The Neutrality of Border Adjustments Revisited

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Abstract

I study the economic effects of border adjustments, which consist in fiscal arrangements that ensure that goods are taxed at destination. This issue has been recently discussed in the context of policy proposals in the United States. I present the classical result that border adjustments are neutral, and revisit two cases in which this proposition may fail. First, border adjustments may distort relative prices when applied to certain indirect tax systems. Second, border adjustments may have effects on government revenue and on the balance of payments. I argue that the first failure of neutrality is not empirically plausible and thus has no place in shaping fiscal policy, but that the second failure may provide a motive to employ border adjustments.

Keywords: Border adjustments, origin- vs. destination-based taxation, commodity taxation, Lerner symmetry

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Resumo

Esta dissertação estuda os efeitos económicos dos ajustamentos de fronteira, instrumentos fiscais que fazem com que os bens sejam tributados no destino. Esta questão foi debatida recentemente no contexto de propostas de partidos políticos nos Estados Unidos. O resultado clássico de que os ajustamentos de fronteira são neutrais é apresentado, e dois casos nos quais esta proposição pode falhar são revisitados. Em primeiro lugar, os ajustamentos de fronteira podem distorcer os preços relativos quando aplicados a determinados sistemas de tributação indirecta. Em segundo lugar, os ajustamentos de fronteira podem ter efeitos na receita do Estado e na balança de pagamentos. Nesta dissertação defende-se que a primeira quebra de neutralidade não é empiricamente plausível e por isso não tem lugar no desenho da política fiscal, mas que a segunda quebra pode ser um motivo para utilizar ajustamentos de fronteira.

Palavras-chave: Ajustamentos de fronteira, tributação na origem ou no destino, tributação de bens, simetria de Lerner

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Contents

1 Introduction

In open economies, goods can be taxed in two ways: according to the origin principle, and according to the destination principle. These alternatives differ in the rate at which goods are taxed. Under origin, all goods are taxed at the rates of the tax system of the country of production. Under destination, all goods are taxed at the rates of the tax system of the country of consumption. A border adjustment, which is the mechanism used to move from origin to destination, consists of a pair of policies: (1) a rebate on exports at the rate that would be applicable if the exported good were sold domestically, and (2) a tax on imports at the rate that would be applicable if the good were bought domestically.

The desirability of border adjustments has been recently discussed, but it is part of an old debate. The public opinion has seen border adjustments as being either pro- or anti-competitive, and many economists have tried to argue that border adjustments are irrelevant[1].

In this dissertation, I study this issue by asking two broad questions in the literature: (1) are border adjustments neutral? and (2) what are the distortions caused by each of the principles?

Because border adjustments are pairs of export subsidies and import tariffs, it would simplify matters to analyze them as trade policies alone. However, the appropriate border adjustments depend on domestic tax rates. Hence, they may be neutral when applied to some tax systems but not neutral when applied to others. This fact implies that their neutrality must be analyzed in the context of specific domestic tax systems.

I review and aim to clarify two propositions in the literature. First, when applied to a uniform pure value-added tax (VAT), which taxes sales net of purchases of intermediates across all stages of production, border adjustments are neutral; both the destination- and the origin-based VAT are equivalent to uniform consumption taxes. This is the classical result on border adjustments, and can be found, for instance, in Feldstein and Krugman (1990). Second, when applied to a uniform sales tax, which only taxes sales of final goods, border adjustments are not neutral. The destination-based sales tax is still equivalent to a consumption tax, but its origin-based counterpart also contains import tariffs and export subsidies on traded intermediates. This result was presented by Grossman (1980).

To revisit these propositions, I use a general equilibrium model with trade in intermediates and in final goods, and where governments levy distortionary taxes on goods and labor income. In answering the question of whether border adjustments are neutral, I follow an equivalence approach. For each of the tax systems, I describe two competitive equilibria: one where goods are taxed under

[1]See Weisbach (2017) and Auerbach et al. (2017) for examples of the recent debate, and see Johnson and Krauss (1970) for an example of the old debate.

the origin principle, and another where goods are taxed under the destination principle. I then try to find prices and transfers such that the allocations in the two equilibria are the same. To answer the question of how the distortions imposed by a tax system may differ according to the principle employed, I derive a set of conditions that the undistorted optimal allocations must satisfy. For each tax system, I then assess the tax wedges imposed by each of the principles.

Regarding the two different neutrality propositions in the literature, I emphasize that they are due to the fact that the imposition of border adjustments consists in trade policy reforms that vary according to the domestic tax system in which they are applied. In the context of a VAT, which taxes all firms, border adjustments increase all international prices faced by a country in the same way, and thus consist of a nominal adjustment along the lines of a currency devaluation. This implies that if the domestic price level rises, all relative prices are preserved. In the context of a sales tax, which solely taxes final goods firms, border adjustments only increase the international prices faced by traders of final goods, and relative prices must be affected. These facts are linked to two results in international trade theory that concern the effects of trade policy reforms: the Lerner and the McKinnon symmetry theorems. I establish neutrality for the case of the VAT using Lerner symmetry, and establish non-neutrality for the case of the sales tax using McKinnon symmetry.

Regarding the distortionary nature of each principle, I show that the destination-based tax systems are both equivalent to a uniform consumption tax because they impose the same distortions as a uniform consumption tax would. I also show that relative to its destination-based counterpart, the origin-based sales tax imposes an additional distortion that would be generated by import tariffs and export subsidies on intermediates.

Note that in the classical literature on the subject, the neutrality of border adjustments for the VAT is established under the assumption that the increase in the price level has no real effects. When households have non-labor income, however, the increase in prices may decrease the real value of assets, and border adjustments can have effects on government revenue and on the balance of payments. These effects are well-documented in the literature, but they are not integrated in a single framework. For instance, for neutrality to obtain in the case where transfers between countries are not possible, both Costinot and Werning (2018) and Barbiero et al. (2018) require that equity is not traded internationally, but while Costinot and Werning do not consider the existence of foreign bonds, Barbiero et al. require that they are denominated in foreign currency. Furthermore, if transfers between governments and households are possible, Auerbach (1997) suggests and Barbiero et al. (2018) show that a border adjustment leads to a non-distortionary increase in government revenue by a proportion of the trade deficit.

I allow for equity and bonds to be traded internationally, as well as for country-specific currencies, and analyze the role of international and domestic transfers in preserving the neutrality of border adjustments when applied to a uniform VAT. I emphasize that the way in which these transfers must be used depends on the nominal price adjustment that ensures that relative prices are preserved, on the existence of trade surpluses, and on the type of international assets to which economies have access.

I argue that while the effects of border adjustments on government revenue and on the balance of payments are relevant, the distortion caused by the origin-based sales tax is empirically irrelevant. Even though the sales tax is a widely used indirect tax system, this distortion is based on the unrealistic assumption that consumption goods can be directly traded by households in the international market. However, there are other reasons why border adjustments may be relevant. In a dynamic context with price flexibility, Chari, Nicolini, and Teles (2018) make the case that the second-best allocation has optimal distortions in consumption, but no distortions in production. They show that time-varying value-added taxes with border adjustments can implement these distortions, but without border adjustments they cannot. This is line with Barbiero et al. (2018). In a dynamic context with price rigidity, they show that a border adjustment must be permanent unanticipated to be neutral.

The model I use in this dissertation relates to the environment in Chari, Nicolini, and Teles (2018). They use a two-country neoclassical growth model with one final good and one intermediate good per country, and trade in intermediates only. The model of this dissertation has less periods - only one - but more goods in each sector. Due to these modifications, the model is also related to Costinot and Werning (2018). In their set-up, there is an arbitrary set of countries, consumers, commodities and multinational firms, as well as general technologies and preferences. Governments levy distortionary taxes that discriminate across agents, commodities, as well as origin and destination countries. Lump-sum transfers are allowed, both between governments and between the government and the households of each country. In this dissertation, governments have the same instruments as in Costinot and Werning (2018).

I proceed as follows. In Section 2, I review the relationship between symmetry theorems and neutrality propositions present in the literature. In Section 3 I present the fundamentals of the economy and solve the first-best problem. In Section 4 I describe a competitive equilibrium that is distorted by *ad valorem* taxes and derive the conditions that characterize its allocations. In Section 5 I show the Lerner and McKinnon symmetry theorems, and in Section 6 I apply them to an analysis of a uniform VAT and a uniform sales tax. In Section 7 I consider the effects of border adjustments on government revenue and on the balance of payments. Finally, in Section 8 I conclude.

2 Symmetry and Neutrality

Symmetry results are closely related with the neutrality of border adjustments. Lerner symmetry, in particular, is explicitly mentioned by Chari, Nicolini, and Teles (2018), Costinot and Werning (2018), and by Barbiero et al. (2018), and it is the implicit neutrality mechanism in Berglas (1974), Grossman (1980), Dixit (1985), Feldstein and Krugman (1990), and Auerbach and Holtz-Eakin (2016).

The Lerner symmetry theorem, as initially shown by Lerner (1936) in a two-good model with perfect competition, one imported good and one exported good, states that import tariffs and export taxes are equivalent; that is, imposing an import tariff on one good has the same effects as imposing an export tax of the same amount on the other good. Costinot and Werning (2018) have generalized this theorem to a wider Arrow-Debreu setting. In their version, Lerner symmetry states that uniformly increasing export subsidies and import tariffs across all traded goods in one country is neutral. This version of the theorem is not an equivalence result, as originally, but a neutrality proposition. in this dissertation, I use the version of Costinot and Werning (2018) because in the context of uniform taxation, a border adjustment is composed of an import tariff together with an export subsidy of the same amount. Therefore, the neutrality of border adjustments can be easily established using the (modified) Lerner symmetry theorem.

The mechanism of Lerner symmetry is the following. Uniformly increasing export taxes and import subsidies on all goods rises the international prices faced by the home country. If prices increase by the same proportion, all relative prices are unchanged. Therefore, this reform is neutral. Note, however, that Lerner symmetry holds only if import tariffs and export subsidies change across all traded goods. If they change in only one set of goods, the international prices faced by each country do not change uniformly, and relative prices are affected.

Reforms that do not change trade taxes - export subsidies and import tariffs - in all goods are the subject of McKinnon symmetry. The McKinnon symmetry theorem, initially shown by McKinnon (1966) and generalized by Kaempfer and Tower (1982), states that increasing trade taxes on one set of goods is equivalent to decreasing them on the remaining goods[2]. It is an equivalence result, but it can be shown using the neutrality proposition of Lerner symmetry. To see this fact, consider a country with many traded goods which can be divided in two sets, 1 and 2. Consider now two trade policy reforms, A and B. Reform A increases trade taxes on the goods in set 1 by a proportion x but keeps trade taxes on the goods in set 2 constant, while reform B keeps trade taxes on set 1 constant but decreases trade taxes on set 2 by a proportion x. All trade taxes in reform B are lower than those in reform A by a proportion x. Then, by Lerner symmetry, moving from one reform to

[2]This corresponds to the version of the theorem in Kaempfer and Tower (1982)

the other is neutral, which implies that the two reforms are equivalent.

Grossman (1980) provides a useful application of these theorems in the context of border adjustments. He presents the basic argument for neutrality by developing a static general equilibrium model with trade in intermediate and in final goods. He first considers the effects of introducing a uniform idealized VAT. He shows that to move from the origin to the destination principle, equal-sized export taxes and import subsidies must be imposed on all goods, increasing the international prices faced by the home country. If domestic prices increase by the same proportion, all relative prices are unchanged. This is a direct application of Lerner symmetry.

Grossman (1980) also considers a uniform sales tax, which only taxes sales of final goods. He shows that under the destination principle, this tax system is still equivalent to a uniform consumption tax. However, under origin it additionally imposes a tariff on the imported intermediate. Lerner symmetry fails to hold when applied to the uniform sales tax because only sales of final goods are taxed. In that case, moving from destination to origin uniformly decreases trade taxes on the final good sector only. By McKinnon symmetry, this reform is equivalent to increasing import tariffs and export subsidies on intermediates.

3 Fundamentals and Optimal Planning

3.1 Fundamentals

There are two countries indexed by $i = 1,2$. There are two final goods indexed by $l = 1,2$, and two intermediate goods indexed by $m = 1,2$. Goods are not country-specific: each country can produce and consume the two final goods and the two intermediates. Preferences and technologies, however, differ across countries. This fact is represented by a superscript $i = 1,2$ in utility functions and transformation frontiers. Factors are also different across countries. Each country has an endowment of capital, and labor is supplied by a representative household in each country. These factors are immobile.

In each country, the representative household derives utility from consumption of the two final goods and labor. The preferences of the household in country i are characterized by the utility function

$$u^i(\mathbf{c}_i, n_i), \tag{1}$$

where $\mathbf{c}_i \equiv [c_i(l)]_{l=1}^2$, $c_i(l)$ is consumption of final good l in country i, and n_i is the labor supply in country i.

