UNIVERSIDADE CATÓLICA PORTUGUESA

Bond return predictability in the frequency domain

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Católica Porto Business School
April, 2024

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Final Assignment in the modality of Dissertation presented to Universidade Católica Portuguesa to obtain the degree of Master in Finance

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Acknowledgments

To my parents and my sister, Rita, for being by my side since day one, for believing in me, and for always providing me with the best experiences. Without your support, this journey would have been much more difficult.

To my grandmothers, who always offered especial affection and unwavering support, and to my grandfathers, that unfortunately are not here with us, but whose pride I hope to have earned with this achievement.

A special thank you extends to the rest of my family, including Francisco, for being there and supporting my academic journey.

To my best friends, Mariana, Rita, and Sofia, for always encouraging and guiding me with their words of motivation and friendship. You were my pillar during this challenge.

To my friends, especially João, who helped me tremendously on this path, for the assistance, motivation, and friendship, and to Luís for always offering to help whenever needed. To all others, thank you for understanding my absence and for always being there, in the best and worst moments.

To my professors, who taught me so much, and a special thank you to my advisors, Professors Gonçalo Faria and Fábio Verona, for always being available to discuss ideas and concerns to achieve the best work possible.

Resumo

Nesta dissertação, propomos analisar o poder de previsão *out-of-sample* dos retornos das obrigações do tesouro dos USA utilizando técnicas de domínio de frequência. Seguimos a investigação de Faria e Verona (2018, 2019, 2020a, b, 2021) e utilizamos uma abordagem baseada em *wavelets* para decompor uma série temporal em diferentes componentes de frequência: alta frequência, frequência do ciclo económico e baixa frequência. Desta forma, será possível revelar informações que as técnicas de domínio de tempo ocultam nas previsões. As variáveis de previsão de Almadi et al. (2014) foram utilizadas para prever os restornos *out-of-sample* das obrigações do tesouro dos USA em dois períodos de amostra diferentes.

Os resultados mostram que o modelo tem uma limitação significativa em termos de previsão, especialmente quando comparando variáveis de previsão bem sucedidas noutros métodos. Quando aplicadas abordagens de domínio de frequência como *wavelets*, a eficácia reduz significativamente. A investigação destaca a vulnerabilidade do modelo a situações de mercado desfavoráveis e fornece informações sobre a eficácia do modelo em períodos de instabilidade financeira e macroeconómica.

Palavras-chave: Previsibilidade, Retornos de obrigações, Previsões *out-of-sample*, Previsão de retornos, Domínio da frequência, Análise multiresolução

Abstract

In this dissertation we analyse the out-of-sample forecast of the long-term government bond yield using frequency domain techniques. We follow Faria and Verona (2018, 2019, 2020a, b, 2021) and use a wavelet-based approach to break down a time series into different component frequencies: high-frequency, business-cycle frequency and low-frequency. In this way it is possible to reveal hidden information that time domain techniques cover in their prediction. Almadi et al. (2014) predictors were used to predict out-of-sample bond returns with frequency domain techniques in two different sample periods.

The results show that the model has a significant shortcoming in terms of bond return forecasting accuracy, especially when compared to other successful models. The efficacy of the predictors decreases when applying frequency domain approaches like wavelets. Our research highlights the model's limitations to unfavourable market situations and provides insight into how the model performs during times of financial and macroeconomic instability.

Keywords: Predictability, Bond returns, Out-of-sample forecasts, Return forecasting, Frequency domain, Multiresolution analysis

Index

Index of Figures

Index of Tables

Introduction

Financial markets serve a pivotal economic function by acting as a channel for the transfer of funds from savers to economic agents requiring capital beyond their available resources, constituting a direct means of funding and financing. According to Deutsch Bank (2015), debt markets, which include bonds, accounted for approximately 77% of the financial market in 2014. Because of this, the significance of studying the debt market and, particularly, the analysis of bonds, is crucial for an investor. However, in contrast with what happens with equity returns, the literature on bond return predictability is still scarce. Therefore, there exists a need for continual refinement and advancement of models designed to capture the available information accessible to participants in the bond market. This process is essential for the construction of a successful bond portfolio.

The research on bond returns is still very scarce but a variety of different methodologies and approaches have been used. Dynamic component analysis was used by Ludvigson and Ng (2009) to investigate the connection between excess bond returns and macroeconomic fundamentals. Thornton and Valente (2012) evaluated the economic significance of out-of-sample forecasts while searching into in-sample predictability models. A unique approach to affine term structure models that include bond excess returns and yields was presented by Sarno et al. (2016). Dynamic Asset Allocation portfolios were assessed by Almadi et al. (2014), who found that projections based on a variety of variables outperformed constant expected returns. Out-of-sample predictions have been focused on the time domain.

The main research question to be addressed is this study is: can the out-of-sample predictability of aggregate bond index returns be improved by using frequency domain techniques? Recent studies about out-of-sample predictability of equity returns (e.g. Bandi et al., 2019, Faria and Verona 2018, 2020, 2021) have shown that there are strong statistical and economic gains when making the forecasting using frequency domain techniques as they can capture the predictors effective forecasting power by eliminating the noise embedded in a time series. As an example, Figures 1-4 we decompose the time series of the long term yield into three different frequency components: high-frequency, business-cycle frequency, and low-frequency. The proposed method analyses what frequencies have the best prediction ability eliminates the one(s) that don't perform as well as the benchmark. We do so by using wavelets, a relatively new technique in economics and finance in the field of frequency domain research.

Figure 1: *Original time series of predictor TMS. Sample period 1973 – 2020, monthly frequency.*

TMS High-frequency component

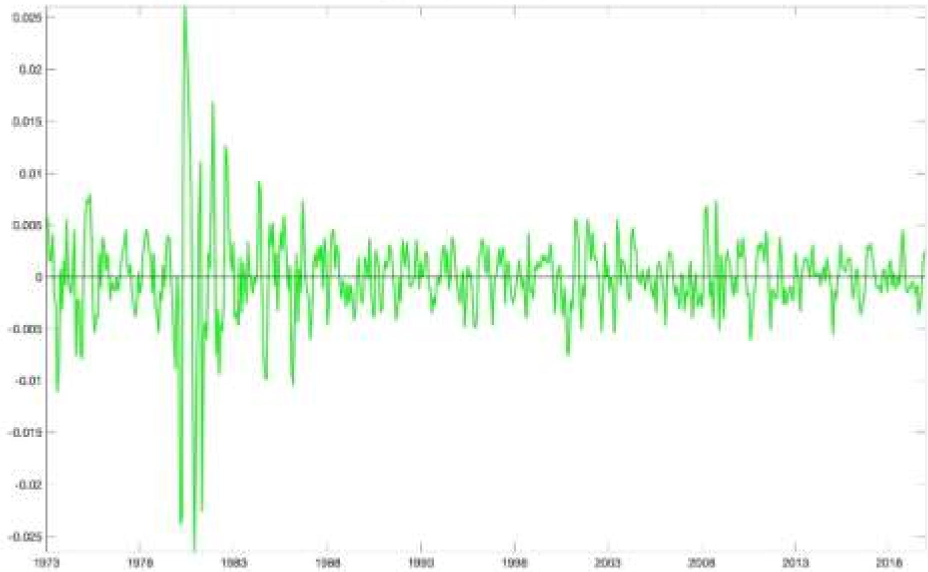

Figure 2: *High-frequency component of predictor TMS. High-frequency captures oscillations of less than 16 months in the sample period of 1987 – 2020, monthly frequency.*

