Optimal Option Portfolio Strategies: Risk or Mispricing?

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Dissertation written under the supervision of Professor José Faias.

Dissertation submitted in partial fulfilment of requirements for the MSc in Finance, at the Universidade Católica Portuguesa, 3 January 2021.

Abstract

Options are known for having return distributions that depart from normality and lacking depth of sample data. These characteristics are the reason why conventional methods of portfolio optimization become obsolete for these securities. Therefore, I analyze the original Optimal Option Portfolio Strategies (OOPS) in Faias & Santa-Clara (2017), proven to tackle option allocation problems properly, in three different frequencies: Monthly, weekly and weekly decomposed. All the strategies provided attractive results in comparison to their corresponding benchmarks, with Sharpe ratios and certainty equivalents ranging between 0.57 and 2.29, and 2.60% and 52.01%, respectively. I find that these strategies (with the exception of the weekly decomposed OOPS) tend to fuel their returns not only on mispricing, but also marginally on jump risk and volatility risk premia, which contradicts past literature.

Key-words: Options, portfolio optimization, Optimal Option Portfolio Strategies (OOPS), Sharpe ratios, certainty equivalents, mispricing, jump risk and volatility risk premia.
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Abstrato

As opções são conhecidas pelas suas distribuições de retornos não normais e falta de dados históricos. Estas características constituem as principais razões que explicam a obsolescência de métodos tradicionais de alocação de capital para este grupo de ativos. Nesse sentido, eu analiso as originais Estratégias Ótimas de Portfolios de Opções (EOPO) em Faias & Santa-Clara (2017), que resolvem este problema adequadamente, em três frequências diferentes: Mensal, semanal e semanal decomposta. Todas as estratégias oferecem resultados atrativos em comparação com o portfolio de referência, atingindo rácios de Sharpe e equivalentes de certeza entre 0.57 e 2.29, e 2.60% e 52.01%, respetivamente. Eu adianto que as estratégias (com a exceção da semanal decomposta) tendem a originar os seus ganhos através, não só de apreciações incorretas, mas também de risco de salto e prémios de risco de volatilidade marginalmente, o que contradiz literatura passada.

Palavras-Chave: Opções, alocação de capital, Estratégias Ótimas de Portfolios de Opções (EOPO), rácios de Sharpe, equivalentes de certeza, apreciações incorretas, risco de salto e prémios de risco de volatilidade.
Title: Estratégias Ótimas de Portfolios de Opções: Risco ou apreciação incorreta?
Author: Lourenço Alves

Acknowledgements

Firstly, I am thankful for the support of my supervisor, Professor José Faias. I am grateful for all the effort made in my behalf and not only for contributing for the steepest learning curve possible, but also for enhancing my interest for this scientific field. Your time and dedication enabled the elevation of my output and most importantly, my knowledge. In addition, I would also like to thank Fundação para a Ciência e Tecnologia (FCT).

Secondly, I am thankful for all the professors that crossed my path and provided the fundamental tools to build this paper. Particularly, I would like to emphasize my gratitude towards Professor Mário Meira, that triggered my passion for finance and with whom I had the pleasure to work with, as a teaching assistant for the course of Finance I.

Thirdly, to all my friends at CLSBE and beyond, thank you for all the great moments and adventures that we have lived together, which certainly enriched my path through this institution. Specially, I thank Johnny Costa, Tiago Sardinha, Mackenzie Ferreira, Luís Frade, Max Werner, Marvin Ulbrich, Rui Ferreira, João Gomes, Vasco Minas, Martim Cruz and my dear group of friends "Piso 3".

Fourthly, I want to express my sense of gratitude towards my girlfriend, Catarina Oliveira. Your unconditional support and care played a crucial role in my success as a student and person. I dearly appreciate your patience and most importantly, your friendship.

Finally, I would like to thank my family and particularly my brother-in-law, not only for being the foundation of who I am today, but also for taking the time to reflect upon my work and giving me thoughtful and sought-for insights.

Contents

List of Tables

List of Figures

1 Introduction

It is known and documented that options hold several benefits to an investor when correctly used. For example, Ross (1976) argues that these contracts help span states of nature. Additionally, Bates (1996), Bakshi et al. (1997), Andersen et al. (2002) and Liu & Pan (2003), among many others, report that these securities provide valuable exposure to priced risk factors such as stochastic volatility and jumps. Despite the mentioned characteristics, options are rarely integrated in investment portfolios. In fact, large institutional investors, like Mutual Funds, find it hard to engage with such securities, since options' volatile nature leads regulators to limit the usage of such assets. The limited Mutual Fund investment in these assets is in fact supported by past literature (Koski & Pontiff (1999), Deli & Varma (2002), Almazan et al. (2004)). The exception lies with hedge funds, that are free to invest in options. Nevertheless, these securities still represent a low percentage of their total invested capital.

It is worthwhile to investigate why securities that can deliver benefits for investment portfolios are often disregarded. In fact, the common portfolio optimization methods are proven to be obsolete for this kind of securities. There are three major explanations to this statement.

Firstly, the option's returns' distribution usually presents substantial skewness and kurtosis, thus departing from a normal distribution. Consequently, means, variances and covariances are not sufficient to make economically sound decisions with these securities.

Secondly, options are a relatively recent financial instrument, meaning that history of option returns is rather limited. This translates into a crippled precision of estimation of their joint distribution, and therefore, into considerable estimation error of the investment portfolio's distribution. Moreover, Michaud & Michaud (2007) report that this issue is particularly concerning since the optimization process leads to increased estimation noise.

Thirdly, the options' market tends to register high transaction costs, with at-the-money (ATM) and out-of-the-money (OTM) options reporting an average relative Bid-Ask spread of 5.00% and 10.00%, respectively. Due to their magnitude, it is very important to consider these costs in the optimization process, which can be challenging to do.

Faias & Santa-Clara (2017) draw inspiration from Brandt et al. (2009), and propose a simple, yet effective optimization strategy: Optimal Option Portfolio Strategies (OOPS). In this thesis, I will test this strategy between January 1996 and December 2019 and through a significant evolution of the option's market's volume and diversity of contracts.

The decision to challenge the OOPS originated in its intriguing solutions to the previously mentioned problems, regarding option portfolio optimization. Firstly, the OOPS considers all the moments of the distribution of portfolio returns, since it maximizes an expected power utility, penalizing negative skewness and high excess kurtosis. Secondly, since the OOPS algorithm's only inputs historical data from the underlying asset, the short option data history does not present an obstacle to the strategy. Finally, the strategy incorporates transaction costs, following Eraker (2013) and Plyakha & Vilkov (2008). To achieve this, Faias & Santa-Clara (2017) decompose each option into a "long option" initiated at the ask quote, and a "short option", initiated at the bid quote, inputting the "short options" in the optimization with a negative sign and establishing a no short selling constraint.

The OOPS is a hold-till-maturity strategy and defines the portfolio's option weights by simulating the respective 1-period options returns and later calculating the capital allocation choice that would average the maximum expected utility possible across all the different simulations, thus arriving at the optimal portfolio weights. Given the strategy, the simulations are built upon the option's quotes and their respective simulated payoffs, originated from the underlying asset's price simulation. The latter simulation is performed using a simple bootstrap method, adequate for index data, since 1950.

In this thesis, the explained algorithm will be used in two distinct sets of securities. Firstly, I will apply Faias & Santa-Clara (2017) method to a portfolio allocation problem, comprising a one-month riskless asset and four European options on the S&P 500 index with one month to maturity. Secondly, I will study a very similar problem, this time composed by a one-week risk-free security and four European options on the S&P 500 with one week to maturity. This analysis was decided upon inspecting the increasing number of options traded in the market. Analyzing the Chicago Board Options Exchange (CBOE) option volume data, it is evident that this market grew at a rapid rate, with shorter maturity options being traded more and more frequently. With this new market segment meeting the asset screening requirements for the OOPS and a possible analysis period of 7 years, I decided to test the strategy in such maturity setting.

With the exception of time to maturity, both groups of assets are very similar. The options belonging to both groups are liquid and can be arranged to generate a wide diversity of payoffs. I will study optimal allocation strategies for both asset groups in a realistic setting, considering that transactions costs will not be neglected, as stated above.

With this in mind, I will firstly analyze the monthly OOPS' performance in an out-of-sample setting, using a rather conservative coefficient of relative risk aversion (CRRA) utility function, between January 1996 and December 2019. For this exercise, the strategy delivers promising results, providing an annualized certainty equivalent of 8.58% and Sharpe ratio of 1.00, comparing to the market's 0.14% and 0.29, for the same period[1]. Even though shorting individual options presents extreme values of negative skewness and positive excess kurtosis, the OOPS' return distribution reports a lower negative skewness and a moderate excess kurtosis, outperforming the former individual option strategy. The

[1]It is relevant to notice that instead of losing value, the recreated OOPS accentuated Faias & Santa-Clara (2017) results, which report a Sharpe ratio of 0.82, for the corresponding strategy.

algorithm takes valid positions on all four options, being that their correspondent optimal weights vary through time. Faias & Santa-Clara (2017) expected some level of exposure to the stock market, but their results proved otherwise, registering a low beta and almost neutral delta.

The results presented in this thesis are much more in line with Faias & Santa-Clara (2017) expectations than with the authors' actual results. In fact, the replicated algorithm registered a portfolio beta of 0.45. This is predictable since the return distribution, that represents a relevant input for the algorithm's capital allocation, is calibrated to market data spanning from 1950 to 2020, which reports an equity Sharpe ratio of 0.48.

Faias & Santa-Clara (2017) argue that there are two reasonable rationales to buy or sell options besides obtaining exposure to equity premium, with these being either to load up on priced risk factors, like volatility and jump risk, or to exploit option mispricing. Despite concluding that the original OOPS derives its performance from leveraging option mispricing, this thesis argues that exploiting pricing inefficiencies does not fully explain the strategy's performance, which seems to be in accordance with the findings of both Brandt et al. (2009) and Andersen et al. (2015), that state that there is limited evidence of option mispricing relative to the respective underlying asset and that the implied volatility dynamics of short-maturity OTM puts, which are a significant component of this strategy, are best explained by jumps, respectively. Additionally, this algorithm also appears to draw gains from market rallies, as the OOPS presents a positive relation with the Fama and French's momentum factor.

Regarding the second asset group, the weekly OOPS performs in a quite impressive way. Surprisingly, the strategy recorded, between January 2012 and December 2019, an annualized certainty equivalent of 52.01% and a Sharpe ratio of 2.29, resultant from an annualized average return and standard deviation of 75.18% and 32.86%. In comparison with the benchmark, it behaves in a very positive manner, since the S&P 500 index registers a certainty equivalent of 8.73% and a Sharpe ratio of 0.91, over the corresponding period. Moreover, comparing to the market, the strategy's return distribution appears to have a more symmetric shape, given its lower skewness, but some relative tail risk, as the OOPS kurtosis presents higher values than the benchmark's.

With these features in mind, how does this strategy outperform the monthly OOPS? The answer resides in frequency of compounding and the naked options summary statistics. In fact, the weekly and monthly strategies allocate weights in a similar way, with the exception of OTM put options. Since the time to maturity is lower, going long on OTM options tends to yield returns less often. Therefore, the weekly OOPS repeatedly shorts these securities more heavily. With this in mind, it is only natural that the weekly strategy registers higher betas and jump risk than the monthly OOPS, averaging a portfolio beta of 1.99 and losing, on average, -5.35% when the underlying asset plummets -5.00%, over the respective week. Exploring the drivers behind such impressive performance, we discover, once again, that its returns are marginally explained by loading up priced risk factors, in addition to mispricing. Despite resulting in the same conclusion, the simple monthly and weekly strategies display different relationships with the mentioned performance drivers.

Lastly, I divide the weekly strategy into four strategies, in which the investor invests in only one week per month and allocates 100.00% of his/her wealth to the riskless asset in the remaining weeks. This is done in order to analyze possible patterns of performance in distinct weeks. We see that the strategies perform well, yielding highly attractive values of annualized mean return, Sharpe ratio and certainty equivalents. In addition, despite the conclusions taken on the simple strategies, the weekly decomposed OOPS performance appear to rely more heavily in mispricing. Moreover, the best performers short out-of-the-money puts more significantly, which can be observed in the first and fourth week of each month. This is intriguing as these contracts present the largest volume amongst the considered weekly option universe. Using volume as a proxy for liquidity, we should witness lower levels of mispricing in this option category. This becomes clear when we find that the best strategies also display a subtle pattern regarding risk exposure. In fact, the success of a strategy tends to be related with better explanatory power of realized volatility on conditional expected returns and greater exposure to jump risk, thus suggesting that the OOPS performs the best when loading on both priced risk factors and mispricing.

Simple option strategies have been studied in a related body of literature. Coval & Shumway (2001) argue that Sharpe ratios close to 1[2] can be obtained by short positions in crash-protected, delta-neutral straddles. Coval & Shumway (2001) and Bondarenko (2003) concludes that shorting naked puts generates high returns, even after considering the high risk associated with the strategy. Later, Driessen & Maenhout (2013) conclude and confirm Coval & Shumway (2001) Sharpe ratio findings.

Despite the literature mentioned above, there is a limited number of papers approaching optimal capital allocation problems with options. Liu & Pan (2003) model stochastic volatility and jump processes and compute the optimal allocation for a CRRA investor with a stock, a 5% OTM put option and cash. Driessen & Maenhout (2007) maximize the average utility of an investor with a stock, and either a 4% OTM put or any crash-neutral option strategy, finding that buying put options is sub-optimal, given historic data. Jones (2006) studies the possibility of exploiting apparent put option mispricing, using a very complex model. Constantinides et al. (2013) study portfolios of both call and put options with a moneyness target. Malamud (2014) studies dynamic capital allocation between OTM options. Both Constantinides et al. (2013) and Malamud (2014) achieve high Sharpe ratios, however along with high kurtosis and negative skewness, translating

[2]The sample period used in Coval & Shumway (2001) is different from the one used in this study, disabling Sharpe ratio comparisons.

into low certainty equivalents. Faias & Santa-Clara (2017) study Optimal Option Portfolio Strategies by maximizing expected utility over a series of option return simulations. Even with their simple empirical model, they find high Sharpe ratios with positive skewness and low kurtosis, and thus, high certainty equivalents. They conclude that the strategy's performance thrives on option mispricing relative to its underlying asset.

