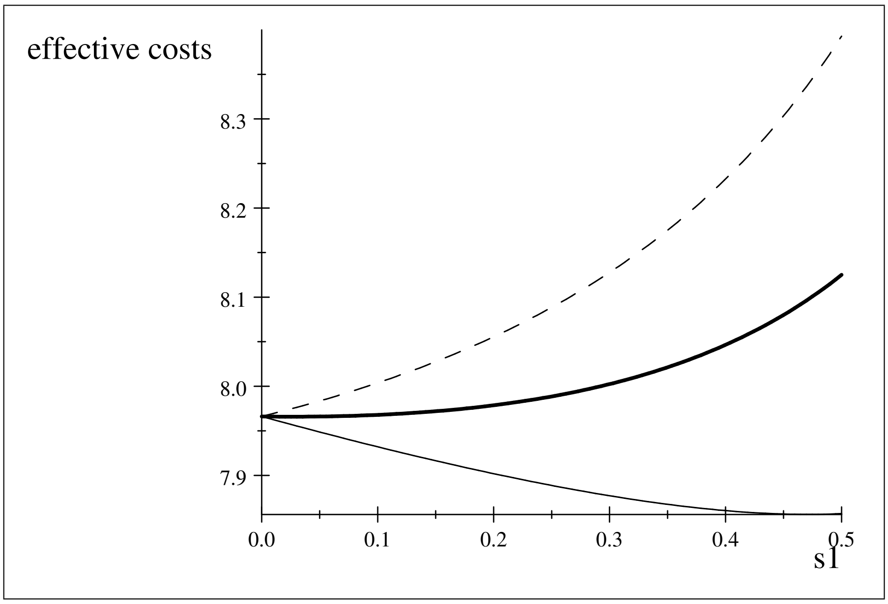

Figure 2: Effective marginal costs under IPD as a function of s_1 when $a = 10$, $b = 1$, $c = 6$, $y = 20$ and $s_2 = 0$. The dashed curve corresponds to firm 2 and the solid curve to firm 1. The thick curve represents the average of the effective costs.

With the same parametrization but assuming an uniform input price, the effective costs and their average are presented in Figure 3. The average cost is in this case decreasing with s_1, which leads to a higher equilibrium input.

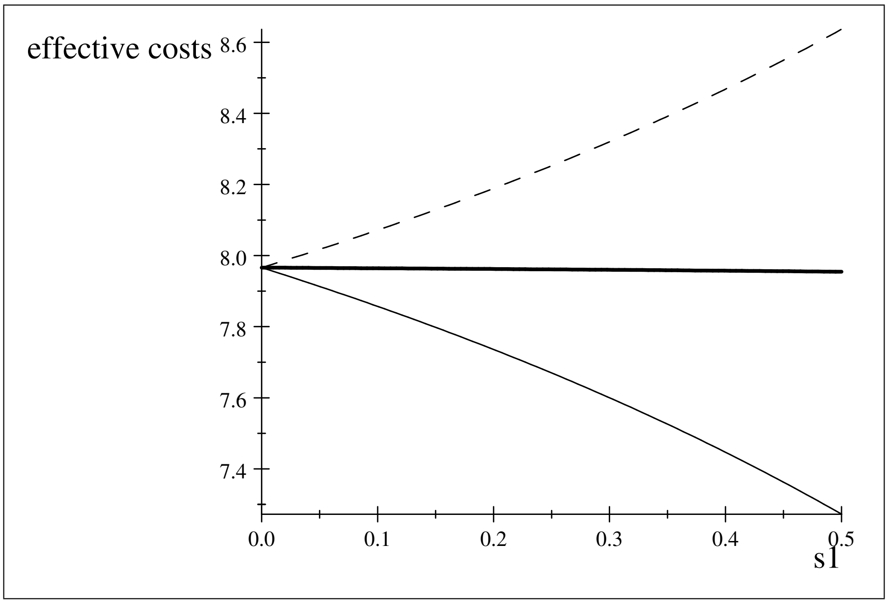

Figure 3: Effective marginal costs under UIP as a function of s_1 when $a = 10$, $b = 1$, $c = 6$, $y = 20$ and $s_2 = 0$. The dashed curve corresponds to firm 2 and the solid curve to firm 1. The thick curve represents the average of the effective costs

As mentioned above, Lestage (2021) analyses the effects of IPD in a similar set up, but without considering the possibility of the input producer investing in order to reduce its marginal cost. The previous case, with $s_2 = 0$, allows us to illustrate another instance in which the introduction of investment changes some results in the literature. With Cournot competition, Lemma 2 in Lestage (2021) states that "Total industry profit and welfare are maximized for all $s_1 = s_2$, under both uniform and discriminatory input pricing." When investment is introduced, this is no longer true as the previous example illustrated.

