Residence Time Distribution of Liquids in a Continuous Tubular Thermal Processing System Part I: Relating RTD to Processing Conditions

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ABSTRACT

A systematic experimental study on residence time distribution (RTD) in tubular flow was performed, covering a wide range of processing conditions. The flow of water in various sections of a tubular continuous thermal processing system was analysed using the classical Danckwerts approach. Methylene blue was used as tracer and different constant temperatures (25–80°C) and flow rates (80–380 l/h) were tested. These conditions yielded mean residence times up to 6 min and Reynolds numbers between 1350 and 9700. Various models were fitted to the experimental data, and the dispersion model showed to yield the best fit. Peak analysis led to both accurate and precise as well as conservative parameters, when compared to other methods of parameter estimation. Results revealed that fluid dispersion in tubular flow (Peclet number) can be related to processing conditions (Reynolds number) by a power law model. Results were compared to published correlations.

NOTATION

c	Concentration (in the time domain)
CST	Continuous stirred tank
d	Diameter (m)
D	Axial dispersion coefficient (m^2/s)

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$\mathscr{D}$	Diffusion coefficient (m^2/s)
Da	Damköhler number ($Da = k\tau$)
DPF	Dispersed plug flow
E	Residence time distribution (s^{-1})
H	Heavyside function
k	Reaction rate constant (min^{-1})
L	Length (m)
LF	Laminar flow
N-CST	N continuous stirred tanks in series
Pe	Peclet number ($Pe = vL/D$)
Pe_a	Axial Peclet number ($Pe = vd/D$)
Q	Flow rate (l/h)
R^2	Regression coefficient of determination
Re	Reynolds number ($Re = \rho vd/\mu$)
RTD	Residence time distribution
Sc	Schmidt number ($Sc = \mu/\rho D$)
SSR	Sum of squares of residuals
t	Time (s)
TF	Turbulent flow
v	Velocity (m/s)
V	Volume (l)
$\alpha\beta\gamma\delta$	Parameters
ρ	Density (kg/m^3)
τ	Mean holding time (s)
μ	Viscosity (poise)

Subscripts

e	Output, exit
i	Input
max	Maximum
pred	Predicted

INTRODUCTION

Aseptic processing is widely applied to pasteurise and sterilise fluid foods (e.g. milk, fruit juices and concentrates, cream, yoghurt, wine, salad dressing, egg and ice cream) and foods containing small discrete particles (e.g. cottage cheese, baby foods, tomato products, soups and rice desserts). Currently major research efforts are being developed in the field of continuous thermal processing of liquid foods containing large particles (Lee *et al.*, 1995; Baptista *et al.*, 1995, 1997). The main advantage of continuous thermal processes versus in-container pasteurisation or sterilisation is the quality improvement of the product due to higher heating and cooling rates (Hersom, 1985). Unfortunately, as previously noted by Lin (1979), the

full potential of continuous thermal processes is yet to be attained due to difficulties in describing accurately the flow characteristics in processing equipment.

The degree of thermal processing of an aseptically processed food product depends not only on the temperature the product is subjected to, but also on the residence time of the product in the processing unit. The knowledge of the residence time distribution (RTD) in the system is therefore essential for an accurate estimation of the degree of processing. Aseptic processes may therefore be optimised (maximising product quality and avoiding overprocessing, while ensuring product safety) by choosing the adequate time/temperature and flow conditions, so that important side deteriorative reactions related to product quality, as well as dispersion, can be minimised (Burton, 1958; Rao & Loncin, 1974*b*; Lund, 1977; Holdsworth & Richardson, 1986; Kessler, 1989; Maesmans *et al.*, 1990).

Experimental RTD determination in aseptic processing, particularly in holding tubes or test rigs, have been reviewed in literature (Rao & Loncin, 1974*a*; Lin, 1979; Burton, 1988; Holdsworth, 1992; Pinheiro Torres & Oliveira, 1998). Water is the fluid most widely used in these studies, and salt the most common tracer. Model food products such as sucrose and guar gum were used by Sancho and Rao (1992), polyethylene glycol by Milton and Zahradnik (1973) and glycerol–water mixtures by Trommelen and Beek (1971). RTD studies on real products were performed on milk by Cerf and Hermier (1973), Nassauer and Kessler (1979), Heppell (1985) and Janssen (1994). No studies were found considering non-isothermal conditions, and most of the reported studies were conducted at room temperature.