Moreover, recent papers support the use of options in investment decisions. First, Wang & Huang (2019) model an option inclusive portfolio's performance via a risk index, arguing that including options in an capital allocation decision adds value to the portfolio. Then, Constantinides et al. (2020) find that a portfolio comprising a fund that follows the S&P 500, European options on the same index and cash, beats the option-free optimal portfolio, registering high returns when the option maturity is short and volatility measures present substantial values.

This thesis is organized in the following order: Section 2 explains the method, section 3 presents the data, section 4 shows and explains the results and section 5 summarizes the thesis' findings.

2 Methodology

2.1 The Algorithm

In this paper, the OOPS algorithm created by Faias & Santa-Clara (2017) was recreated. The algorithm defines a portfolio by allocating different amounts of capital to a risk-free asset and four options, that are not redundant by Put-Call parity and have the same underlying asset (S&P 500 index), namely an at-the-money Call Option, an out-of-the-money Call Option, an at-the-money Put Option and finally an out-of-the-money Put Option.

For notation purposes, all call options will be announced by c and put options by p. Let time be represented by subscript t and simulations indexed by n. At time t, S_t stands for the value of the underlying asset, whilst the exercise price of option i is denoted by $K_{t,i}$. The risk-free rate that prevails from time t to $t+1$ will be represented by rf_t . For each t, weights will be defined by the maximization of the investors expected utility given his/her wealth at time $t+1$, which will be a function of the simulated portfolio returns that derive from individual option returns. Finally, due to the nature of these securities, the option returns depend on the underlying asset's returns. The following steps provide a guide through the OOPS algorithm.

1. N underlying asset's log returns r^n_{t+1}, $n = 1, \ldots, N$ are simulated under the empirical density measure.

2. We use the simulated returns obtained in step 1 to simulate the value of the underlying asset in t+1, given its current value:

$$S^n_{t+1|t} = S_t exp(r^n_{t+1}),$$

where r^n_{t+1}, $n = 1, \dots, N$, and $S^n_{t+1|t}$ stands for the simulated value of the underlying asset in $t+1$ conditional on the available information up to time t, and S_t represents the current observed value of the underlying asset.

3. The options' payoffs are then simulated at their maturity $t+1$ using the observed exercise prices for the calls, $K_{t,c}$, and the puts, $K_{t,p}$, and the simulated 1-period underlying asset's value $S^n_{t+1|t}$ calculated in step 2:

$$C^n_{t+1|t,c} = max(S^n_{t+1|t} - K_{t,c}, 0),$$

$$P^n_{t+1|t,p} = max(K_{t,p} - S^n_{t+1|t}, 0),$$

where $n = 1, \dots, N$. Taking into account the payoffs simulated by the equation above and the known current option prices $C_{t,c}$ and $P_{t,p}$, the option returns are given by:

$$r^n_{t+1|t,c} = \frac{C^n_{t+1|t,c}}{C_{t,c}} - 1,$$

$$r^n_{t+1|t,p} = \frac{P^n_{t+1|t,p}}{P_{t,p}} - 1,$$

where $n = 1, \dots, N$.

4. After the previous steps, portfolio returns are computed according with the following equation:

$$rp^n_{t+1|t} = rf_t + \sum_{c=1}^{C} \omega_{t,c}(r^n_{t+1|t,c} - rf_t) + \sum_{p=1}^{P} \omega_{t,p}(r^n_{t+1|t,p} - rf_t),$$

where $\omega_{t,c}$ and $\omega_{t,p}$ denote the call and put option weights and $n = 1, \dots, N$. The portfolio return is therefore a weighted average of the options' and the risk-free returns, being that only the latter is not simulated.

5. The weights are then decided based on the maximization of the expected utility obtained through the simulated portfolio returns:

$$\max_{\omega} E[U(W_t[1 + rp_{t+1|t}])] \approx \max_{\omega} \frac{1}{N} \sum_{n=1}^{N} U(W_t[1 + rp^n_{t+1|t}]),$$

with the output given by $\omega_{t,c}$ and $\omega_{t,p}$.

6. Finally, the one-period out-of-sample performance is evaluated with actual option returns. The option realized payoffs are given by:

$$C_{t+1,c} = max(S_{t+1} - K_{t,c}, 0),$$

$$P_{t+1,p} = max(K_{t,p} - S_{t+1}, 0),$$

and the corresponding returns are obtained by:

$$r_{t+1,c} = \frac{C_{t+1,c}}{C_{t,c}} - 1$$

$$r_{t+1,p} = \frac{P_{t+1,p}}{P_{t,p}} - 1.$$

To conclude, the one-period out-of-sample portfolio returns are computed, using the weights obtained in step 5:

$$rp_{t+1} = rf_t + \sum_{c=1}^{C} \omega_{t,c}(r_{t+1,c} - rf_t) + \sum_{p=1}^{P} \omega_{t,p}(r_{t+1,p} - rf_t).$$

Summarizing, the OOPS algorithm holds an option's price against its expected payoff and risk, that comes from the assumed probability distribution of the underlying asset, evidenced in the simulation. Taking this into account, if an option price is low relative to the expected payoff and risk, the investor buys it, and if the contrary is true, the investor sells it.

2.2 Return Simulation

The algorithm described in subsection 'The algorithm' requires a simulation of the distribution of the underlying asset returns. Given the fact that the distribution of raw stock market returns is negatively skewed and leptokurtic, therefore departing significantly from a normal distribution (as reported in Table I), the algorithm uses the simulation of the distribution of the underlying asset's standardized returns, denoted as SR, that are expressed as the ratio between raw returns, RR, and their standard deviation, $SDEV$:

$$SR^n_{t+1} = \frac{RR^n_{t+1}}{SDEV_{t+1}}, n = 1, ..., N.$$

Studying the historic SR, we can conclude that this series is closer to a normal distribution, despite presenting a negative skew. Moreover, in support of this methodology, Faias & Santa-Clara (2017), report autoregressive conditional heteroskedasticity (ARCH) effects on the distribution of raw returns, that are no longer significant for the distribution of standardized returns.

Starting in January 1996, different paths for the standardized returns of the underly-

Table I: S&P 500 index returns: Summary Statistics

Table I reports summary statistics (number of observations, skewness, excess kurtosis, 1-month autocorrelation of returns and squared returns) and Ljung & Box (1978) test (with the respective p-values in square brackets) for raw and standardized S&P 500 index returns.

	Raw Returns			Standardized Returns		
Statistics	1950-1995	1996-2020	1950-2020	1950-1995	1996-2020	1950-2020
N. of obs.	552	289	841	552	289	841
Skewness	−0.56	−0.85	−0.67	−0.40	−0.81	−0.47
Excess Kurtosis	3.01	1.69	2.49	0.73	2.27	1.15
$\rho_1(Z)$	0.03	0.05	0.04	0.03	-0.01	0.02
$\rho_1(Z^2)$	0.10	0.22	0.14	−0.08	−0.01	−0.04
$Q_1(Z)$	0.44	0.75	1.14	0.47	0.02	0.35
	[0.51]	[0.39]	[0.28]	[0.49]	[0.88]	[0.55]

ing asset are simulated based on an expanding sample of historical standardized returns starting in January 1950. Using bootstrap, I resample from the standardized empirical distribution, preserving the same moments as the historical distribution by construction, thus following Efron & Tibshirani (1994).

To finally achieve the desired returns, we need to scale up or down the standardized returns by the current realized volatility:

$$r_{t+1}^{n} = sr_{t+1}^{n} \times rv_t, n = 1, ..., N.$$

This metric is calculated based on the last d trading days and scaled by the average number of trading days in a month (21) to get monthly values. We consider several levels for d from 1 to 60 days, being that in each month we only use the realized volatility that maximizes the expected utility in sample. This method is known for incorporating time-varying volatility, thus being similar to the filtered historical simulation of Barone-Adesi et al. (1997), where volatility is estimated based on a parametric method such as a GARCH model. Hull & White (1998), Diebold et al. (1998), and Barone-Adesi et al. (2008) also build on the same idea. Nevertheless, Faias & Santa-Clara (2017) conclude that this method delivers quality results even in simple settings.

2.3 Maximizing Expected Utility

As stated in step 4 of the subsection 'The Algorithm', the conditional expected utility of the next period's wealth is maximized, taking into consideration the budget constraint

$W_{t+1} = W_t(1 + rp_{t+1})$:

$$\max_{\omega_{t,i} \in \mathbb{R}} E[U(W_{t+1})].$$

When returns are normal, investors care only about the mean and standard deviation of portfolio returns, but this is not what an investor normally faces in the real world. In practice, especially for option returns, normality does not hold and therefore investors care about tail risk (kurtosis) and asymmetry (skewness), given that mean and standard deviation do not provide enough information to make capital allocation decisions. Therefore, maximizing expected utility must take into account all the moments of the return distribution.

Faias & Santa-Clara (2017) use the power utility function, evidenced in Brandt (1999). This utility function presents coefficient of relative risk aversion (CRRA) denoted by γ. The function is given by:

$$U(W) = \begin{cases} \frac{1}{1-\gamma} W^{1-\gamma}, if \gamma \neq 1, \\ \ln W, if \gamma = 1. \end{cases}$$

Firstly, the homotheticity property makes the portfolio weights independent of the initial level of wealth, thus meaning that maximizing $E[U(W_{t+1})]$ is the same as maximizing $E[U(1 + rp_t)]$. Secondly, the function penalizes negative skewness and high kurtosis, being that these metrics become relevant as returns do not follow normality. These two characteristics ensure a quality fit for the selected function in the OOPS algorithm[3].

Moreover, Faias & Santa-Clara (2017) set a CRRA parameter of 10[4], despite Bliss & Panigirtzoglou (2004) estimating an empirical risk aversion of 4 for S&P 500 index option data. The decision was done to ensure a conservative asset allocation decision. Selecting a higher level of CRRA than its true value also mitigates in-sample over-fitting. The goal with this choice of CRRA is therefore, to shrink option weights to prevent unreasonable levels, which is often done in practice[5].

2.4 Transaction Costs

Many authors document high transaction costs in the option market, which are believed to be responsible for pricing violations, such as the put-call parity relation (Phillips

[3]Faias & Santa-Clara (2017) use FMINCON in MATLAB to optimize weights. In this thesis, I perform the maximization process with PYTHON's SCIPY OPTIMIZE, which provides a local maximization solution. Therefore, our results vary with the inserted initial guess. Nevertheless, for reasonable initial guesses, the weight selection tends to be robust.

[4]As a robustness test, I run the algorithm with different levels of CRRA, ranging from 7 to 13. The performance of the strategy remains similar

[5]When the algorithm suggests unreasonable weights, I either use the previous month's capital allocation decision, or invest 100.00% of the wealth in the risk-free asset. The former alternative is performed if the algorithm successfully decided the previous month's weights. If this is not the case, the latter alternative is chosen.

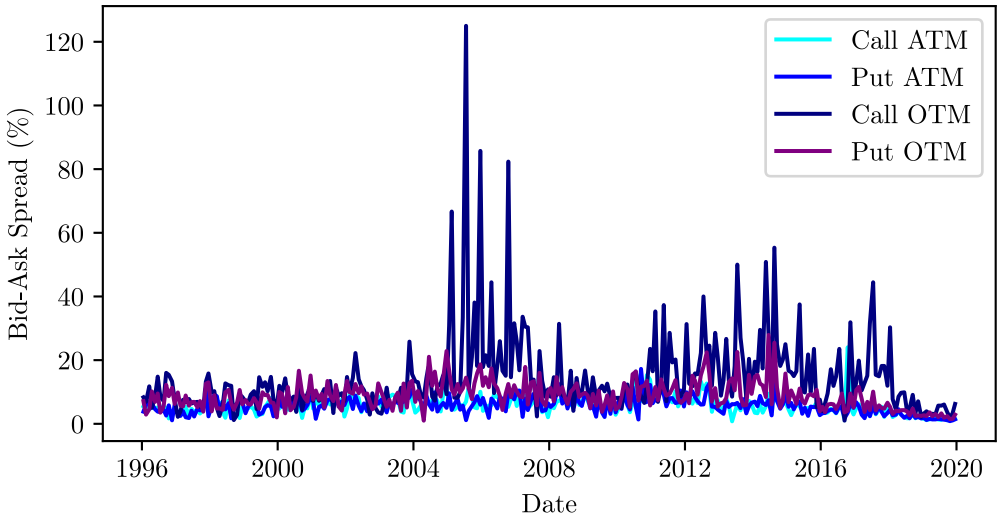
& Smith Jr (1980), Baesel et al. (1983), and Santa-Clara & Saretto (2009)). This body of literature enhances the need to take these market frictions into account. Faias & Santa-Clara (2017) decide to analyze only the Bid-Ask spread as a measure of the transaction costs associated with the strategy.

The relative Bid-Ask spreads for the strategy's options between January 1996 and December 2019 are presented in Figure 1.

To further scrutinize, Table II reports average Bid-Ask spreads of $1.21 across all

Figure 1: Bid-Ask Spreads

Figure 1 presents the monthly observations for Bid-Ask spreads (computed as the ask price less the bid price, divided by the respective mid price) of ATM calls, 5.00% OTM calls, ATM puts and 5.00% OTM puts, between January 1996 and December 2019. Results are provided in percentages.



moneyness segments, translating to an average value of $1.46 and $0.94 for ATM and OTM options, respectively. We relate these values to the option's mid-price, arriving at the option's relative Bid-Ask spreads. Table II presents relative Bid-Ask spreads of 5.80% and 11.58% for ATM and OTM options, respectively. It's important to notice that not only does this measure vary through time, it can also register extreme values for OTM options, since this sample of securities registers a maximum relative Bid-Ask spread of 125%.

Faias & Santa-Clara (2017) incorporate transaction costs into their method by decomposing each option into a bid option and an ask option. This means that long positions are initiated at the ask price and short positions at the bid price, entering short positions with a minus sign in the optimization of expected utility, as suggested by Eraker (2013) and Plyakha & Vilkov (2008). Taking this into account, the algorithm runs under a constraint of no-short-selling, meaning that only one of the ask or bid option is traded in that month.

Table II: Summary Statistics of Options

Table II presents values for option moneyness, price, Bid–Ask spread, relative bid–ask spread, volume, open interest, implied volatility, delta, beta, gamma, and vega for an ATM and OTM calls and puts, relative to the period between Jan. 1996 and Dec. 2019. Moneyness is given by $S/K1$. Price is the option's mid price. Bid–Ask spread is defined as the ask price less the bid price. Relative Bid–Ask spread equals to the bid–ask spread divided by the mid price. Volume is the contract's volume registered one month prior to its maturity. Open interest is recorded on the same day as volume. Implied volatility is equal the annualized volatility given by the Black & Scholes (1973). Delta is defined as the Black & Scholes (1973) delta. Beta is equal to the delta times the ratio of underlying asset's price to the option's price. Gamma is defined as the Black & Scholes (1973) gamma. Vega is defined as the Black & Scholes (1973) vega.