In each country, the production of each sector can be aggregated by constant returns to scale technologies; that is, in each country there is a representative intermediate good firm and a representative final good firm. The representative intermediate good firm in each country produces the two intermediate goods using the country-specific labor and capital according to a technology described by the transformation frontier

$$F^i(\mathbf{y}_i, -n_i, -k_i) \leq 0, \tag{2}$$

where $\mathbf{y}_i \equiv [y_i(m)]_{m=1}^2$, $y_i(m)$ is the production of intermediate m in country i, k_i is the capital stock in country i, and F^i is constant returns to scale. In each country, the two intermediate goods are used by the representative final good firm to produce the two final goods with a technology described by the transformation frontier

$$G^i(\mathbf{z}_i, -\mathbf{x}_i) \leq 0, \tag{3}$$

where $\mathbf{z}_i \equiv [z_i(l)]_{l=1}^2$, and $z_i(l)$ is the supply of final good l in country i. The demand for intermediate m in country i is denoted by $x_i(m)$, and $\mathbf{x}_i \equiv [x_i(m)]_{m=1}^2$. The transformation curve G^i also exhibits constant returns to scale.

There is a government in each country with exogenous government expenditures $\mathbf{g}_i \equiv [g_i(l)]_{l=1}^2$, where $g_i(l)$ denotes consumption of final good l by the government in country i.

All goods are tradable. Therefore, the resource constraints can be written as

$$\sum_{i=1}^{2} [c_i(l) + g_i(l)] \leq \sum_{i=1}^{2} z_i(l),\ l = 1, 2 \tag{4}$$

and

$$\sum_{i=1}^{2} x_i(m) \leq \sum_{i=1}^{2} y_i(m),\ m = 1, 2. \tag{5}$$

3.2 Optimal Planning

I first solve the problem of a benevolent planner that has access to lump-sum transfers to finance the exogenous government expenditures. He can thus decentralize a Pareto-optimal allocation as a competitive equilibrium. The solution to the planning problem consists in a set of conditions that the competitive equilibrium allocations must satisfy in order to be efficient.

Let ω_i be the Pareto weight attributed by the social planner to the utility of the household in country i. The social planner's problem is to choose $\mathbf{c} \equiv [\mathbf{c}_i]_{i=1}^2$, $\mathbf{n} \equiv [n_i]_{i=1}^2$, $\mathbf{y} \equiv [\mathbf{y}_i]_{i=1}^2$, $\mathbf{z} \equiv [\mathbf{z}_i]_{i=1}^2$

and $\mathbf{x} \equiv [\mathbf{x}_i]_{i=1}^2$ to maximize the social welfare function

$$\sum_{i=1}^{2} \omega_i u^i (\mathbf{c}_i, n_i) \tag{6}$$

subject to the constraints (1) to (5). Throughout this dissertation, I assume that the first-order conditions for an interior solution are both necessary and sufficient for optimality. Subscripts in functions denote their derivatives. The conditions that characterize the Pareto frontier are

$$\frac{u_1^1}{u_2^1} = \frac{G_{z(1)}^1}{G_{z(2)}^1}, \tag{7}$$

$$-\frac{u_n^i}{u_1^i} = \left(\frac{F_n^i}{F_1^i}\right)\frac{G_{x(1)}^i}{G_{z(1)}^1}, i = 1, 2, \tag{8}$$

$$\frac{G_{x(1)}^1}{G_{x(2)}^1} = \frac{F_1^1}{F_2^1}, \tag{9}$$

$$\frac{u_1^1}{u_2^1} = \frac{u_1^2}{u_2^2}, \tag{10}$$

$$\frac{G_{z(1)}^1}{G_{z(2)}^1} = \frac{G_{z(1)}^2}{G_{z(2)}^2}, \tag{11}$$

$$\frac{F_1^1}{F_2^1} = \frac{F_1^2}{F_2^2}, \tag{12}$$

$$\frac{G_{x(1)}^1}{G_{x(2)}^1} = \frac{G_{x(1)}^2}{G_{x(2)}^2}, \tag{13}$$

$$\frac{G_{x(1)}^1}{G_{z(1)}^1} = \frac{G_{x(1)}^2}{G_{z(1)}^2}, \tag{14}$$

together with (2) to (5) in equality.

The eight marginal conditions arise for different reasons.

Autarky The marginal conditions (7) to (9) would also be obtained if international trade were not allowed. I call them domestic efficiency conditions.

Equation (7) is a condition for efficiency in the combination of goods. It states that in country

1, the marginal rate of substitution between final goods 1 and 2 is equal to their marginal rate of transformation. Together with (10) and (11), it implies an analogous condition for country 2.

Equation (8) states that in each country, the marginal rate of substitution between labor and final good 1 is equal to their marginal rate of transformation. It is also a condition for efficiency in the combination of goods. Note that the right-hand side is a marginal rate of transformation because labor can produce final good 1 by first producing intermediate 1 at rate F^i_n/F^i_1, which can then be used to produce final good 1 at rate $G^i_{x(1)}/G^i_{z(1)}$.

Equation (9) concerns productive efficiency. It states that in country 1, the marginal rate of technical substitution between intermediates is equal to their marginal rate of transformation. Together with (12) and (13), it implies an analogous condition for country 2.

International Trade Equations (10) to (14) arise due to the possibility of international trade. I call them international efficiency conditions.

Trade in final goods Equations (10) and (11) would be obtained even if there were no trade in intermediates. Equation (10) states that the marginal rates of substitution between final goods are equal across countries, and equation (11) states that the marginal rates of transformation between final goods are also equal across countries. They are, respectively, conditions for efficiency in consumption and in production.

Trade in intermediate goods On the other hand, equations (12) and (13) would be obtained even if there were no trade in final goods. They are both conditions for efficiency in production, and they state, respectively, that the marginal rates of transformation and of technical substitution between intermediates are equal across countries.

Trade in final and in intermediate goods Equation (14) arises because there is trade in both sectors at the same time. It is an equality of the marginal productivities of intermediate 1 in final good 1 in each country. In general, an analogous condition for intermediate m and final good l can be obtained through equations (11), (13) and (14).

4 Competitive Equilibrium

In this section, I describe a competitive equilibrium with *ad valorem* taxes on goods and labor income, but no lump-sum transfers.

In this economy, governments levy taxes on imports and exports. This fact requires that the pattern of trade is made explicit. In this dissertation, however, I am not answering the question of why economies trade, but rather of how taxes distort an open-economy equilibrium. Therefore, I take the trade pattern as given and assume that country 1 exports intermediate 1 and final good 1, and it imports intermediate 2 and final good 2. All goods are also produced domestically.

Taxes vary across country of consumption and production; pre-tax prices and commodities thus need to be indexed by origin and destination.

Pre-tax prices All prices are denominated in a common unit of account. Let $\mathbf{q} \equiv \left[q_{ij}(l)\right]_{i,j,l=1}^{2}$ denote the vector of pre-tax prices of final goods $l = 1,2$ with origin in country $i = 1,2$ and destination in country $j = 1,2$. Let $\mathbf{p} \equiv \left[p_{ij}(m)\right]_{i,j,m=1}^{2}$ be the vector of pre-tax prices of intermediates $m = 1,2$ with origin in country $i = 1,2$ and destination in country $j = 1,2$, and let $\mathbf{w} \equiv [w_i]_{i=1}^{2}$ be the vector of the wage rates in countries $i = 1,2$. $\mathbf{q}$ is partitioned into a vector of domestic prices, $\mathbf{q^d} \equiv [q_{ii}(l)]_{i,l=1}^{2}$, and a vector of international prices, $\mathbf{q}^* \equiv [q_{12}(1), q_{21}(2)]$. Analogously, $\mathbf{p^d} \equiv [p_{ii}(m)]_{i,m=1}^{2}$, and $\mathbf{p}^* \equiv [p_{12}(1), p_{21}(2)]$.

Commodities Let $c_{ji}(l)$ and $g_{ji}(l)$ denote, respectively, the consumption of final good l with origin in country $j = 1,2$ and destination in country i by households and governments in country i. $y_{ij}(m)$ is the supply of intermediate m with origin in country i and destination in country $j = 1,2$, while $z_{ij}(l)$ is the supply of final good l with origin in country i and destination in country $j = 1,2$. $x_{ji}(m)$ is the demand for intermediate m with origin in country $j = 1,2$ and destination in country i. Due to the pattern of trade, $c_{ji}(i) = x_{ji}(i) = y_{ij}(j) = z_{ij}(j) = 0$ with $i \neq j$. Furthermore,

$$c_i(l) = \sum_{j=1}^{2} c_{ji}(l),\ i,l = 1,2 \tag{15}$$

$$y_i(m) = \sum_{j=1}^{2} y_{ij}(m),\ i,m = 1,2 \tag{16}$$

$$z_i(l) = \sum_{j=1}^{2} z_{ij}(l),\ i,l = 1,2 \tag{17}$$

$$x_i(m) = \sum_{j=1}^{2} x_{ji}(m),\ i,m = 1,2 \tag{18}$$

and

$$g_i(l) = \sum_{j=1}^{2} g_{ji}(l),\ i,l = 1,2. \tag{19}$$

4.1 Households

The problem of the household in country i to choose $(\mathbf{c}_i, n_i)$ to maximize (1) subject to the budget constraint

$$\sum_{l=1}^{2} \left[1 + t_i^c(l)\right] q_{ii}(l) c_{ii}(l) + \left[1 + \tau_i^c(j)\right] q_{ji}(j) c_{ji}(j) \leq \left(1 + s_i^n\right) w_i n_i,\ i \neq j, \tag{20}$$

where $1 + t_i^c(l)$ is the (gross) tax levied by country i on the consumption of final good l with origin and destination in country i, $1 + \tau_i^c(j)$ is the (gross) tariff levied by country i on the imports of final good j, and $1 + s_i^n$ is the (gross) labor income subsidy granted by country i. Let $\mathbf{t^c} \equiv \left[1 + t_i^c(l)\right]_{i,l=1}^{2}$, $\boldsymbol{\tau^c} \equiv \left[1 + \tau_i^c(j)\right]_{i \neq j=1}^{2}$, and $\mathbf{s^n} \equiv \left[1 + s_i^n\right]_{i=1}^{2}$. The conditions that characterize the solution to the households' problems are

$$\left[1 + t_i^c(j)\right] q_{ii}(j) = \left[1 + \tau_i^c(j)\right] q_{ji}(j),\ i \neq j = 1,2,$$

$$\frac{u_i^i}{u_j^i} = \left[\frac{1 + t_i^c(i)}{1 + t_i^c(j)}\right] \frac{q_{ii}(i)}{q_{ii}(j)},\ i \neq j = 1,2,$$

and

$$-\frac{u_n^i}{u_i^i} = \left[\frac{1 + s_i^n}{1 + t_i^c(i)}\right] \frac{w_i}{q_{ii}(i)},\ i \neq j = 1,2,$$

together with (20) in equality for $i = 1,2$.

4.2 Intermediate Good Firms

The intermediate good firm in country i chooses $(\mathbf{y}_i, n_i)$ to maximize

$$\pi_i = \sum_{m=1}^{2} \left[1 + s_i^y(m)\right] p_{ii}(m) y_{ii}(m) + \left[1 + \sigma_i^y(i)\right] p_{ij}(i) y_{ij}(i) - \left(1 + t_i^n\right) w_i n_i,\ i \neq j \tag{21}$$

subject to (2), where $1 + s_i^y(m)$ is the (gross) subsidy granted by country i on the supply of intermediate m with common origin and destination in country i, $1 + \sigma_i^y(i)$ is the (gross) subsidy on exports granted by country i on the exports of intermediate i, and $1 + t_i^n$ is the (gross) payroll tax levied by country i. Let $\mathbf{s^y} \equiv \left[1+s_i^y(m)\right]_{i,m=1}^2$, $\boldsymbol{\sigma^y} \equiv \left[1+\sigma_i^y(i)\right]_{i,m=1}^2$, and $\mathbf{t^n} \equiv \left[1+t_i^n\right]_{i=1}^2$. The first-order conditions are

$$\left[1+s_i^y(i)\right]p_{ii}(i) = \left[1+\sigma_i^y(i)\right]p_{ij}(i), i \neq j = 1,2,$$

$$\frac{F_i^i}{F_j^i} = \left[\frac{1+s_i^y(i)}{1+s_i^y(j)}\right]\frac{p_{ii}(i)}{p_{ii}(j)}, i \neq j = 1,2,$$

and

$$\frac{F_n^i}{F_i^i} = \left[\frac{1+t_i^n}{1+s_i^y(i)}\right]\frac{w_i}{p_{ii}(i)}, i = 1,2,$$

together with (2) in equality for $i = 1,2$.

