Evaluating Financial Distress in Portuguese Firms: Revisiting Altman's Z-Score Model

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Dissertation submitted in partial fulfilment of requirements for the MSc in Finance, at the Universidade Católica Portuguesa, September 12th, 2024.

Abstract

This thesis significantly advances the field of financial distress prediction in Portuguese companies by meticulously refining Altman's seminal Z-Score model (1983). With strategic updates to the model's parameters using the latest financial data, this research not only improves predictive accuracy but fundamentally transforms the tool to meet the contemporary needs of Portugal's dynamic economy. By transitioning from multiple discriminant analysis to logistic regression, the study introduces a robust methodological enhancement that substantially increases the model's predictive precision. Furthermore, the integration of macroeconomic indicators such as GDP has revolutionized its predictive capabilities, proving indispensable in today's interconnected financial landscape.

However, the research also unveils limitations; elements such as year dummies, company size, age, and sector-specific factors did not markedly influence the model's effectiveness, prompting a revaluation of traditional assumptions in distress prediction. This thorough analysis of diverse firm characteristics emphasizes the critical need for financial models that are specifically adapted to the distinct economic features of the Portuguese market.

These insights offer invaluable guidance for financial institutions, investors, and policymakers, significantly enhancing the utility and application of distress prediction models across diverse economic environments. Ultimately, the refined model does not just promise better risk management—it guarantees more informed, strategic decision-making for stakeholders within the SME-dominated Portuguese market.

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Keywords: Financial Distress, Z-Score Model, Portuguese Companies, Logistic Regression, Risk Management, Macroeconomic Factors, Industry-Specific Variables, Predictive Accuracy, SMEs (Small and Medium Enterprises), Financial Modelling

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Sumário

Esta tese avança significativamente no campo da previsão de dificuldades financeiras em empresas portuguesas, através de uma meticulosa refinaria do seminal modelo Z-Score de Altman (1983). Com atualizações estratégicas aos parâmetros do modelo utilizando os dados financeiros mais recentes, esta pesquisa não só melhora a precisão preditiva, mas também transforma fundamentalmente a ferramenta para atender às necessidades contemporâneas da dinâmica economia de Portugal. Ao fazer a transição da análise discriminante múltipla para a regressão logística, o estudo introduz uma robusta melhoria metodológica que aumenta substancialmente a precisão preditiva do modelo. Além disso, a integração de indicadores macroeconómicos, como o PIB, revolucionou as capacidades preditivas, provando ser indispensável na paisagem financeira interconectada de hoje.

Contudo, a pesquisa também revela limitações; elementos como dummies anuais, tamanho da empresa, idade e fatores específicos do setor não influenciaram de forma marcante a eficácia do modelo, o que motiva uma reavaliação das suposições tradicionais na previsão de dificuldades. Esta análise minuciosa das diversas características empresariais enfatiza a necessidade crítica de modelos financeiros que sejam especificamente adaptados às características económicas distintas do mercado português.

Estes insights oferecem orientações inestimáveis para instituições financeiras, investidores e formuladores de políticas, melhorando significativamente a utilidade e aplicação de modelos de previsão de dificuldades em ambientes económicos diversos. Em última análise, o modelo refinado não promete apenas uma melhor gestão de riscos — garante uma tomada de decisão mais informada e estratégica para os intervenientes no mercado dominado pelas PME em Portugal.

Palavras-chave: Dificuldades financeiras, Modelo Z-Score, Empresas portuguesas, Regressão logística, Gestão de riscos, Fatores macroeconómicos, Variáveis específicas do sector, Precisão preditiva, PME (Pequenas e Médias Empresas), Modelagem financeira

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List of Abbreviations

Abbreviations	Definition
AUC	Area Under the Curve
EBITTA	Earnings Before Interest and Taxes (EBIT) / Total Assets
FDL	Financial Distress Likelihood
GDP	Gross Domestic Product
ICR	Interest Coverage Ratio
LR	Logistic Regression
LRA	Logistic Regression Analysis
MDA	Multiple Discriminant Analysis
RETA	Retained Earnings / Total Assets
ROC	Receiver Operating Characteristics
SME's	Small and Medium-sized Enterprises
SMOTE	Synthetic Minority Over-sampling Technique
WCTA	Working Capital / Total Assets

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Acknowledgements

I would like to extend my sincere thanks to the professors of Católica’s MSc in Finance program. Your guidance and expertise have been instrumental in helping me complete this thesis. The knowledge and skills you’ve provided have been crucial to my academic development, and I am deeply appreciative of your support.

I am particularly grateful to my supervisor, Professor Ricardo Reis. Your thoughtful feedback and encouragement have been invaluable throughout this research process. Your support has kept me motivated and focused, and I truly appreciate all the time and effort you have dedicated to assisting me.

I also wish to thank my friends and family for their unwavering support. Your encouragement and belief in me have been a constant source of strength. I am grateful for your patience and understanding during this journey.

Thank you all for your significant contributions to this project. Your support has been essential, and I am genuinely thankful for each one of you.

1. Introduction

Research on predicting financial distress has been a crucial topic of study in finance, playing a fundamental role in efficient risk management, well-informed investment choices, and thorough credit assessment. Since its inception in 1968, Altman's Z-Score model has been a very influential and extensively used instrument for predicting organizational insolvency. The model's primary advantage remains in its capacity to consolidate several financial parameters into a unified prediction score, rendering it essential for financial institutions, investors, and policymakers aiming to forecast financial challenges and reduce related risks. Throughout the years, Altman's Z-Score model has been widely modified and improved to increase its usefulness and precision in many sectors, economic situations, and geographical areas.

Portugal exhibits a distinctive economic environment, distinguished by its dependence on small and medium-sized enterprises (SMEs), which make up 99.9% of the nation's enterprises. Not only are these crucial to the national economy in terms of employment and Gross Domestic Product (GDP) contribution, but they also encounter substantial obstacles that set them apart from larger firms. The constraints encompass restricted availability of financial resources, a heavy reliance on debt finance, and heightened susceptibility to both domestic and global economic volatility. The economic downturn caused by the COVID-19 pandemic has highlighted these weaknesses, exposing the fragility of numerous Portuguese companies and the insufficiencies of conventional financial distress models in forecasting failures in such circumstances. Given the difficulties at hand, it is imperative to reexamine and modify existing financial distress models, such as the Z-Score, to accommodate the unique requirements and circumstances of the Portuguese market.

Notwithstanding its historical achievements and extensive adoption, Altman's Z-Score model was initially formulated inside a distinct economic framework, predominantly centred on manufacturing enterprises in the United States throughout the mid-20th century. Despite its robustness, the model may not comprehensively consider the distinctive features of the Portuguese economy and the economic challenges encountered by these smaller organizations. This poses a crucial research inquiry: Is it possible to enhance the predictive accuracy of the Z-Score model in identifying financial difficulties in Portuguese companies, namely by integrating variables and factors that accurately represent the local economic conditions?

The primary aim of this study is to methodically evaluate the effectiveness of the Z-Score model in the Portuguese setting by recalculating its coefficients using up-to-date financial data from Portuguese companies. This study also seeks to ascertain if the incorporation of dummy variables, such as those that indicate business size, age, and industry, can augment the predictive capability of the model. Furthermore, this research aims to compare the modified models with the ones introduced by Altman in his 2016 paper, which will be used as a standard benchmark in this dissertation. Altman's work serves as a crucial benchmark, directing the process of adaptation and facilitating a comparative examination of outcomes, therefore affording a more profound comprehension of the model's suitability and constraints when applied in other economic settings.

This study's methodological approach is supported by an extensive dataset obtained from the SABI database, which contains precise financial information on Portuguese industrial companies. The work utilizes sophisticated statistical methods, such as logistic regression analysis (LRA) and multiple discriminant analysis (MDA), to examine the specific assumptions concerning the adaption of the model. These hypotheses investigate the possible enhancements in the accuracy of the model by using supplementary variables such as organizational size, age, industry-specific characteristics, GDP growth, and the Interest Coverage Ratio (ICR). The evaluation of these models is conducted with great rigor using the Area Under the Curve (AUC) derived from Receiver Operating Characteristics (ROC) curves, which guarantees a reliable assessment of their classification accuracy.

This academic research contributes to a deeper understanding of financial model adaptation, ensuring their ongoing utility in changing economic environments. It sets a precedent for tailoring established models to specific market conditions, potentially improving financial risk assessment worldwide.

The dissertation's structure guides readers through the research journey. It begins by introducing financial distress prediction and its significance. A literature review follows, exploring Altman's Z-Score model and its adaptations across economic landscapes. The methodology section outlines research hypotheses, data selection, and statistical approaches, incorporating dummy variables and macroeconomic indicators. Empirical analysis compares adapted models to Altman's 2016 study, evaluating their effectiveness in Portugal. The discussion interprets findings and their implications for financial prediction. The conclusion summarizes key discoveries, highlights contributions, and proposes future research directions.

This organized approach thoroughly examines each research aspect, from theoretical foundations to practical applications, aiming to refine the Z-Score model for Portugal's economic context.

2. Literature Review

2.1 Financial Distress Prediction Models

Altman's (1968) creation of the initial multivariate bankruptcy prediction model was an important achievement in the realm of financial distress prediction, quickly becoming an essential instrument for experts in several sectors such as finance, banking, and credit risk management. Financial institutions have extensively embraced these models and depend on them to reduce non-performing loans and efficiently handle the risks associated with loan defaults. Capital adequacy standards outlined in frameworks like the Basel Accords further emphasize the significance of precise prediction models.

Altman's Z-Score model not only functioned as a prototype for subsequent prediction models, but it has also become indispensable to the decision-making processes of asset managers and investors. These models serve to reduce the risks linked to poorly performing assets, providing both a protective measure and the possibility of generating substantial profits, especially through tactics like short selling. Moreover, credit rating organizations utilize these models to evaluate the likelihood of default for different businesses and instruments. Furthermore, Altman (1983) proposed that financially troubled companies might employ the Z-Score model as a strategic instrument for achieving financial recovery.

The approaches for predicting bankruptcy have experienced substantial change throughout time. Beaver's (1966, 1968) groundbreaking research in univariate analysis established the basis by distinguishing financial ratios that have robust forecasting powers. Further expanding on this concept, Altman (1968) proposed the multiple discriminant analysis (MDA) model, which is well recognized as the Z-Score. Subsequently, the field has witnessed more progress with the implementation of models such as Ohlson's logit model (1980), Taffler's adaptation of the Z-Score (1984) for the UK market, and Zmijewski's (1984) probit model. These models have all contributed to the continuous improvement and broadening of methodology for predicting financial distress.

2.2 Evolution of Altman's Z-Score Models

First presented in the 1960s, Altman's groundbreaking Z-Score model established the basis for multivariate bankruptcy forecasting. This preliminary model was constructed using a sample of 66 companies, evenly split between those that were insolvent and those that were not. The financially distressed companies, predominantly manufacturers, have declared bankruptcy under Chapter X of the National Bankruptcy Act from 1946 to 1965. With an average asset value of 6.4 million USD, reflecting the relatively small scale typical of that era. To achieve comparison, Altman meticulously chose non-bankrupt companies that were similar in terms of industry and size, albeit having a somewhat bigger asset size than the group of bankrupt firms. The financial data for these companies were sourced from a single reporting period prior to bankruptcy for the bankrupt company, therefore assuring similarity of the data across both groups.

Across domains including liquidity, profitability, leverage, solvency, and operational efficiency, Altman assessed 22 financial ratios. The Z-Score model was developed based on the identification of five fundamental ratios that served as the foundation of the discriminant function. The equation representing the final Z-Score is:

$$Z = 0.012 * X1 + 0.014 * X2 + 0.033 * X3 + 0.006 * X4 + 0.999 * X5 \quad (1)$$

Where:

X1 = Working Capital / Total Assets
X2 = Retained Earnings / Total Assets
X3 = Earnings Before Interest and Taxes (EBIT) / Total Assets
X4 = Market Value of Equity / Total Liabilities
X5 = Sales / Total Assets
Z = Overall Index

In response to the constraints of the initial Z-Score model for non-public companies, Altman suggested modifications, leading to the development of the Z'-Score model. This iteration substituted the market value of equity with the book value of equity, therefore enhancing its applicability to privately held enterprises. The updated equation for the Z'-Score model is as follows:

$$Z' = 0.717 * X1 + 0.847 * X2 + 3.107 * X3 + 0.420 * X4 + 0.998 * X5 \quad (2)$$

Due to the absence of an extensive database for private companies, Altman did not verify the accuracy of the Z'-Score model using a secondary sample. In order to broaden the model's relevance to many sectors, he devised the Z"-Score model, which omits the Sales/Total Assets ratio (X5) due to its susceptibility to industry-specific influences. The following equation with four variables creates the Z"-Score model:

$$Z'' = 3.25 + 6.56 * X1 + 3.26 * X2 + 6.72 * X3 + 1.05 * X4 \quad (3)$$

Within this improved model, the EBIT/Total Assets ratio (X3) continues to be the most effective differentiator between financially troubled and financially stable companies. The versatility of the Z"-Score model enables its application to non-manufacturing firms as well as both private and public corporate entities, extending beyond manufacturing firms. For instance, Altman et al. (2000) effectively implemented the Z"-Score model on a wide range of privately owned companies in different sectors, showing its robust ability to forecast outcomes in non-manufacturing environments. These enhancements have stimulated additional investigation into the efficacy of the model in many settings, including those with distinct economic difficulties, such as Portugal.

