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MIND THE TAILS

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Abstract

Extremes have been gaining importance over time. This dissertation studies the impact of country's tail dependence directly in asset allocation. We propose a new measure of country's extreme dependence aggregating sector information. We use this new variable as a new characteristic in the asset allocation following Brandt and Santa-Clara (2006). The left tail dependence has a larger importance than the right tail, which corroborates findings from previous authors. We show that our OOS Sharpe ratios are of the order of 0.50 for the last two decades, twice as much as Brandt and Santa-Clara (2006). This outperformance is mostly prevalent during crisis periods. The results are robust to different levels of risk aversion.

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Supervisor

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Aknowledgments

Writing a dissertation is like running a marathon with handicap. It is a long and tiring effort but unlike that event you have to discover the way to finish line. Throughout the last months I found several dead ends, and every time I went back to race just to find another hurdle to challenge me. This behavior would not be possible without the help of many people which stood by me.

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1. Introduction

In his seminal paper, Markowitz (1952) creates an optimal rule for the allocation of risky assets in a static setting using a mean-variance utility function. However, the large estimation error in the covariance matrix leads to very extreme weighting and poor out of sample performance.[1]

Brandt and Santa-Clara (2006) present a conditional approach to asset allocation, using assets' characteristics as a tool to define the optimal weights. This technique reduces significantly the number of estimates. In the traditional Markowitz approach, the number of coefficients to estimate increases quadratically with the number of assets, while in Brandt and Santa-Clara's (2006), the method uses linear relationships.[2] Their asset allocation methodology allows for a better and more parsimonious allocation of three assets (T-Bill, Bonds, and Stocks) even though the impact is higher when the number of assets is larger.[3]

Over the past years, a new branch of literature has focused on the impact of extreme events on the underlying characteristics of assets, such as returns, volatility and dependence. Ruenzi and Weigert (2012) prove that investors require a premium to hold portfolios that have very high left tail dependence, to ensure against negative extreme events. Ang and Chen (2002) find an asymmetry in the dependence structure, being 11.6% larger in negative events than the correlation implied by the normal distribution. In the abnormal positive events the authors do not find significant differences in the dependence structure. In a different perspective, Kelly (2011) applies cross-sectional tail risk estimators to predict aggregate returns. Poon et al. (2004) study tail dependence for the largest

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[1] Over the years, several authors improved Markowitz method and introduced different utility functions or methods in order to increase the reliability of the asset allocation. See DeMiguel et al. (2009) which compare several of the different improvements in the asset allocation. The recent

[2] The reason for this difference is related to the estimation of the covariance matrix which is avoided with Brandt and Santa-Clara's (2006) method.

[3] Brandt et al. (2009) apply a similar technique in order to optimally allocate a portfolio using all the S&P 500 stocks.

international markets using extreme value theory (EVT).[4] They show that there are relevant differences in the asymptotic dependence at a country level.

Still, the use of extreme dependence measures has been out of focus in the asset allocation research. The traditional allocation methodologies do not allow for the inclusion of these measures and Brandt and Santa-Clara (2006) do not consider directly extreme dependence in their analysis. One of the exceptions is a paper from Wang et al. (2011) in which the authors use tail dependence as a tool to define the Markov regime switching in periods of high market risk. Overall, tail dependence does not have a direct role in the asset allocation research.

The main contribution of this dissertation is to revisit Brandt and Santa-Clara (2006), introducing tail dependence in the asset allocation problem. The tail dependence estimation will be based on Poon et al. (2004) although presenting a new approach that uses sector data to compute the country's tail dependence. This adequately captures the impact of the asymptotic dependence structure during extreme periods (either positive or negative) which allows for a better asset allocation.

In a first step, the tail dependence is estimated for all pairs of sectors of a particular country using a 20-year rolling window. This information is aggregated in a new time-series measure of tail dependence for the positive and negative joint extremes using all the cross-section information of a country, in a procedure similar to Kelly and Pruitt (2011). This new measure is used as a characteristic in the estimation of the optimal weights of a portfolio, using the method proposed by Brandt and Santa-Clara (2006). The methods that use tail dependence measures in the computation of the portfolio's weights outperform the unconditional and the conditional Brandt and Santa-Clara's (2006) allocations. The specification using the left tail dependence measure is the best performing, which is in line with Ang and Chen's (2002) findings. The OOS Sharpe ratio is 0.51, more than two times larger than Brandt and Santa-Clara's (2006), having also lower kurtosis, skewness and

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[4] Other authors (e.g. Patton (2009)) use Copula functions to model dependence structure. Hilal et al. (2011) argue that copula approach imposes conditions on the dependence structure that are too rigid and the validity of its assumptions was not tested. Still, EVT uses some of the foundations from the Copula approach, though using looser restrictions in the distributions used.

variability in the allocation weights. The improved performance of the specifications using tail dependence measures is attained mostly during crisis periods. This is clearer in the last financial crisis where Brandt and Santa-Clara's (2006) three characteristics are ineffective.

Some robustness checks are performed, including, for instance, other variables in the asset allocation, to capture the differences between the dependence measures, and the tail dependence measures' variations. The performance of several of these specifications beats the benchmarks, reaching Sharpe ratios of around 0.3. Finally, the analysis is extended to other large economies, using data for Germany, UK and Japan. The OOS Sharpe ratios are very distinct in the several specifications, but most of them beat the unconditional asset allocation.

The remainder of the dissertation is organized as follows. Section 2 explains the asset allocation methodology and their data. Section 3 explains the tail dependence, and their data. Section 4 presents the results for the US economy. Some robustness checks are performed in section 5. Section 6 extends the analysis to other major economies. Section 7 presents the concluding remarks.

2. Asset Allocation

2.1 The model

Traditionally, the investor maximizes her expected utility of next period's wealth by choosing the assets' weights $w_{i,t}$:

$$\max_{\{w_{i,t}\}_{i=1}^{N}} \mathrm{E}_t\left[u(1+r_{p,t+1})\right] = \mathrm{E}_t\left[u\left(\sum_{i=1}^{N} w_{i,t}(1+r_{i,t+1})\right)\right]. \tag{1}$$

where the return of each asset i from t to $t+1$ is $r_{i,t+1}$.

Brandt and Santa-Clara (2006) solve the asset allocation problem by parameterizing the portfolio weights $w_{i,t}$ as function of the K assets' characteristics $x_{i,t}$, which are common to the N assets,

$$w_{i,t} = f(x_{i,t};\theta_i) \tag{2}$$

They define a linear function of weights:

$$w_{i,t} = \theta_{i,0} + \theta_i^T x_{i,t} \tag{3}$$

where $\theta_{i,0}$ is a constant, and θ_i^T is a vector of parameters to be estimated.

The advantage of this methodology is that the number of parameters is the same and independent of the number of assets. One only has to estimate the parameters related to each characteristic (and one constant) for each risky asset.[5] For N risky assets, there is a reduction in the variables to estimate from N first and $N(N+1)/2$ second moments of returns to only N functions of portfolio weights, i.e., $(K+1)(N-1)$ parameters.

Following Brandt et al. (2009) a power utility function is used. This function takes into account all moments of the distribution of returns, considering the risk-return trade-off and also penalizing high kurtosis and negative skewness.[6]

The evaluation of the each allocation is made using the annualized Sharpe Ratio, in sample and out of sample. The difference between the two analyses is that the in sample estimation (IS) uses the whole sample to estimate the coefficients θ, while the out of sample (OOS) uses only past information to estimate θ in each moment in time. The initial estimation window uses the first 5 years (60 months) of data. The estimation window is expanded monthly with every reestimation. Brandt et al. (2009) use a similar method but with annual updating.

