UNIVERSIDADE CATÓLICA PORTUGUESA

The diversification benefits of investing in Bitcoin

José Manuel Barroso Martins Salgado Dias

Católica Porto Business School & Lancaster University Management School
September 2022

UNIVERSIDADE CATÓLICA PORTUGUESA

The diversification benefits of investing in Bitcoin

Final Assignment in the modality of Dissertation presented to Universidade Católica Portuguesa and Lancaster University Management School to obtain the Double Degree Master in Finance and Master in Money, Banking and Finance

by

José Manuel Barroso Martins Salgado Dias

under the supervision of
(PhD) Gonçalo Faria

Católica Porto Business School & Lancaster University Management School
September 2022

Acknowledgements

This work could not have been done without the unreserved love from family and friends, who not only supported and encouraged my studies, but also provided the bedrock for my personal growth.

Moreover, I would like to thank the Professors at Católica Porto Business School and Lancaster University Management School, for their role in my academic education and their advice regarding my professional career. Specifically, I would like to thank Professor Gonçalo Faria, who was kind enough to provide me with guidance and encouragement during this whole time.

It has been a massive pleasure to learn and work with all of you.
I deeply hope you enjoyed this research.

Resumo

Nesta dissertação pretendemos estudar os benefícios de investir em Bitcoin no contexto de um portefólio diversificado de ações e obrigações (portfólio clássico 60% ações/40% obrigações). Com base numa análise histórica, avaliámos o impacto de diferentes: (i) períodos de investimento, (ii) frequências de rebalanceamento e (iii) intervalos de alocação de bitcoin na performance global de um dado portfólio. Uma *rolling analysis* foi executada a fim de capturar as dinâmicas temporais entre estes ativos.

Concluímos que, no período amostral em análise, rebalanceamentos trimestrais de alocações em Bitcoin entre 0.5%-7.5% num período mínimo de 3 anos foram sempre vantajosas para o investidor, independentemente do período em que o capital foi aplicado.

Apesar de performance passada não ser garantia de performance futura, acreditámos que as conclusões empíricas desta dissertação têm importantes implicações para investidores institucionais.

Palavras-chave: diversificação, criptomoedas, bitcoin *rolling analysis,* horizonte de investimento, frequência de rebalanceamento, alocação de bitcoin

Número de palavras: 9046

Abstract

In this dissertation we aim to study the diversification benefits of investing in Bitcoin in a context of a well-diversified, balanced portfolio constituted by bonds and stocks (classic 60% equity/40% bonds Traditional Portfolio). Based on historical data, we evaluate the impact of different (i) holding periods of the investment, (ii) rebalancing frequencies of the portfolio and (iii) range of bitcoin allocations, on the overall performance of a given portfolio. A rolling analysis was performed to capture the time-dependent dynamics between these assets.

We conclude that, for the sample period under analysis, quarterly rebalanced bitcoin allocations between 0.5%-7.5% for a minimum holding period of 3 years have always been beneficial for the investor, regardless of the period in which the investment was made.

Although past performance is no guarantee of future performance, we believe that the empirical conclusions presented in this dissertation have significant implications for institutional investors.

Keywords: diversification, cryptocurrencies, bitcoin rolling analysis, holding period, rebalancing period, bitcoin allocation

Word count: 9046

Contents

List of Figures

List of Tables

Introduction

Bitcoin's invention by Satoshi Nakamoto (Nakamoto, 2008), an alias, has reshaped the way the general public, investors, institutions, regulators and even governments think and deal with money. The idea of secure, decentralized, private, peer-to-peer electronic transactions have been lingering in minds of cypherpunks[1] and academics since the 80s, yet prior to the creation of the Bitcoin protocol none of the projects implemented[2] really grasped any traction.

Building on the features of its predecessors, Bitcoin was created in 2008 and has ignited the crypto revolution that we see today. Its unique features quickly took the imagination and interest of market participants, computer scientists and political philosophers. Based on a limited supply of coins and cryptography to protect consumer's data, each transaction is settled resorting to a decentralized ledger technology (a.k.a. the blockchain) and not to a centralized third-party. The Bitcoin blockchain is able to guarantee immutability, transparency and anonymity (Ghiro *et al.*, 2021) through the Proof-of-Work[3] (PoW) consensus mechanism, meaning that the network cannot be breached or hacked, unlike conventional financial transactions.

The breakthrough obtained by Bitcoin was to solve the double spending problem, an issue that arises from the possibility of spending the same currency token more than

[1] See Hughes, 1993, for more details.
[2] See D. Chaum, 1982; Back, 2002; Dwork and Naor, 1993, about forerunners of Bitcoin.
[3] See Lamport, 1983, for a better understanding of the Byzantine Problem. The Byzantine Generals Problem is a game theoretical problem that illustrates the difficulty that decentralized parties have arriving at a consensus without the trust of a centralized authority. Bitcoin was the first realized solution to the Byzantine Generals Problem with respect to money, since the computational cost associated with the use of the PoW mechanism incentivizes participants to be honest, allowing decentralized parties to reach a consensus.

once (note that some authors have explored the idea of a successful double spending attack (Pérez-Solà *et al.*, 2019, Karame *et al.*, 2015)).

Bitcoin's invention promoted the appearance of many altcoins (i.e., any cryptocurrency that exists that it's not Bitcoin), many of which were either developed to solve the pain points of its precursors or designed with entirely different purposes. The potential utility and applications of the technology that supports digital assets, of which cryptocurrencies are only a part of, has been a growing space ever since. The adoption of Bitcoin is expected to be unlike any other in the context of past disruptive technology adoption cycles, being forecasted that by 2030, 10% of the global population will be using Bitcoin (Blockware Solutions, 2022).

Though appealing, traditional finance institutions have been skeptical regarding the future of digital cryptoassets for many reasons[4]. One reason is that traditional fundamental analysis has not been successful in explaining the volatility and price performance of cryptocurrencies. Another important reason is the occurrence of fraudulent events that contaminate the reputation of the crypto space, such as pyramid scams, pump-and-dump schemes and cybercrimes (to name a few), as well an regulatory ambiguity across countries and digital assets (Astrakhantseva, Astrakhantsev and Los, 2021). However, from a finance perspective, the emergence of such a prominent technologic-intensive market can't be ignored, with countless institutional organizations already holding cryptocurrencies in their balance sheets and/or are exposed to cryptocurrency-related products (e.g., CME's Bitcoin Futures, Bitcoin Futures ETFs, Spot Bitcoin ETF, among others).

Hence, in this dissertation we propose to quantify the diversification benefits of holding Bitcoin based on historical data, in a way that is relevant for both academics and financial institutions. Bitcoin (BTC) was chosen because it is the cryptocurrency with the longest track record, the most interesting cryptoasset from an institutional point of view and the world's leading digital currency by market capitalization[5].

[4] See https://bitcoinmagazine.com/markets/jpmorgan-changes-tune-on-bitcoin for more details.
[5] See https://coinmarketcap.com/charts/ for more details.

Additionally, we provide insights on three of the most prominent questions in the minds of investors considering investing in Bitcoin: (i) What is the minimum acceptable holding period for a bitcoin allocation? (ii) What is the best rebalancing frequency for a bitcoin allocation? (iii) How much bitcoin should be added to a traditional 60/40 portfolio?

We believe that this dissertation has relevant implications for institutional investors, banks, regulators, and the wider economy, since it enhances their understanding of the cryptocurrency market. For institutional investors, given that their clients demanding cryptocurrency-related products, obliging portfolio managers to provide concrete evidence of the potential benefits of adopting such strategies. For commercial banks, given that they may provide loans to clients exposed to the crypto market – therefore vulnerable to the idiosyncrasies of the cryptocurrencies' market volatility, liquidity, and technological innovation –, which in turn affects the borrower's ability to repay a loan or meet contractual obligations (credit risk). Moreover, banks themselves may be holding reserves[6] in cryptoassets and need to subsequently account for drawdowns that may affect the compliance with liquidity and capital regulations in place. For regulators, given their role in assuring the global financial stability in a growing crypto space (FATF, 2022). The rapid evolution and international nature of these markets (along with the digital transformation of the financial system) allows for regulatory arbitrage and fragmentation (FSB, 2022), raising the need for international regulatory policies such as AML and KYC requirements to ensure the safety and stability of the crypto market. Nevertheless, the implementation of this policies must be taken without hindering the development of digital assets and affect their attractiveness.

We start by evaluating the comovement between traditional assets and Bitcoin, first through an average for the full sample period and then by a 90-day rolling window.

[6] See (BIS, 2022) for an interesting but skeptical stance toward cryptoassets from the Bank of International Settlement), in which it not only classifies cryptoassets that do and don't comply with BCBS (Basel Committee on Banking Supervision criteria) criteria – Group 1 and 2, respectively –, but also limits a bank's total exposure to Group 2 cryptoassets to 1% of Tier 1 capital.

We find out that, although on average Bitcoin's correlation with traditional assets has been historically low, recent macroeconomic and crypto-specific events have changed this paradigm (e.g., COVID-19 pandemic, supply chain disruptions, war in Ukraine, oil price shocks, VCs investment in the crypto-space, Luna collapse, Celsius bankruptcy, among others). Consequently, investors should be aware of periodical trends that may affect the dynamics between the assets considered.

We then propose a portfolio's construction methodology based on the Investment Time-Horizon, Rebalancing Frequency and Position sizing on BTC, which is built on top of a Rolling Analysis Framework. This allowed for the construction of more than a million different portfolios, each of them built with a unique set of features. The portfolios were then bundled together based on a particular set of characteristics, structured in a way that would allow us to answer the research questions proposed.

Results suggest that, for a small allocation of Bitcoin, all quarterly rebalanced portfolios with a minimum holding period of 3 years have outperformed a baseline 60/40 portfolio (Traditional Portfolio). This is especially significant considering that this finding holds regardless of the period in which the investment was made, both in term of cumulative returns and Sharpe Ratio, and without compelling implications for the annual volatility and maximum drawdown. Moreover, this imply that even when the cryptocurrency space is in a "bear market" – such as in 2018 (when the Bitcoin price fell more than 65%) and in 2022 (it has reached more than a 75% drawdown since the all-time high in 2021) – the implementation of the strategy explored has had successful results, which is exceptional considering the combination of volatility and correlation imbedded in the assets studied.

The rest of the article is organized as follows. We review the existing literature on the main topics in Chapter 2. Chapter 3 presents the data and methodology used to construct the portfolios of interest. Chapter 4 presents empirical results on the diversification role of Bitcoin, by comparing traditional 60/40 portfolios with and without Bitcoin allocation. At last, we conclude and describe the main findings, contribution to the literature and research avenues looking forward in Chapter 5.

Chapter 2
Literature Review

The literature related with cryptocurrencies is recent but growing at a very fast pace (García-Corral *et al.*, 2022; Fang *et al.*, 2022). This is due to the efforts from scholars, investors, and crypto portfolio managers to better understand the features of cryptocurrencies and how they can be valuable. Different articles use different techniques, variables, and time periods, which sometimes results in contradictory results. Moreover, results from articles that were written years ago may change when more recent data is used, due to the evolving dynamic features of the crypto space.

Roughly speaking, the bulk of cryptocurrencies' literature can be divided into two major's categories. The first one regards the empirical properties and stylized characteristics of virtual currency time series, while the second one consists of asset pricing, portfolio theory and diversification benefits associated with cryptocurrencies. This dissertation is built on these two strands, which are reviewed in this section. In specific, in section 2.1. we review the literature on the stylized facts of cryptocurrencies, giving particular emphasis to Bitcoin (as it has been the most studied form of digital currency and the object of study in this research), while in section 2.2. we review the literature regarding Portfolio Theory with and without the inclusion of cryptocurrencies.

2.1. Stylized Facts of Cryptocurrencies

The literature first started by investigating the properties of Bitcoin price process (Kristoufek, 2015; Chu, Nadarajah and Chan, 2015), as well as its relationships with other cryptocurrencies and financial assets (Corbet *et al.*, 2018). Phillip, Chan and Peiris, 2018, show that Bitcoin displays many diverse stylized facts common to most cryptocurrencies, such as long memory, leverage, heteroskedasticity and heavy-tailed distribution. Moreover, several papers investigate the dynamic volatility of cryptocurrencies through a set of statistical tests from which result important implications, such as highly persistent and asymmetric volatility (Dangi, 2020; Katsiampa, 2019; Kakinaka and Umeno, 2021).

