"Testing for Linear and Nonlinear Granger Causality in 0DTE Options Volume and the VIX Index"

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Dissertation written under the supervision of Eva Schliephake

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Abstract

In this thesis I investigate the dynamic temporal relationship between trading volume of 0DTE SPX options and the VIX Index. The analysis is conducted with the traditional linear Granger causality test, and by applying the modified Baek & Brock test proposed by Hiemstra & Jones (1994) to investigate nonlinear Granger causality. Within the context of linear causality, I find strong evidence of unidirectional causality from the VIX to 0DTE volume in all models, apart from the subsample from July 2021 to May 2023. Additionally, there is evidence of bidirectional causality in the linear tests depending on the sample period and model selection. In contrast I find strong evidence of a nonlinear causal effect from 0DTE volume to the VIX in all models, with the full sample and in the period July 2021 to May 2023, with weak evidence for bidirectional causality. This contradicts the findings from linear causality testing. Further, the findings suggest that the significance of the nonlinear relationship parallels with the rise in popularity of 0DTE options. This thesis demonstrates the importance of testing for nonlinear causal relationships in economic and financial data, when investigating their drivers and relationships.

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Resumo

Nesta tese, eu investigo a relação temporal dinâmica entre o volume de negociação 0DTE do SPX e o índice VIX. A análise é efectuada com o tradicional teste de causalidade de Granger linear e aplicando o teste Baek & Brock modificado proposto por Hiemstra & Jones (1994) para investigar a causalidade de Granger não linear. No contexto da causalidade linear, encontro uma forte causalidade unidireccional do VIX para o volume 0DTE em todos os modelos, com excepção da subamostra de Julho de 2021 a Maio de 2023. Além disso, há evidências de causalidade bidireccional nos testes lineares, dependendo do período da amostra e da selecção do modelo. Em contrapartida, encontro fortes indícios de um efeito causal não linear do volume 0DTE para o VIX em todos os modelos, com a amostra completa e no período de Julho de 2021 a Maio de 2023, com indícios fracos de causalidade bidireccional. Este facto contradiz as conclusões dos testes de causalidade linear. Além disso, os resultados sugerem que a importância da relação não linear é paralela ao aumento da popularidade das opções 0DTE. Esta tese demonstra a importância de testar relações causais não lineares em dados económicos e financeiros, investigando os seus factores e relações.

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Introduction

In this thesis, I will examine the temporal relationship between the daily 0DTE trading volume on SPX options and the daily spot VIX index. This relationship is investigated using linear and nonlinear Granger causality tests. Traditional Granger causality tests are widely used and implemented in economics and finance. However, they are not well suited for uncovering nonlinear relations. Baek & Brock (1992) introduced a nonparametric test in order to expand on the traditional Granger causality test by allowing for testing nonlinear causality in a bivariate Vector Autoregressive system.

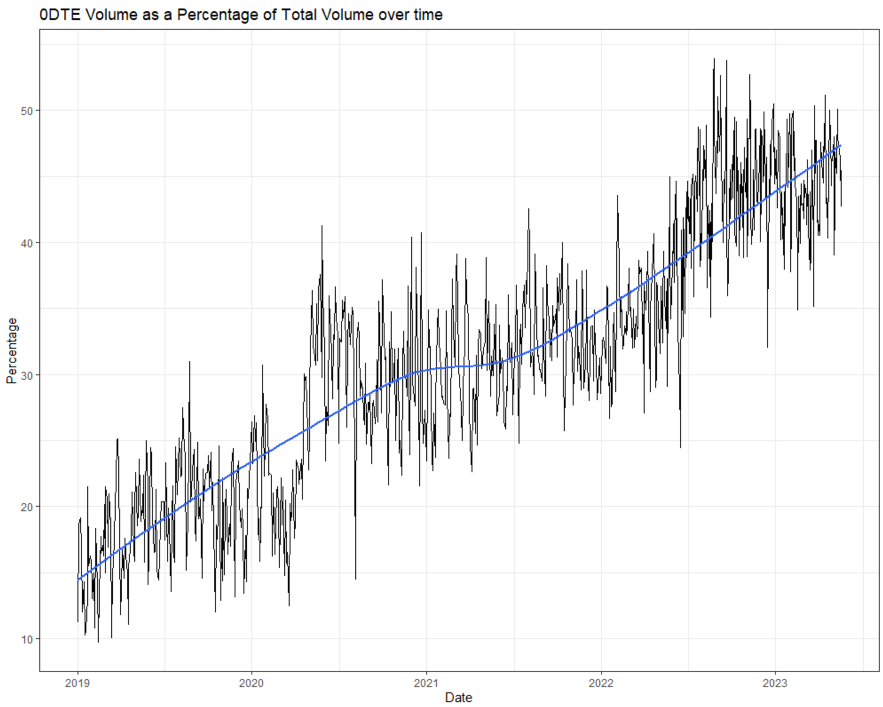
The thesis is structured as follows: Chapter 1 contains a discussion of why the relationship between 0DTE volume and the VIX is important, and some background information. In Chapter 2 I review the research conducted on the topic of linear and nonlinear causality and their findings. In Chapter 3, I review, and present models proposed by Baek & Brock (1992) and Hiemstra & Jones (1994). In Chapter 4, I apply the modified Baek & Brock test proposed by Hiemstra & Jones (1994) on the daily 0DTE trading volume of SPX options and the daily spot VIX Index and present the findings and results from the tests. Finally, chapter 5 summarizes, concludes, and discusses the limitations.

1. Motivation

Zero Days to Expiry (0DTE) options are contracts with less than a day to maturity. These contracts are not necessarily issued the same day, and could have been issued weeks or months before, but are classified as 0DTE on the day they expire. Traditionally, these types of option contracts are used to hedge event risks such as FED rate decisions, inflation numbers, and other economic data releases, by institutional investors (McGeever, 2023). In recent years with the rise of “Wall Street Bets” on Reddit and other such communities, more and more retail investors are exposed to options trading, and 0DTE contracts (Bhansali, n.d.). Studies on the rise of popularity for retail investors have been done by Beckmeyer, Branger & Gayda (2023) and Silva, Smith & So (2022). The former found that more than 75 percent of options trading done by retail investors is in 0DTE contracts. It is widely recognized that 0DTE contracts’ popularity increased significantly during the “meme stock mania” in the second year of COVID as these contracts served as a low-cost vehicle for day trading (*What Are Zero Day to Expiry (0DTE) Options?*, n.d.). After the “meme stock mania” of GameStop and

Tesla, some retail investors switched their day trading activity to indices such as the S&P 500. The CBOE did not issue SPX options expiring Tuesdays and Thursdays before 2022, on April 18, Tuesday-expiring SPX options started trading, and Thursday-expiring options on May 11 (*Cboe to Add Tuesday and Thursday Expirations for SPX Weeklys Options*, n.d.). The introduction of options with expirations every day of the week has led to SPX 0DTE contracts to become the most popular index for 0DTE trading, due to its liquidity and daily expirations (*0DTE Options*, n.d.). As we can see from Figure 1, the volume of 0DTE SPX options as a percentage of the total volume of SPX options increased from approximately 15% to about 46%, in the period January 2019 to May 2023.

Figure 1: 0DTE Volume as a Percentage of Total Volume over Time, Source: Bloomberg



Options with a strike price close to the current market price of the underlying and with short maturity are sensitive to small price changes in the underlying. This makes 0DTE options attractive to traders who want to profit from intraday movements. The time decay of options is measured by Θ (theta). This is known as the rate of decline in value of an option due to time passage. Due to their limited maturity, 0DTE contracts tend to lose their value rapidly. This makes them an attractive option for sellers who seek to capitalize on this decay by selling the contracts and quickly exiting the trade, buying them back at a lower price.

Often 0DTE option contracts are struck far from the OTM strike price, therefore leading to a low likelihood of appreciating as they approach their maturity. Nonetheless, significant intraday market movements can lead to sharp increases in their value resulting in a great risk of substantial losses for the option writers. If many of these writers' rush to buy back these contracts simultaneously, it can cause the price to appreciate even further. In extension to offset their risk, market makers rush to take positions in the underlying security, further increasing the appreciation. If such a unidirectional event happens during times of lower liquidity, analysts fear that market movements could be severe (Ahmed, 2023). Furthermore, analysts from JPMorgan estimate that a significant market move could trigger buying or selling of these positions worth around $30 billion (Reuters, 2023). This could potentially lead to an event similar to the 2018 "Volmageddon" crash where investors lost billions of dollars, due to a substantial volatility shock from a sudden market sell-off (Ahmed, 2023).

The necessity to understand market volatility leads us to the VIX Index which was introduced in 1993. The Cboe Volatility Index (VIX) is considered the world's premier barometer of equity market volatility. It is based on real-time prices of options on the S&P 500 Index (SPX) with maturities between 23 and 37 days and is designed to reflect investors' consensus view of future (30-day) expected stock market volatility. The VIX Index is often referred to as the market's "fear gauge" (*Cboe Global Indices: VIX Index Dashboard*, n.d.). More precisely, the VIX index is computed as the square root of the risk-neutral projection of the accumulated variance of the S&P 500 index (SPX) over the next 30 calendar days (see Appendix 2.). The hypothesis in extension will be that the trading activity in the 0DTE options will affect the price structure of the SPX options with maturities between 23 & 37 days, thus increasing the risk-neutral projection of the variance of SPX over the next 30 days. Conversely, that increased volatility, measured by the VIX, affects the trading volume of 0DTE SPX options.

The goal of this thesis is to analyze the dynamic temporal relationship of 0DTE SPX trading volume and the VIX Index. I thereby aim to identify if there exist causal effects, either linear or nonlinear, between them and if this relationship has been affected by the rise of popularity in day trading 0DTE options.

To do this, I conduct multiple analyses. Primarily, I will perform linear and nonlinear Granger Causality tests, based on the methodology proposed by Hiemstra & Jones (1994), which is based on a nonparametric causality test proposed by Baek & Brock (1992). Secondly, I support my findings with comprehensive analysis of the properties of the time series, graphical inference, and additional correlation analysis.

I find strong evidence of a linear causal effect from the VIX to 0DTE trading volume for all models and samples, with the exception of the SBIC selected VAR in the period 02.07.2021 to 22.05.2023. Furthermore, depending on the model selection I find evidence of a bidirectional causal relationship within the full sample and the period 02.01.2019 to 02.07.2021. Furthermore, when analyzing the impulse response function, we see that a shock in one of the variables leads to a positive shock in the other, thereafter the shock oscillates around zero, with negative and positive signs. When testing for nonlinear effects I find little to no causal effects in the period 02.01.2019 with the exception for 0DTE volume to the VIX with the lag length of 1. However, in the full sample and in the period 02.07.2021 to 22.05.2023 I find very strong evidence of 0DTE Granger cause VIX, with weak evidence of bidirectional causality. This strongly contradicts the results from the linear testing framework. The significance of the causal effect from 0DTE volume to the VIX is strongest in the subsample where the popularity of 0DTE day trading has manifested.

2. Literature Review

To the best of my knowledge, there is little to no research on the relationship between the volume of specific SPX option contracts and the VIX Index. However, there is research done on the causality between the VIX Index, and its derivatives in the options and futures markets by Posselt (2022) and Shu & Zhang (2012). Further, there is well-established research on the causality between stock returns and trading volume changes (see Hiemstra & Jones (1994)). Granger causality tests introduced by Granger (1969), are widely known and used in economics and finance (as well as sciences outside of economics); however, more recent research finds evidence of nonlinear relationships between economic variables, and therefore the importance of testing for nonlinear causality when investigating financial data.

In the article of Hiemstra & Jones (1994), they investigate the relationship between daily aggregate stock prices and trading volume. They find unidirectional causality from stock returns from Dow Jones to trading volumes on NYSE. However, when implementing the modified Baek & Brock test they find evidence of bidirectional causality between the two variables. Their methodology has been adopted by researchers and nonlinear causality has been proved between many different economic variables.

Research by Abhyankar (1998), Yang & Shao, 2020, Shu & Zhang (2012), and Silvapulle & Moosa (1999) find evidence of nonlinear Granger causality in futures markets. Shu & Zhang (2012) study the temporal relationship between the VIX and its futures derivatives. They show that VIX futures lead the VIX Index, using linear Granger causality tests. However, in their research when testing for nonlinear Granger causality they provide further evidence of information efficiency between VIX futures and the VIX. Evidence of nonlinear Granger causality has also been found in exchange rates (Ma & Kanas, 2000) and (Rahimi et al., 2017), stock returns and macroeconomic factors (Hiemstra & Kramer, 1997), money and income (Baek & Brock, 1992), aggregate trading volume (Hiemstra & Jones, 1992). There has also been found evidence of nonlinear dependence in stock returns by (Brock et al., 1991) and (Hsieh, 1991). Hiemstra & Jones (1994), Silvapulle & Choi (1999), LeBaron (1992), and Duffee (1992) also finds evidence of nonlinear Granger causality between trading volume and price returns. These findings stress the importance of testing for nonlinear temporal relations when investigating trading volume and other financial data.

The test first proposed by Baek & Brock (1992) is a general test for nonlinear Granger causality in a bivariate context. Their test was swiftly adopted by fellow researchers Hiemstra & Jones (1994), who proposed a modified Baek & Brock test (hereafter HJ-test). This is the most common methodology to test for nonlinear Granger causality. Alternative tests exist such as Su & White (2008), Bell, Kay & Malley (1996) and the improved HJ-test proposed by Diks & Panchenko (2006) which is further elaborated by Diks & Wolski (2016). Diks & Panchenko (2006) propose a new test statistic which accounts for the over-rejection problem of the HJ-test. However, in this thesis I will focus on the well-established HJ-test.

3. Methodology

3.1 Linear Granger Causality

A Vector Autoregressive model (VAR) is a systems regression model, in other words there is more than one dependent variable, where the regression is on lags of values in the same time series as well as lags of another time series. To test for Granger causality, one must estimate a linear reduced-form VAR:

Equation (1)

$$X_t = A(L)X_t + B(L)Y_t + U_{X,t}$$
$$Y_t = C(L)X_t + D(L)Y_t + U_{X,t}$$

Where *A(L), B(L), C(L)* and *D(L)* are one-sided lag polynomials of orders *a, b, c,* and *d*, in the lag operator *L* with roots outside the unit circle and no roots in common. The regression errors, $\{U_{X,t}\}$ and $\{U_{Y,t}\}$, are assumed to be mutually independent and individually i.i.d. with zero mean and constant variance. The time series $\{X_t\}$ and $\{Y_t\}$ must be strictly stationary which I will come back to in section 3.5. To test for Granger causality from *Y* to *X*, a standard joint test (F- or χ^2-test) of exclusion restrictions is used to test whether lags of *Y* have significant linear predictive power for current value of *X*. The null hypothesis that *Y* does not strictly Granger cause *X* is rejected if the coefficients of the elements in *B(L)* are jointly significantly different from zero. If Granger causality runs in both directions, then bidirectional causality (or feedback) exists and the coefficients on the elements in both *B(L)* and *C(L)* are jointly different from zero. When performing VAR analysis, it is important to choose the lag length of the variables with care to obtain a robust model. There are many approaches to lag selection, one way is to match the frequency of the data (i.e., daily data; 5 lags, monthly data; 12 lags, etc.), more statistical methods include the Akaike (AIC) and Schwarz Bayesian (SBIC) information criterion. Information criteria consist of two components: a term that depends on the residual sum of squares (RSS) and a penalty for the reduction in degrees of freedom resulting from the inclusion of additional parameters, where the object is to choose the number of parameters which minimizes the value of the information criteria (Brooks, 2019, pp. 167, 272, 317). The main difference between AIC and SBIC is how harsh their penalty term is. SBIC is inefficient, but strongly consistent, and AIC is more efficient, but not consistent. AIC will on average select lag lengths resulting in a too large model, while SBIC will asymptotically deliver the correct model, with an infinite amount of data. Conversely, SBIC will deliver greater average variation in the selected model orders from different samples within a given population than with AIC. In conclusion, none of the alternatives are superior to the other. However, AIC might be more appropriate in circumstances where the existence of a definitive model is uncertain, and instead, there are numerous effects of varying magnitudes. In such situations, the primary objective is to achieve an accurate prediction error. As the sample size increases, minor effects become more relevant, and in tune AIC lag selection increases as well. Contrary with previous research (see (Li et al., 2019), (Hiemstra & Jones, 1994), (Scheinkman & LeBaron, 1989) and more), I have chosen to use both the AIC and SBIC for my lag selection, the equation for AIC and SBIC is found in Appendix 3.