4 The Welfare Effects of Input Price Discrimination

In this section we take the ownership structure as given and discuss the effects of IPD on welfare, when compared to uniform input price. In the absence of investment, the literature presents the following result (Lemma 1 in Lestage (2021)), "Under quantity competition, there exists, for all given asymmetric ownership structure, a unique threshold such that discriminatory input pricing improves welfare if the degree of product substitutability is below this threshold, and reduces welfare otherwise." In the case of homogeneous products, the degree of product substitutability exceeds the above mentioned threshold and IPD reduces welfare. Lestage (2021) identifies two effects, the reallocation effect (which is positive) and the output effect (which is negative).[3] The reallocation effect is not present in this model because the product is homogeneous. However, there is an additional effect, the investment effect.

There are thus two relevant effects, the investment effect and the output effect, which are naturally related as the investment affects the equilibrium output. The following corollaries compare investment and output under the two input pricing regimes.

Corollary 1: *Let assumptions 1a and 1b hold. IPD increases investment.*

Corollary 2: *Let assumptions 1a and 1b hold. IPD increases output (and consumer surplus) if and only if* $by(2 - s_1 - s_2) - 2 < 0$.

Proposition 1: *Let assumptions 1a and 1b hold. IPD increases welfare in the presence of investment if and only if*

$$(by)^- < by < (by)^+$$

where the expressions for $(by)^-$ *and* $(by)^+$ *are presented in the appendix.*

Figure 4 presents the parameter range such that IPD increases welfare.

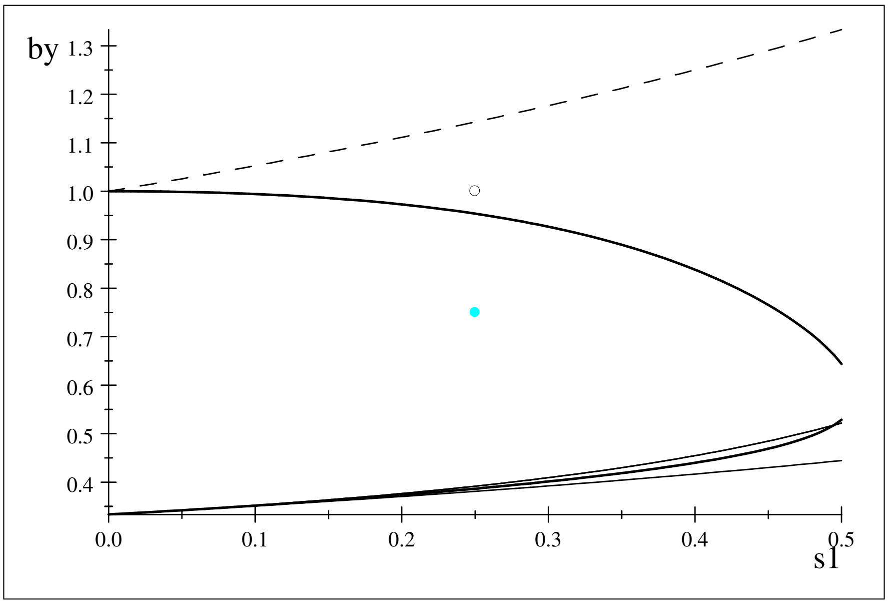

Figure 4: Parameter range such that IPD increases welfare.

[3]Note that with no investment IPD decreases output as mentioned in Lestage (2021):

$$\begin{aligned} & \frac{4(1-s_2)(1-s_1)-(s_1-s_2)^2}{12(1-s_2)(1-s_1)-(s_1-s_2)^2}\frac{a-c}{b} - \frac{a-c}{3b} \\ = & -\frac{2}{3}\frac{(s_1-s_2)^2}{12(1-s_2)(1-s_1)-(s_1-s_2)^2}\frac{a-c}{b} < 0 \end{aligned}$$

As presented in the proof of Proposition 1, the difference in welfare when one moves from uniform pricing to input price discrimination is an inverted parabola in by. The two roots, $(by)^-$ and $(by)^+$ are presented in Figure 4 (thick curves) and, between these two curves, welfare increases with IPD. The dashed curve represents the threshold in Corollary 2, below which output increases with IPD. Increasing output is thus a necessary condition for welfare to be higher with IPD. The other two curves represent the thresholds in assumptions 1a and 1b when $s_2 = 0$ and $a = c$. The real thresholds, when $a > c$, are higher than those represented in Figure 4, but there are values for $a/c > 1$ such that it is still possible that welfare increases with IPD.

As presented in Figure 4, there are three different possibilities of IPD effects on welfare depending on the values of the model parameters:

(i) for values of by inside the interval $(by)^- < by < (by)^+$ which verify assumptions 1a and 1b and the condition in Corollary 2, represented by the dashed line, IPD may increase welfare;

(ii) for values of by outside the interval $(by)^- < by < (by)^+$ which verify assumptions 1a and 1b and the condition in Corollary 2, despite output increasing, IPD decreases welfare;

(iii) for values of by outside the interval $(by)^- < by < (by)^+$ which verify assumptions 1a and 1b but do not meet the condition in Corollary 2, output does no longer increase, and IPD unsurprisingly decreases welfare.

