Investing in Euro equity markets through Parametric Portfolio Policies (PPP)

A feasibility analysis of the PPP approach on principal euro-zone stock markets

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Abstract

Title: Investing in Euro equity markets through Parametric Portfolio Policies (PPP)

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Key words: Parametric portfolio, portfolio optimization, portfolio policies, expected utility, risk aversion

In this paper we exploit the parametric portfolio policy (PPP) approach proposed by Brandt, Santa Clara and Valkanov (2009) on an investable set of only large European stocks, with the goal of accepting its advantages to the portfolio's optimization problem also in the Euro financial markets. Our sample includes all the euro-denominated equities that have been listed between January 1990 and December 2019 on the stock exchange of one of four large European countries: France, Germany, Italy and Spain. The optimal portfolio's weights are determined by a function that considers three stock-specific characteristics: the market capitalization of the company, its book-to-market ratio and its recent 12-month lagged return. The coefficients of this function, corresponding to size, value and momentum, respectively, are established by optimizing the investor's average utility of the portfolio's return over the sample period. We test the model both in-sample and out-of-sample and benchmark the results against the equal-weighted and value weighted portfolios. Also, we extend the base case by including short sales constraints and a sensibility analysis to different coefficients of risk aversion. Overall, we confirm the parametric optimization's improvements to asset-allocation. The optimal portfolios generate robust performances, in-sample and out-of-sample, which are consistently superior to both benchmarks' figures. However, this methodology presents a limitation to its practical use: the large trading activity required. From the coefficients' estimation, we establish that the European investor always prefers value stocks and past winners, while the size preference depends on the risk's tolerance.

Sumário

Titulo: Investir em mercados de acções em euros através da Parametric Portfolio Policies

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Palavras-chave: Carteira paramétrica, optimização de carteira, políticas de carteira, utilidade esperada, aversão ao risco

Neste documento exploramos a abordagem da política de carteira paramétrica (PPP) proposta por Brandt, Santa Clara e Valkanov (2009) sobre um conjunto investável de apenas grandes acções europeias, com o objectivo de aceitar as suas vantagens para o problema de optimização da carteira também nos mercados financeiros do Euro. Os pesos óptimos da carteira são determinados por uma função que considera três características específicas das acções: a capitalização bolsista da empresa, o seu rácio de mercado e o seu rendimento recente de 12 meses atrasado. Os coeficientes desta função, correspondentes ao tamanho, valor e dinâmica, respectivamente, são estabelecidos através da optimização da utilidade média do investidor do rendimento da carteira ao longo do período amostral. Testamos o modelo tanto na amostra como fora da amostra e comparamos os resultados com as carteiras ponderadas por igual e de valor. Além disso, alargamos o caso base incluindo restrições de vendas curtas e uma análise de sensibilidade a diferentes coeficientes de aversão ao risco. Globalmente, confirmamos as melhorias da optimização paramétrica na afectação de activos. As carteiras óptimas geram desempenhos robustos, in-sample e out-of-sample, que são consistentemente superiores aos valores de ambos os parâmetros de referência. No entanto, esta metodologia apresenta uma limitação à sua utilização prática: a grande actividade comercial requerida. A partir da estimativa dos coeficientes, estabelecemos que o investidor europeu prefere sempre as acções de valor e os vencedores passados, enquanto a preferência pela dimensão depende da tolerância do risco.

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I. Introduction

In financial theory, portfolio selection has always challenged investors to search for methodologies to allocate their wealth efficiently. Despite the multitude of asset allocation models proposed by the academic research over the years, there is still not a universal model that produces consistent results upon its different applications. The cause of this conflict stands in the investor's desire to adopt a model that presents theoretical robustness as well as practical efficacy, and that is valid with different inputs sources. As a result, such dilemma has motivated the academic exploration of modern optimization schemes.

Markowitz (1952) provided the first framework of portfolio selection in the literature, known as the mean-variance (MV) approach. He suggested that a rational investor should decide over a portfolio under conditions of risk. For a given target return, the investor should select the portfolio with the lowest risk or, oppositely, for the same level of risk, the investor should pick the portfolio with the highest expected return. This concept entails that the investor has a multitude of portfolios to select amongst, each one with a different risk-return profile, and must pick the ones that best satisfy the investor's risk-reward preference. These portfolios are referred to as efficient portfolios.

While researchers have widely adopted the Markowitz methodology, which led to the creation of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), its accuracy has been questioned in recent decades. Two central critiques have been identified: the first relates to the model assumption that asset returns follow a multivariate normal distribution and that investors have quadratic utility function. According to Quiggin (1982), the quadratic utility function is counter-intuitive, and volatility as a risk measure is incompatible with existing preferences. The second refers to the dimensionality of the model. Under the Markowitz framework, the risk of an individual asset is given by the historical variance of its past returns, while the risk of a portfolio is determined by the variance-covariance (V-C) matrix, constructed from individual asset variances. However, the variance-covariance matrix is not always easy to be correctly estimated, especially if the investable set of assets is large. In fact, when doing so the investor is required to compute N first and $(N^2 + N)/2$ second moments of returns. For instance, for a portfolio of 100 assets, the investor needs to estimate 5050 coefficients. This causes the investor to face a substantial econometric problem.

As the number of assets N to be modelled increase, Michaud (1989) argues that the efficacy of the V-C matrix as estimator of optimal weights decrease. To improve the reliability of the estimated coefficients, different fixes were identified. Ledoit and Wolf (2003) proposed a linear shrinkage of estimates. They construct the V-C matrix by combining a matrix developed on the sample and a target matrix. Furthermore, they propose a factor-model-adjusted structure to the covariance matrix (Ledoit and Wolf, 2017) to provide adaptability to the model. Another improvement relates to imposing different constraints to the portfolio weights, as for instance by dictating a no-short sale constraint. Jagannathan and Ma (2003) show that constraining portfolio weights to be nonnegative leads to improved portfolio performance. Even though these fixes provide clear improvements to portfolio optimization problem, they usually require complex tools to function, such as the tools developed by BARRA, Northfield and other companies. Due to the large efforts needed to implement the Markowitz approach and the inaccuracy of its paradigm, from the 1990's onwards the academics begun to search for alternative variables to explain asset returns.

Rather than forecasting expected stock returns in the time-series, researchers focused on the cross section of individual stock returns. Since then, the market beta - the slope in the regression of a security's return on the market's return – was thought to be sufficient to explain expected stock returns in the cross section. In their work, Fama and French (1992) proved that the market beta does not have much explanatory power in the cross section of average stock returns and, instead, they showed that two easily measured variables, size (price per share times common shares outstanding) and book-to-market ratio (book value per share divided by price per share), capture better the cross-sectional variation in average stock returns. The combination of these two stock-specific characteristics is also described by Hanna and Ready (2005) as a "parsimonious characterization of all of the useful information about expected excess returns".

Following Fama and French (1992) innovative concepts, also Brand and Santa-Clara (2006) studied to skip the return distribution procedure and decided to shape portfolio weights based on firm-specific features. Before their publication, Chan et al. (1999) and Jagannathan and Ma (2003) found that performing portfolio optimization on firm characteristics, which since then was rarely implemented by asset managers, leads to greater portfolio performance. Also, DeMiguel, Garlappi and Uppal (2007) suggested that "exploiting information about the cross-sectional characteristics of assets may be a promising direction to pursue". By determining optimal weights as functions of observable firm characteristics, in fact, the computation of several moments of the return distribution can be avoided. The focus is instead on assessing a scheme of parameters that will define the investor policy. The final objective is then to optimize the policy by maximizing the investor's utility function. In their pioneering work, Brandt and Santa-Clara (2006) performed an optimal allocation of stocks, bonds, and cash by establishing the optimal portfolio weights as functions of some generic and some specific predictors. Specifically, they modelled the weight in each asset class through an objective function that include variables specific to the asset class and a common set of macroeconomic variables valid for all asset classes. While this approach helps circumventing the problem of optimizing portfolios of assets with substantially different characteristics, such as stock, bonds and cash, it was proven to work well also with portfolios of equities that belong to different industries or markets.

Later on, Brandt, Santa Clara and Valkanov (2009) tested this parametric portfolio strategy on a universe of only stocks. In their publication, they determined a set of characteristics explaining the equities' returns in the cross-section and they optimized the portfolio's weights from a sample of all the listed US stocks in the period January 1964 - December 2002. In the research, they use size, value and momentum as firm-specific characteristics. This selection was chosen since Fama and French (1996) showed that these three characteristics are robust proxies for the cross-section of expected returns. Their framework allows for modelling only N weights independently of investor's preferences and the joint distribution of asset returns. Apart from its computational reduction, the results show that this approach leads to robust performances in and out-of-sample. The functioning of this model is improved when the predictors selected capture the asset-specific characteristics. Therefore, a robust selection of factors that can capture dynamics of returns plays a crucial role before applying parametric portfolio policies.