2.3 Comparative Studies

The Z-Score model, established by Altman in 1983, remains a resilient and adaptable instrument for anticipating financial crises, undergoing multiple revisions to retain its usefulness in varied situations, industries, and nations. The significance of reestimating the coefficients of the model to accommodate evolving financial environments has been substantiated by Grice and Ingram (2001) and further reinforced by Altman et al. (2016). These researchers underlined that updating coefficients with the most recent data considerably boosts the model's accuracy, displaying its flexibility to developing economic situations.

One notable breakthrough was made by De Luca and Meschieri in 2017, who changed the Z-Score model by substituting the ratio of working capital to total assets with the ratio of cash and cash equivalents to current liabilities. This modification significantly enhanced the model's ability to produce accurate predictions in Troubled Debt Restructuring situations by 15%. Additionally, Kwak et al. (2005) and Merkevicius et al. (2006) employed sophisticated statistical techniques, such as Multiple Criteria Linear Programming and hybrid artificial neural networks, enhancing the model's accuracy by 12% and achieving an impressive predictive accuracy of 92.35%, respectively.

The full review by Altman and colleagues in 2016 utilizing LRA on a worldwide dataset, which encompassed varied sectors and economic situations, demonstrated the relevance of industry-specific characteristics. These improvements enhanced the model's predictive power by around 10-15%, even in non-manufacturing areas. Further study showed the impact of variables such as the year of bankruptcy and firm size, with older and larger organizations displaying more persistent financial distress patterns. ROC curve analysis was employed to evaluate the re-estimated model's performance, revealing its higher precision across multiple circumstances.

Xu and Zhang (2009) stressed the relevance of incorporating country-specific elements, citing a possible 20% improvement in predicting accuracy when local economic and corporate governance factors are added, matching Altman's findings.

A model specifically designed for U.K. listed companies, integrating accounting, market, and macroeconomic data, was created by Tinoco and Wilson (2013). This model demonstrated an effectiveness rate of 81%. The incorporation of sector-specific variables resulted in a 6% enhancement in prediction accuracy. Also, Pindado et al. (2008) established the Financial Distress Likelihood (FDL) model, which was compared to a re-estimated Z-Score and showed a 10% increase in its ability to categorize situations properly.

In conclusion, the continual revisions and improvements to the Z-Score model emphasize the need for continued adjustment to preserve its pertinence and accuracy in various settings. These adjustments are particularly advantageous in the peculiar economic setting of Portuguese SMEs, where modified predictive models may greatly enhance financial distress prediction accuracy.

2.4 Portuguese Economic Context

Portugal's economy is a complex mix of sectors, with SMEs making up 99.9% of all economic enterprises (PORDATA, 2023). These SMEs are crucial to the economy, contributing significantly to employment and GDP. However, the wider Portuguese business landscape faces unique challenges, such as limited financial reserves and less varied income sources, particularly in sectors like tourism and retail (OECD, 2023).

Access to capital remains a significant obstacle for SMEs, with cautious lending policies benefiting larger corporations. This limits their ability to allocate resources towards technology, expand activities, or penetrate global markets, impacting their competitiveness. Globalization further complicates the competitive environment for Portuguese enterprises, with even small local enterprises competing with larger international corporations. Digital transformation offers opportunities for improving operational effectiveness but also requires substantial investments in technology and skill cultivation (EIF, 2023)

Regulatory compliance remains a significant challenge for smaller firms, as it can impede entrepreneurial drive and innovation. External shocks, such as global economic crises or trade interruptions, provide significant vulnerability to Portuguese enterprises (OECD, 2023). The COVID-19 pandemic highlighted these vulnerabilities, emphasizing the need for enhanced resilience and adaptation within the Portuguese business community.

To address these challenges, the government has implemented measures aimed at fostering innovation, digitalization, and internationalization, including research and development incentives, access to European Union subsidies, and programs enhancing workforce digital skills. These measures are essential for enabling enterprises to compete globally and endure economic risks.

The Portuguese industry mix requires considering sector-specific elements and the wider economic and legislative environment when applying financial distress models.

2.5 Sector-Specific Considerations in Financial Distress Models in Portugal

The distinctive economic makeup of Portugal, characterized by major sectors such as tourism, agriculture, and technology, necessitates customized adjustments to conventional models. Almeida and Santos (2019) emphasized the importance of incorporating sector-specific variables, such as seasonal revenue patterns in the tourism industry and crop production variations in agriculture. Their results demonstrate that models including these factors perform better than conventional models that fail to consider industry disparities. An inherent drawback of sector-specific models is their possible lack of applicability to various sectors. Although carefully calibrated for certain sectors, these models may not exhibit the same level of performance in other businesses that have unique financial features or risk profiles.

To provide more evidence, Teixeira and Gomes (2020) employed generalized linear models to evaluate financial hardship in the Portuguese tourism industry, therefore illustrating its susceptibility to economic volatility. Their study emphasizes the need to include economic and seasonal factors to improve the precision of predictions in the tourism industry. Nevertheless, aligning with the conclusions of Almeida and Santos (2019), the dependence on past data presents a constraint, since it may not comprehensively encompass present or future economic dynamics, especially in swiftly evolving contexts.

Another notable study by Georgiev and Petrova (2015) evaluated the Z-Score model's accuracy in predicting bankruptcy in Portuguese industrial companies. The researchers used Altman's Z-Score model to analyse a subset of manufacturing companies, with a particular emphasis on financial statistics unique to the industry. The research findings indicated that the model effectively differentiated between financially troubled and financially stable companies. Nevertheless, they proposed that the inclusion of further factors, such as the duration of the production cycle and the level of capital investment, would improve the precision. Notwithstanding these enhancements, the concerns of data quality and availability continue to be crucial obstacles. Given the limited availability of complete market-based data in countries such as Portugal, it may still be essential to rely on accounting-based models like the Z-Score, which could potentially restrict the accuracy of predictions.

2.6 Application of the Z-score Model to Portuguese Companies'

Comparative studies have evaluated the Z-Score model against existing bankruptcy prediction models in the Portuguese market. Gordini's (2014) study found that while the Z-Score model showed satisfactory performance, models that integrated market-based data or logistic regression approaches sometimes surpassed it. However, the study also highlighted the limitations in generalizability due to the specific sample used, emphasizing the need to adjust the model to suit local economic circumstances and data availability.

In their 2015 study, Machado and Mata investigated how to make the Z-Score model more accurate by adding GDP and other macroeconomic factors. However, the model may not be practical for small and medium-sized businesses or institutions with limited analytical skills because it is more complicated. The strategy depends on the ongoing accessibility of real-time macroeconomic data, which may not always be possible under unpredictable economic conditions.

3. Research Hypothesis

This thesis aims to improve the accuracy of the Altman Z-Score model in forecasting financial difficulties, particularly in Portuguese companies. The unique economic structure of Portugal, with a significant presence of SMEs and a dependence on debt financing, poses challenges to conventional financial models' prediction precision. The study aims to enhance the model's applicability and precision by specifically examining the Portuguese environment, ensuring it accurately represents the complex financial behaviours and dangers inherent in the local market.

The study evaluates the performance of the original Z-Score model and investigates the influence of several variables on its predicted accuracy. These variables include company size, age, industry, GDP growth, and supplementary indicators of solvency such as the ICR. The research hypotheses aim to evaluate these adjustments and determine their influence on the overall performance of the model.

The results are expected to increase the continuous improvement of financial distress models, increasing their relevance not only in Portugal but also potentially providing valuable insights for other markets with similar economic attributes. The framework does more than just changing the model to fit the needs of a certain area. It also adds to the field of predicting financial distress by showing how predictive models need to be contextualized and constantly revised to stay useful and accurate in a changing economic landscape. The hypotheses examined are both technical and context-specific, offering practical insights into the financial well-being of Portuguese companies.

i) Hypothesis H1

Developed by Altman in 1968, the Z-Score model focuses on financial variables unique to companies as predictors of bankruptcy occurrence. Nevertheless, the existing historical data from the mid-20th century may not accurately represent the current financial environment. In 1980, Ohlson extended existing models for forecasting bankruptcy and suggested that enhancing these models with up-to-date data could refine their accuracy by incorporating changes in economic conditions and corporate activities over time. To ensure the continued relevance and accuracy of bankruptcy prediction, Altman (1983) suggests using the most recent data available.

The significance of recalibrating financial models to encompass current conditions was underscored in a study conducted by Grice and Ingram (2001). The researchers observed that such updates had the potential to greatly improve forecast performance. Furthermore, Beaver, McNichols, and Rhie (2005) showed that the ability of financial ratios to forecast outcomes changes depending on economic circumstances, emphasizing the importance of regularly re-estimating the model.

In more recent work, Altman and his colleagues (2016) revisited the Z-Score model and found that re-estimating the coefficients of the Z´´-Score model using more recent data significantly improved its classification accuracy. This improvement was evident in both domestic (U.S.) and international markets, where updated coefficients yielded better predictive results. These findings underscore the necessity of regularly updating the model's coefficients to ensure it accurately reflects contemporary financial environments.

Considering these factors, the first hypothesis (H1) posits that incorporating new data into the coefficients of the four original variables in the Z"-Score model will enhance its classification performance, particularly for Portuguese firms. The underlying premise of this hypothesis is that if economic conditions and business characteristics change, models for predicting financial distress must likewise adjust to retain their ability to forecast outcomes. Through hypothesis testing, this work seeks to illustrate the need of periodically re-estimating financial distress prediction models to maintain their accuracy and relevance in various markets and time periods. Implementing this procedure is essential for capturing the ever-changing characteristics of financial environments and enhancing the model's capacity to precisely categorize companies as either bankrupt or non-bankrupt.

H1: Re-estimating the coefficients of the Z´´-Score model improves its classification accuracy.

We will test this hypothesis by revising the coefficients of the Z"-Score model using recent financial data and evaluating the performance of the re-estimated model in comparison to the original. A ROC curve will be used to evaluate the performance of the re-estimated model to ascertain if new information can enhance the model's capacity to correctly categorize companies as either failed or non-failed.

ii) Hypothesis H2

Logistic regression analysis (LRA) boasts several significant advantages over multiple discriminant analysis (MDA) when it comes to bankruptcy prediction models. For instance, unlike MDA, LRA does not require three crucial assumptions: multinormality, homoscedasticity, or linearity. This flexibility is a game-changer for complex financial data, which often defies traditional statistical norms.

Previous studies, such as those conducted by Beaver et al. (2005), Bharath and Shumway (2008) and Altman et al. (2016), have consistently demonstrated LRA's superior accuracy in classifying bankrupt firms across diverse datasets. In contrast, MDA was well-suited for the original Z"-Score model's small sample, but it falters when faced with larger, more varied data.

This study sets out to test the hypothesis that a re-estimated Z"-Score using LRA will yield improved classification accuracy compared to MDA. To achieve this, we will compare the predictive power of both models using ROC curves and significance tests, with a particular focus on Portuguese companies. The goal is to determine whether LRA significantly outperforms MDA in financial distress forecasting, providing a valuable tool for businesses, investors, and policymakers seeking to navigate complex financial landscapes.

H2: The prediction accuracy of the logistic regression version of the Z′′-Score model is higher than that of the multiple discriminant analysis version.

iii) Hypothesis H3

The accurate prediction of bankruptcy rests on the shoulders of macroeconomic factors and business cycles. A company's solvency can swiftly change as economic fluctuations shift the fine line between stability and insolvency, rendering models ineffective across different years. Take, for example, the case of a company that was solvent during a period of economic boom but found itself on the brink of bankruptcy during a recession. Altman's Z-Score (1968) was groundbreaking in its time but did not explicitly account for the timing of bankruptcies.

Research by Bharath and Shumway (2008) and Campbell, Hilscher, and Szilagyi (2008) has further shown that incorporating macroeconomic indicators, including the year of bankruptcy, into predictive models can enhance their accuracy. These studies suggest that accounting for temporal variations in economic conditions allows the model to better capture the dynamics that influence a firm's financial stability, thereby improving the model's predictive performance.

Building on this foundation, our study introduces a benchmark Z"-Score LR model that is adapted to the nuances of recent years and various business cycle stages, ensuring its relevance in today's economic landscape. By incorporating the bankruptcy year as a key factor, we hypothesize that classification accuracy will improve, enabling more accurate predictions for Portuguese companies across diverse periods. To put this hypothesis to the test, we will compare the performance of year-specific logistic regression models that account for the bankruptcy year against the Benchmark model, to determine whether accounting for temporal economic variations yields more robust predictions.

H3: The model's prediction accuracy is higher when the effect of the year of bankruptcy is included.

iv) Hypothesis H4

In Portugal, SMEs face unique challenges, such as limited funding and economic vulnerability. The accuracy of financial distress prediction models for these businesses may depend on their size. Larger firms often have different financial traits and risks than smaller ones, affecting model performance.

The Z´´-Score model was built using data from firms with assets between 1 and 25 million USD. Thus, it may not reflect the financial realities of very small or very large firms. Altman (1983) cautioned against using this model universally across all firm sizes. He argued that adjusting the model for different sizes could improve its accuracy. This is because the criteria for bankruptcy differ between small and large firms.