In the first case, IPD may in fact increase welfare when we are in the presence of investment. In order to better explain the effects surrounding this result, we present a specific numeric case, where $a = 10$, $b = 1$, $c = 6$, $s_2 = 0$, $s_1 = \frac{1}{4}$ and $y = \frac{3}{4}$, which meet assumptions 1a and 1b and situates us between the two curves. In Figure 4, this numeric example corresponds to the black dot.

Table 1 presents the equilibrium values taken by the variables of interest, when moving from UIP to IPD.

	π_u	π_1	π_2	CS	W	Q	x
UIP	6.1935	1.4984	0.59938	3.6712	11.963	2.7097	4.129
IPD	6.5561	0.37007	1.3706	3.7846	12.081	2.7512	4.3707

Table 1: Equilibrium values when $a = 10$, $b = 1$, $c = 6$, $s_2 = 0$, $s_1 = \frac{1}{4}$ and $y = \frac{3}{4}$

At first glance, we can easily tell that the introduction of IPD heavily reduces firm 1's operational profits. This is mainly due to the fact that under UIP the upstream firm is not able to capitalize on the asymmetries in the downstream market. When it does, the price of the input charged to firm 1 (w_1) is higher than the previous uniform price (w), resulting in a decrease of output, while the price of input for firm 2 (w_2) is lower, allowing it to produce more. As shown in Table 1 and implied by Corollary 2, the increase in production by firm 2 is larger then the decrease in firm 1's production, and aggregate output increases. This increase in aggregate output is directly associated with the observed increase in consumer surplus. Since production made by both firms changes alongside its input prices, there occurs a reallocation of profits in the downstream market, with firm 1 observing a decrease in its operational profits while firm 2 observes an increase. Overall, this reallocation results in a net negative effect in terms of downstream operational profits, with firm 1's profits decreasing more then firm 2's increase. With the introduction of IPD, the upstream firm is expectedly better off in terms of its operational profits. This increase is slightly higher than the decrease in the downstream market, resulting in industry profits observing an increase. Therefore, welfare is able to increase with the introduction of IPD.

In the second case, the introduction of IPD decreases welfare while output is increasing. The range of parameters used to illustrate this case is the following: $a = 10$, $b = 1$, $c = 6$, $s_2 = 0$, $s_1 = \frac{1}{4}$, $y = 1$, which again meet assumptions 1a and 1b. This corresponds to the white dot in Figure 4

Table 2 presents the equilibrium values taken by the variables of interest, when moving from UIP to IPD for this second example.

	π_u	π_1	π_2	CS	W	Q	x
UIP	4.9231	0.94675	0.37870	2.3195	8.568	2.1538	2.4615
IPD	5.1494	0.22830	0.84555	2.3348	8.5581	2.1609	2.5747

Table 2: Equilibrium values when $a = 10$, $b = 1$, $c = 6$, $s_2 = 0$, $s_1 = \frac{1}{4}$ and $y = 1$

Table 2 now depicts that welfare is decreasing while output and, consequently, consumer surplus both increase. This shows that industry profits must be decreasing, and in a scale which counters the positive effect caused by the increase in consumer surplus. In the downstream market, the same is observed in terms of profits and output. Regarding the latter, the move from UIP to IPD still results in an increase in aggregate output, with firm 2's output expansion being larger than the reduction in firm 1's output. Regarding operational profits, the decrease firm 1 observes still outweighs the increase firm 2 obtains, with downstream operational profits decreasing. The upstream firm is, again, unexpectedly better off in terms of operational profits. However, the magnitude of this increase is smaller than previously, no longer allowing it to outweigh the decrease in profits in the downstream market. This happens because the investment costs are higher than in the previous example. As such, industry profits now decrease and decrease in a magnitude allowing it to counter the positive effects associated with an increase in aggregate output. This results in a decrease in welfare with the introduction of IPD.

In the third and final case, introducing discriminatory input pricing decreases welfare again, with output now decreasing. Since in the previous case welfare was decreasing with output increasing, it is unsurprising that now, with output decreasing as well, welfare falls. As such, the effects are similar to the second case, but since output decreases, the observed decrease in welfare will naturally be larger than previously.

The proof of Proposition 1 also presents what would happen to welfare and consumer surplus in the absence of investment. Both would decrease for all parameter values when moving from UIP to IPD, following the results in Lestage (2021), as only the output effect would prevail.

5 Conclusions

In this dissertation, we have studied the effects of vertical partial backward ownership within a framework that incorporates cost-reducing R&D investments at the upstream level, placing emphasis on the role of input price discrimination which is based on asymmetric backward shareholding. Initially, we analyzed the model under uniform input pricing. Our contribution to the literature lies in the introduction of upstream cost-reducing investments into this context. We found that vertical PCO boosts consumer surplus and welfare when the upstream supplier is able to invest, which contrasts with previous findings in the literature where, in the absence of investment, welfare remained invariant to PCO.