TMS Business-cycle frequency component

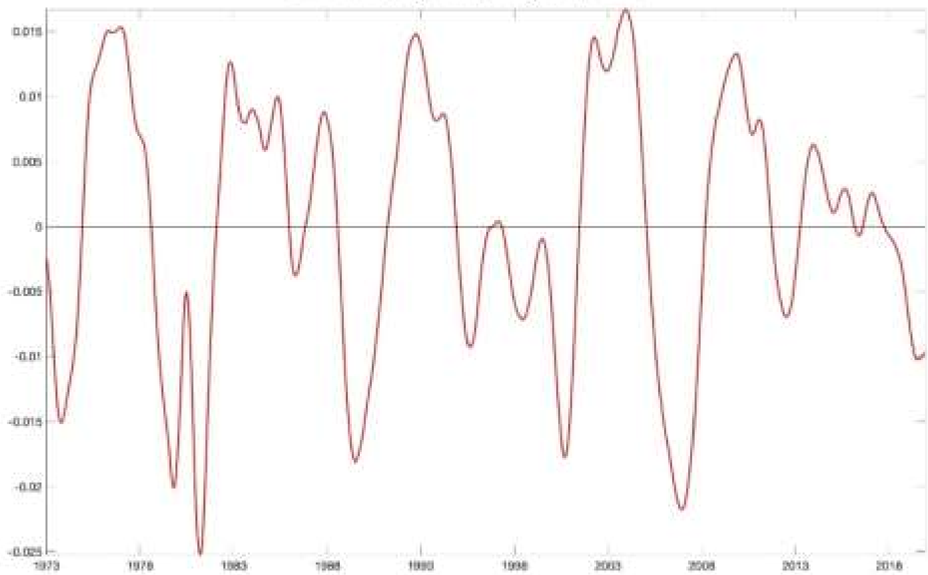

Figure 3: *Business-cycle component of predictor TMS. Business-cycle captures oscillations between 16 and 128 months in the sample period of 1987 – 2020, monthly frequency.*

TMS Low-frequency component

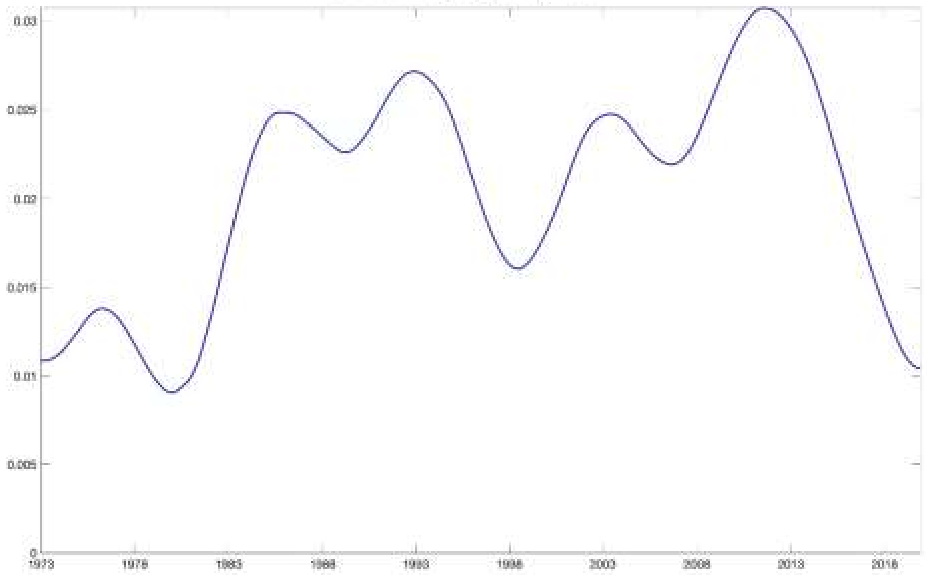

Figure 4: *Low-frequency component of predictor TMS. Low-frequency captures oscillations of more than 128 months in the sample period of 1987 – 2020, monthly frequency.*

In this dissertation we used Almadi et al. (2014) set of macroeconomic and technical predictors and includes additional predictors used by Faria and Verona (2020). We consider two different sample periods in the study, the first runs from January 1973 to December 2022 and the second from January 1973 to December 2020.

Our main finding is that the model in the frequency domain does not perform well in forecasting bond returns. Successful predictors in Almadi et al. (2014) lose predictive ability when using wavelets and this frequency domain method. Despite this limitation, our analysis does shed light on the model's performance during periods of financial and macroeconomic crisis. We find that the model's predictive ability of bond returns declines noticeably in such turbulent circumstances. This demonstrates how vulnerable the model is to unfavourable market circumstances and emphasizes the need for more development to strengthen its resilience in a range of economic scenarios.

This dissertation is organized as follows. In Chapter 1, we review the related literature. In Chapter 2, we present a detailed explanation of the methodology used to construct the predictors and explain frequency domain and wavelets concepts. In Chapter 3, we evaluate, statistically and economically, the results obtained in out-of-sample forecasting and perform a robustness analysis. In Chapter 4, we conclude.

Chapter 1

1.Literature review

In this chapter, we present a review of the existing literature on out-of-sample bond return predictability (subsection 1.1) and a review of the frequency domain and wavelets in economics and finance (subsection 2.2).

1.1 Out-of-sample Bond Return Predictability

Research on bond return predictability has revealed a perplexing disconnect between robust statistical evidence of predictability and the inability to translate return forecasts into actual economic gains. It has been observed that a model's strong predictive performance when tested on the same data it was trained on (in-sample) does not necessarily translate into satisfactory forecasting ability when applied to new, unseen data (out-of-sample) (Kilian et al. (2004; 2006)).

Goyal and Welch (2008) conducted a pivotal study that significantly influenced the evaluation of return predictability in real-time. Their research introduced the concept of out-of-sample (OOS) exercises as a more relevant method for assessing the effectiveness of return prediction models. The focus of their work was on traditional predictors suggested by the academic literature to be good predictors of the equity premium and their performance in OOS forecasts on U.S. data up to 2008. Their key finding was that these conventional predictors exhibited poor performance when applied out-of-sample, indicating that they struggled to accurately predict returns beyond the period they were initially tested on as they were very unstable and would not have helped an investor with access only to available information to profitably time the market. This observation highlighted the need for more robust and reliable forecasting methods to improve the out-of-sample forecast ability of equity premium prediction models.

In the context of bond return predictability, Ludvigson and Ng (2009) used a dynamic factor analysis methodology developed for big datasets to investigate the potential empirical links between predictable variations in excess bond returns and macroeconomic fundamentals. The addition or absence of macroeconomic information in excess bond return projections has a considerable impact on the cyclical patterns found in estimated risk premia for both returns and long-term yields. Notably, in the absence of macroeconomic forces, risk premia exhibit nearly acyclical behaviour. These findings highlight the need of incorporating information beyond what is strictly present in the yield curve. This is useful in highlighting countercyclical differences in bond risk premia at different points in the business cycle, emphasizing the significance of using a broader range of indicators to gain a thorough understanding of these variations.