2.2 Data

2.2.1 Allocation Assets

The asset allocation methodology closely follows Brandt and Santa-Clara's (2006). The same three assets are used: stocks, long term (10-year) and short term (3-month) bonds. Monthly frequency is used in all asset allocations. This allows for a straightforward comparison of our results with theirs.

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[5] In the case of Brandt et al. (2009) the computations are even simpler, since the parameters are the same for all stocks.

[6] The mean-variance utility function is also tested but the results are not very interesting because the allocations use mainly the risk-free asset for low levels of risk aversion, independently of the specification defined.

The US MSCI index is used as a proxy for the stock market. This data is retrieved from Thompson DataStream. The analysis starts in January 1993 and ends in December 2010. The long term bond returns for the US are retrieved from Ibbotson. The short term bonds considered are the 3-month US T-Bill, obtained from FRED. Table I presents the summary statistics for the three assets. The range used is different from the larger ones used in the literature. Those data sets usually start in the decade of 1960 or in years prior to the WWII. In our dataset, the long term bonds are the best performing asset.

Table I - Summary Statistics for the assets to allocate for US

	Stocks		Long term Bonds		Short term Bonds	
	1993-2010	1998-2010	1993-2010	1998-2010	1993-2010	1998-2010
Mean (%)	6.52	2.56	8.21	7.23	3.29	2.81
St. Deviation (%)	15.77	17.33	10.36	10.89	0.55	0.56
Sharpe Ratio	0.20	-0.01	0.48	0.41	-	-

This table presents the descriptive statistics for three assets used in the allocation for US, in the in-sample (Jan 1993 - Dec 2010) and out-of-sample (Jan 1998 - Dec 2010) periods. The results for mean, standard deviation and Sharpe Ratio are annualized.

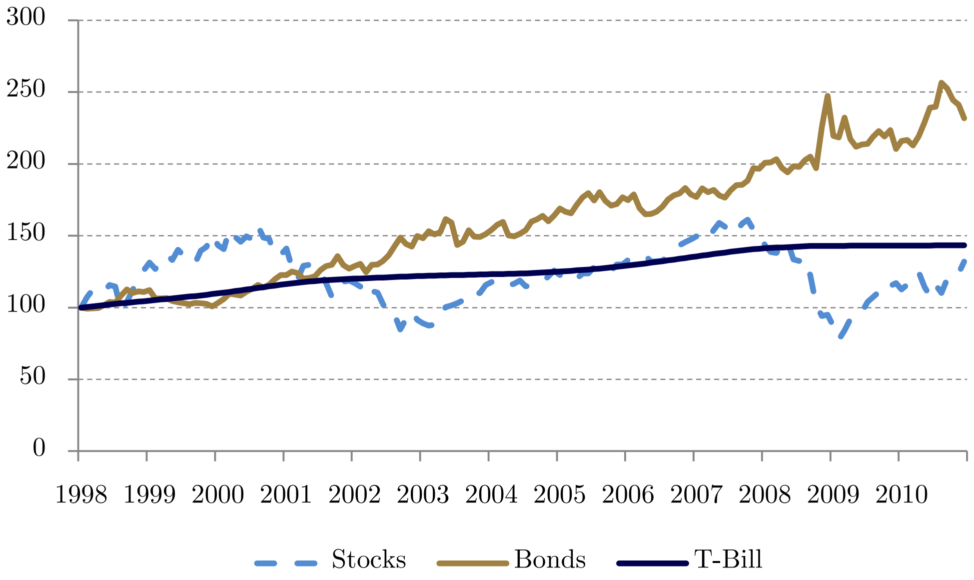

Figure 1 - Performance for the three assets in US

The performance of the three assets has been different over the past years, as one can see from Figure 1. Long term bonds yield larger returns than other assets, having also a low standard deviation, implying a Sharpe ratio of 0.41 in the most recent period. Stocks underperform the remaining assets in the same period, having a Sharpe ratio of -0.01 (table 1). Ex-ante, we might expect a better performance from the specifications that, on average, put a higher weight on long term bonds.

2.2.2 Non dependence characteristics

The characteristics' dataset includes the variables used in Brandt and Santa-Clara (2006) – Dividend-Price Ratio, Term Spread and Default Spread. Term Spread is the difference between the yields of long term bonds (10-year) and short term bonds (3-month) and Default Spread is the difference between the yields of BAA-rated bonds and AAA-rated bonds. These variables are obtained using the dataset from Goyal and Welch (http://www.hec.unil.ch/agoyal/) from December 1992 to December 2010 since dependence measures only start at this time.

Figure 2 presents the evolution of the standardized variables. The three variables have a similar behavior over time, although the correlations are not high, being all the correlation coefficients lower than 0.5. Default Spread and Dividend-Price Ratio peaked in the most recent crisis, which is a result of the higher bankruptcy costs and a sharp reduction in the US stock market.

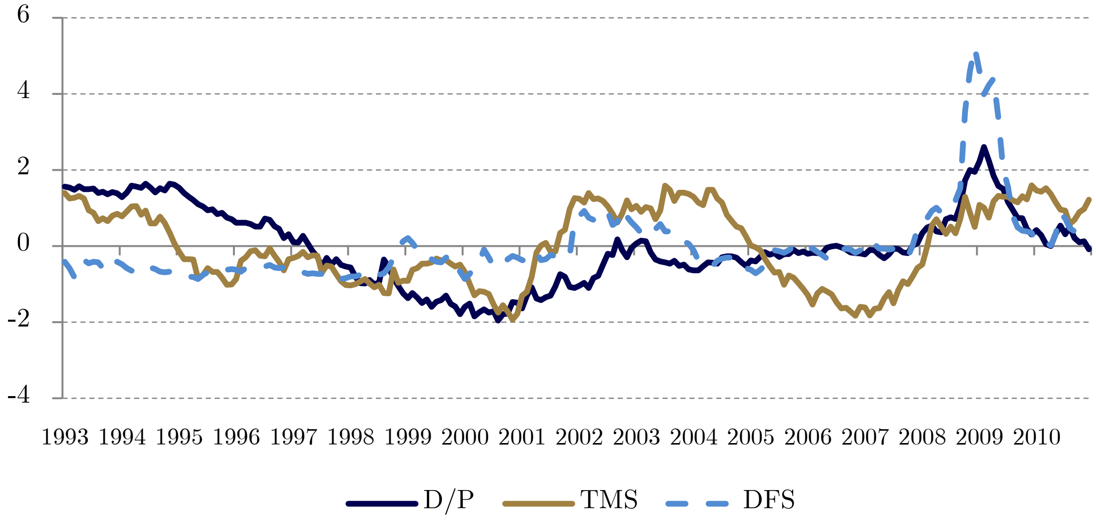

Figure 2 - Standardized non-dependence characteristics for US

3. Extreme Dependence Measures

3.1 Estimation of tail dependence

We use extreme value theory (EVT) to estimate bivariate tail distribution. Considering that only the dependence structure is important in this analysis, one should remove the marginal distributions out of the equation. Following Poon et al. (2004), the bivariate returns (X, Y) are transformed into unit Fréchet marginals (S, T):

$$S = \frac{1}{\log F_X(X)} \text{ and } T = \frac{1}{\log F_Y(Y)} \tag{4}$$

where F_X and F_Y are the respective marginal distribution functions for X and Y.