When introducing IPD, the results become more nuanced. Moving from UIP to IPD, welfare no longer increases unconditionally with vertical PCO and may even decrease for certain parameter ranges. Previous literature established that in a similar framework, excluding upstream investments, welfare and industry profits are maximized with symmetric ownership structures. However, we show that with the inclusion of upstream investments, this conclusion no longer holds.

By isolating on the effects of IPD on welfare, we take the ownership structure as given and study how the introduction of IPD affects total surplus when compared to UIP. Previous literature had set that introducing IPD caused two effects on welfare, a reallocation effect and an output effect. With homogeneous products the prevailing effect was shown to be the output effect with negative consequences on welfare. In our study, we keep the output effect but we lose the reallocation effect since we are working under the assumption of a homogenous product industry. However, we obtain a new effect, which is the investment effect. We show that the introduction of IPD may increase welfare for a specific range of parameters which allows output to increase and industry profits to increase as well. When the latter becomes decreasing, we show that the introduction of IPD may impact welfare negatively, even when output is increasing.

This study contributes to the literature by examining the combined welfare implications of PCO and IPD, particularly in the presence of upstream investments. While prior research has addressed these factors separately, we provide new insights into their interaction and highlight the role of investment dynamics in shaping welfare outcomes. Nonetheless, our analysis is subject to some limitations. For instance, we focus solely on vertical PCO, leaving room for future studies to explore the interaction of both vertical and horizontal cross-ownership structures, or to investigate how incorporating corporate control might alter these dynamics.

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Appendix

Proof of Lemma 1:

At the second stage, the Cournot quantity setting stage, the best response of the downstream firms is given by

$$\begin{aligned}
\frac{\partial V_1}{\partial q_1} &= \frac{\partial\left((a-b(q_1+q_2))q_1 - wq_1 + s_1(w-(c-x))(q_1+q_2) - y\frac{x^2}{2}\right)}{\partial q_1} \\
&= a - 2bq_1 - bq_2 - (w - s_1(w-c+x)) = 0 \\
\frac{\partial V_2}{\partial q_2} &= \frac{\partial\left((a-b(q_1+q_2))q_2 - wq_2 + s_2(w-(c-x))(q_1+q_2) - y\frac{x^2}{2}\right)}{\partial q_2} \\
&= a - bq_1 - 2bq_2 - (w - s_2(w-c+x)) = 0
\end{aligned}$$

that result in the following equilibrium quantities

$$\begin{aligned}
q_1(w,x) &= \frac{a - w + 2s_1(w-c+x) - s_2(w-c+x)}{3b} \\
q_2(w,x) &= \frac{a - w + 2s_2(w-c+x) - s_1(w-c+x)}{3b}
\end{aligned}$$

The demand for the upstream firm is then $Q(w,x) = q_1(w,x) + q_2(w,x) = \frac{2a-2w+(s_1+s_2)(w-c+x)}{3b}$. The upstream firm chooses w and x to maximize V_u which is the same as maximizing its own operational profit:

$$\pi_u = (w-(c-x))\frac{2a-2w+(s_1+s_2)(w-c+x)}{3b} - y\frac{x^2}{2}$$

The first-order conditions for profit maximization are

$$\begin{aligned}
\frac{\partial \pi_u}{\partial w} &= \frac{\partial\left((w-(c-x))\frac{2a-2w+(s_1+s_2)(w-c+x)}{3b} - y\frac{x^2}{2}\right)}{\partial w} \\
&= 2\frac{a+c-2w-x+(s_1+s_2)(w-c+x)}{3b} = 0 \\
\frac{\partial \pi_u}{\partial x} &= \frac{\partial\left((w-(c-x))\frac{2a-2w+(s_1+s_2)(w-c+x)}{3b} - y\frac{x^2}{2}\right)}{\partial x} \\
&= \frac{2a-2w+2(s_1+s_2)(w-c+x)}{3b} - xy = 0
\end{aligned}$$

from where we obtain the equilibrium levels of cost reductions by investing as well as the input price:

$$w = \frac{3by\left(a + c\left(1 - s_1 - s_2\right)\right) - 2a}{3by\left(2 - s_1 - s_2\right) - 2}$$
$$x = \frac{2\left(a - c\right)}{3by\left(2 - s_1 - s_2\right) - 2}$$