The technologies of the intermediate good firms exhibit constant returns to scale, but the capital stock in each country is exogenous. Therefore, profits may be positive in equilibrium. Capital is owned by the household in each country, but it is delivered to firms in exchange for equity. For now, I assume that profits are fully taxed. Therefore, the household in country i only receives labor income $\left(1+s_i^n\right)w_in_i$. In Section 7 I relax this assumption.

4.3 Final Good Firms

The final good firm in country i chooses $(\mathbf{z}_i, \mathbf{x}_i)$ to maximize

$$d_i = \sum_{l=1}^{2}\left[1+s_i^z(l)\right]q_{ii}(l)z_{ii}(l) + \left[1+\sigma_i^z(i)\right]q_{ij}(i)z_{ij}(i) - \\ -\sum_{m=1}^{2}\left[1+t_i^z(m)\right]p_{ii}(m)x_{ii}(m) - \left[1+\tau_i^z(j)\right]p_{ji}(j)x_{ji}(j) \qquad (22)$$

subject to (3) for $i \neq j = 1,2$. $1+s_i^z(l)$ is the (gross) subsidy granted by country i on the supply of final good l with common origin and destination in country i, $1+\sigma_i^z(i)$ is the (gross) subsidy granted by country i on the exports of final good i, $1+t_i^z(m)$ is the (gross) tax levied by country i on the use of intermediate m with common origin and destination in country i, and $1+\tau_i^z(j)$ is the (gross) tariff levied by country i on the imports of intermediate j. Let $\mathbf{s^z} \equiv \left[1+s_i^z(l)\right]_{i,l=1}^2$, $\boldsymbol{\sigma^y} \equiv \left[\sigma_i^z(i)\right]_{i=1}^2$, $\mathbf{t^z} \equiv \left[1+t_i^z(m)\right]_{i,m=1}^2$, and $\boldsymbol{\tau^z} \equiv \left[1+\tau_i^z(j)\right]_{i\neq j=1}^2$. The first-order conditions are

$$\left[1+s_i^z(i)\right]q_{ii}(i) = \left[1+\sigma_i^z(i)\right]q_{ij}(i), i \neq j = 1,2$$

$$\left[1+t_i^z(j)\right]p_{ii}(j) = \left[1+\tau_i^z(j)\right]p_{ji}(j), i \neq j = 1,2$$

$$\frac{G^i_{z(i)}}{G^i_{z(j)}} = \left[\frac{1+s^z_i(i)}{1+s^z_i(j)}\right]\frac{q_{ii}(i)}{q_{ii}(j)}, i \neq j = 1,2,$$

$$\frac{G^i_{x(i)}}{G^i_{x(j)}} = \left[\frac{1+t^z_i(i)}{1+t^z_i(j)}\right]\frac{p_{ii}(i)}{p_{ii}(j)}, i \neq j = 1,2,$$

and

$$\frac{G^i_{z(i)}}{G^i_{x(i)}} = \left[\frac{1+s^z_i(i)}{1+t^z_i(i)}\right]\frac{q_{ii}(i)}{p_{ii}(i)}, i = 1,2.$$

together with (3) in equality for $i = 1,2$.

Because the technologies of the final goods firms exhibit constant returns to scale and (net) outputs are paid their marginal returns, profits are zero in equilibrium.

4.4 Governments, Markets and Balance-of-Payments

The budget constraint of the government in country i is

$$\sum_{j=1}^{2}\sum_{l=1}^{2} q_{ji}(l)\,g_{ji}(l) + \sum_{l=1}^{2} s^z_i(l)\,q_{ii}(l)\,z_{ii}(l) + \sigma^z_i(i)\,q_{ij}(i)\,z_{ij}(i) - \pi_i =$$
$$\sum_{l=1}^{2} t^c_i(l)\,q_{ii}(l)\,c_{ii}(l) + \tau^c_i(j)\,q_{ji}(j)\,c_{ji}(j) + \sum_{m=1}^{2} t^z_i(m)\,p_{ii}(m)\,x_{ii}(m) - \sum_{m=1}^{2} s^y_i(m)\,p_{ii}(m)\,y_{ii}(m) +$$
$$+\tau^z_i(j)\,p_{ji}(j)\,x_{ji}(j) - \sigma^y_i(i)\,p_{ji}(i)\,y_{ji}(i) + \left(t^n_i - s^n_i\right)w_i n_i,\ i \neq j \tag{23}$$

and the following conditions must be satisfied:

$$c_{ji}(l) + g_{ji}(l) = z_{ji}(l),\ i,j,l = 1,2 \tag{24}$$

and

$$x_{ji}(m) = y_{ji}(m),\ i,j,m = 1,2. \tag{25}$$

Together with definitions (15) to (19), these imply the market-clearing conditions

$$\sum_{i=1}^{2}\left[c_i(l) + g_i(l)\right] = \sum_{i=1}^{2} z_i(l),\ l = 1,2 \tag{26}$$

and

$$\sum_{i=1}^{2} x_i(m) = \sum_{i=1}^{2} y_i(m),\, m = 1,2. \tag{27}$$

The budget constraints of the household and government in country i, the expressions for profits of the intermediate and final goods firm in country i, together with conditions (24) and (25) imply

$$[q_{21}(2)\, z_{21}(2) - q_{12}(1)\, z_{12}(1)] + [p_{21}(2)\, y_{21}(2) - p_{12}(1)\, y_{12}(1)] = 0. \tag{28}$$

This equation is a balance-of-payments condition. Trade must be balanced because the intermediate good firms are owned domestically and there are no international transfers.

By Walras' law, out of equations (20) to (28), one is not needed. In the characterization of the competitive equilibrium, the balance-of-payments condition (28) can thus be used instead of the budget constraints of the governments (23).

4.5 Allocations

Because the government levies distortionary taxes, the competitive equilibrium allocations do not, in general, solve the first-best problem. The goal of this section is to derive a set of conditions that the competitive equilibrium allocations need to satisfy and that show how distorting taxes impose wedges on the first-best conditions (7) to (14). To do so, I start with the definition of the distorted competitive equilibrium.

Definition 1. A distorted competitive equilibrium $\mathcal{E}$ is a set of allocations $(\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$, $\boldsymbol{\pi} \equiv [\pi_i]_{i=1}^2$, prices $(\mathbf{p}, \mathbf{q}, \mathbf{w})$, and policies $\mathbf{t} \equiv (\mathbf{t^c}, \mathbf{t^z}, \mathbf{t^n})$, $\mathbf{s} \equiv (\mathbf{s^y}, \mathbf{s^z}, \mathbf{s^n})$, $\boldsymbol{\tau} \equiv (\boldsymbol{\tau^c}, \boldsymbol{\tau^z})$, $\boldsymbol{\sigma} \equiv (\boldsymbol{\sigma^y}, \boldsymbol{\sigma^z})$, such that, given $\mathbf{k} \equiv [k_i]_{i=1}^2$ and $\mathbf{g} \equiv [\mathbf{g}_i]_{i=1}^2$, the following are satisfied:

1. Given $(\mathbf{p}, \mathbf{q}, \mathbf{w})$ and $(\mathbf{t^n}, \mathbf{t^c}, \boldsymbol{\tau^c})$, the representative household in each country chooses $(\mathbf{c}_i, n_i)$ to maximize (1) subject to (20);

2. Given $(\mathbf{p}, \mathbf{w})$, $(\mathbf{t^n}, s^y, \boldsymbol{\sigma^y})$ and $\mathbf{k}$, the intermediate good firm in each country chooses $(\mathbf{y}_i, n_i)$ to maximize (21) subject to (2);

3. Given $(\mathbf{p}, \mathbf{q})$ and $(\mathbf{t^z}, \mathbf{s^z}, \boldsymbol{\sigma^z})$, the final good firm in each country chooses $(\mathbf{z}_i, \mathbf{x}_i)$ to maximize (22) subject to (3);

4. The balance-of-payments condition (28) is satisfied;

5. Markets clear: (26) and (27) are satisfied.

The competitive equilibrium, $\mathcal{E} \equiv \{(\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x}, \boldsymbol{\pi}); (\mathbf{p}, \mathbf{q}, \mathbf{w}); (\mathbf{t}, \mathbf{s}, \boldsymbol{\tau}, \boldsymbol{\sigma})\}$, is thus characterized by the first-order conditions of the problems of the households and firms, together with the balance-of-payments and the market-clearing conditions. These equations can be used to obtain the necessary conditions

$$\frac{u_1^1}{u_2^1} = \left[\frac{1+t_1^c(1)}{1+t_1^c(2)}\right]\left[\frac{1+s_1^z(2)}{1+s_1^z(1)}\right]\frac{G_{z(1)}^1}{G_{z(2)}^1}, \tag{29}$$

$$-\frac{u_n^i}{u_1^i} = \left(\frac{1+s_i^n}{1+t_i^n}\right)\left[\frac{1+s_i^y(1)}{1+t_i^c(1)}\right]\left[\frac{1+s_i^z(1)}{1+t_i^z(1)}\right]\left(\frac{F_n^i}{F_1^i}\right)\frac{G_{x(1)}^i}{G_{z(1)}^i}, \; i = 1, 2, \tag{30}$$

$$\frac{G_{x(1)}^1}{G_{x(2)}^1} = \left[\frac{1+t_1^z(1)}{1+t_1^z(2)}\right]\left[\frac{1+s_1^y(2)}{1+s_1^y(1)}\right]\frac{F_1^1}{F_2^1}, \tag{31}$$

$$\left[\frac{1+s_1^z(1)}{1+t_1^c(1)}\right]\left[\frac{1+\tau_1^c(2)}{1+\sigma_1^z(1)}\right]\frac{u_1^1}{u_2^1} = \left[\frac{1+t_2^c(2)}{1+s_2^z(2)}\right]\left[\frac{1+\sigma_2^z(2)}{1+\tau_2^c(1)}\right]\frac{u_1^2}{u_2^2}, \tag{32}$$

$$\left[\frac{1+s_1^z(2)}{1+t_1^c(2)}\right]\left[\frac{1+\tau_1^c(2)}{1+\sigma_1^z(1)}\right]\frac{G_{z(1)}^1}{G_{z(2)}^1} = \left[\frac{1+t_2^c(1)}{1+s_2^z(1)}\right]\left[\frac{1+\sigma_2^z(2)}{1+\tau_2^c(1)}\right]\frac{G_{z(1)}^2}{G_{z(2)}^2}, \tag{33}$$

$$\left[\frac{1+s_1^y(2)}{1+t_1^z(2)}\right]\left[\frac{1+\tau_1^z(2)}{1+\sigma_1^y(1)}\right]\frac{F_1^1}{F_2^1} = \left[\frac{1+t_2^z(1)}{1+s_2^y(1)}\right]\left[\frac{1+\sigma_2^y(2)}{1+\tau_2^z(1)}\right]\frac{F_1^2}{F_2^2}, \tag{34}$$

$$\left[\frac{1+s_1^y(1)}{1+t_1^z(1)}\right]\left[\frac{1+\tau_1^z(2)}{1+\sigma_1^y(1)}\right]\frac{G_{x(1)}^1}{G_{x(2)}^1} = \left[\frac{1+t_2^z(2)}{1+s_2^y(2)}\right]\left[\frac{1+\sigma_2^y(2)}{1+\tau_2^z(1)}\right]\frac{G_{x(1)}^2}{G_{x(2)}^2}, \tag{35}$$

and

$$\left[\frac{1+s_1^y(1)}{1+t_1^z(1)}\right]\left[\frac{1+\sigma_1^z(1)}{1+\sigma_1^y(1)}\right]\frac{G_{x(1)}^1}{G_{z(1)}^1} = \left[\frac{1+s_2^z(1)}{1+t_2^c(1)}\right]\left[\frac{1+\tau_2^c(1)}{1+\tau_2^z(1)}\right]\frac{G_{x(1)}^2}{G_{z(1)}^2}. \tag{36}$$

These equations illustrate the fact that the first-order conditions to the optimal planning problem are distorted by tax wedges in the competitive equilibrium. Note that equations (29) to (31) show that the domestic efficiency conditions (7) to (9) are distorted only by domestic taxes $(\mathbf{t}, \mathbf{s})$, while equations (32) to (36) show that the first-best international efficiency conditions (10) to (14) are also distorted by trade policies $(\boldsymbol{\tau}, \boldsymbol{\sigma})$.

5 Lerner Symmetry

I show the Lerner symmetry theorem in a way similar to Costinot and Werning (2018). I differ in that I consider a trade policy reform in both countries, and not in one country only.

The exercise consists in considering a competitive equilibrium $\tilde{\mathcal{E}}$, where all trade taxes in each country i are equal to those in $\mathcal{E}$ multiplied by a constant $\eta_i > 0$, for $i = 1, 2$. If $\eta_i > 1$, this reform corresponds to a uniform increase in export subsidies and import tariffs.

Lerner symmetry states that the allocations in $\tilde{\mathcal{E}}$ and $\mathcal{E}$ are the same. The proof is concerned with finding prices $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}})$ that support the competitive equilibrium allocations of $\mathcal{E}$ as competitive equilibrium allocations of $\tilde{\mathcal{E}}$.