To improve accuracy, this study suggests including firm size in the model. It plans to categorize firms as small, medium, or large. Then, it will assess the model's accuracy in each group. The hypothesis is that considering firm size will enhance prediction accuracy.

This idea builds on previous research by Almeida and Santos (2019) and Gordini (2014). They showed the need for tailored models and the benefits of including firm-specific traits like size.

To test this hypothesis, firm size will be added to a logistic regression model. This will allow the model to differentiate firms based on size. The aim is to better predict financial distress across all sizes. The adjusted model's performance will be compared to a standard model that ignores size. The goal is to see if factoring in size improves predictions, especially for Portuguese SMEs.

H4: The model's prediction accuracy is higher when the effect of firm size is included.

v) Hypothesis H5

Firm age is crucial for financial health, yet Altman's Z"-Score model overlooks this vital factor. Although the model's Retained Earnings to Total Assets ratio hints at age, it may not fully capture its impact on bankruptcy risk. For instance, a firm with a high Retained Earnings to Total Assets ratio might still be young and prone to financial troubles.

Studies have repeatedly underscored the significance of age in identifying financial distress. Ohlson's model demonstrated that younger firms are inherently riskier, mainly due to their limited financial history and lack of established relationships with suppliers and customers. Beaver et al. found that incorporating age into financial ratios improves their predictive power, allowing analysts to better identify potential problem areas. Campbell et al. noted that young firms are vulnerable due to poor financial management and unfavourable market positions, making them more susceptible to bankruptcy.

Altman et al.'s 2016 study reinforced the importance of age in assessing risk. By incorporating age into their model, they significantly improved its accuracy. This suggests that adding age to the Z´´-Score model could lead to more accurate predictions, especially when distinguishing between young and old firms. Our proposal is supported by a wealth of research, including studies by Altman et al. (2016), Ohlson (1980), Beaver et al. (2005), and Campbell et al. (2008).

In the context of Portuguese firms, directly considering age-related risks could be a game-changer. By acknowledging the unique challenges faced by young firms, we can develop more targeted strategies to mitigate their risk profile.

H5: The model's prediction accuracy is higher when the effect of firm age is explicitly included.

vi) Hypothesis H6

The original Z´´-Score model used data from manufacturing firms, limiting its broader use. Altman (1983) highlighted the need for models tailored to specific industries, noting different sectors have unique financial traits. For example, the Sales/Total Assets ratio (X5) is crucial in manufacturing but varies in other industries, impacting prediction accuracy.

Later research confirmed the industry's role in predicting financial trouble. Ohlson (1980) argued that industry traits and economic conditions are key. He suggested including these factors in models. Similarly, Beaver, McNichols, and Rhie (2005) found that industry-specific ratios improve prediction. This is especially true when considering different sectors' risks and economic patterns.

Bharath and Shumway (2008) also stressed industry factors in their models. They showed that some industries are more vulnerable to downturns. This highlighted the need for specific models. Meanwhile, Campbell, Hilscher, and Szilagyi (2008) pointed out that industry cycles and competition are crucial in distress, advocating for the inclusion of industry factors.

Based on these insights, our hypothesis suggests that adding industry classification to the Z´´-Score model will enhance its accuracy. By considering the unique traits and behaviours of firms in different sectors, we aim for better predictions of financial distress.

H6: The model’s prediction accuracy is higher when the effect of industry is explicitly included.

vii) Hypothesis H7

Macroeconomic conditions, especially GDP growth, are vital for a firm's health. The Z´´-Score model by Altman (1968) focused on financial ratios. Yet, Altman noted that economic conditions also matter. He suggested including macroeconomic factors in predicting financial distress.

Ohlson (1980) stressed adding GDP growth to models. He argued it significantly affects stability. Later research backed this. Beaver, McNichols, and Rhie (2005) found that adding GDP growth improved the predictive power of financial ratios. This highlights the benefit of combining firm-specific and economic factors.

Bharath and Shumway (2008) also included GDP growth in their models. They showed it aids in understanding a firm's environment. Similarly, Campbell, Hilscher, and Szilagyi (2008) studied the impact of macroeconomic factors on financial distress. They recommended including these factors to better assess firm viability.

Based on these findings, the hypothesis suggests enhancing the Z´´-Score model with GDP growth data. This adjustment aims to improve prediction accuracy, helping to better distinguish between stable and distressed firms.

H7: Including GDP growth improves the model's prediction accuracy.

viii) Hypothesis H8

Altman's original Z"-Score model (1983) focuses on liquidity, profitability, leverage, and operational efficiency. As financial landscapes evolve, especially in places like Portugal, we could improve these models by adding more measures. One such measure is the Interest Coverage Ratio (ICR). The ICR evaluates a firm's ability to cover interest payments with earnings. It's becoming crucial for assessing financial health, particularly for heavily indebted firms.

Bank loans dominate SME financing in Portugal, making debt repayment crucial. Financial vulnerability looms large for these firms, as highlighted by OECD and Banco de Portugal studies. Their reports underscore the precarious balance many small businesses must strike to stay afloat in a loan-dependent ecosystem. They point out high debt levels and a strong reliance on bank funding. The ICR is key here. It shows how well a firm can cover interest payments. This ratio helps identify firms at risk of financial trouble. Moreover, research by Pinho and Tavares (2017) highlights the factors influencing bank loans for SMEs in Portugal. It stresses that being able to service debt is crucial for financial stability.

Adding this variable to the Z"-Score model matches findings from Bharath and Shumway (2008) and Campbell, Hilscher, and Szilagyi (2008). They suggest including both macroeconomic and company-specific factors to better predict bankruptcy. In Portugal, where companies are sensitive to credit and economic changes, the ICR provides deeper insights into financial trouble. This is especially true when traditional ratios fail to reflect the risks of high debt.

Incorporating the ICR into the Z"-Score model may enhance its predictive accuracy for Portuguese firms' financial distress. This hypothesis, H8, posits improved classification performance with ICR inclusion. Testing involves re-estimating the model, adding ICR, and comparing results against the original Z"-Score. The analysis will determine if this additional metric bolsters the model's bankruptcy prediction capabilities, potentially offering a more robust financial health assessment tool for Portugal's economic landscape. Evaluation centers on the modified model's effectiveness in distinguishing between bankrupt and non-bankrupt entities.

H8: The model’s prediction accuracy is higher when the Interest Coverage Ratio is included.

4. Empirical Data and Statistical Methods

4.1 Sample of Firms

SABI's comprehensive database fuelled this study, offering rich financial data on Portuguese firms. Its wide coverage enabled a representative sample across legal forms, regions, and industries. Tailoring the approach to Portugal's SME-dominated economy, the sampling diverged from Altman's 2016 Z´´-Score model. Custom criteria ensured a robust, locally applicable financial distress prediction model. This strategy captured the nuances of Portugal's business landscape, yielding empirical data primed for constructing an accurate predictive framework.

Industrial firms took centre stage in this analysis, excluding financial institutions and non-industrial entities. This choice ensured comparable financial ratios, crucial for the Z''-Score model's application. The study focused on active, corporate businesses, mirroring the broader economic landscape. This approach aligns with financial distress modelling norms, acknowledging the distinct financial structures of industrial and non-industrial firms. Such differences necessitate tailored modelling strategies, making the industrial focus both practical and insightful for the study's objectives.

Limited liability firms formed the study's sole focus, omitting partnerships and sole proprietorships. This choice is restricted from their standardized financial reporting and distinct separation of personal and corporate funds. Such practices yield more dependable financial data, crucial for robust analysis. The exclusion of other business structures ensured consistency in data quality and interpretation across the research sample.

Small firms drive Portugal's economy, making their inclusion vital. Omitting the asset threshold used in the Z''-Score model (2016) ensures broad applicability. This approach captures diverse business sizes, enhancing the model's relevance. By embracing all firm sizes, the study reflects Portugal's unique economic landscape.

Fourth, the study focused exclusively on firms located in Portugal. This study aimed to develop a model tailored to the Portuguese market. Consequently, all qualifying firms from Portugal were included in the analysis. The study did not set a minimum number of failed firms as a criterion for inclusion, recognizing that the sample size would inherently be smaller due to the geographical and economic constraints of focusing on a single country.

The time span for the data collection was set from 2018 to 2022, covering a recent and relevant period for assessing financial distress in the context of contemporary economic conditions. For non-distressed firms, complete financial data were collected for all years within this period to facilitate longitudinal analysis. This approach allows for comprehensive trend analysis, capturing financial patterns that could precede financial distress. Longitudinal data plays a crucial role in modelling financial distress. Laitinen (2005) highlights its significance, noting that such data reveals trends preceding financial troubles. This approach allows analysts to track evolving patterns and anticipate potential crises more effectively. Comprehensive financial datasets yield more robust models by minimizing outlier effects. This enhances prediction stability and reliability - key factors in financial risk assessment and decision-making. As Sun et al. (2021) observe, such thorough data improves model consistency, bolstering accuracy for critical financial analyses and forecasts.

Financial distress signals demand careful analysis. This study focused on pre-bankruptcy data for troubled firms, aligning with Altman's 1968 approach. Companies labelled "Insolvência/Trâmites de Composição" and "inactive" were classified as distressed. The model's predictive power hinges on recognizing financial red flags before collapse. Beaver (1966) and Ohlson (1980) emphasized analysing financial decline preceding bankruptcy. By examining statements up to the bankruptcy year, this research captured the critical financial path to distress. This method ensures the model accurately reflects warning signs, avoiding post-bankruptcy data that could skew results.

After applying all the necessary filters, the final dataset comprises **93,060 firms**, of which **90,603** are non-failed and **2,457** are failed (as shown in Table 1). These filters ensured that only active, corporate businesses with complete financial data were included, providing a consistent and robust analysis.

Table 1- Final Sample

Firm's Classification	*Nº of Observations*
Non-Failed	90,603
Failed	2,457
Total	**93,060**

The dataset is predominantly composed of **SMEs**, representing more than **99%** of the sample (Figure 1), which aligns with the reality of the Portuguese economy (PORDATA, 2023).

By including firms of various sizes and sectors, the sample captures the diversity of the Portuguese economy, enhancing the model's ability to assess financial risks across different types of enterprises.

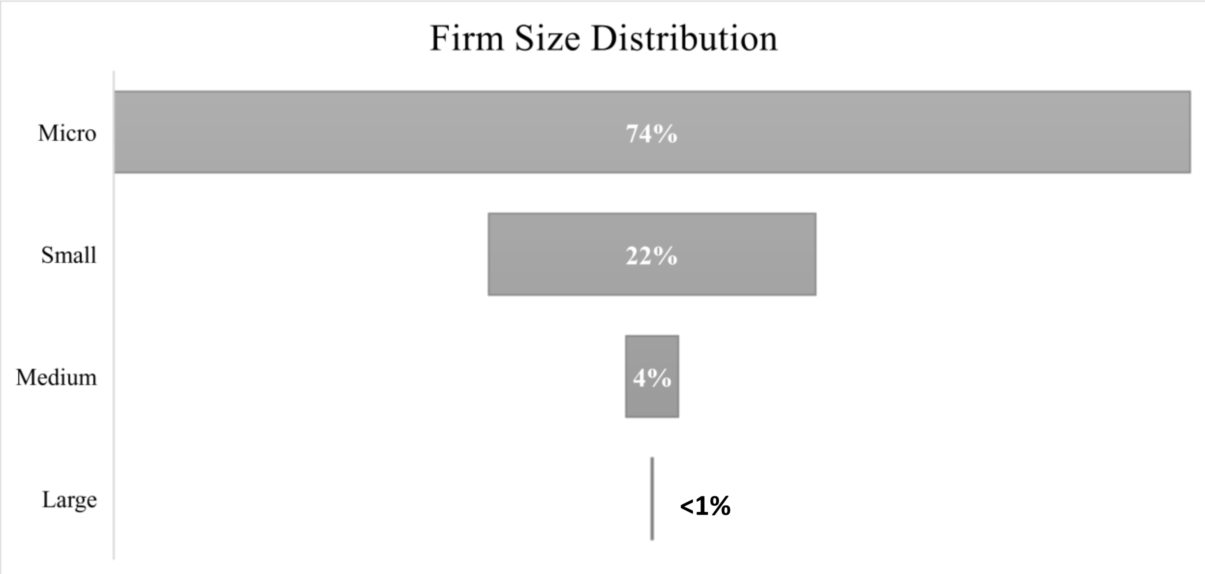

Figure 1- *This chart illustrates the distribution of firms in the sample based on size, categorized according to the European Union's classification criteria. The sample is predominantly composed of SME's, representing more than 99%.*

Splitting the dataset into estimation and test samples enhances the robustness of the model. This division prevents overfitting and validates performance on unseen data. Altman et al. (1977) assert this split assesses generalizability across contexts. Brown & Mues (2012) endorse a 70-30 ratio, balancing training needs with validation rigor. This approach minimizes overfitting risk while enabling robust evaluation of new data, ensuring the model's predictive power extends beyond its training set.

Data cleaning began with removing impossible and infinite values. Outliers were then addressed using winsorization at 1% and 99% levels, as per Altman (2016). This technique focuses the model on central data trends.