Second-order conditions for profit maximization follow from the Hessian matrix,

$$\begin{bmatrix} -\frac{2}{3b}\left(2 - s_1 - s_2\right) & -\frac{2}{3b}\left(1 - s_1 - s_2\right) \\ -\frac{2}{3b}\left(1 - s_1 - s_2\right) & -\frac{3by - 2(s_1 + s_2)}{3b} \end{bmatrix}$$

and require that $-\frac{2}{3b}\left(2 - s_1 - s_2\right) < 0$ and $\frac{2}{9}\frac{3by(2-s_1-s_2)-2}{b^2} > 0$. Moreover, the upstream firm's marginal cost cannot become negative after investment. This means that

$$c - x = c - \frac{2\left(a - c\right)}{3by\left(2 - s_1 - s_2\right) - 2} = \frac{3by\left(2 - s_1 - s_2\right) - 2\frac{a}{c}}{3by\left(2 - s_1 - s_2\right) - 2}c > 0$$

This is implied by Assumption 1a:

$$3by\left(2 - s_1 - s_2\right) - 2\frac{a}{c} > 0$$

This condition also implies the second-order conditions and also that $x > 0$ and $w > 0$.

Plugging the equilibrium input price and cost reduction into the output expressions yields the equilibrium level of output at the downstream level:

$$q_1 = \frac{\left(1 - 2s_2 + s_1\right)}{3by\left(2 - s_1 - s_2\right) - 2}y\left(a - c\right)$$
$$q_2 = \frac{\left(1 - 2s_1 + s_2\right)}{3by\left(2 - s_1 - s_2\right) - 2}y\left(a - c\right)$$
$$Q = \frac{\left(2 - s_1 - s_2\right)}{3by\left(2 - s_1 - s_2\right) - 2}y\left(a - c\right)$$

Finally, it is straightforward to calculate the equilibrium profits of all firms,

$$\pi_u = (w - (c - x))\frac{2a - 2w + \left(s_1 + s_2\right)\left(w - c + x\right)}{3b} - y\frac{x^2}{2} = \frac{y\left(a - c\right)^2}{3by\left(2 - s_1 - s_2\right) - 2}$$
$$\pi_1 = (a - b(q_1 + q_2))q_1 - wq_1 = \frac{\left(1 - 2s_2 + s_1\right)\left(1 - 2\left(s_1 + s_2\right)\right)}{\left(3by\left(2 - s_1 - s_2\right) - 2\right)^2}by^2\left(a - c\right)^2$$
$$\pi_2 = (a - b(q_1 + q_2))q_2 - wq_2 = \frac{\left(1 - 2s_1 + s_2\right)\left(1 - 2\left(s_1 + s_2\right)\right)}{\left(3by\left(2 - s_1 - s_2\right) - 2\right)^2}by^2\left(a - c\right)^2$$

and the consumer surplus and social welfare.

$$\begin{aligned} CS &= \frac{1}{2}\left(\frac{(2-s_1-s_2)}{3by(2-s_1-s_2)-2}y(a-c)\right)^2 b \\ W &= \frac{1}{2}y(a-c)^2\frac{5by(2-s_1-s_2)^2-4}{(3by(2-s_1-s_2)-2)^2}. \end{aligned}$$

If there was no investment in cost reductions, then $x = 0$ and the upstream monopolist's decision referred only to the input price. The first-order conditions for profita maximization are, in this case,

$$\frac{\partial \pi_u}{\partial w} = 2\frac{a+c-2w+(s_1+s_2)(w-c)}{3b} = 0 \Leftrightarrow w = \frac{a+c(1-s_1-s_2)}{2-s_1-s_2}$$

with the second order conditions holding trivially. At this input price, the equilibrium outputs are

$$\begin{aligned} q_1 &= \frac{1-2s_2+s_1}{2-s_1-s_2}\frac{a-c}{3b} \\ q_2 &= \frac{1-2s_1+s_2}{2-s_1-s_2}\frac{a-c}{3b} \\ Q &= \frac{a-c}{3b} \end{aligned}$$

and the three firms obtain profits

$$\begin{aligned} \pi_1 &= \frac{(1-2s_2+s_1)(1-2s_1-2s_2)}{(2-s_1-s_2)^2}\frac{(a-c)^2}{9b} \\ \pi_2 &= \frac{(1-2s_1+s_2)(1-2s_1-2s_2)}{(2-s_1-s_2)^2}\frac{(a-c)^2}{9b} \\ \pi_u &= \frac{1}{(2-s_1-s_2)}\frac{(a-c)^2}{3b} \\ \Pi &= 2\frac{(a-c)^2}{9b} \end{aligned}$$

where Π denotes the industry profit. Finally, in the absence of investment, consumer surplus and welfare are

$$\begin{aligned} CS &= \int_0^Q (a-bz)dz-(a-bQ)Q = \int_0^{\frac{1}{3}\frac{a-c}{b}} (a-bz)dz-(a-b\frac{1}{3}\frac{a-c}{b})\frac{1}{3}\frac{a-c}{b} = \frac{1}{18}\frac{(a-c)^2}{b} \\ W &= \int_0^Q (a-bz)dz-(c)Q = \int_0^{\frac{1}{3}\frac{a-c}{b}} (a-bz)dz-(c)\frac{1}{3}\frac{a-c}{b} = \frac{5}{18}\frac{(a-c)^2}{b} \end{aligned}$$

