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Intra-industry trade, involuntary unemployment and macroeconomic stability[☆]

Antoine Le Riche[a,b], Teresa Lloyd-Braga[c,*], Leonor Modesto[c,d]

[a] School of Economics, Sichuan University, 610065, Chengdu, China
[b] CAC – IXXI, Complex Systems Institute, 69007, Lyon, France
[c] Universidade Católica Portuguesa, Catolica Lisbon School of Business and Economics, Palma de Cima, 1649-023 Lisboa, Portugal
[d] IZA, 53113, Bonn, Germany

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ABSTRACT

We introduce *taste for variety* in a one-sector model of differentiated products with productive labor externalities, considering two OLG countries, one with wage rigidity and the other with full employment. After opening the borders to capital mobility and *intra-industry trade*, steady state output and real wages improve in the full employment country and the saddle path stability, characterizing this country under autarky, will prevail in the globalized world if this economy is big enough. Unemployment increases in the country with wage rigidity and, for intermediate plausible values of both the current propensity to consume and of the labor externality, indeterminacy, which emerges in the rigid wage economy in autarky, will be exported to the world if this country is relatively big. Finally, we show that globalization leads to the appearance of stable deterministic cycles in activity, employment and the trade account, both through flip and Hopf bifurcations, when the world steady state is locally determinate, for empirically plausible low degrees of labor externalities. This implies that trade cycles occur in the absence of shocks to fundamentals, and even without uncertainty in expectations.

1. Introduction

The impact of globalization on unemployment and wages has been a topic of interest for workers and policy makers in developed countries, fostering the attention of economists and the emergence of a growing literature. Several papers have shown that different labor market institutions in the trading partners affect comparative advantage in two-sector models, determining the pattern and the impact of inter-industry trade.[1] However, nowadays, *intra-industry* trade, i.e. trade where each country imports and exports simultaneously different varieties of the same industry, represents a significant and increasing percentage of the trade between developed countries.[2] Accordingly, we consider a one sector model of differentiated goods, where comparative advantage plays no role. This was already advocated in the pioneer works of Krugman (1979, 1980), where he remarks that the prevalence of two-way exchanges of differentiated products among industrial countries casts serious doubts on the ability of comparative cost theory to explain international trade, proposing an alternative framework, anchored on economies of scale, product differentiation and imperfect competition. Here, we focus on the effects of bilateral *intra-industry* trade driven by *taste for variety* on the level and volatility of employment and activity of two countries with different labor market institutions. Although we assume non-stochastic stationary identical preferences and technologies, the setup considered is able to generate endogenous fluctuations driven by self-fulfilling expectations in aggregate output, employment and the trade account. Several studies have already analyzed the link between macroeconomic volatility driven by beliefs and inter-industry trade.[3] However, the literature has not yet addressed the implications of *intra-industry trade*.[4] Our work fills this gap, investigating whether free-trade

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* Corresponding author.
E-mail addresses: antoineleriche@scu.edu.cn (A. Le Riche), tlb@ucp.pt (T. Lloyd-Braga), lrm@ucp.pt (L. Modesto).

1 See Cuñat and Melitz (2012), Boulhol (2011) and Helpman and Itskhoki (2010).

2 Indeed, according to the OECD, more than 60% of U.S. trade, and 65% of European trade, is *intra-industry* trade. See http://www.oecd.org/economy/outlook/2752923.pdf.

3 See Sim and Ho (2007), Nishimura et al. (2014) and Le Riche (2017).

4 On the empirical side, Hayakawa et al. (2017), using OECD data, find that this type of trade is particularly volatile. Ardelean et al. (2018), examining the link between trade and output volatility find that imported varieties make domestic demand more volatile as additional consumption goods of the same industry become available.

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in differentiated goods driven by *taste for variety*, and the liberalization of capital movements,[5] may stabilize (or destabilize) the economies with respect to endogenous deterministic and stochastic fluctuations driven by autonomous volatile changes in expectations,[6] and whether it may bring welfare gains at the country level.

We consider a two-country overlapping generations model with current and future consumption, featuring a sector of differentiated goods, produced out of labor and capital under increasing returns to scale due to a fixed cost and labor externalities. In this sector there is imperfect competition à la Dixit and Stiglitz (1977), and equilibrium is characterized by a constant markup with an endogenous and proyclical number of varieties. We introduce *taste for variety*, according to which an increase of product diversity decreases the aggregate price at the symmetric equilibrium (Bénassy, 1996). The model combines and extends the closed economy framework with *taste for variety* developed in Seegmuller (2008), where current consumption is not considered, and the two-country model with productive labor externalities, international capital mobility but no trade considered in Aloi and Lloyd-Braga (2010).[7] As in the latter, we assume that the two countries only differ in their labor market structure: efficiency wages and involuntary unemployment prevail in one country,[8] whereas in the other there is perfect competition in the labor market and full employment. This framework is particularly well suited to investigate whether the employment and welfare effects of *intra-industry* trade depend on the existing labor market structure. This is an important issue as developed countries, where *intra-industry* trade is particularly relevant, are characterized by different labor market institutions, and it is frequently conjectured that labor market rigidities may hinder the expected benefits of trade.

We start by analyzing autarkic equilibria in each country. We then consider equilibria with free-trade and capital mobility between the two countries, studying the effects of opening the borders on the stability properties and steady state welfare of the world economy. In autarky, as in Aloi and Lloyd-Braga (2010), local indeterminacy, and therefore local sunspots fluctuations, emerge around the unique steady state in the country with efficiency wages and involuntary unemployment, for intermediate values of both the propensity to consume and the degree of externalities. However, in our framework, a higher *taste for variety*, which is not present in Aloi and Lloyd-Braga (2010), reduces the likelihood of local indeterminacy, stabilizing the economy. This contrasts with Seegmuller (2008), where *taste for variety*, instead of labor externalities, is the distortion responsible for indeterminacy. In the country with a perfectly competitive labor market and full employment, the unique steady state is saddle path determinate in autarky. Given the asymmetry across the two countries in terms of local stability properties it is relevant to understand which type of dynamics will prevail under globalization. Will saddle path stability emerge in the globalized world, leading to the absence of expectations driven fluctuations as in the closed full employment country? Or will world indeterminacy and sunspot fluctuations occur as in the rigid wage country under autarky? Will endogenous fluctuations due to supercritical bifurcations occur due to globalization even if the world equilibrium is determinate? These are questions analyzed in this paper.

We find that the effects on stability of opening the borders to both free *intra-industry* trade and capital mobility depend on the existing degree of labor externalities and on the relative size of the two economies. Globalization may bring local saddle path stability to the world, eliminating local indeterminacy in the rigid wage country, in particular if the full employment country is sufficiently big in relative terms. In this case, the local stability properties of the (big) full employment country are exported to the other one, and the world economy is insulated from belief driven local fluctuations. In contrast, if the rigid wage country is big enough, local indeterminacy may prevail at the world level, being exported from the country with rigid wages to the world economy. In this case, the full employment country, which was stable in autarky, will also face local expectation driven fluctuations with origin in the other country. Hence, globalization in economies with different degrees of rigidity in the labor market, may stabilize or destabilize with respect to sunspots fluctuations depending on their relative size. Furthermore, for any country size, indeterminacy, and therefore local sunspots fluctuations at the world level, require a smaller degree of externalities, more likely to be compatible with empirical evidence. Also, bounded deterministic and stochastic cycles associated with Hopf and *flip* bifurcations, which did not exist in autarky, appear after opening the borders, when the world steady state is locally determinate, for empirically plausible low degrees of the productive labor externality. This means that fluctuations in activity, employment and in the balance of trade, occur without shocks to fundamentals, or even in the absence of any uncertainty.[9]

We also show that, for a given degree of labor externalities, a higher *taste for variety* promotes saddle-path stability under free *intra-industry* trade with capital mobility. However, we also find that when *taste for variety* is higher, indeterminacy and Hopf or *flip* bifurcations, and their associated deterministic and/or stochastic fluctuations, become possible with a lower, and more plausible, degree of externalities. Hence, the effects of the degree of *taste for variety* on the macroeconomic stability of the world are ambiguous.

Previous studies on the effects of trade on the stability properties of trading countries obtained distinct results, depending on the framework considered. See, for instance, Nishimura and Shimomura (2002), Iwasa and Nishimura (2014, 2019) and Sim and Ho (2007).[10] When international capital mobility is also introduced the results obtained tend to support the view that trade is destabilizing, as in Nishimura et al. (2010, 2014) in a two-good, two-factor model with infinitely-lived agents and Le Riche (2017)

5 International financial liberalization is a key feature of developed economies, where *intra-industry trade* is particularly relevant. A good example is the case of the European Union. Therefore, it seems important to consider international capital mobility when analyzing the effects of *intra-industry* trade.

6 Although we focus on fluctuations driven by changes in expectations, this does not mean that exogenous productivity and demand shocks play no role in explaining fluctuations in trade and macroeconomic variables. For example, in a two-country two-sector model without unemployment, Engel and Wang (2011) introduce durable goods and productivity shocks, whereas Bai and Ríos-Rull (2015) consider goods market frictions and demand shocks.

7 In Aloi and Lloyd-Braga (2010), without *taste for variety*, there is no trade, the current account balance reflecting only international payments of capital income.

8 Goette et al. (2007) document the pervasiveness of real wage rigidity due to efficiency wages or bargaining power of workers, namely in Germany, Italy and the UK.

9 Moreover, along these fluctuations we obtain a procyclical behavior of exports generated by the existence of *taste for variety* and *intra-industry trade*, in accordance with the pattern exhibited by data for developed economies. See De Bock (2010) and Engel and Wang (2011).

10 Nishimura and Shimomura (2002), considering a two-factor, two-sector, two-country model with infinitely-lived agents, where countries only differ with respect to their initial factor endowments, show that inter-industry trade has no effect on the stability properties of both countries. However, Iwasa and Nishimura (2014, 2019), extending Nishimura and Shimomura (2002) by introducing a consumable capital good, find that endogenous fluctuations may emerge in the world economy. In contrast Sim and Ho (2007), introducing different technologies across countries (different degrees of productive externalities), find that saddle-path stability prevails in the world economy, even if before trade one country exhibits sunspot fluctuations.

and Le Riche (2020) in an overlapping generations framework. However, all these papers assumed inter-industry trade and an inelastic labor supply. In contrast, in this paper we consider *intra-industry* trade with *taste for variety* and unemployment.

In terms of steady state effects of globalization, several natural questions arise. Will unemployment increase in the rigid wage country? Will we observe a displacement of industries and jobs from the rigid to the flexible wage country? What happens to real wages, to the number of varieties, to output and to welfare in the two countries? We show that employment decreases in the country with efficiency wages, i.e. globalization exacerbates unemployment in the rigid wage country. The number of firms, and therefore activity, is also reduced in that country. On the contrary, the number of varieties produced locally and the capital stock increase in the country with a perfectly competitive labor market and full employment. Finally, the steady state welfare of those employed in the country with efficiency wages and involuntary unemployment remains the same, while in the country with a perfectly competitive labor market and full employment citizens are better off. These steady state results operate through the interaction between differences in labor market rigidities and *intra-industry* trade with *taste for variety*.[11] We conclude that *intra-industry* trade may not bring benefits for all countries involved, hurting in particular those with labor market distortions.[12] The same conclusion was obtained by Boulhol (2011), in a static two-sector, two-country model. He finds that unemployment increases in the country with the rigid labor market after opening the borders to bilateral inter-industry trade. Helpman and Itskhoki (2010), considering a static model with no capital, also find an asymmetric impact of trade: the country with lower frictions in the labor market gains proportionately more.

We conclude that in the full employment country steady-state welfare increases with free (*intra-industry*) trade and capital movements, but endogenous macroeconomic fluctuations may become more prevalent, i.e. there is a trade off between steady state welfare gains and (de)stabilization. On the other hand, for the rigid wage country, opening the borders unambiguously reduces steady state welfare, *intra-industry* trade amplifying the effects of labor market rigidities on unemployment, without necessarily diminishing the likelihood of macroeconomic instability. Our work improves therefore our understanding of the effects of globalization on (un)employment, stationary welfare and endogenous fluctuations in the presence of *taste for variety* and labor market distortions.

The rest of the paper is organized as follows. In Section 2 we present the model and obtain the perfect foresight equilibrium for the two economies in autarky, discussing local dynamics. Section 3 provides the analysis of the two-country model. In Section 4 we prove the existence of a unique steady state in the two country model and we discuss the changes in steady state activity, employment and welfare resulting from opening the economies. We analyze local dynamics of the two-country model and present the effects of *intra-industry* trade on stability in Section 5. Finally, Section 6 concludes. Proofs are gathered in Appendix A.

2. Autarky

We consider two infinite horizon discrete time, one-sector economies, country A and country B, that share the same production structure. Both countries have monopolistic competition in the output market and perfect competition in the capital services market, only differing in the functioning of the labor market. In country A there is involuntary unemployment with efficiency wages, while full employment and perfectly competitive wages prevail in country B. Households in both countries live for two periods, work when young and consume at each period a composite good. This composite good is an aggregate of all the differentiated goods (varieties) produced by firms, exhibiting *taste for variety*.

2.1. The model

In both countries population is constant over time and individuals live for two periods. In each period H^j individuals are born in country $j \in \{A, B\}$. In the first period of life, a young employed agent that does not shirk, offers a unit of effort, receiving a wage income, w_t. He uses this income to purchase the composite consumption good, C_t, and to save in the form of capital, $\tilde{K}_{t+1}$, which he rents to firms in the following period. In the second period of life, old retired agents use the rents received to finance consumption, D_{t+1}. As usually done in the literature, the composite good (C_t and D_{t+1}) is defined as an aggregate of the quantities consumed of all the different varieties i produced (respectively, c_{it} and d_{it+1}):

$$C_t = N_t^{1+\beta}\left[\frac{1}{N_t}\sum_{i=1}^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, D_{t+1} = N_{t+1}^{1+\beta}\left[\frac{1}{N_{t+1}}\sum_{i=1}^{N_t} d_{it+1}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

where $\beta \geq 0$, N is the number of varieties, and $\varepsilon > 1$ is the intratemporal elasticity of substitution between varieties.[13]

Following Bénassy (1996), we define the function $t(N)$, which represents the gain from consuming one unit of N different varieties instead of consuming N units of a single variety. It follows from the definition of the composite good given in (1) that $t(N) = N^\beta$. Accordingly, the degree of *taste for variety* is given by the elasticity of $t(N)$:

$$\tau(N) \equiv \frac{Nt'(N)}{t(N)} = \beta. \tag{2}$$

If β is zero, households have no *taste for variety* while if β is higher than zero, there is *taste for variety*.[14]

Agents have preferences defined over consumption in the first period of life, C_t, consumption in the second period of life, D_{t+1}, and young age effort, e_t. A young agent born at period t solves the following dynamic program, taking (1) into consideration:

$$\begin{aligned} \max_{C_t, D_{t+1}, \tilde{K}_{t+1}} \quad & C_t^\alpha D_{t+1}^{1-\alpha} - \nu e_t \\ s.t. \quad & P_t C_t + P_t \tilde{K}_{t+1} = w_t, \\ & P_{t+1} D_{t+1} = r_{t+1}\tilde{K}_{t+1}, \\ & C_t, D_{t+1}, \tilde{K}_{t+1} \geq 0, \end{aligned} \tag{3}$$

where $\nu > 0$ is the disutility of effort, $\alpha \in (0, 1)$ is the propensity to consume when young, $e_t \in \{0, 1\}$ represents the

[11] Indeed, Aloi and Lloyd-Braga (2010) with international capital mobility, but no trade, find no changes in stationary welfare. Their autarkic steady state and the steady state with international capital mobility are identical.

[12] Rodríguez-Clare et al. (2020), considering a quantitative trade model for the US economy, also find welfare losses in response to trade shocks for states characterized by downward nominal wage rigidity.

[13] When $\varepsilon > 1$, the differentiated goods are substitutes.