Thornton and Valente (2012) investigated the predictability of bond returns within the sample data while also evaluating the economic value derived from using these predictive models on out-of-sample data. By assessing both in-sample and out-of-sample performance, the researchers aim to gain a comprehensive understanding of the predictive models' effectiveness in generating meaningful economic returns. They found that the performance of all predictive models based on forward rates deteriorates over time and confirmed the difficulty of improving a simple naive benchmark and that the predictability of bond excess returns found in the literature does not necessarily translate into economic gains for investors.

Almadi et al. (2014) evaluated the effectiveness of Dynamic Asset Allocation (DAA) portfolios constructed using out-of-sample forecasts at various intervals—monthly, quarterly, semi-annual, and annual—for U.S. stock, bond, and bill returns. The investigation shows that stock and bond return forecasts based on major components drawn from a wide range of fundamental, macroeconomic, and technical variables outperform the baseline forecast of constant expected return. To calculate posterior expected returns, the study uses a modified Black-Litterman model with stock, bond, and bill return projections treated as active views. The computation of these returns, as well as DAA portfolio weights for each forecast horizon, assumes a portfolio rebalancing frequency that corresponds to the forecast horizon and addresses transaction costs. The methodology used involves using the multivariate predictive regression model, incorporating information from all N predictors. The conclusion was that monthly rebalancing tends to be most advantageous when transaction costs are low, while annual rebalancing becomes preferable when transaction costs are higher and that optimal rebalancing enhances returns for investors exploiting out-of-sample return predictability through DAA.

Sarno et al. (2016) developed a new estimation approach for affine term structure models that includes both yields and bond excess returns. This integrated methodology enables the production of predictive information that would otherwise be hidden in conventional estimations. The model shows positive economic value primarily during periods of high macroeconomic uncertainty, although showing to negative value in low uncertainty scenarios. The expectations theory remains useful as a standard for guiding bond market investment decisions, especially in low-uncertainty environments. The paper also assesses realized bond excess returns using model risk premia across a variety of horizons and maturities, contributing to an extensive understanding of the model's implications.

Gargano et al. (2017) introduced a three-factor model that includes the Fama and Bliss (1987) forward spread; the Cochrane and Piazzesi (2005) combination of forward rates, yield spreads from Campbell and Shiller (1991), and the Ludvigson and Ng (2009) macro factor. This model significantly improves out-of-sample forecast accuracy compared to a model based solely on the expectations hypothesis. As a result, these enhanced predictions lead to higher risk-adjusted portfolio returns after accounting for estimation error and model uncertainty.

More recently, Bianchi et al. (2020), studied the predictive capabilities of machine learning methodologies, specifically focusing on extreme trees and neural networks, in the context of forecasting bond returns. The study is conducted using two standard methodologies: the first uses yield curve data only, as in Cochrane and Piazzesi (2005), and the second uses a wide range of macroeconomic indicators, as in Ludvigson and Ng (2009). Based on the methodology described by Gu et al. (2020), the study assesses the prediction performance of different machine learning algorithms out-of-sample. To estimate excess Treasury bond returns across a range of maturities, this includes using partial least squares, penalized linear regressions, boosted regression trees, random forests, severely randomized regression trees, and shallow and deep neural networks. The findings underscore the efficacy of modern statistical learning techniques, particularly nonlinear methods like extreme trees and neural networks, in capturing predictable variations in bond excess returns. Importantly, these methods significantly outperform traditional data compression and penalized regression techniques in terms of out-of-sample predictive R^2. Furthermore, the study highlights the added value of incorporating macroeconomic and financial variables, enhancing the predictive accuracy beyond models relying solely on the yield curve or its nonlinear transformations.

Although not all models perform well in terms of bond prediction, such as Thornthon and Valente (2012), which concluded that a model based on forward rates did not bring economic gains to investors, other models, such as Sarno et al. (2016), have been able to achieve economic gains during periods of high instability. Gargano et al. (2017) and Bianchi et al. (2020) have provided valuable insights into bond predictability as well.

1.2 Frequency domain and Wavelets in Economics and Finance

Although there is extensive literature on the out-of-sample predictability of bond returns, most of the forecasting evidence has been traditionally examined in the time domain. This dissertation proposes a novel approach to explore out-of-sample predictability of bond returns in the frequency domain using wavelets.

Ramsey and Lampart (1998) pioneered the application of wavelets to analyse the relationship between macroeconomic variables. Kim and In (2005) introduced a wavelet multiscaling method that systematically decomposes time series data on a scale-by-scale basis, elucidating the intricate relationship between stock returns and inflation and Gencay et al (2005) also used the wavelet multiscaling technique to investigate systematic risk.

Gallegati and Ramsey (2013) investigated the link between bond market Q, stock market Q, and aggregate investments in the United States. Their research not only examined the stability of this relationship across frequencies and time, but also emphasized the need of understanding frequency dynamics when evaluating aggregate investment. Also revisiting the Q theory of investment, Kilponen and Verona (2022) improved the accuracy of short-term forecasts by using wavelet analysis. Their method involved dividing time series into orthogonal components with different frequencies using wavelet multiresolution analysis. Considering the frequency relationship between variables, the Q theory revealed enhanced data fitting.

The contribution of Crowley (2007) to the field by offering another alternative to traditional econometric methods to study economics. It provides literature about wavelets and discusses potential and future applications of wavelet analysis in economics. Aguiar-Conraria and Soares (2011, 2012 and 2014) also had an important role in providing ample literature on applications of wavelets in economics and finance.

Wavelet studies in econometrics by Rua (2010, 2017) are important as they show how well wavelet analysis can be incorporated into forecasting models for economic variables like GDP growth and inflation. Rua highlights the value of frequency-based approaches in contemporary econometric analysis by demonstrating how wavelet techniques may capture different temporal components and enhance both estimate and forecasting accuracy in models.

Ferreira and Santa-Clara (2011) introduced the sum-of-the-parts (SOP) approach. Leveraging the diverse time series persistence of components, this method exploited out-of-sample R-squares, providing a comparative assessment against historical means. This approach was improved by Faria and Verona (2018) using the wavelets (SOPWAV) to frequency decompose stock returns.

In tis study we follow the methodology described by Faria and Verona (2020a, b, 2021) and use a frequency decomposition approach with wavelets within a standard out-of-sample forecasting regression setup. Faria and Verona (2020a, 2021) found that using this strategy to forecast equity premiums resulted in a significant improvement in both statistical and economic performance. Furthermore, the authors applied their knowledge of equity premium and bond predictability to produce promising results in the context of active portfolio management (Faria & Verona, 2020b).

To the best of our knowledge, there is no comparable work applying wavelets for the same purpose of our investigation. Our contribution to literature is to expand Faria and Verona (2020a,b, 2021) research and investigate if and how different frequency components of several macroeconomic and technical indicator variables can capture additional information regarding long-term government bond returns.

Chapter 2

2. Data and Methodology

In this chapter, we present an overview of the data and predictors utilized (subsection 2.1) and offer a comprehensive description of the applied methodology in this dissertation (subsection 2.2).

2.1 Data and predictors

Our target variable is the long-term government bond return (LTR), as predictors, use seven variables taken from the Goyal and Welch (2008) database, each contributing to our analysis.