	Call ATM	Call OTM	Put ATM	Put OTM
Moneyness	−0.01%	−3.29%	0.18%	3.45%
Mid Price	27.08	8.00	25.66	13.08
Bid-Ask Spread	1.51	0.81	1.42	1.07
Relative Bid-Ask Spread	5.84%	14.39%	5.76%	8.79%
Volume	9,255.63	7,295.27	10,236.30	9,179.27
Open Interest ($000)	33,534.73	3,2954.01	29,986.73	47,349.89
Implied Volatility	17.57%	15.52%	17.75%	20.39%
Delta	0.51	0.21	−0.46	−0.25
Beta	30.03	52.27	−29.34	−32.25
Vega	1.65	1.08	1.65	1.28
Gamma	0.67	0.50	0.66	0.44

3 Data

3.1 Securities

In this subsection, I explain the necessary inputs for the calculation of OOPS, following the sequence presented in the section 'Methodology'. Firstly, in order to simulate the underlying asset's returns necessary for the algorithm, I use monthly, weekly and daily log returns from S&P 500 from January 1950 to December 2020. The sample period was chosen to maximize richness of events, thus generating a more accurate bootstrap simulation.

Secondly, this thesis begins with the analysis of optimal portfolio allocation from January 1996 to December 2019, following Faias & Santa-Clara (2017), evolving, in a more advanced stage, to the study of weekly Optimal Option Portfolio Strategies from January 2012 to December 2019. The reason for the lower bounds of both analysis' periods relies on data availability.

The monthly sample period comprises a wide variety of market conditions, such as the 1997 Asian Crisis, the 1998 Russian Crisis and collapse of Long-Term Capital Management, the 2000 NASDAQ peak, the 2001 terrorist attack to the world trade center, the

2006 – 2008 subprime mortgage crisis, the 2011 to 2018 steady market growth generating new highs, the plummeting of the index in the final months of 2018 and even the beginning of the 2019 – 2020 SARS-COV-2 crisis. On the other hand, the weekly analysis' period, despite lacking diversity of events, still comprises bullish and bearish periods, stated in the paragraph above. Nevertheless, this characteristic presents limitations to this analysis, which is important to keep in mind when scrutinizing the respective results.

Regarding the strategy's options, I use data from the OptionMetrics Ivy DB database for European options on the S&P 500, with one month[6] and one week to maturity[7], traded on the CBOE, closely in line with Faias & Santa-Clara (2017). For the monthly options, I select contracts expiring in the third Friday of each month. On the other hand, for the weekly ones, I choose options expiring on each Friday of each individual week. The dataset on these securities comprises the closing bid and ask prices, volume, open interest, implied volatility and the respective option's Greeks from January 1996 to December 2019 and from January 2012 to December 2019, for the monthly and weekly options, respectively.

Finally, in order to improve the quality of results, I eliminate unreliable data. For this purpose, I follow several filters common in past literature. Firstly, the observed options with a bid price lower than \$0.125 or a negative Bid-Ask spread were excluded. Secondly, in order to mitigate the risk of incorporating non-liquid options in the OOPS, I eliminate options with no volume. Lastly, I filter the options that violate arbitrage bounds.

3.2 Construction of Option Returns

The OOPS strategy uses a risk-free asset and a set of risky securities (Faias & Santa-Clara (2017)). The considered riskless asset corresponds to the 1-month and the 1-week Treasury bill (T-bill) for the monthly and weekly strategy, respectively. The risky securities comprise four options with distinct levels of moneyness: an ATM Call, and ATM Put, a 5% OTM Call and a 5% OTM Put. Taking into account the importance of OTM options for kernel spanning (Buraschi & Jackwerth (2001), Vanden (2004)) and the crash risk sensitivity of deep OTM put options, the considered group of assets generates flexible payoffs, despite its simplicity[8].

Furthermore, the mentioned risky assets are options with a month or a week to maturity, depending on the strategy performed, since Buraschi & Jackwerth (2001) mention that S&P 500 index options with less than 30 days to maturity account for the biggest

[6]These options are traded on the CBOE, with the ticker SPX. As of 4 October 2019, the daily volume of SPX options expiring in the third friday of each month, amounted to 72,981 contracts.

[7]Tables and Figures on this group of options are available in the Appendix.

[8]The moneyness intervals are identical to Faias & Santa-Clara (2017). The portfolio has gaps in moneyness in order to mitigate redundancy of options in certain periods. Moreover, a bigger set of options can be used. Nevertheless, its important to notice that, in order to maintain moneyness gaps, the investor would have to invest in deep OTM options, which are often associated with large transaction costs. Therefore, the feasibility of a bigger portfolio, in practical terms, seems crippled.

option trading activity. In addition to this, longer maturity contracts may stop trading if they become deeply in or out of the money. Moreover, the strategy holds the positions till the maturity of the contracts. Therefore, with OOPS, there are only transaction costs at the inception of the trade.

In order to achieve the proposed choice of assets, I closely follow Faias & Santa-Clara (2017), by finding all the contracts with a month and a week to maturity, meaning that I extract and use returns of the Friday before the third Saturday of one month to the Friday before the third Saturday of the next month and returns of the Friday of each week, respectively. The next step is to define moneyness intervals, enabling the categorization of options. Assuming moneyness as the ratio between the underlying and the strike price, minus one ($S/K - 1$), I label options with moneyness between -1.0% and 1.0% as ATM and between 2.0% and 5.0% as OTM. Given the fact that this method results in several potential securities for each month and moneyness segment, the option with the lowest Bid-Ask spread is chosen. In the case of two options with equal spread, the one with the highest amount of open interest remains.

After this option selection process, the synthetic 1-period option returns are built:

$$r_{t|t+1} = \frac{PAYOFF_{t+1}}{PRICE_t} - 1,$$

where $PAYOFF_{t+1}$ is the option's payoff at maturity constructed using the underlying's closing price on the day before the contract's settlement and the option's strike price, and $PRICE_t$ is the option price at the beginning of each investing period. Given the time span of this analysis, I obtain 288 observations for each security in the monthly OOPS, comparing to the 415, observed in the weekly strategy.

Figure 2 presents histograms of returns corresponding to each particular moneyness segment. Taking the figure into account, it is noticeable that monthly option return distributions depart heavily from the normal distribution and reflect a significantly negative tail risk. Moreover, weekly options display features that point towards the same amplified conclusion, observable in Figure 13. Table II further scrutinizes each moneyness category by presenting mean time-series characteristics. Monthly ATM calls and puts have an average moneyness of -0.01% and 0.18%, whilst monthly OTM calls and puts present values of -3.29% and 3.45%, for the respective metric. Taking these numbers into account, we can see that the contracts are somewhat close to the target of each segment. Volume for each contract is around 9,000 and open interest is approximately 30 million. Moreover, mean implied volatility varies with different levels of moneyness, registering values of between 15.52% and 20.39%, evidencing the well-known smile effect. Table XV registers the same analysis, but for weekly options. These options differ from the monthly ones, mainly through tighter gaps of moneyness between ATM and OTM options, lower Bid-Ask spreads, volume and open interest and much higer Black & Scholes (1973) betas.

Figure 2: Densities of Monthly Option Returns

Figure 2 presents the histograms of monthly returns on long at-the-money Calls, out-of-the-money Calls, at-the-money Puts, out-of-the-money Puts over the S&P 500 index. The returns are recorded between January 1996 and December 2019.

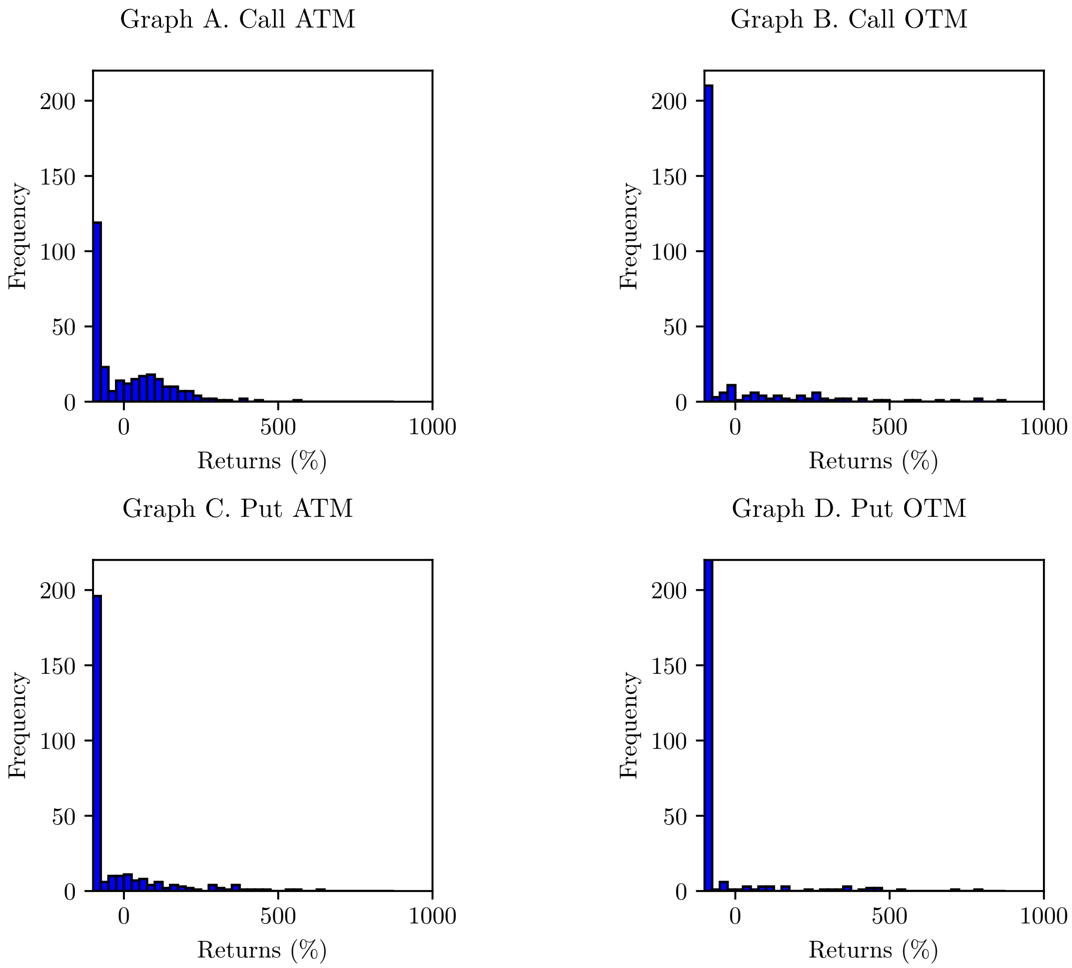


Table III reports further summary statistics of the monthly securities' returns, namely values of annualized mean return, annualized standard deviation, minimum, maximum, skewness, excess kurtosis, annualized Sharpe ratio[9] and annualized certainty equivalent, with the latter being calculated for an investor with a CRRA utility function and a coefficient of relative risk aversion of 4. This statistic is given by:

$$CE = [(1-\gamma)\overline{U}]^{1/(1-\gamma)} - 1,$$

where γ is 4, $\overline{U} = \frac{1}{T}\Sigma_{t=1}^{T} U_t$, and U_t is the CRRA utility for each month t. It is important to notice that the risk aversion coefficient used in this calculation is 4, comparing to the CRRA of 10 used in the optimization. In fact, the usage of an CRRA that is higher than its true value, minimizes in-sample over-fitting. We can see this as a guide, directing the

[9] It is important to consider that the Sharpe ratio takes only the first two moments (mean and standard deviation) into account. However, Broadie et al. (2009) state that several alternative measures suffer from the same handicap.

algorithm into proposing more conservative weights. This is often done in practice.

In Panel A, we can observe S&P 500 index total return's descriptive statistics, being that this index is the benchmark for the risky assets' returns. The S&P 500 index registers an annual average return of 6.49% and standard deviation of 14.98% over the sample period, resulting in a Sharpe ratio of 0.29 and a certainty equivalent of 0.14%. The S&P 500 index returns present a negative skew and positive excess kurtosis. Panel B reports the same summary statistics for the different segments of options. Only ATM calls present a positive value for annualized mean return of 56.28%, whilst ATM puts and OTM calls and puts register significantly negative values for this statistic, adding up to −387.72%, −128.32% and −589.61%, respectively.

Taking this into consideration, buying calls and writing puts during this sample

Table III: Summary Statistics of Returns

Table III presents summary statistics of return for a buy-and-hold strategy in multiple assets, between Jan. 1996 and Dec. 2019, comprising annualized mean returns, annualized standard deviation, minimum, maximum, skewness, excess kurtosis, annualized Sharpe ratio (SR), and annualized certainty equivalent (CE). Panel A presents primitive assets (S&P 500 index and 1-month Treasury Bill). Panel B shows options, namely an ATM call, an OTM call, an ATM put and an OTM put. Panel C register values for delevered options, obtained by investing 104.47% in the risk-free asset and short-selling -4.47% of each option, individually.

	Ann. Mean	Ann. St. Dev.	Min.	Max.	Skew.	Exc. Kurt.	Ann. SR	Ann. CE
Panel A: Primitive Assets								
S&P 500	6.49%	14.98%	−18.56%	10.23%	−0.85	1.56	0.29	0.14%
1-month T-Bill	2.16%	0.60%	0.00%	0.60%	0.49	−1.29	0.00	0.18%
Panel B: Only Options								
Call ATM	56.28%	418.86%	−100.00%	556.55%	1.19	1.40	0.15	−100.00%
Call OTM	−128.32%	803.68%	−100.00%	1845.16%	4.01	20.33	−0.12	−100.00%
Put ATM	−387.72%	464.43%	−100.00%	640.70%	2.50	6.23	−0.84	−100.00%
Put OTM	−589.61%	592.19%	−100.00%	1142.25%	4.38	21.04	−1.00	−100.00%
Panel C: Delevered Options								
Call ATM	−3.57%	19.80%	−26.23%	5.06%	−1.16	1.32	−0.31	−1.24%
Call OTM	−1.43%	41.30%	−93.43%	5.06%	−4.05	20.21	−0.06	−100.00%
Put ATM	17.41%	21.99%	−31.40%	5.06%	−2.52	6.44	0.69	0.36%
Put OTM	25.88%	29.14%	−56.52%	5.06%	−4.38	21.02	0.81	−1.47%

period could have a been a good strategy, yielding Sharpe ratios from 1.26 to 0.12. Nevertheless, one must not forget the substantial tail risk associated with writing options evidenced in Figure 2 and Table III. In fact, writing put options during the sample period would generate returns ranging between a maximum of 100% and a minimum of -640.70% and -1142.25% for ATM and OTM puts, respectively. These characteristics result in a large negative skewness and extremely high excess kurtosis, reaching values as high as 21.04, resulting in a deeply negative certainty equivalent of -100.00%.