[14] Ardelean (2009) estimates consumer's love for variety and suggests that variety matters for both imported and domestically produced goods while Drescher et al. (2008) present evidence on consumers' preferences for variety in food consumption.

effort supplied, r_{t+1} denotes the nominal rental rate of capital, w_t is the nominal wage and P_t represents the price of the composite consumption good.[15]

We adopt a two-stage maximization procedure. First, given a fixed amount of the composite good, C_t and D_{t+1} as defined in (1), a young agent born at period t chooses c_{it} and d_{it}, in order to minimize respective spending:

$$\sum_{i=1}^{N_t} p_{it}c_{it} = P_tC_t \quad \text{and} \quad \sum_{i=1}^{N_t} p_{it+1}d_{it+1} = P_{t+1}D_{t+1}$$

where p_{it} is the price of a variety i. We obtain:

$$c_{it} = N_t^{\beta(\varepsilon-1)-1}\left(\frac{p_{it}}{P_t}\right)^{-\varepsilon}C_t,\ d_{it+1} = N_{t+1}^{\beta(\varepsilon-1)-1}\left(\frac{p_{it+1}}{P_{t+1}}\right)^{-\varepsilon}D_{t+1} \tag{4}$$

so that:

$$P_t = \frac{1}{N_t^{\beta}}\left[\frac{1}{N_t}\sum_{i=1}^{N_t} p_{it}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{5}$$

Second, he determines his intertemporal choice between consumption when young and old and young age effort. Defining the real wage $\omega_t \equiv \frac{w_t}{P_t}$, and the real interest rate $\rho_t \equiv \frac{r_t}{P_t}$ the first-order conditions can be written as:

$$C_t = \alpha\omega_t,\ D_{t+1} = (1-\alpha)\omega_t\rho_{t+1} \text{ and } \tilde{K}_{t+1} = (1-\alpha)\omega_t. \tag{6}$$

Plugging now the demands for the composite goods C_t and D_{t+1} into the utility function defined in (3), we obtain the lifetime indirect utility function of a young agent that supplies a given effort e_t:

$$V(\omega_t, \rho_{t+1}, e_t) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\rho_{t+1}^{1-\alpha}\omega_t - \nu e_t. \tag{7}$$

In the case of a young employed worker supplying $e_t = 1$, the utility becomes

$$V(\omega_t, \rho_{t+1}, 1) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\rho_{t+1}^{1-\alpha}(\omega_t - \bar{\omega}_t) \tag{8}$$

where

$$\bar{\omega}_t \equiv \frac{\nu}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\rho_{t+1}^{1-\alpha}} \tag{9}$$

represents the real reservation wage. Note that, as the indirect utility of an unemployed worker is zero, all youngsters are willing to work when $\omega_t > \bar{\omega}_t$, while for $\omega_t < \bar{\omega}_t$ the labor supply is zero.

As in Dixit and Stiglitz (1977), we assume monopolistic competition in the output market, where there are several firms, each producing a differentiated good of the same industry.[16] In each period $t = 1, \ldots, \infty$, entry and exit are free and the zero profit condition determines the number of firms. Furthermore, we consider the existence of labor externalities in production, as in Aloi and Lloyd-Braga (2010) and in Lloyd-Braga et al. (2007). Each firm produces one variety $i \in \{1, \ldots, N\}$ of output using the following technology:

$$y_{it} = \Theta\left[a_{it}^{s}l_{it}\bar{L}_t^{\gamma} - \phi\right] \tag{10}$$

where s is the share of capital in total income, Θ is the total factor productivity, $a_{it} = k_{it}/l_{it}$ the capital-labor ratio used by firm i, $\bar{L}_t$ is aggregate employment which firms take as given, $\gamma > 0$ represents the degree of the labor externality and $\phi > 0$ a fixed cost.

Aggregate production in period t, P_tY_t, is shared between consumption of young agents, consumption of old agents and investment:

$$P_tY_t = \sum_{i=1}^{N_t} p_{it}y_{it} = [L_tC_t + L_tI_t + L_{t-1}D_t]P_t \tag{11}$$

where L_t denotes employment at period t. We assume full depreciation of capital so that $I_t = \tilde{K}_{t+1}$ and we consider that I_t is defined by an index of varieties similar to the one used for consumption:

$$I_t = N_t^{1+\beta}\left[\frac{1}{N_t}\sum_{i=1}^{N_t} i_{it}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where i_{it} is optimally determined by

$$i_{it} = N_t^{\beta(\varepsilon-1)-1}\left(\frac{p_{it}}{P_t}\right)^{-\varepsilon}I_t. \tag{12}$$

It follows that the aggregate demand for variety i, υ_{it}, is given by $\upsilon_{it} = c_{it}L_t + d_{it}L_{t-1} + i_{it}L_t$, so that using (4) and (12) we obtain:

$$\upsilon_{it} = N_t^{\beta(\varepsilon-1)-1}\left(\frac{p_{it}}{P_t}\right)^{-\varepsilon}[L_t(C_t + I_t) + L_{t-1}D_t]. \tag{13}$$

Country A exhibits labor market rigidity due to the existence of efficiency wages.[17] As in Shapiro and Stiglitz (1984), workers may shirk and in that case the level of effort supplied is zero, i.e. $e_t^A = 0$. A worker who shirks is caught with (ex-ante) probability $\lambda \in (0, 1)$. Firms fire immediately workers who are caught shirking, without any wage, so that they get zero utility. A young agent faces therefore three possibilities: (i) being unemployed, (ii) being employed and not shirking or (iii) being employed and shirking ($e_t^A = 0$, $w_t^A > 0$). Using the indirect utility function given in (7), the utility of an employed worker who shirks is $V^{e,s} = (1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha}(\rho_{t+1}^A)^{1-\alpha}\omega_t^A$. Then, using (8), it is easy to see that employed workers will not shirk ($e_t^A = 1$) if wages are such that $\omega_t^A \geq \frac{\bar{\omega}_t^A}{\lambda}$, i.e. if real wages satisfy the No Shirking Condition (hereafter NSC).

Since the output of a worker who shirks is zero, firms, in order to maximize profits, take into account the NSC, so that the wage chosen induces all works to exert effort. Hence, the problem solved by firms in country A is the following:

$$\begin{aligned} &\max_{w_t^A, l_{it}^A, k_{it}^A \in \Re^3_{++}} \quad p_{it}^A y_{it}^A - w_t^A l_{it}^A - r_t^A k_{it}^A \\ &\quad s.t. \qquad w_t^A \geq \frac{\bar{\omega}_t^A}{\lambda}P_t^A, \end{aligned} \tag{14}$$

where y_{it}^A is given by (10), and p_{it}^A is such that $y_{it}^A = \upsilon_{it}^A$, with the demand function υ_{it}^A given in (13).

The first-order conditions are then given by (10), (13) and

$$\begin{aligned} r_t^A &= \Theta p_{it}^A\left(\tfrac{\varepsilon-1}{\varepsilon}\right)s(a_{it}^A)^{s-1}\left(\bar{L}_t^A\right)^{\gamma}, \\ w_t^A &= \Theta p_{it}^A\left(\tfrac{\varepsilon-1}{\varepsilon}\right)(1-s)(a_{it}^A)^{s}\left(\bar{L}_t^A\right)^{\gamma}, \\ w_t^A &= \tfrac{\bar{\omega}_t^A}{\lambda}P_t^A. \end{aligned} \tag{15}$$

At the symmetric equilibrium $l_{it}^A = l_t^A$, $k_{it}^A = k_t^A$, $a_{it}^A = a_t^A$ and $p_{it}^A = p_t^A$ for all firms in country A and $\bar{L}_t^A = L_t^A = N_t^A l_t^A$. Then, using (5), we can rewrite (15) as:

$$\begin{aligned} \rho_t^A &= \Theta(N_t^A)^{\beta}\left(\tfrac{\varepsilon-1}{\varepsilon}\right)s(a_t^A)^{s-1}\left(L_t^A\right)^{\gamma}, \\ \omega_t^A &= \Theta(N_t^A)^{\beta}\left(\tfrac{\varepsilon-1}{\varepsilon}\right)(1-s)(a_t^A)^{s}\left(L_t^A\right)^{\gamma}, \\ P_t^A &= (N_t^A)^{-\beta}p_t^A. \end{aligned} \tag{16}$$

[15] The reader may note that our results would be the same if we had considered instead a perfectly competitive market of a final good (with price P_t) produced out of the differentiated intermediate products according to (1).

[16] As explained in Krugman (1980): "Because firms can costlessly differentiate their products, and all products enter symmetrically into demand, two firms will never want to produce the same product; each good will be produced by only one firm".

[17] Similar results would apply if we had instead considered monopoly unions or search generated unemployment.

Moreover real wages are set as mark up over the real reservation wage $\omega_t^A = \frac{\bar{\omega}_t^A}{\lambda}$.

On the contrary, in country B, we consider a perfectly competitive labor market with full employment. Therefore, labor demand of firm i is determined by the equality $w_t^B = \Theta p_{it}^B \left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s)(a_{it}^B)^s \left(H_t^B\right)^\gamma$, where, at full employment equilibria $w_t^B > \bar{\omega}_t^B P_t^B$, with $\bar{\omega}_t^B$ given by (9), and employment satisfies $\sum_{i=1}^{N_t^B} l_{it}^B = H^B$. We also have that the rental rate of capital is given by the equality $r_t^B = \Theta p_{it}^B \left(\frac{\varepsilon-1}{\varepsilon}\right) s(a_{it}^B)^{s-1} \left(H_t^B\right)^\gamma$. Note that the markup factor in the differentiated goods market is constant and given by $\varepsilon/(\varepsilon-1)$ in both countries.

At a symmetric equilibrium, $l_{it}^B = l_t^B = \frac{H^B}{N_t^B}$, $k_{it}^B = k_t^B$, $a_{it}^B = a_t^B$ and $p_{it}^B = p_t^B$ for all firms in country B. Then, using (5), we obtain:

$$\begin{aligned} \rho_t^B &= \Theta (N_t^B)^\beta \left(\tfrac{\varepsilon-1}{\varepsilon}\right) s(a_t^B)^{s-1} \left(H^B\right)^\gamma, \\ \omega_t^B &= \Theta (N_t^B)^\beta \left(\tfrac{\varepsilon-1}{\varepsilon}\right) (1-s)(a_t^B)^s \left(H^B\right)^\gamma, \\ P_t^B &= (N_t^B)^{-\beta} p_t^B. \end{aligned} \tag{17}$$

From (16) and (17) the aggregate price P_t^j is equal to p_t^j at the symmetric equilibrium when $\beta = 0$. However, if $\beta > 0$ the aggregate price decreases with the number of varieties, as shown in (5). Moreover, the real interest rate, ρ_t^j, and the real wage, ω_t^j, increase with the number of varieties N_t^j for $j \in \{A, B\}$.

In both countries the free-entry condition is determined by the zero profit condition, $p_t^j y_t^j - k_t^j r_t^j - l_t^j w_t^j = 0$. Hence, using (10), and the expressions obtained above for w and r, in (15) for A and in a similar way for B, we obtain that:

$$\frac{(a_t^j)^s l_t^j \left(\bar{L}_t^j\right)^\gamma}{\varepsilon} = \phi. \tag{18}$$

From (10) and (18), we derive that the production level of each firm is constant at equilibrium and identical in both countries:

$$y_t^j = \Theta[(a_t^j)^s l_t^j \left(\bar{L}_t^j\right)^\gamma - \phi] = \Theta\phi(\varepsilon-1). \tag{19}$$

2.2. Country A - equilibrium with involuntary unemployment

In country A, at a symmetric equilibrium the aggregate demand for capital services is given by $K_t^A = N_t^A k_t^A$. Using the free-entry condition given in (18), and the fact that at the symmetric equilibrium $L_t^A = K_t^A/a_t^A$, we obtain the number of varieties (firms) in country A, N_t^A:[18]

$$N_t^A = \frac{\left(a_t^A\right)^{s-1-\gamma}\left(K_t^A\right)^{1+\gamma}}{\varepsilon\phi} \equiv N^A(a_t^A, K_t^A). \tag{20}$$

Using (16) and (20), the real interest rate and the real wage can also be written as functions of K_t^A and of a_t^A

$$\begin{aligned} \rho_t^A &= \Theta N^A(a_t^A, K_t^A)^\beta \left(\tfrac{\varepsilon-1}{\varepsilon}\right) s \left(a_t^A\right)^{s-1-\gamma} \left(K_t^A\right)^\gamma \equiv \rho(a_t^A, K_t^A), \\ \omega_t^A &= \Theta N^A(a_t^A, K_t^A)^\beta \left(\tfrac{\varepsilon-1}{\varepsilon}\right)(1-s) \left(a_t^A\right)^{s-\gamma} \left(K_t^A\right)^\gamma \equiv \omega(a_t^A, K_t^A). \end{aligned} \tag{21}$$

Employment is determined by the equality $\frac{\bar{\omega}_t^A}{\lambda} = \omega(a_t^A, K_t^A)$, with the real reservation wage given by (9). We assume that the level of employment satisfying this condition verifies $L_t^A = N_t^A l_t^A < H^A$, so that we obtain an equilibrium with unemployment. Indeed, in contrast with perfect competition, wages are set as a mark up over the reservation wage, so that involuntary unemployment emerges. It is worth noting that expectations influence equilibrium through the labor market, since the reservation wage and employment level at period t depend on ρ_{t+1}^A, i.e. on the expectations for the future real interest rate which, under perfect foresight, coincide with its realized value.

In the capital services market, at equilibrium, aggregate demand $K_t^A = N_t^A k_t^A$, must equal aggregate supply, $L_{t-1}^A \tilde{K}_t^A$ so that using (6) we obtain $K_t^A = (1-\alpha)L_{t-1}^A \omega_{t-1}^A$.

The dynamics of the economy are given by the labor market equilibrium condition, $\omega_t^A = \frac{\bar{\omega}_t^A}{\lambda}$, and by the evolution of the capital stock, K_t^A. We then define:

Definition 1. An intertemporal equilibrium with perfect foresight under autarky for the rigid wage country A is a sequence $\left\{a_t^A, K_t^A\right\}_{t=0}^\infty$ which, given the initial capital stock $K_{t=0}^A > 0$, satisfies the capital accumulation equation and the labor market equilibrium condition:

$$K_{t+1}^A = (1-\alpha)\,\omega(a_t^A, K_t^A)\frac{K_t^A}{a_t^A}, \tag{22}$$

$$(1-\alpha)^{1-\alpha}\alpha^\alpha \omega(a_t^A, K_t^A)\rho(a_{t+1}^A, K_{t+1}^A)^{1-\alpha} = \tfrac{\nu}{\lambda}. \tag{23}$$

where $\omega(a_t^A, K_t^A)$ and $\rho(a_{t+1}^A, K_{t+1}^A)$ are given by (21).

Eqs. (22)–(23) rule the dynamics of country A in autarky, and define a two-dimensional dynamic system with one predetermined variable, aggregate capital, which is given by past savings. In contrast, employment in t, and therefore a_t^A, are affected by expectations about the future real interest rate, opening the way for expectations driven fluctuations.

2.2.1. Steady state

A steady state of the dynamic system (22)–(23) is a solution $(a^A, K^A) = (a_t^A, K_t^A)$ for all t, such that

$$a^A = (1-\alpha)\,\omega(a^A, K^A), \tag{24}$$

$$(1-\alpha)^{1-\alpha}\alpha^\alpha \omega(a^A, K^A)\rho(a^A, K^A)^{1-\alpha} = \tfrac{\nu}{\lambda}. \tag{25}$$

This system only has one solution (a^A, K^A) as claimed in the following Proposition.[19]

Proposition 1. *There exists a unique stationary solution (a^A, K^A) of the dynamic system (22)–(23) given by*

$$a^A = \left(\frac{\nu}{\lambda}\right)\frac{(1-\alpha)(1-s)^{1-\alpha}}{\alpha^\alpha s^{1-\alpha}}, \tag{26}$$

$$K^A = \left[\frac{\varepsilon^{1+\beta}\phi^\beta \left(a^A\right)^{(1+\gamma-s)(1+\beta)}}{\Theta(1-\alpha)(1-s)(\varepsilon-1)}\right]^{\frac{1}{\gamma+\beta(1+\gamma)}}. \tag{27}$$

Using (21), we can express the real interest rate as $\rho_t^A = s\omega_t^A/[(1-s)a_t^A]$. Substituting now (24) in this last expression evaluated at the steady state we obtain:

$$\rho^A = \frac{s}{(1-s)(1-\alpha)}. \tag{28}$$

We shall consider that α is high enough so that the real interest rate at equilibrium is positive, i.e. $\rho^A > 1$. In the following Assumption we summarize the restrictions on the parameters' values we will consider from now on.