Proof of Lemma 2:

At the second stage, the Cournot quantity setting stage, the best response of the downstream firms is given by

$$\begin{aligned}
\frac{\partial V_1}{\partial q_1} &= \frac{\partial\left((a-b(q_1+q_2))q_1 - w_1q_1 + s_1(w_1q_1+w_2q_2-(c-x)(q_1+q_2)-y\frac{x^2}{2}\right)}{\partial q_1} \\
&= a-2bq_1-bq_2-w_1+s_1\left(w_1-c+x\right)=0 \\
\frac{\partial V_2}{\partial q_2} &= \frac{\partial\left((a-b(q_1+q_2))q_2 - w_2q_2 + s_2(w_1q_1+w_2q_2-(c-x)(q_1+q_2)-y\frac{x^2}{2}\right)}{\partial q_2} \\
&= a-bq_1-2bq_2-w_2+s_2\left(w_2-c+x\right)=0
\end{aligned}$$

that result in the following equilibrium quantities

$$\begin{aligned}
q_1(w_1,w_2) &= \frac{a-2\left(w_1-s_1\left(w_1-c+x\right)\right)+w_2-s_2\left(w_2-c+x\right)}{3b} \\
q_2(w_1,w_2) &= \frac{a-2\left(w_2-s_2\left(w_2-c+x\right)\right)+w_1-s_1\left(w_1-c+x\right)}{3b} \\
Q(w_1,w_2) &= 2\frac{a-\frac{w_1-s_1(w_1-c+x)+w_2-s_2(w_2-c+x)}{2}}{3b}
\end{aligned}$$

The demand for the upstream firm is derived from these outputs. The upstream firm chooses w_1, w_2 and x to maximize V_u which is the same as maximizing

$$\pi_u = \left(w_1q_1+w_2q_2-(c-x)(q_1+q_2)-y\frac{x^2}{2}\right)$$

The first-order conditions for profit maximization are

$$\begin{aligned}
\frac{\partial \pi_u}{\partial w_1} &= \frac{a+c-x-4w_1\left(1-s_1\right)+w_2\left(2-s_1-s_2\right)+\left(s_2-3s_1\right)\left(c-x\right)}{3b}=0 \\
\frac{\partial \pi_u}{\partial w_2} &= \frac{a+c-x-4w_2\left(1-s_2\right)+w_1(2-s_1-s_2)+\left(s_1-3s_2\right)\left(c-x\right)}{3b}=0 \\
\frac{\partial \pi_u}{\partial x} &= \frac{2a+w_1\left(3s_1-s_2-1\right)-w_2\left(s_1-3s_2+1\right)-2\left(s_1+s_2\right)\left(c-x\right)}{3b}-xy=0
\end{aligned}$$

from where we obtain the equilibrium values

$$
\begin{aligned}
x &= \frac{2(2-s_1-s_2)(a-c)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)} \\
w_1 &= \frac{by\left(a(6-s_1-5s_2)-c\left(11s_1+7s_2+s_1^2+s_2^2-14s_1s_2-6\right)\right)-2a(2-s_1-s_2)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)} \\
w_2 &= \frac{by\left(a(6-5s_1-s_2)-c\left(7s_1+11s_2+s_1^2+s_2^2-14s_1s_2-6\right)\right)-2a(2-s_1-s_2)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)}
\end{aligned}
$$

Second-order conditions for profit maximization follow from the Hessian matrix,

$$
\begin{bmatrix}
-\frac{4}{3}\frac{1-s_1}{b} & \frac{1}{3}\frac{2-s_1-s_2}{b} & \frac{1}{3}\frac{3s_1-s_2-1}{b} \\
\frac{1}{3}\frac{2-s_1-s_2}{b} & -\frac{4}{3}\frac{1-s_2}{b} & -\frac{1}{3}\frac{s_1-3s_2+1}{b} \\
\frac{1}{3}\frac{3s_1-s_2-1}{b} & -\frac{1}{3}\frac{s_1-3s_2+1}{b} & -\frac{1}{3}\frac{-2s_1-2s_2+3by}{b}
\end{bmatrix}
$$

and require that the following three conditions hold:

$$
\begin{aligned}
-\frac{4}{3}\frac{1-s_1}{b} &< 0 \\
\frac{1}{9b^2}\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right) &> 0 \\
\frac{-1}{9b^3}\left(by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)\right) &< 0
\end{aligned}
$$

Moreover, the upstream firm's marginal cost cannot become negative after investment. This means that

$$
\begin{aligned}
c-x &= c-\frac{2(2-s_1-s_2)(a-c)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)} \\
&= \frac{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)\frac{a}{c}}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)}c>0
\end{aligned}
$$

This is implied by Assumption 1b:

$$
\begin{aligned}
by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)\frac{a}{c} &> 0 \\
by &> \frac{2(2-s_1-s_2)\frac{a}{c}}{\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)}
\end{aligned}
$$

This condition also implies that the second-order conditions hold and that $x>0$ and $w_i>0$.