3.2 Nonlinear Granger Causality

Baek and Brock (1992) introduced a nonparametric statistical method to detect nonlinear causal relations that traditional linear causality tests cannot. It has been argued by Granger (2014, p. 215) that the real world is almost certainly nonlinear, therefore in order to properly model it, nonlinear models would be necessary. A nonlinear relationship is one in which the change in one variable is not proportionate to the change in the other variable. Traditional Granger causality tests are effective in detecting linear relationships between time series. However, their ability to uncover nonlinear relationships may be limited due to their lower power in detecting such relationships (see Baek & Brock (1992) and Hiemstra & Jones (1993)). For this reason, it is important to apply additional tests when investigating the temporal relationship between 0DTE Volume and Spot VIX. The HJ-test in short, removes the linear predictive power by using the residuals of the estimated VAR in Equation (1). The method is then simply to test if adding lags of the Y series improves the prediction of X. If adding lags of Y improves the predictive power then Equation (3) will not hold, and since the linear predictive power is removed, this newfound predictive power is considered nonlinear. To test this the HJ-test uses the correlation integral to estimate spatial local correlations of temporal time series which are "embedded" in a high-dimensional space (see (Baek & Brock (1992)). A correlation integral is a measure used to analyze how points in a complex system are related to one another, and stems from the chaos theory branch of mathematics (see (Grassberger & Procaccia, 1983)). Assume you have a set of points that indicate the state of a system at various points in time. The correlation integral estimates the likelihood that two arbitrarily chosen points in this set are close to each other. The correlation integral can help to comprehend the system's structure and how it changes over time. For example, if the likelihood of two points being close to each other is high, it indicates that the system is less chaotic and more predictable. The use of the correlation integral in time series is a normalized count of the sum of pairs in the time series which are within phase space (see (Wolff, 1990)).

The HJ-test as proposed by Hiemstra & Jones (1994) will be derived in this section drawing heavily on the article by Hiemstra & Jones[1]. Consider two strictly stationary and weakly dependent time series $\{X_t\}$ and $\{Y_t\}$, and let X_t^m be the *m*-length lead vector of X_t, and X_{t-Lx}^{Lx} be the *Lx*-length lag vector of X_t. Similarly let Y_{t-Ly}^{Ly} be the *Ly*-length lag vector of Y_t.

Equation (2)

$$\begin{aligned} X_t^m &\equiv (X_t, X_{t+1}, \ldots, X_{t+m-1}), \ m = 1, 2, \ldots, t = 1, 2, \ldots, \\ X_{t-Lx}^{Lx} &\equiv (X_{t-Lx}, X_{t-Lx+1}, \ldots, X_{t-1}), \ Lx = 1, 2, \ldots, t = Lx+1, Lx+2, \ldots, \\ Y_{t-Ly}^{Ly} &\equiv \left(Y_{t-Ly}, Y_{t-Ly+1}, \ldots, Y_{t-1}\right), \ Ly = 1, 2, \ldots, t = Ly+1, Ly+2, \ldots, \end{aligned}$$

For given values of *m*, *Lx*, and *Ly* ≥ 1 and *e* > 0, *Y* does not strictly Granger cause *X* if:

Equation (3)

$$\begin{aligned} &Pr\left(\|X_t^m - X_s^m\| < e \middle| \|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e, \left\|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\right\| < e\right) \\ &= Pr(\|X_t^m - X_s^m\| < e | \|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e) \end{aligned}$$

Where the LHS and RHS of Equation (3)[2] express the joint probability that two arbitrary lead vectors of $\{X_t\}$ are within distance *e* given that either both corresponding lag vectors of $\{X_t\}$ and $\{Y_t\}$ are within distance *e*, or only the lag vectors of $\{X_t\}$ are within distance *e*. A test based on Equation (3) can be conducted by expressing it in terms of the relevant ratios of joint probabilities. These ratios can be defined as,

[1] I will follow the standard convention of denoting random variables in the upper case and their realizations in the lower case.
[2] Where $\Pr(\cdot)$ denotes the probability and $\|\cdot\|$ denotes the maximum norm.

Equation (4)

$$C1(m+Lx,Ly,e) \equiv Pr\left(\|X_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\| < e, \left\|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\right\| < e\right),$$
$$C2(Lx,Ly,e) \equiv Pr\left(\|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e, \left\|Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\right\| < e\right),$$
$$C3(m+Lx,Ly,e) \equiv Pr(\|X_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\| < e),$$
$$C4(Lx,e) \equiv Pr(\|X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\| < e)$$

The condition for strict Granger non-causality in Equation (3) for given values of m, Lx, and $Ly \geq 1$ and $e > 0$, can now be rephrased as follows:

Equation (5)

$$\frac{C1(m+Lx,\ Ly,\ e)}{C2(Lx,\ Ly,\ e)} = \frac{C3(m+Lx,\ e)}{C4(Lx,\ e)}$$

To test the condition in Equation (5), we use correlation-integral estimators of the joint probabilities in Equation (4). Define a kernel, $I(Z_1,\ Z_2,\ e)$, that is equal to 1, when two conformable vectors $Z_1$and Z_2 are within a maximum-norm distance of e from each other, and 0 otherwise. The correlation-integral estimators of the joint probabilities in Equation (4) can then be expressed as follows:

Equation (6)

$$C1(m+Lx,\ Ly,\ e,\ n) \equiv \frac{2}{n(n-1)}\sum_{t<s}\sum I(x_{t-Lx}^{m+Lx} - x_{s-Lx}^{m+Lx},\ e) * I\left(y_{t-Ly}^{Ly} - y_{s-Ly}^{Ly},\ e\right)$$
$$C2(Lx,\ Ly,\ e,\ n) \equiv \frac{2}{n(n-1)}\sum_{t<s}\sum I(x_{t-Lx}^{Lx} - x_{s-Lx}^{Lx},\ e) * I\left(y_{t-Ly}^{Ly} - y_{s-Ly}^{Ly},\ e\right)$$
$$C3(m+Lx,\ Ly,\ e,\ n) \equiv \frac{2}{n(n-1)}\sum_{t<s}\sum I(x_{t-Lx}^{m+Lx} - x_{s-Lx}^{m+Lx},\ e)$$
$$C4(Lx,\ e,\ n) \equiv \frac{2}{n(n-1)}\sum_{t<s}\sum I(x_{t-Lx}^{Lx} - x_{s-Lx}^{Lx},\ e)$$

Where we take the sum of each *t* and *s* where *t* is less than *s,* with the time series of realizations $\{x_t\}$ and y_t, where the defined vectors are within maximum norm distance *e* of each other. The strict Granger non-causality condition in Equation (3) can be tested using the joint probability estimators in Equation (6)[3]. If $\{Y_t\}$ does not strictly Granger cause $\{X_t\}$, then

[3] Where; *t, s = max(Lx, Ly) + 1 to T – m + 1, and n = T + 1 – m – max(Lx, Ly)*

under the assumptions that $\{X_t\}$ and $\{Y_t\}$ are strictly stationary, weakly dependent, and satisfy the mixing conditions of Denker and Keller (1983), for given values of m, Lx, and $Ly \geq 1$ and $e > 0$:

Equation (7)

$$\sqrt{n}\left(\frac{\mathrm{C1}(m+Lx,\ Ly,\ e,\ n)}{C2(Lx,\ Ly,\ e,\ n)} - \frac{\mathrm{C3}(m+Lx,\ e,\ n)}{C4(Lx,\ e,\ n)}\right) \sim^{a} \mathrm{N}\big(0,\ \sigma^2(m,\ Lx,\ Ly,\ e)\big)$$

Where $\sigma^2(m,\ Lx,\ Ly,\ e,\ n)$ and an estimator for it is given in Appendix 1. Equation (7) is the test statistic used in the method, with the null hypothesis being $\{Y_t\}$ does not strictly nonlinearly Granger cause $\{X_t\}$. The difference between HJ-test and Baek & Bock test arise in the calculation of the test-statistic, more specifically the $\sigma^2(m,\ Lx,\ Ly,\ e,\ n)$ element of the test statistic. The HJ-test differs in that it does not assume that the time series being tested are mutually independent and individually independent and identically distributed. Instead, it allows for weak (or short-term) temporal dependence in each series. Any remaining predictive power of the series of residuals from the linear VAR model, will be attributed to the presence of nonlinear relations, as the linear predictive power has been removed. Therefore, the test in Equation (7) is applied to the estimated residual series from the VAR model in Equation (1), namely $\{U_{X,t}\}$ and $\{U_{Y,t}\}$. Under the null hypothesis the test statistic is asymptotically distributed $\mathrm{N}(0,\ 1)$.

3.3 Graphical Analysis

Impulse Response Function and Variance Decomposition Plots

Conducting block F-tests and analyzing causality within a VAR model can provide insight into which variables have statistically significant impacts on the future values of other variables within the system. However, it is important to note that F-test results are limited in their ability to reveal the direction or duration of these relationships. Specifically, F-test results cannot determine whether changes of a given variable have a positive or negative effect on other variables within the system, nor can they indicate the time required for these effects to manifest. To obtain this information, an examination of the VAR’s impulse responses and variance decompositions is necessary (Brooks, 2019, pp. 443–425). *Impulse response analysis* allows for the examination of the responsiveness of dependent variables within a Vector Autoregression (VAR) model to shocks applied to each variable. This is achieved by introducing a unit shock to the error term of each variable in each equation separately and observing the resulting effects on the VAR system over time (Brooks, 2019, pp. 443–425). *Variance decomposition* provides an alternative approach to analyzing the dynamics of a VAR system. This method quantifies the proportion of movements in dependent variables that can be attributed to their own shocks, as opposed to shocks to other variables within the system. A shock to the i^{th} variable will not only directly affect that variable but will also be transmitted to all other variables within the system through the VAR's dynamic structure (Brooks, 2019, pp. 443–425).

Autocorrelation, Partial Autocorrelation, Histogram and Density Plots

Autocorrelation is the correlation between two different values of the same time series, i.e., the time series correlation with itself. Let $\{X_t\}$ denote a time series, then we measure the autocorrelation of the series by: $Corr(X_t, X_{t-k})$, where k = 1, 2,… *The autocorrelation function (ACF)* represents these correlations graphically. This can be useful to analyze the time series, as the graph can help to visually analyze the existence of properties like white-noise process, seasonality, and trend. Note for a time series to be stationary there may not be a presence of a trend or seasonality as well as a constant variance and mean. If there is presence of a white noise process the ACF will display the lags within the significance bands. The presence of a trend would typically mean that the ACF displays large correlations which decay slowly. On the other hand, if there is presence of seasonality typically the ACF will display larger correlations at multiples of the frequency of seasonality. Note the ACF could also contain the presence of both a trend and seasonality (Brooks, 2019, pp. 352–358). *The partial autocorrelation function (PACF)* is used to measure the unique correlation between a given lag k and the current observation, when controlling for observations which are between t and k. In other words, PACF partials out the intervening correlations. Usually, PACF will have non-zero partial autocorrelation coefficients for lags up the to the order of the model, and zero coefficients after (Brooks, 2019, pp. 349–350, 352–358). *Histogram and density plot* could also be a useful tool for analyzing graphically the presence of stationarity. Histogram and density plots visualize the distribution of the time series, where changes in the shape of the distribution over time, may suggest non-stationarity.

3.4 Data Description

The data range for both time series is from 02.01.2019 to 22.05.2023. The data for the VIX Index is collected from the CBOE Market Database and is comprised of end-of-day closing prices. The time series for the daily spot VIX levels are expressed as percentage change, computed as the first difference of the natural logarithm multiplied by 100, as such: $100 * \ln\left(\frac{VIX_t}{\mathrm{VIX}_{t-1}}\right)$. The data for 0DTE SPX trading volume is collected from the Bloomberg Terminal Options Monitor function. The options trading volume dataset consists of the total daily volume and total volume of options with expiry the same day (0DTE), both are calculated at the end-of-day. Similarly, as the VIX, the 0DTE volume is expressed as percentage change, as such: $100 * ln\left(\frac{V_t^{0DTE}}{V_{t-1}^{0DTE}}\right)$. CBOE is the main source of data on the VIX, and Bloomberg is a trusted data provider. When combining the time series, the series for the 0DTE trading volume is dominant, i.e., the daily spot VIX is combined with the series where there are existing values of 0DTE trading volume. This omits some values for the VIX, due to the previously explained lack of "every-day-of-week" expiring SPX options. However, this implies only to the data-range before the Tuesday- and Thursday expiring contracts was introduced in May 2022. Finally, I will conduct tests on subsamples of the total data range in addition to the main data set, which is the full data range. The subsamples are from 02.01.2019 to 02.07.2021 (A), and from 02.07.2021 to 22.05.2023 (B).

3.5 Statistical Properties of Time Series – Stationarity, Cointegration & Homoskedasticity

As previously noted, the data must be strictly stationary to make accurate inferences and achieve robust results. In the fields of mathematics and statistics, stationarity refers to a property of stochastic processes in which the joint probability distribution remains unchanged when subjected to temporal shifts. To test the data for stationarity *Augmented Dickey-Fuller (ADF) tests and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests* are conducted. For robust conclusive result from the stationarity tests, we want the null hypothesis of the Augmented Dickey-Fuller test to be rejected and the null hypothesis of the Kwiatkowski-Phillips-Schmidt-Shin test to not be rejected. If the null hypotheses are rejected in both tests, or if none of the null hypotheses are rejected in the tests, we have conflicting results. The ADF and KPSS tests are shown mathematically in Appendix 4. I found that the daily 0DTE Volume data was non-stationary, with conflicting results on daily VIX level data. However, this is not persistent when expressing the time series in percentage changes as described, this will be discussed further in section Data Analysis.

Cointegration is the concept that two (or more) time series that are non-stationary (i.e., not a constant mean and variance over time), share a common trend. If there exists cointegration between two time series, even though their first differences are stationary, there exists a long-run equilibrium relationship between the two, and deviations in the short-run are considered temporary. Therefore, when conducting VAR analysis, it is important to consider cointegration as well as stationarity. *Johansen Cointegration Tests* (1991) are used to identify if there exists a cointegrating relationship between two time series. Johansen test allows for testing more than one cointegrating relationship, thus rendering it more general than the Engle-Granger test. Johansen's test for cointegration is elaborated on in Appendix 5. I find no evidence of cointegration between the series, if there were cointegration between the time series, taking log first differences is not appropriate.

Finally, to achieve robust and efficient results when testing it is important to consider heteroskedasticity in the estimated models. If there is presence of heteroskedasticity the results will be unbiased, but inefficient. *White's test for heteroskedasticity* (1980) is a general method and involves estimating an auxiliary regression on the squared residuals of the original regression on the original regressors, their cross-products and their squares. The test's core idea is to determine whether a link exists between the variance of the error terms and the values of the independent variables in the model. I find evidence of heteroskedasticity in the estimated VAR regressions. In order to address this problem, *White's Heteroskedasticity-consistent standard errors* (1980) are calculated and used in testing (see Appendix 6.).

4. Results and Discussion

4.1 Data Analysis

The Akaike (1974) and Schwarz Bayesian (1978) Information Criterion are used to determine the optimal lag length of the lag polynomials, this is based on estimating the parameters of each equation and using the residuals of the VAR and the sample size. Table 1 presents the corresponding lag lengths based on information criterion where the maximum lag space search is 40 lags, for the univariate lag lengths, and a space of maximum of 20 lags for the bivariate lag length.

Table 1: Information Criterion

Time Series in Original Format	AIC	SBIC
Spot VIX	11	1
0DTE Total Volume	13	6
Bivariate lag length	10	5

Time Series in Log Changes	AIC	SBIC
Spot VIX (log changes)	2	1
0DTE Volume (log changes)	20	8
Bivariate lag length	14	3

Time Series in Log Changes	AIC	SBIC
Bivariate lag length subsample A	10	2
Bivariate lag length subsample B	9	2

Before I can conduct the Granger causality tests, I need to test for stationarity in the data. As previously noted, the Augmented Dickey-Fuller and Kwiatkowski-Phillips-Schmidt-Shin are used to check for stationarity. Table 2 reports the results from the stationarity tests[4]. For the stationarity tests for the subsamples see Appendix 8.