Poon et al. (2004) present a measure of tail dependence which can be defined as:

$$\bar{\chi} = \lim_{s \to \infty} \frac{2 \log \Pr(S > s)}{\log \Pr(S > s, T > s)} - 1 \tag{5}$$

where $-1 < \bar{\chi} < 1$. This measure is close to the Pearson correlation coefficient in terms of its limits and interpretation. This method captures correctly the asymptotic independence, as it measures the rate that $\Pr(S > s \mid T > s) \to 0$. Values of $\bar{\chi} > 0$, $\bar{\chi} = 0$ and $\bar{\chi} < 0$ loosely correspond, respectively, to when (S, T) are positively associated in the extremes, exactly independent, and negatively associated. Poon et al. (2003) show that in the case of bivariate normal dependence structure, $\bar{\chi}$ is the correlation coefficient.[7]

The maximum likelihood estimator is given by:

$$\hat{\bar{\chi}} = \frac{2}{n_u} \left(\sum_{j=1}^{n_u} \log \left(\frac{z_{(j)}}{u} \right) \right) - 1 \tag{6}$$

with variance is given by:

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[7] Weak assumptions are required to estimate $\bar{\chi}$ and are specified in Poon et al. (2003). A second dependence measure - χ - is defined by Poon et al. (2003) but the results using this measure are qualitatively similar to the ones using $\bar{\chi}$.

$$Var\left(\hat{\bar{\chi}}\right)=\frac{\left(\hat{\bar{\chi}}+1\right)^2}{n_u} \tag{7}$$

where n_u is the number of observations above the threshold u. Throughout this dissertation, n_u will always be fixed to be 5% of n.[8]

3.2 Data

US sector indices are used to compute the tail dependence measures for the ten sectors. The selected sectors are the following: Oil&Gas, Utilities, Financials, Technological, Consumer Goods, Basic Materials, Healthcare, Industrials, Consumer Services and Telecommunications. The data is obtained from Thompson DataStream and spans from January 1, 1973 to December 30, 2010. The Friday closing price is considered for each of the presented indices.

Weekly frequency is preferred over monthly and daily. Hartmann et al. (2004) also make a similar choice of frequency to study tail dependence. In what regards monthly frequency, the reduced number of observations does not allow for a sufficiently large number of joint extreme observations to estimate $\bar{\chi}$. In the case of daily frequency, Poon et al. (2004) warn that heteroskedasticity affects the estimates of $\bar{\chi}$.

3.3 Time series dependence measure

Traditionally, univariate distributions are used to build a time series measure of tail dependence for a country (Kelly (2009); Poon et al. (2004); Ruenzi and Weigert (2011)). These measures are easier to compute but do not capture all the aspects of the tail dependence. Therefore, one can use information from the different sectors of a country to obtain a more complete picture. In this dissertation, a new measure of tail dependence is proposed, combining the information from all the intra-country relations between the sectors.

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[8] Longin and Solnik (2001) use bootstrapping to define the optimal threshold level for several large economies. They find that on average, a level of 4-5% of the total number of observations should be considered as a threshold. A level of 10% is also tested but the results are not economically different from the ones obtained.

Firstly, $\overline{\chi}$ is computed for all pairs of sectors within a country, using weekly returns and a rolling window of 1,040 weeks (20 years). This computation is performed for the two tails of the bivariate distribution, being the positive (negative) extreme joint events considered as the right (left) tail.

Then, a cross-section arithmetic mean of the $\overline{\chi}$ for all the pairs within a country is computed. This is a similar procedure to the one used by Kelly and Pruitt (2011) to aggregate the different estimates in order to forecast returns. The authors argue that the equally weighted aggregation shows stronger performances in practice than other sophisticated weighting systems. If the estimated $\overline{\chi}$ are larger than 1 or smaller than -1, we limit the values to 1 and -1, respectively. The aggregated cross-section measure for the left tail and right tail is defined as Left Tail Mean (LTM) and Right Tail Mean (RTM), respectively. The Pearson correlation coefficient, ρ , is computed as a comparable measure.

Weekly frequency is used in the time-series measures presented above. However, since the asset allocation will be performed in a monthly basis, the variables must be converted into monthly frequency. The last weekly observation of each month is considered.

Figure 3 presents the evolution of the two tail dependence measures and the corresponding measure for Pearson correlation coefficient. The results from Ang and Chen (2002) hold in this time frame, considering that dependence in the left tail is larger than the one using the Pearson correlation coefficient. This trend holds for the entire period. The RTM is almost always between the two previous measures. Also, the distance between the three variables is time-varying, being RTM closer to the Pearson coefficient in normal times and closer to LTM in periods of financial crisis (see shaded areas in Figure 3).[9] This is related to the higher volatility in crisis periods, which leads to positive and negative joint extremes. Finally, the Pearson coefficient has the smoothest pattern of the three variables.

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[9] The two shaded areas correspond to the two crises in the period as defined by NBER (http://www.nber.org/cycles.html). The starting period is the peak and the ending period is the through for the real GDP in US.

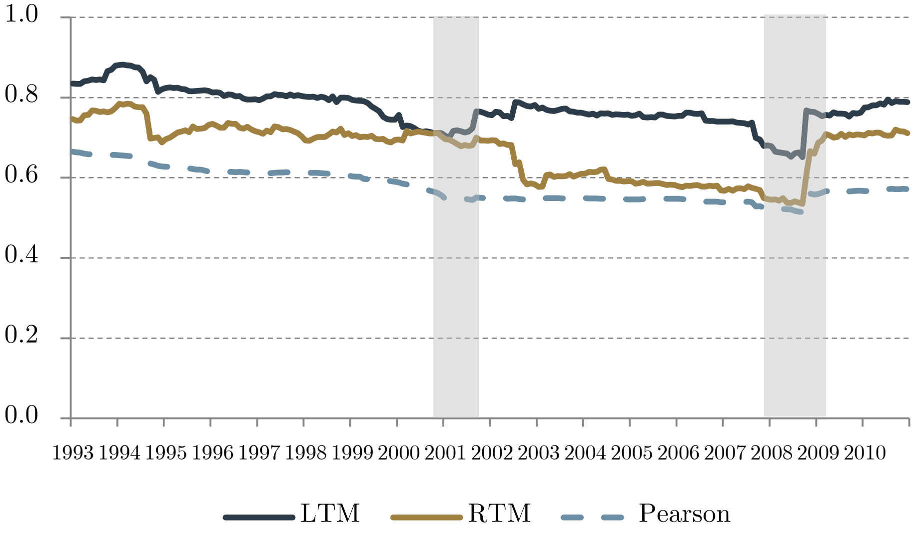

Figure 3 - Dependence measures time series for US

The sectors that lead to the changes during crisis periods are the most traditional sectors such as Oil&Gas, Utilities and Basic Materials. Those sectors had very large shifts in the two shaded areas, being the changes larger in the right tail. In an inverse pattern, Financial and Telecom, have large reductions in the dependence in both tails, but large in the right tail, as well. In the last crisis, this impact is more striking because financial companies were the most affected by the subprime crisis.