18 As usually done in the literature, we refrain from considering the condition that N should be an integer number. However, by choosing a sufficiently small ϕ, we can ensure that the number of varieties is higher than one in both countries.

19 Existence of equilibrium unemployment at the steady state is ensured by assuming that H^A is high enough, so that $L^A = K^A/a^A < H^A$. Then, trajectories that stay close to the steady state also exhibit unemployment.

Assumption 1. $s \in (1/4, 1/2)$, $0 \leq \beta < s/(1-s)$ and $1 > \alpha > \max\{(1-2s)/(1-s), 1/2\} \equiv \underline{\alpha}$.

Under this Assumption, the conditions on s, i.e. on the capital share of output in the economy, ensure that it takes an empirically plausible value. See for example Cecchi and Garcia-Peñalosa (2010). The restriction on β stipulates that *taste for variety* is not too high, in accordance with empirical findings. See Ardelean (2009). Moreover, this restriction allows the equilibrium labor (capital) demand curve to be downward sloping, by guaranteeing that this is the case when $\gamma = 0$. As stated above we suppose that the real interest rate is higher than one, i.e. that $\alpha > (1-2s)/(1-s)$. Finally, we also assume, in accordance with most empirical values obtained from national accounts of OECD countries, that the propensity to consume when young is higher than 1/2.

Before proceeding to analyze the local dynamics, it is interesting to discuss the effects of β - a novel feature of our work - on steady state outcomes. Remark first that β does not influence output per firm, which is constant (see (18) and (19)), nor a^A (see (22)), the real interest rate (see (28)) or the reservation wage (see (9)). Therefore it neither influences the real wage $\omega^A = \frac{\bar{\omega}^A}{\lambda}$. Using now (16) and (18) we conclude that $(N^A)^\beta (L^A)^\gamma$ and $l^A (L^A)^\gamma \equiv \frac{(L^A)^{1+\gamma}}{N^A}$ can not change with β. From the last expression we conclude that changes in L^A and N^A must go in the same direction. Since $(N^A)^\beta$ increases with β for a given N^A higher than one, the product $(N^A)^\beta (L^A)^\gamma$ will not remain constant if both N^A and L^A increase. We conclude therefore that N^A and L^A decrease with β. As *taste for variety* does not influence a^A it holds that the steady state capital stock, $K^A = L^A a^A$, also decreases with β. One would expect that a higher *taste for variety* would result in the production of a higher number of varieties, and therefore in a higher aggregate employment. However, we obtain the opposite result. This happens because of the existence of real wage rigidity: in country A the real wage is only influenced by the variables and parameters that determine indirect utility and the mark-up (and therefore the real reservation wage) and does not respond to changes in any other variables or parameters.[20] Hence, as the interest rate also does not respond to changes in β, from problem (3), we can see that individual demand for the composite goods C, D and $\tilde{K}$, does not change. Noting that, from (1) at the symmetric equilibrium, $C^A = (N^A)^{1+\beta} c^A$, as β increases the same amount of the composite good can be obtained with less demand for each variety. Accordingly, fims reduce production below the treshold given in (19), making losses. Consequently, some producers exit the market, and the number of varieties N^A decreases.

Finally, it is worthwhile to mention the effects of λ on stationary variables. Using (26), (27) and the definition of a^A, it holds that the capital-labor ratio, a^A, the capital stock, K^A, and the level of employment, L^A, decrease with λ. Moreover, from (20), we also get that the number of varieties produced decrease. The rationale for these results is the following. When λ increases the real wage decreases. As the real interest rate does not change (see (28)) firms will substitute capital by labor. Hence, employment at the firm level, l_i, increases and capital intensity, a^A, decreases. Therefore, the marginal productivity of capital at the firm level increases, which would lead *ceteris paribus* to a increase in the real remuneration of capital. However, at the agregate level, the number of varieties produced influences positively the real remuneration of capital through *taste for variety* (see (16)). It turns out that the observed decrease in N^A when λ increases is enough to keep the interest rate constant. Finally note that, although employment at the firm level increases with λ, employment at the economy level decreases with λ because, with *taste for variety* and free entry, the number of firms significantly decreases.

[20] Note that an increase in β acts as an increase in productivity, which we know to have no influence on wages, when there is wage rigidity.

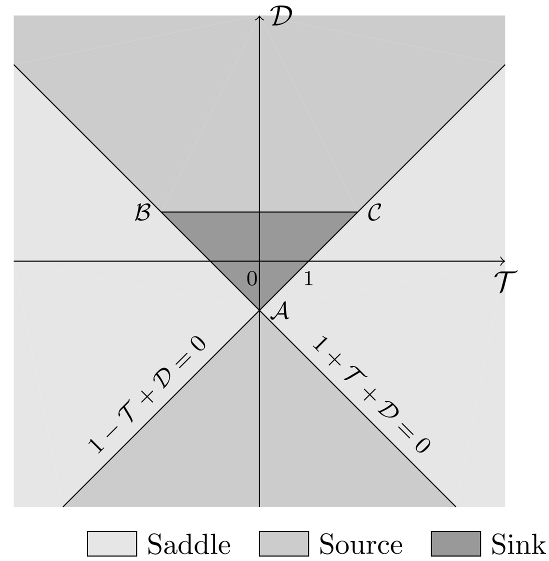

Fig. 1. Stability triangle.

2.2.2. Local dynamics and (in)determinacy

In this subsection we characterize the local dynamics of system (22)–(23). We analyze the role of the propensity to consume when young, α, of the degree of *taste for variety*, β, and of the degree of increasing returns, γ, on the emergence of local indeterminacy and expectation driven fluctuations. Remark that since this system is loglinear, bifurcations are not possible. Denoting percentage deviations from the steady state respectively by $\hat{K}_t^A \equiv \left(K_t^A - K^A\right)/K^A$ and $\widehat{a}_t^A \equiv \left(a_t^A - a^A\right)/a^A$ and loglinearizing (22)–(23) we obtain:

$$\begin{pmatrix} \hat{K}_{t+1}^A \\ \widehat{a}_{t+1}^A \end{pmatrix} = J \begin{pmatrix} \hat{K}_t^A \\ \widehat{a}_t^A \end{pmatrix} \tag{29}$$

where J, given in Appendix A.1, is the Jacobian matrix of the dynamic system. Then, the following Proposition holds.

Proposition 2. *The characteristic polynomial of system (22)–(23) is defined by* $P(\lambda^A) = (\lambda^A)^2 - \lambda^A T + D$, *where the trace,* T, *and the determinant,* D, *are given by:*[21]

$$T = \frac{1-\alpha(1+\beta)(1+\gamma-s)}{(1-\alpha)(1+\gamma-s)(1+\beta)}, \quad D = \frac{s}{(1-\alpha)(1+\gamma-s)} > 0. \tag{30}$$

Proof. See Appendix A.1. □

Following Grandmont et al. (1998), we study the local stability properties of our model, which are determined by the eigenvalues of the characteristic polynomial $P(\lambda^A) = (\lambda^A)^2 - \lambda^A T + D$,[22] by referring to the diagram represented in Fig. 1.

One eigenvalue is equal to 1 on the line AC ($D = T - 1$). On the line AB ($D = -T - 1$) one eigenvalue is equal to -1. On the segment BC the two eigenvalues are complex conjugates

[21] Note that while the propensity to consume when young, α, the degree of increasing returns, γ, and the capital share, s, influence both the trace and the determinant, β, the degree of *taste for variety* only affects the trace. Moreover, as both the trace and the determinant do not depend on the markup, λ, local dynamics are not affected by its value.

[22] Note that T and D correspond respectively to the product and sum of the two roots (eigenvalues) of the associated characteristic polynomial.

with modulus equal to 1. Therefore the steady state is a sink (both eigenvalues with modulus lower than one) when (T, D) is inside the triangle ABC. Since only capital is a predetermined variable, when the steady state is a sink, it is locally indeterminate[23] and there are infinitely many stochastic endogenous fluctuations (sunspots) arbitrarily close to the steady state.[24] The steady state is a source (both eigenvalues with modulus higher than one) if (T, D) is above AB, AC and BC or below AB and AC. It is saddle stable (one eigenvalue with modulus higher than one and one eigenvalue with modulus lower than one) in the remaining cases. In the Proposition below we present conditions on the parameters under which the steady state is a sink, a saddle or a source.

Proposition 3. *Consider Assumption 1 satisfied and define* $\tilde{\alpha} \equiv \frac{1+2s(1+\beta)}{1+3s(1+\beta)}$, $\gamma_1^{aut} \equiv \frac{\alpha(1-s)-(1-2s)}{(1-\alpha)}$ *and* $\gamma_2^{aut} \equiv \frac{2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]}{(1+\beta)(2\alpha-1)}$. *Then, in the country with rigid wages (country A), the following generically holds in autarky:*

⌊i⌋ When $\underline{\alpha} < \alpha < \tilde{\alpha}$, *for* $\gamma < \gamma_1^{aut}$ *the steady state is a source, becomes a sink (locally indeterminate) for* $\gamma_1^{aut} < \gamma < \gamma_2^{aut}$, *becoming a saddle for* $\gamma > \gamma_2^{aut}$.

⌊ii⌋ When $\tilde{\alpha} < \alpha < 1$, *for* $\gamma < \gamma_2^{aut}$ *the steady state is a source, becoming a saddle for* $\gamma > \gamma_2^{aut}$.

Proof. See Appendix A.2. □

This Proposition shows that when α is not too low, nor too high, local indeterminacy is possible. However, a minimal degree of labor externalities, $\gamma > \gamma_1^{aut} > 0$, is also necessary for indeterminacy,[25] as in Farmer and Guo (1994) with infinitely lived agents, and Aloi and Lloyd-Braga (2010) or Lloyd-Braga et al. (2007), with an overlapping generations framework.[26] Note however that, in the presence of empirically plausible values for α which exceed 0.5, local indeterminacy remains possible, in the country with labor market imperfections, for small increasing returns.[27] Proposition 3 also tells us that labor externalities can not be too high for indeterminacy to occur, i.e. $\gamma < \gamma_2^{aut}$.

In order to understand why indeterminacy requires a lower bound and an upper bound on the labor externality, consider that the economy is at the steady state in period t and suppose that agents anticipate a raise in the future interest rate. According to (9), the increase of the expected future interest rate, ρ_{t+1}^A, will decrease the current reservation wage, $\bar{\omega}_t^A$, and the current wage, ω_t^A, so that, considering the labor demand curve is downward sloping, the current level of employment, L_t^A, will increase. Accordingly, current savings also rise implying an increase in the future capital stock, K_{t+1}^A. When $\beta = \gamma = 0$ this increase will unambiguously reduce the future interest rate, see (21), so that expectations can not be fulfilled. However, in the presence of *taste for variety* , $\beta > 0$, and/or labor externalities, $\gamma > 0$, this increase in K_{t+1}^A will shift the labor demand curve to the right, increasing the future level of employment. Note that this increase will be higher for bigger values of γ. In turn, this increase in future employment will increase the future real interest. Therefore, if the positive effect on ρ_{t+1}^A is sufficiently high, i.e. if γ is sufficiently big ($\gamma > \gamma_1^{aut}$), it may overcome the negative effect due to the increase in capital in $t + 1$. As a result, in this last case, expectations can be self-fulfilling. Local indeterminacy also requires a future reversal in the trajectory so that, in the absence of further shocks to expectations, the system will return to the steady state. This implies that we must observe a decrease in the future capital stock, that is the future wage bill must decrease. For this to happen L_{t+1}^A must not increase too much, i.e. γ can not be too big ($\gamma < \gamma_2^{aut}$).

It is worth mentioning the role of *taste for variety*, β, on the local dynamics of the autarkic system. Looking first at the critical bounds of the labor externality, we can see that γ_1^{aut} does not depend on β, while γ_2^{aut} is a decreasing function of β. Second, note also that $\tilde{\alpha}$, the upper bound on α above which indeterminacy does not emerge, decreases with β. Therefore, a higher *taste for variety* reduces the likelihood of local indeterminacy, by shrinking the interval of parameters' values, under which indeterminacy emerges, enlarging the set of parameter values for which the steady state is saddle stable. Our results contrast with those obtained by Seegmuller (2008) who finds that, when the elasticity of substitution between capital and labor is one as in our framework, *taste for variety* facilitates the emergence of local indeterminacy in a closed economy.[28] Note that the indeterminacy mechanism considered in Seegmuller (2008) is totally different from ours. In his framework indeterminacy does not require labor externalities because he does not consider current consumption ($\alpha = 0$), i.e. the real interest rate is negative.

2.3. Country B - equilibrium with full employment

In country B the labor market is perfectly competitive and full employment exists, so that $L^B = H^B$. Using (18) it follows that the number of varieties is only a function of the current capital stock K_t^B:

$$N_t^B = \frac{\left(K_t^B\right)^s \left(H^B\right)^{1+\gamma-s}}{\varepsilon\phi} \equiv N^B(K_t^B). \tag{31}$$

Using (17) the real interest rate and the real wage can also be written as functions of K_t^B:

$$\begin{aligned}\rho_t^B &= \Theta N^B(K_t^B)^\beta \left(\tfrac{\varepsilon-1}{\varepsilon}\right) s \left(K_t^B\right)^{s-1} \left(H^B\right)^{1+\gamma-s} \equiv \rho^B(K_t^B),\\ \omega_t^B &= \Theta N^B(K_t^B)^\beta \left(\tfrac{\varepsilon-1}{\varepsilon}\right)(1-s) \left(K_t^B\right)^{s} \left(H^B\right)^{\gamma-s} \equiv \omega^B(K_t^B).\end{aligned} \tag{32}$$

Therefore, equilibrium dynamics in country B are totally determined by the evolution of capital.

Definition 2. An intertemporal equilibrium with perfect foresight under autarky for the full employment country B is a sequence $\left\{K_t^B\right\}_{t=0}^{\infty}$, which given the initial capital stock $K_{t=0}^B > 0$, satisfies the capital accumulation equation:

$$K_{t+1}^B = (1-\alpha)\,\omega^B(K_t^B)H^B. \tag{33}$$

with $\omega^B(K_t^B)$ given by (32).

This equation defines a one-dimensional system which characterizes the dynamics of country B in autarky.

23 Indeterminacy occurs when the number of eigenvalues strictly lower than one in absolute value is larger than the number of predetermined variables.

24 See also Woodford (1986).

25 Note that, in contrast to Seegmuller (2008) who does not consider current consumption ($\alpha = 0$), positive labor externalities, $\gamma > 0$, are required to obtain local indeterminacy with plausible values for the propensity to consume $\alpha > \underline{\alpha} > 1/2$.

26 As emphasized in Lloyd-Braga et al. (2007), the conditions for indeterminacy are similar in an overlapping generation model with a propensity to current consumption compatible with what is observed in data, and in a model of infinitely lived agents as the one explored in Farmer and Guo (1994). In both set ups considering a more elastic labor supply curve, which in our case is infinitely elastic, reduces the lower bound for γ required for indeterminacy.

27 For example, when the share of capital in total income is 0.3 we have $\underline{\alpha} = 0.571$, and considering $\alpha = 0.6$ we have $\gamma_1^{aut} = 0.05$. Considering a slightly higher $s = 1/3$, and still considering $\alpha = 0.6$, we have $\gamma_1^{aut} = 0.167$.

28 However, Seegmuller (2008) finds a result similar to ours when capital and labor are sufficiently complementary, i.e. with an elasticity of substitution smaller than 1/2.