Theorem 1 (Lerner symmetry)**.** *Consider a competitive equilibrium*

$$\tilde{\mathcal{E}} \equiv \left\{(\tilde{\mathbf{c}}, \tilde{\mathbf{n}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}, \tilde{\mathbf{x}}, \tilde{\boldsymbol{\pi}}); (\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}); \left(\tilde{\mathbf{t}}, \tilde{\mathbf{s}}, \tilde{\tau}, \tilde{\sigma}\right)\right\},$$

and let

$$\frac{1+\tilde{\tau}_i^c(j)}{1+\tau_i^c(j)} = \frac{1+\tilde{\sigma}_i^y(i)}{1+\sigma_i^y(i)} = \frac{1+\tilde{\sigma}_i^z(i)}{1+\sigma_i^z(i)} = \frac{1+\tilde{\tau}_i^z(j)}{1+\tau_i^z(j)} = \eta_i > 0,$$

and

$$\left(\tilde{\mathbf{t}}, \tilde{\mathbf{s}}\right) = (\mathbf{t}, \mathbf{s})$$

for $i \neq j = 1, 2$. Then there are prices $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}})$ *such that* $(\tilde{\mathbf{c}}, \tilde{\mathbf{n}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}, \tilde{\mathbf{x}}) = (\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$.

The proof, which is stated in Appendix A, is instructive because it shows the adjustment in prices that is required for neutrality. The system $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}})$ has unchanged pre-tax international prices, and uniformly higher pre-tax domestic prices in each country; that is,

$$\frac{\tilde{q}_{ii}(l)}{q_{ii}(l)} = \frac{\tilde{p}_{ii}(m)}{p_{ii}(m)} = \frac{\tilde{w}_i}{w_i} = \eta_i,$$

and

$$\tilde{\mathbf{p}}^* = \mathbf{p}^*$$

for $i, l, m = 1, 2$. This fact illustrates that Lerner symmetry is about nominal adjustments. After-tax international prices rise uniformly, and the domestic price level rises by the same proportion. Relative prices are unchanged.

There are two important aspects in this reform. First, import tariffs are paired with export subsidies and rise proportionately for each sector. Second, import tariffs (and export subsidies) on final goods are also paired to import tariffs (export subsidies) on intermediate goods and rise uniformly across all sectors. These facts can be seen from the conditions that characterize the distorted competitive equilibrium. Let Ω_n be the tax wedge caused by the domestic taxes $(\mathbf{t}, \mathbf{s})$ in equation n, where $n = 32, \ldots, 36$. Then equations (32) and (33) become

$$\Omega_{32}\left[\frac{1+\tau_1^c(2)}{1+\sigma_1^z(1)}\right]\frac{u_1^1}{u_2^1}=\left[\frac{1+\sigma_2^z(2)}{1+\tau_2^c(1)}\right]\frac{u_1^2}{u_2^2},$$

and

$$\Omega_{33}\left[\frac{1+\tau_1^c(2)}{1+\sigma_1^z(1)}\right]\frac{G_{z(1)}^1}{G_{z(2)}^1}=\left[\frac{1+\sigma_2^z(2)}{1+\tau_2^c(1)}\right]\frac{G_{z(1)}^2}{G_{z(2)}^2},$$

while equations (34) and (35) become

$$\Omega_{34}\left[\frac{1+\tau_1^z(2)}{1+\sigma_1^y(1)}\right]\frac{F_1^1}{F_2^1}=\left[\frac{1+\sigma_2^y(2)}{1+\tau_2^z(1)}\right]\frac{F_1^2}{F_2^2},$$

and

$$\Omega_{35}\left[\frac{1+\tau_1^z(2)}{1+\sigma_1^y(1)}\right]\frac{G_{x(1)}^1}{G_{x(2)}^1}=\left[\frac{1+\sigma_2^y(2)}{1+\tau_2^z(1)}\right]\frac{G_{x(1)}^2}{G_{x(2)}^2}.$$

The first pair of conditions is not distorted by trade taxes when export subsidies and import tariffs on final goods are equal for each country $i = 1, 2$. The second pair of conditions is not distorted by trade taxes when the same thing happens to the export subsidies and import tariffs on intermediate goods.

Condition (36) states something different. It can be written as

$$\Omega_{36}\left[\frac{1+\sigma_1^z(1)}{1+\sigma_1^y(1)}\right]\frac{G_{x(1)}^1}{G_{z(1)}^1}=\left[\frac{1+\tau_2^c(1)}{1+\tau_2^z(1)}\right]\frac{G_{x(1)}^2}{G_{z(1)}^2}.$$

In this case, the tax wedge caused by trade taxes only vanishes if the exports subsidies, as well as the import tariffs, are equal across sectors. This is implied by McKinnon (1966), whose results state, in the context of this model, that a uniform increase in export subsidies and import tariffs on one sector is equivalent to a decrease in export subsidies and import tariffs on the other sector by the same proportion. It turns out that this result is a consequence of Lerner symmetry.

Corollary 1 (McKinnon symmetry)**.** *Consider two competitive equilibria*

$$\tilde{\mathcal{E}}=\left\{(\tilde{\mathbf{c}},\tilde{\mathbf{n}},\tilde{\mathbf{y}},\tilde{\mathbf{z}},\tilde{\mathbf{x}},\tilde{\boldsymbol{\pi}});(\tilde{\mathbf{p}},\tilde{\mathbf{q}},\tilde{\mathbf{w}});\left(\tilde{\mathbf{t}},\tilde{\mathbf{s}},\tilde{\boldsymbol{\tau}},\tilde{\boldsymbol{\sigma}}\right)\right\}$$

and

$$\hat{\mathcal{E}}=\left\{(\hat{\mathbf{c}},\hat{\mathbf{n}},\hat{\mathbf{y}},\hat{\mathbf{z}},\hat{\mathbf{x}},\hat{\boldsymbol{\pi}});(\hat{\mathbf{p}},\hat{\mathbf{q}},\hat{\mathbf{w}});\left(\hat{\mathbf{t}},\hat{\mathbf{s}},\hat{\boldsymbol{\tau}},\hat{\boldsymbol{\sigma}}\right)\right\}.$$

Suppose that

$$\frac{1+\hat{\sigma}_i^y(i)}{1+\sigma_i^y(i)} = \frac{1+\hat{\tau}_i^z(j)}{1+\tau_i^z(j)} = \eta_i > 0 \quad \textit{and} \quad \frac{1+\hat{\sigma}_i^z(i)}{1+\sigma_i^z(i)} = \frac{1+\hat{\tau}_i^c(j)}{1+\tau_i^c(j)} = 1,$$

while

$$\frac{1+\tilde{\sigma}_i^y(i)}{1+\sigma_i^y(i)} = \frac{1+\tilde{\tau}_i^z(j)}{1+\tau_i^z(j)} = 1 \quad \textit{and} \quad \frac{1+\tilde{\sigma}_i^z(i)}{1+\sigma_i^z(i)} = \frac{1+\tilde{\tau}_i^c(j)}{1+\tau_i^c(j)} = \frac{1}{\eta_i}.$$

Suppose also that

$$\left(\tilde{\mathbf{t}},\tilde{\mathbf{s}}\right) = \left(\hat{\mathbf{t}},\hat{\mathbf{s}}\right) = (\mathbf{t},\mathbf{s}),$$

for $i \neq j = 1,2$. *Then there are price systems* $(\hat{\mathbf{p}},\hat{\mathbf{q}},\hat{\mathbf{w}})$ *and* $(\tilde{\mathbf{p}},\tilde{\mathbf{q}},\tilde{\mathbf{w}})$ *such that* $(\hat{\mathbf{c}},\hat{\mathbf{n}},\hat{\mathbf{y}},\hat{\mathbf{z}},\hat{\mathbf{x}}) = (\tilde{\mathbf{c}},\tilde{\mathbf{n}},\tilde{\mathbf{y}},\tilde{\mathbf{z}},\tilde{\mathbf{x}})$.

Proof. It suffices to note that

$$\frac{1+\hat{\tau}_i^c(j)}{1+\tilde{\tau}_i^c(j)} = \frac{1+\hat{\sigma}_i^y(i)}{1+\tilde{\sigma}_i^y(i)} = \frac{1+\hat{\sigma}_i^z(i)}{1+\tilde{\sigma}_i^z(i)} = \frac{1+\hat{\tau}_i^z(j)}{1+\tilde{\tau}_i^z(j)} = \eta_i > 0,$$

and

$$\left(\hat{\mathbf{t}},\hat{\mathbf{s}}\right) = \left(\tilde{\mathbf{t}},\tilde{\mathbf{s}}\right)$$

for $i \neq j = 1,2$. The result immediately follows from Theorem 1. □

6 Neutrality of Border Adjustments

The purpose of this section is to clarify the results of Grossman (1980) on the choice between an origin- and a destination-based system. I first consider a uniform idealized value-added tax, levied at a single rate in each country on the sales of each firm net of its purchases of intermediates.

6.1 Uniform VAT

Let τ_i^v be the VAT rate in country i. Under the origin principle, the profits of the intermediate good firm in country i are

$$\pi_i = \left(1-\tau_i^v\right)\left[\sum_{m=1}^{2} p_{ii}(m)\, y_{ii}(m) + p_{ij}(i)\, y_{ij}(i)\right] - w_i n_i,$$

for $i \neq j = 1,2$. Sales of intermediates are taxed at the rate of origin, independently of whether they are sold abroad or domestically. The profits of the final goods firm in country i are

$$d_i = \left(1-\tau_i^v\right)\left[\sum_{l=1}^{2} q_{ii}(l)\, z_{ii}(l) + q_{ij}(i)\, z_{ij}(i) - \sum_{m=1}^{2} p_{ii}(m)\, x_{ii}(m) - p_{ji}(j)\, x_{ji}(j)\right].$$

for $i \neq j = 1,2$. Sales of final goods are also taxed at the rate of origin. Since intermediates have already been taxed when exported, they are deducted from the tax base when imported. The budget constraint of the household in country i is

$$\sum_{l=1}^{2} q_{ii}(l)\, c_{ii}(l) + q_{ji}(j)\, c_{ji}(j) \leq w_i n_i.$$

for $i \neq j = 1,2$. Because final goods have already been taxed when exported, they are not taxed when imported.

With border adjustments, the profits of the firms in country i are

$$\pi_i = \left(1-\tau_i^v\right)\sum_{m=1}^{2} p_{ii}(m)\, y_{ii}(m) + p_{ij}(i)\, y_{ij}(i) - w_i n_i,$$

and

$$d_i = \left(1-\tau_i^v\right)\left[\sum_{l=1}^{2} q_{ii}(l)\, z_{ii}(l) - \sum_{m=1}^{2} p_{ii}(m)\, x_{ii}(m)\right] + q_{ij}(i)\, z_{ij}(i) - p_{ji}(j)\, x_{ji}(j)$$

for $i \neq j = 1,2$. Exports of intermediates and of final goods are not taxed. Imports of intermediates are not deducted, and thus they are taxed at the destination rate. The budget constraint of the household in country i is

$$\sum_{l=1}^{2} q_{ii}(l)\, c_{ii}(l) + \frac{1}{1-\tau_i^v} q_{ji}(j)\, c_{ji}(j) \leq w_i n_i.$$

for $i \neq j = 1,2$. Imports of final goods are taxed at the destination rate as they pass borders[3].

Are the allocations different under the two principles? Under both origin and destination, the

[3]For ease of exposition, households are being taxed directly when they are imported. This is not the common practice. In reality, exporters are required to charge the tax rate of destination when selling to consumers. This is a matter of legal incidence, and irrelevant for the case in point.

only distortion is

$$-\frac{u_n^i}{u_1^i} = \left(1-\tau_i^v\right)\left(\frac{F_n^i}{F_1^i}\right)\frac{G_{x(1)}^i}{G_{z(1)}^i}, i = 1,2, \tag{37}$$

implying that border adjustments are irrelevant for the uniform VAT.

Equation 37 shows that a uniform VAT is equivalent to a uniform consumption tax, since a tax system with

$$1+t_i^c(l) = 1+\tau_i^c(j) = \frac{1}{1-\tau_i^v}, i \neq j = 1,2$$

and all other tax rates set to zero would impose the same distortion. To see why, it is useful to consider how prices might adjust to the imposition of the VAT. Consider first that the VAT is levied according to the destination principle. Then, relative to the no-tax situation and keeping pre-tax prices fixed, the after-tax domestic prices faced by the firms in country i, $\left[\left(1-\tau_i^v\right)p_{ii}(m)\right]_{m=1}^2$ and $\left[\left(1-\tau_i^v\right)q_{ii}(l)\right]_{l=1}^2$, decrease by $\left(1-\tau_i^v\right)$. For the household in country i, the after-tax international price, $q_{ji}(j)/\left(1-\tau_i^v\right)$, increases by $1/\left(1-\tau_i^v\right)$ for $i \neq j$. If pre-tax domestic prices rise by $1/\left(1-\tau_i^v\right)$, the relative prices for the firms are unchanged. For households, the relative prices between final goods are also unchanged, but because the price level increases, the real wage decreases. Therefore, there is only a distortion in the labor margin. This is similar to the argument set out by Feldstein and Krugman (1990).

