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Equity Risk Premium Predictability from Cross-Sectoral Downturns

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We illustrate the role of left tail dependence—left tail mean (*LTM*)—in equity risk premium (*ERP*) predictability. *LTM* measures the average of pairwise left tail dependency among major equity sectors incorporating shocks imperceptible at the aggregate level. *LTM*, as well as the variance risk premium, significantly predicts the *ERP* in and out of sample, which is not the case with commonly used predictors. We find this predictability is the result of procyclical shocks' reversals in a stable business cycle. This paper contributes to the ongoing debate on *ERP* predictability. (*JEL* G10, G12, G14)

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Rare events, such as the 2007–2009 global financial crisis, are crucial in the study of asset pricing. Rietz (1988) introduces a disaster-risk-based model to explain the equity premium puzzle. In the subsequent literature, Barro (2006) broadens this model to several countries, and Wachter (2013) shows that investors' perceptions of risk change when rare events occur. If all these

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models display large conditional equity premiums, then the challenge is finding conditional information that best captures this disaster risk and its implied predictability. In general, aggregate-level variables are usually used to predict asset returns. However, seeking and discovering new relations using the nonaggregated quantities of the aggregated phenomenon is intuitive.[1] Indeed, if systemic sectoral shocks hold specific information, they should be used to reflect uncertainty in asset prices. This is even more important when addressing tail comovement, since at the aggregate level tail risk is partially diversified away. Is there a benefit to incorporating systemic sectoral tail shocks when predicting asset returns?

This paper provides a positive answer to the earlier question. We are the first to analyze the joint effect of tail risk and sector heterogeneity to predict asset returns and to show its value in stock return predictability. We combine two strands of the literature. On the one hand, one way to incorporate rare events into finance research is to adopt the use of the extreme value theory (e.g., Longin and Solnik 2001; Bae, Karolyi, and Stulz 2003; Hartmann, Straetmans, and de Vries 2004). For example, Poon, Rockinger, and Tawn (2004) advocate for the use of risk measures based on extreme value theory, rather than for traditional risk measures, such as volatility or value-at-risk. They demonstrate that the latter, which may lead to inaccurate portfolio risk assessment, are unsuitable for measuring tail risk. On the other hand, researchers have shown that a powerful solution when examining aggregate-level variables is the use of sectoral information, because different procyclical shocks can be recognized at the sectoral level but are invisible at the aggregate level (e.g., Horvath 2000; Veldkamp and Wolfers 2007; Comin and Mulani 2009; Holly and Petrella 2012). For example, Hong, Torous, and Valkanov (2007) show that industry interdependencies are essential for the predictability of market returns.

Our first main contribution is to define a new simple and tractable measure of a country's left tail dependency, which has strong and significant predictive power for the U.S. equity premium in sample (IS) and out of sample (OOS). The predictability of left tail mean (*LTM*) is the result of a growing literature on tail dependency (Longin and Solnik 2001; Ang and Chen 2002; Poon et al. 2004) considering that comovements in sector consumption and sector equity prices affect the *ERP*. Our results show that *LTM* has significant IS and OOS predictive power.[2] Based on extreme value theory, we first compute the bivariate sectoral tail dependence for each pair of sectors in a country to

[1] The fields of biology and medicine offer a more straightforward example; for example, just as studying humans at the cellular level will give us information not perceptible by studying humans at the tissue level, organ system level, or, even, as a whole body, so can aggregated behaviors hide information about nonaggregated pieces.

[2] In the Internet Appendix, we define and provide results for an alternative tail dependence measure: left exceedance correlation (*LEC*). The previous literature on exceedance correlations has outlined this measure as the first step to get to *LTM*. It also provides significant IS and OOS predictability.

measure the joint extreme events between the two sectors.[3] Then we compute the average tail dependence between sectors within a country. We designate this average value as left tail mean, LTM. The main intuition is that existing aggregate market tail measures average out important information about tail risk in the economy, while average tail dependency among sectors conveys this information more precisely. We show that the level of the LTM is not only time varying but also quite adaptive, and it reacts quickly and strongly compared with other similar measures. In fact, when compared with other tail measures, LTM reveals (1) returns in the tails are not drawn from a normal distribution; (2) the tails are asymmetric; and (3) the link between the sectors rather than only the risk of each sector or only the overall market.[4]

The source of the ERP predictability by the tail dependence variable, LTM, which is constructed with lagging information, is the result of returns reversals (Cujean and Hasler 2017) with a stable—postwar period—macroeconomic U.S. business cycle. In a setting that assumes no disaster events, a sudden increase of sectoral tail dependence (LTM increases) will push investors to anticipate a disaster and therefore to rebalance all their positions from equity holdings to other assets (e.g., treasuries) in a typical flight-to-quality behavior. This process will reinforce the increased value in the observed LTM that will eventually stop, either when investors realize they are not in a disaster event or when the disaster occurs with all sectors experiencing a downfall that is not necessarily of the same magnitude across sectors but that has the same starting point. The predictability of a similar "fear" behavior is also observed in Bollerslev et al. (2015). By decomposing VRP in terms of the diffusive and jump risk compensations, they show that the predictability of ERP by VRP is the result of jump tail risk premium. They use options data on the market to extract this market information. Moreover, they need to rely on an approximation step to extrapolate tail densities because of a lack of liquidity on deep out-of-the-money options. Our methods are quite distinct since we use granular sector information, which are crucial to our results. In addition, we rely on raw historical return data for our estimator. Our results are in line with those of Lan (2020), who shows that the existence of a business cycle contributes to the predictability of the ERP. In her paper, predictability is detected by studying the business cycle component of aggregate dividend yields; in our case, predictability comes from the intrinsic tail comovements of the different sectors in expectation of a common economic shock embodied in business cycles.

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3 Other authors (e.g., Patton 2009) use copula functions to model dependence structures. Hilal et al. (2011) argue (1) that the copula approach imposes too rigid of conditions on the dependence structure and (2) that the validity of its assumptions have not been tested. However, some of the foundations in the extreme value theory are built on the Copula approach, though they impose looser restrictions in the distributions used.

4 This is in line with studies such as Ang and Chen (2002), who find an asymmetry in their dependence structure that is 12% larger in negative events than the correlation implied by the normal distribution, whereas there are no significant differences in the dependence structure for positive events.

To test the predictability of *LTM* under a controlled environment that considers business cycle shocks, we simulate three different models with procyclical shocks and we compare the predictability of tail dependence variables to other non-business-cycle-related processes (multivariate normal, Student's *t*, dynamic conditional correlation, and regime switching). The results show that the tail predictors, mainly the left tail predictor, *LTM*, can better detect predictability, both IS and OOS. Our results contribute to the previous literature on *tail returns predictability* (e.g., Bali et al. 2014) and *business cycle predictability* (e.g., Lan 2020), considering that we disentangle the origin of the *tail predictability* rooted in the return reversals occurred during the business cycles downturns; therefore, tail predictability is countercyclical. The results of this paper also contribute to the discussion in Neely et al. (2014), as we define and test new economic significant variables that used jointly with macroeconomic business cycle information help understanding *ERP* predictability.

A long dispute about the predictability of several common variables (e.g., Campbell and Thompson 2008; Goyal and Welch 2008; Rapach et al. 2010; Ferreira and Santa-Clara 2011; Li et al. 2015) has persisted. We participate in this debate. We run predictive regressions as in Goyal and Welch (2008). Using a comprehensive set of common variables, we show that only two predictors offer both in- and out-of-sample significance and higher predictive power than the historical average of the equity premium. These two predictors are the *LTM* and the *VRP*.[5] The static and time-varying performance of *LTM* is superior to the *VRP* performance and their unconditional correlation is quite low, indicating a different but valuable impact of these two predictors. We select the new proposed dependence variables as predictors alongside the usual variables, including the variance risk premium, the dividend-price ratio, and the detrended Treasury-bill rate.

Subsequently, we show that *ERP* predictability from *LTM* is due to the sectors' joint shocks. There is no such predictability in the univariate left tail risk of the aggregate market or in the average of the univariate left tail risk of individual sectors. We also present evidence that not all sectors and their left tail joint dependencies are related to future risks in the same way. Nevertheless, using a value-weighted average in *LTM* by the size of each sector leads to the same qualitative conclusions. Another difference from *LTM* to *VRP* is the predictability in business cycles. All predictive power of the former is driven from recession periods, whereas the latter is driven from both periods. This means that *LTM* induce stronger predictability in recession periods than *VRP*, about three times more. *LTM* is able to forecast significantly to horizons of up to 6 months, whereas *VRP* is able to forecast for horizons of up to 3 months with lower predictability. For example, *VRP*

[5] A third predictor that captures the left tail dependence of returns, *LEC*, has in- and out-of-sample significant predictive power. See the Internet Appendix for the definition and results for this predictor.

has a OOS R-squared of 1.04% compared to 5.69% of LTM for a 3-month horizon. All these results support our view that the interdependencies of joint left tail sector shocks are an important source of predictability. Additional robustness tests include time-varying regressions (Dangl and Halling 2012), stock return decomposition (Rapach et al. 2016), and the study of predictability during business cycle recession periods (Henkel et al. 2011).

Our predictability results are consistent with recent findings in the literature; while previous predictors have been attributed to fundamental variables (e.g., the term spread, the dividend-price ratio, the detrended Treasury-bill rate), the latest research about predictability suggests that tail dependency is a key component. Driessen et al. (2009) show that the implied correlation is a good predictor (IS and OOS) of the *ERP* when controlling by fundamental variables, Kelly and Jiang (2014) find that the cross-section of tail returns has predictive power over the aggregate equity market returns, and Bollerslev et al. (2014) discovered that the *ERP* predictability attributed to this variance risk premium in Zhou (2018) is not due to a finite sample data bias.

1. Tail Dependence

This section introduces the tail dependence variables and provides the theoretical economic motivation for the use of these measures in a predictability setting.

1.1 Definition of the tail dependence variables

Traditionally, univariate distributions are used to build a time-series measure of tail dependence for a country (Kelly and Jiang 2014; Poon et al. 2004; Chabi-Yo et al. 2018; Faias 2021). However, these measures do not capture all aspects of the tail dependence. Several papers show that industry interdependencies are important in predictability (Hong et al. 2007; Cohen and Frazzini 2006; Menzly and Ozbas 2010; Rapach et al. 2015). Therefore, one can use information from the different sectors of a country to obtain a more complete picture of that country.

We define a new and simple measure of a country's tail dependence by combining the information from all intracountry tail dependencies between the sectors using extreme value theory (EVT). Considering that only the dependence structure is important in this analysis, we exclude the marginal distributions of this setting. Following Poon et al. (2004), the bivariate returns (X, Y) are transformed into unit Fréchet marginals (S, T):

$$S = \frac{1}{\log F_X(x)}, T = \frac{1}{\log F_Y(y)}, \tag{1}$$

where F_X and F_Y are the respective marginal distribution functions for X and Y. Poon et al. (2004) define the tail dependence measure as

$$\bar{\chi} = \lim_{s \to \infty} \frac{2 \log Pr(S > s)}{\log Pr(S > s, T > s)} - 1, \tag{2}$$

where $-1 < \bar{\chi} < 1$. This method accurately captures the asymptotic independence, since $Pr(S > s | T > s) \to 0$. This measure has the clear advantage of being interpreted loosely as a correlation coefficient. Values of $\bar{\chi} > 0$, $\bar{\chi} = 0$ and $\bar{\chi} < 0$ loosely correspond to when (S, T) are positively associated in the extremes, exactly independent, and negatively associated, respectively. Poon et al. (2004) show that $\bar{\chi}$ is the correlation coefficient in the case of the bivariate Gaussian dependence structure.[6]

Next, we define $Z = min(S, T)$ and rank all its values from $Z(1)$ to $Z(n)$. The maximum likelihood estimator is given by

$$\hat{\bar{\chi}} = \frac{2}{n_u} \sum_{j=1}^{n_u} \log\left(Z_{(j)}/u\right) - 1, \tag{3}$$

where n_u is the number of observations above the threshold u. Throughout this paper, n_u is 5% of n.[7] We interpret this variable as the average log excess returns relative to the threshold u. This is similar to the notion of expected shortfall, but instead of considering the expected return values above a threshold—value-at-risk in this case—our variable uses the expected log returns in excess of a threshold value. This implies that the variable is much more stable through time since we study the distance of each extreme observation from a percentile rather than studying a censored distribution.