Plugging the equilibrium input price and cost reduction into the output expressions yields

$$
\begin{aligned}
q_1(s_1,s_2) &= \frac{y(1-s_1)(s_1-3s_2+2)(a-c)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)} \\
q_2(s_1,s_2) &= \frac{y(1-s_2)(s_2-3s_1+2)(a-c)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)} \\
Q(s_1,s_2) &= \frac{y\left(4(1-s_2)(1-s_1)-(s_1-s_2)^2\right)(a-c)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)}
\end{aligned}
$$

from where the following equilibrium profits can be obtained:

$$
\begin{aligned}
\pi_u &= w_1q_1+w_2q_2-(c-x)(q_1+q_2)-y\frac{x^2}{2} \\
&= \frac{(2-s_1-s_2)\,y\,(a-c)^2}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)} \\
\pi_1 &= (a-b(q_1+q_2))q_1-w_1q_1=\frac{(1-s_1)(2-7s_1-3s_2+8s_1s_2)(2-3s_2+s_1)\,by^2(a-c)^2}{\left(by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)\right)^2} \\
\pi_2 &= (a-b(q_1+q_2))q_2-w_2q_2=\frac{(1-s_2)(2-3s_1-7s_2+8s_1s_2)(2-3s_1+s_2)\,by^2(a-c)^2}{\left(by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)\right)^2}
\end{aligned}
$$

Finally,

$$
\begin{aligned}
CS &= \frac{1}{2}\left(\frac{y\left(4(1-s_2)(1-s_1)-(s_1-s_2)^2\right)(a-c)}{by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)}\right)^2 b \\
W &= y(a-c)^2\,\frac{\begin{array}{c} by\left(20(1-s_2)(1-s_1)-(s_1-s_2)^2\right)\times \\ \times\left(4(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-16(1-s_1-s_2)-4(s_1+s_2)^2\end{array}}{2\left(by\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)-2(2-s_1-s_2)\right)^2}
\end{aligned}
$$

If there was no investment in cost reductions, then $x=0$ and

$$
\begin{aligned}
\frac{\partial \pi_u}{\partial w_1} &= \frac{a+c-4w_1(1-s_1)+w_2(2-s_1-s_2)+(s_2-3s_1)c}{3b}=0 \\
\frac{\partial \pi_u}{\partial w_2} &= \frac{a+c-4w_2(1-s_2)+w_1(2-s_1-s_2)+(s_1-3s_2)c}{3b}=0
\end{aligned}
$$

That result in

$$
\begin{aligned}
w_1 &= \frac{-a\left(s_1+5s_2-6\right)-c\left(11s_1+7s_2+s_1^2+s_2^2-14s_1s_2-6\right)}{12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2} \\
w_2 &= \frac{-a\left(5s_1+s_2-6\right)-c\left(7s_1+11s_2+s_1^2+s_2^2-14s_1s_2-6\right)}{12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2}
\end{aligned}
$$

with second-order conditions following from the Hessian matrix

$$
\begin{bmatrix}
\frac{-4+4s_1}{3b} & \frac{2-s_1-s_2}{3b} \\
\frac{2-s_1-s_2}{3b} & \frac{-4+4s_2}{3b}
\end{bmatrix}
$$

where and $\frac{-4+4s_1}{3b}<0$ the determinant is $\frac{12(1-s_2)(1-s_1)-(s_1-s_2)^2}{9b^2}>0$

The equilibrium quantities that result from these input prices are

$$
\begin{aligned}
q_1 &= \frac{\left(1-s_1\right)\left(2-3s_2+s_1\right)}{12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2}\frac{a-c}{b} \\
q_2 &= \frac{\left(1-s_2\right)\left(2-3s_1+s_2\right)}{12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2}\frac{a-c}{b} \\
Q &= \frac{4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2}{12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2}\frac{a-c}{b}
\end{aligned}
$$

The equilibrium profits are

$$
\begin{aligned}
\pi_1 &= \frac{\left(s_1-1\right)\left(s_1-3s_2+2\right)\left(7s_1+3s_2-8s_1s_2-2\right)}{\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}\frac{\left(a-c\right)^2}{b} \\
\pi_2 &= \frac{\left(s_2-1\right)\left(3s_1-s_2-2\right)\left(-3s_1-7s_2+8s_1s_2+2\right)}{\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}\frac{\left(a-c\right)^2}{b} \\
\pi_u &= \frac{2-s_1-s_2}{12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2}\frac{\left(a-c\right)^2}{b} \\
\Pi &= \frac{8\left(1-s_2\right)\left(1-s_1\right)\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)}{\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}\frac{\left(a-c\right)^2}{b}
\end{aligned}
$$

and

$$
\begin{aligned}
CS &= \int_0^Q (a-bz)dz-(a-bQ)Q=\frac{\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}{\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}\frac{(a-c)^2}{2b} \\
W &= \int_0^Q (a-bz)dz-cQ \\
&= \frac{\left(20\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)}{\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}\frac{(a-c)^2}{2b}
\end{aligned}
$$