Class imbalance posed a challenge, with non-failed firms far outnumbering failed ones. The Synthetic Minority Over-Sampling Technique (SMOTE) addressed this by generating synthetic samples for failed firms, preserving majority class integrity. This balanced approach improved predictive accuracy without additional weighting in logistic regression.

Combining winsorization and SMOTE is a widely accepted approach in financial research, as it adheres to established best practices. By applying these methods, researchers can effectively mitigate the impact of outliers and balance classes, leading to enhanced model robustness and reliability. For instance, studies have shown that winsorization can reduce the influence of extreme values, while SMOTE generates synthetic samples to counter class imbalance. This integration of techniques has been endorsed by reputable researchers, including Chawla et al. (2002) and Fernández et al. (2018). As a result, the refined dataset is optimized for accurate and meaningful analysis, much like a precise navigational tool for charting financial trends.

The performance of each hypothesis-driven model was compared against a benchmark model, specifically the logistic regression model with re-estimated coefficients from Hypothesis 2 (LR Benchmark). This benchmark model serves as the standard for evaluating whether incorporating additional variables or adjusting existing ones improves predictive accuracy.

4.2 Statistical Methods

In this study, eight research hypotheses are drawn for statistical testing. The statistical analysis begins with calculating the original Z"-Score for the firms in the data, as in Equation 3. The classification performance of the original model is assessed by the AUC measure extracted from the ROC curve. AUC has a close connection with the Accuracy Ratio (AR) because AR = 2 AUC– 1. AR equals 0 for a random model, 1 for a perfect model, and 0.5 for a model with an average classification performance. Python was used for all statistical analyses.

The dataset was prepared following established procedures to ensure accuracy and reliability. This included the application of the SMOTE and the division of the data into training and test sets. These steps, detailed in the earlier data preparation section, were critical for addressing class imbalance and preventing overfitting.

The performance of each hypothesis-driven model was compared against the LR benchmark model, specifically the logistic regression model with re-estimated coefficients from Hypothesis 2. This benchmark, which uses less restrictive assumptions than multiple discriminant analysis (Hosmer and Lemeshow, 1989), serves as a standard for evaluating the impact of additional variables or adjustments on predictive accuracy.

The first hypothesis (**H1**) posits that the coefficients of the original Z´´-Score model may be outdated. To assess this, the coefficients were re-estimated using the original statistical method, the MDA. SMOTE was employed to balance the sample, ensuring that non-proportional sampling did not bias the re-estimated model. The performance of the re-estimated model was then evaluated by comparing its AUC from the ROC curve against the AUC of the model with the original coefficients. This comparison aimed to determine whether the re-estimated coefficients offer improved classification accuracy in predicting financial distress.

The second hypothesis (**H2**) tests whether the classification performance of the re-estimated Z”-Score model improves when it is re-estimated using logistic regression analysis (LRA), which is based on less restrictive statistical assumptions than MDA. In this estimation, the dependent variable Y = 0 for non-failed firms and Y = 1 for failed firms. LRA does not require that independent variables be multivariate normal or that groups have equal covariance matrices, which are basic assumptions in MDA (Hosmer and Lemeshow, 1989). LRA creates a logit score L for every firm. It is assumed that the independent variables are linearly related to L. This score or logit is used to determine the conditional probability of failure as follows:

$$(Y = 1|X) = \frac{1}{1 + e^{-L}} = \frac{1}{1 + e^{-(b_0 + b_1 X1 + \cdots + b_4 X_4)}} \tag{4}$$

where bi (i = 0, ... 4) are the coefficients and Xi (i = 1, … 4) are the four independent variables of the original Z”-Model. The effect of this method on classification performance is assessed by testing the statistical significance of the difference between AUCs for this LR model and for the re-estimated MDA model. The resulting model is used as a benchmark (LR benchmark model) for further statistical AUC comparisons because LRA is applied as the principal method in testing the remaining research hypotheses.

The third hypothesis (**H3**) is associated with the performance effect of taking account of the bankruptcy year in the estimation. This hypothesis is tested by estimating an LR model based on the following logit:

$$L = b_0 + \sum_{i=1}^{4} b_i X_i + \sum_{i=1}^{4} c_j D_j \tag{5}$$

where b0 is a constant, Xi (i = 1,..., 4) are the four independent variables of the original Z”-Model, bi (i = 1,..., 4) are their coefficients, cj (j = 1,..., 3) are coefficients of the dummy variables, and D1 = 1 when year = 2018, 0 otherwise; D2 = 1 when year = 2019, 0 otherwise; D3 = 1 when year = 2020, 0 otherwise; D4 = 1 when year 2021, 0 otherwise. The dummy variables do not directly refer to the bankruptcy year, which is not given in the database, but rather to the last available year before bankruptcy. In this model, the year 2022 is the base category. If the AUC of this extended LR model significantly exceeds the AUC of the Z”-Score LR Benchmark model, the evidence supports hypothesis H3.

Hypotheses **H4-H8** are examined using a corresponding methodology to the third hypothesis mentioned earlier. However, for each hypothesis, suitable variables are adopted in place of Year Dummies.

The **H4** hypothesis evaluates the impact of firm size on the predictive accuracy of the Z"-Score model, acknowledging that firm size is a significant factor in assessing financial distress. Two distinct methods were employed to measure firm size and examine its influence on predicting financial distress among Portuguese companies.

The first method measures firm size by the number of employees, treating this metric as a continuous variable integrated directly into the logistic regression model alongside the original Z"-Score variables. This approach presumes a linear relationship between the number of employees and the probability of financial distress, allowing for a direct analysis of how changes in workforce size can impact a firm's financial stability.

Conversely, the second method classifies firms according to legal definitions, grouping them into micro, small, medium, and large enterprises based on a combination of turnover, balance sheet total, and the number of employees. These classifications are then transformed into dummy variables, with micro firms serving as the reference category. The logistic regression model incorporates these variables (D1=1 for small firms, 0 otherwise; D2=1 for medium firms, 0 otherwise; D3=1 for large firms, 0 otherwise), enabling an examination of how various legal classifications of firm size affect the likelihood of financial distress.

Hypothesis **H5** tests whether the classification performance improves when the age of the firm is explicitly considered. When testing this hypothesis, the 15 years or more category is used as the base category, and two dummy variables are incorporated in the LR model (D1: 6-14 years, D2: less than 6 years).

Hypothesis **H6** looks at whether classification performance is affected by the explicit consideration of industry effects. The hypothesis is tested here using dummy variables for seven industries (D1: Hotels and Restaurants; D2: construction; D3: wholesale and retailing; D4: manufacturing; D5: energy and water production; D6: information technology) [No agriculture companies in the final sample data], with all other industries acting as the base category.

GDP growth's potential to enhance the Z"-Score model's predictive power for financial distress in Portuguese firms is under scrutiny. The study maps GDP values from 2018 to 2022 to each company's last financial reporting year. It hypothesizes (**H7**) an inverse relationship between GDP and financial distress likelihood. Higher GDP levels should correlate with lower distress probabilities, manifesting as a negative GDP coefficient in the model. Success hinges on comparing the GDP-enhanced model's AUC to the benchmark Z"-Score. A significant AUC improvement would validate GDP growth's inclusion, offering deeper insights into macroeconomic impacts on Portuguese companies' financial health.

Finally, **H8** explores whether the inclusion of the ICR enhances the predictive accuracy of the Z´´-Score model, the ICR is integrated into the logistic regression model along with the original financial variables from the Z´´-Score model with the same rationale as H7. A significant improvement in the AUC or a reduction in error rates would suggest that the inclusion of the Interest Coverage Ratio enhances the model’s ability to predict financial distress.

The seven LR models with the original four financial ratios and the additional variables specified in the hypotheses are estimated for all data. In addition, an LR model including all additional variables is estimated for all data to assess the simultaneous effect of all variables.

5. Empirical Results

5.1 Statistics

Table 2 provides a comprehensive overview of the descriptive statistics for the four key financial ratios within the Z”-Score model. These statistics offer insights into the financial health of non-failed versus failed firms, reflecting the broader economic and regulatory challenges in Portugal, with an acknowledgment of the additional pressures brought on by the COVID-19 pandemic.

Table 2. *Descriptive Statistics*

Statistic	WCTA *Non-Failed*	WCTA *Failed*	RETA *Non-Failed*	RETA *Failed*	EBITTA *Non-Failed*	EBITTA *Failed*	BVETD *Non-Failed*	BVETD *Failed*
Median	0,32	0,06	0,01	-0,09	0,04	-0,03	0,61	0,05
Mean	0,23	-0,19	-0,20	-0,65	0,04	-0,15	2,44	0,33
Standard Deviation	0,70	1,06	1,39	1,85	0,23	0,41	14,54	7,70
Upper Quartile	0,57	0,32	0,21	0,02	0,11	0,03	1,55	0,28
Lower Quartile	0,06	-0,30	-0,11	-0,52	0,01	-0,20	0,19	-0,24
Maximum	1,00	1,00	0,91	0,91	0,72	0,72	309,98	309,98
Minimum	-6,94	-6,94	-15,17	-15,17	-1,95	-1,95	-0,92	-0,92

***Table 2** - Descriptive Statistics*

Non-failed firms demonstrate better liquidity, as evidenced by their higher WCTA medians [0.32] and means [0.23]. They cleverly handle short-term liabilities in the face of limited credit availability and increasing expenses. Failures exhibit negative means of -0.19, indicating severe cash shortages. Undoubtedly, COVID-19 has intensified these challenges, particularly for companies with tight operational margins. The WCTA values of failed businesses show inconsistent cash management, which the epidemic's effects on the economy further disrupted. This significant difference highlights the crucial importance of strong liquidity in enduring economic downturns and ensuring the survival of a company during unpredictable periods.

Moreover, successful companies retained a small portion of their earnings, suggesting the possibility of intense competition and narrow profit margins. The impact of COVID-19 on incomes was probably negative, but assistance may have mitigated the shock. Insolvent enterprises dissipated funds, maybe as a result of exorbitant expenses, inadequate productivity, and economic difficulties. Their divergent outcomes indicate that some individuals managed more effectively than others. Government assistance may have contributed to the resilience of some individuals during the crisis, while others succumbed to financial strain. The repercussions of the epidemic on retained earnings (RE) underscore the susceptibility of the industry to abrupt disruptions and the need of financial padding.

Firms that failed faced difficulties in generating profits, as evidenced by negative EBITTA figures. Their challenges included diminished output, intense rivalry, and disruptions caused by the social-economic context. Nevertheless, non-failed companies sustained their operational profitability by achieving a positive EBITTA. Such tenacity enabled them to meet expenses and reinvest. They exhibited stability in contrast to the instability of unsuccessful enterprises. While certain firms succeeded, others struggled in response to economic difficulties. The impact of the epidemic underscored the vital importance of adaptation in the survival of businesses.

Additionally, the higher median [0.61] and mean [2.44] values of non-failed enterprises compared to their failed counterparts [median = 0.05; mean = 0.33] show that their equity positions are stronger. This resilient equity buffer provides protection against economic volatility, which is essential in a highly regulated market. Failures, characterized by lesser equity, probably relied on debt- a tactic that flopped during revenue declines caused by the epidemic. The broad distribution of equity among enterprises that did not fail suggests a range of financial strategies, which indicate different levels of access to capital and expertise in leveraging equity during crises. Utilization of leverage became a crucial determinant, particularly since COVID-19 placed more pressure on already fragile stock positions.

Economic volatility has an impact on the ability of financial ratios to predict. The analysis of Z-Score data from 2007-2011 and 2018-2022 in Figure 1 reveals notable differences. Although Altman's 2016 study incorporates the 2007 financial crisis, the present data mostly reflects the impacts of COVID-19. These temporal intervals illustrate how fluctuations in the economy affect the ability of financial indicators to distinguish between profitable and struggling enterprises. The difference of median values of the graph highlights the dynamic variations, revealing insights into the utility of ratios in different economic contexts.

The latest dataset exhibits a notable rise in the median difference of the WCTA ratio (0.266) as compared to Altman's ratio (0.215). These findings indicate that liquidity has become a more important determinant in differentiating the stability of firms in recent years. This may be attributed to companies giving priority to efficient management of cash flow in the face of changing revenue streams during global disruptions. In contrast, the RETA ratio in this study demonstrates a decline in the median difference (0.095) compared to Altman's (0.200). This suggests that RE may not be a dependable measure of financial well-being during periods of economic uncertainty. In such cases, companies may be shifting their reserves towards covering operational expenses instead of saving them.

An increase in the median difference of the EBITTA ratio (0.069 in this study vs. 0.052 in Altman's) indicates that operational efficiency has become more significant as a determinant of financial health. This highlights the imperative for companies to sustain or improve profitability in response to prevailing market difficulties. Furthermore, the BVETD ratio exhibits a larger median disparity (0.564 vs 0.318 in Altman's), underscoring a stronger dependence on the market's assessment of a company's equity worth in relation to its liabilities. This may be attributed to the rise in market volatility and the intensified examination by investors of firm-specific and global economic variables.