First, we compute $\hat{\bar{\chi}}$ for all pairs of the k sectors, for the left (right) tails, within a country using weekly returns and a rolling window of 1,040 weeks (20 years).[8] We censor the values of the estimated $\bar{\chi}$ to a range between -1 and 1. The cross-section arithmetic mean of the left (right) tail $\bar{\chi}$ is defined as the LTM and is given by

$$LTM_t = \binom{n}{2}^{-1} \sum_{i<j} \bar{\chi}^{L}_{i,j,t}. \tag{4}$$

where $\bar{\chi}^{L}_{i,j,t}$ is the left tail risk measure for each pair of sectors i and j at time t, and n is the number of sectors in the country. The cross-section measure for the right tail is the RTM, and is given by

6 Weak assumptions, as specified in Poon et al. (2004), are required to estimate $\bar{\chi}$.

7 Longin and Solnik (2001) use bootstrapping to define the optimal threshold level for several large economies. They find that on average, a level of 4%–5% of the total number of observations should be considered a threshold. We also consider other values of n_u, such as 10% and 20%. In these cases, ERP predictability is achieved but is smaller, confirming the importance of considering tail values.

8 This somewhat-large number of observations is required since the tail dependence measure uses only 5% of the total number, corresponding to 52 observations, a sample size usually assumed to be a large sample for inference.

$$RTM_t = \binom{n}{2}^{-1} \sum_{i<j} \bar{\chi}^R_{i,j,t}. \tag{5}$$

where $\bar{\chi}^R_{i,j,t}$ is the right tail risk measure for each pair of sectors i and j at time t, and n is the number of sectors in the country.

1.2 Economic motivation

The equity risk premium puzzle, defined by Mehra and Prescott (1985), was identified as the inability of an Arrow-Debreu economy without frictions to account for the observed empirical difference between the equity and bond markets. A proposed theoretical solution to the equity risk premium puzzle is due to Rietz (1988) and is defined as an Arrow-Debreu economy with a "rare disaster" state. Later, Barro (2006) extended and empirically tested the Rietz (1988) solution for several countries. Wachter (2013) provide an explanation of the volatility puzzle (excess volatility) using a reduced-form model based on a time-varying probability of the rare disaster of Rietz (1988). All these studies (Mehra and Prescott 1985; Rietz 1988; Barro 2006; Wachter 2013) apply a consumption-based model for the marginal rate of substitution. The limitations of such approaches in explaining equity predictability are in the multivariate dynamics between the consumption sectors of the economy and the equity sectors. Alternatively, we develop and use a less-restricted economic model for the explanation of the empirical results, reproducing the time-varying "rare" disaster characteristics of Rietz (1988), Barro (2006), and Wachter (2013), but without the limitations of being consumption based.

Following Harvey and Siddique (2000), we identify a time-varying "rare" disaster pricing kernel. Let $X_{i,t+1}$ be the ith sector returns at time $t+1$, $X_{M,t+1}$ the market returns at time $t+1$, and $\mathcal{F}_t$ the information set at time t. The traditional pricing kernel is given by $E[(1+X_{i,t+1})m_{t+1}|\mathcal{F}_t]$, where m_{t+1} is the marginal rate of substitution. We incorporate two innovations into this traditional pricing kernel: from (1) Guidolin and Timmermann (2008) we introduce a higher-order moment capital asset pricing model, and from (2) Bawa and Lindenberg (1977) and Ang and Bekaert (2006), we introduce a downside risk (lower partial moment) framework. Define the risk-free interest rate as $X_{f,t}$, the resultant pricing kernel generates a higher-order moment asset pricing model:

$$\begin{aligned} E[X_{i,t+1}|\mathcal{F}_t] - X_{f,t} &= \tilde{\gamma}_{1,t} COV(X_{i,t+1}, X_{M,t+1}|\mathcal{F}_t) \\ &+ \tilde{\gamma}_{2,t} COV(X_{i,t+1}, (X_{M,t+1})^2|\mathcal{F}_t) \\ &+ \tilde{\gamma}_{3,t} COV(X_{i,t+1}, (X_{M,t+1})^3|\mathcal{F}_t), \end{aligned} \tag{6}$$

where $\tilde{\gamma}_{1,t}$, $\tilde{\gamma}_{2,t}$, $\tilde{\gamma}_{3,t}$ are the systematic standard deviation, skewness, and kurtosis of the sector $X_{i,t+1}$. The capital asset pricing model in Equation

(6) nests various models, such as the CAPM, ICAPM, and the conditional skewness asset pricing model of Harvey and Siddique (2000). It can be used to explain the in-sample predictability from *LTM*, yet it has no time-varying "rare" disaster property within. From Ang and Bekaert (2006), we know that the *ERP* of downside returns is higher than the risk premium from upside returns, and this difference is priced by investors. We incorporate this difference into a time-varying rare disaster setup.

The time-varying "rare" disaster property will imply the existence of a state of disaster with an associated threshold under which the equity market returns are considered to hold the qualitative property of a disaster: this qualitative state implies a major transmission of the disaster effect to the real economy (consumption) and visible recessionary periods.

We incorporate this time-varying "rare" disaster property into the model in Equation (6). Define two states of the economy: normal and disaster states. The new pricing kernel is

$$\begin{aligned} E[(1+X_{i,t+1})m_{t+1}|\mathcal{F}_t] &= E[(1+X_{i,t+1})m_{t+1}|\mathcal{F}_t, Pr(X_{M,t+1} < d)]Pr(X_{M,t+1} < d) \\ &\quad + E[(1+X_{i,t+1})m_{t+1}|\mathcal{F}_t, Pr(X_{M,t+1} \geq d)]Pr(X_{M,t+1} \geq d), \end{aligned} \tag{7}$$

where the probability of entering the disaster is $Pr(X_{M,t+1} < d)$, and d is a disaster threshold.

Define the conditional covariance between the ith sector returns and the market returns of k-th order for the case of no disaster

$$C^k_{n,t+1} = COV(X_{i,t+1}, X^k_{M,t+1}|\mathcal{F}_t, Pr(X_{M,t+1} \geq d)]Pr(X_{M,t+1} \geq d),$$

and the conditional covariance between the ith sector returns and the market returns of k-th order for the case of disaster

$$C^k_{d,t+1} = COV(X_{i,t+1}, X^k_{M,t+1}|\mathcal{F}_t, Pr(X_{M,t+1} < d)]Pr(X_{M,t+1} < d).$$

Incorporating the "rare" disaster state to the model in Equation (6), the time-varying "rare" disaster asset pricing model yields

$$\begin{aligned} E[X_{i,t+1}|\mathcal{F}_t] - X_{f,t} &= \tilde{\gamma}_{1,n,t}C^1_{n,t+1} + \tilde{\gamma}_{2,n,t}C^2_{n,t+1} + \tilde{\gamma}_{3,n,t}C^3_{n,t+1} \\ &\quad + \tilde{\gamma}_{1,d,t}C^1_{d,t+1} + \tilde{\gamma}_{2,d,t}C^2_{d,t+1} + \tilde{\gamma}_{3,d,t}C^3_{d,t+1}, \end{aligned} \tag{8}$$

where $\tilde{\gamma}_{1,n,t}, \tilde{\gamma}_{2,n,t}, \tilde{\gamma}_{3,n,t}$ are the systematic standard deviation, skewness, and kurtosis of the sector $X_{i,t+1}$ in the normal state (no disaster), and $\tilde{\gamma}_{1,d,t}, \tilde{\gamma}_{2,d,t}, \tilde{\gamma}_{3,d,t}$ are the corresponding systematic sensitivities in the case of a disaster.

A set of theoretical research supports the existence of two types of systematic sensitivities for the two states of the economy (Longin and Solnik 2001; Chabakauri 2013; Wachter 2013) and empirical evidence (Ang and Bekaert 2002; Ang and Chen 2002; Das and Uppal 2004; Barro 2006).

1.3 Economics of in- and out-of-sample predictability

To link the previous "rare" disaster pricing model (Equation 8) with the predictability of LTM, the tail measure analyzed in this paper, we apply some algebra to the definitions. For simplicity, consider a market with two sectors; then in equilibrium the market portfolio return is $X^{\bigstar}_{M,t} = \omega^{\bigstar}_1 X_{1,t} + (1-\omega^{\bigstar}_1)X_{2,t}$, with $\omega^{\bigstar}_1$ being the equilibrium weight.[9] From Equation (4), the left tail dependence of the sectors of the economy is

$$LTM_t \quad = 2\binom{n}{2}^{-1} \sum_{i<j}\left(\lim_{s\to+\infty}\frac{Pr(S_{I,t}>s, S_{j,t}>s)}{Pr(S_{i,t}>s)} - 1\right), \tag{9}$$

where $S_{i,t}$ is the Fréchet-adjusted sector returns as in Equation (1).

In this economy, the covariance from Equation (6) between the ith sector and the market portfolio is

$$\begin{aligned} COV(X_{i,t}, \omega^{\bigstar}_1 X_{1,t} + (1-\omega^{\bigstar}_1)X_{2,t}|\mathcal{F}_t) \quad &= COV(X_{i,t}, \omega^{\bigstar}_1 X_{1,t}|\mathcal{F}_t) \\ &+ COV(X_{i,t}, (1-\omega^{\bigstar}_1)X_{2,t}|\mathcal{F}_t). \end{aligned} \tag{10}$$

Without loss of generality, set $i=1$, and introduce the "rare" disaster state. The first covariance term of (10) in a disaster state at time t, $C^1_{d,t}$, becomes

$$COV(X_{1,t}, \omega^{\bigstar}_1 X_{1,t}|\mathcal{F}_t, X_{M,t+1} < d) = (\omega^{\bigstar}_1)^2 V(X_{1,t}|\mathcal{F}_t, X_{M,t+1} < d)$$

where $V(\cdot)$ is the variance function, and the second covariance term in (10) translates to

$$\begin{aligned} &(1-\omega^{\bigstar}_1)^2 COV(X_{1,t}, X_{2,t}|\mathcal{F}_t, X_{M,t+1} < d) = \\ &= \frac{CORR(X_{1,t}, X_{2,t}|\mathcal{F}_t, X_{M,t} < d)}{STD(X_{1,t}|\mathcal{F}_t, X_{M,t} < d) STD(X_{2,t}|\mathcal{F}_t, X_{M,t} < d)}. \end{aligned} \tag{11}$$

The source of the in-sample predictability for LTM_t in this two-sector economy is the result of the interaction of the tail dependence with the probability of a disaster: all three covariance terms in the "rare" disaster model (Equation 8) are adjusted by the probability of occurrence of the disaster $Pr(X_{M,t+1} < d)$. By definition, in this two-sector economy:

$$LTM_t = 2\left(\lim_{s\to\infty}\frac{Pr(S_{i,t}>s, S_{j,t}>s)}{Pr(S_{i,t}>s)} - 1\right).$$

Hence, the probability of the equity market entering into a disaster $Pr(X_{M,t} < d)$ at time t can be identified by the probability of the two sectors of the economy falling below a lower bound threshold s, $Pr(S_{i,t} > s, S_{j,t} > s)$

[9] The equilibrium weights maximize the dynamic investment optimization of the pricing kernel in (7).