The fact that cryptocurrencies, in particular Bitcoin, are prone to speculative bubbles (Kakinaka and Umeno, 2021; Cheah and Fry, 2015) has been the subject of many criticisms, regardless the fact that bubbles constitute a necessary component in the process of technology adoption and diffusion. Moreover, since Baek and Elbeck, 2015, referred to Bitcoin as a speculative asset, the issue of informational efficiency has been a matter of interest. Urquhart, 2016, followed by Tiwari *et al.*, 2018, reach the important conclusion that although the Bitcoin market started by being inefficient, it has been moving to achieve efficiency as the market matures. This is consistent with the idea that as more investors trade and analyze Bitcoin, along with its increasing world-wide adoption, Bitcoin will become more efficient.

As an investment, Ram, 2019, shows that Bitcoin represents a unique and distinct asset class, measured by its *investability*, *politico-economic features*, *correlation within and with outside its asset class* and *risk-reward profile*. Moreover, whereas Urquhar tand Zhang, 2019, advocate that Bitcoin has relevant safe haven properties, acting as an intraday hedge and diversifier, Baur, Hong and Lee, 2018, argue that its effects during a financial crisis are inconsistent with this definition. On the other hand, the claim that Bitcoin can be seen as "*the new gold*" is approached by Klein, Pham Thu and Walther,

2018, who argue that Bitcoin has fundamentally different properties from this asset. The authors went a step further to show the effect of including Bitcoin on a portfolio, whose results suggest that Bitcoin is no hedge against equity investments. Besides, based on the assumption that liquidity risk varies over time, Ghabri, Guesmi and Zantour, 2020, show that the inclusion of Bitcoin can lead to potential gains by diversifying risk, as well as strong evidence of downside liquidity risk and hedging effectiveness.

Furthermore, Normative Finance has been struggling with explaining the volatility and price performance of cryptocurrencies through a standard fundamental analysis. Hence, note the role of Behavioral Finance Theory (BPT) in providing a set of explanations for this to be the case, such as the existence and activity of noise traders (Karaa *et al.*, 2021), as well as sentiment analysis (Lee, Guo and Wang, 2018) and price clustering (Urquhart, 2017).

2.2. Portfolio Theory

The Modern Portfolio Theory (MPT) of Harry Markowitz, 1952, 1991, formulizes the popular expression: "*Don't put all your eggs in one basket*" in a model that seeks to maximize the expected return of an asset portfolio for a given level of risk. This is achieved by diversifying a portfolio across different securities from diverse industries, in a way that the gains from low-correlation assets allow to enhance risk-adjusted returns. This theory was further developed by Sharpe, 1964, who significantly advanced the Efficient Frontier and Capital Market Line concepts expressed by Markowitz in his derivation of the CAPM (Capital Asset Pricing Model). Although these concepts built the foundation of mainstream portfolio techniques used today, the lack of empirical support prompted the development of other theories related to investments (e.g., weak, semi-strong and strong versions of the Efficient Market Hypothesis (EMH) by (Fama, 1970), as well as Behavioural Finance Theories (BFT)).

Moreover, note that the implementation of mean-variance optimal portfolios has serious limitations. According to DeMiguel, Garlappi and Uppal, 2009, and DeMiguel *et al.*, 2009, since the estimation of means is more difficult than the estimation of covariances of assets returns (and with a greater out-of-sample penalization (Merton, 1980)), portfolio models that rely solely on covariances are less vulnerable to estimation errors than mean-variance portfolios. The authors go on to show that *allocation mistakes* caused by using weights from an optimizing model – with inputs that have been estimated with error – can cause mean-variance models to significantly underperform[7] the benchmark 1/N model (i.e., 1/N Heuristic or *naive diversification*, related to the portfolio strategy based on an equal weight distribution among all available assets). Furthermore, the *allocation mistakes* are accentuated if (i) the number of available assets (N) is large and if (ii) the assets do not have sufficiently long data history to allow for a precise estimation of moments (note that, in 2022, there are more than 20 000 cryptocurrencies[8], of which the vast majority is limited to no more than a few years of existence).

While there are some empirical papers than support this within a portfolio solely composed with cryptocurrencies (Brauneis and Mestel, 2019; Platanakis and Urquhart, 2019), there are others that consider the allocation to cryptocurrencies to a well-diversified portfolio constituted by traditional and/or alternative assets (Petukhina *et al.*, 2020; Brière, Oosterlinck and Szafarz, 2015; Bakry *et al.*, 2021, Demiralay and Bayracı, 2021). Despite some differences between the scope of analysis between the papers considered (period under analysis, cryptocurrencies studied, assets considered, methodology employed), there is some common ground: cryptocurrencies were found to be far more volatile than traditional assets and with greater average returns; cryptocurrencies historical correlation with traditional assets has been remarkably low; cryptocurrencies are a good diversifier, measured by the difference in risk-adjusted returns (Sharpe Ratio) in portfolios with and without

[7] Note that performance is measured in terms of Sharpe Ratio, Certainty-equivalent Return and Turnover.
[8] See https://coinmarketcap.com/ for more details.

cryptocurrencies. Furthermore, while Letho, Chelwa and Alhassan, 2022, show that cryptocurrencies can enhance risk-adjusted returns in emergent markets, Colombo *et al*., 2021, show how heterogeneous are the diversification marginal effects of adding cryptocurrencies in both developing and developed countries.

Finally, Hougan and Lawant, 2020 and its extension in Hougan and Lawant, 2021, explored the benefits of adding Bitcoin to a well-diversified portfolio employing a different but rather interesting framework. By using a rolling analysis, the authors were able to assess the performance of thousands of portfolios considering a different set of holding periods, rebalancing frequencies and Bitcoin holding positions.

Chapter 3
Data and Methodology

We aim to extend the study of Bitcoin's diversification benefits by using a quantitative approach, , employing the framework used in Hougan and Lawant, 2020 and its extension in Hougan and Lawant, 2021. This dissertation provides an intuitive examination on the impact of adding Bitcoin to a traditional, well-diversified portfolio of stocks and bonds (Traditional Portfolio), allowing us to answer the proposed set of research questions: (i) What is the minimum acceptable holding period for a bitcoin allocation? (ii) What is the best rebalancing frequency for a bitcoin allocation? (iii) How much bitcoin should be added to a traditional 60/40 portfolio?

The expected contribution of this dissertation is not only expanding the period of analysis (capturing the impact of the COVID-19 pandemic and the most recent bear market of 2022), but also to evaluate the investment robustness of the strategies employed under a wider set of portfolio's construction techniques.

3.1. Data Description

We collected data on historical daily adjusted closing prices of Bitcoin – from Bitstamp – and of Vanguard Total Bond Market ETF (BND), as the proxy or bonds, and of Vanguard Total World Stock ETF (VT), as the proxy from equities – both from Yahoo Finance – in USD dollars. Bitsamp is the original global crypto exchange, founded in 2011 (Bitstamp USA, 2022), and Yahoo Finance is a media property that

provides not only financial news but also financial data (Yahoo, 2022). The whole sample period spans from August 18, 2011[9] to June 3, 2022. Data from weekends and American public holidays during this period were excluded, as trading of traditional assets does not occur during these dates[10].

Vanguard Total Bond Market ETF (BND) was chosen to represent the broad bond market, as it tracks the performance of a market-value-weighted portfolio representing all taxable investment-grade corporate U.S. bonds, being exposed to U.S. Treasury, mortgage-backed securities (MBS) and asset-backed securities (ABS). Moreover, Vanguard Total World Stock ETF (VT) was chosen to represent the broad stock market, as it holds a market-capitalization-weighted index designed to measure the market performance of stocks around the world, both from developed and emerging markets[11].

3.2. Descriptive Statistics

This dissertation employs continuously compound returns calculated from daily adjusted closing prices as the natural logarithmic price differences.

Overall, Bitcoin outperforms traditional assets in terms of historical continuous compound daily returns (Table 1). The annualized return of Bitcoin is 0,003 * 252 = 75.6%, which is very high compared with both the stock market (9,51%) and the bond market (1,77%). Moreover, BTC tends to have a return volatility many times higher than BND and VT, with a skewness of -1,026. Although not so clear, it seems that volatility tends to cluster together (Figures 1, 2 and 3), meaning that large changes in prices tend to be followed by large changes and small changes in prices tend to be followed by small changes.

[9] The sample period starts only in 2011 due to data availability.
[10] See https://www.marketbeat.com/stock-market-holidays/ for more details.
[11] See https://advisors.vanguard.com/investments/products/bnd/vanguard-total-bond-market-etf and https://investor.vanguard.com/investment-products/etfs/profile/vt#distributions for more details related to these products.

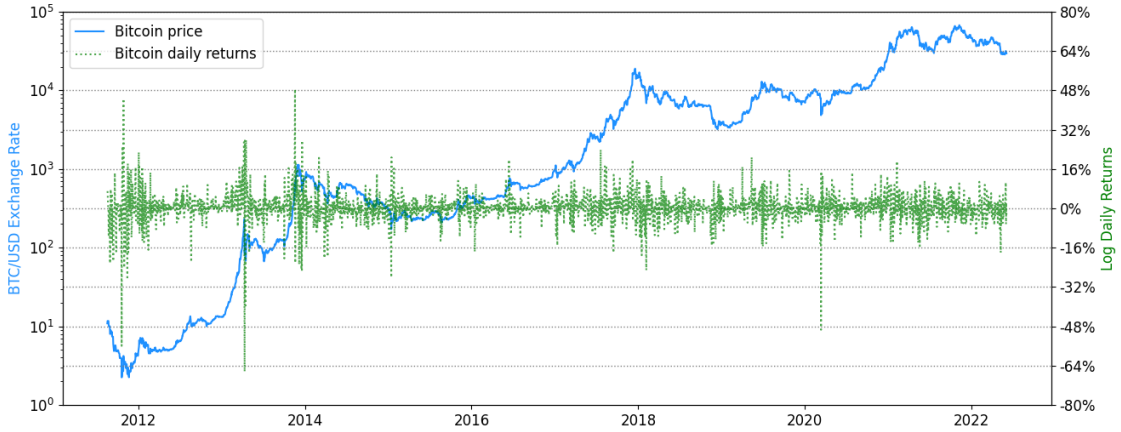

Figure 1: Bitcoin price time series (on the left axis, with a logarithmic scale) and bitcoin historical continuously compound daily returns (on the right axis), from 19th August 2011 to 3rd June 2022.

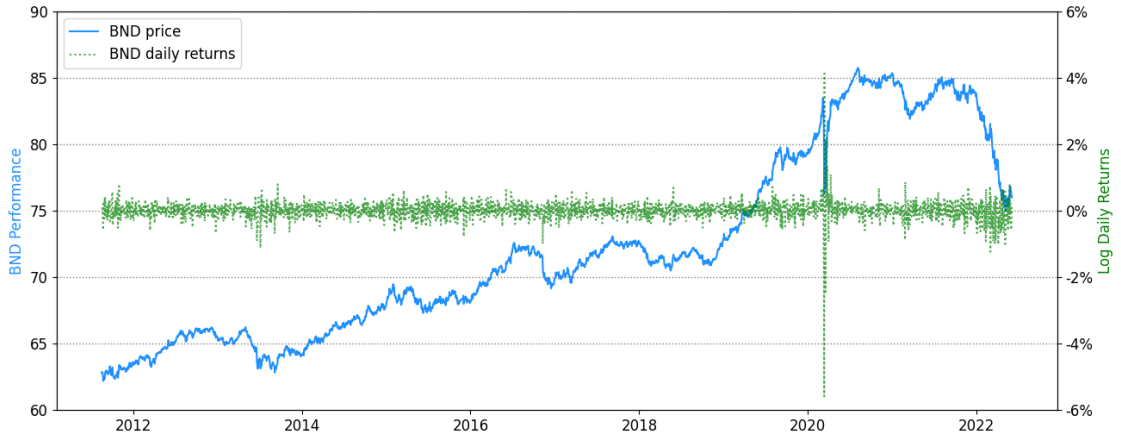

Figure 2: Vanguard Total Bond Market ETF (BND) price time series (on the left axis) and BND historical continuously compound daily returns (on the right axis), from 19th August 2011 to 3rd June 2022.