[4] All test for ADF and KPSS were conducted with a significance level of 1%

Table 2: Stationarity Tests

Stationarity Test Results for 0DTE Volume

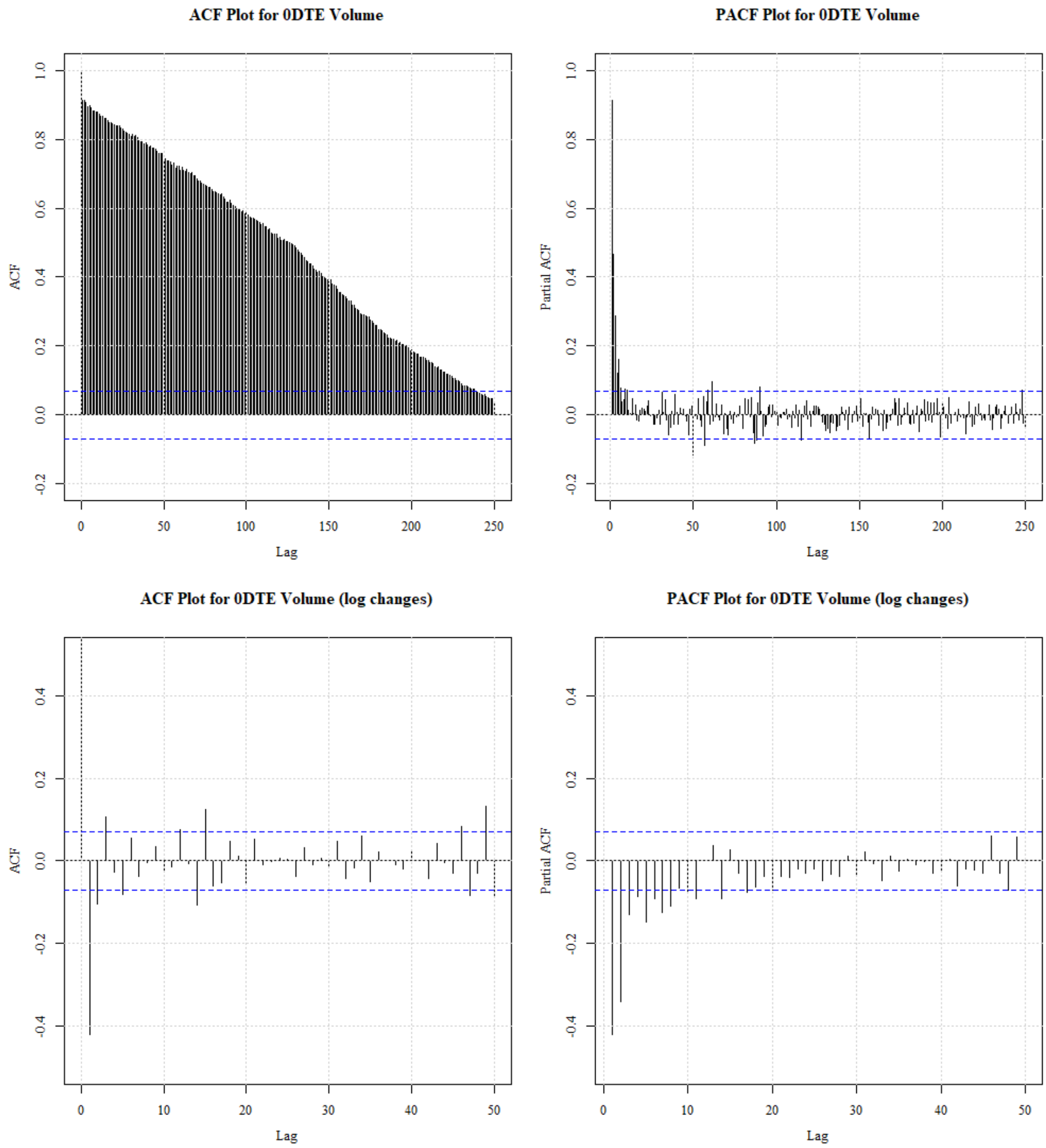
		Augmented Dickey-Fuller		KPSS	
Data	Lag Length	Test Statistic	Result	Test Statistic	Result
Total Volume	14	-2.60188	Non-Stationary	4.91792	Non-Stationary
Total Volume	13	-2.745	Non-Stationary	5.25908	Non-Stationary
Total Volume	10	-3.06093	Non-Stationary	6.65059	Non-Stationary
Total Volume	6	-4.04563	Stationary	10.32991	Non-Stationary
Total Volume	5	-4.31382	Stationary	12.00518	Non-Stationary
Total Volume	3	-5.67542	Stationary	17.8109	Non-Stationary
Log Changes	20	-9.70297	Stationary	0.01974	Stationary
Log Changes	14	-10.01416	Stationary	0.01412	Stationary
Log Changes	8	-14.73273	Stationary	0.00907	Stationary
Log Changes	3	-20.10676	Stationary	0.00425	Stationary

Stationarity Test Results for VIX Index

		Augmented Dickey-Fuller		KPSS	
Data	Lag Length	Test Statistic	Result	Test Statistic	Result
Spot	14	-3.58216	Non-Stationary	0.31971	Stationary
Spot	11	-4.17858	Stationary	0.38159	Stationary
Spot	10	-4.08925	Stationary	0.41	Stationary
Spot	5	-4.23276	Stationary	0.69878	Stationary
Spot	3	-3.87471	Non-Stationary	1.01858	Non-Stationary
Spot	1	-4.20613	Stationary	1.97005	Non-Stationary
Log Changes	14	-8.27862	Stationary	0.02835	Stationary
Log Changes	3	-15.78411	Stationary	0.02227	Stationary
Log Changes	2	-17.79334	Stationary	0.02124	Stationary
Log Changes	1	-22.33223	Stationary	0.01954	Stationary

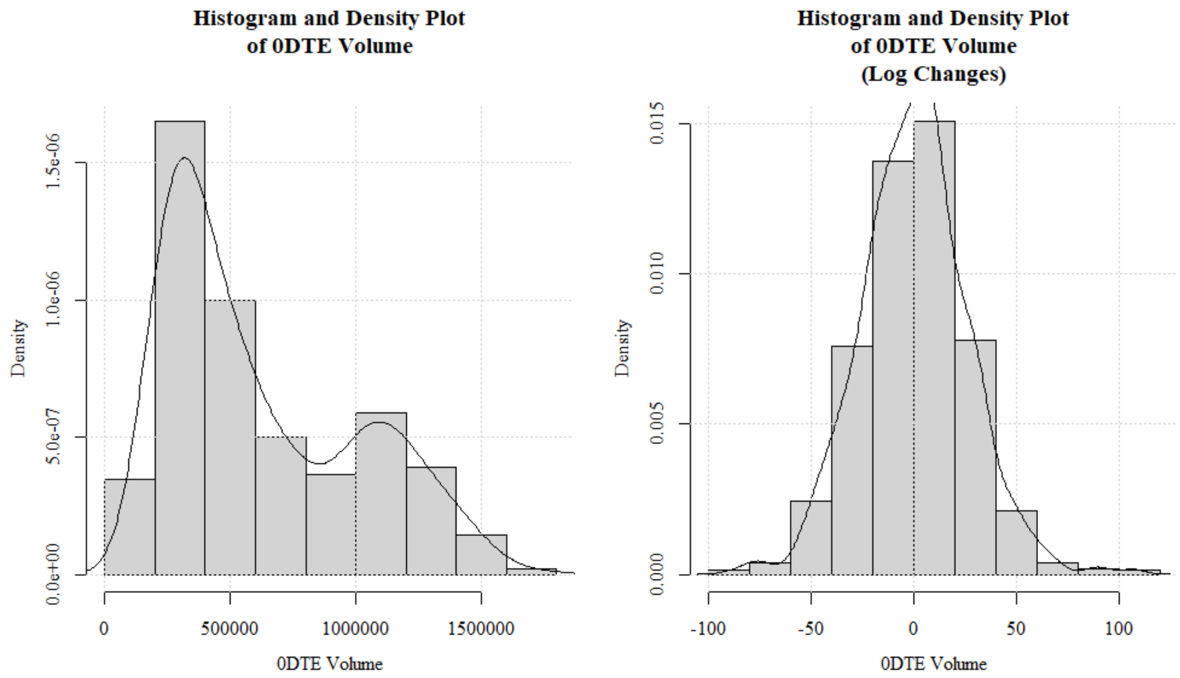
The results in Table 2 show that the data for 0DTE Volume is non-stationary for all KPSS tests, and for three of the six ADF tests conducted when represented in total volume. The spot VIX indicates conflicting results depending on lag lengths. Taking log changes results in stationarity in all tests for both time series. Moreover, I do graphical inference, that supports the results in Table 2. Figure 2 and Figure 4 plots the full sample ACF and PACF for 0DTE volume and VIX respectively, the plots for subsamples are found in Appendix 9. Additionally the plots in Figure 3 Figure 5 show the histogram and density of 0DTE volume and the VIX over the full sample period, for the subsamples see Appendix 7.

Figure 2: ACF & PACF for 0DTE Volume



The PACF (Figure 2) for 0DTE declines geometrically with all significant lags being positive for the data in volume. They are significant up to around the 5th lag. On the other hand, the PACF for log changes has a similar pattern, but with negative lags being significant up to around the 8th lag. The ACF for the volume data is slowly decaying, indicating that shocks are persistent, up to almost the 250th lag. This indicates non-stationarity in the series. When I take log changes the ACF function for 0DTE volume as shown in Figure 2 have a damped sinusoidal pattern, the lags become insignificant after the 5th lag, with some outliers. These findings support the results of the ADF and KPSS tests, that the time series of 0DTE volume is non-stationary while 0DTE in log changes is stationary.

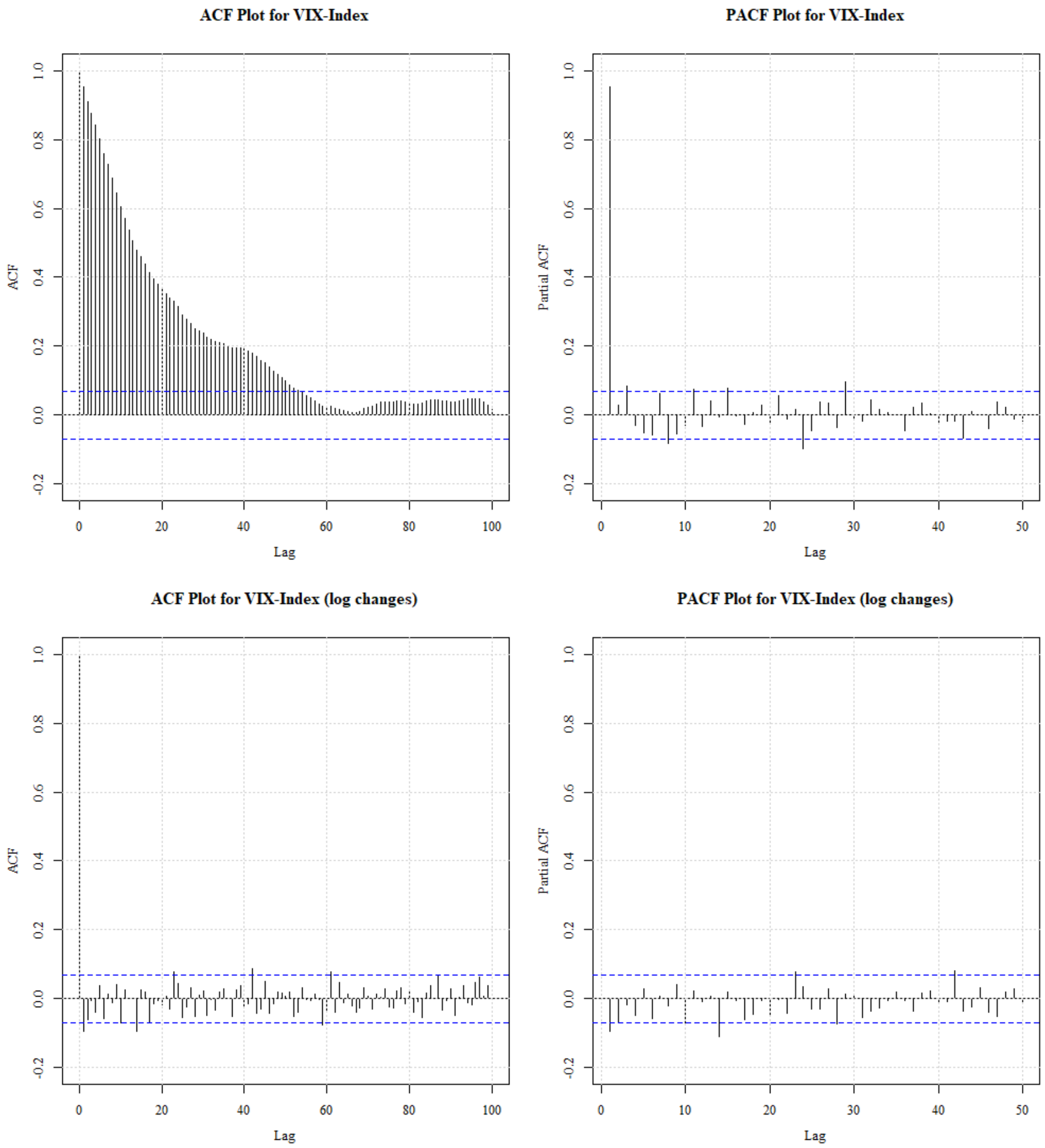
Figure 3: Histogram and Density Plot of 0DTE Volume



Analyzing the distribution of the data using histogram and density plots, will give some additional insight. The histogram and density plot for 0DTE volume in Figure 3 is indicating that the distribution of the series is not stationary. However, looking at the plots for log changes we can see that the series has a bell-shape form for the distribution. White noise process has a mean of zero and a constant variance. The histogram and density plot for 0DTE does not prove that the series is stationary, however it confirms that the distribution is similar to a white noise process graphically. In conclusion, the results from the different methods of stationarity testing suggest that I should take log changes for the 0DTE as it is now stationary.

Moving on to the time series for the VIX Index, I start by looking at the ACF and PACF plots (shown in Figure 4). We see that the PACF rapidly declines for the spot VIX and for log changes, where the PACF for log changes alternates between positive and negative signs, both plots are mostly insignificant after the 3rd lag, but there exist some outliers. The ACF plot for spot VIX shows that the lags are slowly decaying, by closer inspection they are significant up to lags between 50 and 60. The persistency of shocks is not as dramatic as with the ACF for 0DTE volume in Figure 2, nonetheless this might suggest that the data is non-stationary, as a shock is persistent for a relatively long period of time. Inspecting the ACF for VIX in log changes shows patterns associated with stationarity.

Figure 4: ACF and PACF plots for Spot VIX



The plots in Figure 5 indicate that in spot levels the VIX has a large positive skewness this is no clear evidence for or against stationarity, however large positive skewness is often a property of non-stationary data. On the other hand, the plot for changes in VIX has the familiar bell shape, with lower positive skewness. Nonetheless when considering the distribution shape in combination with the ACF and PACF plots as well as the ADF and KPSS test results, I conclude that the VIX in its spot levels is non-stationary.

Figure 5: Spot VIX Histogram and Density Plot

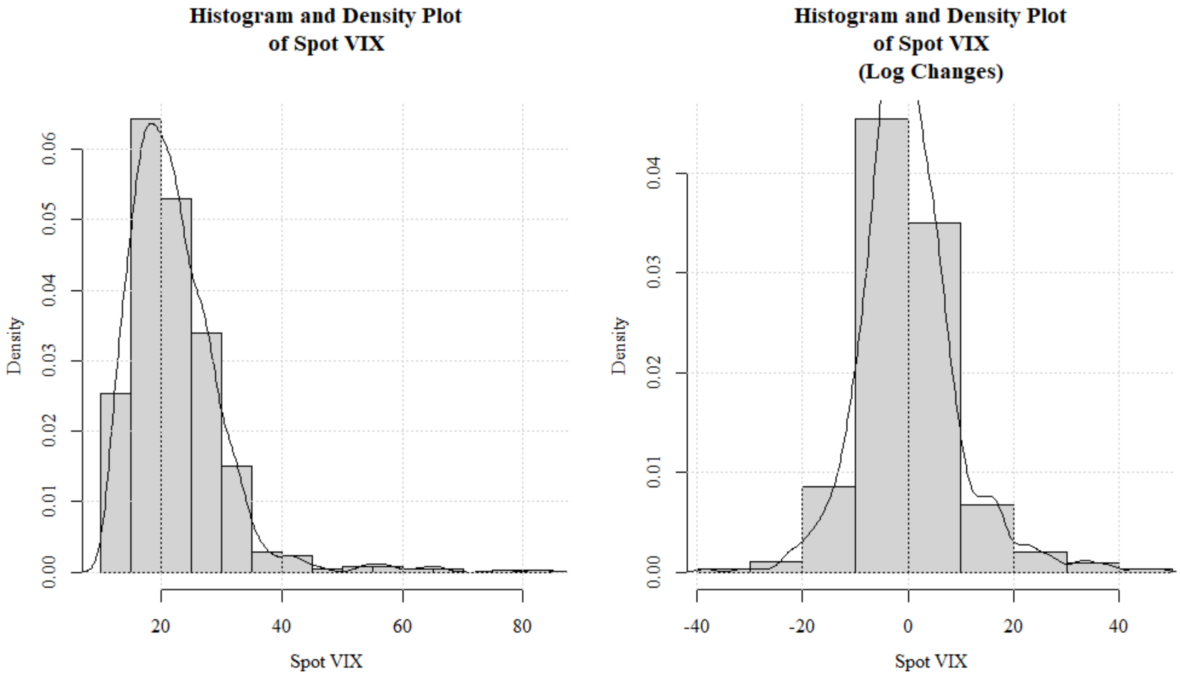


Table 3: Johansen Cointegration Tests

H0: Cointegration of Rank 0				H0: Cointegration of Rank 1			
Type	Lag Length	Test Statistic	Critical Value	Type	Lag Length	Test Statistic	Critical Value
Eigen	14	10.6	19.19	Eigen	14	0.8	11.65
Eigen	10	13.47	19.19	Eigen	10	1.06	11.65
Eigen	5	12.96	19.19	Eigen	5	3.27	11.65
Eigen	3	14.67	19.19	Eigen	3	5.48	11.65
Trace	14	11.41	23.52	Trace	14	0.80	11.65
Trace	10	14.54	23.52	Trace	10	1.06	11.65
Trace	5	16.23	23.52	Trace	5	3.27	11.65
Trace	3	20.15	23.52	Trace	3	5.48	11.65

When conducting the Johansen cointegration tests on the two non-stationary time series (i.e., not in their log differences) there is no evidence of cointegration. The result from the tests is shown in Table 3, indicating that it is appropriate to use log first differences to conduct the causality testing (see (Diks & Wolski, 2016)). For Johansen cointegration test results for the subsamples see Appendix 8. In conclusion, the time series in Spot VIX and 0DTE Volume is non-stationary, and further investigation indicates that taking log first differences is an appropriate approach to eliminate this problem. There is evidence from all tests and plots that the data is stationary when taking log changes.