The goal of this paper is to evaluate the Brandt, Santa Clara and Valkanov (2009) parametric portfolio approach on a dataset of European stocks. We select uniquely euro-denominated equities from four large European economies – Germany, France, Italy, and Spain – and we construct a portfolio following the parametric methodology. We aim at establishing if such asset allocation model would produce robust performances also if implemented by a European investor that narrows the investable universe to the four financial markets aforementioned. We exclude the United Kingdom to avoid currency fluctuations to play a role in stock price changes. We compare the performance of the optimized portfolio with two benchmark portfolios: the equal-weighted (EW) portfolio and the value-weighted (VW) portfolio. These two are constructed from the same pool of equities of the optimized portfolio so as to reduce the differences in performance to only the asset weighting policy. While the inclusion of the market portfolio (i.e., VW) is clearly intuitive as it reflects the way national indexes are constructed, the involvement of the naïve portfolio (i.e., EW) is justified by the literature. DeMiguel, Garlappi and Uppal (2007) demonstrate that the 1/N rule, where N is the number of investable assets, consistently beats 14 portfolio models constructed across seven empirical datasets in terms of Sharpe ratio, and certainty equivalent.

The remaining of the paper is structured as follows: Section II describes the data and outlines the methodology applied to construct the optimized portfolio and to account for its extensions, Section III presents the empirical results obtained by the portfolio in its base case and in the extended cases and, finally, Section IV concludes.

II. Data Description and Methodology

2.1 Data

We aim at evaluating the robustness of the parametric portfolio optimization approach on a sample of European equities, denominated in Euro. In detail, we focus on a selected group of countries, which represent the four largest economies in the Euro-zone. These are France, Germany, Italy and Spain. All the market data, on a monthly basis, is sourced from the Thomson Reuters DataStream data base and is processed and elaborated through the programming language Python. Given the shortage of data on European companies before 1990, we select a shorter time span than Brand, Santa-Clara and Valkanov (2009). We use a 29 years period ranging from January 1990 until December 2019. We do not include the year 2020 in the analysis as it lacks updates on data relevant for our study. For each country, we select all the companies that have been listed on the national stock exchange during the period, including the ones that are currently delisted or dead as a consequence of mergers, privatizations, bankruptcies, etc. The inclusion of such companies is necessary in order to mitigate the effects of survivorship biases which could lead to an overestimation of the historical performance of the constructed portfolios. From the pool of equities available, we provide some adjustments to improve the relevance of our data. We exclude REITs, closed-end funds, and listed companies with price inferior to €1. At each point in time, we exclude the 5% smallest stocks by market capitalization.

For each company, we proceed to have the following monthly data:

- Closing prices in Euro, adjusted for stock splits and dividend payments
- Number of common shares outstanding
- Price-to-book (ptb) ratio, defined as the company's closing price per share divided by its book value per share (BVPS)

We replicate the Brand, Santa-Clara and Valkanov (2009) methodology in the construction of the firms' characteristics. To account for size, the log of the firm's market equity *(me)* is computed by multiplying the log of the price per share times the number of shares outstanding. We use the firm's book-to-market ratio *(btm)*, defined as the log of 1/ price-to-book as the value indicator. Finally, the lagged one-year return compounded from t-13 to t-2 reflects the momentum *(mom)* effect. In the construction of momentum characteristic t-1 is not considered in order to avoid the one-month reversal effect. As risk-free rate, we proceed to take the time-series average of the 10-year governmental bond yield, on a monthly basis, of each country for the period January 1990 to December 2019. We then compute the average of the four countries' averages. The risk-free asset is not included in the investable set of securities, it is only adopted to compute the portfolios' Sharpe Ratio (SR) and Certainty equivalent (CE). For all the results displayed in the Tables, the average monthly risk-free rate considered in the sample is 3.54%. For a given point in time, we only consider equities for which all the three firm's characteristics – size, value and momentum - are available. If this condition is not met, the stock is exempted from the analysis for the given date.

The size of our investable set of equities, comprehensive of companies from the four different regions, is generally showing an upward trend over the sample period, with sharp increases before crisis times and a general stabilization in the last years of the sample. The evolution in the number of companies entering the analysis is showed in Figure 1. The aggregate sample grows at 0.173% per month, 2.087% per year, and the average sample size is of 1359 stocks. The minimum is registered at the beginning of our observable period, February 1996 where data from 931 firms is available, and the maximum of 1537 firms is recorded in December 2008. The largest contribution in number of companies is provided by France with an average sample of 531 firms. Germany follows with 481, then Italy (198), and Spain (149).

Figure 1 - Number of stocks over the sample period

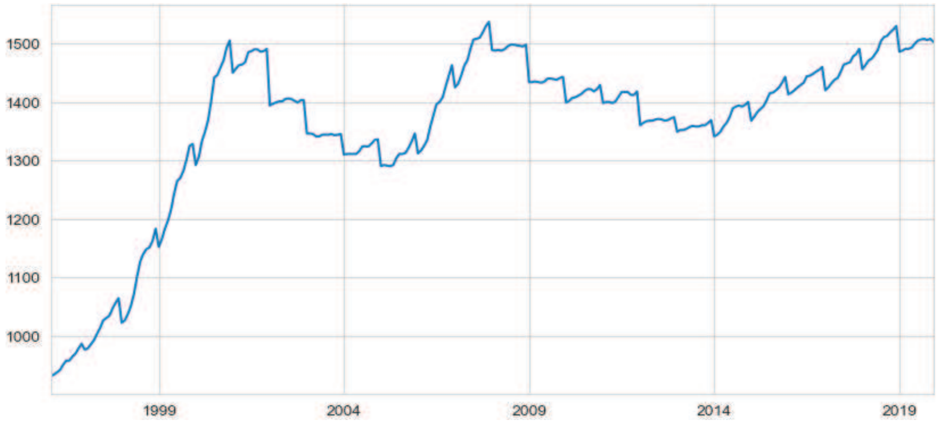


Table 1 provides the summary statistics of the three firm's characteristics – value, size, and momentum. For each variable, we present the mean, median, standard deviation, kurtosis and skewness. The values showed reflect the time-series averages of the monthly cross-sectional statistics.

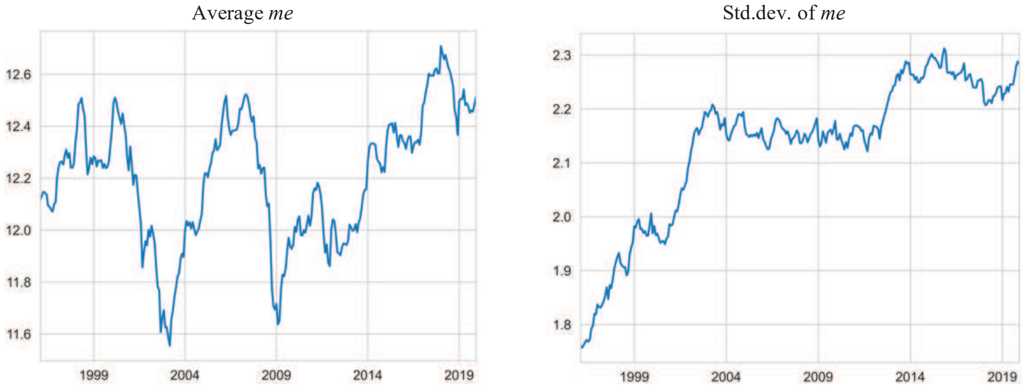
Table 1 - Summary statistics of firm's characteristics

	Size	Value	Momentum
Mean	11,865	0,512	0,075
Median	11,926	0,509	0,076
Standard deviation	0,243	0,074	0,191
Skewness	-0,533	0,254	-0,002
Kurtosis	-0,313	-0,412	-0,576

Figure 2, instead, describes the mean and standard deviation of characteristics through time. The column of figures on the left presents the evolution of the cross-sectional means through the sample period of the non-standardized characteristics. The column of figures on the right, instead, shows the cross-sectional standard deviations. It is worth to mention that the characteristics, as described in Table 1 and showed in Figure 2, are in a non-standardized form. When applied in the portfolio weights optimization function, instead, they are standardized to have zero mean and unit standard deviation. This ensures the comparability of characteristics and allows the portfolio weights, at each point in time, to sum up to 1. We are going to deepen this concept in the next section (2.2).