Analyzing the statistics of these extremely levered securities, we conclude that these levered options generate unreasonable returns, which seem at least unappealing for the average investor. Therefore, I follow Faias & Santa-Clara (2017) by de-leveraging these securities. This is performed by undertaking a long position of 104.47% on the riskless asset and a short position of 4.47% on the option at stake. The riskless asset's weight corresponds to the average one-month T-bill weight in OOPS, being that the option weight corresponds to the one, less the riskless asset's weight.

Despite presenting much lower annualized mean returns, standard deviations and ranges between minimums and maximums, the options still deliver negatively skewed returns with relatively high tail risk and mostly negative certainty equivalents, thus proving that the strategy is not attractive to risk averse investors.

Additionally, Table XVI displays the same summary statistics for the weekly setting. These securities register intriguing features that are not witnessed for their monthly counterparts. Firstly, we can observe that the weekly naked options have much more aggressive mean annualized returns and standard deviations and naturally, report much larger maximums and minimums. Moreover, they present extremely high levels of skewness and excess kurtosis, being that the latter statistic reports a value of 86.84 for OTM puts. This extreme tail risk is also evidenced in Figure 13

4 Results

4.1 Out-of-Sample OOPS Returns

4.1.1 Simple Monthly OOPS

The monthly OOPS out-of-sample returns report several attractive features, not present in the individual option's returns. Figure 3 presents the return distribution for the respective strategy. The OOPS's distribution of returns evidences a symmetric shape and low tail risk, thus being much closer to a normal distribution than the previously analyzed distributions, relative to the risky assets that compose this strategy. With Table IV registering the summary statistics for the strategy and the S&P 500 index returns (as a benchmark for comparison) for the period between January 1996 and December 2019, we observe that the monthly OOPS out-of-sample returns yield an average annual return of 11.14% after transaction costs, with a standard deviation of 11.17%, in contrast with the S&P 500 index's values of 6.48% and 14.98%, for the same statistics, respectively.

These values account for a Sharpe ratio of 1.00, more than three times the S&P 500 index's Sharpe ratio of 0.29, and an annualized certainty equivalent of 8.58%, despite presenting moderate levels of tail risk, which compares well with the S&P 500 index's

Figure 3: Monthly OOPS out-of-sample return distribution

Figure 3 presents the monthly OOPS out-of-sample return distribution, between January 1996 and December 2019. OOPS returns are after transaction costs.

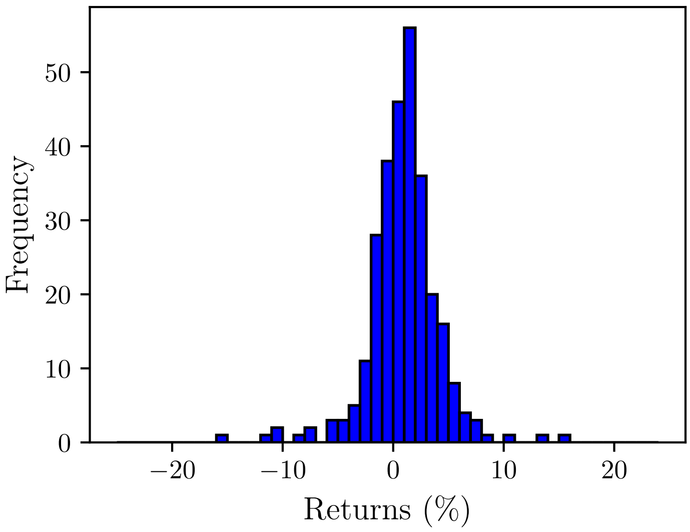


Table IV: Monthly OOPS out-of-sample returns

Table IV presents summary statistics of the monthly OOPS out-of-sample returns and a benchmark (S&P 500 index returns), between Jan. 1996 and Dec. 2019, comprising annualized mean returns, annualized standard deviation, minimum, maximum, skewness, excess kurtosis, annualized Sharpe ratio (SR), and annualized certainty equivalent (CE). The OOPS returns are after transaction costs.

	Ann. Mean	Ann. St. Dev.	Min.	Max.	Skew.	Exc. Kurt.	Ann. SR	Ann. CE
OOPS	11.14%	11.17%	−15.51%	15.87%	−0.45	5.71	1.00	8.58%
S&P 500	6.49%	14.98%	−18.56%	10.23%	−0.85	1.56	0.29	0.14%

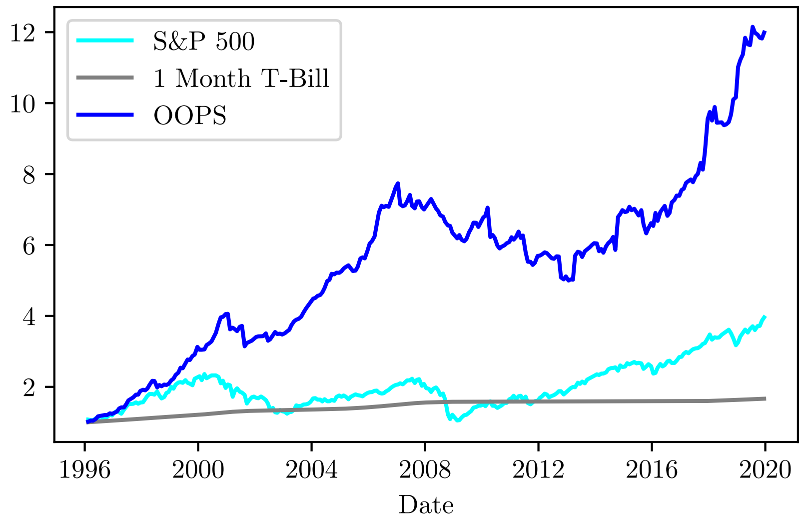
certainty equivalent of 0.14%. Furthermore, despite individual options showing extreme negative skewness and high kurtosis, the OOPS registers a low negative skewness of -0.45 and a moderate excess kurtosis of 5.71. Moreover, the strategy has a minimum return of -15.51% and a maximum return of 15.87%, comparing to the benchmark's minimum return of -18.56% and 10.23%.

For further visibility of the strategy's performance, Figure 4 depicts the evolution of a hypothetical amount of $1.00 invested in a riskless asset, the S&P 500 index and the monthly OOPS, throughout the analysis period, beginning in January 1996 and December 2019.

Not only does the strategy provide a stable growth through the years but it significantly outperforms the S&P 500 index, yielding a total realized return of around 1,100%, versus the 300% given by the S&P 500 index. Scrutinizing further the OOPS performance, we can relate its biggest plummets to specific events. For example, the biggest loss reg-

Figure 4: Monthly OOPS cumulative Returns

Figure 4 presents the monthly OOPS out-of-sample cumulative returns, between January 1996 and December 2019. OOPS returns are after transaction costs.



istered by the OOPS presents a magnitude of -15.51% and coincides with the infamous terrorist attack targeting the world trade center in New York. On the other hand, the maximum return achieved by the strategy happens in the November 2014 after one of the best Novembers the index has ever seen.

Table V registers the average, minimum and maximum net weights for each of the

Table V: Monthly OOPS weights

Table V presents the time-series mean, minimum and maximum weights for the assets comprising the monthly OOPS, between January 1996 and December 2019. ATM stands for at-the-money, OTM stands for out-of-the-money and RF stands for the risk-free asset.

	Call ATM	Call OTM 5%	Put ATM	Put OTM 5%	Risk-Free
Mean	−0.45%	1.05%	1.33%	−1.50%	99.56%
Minimum	−5.00%	−2.05%	−1.25%	−4.96%	93.67%
Maximum	2.76%	4.64%	5.35%	1.36%	105.55%

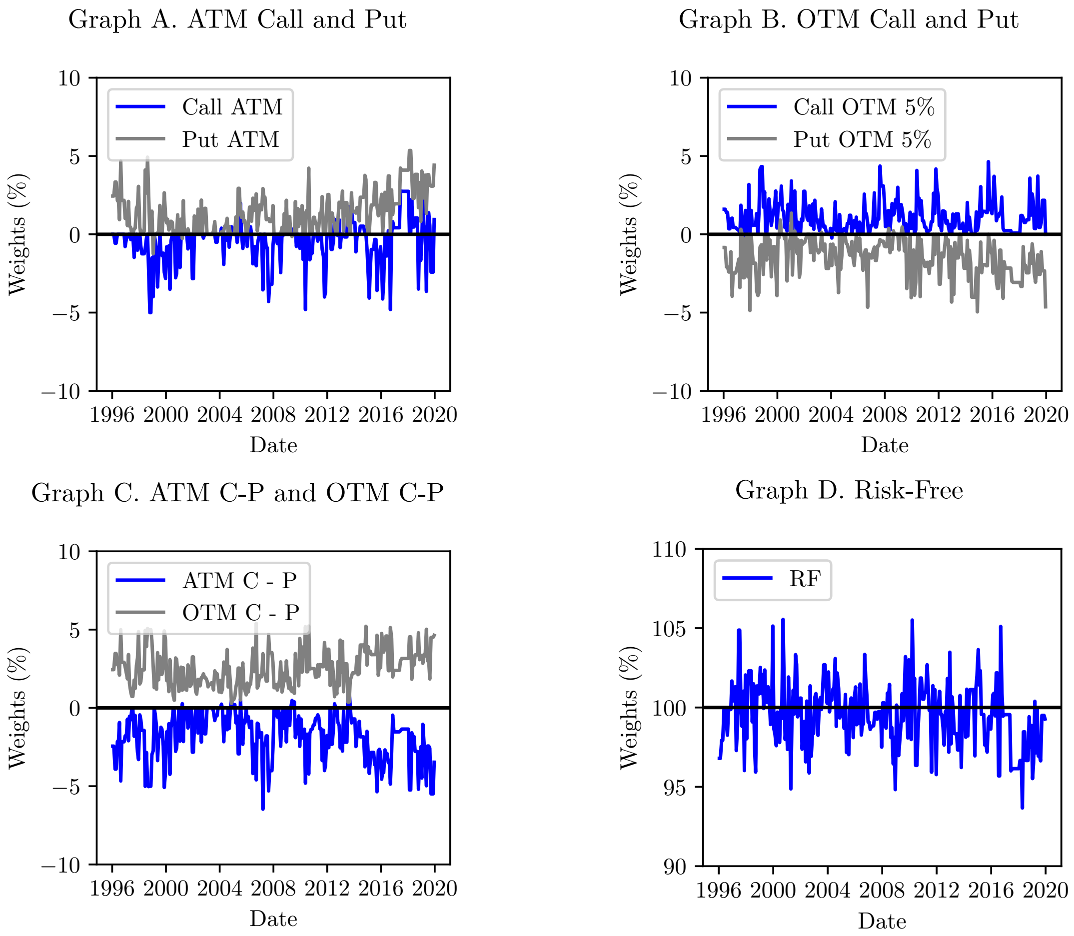
options used in the strategy. We can observe that the ATM call and put option's optimal weights present a wider range of values, between -5.00% and 5.35%, whilst OTM calls and puts have a slightly tighter interval, between -4.96% and 4.64%. This is somewhat in line with expectations as risk averse investors avoid high amounts of tail risk, present in OTM options. Moreover, the OOPS perfectly balances long and short positions, being neither a net seller nor a net buyer. Furthermore, put options have generally larger long or short positions comparing to call options. Figure 6 presents the evolution and relationships between option weights through time. After this analysis, we can evidence that the options' positions have particular periods where they're positive, thus indicating long positions. Moreover, there is an offsetting effect within ATM and OTM options.

Figure 5, Graph D describes the evolution of the riskless asset's weight. The average risk-free weight is 99.56%, registering a minimum value of 93.67% and a maximum of 105.55%. As previously mentioned, OOPS tends to balance long and short positions in options, being that in 61.46% of the analyzed months the strategy buys options in net terms, with an average net position of 1.68% in options and risk-free's weights averaging 98.32% in these months. Furthermore, the strategy has months on which OTM puts are bought, like in February of 2001.

This contradicts past literature, where it is shown that CRRA investors always prefer

Figure 5: OOPS time-series weights

Figure 5 presents the monthly OOPS' weights, between January 1996 and December 2019. Graph A presents the weights for ATM Calls and Puts. Graph B shows the weights for OTM Calls and Puts. Graph C presents the difference in Call and Put weights for the two levels of moneyness. Graph D show the monthly weights for the riskless asset.



to short-sell OTM puts, with limited exceptions (Driessen & Maenhout (2007)). Given that the Driessen & Maenhout (2007) setting is static, meaning they would apply constant weights to each option throughout the analysis window, and that their opportunity set was smaller, with only one option strategy at a time, we can conclude that the contradiction is justified, since the OOPS does not always simply sell volatility. In fact, there is a strong correlation between the sum of call weights and put weights. Moreover, the OOPS usually (85.76% of months) shorts OTM puts, and partially hedges this position with the other options comprising the portfolio.