The full sample period, with monthly data, runs from January 1973 and December 2022. The variables are the following:

- Long-term yield (LTY): long-term government bond yield.
- Term spread (TMS): difference between the long-term government bond yield and the T-bill.
- Default yield spread (DFY): difference between Moddy's BAA- and AAA- rated corporate bond yields.
- Default return spread (DFR): difference between long-term corporate bond and long-term government bond returns.
- Inflation rate (INFL): calculated from the Consumer Price Index (CPI) for all urban consumers.
- Output gap (OG): deviation of the log of industrial production from a quadratic trend. The output gap predictor was calculated based on information from Industrial Production available at St. Louis Federal Reserve Economic Data (FRED), employing the one-sided Hodrick-Prescott filter.
- MOMBY (6): dummy variable equal to -1 (1) if the bond yield is more than 5 basis points above (below) its 6-month moving average, and 0 otherwise.

- MOMBY (12): dummy variable equal to -1 (1) if the bond yield is more than 5 basis points above (below) its 12-month moving average, and 0 otherwise.

For the set of predictors, we follow Almadi et al. (2014) and include also other predictors commonly used in this area of research from Goyal and Welch (2008) database.

2.2 Methodology

This section contains a detailed explanation of frequency domain and wavelets (2.2.1) and out-of-sample forecasting (subsection 2.2.2). After that, the approach used to evaluate results is explained by using the OOS R-Squared and the CER gains analysis (2.2.3).

2.2.1 Frequency domain and Wavelets

Joseph Fourier's fundamental insight into signals being functions of frequencies initiated the foundation for frequency domain. The basis for frequency domain analysis lies in the ability to represent a signal in the time domain within the frequency domain through its decomposition into distinct components. In the time domain, a signal's behaviour over time is depicted, while the frequency domain illustrates how the signal behaves across various frequencies within the original time series, but it falls short on efficiently representing abrupt changes due to its reliance on sine waves that lack localization in time or space. Historically, the Fourier and spectral analysis stood as the conventional transform methods.

More recently, wavelets have emerged as a valuable alternative. Wavelets offer enhanced adaptability, addressing limitations of traditional methods and providing a better approach to capturing both time and frequency features in a time series just by varying the time window size. To show a high-frequency (low-frequency) feature of the time series one should use a short (long) time window. A wavelet is defined as a function of finite length with oscillatory behaviour around a time axis. Its efficacy decreases as it moves away from the center. According to Crowley (2007), wavelets prove exceptionally useful when a variable displays varying behaviour at different time intervals (jumps or breaks) or exhibits time-varying volatility.

The discrete wavelet transform (DWT) multiresolution analysis (MRA) enables the decomposition of a time series into its various multiresolution (frequency) components. There are two categories of wavelets: father wavelets (ϕ), capturing the smooth and low-frequency part of the series, and mother wavelets (ψ), capturing the high-frequency components of the series, where $\int \phi(t)dt = 1$ and $\int \psi(t)dt = 0$.

Given a time series y_t with a certain number of observations N, its wavelet multiresolution representation is given by:

$$y_t = \sum_k s_{J,k}\ \phi_{J,k}(t) + \sum_k d_{J,k}\ \psi_{J,k}(t) + \sum_k d_{J-1,k}\ \phi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\ \psi_{1,k}(t)$$

where J represents the number of multiresolution levels (or frequencies), k defines the length of filter $\phi_{J,k}\,(t)$ and $\psi_{J,k}(t)$ are the wavelet functions, and $s_{J,k}$, $d_{j,k}$, $d_{J-1,k}$, …, $d_{1,k}$ are the wavelet coefficients.

The wavelet functions are generated from the father and mother wavelets through scaling and translation as follows:

$$\phi_{J,k}(t) = 2^{-\frac{J}{2}}\phi(2^{-J} - k)$$
$$\psi_{J,k}(t) = 2^{-\frac{J}{2}}\psi(2^{-j}t - k)$$

And the wavelet coefficients are given by

$$s_{J,k} = \int \gamma t\, \phi_{J,k}(t)d(t)$$

$$d_{j,k} = \int \gamma t \, \psi_{j,k}(t) d(t)$$

Where $j = 1, 2, 3, \dots, J$.

The wavelet multiresolution decomposition of a time series y_t can be rewritten as:

$$y_t = \sum_{j=1}^{J} y_t^{D_j} + y_t^{S_J}$$

Where $y_t^{D_j}$, $j = 1, 2, 3, \dots, J$ are the J wavelet detail components and $y_t^{S_j}$, is the smooth component. The smaller j detail components show the high-frequency characteristics of a time series, capturing its short-term dynamics. While, the higher j detail components represent the dynamics related to the business cycle. Finally, the smooth component includes the lowest frequency, reflecting the long-term dynamics. The resulting filtered time series becomes smoother as the frequency increases (decreases).

We use for the maximal overlap discrete wavelet transform multiresolution analysis (MODWT MRA) as our method. As pointed out by Faria and Verona (2018), this methodology offers three key advantages with respect to the DWT:

i. It is not constrained by a specific sample size.
ii. It is translation-invariant, meaning it is not influenced by the choice of the starting point for the examined time series.
iii. It does not introduce phase shifts in the wavelet coefficients.

This last feature is particularly significance in an out-of-sample forecasting exercise.

Following Faria and Verona (2018, 2020a, b, 2021) we use the MODWT MRA and use the Haar Wavelet filter[1] with reflective boundary conditions.

In line with Faria and Verona (2020) we use three frequency components as potential predictors for long-term government bond return. By employing frequency-domain filtering techniques, we expand an initial set of predictors to incorporate additional predictors. The frequencies used were high-frequency (HF), the business cycle frequency (BCF) and the low-frequency (LF) components of each time series of the original predictors.

We consider a $J = 6$ level maximal overlap discrete wavelet transform multiresolution analysis (MODWT MRA) for each of the original predictors. The determination of the number of frequency bands is contingent on the number of observations in the in-sample period (N). Given that N=192, we ensure that J is such that $J \leq log_2$ N $\simeq$ 8. The MRA decomposition thus delivers seven time-frequency series: six wavelet detail components ($y_t^{D1}$to y_t^{D6}) and the wavelet smooth component (y_t^{S6}). As we use monthly data, the first detail component y_t^{D1} captures oscillations between 2 and 4 months, while detail components y_t^{D2}, y_t^{D3}, y_t^{D4}, y_t^{D5} and y_t^{D6} capture oscillations with a period of 4 - 8, 8 - 16, 16 - 32, 32 - 64 and 64 - 128 months, respectively. Finally, the smooth component y_t^{S6}, which in what follows we re-denote y_t^{D7}, captures oscillations with a period longer than 128 months (10.6 years). The high-frequency component captures oscillations with a period between 2 and 32 months ($y_{HF,t} = \sum_{j=1}^{3} y_t^{D_j}$). The business cycle component ranges from 32 to 128 months ($y_{BCF,t} = \sum_{j=4}^{6} y_t^{D_j}$) and the low-frequency component ($y_{LF,t} = y_t^{S6}$) captures the oscillations for no longer than 128 months.