Mathematical models for fluid flow in tubular systems (such as holding tubes) at isothermal conditions were presented by Levenspiel and Bischoff (1963), Levenspiel (1972), Rao and Loncin (1974*a*) and Wen and Fan (1975). These models are often based on the classical RTD Danckwerts' approach (Danckwerts, 1953). Ideal RTD models like plug flow (PF) and continuous stirred tank (CST) rarely reflect a real situation close enough. Non-ideal models are therefore applied, accounting for the deviation in real systems. These models vary in complexity, but one parameter models are often adequate enough to represent tubular vessels (Levenspiel & Bischoff, 1963). The model for the axial dispersed plug flow (DPF), or simply the dispersion model, has been widely applied by chemical engineers to describe the flow in tubes, and is the most frequently selected to simulate flow in holding tubes in aseptic processing (Aiba & Sonoyama, 1965; Nassauer & Kessler, 1979; Wennerberg, 1986; Sancho & Rao, 1992). The N-CST model has also been applied to describe flow in tubes (Levenspiel & Bischoff, 1963; Wen & Fan, 1975) and to aseptic processes (Roig *et al.*, 1976; Wennerberg, 1986; Malcata, 1991; Sancho & Rao, 1992). Further models proposed for describing the experimental results were the velocity distribution model (Aiba & Sonoyama, 1965; Nassauer & Kessler, 1979) and the N-CST model with a PF in series (Veerkamp *et al.*, 1974).

Quantitative analysis of the effect of process parameters in the RTD of holding tubes was not addressed in most studies. Sancho and Rao (1992) reported that RTD, expressed in terms of a dispersion coefficient, was higher for laminar than for turbulent flow, independently of the nature of the liquid.

The purpose of this work was: (1) to model fluid flow in a tubular continuous processing system by application of the RTD Danckwerts analysis, (2) to assess the influence of processing conditions (temperature, flow rate and tube length) on the model parameters and (3) to develop dimensionless correlations that might be used to estimate RTD for different processing conditions.

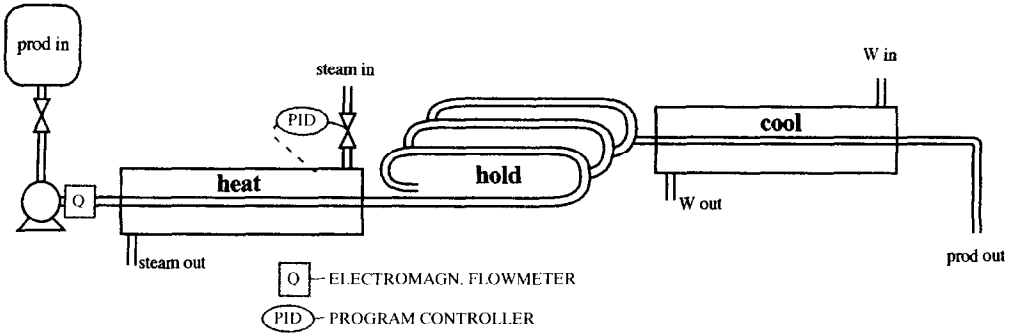

Fig. 1. Schematic representation of the thermal processing unit.

MATERIALS AND METHODS

The continuous thermal processing system

Residence time experiments were conducted in a continuous thermal processing pilot plant (Pinheiro Torres & Oliveira, 1993). This system (Fig. 1) includes: feed tanks, a positive displacement pump (Eco Gearcham G4, Pulsafeeder), concentric double tube heat exchangers for heating and cooling the product, and a holding tube with variable length and the required upward inclination of 1 in/ft (all tubular sections with 2·2 cm ID). This system was, however, always operated at constant temperature in the different sections. Water was run at various temperatures (25–80°C) and flow rates (80–380 l/h) giving Reynolds numbers (*Re*) from 1350 to 9700, thus covering a range of laminar, transient and turbulent fluid flow. This system was coupled to a tracer injection system and an on-line tracer detection system, consisting of a continuous sampler connected to a spectrophotometer with a data acquisition system specially developed for this purpose.

Experimental method

Residence time experiments were conducted using methylene blue as tracer. A pulse of tracer was injected into the flow through a syringe, which was mounted on an independent piece of tubing added in the desired injection point. Preliminary experiments showed that best input signals could be achieved with 5 ml samples. A net (mesh size 0·8 mm) was inserted in the cross section of the tube, immediately after injection, in order to break the velocity profile and improve tracer distribution. The ideality of the injection was checked by measuring the response shortly after injection. Continuous on-line sampling was made at the outlet of the section under analysis, and absorbance recorded every second by a spectrophotometer (Spectronic 6820), at wavelength of 663 nm, connected to a PC (Bull Micral through an RS232 port). This continuous sampling device was found to give results equivalent to cup sampling. Tracer concentration was selected for each flow rate to give an output signal of a maximum of 1·0 in absorbance, thus guaranteeing that absorbance would fall in the range where the Lambert-Beer's law is valid.