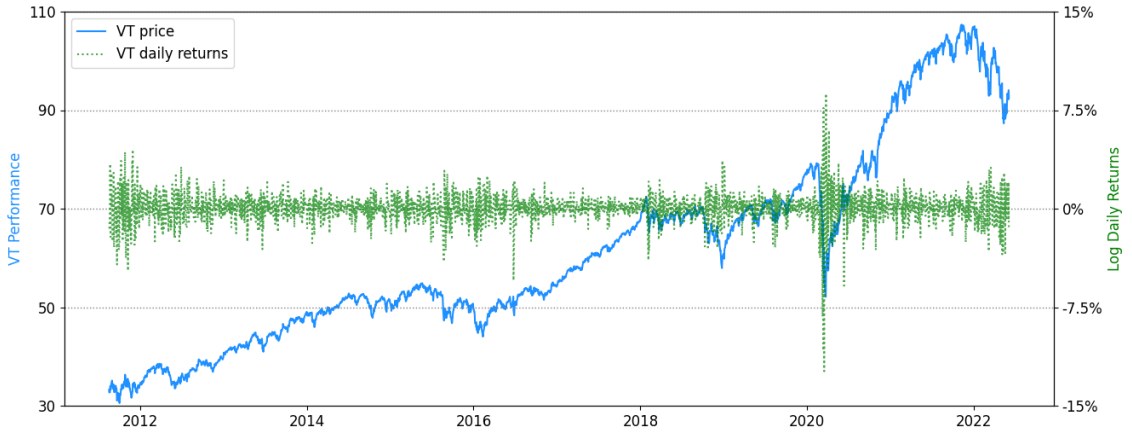

Figure 3: Vanguard Total World Stock ETF (VT) price time series (on the left axis) and VT historical continuously compound daily returns (on the right axis), from 19th August 2011 to 3rd June 2022.

In case of kurtosis, all financial assets return distribution greatly deviates from the normal distribution. Interestingly, the asset with a higher kurtosis is BND, which is justified by a negative return swing following the burst of the COVID-19 pandemic, approximately 14 times higher than its standard deviation.

Variables	Mean	Standard Deviation	Minimum	Median	Maximum	Skewness	Excess Kurtosis
BND	0,000	0,003	-0,056	0,000	0,041	-2,348	89,215
VT	0,000	0,011	-0,124	0,001	0,087	-1,053	15,760
BTC	0,003	0,057	-0,664	0,002	0,485	-1,026	19,535

Table 1: Summary Statistics of the daily log returns of BND (Vanguard Total Bond Market ETF), VT (Vanguard Total World Stock ETF) and BTC (Bitcoin protocol). The presented statistics are computed using daily observations from 19th August 2011 to 3rd June 2022, accounting for 2,717 observations for each variables.

3.3. Correlation Analysis

Average correlations for the full sample period are reported in Table 2, where we find that the highest correlation is between BTC and VT and the lowest is between VT and BND (this seems to suggest that bonds and stocks are often viewed as substitutes).

Correlation Matrix	BND	VT	BTC
BND	1,000		
VT	-0,019	1,000	
BTC	0,081	0,129	1,000

Table 2: Correlation matrix of the daily log returns of the variables employed in this study.

Since Bitcoin and the stock market seem to traditionally have a low correlation, we would expect significant diversification benefits of allocating capital to this new asset class. The low correlation between these assets can be explained by their different

value drivers: while the performance of the former depends on its own idiosyncratic risks (market adoption, liquidity pressure, political and regulatory constraints, among others), the performance of the latter depends on macroeconomic factors (supply and demand frictions, fiscal and monetary policy, technological innovation, corporate and government performance, among others).

Nevertheless, note that the correlation between two given assets changes over time, particularly in periods of financial turmoil. To capture this dynamic, a rolling correlation analysis was computed using the correlation of two time series on a rolling window (90 days). Hence, we compute the correlation between BTC and VT, BTC and BND and VT and BND.

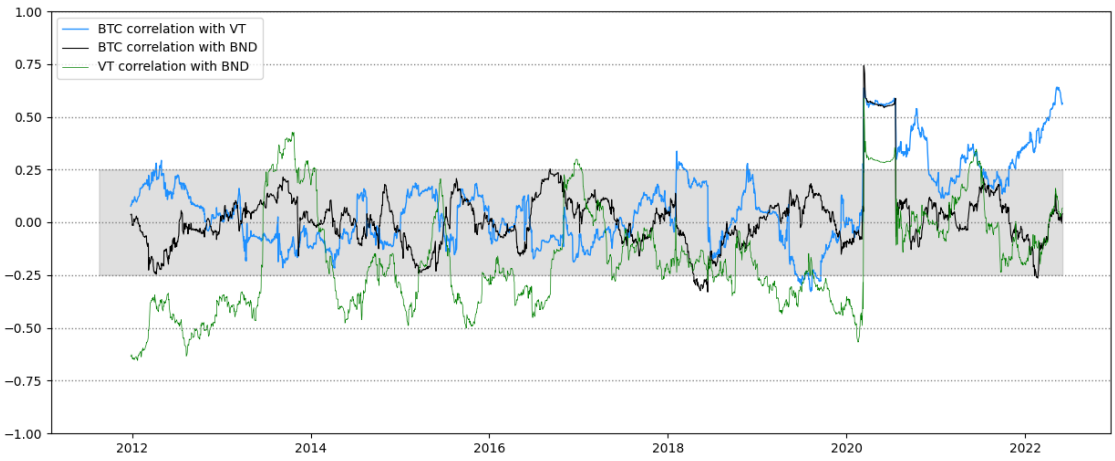

Figure 4: Rolling 90-Day correlation coefficient.

In fact, it's possible to analyze that since the COVID-19 pandemic started, the correlation between Bitcoin and the stock market soared, therefore questioning the popular assertion in the crypto market that cryptocurrencies can be seen as a safe haven (Baur, Hong and Lee, 2018). Furthermore, this also provides support to the idea that connectedness via negative returns is much stronger than via positive ones, exhibiting an increase trend in recent periods (Zeng, Yang and Shen, 2020), despite the fact that COVID-19 does not cause a significant shock on herding in the cryptocurrency markets (Yarovaya, Matkovskyy and Jalan, 2021).

3.4. Methodology

The baseline Traditional Portfolio features 60% allocation to the Vanguard Total World Stock ETF (VT) and 40% allocation to the Vanguard Total Bond Market ETF (BND), representing the benchmark against which the performance of portfolios exposed to Bitcoin are assessed. Aiming in being as comprehensive as possible in this analysis, the BTC's portfolios construction is dictated by three factors:

- Investment Time-Horizon.

Investment Time-Horizon is related to the length of time through which the capital will be allocated and therefore subject to price fluctuations. Given the limitations regarding the length of our dataset (less than 11 years), we considered rolling periods of 1, 2 and 3 years. Using this setup, we were able to compute 2466 portfolios with 1 year-horizon, 2214 portfolios with 2 years-horizon and 1962 portfolios with 3 years-horizon. Please note that as the time frame expands there are less portfolios that can be assessed, as for 1 year-horizon we start having results on August 17th 2012, for 2 year-horizon only on August 21st 2013 and for 3 year-horizon only on August 21st 2014.

- Rebalancing Frequency.

Rebalancing Frequency is related to the periodicity in which the assets' weight in the portfolio is restored, regardless of the market direction or future expectations. Although this strategy may be questionable (since it is related to trimming down "winning positions" and allocating capital to underperforming assets, which may be associated with lower returns), this strategy and its "ideal" frequency is of the most importance since it takes advantage of well-balanced diversified portfolios that may result in lower drawdowns and risk.

In other words, by *harvesting volatility* (Bouchey *et al.*, 2012), the investor is akin to benefit from rebalancing volatile assets over multiple periods of time without the costs of acquiring skills on forecasting, security selection or informational advantages.

In this dissertation, we considered the following rebalancing frequencies: no rebalance, monthly, quarterly, and yearly.

- Position sizing on BTC.

The percentage allocated to Bitcoin is crucial, as it affects its risk profile and therefore its cumulative returns. To properly understand the risk-adjusted returns dynamic, we considered BTC allocations between 0% (Traditional Portfolio) and 9.5% with increments of 0.5%, as well as BTC allocations between 10% and 100% (invest solely on Bitcoin) with increments of 5%. For each portfolio, the percentage not allocated to Bitcoin follows the 60/40 baseline Traditional Portfolio.

First, we begin the study by considering the impact of a modest Bitcoin allocation to a classic 60/40 portfolio for the full sample period. This analysis illustrates an intuitive framework that show how a small BTC allocation can scale the overall performance of a portfolio. However, we avoid fixing on a specific (arbitrary) period of history by performing a Rolling Analysis, following the methodology of Hougan and Lawant, 2021.

A Rolling Analysis enables us to appraise the stability of the portfolio's construction techniques employed. By considering the evolution of the performance metrics studied over a rolling window of a fixed size through the sample, we are able to take into account changes in the economic environment that may result in unstable parameters (Zivot and Wang, 2003). Moreover, note that it's possible to eliminate concerns about cherry-picking specific time periods as we analyze the full sample period through a constant rolling holding period window (e.g., one year, two years or three years). As such, it's possible to characterize which portfolio's construction techniques are associated with better risk-adjusted returns by assessing the evolution of the performance metrics employed: Cumulative and Annualized returns, Annualized standard deviation, Sharpe ratio and Maximum drawdowns (Appendix I provides a brief explanation on how these metrics were constructed). Moreover, note that for the Rolling Analysis we also considered the Cumulative Return Contribution

and Sharpe Ratio Contribution, Win Rate and Loss Rate, which show the benefits and drawbacks of allocating capital to BTC *vis-à-vis* to a Traditional Portfolio.

In particular:

- **Cumulative Return Contribution** allows to capture the difference between the aggregate return of a portfolio with and without (Traditional Portfolio) Bitcoin Allocation for the same period. This value can be positive or negative.
- **Sharpe Ratio Contribution** allows to capture the difference between the Sharpe Ratio of a portfolio with and without (Traditional Portfolio) Bitcoin Allocation for the same period. This value can be positive or negative.
- **Win Rate**, which can be applied to the Cumulative Return Contribution or Sharpe Ratio Contribution**,** allows to compute the percentage of times each indicator has been positive (i.e., percentage of times the portfolio with Bitcoin allocation outperformed the Traditional Portfolio).
- **Loss Rate,** which can be applied to the Cumulative Return Contribution or Sharpe Ratio Contribution**,** allows to compute the percentage of times each indicator has been negative (i.e., percentage of times the portfolio with Bitcoin allocation underperformed the Traditional Portfolio).

Within the Rolling Analysis Framework, this methodology allows us to explore the performance of 1 036 152 individual portfolios[12], each of them with a different set of features: day of investment, time-horizon, rebalance frequency and position sizing on BTC. In addition, note that we are only considering the Bitcoin price dynamic when assessing its impact on a Traditional Portfolio, excluding the impact of hard forks[13] and airdrops[14] over the study period.

[12] Entire set of results available upon request.
[13] Hard Forks are radical changes in the protocol of a cryptocurrency that result in separating a blockchain into two separate branches that follow a different set of rules. Bitcoin has had different hard forks throughout its history (Bitcoin XT, BCH, BTG, among others), some of which that have had an interesting price trajectory.
[14] Airdrops are unsolicited distributions of tokens that aim to promote the adoption of a specific network.

Chapter 4
Findings and Results

We started by considering the impact of a modest Bitcoin allocation to a classic 60/40 portfolio for the full sample period. While Figure 5 illustrates the cumulative log returns evolution, Table 3 summarizes the performance metrics of interest.