To achieve robust results when conducting the linear and nonlinear causality tests, we also test the regressions for heteroskedasticity. This is done with White's test for heteroskedasticity shown in Table 4[5]. In accordance with these results, we would need to calculate heteroskedasticity consistent standard errors, when testing for Granger causality. When conducting linear Granger causality tests, I will use White's heteroskedasticity-consistent standard errors. However, this is not necessary in the context of the nonlinear tests as the test statistic by construction addresses the issue of heteroskedasticity (see Appendix 1.).

Table 4: White's Test for Heteroskedasticity

H0: Homoskedasticity in VAR regression for V_t^{0DTE}			H0: Homoskedasticity in VAR regression for VIX_t		
Data Range 02.01.2019 – 22.05.2023					
Lag Length	P-Value	Test Statistic	Lag Length	P-Value	Test Statistic
14	0.0485	484.047 **	14	0.0002	547.78 ***
3	0.0014	54.2403 ***	3	0.0000	93.9545 ***
Data Range 02.01.2019 – 02.07.2021					
Lag Length	P-Value	Test Statistic	Lag Length	P-Value	Test Statistic
10	0.2334	245.2688	10	0.0235	274.5439 **
2	0.4006	14.6765	2	0.0001	41.1227 ***
Data Range 02.07.2021 – 22.05.2023					
Lag Length	P-Value	Test Statistic	Lag Length	P-Value	Test Statistic
9	0.9999	118.4115	9	0.0027	247.5491 ***
2	0.8386	8.8897	2	0.0838	21.7546 *

[5] For the results * indicates significance at the 10% level, ** indicates significance at the 5% level, and *** indicates significance at the 1% level.

4.2 Correlation Analysis

To further understand the relationship between the trading volume of 0DTE SPX options and the VIX Index, we can look at the correlation between the time series. This will be conducted by looking at the rolling correlation across time, and the cross-correlation between the time series. Since we have identified a nonlinear relationship between the series, Pearson correlation may suffer in accuracy. Presented in Table 5 are Pearson correlation, as well as the rank-based methods of Spearman and the more conservative Kendall.

Table 5: Correlation Estimates

Data Range 02.01.2019 – 22.05.2023

Method	Estimate	P-Value
Pearson	0.19633 ***	0.00000
Kendall	0.11356 ***	0.00000
Spearman	0.17122 ***	0.00000

Data Range 02.01.2019 – 02.07.2021

Method	Estimate	P-Value
Pearson	0.19854 ***	0.00007
Kendall	0.11926 ***	0.00038
Spearman	0.18054 ***	0.00029

Data Range 02.07.2021 – 22.05.2023

Method	Estimate	P-Value
Pearson	0.19279 ***	0.00011
Kendall	0.10369 ***	0.00198
Spearman	0.15668 ***	0.00171

We can see from Table 5 that for all methods in all samples there is a correlation between VIX and 0DTE volume with significance levels below the 1 percent level[6]. Furthermore, the correlation coefficient is positive, indicating that an increase in trading volume will result in an increase in the VIX Index, and vice versa. We can also see that the correlation coefficient estimates are relatively close in the subsamples to the full data range. Looking at the plot in Figure 6 we can see that the correlation oscillated around 0.20 up till 2022, when a downward trend of the correlation started. Coincidentally, this is the period where the CBOE introduced SPX options for every day of the week.

[6] For the results * indicates significance at the 10% level, ** indicates significance at the 5% level, and *** indicates significance at the 1% level.

Figure 6: 90-Days Rolling Correlation

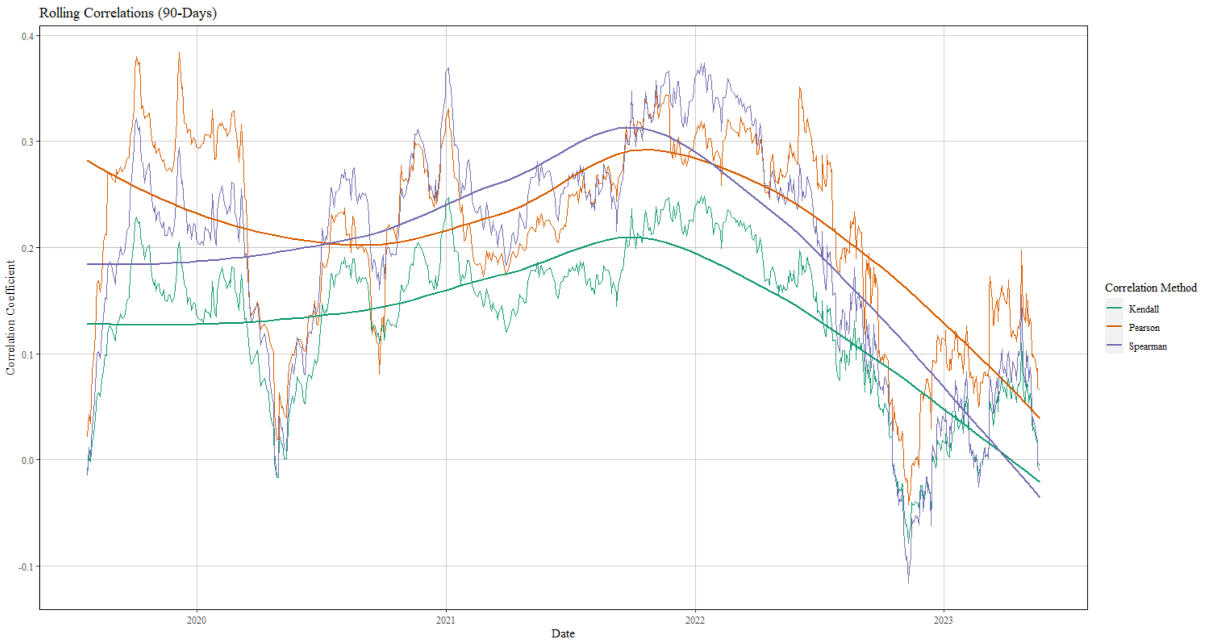
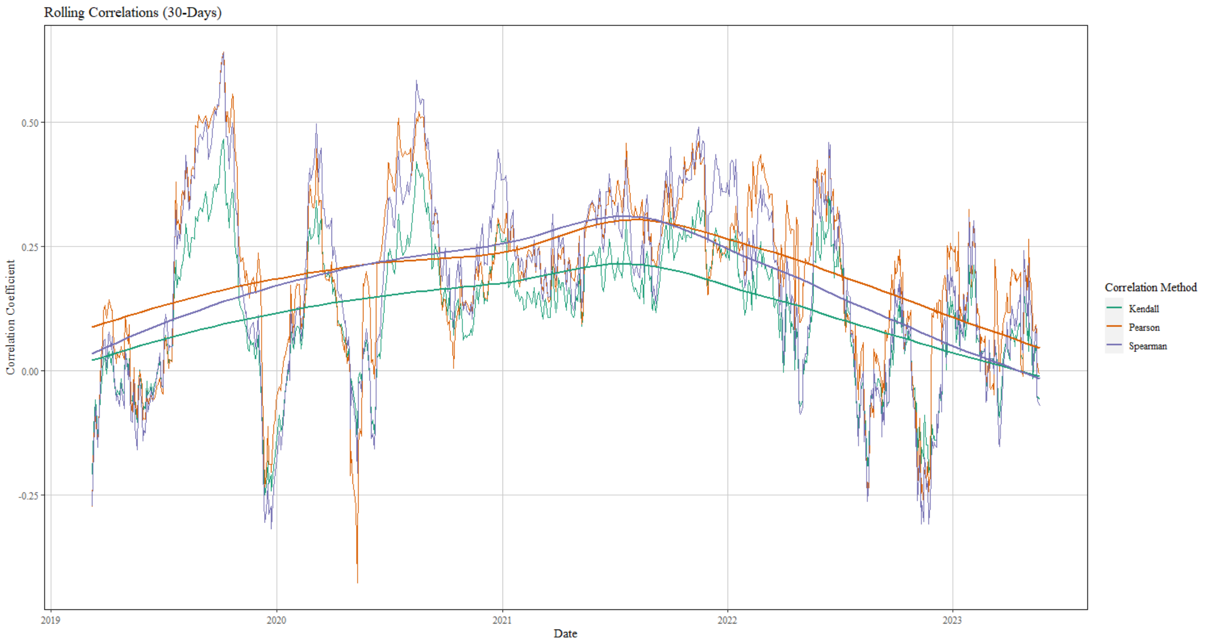
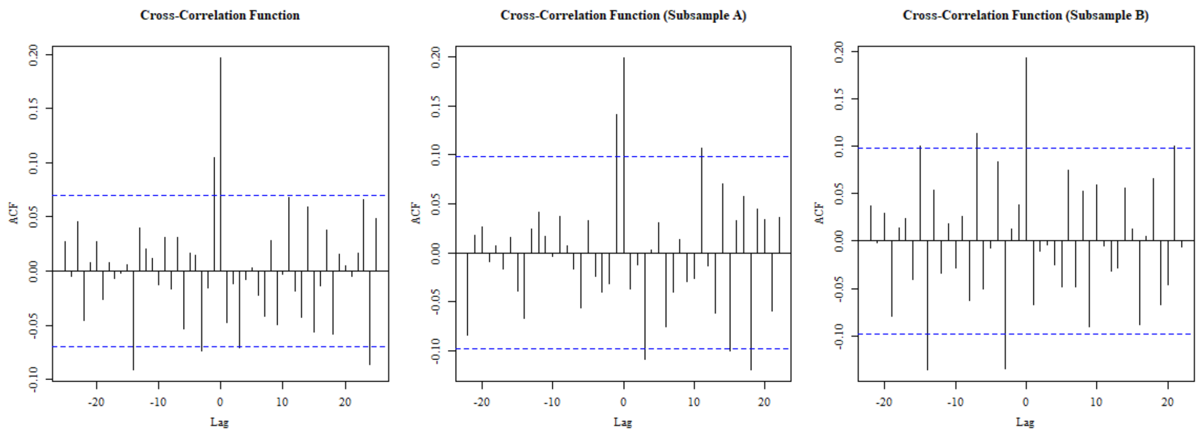


Figure 7: 30-Days Rolling Correlation



In Figure 7 we see the 30-days rolling correlation, the downward trend in a shorter rolling window seems to be less significant. Nonetheless, we still see a decline in the correlation starting in mid-2021. During the meme-stock-hysteria of 2021, the correlation seems consistent around 0.2 to 0.3, interestingly the correlation coefficients seem to trend downward in the period where SPX options for every day of the week were introduced.

Figure 8: Cross-Correlation Function



When looking at the Cross-Correlation plots, we can see significant values for all three plots (Figure 8). Significant lags on the left-hand side of the plot indicate that lags of VIX have an impact on present 0DTE volume. While significant lags on the right-hand side indicate that lags of 0DTE volume have an impact on the present VIX. This provides further evidence of a relationship between the time series. Whereas we do see that there exist bidirectional significant lags, there is little evidence as of how many lags would be optimal in fitting a model for the time series.

4.3 Linear Granger Causality Test Results

First, I need to estimate a VAR model of the 0DTE volume and spot VIX as specified in Equation (1) utilizing OLS per equation, I then calculate the White's heteroskedasticity-consistent standard errors. This VAR is then expressed as such,

Equation (8)

$$\mathrm{V_t}^{\mathrm{0DTE}} = \mathrm{A(L)V_t^{0DTE}} + \mathrm{B(L)VIX_t} + \mathrm{U}_{V^{0DTE},\,t}$$
$$\mathrm{VIX_t} = \mathrm{C(L)V_t^{0DTE}} + \mathrm{D(L)VIX_t} + \mathrm{U}_{VIX,\,t}$$

The results from the linear Granger causality test are reported in Table 6 and Table 7. The tables include the lag lengths, test statistics and p-values, for each null hypothesis.

Table 6: Linear Granger Causality Test Results with Full Data Range

Data Range: 02.01.2019 – 22.05.2023

H0: 0DTE Volume Changes Do Not Cause VIX Level Changes				H0: VIX Level Changes Do Not Cause 0DTE Volume Changes			
Type	Lag Length	Test Statistic	P-Value	Type	Lag Length	Test Statistic	P-Value
AIC	14	1.3854	0.1520	AIC	14	4.0923 ***	0.0000
SBIC	3	5.7939 ***	0.0006	SBIC	3	8.6031 ***	0.0000

The results in Table 6 using AIC, show clear evidence of causality between the changes in spot VIX on the percentage changes in 0DTE volume, with a close to zero significance level[7]. In contrast, there is no evidence for causality from percentage 0DTE volume changes on the changes in spot rates for VIX. The test results in Table 6 using SBIC contradicts the results obtained with AIC. There is still strong evidence for VIX Granger causality to 0DTE, with significance close to zero. However, there is also strong evidence of the reverse order, with significance at the 0.06 percent level. In other words, there is evidence of bidirectional causality between the series, which contradicts the findings using the AIC fitted model.

Table 7: Linear Granger Causality Test Results with Subsamples

Data Range: 02.01.2019 – 02.07.2021

H0: 0DTE Volume Changes Do Not Cause VIX Level Changes				H0: VIX Level Changes Do Not Cause 0DTE Volume Changes			
Type	Lag Length	Test Statistic	P-Value	Type	Lag Length	Test Statistic	P-Value
AIC	10	1.5416	0.1202	AIC	10	5.6093 ***	0.0000
SBIC	2	6.1636 ***	0.0022	SBIC	2	5.4569 ***	0.0044

Data Range: 02.07.2021 – 22.05.2023

H0: 0DTE Volume Changes Do Not Cause VIX Level Changes				H0: VIX Level Changes Do Not Cause 0DTE Volume Changes			
Type	Lag Length	Test Statistic	P-Value	Type	Lag Length	Test Statistic	P-Value
AIC	9	1.1371	0.3336	AIC	9	2.1424 **	0.0242
SBIC	2	1.8027	0.1655	SBIC	2	1.5350	0.2161

The results in Table 7 present the findings where the data range for fitting and estimating the VARs are split into two subsamples. We can see that in the period from 02.01.2019 to 02.07.2021, there is evidence of bidirectional causality only for the SBIC selected VAR. In

[7] For the results * indicates significance at the 10% level, ** indicates significance at the 5% level, and *** indicates significance at the 1% level.

the AIC selected VAR there is strong evidence of Granger causality from VIX to 0DTE volume, but no evidence of bidirectional causality. Furthermore, when assessing the period of 02.07.2021 to 22.05.2023 there are contradictory results. There is only significance for VIX Granger causality to 0DTE in the AIC model, at the 2.4 percent significance level. Nonetheless, the results in Table 6 and Table 7 are strongly contrasted with the results of the nonlinear Granger causality test results presented next.

4.4 Nonlinear Granger Causality Test Results

The nonlinear Granger causality test is conducted on the estimated residual series from the estimated VAR in Equation (8), namely $\{\widehat{U}_{V^{0DTE},t}\}$ and $\{\widehat{U}_{VIX,t}\}$, corresponding to percentage 0DTE volume changes and percentage spot VIX changes. Before I start the implementation of the test, each series of estimated residuals is standardized, such that the two series share the same standard deviation, i.e., σ = 1, and therefore they share a common scale parameter. To conduct a test on these estimated residuals, I first need to define values for m, Lx, Ly, and e. In contrast to the procedures for linear causality tests, there is no literature on the appropriate method for choosing optimal values for the scale parameter e and the lag lengths Lx and Ly. Therefore, I will have to rely on previous research on the method when choosing these values. Lag lengths of 1 to 4 is used by (Abhyankar, 1998) and (Shu & Zhang, 2012), while lag lengths of 1 to 9 is used by (Silvapulle & Choi, 1999) and (Rahimi et al., 2017) uses lag lengths of 1 to 12. Hiemstra & Jones (1994), choose the values based on their Monte Carlo results in Hiemstra & Jones (1993) and these are the parameter values I too will choose. For all cases they set m = 1, and $Lx = Ly$, using lag length of 1 to 8, and the common scale parameter $e = 0.5\sigma$, $e = 1\sigma$, and $e = 1.5\sigma$, where $\sigma = 1$ denotes the standard deviation of the standardized time series[8]. The results from the HJ-test are reported in Table 8 and Table 9 for the full sample and in Table 10 and Table 11 for the subsamples:

Table 8: Nonlinear Granger Causality Tests Results (AIC)

[8] They also use $e = 0.5\sigma$ and $e = 1\sigma$, but only report the findings for $e = 1.5\sigma$. I will only apply $e = 1.5\sigma$.

	H0: 0DTE Volume Changes Do Not Cause VIX Level Changes			H0: VIX Level Changes Do Not Cause 0DTE Volume Changes	
Lx = Ly	CS	TVAL	Lx = Ly	CS	TVAL
1	0.00436	3.43865 ***	1	0.00475	2.55112 ***
2	0.00104	0.47974	2	0.00692	1.62919 *
3	0.00664	2.48774 ***	3	0.00918	1.41862 *
4	0.00729	2.38281 ***	4	0.01229	1.54263 *
5	0.00944	2.86928 ***	5	0.00684	0.75027
6	0.00728	2.12347 **	6	0.01253	1.23723
7	0.00568	1.66782 **	7	0.01682	1.45592 *
8	0.00408	1.16437	8	0.02232	1.81485 **

Where in Table 8, *Lx* = *Ly* denotes the number of lags on the unconditionally standardized residuals series, *m* and *e* is not reported as these values are constantly set as 1, and 1.5 respectively. CS denotes the difference between the two conditional probabilities in Equation (5), and TVAL denotes the standardized test statistic in Equation (7) [9] .