2.3.1. Steady state

A steady state of the dynamic system (33) is a solution $K^B = K_t^B = K_{t+1}^B$ for all t, such that

$$K^B = (1-\alpha)\,\omega^B(K^B)H^B. \tag{34}$$

We can easily prove that:[29]

Proposition 4. *K^B is a unique stationary solution of the dynamic system (33). The value of K^B is given by*

$$K^B = \left[\frac{\Theta(1-\alpha)(1-s)(\varepsilon-1)(H^B)^{(1+\gamma-s)(1+\beta)}}{\varepsilon^{1+\beta}\phi^{\beta}}\right]^{\frac{1}{1-s(1+\beta)}}. \tag{35}$$

The number of varieties evaluated at the steady state is given by

$$N^B = \left[\frac{\Theta(1-\alpha)(1-s)(\varepsilon-1)\left(H^B\right)^{\frac{1+\gamma-s}{s}}}{\varepsilon^{\frac{1}{s}}\phi^{\frac{1-s}{s}}}\right]^{\frac{s}{1-s(1+\beta)}}. \tag{36}$$

From this last expression we can see that, as expected, the number of varieties in country B increases with *taste for variety*. This contrasts with what happens in country A, where the number of varieties (firms) decreases with β due to wage rigidity as explained before.

Note also that the steady state real interest rate in country B is identical to the steady state real interest rate of country A given in (28). Using (32), we can express country B's real interest rate as $\rho_t^B = s\omega_t^B H^B/[(1-s)K_t^B]$. Substituting now (34) in this last expression evaluated at the steady state we obtain:

$$\rho^B = \rho^A = \frac{s}{(1-s)(1-\alpha)}. \tag{37}$$

2.3.2. Local dynamics and (in)determinacy

Differentiating the capital accumulation given in (33) we obtain:

$$\frac{dK_{t+1}^B}{K_{t+1}^B} = s\,(1+\beta)\,\frac{dK_t^B}{K_t^B}. \tag{38}$$

We can immediately see that, as the dynamic system is loglinear, bifurcations are not possible. Moreover, under Assumption 1, we can state the following.

Proposition 5. *Consider Assumption 1 satisfied. Then, the steady state of the full employment country B is stable as $s\,(1+\beta) < 1$.*

3. The two-country model

We consider a world economy with two countries, A and B, which differ only in the functioning of their labor markets. We suppose that capital is mobile across countries, which implies that the nominal interest rates are equalized, i.e. $r_t^A = r_t^B$, while labor is internationally immobile. Furthermore, goods are freely traded, so that households, both from country A and country B, have access to all N_t^W varieties existing in the world, some produced in country A and others in country B i.e. $N_t^W = N_t^A + N_t^B$.[30]

Hence, in the world economy, the composite goods C_t^j and D_{t+1}^j and investment I_t^j in country $j \in \{A, B\}$ are defined as:

$$C_t^j = (N_t^W)^{1+\beta}\left[\frac{1}{N_t^W}\sum_{i=1}^{N_t^W}(c_{it}^j)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$
$$D_{t+1}^j = (N_{t+1}^W)^{1+\beta}\left[\frac{1}{N_{t+1}^W}\sum_{i=1}^{N_{t+1}^W}(d_{it+1}^j)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{39}$$

$$I_t^j = (N_t^W)^{1+\beta}\left[\frac{1}{N_t^W}\sum_{i=1}^{N_t^W}(i_{it}^j)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{40}$$

The price of each variety is identical for all agents. Moreover, all individuals share the same *taste for variety* and buy all varieties. Hence, the price of the composite good, given in (5), which we denote by P_t^W, is now defined over all varieties $i = 1, \ldots, N_t^W$, and is identical for all households in the world. It follows that the demand for variety i at the world level is now given by

$$\upsilon_{it}^W = \left(\frac{p_{it}}{P_t^W}\right)^{-\varepsilon}\sum_{j=A,B}(N_t^W)^{\beta(\varepsilon-1)-1}\left[L_t^j\left(C_t^j + I_t^j\right) + L_{t-1}^j D_t^j\right]. \tag{41}$$

Therefore, the solution of the problem faced by the producer of each differentiated good, in both countries A and B, is similar to that under autarky. Indeed, producers of each variety face the same aggregate demand, and produce an identical quantity at the free-entry equilibrium, $y_t^j = \Theta\phi(\varepsilon-1)$ as given in (19). Equilibrium in the world output market is symmetric, $p_{it} = p_t$. Taking into account this last relation, together with international capital mobility, $r_t^A = r_t^B$, using (15) for country A and country B, we obtain the following relation at equilibrium:

$$\left(a_t^A\right)^{s-\gamma-1}\left(K_t^A\right)^{\gamma} = \left(K_t^B\right)^{s-1}\left(H^B\right)^{1+\gamma-s} \tag{42}$$

where K_t^j denotes capital used in production in country $j \in \{A, B\}$.

We denote by K_t^W the world capital stock $K_t^W = K_t^A + K_t^B$. Then, using (42) with $K_t^B = K_t^W - K_t^A$, it is possible to express a_t^A as a function of K_t^A and of K_t^W:

$$a_t^A = \frac{(K_t^W - K_t^A)^{\frac{1-s}{1+\gamma-s}}\left(K_t^A\right)^{\frac{\gamma}{1+\gamma-s}}}{H^B} \equiv a^A(K_t^A, K_t^W). \tag{43}$$

Using (18) as under autarky, (20) and (31) still apply. Taking into account that $N_t^W = N_t^A + N_t^B$, the number of varieties in the world can be written as a function of K_t^A and of K_t^W:

$$N_t^W = \frac{\left[a^A(K_t^A, K_t^W)\right]^{s-1-\gamma}(K_t^A)^{1+\gamma} + \left(H^B\right)^{1+\gamma-s}(K_t^W - K_t^A)^s}{\varepsilon\phi}$$
$$\equiv N^W(K_t^A, K_t^W). \tag{44}$$

As $p_{it} = p_t$ we obtain $P_t^W = (N_t^W)^{-\beta}p_t$. This relation, together with international capital mobility, implies that real interest rates have to be identical across countries. Using (16) and (17) together with (44), the world equilibrium real interest rate is given by:

$$\rho^W(K_t^A, K_t^W) = \Theta N_t^W(K_t^A, K_t^W)^{\beta}\left(\tfrac{\varepsilon-1}{\varepsilon}\right)s\left[a^A(K_t^A, K_t^W)\right]^{s-1-\gamma}(K_t^A)^{\gamma}$$
$$= \Theta N_t^W(K_t^A, K_t^W)^{\beta}\left(\tfrac{\varepsilon-1}{\varepsilon}\right)s(K_t^W - K_t^A)^{s-1}\left(H^B\right)^{1+\gamma-s} \tag{45}$$

However, real wages are different across countries:

$$\omega^{A,W}(K_t^A, K_t^W) = \Theta N_t^W(K_t^A, K_t^W)^{\beta}\left(\tfrac{\varepsilon-1}{\varepsilon}\right)(1-s)\left[a^A(K_t^A, K_t^W)\right]^{s-\gamma}(K_t^A)^{\gamma},$$
$$\omega^{B,W}(K_t^A, K_t^W) = \Theta N_t^W(K_t^A, K_t^W)^{\beta}\left(\tfrac{\varepsilon-1}{\varepsilon}\right)(1-s)(K_t^W - K_t^A)^s\left(H^B\right)^{\gamma-s} \tag{46}$$

29 To guarantee that full employment exists at the steady state we ensure that $\omega^B > \bar{\omega}^B$ at the steady state by choosing a sufficiently small ν. Therefore along trajectories sufficiently close to the steady state $\omega_t^B > \bar{\omega}_t^B$.

30 Since each firm produces a specific differentiated product, varieties produced in A are different from the ones produced in B. Note that the same happened in autarky. Indeed, as explained before, we consider the existence in each economy of a single productive sector featuring horizontally differentiated goods, i.e. varieties with different attributes (quality, location, color, brand and so on).

although they are linked by the following relationship:

$$\omega^{B,W}(K_t^A, K_t^W) = \frac{K_t^W - K_t^A}{a^A(K_t^A, K_t^W)H^B}\omega^{A,W}(K_t^A, K_t^W). \tag{47}$$

3.1. Equilibrium

The world equilibrium is given by two dynamic equations describing respectively the world capital accumulation that prevails under international capital mobility and the labor market equilibrium in country A.

Capital accumulation in the world is driven by the sum of savings in both countries, i.e.:

$$K_{t+1}^W = (1-\alpha)\left[\omega^{A,W}\left(K_t^A, K_t^W\right)\frac{K^A}{a_t^A(K_t^A, K_t^W)} + \omega^{B,W}\left(K_t^A, K_t^W\right)H^B\right] \tag{48}$$

and the labor market equilibrium in country A is given by

$$(1-\alpha)^{1-\alpha}\alpha^\alpha\omega^{A,W}(K_t^A, K_t^W)\left[\rho^W(K_{t+1}^A, K_{t+1}^W)\right]^{1-\alpha} = \frac{\nu}{\lambda} \tag{49}$$

with $a_t^A\left(K_t^A, K_t^W\right)$, $\rho^W(K_t^A, K_t^W)$, $\omega^{A,W}\left(K_t^A, K_t^W\right)$ and $\omega^{B,W}\left(K_t^A, K_t^W\right)$ given in (43), (45) and (46).

Definition 3. An intertemporal equilibrium with perfect foresight of the world economy is a sequence $\left\{K_t^A, K_t^W\right\}_{t=0}^{\infty}$ which satisfies (48)–(49), given the initial world capital stock $K_{t=0}^W > 0$.

Eqs. (48)–(49) define a two-dimensional dynamic system with one predetermined variable, the world aggregate capital which is given by past savings. However, capital used in production in each country is a non predetermined variable as capital moves freely across countries.[31]

3.2. Trade balance and capital flows

The trade balance of country $j \in \{A, B\}$ in real terms, $\mathcal{TB}_t^j$, is defined as the country's excess supply of goods. From the aggregate production given in (11) we get that:

$$\mathcal{TB}_t^j = Y_t^j - \left(L_t^j C_t^j + L_t^j \tilde{K}_{t+1}^j + L_{t-1}^j D_t^j\right).$$

When the trade balance of country $j \in \{A, B\}$, $\mathcal{TB}_t^j$, is positive at equilibrium, the country $j \in \{A, B\}$ is a net exporter of goods as output is higher than domestic demand. Of course, in equilibrium, the sum of the two countries' trade balances must be zero, i.e. $\mathcal{TB}_t^A + \mathcal{TB}_t^B = 0$.

Using the budget constraint of households we can rewrite the balance of trade as:

$$\mathcal{TB}_t^j = Y_t^j - L_t^j\omega_t^{j,W} - \rho_t^W L_{t-1}^j \tilde{K}_t^j.$$

Recall that firms are subject to a free-entry condition so that $Y_t^j = \rho_t^W K_t^j + \omega_t^{j,W} L_t^j$. It follows that:

$$\mathcal{TB}_t^j = \rho_t^W(K_t^j - L_{t-1}^j\tilde{K}_t^j)$$

i.e. a surplus of the trade balance of country j implies that capital used in production in that country, K_t^j, exceeds savings of its residents, that were accumulated in the past, $L_{t-1}^j\tilde{K}_t^j$. Denoting by $\eta_t^j = K_t^j - L_{t-1}^j\tilde{K}_t^j$ the inflows of capital services to country $j \in \{A, B\}$ in period t, we obtain

$$\mathcal{TB}_t^j = \rho_t^W\eta_t^j. \tag{50}$$

[31] Without international capital mobility, factor prices still depend on the number of varieties in the world, but, as nominal interest rates are not equalized, the traditional arbitrage condition (42) does not hold anymore. It follows that the dynamic system would become three-dimensional, being characterized by the capital accumulation of each country and the labor market equilibrium in country A, with two-predetermined variables, the capital stock in each country.

4. Steady state of the world economy

A steady state of the dynamic system (48)–(49) of the world economy is a sequence $(K_t^A, K_t^W) = (K^A, K^W)$ for all t satisfying

$$K^W = (1-\alpha)\left[\omega^{A,W}\left(K^A, K^W\right)\frac{K^A}{a^A(K^A, K^W)} + \omega^{B,W}\left(K^A, K^W\right)H^B\right] \tag{51}$$

$$(1-\alpha)^{1-\alpha}\alpha^\alpha\omega^{A,W}(K^A, K^W)\rho^W(K^A, K^W)^{1-\alpha} = \frac{\nu}{\lambda}. \tag{52}$$

The following Proposition establishes existence and uniqueness of the steady state with open borders.

Proposition 6. *Consider that Assumption 1 is satisfied. Then, there exists a unique stationary solution (K^A, K^W) of the dynamic system of the world economy (48)–(49) given by*

$$K^W - K^A = \left[\frac{\Theta(\varepsilon-1)(1-s)(1-\alpha)}{\varepsilon(\varepsilon\phi)^\beta}(H^B)^{(1+\beta)(1+\gamma-s)}\right]^{\frac{1}{(1+\beta)(1-s)}} \times (K^W)^{\frac{\beta}{(1+\beta)(1-s)}} \tag{53}$$

where K^A is the unique solution of

$$\frac{\varepsilon^{1+\beta}\phi^\beta\left(a^A\right)^{(1+\gamma-s)(1+\beta)}}{\Theta(\varepsilon-1)(1-s)(1-\alpha)} = \left[\left(K^A\right)^{\frac{\gamma+(1+\gamma)\beta}{\beta}} + \left(H^B a^A\right)^{\frac{1+\gamma-s}{1-s}}\left(K^A\right)^{\frac{\gamma(1-s-\beta s)}{\beta(1-s)}}\right]^\beta \tag{54}$$

with

$$a^A = \left(\frac{\nu}{\lambda}\right)\frac{(1-\alpha)(1-s)^{1-\alpha}}{\alpha^\alpha s^{1-\alpha}}. \tag{55}$$

Moreover, the steady state world real interest rate is identical to the autarkic ones in both countries:

$$\rho^W = \rho^A = \rho^B = \frac{s}{(1-s)(1-\alpha)}. \tag{56}$$

Proof. See Appendix A.4 □

Before opening the borders the steady state real interest rates were already identical in both countries. Therefore there will be no adjustments in savings/investment decisions and no incentives for international capital movements once they are liberalized. Naturally, the absence of capital movements at the steady state implies that the world interest rate will not adjust, keeping a value identical to the one observed under autarky. It also means that, at the steady state, capital used in production equals savings in both countries, so that capital inflows are zero, $\eta^j = K^j - L^j\tilde{K}^j = 0$, $j \in \{A, B\}$. Indeed, steady state savings in country A are given by $(1-\alpha)\omega^{A,W}L^A$. From (64) we have that $\omega^{A,W} = a^A/(1-\alpha)$. Substituting this in the expression for savings we immediately have that $(1-\alpha)\omega^{A,W}L^A = a^A L^A = K^A$, i.e. capital accumulation in country A equals savings in that country. Of course, as world capital accumulation is equal to world savings, this implies that the same happens in country B. Using (50) we also have that at the stationary state the trade account is balanced.

Since each variety is only produced in one country (in one firm) but, due to *taste for variety*, consumed in both countries, each country exports the varieties it produces, importing the varieties produced abroad. Therefore bilateral trade exists, due to the existence of *taste for variety*, even at the steady state. However, at the steady state the value of imports is identical to the value of exports so that the steady state is characterized by no net *intra-industry* trade. Nevertheless, as discussed in the next section, non-steady state equilibria exhibit net trade. Also, net trade would have been obtained at the steady state if the degree of *taste* *for variety* was different across countries.[32] We chose instead to consider identical fundamentals (preferences and technologies) across countries because this simpler framework is sufficient to explain the existence of bilateral trade, both at the steady state and along deterministic and stochastic fluctuations.

It is also interesting to analyze the effects of the degree of *taste for variety*, β, on steady state equilibrium. Using (55) it is easy to see that a^A is not affected by β. In Appendix A.5, we prove that steady state capital stock and employment in country A after opening the borders are smaller when β is higher. The same happens with the number of varieties produced in country A at the steady state with trade and capital mobility, which is a decreasing function of β.[33] As in the autarkic equilibrium this is due to the existence of wage rigidity. In contrast, in country B, after opening the borders, the steady state capital stock and varieties produced increase with β.[34] This implies that N^A/N^B decreases with β. Denoting the share of varieties produced in country A at the steady state by $n^A \equiv N^A/N^W = \frac{N^A/N^B}{1+N^A/N^B}$ we conclude that this share also decreases with β. As proved in Appendix A.3, n^A is also identical to K^A/K^W, the share of capital used in country A in total capital used in the world at the steady state. We use this share as a proxy of the relative economic size of country A and, as we shall see, it influences our dynamic results.[35]

4.1. Steady state effects of opening the borders

In this section we analyze the steady state effects of opening the borders to *intra-industry* trade and to capital flows in both countries. Our main results are summarized in Proposition 7 below.