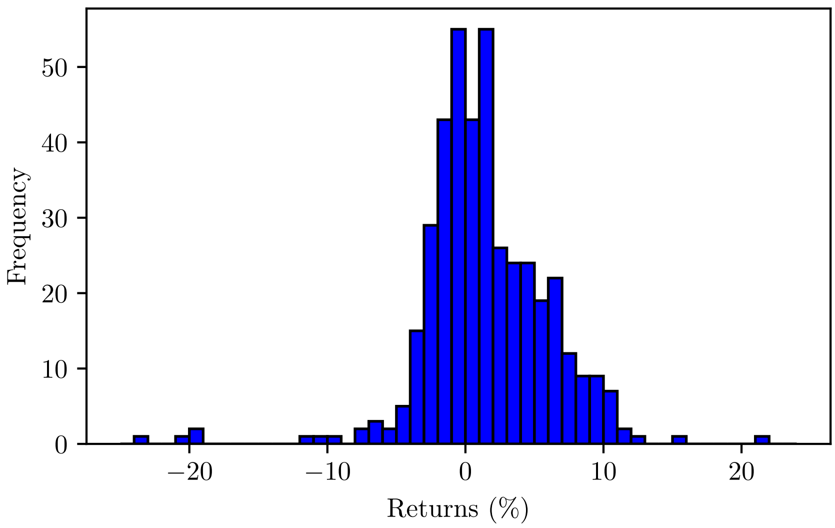
4.1.2 Simple Weekly OOPS

Just like the monthly strategy, the weekly OOPS out-of-sample returns presents some appealing characteristics that are not present in the individual option's returns. For visibility purposes, I show the weekly OOPS return distribution in Figure 6. The strategy's return distribution evidences a fairly symmetric shape, registering a skewness of -0.63 (a value slightly lower than the one reported by the monthly OOPS), and a relatively significant amount of tail risk, with excess kurtosis reaching 5.45, comparing with a kurtosis of 5.71 for the monthly strategy. Therefore, we conclude that in terms of distribution asymmetry and tail risk, both strategies are similar, with their distribution of returns converging closer to a normal distribution than the ones of their individual components'.

Table VI registers the summary statistics for the weekly OOPS and its respective

Figure 6: Weekly OOPS out-of-sample return distribution

Figure 3 presents the weekly OOPS out-of-sample return distribution, between January 2012 and December 2019. OOPS returns are after transaction costs.



benchmark (S&P 500 index weekly returns) for the period between January 2012 and December 2019. We can observe that the weekly OOPS out-of-sample returns yields an average annual return of 75.18% after transaction costs, with a standard deviation of 32.86%, in contrast with the S&P 500 index's values of 11.77% and 12.21%, for the same statistics, respectively.

These values account for a weekly OOPS Sharpe ratio of 2.29, more than two times the S&P 500 index's Sharpe ratio of 0.91. Regarding these parameters, the weekly OOPS

Table VI: Weekly OOPS out-of-sample returns

Table VI presents summary statistics of the weekly OOPS out-of-sample returns and a benchmark (S&P 500 index returns), between Jan. 2012 and Dec. 2019, comprising annualized mean returns, annualized standard deviation, minimum, maximum, skewness, excess kurtosis, annualized Sharpe ratio (SR), and annualized certainty equivalent (CE). The OOPS returns are after transaction costs.

	Ann. Mean	Ann. St. Dev.	Min.	Max.	Skew.	Exc. Kurt.	Ann. SR	Ann. CE
OOPS	75.18%	32.86%	−23.66%	21.61%	−0.63	5.45	2.29	52.01%
S&P 500	11.77%	12.21%	−7.31%	4.73%	−0.79	2.09	0.91	8.73%

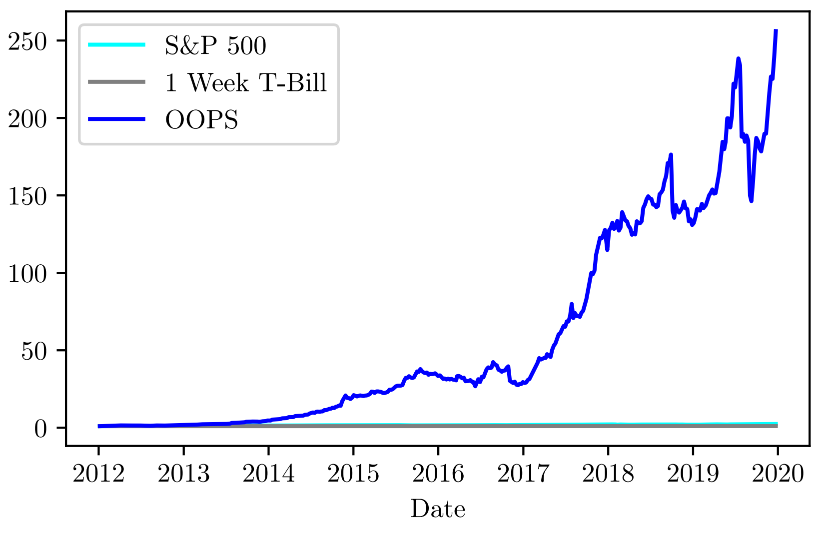
clearly outperforms the monthly strategy for the same period. This is believed to happen due to the higher frequency of investment and compounding, and the summary statistics of the options that comprise the strategy. In fact, if we observe Table XVI, we can conclude that OTM puts, which are significantly shorted in this strategy, tend to yield negative returns more often, contributing to its extreme levels of skewness and excess kurtosis.

Moreover, the weekly strategy reports a more favorable skewness but worse kurtosis, comparing to the benchmark, with a minimum return of -23.66% and a maximum return of 21.61%, versus the benchmark's minimum return of -7.31% and 4.73%, which corroborate the former statement about the strategy's return distribution's symmetry and tail risk.

Figure 7 depicts the evolution of a hypothetical amount of $1.00 invested, with a

Figure 7: Weekly OOPS cumulative Returns

Figure 7 presents the monthly OOPS out-of-sample cumulative returns, between January 2012 and December 2019. OOPS returns are after transaction costs.



weekly frequency, in a riskless asset, the S&P 500 index and the weekly OOPS, throughout the analysis period, beginning in January 2012 and December 2019. The weekly OOPS provides a realized return of around 25500.00%, versus the benchmark's 140.00%, enabling the conclusion that the strategy severely outperforms the market.

Scrutinizing further the OOPS performance, we can relate its biggest plummets and rallies to specific events. The biggest losses relate to significant market drops. For example, the strategy's minimum return of -23.66% coincided with a substantial loss in the market during 2016, as oil prices dropped. On the other hand, the maximum return of the weekly OOPS, reaching 21.61%, takes place after one of the best Novembers the S&P 500 has seen over the years. There is therefore a clear pattern, in which the strategy thrives as the underlying asset's price rallies and loses when the underlying asset's prices plummet. This happens given the moderate short selling of OTM puts. Moreover, the weights allocated to the weekly strategy follow the same patterns and characteristics as the monthly strategy, displayed and explained in previous subsections.

4.1.3 Weekly Decomposed OOPS

Additionally, I study four different weekly decomposed strategies, in which the investor invests in options only in one week per each month, allocating 100% of his/her wealth in the riskless assets in the remaining weeks in order to study and disclose any performance patterns within these strategies. In fact, these strategies proved to behave in a rather positive way.

Table VII presents summary statistics on each individual strategy. Most of the strategies' return distributions present attractive symmetry features. For instance, weeks one, three and four record positive skews between 0.71 and 3.88, versus the market's −0.79. However, they also present some severe tail-risk, registering excess kurtosis between 9.74

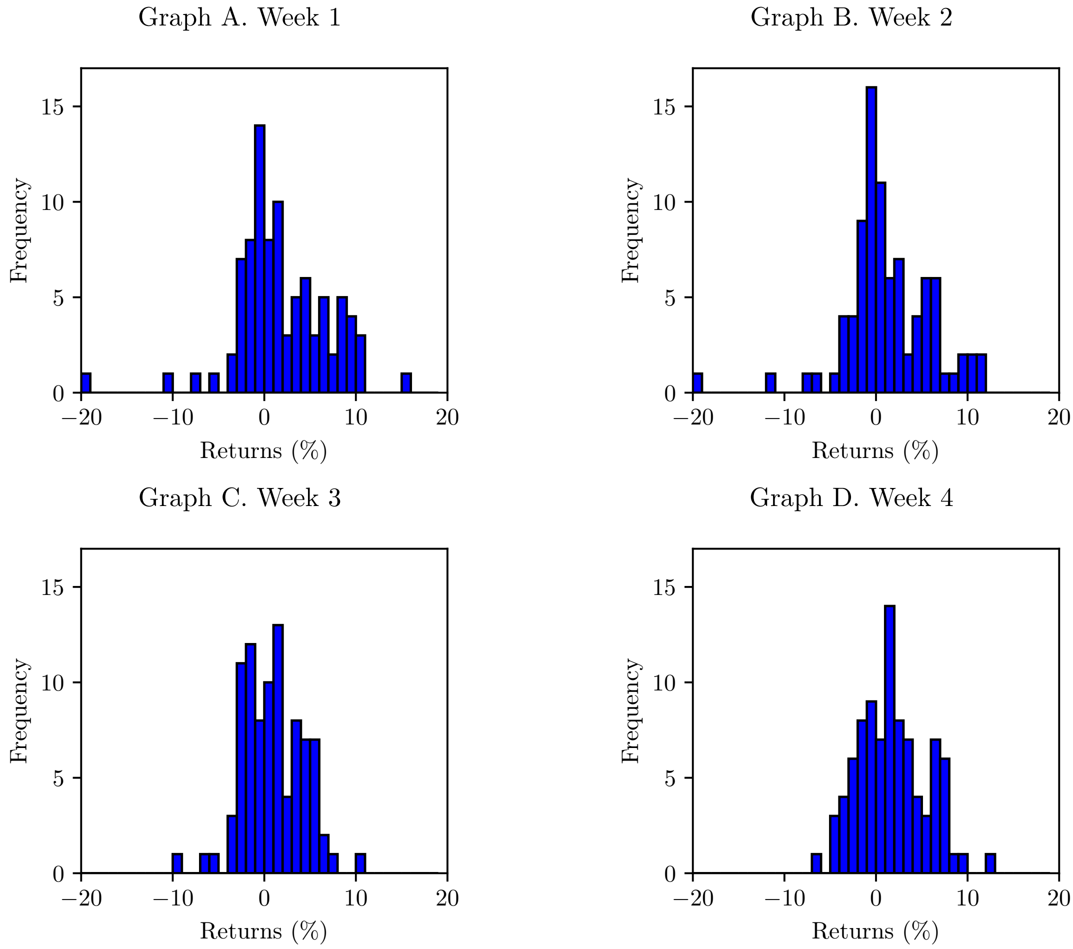
Table VII: Weekly decomposed OOPS out-of-sample returns

Table VII presents summary statistics of the each weekly decomposed OOPS out-of-sample returns and a benchmark (S&P 500 index returns), between January 2012 and December 2019, comprising annualized mean returns, annualized standard deviation, minimum, maximum, skewness, excess kurtosis, annualized Sharpe ratio (SR), and annualized certainty equivalent (CE). The OOPS returns are after transaction costs. Kurtosis is not representative since weeks were only the risk free is bought induce high kurtosis to the distributions.

	Ann. Mean	Ann. St. Dev.	Min.	Max.	Skew.	Exc. Kurt.	Ann. SR	Ann. CE
OOPS Week 1	24.28%	18.19%	−19.70%	15.77%	0.71	17.03	1.30	17.69%
OOPS Week 2	12.16%	20.10%	−23.66%	11.53%	−2.42	28.07	0.57	2.60%
OOPS Week 3	12.99%	13.90%	−9.36%	21.61%	3.88	39.92	0.89	9.51%
OOPS Week 4	21.03%	14.00%	−6.98%	12.33%	2.37	9.74	1.45	17.40%
S&P 500	11.77%	12.21%	−7.31%	4.73%	−0.79	2.09	0.91	8.73%

Figure 8: Weekly decomposed OOPS return distribution

Figure 8 presents the weekly decomposed OOPS' out-of-sample return's distribution for each individual strategy (excluding the weeks with 100% of wealth invested in the risk free asset), between January 2012 and December 2019. OOPS returns are registered after considering transaction costs.



and 39.92. Nevertheless, one must not forget that these values are not representative since each strategy invests in 3 out of 4 weeks in the riskless asset, thus causing a ramp up effect on the returns' distributions' excess kurtosis. If we consider Figure 8, we can conclude that the strategies' return distributions appear to have excess kurtosis levels bellow the ones stated in Table VII.

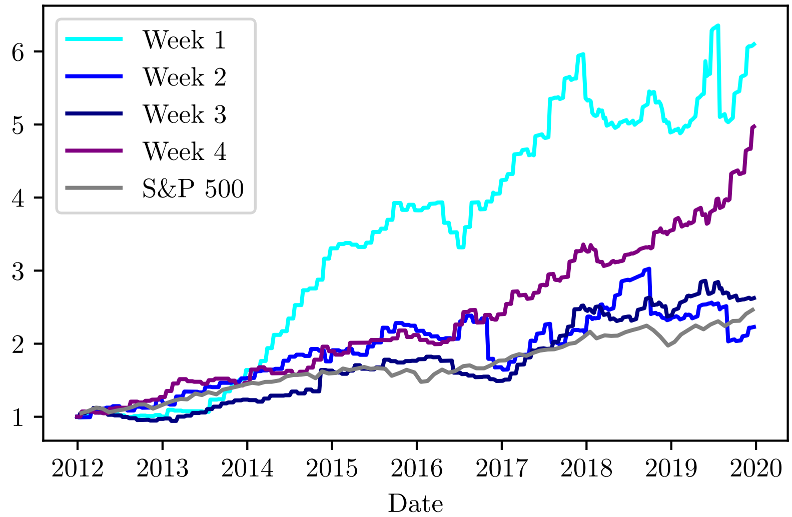
Moreover, the strategies report average annualized returns between 12.15% and 24.28% and standard deviations ranging from 13.90% to 20.10%, comparing to the benchmark's annualized mean return of 11.77% and standard deviation of 12.21%. Moreover, these statistics relate to high annualized Sharpe ratios and certainty equivalents. In fact, the strategies show Sharpe ratios between 0.57 and 1.45 and certainty equivalents ranging from 2.60% to 17.68%, beating, in their majority, the market's statistics.

To further scrutinize, the best OOPS over the period beginning in January 2012 and ending in December 2019, are the strategies that invest in options in the first and fourth week of each month and in the risk-free asset in the remaining weeks. These strategies register, respectively, annualized mean returns of 24.28% and 21.03% and standard deviations of 18.19% and 14.00%, resulting in Sharpe ratios of 1.30 and 1.45 and certainty equivalents of 17.69% and 17.40%, even after accounting for the considerable excess kurtosis present in each of their respective return distribution. Based on the previous statements, we conclude that these strategies outperform the market in every statistic with the exception of tail risk.

For further visibility of the strategy's performance, Figure 9 depicts the evolution of

Figure 9: Weekly OOPS cumulative Returns

Figure 9 presents each weekly decomposed OOPS out-of-sample cumulative returns, between January 2012 and December 2019. OOPS returns are after transaction costs.



a hypothetical amount of $1.00 invested in the S&P 500 index and in each weekly OOPS, throughout the analysis period, beginning in January 2012 and December 2019. Here, we can see what was stated above, with the week one and four OOPS outperforming the remaining strategies. It is also important to notice that, on average, the weeks tend to perform, at least as well as the market, being that strategy one and four severely outperform the benchmark, yielding a realized return, over the analyzed period, of around 500.00% and 400.00%, versus the market's 200.00%.