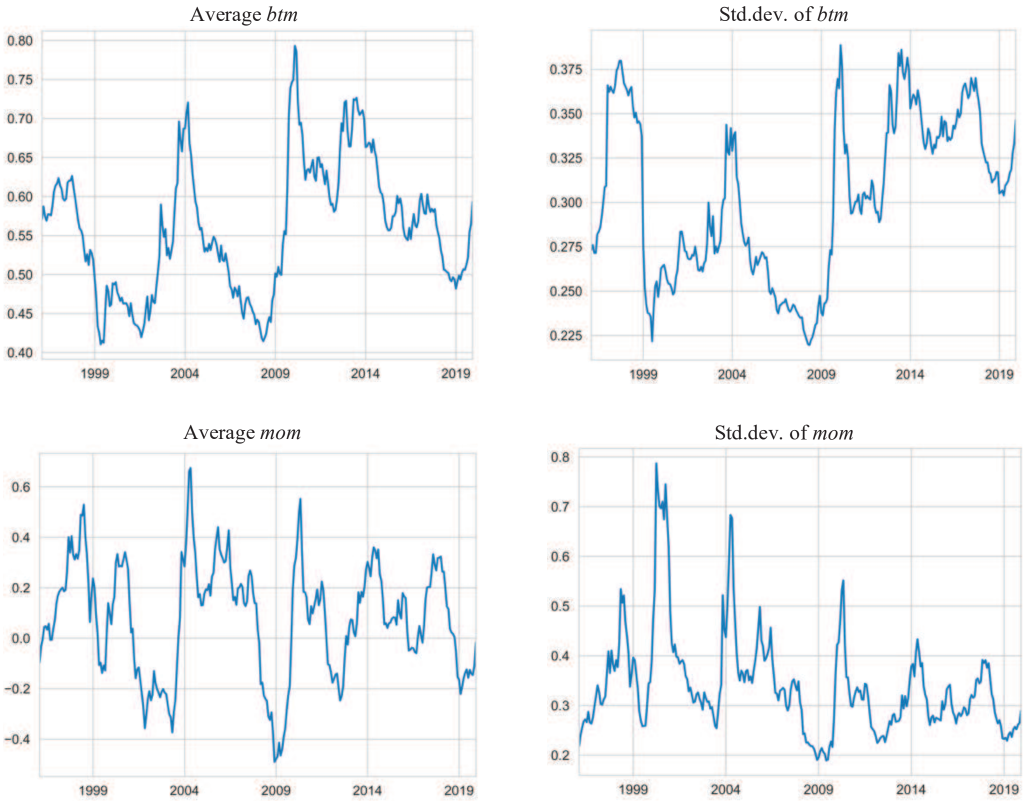
Figure 2 - Mean and Standard deviation of firm's characteristics

The figure presents the cross-sectional non-standardized means and standard deviations of the firm characteristics *me, btm,* and *mom* for the period January 1996 to December 2019.



Average *me*

Std.dev. of *me*


Average *btm*

Std.dev. of *btm*

Average *mom*

Std.dev. of *mom*

2.2 Methodology

The fundamental idea behind the Santa-Clara and Valkanov (2009) approach is to choose the portfolio weights in *t* that maximize the investor's utility in *t*+1. Unlike the traditional Markowitz (1952) approach, for which the investor must estimate each period the optimal weights for each asset in the portfolio, the parametric approach finds optimal weights depending on a single function of parameters, constant for all stocks and through time. The combination of these parameters determines the investor's policy. For instance, an investor's policy with a negative size coefficient and a positive value coefficient, it would indicate that the investor over allocates stocks showing low market capitalization (small companies) and high book-to-market ratios (value firms). Consequently, the same investor under allocates stocks with high market capitalization (large companies) and with low book-to-market ratios (growth firms). The magnitude of under allocation and over allocation is given by the characteristic's coefficient, which determines the deviation of the optimal weights from the weight the same stock has in the benchmark portfolio, in our case the capitalization-weighted portfolio, or also known as the value-weighted or market portfolio.

The optimal portfolio weights are found as a function of the investor's policy and the stock's characteristics, as presented by the following function:

$$(1)\quad w_{i,t} = f(x_{i,t};\ \theta)$$

We determine our optimal portfolio weights as a linear function of the stock's characteristics. The function is presented as follows:

$$(2)\quad w_{i,t} = \overline{w_{i,t}} + \frac{1}{N_t}\ \theta^T\ \hat{x}_{i,t}$$

where $\overline{w_{i,t}}$ is the weight of stock i at time t in the benchmark portfolio, the value-weighted (VW) portfolio. θ^T is a vector for the parameters associated with the characteristics and $\hat{x}_{i,t}$ is the vector containing the cross-sectionally standardized characteristics of stock i. One crucial feature of the parametric approach, which makes it substantially different from the Markowitz approach, is that the vector of parameters determined by θ^T is constant over time and across stocks. This entails that the optimal weight of a stock within the portfolio is exclusively given by the characteristics shown by the stock itself, while the pattern of its historical returns has no importance. In other words, investors allocate similarly amongst stocks that present similar characteristics regardless of the differences in their historical returns.

As Brand, Santa-Clara and Valkanov (2009) suggest, the standardization of characteristics allows to have the factor $\theta^T\ \hat{x}_{i,t}$ with mean zero and unit standard deviation. A zero cross sectional mean ensures that the sum of the deviations of optimized weight from the benchmark weights equal to zero, which confirms that the sum of stocks' weights, at each point in time, equals to 1. The unit standard deviation, instead, guarantees a stationary distribution of the characteristics, which would tend to be non-stationary if we use raw $x_{i,t}$. Finally, the term 1/ N_t allows the portfolio weight function to be scalable as the number of stocks in the sample changes over time. If we would omit this term, an increase in N, without changing the cross-sectional distribution of characteristics, would lead the investors to increase their leverage, reflecting more aggressive allocations of wealth on short positions.

As mentioned above, investors allocate by maximizing their utility in *t*+1. The utility function (3), presented below, incorporates the investors' degree of risk's tolerance and, thus, the investor's preferrable risk-reward profile. This profile is determined by the investor's coefficient of risk aversion γ. In order to maintain the framework proposed by Brand, Santa-Clara and Valkanov (2009), we assume that our hypothetical European investor has a CRRA (Constant Relative Risk Aversion) preference of five. In an extension to the base case, γ is scaled, upward and downward, to study how the investor's allocation policy changes when subject to different risk's tolerances. The investor's power utility function is given by:

$$(3) \quad U_t = \frac{(1 + monthly\, returns_t)^{1-\gamma}}{1 - \gamma}$$

By comprehending Equation (2) and Equation (3), we can now outline the utility maximization function displayed in Equation (4). In summary, the investor's problem is to choose the portfolio weights $w_{i,t}$ that maximize the conditional expected utility of the portfolio's return $r_{p,t+1}$:

$$(4) \quad \max_{\{w_{i,t}\}_{i=1}^{N_t}} E_t\left[u(r_{p,t+1})\right] = E_t\left[u\left(\sum_{i=1}^{N_t} w_{i,t}\, r_{i,t+1}\right)\right]$$

As previously stated, one key aspect of the parametric approach is that the coefficients θ remain constant for all stocks and for the entire investable period. Constant coefficients over time imply that the coefficients that maximize the investor's conditional expected utility at a given time are the same for the entire period, meaning that they also maximize the investor's unconditional expected utility. Given this implication, we can rewrite Equation (4) with respect to the coefficients θ. With the linear function (2), the optimization problem becomes:

$$(5) \quad \max_{\theta} \frac{1}{T} \sum_{t=0}^{T-1} u\left(\sum_{i=1}^{N_t} (\overline{w_{i,t}} + \frac{1}{N_t}\, \theta^T\, \hat{x}_{i,t})\right) r_{i,t+1}$$

Brandt *et al.* (2009) consider their parametric approach to have two great advantages over the other portfolio optimization solutions: to be "computationally simple" and "easily modifiable and extendable". While we have just proven the first advantage to be correct, we are now going to test if the second statement is also accurate.

In its base form, the optimization problem allows portfolio weights to be negative, thus allowing for short sales. However, this implies that investors can take short positions at no cost of borrowing, which is an inaccurate assumption. In an extension of the base case, we construct weight-constrained optimal portfolios. We propose two different perspectives. First, we constrain the portfolio weights to only positive values (long-only portfolio). Second, we allow them to be negative to a certain extent. Under this setting, the short positions, as a percentage of total wealth, are capped according to a leverage limit.