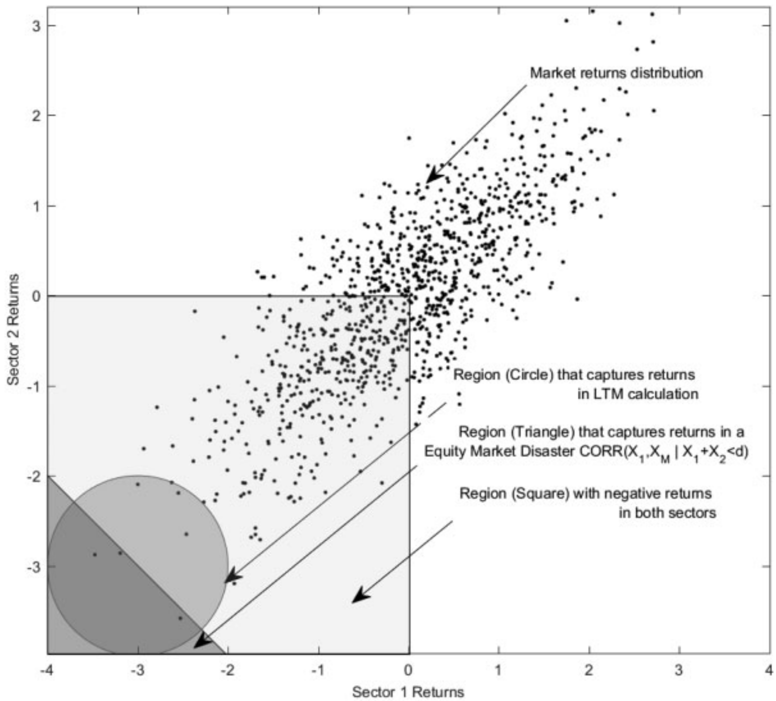

Figure 1
Source of the predictability: The tail dependence predictors in a two-sector economy
This figure presents the interaction between the joint sector returns, the market rare disaster region and the *LTM* region (tail dependence measure). The joint negative returns correspond to the shaded squared region, the rare disaster region corresponds to the shaded lower triangle region, and the *LTM* region corresponds to the circle region. All these regions overlap, showing that joint returns that report a higher *LTM* value, will be close to the rare disaster region. The sample is generated from a MVN distribution with high correlation (0.8) as an example.

at a time t, if the threshold s is appropriately selected. Figure 1 explains this result.

Figure 1 shows the market returns in an economy with two sectors with equilibrium weights ω_1^* and $1 - \omega_1^*$. In this two-sector economy, the joint sector returns are plotted as dots. The triangle region defines the market disaster region; any bivariate sector return inside this region will imply a disaster. The circle area represents the region with the joint sector returns measured by the LTM_t variable.[10] By decreasing the number of joint returns considered in the LTM_t calculation (5% in our case, see Figure 3), the circle region reduces the size of the disaster region (triangle) that it intersects.

The source of the out-of-sample predictability of the LTM_t is coming from the correlation (and covariance) clustering "stylized fact" of the equity

[10] It is not necessarily a circle but a curved area; however, we draw a circle for illustration purposes.

market (Ang and Chen 2002): the covariance during normal times at t, $C_{n,t}^k = COV(X_{i,t}, X_{M,t}^k | \mathcal{F}_t, Pr(X_{M,t} \geq d)]Pr(X_{M,t} \geq d)$, is dependent on the lagged covariance $C_{n,t-1}^k$; then, there exists time dependence in the probability of staying in the recessionary period, a "rare" disaster probability cluster effect. This "rare" disaster asset pricing model, in addition to the disaster correlation and disaster probability cluster effect provides the econometric explanation for the source of the tail predictability in equilibrium.

2. Data

The main analysis uses end-of-month U.S. observations starting in January 1990 and ending in December 2020 since dependence variables require 20 years of data to initialize (i.e., we use data starting in January 1970, to initialize the dependence variables). This analysis period is analogous to those used by many other papers, such as Rapach et al. (2016).

Sector data at the weekly level are used to construct the dependence variables LTM and RTM. The Friday closing price is considered for each of the target indexes. The 17 selected sectors are the following: food (FD), mining and minerals (MNS), oil & petroleum (OIL), textiles, apparel, & footwear (CLT), consumer durables (DUR), chemicals (CHM), consumer (CNM), construction and construction materials (CNT), steel works (STL), fabricated products (FAB), machinery and business equipment (MAC), automobiles (CAR), transportation (TRA), utilities (UTI), retail stores (RTL), financial (FIN), and other (OTH).[11] These weekly data are obtained from Kenneth R. French's website and span January 1970 to December 2020. Weekly frequency is preferred over monthly and daily frequencies. Hartmann et al. (2004) also make a similar choice of frequency when studying tail dependence. The choice of weekly frequency rather than daily avoids the problems of nonsynchronous trading and heteroscedasticity, which affect the estimates of tail dependence (Poon et al. 2004). The choice of weekly observations rather than monthly implies a fourfold increase in sample size, which is important in this setting. Weekly frequency is used in the time-series measures. However, because predictability is performed monthly, the variables had to be converted to a monthly frequency. Here, the monthly measure is the average of weekly values within each month.[12]

As a benchmark, we also compute the same type of tail measures using the traditional Pearson correlations. The Pearson correlation measures the average of deviations from the mean without making any distinction between

[11] Seventeen sectors correspond to the 136 pairs computing LTM, RTM, and $CORR$. Later on, we will also make a comparison with a different number of sectors, and the qualitative results are robust to the chosen number of sectors when a minimum disaggregation of sectors is selected.

[12] We also use the last weekly observation of each month, and the results are of similar magnitude. They are presented as robustness results.

negative and positive returns. The cross-section measure for the Pearson correlation is designated by the $CORR$ and is given by

$$CORR_t = \binom{n}{2}^{-1} \sum_{i<j} \rho_{i,j,t}, \tag{12}$$

where $\rho_{i,j,t}$ is the Pearson correlation measure for each pair of sectors i and j at time t, and n is the number of sectors in the country.

Additionally, we consider two univariate tail risk variables. The first one is the aggregated market univariate measure for the left tail ($ALTM$) and is given by

$$ALTM_t = \bar{\chi}^L_{M,M,t}. \tag{13}$$

where $\bar{\chi}^L_{M,M,t}$ is the univariate left tail risk measure for the market at time t. The second measure is the univariate sectors' left tail mean ($SLTM$) and is given by

$$SLTM_t = \frac{1}{n}\sum_i \bar{\chi}^L_{i,i,t}. \tag{14}$$

where $\bar{\chi}^L_{i,i,t}$ is the univariate left tail risk measure for each sector i at time t, and n is the number of sectors in the country.

Panel A of Table 1 presents the descriptive statistics of the five dependence variables: LTM, RTM, $CORR$, $ALTM$, and $SLTM$. Figure 2 presents their standardized evolution to understand better the implication of the variables in the predictive regressions.[13] The standardization uses the first two unconditional moments. Looking at $\rho(1)$, all variables are quite persistent which would be expected by their definition. However, this high serial correlation is also standard in the used traditional predictor variables. In a different setup, Ang and Chen (2002) find that negative tails deviate more from the normal distribution than right tails. We observe this stylized fact in the difference between $CORR$ and LTM and $CORR$ and RTM. LTM reacts more strongly, and it is quite adaptive in several episodes, such as in the periods between 2001 and 2002 and between 2008 and 2009. Notably, the univariate measures, $ALTM$ and $SLTM$, are almost flat after 2011, which reveals their inadequacy in capturing changes in the ERP.

2.1 Other predictor variables and equity risk premium

We use traditional predictors and the new proposed dependence variables to study the predictability of the stock market equity premium. All variables lag the stock market equity premium by one month. At the start of each month,

[13] The three shaded areas in Figure 2 represent the three recessionary periods, as defined by NBER (http://www.nber.org/cycles.html). The starting period is the peak, and the ending period is the trough for real gross domestic product in the United States.

Table 1
Summary statistics

	Mean	SD	Skew	Kurt	Min	Max	$\rho(1)$	$\rho(LTM_t)$	$\rho(ERP_{t+1})$
A. Dependence predictors									
CORR	0.62	0.06	0.09	2.06	0.52	0.73	0.99	0.56	0.13
LTM	0.82	0.05	−1.38	7.05	0.65	0.92	0.97	1.00	0.23
RTM	0.71	0.10	−1.02	2.68	0.48	0.87	0.99	0.51	0.14
ALTM	0.30	0.03	−0.69	2.54	0.23	0.36	0.98	0.41	0.13
SLTM	0.30	0.02	0.26	1.46	0.27	0.34	0.99	0.10	0.10
B. Traditional predictors									
DFS (%)	0.96	0.38	3.17	17.16	0.55	3.38	0.96	−0.30	−0.01
TMS (%)	1.76	1.10	−0.03	1.96	−0.53	3.76	0.98	0.05	−0.01
DP	−3.92	0.27	0.23	2.91	−4.52	−3.24	0.98	0.17	0.10
TBILL (%)	2.63	2.22	0.33	1.84	0.01	7.90	0.99	0.03	−0.05
BM	0.29	0.08	0.23	2.95	0.12	0.52	0.97	0.09	0.07
DY	−3.84	0.28	−0.23	3.19	−4.57	−3.21	0.98	0.38	0.11
DE	−0.80	0.40	2.77	13.91	−1.24	1.38	0.98	−0.03	0.03
EP	−3.12	0.35	−2.13	10.09	−4.84	−2.57	0.97	0.16	0.04
SV (%)	0.26	0.45	7.23	75.41	0.02	5.81	0.69	−0.21	−0.11
NTS	0.00	0.02	−0.30	2.90	−0.06	0.05	0.98	0.50	0.04
INFL (%)	0.20	0.33	−0.89	8.16	−1.92	1.22	0.47	−0.08	−0.02
LTY (%)	4.87	2.00	0.05	2.21	0.62	9.24	0.98	0.04	−0.07
VRP (%)	15.90	20.49	−3.08	50.55	−218.56	115.85	0.24	0.10	0.18
C. Equity risk premium									
ERP(%)	0.71	4.39	−0.59	4.26	−17.23	13.65	0.05	0.19	0.05

This table reports summary statistics for the dependence predictors (panel A), the traditional predictors (panel B), and the equity risk premium (panel C). The dependence predictors are *CORR* (correlation sectors' mean), *LTM* (bivariate left tail sectors' mean), *RTM* (bivariate right tail sectors' mean), *ALTM* (univariate left tail of the market), and *SLTM* (univariate left tail sectors' mean). The traditional predictors are *DFS* (default spread), *TMS* (term spread), *DP* (dividend-price ratio), *TBL* (detrended Treasury-bill rate), *BM* (book-to-market ratio), *DY* (dividend yield), *DE* (dividend payout ratio), *EP* (earnings-to-price ratio), *SV* (realized stock variance), *NTS* (net equity expansion), *INFL* (inflation), *LTY* (long-term yield), and *VRP* (variance risk premium). The *VRP* is from Hao Zhou's website. The remaining variables are from Amit Goyal's website. Section 2 defines all variables. For each variable, the time-series average (Mean), standard deviation (SD), skewness (Skew), excess kurtosis (Kurt), minimum (Min), maximum (Max), first-order unconditional autocorrelation ($\rho(1)$), contemporaneous unconditional correlation with *LTM* ($\rho(LTM_t)$), and correlation with 1-month-ahead *ERP* ($\rho(ERP_{t+1})$) are reported. The sample period is from January 1990 to December 2020.

the investor can choose from 17 variables. The set of traditional variables are the common variables used in the literature (e.g., Goyal and Welch 2008) that are related to stock market characteristics, interest rates, and broad macroeconomic indicators. The default spread (*DFS*) is the difference between the returns of BAA-rated and AAA-rated bonds. The term spread (*TMS*) is the difference between long-term bond returns (10-year) and Treasury-bill returns. The dividend-price (*DP*) ratio is defined as the difference between the log of the 12-month moving sum of dividends paid on the S&P 500 index and the log of prices. The detrended Treasury-bill (*TBL*) rate is the Treasury-bill rate reduced by the 12-month backward moving average. The book-to-market (*BM*) ratio is the book-to-market ratio of the Dow Jones Industrial Average. Dividend yield (*DY*) is the difference between the log of the 12-month moving sum of dividends paid on the S&P 500 index and the log of lagged prices. The dividend payout (*DE*) is the difference between the log of the 12-month moving sum of dividends paid on the S&P 500 index and the