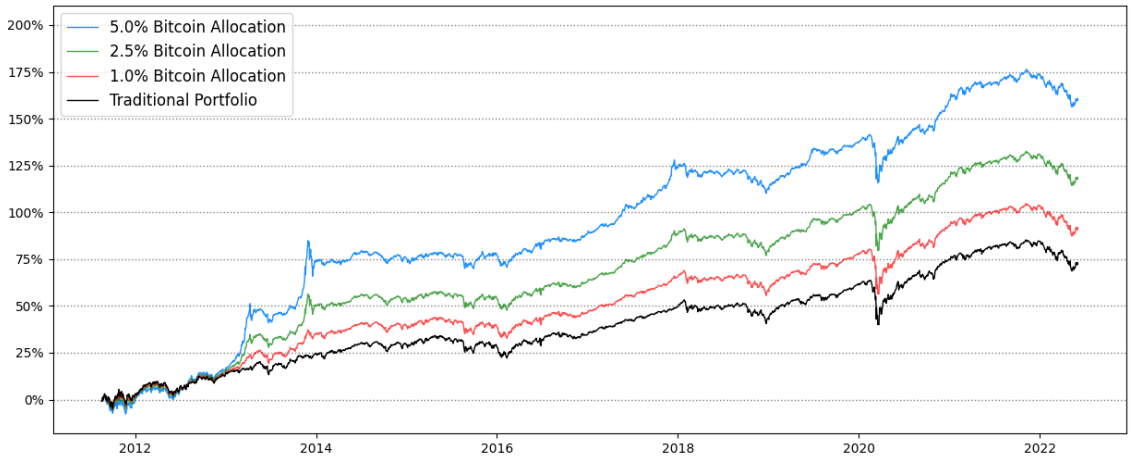

Figure 5: Cumulative log returns on quarterly rebalanced portfolios, invested on 18th August 2011 to 3rd June 2022, with bitcoin allocations of 0.0% (Traditional Portfolio), 1.0%, 2.5% and 5.0%.

Traditional Portfolio with bitcoin allocation	Cumulative return	Annualized return	Volatility (Annualized Std. Dev.)	Sharpe Ratio	Maximum Drawdown
0.0%	72,181%	4,632%	10,296%	0,450	23,666%
1,0%	90,875%	5,535%	10,448%	0,530	23,935%
2,5%	117,743%	6,699%	11,129%	0,602	24,553%
5,0%	159,750%	8,280%	12,984%	0,638	25,738%

Table 3: Performance Metrics on quarterly rebalanced portfolios, invested on 18th August 2011 to 3rd June 2022, with bitcoin allocations of 0.0% (Traditional Portfolio), 1.0%, 2.5% and 5.0%, respectively.

The results show that adding any percentage of Bitcoin to a quarterly rebalanced 60/40 baseline portfolio can significantly scale the annualized return without blowing volatility and the maximum drawdown, for any level of BTC allocation. For instances, a 2.5% BTC allocation would increase the annualized return by approximately 2.07 pp[15] and volatility by 0.83 pp, resulting in a Sharpe Ratio increase of 0.15 points. At the same time, the maximum drawdown would increase by less than 1 pp. It's remarkable how a small allocation on BTC with periodical rebalances can significantly increase the overall performance of a portfolio.

Let's now aim to answer the proposed research questions by considering the Rolling Analysis Framework, which will boost our understanding of rolling returns periods. Please note that all data presented in sections 4.1., 4.2. and 4.3. results from the author's computations. The graphs from Figures 6-11 represent the Cumulative Return and the Cumulative Sharpe Ratio for all portfolios discussed in this dissertation for a given time-horizon, rebalance frequency and BTC allocation. Moreover, each data point in each graph represents the performance of an individual portfolio with an end date on that point. For instances, in Figure 6, the bottom-left graph "5.0% Bitcoin Allocation/ 1-Year Window/ Quarterly Rebalanced" analyses the cumulative return of all portfolios quarterly rebalanced from 18th August 2011 to 3rd June 2022., considering a 5.0% BTC weight and a time-horizon of 1 year. Note that since we are using a 1-year rolling window, the first data point corresponds to the cumulative return of a portfolio with a start date on 18th August 2011 and an end date of August 17th 2012; conversely, the last data point corresponds to the cumulative return of a portfolio with a start date on June 4th 2021 and an end date of June 3rd 2022. The other data points represent all 1-year windows between these two.

[15] The contribution of a bitcoin allocation is hereby measured in the actual amount (i.e., the difference between two percentages) instead of the rate of change. For example: if the Traditional Portfolio returned 1% while the same portfolio with a bitcoin allocation returned 2%, the impact will be presented as a 1 percentage point (pp) positive impact instead of as a 100% boost in returns; conversely, if the Traditional Portfolio has a Sharpe ratio of 1.00 while the same portfolio with a bitcoin allocation has a Sharpe ratio of 1.20, the impact will be presented as 0.20 point contribution instead of 20% increase.

Common to all graphs, while the black line represents the performance of the Traditional Portfolio, the green and red shade show the positive and negative contribution that a Bitcoin allocation brings, respectively.

4.1. What is the minimum acceptable holding period for a bitcoin allocation?

Aiming in answer to this question, we designed a Rolling Analysis Framework in which we could evaluate the impact of different Investment Time-Horizons (1, 2 and 3 years) for a portfolio with a constant Rebalance Frequency and a given Position Size on BTC. Assuming quarterly Rebalance Frequency and BTC allocations of 1.0%, 2.5% and 5.0%, consider the impact of different Investment Time-Horizons on the Rolling Cumulative Return Contribution of Bitcoin.

Rolling Cumulative Return Contribution						
Rolling Period (years)	**Bitcoin Allocation**	**Maximum**	**Median**	**Minimum**	**Win Rate**	**Loss Rate**
1	**1,00%**	13,144 pp	1,330 pp	(1,238) pp	80,70%	19,30%
	2,50%	31,408 pp	3,288 pp	(3,105) pp	80,70%	19,30%
	5,00%	58,680 pp	6,415 pp	(6,242) pp	80,66%	19,34%
2	**1,00%**	14,876 pp	3,121 pp	(0,714) pp	97,56%	2,44%
	2,50%	35,743 pp	7,673 pp	(1,803) pp	97,43%	2,58%
	5,00%	67,343 pp	14,960 pp	(3,662) pp	97,25%	2,76%
3	**1,00%**	11,849 pp	4,579 pp	0,215 pp	100,00%	0,00%
	2,50%	28,741 pp	11,196 pp	0,508 pp	100,00%	0,00%
	5,00%	54,861 pp	21,683 pp	0,912 pp	100,00%	0,00%

Table 4: Rolling Cumulative Return Contribution on quarterly rebalanced portfolios with bitcoin allocations of 1.0%, 2.5% and 5.0% and rolling windows of 1, 2 and 3 years. Negative values in brackets.

Taking as the base case a quarterly rebalanced portfolio with 2.5% BTC allocation, Table 4 shows that the median Cumulative Return Contribution is 3.288 pp, 7.673 pp and 11.196 pp for holding periods of 1, 2 and 3 years, respectively. Although there are negative results associated with a holding period on BTC for 1 and 2 years, note that its impact is asymmetrical. Moreover, for a holding period of 3 years, all portfolios have a Cumulative Return superior to the baseline 60/40 Traditional Portfolio (100% Win Rate). Consider now the impact of different Investment Time-Horizons on the Rolling Sharpe Ratio Contribution of Bitcoin.

Rolling Sharpe Ratio Contribution						
Rolling Period (years)	**Bitcoin Allocation**	**Maximum**	**Median**	**Minimum**	**Win Rate**	**Loss Rate**
1	**1,00%**	0,104 p	0,012 p	(0,014) p	80,049%	19,951%
	2,50%	0,151 p	0,024 p	(0,036) p	78,710%	21,290%
	5,00%	0,212 p	0,037 p	(0,072) p	75,547%	24,453%
2	**1,00%**	0,104 p	0,024 p	(0,007) p	97,606%	2,394%
	2,50%	0,174 p	0,039 p	(0,019) p	97,290%	2,710%
	5,00%	0,185 p	0,051 p	(0,046) p	93,044%	6,956%
3	**1,00%**	0,085 p	0,038 p	0,003 p	100,000%	0,000%
	2,50%	0,145 p	0,075 p	0,006 p	100,000%	0,000%
	5,00%	0,207 p	0,082 p	0,006 p	100,000%	0,000%

Table 5: Rolling Sharpe Ratio Contribution on quarterly rebalanced portfolios with bitcoin allocations 1.0%, 2.5% and 5.0% and rolling windows of 1, 2 and 3 years. Negative values in brackets.

Table 5 shows that, for a quarterly rebalanced portfolio with 2.5% BTC allocation, there is a median contribution of 0.024 points, 0.039 points and 0.075 points for holding periods of 1, 2 and 3 years, respectively. For a holding period of 3 years, the fact that the gains related to the increase performance of the portfolio (i.e., cumulative returns) are greater than the increase in risk (i.e., volatility) for each individual portfolio under analysis results in a 100% Win Rate.

Hence, we reach the conclusion that regardless of the percentage allocated to BTC, assuming a quarterly rebalance frequency, 100% of the historical portfolios have benefited from being exposed to Bitcoin for a minimum holding period of 3 years, both in the case of the Cumulative Return and Sharpe Ratio. In other words, the shorter the time horizon, the higher the risk of a negative result (consistently with theories from both Normative and Behavioral Finance).

Finally, note that Figures 6-8 show a graphical representation of BTC positive and negative contribution for the different holding periods and BTC positions considered, assuming quarterly rebalancing.

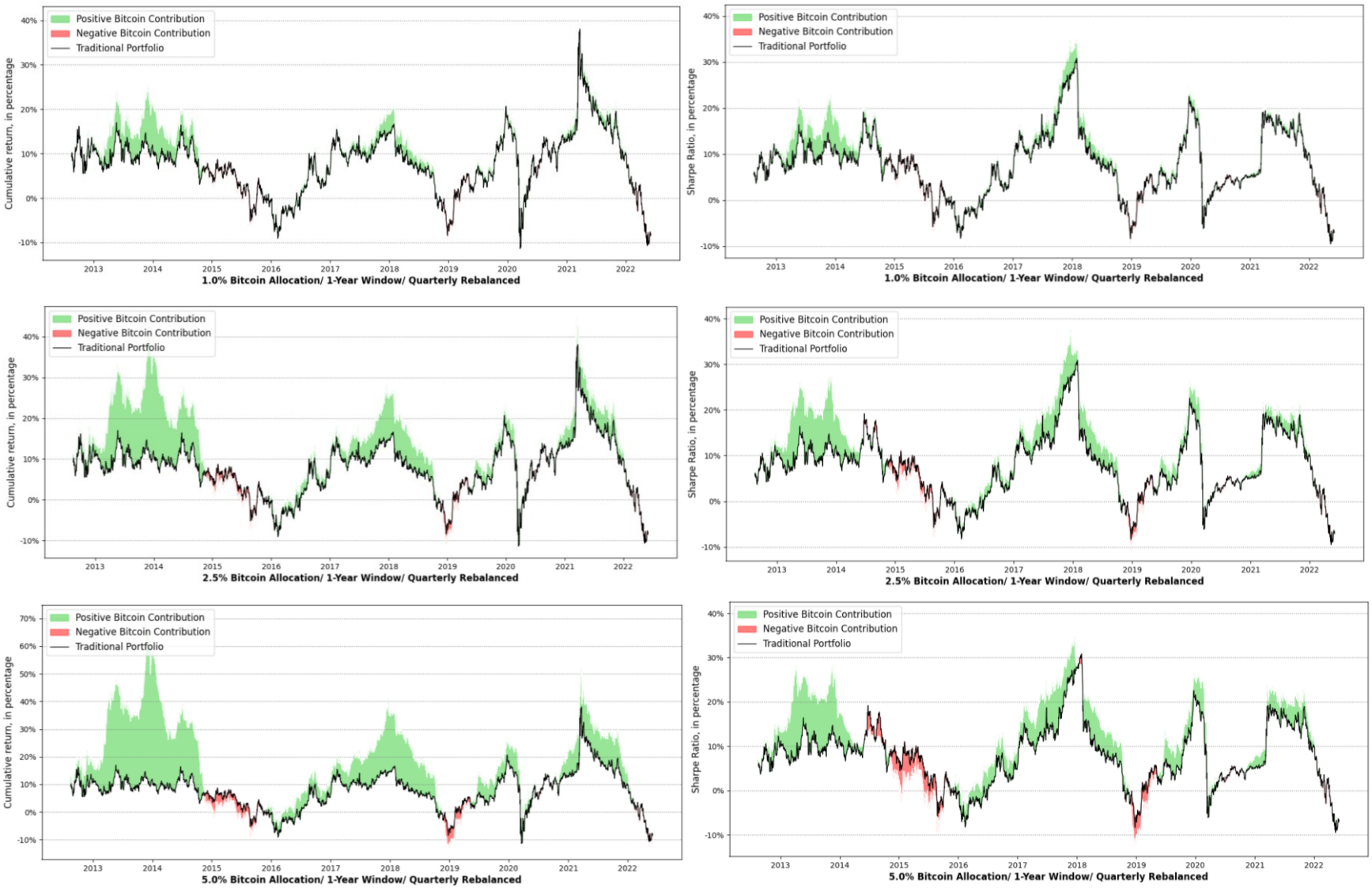

Figure 6: Rolling Cumulative Return Contribution (on the left) and Rolling Sharpe Ratio Contribution (on the right) on quarterly rebalanced portfolios with a rolling window of 1 year, considering Bitcoin allocations of 1.0%, 2.5% and 5.0%.