4. Results

4.1 Base specifications – simple dependence measures

The method presented in section 2.1 is used to allocate the three assets. Several specifications are presented, using dependence measures as a single characteristic or in combination with other variables. Table II shows the different specifications used and contrasts to two baseline specifications: Brandt and Santa-Clara's (2006) and the unconditional asset allocation. In the latter, no characteristics are used in the maximization of the utility function, and the results are shown in the first column of Table II. This allocation puts a larger weight on bonds (slightly above 100% in IS and OOS) and a negative weight on T-Bills. This is a trend for most specifications in the US. Column (1) considers the characteristics proposed by Brandt and Santa-Clara (2006) – Dividend-Price

Ratio, Default Spread and Term Spread. The performance using these three characteristics is better than the one using (0) both IS and OOS, being the improvement larger in sample.

Table II - Comparison of the different specifications of the asset allocation for US

	(0)	(1)	(2a)	(2b)	(2c)	(3a)	(3b)	(3c)	(4a)
D/P, TMS, DFS		X				X	X	X	
LTM			X			X			X
RTM				X			X		X
Pearson					X			X	
					In Sample Returns				
Mean (%)	10.23	25.97	26.27	13.88	26.10	36.98	29.71	29.82	26.34
St. Deviation (%)	12.86	23.34	24.47	16.22	24.23	27.53	24.78	24.72	24.02
Sharpe Ratio	0.58	0.99	0.96	0.68	0.96	1.24	1.09	1.09	0.98
Skewness	-0.36	-0.11	0.73	0.02	0.57	-0.45	-0.28	-0.36	0.28
Excess Kurtosis	2.98	1.01	5.33	1.20	4.26	0.79	1.00	0.87	2.89
Equalization Fee (%)	-	9.69	9.33	1.71	9.30	18.28	12.60	12.74	9.67
					In Sample Average Weights				
T-Bill (%)	-51	-81	-80	-46	-88	-114	-68	-76	-91
Bond (%)	119	97	102	111	110	96	73	74	107
Stocks (%)	32	84	79	35	77	118	95	101	85
					Out of Sample Returns				
Mean (%)	6.17	11.73	21.91	10.67	11.13	1.12	11.35	-7.99	12.00
St. Deviation (%)	23.90	44.06	37.24	44.22	33.16	48.90	55.85	43.41	43.70
Sharpe Ratio	0.14	0.20	0.51	0.18	0.25	-0.03	0.15	-0.25	0.21
Skewness	-1.62	1.14	-0.68	-1.02	-1.99	-0.63	-0.14	-0.57	-1.15
Excess Kurtosis	9.44	5.75	3.45	8.47	10.89	4.18	2.02	2.75	5.95
Equalization Fee (%)	-	2.71	13.86	1.63	3.65	-8.57	0.68	-16.91	3.03
					Out of Sample Average Weights				
T-Bill (%)	-112	63	-215	-304	-42	-290	-382	-108	-318
Bond (%)	132	36	277	274	130	256	228	93	306
Stocks (%)	79	1	38	130	12	134	254	115	111

This table presents the results of the estimation of different specifications of characteristics. *D/P* measures the Dividend-Price Ratio, *TMS* measures the term structure defined as the difference between long term (10-year) bonds and T-Bills for the US market, *DFS* measures the Default Spread defined as the difference between the returns of BAA and AAA bonds, *LTM* measures the tail dependence on the left tail, *RTM* is the equivalent measure for the right tail and *Pearson* measures the dependence using the Pearson correlation coefficient. The dependence measures were computed by the author, the remaining were obtained from Goyal's database. The portfolio allocation uses monthly data starting in January 1993 and ending in December 2010, using 5 years of data for the initial estimation period for the OOS. The coefficient of risk aversion considered in all optimizations is 4. The equalization fee is defined as the yearly fee that an investor would be willing to pay to be able to use the conditional instead of the unconditional policy. The results for mean, standard deviation and Sharpe Ratio are annualized.

These results confirm the findings of Brandt and Santa-Clara (2006) in a different time frame, since their analysis starts in 1945 and ends in 2000. The OOS average weight allocation is the least extreme of all, investing mainly on T-Bills, although the weights variation is very large, as can be seen ahead in this section. Still, the mean and standard deviation are almost twice as large when comparing to the unconditional case. In order to better assess the relative performance of each specification, the equalization fee is computed. This is the yearly fee that an investor is willing to pay to choose a specific allocation strategy over the unconditional methodology.

The remaining specifications either use only dependence measures computed as shown in section 2.3 or a combination of dependence measures with other state variables. Specification (2) uses one dependence measure at a time to allocate the different assets. The use of only one variable usually leads to better OOS results, as the inclusion of more variables may add estimation noise. Conversely, the IS results are worse, since the use of more variables increases the fit of the allocation. In this case, the IS results for the three dependence measures are lower or similar than the ones from (1), while the OOS are better or similar. The best performing specification uses the LTM ((2a)) and yields an OOS Sharpe Ratio of 0.51, more than two times larger than the baseline specifications. These results highlight the importance of using tail dependence measures in asset allocation, especially in the negative extremes. The remaining specifications from (2) have very similar results to the ones using the characteristics proposed by Brandt and Santa-Clara (2006). Therefore, an investor allocating using these types of measures would not be statistically worse off than using their method.

The positive impact of extreme dependence measures disappears when combining the dependence measures with the characteristics from (1). The IS Sharpe ratios from (3) are improved, but the OOS results are clearly worse than the ones from (1) and (2).[10]

Finally, a combination of dependence measures is tested in (4). The last column presents a combination of LTM and RTM. The results are similar to the ones obtained in (1)

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[10] The inclusion of two dependence measures instead of only one produces similar results.

although the weight distribution across the three assets is different resulting on a higher leverage in (4a).[11]

Figure 4 compares cumulative performance for some of the specifications from table II. This time series analysis allows for a better understanding of when the best specifications outperform.

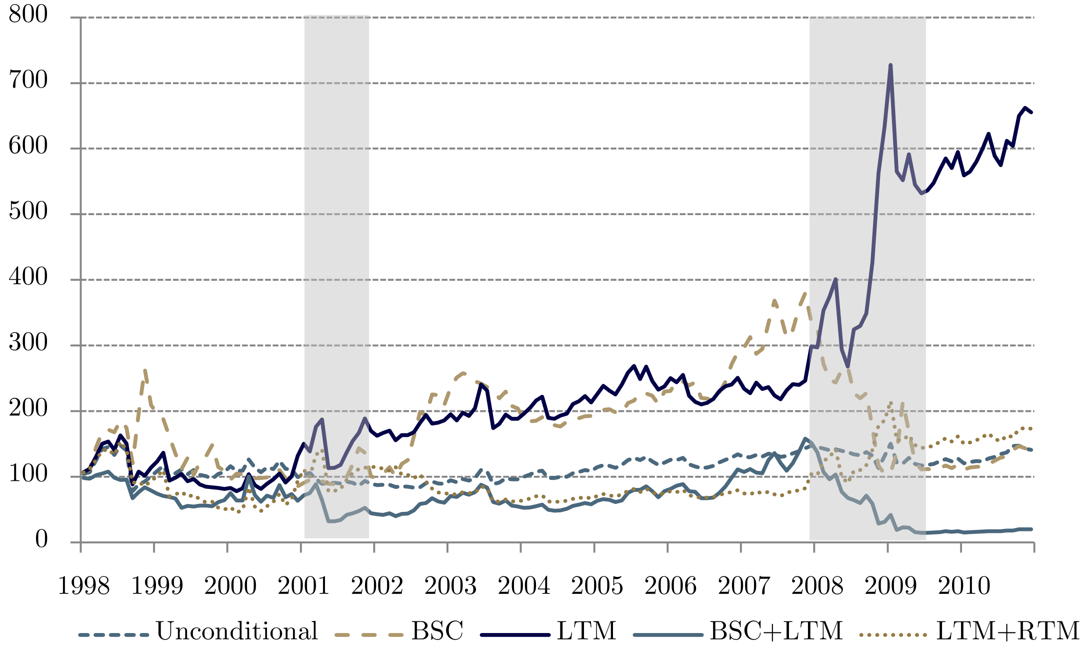

Figure 4 - Comparison of OOS cumulative performance of selected specifications for US