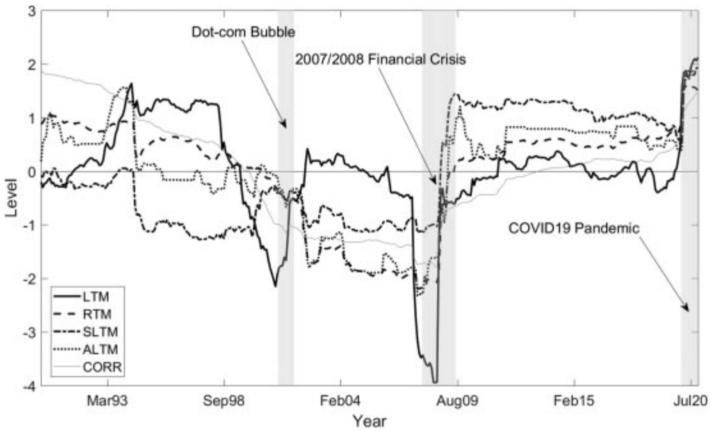

Figure 2
Evolution of the dependence measures
This figure presents the evolution of the dependence measures *LTM* (bivariate left tail sectors' mean), *RTM* (bivariate right tail sectors' mean), *SLTM* (univariate left tail sectors' mean), *ALTM* (univariate left tail of the market), and *CORR* (sectors' correlation mean). Standardization is performed using the unconditional moments. The gray vertical bands represent NBER-defined recessionary periods.

log of the 12-month moving sum of earnings on the S&P 500 index. The earnings-to-price (*EP*) ratio is the difference between the log of the 12-month moving sum of earnings on the S&P 500 index and the log of prices. The realized stock variance (*SV*) is the sum of squared daily returns on the S&P 500 index during a month. Net equity expansion (*NTS*) is the ratio of the 12-month moving sums of net equity issues by NYSE listed stocks to the total end-of-year market capitalization of NYSE stocks. Inflation (*INFL*) is the Consumer Price Index provided by the Bureau of Labor Statistics. The long-term yield (*LTY*) is the long-term U.S. government bond yield. The variance risk premium (*VRP*) is the difference between the expected 1-month-ahead stock return variance under the risk-neutral measure and the expected 1-month-ahead variance under the physical measure. *VRP* is obtained from Hao Zhou's website.[14] The remaining variables, which come from Amit Goyal's website, range from January 1990 to December 2020, the period during which they first became available.

Panel B of Table 1 presents the summary statistics for these variables. All summary statistics are generally consistent with those found in the literature. Many of these economic variables often exhibit near-unit-root persistence. In the Internet Appendix, we report the correlation matrix of these variables and the dependence variables. The findings reveal some typical connections between the variables. *DY* and *DP* have a strong and positive correlation of

[14] We remove two observations in March 2020. The z-scores for the values of *VRP* and *SV* are of the order of 10, making them totally different from the remaining values for the time series of all variables. No investor would use these values to predict the *ERP* even in real time.

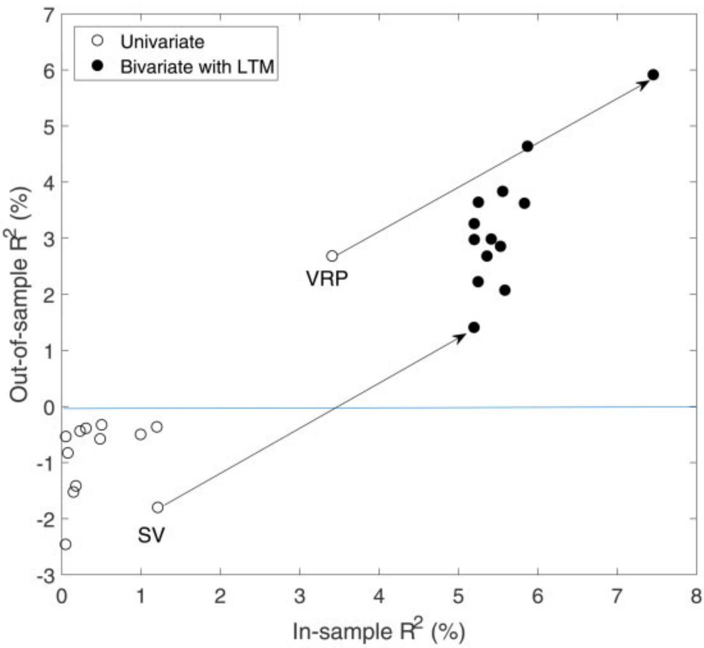

Figure 3
Predictability: The incremental joint effect
This figure presents the IS *R*-squared (horizontal axis) and the OOS *R*-squared (vertical axis) for univariate and bivariate predictive regressions. The bivariate regressions combine each variable of the univariate predictive regressions with the *LTM*. Table 1 defines all variables. The time span is from January 1990 to December 2020.

0.84. *DE* and *EP* have a strong and negative correlation of -0.75. The *BM* and *DP* have a positive strong correlation of 0.79. Note that *DFS*, *TMS*, *DP*, and *TBL* are weakly correlated with a maximum absolute value of correlation of 0.44. *DFS* and *DY* have a correlation greater than 0.40 with many variables. It is remarkable that *INFL* and the *VRP* are weakly correlated with any of the other variables. The dependence variables are strongly unconditionally correlated: the correlation between the *RTM* and the *CORR* is 0.87; between the *LTM* and the *CORR* is 0.56; and between the *LTM* and the *RTM* is 0.51. It is remarkable to see that the *LTM* strongly correlates with *NTS* and it is somewhat correlated to *DY*. In untabulated results, we also compute the contemporaneous 24-month rolling window correlation between the *LTM* and each of these variables. A remarkable volatile movement in the contemporaneous correlation between *LTM* and many variables presents quite a few sign changes that turn out to be significant, and there are moments in time with a correlation of zero, although most of the time, the correlation is of course positive and significant. As an example, we analyze the case of *NTS* and *DP*. Notably, the correlation achieves values of -0.70 in 1997 and 2012. For *DP*, the correlation is below −0.70 in 1992, 1993, 1994, 2005, 2008, 2011, 2016, 2018, and 2019. Thus, a broad selection of effects is captured. The RTM is strongly correlated to the $CORR$. The $CORR$ is somewhat related to DY, NTS, and LTY. This shows that all these dependence variables actually capture different effects from the economy.

The equity premium is simply the difference between the stock market returns and the short-term rate. The CRSP VW index, which is retrieved from Kenneth R. French's website, is used to proxy for the stock market. The short-term bond is proxied for by the 3-month U.S. Treasury bill and is obtained from the Federal Reserve Economic Data (FRED). Panel C of Table 1 presents the summary statistics for the ERP for the period from January 1990 to December 2020, which are well known and similar to the previous literature.

3. Predictability

In this section, we are mainly interested in testing the predictability of the ERP using LTM. We care about both in- and out-of-sample results. Then, we contrast these results with the ones using the traditional predictors and other measures of tail dependence. Next, we demonstrate that ERP predictability is robust to different definitions of the LTM. At the end of this section, we also present the incremental value of the LTM by combining with other predictors.

3.1 Methodology

We apply the widely used methodology of comparing the sum of squared errors (SSE) of the predictive regression with the SSE of the average historical ERP (e.g., Goyal and Welch 2008; Ferreira and Santa-Clara 2011; Campbell and Thompson 2008; Rapach et al. 2010; Li et al. 2015).

First, we obtain in-sample (IS) results. We run a predictive regression for the entire sample of available data in the following form:

$$ERP_t = \alpha + \beta x_{t-1} + \epsilon_t \tag{15}$$

where x_{t-1} is the predictor at time $t-1$ and ERP_t is the equity risk premium at time t. Then, we compute the R-squared of this regression as

$$R^2_{IS} = 1 - \frac{\sum_{t=2}^{T} (ERP_t - \widehat{ERP}_t)^2}{\sum_{t=2}^{T} (ERP_t - E\bar{R}P_t)^2}, \tag{16}$$

where T is the size of the sample, $\widehat{ERP}_t$ is the predicted value from Equation (15) and $E\bar{R}P_t$ is the sample average of the risk premium using an expanding window until time t. If the R-squared is positive, then the predictor forecasts the value of the equity risk premium better than the historical risk premium average. As the R-squared increases, the quality of the forecast improves.

We also evaluate the out-of-sample (OOS) predictive power, which is closer to real-time forecasting. To predict the value of the risk premium OOS at time $t+1$, we only use the data available until time t instead of the entire available sample. Hence, the regression is reestimated before every prediction. The OOS R-squared is given by

$$R^2_{OOS} = 1 - \frac{\sum_{t=m+1}^{T} (ERP_t - \widehat{ERP}_t)^2}{\sum_{t=m+1}^{T} (ERP_t - E\bar{R}P_t)^2} \tag{17}$$

For the OOS forecast, we require m periods for the initial estimation period for the first prediction, and we then either roll over the estimation period (rolling window) or expand it for the next forecasts (recursive or expanding window), allowing us to obtain $q = T - m$ OOS observations. Consistent with Goyal and Welch (2008), we use an expanding window with an initial estimation period of 5 years preserving a 20-year rolling window for the estimation of $E\bar{R}P_t$.[15] To test the statistical significance of the IS and OOS predictions, we use the Clark and West (2007) test of equal forecast ability. The test helps to identify whether the mean squared percentage errors (MSPE) of prediction is significantly lower than MSPE of the historical equity risk premium average. In practice, this is identical to testing the null hypothesis of $R^2_{OOS} < 0$ against the alternative hypothesis of $R^2_{OOS} > 0$. We apply Hodrick (1992) standard error correction for overlapping data using 12 lags.[16]

3.2 Results

Panels A and B of Table 2 present the in- and out-of-sample results. All predictors present positive in-sample R-squared, although only a few are statistically significant: DY, SV, VRP, $CORR$, LTM, RTM, $ALTM$, and $SLTM$. Notably, each bivariate second-order dependence measure present a positive and significant R-squared. The LTM has the highest (5.20%), the VRP the second highest (3.41%) and the RTM the third highest (1.99%). Next, we evaluate the out-of-sample predictability. As expected, most of the predictors exhibit a significant reduction in R-squared and lose significance when compared to the in-sample results. The only variables with positive and

[15] We also run different tests for this assumption and present the results in the Internet Appendix, but the results for LTM are not so sensitive to these choices. The results for VRP are sensitive to this choice.

[16] Richardson and Smith (1991) argue that overlapping return observations produce a moving average structure in the errors of the forecast that jeopardizes the reliability of the tests based on ordinary least squares (OLS) and, even, Newey and West (1987) standard errors. According to Ang and Bekaert (2007), the Hodrick (1992) standard error correction yields the most conservative test results.