If weights are constrained to be only greater than zero, the sum of the optimal portfolio weights no longer equals to 1, but instead it would exceed one. To address this issue, we are going to follow the procedure of Brandt *et al.* (2009), by which they renormalize the portfolio weights in the following manner:

$$(6) \quad w_{i,t}^{+} = \frac{max\ [0, w_{i,t}]}{\sum_{j=1}^{N_t} max\ [0, w_{j,t}]}$$

The main idea behind the renormalization of weights is to treat the negative weights as 0 and divide all weights by the sum of all nonnegative weights.

In order to test the robustness of the results shown by the optimal portfolio, we adopt two frameworks of analysis: the in-sample (IS) and the out-of-sample (OOS). For the in-sample analysis, we estimate the parameters by maximizing the utility function of the investor across the entire period. Thus, the parameters estimated in the in-sample are constant through time and across stocks. For the out-of-sample, instead, we use the first 5 years of data available (corresponding to 60 months), from January 1990 to December 1995, to estimate the coefficients of the initial portfolio policy. These coefficients, then, are used to construct all the monthly portfolios of 1996. In January 1997, we re-estimate the coefficients by incorporating all the information seen up to that point. Indeed, we adopt an expanding window that, with annual frequency, increases in size until the end of the sample data.

Since the performance of the optimal portfolio by itself does not give an idea of the strength of the parametric approach, we benchmark its performance against two market portfolios. Both portfolios are constructed from the same sample data used for the parametric application. The first is the naïve portfolio or equal-weighted (EW) portfolio. In the EW, every stock has the same portfolio's weights regardless of any company's characteristics. The second benchmark portfolio is the value-weighted (VW) portfolio, which allocates the weights according to the company's market capitalization (price per share times shares outstanding). Companies with large market capitalization have higher portfolio weights compared to low-cap companies.

In order to determine if the optimal portfolio contributes to improve the performance of the benchmark portfolios, we look at its cumulative compounded return over the sample period as well as we investigate three different performance metrics: Sharpe ratio (SR), Certainty equivalent (CE) and Turnover.

- **Sharpe Ratio (SR)**

  Proposed by Sharpe (1966), the Sharpe ratio is often used by investors to quantify the trade-off between risk and return of a portfolio. It takes the portfolio's average excess return, relative to the risk-free rate, and divide it by the average volatility of the portfolio. It is defined by the following formula:

$$(7)\quad SR = \frac{\mu_p - r_f}{\sigma_p}$$

- **Certainty equivalent (CE)**

  The certainty equivalent gives the guaranteed return an investor would be willing to immediately accept, hence a risk-free investment, rather than obtaining a higher return from a riskier portfolio. It is represented as:

$$(8)\quad CEQ = (\mu_p - r_f) - \frac{\gamma}{2}\sigma^2$$

  where the first term, in parenthesis, represents the excess portfolio's return over the risk-free rate, γ is the coefficient of risk aversion and σ^2 is the portfolio's variance.

- **Turnover**

  In the scenario where transaction costs are not considered, turnover is a fundamental measure since it measures the absolute change in weight between periods. We adopt the definition of turnover proposed by De Miguel, Garlappi, and Uppal (2007), which is the following:

$$(9) \quad Turnover = \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{N} (|w_{j,t+1} - w_{j,t}|)$$

where T is the number of periods in the sample and N is the number of stocks present in the portfolio. In brackets we see that the absolute value is considered. We proceed to always annualize the portfolio's turnover.

III. Empirical results

The following chapter presents the results of the parametric portfolio optimization strategy, implemented by Brandt, Santa Clara and Valkanov (2009) on the US stock market, and tested in this paper on four European stock markets, jointly. For each of the scenarios considered – base case and its extensions – we display the results of the optimal portfolio and of two benchmark portfolios: the equal-weighted portfolio and the value-weighted portfolio.

The results of the portfolios are shown in four different tables - from Table 2 to Table 5 - corresponding to four different frameworks of analysis: Table 2 refers to the results of the base or unconstrained scenario, Table 3 shows the results of our portfolio when only long positions are allowed (i.e., no short sales). Tables 4 presents the results of the optimal portfolio constrained to a maximum level of leverage (in our case is considered to be 50% on total wealth). Finally, in Table 5 and in Table 6, we see the results of our portfolio when different coefficients of risk aversion are considered. We must recall that, unless in this last extension, in all other cases the investor is considered to have a CRRA (Constant Relative Risk Aversion) preference equal to five.

Each table is structured in three distinct sections: (i) the first three rows present the estimates of the parameters for each characteristic – size, value, and momentum. Since the firm characteristics are standardized in the cross section, the magnitudes of the estimated parameters can be compared each other. While the coefficients estimated in the in-sample analysis are valid for all stocks and all periods (i.e., they are constant), in the out-of-sample analysis they are still constant across stocks, but they vary with time. Consequently, the out-of-sample coefficients shown in Table 2, as well as in all the following tables, are the time-series average of the parameters estimated with annual frequency. As stated previously, we use the first 5 years of our sample, from January 1991 to December 1995, to estimate the initial coefficients of the portfolio policy as of January 1996. We adopt the estimated coefficients to assign our initial portfolio's weights and use them to form the monthly portfolios for each month of the year 1996. After one year, we estimate again the coefficients necessary to construct the monthly portfolios for 1997 by enlarging the sample. The expanding window method is adopted until the end of the sample.

The second group of rows (ii) provides information on the allocation of weights across stocks. Specifically, five different values are presented: the average absolute portfolio weight, the average minimum weight, the average maximum weight, the average sum of negative positions, the average fraction of negative positions on the total invested positions, and, in conclusion, the portfolio turnover. All values are presented in percentage terms. Except for the portfolio turnover, which has a different computational methodology explained by Equation (9), the averages presented are the time-series average of the cross-sectional monthly averages. Finally, the last four rows (iii) present some performance statistics of the portfolios we consider important to display, all in annual terms. We give the portfolio's average annualized return, its annualized average volatility, and its total cumulative compounded return. While the first two are indicative of the return-risk profile of the portfolio over the sample period, the cumulative return, instead, gives how much a unit of euro invested in January 1996 would be worth as of December 2019. Inflation is already incorporated in the level of prices. Furthermore, we consider the Sharpe ratio, explained by Equation (7), and the Certainty Equivalent (CE), explained by Equation (8). Since we use the first 5 years of data to estimate the initial coefficients for the out-of-sample analysis, the results shown in the Tables, as well as in the Figures, are representative of the time period between January 1996 and December 2019. In order to ensure comparability, this reduction in the data time span is conducted also in the in-sample results, as well as in the two benchmark portfolios, EW and VW.

3.1 Base case (Unconstrained case)

Table 2 collects the results of the two benchmark portfolios (first two columns, EW and VW), of the in-sample parametric portfolio policy (third column) and of the out-of-sample parametric portfolio policy (fourth column).

The fundamental concept behind the parametric portfolio policy strategy is that the optimal weight allocated to a stock reflects the deviation the same stock has from its weight in the benchmark portfolio – in our case it is the value-weighted (VW) portfolio. The extent of this deviation is impacted by two factors: the stock-specific characteristics and the coefficient loading assigned to each characteristic from the optimization.

In the in-sample analysis, we show that the deviations of the optimal weights from the market-capitalization weights expand with the firm's book-to-market ratio (value) and the firm's lagged one-year return (momentum) and decline with the firm's market capitalization (size).

From this, we can say that the European investor with wealth in euros tends to over allocate value firms, short-term past winners and to a smaller extent small firms. Oppositely, the same investor is likely to under allocate growth firms, short-term past losers, and big firms.

The ranking of the coefficients' magnitudes – value first (4.673), momentum second (1.709), and size third (-0.418) – tells us that a value stock (high *btm*) has the largest overweighting. A high momentum follows, and finally a low market capitalization generates the smallest overweighting. Although our European application, the signs of the coefficients are in line with Brandt, Santa Clara and Valkanov (2009) findings on the US financial markets.