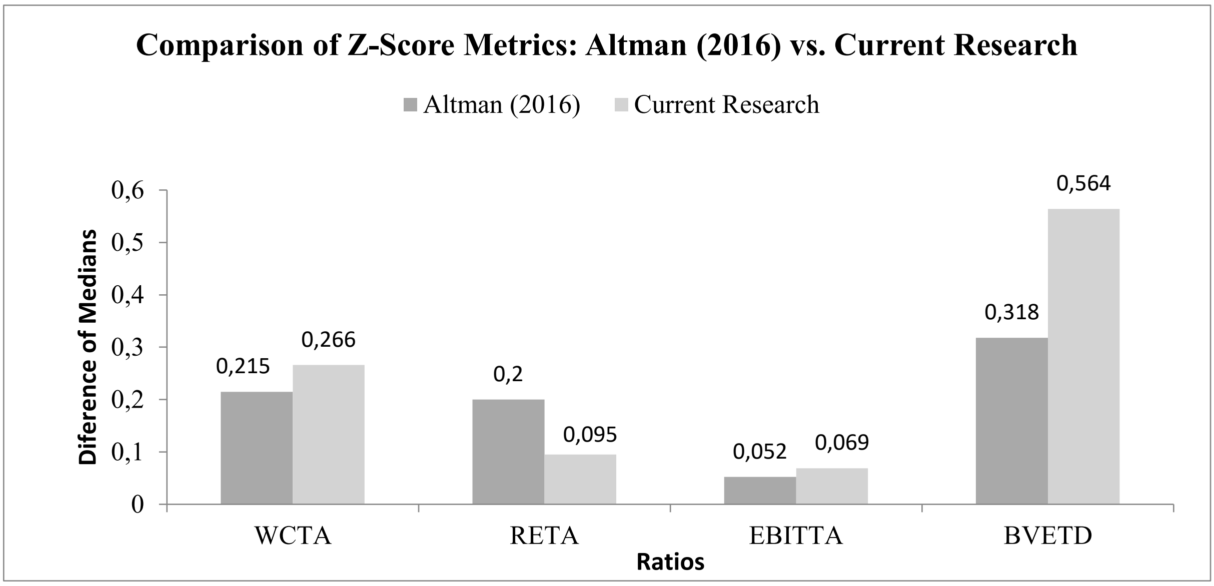

***Figure 2**- The chart depicts the differences in median values of key financial ratios between failed and non-failed firms, incorporating findings from Altman's 2016 study. Notable, the values from Altman's study represent the difference in medians for firms in Portugal, providing a regional context to the international application of the Z''-Score model.*

The aforementioned changes emphasize the necessity of regularly readjusting financial distress prediction models in order to preserve their pertinence and precision. Collectively with the changing financial environment, the methodologies we employ to forecast the stability of firms must also adapt. Consistent updates, including the most recent economic data and maybe more dynamic real-time benchmarks, will guarantee the continued effectiveness of these models in a rapidly evolving global economy. This analysis not only validates the necessity for revised models but also underscores the need for refining these models to suit present socio-economic contexts, therefore guaranteeing their continued effectiveness as reliable instruments for evaluating financial risk.

5.2 Coefficients / Regressions

Firm failure prediction hinges on financial ratios, as Table 3's significant coefficients reveal. Surprisingly, RETA's positive impact contradicts Altman's models. This reversal likely stems from unique conditions in the 2018-2022 Portuguese market. The pandemic's economic turmoil potentially amplified financial strains, altering traditional ratio interpretations. These findings reinforce the need for context-specific analysis when applying financial models to predict corporate distress.

The WCTA ratio also shows a diminished influence in these models compared to the original Z"-Score, although the coefficients remain significant. This reduction likely reflects the pervasive liquidity challenges in Portugal, which affect both failed and non-failed firms, thereby reducing WCTA's ability to differentiate between these groups effectively. The financial constraints during and after the COVID-19 crisis likely exacerbated these liquidity issues, underscoring that while WCTA remains important, its influence is somewhat muted in the face of extraordinary pressures on cash flow.

Profitability remains crucial for firm survival in Portugal's post-COVID economy. The EBITTA ratio's strong negative influence on failure aligns with Altman's Z"-Score and later research. Operational earnings before interest and taxes prove vital, with significant negative coefficients reinforcing their predictive power for financial health. This consistency highlights profitability's enduring role as a key determinant in challenging economic landscapes.

High debt levels, common in Portugal, weaken the BVETD ratio's predictive power for failure. While still significant, its influence diminishes in my models compared to the original Z"-Score. Leverage remains important but loses potency when excessive indebtedness becomes the norm. Non-failed Portuguese firms show high variability in leverage, affecting the ratio's effectiveness.

Table 3. The Coefficients of the Different Models Estimated for all Data

		Coefficients for different statistical models									
Variable	*Z''Score*	*Model 1*	*Model 2*	*Model 3*	*Model 4*	*Model 5*	*Model 6*	*Model 7*	*Model 8*	*Model 9*	*Model "All"*
Constant	3,25	-0,026***	0,207***	-0,857***	0,138***	-0,125***	-0,146***	0,345***	89,926***	0,249***	10650
WCTA	6,56	-0,309***	-0,336***	-0,342***	-0,330***	-0,251**	-0,312***	-0,374***	-0,338***	-0,389***	-0,027
RETA	3,26	0,068***	0,109***	0,019*	0,127***	0,043***	0,058***	0,117***	0,157***	0,142***	0,188***
EBITTDA	6,72	-1,560***	-2,129***	-1,579***	-2,235***	-2,228***	-2,203***	-2,185***	-1,946***	-2,020***	-0,945***
BVETD	1,05	-0,016***	-0,321***	-0,167***	-0,347***	-0,304***	-0,263***	-0,330***	-0,278***	-0,383***	-1,869***
Year Dummies											
Year 2018				197,496							1,076
Year 2019				53,578							-110,302
Year 2020				22,014							16,229
Year 2021				44,888							-69,581
Size- Nº of Employees					0,006***						-
Size - Legal Definition											
Small						0,804***					1,076***
Medium						0,948***					0,880***
Large						-33,037					-17,004
Age Dummies											
6 to 15 years							0,395***				0,074*
less than 6 years							0,883***				0,447
Industry Dummies											
Restaurants & Hotels								-0,056*			-2,500***
Construction								-0,706***			-0,644***
Wholesale and Retailing								-0,299***			-2,511***
Manufacturing								-2,819***			-1,249***
Energy and Water Production								-0,257*			-2,353*
Information Technology								-0,927***			-3,300***
Country Specific Dummy											
GDP									-0,353***		-41,446
New Variable											
ICR										-0,000003**	-

Table 3- *Significance:* ***p < 0.10 (*), p < 0.05 (**), p < 0.01 (***)****; **Models:** Model 0 = Original Altman (1983) Z"-Score Model coefficients; Model 1 = MDA model; Model 2 = LR Benchmark model; Model 3 = LR model estimated for all data with year dummies; Model 4 & 5 = LR model estimated for all data with size variables; Model 6 = LR model estimated for all data with age category dummies; Model 7 = LR model estimated for all data with industry dummies; Model 8 = LR model estimated for all data with macroeconomic (GDP) variable; Model 9 = LR model estimated with a new variable (ICR); Model All: Include all variables. But ICR and nº of employees*

5.3 Control Variables

Having established the significance and implications of the core financial ratios, it's important to examine how additional factors such as temporal events, firm-specific characteristics, and industry-specific factors influence the likelihood of financial distress.

Model 3: Year Dummies

Model 3's year dummy coefficients lack statistical significance. This implies no meaningful variation in firm failure likelihood across years, once key financial ratios are considered. The base year's probability remains consistent throughout the period analysed pre-pandemic years 2018 and 2019 mirror firm failure rates of COVID-19 era. This intriguing revelation challenges assumptions about pandemic's impact on business closures. Despite expectations, no significant differences emerged across these periods. This may imply that, in Portugal, firms' financial health is a bigger factor in failure than the economic conditions of a given year.

Altman's 2016 study found year dummies significant, especially for the 2008 crisis. My model's results differ, suggesting our financial ratios capture distress across economic conditions, diminishing year-specific effects. The insignificance of pre-pandemic years (2018-2019) reinforces this notion. Systemic challenges and firm-specific health emerge as more critical failure determinants than yearly macroeconomic events.

Model 4: Number of Employees

SMEs dominate Portugal's economy, particularly in tourism and retail. These sectors, often labour-intensive, face unique financial pressures. Analysis shows larger workforces correlate with increased distress risk. This counterintuitive link likely stems from operational inefficiencies plaguing low-productivity industries. Portugal's economic resilience hinges on addressing these sector-specific challenges.

Labour costs loom large for Portuguese firms, especially in less productive sectors. As employee numbers become a critical indicator, the nation's labour productivity lags European peers Their financial health hinges on workforce size, revealing operational efficiency's critical role. This local sensitivity challenges Altman's universal approach to financial distress models. Economic conditions demand tailored analysis, balancing efficiency with workforce needs. Larger teams pose risks, while lean operations often signal fiscal strength. Portugal's case underscores the nuanced interplay between staffing and financial stability.

Model 5: Firm Size

Firm size significantly influences financial distress risk in Portuguese businesses. Small and medium enterprises face higher odds compared to micro-entities. This stems from their increased operational complexity and fixed costs. As companies scale up, they encounter greater challenges: lower productivity, limited financing options, and heightened economic vulnerability. These factors make SMEs particularly susceptible to financial turmoil. The analysis reveals a clear pattern: larger firms within the SME spectrum grapple with more financial hurdles than their tiny counterparts, highlighting the unique struggles of growing businesses in Portugal's economic landscape.

SMEs in Portugal struggle with unique obstacles, notably constrained financing and economic volatility. Firm size critically influences financial stability, especially in SME-dominated markets, as Almeida and Santos (2019) demonstrated. Our results align with these insights. Large firms show a negative but statistically insignificant coefficient, likely stemming from their minimal 0.2% representation in the dataset, hampering the model's capacity to discern meaningful effects for this group.

Portugal's business landscape, dominated by small and medium enterprises, yields sparse data on large firms. This scarcity delays meaningful statistical analysis for bigger companies. The sample's limited representation of large enterprises likely explains the lack of significant results, echoing Gordini's 2014 observation. He noted that financial models' predictive power often fluctuates with firm size, especially in SME-centric economies. To grasp larger Portuguese companies' financial dynamics, more focused data collection is essential.

Model 6: Firm Age

In the analysis of Model 6, the results indicate that younger firms face a higher risk of financial instability. Specifically, firms less than 6 years old show a positive but often non-significant coefficient, while those in the 6 to 15 years age bracket consistently display a significant positive coefficient. This suggests that relative to firms older than 15 years (the base category), younger firms are more vulnerable to financial distress.

The heightened risk among younger firms can be attributed to factors such as limited access to credit, inexperience in management, and lower financial resilience. These challenges are particularly pronounced in Portugal's economic environment, where SMEs dominate and often operate with tighter financial margins.

Compared to Altman’s study, which filtered firms based on a minimum asset threshold, this study's inclusion of a broader range of firms without such restrictions has highlighted the significance of firm age in predicting financial distress. Altman’s focus on more established firms likely downplayed the role of age, whereas this study's approach reveals that younger firms in Portugal are particularly at risk, underscoring the need for models that account for the specific dynamics of the local market.

Model 7: Industry Sectors

Financial distress likelihood varies significantly across different industries, and each sector's unique characteristics play a significant role in determining its susceptibility to financial difficulties. For instance, the energy and water production sectors have proven to be exceptionally stable, thanks to the consistent demand for their services and the support they receive from governments. In contrast, restaurants and hotels, which were severely impacted by the pandemic, have shown remarkable resilience by adopting adaptive strategies and benefiting from targeted aid. The construction sector, on the other hand, remains vulnerable to economic fluctuations, while wholesale and retail industries are more stable due to their steady cash flow. The manufacturing sector faces significant risks due to supply chain disruptions, and the IT sector is under pressure to constantly innovate and keep pace with rapid technological advancements.

The findings of Model 7, which considered these sector-specific factors through dummy variables, highlight the complex interplay between industry characteristics, economic conditions, and policy interventions that shape financial distress risks. For example, the model reveals that government support can significantly mitigate financial distress risks in certain industries, while supply chain disruptions can have devastating effects on others. Understanding these nuances is crucial for developing targeted strategies to address financial vulnerabilities in different sectors of the economy. By recognizing the unique challenges and opportunities of each sector, policymakers and business leaders can work together to create more resilient and sustainable economies.

Model 8: GDP

Economic prosperity, measured by GDP, reduces financial distress risk. Firms thrive when the economy grows, boosting revenues and stability. Model 8 exposed a tight inverse relationship between economic growth and business instability. As GDP rises, corporate financial upheaval plummets. This robust connection underscores how macroeconomic health directly impacts company stability. This finding validates the intuitive connection between macroeconomic health and business success.

From pandemic-induced recession to recovery, Portugal's GDP rollercoaster mirrored firms' financial health. The 2020 downturn hit SMEs hardest, exposing their vulnerability to economic shocks. Financial strain expanded rapidly as GDP declined sharply. Yet the rebound, fuelled by domestic demand and EU support, eased distress risks. Marques and Pereira's 2021 study exposes economic upheaval's harsh impact on companies. Economic turbulence reveals the vital link between GDP fluctuations and company health. This research spotlights how national output impacts business finances, especially in volatile times.

Model 8's findings match the broader literature: macroeconomic stability is key for firm performance (Costa and Reis, 2018). Stable GDP growth helps businesses by providing better access to finance, predictable demand, and investor confidence. In short, GDP is crucial for predicting financial distress. Stronger economic conditions reduce distress, but firm-specific factors also matter.