Table 2
Predictability

A. By the traditional predictors

	DFS	TMS	DP	TBL	BM	DY	DE
IS R^2 (%)	0.05	0.06	1.00	0.31	0.51	1.20*	0.15
OOS R^2 (%)	−2.47	−0.54	−0.51	−0.40	−0.34	−0.37	−1.53

	EP	SV	NTS	INFL	LTY	VRP
IS R^2 (%)	0.18	1.22*	0.24	0.08	0.49	3.41*
OOS R^2 (%)	−1.42	−1.81	−0.45	−0.84	−0.59	2.67*

B. By the tail dependence predictors

	CORR	LTM	RTM	ALTM	SLTM
IS R^2 (%)	1.65*	5.20*	1.99*	1.62*	0.99*
OOS R^2 (%)	0.40	4.55*	0.47	0.83	−0.05

C. By each sector tail dependence in LTM

	FD	MNS	OOIL	CLT	DUR	CHM	CN	CNT	STL
IS R^2 (%)	0.56	0.09	1.30*	0.77*	1.08*	1.97*	0.72	0.16	0.18
OOS R^2 (%)	−0.31	−1.42	0.69	−1.12	−1.47	0.50	−0.73	−1.00	−1.27

	FBP	MAC	CAR	TRA	UTI	RTL	FIN	OTH
IS R^2 (%)	0.91	0.49	4.98*	0.11	4.53*	1.07	0.84	2.10*
OOS R^2 (%)	−0.62	−1.03	4.02*	−1.36	3.20*	0.15	−1.14	0.66

D. By different versions of LTM

	FF5	FF10	FF17	FF38	FF48	VW
IS R^2 (%)	0.14	4.37*	5.20*	3.42*	3.48*	3.57*
OOS R^2 (%)	−1.68	2.98*	4.55*	3.40*	1.94*	2.64*

E. Adjusted by realized volatility (Johnson 2018)

	VRP	LTM	LTM VW
IS R^2 (%)	0.80	3.32*	2.36*

This table reports the R-squared of in- and out-of-sample predictive regressions for a single predictor of 1-month-ahead equity risk premium. The following predictive regression is used: $ERP_t = \alpha + \beta x_{t-1} + \epsilon_t$, where x_{t-1} is the predictor at time $t-1$, and ERP_t is the equity risk premium at time t. For the IS analysis in which we use the full period, we compute the R-squared of this regression as $R^2_{IS} = 1 - \frac{\sum_{t=2}^{T}(ERP_t - \widehat{ERP}_t)^2}{\sum_{t=2}^{T}(ERP_t - \bar{ERP}_t)^2}$, where T is the size of the sample, $\widehat{ERP}_t$ is the predicted value from the predictive regression, and $\bar{ERP}_t$ is the sample average of the risk premium using a rolling window until time t. For the OOS analysis in which only information until time t is taken, the following R-squared is computed $R^2_{OOS} = 1 - \frac{\sum_{t=m+1}^{T}(ERP_t - \widehat{ERP}_t)^2}{\sum_{t=m+1}^{T}(ERP_t - \bar{ERP}_t)^2}$. We use an expanding window with an initial estimation period of 5 years preserving a 20-year rolling window for the estimation of $\bar{ERP}_t$. Panel A presents the results for traditional predictors. Panel B presents the results for tail dependence predictors. Panel C presents the results for each sector in LTM. Panel D presents the results for different versions of LTM. Panel E presents the IS results adjusting the R-squared by the realized volatility as in Johnson (2018). Table 1 defines all variables. The data and the definition of all sectors have been extracted from Kenneth R. French's website (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The time span is from January 1990 to December 2020.
*denotes a significant predictor at a 5% significance level.

significant results are the LTM and VRP. The out-of-sample R-squared values are 4.55% and 2.67%, respectively. All of these are considered very high levels of predictability.[17] Harvey et al. (2016) claim that given extensive data mining in the current literature, it does not make any economic or statistical sense to use the usual significance criteria for a newly discovered factor, for example, a t-ratio greater than two. Instead, they suggest that a newly variable needs to clear a much higher hurdle, with a t-ratio greater than 3.0. We investigate the statistic value of the slope of the predictive regression and the R-squared statistic for LTM. The t-statistic of the IS slope of the predictive regression is -4.29 and the IS R-squared statistic is 5.01, The values are similar for OOS predictive regressions. All this is clear evidence that this is a significant effect. The Internet Appendix provides results confirming, as one would expect, that much of this predictability is derived from recession periods. The results also support the view that valuation ratios have lost their predictive power over time.[18]

Next, panel B of Table 2 presents the in- and out-of-sample R-squared for different alternatives. First, we present the results for the aggregate measure of left tail risk, $ALTM$. There is in-sample predictability but no out-of-sample predictability by the univariate market left tail risk. The IS R-squared is 1.62% and statistically significant, and the OOS R-squared is 0.83%, but not significant. This is evidence that seeing the shocks at the sector level is important. We also compute the measure using univariate left tail risk for each sector, $SLTM$. The IS R-squared is 0.99% and is statistically significant, but the OOS R-squared is -0.05%. The conclusion is the same: there is no predictability using this variable. This demonstrates that joint sectoral shocks, that is, their interdependencies in the tails, are the most important factor, not shocks to individual sectors. We then disaggregate the LTM to each sector contribution. We compute the joint left tail risk measure for all the pairs that contain a specific sector and average these 16 pairs so that we

[17] We extend the same analysis for the period ending in April 2021. The results are even more impressive. For LTM, the IS R-squared is 5.36% and the OOS R-squared is 4.78%, both of which are significant. Unfortunately, we cannot use this time span at this time since the other predictors are not available for this extended time frame.

[18] In the Internet Appendix, we also run forward-looking implicit correlation measures of Driessen, Maenhout and Vilkov (2009, 2012) and Buss and Vilkov (2012) for the period available, which is between 1996 and 2020. Some of their measures, such as implied correlation (IC) from options with maturities from a quarter to a year, are significant IS and OOS. They achieve a R-squared of magnitude 5.40% in the IS exercises and 4.72% in the OOS exercises in this time frame. In this shorter period, LTM achieves a R-squared of 6.67% and 6.42%, respectively. Notice that the OOS predictability is about 50% more than the one from the best IC measure. This is evidence that predictability is not only present in forward-looking measures and incorporating historical data aggregating by sectors may also add information to the investor. Our measure also incorporates some sort of forward-looking information since business cycles are cyclical and our measure is connected to procyclical shocks in a stable business cycle model. The literature has evolved to include nonparametric conditional market risk premium estimators. For example, Martin (2017) develops a lower bound on expected market returns from particular option portfolios. We use their daily measure into our setting through either an average within month or taking the last value of the month, but find no predictability at any horizon. We also use the technical indicators Moving Average and Momentum with different window lengths from Neely et al. (2014) and no predictability is found. Finally, we try using factors from Lettau and Pelger (2020) and again find no predictability results. All these results are available in the Internet Appendix.

obtain the equivalent *LTM* for each sector. We present the results in panel C of Table 2. All sectors present positive IS *R*-squared, and many present significant results: OOIL, CLT, DUR, CHM, CAR, UTI, and OTH. However, only the sectors CAR and UTI present a positive and significant OOS *R*-squared. A way to incorporate the importance of each sector (composition effects) through time is to consider their average size at each point in time. Thus, we construct the variable *LTM* using a value-weighted average rather than an arithmetic average. The in-sample *R*-squared is positive and statistically significant, 3.57%, and the out-of-sample *R*-squared is positive and statistically significant, 2.64%. These results are weaker than the equal-weighted but still significant and show that the *LTM* measure calculated with the sector's relative importance does not cancel the predictability.[19] We also investigate the role of having less or more sectors in the definition of *LTM*. We present the results for 5, 10, 17, 38, and 48 sectors using Fama-French industry classifications. Panel D of Table 2 presents the results. As expected, there is no predictability when using a small number of sectors. For five sectors, the IS *R*-squared is only 0.14% and the OOS *R*-squared is -1.68%. For 10 sectors, we observe that predictability starts to materialize: the in- and out-of-sample numbers are 4.37% and 2.98%, respectively.[20] For 38 sectors, the numbers are 3.42% and 3.40% and statistically significant, respectively, which is clear evidence that increasing the number of sectors improves predictability results. The robustness of the variance risk premium predictability has itself been called into question because OLS is inefficient in the presence of time-varying volatility. Johnson (2018) suggests incorporating heteroscedasticity into returns predictability regressions using the generalized least squares and argues that variance risk premium predictability is likely a "false positive." If that is true for the variance risk premium it is likely true for *LTM*. We address this in panel E of Table 2. As can be seen, *VRP* is still significant in this timeframe but its performance deteriorates by 77% of the baseline regression, and is not even significant with this constraint. In the case of *LTM*, it deteriorates by only 35%, and it is still positive and significant. It does not seem to play an important role in *LTM* predictability.

Panels A and B of Table 3 report the robustness results for the alternative definitions that can be used to calculate the tail predictors rather than using the arithmetic average. We use the median and the 95% truncated mean as in Rapach et al. (2010). The results by *LTM* are quite resilient. Outliers do not play a role. Our definition of *LTM* uses the average within month of the weekly estimates. The results in panel C show that assuming just the last week does not change much the found results. Another arguable assumption is that

19 In cross-sectional asset pricing, equal-weighted portfolios usually produce stronger results than value-weighted ones. We predict sector *ERP* (value-weighted portfolios vs. equally weighted portfolios) using sector *LTM*. In the Internet Appendix, we show that this is true for 12 of 17 sectors.

20 We also ran this regression in a previous version of the paper with MSCI data for 10 sectors until 2013, and the results were of the same magnitude.

Table 3
Predictability: Different Definitions of Tail Dependence Variables

A. Median (50th percentile)

	CORR	LTM	RTM	ALTM	SLTM
IS R^2 (%)	1.20*	5.32*	2.17*	1.62*	0.72*
OOS R^2 (%)	−0.29	4.73*	0.43	0.83	−0.29

B. 95% truncated mean

	CORR	LTM	RTM	ALTM	SLTM
IS R^2 (%)	1.21*	5.39*	2.15*	1.62*	0.72*
OOS R^2 (%)	−0.31	4.81*	0.40	0.83	−0.29

C. Last week of the month

	CORR	LTM	RTM	ALTM	SLTM
IS R^2 (%)	1.71*	5.22*	2.08*	1.94*	1.12*
OOS R^2 (%)	0.47	4.56*	0.59	1.18	0.07

D. Only top-16 cross-pair sectors

	CORR	LTM	RTM	ALTM	SLTM
IS R^2 (%)	1.60*	5.13*	1.97*	1.62*	0.93*
OOS R^2 (%)	0.22	4.56*	0.38	0.83	−0.12

E. Only bottom-16 cross-pair sectors

	CORR	LTM	RTM	ALTM	SLTM
IS R^2 (%)	1.63*	4.03*	2.07*	1.62*	1.60*
OOS R^2 (%)	0.88	2.47*	1.01	0.83	0.66

This table reports the R-squared of in- and out-of-sample predictive regressions for a single predictor of 1-month-ahead equity risk premium using different ways to compute each dependence variable. Table 2 explains in detail the computation of the R-squared values. This table replicates panel B of Table 2 but changing marginally the variables definition. Panel A presents the results for each dependence predictor using the median rather than the mean. Panel B presents the results using a 95% truncated mean. Panel C presents the results using the last week of the month rather than the average across the 4 weeks of the month. Panels D and E presents the results using the top- and bottom-16 cross-pair sectors in terms of the magnitude of the measure, respectively. Table 1 defines all variables. The time span is from January 1990 to December 2020.
*denotes a significant predictor at a 5% significance level.

the results may be coming just from the top or bottom industry pairs dependence measures. Thus, we consider a subset of all the pairs by assuming either the top-16 or the bottom-16 bivariate cross-pair sectors (from a total of 136 possible pair combinations). The results presented in panel E show that the top values are more important to predictability; we interpret this to mean that the strength of the connection during crisis events between sectors may drive the *ERP*.