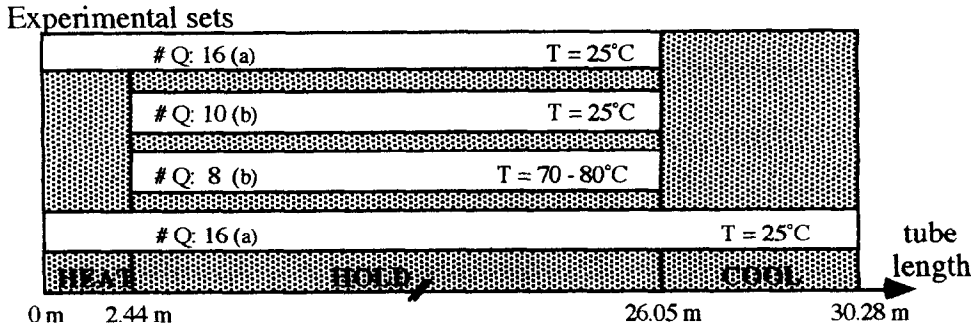

Fig. 2. Experimental conditions tested (number of flow rates tested in the range of (a) 80–380 l/h, and (b) 100–250 l/h).

Overall, 50 experiments were performed, covering different combintions of temperature and flow rate (Fig. 2). For each set of processing conditions, two replicates were conducted.

Raw data treatment

RTD curves were standardised as follows:

$$E(t) = \frac{c_e(t)}{\int_0^{\infty} c_e(t)\mathrm{d}t} \tag{1}$$

Mean holding times were calculated as the ratio between the volume of the corresponding test section (V) and the flow rate (Q):

$$\tau = \frac{V}{Q} \tag{2}$$

Mean residence times were also estimated by statistical analysis of the RTD curves. Experimental results where the mean holding time τ differed significantly from the mean residence time obtained from the RTD curve (above 10%) were rejected, as this indicated that a channelling in the fluid circuit has occurred. *Re* was calculated for each set of processing conditions.

RESULTS AND DISCUSSION

Figure 3 shows selected curves, that allow us to analyse the effect of flow rate (Fig. 3a), temperature (Fig. 3b) and tube length (Fig. 3c) on RTD. As would be expected, dispersion and tailing effects increase as flow rate and/or temperature decrease (lower *Re*). When comparing different test sections at ambient temperature (Fig. 3c), it can be concluded that dispersion increases with tube length.

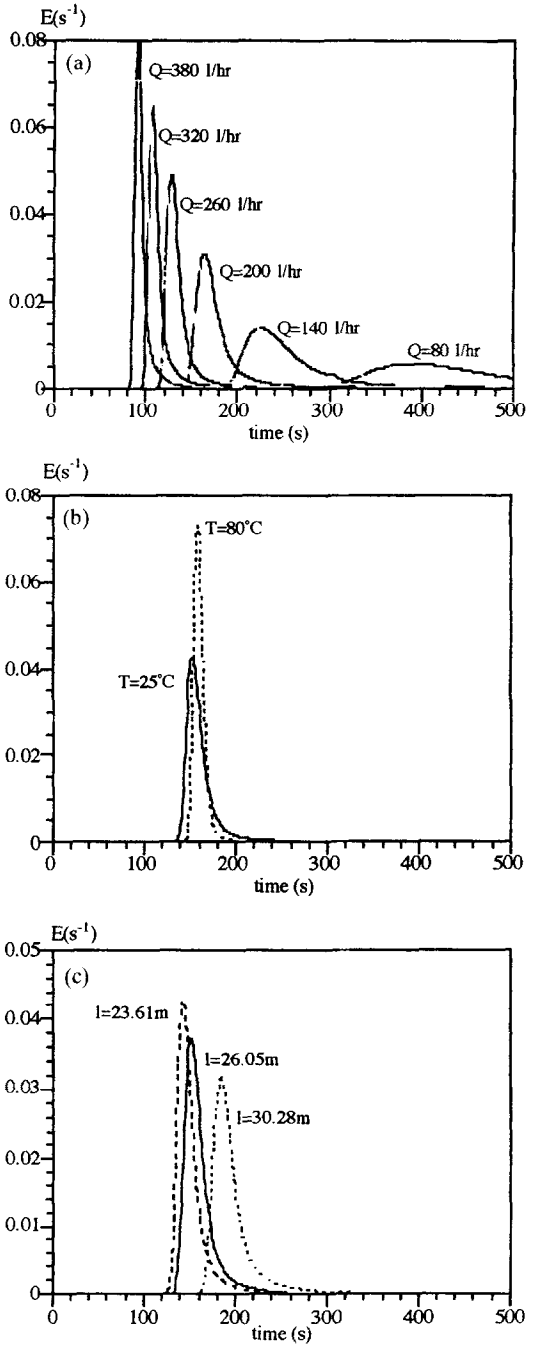

Fig. 3. Effect of operating conditions on typical RTD curves: (a) effect of flow rate, (b) effect of temperature and (c) effect of tube length.