In normal times, the difference between the five specifications is not large, as can be seen from the performance in the periods prior to 2000 and between the two crises. However, in a crisis period, the cumulative performance turns out to be different. In the 2000-2002 crisis, the specifications using the LTM start to move away from the baseline. This behavior is repeated in the most recent financial crisis, with a particularly great performance by LTM specification. This difference is even clearer when comparing LTM with Brandt and Santa-Clara's (2006) specification. The performance during normal times is very similar, but in the two crises LTM performs much better.

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[11] A combination of tail dependence measures with the Pearson correlation coefficient was also tested ((4b) and (4c)). The results are worse than the ones from (1) and (4a).

The worst performer is the specification that combines LTM with Brandt and Santa-Clara's (2006). This particular specification combines the negative performance of Brandt and Santa-Clara's (2006) characteristics during crisis periods with the poor performance of dependence characteristics in normal periods. The unconditional allocation has the smoothest behavior in the analyzed period, being slightly affected by the two crises.

Investors also care about the turnover of their asset allocation due to transaction costs. Figure 5 presents the evolution of the OOS weights allocated to each asset, in three specifications.

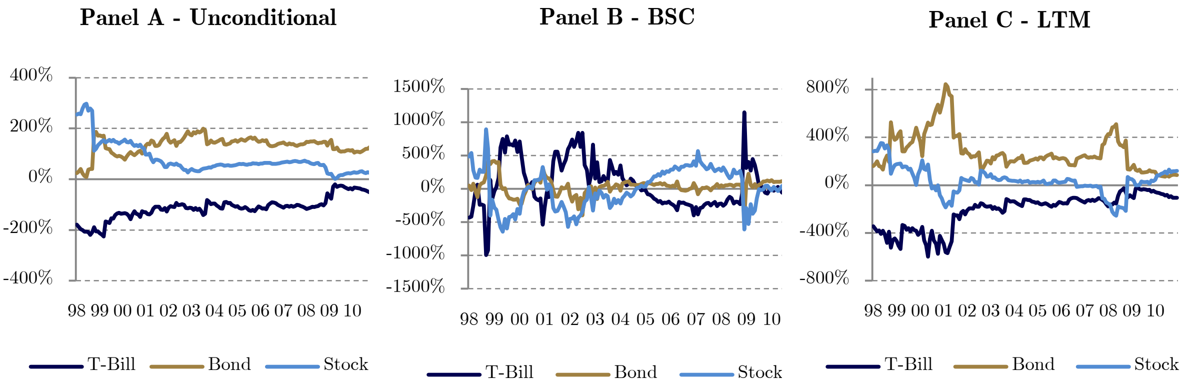
Panel A - Unconditional **Panel B - BSC** **Panel C - LTM**

Figure 5 - Performance OOS - Allocation weights for US

The unconditional allocation is the less leveraged and more stable over time. In contrast, Brandt and Santa-Clara (2006) has the highest volatility in the asset allocation weights. In some periods, some assets have weights close to 1,000%, with an extreme variation in the position on T-Bills, having very extreme long and short positions in the analyzed period. The use of LTM, allows for a much lower variation of the assets' weights when comparing to Brandt and Santa-Clara (2006), which is another advantage in terms of performance. It should be highlighted that the T-Bill has a relatively stable weight over time, being most of the variation held on Stocks and Bonds. LTM and the Unconditional put a larger weight on Bonds, while Brandt and Santa-Clara (2006) do not have a clear emphasis on a particular asset.

4.2 Improving the performance using VIX

The different specifications perform better in particular periods as can be seen in Figure 4. For instance, LTM excels during crisis periods while Brandt and Santa-Clara's (2006) performs well during non-crisis periods. Therefore, an investor would choose the best performing specification in each period of time.

The problem is to define an ex-ante indicator of a financial crisis. Schwert (1989) show that market volatility increases during crisis periods, therefore this indicator might be a good proxy to make the separation between the two types of periods. We use VIX as a measure of implied volatility in the stock market which is retrieved from Bloomberg.[12] In table III, two different strategies are presented, one using Brandt and Santa-Clara's (2006)

Table III - Combination of asset allocation methods using the VIX for US

	A)				**B)**			
Non-Crisis	**D/P, TMS, DFS**				**Unconditional**			
Crisis	**LTM**				**LTM**			
VIX threshold	**25**	**30**	**35**	**40**	**25**	**30**	**35**	**40**
Months in crisis period (%)	30	13	7	4	30	13	7	4
	Out of Sample Returns							
Mean (%)	11.29	10.60	8.23	6.54	4.37	8.06	8.74	5.08
St. Deviation (%)	35.68	34.40	35.12	35.66	30.86	25.92	25.54	24.22
Sharpe Ratio	0.24	0.23	0.15	0.10	0.05	0.20	0.23	0.09
Skewness	-0.47	0.09	0.03	-0.14	-1.37	-0.78	-0.84	-1.42
Excess Kurtosis	2.75	1.46	1.45	1.58	6.99	8.59	9.25	9.12
	Out of Sample Average Weights							
T-Bill (%)	-57	15	38	45	-146	-120	-116	-115
Bond (%)	105	55	46	44	177	138	137	137
Stocks (%)	52	30	16	11	69	82	79	78

This table presents two strategies to improve the allocation using the VIX. *D/P* measures the Dividend-Price Ratio, *TMS* measures the term structure defined as the difference between long term (10-year) bonds and T-Bills for the US market, *DFS* measures the Default Spread defined as the difference between the returns of BAA and AAA bonds, *LTM* measures the tail dependence on the left tail. This dependence measure was computed by the author, the remaining were obtained from Goyal's database. VIX was obtained from Bloomberg. The portfolio allocation uses monthly data starting in January 1998 and ending in December 2010, using OOS estimation. The number of months in crisis are defined as a percentage of the total number of out of sample observation, using each threshold value. The results for mean, standard deviation and Sharpe Ratio are annualized.

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[12] The use of VIX as characteristic in the asset allocation does not lead to a positive OOS Sharpe ratio. However, when combined with LTM, an OOS Sharpe ratio of 0.32 is attained.

allocation and the other using the unconditional during non-crisis periods, and the allocation with LTM during crises. The use of the crisis specification is based on the value of VIX in the previous period. If VIX is above a specific threshold we perform the allocation OOS using only LTM. If this variable is below the same threshold, the non-crisis specification is used. Several threshold values are presented, changing the size of the crisis period.

The use of this alternative method of allocation does not improve the results when comparing to the LTM specification. However, the inclusion of dependence measures during crisis periods improves slightly the allocation when comparing to Brandt and Santa-Clara's (2006) and unconditional allocations, for some threshold levels. The use of higher thresholds reduces the number of crisis months, therefore the use of LTM is reduced. This leads to lower OOS Sharpe ratios for the most extreme thresholds.