Model 9: Interest Coverage Ratio (ICR)

The Interest Coverage Ratio's negative coefficient in Model 9 suggests firms with higher ICRs face less financial distress, echoing Altman's and Mateus and Pimentel's findings. Yet its modest impact pales against profitability and liquidity ratios. Silva et al. note ICR works best combined with other metrics.

Integrating firm size, industry sectors, and macroeconomic indicators like GDP provides nuanced insight into financial distress factors. This approach shows traditional ratios remain crucial, but contextual variables enhance predictive accuracy. The models now better reflect Portugal's unique economic landscape.

Model "All": All Variables Included

Model "All" incorporates all variables except for two, as I opted for Approach 2 instead of one to avoid redundancy and nearly negligible discriminatory power (ICR coefficient). Specific variables undergo changes in their significance. Examples of metrics are firm age, GDP, and WCTA. This shifting may be attributed to the interaction of various financial variables and the unique economic environment of Portugal. The relevance of WCTA diminishes as several Portuguese SMEs consistently face liquidity challenges, and their financial stability is contingent upon other variables. Due to the dominance of firm-specific and industry-related risks, the significance of GDP diminishes in macroeconomic conditions. When evaluating several financial indicators collectively, the age of a corporation becomes less significant. Among Portuguese companies, which are predominantly family-owned, the age of a firm has less significance in determining its financial well-being.

5.4 Model’s AUC performances

After analysing the coefficients and adding dummy variables in the preceding sections, these models must be evaluated for financial distress prediction. AUC values from each model will be compared for this evaluation. Table 4's AUC and the importance of each model's variables will help me assess the predictions' robustness, accuracy, and empirical support for the tested hypotheses. The AUC, Accuracy Ratio, and appropriate thresholds metrics show substantial variations in logistic regression models that predict Portuguese business financial hardship.

Table 4. Models’ Performance

Models	***AUC***	***Accuracy Ratio (AR)***	*Performance Classification*
Model 0	**0,75**	0,50	Moderate Performance
Model 1	**0,77**	0,54	Moderate Performance
Model 2	**0,81**	0,62	Good Performance
Model 3	**0,93**	0,86	Excellent Performance
Model 4	**0,81**	0,62	Good Performance
Model 5	**0,80**	0,60	Moderate Performance
Model 6	**0,79**	0,58	Moderate Performance
Model 7	**0,79**	0,58	Moderate Performance
Model 8	**0,88**	0,76	Good Performance
Model 9	**0,81**	0,62	Good Performance
Model “All”	**0.93**	0,86	Excellent Performance

Table 4*- The results of the models were evaluated based on metrics such as AUC and AR, as presented in Table 4. The performance classification was assigned based on predefined value ranges, as detailed in Table 1 in the Appendix, which provides the specific criteria for each performance category.*

The baseline model, Model 0, has an AUC of 0.75 and an Accuracy Ratio of 0.50, indicating modest efficacy. This model's mediocre classification performance shows the limits of the original Z-Score technique, especially in Portugal, where SMEs dominate commerce. The model's high Type I/Type II error ratio of 85.32 is due to the extremely conservative optimal threshold of 0.998 (Table 6), which makes it more likely to miss distressed firms (high false negatives) than false positives, which could be problematic in contexts where early distress detection is crucial. Due to their low financial buffers and fewer diversified income sources, these SMEs are more susceptible to economic volatility and require a more specialized strategy to financial distress prediction.

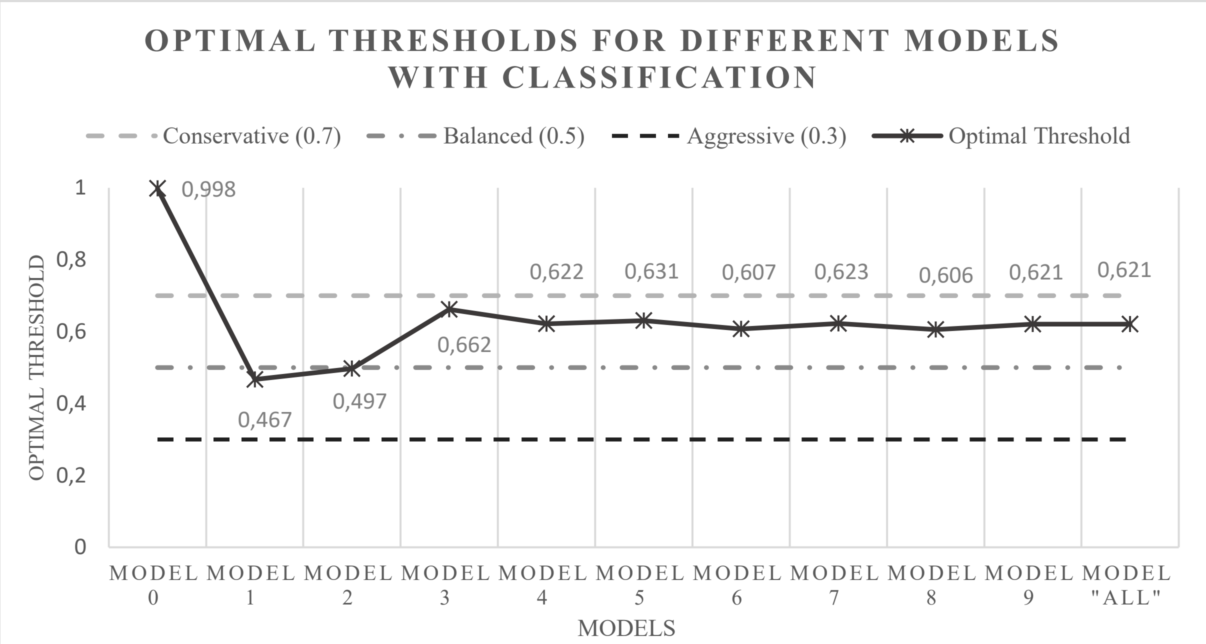

Figure 3 *– Optimal Thresholds for different Models with classification*

Model 1 has an AUC of 0.77 and an AR of 0.54, somewhat better than Model 0. Hypothesis H1, that recalculating the Z"-Score model's coefficients improves performance, is poorly supported. Model 1 has a high Type I/Type II error ratio of 144.99 due to its aggressive lower threshold of 0.467. This shows that the algorithm flags businesses as distressed too aggressively, resulting in many false positives. This minimizes the probability of overlooking a troubled business, but it may prejudice stakeholder decisions for healthy enterprises.

Model 2, which switches from MDA to LR, improves significantly with an AUC of 0.81 and AR of 0.62. This validates Hypothesis H2, that switching from MDA to LR improves model performance. This model has a moderate Type I/Type II error ratio due to its balanced optimum threshold of 0.497. This balanced method lets the model trade-off erroneous positives and negatives, making it more trustworthy for Portuguese use. This shows that logistic regression is a superior financial distress prediction tool in this situation, where precise identification of distressed and non-distressed enterprises is crucial for financial stability. The result matches Das et al. (2009) and Altman (2016) study.

Model 3, including year dummies, has "Excellent Performance." AUC = 0.93 and Accuracy Ratio = 0.86. The high AUC shows great prediction accuracy, however the comparison with the LR Benchmark Model raises concerns about overfitting owing to year-specific characteristics. The conservative optimum threshold of 0.662 yields the lowest Type I/Type II error ratio of 12.09 among the models, demonstrating a significant focus on eliminating false positives at the expense of erroneous negatives. This cautious approach works well in stable economies but may overlook weak enterprises in unpredictable ones. Portugal is especially vulnerable to economic oscillations and exogenous shocks like the COVID-19 epidemic, which affect tourism and retail. Thus, the model's performance supports Hypothesis H3, showing that year-specific features can improve model accuracy but must be used judiciously.

Table 5.

Models	*Type I Error*	*Type 2 Error*	*Ratio Type 1/Type 2*
Model 0	11006	129	85,32
Model 1	4583	32	144,99
Model 2	3890	120	32,42
Model 3	1294	107	12,09
Model 4	3661	124	29,52
Model 5	3161	189	16,72
Model 6	5606	91	61,60
Model 7	4167	125	33,34
Model 8	1921	142	13,53
Model 9	4223	101	41,81
Model “All”	1153	105	10,98

Table 5 *- This table presents the Type I and Type II error counts for each model, along with the corresponding Type I/Type II error ratios. The ratios provide insight into the balance each model strikes between false positives (Type I errors) and false negatives (Type II errors), highlighting the varying degrees of conservatism or aggressiveness in the models' classifications of financial distress.*

Models 4 and 5, which focus on firm size, offer fascinating insights. Model 4 has "Good Performance," like the Logit Benchmark Model, with an AUC of 0.81 and an Accuracy Ratio of 0.62. Hypothesis H4 is not supported by adding firm size (1st method) to our study, which does not improve model accuracy beyond the benchmark. This shows that business size is important but does not boost predictive power in our setting. Despite outperforming Altman's study, Model 5's AUC and Accuracy Ratio of 0.80 and 0.60, respectively, show that these models cannot outperform our logistic regression benchmark in predicting financial hardship. In contrast, Pindado et al. (2008) found that operational characteristics strongly influence financial distress projections.

Model 6's AUC of 0.79 and Accuracy Ratio of 0.58 indicate "Moderate Performance." This model's solid age assessment does not improve forecast accuracy over our LB Benchmark Model. It does not support H5, which held that firm age would considerably increase model performance. This contradicts Beaver, McNichols, and Rhie (2005), who found that age might predict financial difficulty. This shows that financial measures alone may be more predictive than age in Portuguese context.

AUCs of 0.79 and ARs of 0.58 indicate reasonable performance for Models 7. These models include Portuguese-specific industry characteristics, which are crucial. Local research like Almeida and Santos (2019) and Teixeira and Gomes (2020) have shown that sector-specific characteristics like tourism's seasonal income patterns and technology's economic sensitivity can improve model accuracy. Despite these sector-specific improvements, these models perform marginally worse than the LR Benchmark Model and fail to implement H6.

Model 8, which integrates GDP, performs well with an AUC of 0.88 and an Accuracy Ratio of 0.76, earning "Good Performance." GDP improves prediction accuracy in logistic regression, as seen by its 13.53 error ratio. It supports Hypothesis H7 that macroeconomic stability is essential to financial distress prediction models, supporting economic literature (Machado and Mata, 2015) that recommends integrating macroeconomic variables like GDP to improve predictive accuracy. GDP is essential in Model 8, but is less important in Model "All" since it loses statistical significance. This shows that other key predictors may dilute GDP's influence when all factors are included. This result shows the difficulty of financial distress prediction and the necessity to carefully assess each variable's contribution in a properly described model.

Finally, Model 9 includes the ICR, which is statistically significant but has practically nil discriminant power, raising issues about its practical influence on model performance. The model's usefulness is confirmed by the Type I/Type II error ratio of 41.81, but the identical AUC to the benchmark model implies that while the ICR's inclusion is statistically legitimate, it may not improve performance. Under these conditions, Hypothesis H8 is not supported, indicating that integrating the ICR does not significantly improve the model beyond logistic regression.

5.5 Comparison with Altman's' Findings

As shown in Table 7, our study and Altman's 2016 work demonstrate the benefits and drawbacks of adjusting financial distress prediction models to diverse circumstances. Most models in this study had higher AUC values than Altman's, especially the comprehensive "All" model, which had 0.93 compared to Altman's 0.80. This shows that adding variables customized to local economic conditions to represent the Portuguese market improved the model's forecast performance. Given data treatment and model design discrepancies, these results should be interpreted cautiously.

Altman's vs This Paper

<table>
<tr><th></th><th>Altman 2016</th><th>This Paper</th></tr>
<tr><td>LR International Data (Altman's)</td><td colspan="2">0,741</td></tr>
<tr><td>Model 1 MDA</td><td>0,75</td><td>0,77</td></tr>
<tr><td>Model 2 LR</td><td>0,76</td><td>0,81</td></tr>
<tr><td>Model 3 (Year Dummy)</td><td>0,76</td><td>0,93</td></tr>
<tr><td>Modal 4 (nº employees)</td><td rowspan="2">0,78</td><td>0,81</td></tr>
<tr><td>Model 5 (Legal Definition)</td><td>0,8</td></tr>
<tr><td>Model 6 (Age Dummy)</td><td>0,76</td><td>0,79</td></tr>
<tr><td>Model 7 (Industry Dummies)</td><td>0,77</td><td>0,79</td></tr>
<tr><td>Model "All"</td><td>0,8</td><td>0,93</td></tr>
</table>

Table 6*- Comparison of Model Performances between Altman's Research and my Academic work*

Firm size and dataset inclusion criteria are important methodological differences. Altman's method focused on larger, possibly more stable organizations by requiring them to declare at least €100,000 in assets at least once over the period. The uniform importance of variables in his models may have been due to a more homogenous dataset from these criteria. This study included a larger, more representative sample of the Portuguese economy, which is mostly SMEs, without size limits. While making the dataset more representative of local business, this approach increased model variability and complexity.

Inclusion of businesses without size restrictions may explain why this study's models have greater AUC values. By incorporating more business sizes and financial health profiles, the models may better understand Portugal's distinctive financial crisis dynamics. This technique has drawbacks, as several variables, especially in the "All" model, were not statistically significant. The lack of significance in these factors implies that while the model is successful at categorization, several predictors of financial hardship are inconsistent in Portugal.