TABLE 1
Models Tested for Describing RTD Curves with Tailing Effects

Type of model	*Model details*
1. Classical RTD models applied to tubular systems	1a. DPF
	1b. N-CST
2. Classical RTD models predicting long tails	2a. DPF with stagnant zone
	2b. N-CST with stagnant zone — model G (Levenspiel, 1972)
3. Other/new models	3a. DPF with outer annulus in laminar flow
	3b. DPF in paralle with LF
	3c. DPF in parallel with TF
	3d. DPF in parallel with PF followed by a 2nd DPF
	DPF in parallel with PF followed by 2 parallel CSTs

Modelling of the RTD curve

Tailing effects are common in published results and may be due to improper injection (Milton & Zahradnik, 1973) or indicate laminar, transient or low turbulent flow regimes. These effects are often difficult to include in theoretical models but, according to Levenspiel (1972), and with regard to conversion, the tail correction usually does not justify the extra modelling work. In a previous work (Pinheiro Torres *et al.*, 1993) we have applied different flow models to account for this tailing (Table 1). Models were fitted to experimental data by non-linear regression and compared in terms of the sum of squares of the residuals (SSR) between experimental data and the model, for three typical experimental curves. The dispersed plug flow model (DPF) was found to yield the smallest SSR, although not predicting the tailing effect. Furthermore, it has the advantage of requiring the estimation of only one parameter, the Peclet number (Pe):

$$E(t) = \sqrt{\frac{Pe}{4\pi\tau t}} \exp\left\{ - \frac{Pe(t-\tau)^2}{4t\tau} \right\} \tag{3}$$

where Pe is a dimensionless measure of the axial dispersion, D:

$$Pe = \frac{vL}{D} \tag{4}$$

Various techniques were proposed by different authors to estimate this parameter (Pinheiro Torres & Oliveira, 1998). These include (i) analysis of specific characteristics of the RTD curve (Levenspiel, 1972), such as peak time, peak height, inflection points and difference of residence times of the middle 68% portion of the RTD curve, (ii) statistical analysis (Levenspiel & Bischoff, 1963), (iii) curve fitting

by non-linear optimisation (Fahim & Wakao, 1982; Rangaiah & Krishnaswamy, 1990), (iv) Laplace domain curve fitting (Hopkins *et al.*, 1969) and (v) analysis of RTD curves obtained from non-ideal input functions in the Laplace domain (Østergaard & Michelsen, 1969).

Analysis of specific characteristics of the RTD curve (except for peak height) and statistical analysis were found to yield inconsistent and inaccurate results in our study. The remaining methods for parameter estimation were then compared, considering both the precision of the parameter and its ability to predict accurately and conservatively the extension of a first order reaction in the system (non-linear optimisation and Laplace domain curve fitting could, however, only be tested when ideal injection was observed). The final or exit concentration of a quality factor, c_e, for first order reactions under isothermal conditions, may be calculated as (Levenspiel & Bischoff, 1963):

$$\frac{c_e}{c_i} = \int_0^\infty e^{-kt} \times E(t)dt \tag{5}$$

where c_i is the initial or input concentration and k the reaction rate constant. For the DPF model, this yields (Wehner & Wilhelm, 1956):

$$\left(\frac{c_e}{c_i}\right) = f(Pe,Da) = \frac{4\alpha e^{Pe/2}}{(1+\alpha)^2 e^{\alpha Pe/2} - (1-\alpha)^2 e^{-\alpha Pe/2}} \tag{6}$$

with:

$$\alpha = \sqrt{1+4Da/Pe} \tag{7}$$

where the Damköhler number ($Da = k\tau$) is a dimensionless form of the reaction rate constant.

Accuracy was defined as the difference between the reaction extension estimated with eqn (5) and eqn (6). Conservativeness implies that this difference should be positive. A reference reaction rate of $0{\cdot}01\ s^{-1}$ was chosen so that a significant reaction (conversion from 0·6 to 0·9) would occur in the range of the processing conditions tested. Figure 4 shows that the different procedures tested led to acceptable results, with errors in general lower than 10%. Methods using the Laplace domain analysis yielded the most accurate results, though systematically non-conservative. Both peak analysis and non-linear optimisation led to conservative predictions, except for *Re* below 3200. Peak analysis was chosen for further studies, as it was found to be more accurate than non-linear optimisation. Furthermore, this is also the simplest and fastest method for parameter estimation, and does not require an ideal injection.

Pe was therefore estimated from the maximum value of the *E*-curve (E_{max}) as follows:

$$Pe = 4\pi\tau^2 E_{max}^2 \tag{8}$$

Figure 5 shows an example of the fit between the DPF model and the experimental data, both for small and large dispersions. It was found that good fits were obtained for turbulent flow.

Relating Peclet to operating conditions

Several correlations relating axial diffusion ($\mathscr{D}$) to Re were found in literature for tube flow of Newtonian fluids (Table 2). Correlations in laminar flow also include the Schmidt number ($Sc = \mu/\rho\mathscr{D}$). All the correlations were developed in terms of the axial Peclet number (Pe_a):

$$Pe_a = Pe \times \frac{d}{L} = \frac{vd}{D} \tag{9}$$

It should be stressed that these correlations were developed for other engineering processes. For laminar flow, the models were based on fluid mechanics theory