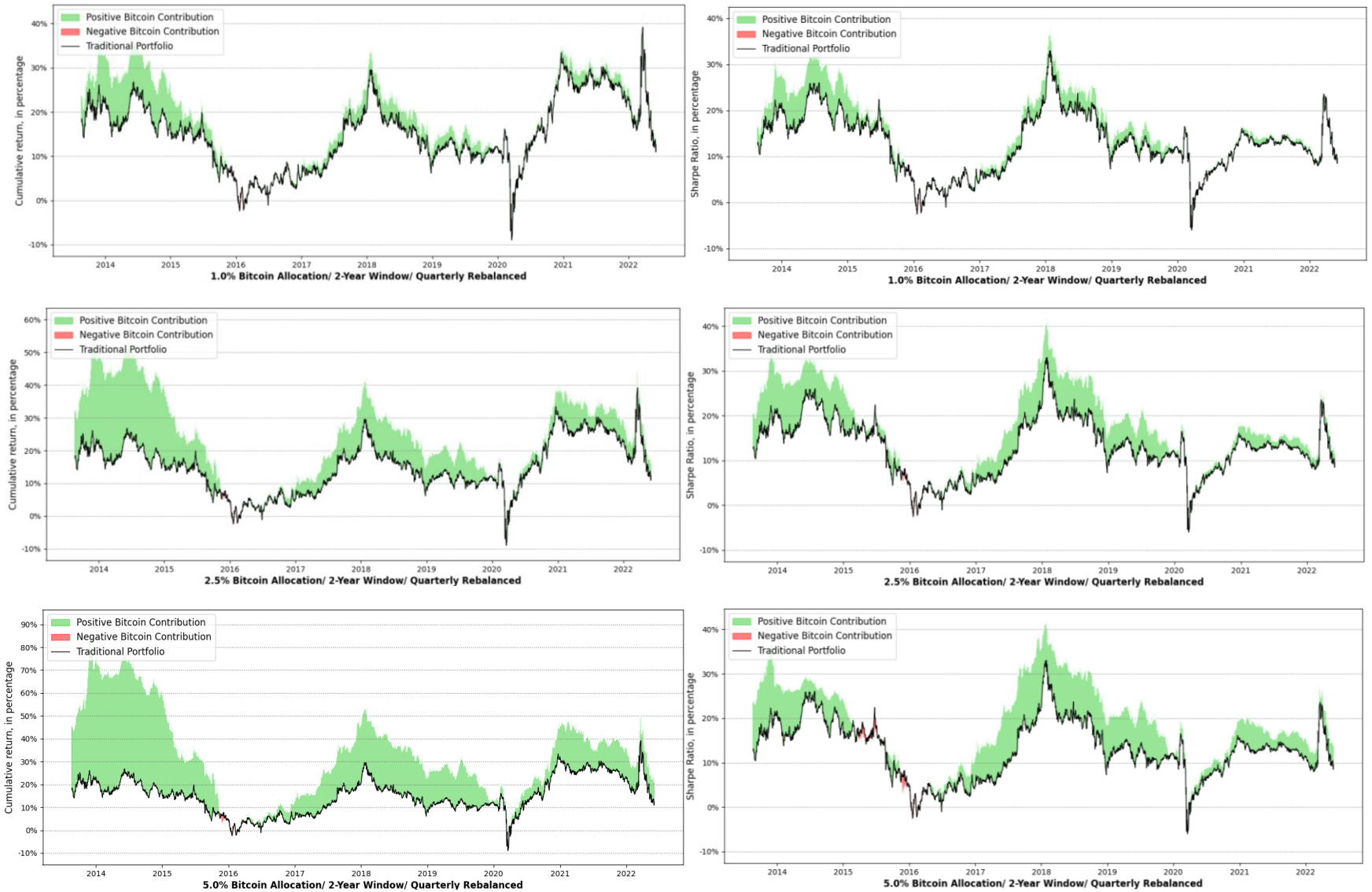

Figure 7: Rolling Cumulative Return Contribution (on the left) and Rolling Sharpe Ratio Contribution (on the right) on quarterly rebalanced portfolios with a rolling window of 2 years, considering Bitcoin allocations of 1.0%, 2.5% and 5.0%.

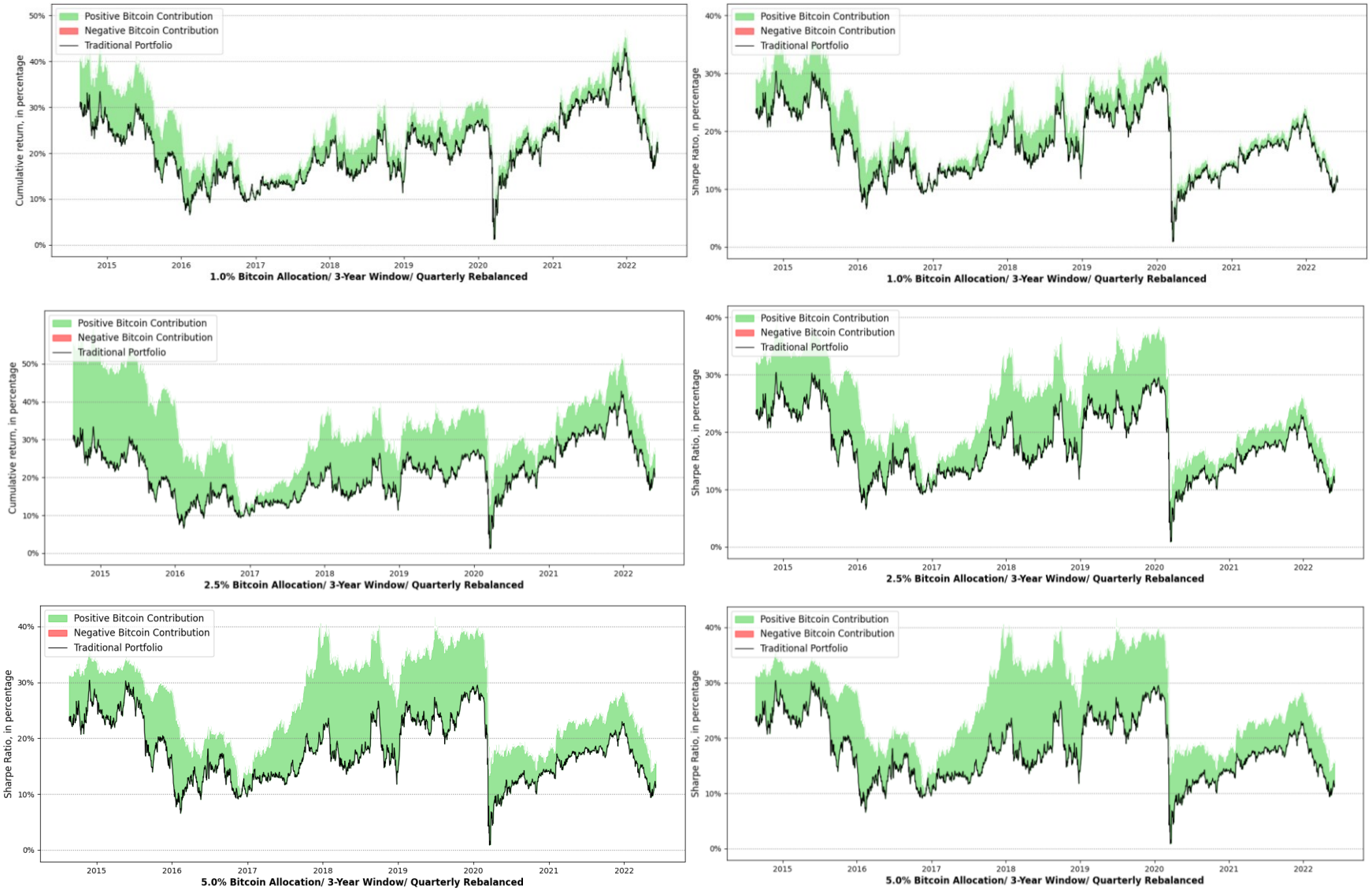

Figure 8: Rolling Cumulative Return Contribution (on the left) and Rolling Sharpe Ratio Contribution (on the right) on quarterly rebalanced portfolios with a rolling window of 3 years, considering Bitcoin allocations 1.0%, 2.5% and 5.0%.

4.2. What is the best rebalancing frequency for a bitcoin allocation?

Although it may be appealing to be exposed to BTC's historical returns, without a rebalancing strategy in place a volatile asset such as Bitcoin could "grow" to dominate a portfolio and dictate its performance (being harmful for its risk-adjusted returns).

Aiming in answer to this question, we designed a Rolling Analysis Framework in which we could evaluate the impact of different Rebalancing Frequencies (no rebalance, monthly, quarterly, and yearly rebalance) for a given portfolio with constant Investment Time-Horizon and Position on BTC. Assuming a 3 year-horizon, and BTC allocations of 1.0%, 2.5% and 5.0%, the following table represents the Rolling Cumulative Return Contribution of the different Rebalancing Frequencies considered.

Rolling Cumulative Return Contribution						
Rebalancing Frequency	**Bitcoin Allocation**	**Maximum**	**Median**	**Minimum**	**Win Rate**	**Loss Rate**
No Rebalance	**1,00%**	82,960 pp	6,525 pp	(0,414) pp	98,216%	1,784%
	2,50%	144,244 pp	15,578 pp	(1,038) pp	98,216%	1,784%
	5,00%	200,983 pp	29,053 pp	(2,086) pp	98,216%	1,784%
Monthly	**1,00%**	9,315 pp	3,395 pp	(0,002) pp	99,949%	0,051%
	2,50%	22,846 pp	8,436 pp	(0,023) pp	99,949%	0,051%
	5,00%	44,349 pp	16,698 pp	(0,111) pp	99,949%	0,051%
Quarterly	**1,00%**	11,849 pp	4,579 pp	0,215 pp	100,000%	0,000%
	2,50%	28,741 pp	11,196 pp	0,508 pp	100,000%	0,000%
	5,00%	54,861 pp	21,683 pp	0,912 pp	100,000%	0,000%
Yearly	**1,00%**	41,341 pp	9,320 pp	0,023 pp	100,000%	0,000%
	2,50%	84,130 pp	21,840 pp	0,040 pp	100,000%	0,000%
	5,00%	132,729 pp	39,952 pp	0,020 pp	100,000%	0,000%

Table 6: Rolling Cumulative Return Contribution on 3 years rolling windows portfolios with bitcoin allocations of 1.0%, 2.5% and 5.0%, considering no rebalance, monthly rebalance, quarterly and yearly rebalance. Negative values in brackets.

Considering as the base case a 3-year rolling period with 2.5% BTC allocation, Table 7 shows that the median Cumulative Return Contribution is 15.578 pp, 8.436 pp, 11,196 pp and 21,840 pp for no rebalance, monthly, quarterly, and yearly rebalancing frequencies, respectively.

Moreover, for quarterly and yearly rebalancing frequencies, 100% of the portfolios with BTC allocation had superior cumulative returns than the Traditional Portfolio.