5. Robustness

5.1 Risk aversion coefficient

Throughout this dissertation, the coefficient of risk aversion, γ, considered is 4. The choice of coefficient of risk aversion is never a unanimous decision. Several economists try to estimate the value of risk aversion that would explain the equity premium from stocks and bonds. The estimates for the coefficient of risk aversion computed from option prices vary between 2 and 8 (Rosenberg and Engle (2002), Tarashev and Tsatsaronis (2006)). The value defined for this study is within this interval.[13] However, it is important to check if changes in this parameter affect the allocation outcome. Table IV presents the results for three specifications for different levels of γ.

The behavior is similar across the different specifications. As one would expect, the larger the coefficient of risk aversion, the less riskier are the allocations. For all specifications the volatility and mean of returns is reduced with a choice of a larger level of risk aversion, both IS and OOS. In sample, the Sharpe ratios are not affected by the level of risk

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[13] Brandt and Santa-Clara (2006) use a coefficient of risk aversion of 5. However, the results of the asset allocation do not change meaningfully by using a value of 5 instead of 4.

aversion while OOS Sharpe ratios vary slightly with the changes in the risk aversion level. As expected, the allocations with a higher risk aversion level are less exposed to risky assets, having a much lower short position on risk free assets than the allocations obtained by a lower risk aversion coefficient. Skewness and kurtosis do not change significantly with the different levels of risk aversion.

Table IV - Impact of different levels of risk aversion in the asset allocation for US

	(1)			(2a)			(4a)		
	D/P, TMS, DFS			**LTM**			**LTM, RTM**		
	$\gamma=4$	$\gamma=7$	$\gamma=10$	$\gamma=4$	$\gamma=7$	$\gamma=10$	$\gamma=4$	$\gamma=7$	$\gamma=10$
				In Sample Returns					
Mean (%)	25.97	16.01	12.21	26.27	16.32	12.43	26.34	16.55	12.59
St. Deviation (%)	23.34	13.29	9.33	24.47	13.96	9.78	24.02	13.79	9.67
Sharpe Ratio	0.99	0.99	1.01	0.96	0.97	0.98	0.98	1.00	1.01
Skewness	-0.11	-0.13	-0.14	0.73	0.67	0.63	0.28	0.23	0.20
Excess Kurtosis	1.01	1.05	1.04	5.33	4.92	4.72	2.89	2.66	2.56
				In Sample Average Weights					
T-Bill (%)	-81	-4	27	-80	-5	26	-91	-11	22
Bond (%)	97	51	36	102	61	43	107	62	43
Stocks (%)	84	53	37	79	44	31	85	49	35
				Out of Sample Returns					
Mean (%)	11.73	3.86	4.88	21.91	13.84	10.55	12.00	8.52	6.40
St. Deviation (%)	44.06	25.65	17.29	37.24	21.42	15.03	43.70	24.96	17.85
Sharpe Ratio	0.20	0.04	0.12	0.51	0.51	0.52	0.21	0.23	0.20
Skewness	1.14	1.31	1.22	-0.68	-0.66	-0.65	-1.15	-0.90	-1.06
Excess Kurtosis	5.75	7.36	5.61	3.45	3.36	3.30	5.95	5.60	5.61
				Out of Sample Average Weights					
T-Bill (%)	63	85	89	-215	-82	-28	-318	-140	-68
Bond (%)	36	15	12	277	159	112	306	175	123
Stocks (%)	1	1	-1	38	22	16	111	66	46

This table presents the impact of the use of different coefficients of risk aversion for different specifications of characteristics. *D/P* measures the Dividend-Price Ratio, *TMS* measures the term structure defined as the difference between long term (10-year) bonds and T-Bills for the US market, *DFS* measures the Default Spread defined as the difference between the returns of BAA and AAA bonds, *LTM* measures the tail dependence on the left tail, while *RTM* measures the tail dependence on the right tail. The dependence measures were computed by the author, the remaining were obtained from Goyal's database. The portfolio allocation uses monthly data starting in January 1993 and ending in December 2010, using 5 years of data for the initial estimation period for the OOS. The results for mean, standard deviation and Sharpe Ratio are annualized.

5.2 Alternative scenario – dependence measures

5.2.1 Differences in dependence measures

The differential between the dependence measures may help to predict the behavior of the different assets. Left minus Right (LMR) aims at capturing the different dependence structures between the two tails, in a similar method to Jondeau and Rockinger (2003). If this measure is positive, the tail dependence of extreme joint negative events is larger than the one of extreme joint positive events. Left minus Pearson correlation coefficient (LMP) and the correspondent measure for the right tail (RMP) measures the differential between the tail dependence measures and the dependence for the whole distribution measured by Pearson correlation coefficient which can target the distance of magnitude to the normal distribution case.

The use of LMP alone outperforms the OOS performance of Brandt and Santa-Clara (2006), yielding a Sharpe ratio of 0.26, while LMR and RMP performs slightly worse. As in table II, the combination of these measures with Brandt and Santa-Clara's (2006) characteristics does not perform well, yielding OOS Sharpe ratios close to zero.

The inclusion of a combination of difference measures also worsens the results OOS. The use of differences between the dependence measures has a lower impact in improving the asset allocation that the use of the measures by themselves. Therefore, only the LMP presents a clear positive performance, which is a direct consequence of the behavior of LTM, as shown in Table II.

5.2.2 Variations in dependence measures

The three dependence measures have a relative smooth pattern if we exclude the two financial crises: dot-com burst (2000-2002) and subprime (2008-2009). Therefore, the use of variations in each measure should allow for a larger variability in the time-series, which can improve the asset allocation. Boyer et al. (1999) address the changes in the correlation in extreme periods or "correlation breakdowns" and warn about its importance for traders.

The results IS are generally worse than the ones using the absolute dependence measures. However, the results OOS show that some specifications produce positive results. There is an inversion in the results when comparing to Table II, being ΔRTM and ΔPearson the best performing specifications. The correspondent measure for the left tail, ΔLTM, does not perform well in asset allocation, which is against the initial intuition. The Sharpe ratio of this specification is negative. The combination of the dependence measures between them and with non-dependence characteristics yield results similar to the ones attained by Brandt and Santa-Clara's (2006).

6. International evidence – UK, Japan and Germany

It is important to verify if the results obtained for US are similar to the ones of other three major economies. For this reason, the same methodology is applied to UK, Japan and Germany, although the time frames are not always the same due to data constraints.[14]

The same three assets used in US are chosen for the three countries. The data regarding UK, Japanese and German bonds (3-month and 10-year) is retrieved from Bloomberg database, using the maximum period available. In the case of UK and Japan, the analyzed period starts in May 1993 and ends in December 2010. For Germany, the analysis starts in September 1999 and ends in December 2011, in order to obtain a comparable estimation window length.

Table V presents the summary statistics of the assets to allocate in the three countries. In UK and Japan, the performance is similar to the US, being long term bonds the best performing asset. The relatively positive performance of bonds should benefit the allocations that are long on bond market and short on equities. The case of Germany is different since the time frame is not the same and both bonds and stocks have a poor performance.

Jondeau and Rockinger (2008) show that tail dependence is different across markets. Some countries have very distinct behaviors in the right and left tail, while in others the difference is not as large. Figure 6 presents the time series of dependence measures for the three countries and corroborates their findings.