The test results in Table 8 show strong evidence of causality from percentage changes in 0DTE volume to the changes in spot VIX, for all lag lengths except the lag length of 2 and 8. Lag lengths of 1, 3, 4 and 5 are significant at the 1 percent level, with 6 and 7 at the 5 percent significance level. Conversely for the tests on spot VIX changes to 0DTE volume changes the tests show evidence of a causal effect, but with weaker significance. The null hypothesis is rejected at the 1 percent significance level for lags of 1, and at the 5 percent level for lag length of 8. Furthermore, at lag lengths of 2, 3, 4 and 7 the null is rejected at the 10 percent significance level. Even though the significance for VIX Granger cause 0DTE is weaker than the reverse order, this coincides with the results from the linear tests with the SBIC selected VARs in the full sample and subsample A, i.e., bidirectional causal relationship. Nonetheless in the context of comparing results from the AIC fitted VARs, the nonlinear test contradicts the findings of the linear tests. This coincides with results from prior research into stock returns and trading volume. Where evidence of unidirectional causality from returns to volume is found in the linear test, but bidirectional causality is found in the nonlinear tests (see Hiemstra & Jones (1994)).

[9] For the results * indicates significance at the 10% level, ** indicates significance at the 5% level, and *** indicates significance at the 1% level.

Table 9: Nonlinear Granger Causality Test Results (SBIC)

	H0: 0DTE Volume Changes Do Not Cause VIX Level Changes			H0: VIX Level Changes Do Not Cause 0DTE Volume Changes	
Lx = Ly	CS	TVAL	Lx = Ly	CS	TVAL
1	0.00682	5.3637 ***	1	0.00464	2.2564 **
2	0.00743	3.35635 ***	2	0.00961	2.15388 **
3	0.00856	3.22226 ***	3	0.00737	1.16634
4	0.00795	2.53901 ***	4	0.01103	1.45853 *
5	0.01238	3.70445 ***	5	0.00743	0.89341
6	0.00909	2.65376 ***	6	0.01008	1.11518
7	0.00904	2.62993 ***	7	0.0074	0.7288
8	0.00969	2.70155 ***	8	0.00772	0.71125

The test results shown in Table 9 based on the estimated VAR with SBIC lag selection, show weak evidence of bidirectional causality[10]. However, the results show strong evidence of causality from percentage changes in trading volume to spot VIX. Where all tests are significant at the 1 percent level. On the other hand, there is some weak evidence from the test of VIX Granger cause 0DTE. They are significant at the 5 percent level for lag length of 1 and 2, and at the 10 percent level for lag length of 4. In contrast to the linear test's findings for SBIC with full sample, it is now the direction of 0DTE volume to spot VIX that is most significant, with only weak evidence of spot VIX to 0DTE volume.

[10] For the results * indicates significance at the 10% level, ** indicates significance at the 5% level, and *** indicates significance at the 1% level.

Table 10: Nonlinear Granger Causality Test Results with Subsamples (AIC)

Data Range: 02.01.2019 – 02.07.2021

	H0: 0DTE Volume Changes Do Not Cause VIX Level Changes			H0: VIX Level Changes Do Not Cause 0DTE Volume Changes	
Lx = Ly	CS	TVAL	Lx = Ly	CS	TVAL
1	0.00683	4.12422 ***	1	-0.00107	-0.43677
2	-0.00384	-1.54842	2	0.00012	0.02154
3	-0.00091	-0.33417	3	0.00160	0.18363
4	-0.00366	-1.18128	4	-0.00015	-0.01432
5	-0.00591	-1.87219	5	-0.00930	-0.81363
6	-0.01859	-5.42507	6	-0.15800	-1.27535
7	-0.02484	-7.13477	7	-0.03196	-2.51165
8	-0.05057	-14.87191	8	-0.02297	-1.89635

Data Range: 02.07.2021 – 22.05.2023

	H0: 0DTE Volume Changes Do Not Cause VIX Level Changes			H0: VIX Level Changes Do Not Cause 0DTE Volume Changes	
Lx = Ly	CS	TVAL	Lx = Ly	CS	TVAL
1	0.00671	3.89279 ***	1	0.00257	1.00729
2	0.01328	4.33900 ***	2	0.00466	0.77233
3	0.01038	2.84689 ***	3	0.00772	0.83530
4	0.01925	4.77633 ***	4	0.01071	0.87509
5	0.01807	4.98709 ***	5	0.01834	1.22029
6	0.01563	4.79421 ***	6	0.01691	0.96809
7	0.00874	3.0659 ***	7	0.01816	0.92425
8	-0.00192	-0.72099	8	0.01775	0.82148

Table 10 presents the results for the HJ-test using AIC selected VARs in the subsamples[11]. In subsample A we find no evidence of bidirectional causality. However, for a lag length of 1, there is significance at the 1 percent level. This strictly contradicts the findings for the full sample tests and the linear tests. On the other hand, when looking at the results from subsample B, we find strong evidence of 0DTE volume having a causal effect on the VIX, but no bidirectional causality. This again contrasts the findings for the full sample.

[11] For the results * indicates significance at the 10% level, ** indicates significance at the 5% level, and *** indicates significance at the 1% level.

Table 11: Nonlinear Granger Causality Test Results with Subsamples (SBIC)

Data Range: 02.01.2019 – 02.07.2021

	H0: 0DTE Volume Changes Do Not Cause VIX Level Changes			H0: VIX Level Changes Do Not Cause 0DTE Volume Changes	
Lx = Ly	CS	TVAL	Lx = Ly	CS	TVAL
1	0.00604	3.67601 ***	1	0.00457	1.7579 **
2	0.0031	0.11873	2	0.00823	1.44689 *
3	-0.00191	-0.63759	3	0.00650	0.85061
4	-0.00915	-2.63572	4	0.00963	1.10452
5	-0.00551	-1.51249	5	0.00593	0.64634
6	-0.01838	-4.83363	6	0.00123	0.12072
7	-0.02028	-5.46459	7	-0.00914	-0.87358
8	-0.02941	-7.76357	8	-0.00855	-0.78736

Data Range: 02.07.2021 – 22.05.2023

	H0: 0DTE Volume Changes Do Not Cause VIX Level Changes			H0: VIX Level Changes Do Not Cause 0DTE Volume Changes	
Lx = Ly	CS	TVAL	Lx = Ly	CS	TVAL
1	0.00683	4.19731 ***	1	0.00466	1.69926 **
2	0.01717	5.96446 ***	2	0.00710	1.14061
3	0.11000	3.28079 ***	3	0.01038	1.17582
4	0.01854	5.29786 ***	4	0.01526	1.32100 *
5	0.01601	5.32692 ***	5	0.01576	1.18318
6	0.00850	3.32903 ***	6	0.02385	1.67275 **
7	0.00528	2.42822 ***	7	0.01627	1.10435
8	0.0067	3.51605 ***	8	0.01357	0.86446

The findings from the SBIC selected VARs in the subsample also have mixed results[12]. For subsample A in Table 11, there is again contradicting results from the full sample tests. However, in contrast to the AIC selected VARs there is now evidence of VIX Granger causality to 0DTE volume at the 5 percent significance level and 10 percent level, for lag lengths of 1 and 2 respectively. Furthermore, the results for subsample B show strong unidirectional causality from 0DTE volume to the VIX, where all are significant at the 1 percent level. For VIX Granger cause 0DTE volume the tests are significant at lag lengths of 1, 4 and 6, with weaker significance than the reverse order.

It is important to note that it is not possible to conclude if the nonlinear causality found in the

[12] For the results * indicates significance at the 10% level, ** indicates significance at the 5% level, and *** indicates significance at the 1% level.

nonlinear Granger causality tests are positive or negative, adding to the complexity of the results. The lack of significance in subsample A could be attributed to the fact that the trading activities in 0DTE options were substantially lower than at a more recent time. Furthermore, there were only SPX contracts expiring on Mondays, Wednesdays, and Fridays. In this period, for the linear tests, we see strong evidence of causality from the VIX to the trading volume in all models, with bidirectional causality in the SBIC selected VAR models. The results are consistent with the results from the full period linear tests. These findings could be capturing the event risk hedging activities which traditionally were the primary usage for 0DTE contracts. Nonetheless, I find evidence of trading volume with lags up to 8 days having a significant effect on today's VIX Index, when investigating the full sample and subsample B. The reason for this causal effect on VIX could be due to persistency in prices of SPX options with maturities of 23-37 days, when the volume of 0DTE increases, or reverse. Further investigation into the relationship of 0DTE volume and SPX options prices for 23-27 maturities would have to be conducted. Conversely, the causal effect from the VIX Index to 0DTE volume seems to be strongest in the first lags. This could be due to increased trading opportunities for day-traders, both retail investors and professionals, when there is more volatility in the market. Since 0DTE options with strike prices close to the underlying market price will be sensitive to small movements, days with high volatility could result in a higher frequency of day-trading. Furthermore, if there is increased volatility traders and investors could be more likely to hedge their positions, further increasing the volume. This could further be fueled by a feedback loop between the VIX and the trading volume. This could be further analyzed with intraday data for the VIX Index and trading volume of 0DTE options. Interestingly, when testing for nonlinear causality in subsamples I find that for the period from 02.01.2019 to 02.07.2021 there are little to no causal effects between VIX and 0DTE volume, with the exception for the first lags. While in the period from 02.07.2021 to 22.05.2023 there is strong evidence of 0DTE volume having a causal effect on the VIX Index, with little evidence of bidirectional causality. A plausible explanation could be the aforementioned increased popularity of 0DTE options amongst retail investors and therefore also professional investors, as well as the introduction of every day of week contracts. However, further analyses would have to be conducted to evaluate if this effect is indeed due to retail investors and their increased trading activity in 0DTE options.

4.5 Impulse Response Function and Variance Decomposition

To gain more insight into the linear Granger causality test results we look at the impulse response function and variance decomposition as described in section 3.3. Figure 9 and Figure 10 show the impulse response function plots with AIC and SBIC respectively, and the variance decomposition plots shown in Figure 11. Where Figure 12 is a reverse ordering of the VAR as a robustness check of Figure 11. All the plots presented here corresponds to the VARs selected in the full sample, the corresponding plots for the subsamples are found in Appendix 10. The plots of most interest in Figure 9 and Figure 10, are where we can see the response in one variable to a shock in the other variable (i.e., shock of 0DTE to VIX or vice versa), these plots allows us to look at how the data reacts to unit root shocks, and how long they persist.

Figure 9: Impulse Response Function (AIC)

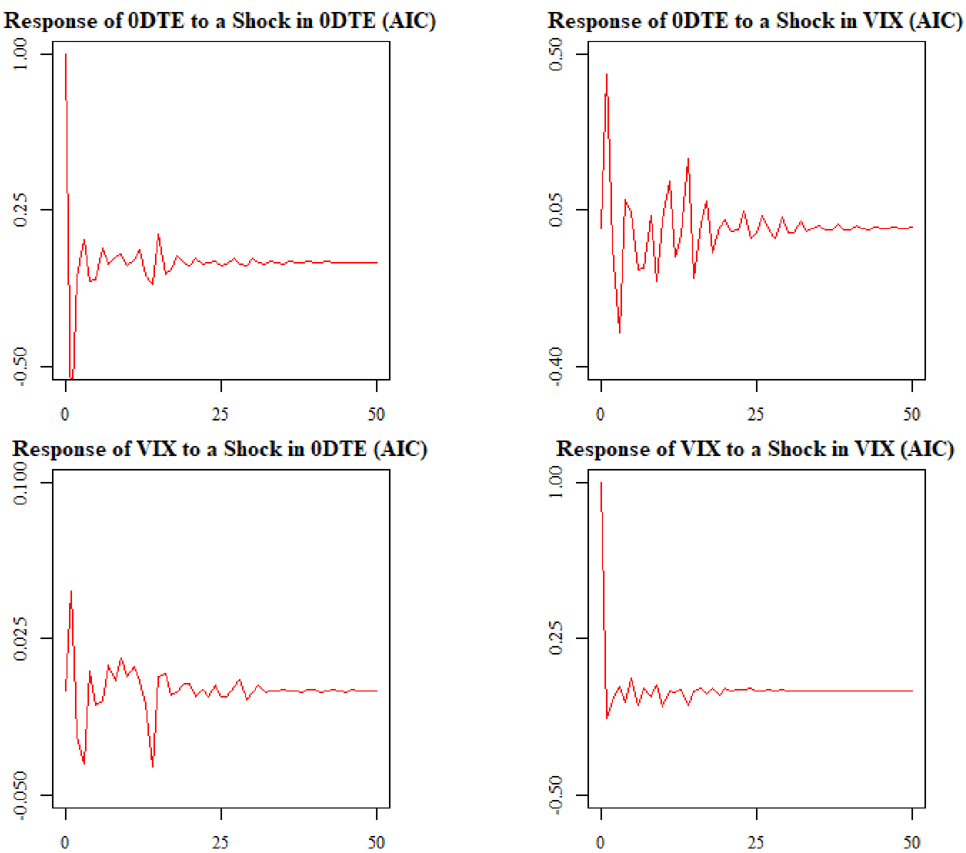


Figure 10: Impulse Response Function (SBIC)