Table 2 - Simple linear Portfolio Policy

	EW	VW	In sample PPP	Out of sample PPP
θ *(size)*	-	-	-0,418	-0,266
θ *(btm)*	-	-	4,673	5,216
θ *(mom)*	-	-	1,709	1,336
Absolute w (%)	0,063	0,062	0,921	1,431
Max w (%)	0,063	3,629	4,949	6,464
Min w (%)	0,063	0,000	-2,672	-4,137
Sum w < 0 (%)	-	-	-179,1	-280,5
Fraction w < 0	-	-	0,390	0,419
Turnover (%)	12,64	69,31	1014,3	949,0
Certainty equivalent (CE) (%)	1,997	2,642	10,32	8,336
Average annual portfolio return (%)	12,67	13,27	37,11	31,94
Average annual portfolio volatility (%)	16,90	16,84	30,49	32,90
Total cumulative return	2,52 x	3,06 x	8,88 x	7,64 x
Sharpe Ratio	0,748	0,786	1,216	0,970

Since the parametric portfolio strategy embodies the concept of active portfolio management, we expect that the optimal portfolio's weights are distributed across stocks less uniformly than the benchmark portfolio's ones. This is confirmed by our results. We find that, in absolute terms, the investor takes more extreme positions than if investing in the benchmark portfolio (0.92% compared to 0.062% in the VW). Partially, this is due to the nature of the portfolios. The two benchmarks are long-only, the optimal portfolio, instead, allows for short selling activity. This discrepancy in weight allocation's intensity is also reflected by the time-series average of maximum and minimum portfolio's weight. For the optimal portfolio, the weights, on average, range between 4.95% and -2.67%, compared to a smaller gap for the market portfolio, where they range between 3.63% and 0.00%.

The average sum of negative positions is -179%, which points out that every month the investor borrows 1.79 times the wealth invested. It follows that the average sum of positive positions is

279%. Out of the total positions invested in the portfolio, the average fraction of shorted stocks is 0.39.

It is arguable that the parametric portfolio policy is difficult to be implemented by portfolio managers because of the extensive trading activity required, which would generate unsustainable transaction costs. To a certain extent, this critique is applicable to our analysis, given that the optimized portfolio has an average annual turnover of 1014%, compared to 69% for the value-weighted and 13% for the equal-weighted. The cause of the large difference stands in the events impacting one or the other. In fact, while the turnover of the equal-weighted portfolio is is mainly influenced by the new listings and delistings and of the value-weighted by the changes in the market-capitalization distribution, the turnover of the optimal portfolio reflects the variability of the characteristics over time.

Finally, the last set of rows in Table 2 characterize the performance statistics of the different strategies. The values displayed are all annualized. Over the sample period, the optimal portfolio presents an annual average return of 37.11%, almost three times higher than the market portfolio (13.27%). However, this superior performance is paired with higher returns' volatility (30.49% of the optimal portfolio versus 16.84% of the value-weighted). This interesting risk-return profile, combined with the risk-free rate according to Equation (7), leads to a Sharpe ratio of 1.22 for the optimal portfolio, far preferrable than the ratios of the market and of the naïve portfolios, which are 0.79 and 0.75, respectively. As regards the certainty equivalent (CE), which captures the risk-free rate our investor would require to exchange the optimal portfolio's return for a riskless strategy, the optimal portfolio also exceeds the benchmarks. For not investing in the optimal portfolio, the investor requires a guaranteed return of 10.32%, while for the market portfolio would be of only 2.64%. In conclusion, we present the total cumulative compounded return of the portfolios, as to determine how much €1, invested in January 1996, would be worth as of December 2019 if invested in the three portfolios. For the optimal portfolio, the cumulative return is 8.88 times the initial wealth, compared to 2.52 and 3.06 times for the EW and VW, respectively. This statistic is shown in Figure 3 below.

Our analysis shows that the optimal portfolio outperforms both the equal-weighted and, more interestingly, the market portfolio. However, so far, the analysis was performed on a in-sample optimization scheme, which it is not new to exceed the market's performance. In order to assess if these results are robust, we need to perform the out-of-sample analysis.

From the out-of-sample results, displayed in the fourth column of Table 2, we see that the investor's preferences do not suffer any substantial change. The signs of the estimated coefficients are unaffected, their loading slightly changes. Both size and momentum mitigate their effect (down to -0.266 and 1.336, respectively), while value increases its influence (up to to 5.126).

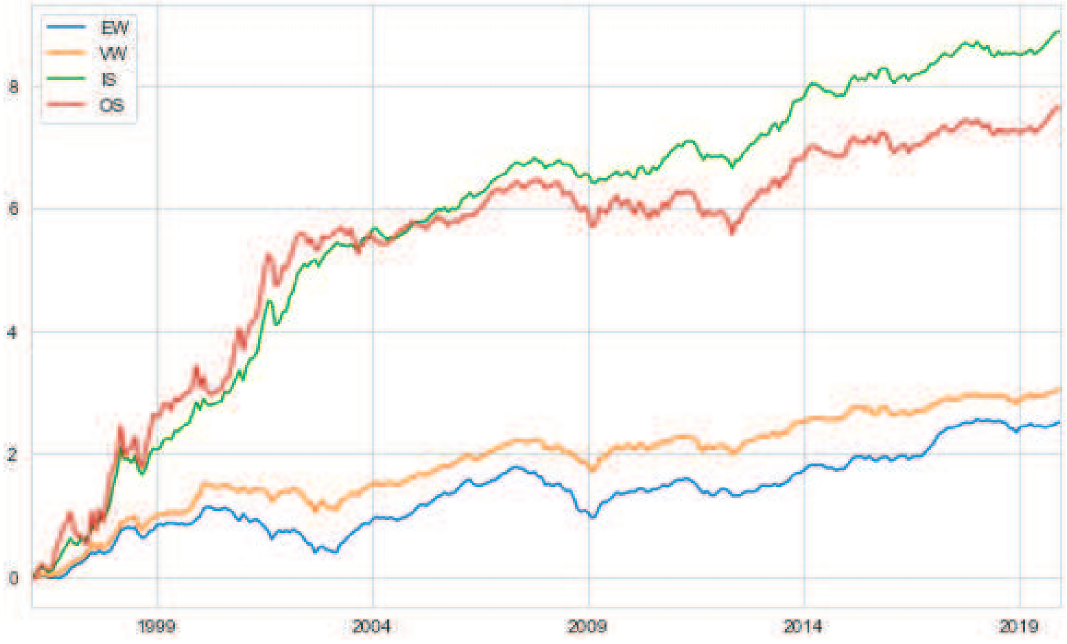
From the statistics on the distribution of weights, we can notice that the weight allocation policy is more aggressive, compared to all the other three portfolios. The average absolute weight increases to 1.43%, while the average maximum and minimum weight magnify to 6.46% and -4.14%, respectively. Also, the average sum and the average fraction of negative positions confirm this trend, with the first being -280% (vs. 179% in the in-sample) and the latter 0.42 (vs. 0.39 in the in-sample). Surprisingly, the turnover slightly decreases from 1014% to 949%. However, this level of turnover is probably still too high to consent an actual implementation of the parametric portfolio policy in the Euro stock markets.

In terms of portfolio's performance, the optimal portfolio yet generates above-benchmark annual average returns (31.94%), but it shows a modest deterioration from the in-sample (down by 7 p.p.). The average return's volatility increases to 32.90%. Following such changes, the Sharpe ratio decreases from 1.22 of the in-sample optimization to 0.97. The largest worsening is given by the certainty equivalent figure, which declines to 4.34%, implying that the European investor would hardly invest in this portfolio, with the risk-free asset available in the market. In conclusion, the out-of-sample portfolio cumulative return is 7.64, above both benchmarks but marginally below the in-sample, as shown in Figure 4.

Figure 4 compares the total cumulative return of the four portfolios – EW, VW, in-sample and out-of-sample optimal portfolios – over the investable period, from January 1996 to December 2019. From this metric, two annotations can be made. First, it is evident that the parametric portfolio policy provides major cumulative portfolio performance, compared to the benchmark portfolios, at a cost of higher volatility. This is especially shown by the accentuated drops and rises of the out-of-sample optimal portfolio during the first years of portfolio's holding. The reason can be found in the particular crisis time following the 2000 dot com bubble, as well as in the restricted sample used to optimize portfolio's weights. In fact, with the enlargement of the sample the out-of-sample optimal portfolio's returns seem more stable than in the beginning. Second, the pattern of returns of the optimal portfolios reveals a low correlation with the benchmark portfolios' returns. This entails that the parametric portfolio strategy could be interesting to be implemented with the market index.

Figure 3 - Cumulative performance of unconstrained portfolios

The figure displays the cumulative compounded return of the two benchmark portfolios and of the in-sample and out-of-sample parametric portfolios under the base or unconstrained setting. The period considered ranges from January 1996 to December 2019.



3.2 Portfolio weights constraints

The first extension to the base case we propose addresses an issue that many portfolio managers encounter in their investment decisions: constraints on short positions. In the following section, we present the results of the parametric portfolio strategy under negative weights restrictions. Specifically, we test two different scenarios. First, we impose a no-short sale policy and construct a long-only optimal portfolio. Secondly, we allow for a minimum level of leverage (i.e., a leverage limit) and optimize the portfolio's weights under this condition. We consider that the European investor cannot borrow more than 50% of its wealth, which is in line with the maximum leverage a vast number of brokerage firms can employ. As in the unconstrained case, we compare the weight-constrained portfolios' results to the two benchmark portfolios, equal-weighted and value-weighted. The statistics of the long-only portfolio and of the leverage-constrained portfolio are presented in Table 3 and in Table 4, respectively.