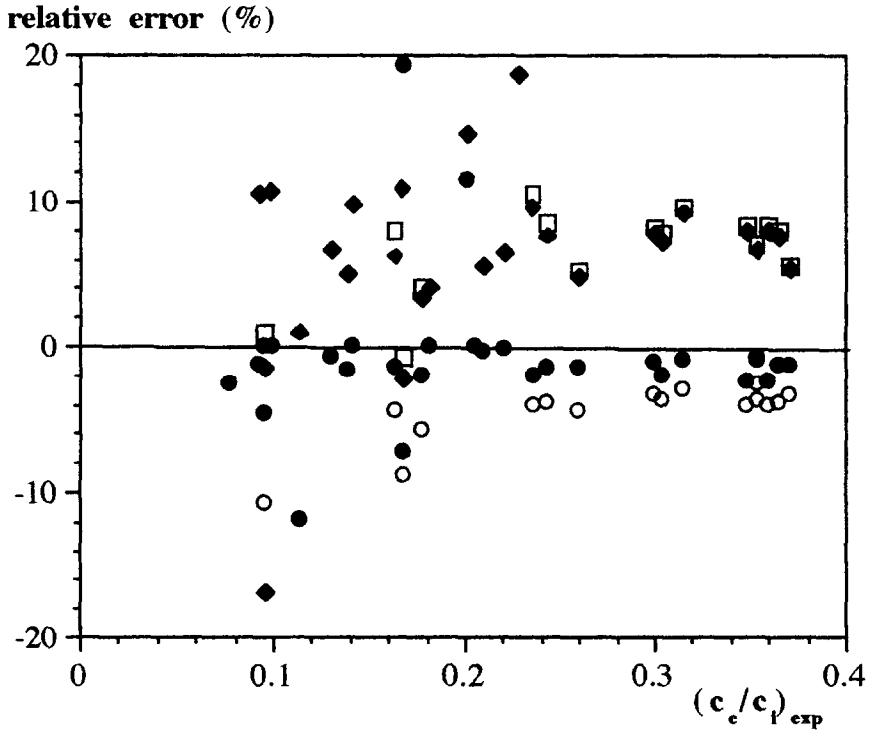

Fig. 4. Effect of the method of parameter estimation on the accuracy of predictions of concentration decay for a first order reaction with rate constant of 0·01 s^{-1}, for $Re > 3200$: (◆) peak analysis, (□) time domain curve fitting, (○) Laplace domain curve fitting, assuming ideal injection of tracer, and (■) Laplace domain curve fitting, assuming nonideal injection of tracer.

(Levenspiel, 1958; Levenspiel & Bischoff, 1963; Wen and Fan, 1975); predicted Pe_a values were compared to experimental data (Wen & Fan, 1975), showing relative errors from -76% to 40%, for $Re \times Sc$ ranging from 1 to 2000. For turbulent flow, the models of Taylor (1954) and Wen and Fan (1975) were also based on fluid mechanics theory, considering axial dispersion. The latter model includes an additional term that accounts for axial diffusion (Aris, 1959). Further discussion on the theoretical basis of these models may be found in Levenspiel and Bischoff (1963). The remaining two models are empirical: Nassauer and Kessler's model is identical to Taylor's model, although with different values for the parameters; Sjenitzer's model is similar to Taylor's model, but includes the effect of the ratio between tube length and diameter. Using the concept of nested models, Sjenitzer's can be seen as a full model, whereas Taylor and Nassauer and Kessler's models are partial models. We have analysed the data used by Sjenitzer (1958), finding that the regression coefficient of determination was considerably low ($R^2 = 0{\cdot}61$).

Experimental data reported in literature (Pe_a vs Re) show a large scatter (Levenspiel & Bischoff, 1963; Levenspiel, 1958; Sjenitzer, 1958; Wen & Fan, 1975). Taylor's model was found to be, in general, relatively adequate to describe these results (Levenspiel, 1958), but it was unable to describe an experimentally observed sharp increase in Pe_a for $Re < 10000$ (Levenspiel & Bischoff, 1963).

Figure 6 shows how Pe_a increases with Re, for all the experiments, conducted in our work. A coefficient of determination of 0·25 was found between the variables L