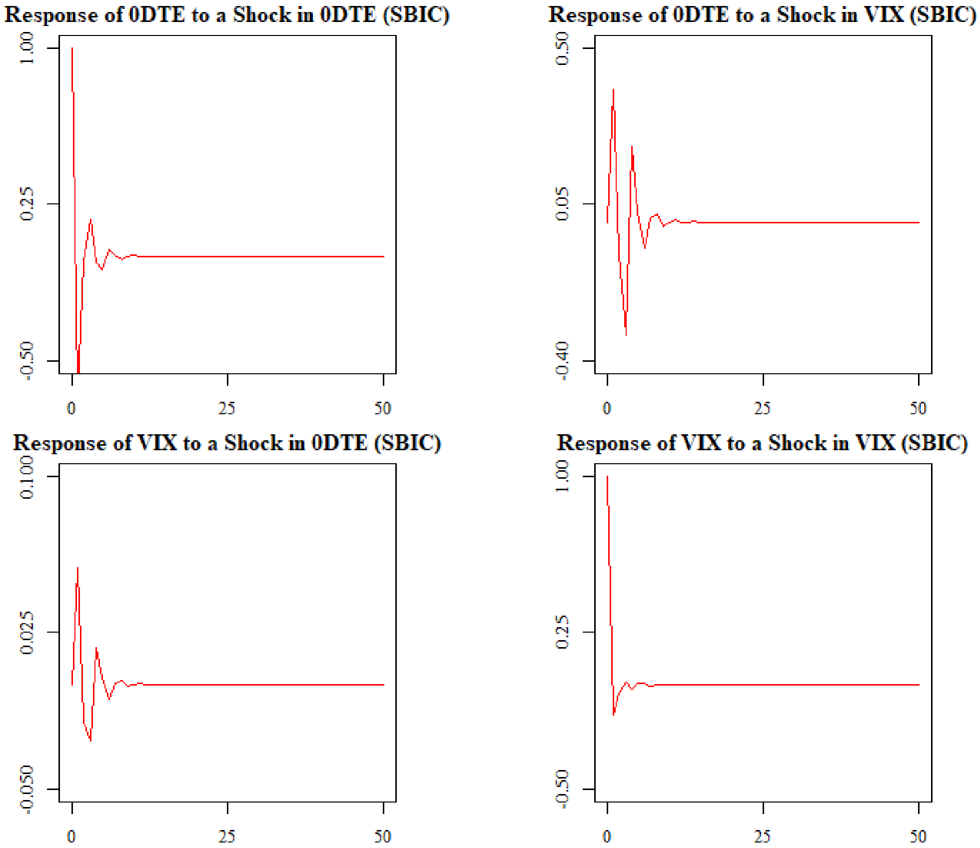


Figure 11: Variance Decomposition Plots

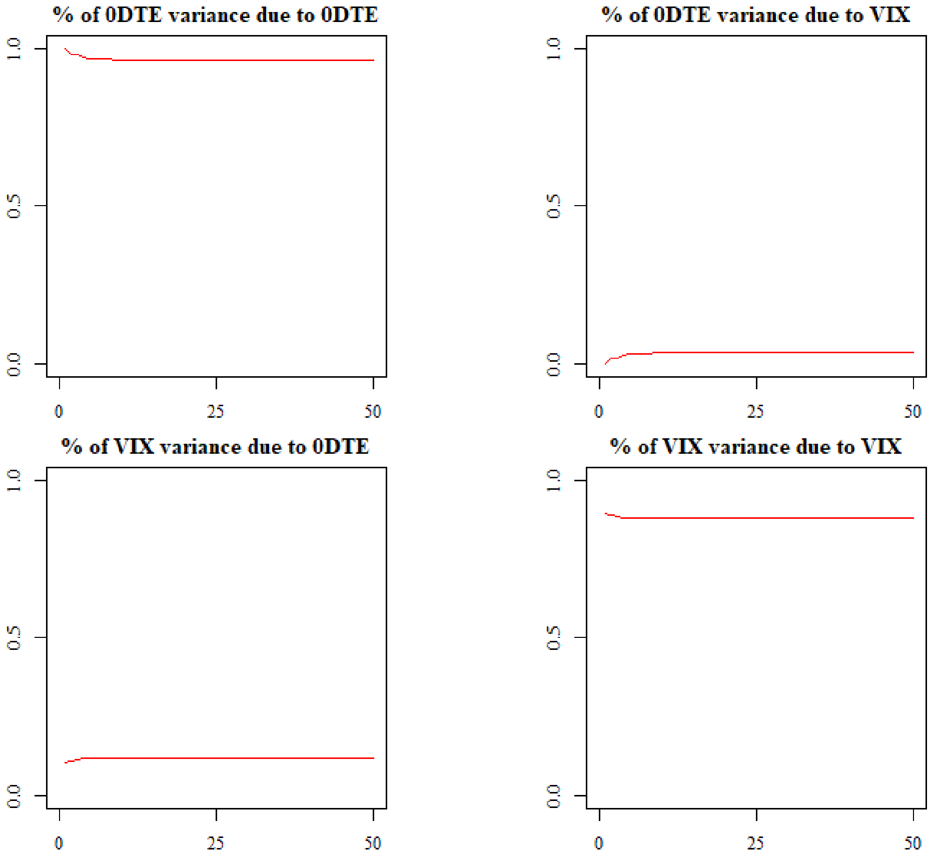
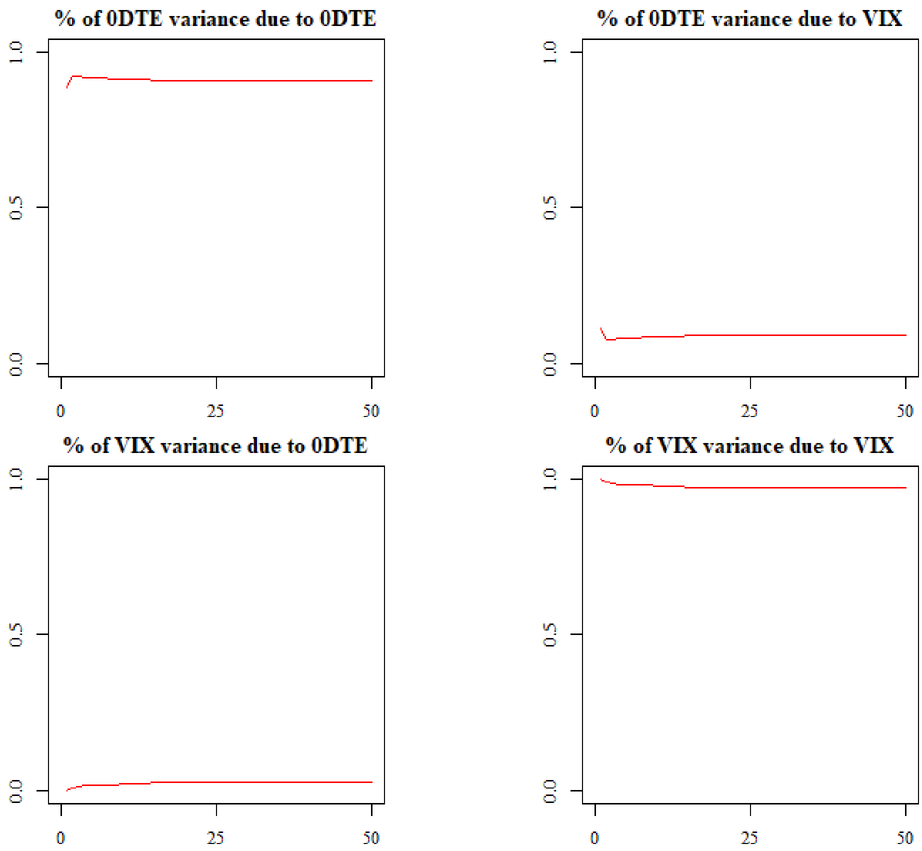


Figure 12: Reverse Ordering Variance Decomposition Plots



Looking at the response of 0DTE volume to a shock in the VIX with AIC (Figure 9), there is a large spike in the first time-step, after that the response starts to oscillate around zero, with mostly negative values, and some positive spikes farther out. This could indicate that when the VIX has a sharp increase this leads to a spike in trading volume, due to increased volatility in the market and increased trading opportunities. Similarly, the same plot when SBIC is used (Figure 10), there is alternating positive and negative values, the shock is also not as persistent as with AIC. Conversely, when looking at the response of the VIX, when there is a shock in 0DTE volume we see (for both AIC and SBIC) that there is a positive spike followed by a sharp negative spike. This could indicate a short-term increase in volatility when there are sudden large trading activities in 0DTE options. The response of the VIX to itself, there is a large spike which quickly drops to zero. While for 0DTE on itself there is a large positive spike, followed by a sharp negative spike. Indicating that when there is a day of high volume the next day will be less than normal.

Figure 11 show that a relatively significant portion of the variance in the VIX could be attributed to the 0DTE volume. With a small percentage of 0DTE volume variance due to the

VIX. However, when looking at the same plots in Figure 12, where the VAR model is reversed in ordering of the variables, the findings in Figure 11 do not hold. Now it is a significant (but still smaller than the portion for percentage VIX variance due to 0DTE volume) portion of the trading volume variance that is due to the VIX. When examining the opposite way, i.e., portion of VIX variance, there is little to no impact from 0DTE volume. This renders the variance decomposition plots harder to analyze, as there are mixed results. To conclude, we see that shocks in each variable translate to initial positive spikes in the other series, followed by an oscillating pattern of alternating negative and positive signs. In other words, the linear Granger causality is initially positive, with alternating signs thereafter.

5. Summary and Conclusion

By applying the traditional Granger causality test and the HJ-test I have found significant evidence of causality between 0DTE trading volume and the VIX. In the context of linear causality, the evidence is in favor of VIX Granger causing 0DTE trading volume. Elevated levels of volatility in financial markets can present opportunities for day traders, who are among the primary users of 0DTE contracts. This can result in an increase in the trading volume of 0DTE options. However, it is important to note that heightened volatility also introduces greater risk into the market. As such, institutional traders may seek to hedge their exposure when volatility rises significantly, further contributing to an increase in trading volume. I found the relationship between the VIX and 0DTE volume to be initially positive in the linear context, with altering negative and positive effect thereafter. Conversely, when volatility has increased leading to heightened trading activity this can further increase the volatility, feeding into a feedback loop. The results from the nonlinear tests might suggest that this is the case in the relationship between 0DTE volume and the VIX Index. In the HJ-test I found significant evidence of causality from the trading volume of 0DTE options to the VIX. The most significant results are found in the period from 02.07.2021 to 22.05.2023. This is the same time period that the CBOE introduced SPX options for every day of the week, and also the same period that retail investors' share of trading 0DTE contracts gained traction. Furthermore, I also found evidence of bidirectional causality in the nonlinear context, i.e., indicating that there is a feedback loop between the time series. Additionally, depending on model selection and data sample, there is also evidence that 0DTE volume Granger causes VIX in the linear testing. This might suggest that when the VIX increases there is a feedback loop of increased trading volume, which again increases the VIX, and vice versa. However, further analysis into intraday data could be conducted to test this dynamic more accurately. Nonetheless, I have proven that the nonlinear causality test proposed by Hiemstra & Jones (1994) has captured a dynamic which the traditional Granger causality testing framework could not. These results stress the importance of testing for both linear and nonlinear temporal relationships, when investigating financial and economic variables. It is also worth noting that it is not possible to conclude if the nonlinear causality found in the nonlinear Granger causality tests are positive or negative, increasing the complexity of the test inferences. Even so, I have found strong evidence of 0DTE trading volume affecting the changes in the VIX Index. This could improve forecasting of the VIX Index in the future. For further investigation of the dynamic temporal relationship between 0DTE trading volume and the VIX, the following could be investigated:

The use of *Intraday Data* over time for 0DTE volume and spot VIX could improve the inference, as we could investigate more swiftly the rapid changes in spot VIX for spikes in trading volume, and vice versa. However, this is beyond the scope of the thesis, and the data could be troublesome to obtain. *1-day VIX Index* in the middle of writing this thesis the CBOE announced the introduction of a 1-day VIX Index. As the analysis is of 0DTE options volume this could improve the significance as the time series would be aligned in their time horizon. As previously noted *Diks & Panchenko Test Statistic* (2006) argue that the Modified Baek & Brock test can over-reject if the null hypothesis is true. They proposed a new improved test statistic to tackle this problem in their article. For a more comprehensive analysis it would be recommended to test the results of this thesis against the results of the test proposed by Diks & Panchenko. *Bootstrapped Critical Values*; Hiemstra & Jones (1994) argue that the critical values for the test statistic should be bootstrapped when implementing the Baek & Brock test or the modified version of it. This is due to the evidence from Hiemstra & Jones (1993) and Baek & Brock (1992), where they find that the finite-sample size of the original Baek & Brock test, which assumes that the series to which the test is applied are mutually independent and individually i.i.d., can be considerably larger than its nominal size. Through the utilization of Monte Carlo simulations, Hiemstra and Jones (1993) discovered that the modified Baek and Brock test exhibits exceptional finite-sample size and power properties against a diverse range of linear and nonlinear Granger causal and noncausal relationships. Nonetheless, they recommend the use of bootstrapped critical values for tests. This is beyond the scope of this thesis. Finally, the relationship between 0DTE volume and SPX prices for maturities of 23-27 days could give some more insight into the effects of 0DTE volume on the

VIX. This is due to the VIX being a function of these SPX options prices, and therefore the study of the VIX is indirectly a study of the mentioned SPX contracts. *Asymmetric Lag Lengths in VAR*, the linear and nonlinear Granger causality tests could be more robust in their estimations if the chosen lag length were asymmetric as the ones chosen by Hiemstra & Jones (1994). Studies have shown that asymmetric VARs could improve the robustness of the results (see (Keating, 1993, 1994). The selection of lag lengths is important for VAR and Granger causality analysis, overfitted lag length VAR models tend to lead to over rejection of the null hypothesis. However, this would require additional analysis in lag selection and model fitting and is beyond the scope of this thesis. *Dynamic Conditional Correlation* testing in the GARCH framework could help to identify more accurately the correlation between the two nonlinearly related time series.

Appendix

Appendix 1.

The Variance of the HJ-Test

The variance estimator used in the HJ-test is derived as such proposed by Hiemstra & Jones (1994):

Appendix Equation (1)

$$h1_{C1}\left(x_{t-Lx}^{m+Lx},\ y_{t-Ly}^{Ly},\ e\right) \equiv Pr\left(\left\|x_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\right\| < e,\ \left\|y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\right\| < e\right),$$
$$h2_{C2}\left(x_{t-Lx}^{Lx},\ y_{t-Ly}^{Ly},\ e\right) \equiv Pr\left(\left\|x_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\right\| < e,\ \left\|y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly}\right\| < e\right),$$
$$h3_{C3}\left(x_{t-Lx}^{m+Lx},\ e\right) \equiv Pr\left(\left\|x_{t-Lx}^{m+Lx} - X_{s-Lx}^{m+Lx}\right\| < e\right),$$
$$h4_{C4}\left(x_{t-Lx}^{Lx},\ e\right) \equiv Pr\left(\left\|x_{t-Lx}^{Lx} - X_{s-Lx}^{Lx}\right\| < e\right).$$

By applying the delta method as described by (Serfling, 2009, pp. 122–125) to Appendix Equation (1), and assuming that the underlying series are strictly stationary, weakly dependent, and satisfy the mixing conditions outlined by Denker and Keller (1983), it is possible to derive an expression for the variance of the Baek and Brock test presented in Equation (7);

Appendix Equation (2)

$$\sigma^2(m,\ Lx,\ Ly,\ e) = \boldsymbol{d}\Sigma\boldsymbol{d}',$$

where

Appendix Equation (3)

$$d = [\mathrm{d_i}], i = 1, \dots, 4 = \left[\frac{1}{C2(Lx, Ly, e)}, \frac{-C1(m + Lx, Ly, e)}{C2^2(Lx, Ly, e)}, \frac{-1}{C4(Lx, e)}, \frac{C3(m + Lx, e)}{C4^2(Lx, e)}\right]$$

Appendix Equation (4)

$$\Sigma = \left[\Sigma_{i,j}\right], i, j = 1, \dots, 4$$

$$\Sigma = \left[4 * \sum_{k \geq 1} \omega_k(n)\, E\left(A_{i,t} * A_{j,t-k+1}\right)\right], \omega_k = \begin{cases} 1, & if\ k = 1 \\ 2, & otherwise \end{cases}$$

Appendix Equation (5)

$$A_{1,t} = h1_{C1}\left(x_{t-Lx}^{m+Lx},\ y_{t-Ly}^{Ly},\ e\right) - C1(m + Lx,\ Ly,\ e)$$
$$A_{2,t} = h2_{C2}\left(x_{t-Lx}^{Lx},\ y_{t-Ly}^{Ly},\ e\right) - C2(Lx,\ Ly,\ e)$$
$$A_{3,t} = h3_{C3}\left(x_{t-Lx}^{m+Lx},\ e\right) - C3(m + Lx,\ Ly,\ e)$$
$$A_{4,t} = h4_{C4}\left(x_{t-Lx}^{Lx},\ e\right) - C4(Lx,\ e)$$

Where E denotes expected value, and $C_i(\cdot)$ terms are defined in Equation (4)[13]. By applying the findings of Denker and Keller (1983) and Newey and West (1987), it is possible to derive a consistent estimator for the $\Sigma_{i,j}$ elements presented in Appendix Equation (2);

Appendix Equation (6)

$$\hat{\Sigma}_{i,j}(\mathrm{n}) = 4 * \sum_{k=1}^{K(n)} \omega_k(n) \left[\frac{1}{2(n-k+1)} \sum_t \left(\hat{A}_{i,t}(n) * \hat{A}_{j,t-k+1}(n) \hat{A}_{i,t-k+1}(n) * \hat{A}_{j,t}(n) \right) \right]$$

$$t = max(Lx, Ly) + k, \ \dots, T - m + 1$$
$$n = T + 1 - m - max(Lx, Ly)$$
$$n = T + 1 - m - max(Lx, Ly)$$
$$K(n) = (int) n^{(1/4)}$$
$$\omega_k(n) = \begin{cases} 1, & if\ k = 1, \\ 2(1 - [(k-1)/K(n)]), & otherwise \end{cases}$$

where

Appendix Equation (7)

$$\hat{A}_{1,t}(n) = \frac{1}{n-1} \left(\sum_{s \neq t} I(x_{t-Lx}^{m+Lx} - x_{s-Lx}^{m+Lx}, e) * I\left(y_{t-Ly}^{Ly} - y_{s-Ly}^{Ly}, e\right) \right) - C1(m + Lx, Ly, e, n)$$
$$\hat{A}_{2,t}(n) = \frac{1}{n-1} \left(\sum_{s \neq t} I(x_{t-Lx}^{Lx} - x_{s-Lx}^{Lx}, e) * I\left(y_{t-Ly}^{Ly} - y_{s-Ly}^{Ly}, e\right) \right) - C2(Lx, Ly, e, n)$$
$$\hat{A}_{3,t}(n) = \frac{1}{n-1} \left(\sum_{s \neq t} I(x_{t-Lx}^{m+Lx} - x_{s-Lx}^{m+Lx}, e) \right) - C3(m + Lx, Ly, e, n)$$
$$\hat{A}_{4,t}(n) = \frac{1}{n-1} \left(\sum_{s \neq t} I(x_{t-Lx}^{Lx} - x_{s-Lx}^{Lx}, e) \right) - C4(Lx, e, n)$$
$$t, s = max(Lx, Ly) + 1, \dots, T - m + 1$$

where the $C_i(\cdot, n)$ correlation integrals are defined in Equation (6) ,and the $I(\cdot)$ indicators are described in Nonlinear Granger Causality. The estimator for $\boldsymbol{d}$ can be written as;

Appendix Equation (8)

$$\widehat{\boldsymbol{d}}(\mathrm{n}) = \left[\frac{1}{C2(Lx, Ly, e, n)}, \frac{-\mathrm{C1(m + Lx, Ly, e, n)}}{C2^2(Lx, Ly, e, n)}, \frac{-1}{C4(Lx, e, n)}, \frac{\mathrm{C3(m + Lx, e, n)}}{C4^2(Lx, e, n)} \right]$$

[13] Where E denotes expected value

Finally, a consistent estimator for $\sigma^2(\mathrm{m, Lx, Ly, e, n})$ can be written as;

Appendix Equation (9)

$$\hat{\sigma}^2(\mathrm{m, Lx, Ly, e, n}) = \widehat{\boldsymbol{d}}(\mathrm{n}) * \hat{\Sigma}(\mathrm{n}) * \widehat{\boldsymbol{d}}(\mathrm{n})'$$

The test statistics for the nonlinear Granger causality tests use this variance estimator, see Hiemstra & Jones (1993) and Hiemstra & Jones (1994) for more detailed discussions of the HJ-test.

Source: (Hiemstra & Jones, 1994)

Appendix 2.