Proposition 7. *Consider that Assumption 1 is satisfied and* $\beta > 0$*. Then, the following results hold:*

$\lfloor i \rfloor$ *In country A at the free-trade steady state with capital mobility, the capital stock, the number of varieties produced and employment are lower than their respective values at the autarkic steady-state. The steady state real wage rate and the real interest rate are identical at both steady-states. The world share of varieties produced in country A, the world share of the capital stock of country A and the world share of savings of country A are smaller than their respective values at the autarkic steady state;*

$\lfloor ii \rfloor$ *In country B at the free-trade steady state with capital mobility, the capital stock, the number of varieties produced, and the real wage are higher than their respective values at the autarkic steady-state. The real interest rate is identical at both steady-states. The world share of varieties produced in country B, the world share of the capital stock of country B and the world share of savings of country B are higher than their respective values at the autarkic steady state;*

$\lfloor iii \rfloor$ *The total number of varieties consumed in each country at the free-trade steady state with capital mobility is higher than the number of the varieties consumed in autarky in each country;*

Proof. See Appendix A.6 and the paragraphs below. □

As referred before, varieties produced in A are different from the ones produced in B, both under autarky and under free-trade. Following openness, as stated in Proposition 7 above, *intra-industry* trade destroys employment and firms (varieties produced) in the rigid wage country while it increases the number of firms (varieties produced) in the full employment country. However, although in country A the number of varieties produced with free trade is smaller than in autarky, households in A consume more varieties after opening the borders as the total number of varieties produced in the globalized world is higher than the number of varieties produced in country A under autarky. See Proposition 7 $\lfloor iii \rfloor$. Moreover, as typically happens in models of *intra-industry* trade where preferences and technologies are identical across countries, in our model the pattern of trade is indeterminate. See Krugman (1980). We are not able to determine which varieties are produced in each country.

Aloi and Lloyd-Braga (2010), considering a similar framework, but without *taste for variety* ($\beta = 0$) and no trade, find that the autarkic steady state and the steady state with perfect capital movements coincide. It follows that all the steady state effects stated in Proposition 7 above are due to the presence of *intra-industry trade* with *taste for variety*, $\beta > 0$, and not associated with capital mobility.[36] The intuition is as follows. With *taste for variety,* the number of varieties households are able to consume with trade is naturally higher than under autarky. See Appendix A.6. This would lead, if nothing else changed, to an increase in real wages in both countries. See (46). However, in country A, due to real wage rigidity this increase cannot happen.[37] Since a^A is the same before and after opening the borders, and real wages are increasing in capital, capital must decrease in A to compensate for the increase in varieties. See (46). This in turn implies a lower level of employment. In country B, in contrast, real wages increase with the number of varieties. However, the interest rate remains identical to its steady state level under autarky. See Proposition 6. Since the observed increase in the number of varieties consumed after opening the borders pushes the interest rate up, to counteract this increase, and since the real interest rate is decreasing in capital, capital must increase in B, see (45), which reinforces the initial increase in wages in this country. See (46). Note that in the absence of *taste for variety*, $\beta = 0$, the number of varieties no longer plays a role, and all these effects would vanish.

Although our model is quite stylized, it is able to capture some fears commonly associated with globalization/free-trade agreements, that are based on the belief that opening the borders

32 Suppose that *taste for variety* increases in country A. Then, as shown below, N^A decreases. Country B imports less varieties, and becomes a net exporter of goods, which corresponds to a deficit in the trade account of country A. Indeed, when β is different across countries, the price of the composite good, and hence the real interest rate, are no longer equal across countries. A higher β in country A means that the price of the composite good in this country is lower than in country B. Hence, the real interest rate will be higher in A. It can be shown that this leads to an excess of savings over and above the amount of capital used in production in this country. We then observe an outflow of capital from country A, which compensates its deficit in the trade account.

33 Using (20), we can see that if capital decreases with β the same happens with N^A, as a^A does not vary with β.

34 Indeed from (42) we can see that, as a^A is not affected by β and as K^A decreases, K^B must increase with β.

35 Finally, we mention the effects of λ on stationary variables. For country A using (20), (54), (55) and the definition of a^A, we find that, as in autarky, the capital-labor ratio, a^A, the capital stock, K^A, the level of employment, L^A, and the number of varities produced, N^A, decrease with λ. The rationale is the same as in autarky. See Section 2.2.1. However, in the globalized economy, the change in N^A, induced by a change in λ, also influences the steady state values of country B. More precisely, the resulting contraction in economy A is exported to country B. Using (20), (31), (45), (55), (70) and the definitions of N^W and a^A, it follows that the capital stock, K^B, and the number of varieties produced, N^B, also decrease with λ. Therefore the total number of varieties existing in the world, N^W, also decreases.

36 Indeed, without capital mobility steady state real interest rates are still identical across countries. They are still given by (45), only depending on the propensity to save and on the capital share of output, which are the same in the two countries. Therefore, with or without capital mobility, there will be no capital flows at the steady state. Hence, steady state effects without capital mobility would be identical to those obtained with capital mobility.

37 Since the steady state real interest rate does not change after opening the borders, the reservation wage remains the same and so does the real wage, which is a constant markup over the reservation wage.

will displace industries and jobs abroad, increasing unemployment. Indeed, in our framework this happens in the rigid wage country.[38] The reverse implication of this mechanism, is that a country without significant labor market rigidities, will suffer drastic losses by reverting to an autarkic regime if most of the trade is *intra-industry*. Egger et al. (2011), in a static model, show that free trade and capital mobility lead to a higher number of varieties produced abroad when labor market rigidities increase in the home country, a result consistent with ours, according to which globalization leads to an increase in the number of varieties produced in the flexible full employment economy.

It is also interesting to analyze the effects of trade on capital intensity at the firm level in both countries. In country *B*, as aggregate employment is constant and the number of firms increases, employment at the firm level decreases. However, as production per firm is constant, (see (19)) capital per firm increases. Therefore, firms in country *B* become more capital intensive. Indeed, since in this country, ω/ρ, the ratio between real rages and the real interest rate increases, firms will substitute labor for capital. In contrast in country *A*, as ω/ρ does not change, capital intensity at the steady state remains unchanged after opening the borders.

4.1.1. Stationary welfare

We now compare steady state welfare in the two countries before and after the opening of the borders. In country *A*, as the real interest rate and the real wage are identical before and after trade, the utility of a worker that keeps its job when there is trade is the same in the two steady states. However, as employment is smaller in the steady state with free trade and perfect capital mobility, and the utility of an unemployed worker is zero, it follows that, under an utilitarian social welfare function, aggregate utility decreases. In contrast, in country *B*, there is full employment before and after opening the borders. As the real wage is higher in the world steady state and the real interest rate does not change, we conclude that individual and aggregate utility increase with trade. The following Proposition summarizes these results.

Proposition 8. *Consider that Assumption 1 is satisfied. Then, the following results hold at the steady state:*

⌊*i*⌋ *In country A, the utility level of a worker that keeps its job when there is trade is the same as in autarky. However, those workers that lose their jobs are worse off with free trade.*

⌊*ii*⌋ *All agents in country B gain from trade.*

The full employment country is the one that unambiguously benefits in terms of steady state welfare from free *intra-industry* trade. In contrast, in country *A* we observe, due to the existence of labor market distortions, an aggregate reduction in steady state welfare after opening to trade. In economies with distortions such as ours, benefits from trade are not guaranteed for all countries (see Helpman and Krugman, 1985) so that this result should not surprise us. However, Helpman and Itskhoki (2010), considering a static model with no capital, but with both intra and inter industry trade between two countries that also differ in the degree of labor market rigidities, find that both countries gain from trade in welfare terms. Nevertheless, like us, they also find an asymmetric impact of trade: the country with lower frictions in the labor market gains proportionately more.

As it is well known, *intra-industry* trade influences welfare through two channels: the scale effect and the variety effect. The first one emerges because trade, increasing market size, allows firm to produce more, benefitting from scale economies. Moreover, with trade, each country gains access to a larger number of varieties which increases utility in the presence of *taste for variety*. In our framework, the scale of production at the firm level is constant (see (19)), so that the scale effect is absent. Hence, we are able to ensure that all the effects of *intra-industry* trade on welfare operate via *taste for variety*.

5. Dynamics in the two-country model

We start by providing a full characterization of the local stability properties around the unique steady state equilibrium. We first loglinearize system (48)–(49). Denoting percentage deviations from the steady state respectively by $\widehat{K}_t^W \equiv \left(K_t^W - K^W\right)/K^W$ and $\widehat{K}_t^A \equiv \left(K_t^A - K^A\right)/K^A$ we have that

$$\begin{pmatrix} \widehat{K}_{t+1}^W \\ \widehat{K}_{t+1}^A \end{pmatrix} = J^W \begin{pmatrix} \widehat{K}_t^W \\ \widehat{K}_t^A \end{pmatrix} \tag{57}$$

where J^W, given in Appendix A.7, is the Jacobian matrix of the dynamic system. The following Proposition gives the characteristic polynomial.

Proposition 9. *The trace, T^W, and determinant, D^W, of matrix J^W, given below, correspond respectively to the sum and product of the two roots (eigenvalues) of the associated characteristic polynomial $P^W(\lambda^W) \equiv (\lambda^W)^2 - \lambda^W T^W + D^W$:*

$$T^W = 1 - \frac{\gamma - n^A(1+\gamma-s)[(1+\beta)s-\beta]}{(1-\alpha)n^A(1+\beta)(1+\gamma-s)(1-s)},\ D^W = -\frac{s[\gamma - n^A(1+\gamma-s)]}{(1-\alpha)n^A(1+\gamma-s)(1-s)} \tag{58}$$

where $n^A = N^A/N^W = K^A/K^W$.[39]

Proof. See Appendix A.7. □

As in autarky, we refer to Grandmont et al. (1998) in order to appraise the local stability properties of the dynamic system defined by (48)–(49). Note that, in contrast to what happened in autarky, the dynamic system with trade and capital mobility is not loglinear. Therefore, bifurcations are now possible. We can use Fig. 1 to study local bifurcations. When a (bifurcation) parameter is made to vary continuously in its admissible range, if the values of T^W and D^W cross the interior of the segment *BC*, a pair of complex conjugate eigenvalues crosses the unit circle and a Hopf bifurcation generically occurs. In this case there are deterministic cycles describing orbits that lie over an invariant closed curve, surrounding the steady state, in the state space. If the Hopf bifurcation is subcritical, this curve emerges when the steady state is a sink. When the Hopf bifurcation is supercritical the invariant closed curve appears when the steady state is determinate, a source, and although sunspot equilibria that stay arbitrarily close to the steady state do not exist, there are nevertheless infinitely many equilibria exhibiting bounded stochastic fluctuations around the invariant closed curve. Moreover, when T^W and D^W cross the *AB* line, a *flip* bifurcation (supercritical or subcritical) generically occurs, leading to the appearance of deterministic cycles of period two. Moreover, a cascade of period doubling cycles is expected to occur as the bifurcation parameter moves further away from its bifurcation value, eventually leading to the appearance of bounded aperiodic equilibrium trajectories.

In Proposition 10 we present our results, considering γ as our bifurcation parameter. As usually done in the literature, we consider the normalized steady state in country *A* with $a^A = 1 = K^A$, by fixing the parameters λ and Θ at the appropriate level.

38 Boulhol (2011), in a static two-sector, two-country model also finds that unemployment increases in the rigid wage country with bilateral inter-industry trade.

39 As in autarky β does not influence the determinant.

Then we use the parameter H^B to ensure that n^A does not vary with the other parameters that influence directly the trace and the determinant given in (58).[40]

Proposition 10. *Consider that Assumption 1 is satisfied and define* $\gamma_1^W \equiv \frac{n^A(1-s)[\alpha(1-s)-(1-2s)]}{s-n^A[\alpha(1-s)-(1-2s)]}$, $\gamma_2^W \equiv \frac{n^A(1-s)[2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]]}{1+s(1+\beta)-n^A[2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]]}$ *and* $\tilde{\alpha} \equiv \frac{1+2s(1+\beta)}{1+3s(1+\beta)}$. *Then, the following generically holds at the world level:*

⌊i⌋ When $\underline{\alpha} < \alpha < \tilde{\alpha}$, *for* $\gamma < \gamma_1^W$ *the steady state is a source, undergoes a Hopf bifurcation when* γ *crosses the critical threshold* γ_1^W, *becomes a sink (locally indeterminate) for* $\gamma_1^W < \gamma < \gamma_2^W$, *undergoes a flip bifurcation when* γ *crosses the critical threshold* γ_2^W, *becoming a saddle for* $\gamma > \gamma_2^W$.

⌊ii⌋ When $\tilde{\alpha} < \alpha < 1$, *for* $\gamma < \gamma_2^W$ *the steady state is a source, undergoes a flip bifurcation when* γ *crosses the critical threshold* γ_2^W, *becoming a saddle for* $\gamma > \gamma_2^W$.

Proof. See Appendix A.8. □

This Proposition shows that, in the presence of *intra-industry* trade and free international capital flows, the world economy, i.e. not only country A, but also country B, can exhibit local fluctuations driven by changes in expectations.[41] This will occur through bifurcations, that were not possible in autarky, and/or when the world equilibrium is locally indeterminate (a sink). Indeterminacy, as in autarky, requires intermediate values of the propensity to consume of a young agent, $\underline{\alpha} < \alpha < \tilde{\alpha}$ and a lower and an upper bound for the labor externality, γ_1^W and γ_2^W respectively. Although the bounds on the propensity to consume are the same as the ones in autarky, the bounds on the labor externality are different, depending on the value of n^A. It follows that the effects of opening the economies on local stability, can be studied by comparing the critical values for γ, γ_1^W and γ_2^W, with the relevant critical values in autarky, γ_1^{aut} and γ_2^{aut}. Concentrating in the case $\underline{\alpha} < \alpha < \tilde{\alpha}$, under Assumption 1, $\gamma_1^W > 0$ is an increasing function of n^A, becoming identical to γ_1^{aut} for $n^A = 1$. Therefore we have $\gamma_1^W < \gamma_1^{aut}$. Similarly $\gamma_2^W > 0$ is an increasing function of n^A, becoming identical to γ_2^{aut} for $n^A = 1$. Therefore we have $\gamma_2^W < \gamma_2^{aut}$. However, γ_2^W can be higher or lower than γ_1^{aut}, depending on the value of n^A. Accordingly we have the following Lemma.

Lemma 1. *Assume that* $\underline{\alpha} < \alpha < \tilde{\alpha}$ *and consider Assumption 1 satisfied. Then, defining* $n_*^A \in (0,1)$

$$n_*^A \equiv \frac{[1+s(1+\beta)][\alpha(1-s)-(1-2s)]}{s\,[2(1+\beta)[(1-\alpha)(1-s)+s]-\beta]}$$

we have:

⌊i⌋ For $n^A < n_*^A$, $0 < \gamma_1^W < \gamma_2^W < \gamma_1^{aut} < \gamma_2^{aut}$;

⌊ii⌋ For $n^A > n_*^A$, $0 < \gamma_1^W < \gamma_1^{aut} < \gamma_2^W < \gamma_2^{aut}$.

5.1. Effects of opening the economies on stability

From Proposition 10 and Lemma 1, it follows that the relative size of the two countries will influence the results. To facilitate the analysis we present in Fig. 2, in the space (n^A, γ), the critical values of γ delimiting the regions where the steady state is locally a source, sink and saddle, both under autarky (for country A) and after opening the borders (in the World, W), considering $\underline{\alpha} < \alpha < \tilde{\alpha}$. To further ease the discussion we provide a numerical illustration. In accordance with Assumption 1, we consider $s = 1/3$, a sufficiently small value for $\beta = 0.01$, and $\alpha = 0.6 \in (\underline{\alpha} = 0.5, \tilde{\alpha} = 0.833)$, so that $\gamma_1^{aut} = 0.167$, $\gamma_2^{aut} = 5.95$ and $n_*^A = 0.222$. In order to concentrate the discussion on empirically plausible values for the parameters, in Fig. 2 we will only consider values for γ below $s = 1/3$,[42] i.e. γ_2^{aut} will not be depicted. Moreover, we denote by $n_{**}^A \in (n_*^A, 1)$ the value of n^A such that $\gamma_2^W = s$. With our parametrization we have $n_{**}^A = 0.371$.