[1] The Haar filter is a simple and widely used filter (Manchaldore et al., 2010; Malagon et al., 2015) and makes a neat connection to temporal aggregation as the wavelet coefficients are simply differences of moving averages (Ortu et al., 2013; Bandi et al., 2016)

Figures 5 and 6 show the graphs for the different predictors, where we can see the different components. The LF component exhibits a smooth line, the BCF component contains more oscillatory behaviour, and the HF component is extremely detailed.

LTY and its frequency components

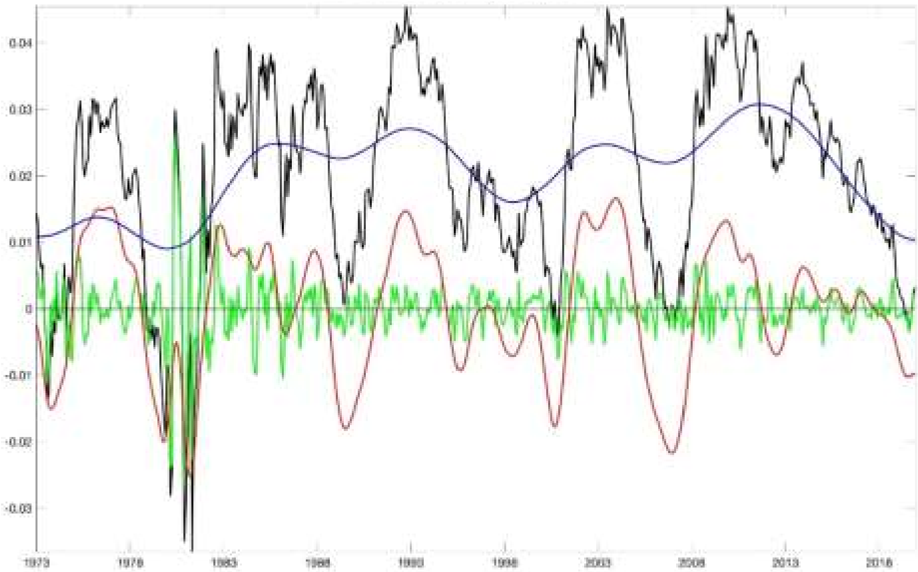

Figure 5: *LTY wavelets decomposition. Graph shows the original time series of each predictor (black line), and its 3 frequency components obtained through wavelets decomposition: high-frequency component captures oscillations of less than 16 months (green line), business-cycle frequency component between 16 and 128 months (red line), and low frequency component more than 128 months (blue line).*

TMS and its frequency components

Figure 6: *TMS wavelets decomposition. Graph shows the original time series of each predictor (black line), and its 3 frequency components obtained through wavelets decomposition: high-frequency component captures oscillations of less than 16 months (green line), business-cycle frequency component between 16 and 128 months (red line), and low frequency component more than 128 months (blue line).*

2.2.2 Out-of-sample Forecasting

The OOS forecasts of the long-term government bond return are generated using a sequence of expanding windows. The initial sample used to make the OOS forecast was the in-sample period of January 1973 to December 1989. This sample is then increased by one observation and a new one-step-ahead OOS forecast is produced until the end of the sample, obtaining a sequence of 396 one-step-ahead observations. The full OOS period spans the period from January 1990 and December 2022.

2.2.3 Forecast evaluation

The Campbell and Thompson's (2008) R_{OS}^2 statistic was used to evaluate the forecasting ability of predictive performance of time series and wavelet-based models. The R_{OS}^2 metric indicates the proportional reduction in mean squared forecast error for the predictive model (MSFEPRED) compared to the historical mean (MSFEHM). Our benchmark model is the historical mean (HM), which is widely used in frequency domain literature. The R_{OS}^2 statistic is provided by:

$$R_{OS}^2 = 100\left(1 - \frac{MSFE_{PRED}}{MSFE_{HM}}\right) = 100\left[1 - \frac{\sum_{t=t_0}^{T-1}(r_{t+1} - \hat{r}_{t+1})^2}{\sum_{t=t_0}^{T-1}(r_{t+1} - \bar{r}_{t+1})^2}\right]$$

where $\hat{r}_{t+1}$ is the LTR forecast for $t+1$ from the predictive model under analysis, and r_{t+1} is the realized LTR from t to $t+1$. So, a positive R_{OS}^2 indicates that our predictive regression has a lower MSFE compared to the historical average benchmark forecast. Consequently, in the context of MSFE, the predictive regression outperforms the benchmark model.

To assess the degree of statistical significance improvement in MSFE, we employ a standard statistic introduced by Clark and West (2007), known as the MSFE adjusted statistic. This statistic tests the null hypothesis, H_0, suggesting that the MSFE with the historical mean is less than or equal to the MSFE with the predictive regression, against the alternative hypothesis, H_1, indicating that the MSFE with the historical mean is greater than the MSFE with the predictive regression. Mathematically, this can be expressed as:

$$\begin{cases} H_0 = R_{OS}^2 \leq 0 \\ H_1 = R_{OS}^2 > 0 \end{cases}$$

If the computed t-statistic exceeds the crucial value, we can reject the null hypothesis (H_0). In this scenario, we may conclude that Mean Squared Forecast

Error (MSFE) with predictive regression outperforms MSFE with the historical mean at a significance level of 1%, 5%, or 10%.

2.2.4 Certainty Equivalent Return

To examine the economic advantages of using the predictive model and frequency decomposition in the context of bond returns, we conduct an economic analysis. At the end of month t, the investor optimally allocates to bond a share:

$$w_t = \frac{1}{\gamma}\frac{\hat{r}_{t+1}}{\hat{\sigma}_{t+1}}$$

Where γ represents the investor's relative risk aversion coefficient, $\hat{r}_{t+1}$ is the forecasted bond return at time t for $t+1$ and $\hat{\sigma}^2_{t+1}$ is the out-of-sample forecast of the variance of the bond return. We used a ten-year moving window of past excess returns to estimate the variance of bond return. Additionally, we set the relative risk aversion coefficient of three and limited the weights (w_t) to be greater than -0.5 and smaller than 1.5 to limit possibilities of leveraging or short-selling the portfolio as used by Rapach et al. (2016), Fan et al (2018) and Faria and Verona (2018).

The certainty equivalent return (CER) is "the annual portfolio management fee that an investor is willing to pay for access to the alternative forecasting model versus the historical average forecast" (Faria and Verona (2019)) and can be computed as:

$$CER = \bar{R}_\rho - 0{,}5\gamma\sigma^2$$

Where $\bar{R}_\rho$ is the sample mean and σ^2 the variance of the portfolio return. Higher CER gains imply larger economic benefit for the investor.

Chapter 3

3. Results

In this chapter we present the results obtained in the Out-of-sample Forecasts (subsection 3.1) and a Robustness Analysis (subsection 3.2) that involves a different forecast horizon and annualized certainty equivalent return gains (CER gains) with different Investor's Relative Risk Aversion.

3.1 Out-of-sample Performance

Statistical findings comparing the out-of-sample performance versus the historical mean with our method are showed in Table 1 below.