3.2.1 Long-only optimal portfolio

In Table 3, we display the results of the long-only portfolio, again in the in-sample and out-of-sample setting.

Table 3 - Portfolio Policy under no-short sales regime

	EW	VW	In sample PPP	Out of sample PPP
θ *(size)*	-	-	0,276	0,691
θ *(btm)*	-	-	2,562	1,735
θ *(mom)*	-	-	1,231	0,944
Absolute w (%)	0,063	0,062	0,294	0,234
Max w (%)	0,063	3,629	1,240	0,836
Min w (%)	0,063	0,000	0,006	0,006
Sum w < 0 (%)	-	-	-	-
Fraction w < 0	-	-	-	-
Turnover (%)	12,64	69,31	446,0	279,0
Certainty equivalent (CE) (%)	1,997	2,642	5,948	3,319
Average annual portfolio return (%)	12,67	13,27	15,55	12,97
Average annual portfolio volatility (%)	16,90	16,84	15,57	15,63
Total cumulative return	2,52 x	3,06 x	3,72 x	2,79 x
Sharpe Ratio	0,749	0,787	0,996	0,827

From the unconstrained scenario, there is a switch in the sign of the size coefficient, which turns positive. This suggests that the deviations of the optimal weights from the benchmark weights increase with all three characteristics – size, value and momentum. Overall, the deviations are less heavy compared to the base case, as all the coefficients have a lower loading, but same ranking. Thus, the long-only investor is more influenced by stocks that show high value (0.562), to a smaller extent by stocks with large positive momentum (1.231), and finally by large stocks (0.276). This last, although positive, generates the littlest deviations from the benchmark portfolio's weights.

This mitigation in the investor behavior is also shown in the weight allocation's statistics. The long-only investor tends to distribute the wealth more equally among the investable set. The average weight, in absolute terms, decreases from 0.92% in the base case to 0.29%. Similarly, the average maximum weight decreases from 4.95% to 1.24%, also lower than that of the value-weighted portfolio (3.63%). Portfolio's turnover decreases substantially from the base case, from 1014% to 446%, but still remains too high for an investment environment with transaction costs.

Despite the absence of short-selling, so the impossibility to benefit from negative stock news, the annual average portfolio's return persists to beat the benchmark portfolios' average return: 15.55% for the long-only versus 13.27% for the value-weighted. It is even more surprising if we consider the lower volatility: 15.57% vs. 16.84%. This translates into a Sharpe ratio of 1.00 and a certainty equivalent of 5.95%, quite high considering the restriction on portfolio's weights. Finally, the long-only portfolio generates a cumulative return of 3.72, above both benchmarks.

Such results confirm that the parametric portfolio policy generates superior performance also in a no-short sales setting. As always, the out-of-sample analysis is essential to understand if the strategy's results are robust, and thus if they are practically replicable.

When testing the out-of-sample robustness of our in-sample findings, we see that the parametric portfolio's statistics degrade lightly, as expected.

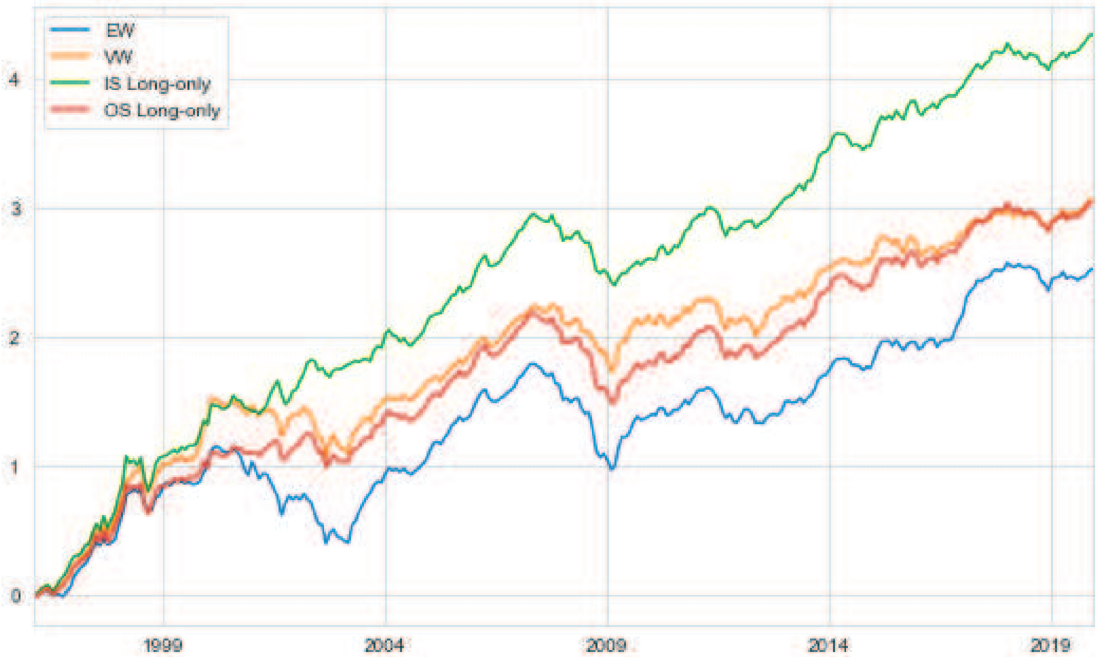
The characteristics' coefficients are more uniform than in the in-sample, generating very similar deviations to stocks showing different characteristics. Also, the weight allocation is more parsimonious, with a lower average absolute weight (0.23%), a lower average maximum weight (0.84%), and a lower and feasible turnover (279%). In terms of performance, the resulting average portfolio's return falls to 12.97%, below the market portfolio's return, while the average returns' volatility does not suffer a large increase, from 15.57% in the in-sample to 15.63% in the out-of-sample. Especially because of the average return's deterioration, the Sharpe ratio declines to 0.83, below the in-sample, but above both the equal-weighted and value-weighted ratios. The certainty equivalent sets at 3.32%, making the optimal portfolio more appealing for a long-only investor than the benchmark portfolios (CE of 2.64% for the VW). Finally, the out-of-sample optimization provides a cumulative return of 2.79.

The gains of carrying out a long-only policy in constructing a parametric portfolio are visible in the in-sample, but not extremely implementable as suggested by the out-of-sample analysis. The main drawback is given by the higher volatility in light of the deterioration of average portfolio's returns. We conclude that the long-only investor would probably prefer to invest in the market portfolio rather than adopting a parametric portfolio strategy. Thus, in the following section, we want to understand if the portfolio's performance improves when allowing for a certain limit in the short-selling activity.

In Figure 4, we present the cumulative return performance of the two benchmark portfolios (equal to the base case) and of the in-sample and out-of-sample optimized long-only portfolios.

Figure 4 - Cumulative performance of long-only portfolios

The figure displays the cumulative compounded return of the two benchmark portfolios and of the in-sample and out-of-sample parametric portfolios subject to a no-short sales policy. The period considered ranges between January 1996 and December 2019.



3.2.2 Optimal portfolio with leverage limit

Table 4 displays the results of the parametric portfolio policy when short selling is constrained to a leverage limit. In our case, we consider that the European investor might borrow up to 50% of the total wealth invested, for any given point in time. Looking at the signs of the coefficients, a low-market capitalization of a stock returns to be a desirable characteristic. As in the base case, in fact, the deviations of optimal weights from the benchmark weights decrease with size and increase with both value and momentum. Compared to the long-only optimal portfolio, the coefficients of the three parameters present higher loadings. Value (3.414) generates the largest deviations, momentum (1.350) follows, and size (-0.149) is the least impacting characteristic.