Furthermore, note that despite the significant (maximum and median) cumulative returns contribution obtained without rebalancing for BTC allocation, almost 2% of the portfolios considered had lower cumulative returns than the baseline 60/40 Traditional Portfolio. Consider now the impact of different Rebalancing Frequencies on the Rolling Sharpe Ratio Contribution of Bitcoin.

Rolling Sharpe Ratio Contribution						
Rebalancing Frequency	**Bitcoin Allocation**	**Maximum**	**Median**	**Minimum**	**Win Rate**	**Loss Rate**
No Rebalance	**1,00%**	0,200 p	0,010 p	(0,181) p	68,552%	31,448%
	2,50%	0,216 p	0,008 p	(0,191) p	59,123%	40,877%
	5,00%	0,200 p	0,004 p	(0,190) p	53,670%	46,330%
Monthly	**1,00%**	0,072 p	0,033 p	0,001 p	100,000%	0,000%
	2,50%	0,149 p	0,072 p	0,001 p	100,000%	0,000%
	5,00%	0,207 p	0,113 p	(0,003) p	99,847%	0,153%
Quarterly	**1,00%**	0,085 p	0,038 p	0,003 p	100,000%	0,000%
	2,50%	0,145 p	0,075 p	0,006 p	100,000%	0,000%
	5,00%	0,207 p	0,082 p	0,006 p	100,000%	0,000%
Yearly	**1,00%**	0,125 p	0,025 p	(0,052) p	90,010%	9,990%
	2,50%	0,205 p	0,046 p	(0,093) p	85,372%	14,628%
	5,00%	0,227 p	0,055 p	(0,111) p	84,506%	15,494%

Table 7: Rolling Sharpe Ratio Contribution with a rolling window of 3 years on no rebalanced, monthly, quarterly, and yearly rebalanced portfolios with bitcoin allocations of 1.0%, 2.5% and 5.0%. Negative values in brackets.

As shown in Table 8, only quarterly rebalanced portfolios have had consistently higher Sharpe Ratios compared with the Traditional Portfolios, regardless of the BTC allocation (assuming 3-years holding period). Moreover, for portfolios with the "no rebalance" strategy, Loss Rates were above 30% for every BTC allocation, meaning that the gains related to the increase performance of the portfolio (i.e., cumulative returns) are not persistently greater than the increase in risk (i.e., volatility) associated.

Hence, we reach the conclusion that regardless of the percentage allocated to BTC, assuming a 3-year holding period, 100% of the historical portfolios have benefited from being exposed to Bitcoin for an "ideal" quarterly rebalance frequency.

Finally, note that Figures 8-11 show a graphical representation of BTC positive and negative contribution for the different rebalancing frequencies and BTC positions considered, assuming a 3-year holding period.

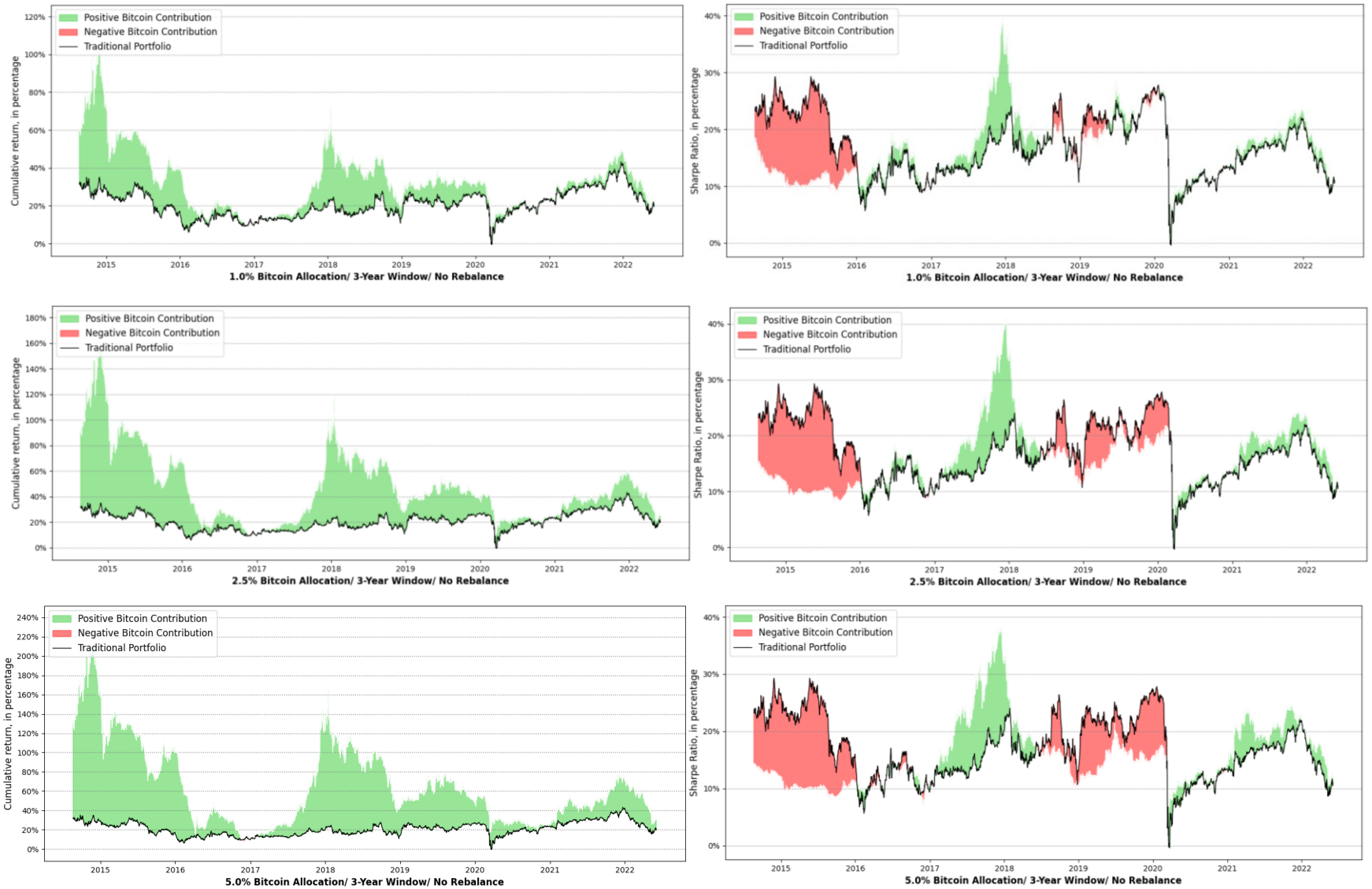

Figure 9: Rolling Cumulative Return Contribution (on the left) and Rolling Sharpe Ratio Contribution (on the right) on no rebalanced portfolios with a rolling window of 3 years, considering Bitcoin allocations 1.0%, 2.5% and 5.0%.

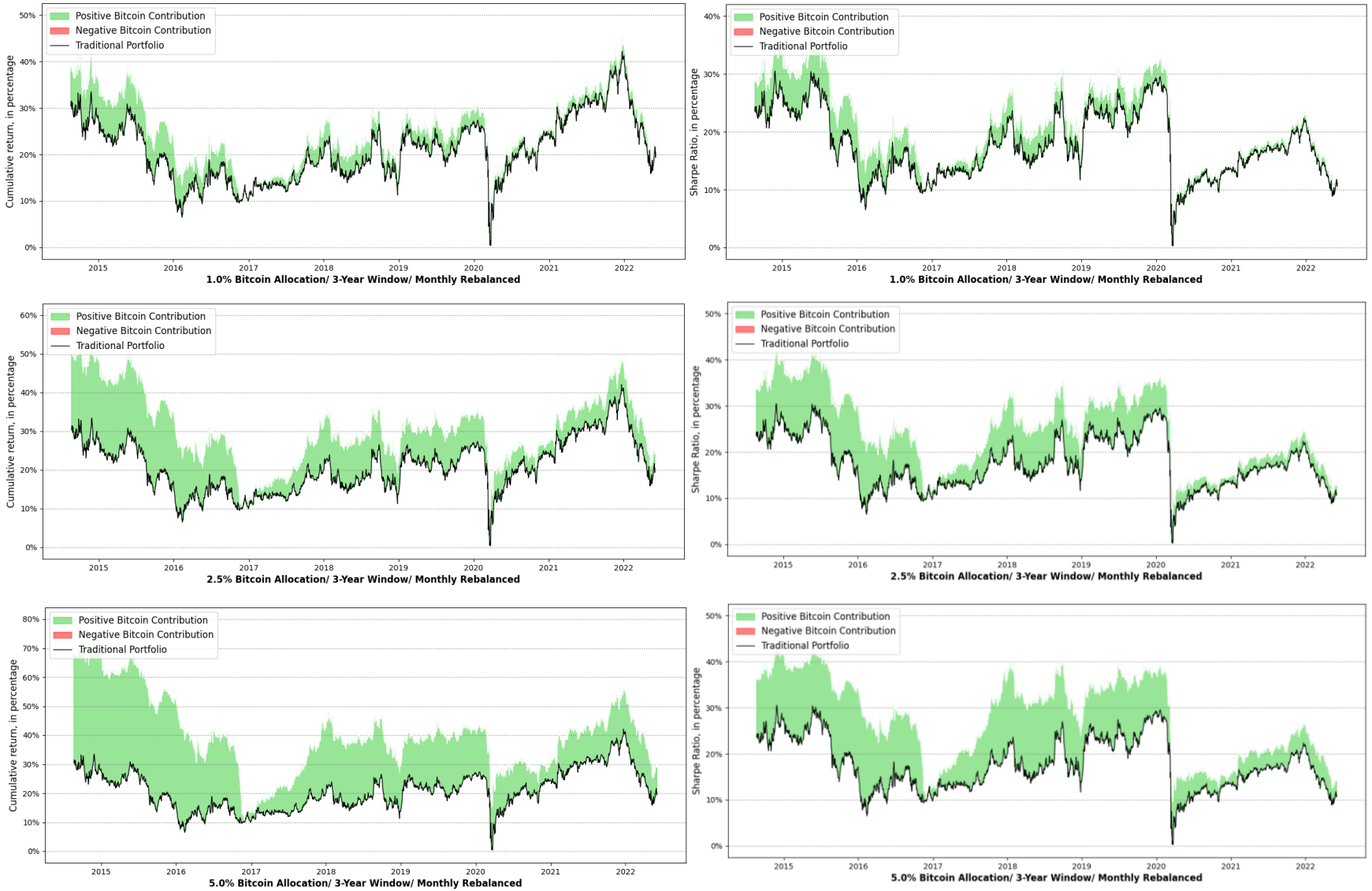

Figure 10: Rolling Cumulative Return Contribution (on the left) and Rolling Sharpe Ratio Contribution (on the right) on monthly rebalanced portfolios with a rolling window of 3 years, considering Bitcoin allocations 1.0%, 2.5% and 5.0%.