The origin-based VAT is also equivalent to a uniform consumption tax because removing the border adjustment from the destination-based system consists in the reform described in Theorem 1 with $\eta_i = 1-\tau_i^v$. This implies that relative to its destination counterpart, the origin-based VAT has lower domestic prices and nominal wages. The only difference between the systems is thus in nominal prices. Relative to the situation without taxes, the origin principle leads to a decrease in the nominal wage, while the destination principle leads to an increase in the nominal domestic prices of final goods and intermediates. This fact actually provides a rational for employing border adjustments: origin- and destination-based systems may be able to avoid price and wage rigidities.

6.2 Uniform Sales Tax

A uniform sales tax differs from the VAT in its treatment of intermediate goods, which do not go through the tax system. Let τ_i^s be the single sales tax rate in country i. Regardless of border adjustments, the profits of the intermediate goods firm in country i are

$$\pi_i = \sum_{m=1}^{2} p_{ii}(m)\, y_{ii}(m) + p_{ij}(i)\, y_{ij}(i) - w_i n_i,$$

for $i \neq j = 1,2$. The profits of the final goods firm in country i are

$$d_i = \left(1-\tau_i^s\right)\left[\sum_{l=1}^{2} q_{ii}(l)\,z_{ii}(l) + q_{ij}(i)\,z_{ij}(i)\right] - \sum_{m=1}^{2} p_{ii}(m)\,x_{ii}(m) - p_{ji}(j)\,x_{ji}(j),$$

Differently from the VAT, purchases of intermediates are not deducted, but final goods are treated in the same way. Under origin, exports of final goods are taxed at the origin rate, and imports of final goods are free of tax. The budget constraint of the household in country i is the same as in the origin-based VAT. Under destination, exports of final goods are not taxed, and imports of final goods are taxed at the destination rate. The profits of the final goods firm in country i are

$$d_i = \left(1-\tau_i^s\right)\sum_{l=1}^{2} q_{ii}(l)\,z_{ii}(l) + q_{ij}(i)\,z_{ij}(i) - \sum_{m=1}^{2} p_{ii}(m)\,x_{ii}(m) - p_{ji}(j)\,x_{ji}(j),$$

and the budget constraint of the household in country i is

$$\sum_{l=1}^{2} q_{ii}(l)\,c_{ii}(l) + \frac{1}{1-\tau_i^s} q_{ji}(j)\,c_{ji}(j) \leq w_i n_i.$$

Under the destination principle, the sales tax is still equivalent to a uniform consumption tax; it only imposes a tax wedge on the labor margin:

$$-\frac{u_n^i}{u_1^i} = \left(1-\tau_i^s\right)\left(\frac{F_n^i}{F_1^i}\right)\frac{G_{x(1)}^i}{G_{z(1)}^i},\ i = 1,2.$$

Under the origin principle, however, the sales tax imposes the additional distortion

$$\left(1-\tau_1^s\right)\frac{G_{x(1)}^1}{G_{z(1)}^1} = \left(1-\tau_2^s\right)\frac{G_{x(1)}^2}{G_{z(1)}^2}. \tag{38}$$

These two distortions could alternatively be generated with a tax system that consists of a uniform consumption tax in each country i equal to $1/\left(1-\tau_i^s\right)$,

$$1 + t_i^c(l) = 1 + \tau_i^c(j) = \frac{1}{1-\tau_i^s},\ i \neq j, l = 1,2$$

together with an export subsidy and an import tariff on intermediates of the same size,

$$1+\sigma_i^y(i)=1+\tau_i^z(j)=\frac{1}{1-\tau_i^s},\ i\neq j=1,2,$$

and all other tax rates set to zero. Thus, the origin-based sales tax is equivalent to the destination-based sales tax together with import tariffs and export subsidies on traded intermediates. This is the result of Grossman (1980). Border adjustments are relevant in this case.

The argument for the equivalence between the destination-based sales tax and a uniform consumption tax follows along the lines of the argument for the destination-based VAT. The sole difference is that relative to the no-tax situation, only the domestic pre-tax prices of final goods need to adjust.

To see why there are no prices that can support the same allocation under origin and destination, note that in this case, moving from the destination to the origin principle is not a Lerner symmetry exercise, as described in Theorem 1, but a McKinnon symmetry exercise, as described in Corollary 1. Using the terminology of Section 5, suppose that the policies $(\mathbf{t},\mathbf{s},\boldsymbol{\tau},\boldsymbol{\sigma})$ in a competitive equilibrium $\mathcal{E}$ consist in a destination-based sales tax system. In the final good sector, those policies include export subsidies equal to zero and import tariffs equal to $\left[1/\left(1-\tau_i^s\right)\right]$ in each country $i=1,2$. The origin-based sales tax system can be seen as the set of policies $\left(\hat{\mathbf{t}},\hat{\mathbf{s}},\hat{\boldsymbol{\tau}},\hat{\boldsymbol{\sigma}}\right)$ that are part of the competitive equilibrium $\hat{\mathcal{E}}$ described in Corollary 1. Relative to $\mathcal{E}$, the export subsidies and import tariffs on final goods decrease by a proportion $\eta_i=\left(1-\tau_i^s\right)$ in each country $i=1,2$. By McKinnon symmetry, the allocations $(\hat{\mathbf{c}},\hat{\mathbf{n}},\hat{\mathbf{y}},\hat{\mathbf{z}},\hat{\mathbf{x}})$ in $\hat{\mathcal{E}}$ are equal to the allocations $(\tilde{\mathbf{c}},\tilde{\mathbf{n}},\tilde{\mathbf{y}},\tilde{\mathbf{z}},\tilde{\mathbf{x}})$ in $\tilde{\mathcal{E}}$. $\tilde{\mathcal{E}}$ has policies which, relative to $\mathcal{E}$, include import tariffs and export subsidies on intermediates equal to $(1/\eta_1)=\left[1/\left(1-\tau_i^s\right)\right]$ in each country $i=1,2$.

In fact, the origin-based sales tax is not really origin-based. Note that intermediates are not taxed, and regardless of whether they are sold domestically or abroad, they are not deducted. They are thus always taxed in their destination country at the rate that is applicable to final goods. When final goods are taxed at the origin, there is an asymmetry of fiscal treatment - destination for intermediates and origin for final goods - that imposes a distortion on the first-best condition (14).

What is the mechanism of this distortion? Equation (38) is an arbitrage condition. It states that intermediate 1 can produce final good 1 for consumption in country 1 in two ways. The first way is to do it directly in country 1, corresponding to the left-hand side of equation (38), and the second way is to do it indirectly through trade, corresponding to the right-hand side of the same equation. If the intermediate is used domestically, its marginal productivity is reduced by the domestic tax rate and becomes $\left(1-\tau_1^s\right)\left(G^1_{x(1)}/G^1_{z(1)}\right)$. If the intermediate is exported and used abroad, and the final good is then imported, the marginal productivity of the intermediate is reduced by the foreign tax rate and becomes $\left(1-\tau_2^s\right)\left(G^2_{x(1)}/G^2_{z(1)}\right)$ if final goods are taxed at the origin, or becomes $\left(1-\tau_1^s\right)\left(G^2_{x(1)}/G^2_{z(1)}\right)$ if they are taxed at the destination. At the optimum, the effective marginal returns must be the same:

$$\left(1-\tau_1^s\right)\frac{G^1_{x(1)}}{G^1_{z(1)}}=\left(1-\tau_2^s\right)\frac{G^2_{x(1)}}{G^2_{z(1)}}$$

in the case of origin-based taxation, or

$$\left(1-\tau_1^s\right)\frac{G^1_{x(1)}}{G^1_{z(1)}}=\left(1-\tau_1^s\right)\frac{G^2_{x(1)}}{G^2_{z(1)}}$$

in the case of destination-based taxation.

Without border adjustments, arbitrage thus prevents marginal productivities from being equalized. With border adjustments, the first-best condition (14) is in any case satisfied.

6.3 Practical Relevance of Border Adjustments

The goal of this subsection is to assess whether the above effects of border adjustments have any consequences for fiscal policy.

The sales tax is the indirect tax system that is applied at the state level in the United States. It is also equivalent to the European VAT. This system differs from the idealized, pure VAT I have considered in how the tax liability is computed. The idealized VAT first subtracts purchases of intermediates from sales, and then applies the tax rate; the European VAT first taxes sales, and then grants a credit of the size of the tax already paid on intermediates[4]. The European VAT is equivalent to a sales tax because producer and user prices of intermediates coincide. It taxes sales of intermediates at a given rate, and grants a subsidy to purchases of intermediates at the same rate. This is the same as doing nothing; it is as if the intermediate good sector did not go through the tax system, as in the sales tax.

The fact that the sales tax is widely used both in the United States and in the European Union does not imply that the distortion found by Grossman (1980) is of practical relevance. This distortion only arises because there is trade in both the intermediate and the final good sectors; if there is no trade in final goods, there is no distortion. Indeed, there are good reasons to think that consumption goods are really non-tradable. When goods hit the shelves of the retailer, they become

[4]For this reason, the idealized VAT is called a subtraction-method VAT, and the European VAT is called a credit-method VAT. See Due (1965), Shoup (1969) and Shoup (1986).

different goods than what they were at the warehouse of the producer; there is an importer in the middle. One apparent exception is e-commerce, as nowadays consumers order goods online. However, online trade between businesses seems to have much more weight in e-commerce than trade between firms and consumers. Furthermore, whenever an online retailer acquires significant presence in a country or state, it must register in that jurisdiction and levy the tax at the destination rate; the e-seller thus works as a domestic operator[5]. These legal arrangements imply that trade in consumption goods does not really exist for tax purposes. Therefore, the distortion of the origin-based sales tax described here should have no practical relevance.

The VAT I have considered is idealized, and never used in practice; McLure (1987) points out the many difficulties in implementing it. However, when coupled with a payroll subsidy of the same amount, it is equivalent to a corporate income tax. This is relevant because the recent discussion on border adjustments surrounds a type of corporate income tax, the destination-based cash flow tax (DBCFT), proposed by Auerbach et al. (2017). It is destination-based because it contains a border adjustment that exempts exports and taxes imports. The idealized VAT can thus be used to analyze the effects of border adjustments on the DBCFT.

I have considered uniform taxation. Uniform tax systems may be relevant in the context of a Ramsey planner who is restricted to using linear taxes on goods and labor income to finance exogenous government expenditures. Appendix (C) shows the classical result that if the conditions for the productive efficiency theorem of Diamond and Mirrlees (1971) are satisfied, and for widely used preferences described by Atkinson and Stiglitz (1972), uniform commodity taxation is optimal and production should not be distorted.

7 Government Revenue and Balance of Payments

The possible effects of border adjustments on government revenue and on the balance of payments depend on the existence of non-labor income in each country, international assets, and on the absence of a common numeraire.

Let e, the exchange rate, denote the price of the currency of country 2 in terms of the currency of country 1. Let T_i be the domestic lump-sum transfer from the government in country i to the household in country i, and let UT_i be the international lump-sum transfer from the government in country j to the government in country i, with $i \neq j = 1, 2$. Let $\mathbf{T} \equiv [T_i]_{i=1}^2$ and $\mathbf{UT} \equiv [UT_i]_{i=1}^2$. θ_{ji} is the share of the intermediate goods firm in country j held by the household in country i, for

[5] Regarding the weight of online trade between firms and consumers in e-commerce, see UNCTAD (2016). Regarding the current legal arrangements, see the Council Directive 2006/112/EC of 28 November 2006 for the European Union and EY (2018) for the United States.

$i, j = 1, 2$. f_i^1 is a claim held by the household in country i against the household in country j, and denominated in the currency of country 1, for $i \neq j = 1, 2$. f_i^2 has the analogous definition, but it is denominated in the currency of country 2. $[p_{ii}(m)]_{m=1}^2$, $[q_{ii}(l)]_{l=1}^2$, w_i , π_i, and d_i are denominated in the currency of country i for $i = 1, 2$, but $\mathbf{q}^* \equiv [q_{12}(1), q_{21}(2)]$, $\mathbf{p}^* \equiv [p_{12}(1), p_{21}(2)]$ and $\mathbf{UT}$ are denominated in the currency of country 2. Private wealth is no longer taxed.