The VIX

Appendix Equation (10)

$$\sigma^2 = \left(\frac{2}{T}\right)\sum_t \left(\frac{\Delta K_i}{{K_i}^2}\right) * e^{(RT)} * Q(K_i) - \left(\frac{1}{T}\right) * \left[\left(\frac{F}{K^0}\right) - 1\right]^2$$

$$where \sigma = (VIX/100) \Longrightarrow VIX = \sigma * 100,$$

$$F\ is\ the\ forward\ index\ level\ derived\ from\ index\ option\ prices,$$

$$T\ is\ the\ time\ to\ expiration,$$

$$R\ is\ the\ risk-free\ interest\ rate\ to\ expiration,$$

$$K^0 is the first strike below the forward index level F,$$

$$K_i is the strike price of the i^{th} out-of-the-money option; a call if K_i > K^0 and put if$$

$$K_i < K^0; both put and call if K_i = K^0,$$

$$\Delta K_i\ is\ the\ interval\ between\ strike\ prices$$

$$-\ half\ the\ difference\ between\ the\ strike\ on\ either$$

$$side\ of K_i\text{:}\ \Delta K_i = ({K_i}^{+1} - {K_i}^{-1})/2,$$

$$Q(K_i) is the midpoint of the bid-ask spread for each option with strike K_i$$

Source: (*Cboe VIX FAQ*, n.d.).

Appendix 3.

Information Criterion

Appendix Equation (11)

$$AIC(n) = lndet\left(\Sigma_{\mathrm{u}}^{\sim}(n)\right) + (2/T) * n * K^2$$
$$SBIC(n) = lndet\left(\Sigma_{\mathrm{u}}^{\sim}(n)\right) + (ln(T)/T) * n * K^2$$
$$with\ \ \Sigma_{\mathrm{u}}^{\sim}(n) = T^{-1} \sum_{t=1}^{T} \hat{u}_{\mathrm{t}} \hat{u}_{\mathrm{t}}'$$

$and\ n *\ is\ the\ total\ number\ of\ parameters\ in\ each\ equation\ and\ n\ assigns\ the\ lag\ order$

Sources: (Akaike, 1974) & (Schwarz, 1978)

Appendix 4.

Augmented Dickey-Fuller and Kwiatkowski-Phillips-Schmidt-Shin Tests

ADF-testing procedure:

Appendix Equation (12)

$$\Delta \mathrm{y}_t = \alpha + \beta_t + \gamma y_{t-1} + \delta_1 \Delta \mathrm{y}_{t-1} + \cdots + \delta_{p-1} \Delta \mathrm{y}_{t-p+1} + \varepsilon_t$$
$$DF_{\tau} = \frac{\hat{\gamma}}{SE(\hat{\gamma})}$$

For more elaboration see (Dickey & Fuller, 1979, 1981).

KPSS-testing procedure:

Appendix Equation (13)

$$\mathrm{x}_{\mathrm{t}} = \alpha\ *\ \mathrm{t} + \mathrm{u}_{\mathrm{t}} + \mathrm{e}_{\mathrm{t}}$$
$$where\ u_t\ satisfies\ u_t = u_{t-1} + \varepsilon_t,\ and\ \varepsilon_t\ are\ i.i.d.\ (o,\ \sigma^2)$$
$$where\ \varepsilon_t\ represent\ a\ white\ noise\ term$$

For more elaboration see (Kwiatkowski et al., 1992).

Appendix 5.

Johansen Cointegration Test

Given a general VAR:

Appendix Equation (14)

$$\mathrm{X_t = \Pi_1 X_{t-1} + \cdots + \Pi_k X_{t-k} + \mu + \Phi D_t + \varepsilon_t,\;\; t = 1, \dots, T}$$

Then VECM exists:

Appendix Equation (15)

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_k \Delta X_{t-k} + \Pi_k X_{t-k} + \mu + \Phi D_t + \varepsilon_t$$
$$where\ if\ long - run\ impact, \qquad \Gamma_i = -(\mathrm{I} - \Pi_1 - \cdots - \Pi_\mathrm{i}), i = 1, \dots, k-1$$
$$where\ else\ if\ transatory\ impact, \quad \Gamma_i = -(\Pi_{\mathrm{i}+1} + \cdots + \Pi_\mathrm{k}), i = 1, \dots, k-1$$
$$and, \Pi = -(\mathrm{I} - \Pi_1 - \cdots - \Pi_k)$$

For more elaboration see (Johansen, 1988, 1991; Johansen & Juselius, 1990)

Appendix 6.

White's Heteroskedasticity-consistent Standard Errors and Test

Auxiliary regression in general form:

Appendix Equation (16)

$$\mathrm{\hat{u}^2 = \delta_0 + \delta_1 X_1 + \delta_2 X_2 + \cdots + \delta_k X_k + \delta_{k+1} X_1^2 + \delta_{k+2} X_2^2 + \cdots}$$
$$\ldots + \delta_{2k} X_k^2 + \delta_{2k+1} X_1 * X_2 + \cdots + \delta_{2k+k(k-1)/2} X_{k-1} * X_k$$
$$where\ \hat{u}\ represents\ the\ residual\ series\ from\ the\ original\ regression$$
$$where\ the\ test\ statistic\ is\ given\ by \quad WT = TR^2, where\ T\ is\ the\ sample\ size$$

White's Heteroskedasticity-consistent standard errors:

Appendix Equation (17)

$$\mathrm{HC = (X'X)^{-1} X' \Omega X (X'X)^{-1}}$$

For more elaboration on the calculations of the test or standard errors see (White, 1980).

Appendix 7.

Stationarity Tests of Subsamples

All lag lengths are chosen with AIC and SBIC, for univariate (lag space of 40) and bivariate (lag space of 20). All stationarity tests were conducted at the 1 percent significance level.

Appendix Table 1: Stationarity Test Results for Subsample (A)

		Stationarity Test Results for 0DTE Volume			
		Augmented Dickey-Fuller		KPSS	
Data	Lag Length	Test Statistic	Result	Test Statistic	Result
Total Volume	15	-4.50544	Stationary	1.88428	Non-Stationary
Total Volume	10	-4.58006	Stationary	2.51542	Non-Stationary
Total Volume	3	-7.27065	Stationary	5.26442	Non-Stationary
Log Changes	14	-6.91575	Stationary	0.02159	Stationary
Log Changes	10	-9.88172	Stationary	0.01675	Stationary
Log Changes	2	-16.84304	Stationary	0.00576	Stationary

		Stationarity Test Results for VIX Index			
		Augmented Dickey-Fuller		KPSS	
Data	Lag Length	Test Statistic	Result	Test Statistic	Result
Spot	10	-2.87005	Non-Stationary	0.69553	Stationary
Spot	6	-2.81364	Non-Stationary	1.03372	Non-Stationary
Spot	3	-2.63585	Non-Stationary	1.74004	Non-Stationary
Spot	1	-2.84146	Non-Stationary	3.37902	Non-Stationary
Log Changes	10	-6.04473	Stationary	0.01675	Stationary
Log Changes	2	-12.01111	Stationary	0.05706	Stationary
Log Changes	1	-15.46749	Stationary	0.05317	Stationary

Appendix Table 2: Stationarity Test Results for Subsample (B)

		Stationarity Test Results for 0DTE Volume			
		Augmented Dickey-Fuller		KPSS	
Data	Lag Length	Test Statistic	Result	Test Statistic	Result
Total Volume	10	-2.88436	Non-Stationary	3.35806	Non-Stationary
Total Volume	5	-3.8593	Non-Stationary	5.96005	Non-Stationary
Total Volume	3	-5.32205	Stationary	8.72844	Non-Stationary
Log Changes	9	-9.32332	Stationary	0.04373	Stationary
Log Changes	5	-12.56431	Stationary	0.02775	Stationary
Log Changes	2	-18.0545	Stationary	0.01449	Stationary

		Stationarity Test Results for VIX Index			
		Augmented Dickey-Fuller		KPSS	
Data	Lag Length	Test Statistic	Result	Test Statistic	Result
Spot	5	-3.28075	Non-Stationary	0.9559	Non-Stationary
Spot	3	-3.37966	Non-Stationary	1.36771	Non-Stationary
Spot	2	-3.53872	Non-Stationary	1.77566	Non-Stationary
Spot	1	-3.77245	Non-Stationary	2.58279	Non-Stationary
Log Changes	9	-7.09563	Stationary	0.04373	Stationary
Log Changes	2	-13.43289	Stationary	0.0632	Stationary
Log Changes	1	-16.2133	Stationary	0.0569	Stationary

The results in both Appendix Table 1 and Appendix Table 2 show clear evidence of non-stationarity in the data when in their original levels, and stationarity when log first differences are taken.

Appendix 8.

Johansen Tests for Subsamples

Appendix Table 3: Johansen Test for Subsample (A)

H0: Cointegration of Rank 0				H0: Cointegration of Rank 1			
Type	Lag Length	Test Statistic	Critical Value	Type	Lag Length	Test Statistic	Critical Value
Eigen	10	8.28	19.19	Eigen	10	5.48	11.65
Eigen	2	60.49 ***	19.19	Eigen	2	6.04	11.65
Eigen	3	32.59 ***	19.19	Eigen	3	5.18	11.65
Trace	10	13.76	23.52	Trace	10	5.48	11.65
Trace	3	66.53 ***	23.52	Trace	3	6.04	11.65
Trace	2	37.77 ***	23.52	Trace	2	5.18	11.65

Appendix Table 4: Johansen Test for Subsample (B)

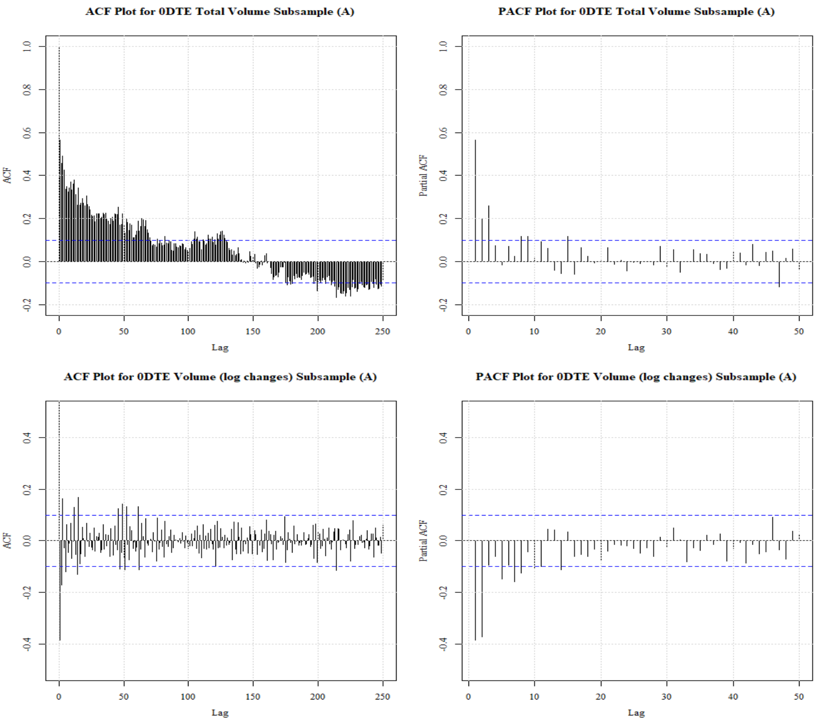
H0: Cointegration of Rank 0				H0: Cointegration of Rank 1			
Type	Lag Length	Test Statistic	Critical Value	Type	Lag Length	Test Statistic	Critical Value
Eigen	9	7.29	19.19	Eigen	9	0.78	11.65
Eigen	5	10.79	19.19	Eigen	5	2.09	11.65
Eigen	2	15.82	19.19	Eigen	2	9.12	11.65
Eigen	3	12.84	19.19	Eigen	3	4.31	11.65
Trace	9	8.07	23.52	Trace	9	0.78	11.65
Trace	5	12.88	23.52	Trace	5	2.09	11.65
Trace	2	24.94 ***	23.52	Trace	2	9.12	11.65
Trace	3	17.15	23.52	Trace	3	4.31	11.65

There is some evidence of cointegration in the subsample period from 02.01.2019 to 02.07.2021 (Appendix Table 3), it would be appropriate to apply a VECM when conducting linear and nonlinear causality testing. For the subsample 02.07.2021 to 22.05.2023 (Appendix Table 4) there is however more evidence of no cointegration, and therefore appropriate to use log first order differences.

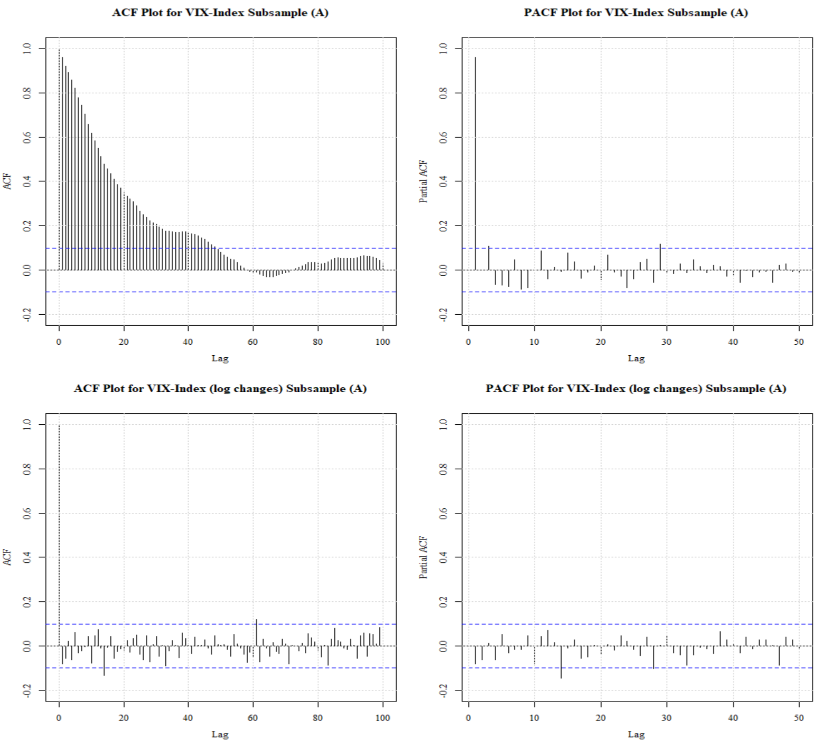
Appendix 9.

ACF & PACF and Histogram and Density plots for subsamples

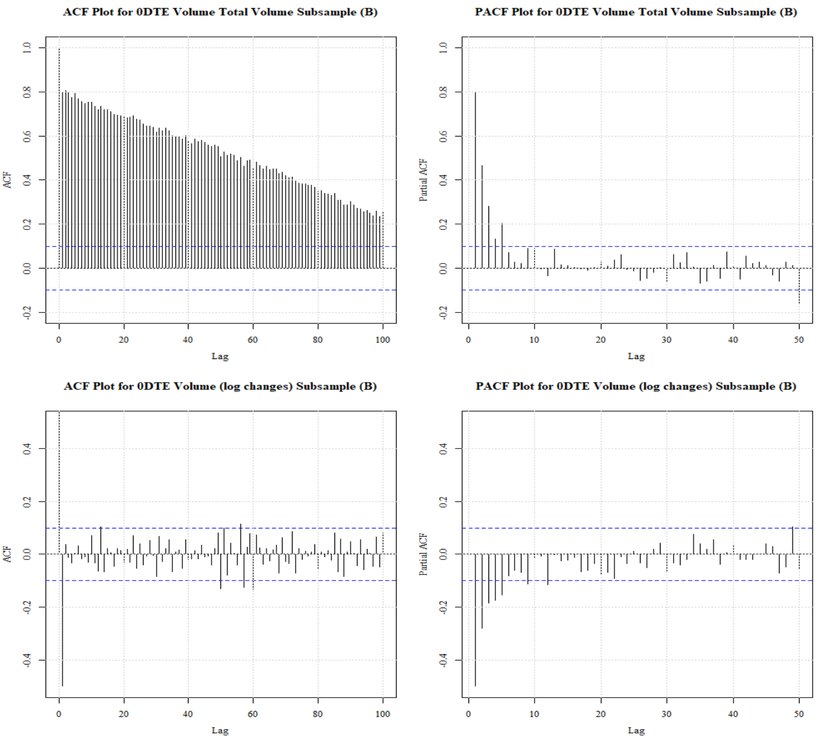
Appendix Figure 1: ACF and PACF for 0DTE Volume in Subsample (A)



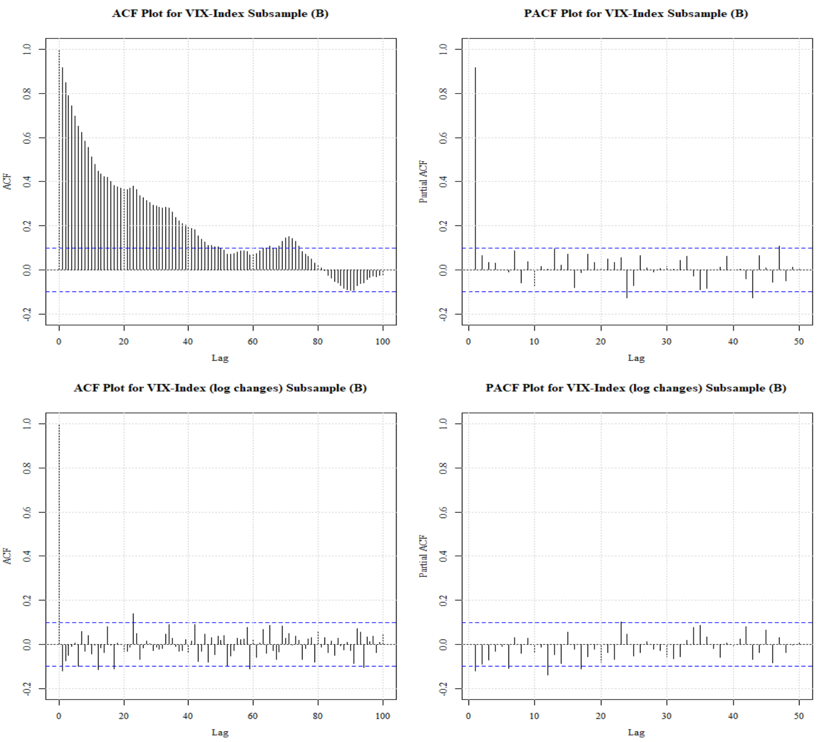
Appendix Figure 2: ACF and PACF for VIX Index in Subsample (A)