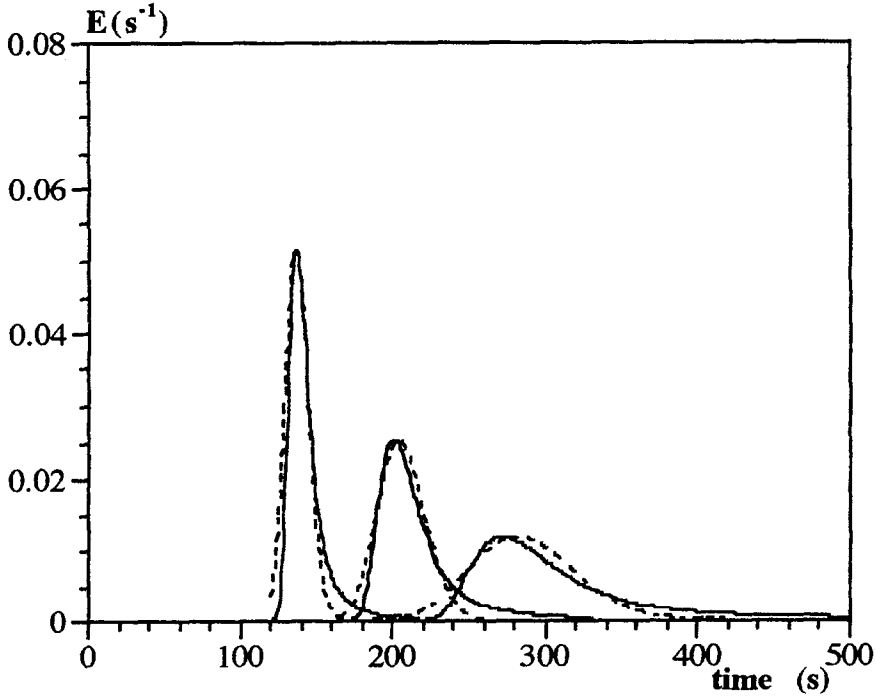

Fig. 5. Typical experimental (full line) and predicted (dotted line) RTD curves at different flow rates for the holding section.

and Pe_a, meaning that there is statistical evidence that L contributes 25% to the variability of Pe_a, and should therefore be included in the model.

Predictions for laminar flow were analysed, considering a diffusion coefficient of $D = 2{\cdot}3 \times 10^{-9}$ m^2/s, estimated from a published correlation (Geankoplis, 1983). Estimated Pe_a values were however 2–4 orders of magnitude lower than our experimental results, probably because of the range of $Re \times Sc$ tested ($5{\cdot}0 \times 10^5$ to $7{\cdot}8 \times 10^5$).

Figure 6 includes the curves predicted for turbulent flow (Table 2). It can be seen that the model of Wen and Fan (1975) best approaches our results, although clearly showing a biased pattern, overpredicting the Pe_a values. The other models show very poor predictions. We have then, for turbulent flow ($Re > 3000$), estimated the parameters (α and β; α, β and γ; and α, β, γ and δ, depending on the model — see Table 3) that would lead to the best fit of the models described in Table 2 to our data. We have also considered the case of $\gamma = -1$ for simplification of Sjenitzer's model, thus creating a partial model including the effect of L. This power model suggests a different interpretation of the relation between fluid dispersion and Re (axial disper-

TABLE 2
Published Correlations Relating Dimensionless Fluid Dispersion to Processing Conditions

Reference	*Correlation*	*Restrictions*
	Laminar flow: $1/Pe_a = f(Re, Sc)$	
Levenspiel (1958)	$\frac{1}{Pe_a} = \frac{Re \times Sc}{192}$	$Re < 2100$
Wen & Fan (1975)	$\frac{1}{Pe_a} = \frac{1}{Re \times Sc} + \frac{Re \times Sc}{192}$	$Re < 2000$
	Turbulent flow: $1/Pe_a = f(Re, L/d)$	
Taylor (1954)	$\frac{1}{Pe_a} = \frac{2{\cdot}010}{Re^{0{\cdot}125}}$ *	$Re > 3000$
Nassauer & Kessler (1979)	$\frac{1}{Pe_a} = \frac{0{\cdot}766}{Re^{0{\cdot}1}}$ *	$Re > 3000$
Sjenitzer (1958)	$\frac{1}{Pe_a} = \frac{1429}{Re^{0{\cdot}9}} \left(\frac{L}{d}\right)^{0{\cdot}141}$ *	$Re > 3000$
Wen & Fan (1975)	$\frac{1}{Pe_a} = \frac{3{\cdot}0\text{E}7}{Re^{2{\cdot}1}} + \frac{1{\cdot}35}{Re^{0{\cdot}125}}$	$Re > 3000$

*Altered using the Blasius equation, which relates the friction factor to Re (Bird *et al.*, 1960), for $3000 < Re < 100\,000$ and smooth pipes.

sion being proportional to tube length — through Pe, and not to the tube diameter — through Pe_a).

Results from optimisation were analysed with respect to lack of fit (ANOVA), adjusted correlation coefficient (R^2_{adj}), analysis of residuals in terms of their normality (Shapiro and Wilk test), independence (correlation to the estimated values), zero mean and detection of outsiders. Parameters were evaluated towards the null hypothesis. Nested models were confronted through an extra SSR analysis (Bates & Watts, 1988). Table 3 summarises the models tested, the parameters estimates, as well as the most relevant statistical data.