The budget constraints of the households in countries 1 and 2 are, respectively,

$$\sum_{l=1}^{2} \left[1 + t_1^c(1)\right] q_{11}(l) c_{11}(l) + \left[1 + \tau_1^c(2)\right] e q_{21}(2) c_{21}(2) \leq$$
$$\left(1 + s_1^n\right) w_1 n_1 + \theta_{11}\pi_1 + \theta_{21} e \pi_2 + f_1^1 + e f_1^2 + T_1,$$

and

$$\sum_{l=1}^{2} \left[1 + t_2^c(l)\right] q_{22}(l) c_{22}(l) + \left[1 + \tau_2^c(1)\right] q_{12}(1) c_{12}(1) \leq$$
$$\left(1 + s_2^n\right) w_2 n_2 + \theta_{22}\pi_2 + \theta_{12}\frac{\pi_1}{e} + \frac{f_2^1}{e} + f_2^2 + T_2.$$

The profits of firms in country 1 are

$$\pi_1 = \sum_{m=1}^{2} \left[1 + s_1^y(m)\right] p_{11}(m) y_{11}(m) + \left[1 + \sigma_1^y(1)\right] e p_{12}(1) y_{12}(1) - \left(1 + t_1^n\right) w_1 n_1,$$

and

$$d_1 = \sum_{l=1}^{2} \left[1 + s_1^z(l)\right] q_{11}(l) z_{11}(l) + \left[1 + \sigma_1^z(1)\right] e q_{12}(1) z_{12}(1)$$
$$- \sum_{m=1}^{2} \left[1 + t_1^z(m)\right] p_{11}(m) x_{11}(m) - \left[1 + \tau_1^z(2)\right] e p_{21}(2) x_{21}(2),$$

while the profits of the firms in country 2 are

$$\pi_2 = \sum_{m=1}^{2} \left[1 + s_2^y(m)\right] p_{22}(m) y_{22}(m) + \left[1 + \sigma_2^y(2)\right] p_{21}(2) y_{21}(2) - \left(1 + t_2^n\right) w_2 n_2.$$

and

$$d_2 = \sum_{l=1}^{2} \left[1 + s_2^z(l)\right] q_{22}(l) z_{22}(l) + \left[1 + \sigma_2^z(2)\right] q_{21}(2) z_{21}(2) -$$
$$- \sum_{m=1}^{2} \left[1 + t_2^z(m)\right] p_{22}(m) x_{22}(m) - \left[1 + \tau_2^z(1)\right] p_{12}(1) x_{12}(1).$$

Since

$$\sum_{i=1}^{2} f_i^1 = \sum_{i=1}^{2} f_i^2 = 0,$$

the balance-of-payments conditions can be written as

$$[q_{21}(2) z_{21}(2) - q_{12}(1) z_{12}(1)] + [p_{21}(2) y_{21}(2) - p_{12}(1) y_{12}(1)] = \frac{NFA_1}{e} + UT_1$$

and

$$UT_1 + UT_2 = 0,$$

where

$$NFA_1 \equiv \theta_{21} e \pi_2 - \theta_{12} \pi_1 + f_1^1 + e f_1^2.$$

These equations can be used to characterize a competitive equilibrium

$$\mathcal{E}^* \equiv \{(\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x}, \boldsymbol{\pi}); (\mathbf{p}, \mathbf{q}, \mathbf{w}, e); (\mathbf{t}, \mathbf{s}, \boldsymbol{\tau}, \boldsymbol{\sigma}, \mathbf{T}, \mathbf{UT})\}.$$

I proceed almost as in Section 5. I consider a competitive equilibrium $\tilde{\mathcal{E}}^*$ where all trade taxes in country 1 are equal to those in $\mathcal{E}^*$ multiplied by a constant $\eta = 1/\left(1 - \tau_1^v\right) > 0$. This exercise is a unilateral reform in country 1 that implements a border adjustment of size $\tau_1^v = (\eta - 1)/\eta$ in a uniform VAT. I then find prices $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}, \tilde{e})$ and transfers $\left(\tilde{\mathbf{T}}, \tilde{\mathbf{UT}}\right)$ such that the allocations $(\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$ in $\mathcal{E}^*$ are equal to the allocations $(\tilde{\mathbf{c}}, \tilde{\mathbf{n}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}, \tilde{\mathbf{x}})$ in $\tilde{\mathcal{E}}^*$.

This reform changes import tariffs and export subsidies uniformly in country 1. Therefore, $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}, \tilde{e})$ can still be constructed so that relative prices are unchanged. However, because households receive non-labor income and there are international assets, lump-sum transfers need to be used to preserve neutrality. These adjustments in transfers can be used to discuss the effects of border adjustments on government revenue and on the balance of payments.

With different currencies, $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}, \tilde{e})$ can be constructed in two ways: either the domestic price level rises, as before, or the exchange rate appreciates. These two possibilities are accompanied with different adjustments in transfers. These facts are shown in the following theorem, the proof of which is presented in Appendix B.

Theorem 2. *Consider a competitive equilibrium*

$$\tilde{\mathcal{E}}^* \equiv \left\{ (\tilde{\mathbf{c}}, \tilde{\mathbf{n}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}, \tilde{\mathbf{x}}, \tilde{\boldsymbol{\pi}}) ; (\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}, \tilde{e}) ; \left(\tilde{\mathbf{t}}, \tilde{\mathbf{s}}, \tilde{\boldsymbol{\tau}}, \tilde{\boldsymbol{\sigma}}, \tilde{\mathbf{T}}, \tilde{\mathbf{UT}} \right) \right\}.$$

Let

$$\frac{\left[1 + \tilde{\tau}_1^c(2)\right]}{\left[1 + \tau_1^c(2)\right]} = \frac{\left[1 + \tilde{\tau}_1^z(2)\right]}{\left[1 + \tau_1^z(2)\right]} = \frac{\left[1 + \tilde{\sigma}_1^y(1)\right]}{\left[1 + \tau_1^y(1)\right]} = \frac{\left[1 + \tilde{\sigma}_1^y(1)\right]}{\left[1 + \tau_1^y(1)\right]} = \eta,$$

and

$$\left(\tilde{\mathbf{t}}, \tilde{\mathbf{s}}\right) = (\mathbf{t}, \mathbf{s}),$$

with $\eta > 0$. Then there are two price systems $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}, \tilde{e})$ with transfers $\left(\tilde{\mathbf{T}}, \tilde{\mathbf{UT}}\right)$ such that $(\tilde{\mathbf{c}}, \tilde{\mathbf{n}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}, \tilde{\mathbf{x}}) = (\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$. Namely, either

1. $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}, \tilde{e})$ *satisfy*

$$\frac{\tilde{p}_{11}(m)}{p_{11}(m)} = \frac{\tilde{q}_{11}(l)}{q_{11}(l)} = \frac{\tilde{w}_1}{w_1} = \frac{1}{1 - \tau^v}, \; m, l = 1, 2,$$

$$\frac{\tilde{p}_{22}(m)}{p_{22}(m)} = \frac{\tilde{q}_{22}(l)}{q_{22}(l)} = \frac{\tilde{w}_2}{w_2} = 1, \; m, l = 1, 2,$$

$$\tilde{e} = e,$$

$$\tilde{\mathbf{p}}^* = \mathbf{p}^*,$$

and $\left(\tilde{\mathbf{T}}, \tilde{\mathbf{UT}}\right)$ *satisfy*

$$\frac{\tilde{T}_1}{\tilde{q}_{11}(1)} = \frac{T_1}{q_{11}(1)} + \tau^v \left[\frac{NFA_1}{q_{11}(1)} + \frac{\theta_{12}\pi_1}{q_{11}(1)} \right],$$

$$\tilde{T}_2 = T_2 - \left(\frac{\tau^v}{1 - \tau^v} \right) \theta_{12} \frac{\pi_1}{e},$$

$$\tilde{UT}_1 = UT_1 + \left(\frac{\tau^v}{1 - \tau^v} \right) \theta_{12} \frac{\pi_1}{e},$$

$$\tilde{UT}_2 = -\tilde{UT}_1,$$

or

2. $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}, \tilde{e}) = (\mathbf{p}, \mathbf{q}, \mathbf{w}, (1 - \tau^v) e)$, *and* $\left(\tilde{\mathbf{T}}, \tilde{\mathbf{UT}}\right)$ *satisfy*

$$\frac{\tilde{T}_1}{\tilde{q}_{11}(1)} = \frac{T_1}{q_{11}(1)} + \tau^v \left[\frac{NFA_1}{q_{11}(1)} + \frac{\theta_{12}\pi_1}{q_{11}(1)} - \frac{f_1^1}{q_{11}(1)} \right],$$

$$\tilde{T}_2 = T_2 - \left(\frac{\tau^v}{1-\tau^v}\right)\left(\theta_{12}\frac{\pi_1}{e} - \frac{f_1^1}{e}\right),$$

$$\tilde{UT}_1 = UT_1 + \left(\frac{\tau^v}{1-\tau^v}\right)\left(\theta_{12}\frac{\pi_1}{e} - \frac{f_1^1}{e}\right),$$

and

$$\tilde{UT}_2 = -\tilde{UT}_1.$$

Regardless of the adjustment in prices, real domestic lump-sum transfers in country 1 must increase by a proportion τ^v of net foreign assets[6]. With a border adjustment, the government receives a higher proportion of the value of imports, but also pays a higher proportion of the value of exports. A higher fraction of the value of the trade deficit thus goes to the government. This is shown in Barbiero et al. (2018). This increase in revenue is proportional to net foreign assets because in a one-period model, net foreign assets are equal to the trade deficit.

Depending on the adjustment in prices, there are different valuation effects on the balance of payments. When the price level in country 1 rises, the profits of the intermediate goods firm in country 1 rise with it. Since these are held internationally, the foreign liabilities of country 1 increase. Unilateral transfers to country 1 must increase by $[\tau^v/(1-\tau^v)]\theta_{12}(\pi_1/e)$ to offset this effect. This is shown in Costinot and Werning (2018).

When the exchange rate appreciates, the profits of country 1 held by the household in country 2 increase when converted to the currency of country 2. Unilateral transfers to country 1 must thus still increase by $[\tau^v/(1-\tau^v)]\theta_{12}(\pi_1/e)$. However, the value of the claims denominated in domestic currency, f_1^1/e, also changes because f_1^1 is fixed. If country 1 has credits in domestic currency, with $f_1^1 > 0$, its foreign assets increase. To undo this effect, unilateral transfers to country 1 must additionally decrease by $[\tau^v/(1-\tau^v)]\left(f_1^1/e\right)$. This valuation effect is a reason why Barbiero et al. (2018) establish that neutrality requires that debt cannot be held in the currency of the country that implements a border adjustment.

Note that these effects are not necessarily linked to unbalanced trade. Indeed, how much extra revenue the government receives depends on the net foreign asset position. This is stated in Barbiero et al. (2018). However, how much the balance of payments deteriorates or improves depends on gross assets and liabilities. This is stated in Costinot and Werning (2018), and linked to the breakdown of Lerner symmetry described by Blanchard (2009).

These results are also related to Keynes (1931) and Farhi et al. (2014) on fiscal devaluations.

[6] I write the domestic lump-sum transfers in terms of final good 1 for comparability between the two nominal adjustments.

Keynes (1931) argued that a currency devaluation would lead to a deterioration of the balance of payments in the United Kingdom, given that the country's loans to the rest of the world were mainly denominated in sterling at the time. He suggested that uniform import tariffs and export subsidies across all traded goods would be preferable to a currency devaluation, since this reform would have the same competitive effects as a currency devaluation, but would not lead to a decrease in foreign assets. Farhi et al. (2014) formalize this idea. They show that a uniform import tariff together with an export subsidy of the same amount may constitute a fiscal devaluation in the sense that they can mimic an unanticipated one-time permanent currency devaluation. However, if bonds are held in domestic currency, a haircut on debt is required, and if equity is traded internationally, dividend taxes must be used.

8 Conclusion

The neutrality of border adjustments is a classical proposition in international trade theory. Grossman (1980) argues that it is correct in the context of uniform value-added taxation, but it is incorrect when applied to a uniform sales tax; without a border adjustment, trade would be distorted. The main goal of this dissertation is to clarify this and related results.

I use a static Arrow-Debreu model with traded intermediates and final goods, and discuss the general equilibrium effects of two tax systems - a uniform VAT and a uniform sales tax - under two principles of international taxation - origin and destination. In this set-up, the argument in Grossman (1980) can be restated as follows. A uniform VAT levies a tax on the value-added by both sectors, while a uniform sales tax only taxes sales of final goods. A border adjustment, which is used to move from an origin- to a destination-based system, consists in a rebate to exports by the size of the domestic tax rate, together with an import tariff of the same amount. Therefore, when it is applied to the VAT, the border adjustment increases all international prices faced by a country, and if the domestic price level or the exchange rate adjusts, all relative prices are unchanged. However, when it is applied to the sales tax, the border adjustment only increases the international prices of final goods, having effects on the relative prices of intermediates. This creates a wedge in the equality of the marginal productivities of intermediates across countries.