Based on Figure 9 and table VII, we can see that the extreme results obtained in the simple weekly strategy are due to investing frequency and compounding. With shorter maturities, the investor can crystalize returns of similar magnitudes in a much faster way, thus ramping up returns to the levels reflected in the previous sections. However, when we analyze Figure 9 and Table VII we can see that without such a high frequency of investment and compounding, we attain more conservative levels of realized return for the strategies.

Table VIII presents the options net weights for the respective strategies. In this table, we can observe that the weights applied to each option vary mildly for each weekly

Table VIII: Weekly decomposed OOPS weights

Table VIII presents the time-series mean, minimum and maximum weights for the assets comprising the weekly decomposed OOPS, between January 2012 and December 2019. ATM stands for at-the-money, OTM stands for out-of-the-money and RF stands for the risk-free asset.

	Call ATM	Call OTM	Put ATM	Put OTM	RF
Week 1 OOPS					
Mean	−0.42%	0.33%	−0.10%	−2.02%	102.22%
Maximum	3.86%	4.00%	3.98%	0.00%	110.01%
Minimum	−4.90%	−4.00%	−6.58%	−4.00%	93.18%
Week 2 OOPS					
Mean	−0.40%	0.50%	0.25%	−1.66%	101.32%
Maximum	3.07%	3.52%	6.00%	0.33%	110.00%
Minimum	−8.00%	−4.00%	−5.39%	−4.00%	93.39%
Week 3 OOPS					
Mean	0.05%	0.48%	0.63%	−1.40%	100.24%
Maximum	5.85%	2.65%	4.16%	0.58%	110.00%
Minimum	−4.81%	−4.00%	−7.84%	−4.00%	93.61%
Week 4 OOPS					
Mean	−0.22%	0.49%	0.72%	−1.89%	100.91%
Maximum	4.46%	4.00%	4.74%	0.05%	110.00%
Minimum	−8.00%	−4.00%	−4.31%	−4.00%	91.58%

strategy. The strategies tend to be net sellers of options. We can also notice that the strategies that perform the best tend to be the ones that short OTM puts the most, thus being in line with the conclusions taken by Driessen & Maenhout (2007).

4.2 Risk or Mispricing?

OOPS's performance can occur based on two distinct drivers, with the first being the load in priced risk factors and the second being the exploitation of option mispricing. Several bodies of literature support both hypotheses. Supporting the former, Bates (1996), Bakshi et al. (1997), Chernov & Ghysels (2000), Duffie et al. (2000), Pan (2002), Liu & Pan (2003), Eraker (2004), Broadie et al. (2007), Todorov (2010), and Christoffersen et al. (2013) argue that an option's price is a function of priced volatility and crash risk factors. On the other hand, Jackwerth (2000), Coval & Shumway (2001), Jones (2006), Goyal & Saretto (2009), and Constantinides et al. (2013) favor the latter, showing that exposure to priced risk factors does not explain option prices completely. Following Faias & Santa-Clara (2017), this thesis will explore both.

To approach this question, the performance of the strategy was held against two analysis. First, we will explore the strategies' exposure to priced risk factors through two distinct risk measures, comprising Greeks from Black & Scholes (1973), namely the OOPS's beta and jump risk. An option's beta is calculated by multiplying the Black & Scholes (1973) delta[10] by the ratio between the underlying asset and the option value. On the other hand, jump risk represents the change in the portfolio's value with a sudden drop of 5.00% in the S&P 500 index. Therefore, this risk factor is computed at time t as $O_t^{BS}(S,\sigma) - O_t^{BS}(0.95S,\sigma)$, where O_t^{BS} is the Black & Scholes (1973) option value for each security comprised in the strategy. We then aggregate both risk factors for the OOPS portfolio in each investing period.

Then, to further comprehend the risk exposure of the strategy, the OOPS' ex-post risk vulnerability is analyzed, by performing explanatory and predictive linear regressions, between the dependent variable, being the OOPS excess returns, and several independent variables, common in equities and options literature.

Regarding the explanatory analysis, we run regressions of out-of-sample OOPS excess returns, $r_t - rf_t$, equivalent to the strategy's returns minus the risk-free rate on distinct risk variables, X_t:

$$r_t - rf_t = \alpha + \beta X_t + \varepsilon_t$$

Several simple linear regressions are done on traditional risk factors. Firstly, I regress OOPS Excess Returns on Market Excess Return ($r_{(S\&P500)} - r_f$) which is defined as the difference between the market return and the risk-free rate. Secondly, the relationship with Jump Risk ($JUMP$) is considered. $JUMP$ is similar to a dummy variable, equating to the S&P 500 index return when the monthly return of the index registers values below -5.00%, and 0.00% otherwise. Thirdly, two volatility measures were taken into account: realized volatility at the end of the previous and current month using daily data, and the VIX at the end of both periods as well. OOPS excess returns are therefore regressed on the realized volatility variable (ΔRV) and the VIX variable (ΔVIX), which are expressed as the change of the respective volatility measure over each investing period.

Then, several distinct predictive regressions of out-of-sample OOPS excess returns, $r_t - rf_t$, equivalent to the strategy's returns minus the risk-free rate, are performed on four distinct lagged variables, Z_{t-1}:

$$r_t - rf_t = \alpha + \beta Z_{t-1} + \varepsilon_t$$

The predictive regressions analyze the conditional expected return of OOPS. Therefore, it's only natural to use variables that proxy conditional risk premia. Firstly, the same two volatility measures taken in the former regressions, are considered. The regressions are performed based on the realized volatility (RV) and VIX (VIX) variables, which are the value of each measure, observed at the end of the previous period. Secondly, the relationship between volatility risk premium and the dependent variable is taken into

[10] Faias & Santa-Clara (2017) mention that the delta time-series distribution is narrow and close to 0.

account by using the VRP variable, presented as the spread between the implied and realized volatility, taking values from the measures considered in the calculation of the former variables. Thirdly, the Jump risk ($JUMP$) variable's relationship with the strategy's excess returns is studied. In this setting, $JUMP$ is identical to the one used in the explanatory regression, with the exception that this variable is registered at the end of the previous period.

Additionally, in an effort to further explore potential drivers and predictors of the strategy's performance, I regress the monthly OOPS out-of-sample excess returns, $r_t - rf_t$, on the current and lagged 4 Fama and French factors, respectively. This analysis is only performed for the monthly OOPS due to data availability.

Regarding the former analysis, I regress the dependent variable, $r_t - rf_t$, on the 4 Fama and French factors, following the linear regression bellow:

$$r_t - rf_t = \alpha + \beta_1 Mkt - RF_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_1 MOM_t + \varepsilon_t.$$

For the latter study, I regress the dependent variable, $r_t - rf_t$, on the lagged 4 Fama and French, following the subsequent linear regression:

$$r_t - rf_t = \alpha + \beta_1 Mkt - RF_{t-1} + \beta_2 SMB_{t-1} + \beta_3 HML_{t-1} + \beta_1 MOM_{t-1} + \varepsilon_t.$$

The following sub-subsections present and dissect the results of the analysis described above, for the entire collection of strategies, thus analyzing firstly the simple monthly OOPS, secondly the simple weekly OOPS and finally the weekly decomposed OOPS.

4.2.1 Simple Monthly OOPS

In Figure 7, the time-series evolution between January 1996 and December 2019 of the previously mentioned risk measures is displayed. Analyzing the evolution of the OOPS's betas, we can evidence that the strategy has a positive and low beta, registering an average of 0.45. Nevertheless, the strategy is occasionally exposed to the market's systematic risk, registering a minimum of -0.006 and a maximum of 1.65. Furthermore, we can evidence in Figure 11 Graph A that OOPS has an increasing beta through time. If we analyze the period from 1996 to 2012 separately, we register a smaller average beta of 0.37, with a minimum of 0.11 and a maximum of 0.95. In contrast, the remaining period reports an average beta of 0.64, with a minimum of -0.006 and a maximum of 1.65.

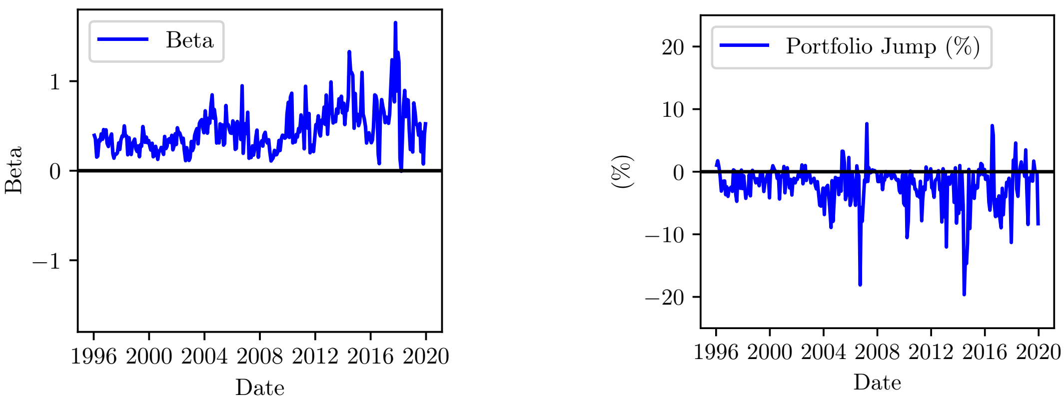
Additionally, analyzing jump risk, we see that our portfolio reacts to a sudden market plummet of -5.00% by -2.18%, with a minimum of -19.64% and a maximum of 7.68%, indicating that the strategy tends to lose when the market drops. Nevertheless, the monthly OOPS still reports a low portfolio beta and an average jump risk smaller than 5.00% between January 1996 and December 2019, therefore proving its small but still present

Figure 10: Monthly OOPS time-series risk measures

Figure 11 presents the monthly OOPS' risk measures according to the Black & Scholes (1973) model, between January 1996 and December 2019. Graph A presents the portfolio's beta, which represents the change when the underlying changes by 1.00%. Graph B shows the portfolio's jump risk, which represents the percentage change when the underlying drops suddenly by -5.00%

Graph A. Beta

Graph B. Market Jump of -5.00%



exposure to the mentioned risk factors and proving that the high OOPS returns can be marginally explained as a compensation for systematic and jump risk. This converges with the strategy's low but negative skewness and moderate kurtosis, stated in the previous subsection.

Following the description detailed in the subsection 'Risk or Mispricing?', I analyze the monthly OOPS' ex-post risk vulnerability. Table IX reports the results for the explanatory regressions. Faias & Santa-Clara (2017) find that the strategy is resilient regarding these variables, which register low coefficients and a maximum $\bar{R}^2$ of 4.99% across all the regressions. My findings are somewhat contradictory to theirs. In fact, the market coefficient is 0.23, being significant at the 1.00% level, enabling the conclusion that the strategy's returns are positively related to the market. Moreover, the jump risk coefficient is positive and significant at the 1.00% level, registering a value of 0.48, implying that the strategy loses with negative market jumps. Moreover, the coefficient for change in VIX is negative, registering values of -0.0016, significant at a 1.00% level, therefore being negatively related to the out-of-sample OOPS returns. Finally, the change in realized volatility variable presents a negative and low coefficient, not significant at a 10.00% level, and a $\bar{R}^2$ of 0.30%, therefore suggesting that the strategy's performance is independent of this risk factor.

It is important to notice that the majority of the coefficients' magnitude is relatively small, being that the largest coefficient, related to jump risk, is still lower than 1.00, implying that the strategy loses less than the market, when a contemporaneous market jump occurs. Moreover, all the significant variables reach the plateau of statistical significance, being significant at the 1.00% level, and present relatively high $\bar{R}^2$, ranging from 7.20% to 20.90%.

Table IX: Monthly explanatory regressions

Table IX presents the coefficients and p-values, calculated with Newey-West standard errors, are in square brackets and the adjusted $\bar{R}^2$ statistics from four explanatory regressions. The dependent variable is the weekly OOPS excess returns, and the independent variables are listed in the column 'Variables'. ** and *** indicate significance at the 5% and 1% levels, respectively. The estimation period is between January 1996 and December 2019, which translates to 288 observations.

Variables	OOPS excess returns				
	1	2	3	4	5
$r_{S\&P500} - rf$	0.2302***				
	[0.00%]				
$JUMP$		0.4797***			
		[0.00%]			
ΔRV			−0.0813		
			[52.00%]		
ΔVIX				−0.0016***	
				[0.50%]	
$Mkt - RF$					−0.0101
					[79.10%]
SMB					0.0638
					[30.20%]
HML					−0.0292
					[65.50%]
MOM					0.1199**
					[3.50%]
$\bar{R}^2$	10.90%	20.90%	0.30%	7.20%	3.00%

These findings imply low but significant economic risk, as measured by the regressions' variables' magnitude and $\bar{R}^2$, with market jump presenting the strongest relationship with the OOPS' returns, which confirms our ex-ante conclusions on OOPS risk exposure. These results point towards the possibility that the OOPS' exposure to priced risk factors, despite being low, partially explains the monthly OOPS out-of-sample returns, given that $\bar{R}^2$ can reach 20.90% for some variables.

Regarding the regression on the 4 Fama and French factors, we see that the majority of coefficients are not statistically significant with the exception of momentum. This factor presents a positive coefficient of 0.12, significant at a 5.00% level. This coincides with what was seen in Figure 5, where the monthly OOPS seemed to perform the best in periods of S&P 500 index prosperity. Nevertheless, this regression presents a low $\bar{R}^2$, implying that the 4 Fama and French factors do not explain the strategy's returns. To finish the proposed analysis, we perform the previously explained predictive regressions. Table X reports the results for these regressions. Once again, Faias & Santa-Clara (2017) argue that the analyzed variables are not able to predict the OOPS' performance. Re-

Table X: Monthly predictive regressions

Table X presents the coefficients and p-values, calculated with Newey-West standard errors, are in square brackets and the adjusted $\bar{R}^2$ statistics from four predictive regressions. The dependent variable is the weekly OOPS excess returns, and the lagged independent variables are listed in the column 'Variables'. ** and *** indicate significance at the 5% and 1% levels, respectively. The estimation period is between January 1996 and December 2019, which translates to 288 observations.