Bollerslev et al. (2009) is an early result on the so-called "short-run predictability" (1–6 months) of market excess returns by the equity variance risk premium. These authors show that the market variance risk premium has strong predictive power for aggregate returns, displaying a hump-shaped

Table 4
Predictability: Time persistence

A. Predictability by the traditional predictors

	DFS	TMS	DP	TBL	BM	DY	DE	EP	SV	NTS	INFL	LTY	VRP
							1-month ahead						
IS R^2 (%)	0.05	0.06	1.00	0.31	0.51	1.20*	0.15	0.18	1.22*	0.24	0.08	0.49	3.41*
OOS R^2 (%)	−2.47	−0.54	−0.51	−0.40	−0.34	−0.37	−1.53	−1.42	−1.81	−0.45	−0.84	−0.59	2.67*
							1-quarter ahead						
IS R^2 (%)	0.39	0.05	1.16*	0.30	1.14*	1.10*	0.54	0.05	0.06	0.65	1.63*	0.37	1.70*
OOS R^2 (%)	−2.62	−0.55	−0.19	−0.39	0.66	−0.18	−0.41	−1.22	−2.80	−0.06	−0.66	−0.59	1.04*
							1-semester ahead						
IS R^2 (%)	0.37	0.18	0.88*	0.41	1.12*	0.97*	0.33	0.05	0.43	0.26	1.22	0.23	0.35
OOS R^2 (%)	−1.18	−0.55	−0.27	−0.24	0.63	−0.66	−0.51	−0.93	−1.97	−0.64	0.42	−0.91	−0.73
							1-year ahead						
IS R^2 (%)	0.34	0.81	1.61*	0.69	0.83*	1.29*	0.48	0.12	0.11	0.18	0.40	0.20	0.77*
OOS R^2 (%)	−0.69	−0.00	0.06	0.04	0.01	−0.13	−1.20	−1.39	−1.36	−0.92	−0.33	−0.89	0.03

B. Predictability by the dependence predictors

	CORR	LTM	RTM	ALTM	SLTM
			1-month ahead		
IS R^2 (%)	1.65*	5.20*	1.99*	1.62*	0.99*
OOS R^2 (%)	0.40	4.55*	0.47	0.83	−0.05
			1-quarter ahead		
IS R^2 (%)	1.47*	6.11*	1.73*	1.01*	0.87
OOS R^2 (%)	0.28	5.69*	0.13	0.28	−0.11
			1-semester ahead		
IS R^2 (%)	0.96*	4.27*	1.14*	0.89*	0.57
OOS R^2 (%)	−0.48	3.34*	−0.61	0.18	−0.72
			1-year ahead		
IS R^2 (%)	0.81*	1.65	0.86	0.87	0.58
OOS R^2 (%)	−0.74	−0.03	−0.80	−0.00	−0.79

This table reports the R-squared of in- and out-of-sample predictive regressions for a single predictor of a month-, a quarter-, a semester, and a year-ahead equity risk premium. Panel A presents the results for traditional predictors, and panel B presents the results for dependence measures. Table 2 explains the computation of the R-squared values in detail. This table replicates the results of panels A and B of Table 2 but analyzes different horizons. Table 1 defines all variables. The time span is from January 1990 to December 2020.
*denotes a significant predictor at a 5% significance level.

pattern in R-squared's around 3- to 6-month horizons. Table 4 reports predictability results for different horizons from 1 month to 1 year. In our sample *VRP* is persistent up to only 3 months, but *LTM* predictability is quite persistent for horizons up to 6 months. Also here, *LTM* has a different implication for *ERP* predictability. In sum, all these results show that using a variable with a joint left tail sectoral shock is very important for predictability, but the predictability is stronger when considering all sectors simultaneously, as in the *LTM*.

Finally, we aim to understand the incremental predictability value of *LTM*. Therefore, we compare univariate against bivariate predictability for each of the 13 classical predictor variables. In the case of bivariate predictive regressions, we combine the *LTM* with each one of the 13 alternative predictors. As expected (because of an additional variable), the IS *R*-squared increases for all variables. More notable is the fact that the OOS *R*-squared also increases for all predictive regressions. Figure 3 confirms this. The plot shows a northeast shift of all the observations. For example, when combining the *LTM* with the *VRP*, the IS *R*-squared improves from 3.41% (univariate regression) to 7.46% (bivariate regression), and the OOS *R*-squared moves from 2.67% (univariate regression) to 5.90%. This is surprising since it is common that, when one adds an additional predictor in the predictive regression, the IS results would improve but OOS results would drop due to an increased estimation error. For all of the predictors used, on average, the IS *R*-squared increases by 4.91 percentage points (pp) and the OOS *R*-squared increases by 3.89 pp. In sum, the *LTM* not only predicts the *ERP* on its own but also improves the predictability of the traditional variables when jointly used.

Wachter (2013) shows that a continuous-time endowment model in which there is a time-varying risk of a rare disaster can explain the *ERP* without assuming a high value of risk aversion. This model, however, with no sectors, uses an implied disaster-risk measure based on simulations designated by implied disaster probability (*IDP*). In complete contrast, we provide a direct, easy, and tractable measure, *LTM*. We check the predictability of *ERP* by *IDP* in our time span until 2010.[21] There is no such predictability. The in-sample *R*-squared is 0.09% and the out-of-sample *R*-squared is -2.85%. Neither is significantly different from zero for a significance level of 5%. Wachter's measure is implied from roughly the actual earnings-to-price ratio. In our time span, the earnings-to-price ratio has no significant predictability in sample and out of sample as seen in panel A of Table 2. In fact, Wachter (2013) uses the smoothed earnings-to-price ratio from Shiller (1989). The correlation between *IDP* and the smoothed earnings-to-price ratio is -0.68. This is a strong negative correlation value. We also run the predictability regressions using this ratio. The in-sample *R*-squared is 0.69% and the out-of-sample *R*-squared is -1.33%. Neither is significantly different from zero for a significance level of 5%. We also get the orthogonal component of *IDP* from the smoothed earnings-to-price ratio by computing the residuals of the former on the later variable. Even the residuals cannot predict the *ERP*. The in-sample *R*-squared is 0.22% and the out-of-sample *R*-squared is -2.38%, both of which are insignificant. These results show that there is no predictability from *IDP* directly, or indirectly through the original time series that originated it, that is, the smoothed or actual price-earnings ratio or the

[21] We thank Jessica Wachter for providing these data, which are only available until 2010.

orthogonal component of smoothed earnings-to-price ratio to *IDP*. In addition, *IDP* presents the value of zero during 59% of the months in our time span. This is a very stale time series. We repeat the previous analysis analyzing only the nonzero *IDP* months. Our conclusions remain. It is important to stress that the correlation between *IDP* and *LTM* is extremely low, at -0.01. Accordingly, our paper shows that sectoral considerations lead to a better empirical measurement of time-varying disaster risk than a model with no such considerations.

3.3 Predictability in the extremes

Tail predictors share a common parameter: the threshold that splits what is considered the tail and the body of the distribution. Section 3.2 used a standard threshold found in the literature ($n_u = 5\%$). Nevertheless, changes to this parameter might affect the predictability's results. Then, it might be of interest to analyze the behavior of the measures when the threshold parameter is moved toward the tail.

Figure 4 presents the IS and OOS *R*-squared predictability for different thresholds. The *x*-axis represents the increasing n_u threshold for calculating *LTM*, moving from a narrower tail in the left to a wider tail in the right of the axis. The predictability is stable but decreases a bit below a threshold of 4%. Nevertheless, all thresholds tested had significant IS and OOS predictability.

3.4 Time-varying predictability

The previous section tested the in- and out-of-sample *ERP* predictability using the full time span. There is a concern that this predictability holds only for the chosen window. Henkel et al. (2011) find that traditional predictors, such as short-term interest rate (*TBL*) and dividend yield (*DY*), have no predictive power during economic expansions in a sample of the G7 countries but they do during contraction periods. Dangl and Halling (2012) find that *ERP* static-time regressions underestimate the predictive ability of some variables during particular periods of time, such as crises, which are rare disasters. They develop a time-varying regression framework under which the estimated parameters α and β of the regression in Equation (15) are time varying: α_t and β_t. They report up to 5.8% more profits in an asset management exercise than when using static regressions.

We follow the same idea and run simple time-varying regression tests in a setup similar to backtesting. The parameters of the regressions α_t, β_t, are calculated using always the same final point, December 2020. The first starting point is January 1990, and each month, we will move one month ahead until December 2000.[22] The first window is from January 1990 to December 2020, and the last window is from December 2000 to December 2020. This

[22] For convergence stability, we need a large sample period with different cycles of the economy.

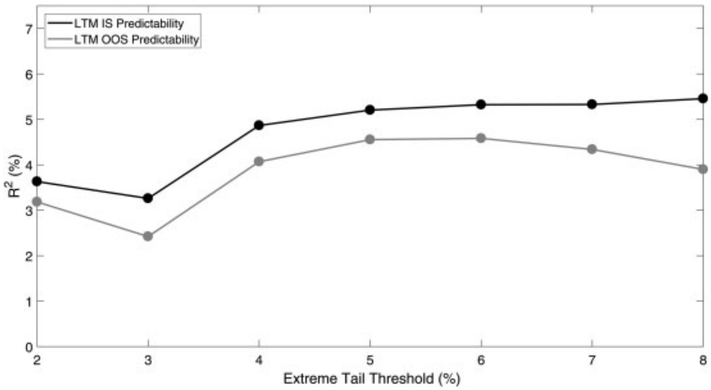

Figure 4
Predictability of tail dependence measures
This figure presents the *R*-squared of in- and out-of-sample predictive regressions of the 1-month-ahead *ERP* using *LTM* as predictor. The *x*-axis represents the tail threshold (n_u) used for the *LTM* calculation. The solid lines represent *LTM* predictability at a 5% significance level. Significant predictability is represented with a dot.

will allow us to determine the robustness of our previous results. Figure 5 presents the IS (panel A) and OOS (panel B) results for the best six unconditional IS predictors. IS *R*-squared for *LTM* fluctuates between 5.2% and 8.4%, being always significant. In fact, our baseline window (January 1990 to December 2020) delivers the worse performance from all windows. Notice also that this is the most important predictor in all months. OOS *R*-squared for *LTM* fluctuates between 3.2% and 7.1% and again always significant. It is interesting to see the persistence of *LTM* in delivering *ERP* predictability. Notice that most of the other predictors deliver volatile *R*-squared IS and OOS. For example, *VRP* delivers a OOS *R*-squared ranging between about -1.5% and 3%. The predictability stemming from *LTM* is resilient, positive, and significant, and the static performance seems to be a lower bound of the time-varying performance.

3.5 Stock return decomposition

In this section, we analyze whether the predictability of *LTM* is derived from the discount rate and/or the cash flow channels. We use the framework in Rapach et al. (2016); the authors borrow from Campbell (1991) and Campbell and Ammer (1993). They use a VAR framework to extract the cash flow and discount rate news components of stock return innovations using the log return, log dividend-price ratio, and the first three principal components extracted from 14 popular predictors of Goyal and Welch (2008). They also show that using either the first three principal components or each individual predictor yields similar qualitative results. Then they run predictive regressions of each component—expected return (ER), discount

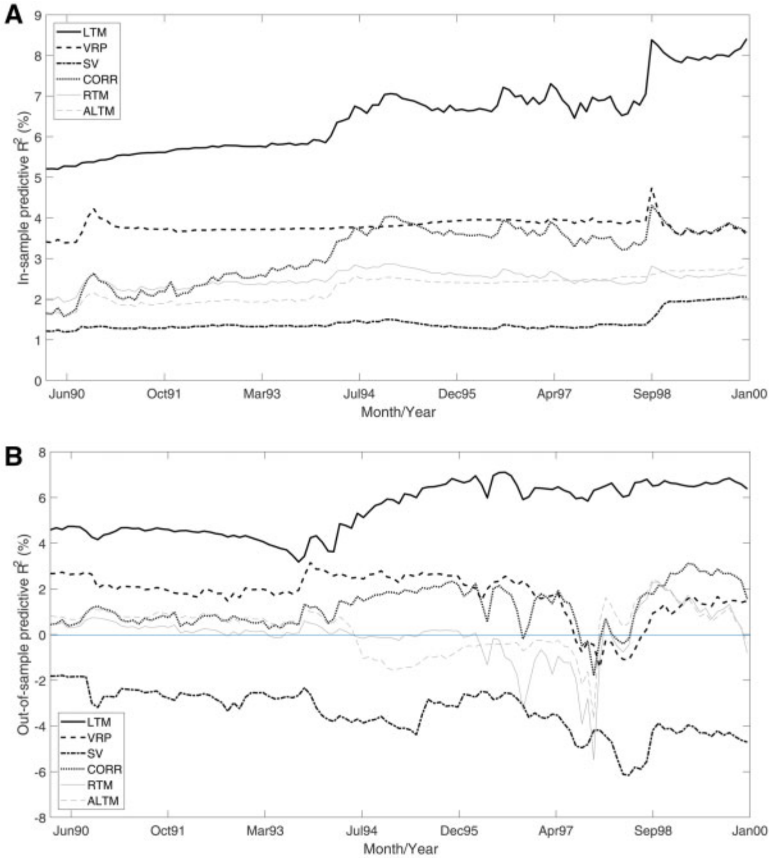

Figure 5
Time-varying predictability
This figure presents the *R*-squared of in- and out-of-sample time-varying 1-month-ahead *ERP* predictive regressions for a single variable. Table 2 explains in detail the computation of the *R*-squared values. Each point represents the value of the *R*-squared for a time-varying window with the same final point (December 2020) and an initial point corresponding to that month. Panels A and B present the in-sample and the out-of-sample results, respectively. We present only the best six unconditional IS predictors of the 1-month-ahead *ERP*.

rate news (DR), and cash flow news (CF)—on short interest index and show that short interest index is relevant for future aggregated cash flows.