I also discuss the effects of a border adjustment on government revenue and on the balance of payments. Because a border adjustment is a uniform tax on the trade deficit, it leads to a transfer from households to governments by a proportion of net foreign assets. Depending on the nominal adjustment of the economy, it also leads to transfers between countries. If there is a rise in the domestic price level, the value of internationally-held equity increases. If there is a currency appreciation, the value of bonds held in domestic currency also increases. This is a review of some of the results in Barbiero et al. (2018) and Costinot and Werning (2018).

Even though the sales tax is widely used in the United States and it is equivalent to the European VAT, the non-neutrality suggested by Grossman (1980) is not likely to be relevant. It depends on the existence of trade in both sectors, and trade in final goods is largely a theoretical artifice. On the other hand, the non-neutrality on government revenue and on the balance of payments seems to have more practical importance. For countries with trade deficits, a border adjustment may represent an extra source of revenue for governments. However, it may also lead to a deterioration of the international investment position, depending on the type of internationally-traded assets.

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A Proof of Theorem 1

Proof. The proof is by construction. Suppose that $(\tilde{\mathbf{c}}, \tilde{\mathbf{n}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}, \tilde{\mathbf{x}}) = (\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$, and let

$$\frac{\tilde{q}_{ii}(l)}{q_{ii}(l)} = \frac{\tilde{p}_{ii}(m)}{p_{ii}(m)} = \frac{\tilde{w}_i}{w_i} = \eta_i$$

and

$$\tilde{\mathbf{p}}^* = \mathbf{p}^*,$$

for $i \neq j, l, m = 1, 2$. The profits of the intermediate goods firms are

$$\begin{aligned}
\tilde{\pi}_i &= \sum_{m=1}^{2} \left[1 + \tilde{s}_i^y(m)\right] \tilde{p}_{ii}(m)\, \tilde{y}_{ii}(m) + \left[1 + \tilde{\sigma}_i^y(i)\right] \tilde{p}_{ij}(i)\, \tilde{y}_{ij}(i) - \left(1 + \tilde{t}_i^n\right) \tilde{w}_i \tilde{n}_i \\
&= \sum_{m=1}^{2} \left[1 + s_i^y(m)\right] \eta_i p_{ii}(m)\, y_{ii}(m) + \eta_i \left[1 + \sigma_i^y(i)\right] p_{ij}(i)\, y_{ij}(i) - \eta_i \left(1 + t_i^n\right) w_i \tilde{n}_i \\
&= \eta_i \pi_i,
\end{aligned}$$

for $i \neq j = 1, 2$, while the profits of the final goods firms are

$$\begin{aligned}
\tilde{d}_i &= \sum_{l=1}^{2} \left[1 + \tilde{s}_i^z(l)\right] \tilde{q}_{ii}(l)\, \tilde{z}_{ii}(l) + \left[1 + \tilde{\sigma}_i^z(i)\right] \tilde{q}_{ij}(i)\, \tilde{z}_{ij}(i) - \\
&- \sum_{m=1}^{2} \left[1 + \tilde{t}_i^z(m)\right] \tilde{q}_{ii}(m)\, \tilde{x}_{ii}(m) - \left[1 + \tilde{\tau}_i^z(j)\right] \tilde{p}_{ji}(j)\, \tilde{x}_{ji}(j) = \\
&= \sum_{l=1}^{2} \left[1 + s_i^z(l)\right] \eta_i q_{ii}(l)\, z_{ii}(l) + \eta_i \left[1 + \sigma_i^z(i)\right] q_{ij}(i)\, z_{ij}(i) - \\
&- \sum_{m=1}^{2} \left[1 + t_i^z(m)\right] \eta_i q_{ii}(m)\, x_{ii}(m) - \eta_i \left[1 + \tau_i^z(j)\right] p_{ji}(j)\, x_{ji}(j) = \eta_i d_i
\end{aligned}$$

for $i \neq j = 1, 2$. Since profits in $\tilde{\mathcal{E}}$ are the profits in $\mathcal{E}$ multiplied by a constant, $(\mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$ solves the problem of the firms. The budget constraint of the household is

$$\sum_{l=1}^{2} \left[1 + \tilde{t}_i^c(l)\right] \tilde{q}_{ii}(l)\, \tilde{c}_{ii}(l) + \left[1 + \tilde{\tau}_i^c(j)\right] \tilde{q}_{ji}(j)\, \tilde{c}_{ji} = \left(1 + \tilde{s}_i^n\right) \tilde{w}_i \tilde{n}_i$$

$$\Longleftrightarrow$$

$$\sum_{l=1}^{2}\left[1+t_i^c(l)\right]\eta_i q_{ii}(l)c_{ii}(l)+\eta_i\left[1+\tau_i^c(j)\right]q_{ji}(j)c_{ji}=\left(1+s_i^n\right)\eta_i w_i n_i$$

$$\Longleftrightarrow$$

$$\sum_{l=1}^{2}\left[1+t_i^c(l)\right]q_{ii}(l)c_{ii}(l)+\left[1+\tau_i^c(j)\right]q_{ji}(j)c_{ji}=\left(1+s_i^n\right)w_i n_i$$

$i \neq j = 1,2$. Because the budget constraints of the households are unchanged, $(\mathbf{c},\mathbf{n})$ solves the utility-maximization problems. Because international prices do not change, the balance-of-payments condition is trivially satisfied. The market-clearing conditions are also trivially satisfied. Therefore, $(\tilde{\mathbf{c}},\tilde{\mathbf{n}},\tilde{\mathbf{y}},\tilde{\mathbf{z}},\tilde{\mathbf{x}})=(\mathbf{c},\mathbf{n},\mathbf{y},\mathbf{z},\mathbf{x})$. □

B Proof of Theorem 2

Proof. The proof follows along the lines of the proof of Theorem 1. Suppose that $(\tilde{\mathbf{c}},\tilde{\mathbf{n}},\tilde{\mathbf{y}},\tilde{\mathbf{z}},\tilde{\mathbf{x}})=(\mathbf{c},\mathbf{n},\mathbf{y},\mathbf{z},\mathbf{x})$.

1. Consider that $(\tilde{\mathbf{p}},\tilde{\mathbf{q}},\tilde{\mathbf{w}},\tilde{e})$ satisfy

$$\frac{\tilde{p}_{11}(m)}{p_{11}(m)}=\frac{\tilde{q}_{11}(l)}{q_{11}(l)}=\frac{\tilde{w}_1}{w_1}=\eta,\ m,l=1,2,$$

$$\frac{\tilde{p}_{22}(m)}{p_{22}(m)}=\frac{\tilde{q}_{22}(l)}{q_{22}(l)}=\frac{\tilde{w}_2}{w_2}=1,\ m,l=1,2,$$

$$\tilde{e}=e,$$

and

$$\tilde{\mathbf{p}}^*=\mathbf{p}^*,$$

and $\left(\tilde{\mathbf{T}},\tilde{\mathbf{UT}}\right)$ satisfy

$$\tilde{T}_1=\eta T_1+(\eta-1)NFA_1+(\eta-1)\theta_{12}\pi_1,$$

$$\tilde{T}_2=T_2-(\eta-1)\theta_{12}\frac{\pi_1}{e},$$

$$\tilde{UT}_1=UT_1+(\eta-1)\theta_{12}\frac{\pi_1}{e},$$

and

$$\tilde{UT}_2=-\tilde{UT}_1.$$

Then

$$\frac{\tilde{\pi}_1}{\pi_1} = \frac{\tilde{d}_1}{d_1} = \eta$$

and

$$\frac{\tilde{\pi}_2}{\pi_2} = \frac{\tilde{d}_2}{d_2} = 1.$$

This implies that $(\mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$ solves the problem of the firms. With

$$\tilde{T}_1 = \eta T_1 + (\eta - 1) NFA_1 + (\eta - 1) \theta_{12} \pi_1$$

and

$$\tilde{T}_2 = T_2 - (\eta - 1) \theta_{12} \frac{\pi_1}{e},$$

the budget constraints of the households are unchanged. Therefore $(\mathbf{c}, \mathbf{n})$ solves the problem of the households. With

$$\tilde{UT}_1 = UT_1 + (\eta - 1) \theta_{12} \frac{\pi_1}{e}$$

and

$$\tilde{UT}_2 = -\tilde{UT}_1,$$

the balance-of-payments conditions are also satisfied. Finally, the allocation also satisfies the market-clearing conditions. Thus $(\tilde{\mathbf{c}}, \tilde{\mathbf{n}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}, \tilde{\mathbf{x}}) = (\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$.

2. Consider now that $(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}, \tilde{\mathbf{w}}, \tilde{e}) = \left(\mathbf{p}, \mathbf{q}, \mathbf{w}, \frac{1}{\eta} e\right)$ and that $\left(\tilde{\mathbf{T}}, \tilde{\mathbf{UT}}\right)$ satisfy

$$\tilde{T}_1 = T_1 + \left(\frac{\eta - 1}{\eta}\right) NFA_1 + \left(\frac{\eta - 1}{\eta}\right) \left(\theta_{12} \pi_1 - f_1^1\right),$$

$$\tilde{T}_2 = T_2 - (\eta - 1) \left(\theta_{12} \frac{\pi_1}{e} - \frac{f_1^1}{e}\right),$$

$$\tilde{UT}_1 = UT_1 + (\eta - 1) \left(\theta_{12} \frac{\pi_1}{e} - \frac{f_1^1}{e}\right),$$

and

$$\tilde{UT}_2 = -\tilde{UT}_1.$$

Then

$$\frac{\tilde{\pi}_1}{\pi_1} = \frac{\tilde{d}_1}{d_1} = \frac{\tilde{\pi}_2}{\pi_2} = \frac{\tilde{d}_2}{d_2} = 1.$$

This implies that $(\mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$ solves the problem of the firms. With

$$\tilde{T}_1 = T_1 + \left(\frac{\eta - 1}{\eta}\right) NFA_1 + \left(\frac{\eta - 1}{\eta}\right)\left(\theta_{12}\pi_1 - f_1^1\right)$$

and

$$\tilde{T}_2 = T_2 - (\eta - 1)\left(\theta_{12}\frac{\pi_1}{e} - \frac{f_1^1}{e}\right)$$

the budget constraints of the households are unchanged, as $f_1^1 = -f_2^1$. Therefore $(\mathbf{c}, \mathbf{n})$ solves the problem of the households. With

$$\tilde{UT}_1 = UT_1 + (\eta - 1)\left(\theta_{12}\frac{\pi_1}{e} - \frac{f_1^1}{e}\right)$$

and

$$\tilde{UT}_2 = -\tilde{UT}_1,$$

the balance-of-payments conditions are also satisfied. Finally, the allocation also satisfies the market-clearing conditions. Thus $(\tilde{\mathbf{c}}, \tilde{\mathbf{n}}, \tilde{\mathbf{y}}, \tilde{\mathbf{z}}, \tilde{\mathbf{x}}) = (\mathbf{c}, \mathbf{n}, \mathbf{y}, \mathbf{z}, \mathbf{x})$.

□

C Ramsey Problem

In this section, I solve the problem of a planner who is subject to financing exogenous government expenditures through the distorting linear taxes described in Section 4.

In this Appendix, I assume that there are transfers between governments. Together with the fact that technologies exhibit constant returns to scale and that profits are fully taxed, this possibility ensures that the conditions for the productive efficiency theorem of Diamond and Mirrlees (1971) are satisfied.

There are then two balance-of-payments conditions:

$$[q_{21}(2)z_{21}(2) - q_{12}(1)z_{12}(1)] + [p_{21}(2)y_{21}(2) - p_{12}(1)y_{12}(1)] = UT_1. \tag{39}$$

and

$$UT_1 + UT_2 = 0. \tag{40}$$

Suppose also that the utility functions u^i can be written as

$$u^i(\mathbf{c}_i, n_i) = u^i\left[v^i\left(h^i(\mathbf{c}_i)\right), n_i\right], \tag{41}$$

where $v_h^i > 0$ and h^i is linear homogeneous. These preferences are homothetic in the consumption goods and weakly separable in consumption and labor. These are conditions for the optimality of uniform commodity taxation, as shown by Atkinson and Stiglitz (1972).