The first result we highlight is that, after opening the borders, country A may become saddle determinate for empirically plausible values of the parameters. In terms of Fig. 2, this will occur in the region to the left and above the red line representing γ_2^W. Since γ_2^W decreases as n_A decreases, we conclude that the bigger the size of country B, the more likely is local saddle determinacy in a globalized world. In this case the local stability properties of country B are exported to country A. In contrast, in the region to the right of the γ_2^W schedule, country B may become locally indeterminate, or even a source. In this case the local stability properties of country A, are exported to country B.

Let us now consider values of $\gamma > \gamma_1^{aut} = 0.167$ so that in autarky the steady state was locally indeterminate in country A. We find that, after opening the borders, the set of values of γ under which indeterminacy emerges shrinks as the steady state in both countries becomes a saddle for $\gamma > \gamma_2^W$. Moreover if n^A is sufficiently small, $n^A < n_*^A = 0.222$, local indeterminacy and therefore sunspots fluctuations are totally eliminated in the world.[43] However, if n^A is sufficiently big, namely $n^A > n_{**}^A = 0.371$, local indeterminacy prevails in the world. In this case there exists a transmission of local indeterminacy from country A to country B, so that expectation driven fluctuations, with origin in country A will be exported to country B, which was stable in autarky.[44]

We consider now lower and more plausible values of $\gamma < \gamma_1^{aut} = 0.167$. After opening the borders, indeterminacy which was not possible in autarky can now emerge. Indeed, the lower bound on γ required for indeterminacy is lower in a globalized world, i.e. $\gamma_1^W < \gamma_1^{aut}$. Therefore, with free-trade and capital movements it is possible to obtain fluctuations driven by self-fulfilling volatile expectations with small values of labor externalities consistent with empirical evidence.[45] To obtain γ_1^W and γ_2^W we will consider two values for n^A, $n^A = 0.1$ and $n^A = 0.3$, respectively below and above n_*^A. For $n^A = 0.1$, we obtain $\gamma_1^W = 0.0136$ and $\gamma_2^W = 0.0658$, while for $n^A = 0.3$ we obtain $\gamma_1^W = 0.043$, and $\gamma_2^W = 0.246$. We confirm therefore

40 Using (55) and $a^A = 1$, we get $\lambda = \nu \frac{(1-\alpha)(1-s)^{1-\alpha}}{\alpha^\alpha s^{1-\alpha}} \in (0,1)$ provided $\nu < \frac{\alpha^\alpha s^{1-\alpha}}{(1-\alpha)(1-s)^{1-\alpha}}$. Using (54) together with $K^A = 1$ and $a^A = 1$ we obtain $\Theta = \left\{\varepsilon^{1+\beta}\phi^\beta\right\} / \left\{(\varepsilon-1)(1-\alpha)(1-s)\left[1+\left(H^B\right)^{(1+\gamma-s)/(1-s)}\right]^\beta\right\}$. Then, using (20), (42) and (44), as at the normalized steady state, from (42), we have that $H^B = \left(K^B\right)^{(1-s)/(1+\gamma-s)}$, the value of the parameter $n^A \equiv N^A/N^W = K^A/K^W = 1/(1+K^B)$ can be fixed at a certain value n^A by setting H^B as $H^B = \left[(1-n^A)/n^A\right]^{\frac{1-s}{1+\gamma-s}}$. Hence, as γ is made to vary, n^A is kept constant as long H^B takes values in accordance with the last expression. Finally, with the normalization procedure followed neither λ and Θ nor H^B influence the local stability properties of the model.

41 Indeed, as discussed below, changes in expectations in country A will trigger changes in the number of varieties produced and in the capital used by firms in A, which through trade and capital mobility will also influence economic activity in country B.

42 Most of empirical estimates for the degree of increasing reurns to scale point to small values only slightly higher than zero, and values higher that 1/3 are usually considered highly implausible. See Basu and Fernald (1997) and Burnside (1996).

43 The same result has been found in Sim and Ho (2007) who consider inter-industry trade and different technologies across countries.

44 The same result was obtained by Nishimura et al. (2010) who, using a two-country, two-good, two-factor general equilibrium model with sector specific externalities, found that some country's expectation-driven fluctuations can spread throughout the world once inter-industry trade opens, even if the other country has determinacy under autarky.

45 This result was also emphasized in Aloi and Lloyd-Braga (2010) with perfect capital mobility but no trade.

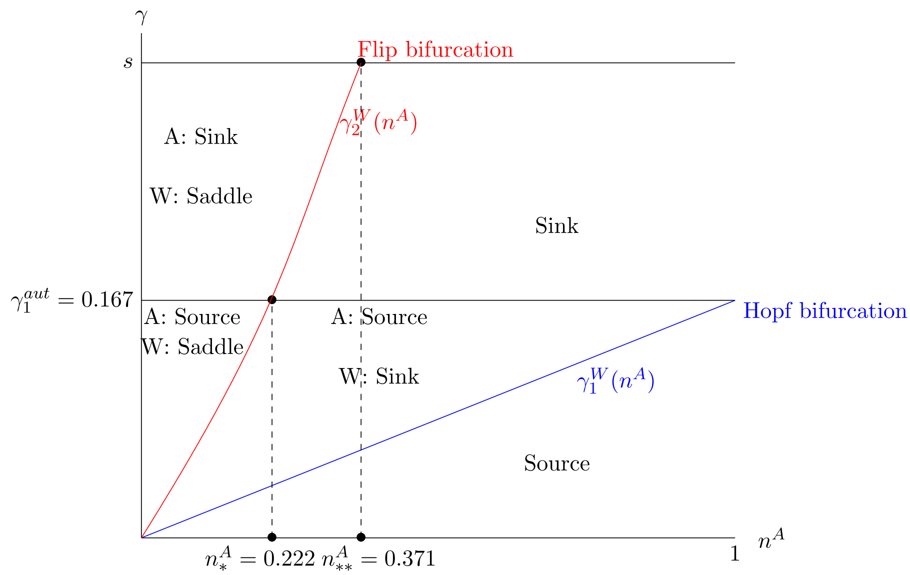

Fig. 2. Local dynamics for $\alpha \in (\underline{\alpha}, \tilde{\alpha})$, $\beta = 0.01$, $\alpha = 0.6$ and $s = 1/3$.

that, after opening the borders to *intra-industry* trade and capital mobility, indeterminacy becomes possible for lower values of γ, in accordance with empirical evidence.

Another important result is that, after opening the borders, bifurcations, which in our framework did not occur in autarky, become possible. When γ crosses the critical value γ_1^W a Hopf bifurcation occurs, whatever the relative size of the two countries provided $\underline{\alpha} < \alpha < \tilde{\alpha}$. In all our simulations the Hopf bifurcation is supercritical, so that the invariant closed curve appears when the steady state is a source. With $n^A = 0.1$, the Hopf bifurcation occurs when $\gamma = \gamma_1^W = 0.0136$, and for $\gamma = 0.0132$ we obtain an invariant closed curve surrounding the steady state, which we depict in Fig. 3.[46] This means that non-explosive deterministic and stochastic[47] fluctuations become possible in the world economy for small and plausible values of γ. To our knowledge, ours is the first paper highlighting that, by opening the economy, fluctuations due to a Hopf bifurcation emerge.[48]

Moreover, for $n^A < n^A_{**}$, when γ crosses the critical value γ_2^W, a *flip* bifurcation occurs.[49] In our simulations the *flip* bifurcation is supercritical. With $n^A = 0.1$, the *flip* bifurcation occurs when $\gamma = \gamma_2^W = 0.0658$.[50] In Fig. 4 we depict the corresponding bifurcation diagram for values of γ sufficiently close but above γ_2^W, i.e. in the saddle region. We can observe a cascade of doubling periodic cycles.[51]

It is also important to discuss the role of *taste for variety*, β, on stability in a globalized world. While γ_1^W does not directly depend on β, γ_2^W is a decreasing function of it, so that in Fig. 2 we would observe a rightward shift of the γ_2^W schedule when β increases. We can also see that the critical value n^A_{**} increases with β. This would imply, other things being equal, that a higher *taste for variety* shrinks the sink region, which would reduce the likelihood of local sunspots fluctuations, and enlarges the region where we obtain saddle path stability, exerting therefore a (local) stabilizing effect. Although the normalization used keeps constant the value of n^A when β varies, in Section 4 we found that n^A decreases with β. Considering a fixed value of γ, this last indirect effect, moving in Fig. 2 the relevant point to the left, reinforces the direct effect as the likelihood of falling into the saddle region increases. However, an increase in β decreases γ_1^W (through n^A only) and γ_2^W (directly and also through n^A). Therefore, with *intra-industry* trade, an increase in *taste for variety* makes indeterminacy and Hopf or *flip* bifurcations, and their associated deterministic and stochastic cycles, possible with lower and more plausible values of γ. Hence, the effects of *taste for variety* on stability are ambiguous.

Finally, we address the case where $\tilde{\alpha} < \alpha < 1$, so that Proposition 10 $\lfloor ii \rfloor$ applies. We can see that after opening the borders, country *A* may become saddle determinate for plausible and sufficiently small values of γ. However, *flip* bifurcations occur. In our simulations these bifurcations were subcritical, so that endogenous fluctuations appear when the steady state is a saddle. Therefore we can not simply conclude that opening the borders exerts a stabilizing influence.

Summarizing, although indeterminacy that existed under autarky in country *A* can be eliminated after opening the borders, the steady state becoming saddle stable, this only happens if country *B* is sufficiently large. On the other hand, for lower and plausible values of γ, local indeterminacy and sunspots arbitrarily near the steady state can now emerge. Also, bounded deterministic and stochastic fluctuations associated with a supercritical Hopf bifurcation are now possible. Furthermore, for higher values of γ, but still within the plausible range, deterministic and stochastic fluctuations when the steady state is a saddle, due to a supercritical *flip* bifurcation also occur. We further notice that, after opening the borders, local indeterminacy and *flip*/Hopf

46 For $n^A = 0.3$ the supercritical Hopf occurs for $\gamma = \gamma_1^W = 0.043$ and the invariant closed curve appears for $\gamma = 0.0419$.

47 Around the invariant closed curve, there exist infinitely many equilibria exhibiting bounded stochastic fluctuations. See Grandmont et al. (1998).

48 Note that fluctuations along the invariant closed curve appear near unit roots through the Hopf bifurcation, and that equilibria along the invariant closed curve can be represented by quasi periodic orbits. Hence, Hopf bifurcations lead to persistent and irregular fluctuations, features exhibited by real data on output business cycles.

49 Nishimura et al. (2014), with inter-industry trade and capital movements, also obtain a *flip* bifurcation, in a two-factor, two-sector, two-country model with decreasing returns to scale technologies. However, they do not have local indeterminacy.

50 With $n^A = 0.3$, the *flip* bifurcation occurs when $\gamma = \gamma_2^W = 0.246$.

51 Stochastic bounded fluctuations around the periodic cycles also appear. See Grandmont et al. (1998).

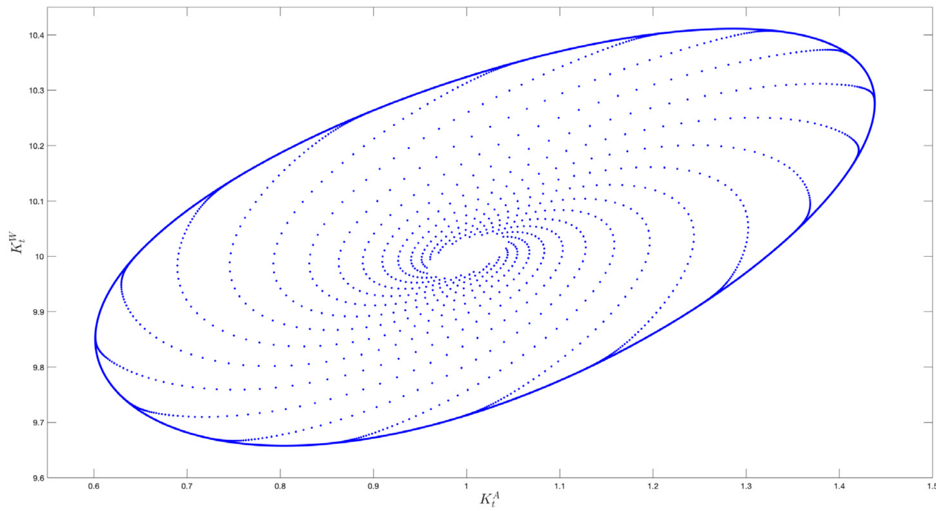

Fig. 3. Invariant closed curve surrounding the steady state for $\gamma = 0.0132$.

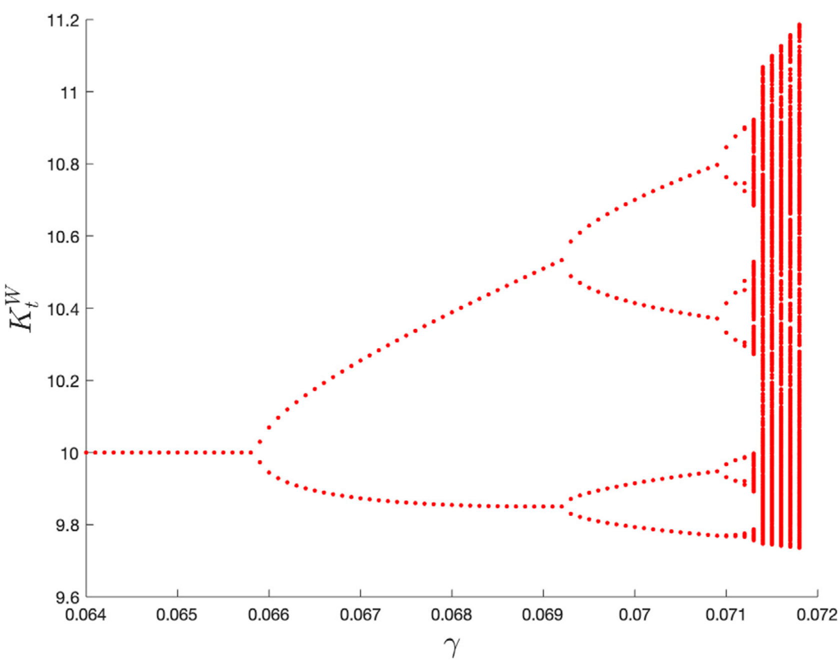

Fig. 4. Bifurcation diagram: Supercritical *flip* bifurcation at $\gamma = 0.0658$.

bifurcations may also appear in country B, triggering endogenous fluctuations that could not exist under autarky.

We now highlight that net trade and the creation/destruction of new varieties emerge along trajectories that exhibit endogenous fluctuations. In the following we provide an example associated with indeterminacy where local fluctuations driven by shocks to expectations exist. At the steady state, in each country, households save in capital what is needed to use in production. Hence, as we have seen, there are no capital movements across countries at the steady state, and net trade is zero as well. However, changes in expectations in the rigid wage country lead to fluctuations in activity, inducing net trade and changes in the remuneration of factors in both countries. Departing from the steady state, suppose that, suddenly, expectations of the future real interest rate increase. In country A, the reservation wage decreases (see (9)) and so does the real wage. For a given level of the capital stock, K_t^A, and of the number of varieties produced in both countries, N_t^A and N_t^B, and considering that at a symmetric equilibrium the labor demand is downward sloping (which occurs with γ small), employment at the firm level in country A, l_t^A, increases.[52] Hence, ceteris paribus, average costs decrease and profits increase at the firm level. This induces the entry of new firms and therefore the production of more varieties in country A,[53] a part of which is exported. Country A becomes therefore a net exporter of goods. At the same time, the observed increase in l_t^A leads, if everything else is equal, to an increase in the marginal productivity of capital in country A, and consequently in r_t^A, the current interest rate in country A. This in turn triggers inflows of capital from country B until interest rates are equalized in the two countries, which leads to fluctuations in wages and activity in country B as well. Finally, it is also worth emphasizing that our

52 To see this note that adapting (16) to the case where N_t^W varieties are consumed we obtain $\omega_t^{A,W} = \Theta(N_t^W)^\beta \left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s)(K_t^A)^s \left(N_t^A l_t^A\right)^{\gamma-s}$.