	R^2_{OS} (%)			
Variable	TS	HF	BCF	LF
Panel A: Macroeconomic variables				
TB	0.07	-3.4	-0.28	-0.08
LTY	-0.22*	-17.36	1.31***	0.2
INFL	-0.32	-0.46	-3.19	-1.36
TMS	0.17	0.2	-0.28	-0.89
DFY	-1.21	-2.53	-2.31	-0.41
OG	-0.63	-33.15	-1.45	-0.62
DFR	-0.6	-1.66	-3.23	-2.4
Panel B: Technical indicators variables				
MOMBY (6)	0.71*	0.13	-0.25	-0.81
MOMBY (12)	-0.59	-0.56	-9.43	-2.77

Table 1: *Out-of-sample R-Squared period 1973-2022. This table reports out-of-sample results for individual forecasts of bond returns based on 7 macroeconomic variables (Panel A) and 2 technical indicator variables (Panel B). For each predictor, we report the R-Squared for the original time series (time domain) (TS) and the R-Squared for 3 different frequency components obtained through wavelets decomposition (frequency domain):*

*high-frequency (HF), business-cycle frequency (BCF), and low-frequency (LF) components. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.*

All macroeconomic variables, except for the TB and TMS variables, have negative R-Squared values, meaning that they fail to outperform the historical mean when tested in time series. Regarding technical indicators variables, MOMBY(12) also produces negative results in time series. Even while these results are negative in time series testing, they do not improve much when analysed at decomposition frequencies.

In high-frequency, only the variables TMS and MOMBY (6) can outperform the historical mean. That means that when tested in high-frequency, TMS loses predictive power.

In business-cycle frequency component and low-frequency component, the only variable that has a positive R-Squared is LTY achieving a result of 1.31% at BCF with a level of significance of 1%.

Our analysis suggests that the proposed approach does not significantly influence the prediction of bond returns, rather, the results indicate inconsistency in this regard. Additionally, there is a potential issue of testing the model during a macroeconomic crisis period, which could bias the outcomes. These results emphasize the need for additional investigation into the model's robustness in various economic circumstances.

3.2 Robustness Analysis

3.2.1 Different forecast horizon

The out-of-sample analysis conducted earlier was undertaken during a period of considerable macroeconomic instability, notably marked by the onset of the COVID-19 crisis and the Russia-Ukraine conflict. These events exerted profound impacts on global markets, introducing unprecedented volatility and uncertainty. Recognizing the potential influence of such tumultuous occurrences on the overall model results, evaluate our model using a sample that runs from 1973 to 2020. By including data up to 2020, we seek to mitigate the disproportionate influence of the two years characterized by heightened instability. This adjustment enables us to conduct a more comprehensive assessment of the model's effectiveness in forecasting bond returns across diverse economic environments. Such an approach allows us to discern underlying patterns or trends in the model's performance, independent of short-term disruptions or crisis.

In Table 2 below, we present the new out-of-sample performance compared to the historical mean within the out-of-sample period ending in 2020.

	R^2_{OS} (%)			
Variables	TS	HF	BCF	LF
Panel A: Macroeconomic variables				
TB	0.28	-9.66	-0.71	0.32*
LTY	1.26***	-30.21	-0.57	0.8**
INFL	0.01	-0.08	0.05	-0.13
TMS	0.45*	-1.88	0.32	0.25
DFY	0.02	-0.24	-0.5	0.1
OG	-0.43	-26.96	-1.23	-0.38
DFR	-0.53	-0.91	-1.91	-0.59
Panel B: Technical indicators variables				
MOMBY (6)	0.71*	0.13	-0.25	-0.81
MOMBY (12)	-0.59	-0.56	-9.43	-2.77

Table 2: *Out-of-sample R-Squared period 1973-2020. This table reports out-of-sample results for individual forecasts of bond returns based on 7 macroeconomic variables (Panel A) and 2 technical indicator variables (Panel B). For each predictor, we report the ROS 2 for the original time series (time domain) (TS) and the ROS 2 for 3 different frequency components obtained through wavelets decomposition (frequency domain): high-frequency (HF), business-cycle frequency (BCF), and low-frequency (LF) components. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.*

Overall, the out-of-sample analysis performed better in this new period excluding 2021 and 2022. The exclusion of these two years accounts for the potential bias resulting from significant macroeconomic shocks during that period. Thus, the adjustment in the timeframe helps making more accurate forecast by mitigating the influence of these extraordinary events.

In time series analysis, almost all variables end up outperforming the historical mean, except OG, DFR, and MOMBY (12) that show negative R-Squared results. The previously negative LTY variable now exhibits a result that outperforms the historical mean by 1.26% with a level of significance of 1%. This suggests that the analysis may have been impacted by the sample results that included the years of financial crisis.

When we move on to decomposed frequencies, we find that this sample still produces worse results at high frequency since the TMS variable no longer performs better than the historical mean. The only one with a positive R-Squared is MOMBY (6). The once-outperforming LTY variable is now showing negative results in the business-cycle frequency, whereas INFL and TMS, with R-Squared values of 0.05% and 0.3%, respectively, are able to outperform the historical mean. At low frequency, four variables manage to outperform the historical mean, namely TB, LTY, TMS, and DFY. Notably, the LTY variable achieves the most significant improvement, outperforming the historical mean by 0.8%,with a level of significance of 5%, compared to 0.20% in the previous sample where it was the only explanatory variable. TB also has a good result in this frequency, demonstrating a superior performance than time series, with a level of significance of 10%.

When comparing time series and decomposed frequencies, this analysis reveals that decomposed frequencies are more relevant in Business-Cycle Frequency (BCF) with the INFL variable and in Low Frequency (LF) with TB and

DFY. However, this difference does not appear highly significant, mainly due to the small percentage variations and the condition that the decomposed frequency model adds value only if the variable exhibits a positive R-Squared in more than one frequency and if it genuinely contributes informational relevance compared to time series, which does not seem to be the case. Also, the majority of the variables do not seem to outperform in the same frequency the historical mean which can difficult the analysis of the model. Consequently, overall, the model fails to perform well if relying solely on one decomposed frequency to explain the entire variable.

3.2.2 Economic Analysis for Different Investor's Relative Risk Aversion

We now proceed with the annualized certainty equivalent return gains (CER gains). As explained earlier, these gains represent the difference between the CER for an investor utilizing the predictive model for forecasting bond returns and the CER for an investor employing the historical mean model for the same purpose. For this analysis we used the sample that generated better results out-of-sample, period from 1973 to 2020.

In Table 3 we report the CER gains considering an investor with a relative risk aversion coefficient of 3.

	CER Gains			
Variable	TS	HF	BCF	LF
Panel A: Macroeconomic variables				
TB	0.09	-1.99	-0.06	-0.06
LTY	1.15	-4.07	-0.50	0.52
INFL	-0.16	-0.06	0.24	-0.22
TMS	0.33	-0.53	0.50	0.44
DFY	-0.04	-0.24	-0.49	0.11
OG	-0.27	-0.84	-1.51	-0.31
DFR	-0.63	-1.10	-0.65	-0.25
Panel B: Technical indicators variables				
MOMBY (6)	0.65	0.44	0.02	0.04
MOMBY (12)	-0.10	0.19	-1.79	-0.51

Table 3: *Annualized CER gains period 1973-2020. This table reports CER gains for individual forecasts of bond returns with investor with a relative risk aversion coefficient of 3, based on 7 macroeconomic variables and 2 technical indicator variables. For each predictor, we report the results for the original time series (time domain) (TS) and for the 3 different frequency components obtained through wavelets decomposition (frequency domain): high-frequency (HF), business-cycle frequency (BCF), and low-frequency (LF) components.*

In this scenario where an investor has an intermediate risk profile, the results show that in times series four variables have positive values such as TB, LTY, TMS, MOMBY(6). The best result in this scenario is from the LTY variable in time series (1.15 basis points)

In high frequency, technical variables stand out being the ones where the predictive model can outperform the historical mean an with MOMBY(6) delivering CER gains of 44 basis points.