We follow this setting and use *LTM* and *VRP* instead of short interest index. Table 5 shows the estimated results for the slope of the three predictive regressions with the dependent variables given by ER, DR, and CF. The ability of *LTM* to anticipate cash flow news is clearly the most economically important source of *LTM*'s predictive power for stock returns. The estimate is 0.83 and highly significant with a *t*-statistic of 2.91. Expected return is also positive and significant, but the magnitude of the parameter is approximately

Table 5
Decomposition into cash flow and discount rate components

		ER	CF	DR
LTM	**Slope**	0.34*	0.83*	0.01
	***t*-statistic**	4.57	2.91	0.03
VRP	**Slope**	−0.11	−0.74*	0.11
	***t*-statistic**	−0.55	−1.99	0.55

This table reports the ordinary least squares estimate of the slope of a predictive regression of expected returns (ER), cash flow news (CF), and discount rate news (DR) on the lagged variable presented in the left column, *LTM* or *VRP*. Table 1 defines all variables. These three components (ER, CF, and DR) are estimated using the Campbell (1991) and Campbell and Ammer (1993) vector autoregression (VAR) framework comprising log returns, the log dividend price-ratio, and the first three principal components extracted from 14 popular predictors from Goyal and Welch (2008). Below each slope estimate, we report the heteroscedasticity- and autocorrelation-robust *t*-statistics. The sample period is from January 1990 to December 2020.
*denotes a significant predictor at a 5% significance level.

one-third that of cash flow. Discount rate news presents a negligible and clearly insignificant estimate. *VRP* also presents similar results for cash-flow news although with the opposite sign. In previous literature, most previous predictors anticipate discount rate news. We find that the differential information in *LTM* is relevant for future aggregated cash flows and aggregated expected returns but mainly the former.

4. Economic Added Value

The previous analysis shows the outstanding predictability of our proposed predictor. Is this able to translate to value? In that regard, we construct a trading strategy based on the traditional approach, such as the one in Ferreira and Santa-Clara (2011). We follow their methodology for the monthly frequency, and we use the OOS forecasts to determine the mean-variance weights for allocating wealth between the risky asset (the market) and riskless asset (the Treasury-bill rate). As an estimate of the expected returns, we use the estimate resulting from a predictive regression using either *LTM* or *VRP*. As a measure of the variance of stock market returns, we estimate the sample variance of the available returns using a rolling window of the previous 5 years at the time of the portfolio computation. Then, we compute the Sharpe ratio of the strategy and the certainty equivalent return that a risk-averse investor is willing to pay for switching from the benchmark historical mean to the proposed model. We set the risk aversion parameter equal to two as in Ferreira and Santa-Clara (2011). Table 6 reports the Sharpe ratio and certainty equivalent levels and gains (annualized and in percentage) from using our predictor relative to investing based on the historical mean. Panel A constrains the stock market weight between -100% and 250%, and panel B constrains this weight to be between 0% and 150%. Compared to using the historical estimate, the strategy based on a predictive regression using *LTM* gives an incremental annualized Sharpe ratio of about

Table 6
Trading strategy

A. Stock market weight between -100% and 250%

	SR	ΔSR (%)	CE (%)	ΔCE (pp)
Hist mean	0.52		6.29	
VRP	0.60	15.55	10.22	3.93
LTM	0.79	52.89	19.50	13.22

B. Stock market weight between 0% and 150%

	SR	ΔSR (%)	CE (%)	ΔCE (pp)
Hist mean	0.54		9.50	
VRP	0.62	14.73	12.39	2.89
LTM	0.73	34.96	16.38	6.89

This table reports the out-of-sample portfolio choice results for a monthly frequency. The numbers are the level and percentage change in the Sharpe ratio and the level and percentage point change in the certainty equivalent of a trading strategy timing the market with a return forecast from either *LTM* and *VRP*, and comparing with a return forecast from the historical average. The variance is estimated in all cases by a historical 5-year variance estimate. All numbers are annualized. The sample period is from January 1990 to December 2020. The forecasts begin 5 years after the sample start. We use a mean-variance utility setting for the optimization with a risk aversion parameter of two.

50% in panel A and 35% in panel B and the certainty equivalent increases by about 13 pp in panel A and 7 pp in panel B when the constraints are looser (tighter). In either case, this increase is twice and three times more than the one derived from a predictive regression based on *VRP*. To understand the magnitude of our results, the average annualized risk-free rate during the OOS period was 2.22%. The certainty equivalent from the strategy using *LTM* estimates is about sixfold this magnitude. We also run a strategy assuming a risk aversion coefficient of five. This strategy delivers even better performance measures: an incremental Sharpe ratio of 38% and an incremental certainty equivalent of 8.60 pp for the case when the stock market is constrained between 0% and 150%.

All in all, the mean-variance trading strategy based on *LTM* provides tangible economic gains to a risk-averse investor who implements the proposed model for OOS forecasting of the equity risk premium.

5. Source of the Predictability

In this section, we study the drivers of the *LTM* predictability. We disentangle the factors that characterize these tail variables and that are innovations of the traditional *ERP* predictors:

1. *The multivariate nature of the variables*: using an average of multivariate return components through the sectoral returns rather than an aggregate market variable.

2. *The inclusion of extreme events into the prediction*: while other predictors have a multivariate component, such as the Pearson correlation, coskewness, and cokurtosis, all of which are an "average" function of extreme returns; instead, LTM is sensitive to small movements in the tail.

5.1 Multivariate nature of LTM

The predictability of LTM can be explained by a growing literature on tail dependency (Longin and Solnik 2001; Ang and Chen 2002; Poon et al. 2004) considering that comovements in sectoral returns affect the ERP. Chabi-Yo et al. (2018) show that investors require a premium to hold portfolios with high left tail dependence as insurance against negative extreme events.

Guidolin and Timmermann (2008) develop a four-moment asset pricing model to quantify the premium of the tails: they approximate the distribution of the investor wealth by a Taylor expansion, from which they can incorporate the effects of higher-order moments of the investor's wealth distribution into the asset pricing equation. On the other end, Das and Uppal (2004) developed a reduced-form multivariate asset pricing model that incorporates multivariate jumps into asset returns, jumps that affect the distribution of the wealth and finally the utility function. Both asset pricing models (Guidolin and Timmermann 2008; Das and Uppal 2004) can be used to explain the first source of predictability of the LTM variable: disaggregated variables contribute with additional information to the predictability and this is the reason that the use of multivariate higher-order moments, such as coskewness and cokurtosis, provide value in the pricing of the assets. Positive coskewness will increase the ERP, while positive cokurtosis will decrease the ERP. In an aggregated univariate model, positive coskewness contributions to premiums are canceled with negative coskewness contributions, and similar effect might apply in the case of the cokurtosis contributions.

5.2 Sensitivity to extreme events

Asset pricing equations, such as the ones in Guidolin and Timmermann (2008), have their origin in the consumption-investment equilibrium of a rational investor (Sharpe 1964; Harvey and Siddique 2000). The ERP puzzle appears to be a consequence of the failure of traditional consumption models to explain the historical high difference between the equity and the government bond returns. In the classic formulation of the ERP puzzle, Mehra and Prescott (1985) recognize that for an Arrow-Debreu economy, the difference between equity and Treasury-bill returns was excessively large, implying that they could explain the large equity premium only when considering frictions in the economy. Nevertheless, recent evidence from rare disaster models, such as Barro (2006), proposes that the puzzle is solved in a frictionless economy when large consumption drawdowns are included in a model. Our empirical results strengthen the idea that equity premium is predictable, and this predictability is associated with the proximity of a consumption disaster.[23] The second predictability feature of the *LTM* is its incorporation of time-varying rare disasters events into a more sensitive and responsive approach than other variables, such as exceedance comoments.[24]

5.3 Statistical source of the predictability

In this section, we provide insights about the statistical nature from the *LTM* in- and out-of-sample *ERP* predictability.

Simulations: Testing predictability from a single path can be misleading. In a setting with a large number of simulations, nonpredictable processes might "randomly" generate a sample process that could deceive the in- and out-of-sample predictability tests in Section 3.1. Hence, we generate simulations from multivariate normal (MVN) and multivariate Student's *t* (MST) returns independent over time (nonpredictable process) calibrated from historical returns, and we test the statistical difference against the generated predictable sample simulations from four different processes: (1) dynamic conditional correlation (DCC) process calibrated with the data, (2) bootstrapping process generated with a random selection of the historical *ERP* values, (3) regime switching correlation process calibrated with empirical data, and (4) jump diffusion process calibrated as in Das and Uppal (2004). In the regime switching process and jump diffusion process group, we include additional simulations where the reversal effect is modeled (Cujean and Hasler 2017). We calculate the tail measures—*LTM* and *RTM*—and then we test the in- and out-of-sample *ERP* predictability. We consider a random walk will have a MVN distribution; for instance, we test the null hypothesis that the average of the predictability of the simulated process to be tested to be equal to the average of the predictability of the MVN distribution. If the null hypothesis of the MVN sample and the alternative process being equal is rejected, the average value of the variable predictor is considered significant. One thousand random simulation paths are generated for each case.

The first two columns of Table 7 displays the results of the percentage of samples producing significant in-sample predictability with positive *R*-squared values. On average, nonpredictable processes—MVN and MST—produce 14% of their samples as predictable by any of the tail predictors. Measurement of the predictability of the DCC and the Bootstrapped

[23] Hansen and Singleton (1983) study restrictions to the modeling of the joint distribution of consumption and asset returns. In their modeling, they find that when consumption is lognormally distributed by a random walk, the asset returns will be serially uncorrelated. However, asset returns will have predictable components when consumption growth has "nontrivial predictable" components.

[24] This statement is confirmed when measuring the predictability's power of exceedance correlation and exceedance covariance that can be provided by the authors on request.

Table 7
Predictability: Simulations

	IS LTM	RTM	OOS LTM	RTM
Processes with no predictable component				
MVN	13.60	13.10	1.40	1.20
MVT	14.50	12.60	2.00*	2.10*
Processes with some predictable component				
DCC	10.40	10.90	0.80*	1.40*
Bootstrapping	13.80	11.70	1.50*	1.30*
Regime switching correlation				
R-S positive return shock	21.90	21.70	3.00*	3.50*
R-S negative return shock	23.30	24.20	3.30	3.90
R-S bootstrapping 10S positive return	21.50	23.40	3.70*	3.20*
R-S bootstrapping 10S negative return	23.30	21.20	4.10	3.20*
Jump Diffussion				
Correlated	13.10	13.90	1.00	1.50
Systemic shock	14.40	15.00	2.00	1.70
Systemic big positive shock	18.00*	16.20*	8.30	8.30*
Systemic big negative shock	20.10*	19.70*	12.60*	12.00
Procyclical shock	22.20	19.80	2.80	1.80*
Procyclical big positive shock	46.30*	34.50*	12.20*	5.20
Procyclical big negative shock	29.40*	36.00*	5.40*	8.70*

This table reports the percentage of the samples with a positive R-squared of in- and out-of-sample predictive regressions of simulated 1-month-ahead *ERP* using dependence predictors, that have a t-statistic greater than two. Table 1 defines all variables. The time span is from January 1990 to December 2020.
*denotes that the predictor applied over a non-MVN distribution has a statistically significant different sample compared with the predictors applied to a MVN sample for a 5% significance level in the first row (Student's t two-tailed test of two samples).

processes using tail measures reports no increase in comparison with the MVN and MST processes (they are even smaller in proportion). Regime switching correlation processes double the number of samples with significant in-sample predictability to 20% for almost all predictors. Switching correlation from positive to negative returns (contagion) and from negative to positive returns (reversal) was tested. While studying the correlation of the sectors, we discovered a strong negative correlation the next month after a downturn of all 10 sectors. We simulate a process with these characteristics (regime-switching bootstrapping after a 10-sector downturn) from positive to negative returns (contagion) and from negative to positive returns (reversal). The last type of process considered was the jump diffusion. We tested a simple correlated jump diffusion (Merton 1976), a systemic shock jump diffusion (Das and Uppal 2004), and a procyclical systemic shock jump diffusion, where the shock is generated when the process enters the recession region. We additionally tested increases of two mean jump returns (big positive and big negative shock) during the systemic/procyclical shock. The results show that tail measures are ineffective for detecting predictability from the systemic shock alone. On the other hand, the tail measures did predict better ($LTM =$ 22% of the samples) when the shock is procyclical. In a procyclical jump diffusion process with a large and persistent positive reversal shock, the tail measures determined that 46% of the paths were predictable in sample.