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[14] Those countries were also studied by Poon et al. (2004) and Jondeau and Rockinger (2003).

Table V - Descriptive statistics of the three assets for UK, Germany and Japan

	Stocks		Long term Bonds		Short term Bonds	
	UK					
	1993-2010	1998-2010	1993-2010	1998-2010	1993-2010	1998-2010
Mean (%)	4.91	0.98	7.86	7.28	3.30	2.75
St. Deviation (%)	14.77	15.73	10.44	11.02	0.55	0.56
Sharpe Ratio	0.11	-0.11	0.44	0.41	-	-
	Germany					
	1999-2011	2004-2011	1999-2011	2004-2011	1999-2011	2004-2011
Mean (%)	-1.23	2.82	2.63	3.16	2.77	2.38
St. Deviation (%)	20.89	18.04	6.71	7.05	0.39	0.42
Sharpe Ratio	-0.19	0.02	-0.02	0.11	-	-
	Japan					
	1993-2010	1998-2010	1993-2010	1998-2010	1993-2010	1998-2010
Mean (%)	-2.72	-2.10	2.32	0.84	0.68	0.33
St. Deviation (%)	18.20	18.52	6.67	5.40	0.24	0.09
Sharpe Ratio	-0.19	-0.13	0.25	0.09	-	-

This table presents the descriptive statistics for three assets used in the allocation for UK, Germany and Japan, in the in sample (first column of each asset) and out of sample (second column) periods. The results for mean, standard deviation and Sharpe Ratio are annualized.

However some similarities can be found in the dependence structures. There is a reduction in the gap between the right and left tail, being the left tail dependence generally larger than any other dependence measure. The oddest behavior is the one from UK (Panel B), where right and left tail dependence measures have significant changes over time, also shifting their relative position in this period.

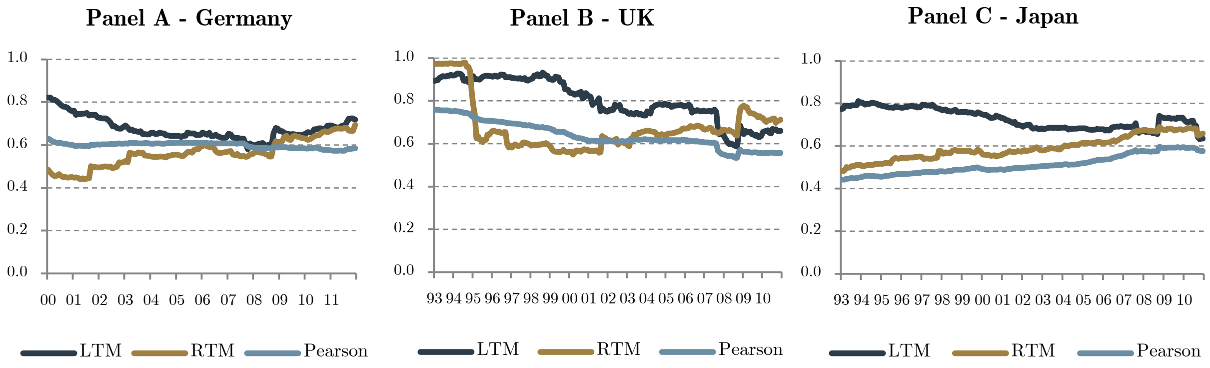

Figure 6 - Dependence measures time series - International evidence

The dependence measures in Japan are the ones with less variability, although the most recent crisis clearly affects the left tail dependence measure. The crisis in the end of 1990's did not affect the dependence between the Japanese sectors.

Tables VI, VII and VIII present a similar analysis to Table II for Germany, UK and Japan, respectively. However, for the three countries, there is no available data for most of the variables used in US case. Consequently, the variables used as non-dependence characteristics for these countries are Book-to-Market and Term Spread in UK and Germany, and Dividend-Price ratio and Term Spread in Japan. The information for these variables is retrieved from Bloomberg.[15]

Table VI - Comparison of the different specifications of the asset allocation for Germany

	(0)	(2a)	(2b)	(2c)	(4a)	(4b)	(4c)	(5)	(6a)	(6b)	(6c)
B/M, TMS								X	X	X	X
LTM		X			X	X			X		
RTM			X		X		X			X	
Pearson				X		X	X				X
	Out of Sample Returns										
Mean (%)	-0.44	29.42	-3.85	10.82	29.16	41.72	11.31	53.44	60.64	48.06	38.61
St. Deviation (%)	10.19	46.74	16.42	29.28	59.96	77.44	31.36	56.46	75.46	60.20	69.35
Sharpe Ratio	-0.28	0.58	-0.38	0.29	0.45	0.51	0.28	0.90	0.77	0.76	0.52
Skewness	0.60	3.15	-2.08	1.91	1.70	3.38	0.07	2.01	2.82	0.81	2.31
Excess Kurtosis	1.34	17.87	9.05	10.92	12.98	15.80	7.83	6.80	12.67	2.77	8.66
Equalization Fee	-	39.99	-1.68	16.55	43.38	60.78	17.61	66.70	79.15	62.35	55.43
	Out of Sample Average Weights										
Ger 3m Bond (%)	-112	379	15	167	354	345	-55	486	475	410	274
Ger 10y Bond (%)	132	-224	35	-55	-264	-159	68	-326	-296	-328	-106
MSCI Ger (%)	79	-55	50	-11	10	-85	87	-60	-79	18	-68

This table presents the results of the estimation of different specifications of characteristics. *B/M* measures the Book to Market Ratio, *TMS* measures the term structure defined as the difference between long term (10-year) bonds and the 3-month Bond for Germany, *LTM* measures the tail dependence on the left tail, *RTM* is the equivalent measure for the right tail and *Pearson* measures the dependence using the Pearson correlation coefficient. The dependence measures were computed by the author, the remaining were obtained from Bloomberg. The portfolio allocation uses monthly data starting in September 2004 and ending in December 2011, using 5 years of data for the initial estimation period for the OOS. The coefficient of risk aversion considered in all optimizations is 4. The equalization fee is defined as the yearly fee that an investor would be willing to pay to be able to use the conditional instead of the unconditional policy. The results for mean, standard deviation and Sharpe Ratio are annualized.

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[15] In the case of UK, three sectors were excluded due to insufficient historical data (Telecommunications, Technological and Utilities). For Germany, two sectors were excluded for the same reason (Oil & Gas and Technological).

The three countries the unconditional model does not perform well. In the case of Germany, the addition of characteristics improves the allocation in all specifications but one (RTM). Also, the results using left tail dependence measure are very positive, which indicates that the findings from US hold in Germany, especially the ones related to the dependence to extreme negative events. The results from this allocation are, on average, short both on long term bonds and stocks, however it performs exceptionally well during crisis periods. This is not the case in the allocation using right tail dependence. The use of non-dependence characteristics has a similar performance to the specification using only the left tail measure, having a much higher short position and more extreme volatility in returns.