Wen and Fan's model converged to new parameter values which showed to accept the null hypothesis, turning to Taylor's model, and was thus rejected. For the remaining models there was no statistical evidence that the lack of fit should be accepted, thus suggesting a valid relationship between Pe_a and Re. Also, no outsiders were found. The best fit was obtained for both Sjenitzer's and the power models, though extra SSR analysis for nested models showed that there is no statistical evidence that Sjenitzer's model describes better our results than the power model. Additionally, the null hypothesis was accepted for the parameter α in the former model, and a correlation of $-0{\cdot}97$ was found between parameters α and γ. Therefore, the parameters and corresponding standard errors of the power model, for $Re > 3000$, are:

$$Pe_a = (0{\cdot}015 \pm 0{\cdot}003)(d/L)Re^{(1{\cdot}26 \pm 0{\cdot}22)} \tag{10}$$

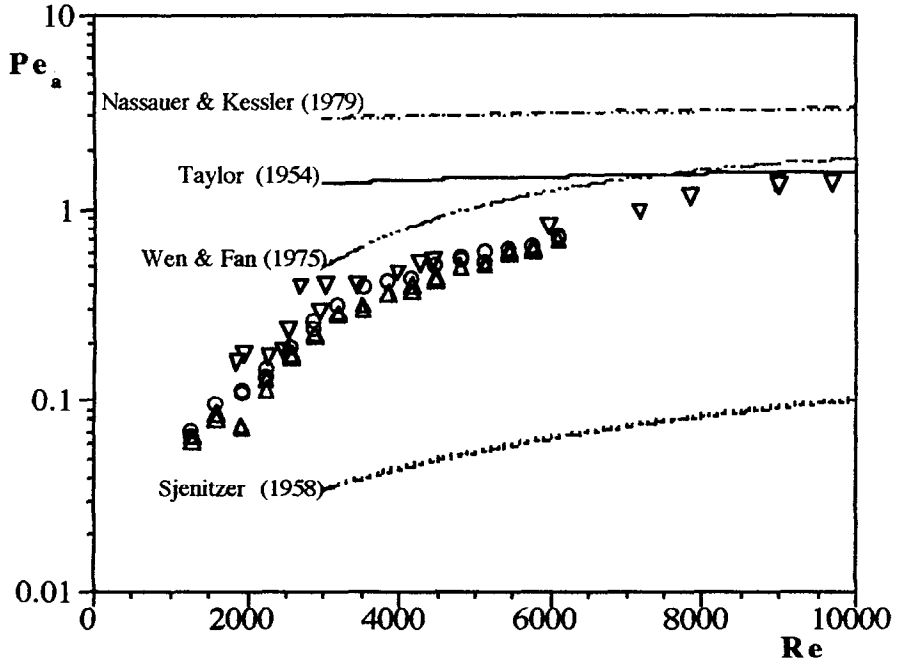

Fig. 6. Experimental Pe_a and values obtained from published correlations, for different tube lengths: (▽) L = 23·61 m, (○) L = 26·05 m, and (△) L = 30·28 m.

TABLE 3
Summary of Results from the Fit of Various Models to Experimental Data (for $Re > 3000$)

Basis model for optimisation	$R^2_{adj.}$	*Analysis of residuals**	*Extra SSR analysis for nested models***	*Parameters (significance level of H_0)*
Wen & Fan (1975): $Pe_a = (\alpha\, Re^{\beta} + \gamma\, Re^{\delta})^{-1}$	96·68%	No	–	H_0 accepted (turns into Taylor's model)
Taylor (1954)/Nassauer & Kessler (1979): $Pe_a = \alpha\, Re^{\beta}$	99·42%	No	Partial model not accepted	H_0 rejected: $\alpha = 2{\cdot}3\text{E-}6$ H_0 rejected: $\beta = 1{\cdot}45$
Sjenitzer (1958): $Pe_a = \alpha\, Re^{\beta}\,(L/d)^{\gamma}$	99·78%	Yes	Full model	H_0 accepted: $\alpha = 2{\cdot}9\text{E-}3$ (78%), $\alpha = 0$ (22%) H_0 rejected: $\beta = 1{\cdot}29$ H_0 rejected: $\gamma = -0{\cdot}81$
Power: $Pe_a = \alpha Re^{\beta}\,(L/d)^{-1}$	99·77%	Yes	Partial model accepted	H_0 rejected: $\alpha = 0{\cdot}015$ H_0 rejected: $\beta = 1{\cdot}26$

*Zero mean, normal distribution (Shapiro & Wilk test for normality) and independency of residuals (no significant correlation with predicted independent variable).
**Bates & Watts (1988).

Equation (10) can be simplified using eqn (9), and leads to a relation which, unlike all other models proposed in literature (Table 2), relates Pe, and not Pe_a, to Re:

$$Pe = 0{\cdot}015 Re^{1{\cdot}26} \text{ for } Re > 3000 \tag{11}$$

The fit of this model to our experimental data is presented in Fig. 7, and obviously describes better the experimental results than the published correlations. Note that the power model is here linearised by using a log–log scale, and that, unlike in Fig. 6, the y-axis corresponds to the Pe, and not to the Pe_a number. The histogram of the residuals shows a normal tendency of the residuals for $Re > 3000$.