Table 4 - Portfolio Policy under leverage limit regime

	EW	VW	In sample PPP	Out of sample PPP
θ *(size)*	-	-	-0,149	-0,086
θ *(btm)*	-	-	3,414	3,285
θ *(mom)*	-	-	1,350	0,847
Absolute w (%)	0,063	0,062	0,488	0,303
Max w (%)	0,063	3,629	2,143	1,190
Min w (%)	0,063	0,000	-1,216	-0,603
Sum w < 0 (%)	-	-	-47,12	-40,76
Fraction w < 0	-	-	0,293	0,262
Turnover (%)	12,64	69,31	833,3	278,9
Certainty equivalent (CE) (%)	2,042	2,219	10,50	5,19
Average annual portfolio return (%)	12,67	13,27	22,96	15,14
Average annual portfolio volatility (%)	16,90	16,84	18,89	16,02
Total cumulative return	2,52 x	3,06 x	5,49 x	3,38 x
Sharpe Ratio	0,749	0,787	1,214	0,943

The concession of some leverage induces the investor to a more dynamic weight allocation. The average absolute weight increases to 0.49%, versus 0.30% in the long-only scenario. As well, the average maximum and minimum weights are more distant, with the first being 2.14% and the latter -1.22%. The times-series average sum of negative positions is -47.12%, implying that the investor does not always borrow the maximum allowed, but, instead, in some months optimizes weights with a lower level of borrowing. From the base case, the average fraction of negative weights on the total positions taken decreases from 0.39 to 0.29, reflecting the limited usage of leverage. Also, turnover decreases from 1014% in the base case to 833% but remains quite unsustainable for an environment of high transaction fees.

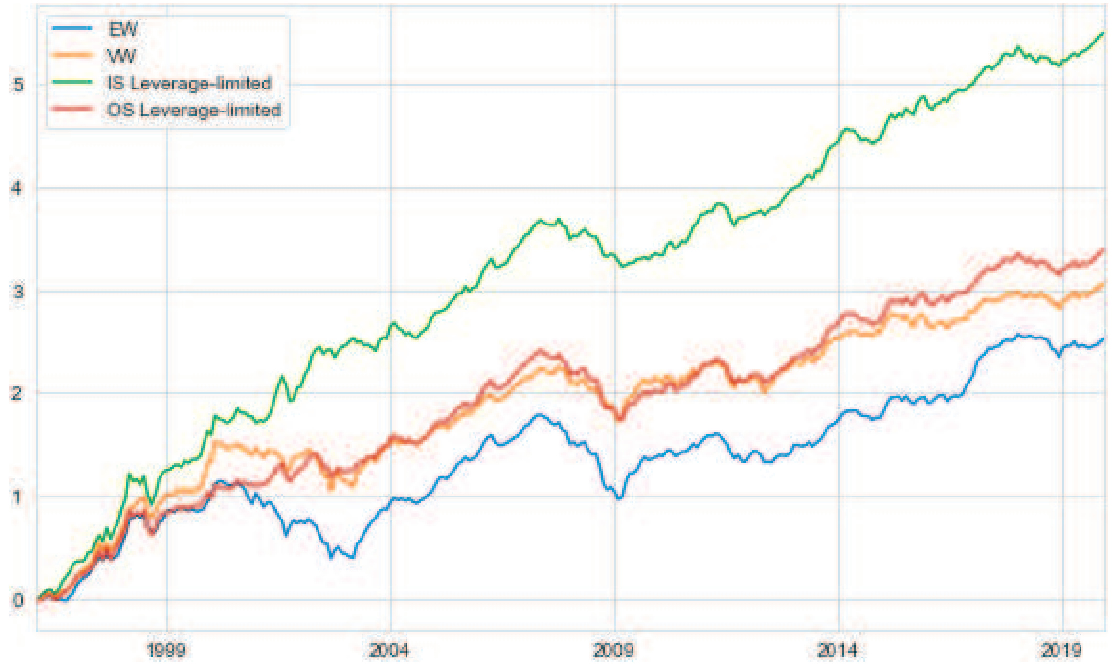
In terms of performance, the weight-constrained portfolio shows remarkable metrics. On average, our optimal portfolio consistently outperforms the market, with a controlled level of portfolio's volatility. It generates an annualized return of 22.96% (vs. 13.27% of the market portfolio) and a standard deviation of returns of 18.89% (vs. 16.84% of the market portfolio). This attractive portfolio's risk and return profile translates into a certainty equivalent of 10.50%, a Sharpe ratio of 1.21 and a total cumulative return of 5.49 times per unit invested at the beginning of the sample period.

In the out-of-sample analysis, the coefficients associated with all three characteristics diminish in magnitude. The size coefficient remains negative, but almost approaches zero (-0.086), implying that the investor's optimal allocation is scarcely impacted by the firm's size. It is instead predominantly influenced by the firm's value (3.285), and to a still significant extent by the firm's momentum (0.847). Also, the investor allocates more lightly than in the in-sample. The average portfolio weight, in absolute terms, is 0.30%, with an average maximum of 1.19% and an average minimum of -0.60%. The average sum of negative positions decreases to 40.76%, and the average fraction to 0.26. Finally, a less intensive weight allocation policy reduces the portfolio's turnover to 279%.

Compared to the long-only portfolio, the performance statistics do not suffer large deterioration in the out-of-sample. The weight-constrained portfolio produces on average 15.14% of annual returns, a clear reduction from the in-sample, but above both benchmark portfolios' figures. The combination with a low return's volatility (16.02%) leads to a Sharpe ratio of 0.94 and a certainty equivalent of 5.19%. Over the sample period, the portfolio presents a cumulative return of 3.38, marginally larger than that of the value-weighted portfolio (3.06)

Figure 5 - Cumulative performance of portfolios constrained to leverage limit

The figure displays the cumulative compounded return of the two benchmark portfolios and of the in-sample and out-of-sample parametric portfolios subject to a leverage limit of 50% on total wealth invested. The period considered ranges between January 1996 and December 2019.



In figure 5, we display the cumulative return performance of the two benchmark portfolios – EW and VW – and of the in-sample and out-of-sample optimal portfolios constrained to a maximum level of leverage throughout the investable period of 50% the wealth invested. Differently from the base case, in Figure 5 we can observe that the optimal portfolios follow closely the pattern of return of the benchmark portfolios, in a more evident way in the last years of the sample than at the beginning. We conclude that, as we reduce the available leverage, the characteristics have lower coefficient loadings, which reduce the size of the optimal weights' deviations. Thus, optimal weights are closer to the benchmark weights and portfolios' returns come more similar.

3.3 Varying risk aversion

One central assumption of the Brandt, Santa Clara and Valkanov (2009) framework is that investors shape their optimal portfolio policy based on a Constant Relative Risk Aversion (CRRA) power utility function. In both the unconstrained and constrained case, the coefficient of risk aversion is always set equal to 5. However, it is interesting to study the optimal portfolio's statistics under different risk tolerance levels and sensitivities to losses. Specifically, we consider five different scenarios for which γ takes five different values: $\gamma = 1$ (minimum level of risk aversion), $\gamma = 3$, $\gamma = 5$ (base case), $\gamma = 7$, and $\gamma = 10$ (high risk aversion). When modelling the utility function of the investor, defined in Equation (3), the coefficient of risk aversion γ strongly impacts how wealth is distributed among the investable universe. The goal of this extension is to understand how the coefficient loadings, the distribution of weights and the performance metrics change when investors with different risk tolerances are considered. Table 6 and Table 7 are constructed similarly to the tables above, with the only difference that the five columns correspond the different values given to γ. While Table 6 displays the results of the in-sample optimal portfolio, Table 7 presents the results of the out-of-sample optimization.

For the different degrees of risk aversion, the signs of the coefficients related to value and momentum characteristics remain positive, while their magnitudes increase as γ decreases. The size coefficient, instead, amplifies its negative value with a lower risk aversion, and it turns positive with higher risk aversion coefficients. With the lowest γ, the investor overweighs small firms, value firms and past winners. With the highest γ, the investor favors small firms to large firms, so a more conservative approach. These results lead to one consideration. The European investor, when increasing the risk aversion, is less likely to allocate assets far from their market-capitalization weights because recognizes the risks associated with the characteristics, especially with size and momentum. This entails a relation between the characteristics and mean returns and volatility.