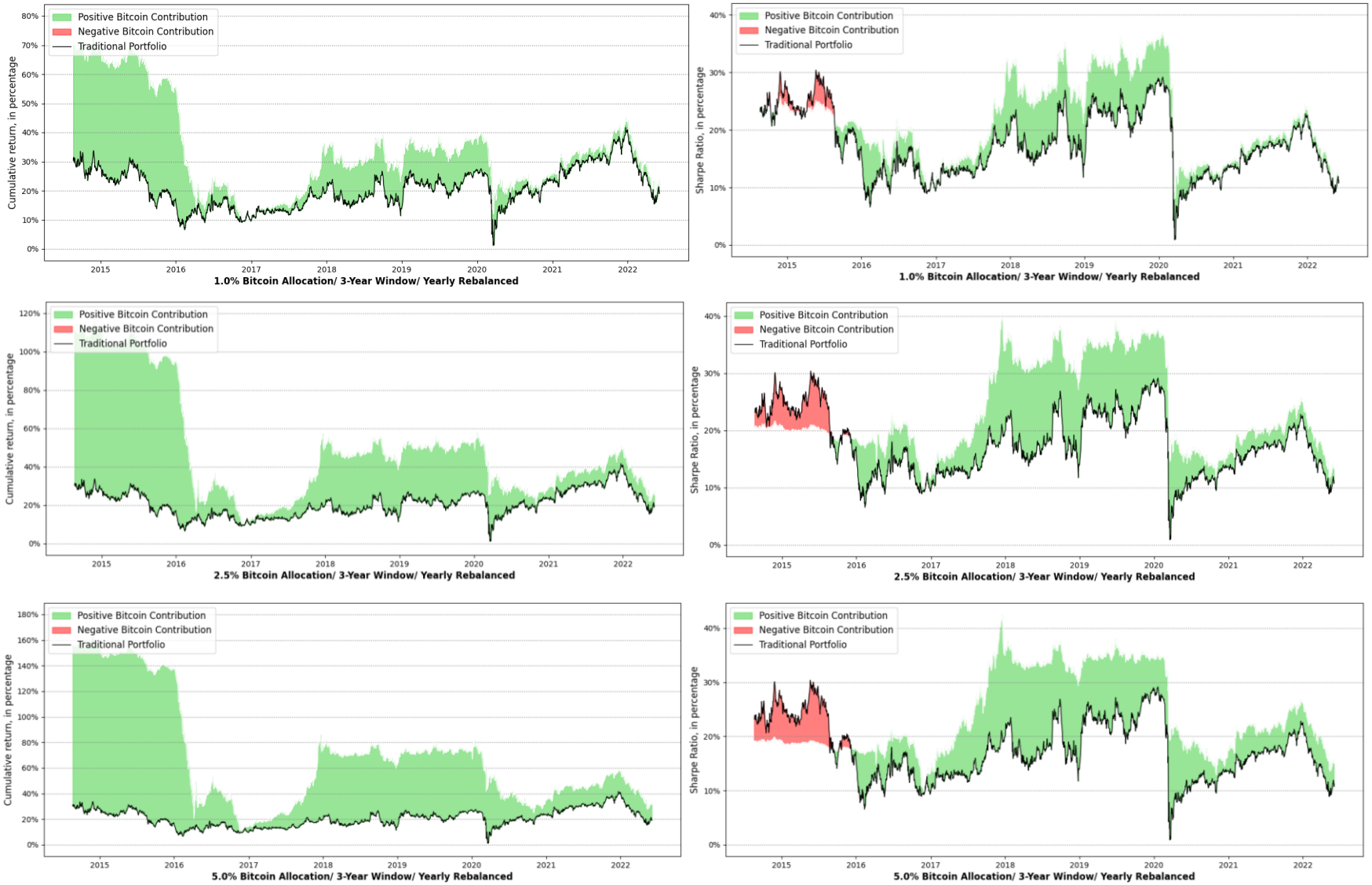

Figure 11: Rolling Cumulative Return Contribution (on the left) and Rolling Sharpe Ratio Contribution (on the right) on yearly rebalanced portfolios with a rolling window of 3 years, considering Bitcoin allocations 1.0%, 2.5% and 5.0%.

4.3. How much bitcoin should be added to a traditional 60/40 portfolio?

This is probably the most important question in the mind of an investor considering allocating Bitcoin to their portfolios. So far, we know that the optimal strategy to maximize risk-adjusted returns comprehends a minimum 3-year holding period with a quarterly rebalancing frequency. Aiming in answer to this question, we designed a Rolling Analysis Framework in which we could evaluate the impact of different BTC positions – between 0% (Traditional Portfolio) and 100% (invest solely on Bitcoin) with increments of 5% –, on the overall Cumulative Return, Standard Deviation, Sharpe Ratio and Maximum Drawdown of the portfolios under analysis.

On each figure presented below (Figure 12-17), note that the X-axis represents the BTC position size – from 0% (on the left) to 100% (on the right) – and the Y-axis represents the portfolio performance metric discussed. Moreover, note that the vertical line is composed by a series of dots that represent the performance of the rolling portfolios constructed. In other words, each vertical line is composed with 1962 dots that represent the performance of all the quarterly rebalanced portfolios with a 3-year horizon, starting on August 21st 2014 until June 3rd 2022. This analysis allows us to visualize how similar the performance of the portfolios throughout the rolling analysis really is, revealed by the concentration and dispersion of the dots presented.

Moreover, note that the dashed green line highlights the mean result of each BTC position considered.

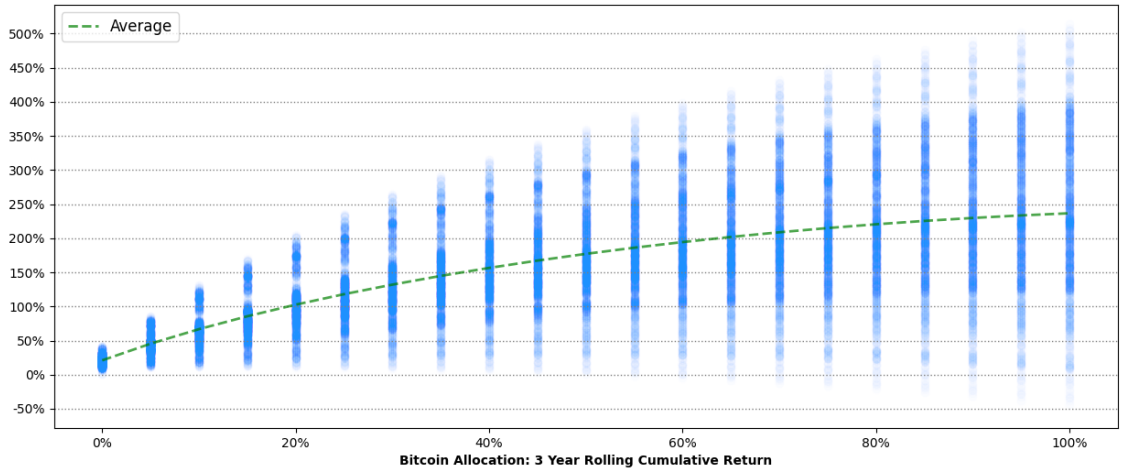

Figure 12: Rolling Cumulative Return by percentage of bitcoin allocation (between 0% and 100%) and its average, assuming quarterly rebalancing with a 3-year horizon.

Figure 12 shows that there is a clear relationship between the BTC position size and the average Cumulative Returns associated: "the more, the better". However, note that a high BTC position allocation is associated with a higher dispersion of returns (i.e., volatility) and with the possibility of obtaining negative cumulative returns. Moreover, on average, there are diminishing returns in increasing the BTC position size (i.e., decrease in the marginal cumulative returns as the BTC position size incrementally increases, holding all other factors equal (*ceteris paribus*)). The following figure shows the impact of different BTC allocations on standard deviation.

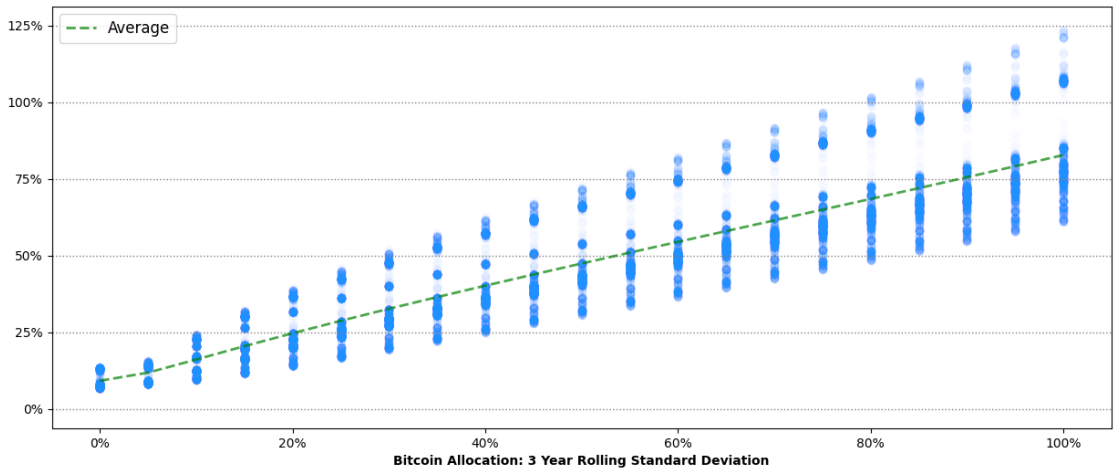

Figure 13: Rolling Volatility by percentage of bitcoin allocation (between 0% and 100%) and its average, assuming quarterly rebalancing with a 3-year horizon.

Conversely, Figure 13 shows that there seems to exist a nearly linear relationship between the amount of bitcoin added to the portfolio and its average standard deviation. This is to be expected, considering the gap between the historical volatilities of Bitcoin and the traditional assets considered.

Figure 14 combines the previous two by examining the Sharpe Ratio.

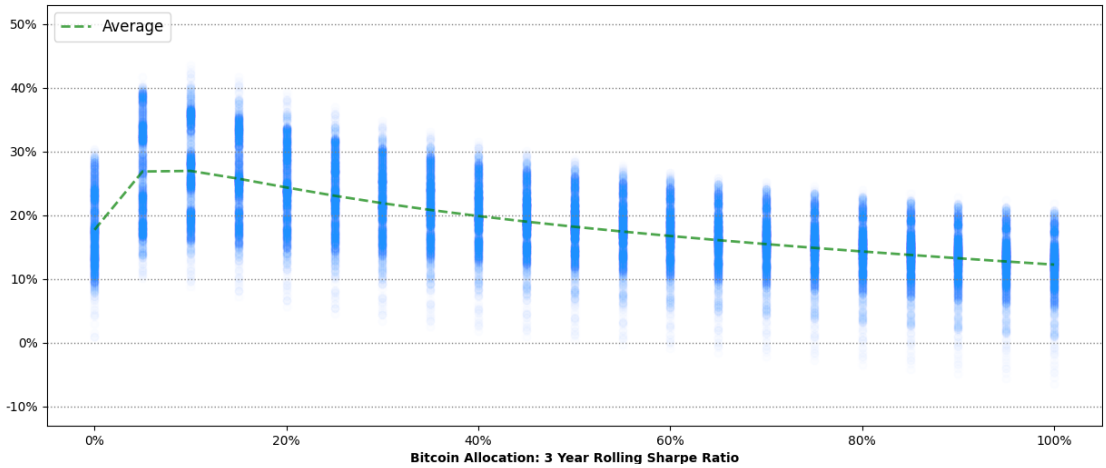

Figure 14: Rolling Sharpe Ratio by percentage of bitcoin allocation (between 0% and 100%) and its average, assuming quarterly rebalancing with a 3-year horizon.

While the average cumulative returns have a concave shape (Figure 12), rising sharply at first and then flattening out as the BTC position size increases, the average standard deviation seems to increase in a somewhat proportional way (Figure 13). This combination of factors results in a concave shape for the average Sharpe Ratio (Figure 14), meaning that the initial benefits (i.e., increase cumulative returns) of allocating a higher position to BTC are surpassed by the rising volatility, resulting in diminishing risk-adjusted returns. Since this function seems to peak somewhere where the BTC position size is between 0% and 10%, the following picture shows the Rolling Sharpe Ratio by percentage of bitcoin allocation within this interval.

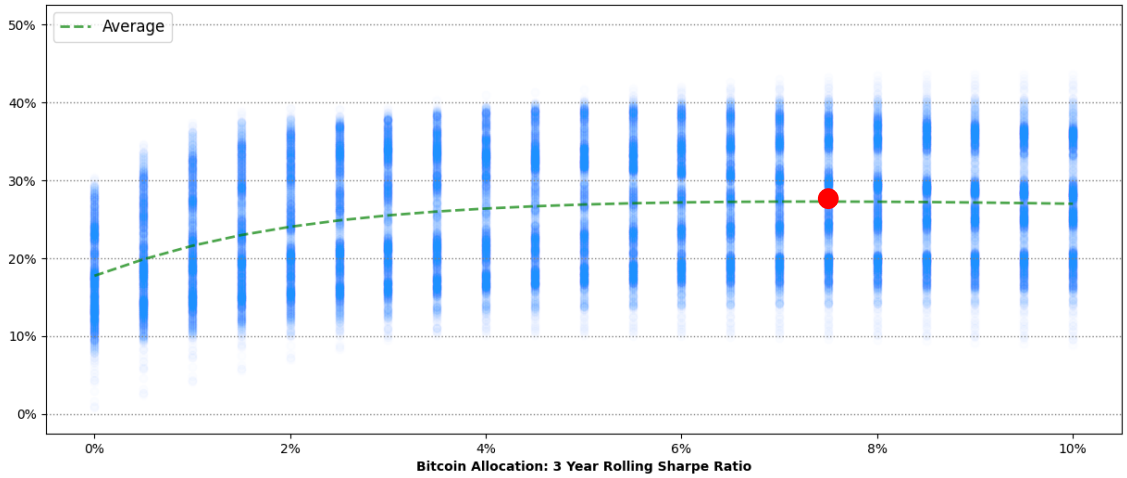

Figure 15: Rolling Sharpe Ratio by percentage of bitcoin allocation (between 0% and 100%) and its average, assuming quarterly rebalancing with a 3-year horizon.