Business-cycle frequency and low frequency both exhibit positive results in variables such as TMS and MOMBY (6).

To assess the model's sensitivity to varying levels of risk aversion, we explore two additional scenarios representing different risk aversion profiles: a "risk-neutral" investor (y=1) and a risk averse investor (y=5). By examining the outcomes for these contrasting risk aversion profiles, we aim to understand how the model behaves under different risk preferences. The results of these scenarios are summarized in Table 4.

	Variable	CER Gains (%)			
		TS	HF	BCF	LF
Scenario γ=1	TB	-0.84	-3.35	-0.74	-0.70
	LTY	0.15	-5.57	-1.76	-0.55
	INFL	-0.11	0.00	-0.35	-0.05
	TMS	-0.25	-1.54	-0.57	-0.76
	DFY	0.00	-0.01	-0.23	0.00
	OG	-0.79	-2.16	-1.07	-0.79
	DFR	-0.97	-1.59	-0.91	-0.44
	MOMBY (6)	0.59	-0.05	0.00	0.00
	MOMBY (12)	0.00	-0.09	-1.82	-0.02
Scenario γ=5	TB	0.17	-1.41	-0.22	0.22
	LTY	1.29	-3.35	-0.24	0.67
	INFL	-0.04	-0.12	0.22	-0.14
	TMS	0.43	-0.27	0.36	0.72
	DFY	-0.27	-0.15	-0.53	0.04
	OG	-0.29	-0.52	-1.36	-0.28
	DFR	-0.58	-1.03	-0.77	-0.29
	MOMBY (6)	1.08	0.83	0.22	0.02
	MOMBY (12)	0.14	0.32	-1.97	-1.41

Table 4: *Annualized CER gains for period 1973-2020 with "risk-neutral" investors and for conservative investors. This table reports CER gains for individual forecasts of bond returns with investor with a relative risk aversion coefficient of 1 and 5, based on 7 macroeconomic variables and 2 technical indicator variables. For each predictor, we report the results for the original time series (time domain) (TS) and for the 3 different frequency components obtained through wavelets decomposition (frequency domain): high-frequency (HF), business-cycle frequency (BCF), and low-frequency (LF) components.*

Analysing the table above, we can conclude that, in all scenarios, there is a predominance of negative values, indicating that there are no economic gains for the investor when using the explained predictive model instead of the historical mean model for the same purpose.

Looking into the different scenarios more thoroughly, starting with the "risk-neutral" investor scenario, gains are only observed in time series and for two variables, LTY and MOMBY(6).

The scenario of a risk averse investor closely resembles the baseline scenario (the first one where the coefficient of risk aversion was of 3) where we find the same variables with positive CER gains, except for MOMBY(12), which in this scenario is also positive in time series. LF stands out in this scenario with the variables TB, LTY, TMS, DFY, and MOMBY(6) showing positive CER gains.

In general, we observe that positive CER gains coincide with variables that demonstrate the ability to outperform the historical mean, as evidenced by positive out-of-sample R-squared values. This suggests that variables with higher explanatory power may enhance the effectiveness of our methodology in predicting bond returns. By identifying the variables that contribute most significantly to positive CER gains and robust out-of-sample R-Squared values, we gain insights into the potential avenues for improving the predictive accuracy of our model. Furthermore, exploring additional variables with even stronger explanatory power could lead to further enhancements in predictive performance, potentially unlocking greater CER gains and strengthening the overall effectiveness of our methodology in forecasting bond returns. Thus, this observation underscores the importance of continuous refinement and augmentation of our predictive model to capitalize on variables with superior predictive capabilities and optimize investment strategies accordingly.

The main conclusion derived from this analysis is that wavelet decomposition does not demonstrate superior performance compared to time series when evaluating economic performance across any frequency.

Conclusion

Bond returns were predicted using the wavelet-based method that decomposes the times series into three frequencies as high-frequency, business-cycle frequency, and low frequency components, while preserving the original characteristics of the time series. The assessment of the out-of-sample results is done by the R-Squared perspective. The analysis is conducted over two periods, from 1973 to 2022 and from 1973 to 2020, with the intention of mitigating the effects of the COVID-19 pandemic and the Ukraine-Russia conflict, which induced substantial macroeconomic instability that could potentially distort the results. This analysis also contains an annualized certainty equivalent return gains perspective with three levels of investor's relative risk aversion coefficient.

In the first sample of data from 1973 to 2022, the extensive evaluation of macroeconomic and technical indicator variables for predicting bond returns indicates limited success, with most variables failing to outperform historical means in time series testing and in decomposed frequencies. No variable can outperform the historical mean in two decomposed frequencies simultaneously, and even when examining just one decomposed frequency at a time, we only have two variables with positive results in high-frequency. The model does not consistently support the notion that the chosen approach significantly influences bond return predictions.

The results in the sample from 1973 to 2020 showed improved performance. Most variables outperformed the historical mean in time series. Notably, LTY, previously negative, now outperforms the historical mean by 1.26%, potentially influenced by financial crisis years. In decomposed frequencies, the new sample demonstrated worse high-frequency results, except for MOMBY (6). LTY now shows negative results in business cycle frequency, while INFL and TMS outperform the historical mean. At low frequency, four variables surpassed the historical mean, with LTY achieving a notable 0.79% improvement. Comparing time series and decomposed frequencies, the latter is more relevant in Business Cycle Frequency (BCF) and Low Frequency (LF). However, the model loses strength when relying solely on one decomposed frequency.

When analysing the economic gains for different investor's relative risk aversion, in all scenarios, we can see a predominance of negative values that indicates no effective CER gains for investors. An investor with an intermediate risk profile sees positive values in both time series and decomposed frequencies. The conservative investor scenario mirrors the intermediate scenario, with positive CER gains in LF.

Our main findings highlight a significant discrepancy in the model's bond return forecasting accuracy, especially when compared to predictors found to be effective in previous studies, such as the one carried out by Almadi et al. (2014). Interestingly, we find that when using wavelets and this frequency domain technique, predictors that were previously considered effective no longer have the same predictive power. This surprising outcome draws attention to the complexity involved in correctly predicting bond returns and demands a rigorous analysis of the model's underlying mechanics. Despite this constraint, our investigation explores the model's behaviour during times of financial and macroeconomic crisis, providing important light on how it behaves in volatile market environments. Nevertheless, despite these obstacles, there is some cause for optimism because our study points to the methodology's possible efficacy with variables that have greater explanatory power and during periods characterized by low instability in financial markets.

For future research it might be worth to investigate new approaches for bond return prediction. Expanding on the strong statistical support for bond return predictability offered by machine learning techniques, particularly extreme trees, and neural networks, as demonstrated by the results of Bianchi et al. (2020), a viable path would be to apply the same model to forecast bond returns.