As in Chari and Kehoe (1999), I follow the primal approach to the second-best problem. This consists of optimally choosing implementable allocations instead of after-tax prices. Any implementable allocation must satisfy

$$\frac{u_i^i}{u_j^i} = \left[\frac{1+t_i^c(i)}{1+t_i^c(j)}\right]\frac{q_{ii}(i)}{q_{ii}(j)}, \; i \neq j = 1,2, \tag{42}$$

$$-\frac{u_n^i}{u_i^i} = \left[\frac{1+s_i^n}{1+t_i^c(i)}\right]\frac{w_i}{q_{ii}(i)}, \; i \neq j = 1,2, \tag{43}$$

$$\sum_{l=1}^{2}\left[1+t_i^c(l)\right] q_{ii}(l)\, c_i(l) = \left(1+s_i^n\right) w_i n_i, \; i \neq j, \tag{44}$$

$$\frac{F_i^i}{F_j^i} = \left[\frac{1+s_i^y(i)}{1+s_i^y(j)}\right]\frac{p_{ii}(i)}{p_{ii}(j)}, \; i \neq j = 1,2, \tag{45}$$

$$\frac{F_n^i}{F_i^i} = \left[\frac{1+t_i^n}{1+s_i^y(i)}\right]\frac{w_i}{p_{ii}(i)}, \; i = 1,2, \tag{46}$$

$$\frac{G_{z(i)}^i}{G_{z(j)}^i} = \left[\frac{1+s_i^z(i)}{1+s_i^z(j)}\right]\frac{q_{ii}(i)}{q_{ii}(j)}, \; i \neq j = 1,2, \tag{47}$$

$$\frac{G_{x(i)}^i}{G_{x(j)}^i} = \left[\frac{1+t_i^z(i)}{1+t_i^z(j)}\right]\frac{p_{ii}(i)}{p_{ii}(j)}, \; i \neq j = 1,2, \tag{48}$$

$$\frac{G_{z(i)}^i}{G_{x(i)}^i} = \left[\frac{1+s_i^z(i)}{1+t_i^z(i)}\right]\frac{q_{ii}(i)}{p_{ii}(i)}, \; i = 1,2. \tag{49}$$

$$\left[\frac{1+s_i^z(i)}{1+\sigma_i^z(i)}\right] q_{ii}(i) = \left[\frac{1+t_j^c(i)}{1+\tau_j^c(i)}\right] q_{jj}(i), \; i \neq j = 1,2, \tag{50}$$

$$\left[\frac{1+s_i^y(i)}{1+\sigma_i^y(i)}\right]p_{ii}(i)=\left[\frac{1+t_j^z(i)}{1+\tau_j^z(i)}\right]p_{jj}(i),\ i\neq j=1,2, \tag{51}$$

$$\sum_{i=1}^{2}\left[c_i(l)+g_i(l)\right]=\sum_{i=1}^{2}z_i(l),\ l=1,2, \tag{52}$$

$$\sum_{i=1}^{2}x_i(m)=\sum_{i=1}^{2}y_i(m),\ m=1,2, \tag{53}$$

$$F^i(\mathbf{y}_i,-n_i,-k_i)=0,\ i=1,2, \tag{54}$$

and

$$G^i(\mathbf{z}_i,-\mathbf{x}_i)=0,\ i=1,2. \tag{55}$$

These conditions are also sufficient to describe a competitive equilibrium allocation because international prices can always be chosen to satisfy the arbitrage conditions of the problem of each agent and unilateral transfers can be chosen to satisfy the balance-of-payments conditions. The following Lemma further simplifies this set of equations.

Lemma 1. *Any allocation* $(\mathbf{c},\mathbf{n},\mathbf{y},\mathbf{z},\mathbf{x})$ *that is implementable as a competitive equilibrium through a choice of prices and policies satisfies equations (52), (53) and the implementability conditions*

$$\sum_{l=1}^{2}u_l^i c_i(l)+u_n^i n_i=0,\ i=1,2 \tag{56}$$

together with (54) and (55) for $i\neq j=1,2$. *Conversely, any allocation* $(\mathbf{c},\mathbf{n},\mathbf{y},\mathbf{z},\mathbf{x})$ *that satisfies those equations is implementable as a competitive equilibrium.*

Proof. To show that these equations are necessary, use the first-order conditions (42) and (43) to substitute the after-tax price of final good 2 and the after-tax wage rate in the budget constraint of each household. To show that they are sufficient, I note that all other equations can be satisfied by an appropriate choice of prices and taxes. Namely, (42) determines $q_{ii}(i)$, (45) determines $p_{ii}(i)$ and (46) determines w_i. Arbitrage conditions (50) and (51) determine $q_{jj}(i)$ and $p_{jj}(i)$ respectively. (43) determines s_i^n, and (47) determines $s_i^z(j)$. (49) determines $t_i^z(i)$, while (48) determines $t_i^z(j)$. (44), (52), (53) and (54) and (55) are imposed. □

The problem of the Ramsey planner is to choose an allocation $(\mathbf{c},\mathbf{n},\mathbf{y},\mathbf{z},\mathbf{x})$ to maximize the social welfare function (6) subject to the constraints (52) to (55) and the implementability conditions (56). Let φ_i denote the Lagrange multiplier associated with the implementability condition of country i.

The conditions that characterize the Ramsey allocation are

$$\frac{u_1^1}{u_2^1} = \frac{G_{z(1)}^1}{G_{z(2)}^1},$$

$$\frac{u_n^i}{u_1^i}\left[\frac{\omega_i + \varphi_i\left(1 + \frac{u_{vn}^i}{u_n^i} v_h^i h^i + \frac{u_{nn}^i}{u_n^i} n_i\right)}{\omega_i + \varphi_i\left(1 + A^i + \frac{u_{vn}^i}{u_v^i} n_i\right)}\right] = -\frac{G_{y(1)}^i}{G_{z(1)}^i}\frac{F_n^i}{F_1^i}, i = 1, 2,$$

$$\frac{G_{x(1)}^1}{G_{x(2)}^1} = \frac{F_1^1}{F_2^1},$$

$$\frac{u_1^1}{u_2^1} = \frac{u_1^2}{u_2^2},$$

$$\frac{G_{z(1)}^1}{G_{z(2)}^1} = \frac{G_{z(1)}^2}{G_{z(2)}^2},$$

$$\frac{F_1^1}{F_2^1} = \frac{F_1^2}{F_2^2},$$

$$\frac{G_{x(1)}^1}{G_{x(2)}^1} = \frac{G_{x(1)}^2}{G_{x(2)}^2},$$

$$\frac{G_{x(1)}^1}{G_{z(1)}^1} = \frac{G_{x(1)}^2}{G_{z(1)}^2},$$

together with conditions (52) to (56), where

$$A^i \equiv \frac{u_{11}^i}{u_1^i} c_i(1) + \frac{u_{12}^i}{u_1^i} c_i(2) = \frac{u_{21}^i}{u_2^i} c_i(1) + \frac{u_{22}^i}{u_2^i} c_i(2)$$

This is summed up in to the following proposition.

Proposition 1. *The Ramsey allocation satisfies the productive efficiency conditions (11)-(14), as well as the consumption efficiency condition (10). There is only one tax wedge on condition (8), the labor margin.*

Proof. The marginal conditions that characterize the second-best allocation are

$$\frac{u_1^1\left\{\omega_1+\varphi_1\left(1+\frac{u_{11}^1}{u_1^1}c_1(1)+\frac{u_{21}^1}{u_1^1}c_1(2)+\frac{u_{n1}^1}{u_1^1}n_1\right)\right\}}{u_2^1\left\{\omega_1+\varphi_1\left(1+\frac{u_{22}^1}{u_2^1}c_1(2)+\frac{u_{12}^1}{u_2^1}c_1(1)+\frac{u_{n2}^1}{u_2^1}n_1\right)\right\}}=\frac{G_{z(1)}^1}{G_{z(2)}^1},$$

$$\frac{u_n^i\left\{\omega_i+\varphi_i\left(1+\frac{u_{nn}^i}{u_n^i}n_i+\frac{u_{1n}^i}{u_n^i}c_i(1)+\frac{u_{2n}^i}{u_n^i}c_i(2)\right)\right\}}{u_1^i\left\{\omega_i+\varphi_i\left(1+\frac{u_{11}^i}{u_1^i}c_i(1)+\frac{u_{21}^i}{u_1^i}c_i(2)+\frac{u_{n1}^i}{u_1^i}n_i\right)\right\}}=-\frac{G_{y(1)}^i}{G_{z(1)}^i}\frac{F_n^i}{F_1^i},\ i=1,2,$$

$$\frac{G_{x(1)}^1}{G_{x(2)}^1}=\frac{F_1^1}{F_2^1},$$

$$\frac{u_1^1\left\{\omega_1+\varphi_1\left(1+\frac{u_{11}^1}{u_1^1}c_1(1)+\frac{u_{21}^1}{u_1^1}c_1(2)+\frac{u_{n1}^1}{u_1^1}n_1\right)\right\}}{u_2^1\left\{\omega_1+\varphi_1\left(1+\frac{u_{22}^1}{u_2^1}c_1(2)+\frac{u_{12}^1}{u_2^1}c_1(1)+\frac{u_{n2}^1}{u_2^1}n_1\right)\right\}}=\frac{u_1^2\left\{\omega_2+\varphi_2\left(1+\frac{u_{11}^2}{u_1^2}c_2(1)+\frac{u_{21}^2}{u_1^2}c_2(2)+\frac{u_{n1}^2}{u_1^2}n_2\right)\right\}}{u_2^2\left\{\omega_2+\varphi_2\left(1+\frac{u_{22}^2}{u_2^2}c_2(2)+\frac{u_{12}^2}{u_2^2}c_2(1)+\frac{u_{n2}^2}{u_2^2}n_2\right)\right\}},$$

$$\frac{G_{z(1)}^1}{G_{z(2)}^1}=\frac{G_{z(1)}^2}{G_{z(2)}^2},$$

$$\frac{F_1^1}{F_2^1}=\frac{F_1^2}{F_2^2},$$

$$\frac{G_{x(1)}^1}{G_{x(2)}^1}=\frac{G_{x(1)}^2}{G_{x(2)}^2},$$

and

$$\frac{G_{x(1)}^1}{G_{z(1)}^1}=\frac{G_{x(1)}^2}{G_{z(1)}^2}.$$

Homotheticity implies that,

$$\frac{u_{11}^i}{u_1^i}c_i(1)+\frac{u_{12}^i}{u_1^i}c_i(2)=\frac{u_{21}^i}{u_2^i}c_i(1)+\frac{u_{22}^i}{u_2^i}c_i(2)\equiv A^i,$$

for $i=1,2$. From separability, for $i,l=1,2$,

$$u_l^i=u_v^i v_l^i$$

and

$$u^i_{ln} = u^i_{vn} v^i_l.$$

Therefore,

$$\frac{u^i_{nl}}{u^i_l} n_i = \frac{u^i_{vn} v^i_l}{u^i_v v^i_l} n_i = \frac{u^i_{vn}}{u^i_v} n_i.$$

It follows that

$$\frac{u^1_1}{u^1_2} = \frac{G^1_{z(1)}}{G^1_{z(2)}},$$

and

$$\frac{u^1_1}{u^1_2} = \frac{u^2_1}{u^2_2}.$$

The optimal tax wedges on the labor margin become

$$\frac{\omega_i + \varphi_i \left(1 + \frac{u^i_{vn} v^i_1}{u^i_n} c_i(1) + \frac{u^i_{vn} v^i_2}{u^i_n} c_i(2) + \frac{u^i_{nn}}{u^i_n} n_i\right)}{\omega_i + \varphi_i \left(1 + A^i + \frac{u^i_{vn}}{u^i_v} n_i\right)} = \frac{\omega_i + \varphi_i \left\{1 + \frac{u^i_{vn}}{u^i_n} \left[v^i_1 c_i(1) + v^i_2 c_i(2)\right] + \frac{u^i_{nn}}{u^i_n} n_i\right\}}{\omega_i + \varphi_i \left(1 + A^i + \frac{u^i_{vn}}{u^i_v} n_i\right)}.$$

Since v^i is an increasing function of a linear homogeneous function $h^i(c_i(1), c_i(2))$,

$$v^i_1 c_i(1) + v^i_2 c_i(2) = v^i_h h^i_1 c_i(1) + v^i_h h^i_2 c_i(2) = v^i_h h^i,$$

and

$$\frac{u^i_n}{u^i_1} \left[\frac{\omega_i + \varphi_i \left(1 + \frac{u^i_{vn}}{u^i_n} v^i_h h^i + \frac{u^i_{nn}}{u^i_n} n_i\right)}{\omega_i + \varphi_i \left(1 + A^i + \frac{u^i_{vn}}{u^i_v} n_i\right)}\right] = -\frac{G^i_{y(1)}}{G^i_{z(1)}} \frac{F^i_n}{F^i_1}.$$

□

Note that a uniform VAT and a destination-based sales tax can implement the second-best allocation with

$$1 - \tau^v_i = 1 - \tau^s_i = \frac{\omega_i + \varphi_i \left(1 + \frac{u^i_{vn}}{u^i_n} v^i_h h^i + \frac{u^i_{nn}}{u^i_n} n_i\right)}{\omega_i + \varphi_i \left(1 + A^i + \frac{u^i_{vn}}{u^i_v} n_i\right)}$$

for $i = 1, 2$. An origin-based sales tax cannot implement the second best allocation because it imposes a tax wedge on the condition

$$\frac{G^1_{x(1)}}{G^1_{z(1)}} = \frac{G^2_{x(1)}}{G^2_{z(1)}}.$$