Variables	OOPS excess returns				
	1	2	3	4	5
RV	−0.0015***				
	[0.20%]				
VIX		−0.0003			
		[5.70%]			
VRP			−0.0002		
			[32.20%]		
$JUMP$				0.0752**	
				[5.00%]	
$Mkt - RF$					−0.0057
					[88.90%]
SMB					−0.054
					[50.10%]
HML					−0.0354
					[56.30%]
MOM					0.1067**
					[1.60%]
$\bar{R}^2$	1.70%	0.50%	0.20%	0.50%	2.30%

garding this particular study, we reach a similar, but distinct conclusion. Despite the fact that the highest $\bar{R}^2$ amongst all the regression is 1.70%, we find some statistically significant variables, thus implying the existence of variables that impact the expected OOPS out-of-sample returns, regardless of not carrying explanatory power.

Contemplating each individual variable, we observe that realized volatility presents a coefficient of -0.0015, significant at the 1.00% level, with a $\bar{R}^2$ of 1.70%. The VIX variable reports a negative coefficient of -0.0003, significant at a 10.00% level, and a $\bar{R}^2$ of 0.50%. Furthermore, jump risk presents a coefficient of 0.0752, significant at a 5.00% level, and a $\bar{R}^2$ of 0.50%. Finally, the variance risk premium variable is not significant and presents a $\bar{R}^2$ of 0.20%.

Taking these results into account, we observe very low coefficients for each variable, implying the low economic risk mentioned in the previous group of regressions. Moreover, we are able to confirm that, notwithstanding the significance of some variables, the exposure to priced risk factors is not able to explain the expected OOPS out-of-sample returns. In fact, the results obtained through ex-ante risk measures and ex-post linear regressions, show that the conditional expected out-of-sample OOPS returns stem from index mispricing, whilst the strategy's unexpected returns are also driven by loading on priced risk factors like equity risk premia and jump risk.

Regarding the predictive regressions on the 4 Fama and French factors we find once again that all the factors present insignificant coefficients with the exception of momentum. In fact, this variable presents a positive coefficient of 0.11, significant at a 5.00% level. This implies that periods of positive momentum tend to help OOPS in materializing positive returns, which is in line with the conclusions withdrawn from the explanatory regressions. Nevertheless, this regression results in a $\bar{R}^2$ of 2.30% which leads to the conclusion that the 4 Fama and French factors are not able to explain the expected OOPS out-of-sample returns. The same conclusion can be taken regarding the corresponding explanatory regressions, thus deciding that despite impacting the expected and unexpected returns, the 4 Fama and French factors cannot explain the monthly OOPS performance.

4.2.2 Simple Weekly OOPS

In this sub-subsection, I study the drivers of the simple weekly OOPS, as performed for the simple monthly OOPS. In accordance with this, I begin by studying ex-ante risk measures, such as portfolio beta and jump risk, and end with OOPS excess returns simple linear regressions, on the several risk factors specified in subsection 'Risk or Mispricing?'.

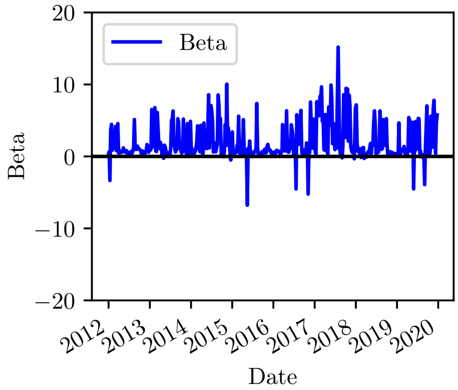
Figure 11, presents the time-series evolution of portfolio beta and Jump risk, for the simple weekly strategy. We find that the weekly OOPS displays a much more positive average beta, when compared to the monthly strategy. In fact, the weekly strategy registers a beta of 1.99 versus the 0.49 reported by the monthly OOPS, implying a much stronger connection to the market's systematic risk, with periods recording betas of around 15. Nevertheless, the strategy still presents negative betas, with the respective risk measure time-series registering a minimum of -6.78. Moreover, this parameter seems to vary around its mean in a constant manner through time. Secondly, analyzing jump risk, we see that the weekly portfolio reacts in a more aggressive way to sudden market drops of 5.00%, comparing to the monthly strategy. In fact, the average impact of a 5.00% market plummet averages -5.35%, with values ranging between -72.69% (corresponding to the period where the portfolio registers a beta of around 15) and 12.62%, evidencing once again the emphasis on short selling OTM put options, which might not satisfy risk averse investors' needs. This analysis has proven that the weekly OOPS is substantially more exposed to the studied risk measures than the monthly OOPS. This exposure can be the driver of the strategy's aggressively positive summary statistics, given that such characteristics tend to benefit from majorly bullish periods, such as the one on which this strategy was analyzed.

Once again, the two groups of regressions performed for the monthly OOPS are done

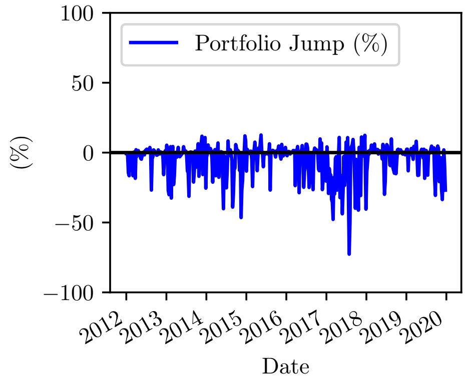
Figure 11: Weekly OOPS time-series risk measures

Figure 11 presents the weekly OOPS' risk measures according to the Black & Scholes (1973) model, between January 2012 and December 2019. Graph A presents the portfolio's beta, which represents the change when the underlying changes by 1.00%. Graph B shows the portfolio's jump risk, which represents the percentage change when the underlying drops suddenly by -5.00%.

Graph A. Beta



Graph B. Market Jump of -5.00%



on this setting, in order to provide clarity over the weekly strategy's ex-post risk exposure.

Table XI reports the results for the explanatory regressions. The explanatory regres-

Table XI: Weekly explanatory regressions

This table presents the coefficients and *p*-values, calculated with Newey-West standard errors, are in square brackets and the adjusted $\bar{R}^2$ statistics from four explanatory regressions. The dependent variable is the weekly OOPS excess returns, and the independent variables are listed in the column 'Variables'. ** and *** indicate significance at the 5% and 1% levels, respectively. The estimation period is January 2012 and December 2019; the number of observations is 416.

Variables	OOPS excess returns			
	1	2	3	4
$r_{\text{S\&P500}} - rf$	0.2825			
	[13.10%]			
$JUMP$		0.9031		
		[7.50%]		
ΔRV			0.6697	
			[8.30%]	
ΔVIX				−0.0005
				[74.20%]
$\bar{R}^2$	0.90%	1.90%	1.10%	−0.20%

sions present a story which is not displayed in the previous strategy. First, the market excess returns and changes in VIX present relatively low and statistically insignificant coefficients, with $\bar{R}^2$ of 0.90% and −0.20%, respectively. Then, jump risk and changes in RV, present substantially positive coefficients of 0.9031 and 0.6697, respectively. Both values are significant at a 10.00% level and present a $\bar{R}^2$ of 1.90% and 1.10%, respectively.

Based on such results, we see that both market excess returns and jump risk impact positively the dependent variable, matching with the monthly strategy. On the other hand, in contrast with the monthly strategy, changes in realized volatility tend to impact the the weekly OOPS excess returns quite positively. Additionally, changes in VIX maintains its negative signal and therefore, presents no surprises.

Taking these results into account, we can conclude that the weekly strategy thrives on underlying asset prosperity and loses significantly when the S&P 500 index return plummets by 5.00%. Likewise, the strategy tends to perform better with increases in realized volatility. This can happen as the shorter option maturity demands higher price fluctuation to materialize returns.

Summarizing the obtained results in the explanatory regressions, the weekly OOPS appears to have different signals and magnitudes of impact from the several explanatory variables. In fact, the jump risk and changes in realized volatility present higher coefficients for this strategy, when compared to the monthly OOPS, significantly impacting the dependent variable, which is in line with the ex-ante analysis. On the other hand, the performed regressions do not present substantial $\bar{R}^2$, enabling the conclusion that the exposure to these priced risk factors cannot explain the weekly OOPS performance, pointing towards the hypothesis that, notwithstanding the larger exposure to ex-ante and ex-post risk factors, mispricing is the main excess returns driver for this setting, reinforcing the differences between the weekly and monthly strategies.

Finally, several distinct predictive regressions of out-of-sample OOPS excess returns are analyzed. Table XII reports the results for these regressions. The predictive re-

Table XII: Weekly predictive regressions

Table XII presents the coefficients and p-values, calculated with Newey-West standard errors, are in square brackets and the adjusted $\bar{R}^2$ statistics from four predictive regressions. The dependent variable is the weekly OOPS excess returns, and the lagged independent variables are listed in the column 'Variables'. ** and *** indicate significance at the 5% and 1% levels, respectively. The estimation period is January 2012 and December 2019; the number of observations is 416.

Variables	OOPS excess returns			
	1	2	3	4
RV	−0.0151***			
	[0.00%]			
VIX		−0.0023***		
		[0.00%]		
VRP			−0.0019***	
			[1.00%]	
$JUMP$				0.0909
				[32.40%]
$\bar{R}^2$	9.10%	3.30%	1.50%	-0.20%

gressions performed report mostly significant coefficients, which is not witnessed in the predictive regressions for monthly OOPS and explanatory regression of the weekly strategy.

Realized volatility registers a negative and low coefficient of -0.0151, significant at a 1.00% level, presenting an adjusted $\bar{R}^2$ of 9.10%. This implies that the strategy performs in a worse manner when the realized volatility from the previous period $(t-1)$ reports higher values. The lagged VIX and variance risk premium present negative, low and significant coefficients at the 1.00% level, with $\bar{R}^2$ of 3.30% and 1.50%, respectively. The variables register coefficients of -0.0023 and -0.0019, implying once again that the strategy tends to perform worse when the VIX and the VRP register high values at the beginning of the investing period, matching the conclusion retrieved from the realized volatility variable. On the other hand, the lagged jump variable displays a small and positive coefficient, which is statistically insignificant. Moreover, the regression shows a $\bar{R}^2$ of -0.20%.

Taking these results into account, we are able to say that the weekly OOPS conditional expected excess returns are significantly impacted by volatility related risk factors, with realized volatility, VIX and variance risk premium presenting statistical significance at a 1.00% level. Nevertheless, the majority of regressions present low $\bar{R}^2$, implying the lack of explanatory power of such risk factors. However, since the realized volatility regression presents a substantial $\bar{R}^2$ of 9.20%, we cannot say that the strategy's performance, despite being mainly explained by mispricing, is not justified by loading on priced risk factors given that realized volatility appears to somewhat define the weekly OOPS excess returns.

4.2.3 Weekly Decomposed OOPS

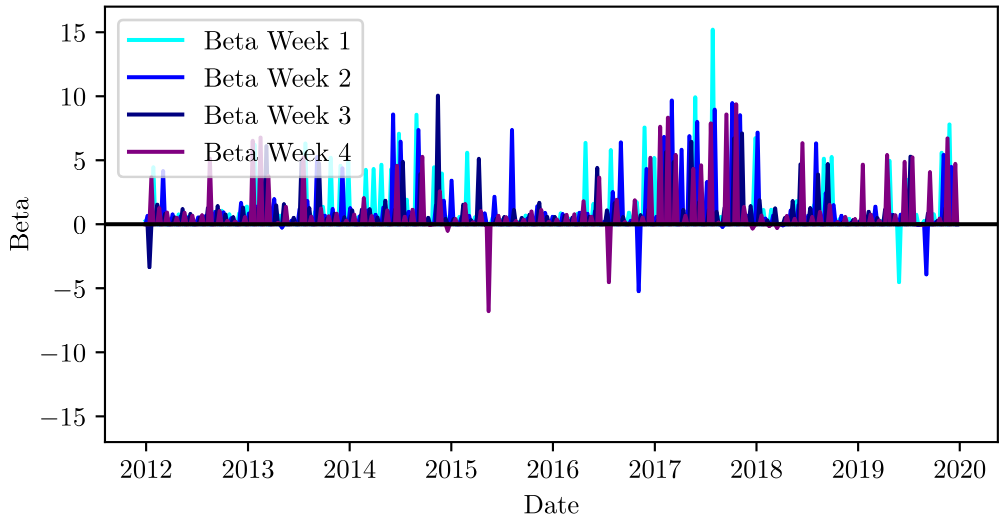
Following the rationale in the previous sub-subsections, we begin by analyzing the strategies' ex-ante risk measures. Firstly, since the strategies are composed by investing in options in only one week per month and investing in the riskless asset in the remaining weeks, the average portfolio betas and jump risk are much more moderate than the simple weekly strategy, despite maintaining the same maximums and minimums seen in the weekly strategy's time-series for the mentioned risk measures. Figure 12 Graph A presents the portfolios' betas for each strategy. The average values for this measure hovered around 0.37 and 0.54, which are much smaller than the value registered by the simple weekly OOPS. This implies a lower exposure to systematic risk, mainly because in three fourths of the strategies' observations, the investor invests 100.00% of their wealth on the risk-free asset.