References

Aguiar-Conraria, L., & Joana Soares, M. (2011). Business cycle synchronization and the Euro: A wavelet analysis. *Journal of Macroeconomics*, *33*(3), 477–489. https://doi.org/10.1016/J.JMACRO.2011.02.005

Aguiar-Conraria, L., Martins, M. M. F., & Soares, M. J. (2012). The yield curve and the macro-economy across time and frequencies. *Journal of Economic Dynamics and Control*, *36*(12), 1950–1970.
https://doi.org/10.1016/J.JEDC.2012.05.008

Aguiar-Conraria, L., & Soares, M. J. (2014). The continuous wavelet transform: Moving beyond uni- and bivariate analysis. *Journal of Economic Surveys*, *28*(2), 344–375.
https://doi.org/10.1111/joes.12012

Almadi, H., Rapach, D. E., & Suri, A. (2014). Return Predictability and Dynamic Asset Allocation: How Often Should Investors Rebalance? *London: Journal of Portfolio Management.*
http://doi.org/10.3905/jpm.2014.40.4.016

Bianchi, D., Büchner, M., & Tamoni, A. (2020). Bond Risk Premia with Machine Learning. *WBS Finance Group Research Paper No. 252.*
https://dx.doi.org/10.2139/ssrn.3232721

Campbell, J. Y., & Shiller, R. J. (1991). Yield Spreads and Interest Rate Movements: A Bird's Eye View. *The Review of Economic Studies, 58(3), 495–514.* https://doi.org/10.2307/2298008

Campbell, J. Y., & Thompson, S. B. (2008). Predicting excess stock returns out of sample: Can anything beat the historical average? *Review of Financial Studies, 21(4), 1509–1531.*
https://doi.org/10.1093/rfs/hhm055

Cheng, T., Jiang, S., Zhao, A. B., & Jia, Z. (2023). Complete subset averaging methods in corporate bond return prediction. Finance Research Letters, 54, 103727. https://doi.org/10.1016/J.FRL.2023.103727

Clark, T. E., & West, K. D. (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics, 138(1), 291–311.* https://doi.org/10.1016/j.jeconom.2006.05.023

Cochrane, J. H., Piazzesi, M., Pi-Azzesi:, N. ;, We, N., Bekaert, G., Brandt, M., Collin-Dufresne, P., Hansen, L., Hodrick, B., Kocherlakota, N., Santa-Clara, P., Schneider, M., & Singleton, K. (2005). Bond Risk Premia.

Crowley, P. M. (2007). A Guide to Wavelets for Economists. Journal of Economic Surveys, 21(2).

Fama, E. F., & Bliss, R. R. (1987). The Information in Long-Maturity Forward Rates. The American Economic Review, 77(4), 680–692.

Fan, Z., Xiao, X., & Zhou, H. (2018). Variance Risk Premium, Higher Order Risk Premium, and Expected Stock Returns. SSRN Electronic Journal. https://doi.org/10.2139/ssrn.3120260

Faria, G., & Verona, F. (2017). Forecasting the equity risk premium with frequency-decomposed predictors. Research Discussion Paper No. 1/2020, Bank of Finland.

Faria, G., & Verona, F. (2018). Forecasting stock market returns by summing the frequency-decomposed parts. Journal of Empirical Finance, 45, 228–242. https://doi.org/10.1016/j.jempfin.2017.11.009

Faria, G., & Verona, F. (2020a). Frequency-domain information for active portfolio management. Research Discussion Paper No. 2/2020, Bank of Finland.

Faria, G., & Verona, F. (2020b). The yield curve and the stock market: Mind the long run. Journal of Financial Markets, 50. https://doi.org/10.1016/j.finmar.2019.100508

Faria, G., & Verona, F. (2021). Time-frequency forecast of the equity premium. Quantitative Finance, 21(12), 2119–2135. https://doi.org/10.1080/14697688.2020.1820071

Ferreira, M. A., & Santa-Clara, P. (2011). Forecasting stock market returns: The sum of the parts is more than the whole. Journal of Financial Economics, 100(3), 514–537. https://doi.org/10.1016/J.JFINECO.2011.02.003

Gallegati, M., & Ramsey, J. B. (2013). Bond vs stock market's Q: Testing for stability across frequencies and over time. *Journal of Empirical Finance*, *24*, 138–150. https://doi.org/10.1016/j.jempfin.2013.10.003

Gargano, A., Pettenuzzo, D., & Timmermann, A. (2019). Bond return predictability: Economic value and links to the macroeconomy. *Management Science*, *65*(2), 508–540. https://doi.org/10.1287/mnsc.2017.2829

Gençay, R., Selçuk, F., & Whitcher, B. (2005). Multiscale systematic risk. *Journal of International Money and Finance*, *24*(1), 55–70. https://doi.org/10.1016/J.JIMONFIN.2004.10.003

Inoue, A., & Kilian, L. (2004). In-sample or out-of-sample tests of predictability: Which one should we use? *Econometric Reviews*, *23*(4), 371–402. https://doi.org/10.1081/ETC-200040785

Kilponen, J., & Verona, F. (2017). Testing the Q theory of investment in the frequency domain. Working Paper 2017-01, cef.up.

Kim, S., & In, F. (2005). The relationship between stock returns and inflation: new evidence from wavelet analysis. *Journal of Empirical Finance*, *12*(3), 435–444. https://doi.org/10.1016/J.JEMPFIN.2004.04.008

Lin, H., Wu, C., & Zhou, G. (2018). Forecasting corporate bond returns with a large set of predictors: An iterated combination approach. *Management Science*, *64*(9), 4218–4238.

https://doi.org/10.1287/mnsc.2017.2734

Ludvigson, S. C., & Ng, S. (2009). Macro factors in bond risk premia. *Review of Financial Studies*, *22*(12), 5027–5067. https://doi.org/10.1093/rfs/hhp081

Ramsey, J. B., & Lampart, C. (1998). Decomposition of Economic Relationships by Timescale using Wavelets. Macroeconomic Dynamics, 2(1), 49–71. https://doi.org/https://doi.org/10.1017/S1365100598006038

Rapach, D. E., Ringgenberg, M. C., & Zhou, G. (2016). Short interest and aggregate stock returns. Journal of Financial Economics, 121(1), 46–65. https://doi.org/10.1016/j.jfineco.2016.03.004

Rua, A. (2011). A wavelet approach for factor-augmented forecasting. Journal of Forecasting, 30(7), 666–678. https://doi.org/10.1002/for.1200

Rua, A. (2017). A wavelet-based multivariate multiscale approach for forecasting. International Journal of Forecasting, https://doi.org/10.1016/j.ijforecast.2017.01.007

Sarno, L., Schneider, P., & Wagner, C. (2016). The economic value of predicting bond risk premia. *Journal of Empirical Finance*, *37*, 247–267. https://doi.org/10.1016/j.jempfin.2016.02.001

Thornton, D. L., & Valente, G. (2012). Out-of-sample predictions of bond excess returns and forward rates: An asset allocation perspective. In *Review of Financial Studies* (Vol. 25, Issue 10, pp. 3141–3168). https://doi.org/10.1093/rfs/hhs069

Welch, I., & Goyal, A. (2008). A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies*, *21*(4), 1455–1508. https://doi.org/10.1093/rfs/hhm014