Table 5 - In-sample Portfolio Policy under varying risk aversion

	γ = 1	γ = 3	γ = 5	γ = 7	γ = 10
θ *(me)*	-2,620	-1,701	-0,418	0,252	0,441
θ *(btm)*	10,573	7,489	4,673	3,324	2,347
θ *(mom)*	3,903	2,087	1,709	1,883	1,788
Absolute w (%)	2,014	1,432	0,921	0,698	0,536
Max w (%)	10,78	7,741	4,949	3,703	2,799
Min w (%)	-5,965	-4,230	-2,672	-1,965	-1,428
Sum w < 0 (%)	-452,1	-306,6	-179,1	-123,4	-83,20
Fraction w < 0	0,450	0,430	0,390	0,355	0,311
Turnover (%)	1724,1	1364,9	1014,3	854,4	740,4
Certainty equivalent (CE) (%)	39,89	16,12	10,32	6,468	1,986
Average annual portfolio return (%)	62,60	50,08	37,11	30,88	26,15
Average annual portfolio volatility (%)	61,93	45,04	30,49	24,42	20,31
Total cumulative return	14,97 x	11,98 x	8,88 x	7,39 x	6,25 x
Sharpe Ratio	1,010	1,111	1,216	1,263	1,286

As expected, the change in behavior is also reflected in the statistics on the distribution of weights. In fact, with the minimum level of risk aversion, the investor's allocation policy is more aggressive. Compared to the base case, where γ equals 5, the investor holds both larger long and short positions. The absolute weight more than doubles, from 0.92% to 2.01%, and the average maximum weight reaches the double-digit (10.78%). The average fraction of negative weights increases to 0.45, and the average sum of borrowed wealth is 4.52 times the capital invested. The average turnover rises to 1724%. With the increase in the risk aversion, the results show that the investor tends to marginally use less leverage. All six statistics outline a decreasing trend as we move from left to right in Table 6. When considering $\gamma = 10$, the average minimum weight held in the portfolio is -1.43%, the average sum of negative positions is only -83.20% and the average fraction of shorted stocks is 0.31.

In terms of performance, Table 6 presents a clear pattern: the average annualized return and standard deviation decrease at different speeds, causing the Sharpe ratios to improve with higher coefficients of risk aversion. The greatest performance is in fact furnished by the portfolio constructed with highest coefficient of risk aversion (Sharpe ratio of 1.29 with $\gamma = 10$ vs. 1.01 with $\gamma = 1$). This trend implies that the investor receives a higher return per unit of risk taken.

Table 6 - Out-of-sample Portfolio Policy under varying risk aversion

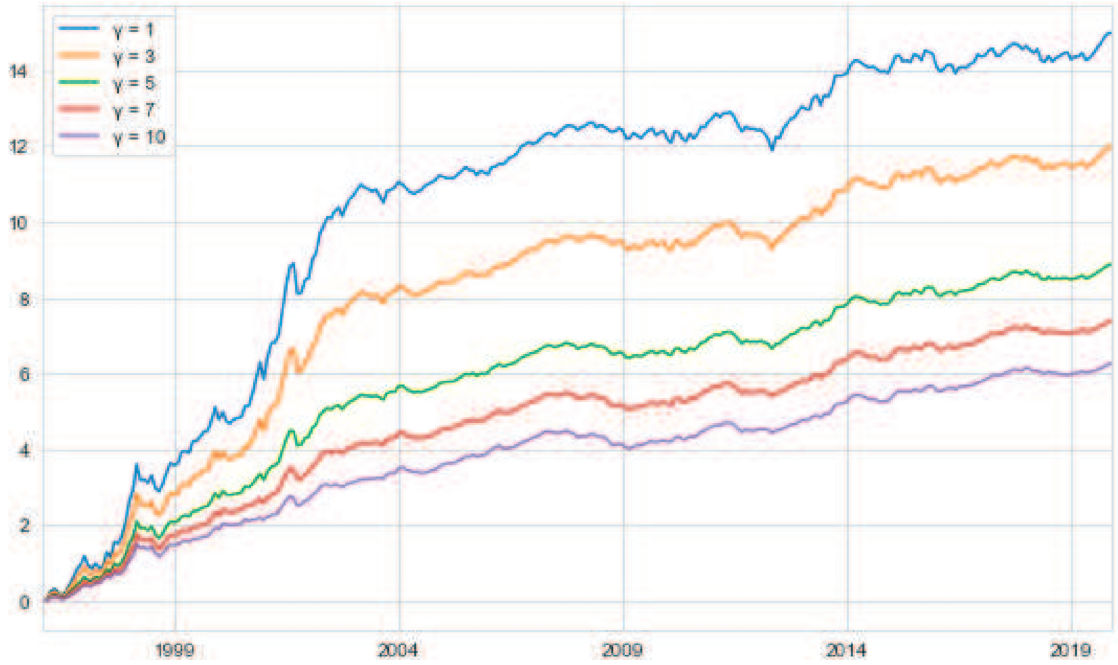
	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$
θ *(me)*	-1,794	-1,247	-0,266	-0,193	1,287
θ *(btm)*	11,960	8,891	5,216	3,582	2,167
θ *(mom)*	1,697	1,551	1,336	0,699	0,966
Absolute w (%)	3,464	2,390	1,431	1,009	0,700
Max w (%)	15,71	10,83	6,464	4,539	3,112
Min w (%)	-10,43	-7,084	-4,137	-2,855	-1,910
Sum w < 0 (%)	-744,4	-499,2	-280,5	-184,3	-114,1
Fraction w < 0	0,466	0,451	0,419	0,387	0,340
Turnover (%)	2301,6	1566,6	949,0	708,5	569,3
Certainty equivalent (CE) (%)	14,89	9,098	4,336	-16,55	-13,97
Average annual portfolio return (%)	58,76	45,03	31,94	27,96	21,50
Average annual portfolio volatility (%)	89,82	46,47	32,90	34,21	25,27
Total cumulative return	14,05 x	10,77 x	7,64 x	6,21 x	5,14 x
Sharpe Ratio	0,654	0,968	0,970	0,816	0,849

By conducting the out-of-sample analysis, we confirm the trends outlined in the in-sample. From Table 6, we notice that the characteristics' coefficients lose loading as we increase the coefficient of risk aversion, but they keep the same ranking. Value regularly provokes the largest deviations of optimal weights from the benchmark weights, followed by size and momentum together. The main difference from the in-sample framework lies in the pattern of the Sharpe ratio statistic, which reaches its maximum (0.97) in the base case portfolio, where $\gamma = 5$. Especially in terms of performance statistics, we note that the parametric portfolio policy works better in the in-sample rather than in the out-of-sample. This is mostly visible in the certainty equivalent metrics and in the larger volatility of portfolios constructed in the out-of-sample.

In Figure 6, we display the in-sample cumulative return of the five portfolios corresponding to five different risk aversion coefficients. The plot confirms the findings mentioned above: lower risk aversion leads to larger investor's bets, which consequently generate larger cumulative returns at a cost of higher volatility.

Figure 6 - Cumulative performance of portfolios with varying risk aversion

The figure displays the cumulative compounded return of the in-sample parametric portfolio policy under different coefficients of risk aversion. The period considered ranges between January 1996 and December 2019.



IV. Conclusion

The parametric portfolio policy approach, proposed by Brandt, Santa Clara and Valkanov (2009), has earned considerable curiosity from academics and practitioners since it solves the problem of optimizing large-scale equity portfolios by providing an attractive reduction technique. The authors convey that their approach is "computationally simple, easily modified and extended, produces sensible portfolio weights, and offers robust performance in and out of sample". The aim of this paper is to assess if these arguments hold also on a large-scale equity portfolio of exclusively euro-denominated stocks, trying also to investigate the European investor's investment preferences.

We model the optimal portfolio's weights as a function of three firm-specific characteristics: the market capitalization (size), the book-to-market ratio (value) and the one-year lagged return (momentum). We estimate the coefficients of size, value and momentum by optimizing the investor's average utility of the portfolio's return over the sample period. Our empirical application comprehends an average of 1359 companies listed either in France, Germany, Italy or Spain in the period between January 1990 and December 2019. We benchmark the results of the optimal portfolio against the equal-weighted and the value-weighted portfolios.

Overall, our findings are in line with the literature. In the base case, we show that the European investor has a tendence of overweighting small firms, value firms and past winners. With respect to the weights in the market portfolio, in fact, the optimal portfolio's weights are positively deviated by low market capitalization, high book-to-market ratio, and considerably large last year compounded return. Similar results are achieved with short-sale restrictions (i.e., long-only optimal portfolio) and when imposing a leverage limit of 50%. All these strategies, tested in-sample and out-of-sample, consistently perform better than the equal-weighted portfolio and the value-weighted portfolio. The superior performance lies in larger certainty-equivalent returns and above-benchmarks Sharpe ratios. However, the main drawback is given by the high turnover, which undermines the feasibility of implementing this approach in real investments. Finally, we test the parametric policy with varying risk aversion coefficients, and we find that the strategy's robustness deteriorates when allowing for different risk's tolerances. Also, we establish a relation between the size and momentum characteristics and risk: the more risk-averse the investor is, the less the overweighting of small firms and past winners, being them more unpredictable.

We conclude that the parametric optimization provides a substantial improvement, in terms of methodology and performance, to the past portfolio's weights optimization techniques. Since this is confirmed also with a European application, we consider our contribution to make parametric portfolio policies more interesting to be studied and possibly implemented by quantitative funds.

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