For Re below 3000, Fig. 7 suggests also a power model, but with different parameters. These were found to be:

$$Pe = (1{\cdot}55 \pm 0{\cdot}12) \times 10^{-4} \times Re^{(1{\cdot}81 \pm 0{\cdot}10)} \text{ for } Re < 3000 \tag{12}$$

This shows that for laminar flow the ratio between hydrodynamic and diffusional flow is rather small and the effect of increasing Re is greater, as might be expected. If one wants to rewrite this equation including Sc, then:

$$Pe = 3{\cdot}45 \times 10^{-9} \times (Sc \times Re)^{1{\cdot}81} \text{ for } Re < 2100 \tag{13}$$

It should however be stressed that all our experiments in this range of Re were conducted in conditions such that Sc is equal to 372, and thus no conclusions can be taken in relation to the effect of Sc on RTD.

Finally, we have analysed if there was a statistical evidence that the transition between these two zones is around $Re = 3000$. This was tested by performing a joint optimisation of parameters for all the range of Re, according to the following equation:

$$Pe = H(Re - Re_{trans}) \times \alpha Re^{\beta} + H(Re_{trans} - Re) \times \gamma Re^{\delta} \tag{14}$$

where Re_{trans} is the value of Re of transition and H is the Heavyside function.The best fit was indeed found when considering the transition around the considered

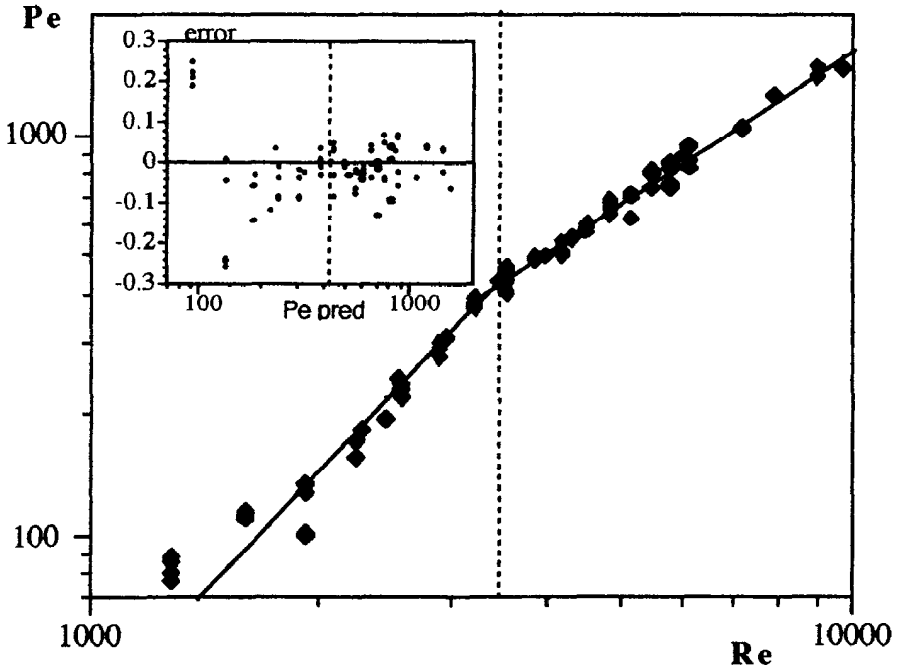

Fig. 7. Fit of the power law model to the experimental data for different flow regimes and corresponding histogram of residuals.

value of $Re_{trans} = 3000 (R^2 = 99{\cdot}75\%)$, thus validating the results presented in eqn (11) and eqn (12). However this function is not continuous in the transition between both equations. By optimising the same eqn (14), subjected to the constraint of being continuous at $Re = Re_{trans}$, results were slighly different ($R^2 = 99{\cdot}71\%$):

$$Pe = \begin{cases} (2{\cdot}9 \pm 2{\cdot}8) \times 10^{-5} \times Re^{(2{\cdot}03 \pm 0{\cdot}12)} & Re < Re_{trans} \\ (1{\cdot}6 \pm 0{\cdot}3) \times 10^{-2} \times Re^{(1{\cdot}25 \pm 0{\cdot}03)} & Re > Re_{trans} \end{cases} \tag{15}$$

$$\text{with } Re_{trans} = 3300 \pm 170$$

Again, if one wants to rewrite the equation including Sc, for laminar flow, then:

$$Pe = 1{\cdot}8 \times 10^{-10} \times (Sc \times Re)^{2{\cdot}03} \text{ for } Re < 2100 \tag{16}$$

The considerations taken in eqn (13) on the effect of Sc on Pe hold for eqn (16).

CONCLUSIONS

Quantitative analysis and modelling of RTD on model foods is required, before extending conclusions to real fluid foods, and applying systematically RTD to the

evaluation and optimisation of continuous thermal processing. For the processing conditions tested in this work, RTD was adequately described by the dispersion model. Results also revealed that, in the range studied, fluid dispersion in tubular flow (through *Pe*) was related to processing conditions (through *Re*) by a power law model, with *Re* showing a greater effect in the laminar regime. These results may allow the establishment of processing conditions that guarantee product safety, with a lower detrimental impact of product quality, when compared to the traditional calculations that assume laminar flow.

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