Proof of Corollary 1: The proof follows from the difference in the equilibrium investment levels presented in lemma 1 and lemma 2:

$$
\begin{aligned}
&\frac{2\left(2-s_1-s_2\right)(a-c)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)}-\frac{2(a-c)}{3by\left(2-s_1-s_2\right)-2} \\
= &\frac{8by\left(s_1-s_2\right)^2(a-c)}{\left(by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)\right)\left(3by\left(2-s_1-s_2\right)-2\right)}>0
\end{aligned}
$$

Proof of Corollary 2: The proof follows from the difference in the equilibrium output levels presented in lemma 1 and lemma 2:

$$
\begin{aligned}
&\frac{y\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)(a-c)}{by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)}-\frac{\left(2-s_1-s_2\right)}{3by\left(2-s_1-s_2\right)-2}y(a-c) \\
= &\frac{-2y\left(s_1-s_2\right)^2\left(by\left(2-s_1-s_2\right)-2\right)(a-c)}{\left(3by\left(2-s_1-s_2\right)-2\right)\left(by\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)-2\left(2-s_1-s_2\right)\right)}
\end{aligned}
$$

Proof of Proposition 1: Let $s_1 \neq s_2 = 0$. From lemma 1 and lemma 2, welfare increases with IPD if and only if

$$
y(a-c)^2\frac{by\left(20-s_1^2-20s_1\right)\left(4-s_1^2-4s_1\right)-16\left(1-s_1\right)-4s_1^2}{2\left(by\left(12\left(1-s_1\right)-s_1^2\right)-2\left(2-s_1\right)\right)^2}>\frac{1}{2}y(a-c)^2\frac{5by\left(2-s_1\right)^2-4}{\left(3by\left(2-s_1\right)-2\right)^2}
$$

which is equivalent to

$$
-b^2y^2\left(24-24s_1-s_1^2\right)\left(2-s_1\right)^2+8by\left(16-30s_1+11s_1^2+s_1^3\right)-4\left(8-12s_1+s_1^2\right)>0
$$

or

$$
(by)^- < by < (by)^+
$$

with

$$
\begin{aligned}
(by)^{-} &= \frac{4\left(16-30s_1+11s_1^2+s_1^3\right)-2\sqrt{256-1152s_1+1680s_1^2-768s_1^3-104s_1^4+96s_1^5+5s_1^6}}{(2-s_1)^2\left(24-24s_1-s_1^2\right)} \\
(by)^{+} &= \frac{4\left(16-30s_1+11s_1^2+s_1^3\right)+2\sqrt{256-1152s_1+1680s_1^2-768s_1^3-104s_1^4+96s_1^5+5s_1^6}}{(2-s_1)^2\left(24-24s_1-s_1^2\right)}
\end{aligned}
$$

There may be values for by in the interval above that verify assumptions 1a and 1b when $s_2 = 0$, which become:

Assumption 1a: $by > \frac{2}{3(2-s_1-s_2)}\frac{a}{c} > \frac{2}{3(2-s_1-0)}$

Assumption 1b: $by > \frac{2(2-s_1-s_2)}{\left(12(1-s_2)(1-s_1)-(s_1-s_2)^2\right)}\frac{a}{c} > \frac{2(2-s_1-0)}{\left(12(1-0)(1-s_1)-(s_1-0)^2\right)}$.

In the absence of investment, we the change in welfare from moving from UIP to IPD would be

$$
\begin{aligned}
&\frac{\left(20\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)}{\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}\frac{(a-c)^2}{2b}-\frac{5}{18}\frac{(a-c)^2}{b} \\
= \; & -\frac{\left(24\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)\left(s_1-s_2\right)^2}{\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}\frac{2(a-c)^2}{9b} < 0
\end{aligned}
$$

and IPD would always lower welfare.

As far as consumer surplus is concerned, in the presence of investment, IPD can lead to higher or lower levels of consumer surplus depending on the condition presented in Corollary 2. If there was no investment, the effect of IPD on consumer surplus is negative if $s_1 \neq s_2$:

$$
\begin{aligned}
\frac{\left(4\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}{\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2}\frac{(a-c)^2}{2b}-\frac{1}{18}\frac{(a-c)^2}{b} &= \\
-\frac{4\left(6\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)\left(s_1-s_2\right)^2(a-c)^2}{9b\left(12\left(1-s_2\right)\left(1-s_1\right)-\left(s_1-s_2\right)^2\right)^2} &< 0
\end{aligned}
$$