The last two columns of Table 7 show the corresponding results for the out-of-sample case. Tail measures detect OOS predictability from highly predictable processes that have a positive reversal effect, strengthening our hypothesis and empirical results that predictability is the result of strong positive reversals after negative shocks, or a procyclical positive reversal shock.

6. Conclusion

This paper examines the predictability of the *ERP* by a sectors' multivariate tail dependency and shows that stock market predictability is not dead but rather it still has economic value for investors. We propose a new measure of country left tail dependence that is based on the cross-sectional left tail behavior of its pairs of sectors (*LTM*). This measure is derived by combining two types of information sets: the information in the tails and the intracountry sector relation. We provide evidence of the predictability of stock market premiums using this measure in exercises in sample and out of sample.

The results of *LTM* are the consequence of two joint characteristics: *LTM* is (1) disaggregated/multivariate and (2) a tail-dependent variable. Multivariate left tail dependence is a key factor in predictability. No other variable, except for variance risk premium, can predict the stock market risk premium significantly better than the historical average. We also show that this significance is not a result of just a data mining exercise. We show that the information at the industry level and their dependencies in the tails are crucial for this outcome. Moreover, the new tail measure *LTM*, which is constructed from more granular information about the dependency of sectors, is superior to aggregate univariate tail measures. In addition, we show that the economic source of *LTM*'s predictive power predominantly stems from a cash flow channel, and its predictability, although time varying, is resilient, which is not the case with all other common predictors.

In our setting, out-of-sample equity risk premium predictability by the *LTM* is the result of an optimal hedging strategy in which the investor is searching for the "timing" of rare consumption disasters, which substantially affect their equity assets. Investors first observe the aggregate variable; then, a market sectoral joint downturn movement is a strong signal that a systemic event is under way. *LTM* is a good descriptor of sectoral tail dependency, and we find that an increase in sectoral tail dependency precedes a disaster.

A full statistical assessment of the *LTM* predictability is provided here. This statistical analysis reveals that the source of the predictability takes place during "rare" disaster events, or events that might lead to a crisis. The predictability test regressors' coefficients are flat in normal times and become positive during a crisis. The puzzling question of the origin of predictability by trailing historical variables is answered by looking into the timing of the predictors: the stability of the business cycle is strongly associated with predictability; then prewar periods and periods with a highly unstable business cycle might lead to a reduced ability in the predictive power of the tail dependence measures.

All in all, joint left tail sector relations play an important role in stock market predictability.

References

Ang, A., and G. Bekaert. 2002. International asset allocation with regime shifts. *Review of Financial Studies* 15:1137–87.

———. 2006. Downside risk. *Review of Financial Studies* 19:1191–239.

———. 2007. Stock return predictability: Is it there? *Review of Financial Studies* 20:651–707.

Ang, A., and J. Chen. 2002. Asymmetric correlations of equity portfolios. *Journal of Financial Economics* 63:443–94.

Bae, K.-H., G. A. Karolyi, and R. M. Stulz. 2003. A new approach to measuring financial contagion. *Review of Financial Studies* 16:717–63.

Bali, T. G., N. Cakici, and R. F. Whitelaw. 2014. Hybrid tail risk and expected stock returns: When does the tail wag the dog? *Review of Asset Pricing Studies* 4:206–46.

Barro, R. 2006. Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics* 121:823–66.

Bawa, V., and E. Lindenberg. 1977. Capital market equilibrium in a mean-lower partial moment framework. *Quarterly Journal of Economics* 5:189–200.

Bollerslev, T., J. Marrone, L. Xu, and H. Zhou. 2014. Stock return predictability and variance risk premia: Statistical inference and international evidence. *Journal of Financial and Quantitative Analysis* 49:633–61.

Bollerslev, T., G. Tauchen, and H. Zhou. 2009. Expected stock returns and variance risk premia. *Review of Financial Studies* 22:4463–92.

Bollerslev, T., V. Todorov, and L. Xu. 2015. Tail risk premia and return predictability. *Journal of Financial Economics* 118:113–34.

Buss, A., and G. Vilkov. 2012. Measuring equity risk with option-implied correlations. *Review of Financial Studies* 25:3113–40.

Campbell, J. 1991. A variance decomposition for stock returns. *Economic Journal* 101:491–527.

Campbell, J., and J. Ammer. 1993. What moves the stock and bond markets? A variance decomposition for long-term asset returns. *Journal of Finance* 48:3–37.

Campbell, J., and S. Thompson. 2008. Predicting the equity premium out of sample: Can anything beat the historical average? *Review of Financial Studies* 21:1456–508.

Chabakauri, G. 2013. Dynamic equilibrium with two stocks, heterogeneous investors, and portfolio constraints. *Review of Financial Studies* 26:3104–41.

Chabi-Yo, F., S. Ruenzi, and F. Weigert. 2018. Crash sensitivity and the cross section of expected stock returns. *Journal of Financial and Quantitative Analysis* 53:1059–100.

Clark, T., and K. West. 2007. Approximately normal tests for equal predictive accuracy in nested models. *Journal of Econometrics* 138:291–311.

Cohen, L., and A. Frazzini. 2006. Economic links and predictable returns. *Journal of Finance* 63:1977–2011.

Comin, D., and S. Mulani. 2009. A theory of growth and volatility at the aggregate and firm level. *Journal of Monetary Economics* 56:1023–42.

Cujean, J., and M. Hasler. 2017. Why does return predictability concentrate in bad times? *Journal of Finance* 72:2717–58.

Dangl, T., and M. Halling. 2012. Predictive regressions with time-varying coefficients. *Journal of Financial Economics* 106:157–81.

Das, S., and R. Uppal. 2004. Systemic risk and international portfolio choice. *Journal of Finance* 59:2809–34.

Driessen, J., P. J. Maenhout, and G. Vilkov. 2009. The price of correlation risk: Evidence from equity options. *Journal of Finance* 64:1377–406.

———. 2012. Option-implied correlations and the price of correlation risk. Working Paper, Tilburg University School of Economics and Management.

Faias, J. 2021. Predicting equity risk premium using the smooth cross-sectional tail risk. Working Paper, Universidade Católica Portuguesa. Católica Lisbon School of Business and Economics.

Ferreira, M., and P. Santa-Clara. 2011. Forecasting stock market returns: The sum of the parts is more than the whole. *Journal of Financial Economics* 100:514–37.

Goyal, A., and I. Welch. 2008. A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies* 21:1455–508.

Guidolin, M., and A. Timmermann. 2008. International asset allocation under regime switching, skew, and kurtosis preferences. *Review of Financial Studies* 21:889–935.

Hansen, L., and K. Singleton. 1983. Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Journal of Political Economy* 91:249–65.

Hartmann, P., S. Straetmans, and C. G. de Vries. 2004. Asset market linkages in crisis periods. *Review of Economics and Statistics* 86:313–26.

Harvey, C., Y. Liu, and H. Zhu. 2016. …and the cross-section of expected returns. *Review of Financial Studies* 29:5–68.

Harvey, C., and A. Siddique. 2000. Conditional skewness in asset pricing tests. *Journal of Finance* 55:1263–95.

Henkel, S., J. Martin, and F. Nardari. 2011. Time-varying short-horizon predictability. *Journal of Financial Economics* 99:560–80.

Hilal, S., S.-H. Poon, and J. Tawn. 2011. Hedging the black swan: Conditional heteroskedasticity and tail dependence in S&P500 and VIX. *Journal of Banking and Finance* 35:2374–87.

Hodrick, R. 1992. Dividend yields and expected stock returns: Alternative procedures for inference and measurement. *Review of Financial Studies* 5:357–86.

Holly, S., and I. Petrella. 2012. Factor demand linkages, technology shocks, and the business cycle. *Review of Economics and Statistics* 94:948–63.

Hong, H., W. Torous, and R. Valkanov. 2007. Do industries lead stock markets? *Journal of Financial Economics* 83:367–96.

Horvath, M. 2000. Sectoral shocks and aggregate fluctuations. *Journal of Monetary Economics* 45:69–106.

Johnson, T. L. 2018. A fresh look at return predictability using a more efficient estimator. *Review of Asset Pricing Studies* 9:1–46.

Kelly, B., and H. Jiang. 2014. Tail risk and asset prices. *Review of Financial Studies* 27:2841–71.

Lan, C. 2020. Stock price movements: Business-cycle and low-frequency perspectives. *Review of Asset Pricing Studies* 10:335–95.

Lettau, M., and M. Pelger. 2020. Factors that fit the time series and cross-section of stock returns. *Review of Financial Studies* 33:2274–325.

Li, J., I. Tsiakas, and W. Want. 2015. Predicting exchange rates out of sample: Can economic fundamentals beat the random walk? *Journal of Financial Econometrics* 13:293–341.

Longin, F., and B. Solnik. 2001. Extreme correlation in international equity markets. *Journal of Finance* 56:649–76.

Martin, I. 2017. What is the expected return on the market? *Quarterly Journal of Economics* 132:367–433.

Mehra, R., and E. Prescott. 1985. The equity premium: A puzzle. *Journal of Monetary Economics* 15:145–61.

Menzly, L., and O. Ozbas. 2010. Market segmentation and the cross-predictability of returns. *Journal of Finance* 65:1555–80.

Merton, R. C. 1976. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* 3:125–44.

Neely, C. J., D. E. Rapach, J. Tu, and G. Zhou. 2014. Forecasting the equity risk premium: The role of technical indicators. *Management Science* 60:1772–91.

Newey, W., and K. West. 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55:703–708.

Patton, A. 2009. Copula-based models for financial time series. *Handbook of Financial Time Series* 29:767–85.

Poon, S.-H., M. Rockinger, and J. Tawn. 2004. Extreme value dependence in financial markets: Diagnostics, models, and financial implications. *Review of Financial Studies* 17:581–610.

Rapach, D. E., M. C. Ringgenberg, and G. Zhou. 2016. Short interest and aggregate stock returns. *Journal of Financial Economics* 121:46–65.

Rapach, D. E., J. K. Strauss, J. Tu, and G. Zhou. 2015. Industry interdependencies and cross-industry return predictability. Working Paper, Lee Kong Chian School Of Business.

Rapach, D. E., J. K. Strauss, and G. Zhou. 2010. Out-of-sample equity premium prediction: Combination forecasts and links to the real economy. *Review of Financial Studies* 23:821–62.

Richardson, M., and T. Smith. 1991. Tests of financial models in the presence of overlapping observations. *Review of Financial Studies* 4:227–54.

Rietz, T. 1988. The equity risk premium: A solution. *Journal of Monetary Economics* 22:117–31.

Sharpe, W. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* 19:425–42.

Veldkamp, L., and J. Wolfers. 2007. Aggregate shocks or aggregate information? Costly information and business cycle comovement. *Journal of Monetary Economics* 54:37–55.

Wachter, J. 2013. Can time-varying risk of rare disasters explain aggregate stock market volatility? *Journal of Finance* 68:987–1035.

Zhou, H. 2018. Variance risk premia, asset predictability puzzles, and macroeconomic uncertainty. *Annual Review of Financial Economics* 10:481–97.