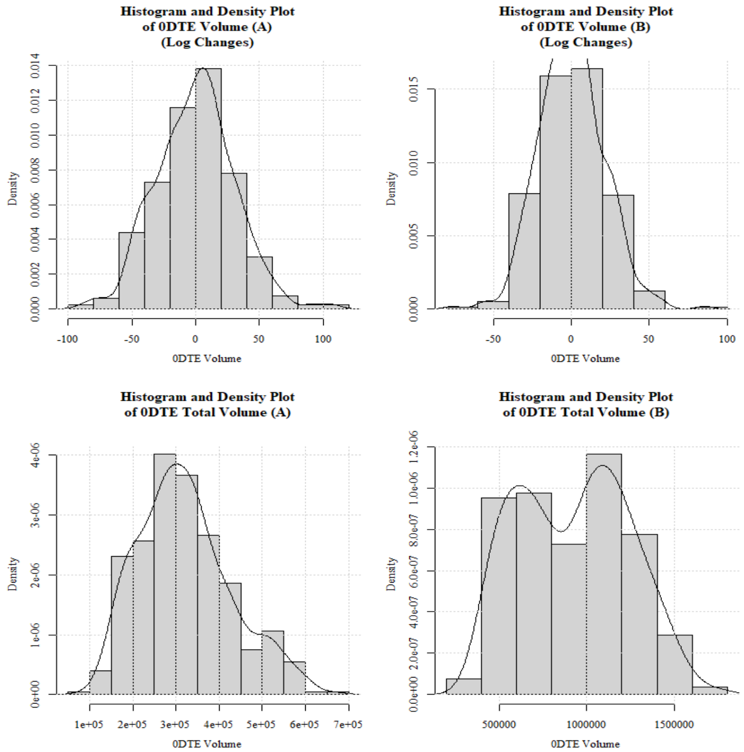
Appendix Figure 3: ACF and PACF for 0DTE Volume in Subsample (A)



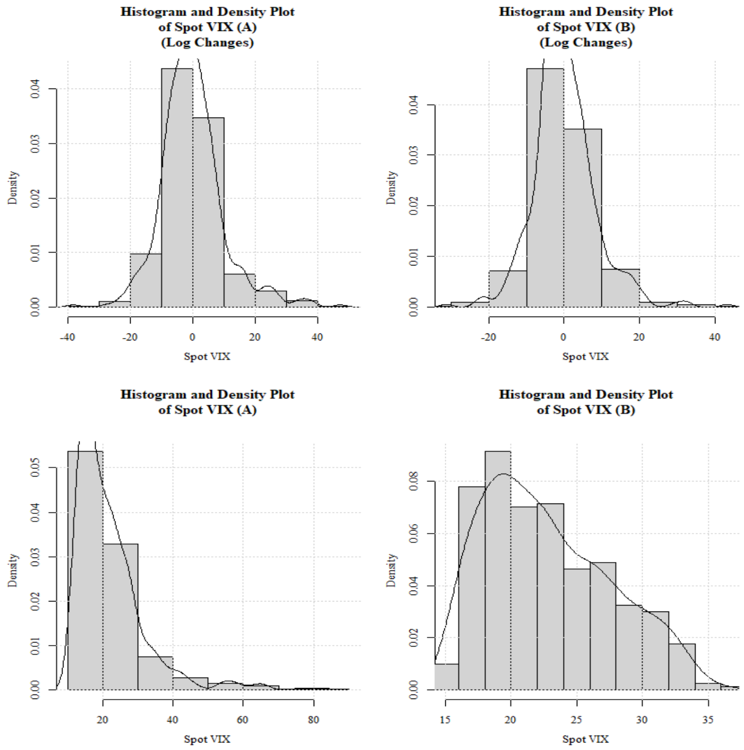
Appendix Figure 4: ACF and PACF for VIX Index in Subsample (B)



Appendix Figure 5: Histogram and Density for 0DTE Volume in Subsamples



Appendix Figure 6: Histogram and Density for VIX Index in Subsamples

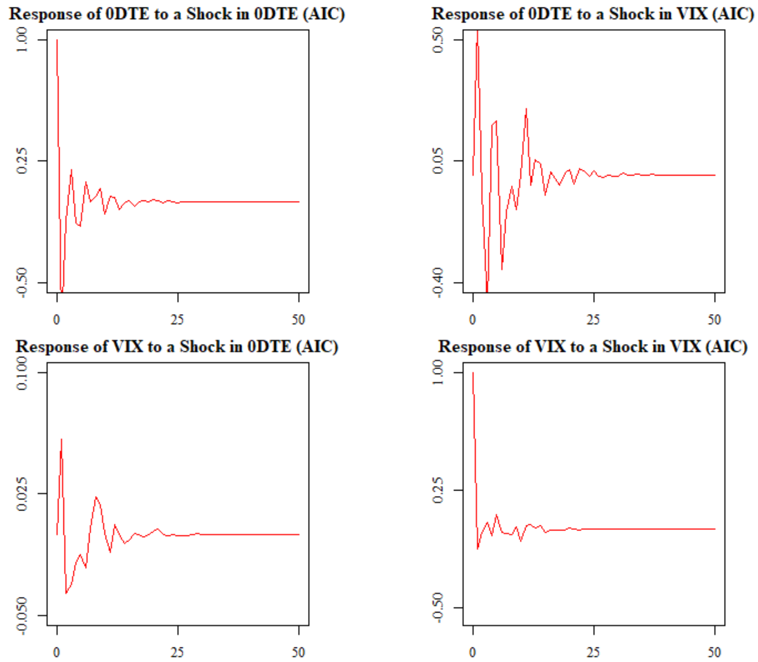


Appendix 10.

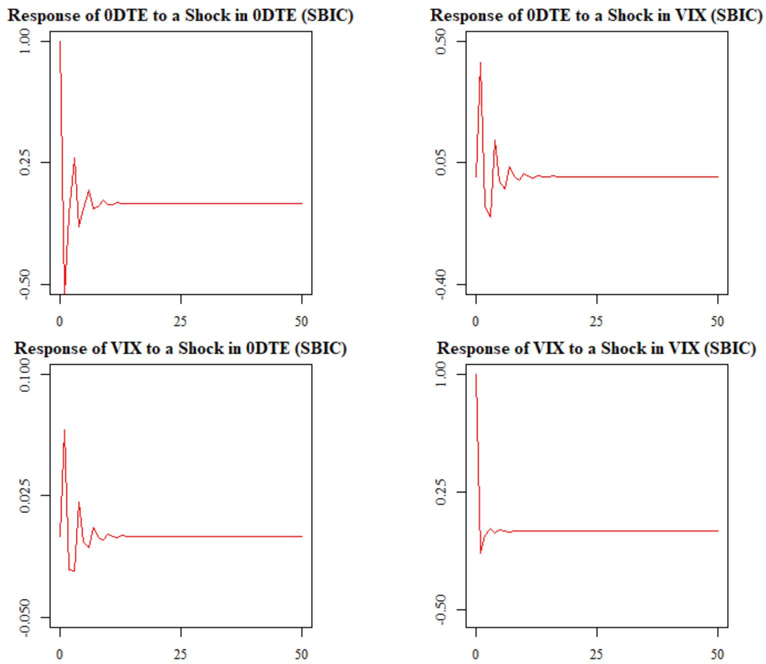
Impulse Response Function and Variance Decomposition of Subsample VARs

We start with subsample of 02.01.2019 – 02.07.2021 (A).

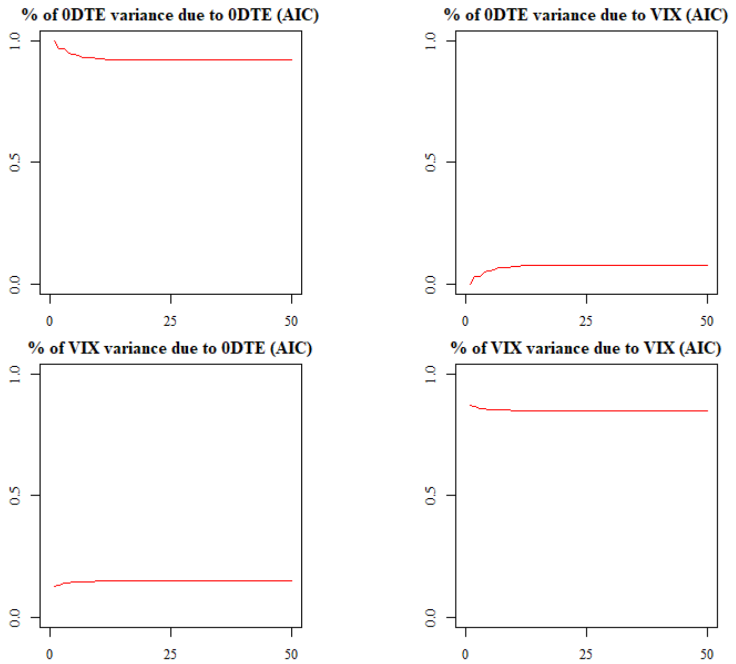
Appendix Figure 7: Impulse Response Function (AIC) with Subsample A



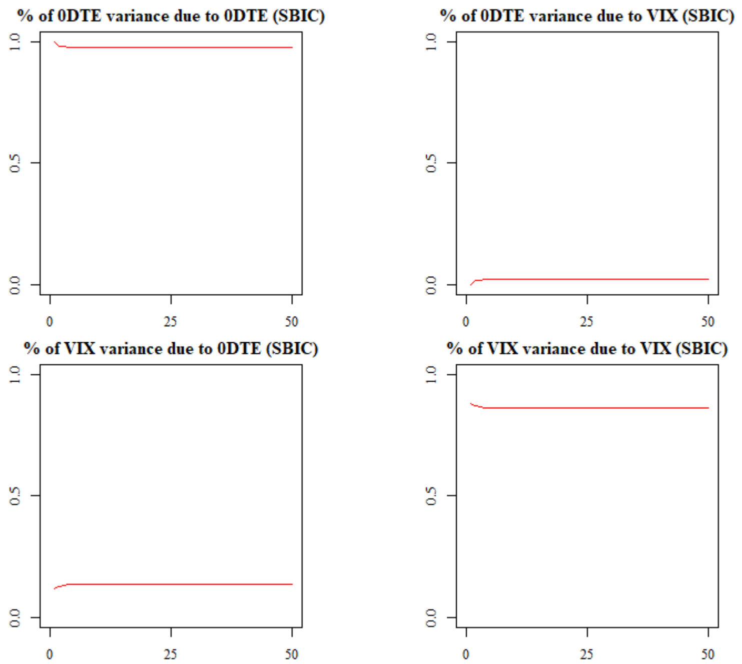
Appendix Figure 8: Impulse Response Function (SBIC) with Subsample A



Appendix Figure 9: Variance Decomposition (AIC) with Subsample A

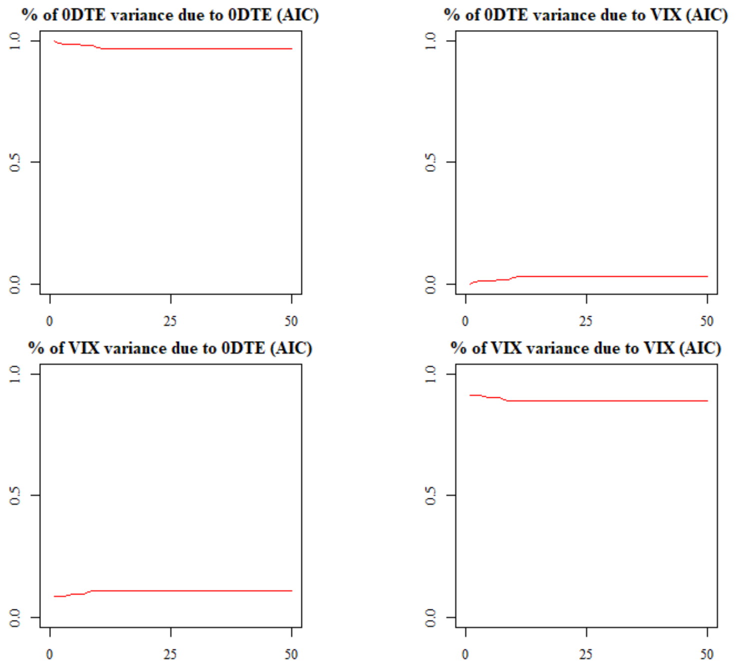


Appendix Figure 10: Variance Decomposition (SBIC) with Subsample A

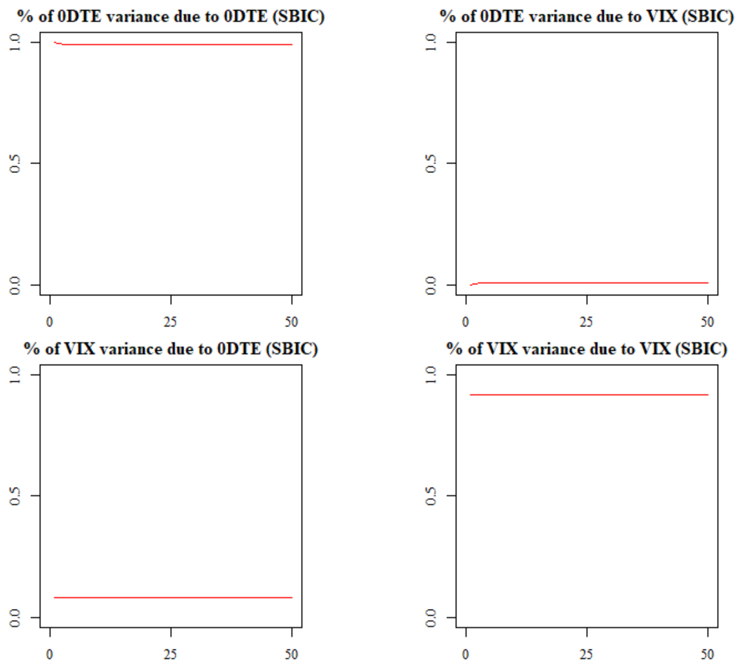


The next plots are for corresponding plots for the subsample 02.07.2021 – 22.05.2023 (B).

Appendix Figure 11: Variance Decomposition (AIC) with Subsample B

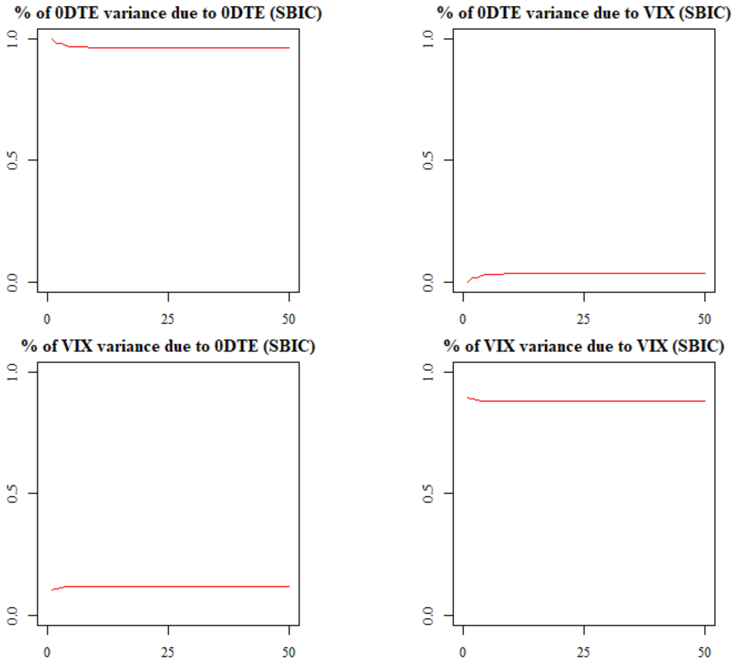


Appendix Figure 12: Variance Decomposition (SBIC) with Subsample B



Lastly the plot for variance decomposition with the full sample size using SBIC.

Appendix Figure 13: Variance Decomposition (SBIC) with Full Data Range



Appendix 11.

Robustness check of Hiemstra & Jones Test

To be sure of the self-built script in R used to the apply the HJ-test, I ran the nonlinear test where both variables were the same time series. This resulted in a difference in Equation (5) of zero. The results are shown in Appendix Table 5.

Appendix Table 5: Robustness Check of HJ-test Model

	H0: 0DTE Volume Changes Do Not Cause 0DTE Volume Changes			H0: VIX Level Changes Do Not Cause VIX Level Changes	
Lx = Ly	CS	TVAL	Lx = Ly	CS	TVAL
1	0	0	1	0	0
2	0	0	2	0	0
3	0	0	3	0	0
4	0	0	4	0	0
5	0	0	5	0	0
6	0	0	6	0	0
7	0	0	7	0	0
8	0	0	8	0	0

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