This analysis also found no statistical significance in the year dummy variables, demonstrating that economic circumstances within individual years did not greatly impact company failure when the key financial ratios were included. In contrast, Altman found that year dummies were more important, perhaps due to his bigger, more stable dataset, which better reflected global economic events. The non-significance of year dummies in this study suggests that the high model performance may be due to the Portuguese data rather than a stronger prediction model.

Comparing the overall performance of models at the country level against larger worldwide data shows that models tailored to the Portuguese market perform better. This study's higher AUC values, displayed in Table 7, emphasize the need for context-specific model changes, especially in SMEs-dominated countries like Portugal.

In conclusion, this study's models exceed Altman's 2016 work in AUC values, however this does not mean they are better. The lack of size limits in the methodology may have made the models more adaptive to the Portuguese environment but also more variable. These findings underline the need to refine prediction models to preserve accuracy and relevance, especially in specific economic circumstances. The contrasts between this study and Altman's show how methodological considerations, notably data inclusion criteria, can affect model performance and financial hardship prediction outcomes.

6. Conclusion

In conclusion, this research examined the changes needed to integrate Altman's Z-Score model in the Portuguese economy, considering the COVID-19 epidemic. This study considerably enhanced the traditional model to appropriately describe Portuguese firms' current financial issues using a recent dataset. With this improved model, the study revealed complicated financial ratio dynamics in the current economic scenario. Liquidity and operational profitability remain essential measures of financial health, but the study found that retained earnings and equity positions are no longer as predictive. The statistical significance of the X1-X4 coefficients supported these results, however these financial ratios had a somewhat different influence than the Z''-Score.

These disparities emphasize Portugal's unique economic climate and the COVID-19 pandemic's anticipated impact on liquidity issues. This study found very moderate gains in financial stability prediction after recalibration. These tiny improvements demonstrate that even small tweaks in financial models may yield useful insights, especially in dynamic economic contexts. This discovery paved the way for more fundamental methodological changes, such as the switch from multiple discriminant analysis (MDA) to logistic regression (LR), which better predicted financial distress. This methodological shift shows how adjusting predictive methodologies to modern economic situations may increase their accuracy and usefulness, especially in the Portuguese market.

The inclusion of GDP and other economic variables contributed to these gains. The model's predictive power increased by include macroeconomic data, highlighting their importance in business stability assessment. This shows the significance of knowing how national output and market volatility affect business financial health, especially amid economic uncertainty like the COVID-19 epidemic.

The examination of temporal elements, such as year-specific variables, found that economic variables typically outweighed them. This shows that time-related parameters in financial models must be balanced with more important economic data to avoid overfitting and keep the model current and accurate. Certain firm-level factors, such as size and age, did not indicate financial hardship as expected, adding to this complexity. Industry-specific characteristics were notably less important than predicted, suggesting that these variables may not completely represent the intricacies of financial hardship across industries. This emphasizes the need to combine macroeconomic, firm-specific, and industry-level information to grasp financial distress's complexity.

This research concludes that model modification and more economic and sector-specific data are needed. When altered to reflect current economic realities, financial distress models become more predictive, making them useful in fast changing contexts like the Portuguese market. To capture the varied and complicated elements that affect financial stability, a dynamic and developing methodology is essential

6.1 Limitations and Future Research

This thesis examined logistic regression models for forecasting financial hardship in Portuguese enterprises, highlighting the effects of economic and demographic factors. Despite these advances, significant restrictions must be considered to fully understand the findings' breadth and usefulness. The core dataset utilized in this study does not include socio-economic developments over the last two years, a period of major worldwide economic turmoil. The models may not correctly reflect Portuguese enterprises' current economic reality, limiting their usefulness to current situations. Real-time assessments are limited by outdated data sources, which may limit the prediction models' practicality in addressing current economic concerns. The COVID-19 pandemic, global supply chain adjustments, and other recent economic disturbances may have changed business finance structures, which is not represented in the current statistics. Despite not using a firm size filter to capture a broader perspective of the Portuguese economy, which is dominated by SMEs (almost 99% of the sample), future research might examine how financial crisis impacts different business sizes. This might assist pinpoint the risks encountered by enterprises of different sizes and increase the model's accuracy for each category.

Recent research has demonstrated that machine learning algorithms like random forests, support vector machines (SVM), and gradient boosting improve financial distress predictions compared to logistic regression. These approaches can handle high-dimensional datasets and capture complicated, non-linear variable connections. Random forests are good at regulating predictor interactions and reducing overfitting by averaging decision tree outputs. When working with huge, unstructured datasets or when linear models cannot capture variable connections, machine learning approaches have been shown to perform better.

These strategies may improve forecast accuracy in future studies, especially as enterprises encounter more complicated economic circumstances. Addressing these restrictions allows further study. With real-time data streams or frequently updated datasets, future research might improve model robustness by include current economic data. This would keep models current and effective in shifting economic conditions. Systematic research can improve financial distress prediction tools, helping stakeholders make better decisions to promote economic resilience and stability in an increasingly uncertain global market.

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8. Appendix

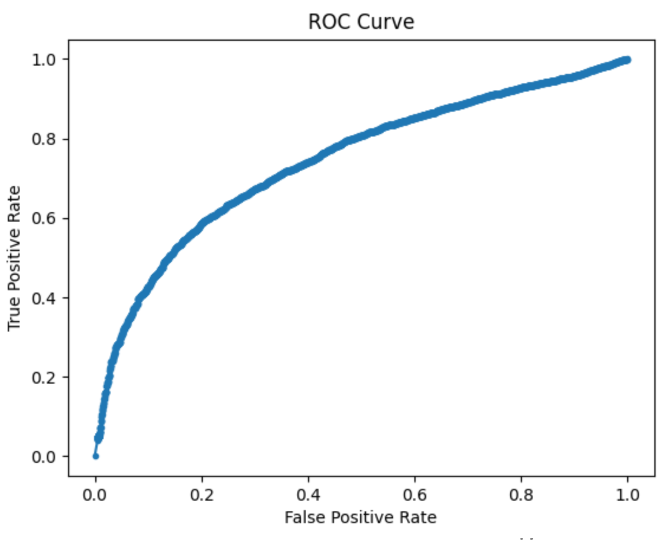
Table 1 – Performance Classification Metrics

AR Range	*Classification*	***Description***
AR = 1	Perfect Model	The model has perfect classification accuracy.
0.8 ≤ AR < 1	Excellent Performance	The model is highly accurate in distinguishing between classes.
0.6 ≤ AR < 0.8	Good Performance	The model performs well but is not perfect.
0.4 ≤ AR < 0.6	Moderate or Fair Performance	The model has some predictive power, better than random but with room for improvement.
0.2 ≤ AR < 0.4	Weak Performance	The model is slightly better than random guessing.
0 ≤ AR < 0.2	Poor Performance	The model's accuracy is close to random guessing.
AR = 0	No Discrimination	The model performs as well as random guessing (AR = 0 corresponds to AUC = 0.5).
AR < 0	Worse than Random	The model performs worse than random guessing, indicating potential issues such as overfitting.

ROC curves for each Model:

- **Model 0**

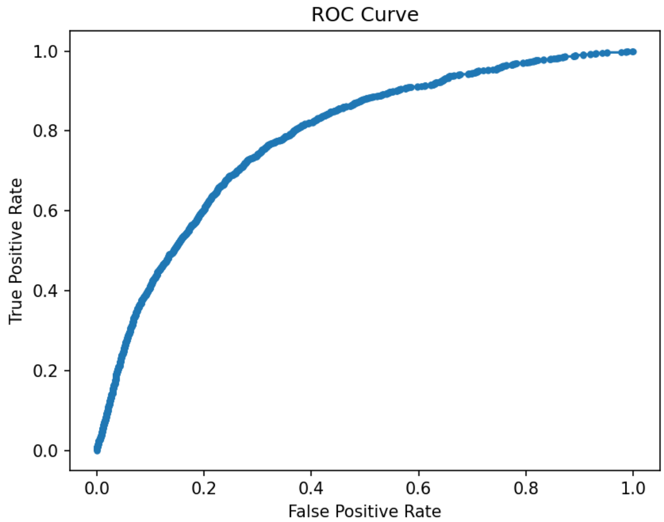
Optimal Threshold	0,998
AUC	0,75
Type I Error Count	129
Type II Error Count	11006
Ratio Type I/Type II	0,01



- **Model 1**

Optimization Model 1	
	Coef
const	-0,0258
WCTA	-0,3089
RETA	0,0681
EBITTA	-1,5599
BVETD	-0,0160

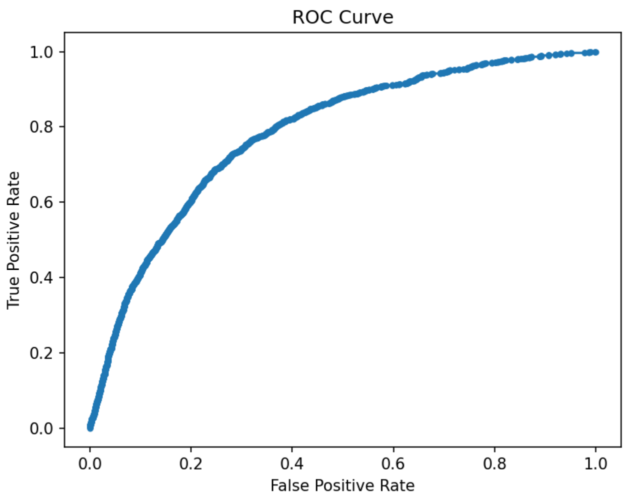
Optimal Threshold	0,467
AUC	0,77
Type I Error Count	4583
Type II Error Count	145
Ratio Type I/Type II	31,61



- **Model 2**

Optimization - Model 2							
	Coef	Std err	Z	P>|z|	[0.025	0.975]	Coefficient
const	0,207	0,010	20,468	0,000	0,187	0,227	0,207***
WCTA	-0,336	0,018	-18,773	0,000	-0,371	-0,301	-0,336***
RETA	0,109	0,009	12,342	0,000	0,092	0,126	0,109***
EBITTA	-2,129	0,048	-44,586	0,000	-2,223	-2,036	-2,129***
BVETD	-0,321	0,009	-36,156	0,000	-0,338	-0,303	-0,321***

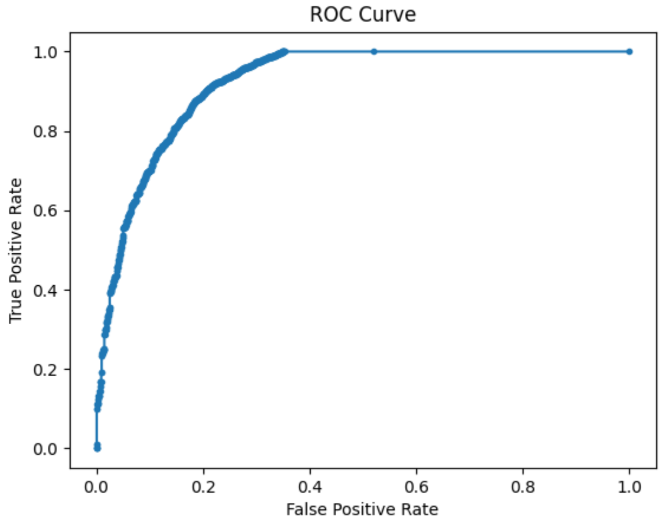
Optimal Threshold	0,497
AUC	0,81
Type I Error Count	3890
Type II Error Count	120
Ratio Type I/Type II	32,42



- **Model 3**

Optimization - Model 3						
	Coef	Std err	Z	P>|z|	[0.025	0.975]
const	-0,857	0,013	-64,823	0,000	-0,883	-831,000
WCTA	-0,342	0,020	-17,425	0,000	-0,380	-0,303
RETA	0,019	0,010	2,005	0,045	0,000	0,038
EBITTA	-1,579	0,049	-32,349	0,000	-1,674	-1,483
BVETD	-0,167	0,009	-18,602	0,000	-0,185	-0,149
year_2018	197,4957	3,13E+31	6,32E+30	1	-6,13E+31	6,13E+31
year_2019	53,5778	4,80E+09	1,12E+08	1	-9,40E+09	9,40E+09
year_2020	22,0142	5,64E+02	3,90E-02	0,969	-1084,346	1,13E+03
year_2021	44,8876	3,99E+07	1,12E+07	1	-7,82E+07	7,82E+07

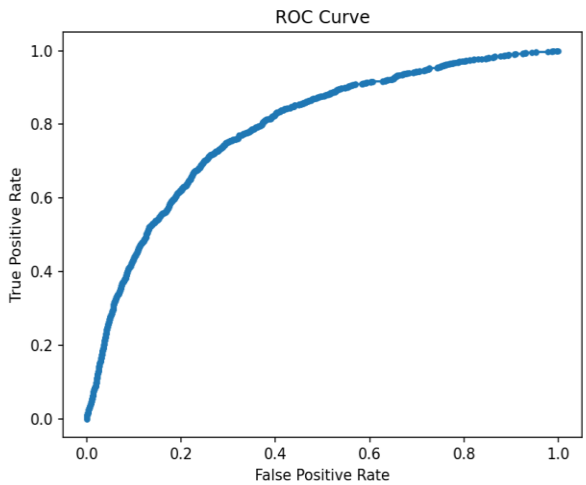
Optimal Threshold	0,66
AUC	0,93
Type I Error Count	1294
Type II Error Count	107
Ratio Type I/Type II	12,09