Table VII - Comparison of the different specifications of the asset allocation for UK

	(0)	(2a)	(2b)	(2c)	(4a)	(4b)	(4c)	(5)	(6a)	(6b)	(6c)
B/M, TMS								X	X	X	X
LTM		X			X	X			X		
RTM			X		X		X			X	
Pearson				X		X	X				X
						Out of Sample Returns					
Mean (%)	3.95	1.53	0.82	5.60	0.60	0.67	0.01	51.22	54.61	67.93	78.43
St. Deviation (%)	16.05	22.43	49.74	32.87	48.91	47.24	54.14	66.53	73.31	134.94	108.80
Sharpe Ratio	0.08	-0.05	-0.04	0.09	-0.04	-0.04	-0.05	0.76	0.71	0.48	0.71
Skewness	-0.45	-1.26	-2.67	-0.86	-2.59	0.93	-2.37	0.29	-0.08	-0.48	0.48
Excess Kurtosis	2.83	4.53	22.39	1.93	23.13	5.09	17.15	2.65	4.95	9.83	9.09
Equalization Fee (%)	-	-2.91	-5.69	0.37	-5.85	-5.65	-6.83	43.44	46.32	54.98	67.45
						Out of Sample Average Weights					
UK 3m Bond (%)	-71	-180	-97	-160	-52	329	-115	-348	-360	-306	-152
UK 10y Bond (%)	127	181	184	245	131	-51	191	399	416	469	383
MSCI UK (%)	44	99	13	16	22	-178	24	49	43	-63	-131

This table presents the results of the estimation of different specifications of characteristics. *B/M* measures the Book to Market Ratio, *TMS* measures the term structure defined as the difference between long term (10-year) bonds and the 3-month Bond for UK, *LTM* measures the tail dependence on the left tail, *RTM* is the equivalent measure for the right tail and *Pearson* measures the dependence using the Pearson correlation coefficient. The dependence measures were computed by the author, the remaining were obtained from Bloomberg. The portfolio allocation uses monthly data starting in June 1998 and ending in December 2010, using 5 years of data for the initial estimation period for the OOS. The coefficient of risk aversion considered in all optimizations is 4. The equalization fee is defined as the yearly fee that an investor would be willing to pay to be able to use the conditional instead of the unconditional policy. The results for mean, standard deviation and Sharpe Ratio are annualized.

In the UK case, the inclusion of tail dependence measures does not improve the OOS results. In this case, similarly to Germany, the specification using Book to Market and Term Spread performs very well. Also, the combination of these measures with tail dependence produces better IS results but slightly poorer OOS Sharpe ratios. The OOS

Sharpe ratios using the specifications (14) and (15) perform much better than the comparable specification (0). This provides some evidence that the use of characteristics to allocate between assets is beneficial for an investor, although exposing it to very high variability in returns.

Table VIII - Comparison of the different specifications of the asset allocation for Japan

	(0)	(2a)	(2b)	(2c)	(4a)	(4b)	(4c)	(7)	(8a)	(8b)	(8c)
D/P, TMS								X	X	X	X
LTM		X			X	X			X		
RTM			X		X		X			X	
Pearson				X		X	X				X
Out of Sample Returns											
Mean (%)	-0.28	-4.35	-2.38	-2.41	-3.67	4.99	-1.21	87.31	153.43	158.78	164.60
St. Deviation (%)	9.37	24.83	25.42	26.88	33.99	40.45	31.62	113.11	158.51	152.83	151.60
Sharpe Ratio	-0.06	-0.19	-0.11	-0.10	-0.12	0.12	-0.05	0.77	0.97	1.04	1.08
Skewness	-1.17	-1.14	-1.66	-1.35	-0.93	-0.46	-0.81	1.60	1.76	1.73	1.69
Excess Kurtosis	6.52	10.51	17.38	11.68	4.93	2.75	6.39	5.65	4.98	4.83	4.76
Equalization Fee (%)	-	-3.07	-1.06	-0.99	-1.79	7.28	0.51	94.32	163.38	168.37	174.10
Out of Sample Average Weights											
Japan 3m Bond (%)	23	-22	-65	-23	-68	-67	37	-486	-839	-691	-723
Japan 10y Bond (%)	103	133	130	74	159	133	14	862	1218	1045	989
MSCI Japan (%)	-26	-10	35	50	9	34	49	-276	-279	-254	-166

This table presents the results of the estimation of different specifications of characteristics. *D/P* measures the Dividend-Price Ratio, *TMS* measures the term structure defined as the difference between long term (10-year) bonds and T-Bills for Japan, *LTM* measures the tail dependence on the left tail, *RTM* is the equivalent measure for the right tail and *Pearson* measures the dependence using the Pearson correlation coefficient. The dependence measures were computed by the author, the remaining were obtained from Bloomberg. The portfolio allocation uses monthly data starting in May 1993 and ending in December 2010, using 5 years of data for the initial estimation period for the OOS. The coefficient of risk aversion considered in all optimizations is 4. The equalization fee is defined as the yearly fee that an investor would be willing to pay to be able to use the conditional instead of the unconditional policy. The results for mean, standard deviation and Sharpe Ratio are annualized.

The results in the Japanese case are affected by the poor performance of most of its assets. The allocations that use dependence-related characteristics do not differ significantly from the unconditional case. This is related to the fact that for most of the analyzed period the tail dependence measures seem to be flat. Interestingly, the use of these characteristics worsens both the mean and the standard deviation. However, the use of tail dependence characteristics improves the results from the allocation when combined with non-dependence characteristics. These allocations are extremely volatile and put an enormous weight on the bond market, which is the best performing asset in this period. The fact that Japan has been in a financial crisis for almost the whole OOS period may explain the results.

It is clear that Germany is closest to US. Germany has a similar tail dependence structure, having always larger left tail dependence and a reduction in the differential between right and left tails in crisis periods. However, results from the remaining two countries do not show the superiority of using tail dependence measures. In the two countries the behavior of tail dependence is distinct. In UK, there is a higher variability in the two tail dependence measures, while in Japan, those measures are almost flat. Therefore, the variability in the two countries dependence structure might account for the differences in the results.

7. Concluding Remarks

The primary goal of this dissertation is to introduce the concept of extreme dependence into the asset allocation framework. In order to achieve this goal, two distinct approaches are connected using a new concept of intra-country sector relationship. A new measure is constructed to evaluate the tail dependence of a country over time for positive and negative extremes, based on cross-section behavior of its pairs of sectors.

The use of the measure improves the performance of the asset allocation when comparing to the unconditional model and Brandt and Santa-Clara's (2006) specification. The OOS Sharpe ratios can be as high as 0.51 for US, which is more than two times larger than the comparable specifications. The OOS performance is superior when only dependence measures are used, being the IS results similar to the benchmarks used. Results are better for the left tail than the right tail. This follows Ang and Chen (2006) and Ruenzi and Weigert (2011) which show that dependence of negative extreme events have a larger impact on returns. This better performance is obtained fundamentally during crisis periods, where the OOS returns are larger on average. Results are robust to changes in the risk aversion coefficient, for the US. So, a risk averse investor (higher risk aversion coefficient) would obtain similar Sharpe ratios.

The same methodology is tested in three other major economies. Overall, the use of these measures leads to a better allocation than the unconditional model, IS and OOS. In the Germany case, the results are similar to the ones obtained for US. The left tail measure has an OOS Sharpe ratio of 0.58. However in the remaining two countries this effect is not present. In the analyzed period the dependence structure of those two countries is different from those in the US and Germany.

The use of dependence measures in the asset allocation has positive results overall. The good performance in crisis periods can give asset managers an alternative to conventional allocation methodologies in those periods. The methodology is also simple enough to be adapted to accommodate different levels of risk aversion and transaction costs.

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