Figure 12 Graph B presents the jump-risk measure for each OOPS. In line with what was previously said, this measure also presents much more reasonable average values for these strategies, ranging between -1.76% and -0.70%, despite reporting the extreme

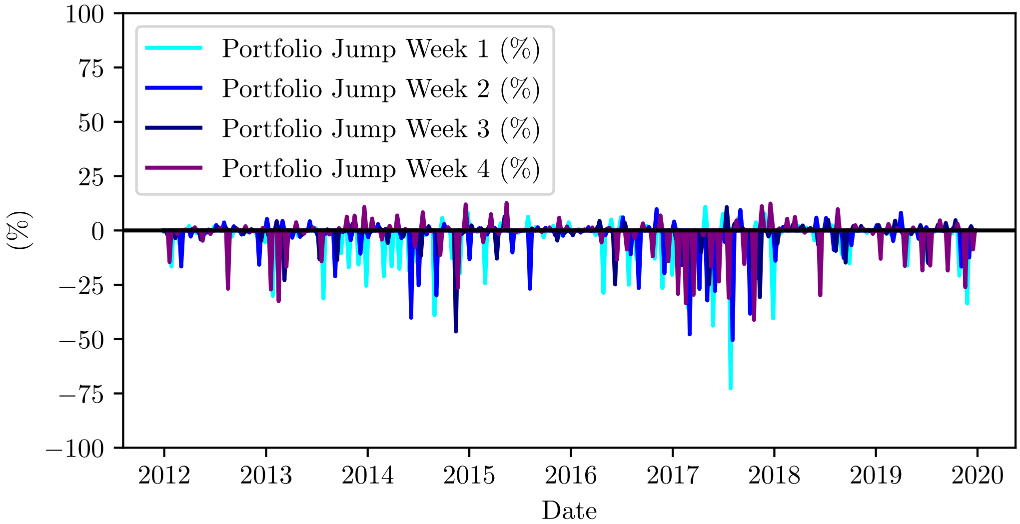
Figure 12: Weekly decomposed risk measures

Figure 12 presents the weekly decomposed OOPS' risk measures according to the Black & Scholes (1973) model, between January 2012 and December 2019. Graph A presents the portfolio's beta, which represents the change when the underlying changes by 1.00%. Graph B shows the portfolio's jump risk, which represents the percentage change when the underlying drops suddenly by -5.00%

Graph A. Betas



Graph B. Market Jump −5.00%



maximums and minimums displayed in the simple weekly OOPS. In fact, these averages of ex-ante risk measures are much more appealing than the ones presented in the previous sub-subsection, since the exposure to systematic risk is around four times smaller and the strategies lose less on average, than the simple weekly OOPS, when the market plummets by 5.00%.

Summarizing the stated results, these strategies display lower averages for portfolio beta and jump risk, implying low but relevant average levels of economic vulnerability, despite reaching extreme occasional peaks and depressions. Moreover, when comparing weekly decomposed strategies, we can witness a slight pattern of risk exposure, where the best strategies tend to have the highest vulnerability to jump risk.

With the ex-ante risk measures analysis complete, we move to the next step. Table XIV presents the coefficients with the corresponding p-values and $\bar{R}^2$ of the explanatory regressions.

The market excess return variable reports much lower coefficients comparing to the

Table XIII: Weekly decomposed explanatory regressions

Table XIII presents the coefficients (p-values, calculated with Newey-West standard errors, are in square brackets) and the adjusted $\bar{R}^2$ estimated in the explanatory regressions. The dependent variable is the weekly OOPS excess returns, and the independent variable is presented in the column 'Variable'. ** and *** indicate significance at the 5% and 1% levels, respectively. The estimation period is between January 2012 and December 2019, which translates to 416 observations.

Variables	OOPS excess returns			
	Week 1	Week 2	Week 3	Week 4
$r_{S\&P500} - rf$	0.0822	0.0965	0.1197	−0.0422
	[46.50%]	[43.70%]	[5.10%]	[40.10%]
$\bar{R}^2$	0.10%	0.40%	1.50%	0.20%
$JUMP$	0.5948	0.0999	0.0499	0.1117**
	[21.70%]	[23.40%]	[83.80%]	[2.00%]
$\bar{R}^2$	3.10%	−0.20%	−0.20%	0.00%
ΔRV	0.1470	−0.2076	0.2611***	0.2526***
	[33.60%]	[47.50%]	[0.20%]	[0.80%]
$\bar{R}^2$	0.00%	0.10%	1.50%	1.00%
ΔVIX	−0.0005	−0.0002	−0.0005	0.0006
	[55.00%]	[83.50%]	[26.90%]	[8.20%]
$\bar{R}^2$	0.00%	0.00%	0.60%	0.60%

simple strategies, ranging between -0.0422 and 0.1197. However, these coefficients tend to not be statistically significant and present rather low $\bar{R}^2$. When analyzing this variable across strategies, we now evidence that the two best strategies (first and fourth week) present the lowest coefficients, which are statistically insignificant. Regarding Jump risk, we witness, on average, lower values in comparison to the simple strategies. Moreover, this variable presents mainly positive and insignificant coefficients, with the exception of week 4, suggesting that this variable does not impact the weekly decomposed OOPS in a significant way. Additionally, the $\bar{R}^2$ associated with these regressions tends to be very small. Likewise, changes in realized volatility present lower coefficients, relative to the weekly OOPS, hinting towards lower exposure to this risk factor. Moreover, it presents mainly positive values and low $\bar{R}^2$. However, Week 3 and Week 4 present significant coefficients at a 1.00% level. Finally, the regressions on changes in VIX record very low and statistically insignificant coefficients, relating to weak $\bar{R}^2$.

Taking these results into account, we see variables that occasionally impact the dependent variable significantly. However, the average strategy reports low relative economic risk, measured by the magnitude of coefficients, when compared to the simple weekly OOPS. Moreover, the highest registered $\bar{R}^2$ records values of 3.10%, thus implying that, despite the low, significant and occasional impact of the analyzed variables, the weekly decomposed OOPS relies on mispricing to drive its performance. Finally, when comparing regressions across strategies, we see that the best strategies (Week 1 and Week 4) tend to have milder relationships with the market's excess returns and higher exposures to jump risk, which converges with the analysis made on the ex-ante study. Concluding

Table XIV: Weekly decomposed predictive regressions

Table XIV presents the coefficients (p-values, calculated with Newey-West standard errors, are in square brackets) and the adjusted $\bar{R}^2$ estimated in the predictive regressions. The dependent variable is the weekly OOPS excess returns, and the lagged independent variable is presented in the column 'Variable'. ** and *** indicate significance at the 5% and 1% levels, respectively. The estimation period is between January 2012 and December 2019, which translates to 416 observations.

Variables	OOPS excess returns			
	Week 1	Week 2	Week 3	Week 4
RV	−0.0052***	−0.0008	−0.0024***	−0.0040***
	[0.00%]	[60.80%]	[0.70%]	[0.00%]
$\bar{R}^2$	3.50%	−0.20%	1.70%	3.70%
VIX	−0.0006**	−0.0007	−0.0004**	−0.0004
	[3.30%]	[7.60%]	[5.80%]	[10.20%]
$\bar{R}^2$	0.50%	0.80%	0.70%	0.40%
VRP	0.0675***	−0.0009	−0.0004	−0.0002
	[0.10%]	[9.40%]	[12.30%]	[46.90%]
$\bar{R}^2$	−0.20%	1.00%	0.30%	−0.10%
$JUMP$	−0.0004	−0.0019	−0.0631	0.0286
	[22.30%]	[95.50%]	[47.40%]	[45.20%]
$\bar{R}^2$	0.00%	−0.20%	−0.20%	−0.20%

the section 'Results', we'll scrutinize the predictive regressions for the weekly decomposed OOPS. Table XIV shows the results for these regressions.

Realized volatility presents low, negative and mainly significant coefficients, displaying $\bar{R}^2$ ranging from -0.20% to 3.70%. The VIX regressions tell a similar, but less significant story. This variable presents very low and negative values that gravitate towards being significant at, at least, a 10.00%. Contrary to the realized volatility regressions, the VIX variable favors an association with very low $\bar{R}^2$. Additionally, we observe mostly modest, negative and insignificant values for the variance risk premium regressions' coefficients, with the exception of Week 1, where this variable present a much larger, positive and significant coefficient. Nevertheless, these regressions present limited $\bar{R}^2$ across all weekly decomposed strategies. Finally, jump risk presents meager, statistically insignificant and negative coefficients. In addition, when comparing the predictive regressions across the distinct strategies, we witness that the best performing weekly decomposed strategies appear to have a closer relationship with the lagged realized volatility variable.

With the outputs mentioned above, we are able to state that, notwithstanding the existence of significant impacts of risk factors on the dependent variable, the vulnerability of the strategy to the analyzed variables remains small, given the limited magnitude of coefficients. Moreover, since the regressions' $\bar{R}^2$ reach a plateau of 3.70%, we conclude that the reduced exposure to such risk factors is not able to explain the weekly decomposed OOPS performance. This coincides with the analysis performed on the explanatory regressions, where we conclude that mispricing appears to be the main driver of the strategies' excess returns.

5 Conclusion

In this thesis, I study the portfolio optimization method created by Faias & Santa-Clara (2017). The created strategy takes into account a complex option return's distribution and risk profile related investor preferences, therefore accounting for higher order moments. I scrutinize the mentioned method on three separate settings. I start by analyzing the simple monthly OOPS, evolving into the study of the simple weekly OOPS and finish by scrutinizing the weekly decomposed strategy.

Regarding the simple monthly OOPS, I present positive summary statistics. This strategy yields an annualized mean return, Sharpe ratio and certainty equivalent that beat the benchmark's, thus proving its attractiveness, even to risk averse investor, despite presenting bigger tail risk. Moreover, I study the drivers of this strategy's performance and find out that, in contrast to Faias & Santa-Clara (2017) findings, the OOPS thrives not only on option mispricing but also by loading in priced risk factors, since, despite lacking explanatory power for the conditional expected returns, they seem to partially explain the monthly OOPS unexpected returns.

On the other hand, the simple weekly strategy displayed outstanding summary statistics, with extremely high levels of annualized average return, Sharpe ratio and certainty equivalent, maintaining the same level of excess kurtosis as its monthly equivalent. These metrics are believed to be a cause of both greater frequency of investment and compounding, and the unique weekly naked options' features. Naturally, the extreme value of annualized mean return relates directly to higher exposure to ex-ante risk measures and several priced risk factors. However, despite being fundamentally similar, the simple weekly and monthly OOPS present distinct relationships with the same risk factors. Notwithstanding these differences, the weekly OOPS out-of-sample returns appear to be explained not only by mispricing, but also by vulnerability to priced risk factors, though marginally, such as the monthly strategy. Even though the conclusion is the same, the monthly and weekly OOPS are explained in a distinct manner.

Finally, the weekly decomposed OOPS represents the middle ground between the two strategies mentioned above. In fact, the majority of these weekly decomposed strategies present very positive summary statistics, beating the benchmark in the majority of the analyzed metrics. Moreover, these weekly decomposed OOPS appear to have a much lower exposure to risk measures and priced risk factors, whilst maintaining similar relationships with the studied ex-post variables in comparison to the simple weekly strategy. Nonetheless, these strategies display a feature that is not witnessed in the simple strategies. Actually, based on the ex-ante risk measures and ex-post regressions, we conclude that exposure to priced risk factors cannot explain the weekly decomposed OOPS performance. Furthermore, when studying the performance across weekly decomposed OOPS, a slight pattern can be observed. The best performing strategies gravitate towards higher exposure to jump risk, whilst the strategy's corresponding conditional expected return appears to be relatively better explained by realized volatility.

To conclude this study, all the analyzed strategies display attractive characteristics and pose a viable alternative for risk averse investors to invest in options, given the strategies' high certainty equivalents. Moreover, the options weights tend to be very small and thus demand a very high amounts of capital, implying that this strategy is most suited to high net worth individuals and institutional investors. Additionally, it is important to notice the limitations implicit to this thesis. The fact that OptionMetrics Ivy DB database only displays data till December 2019, cripples the ability to study this strategy in a COVID-19 setting, which would be highly interesting. Moreover, this study does not take into account margin calls, which can severely impact the strategies returns, since the OOPS presents a substantial number of periods where it categorizes as an net option seller. This can present an important obstacle to the implementation of the proposed strategies, which might be worthwhile to investigate.

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A Appendix

Table XV: Summary Statistics of Weekly Options

Table XV presents values for weekly option moneyness, price, bid–ask spread, relative bid–ask spread, volume, open interest, implied volatility, delta, beta, gamma, and vega for an ATM and OTM calls and puts, relative to the period between Jan. 2012 and Dec. 2019. Moneyness is given by $S/K1$. Price is the option's mid price. Bid–ask spread is defined as the ask less the bid price. Relative bid–ask spread equals to the bid–ask spread divided by the mid price. Volume is the contract's volume registered one month prior to its maturity. Open interest is recorded on the same day as volume. Implied volatility is equal the annualized volatility given by the Black & Scholes (1973). Delta is defined as the Black & Scholes (1973) delta. Beta is equal to the delta times the ratio of underlying asset's price to the option's price. Gamma is defined as the Black & Scholes (1973) gamma. Vega is defined as the Black & Scholes (1973) vega.

	Call ATM	Call OTM	Put ATM	Put OTM
Moneyness	−0.32%	−1.94%	0.37%	3.48%
Mid Price	10.86	1.81	11.80	2.86
Bid-Ask Spread	0.61	0.22	0.66	0.28
Relative Bid-Ask Spread	6.44%	19.74%	6.09%	14.32%
Volume	7062.54	7467.51	7271.62	11126.80
Open Interest ($000)	15,782.42	15,410.71	14,109.42	21,745.38
Implied Volatility	11.80%	10.81%	12.86%	18.01%
Delta	0.41	0.10	−0.41	−0.10
Beta	104.64	169.82	−89.65	−96.21
Vega	1.06	0.47	1.10	0.46
Gamma	0.01	0.01	0.01	0.00

Table XVI: Summary Statistics of Returns (Weekly)

Table XVI presents summary statistics of return for a buy-and-hold strategy in multiple weekly assets, between Jan. 2012 and Dec. 2019, comprising annualized mean returns, annualized standard deviation, minimum, maximum, skewness, excess kurtosis, annualized Sharpe ratio (SR), and annualized certainty equivalent (CE). Panel A presents primitive assets (S&P 500 index and 1-week Treasury Bill). Panel B shows options, namely an ATM call, an OTM call, an ATM put and an OTM put.

	Ann. Mean	Ann. St. Dev.	Min.	Max.	Skew.	Exc. Kurt.	Ann. SR	Ann. CE
Panel A: Primitive Assets								
S&P 500	11.77%	12.21%	−7.31%	4.73%	−0.79	2.09	0.91	8.73%
1 Week T-Bill	0.69%	0.12%	0.00%	0.05%	1.01	−0.58	0.00	0.67%
Panel B: Only Options								
Call ATM	594.35%	1173.78%	−100.00%	1006.41%	1.98	5.25	0.51	−100.00%
Call OTM	−793.12%	2341.16%	−100.00%	3237.06%	6.56	53.04	−0.34	−100.00%
Put ATM	−1331.37%	1261.74%	−100.00%	1234.74%	3.56	15.60	−1.06	−100.00%
Put OTM	−2939.26%	1987.26%	−100.00%	3295.48%	8.82	86.84	−1.48	−100.00%

Figure 13: Densities of Weekly Option Returns

Figure 13 presents the histograms of weekly returns on long at-the-money Calls, out-of-the-money Calls, at-the-money Puts, out-of-the-money Puts over the S&P 500 index. The returns are recorded between January 2012 and December 2019.