53 Since output per firm is constant, this implies that aggregate output in country A unambiguously increases.

model's features make it appropriate to describe what happens in developed countries, where *intra-industry* trade is more prevalent and where we observe diversified labor market institutions and where, as shown in De Bock (2010) and Engel and Wang (2011), and in contrast to what happens in developing countries, exports are procyclical. Indeed, *intra-industry* trade seems to be appropriate to explain this last characteristic. In fact, as seen above, output increases when the number of varieties increases (since the quantity produced by each variety or firm is constant). As with *taste for variety* these varieties are always imported by the partner country, we conclude that one country's exports naturally increase with the number of varieties it produces, and therefore with the level of its output.[54]

6. Concluding remarks

In this paper we consider a two-country, two-factor, overlapping generations model with *taste for variety*, imperfect competition and increasing returns to scale. We also assume that the two countries have different labor market characteristics: in one country, A, there are efficiency wages and unemployment, while in the other country, B, there exists full employment. We first show that in autarky country B is locally stable, while in country A local indeterminacy, and therefore belief driven fluctuations, may emerge, provided the propensity to consume and the degree of increasing returns to scale take intermediate values, although bifurcations are not possible. When trade and capital movements are liberalized, the effects on stability depend on the relative size of the countries and on the existing degree of increasing returns to scale. Considering a parameterization under which indeterminacy existed in country A in autarky, we show that, if country A is sufficiently big, it will export local fluctuations to the full employment country B, globalization inducing local macroeconomic instability in the world. In contrast, provided country B is big enough, local indeterminacy that existed in autarky in country A is eliminated, globalization having in this case a (local) stabilizing effect in the world economy. However, whatever the relative size of the two countries, indeterminacy, and therefore local sunspots fluctuations at the world level, require a degree of externalities smaller than the one needed in autarky. Also bounded deterministic and stochastic fluctuations associated with Hopf and *flip* bifurcations, which did not exist in autarky, become possible in the world economy for sufficiently small values of increasing returns consistent with empirical estimates. This means that fluctuations in economic activity and in *intra-industry* trade emerge without shocks to fundamentals, and even without uncertainty, for empirically plausible values of the parameters. In terms of steady state welfare, we prove that the full employment country unambiguously gains from opening its borders, while unemployment increases in the country with labor market rigidities, reducing country welfare. Furthermore, we show that *intra-industry* trade alone is responsible for these welfare gains and losses.

Very few papers in the literature have simultaneously addressed the effects of trade on welfare and on stability properties. Two examples are Nishimura et al. (2010) and Le Riche (2020). However, they consider inter-industry trade and no unemployment. Moreover, the models used and the mechanisms emphasized are different from ours. They consider perfectly competitive labor markets in both countries and assume that countries have different technologies. Both papers find that opening to inter-industry trade with capital mobility increases the likelihood of local indeterminacy, that one country will gain in terms of stationary welfare while the other country always looses, although Nishimura et al. (2010) also show that at the world level steady state welfare increases. Considering instead *intra-industry* trade and introducing labor market imperfections in one local market, our findings, while mostly supporting these previous insights, highlight the role of the relative size of the countries and of the degree of increasing returns to scale on shaping the effects of globalization on stability. Therefore, a fruitful extension of the model could be to understand how the interaction between comparative advantage (inter-industry trade), increasing returns to scale and *taste for variety* (*intra-industry* trade) affect the stability and the welfare of the trading economies.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

A.1. Proof of Proposition 2

From (20) and (21) we obtain:

$$\frac{d\omega}{dK^A} = \frac{\beta(1+\gamma)+\gamma}{K^A}\omega,\ \frac{d\omega}{da^A} = \frac{s-\gamma-\beta(1+\gamma-s)}{a^A}\omega \tag{59}$$

$$\frac{d\rho}{dK^A} = \frac{\beta(1+\gamma)+\gamma}{K^A}\rho,\ \frac{d\rho}{da^A} = -\frac{(1+\beta)(1+\gamma-s)}{a^A}\rho. \tag{60}$$

Substituting Eqs. (20) and (21) into the dynamic system (22)–(23), linearizing it and using (59) and (60) we obtain

$$\widehat{K}^A_{t+1} = \underbrace{(1+\beta)(1+\gamma)}_{z_1}\widehat{K}^A_t \ \underbrace{-(1+\beta)(1+\gamma-s)}_{z_2}\widehat{a}^A_t$$

and

$$\underbrace{(1-\alpha)[\beta(1+\gamma)+\gamma]}_{x_1}\widehat{K}^A_{t+1} \underbrace{-(1-\alpha)(1+\beta)(1+\gamma-s)}_{x_2}\widehat{a}^A_{t+1} = \underbrace{-[\beta(1+\gamma)+\gamma]}_{z_3}\widehat{K}^A_t + \underbrace{[\beta(1+\gamma-s)-(s-\gamma)]}_{z_4}\widehat{a}^A_t$$

where $\widehat{K}^A_t$ and $\widehat{a}^A_t$ denote percentage deviations of K^A and a^A from the steady state. We now rewrite the linear system above in matrix form

$$\underbrace{\begin{bmatrix} 1 & 0 \\ x_1 & x_2 \end{bmatrix}}_{J_1}\begin{bmatrix} \widehat{K}^A_{t+1} \\ \widehat{a}^A_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix}}_{J_0}\begin{bmatrix} \widehat{K}^A_t \\ \widehat{a}^A_t \end{bmatrix}.$$

The Jacobian matrix, J, is then

$$J = J_1^{-1}\cdot J_0 = \begin{bmatrix} 1 & 0 \\ -\frac{x_1}{x_2} & \frac{1}{x_2} \end{bmatrix}\begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 \\ \frac{z_3-z_1x_1}{x_2} & \frac{z_4-z_2x_1}{x_2} \end{bmatrix}.$$

The trace, T, and determinant, D, of matrix J, correspond respectively to the sum and product of the two roots (eigenvalues) of the associated characteristic polynomial $P(\lambda) \equiv \lambda^2 - \lambda T + D$.

Results follow.

[54] This also happens when simulating the model along deterministic trajectories. The correlation between output and exports obtained along the invariant closed curve ($\gamma = 0.0132$) and along a period four cycle ($\gamma = 0.07$) were 0.98 and 0.67 respectively. In these simulations we considered as above $s = 1/3$, $\beta = 0.01$, $\alpha = 0.6$ and $n^A = 0.1$.

A.2. Proof of Proposition 3

Local indeterminacy emerges when the steady state is a sink, i.e. when $D < 1$, $1+T+D > 0$ and $1-T+D > 0$. Local determinacy will arise for any other configuration. In particular, as $D > 0$, the steady state is a saddle when $1-T+D < 0$ or $1+T+D < 0$. In any other configuration the steady state will be a source.

From (30), we get that the determinant is lower than one if and only if $\gamma > [\alpha(1-s)-(1-2s)]/(1-\alpha) \equiv \gamma_1^{aut}$. Such a threshold is positive under Assumption 1. Furthermore, from (30), we can compute $1-T+D$ and $1+T+D$:

$$\begin{aligned} 1-T+D &= \frac{\gamma+\beta(1+\gamma)}{(1-\alpha)(1+\gamma-s)(1+\beta)} > 0, \\ 1+T+D &= \frac{1+(1+\beta)[1+\gamma-2\alpha(1+\gamma-s)]}{(1-\alpha)(1+\gamma-s)(1+\beta)} \gtrless 0. \end{aligned} \tag{61}$$

We have $1+T+D > 0$ when $\gamma < \gamma_2^{aut} \equiv \frac{2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]}{(1+\beta)(2\alpha-1)} > 0$ under Assumption 1.[55] As $1-T+D > 0$, when $\gamma < \min\{\gamma_1^{aut}, \gamma_2^{aut}\}$ we get $D > 1$ and $1+T+D > 0$, and thus the steady state is a source. When $\gamma_1^{aut} < \gamma < \gamma_2^{aut}$, we obtain $D < 1$ and $1+T+D > 0$. It follows that in this configuration the steady state is a sink. When $\gamma > \gamma_2^{aut}$, we get $1+T+D < 0$ so that the steady state is a saddle. Noting that $\gamma_1^{aut} < \gamma_2^{aut}$ requires $\alpha < \tilde{\alpha} \equiv \frac{1+2s(1+\beta)}{1+3s(1+\beta)}$, results follow.

A.3. Proof that $n^A = \chi^A = S^A$

Denote by $n^A \equiv N^A/N^W$ the percentage of varieties produced in country A at the steady state, by $S^A \equiv (1-\alpha)\omega^{A,W}L^A/K^W$ the steady state share of country A savings in world savings, and by $\chi^A \equiv K^A/K^W$ the share of capital used in country A in total capital used in the world at the steady state. We will start by showing that $n^A = \chi^A$. First, substituting (42) in (44) evaluated at the steady state we obtain

$$N^W = \underbrace{\frac{(a^A)^{s-1-\gamma}\left(K^A\right)^{1+\gamma}}{\varepsilon\phi}}_{N^A}\left(1+\frac{K^B}{K^A}\right). \tag{62}$$

As $\chi^A = \frac{K^A}{K^W}$ we can rewrite (62) as $N^W = N^A\left(1+\frac{1-\chi^A}{\chi^A}\right)$, which, as $n^A = \frac{N^A}{N^W}$, gives $n^A = \chi^A$.

We now show that $n^A = S^A$. Using (20), (21), (31), and (32) at the steady state we have that :

$$\begin{aligned} \frac{N^A}{N^B} &= \frac{\left(a^A\right)^{s-1-\gamma}\left(K^A\right)^{1+\gamma}}{\left(H^B\right)^{1+\gamma-s}\left(K^B\right)^{s}} = \frac{\omega^{A,W}\frac{K^A}{a^A}}{\omega^{B,W}H^B} \\ &= \frac{\omega^{A,W}L^A}{\omega^{B,W}H^B} = \frac{(1-\alpha)\omega^{A,W}L^A}{K^W}\frac{K^W}{(1-\alpha)\omega^{B,W}H^B} = \frac{S^A}{1-S^A} \end{aligned} \tag{63}$$

As $\frac{N^A}{N^B} = \frac{n^A}{1-n^A}$ this obviously means that $n^A = S^A$ so that the claim $n^A = \chi^A = S^A$ is true.

A.4. Proof of Proposition 6

Denote by $S^A \equiv (1-\alpha)\omega^{A,W}L^A/K^W$ the steady state share of country A savings in world savings, and by $\chi^A \equiv K^A/K^W$ the share of capital used in country A in total capital used in the world at the steady state. As $S^A \equiv \frac{(1-\alpha)\omega^{A,W}L^A}{K^W} = \frac{(1-\alpha)\omega^{A,W}K^A}{a^A K^W} = \frac{(1-\alpha)\omega^{A,W}}{a^A}\chi^A$ and from Appendix A.3 we have $S^A = \chi^A$, we conclude that $\frac{\omega^{A,W}}{a^A} = \frac{1}{1-\alpha}$. Also as $1-S^A = \frac{(1-\alpha)\omega^{B,W}H^B}{K^W} = 1-\chi^A = \frac{K^B}{K^W}$ we conclude that $\frac{\omega^{B,W}H^B}{K^B} = \frac{1}{1-\alpha}$ implying that

$$\frac{\omega^{B,W}H^B}{K^B} = \frac{\omega^{A,W}}{a^A} = \frac{1}{1-\alpha}. \tag{64}$$

Combining (45) and (46), and using the last expression we have that at the steady state

$$\rho^W = \frac{s\omega^{A,W}}{(1-s)a^A} = \frac{s}{(1-s)(1-\alpha)} \tag{65}$$

so that $\rho^W = \rho^A = \rho^B$. Now, using also (52), we obtain the steady state value of the capital-labor ratio in country A given by (55). Note that it is identical to the value obtained in autarky.

We now prove uniqueness of the steady state. First, substituting Eqs. (44), (45) and (46) into (48)–(49) we rewrite our dynamic system in terms of K^A and K^W:

$$\begin{aligned} K_{t+1}^W &= \Psi_1 (K_t^W)^{(1+\beta)}(K_t^W-K_t^A)^{(s-1)(1+\beta)} \\ (K_{t+1}^W-K_{t+1}^A) &= \Psi_2\Big[(K_t^W)^{\beta}(K_t^W-K_t^A)^{\frac{(s-1)[s-\gamma+\beta(s-1-\gamma)]}{(s-1-\gamma)}} \\ &\quad \times (K_t^A)^{\frac{-\gamma}{(s-1-\gamma)}}(K_{t+1}^W)^{\beta(1-\alpha)}\Big]^{\frac{1}{(1-\alpha)(1+\beta)(1-s)}} \end{aligned} \tag{66}$$

where

$$\Psi_1 \equiv \frac{\Theta(\varepsilon-1)(1-s)(1-\alpha)}{\varepsilon(\varepsilon\phi)^{\beta}}(H^B)^{(1+\beta)(1+\gamma-s)}$$

$$\Psi_2 \equiv \left\{\frac{\lambda}{\nu}\frac{\alpha^{\alpha}s^{1-\alpha}}{(1-\alpha)(1-s)^{1-\alpha}}\left[\frac{\Theta(\varepsilon-1)(1-s)(1-\alpha)}{\varepsilon(\varepsilon\phi)^{\beta}}\right]^{(2-\alpha)} \times (H^B)^{[(1+\gamma-s)[\beta+(1+\beta)(1-\alpha)]+\gamma-s]}\right\}^{\frac{1}{(1-\alpha)(1+\beta)(1-s)}}.$$

At the steady state system (66) becomes:

$$K^W - K^A = \Psi_1^{\frac{1}{(1+\beta)(1-s)}} K^{W\,\frac{\beta}{(1+\beta)(1-s)}} \tag{67}$$

$$(K^W-K^A)^{\frac{(s-1-\gamma)[(1-\alpha)(1+\beta)+\beta]+s-\gamma}{(s-1-\gamma)(1-\alpha)(1+\beta)}} = \Psi_2 K^{W\,\frac{\beta(2-\alpha)}{(1-\alpha)(1+\beta)(1-s)}} \times K^{A\,\frac{\gamma}{(1+\gamma-s)(1-\alpha)(1+\beta)(1-s)}}. \tag{68}$$

Substituting the first equation in the second we obtain

$$K^W = \left(\frac{\Psi_3}{(K^A)^{\gamma(1+\beta)}}\right)^{\frac{1}{\beta}}$$

where

$$\Psi_3 \equiv \Psi_1^{(1+\gamma-s)[(1-\alpha)(1+\beta)+\beta]-s+\gamma}\Psi_2^{-(1-\alpha)(1+\beta)^2(1+\gamma-s)(1-s)}$$

Substituting now this last expression in (67) we obtain:

$$\left[\left(K^A\right)^{\frac{\gamma+(1+\gamma)\beta}{\beta}} + [\Psi_1\Psi_3]^{\frac{1}{(1+\beta)(1-s)}}\left(K^A\right)^{\frac{\gamma(1-s-\beta s)}{\beta(1-s)}}\right]^{\beta} = \Psi_3$$

Substituting Ψ_1 and Ψ_3 in the previous expression, and using (55), we finally obtain Eq. (54) that we rewrite below:

$$\begin{aligned} \frac{\varepsilon^{1+\beta}\phi^{\beta}\left(a^A\right)^{(1+\gamma-s)(1+\beta)}}{\Theta(\varepsilon-1)(1-s)(1-\alpha)} &= H(K^A) \\ &\equiv \left[\left(K^A\right)^{\frac{\gamma+(1+\gamma)\beta}{\beta}} + \left(H^B a^A\right)^{\frac{1+\gamma-s}{1-s}}\left(K^A\right)^{\frac{\gamma(1-s-\beta s)}{\beta(1-s)}}\right]^{\beta}. \end{aligned}$$

Under Assumption 1 the derivative of the *RHS* of the previous expression is unambiguously positive. Moreover, the *RHS* tends to 0 when K^A tends to 0 and it tends to $+\infty$ when K^A tends to $+\infty$, implying that the *RHS* is an increasing function going from 0 to $+\infty$. Since the *LHS* is a positive constant, it follows that there

55 Indeed the numerator of γ_2^{aut} is positive under Assumption 1. Either $2\alpha(1-s) < 1$, so that the numerator is positive or since Assumption 1 implies $\beta < 1$, we have $2[1-\alpha(1-s)]/[2\alpha(1-s)-1] > 1$.

exists a unique K^A solution of that equation. As from (67) we have that K^W is uniquely determined by K^A we conclude that the steady state is unique.