- **Model 4**

Optimization - Model 4						
	Coef	Std err	Z	P>|z|	[0.025	0.975]
const	0,138	0,011	12,140	0,000	0,116	0,160
WCTA	-0,330	0,019	-17,763	0,000	-0,367	-0,294
RETA	0,127	0,009	14,041	0,000	0,110	0,145
EBITTA	-2,235	0,050	-44,568	0,000	-2,334	-2,137
BVETD	-0,347	0,009	-36,860	0,000	-0,365	-0,328
n_employees	0,0059	0	18,093	0,000	0,005	0,007

Optimal Threshold	0,622
AUC	0,81
Type I Error Count	3661
Type II Error Count	124
Ratio Type I/Type II	29,52

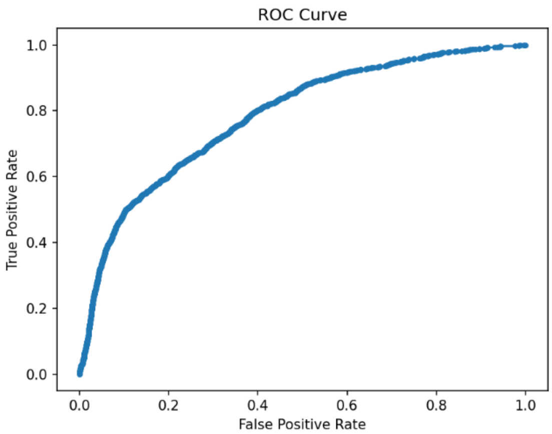


- **Model 5**

Optimization - Model 5						
	Coef	Std err	Z	P>|z|	[0,025	0,975]
const	-0,125	0,012	-9,993	0,000	-0,149	-0,100
WCTA	-0,251	0,018	-13,963	0,005	-0,286	-0,215
RETA	0,043	0,009	4,772	0,000	0,025	0,061
EBITTA	-2,228	0,049	-45,525	0,000	-2,324	-2,132
BVETD	-0,304	0,009	-33,929	0,000	-0,321	-0,286
small	0,804	0,019	42,900	0,000	0,767	0,841
medium	0,948	0,038	24,744	0,000	0,873	1,023
large	-33,037	2,88E-06	-1,15E-05	1,000	-5,64E-06	5,64E-06

Optimal Threshold	0,631
AUC	0,80
Type I Error Count	3161
Type II Error Count	189
Ratio Type I/Type II	16,72

Company Size					
	Micro	Small	Medium	Large	Total
Active	74,34%	22,03%	3,49%	0,14%	100,00%
Inactive	55,02%	36,84%	8,08%	0,06%	100,00%

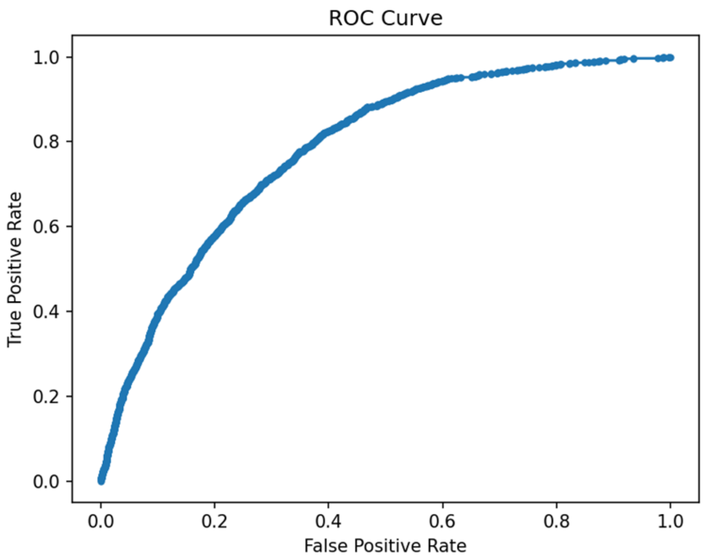


- **Model 6**

Optimization - Model 6						
	Coef	Std err	Z	P>|z|	[0.025	0.975]
const	-0,146	0,014	-10,190	0,000	-0,174	-0,118
WCTA	-0,312	0,018	-17,129	0,000	-0,348	-0,276
RETA	0,058	0,009	6,409	0,000	0,041	0,076
EBITTA	-2,203	0,049	-44,524	0,000	-2,300	-2,106
BVETD	-0,263	0,009	-30,654	0,000	-0,280	-0,246
6_to_15y	0,3954	0,019	20,376	0,000	0,357	0,433
less_than_6y	0,8834	0,023	39,197	0,000	0,839	0,928

Optimal Threshold	0,607
AUC	0,79
Type I Error Count	5606
Type II Error Count	91
Ratio Type I/Type II	61,6

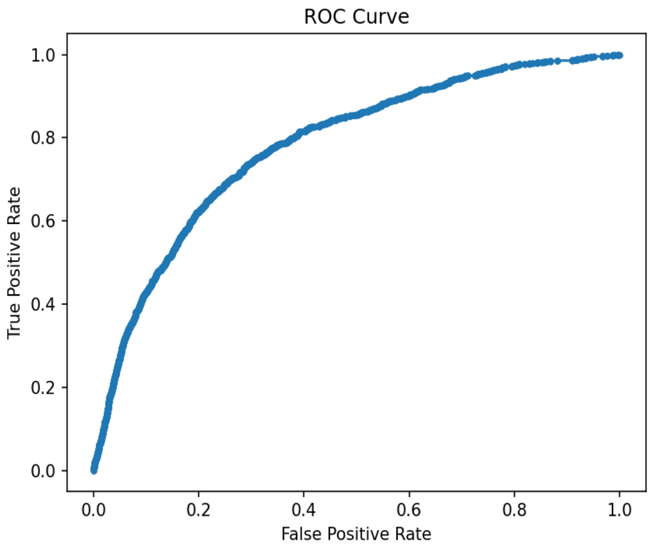
Company Age			
	< 6 Y	6 to 15 Y	> 15 Y
Active	15,14%	27,53%	57,33%
Inactive	33,18%	27,87%	38,95%



- **Model 7**

Optimization - Model 7						
	Coef	Std err	Z	P>|z|	[0.025	0.975]
const	0,345	0,012	29,785	0,000	-0,279	-0,251
WCTA	-0,374	0,018	-20,208	0,000	0,002	0,008
RETA	0,117	0,009	12,931	0,000	-0,004	-0,001
EBITTA	-2,185	0,049	-44,412	0,000	-0,010	-0,003
BVETD	-0,330	0,009	-36,524	0,000	0,251	0,265
construction	-0,056	0,027	-2,049	0,041	0,865	0,988
wholesale and retailing	-0,706	0,047	-15,034	0,000	-0,171	-0,098
manufacturing	-0,299	0,038	-7,760	0,000	0,504	0,632
energy and water production	-2,819	0,654	-4,311	0,000	-0,195	-0,085
information technology	-0,257	0,120	-2,142	0,032	1,855	4,180
Restaurants and Hotels	-0,927	0,031	-29,634	0,000	-0,045	0,266

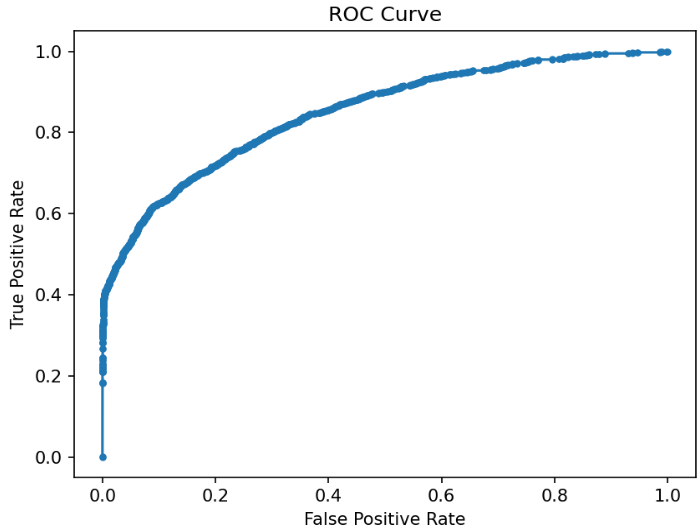
Optimal Threshold	0,623
AUC	0,79
Type I Error Count	4167
Type II Error Count	125
Ratio Type I/Type II	33,34



- **Model 8**

Optimization- Model 8						
	Coef	Std err	Z	P>|z|	[0.025	0.975]
const	89,926	2,054	43,783	0,000	85,901	93,952
WCTA	-0,338	0,020	-17,219	0,000	-0,376	-0,299
RETA	0,157	0,011	14,748	0,000	0,136	0,178
EBITTA	-1,946	0,051	-37,981	0,000	-2,046	-1,845
BVETD	-0,278	0,010	-29,141	0,000	-0,297	-0,259
GDP	-0,3534	0,008	-43,901	0,000	-0,369	-0,338

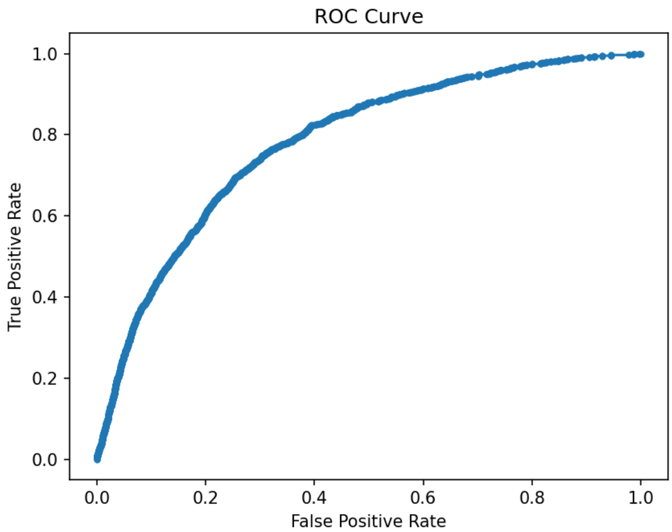
Optimal Threshold	0,606
AUC	0,88
Type I Error Count	1921
Type II Error Count	142
Ratio Type I/Type II	13,53



- **Model 9**

Optimization -Model 9						
	Coef	Std err	Z	P>|z|	[0,025	0,975]
const	0,249	0,010	24,215	0,000	-0,632	-0,593
WCTA	-0,389	0,019	-20,237	0,000	-0,012	0,006
RETA	0,142	0,009	15,261	0,000	-0,014	-0,005
EBITTA	-2,020	0,046	-44,261	0,000	-0,008	0,015
BVETD	-0,383	0,010	-38,804	0,000	1,472	1,538
Int Cov	-2,63E-06	8,96E-07	-2,936	0,003	1,54E-06	4,95E-06

Optimal Threshold	0,621
AUC	0,81
Type I Error Count	4223
Type II Error Count	101
Ratio Type I/Type II	41,81



- **Model All**

Optimization – Model All							
	Coef	Std err	Z	P>|z|	[0,025	0,975]	
const	1,07E+04	2,57E+05	0,041	0,967	-4,93E+05	5,14E+05	10650,000
WCTA	-0,03	0,023	-1,167	0,243	-0,072	0,018	-0,027
RETA	0,19	0,013	14,633	0,000	0,163	0,213	0,188***
EBITTA	-0,95	0,057	-16,535	0,000	-1,057	-0,833	-0,945***
BVETD	-1,87	0,034	-55,538	0,000	-1,935	-1,803	-1,869***
year_2019	-669,10	3,93E+04	-0,017	0,986	-7,77E+04	7,63E+04	1,076
year_2020	-110,30	3,80E+04	-0,029	0,977	-7,55E+04	7,33E+04	-110,302
year_2021	16,23	3,00E+07	-5,41E+07	1,000	-5,88E+04	5,88E+07	16,229
year_2022	-69,58	9,29E+03	-0,007	0,994	-1,83E+04	1,81E+04	-69,581
small	1,08	0,03	37,046	0,000	1,019	1,133	1,076***
medium	0,88	0,061	14,302	0,000	0,759	1,000	0,880***
large	-17,00	1657,262	-0,01	0,992	-3265,178	3231,17	-17,004
6_to_15y	0,07	0,03	2,459	0,015	0,015	0,133	0,074*
less_than 6y	0,45	0,035	12,913	0,379	0,379	0,515	0,447
restaurants	-2,50	0,072	-34,825	0,000	-2,641	-2,359	-2,500***
construction	-0,64	0,045	-14,203	0,000	-0,733	-0,555	-0,644***
wholesale	-2,51	0,144	-17,407	0,000	-2,794	-2,228	-2,511***
manufacturing	-1,25	0,072	-17,344	0,000	-1,39	-1,108	-1,249***
energy	-2,35	0,768	-3,064	0,020	-3,858	-0,848	-2,353*
information	-3,30	0,597	-5,832	0,000	-4,469	-2,131	-3,300***
gdp	-41,45	1009,191	-0,041	0,967	-2019,425	1936,532	-41,446