A.5. Proof that K^A and L^A decrease with β

We can rewrite (54) as:

$$\frac{\varepsilon\left(a^A\right)^{(1+\gamma-s)}}{\Theta(\varepsilon-1)(1-s)(1-\alpha)} = \left(K^A\right)^{\gamma}\left[\frac{\left(K^A\right)^{1+\gamma}}{\varepsilon\phi\left(a^A\right)^{(1+\gamma-s)}}\left(1+\left(\frac{H^B a^A}{K^A}\right)^{\frac{1+\gamma-s}{1-s}}\right)\right]^{\beta}.$$

We have proved above that the *RHS* of this expression is an increasing function of K^A. Therefore the steady state level of capital in country A is determined by the intersection of this increasing function with the constant on the *LHS*. We can show that $\frac{\left(K^A\right)^{1+\gamma}}{\varepsilon\phi\left(a^A\right)^{(1+\gamma-s)}} > 1$. Indeed, using (20) this inequality can be rewritten as $N^A > 1$, which is always satisfied. We conclude that the term in square brackets in the *RHS* is higher than one. Therefore, the increasing function of K^A in the *RHS* of this last expression shifts up when β increases. This implies that K^A decreases with β. As a^A is not affected by β, L^A also decreases with β.

A.6. Proof of Proposition 7

With *intra-industry* trade and free capital movements the number of varieties in the world which are consumed by residents in countries A and B at the steady state, $N^W = N^A + N^B = N^A\left[1+\frac{N^B}{N^A}\right]$, is given by (44) evaluated at steady state, that, using (42), we can rewrite as:

$$N^W = \underbrace{\frac{\left(a^A\right)^{s-1-\gamma}\left(K^A\right)^{1+\gamma}}{\varepsilon\phi}}_{N^A}\left[1+\left(\frac{H^B}{L^A}\right)^{\frac{1+\gamma-s}{1-s}}\right] \tag{69}$$

so that

$$\frac{N^B}{N^A} = \left(\frac{H^B}{L^A}\right)^{\frac{1+\gamma-s}{1-s}}. \tag{70}$$

Let N^B_{aut} denote the number of varieties produced at the steady state in country B in autarky. In country B, as $N^B > N^B_{aut}$, we immediately conclude that $N^W > N^B_{aut}$, i.e. at the steady state residents in country B consume more varieties after opening its borders to free trade and capital movements. The number of varieties in country A under autarky is given by (20). Since a^A is the same in autarky and after opening the borders, and denoting by N^A_{aut} the number of varieties produced at the steady state in country A in autarky and by K^A_{aut} the steady state capital stock of country A in autarky, we have that

$$N^A_{aut} = \left(\frac{K^A_{aut}}{K^A}\right)^{1+\gamma} N^A.$$

Now, using (27) we can rewrite (54) as:

$$\frac{K^A_{aut}}{K^A} = \left[1+\left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]^{\frac{\beta}{\gamma+(1+\gamma)\beta}}. \tag{71}$$

Note that as the *RHS* is higher than one it follows that the steady state capital stock of country A decreases with trade. Moreover using the previous expression we obtain

$$N^A_{aut} = N^A\left[1+\left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]^{\frac{\beta(1+\gamma)}{\gamma+(1+\gamma)\beta}},$$

i.e. the number of varieties produced in A decreases with free *intra-industry* trade. Combining now (69) and the previous expression, we obtain:

$$\frac{N^W}{N^A_{aut}} = \frac{\left[1+\left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]}{\left[1+\left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]^{\frac{\beta(1+\gamma)}{\gamma+(1+\gamma)\beta}}} = \left[1+\left(\frac{H^B}{L^A}\right)^{1+\frac{\gamma}{1-s}}\right]^{\frac{\gamma}{\gamma+(1+\gamma)\beta}} > 1.$$

We conclude that, although the number of varieties produced in A decreases with free *intra-industry* trade and capital mobility, the residents of country A have access and consume more varieties at the steady state after opening the borders.

Finally, we have that $N^A/N^B = n^A/(1-n^A)$. As N^A decreases with trade and N^B increases with trade, N^A/N^B decreases with trade. Since N^A/N^B is an increasing function of n^A, it follows that n^A decreases and $n^B = 1-n^A$ increases with trade. As $n^A = S^A = \chi^A$ (see Appendix A.3) this also means that the steady state share of savings of country A, S^A, and the steady state share of capital of country A, χ^A, decrease, while the steady state share of saving of country B, $1-S^A$, and the steady state share of capital stock of country B, $1-\chi^A$, increase.

A.7. Proof of Proposition 9

Linearizing the dynamic system (66) we obtain

$$\widehat{K}^W_{t+1} = \underbrace{\frac{(1+\beta)(s-n^A)}{1-n^A}}_{z^W_1}\widehat{K}^W_t + \underbrace{\frac{(1+\beta)(1-s)n^A}{1-n^A}}_{z^W_2}\widehat{K}^A_t$$

and

$$\underbrace{\frac{1-s-\beta s+\beta n^A}{(1-n^A)(1+\beta)(1-s)}}_{x^W_1}\widehat{K}^W_{t+1} + \underbrace{-\frac{n^A}{1-n^A}}_{x^W_2}\widehat{K}^A_{t+1} = \underbrace{\frac{\beta(1+\gamma-s)(s-n^A)+(1-s)(s-\gamma)}{(1-n^A)(1+\gamma-s)(1-\alpha)(1+\beta)(1-s)}}_{z^W_3}\widehat{K}^W_t + \underbrace{\frac{\gamma-n^A\{\gamma+(1-s)[s-\gamma-\beta(1+\gamma-s)]\}}{(1-n^A)(1+\gamma-s)(1-\alpha)(1+\beta)(1-s)}}_{z^W_4}\widehat{K}^A_t$$

where $\widehat{K}^W_t$ and $\widehat{K}^A_t$ denote percentage deviations of K^W and K^A from the steady state.

We now rewrite the linear system above in matrix form

$$\underbrace{\begin{bmatrix} 1 & 0 \\ x^W_1 & x^W_2 \end{bmatrix}}_{J^W_1}\begin{bmatrix} \widehat{K}^W_{t+1} \\ \widehat{K}^A_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} z^W_1 & z^W_2 \\ z^W_3 & z^W_4 \end{bmatrix}}_{J^W_0}\begin{bmatrix} \widehat{K}^W_t \\ \widehat{K}^A_t \end{bmatrix}.$$

The Jacobian matrix, J^W, is then

$$J^W = \left(J^W_1\right)^{-1}\cdot J^W_0 = \begin{bmatrix} 1 & 0 \\ -\frac{x^W_1}{x^W_2} & \frac{1}{x^W_2} \end{bmatrix}\begin{bmatrix} z^W_1 & z^W_2 \\ z^W_3 & z^W_4 \end{bmatrix}$$

$$= \begin{bmatrix} z_1^W & z_2^W \\ \frac{z_3^W - z_1^W x_1^W}{x_2^W} & \frac{z_4^W - z_2^W x_1^W}{x_2^W} \end{bmatrix}.$$

The trace, T^W, and determinant, D^W, of matrix J^W, correspond respectively to the sum and product of the two roots (eigenvalues) of the associated characteristic polynomial $P^W(\lambda^W) \equiv (\lambda^W)^2 - \lambda^W T^W + D^W$.

Results follow.

A.8. Proof of Proposition 10

The steady state is a sink when, at the same time, $D^W < 1$, $1 + T^W + D^W > 0$ and $1 - T^W + D^W > 0$. In that case local indeterminacy emerges. Local determinacy will arise in the remaining cases. In particular, the steady state is a saddle when, simultaneously, $1 + T^W + D^W > 0$ and $1 - T^W + D^W < 0$ or $1+T^W+D^W < 0$ and $1-T^W+D^W > 0$. In any other configurations, the steady state will be a source.

Using (58) we have that D^W is lower than one if and only if $\gamma > \frac{n^A(1-s)[\alpha(1-s)-(1-2s)]}{s-n^A[\alpha(1-s)-(1-2s)]} \equiv \gamma_1^W$, where γ_1^W is the value of γ for which $D^W = 1$. Such a threshold is positive under Assumption 1.[56] Furthermore, from (58), we can compute

$$1 - T^W + D^W = \frac{n^A\beta(1+\gamma-s) + \gamma[1-s(1+\beta)]}{(1+\gamma-s)(1+\beta)(1-s)(1-\alpha)n^A} > 0$$

under Assumption 1

and

$$1 + T^W + D^W = \frac{n^A(1+\gamma-s)\{2(1+\beta)[(1-\alpha)(1-s)+s]-\beta\} - \gamma[1+s(1+\beta)]}{(1+\gamma-s)(1+\beta)(1-s)(1-\alpha)n^A}.$$

Denote $\gamma_2^W \equiv \frac{n^A(1-s)[2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]]}{1+s(1+\beta)-n^A[2(1-\alpha+\alpha s)-\beta[2\alpha(1-s)-1]]} > 0$.[57] We have that $1 + T^W + D^W > 0$ when $\gamma < \gamma_2^W$ under Assumption 1. Remark that γ_2^W is the value of γ for which $D^W = -1 - T^W$. As we always have $1 - T^W + D^W > 0$, we conclude that:

(i) the steady state is a source when $D^W > 1$ and $1 + T^W + D^W > 0$, i.e. when $\gamma < \min\{\gamma_1^W, \gamma_2^W\}$;

(ii) the steady state is a sink when $D^W < 1$ and $1+T^W+D^W > 0$, i.e. when $\gamma_1^W < \gamma < \gamma_2^W$;

(iii) the steady state is a saddle when $1 + T^W + D^W < 0$, i.e. when $\gamma > \gamma_2^W$.

Noting that $\gamma_1^W < \gamma_2^W$ requires $\alpha < \frac{1+2s(1+\beta)}{1+3s(1+\beta)}$, results follow.

References

Aloi, M., Lloyd-Braga, T., 2010. National labor markets, international factor mobility and macroeconomic instability. Econom. Theory 43 (3), 431–456.

Ardelean, A., 2009. How Strong is the Love of Variety?. Santa Clara University, Mimeo.

Ardelean, A., León-Ledesma, M., Puzzello, L., 2018. Industry volatility and international trade. In: Working Paper.

Bai, Y., Ríos-Rull, J.-s.-V., 2015. Demand shocks and open economy puzzles. Amer. Econ. Rev. 105 (2), 644–649.

Basu, S., Fernald, J.G., 1997. Returns to scale in U.S. production: estimates and implications. J. Polit. Econ. 105 (2), 249–283.

Bénassy, J.-P., 1996. Taste for variety and optimum production patterns in monopolistic competition. Econom. Lett. 52, 41–47.

Boulhol, H., 2011. Unemployment and relative labor market institutions between trading partners. J. Int. Econ. 83, 83–91.

Burnside, C., 1996. Production function regressions, returns to scale, and externalities. J. Monetary Econ. 37 (2), 177–201.

Cecchi, D., Garcia-Peñalosa, C., 2010. Labour market institutions and the personal distribution of income in the OECD. Economica 77, 413–450.

Cuñat, A., Melitz, M.J., 2012. Volatility, labor market flexibility, and the pattern of comparative advantage. J. Eur. Econom. Assoc. 10 (2), 225–254.

De Bock, R., 2010. The composition and cyclical behavior of trade flows in emerging economies. In: International Monetary Fund Working Paper 10/46.

Dixit, A., Stiglitz, J.E., 1977. Monopolistic competition and optimum product diversity. Amer. Econ. Rev. 67 (3), 297–308.

Drescher, L., Thiele, S., Weiss, S., 2008. The taste for variety: a hedonic analysis. Econom. Lett. 101, 66–68.

Egger, P., Greenway, D., Seidel, T., 2011. Rigid labour markets with trade and capital mobility: theory and evidence. Canadian J. Econ. 44 (2), 509–540.

Engel, C., Wang, J., 2011. International trade in durable goods: Understanding volatility, cyclicality, and elasticities. J. Int. Econ. 83, 37–52.

Farmer, R.E.A., Guo, J.-T., 1994. Real business cycles and the animal spirits hypothesis. J. Econom. Theory 63 (1), 42–72.

Goette, L., Sunde, U., Bauer, T., 2007. Wage rigidity: Measurement, causes and consequences. Econ. J. 117, F499–F507.

Grandmont, J.-M., Pintus, P., de Vilder, R., 1998. Capital-labor substitution and competitive nonlinear endogenous business cycles. J. Econom. Theory 80 (1), 14–59.

Hayakawa, K., Ito, T., Okubo, T., 2017. On the stability of intra-industry trade. J. Japanese Int. Econ. 45, 1–12.

Helpman, E., Itskhoki, O., 2010. Labour market rigidities, trade and unemployment. Rev. Econom. Stud. 77 (3), 1100–1137.

Helpman, E., Krugman, P.R., 1985. Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition and International Economy. MIT Press, Cambridge, MA.

Iwasa, K., Nishimura, K., 2014. Dynamic two-country Heckscher-Ohlin model with externality. Int. J. Econ. Theory 10 (1), 53–74.

Iwasa, K., Nishimura, K., 2019. Trade and indeterminacy revisited. Int. J. Econ. Theory 15 (1), 37–51.

Krugman, P.R., 1979. Increasing resturns, monopolistic competition, and international trade. J. Int. Econ. 9, 469–479.

Krugman, P.R., 1980. Scale economies, product differentiation, and the pattern of trade. Amer. Econ. Rev. 70 (5), 950–959.

Le Riche, A., 2017. Macroeconomic volatility and trade in OLG economies. Int. J. Econ. Theory 13 (4), 401–425.

Le Riche, A., 2020. The instability of open Pareto efficient economies. Macroecon. Dyn. http://dx.doi.org/10.1017/S1365100520000310.

Lloyd-Braga, T., Nourry, C., Venditti, A., 2007. Indeterminacy in dynamic models: When Diamond meets Ramsey. J. Econom. Theory 134 (1), 513–536.

Nishimura, K., Shimomura, K., 2002. Trade and indeterminacy in a dynamic general equilibrium model. J. Econom. Theory 105 (1), 244–260.

Nishimura, K., Venditti, A., Yano, M., 2010. Expectation-driven fluctuations and welfare loss under free trade in two-country models. Int. J. Econ. Theory 6 (1), 97–125.

Nishimura, K., Venditti, A., Yano, M., 2014. Destabilization effect of international trade in a perfect-foresight dynamic general equilibrium model. Econom. Theory 55 (2), 357–392.

Rodríguez-Clare, A., Ulate, M., Vásquez, J.P., 2020. Rigid labour markets with trade and capital mobility: theory and evidence. In: NBER Working Paper 27905.

Seegmuller, T., 2008. Taste for variety and endogenous fluctuations in a monopolistic competition model. Macroecon. Dyn. 12 (4), 561–577.

Shapiro, C., Stiglitz, J.E., 1984. Equilibrium unemployment as a worker discipline device. Amer. Econ. Rev. 74 (3), 433–444.

Sim, N.C.S., Ho, K.-W., 2007. Autarky indeterminacy and trade determinacy. Int. J. Econ. Theory 3 (4), 151–168.

Woodford, M., 1986. Stationary Sunspot Equilibria. The Case of Small Fluctuations Around a Deterministic Steady State. Mimeo, University of Chicago and New York University.

56 The denominator of γ_1^W is positive under Assumption 1, as the term in square brackets is positive and $\frac{s}{\alpha(1-s)-(1-2s)} > 1$.

57 Note that the numerator of γ_2^W is identical to the numerator of γ_2^{aut} (which is positive under Assumption 1) multiplied by n^A. The denominator of γ_2^W is also positive under Assumption 1. Indeed the term in square brackets is positive and $\frac{1+s(1+\beta)}{2(1-\alpha+s)+\beta[1-2\alpha(1-s)]} > 1$.