As shown in Figure 15, the optimal BTC position size that historically maximizes average risk-adjusted returns is 7.5%, assuming quarterly rebalancing with a 3-year horizon. Furthermore, any portfolio with a BTC allocation between 0.5% and 7.5% is associated with a higher overall cumulative and risk-adjusted performance than the Traditional Portfolio for the same period, measured by a 100% Rolling Cumulative Returns and Rolling Sharpe Ratio Win Rate, respectively.

Finally, Figure 16 shows the impact of different BTC allocations on the maximum drawdown.

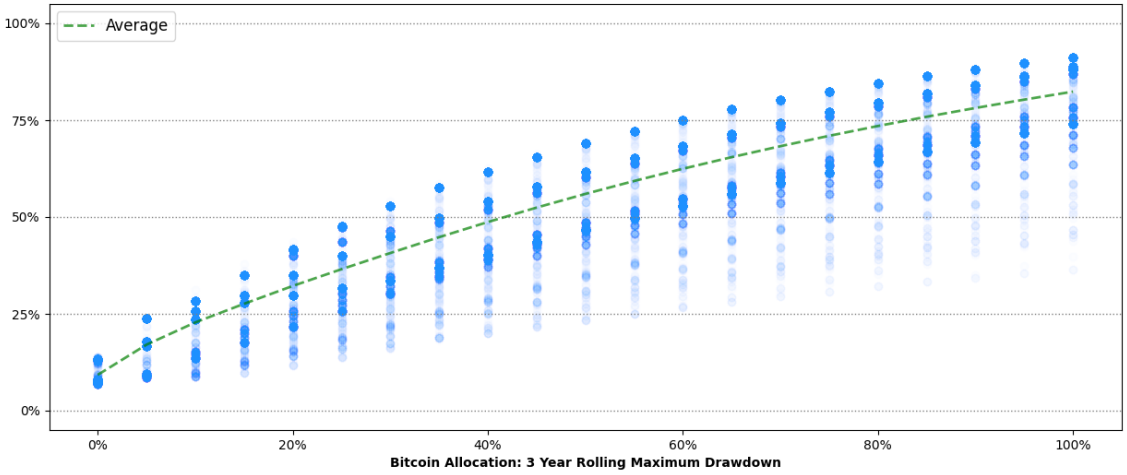

Figure 16: Rolling Maximum Drawdown by percentage of bitcoin allocation (between 0% and 100%) and its average, assuming quarterly rebalancing with a 3-year horizon.

On average, Bitcoin becomes a major driver of maximum drawdowns, reaching more that 50% average losses for the majority of BTC positions considered, assuming 3-year rolling maximum drawdowns quarterly rebalanced. Note that this metric only indicates the largest drawdown, without consideration for the time it took to recover the lost capital (if it was recovered). Hence, it's understandable that risk-averse investors would feel uncomfortable with such losses, meaning that they should exercise caution by limiting their exposure to Bitcoin in their portfolios. For this reason, the following figure represents rolling maximum drawdowns for BTC positions between 0% and 10%.

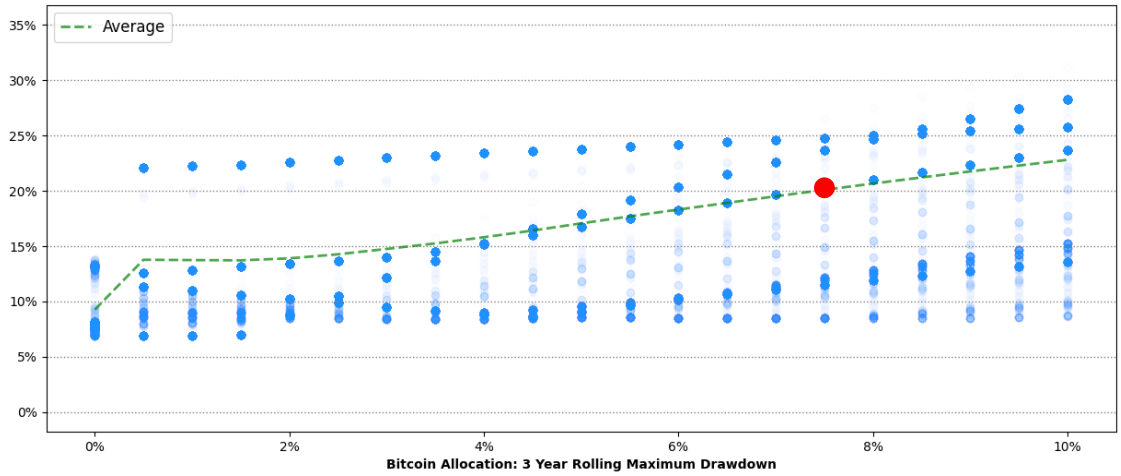

Figure 17: Rolling Maximum Drawdown by percentage of bitcoin allocation (between 0% and 10%) and its average, assuming quarterly rebalancing with a 3-year horizon.

On average, there is a sharply increase in the maximum drawdown once BTC is considered for a portfolio. Furthermore, note that the BTC position that maximizes the rolling Sharpe Ratio (7.5%) is associated with an average maximum drawdown of approximately 20%.

Finally, Tables 9-10 summarize the impact of different bitcoin allocations on all four of the key portfolio metrics over the three-year period studied.

BTC Allocation.	Cumulative return			Sharpe Ratio			Standard Deviation			Maximum Drawdown		
	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
0,0%	1,193%	21,027%	42,786%	0,009	0,177	0,304	6,91%	9,26%	13,81%	6,84%	13,81%	21,99%
0,5%	3,475%	23,636%	44,816%	0,025	0,199	0,346	6,90%	9,29%	13,90%	6,89%	13,77%	22,11%
1,0%	5,729%	26,203%	46,828%	0,041	0,216	0,373	6,94%	9,40%	13,99%	6,95%	13,74%	22,24%
1,5%	7,954%	28,731%	50,309%	0,056	0,230	0,388	7,03%	9,58%	14,10%	7,00%	13,72%	22,37%
2,0%	10,153%	31,221%	55,711%	0,070	0,240	0,393	7,17%	9,82%	14,23%	8,48%	13,90%	22,57%
2,5%	10,270%	33,674%	61,006%	0,083	0,249	0,391	7,35%	10,11%	14,36%	8,44%	14,26%	22,77%
3,0%	10,359%	36,091%	66,197%	0,095	0,255	0,393	7,57%	10,43%	14,50%	8,40%	14,75%	22,97%
3,5%	10,444%	38,474%	71,289%	0,102	0,260	0,402	7,82%	10,78%	14,65%	8,36%	15,25%	23,18%
4,0%	10,525%	40,823%	76,290%	0,102	0,264	0,409	8,01%	11,15%	14,81%	8,33%	15,81%	23,38%
4,5%	10,602%	43,139%	81,254%	0,101	0,267	0,414	8,08%	11,53%	14,98%	8,49%	16,41%	23,58%
5,0%	10,675%	45,424%	86,131%	0,101	0,269	0,417	8,15%	11,93%	15,75%	8,59%	17,06%	23,78%
5,5%	10,744%	47,678%	90,924%	0,100	0,271	0,418	8,24%	12,34%	16,63%	8,54%	17,69%	23,98%
6,0%	10,809%	49,903%	95,636%	0,100	0,272	0,420	8,34%	12,75%	17,50%	8,50%	18,31%	24,19%
6,5%	10,869%	52,099%	100,270%	0,099	0,272	0,426	8,45%	13,17%	18,38%	8,46%	18,91%	24,39%
7,0%	10,926%	54,266%	104,828%	0,098	0,273	0,430	8,58%	13,60%	19,25%	8,47%	19,50%	25,43%
7,5%	**10,979%**	**56,406%**	**109,314%**	**0,096**	**0,273**	**0,432**	**8,71%**	**14,03%**	**20,13%**	**8,49%**	**20,08%**	**26,51%**
8,0%	11,027%	58,519%	113,731%	0,095	0,272	0,434	8,85%	14,46%	21,03%	8,50%	20,65%	27,54%
8,5%	11,072%	60,607%	118,079%	0,094	0,272	0,436	9,00%	14,90%	21,92%	8,51%	21,21%	28,52%
9,0%	11,113%	62,669%	122,362%	0,092	0,271	0,436	9,15%	15,33%	22,79%	8,53%	21,75%	29,45%
9,5%	11,150%	64,71%	126,58%	0,091	0,271	0,436	9,32%	15,77%	23,66%	8,60%	22,28%	30,34%

Table 8: Key Portfolio Metrics by Bitcoin Allocation (between 0% and 9.5%, with 0.5% increments), assuming a 3-year rolling window quarterly rebalanced. BTC allocation that maximizes the average Sharpe Ratio highlighted.

BTC	Cumulative return			Sharpe Ratio			Standard Deviation			Maximum Drawdown		
Allocation.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
10,0%	11,182%	66,719%	130,741%	0,089	0,270	0,436	9,49%	16,21%	24,51%	8,70%	22,80%	31,18%
15,0%	11,295%	85,632%	170,084%	0,073	0,258	0,418	11,54%	20,55%	32,47%	9,83%	27,60%	37,90%
20,0%	11,023%	102,621%	204,929%	0,058	0,244	0,393	13,97%	24,76%	39,55%	11,65%	32,21%	43,51%
25,0%	10,371%	118,003%	236,213%	0,045	0,231	0,370	16,59%	28,82%	45,98%	13,51%	36,54%	49,04%
30,0%	9,347%	132,003%	264,618%	0,035	0,219	0,349	19,33%	32,74%	51,95%	15,84%	40,71%	54,05%
35,0%	7,955%	144,794%	290,802%	0,026	0,209	0,331	22,12%	36,55%	57,55%	18,29%	44,80%	58,53%
40,0%	6,199%	156,506%	315,453%	0,018	0,199	0,316	24,96%	40,27%	62,87%	19,96%	48,71%	62,54%
45,0%	4,079%	167,244%	338,343%	0,011	0,190	0,302	27,83%	43,91%	67,97%	21,68%	52,44%	66,18%
50,0%	1,597%	177,089%	359,688%	0,004	0,182	0,289	30,73%	47,50%	72,89%	23,42%	55,97%	69,50%
55,0%	(1,248)%	186,111%	379,658%	(0,003)	0,175	0,278	33,64%	51,05%	77,66%	25,06%	59,30%	72,56%
60,0%	(4,456)%	194,363%	398,391%	(0,009)	0,168	0,268	36,56%	54,57%	82,30%	26,62%	62,46%	75,38%
65,0%	(8,030)%	201,890%	415,999%	(0,015)	0,161	0,259	39,51%	58,06%	86,92%	28,08%	65,45%	77,98%
70,0%	(11,974)%	208,729%	432,576%	(0,022)	0,155	0,252	42,47%	61,56%	91,76%	29,48%	68,29%	80,38%
75,0%	(16,293)%	214,910%	448,198%	(0,028)	0,149	0,246	45,46%	65,05%	96,73%	30,80%	70,97%	82,60%
80,0%	(20,993)%	220,457%	462,930%	(0,034)	0,143	0,240	48,49%	68,55%	101,86%	32,05%	73,50%	84,64%
85,0%	(26,080)%	225,388%	476,825%	(0,041)	0,138	0,234	51,56%	72,08%	107,09%	33,24%	75,89%	86,51%
90,0%	(31,565)%	229,717%	489,929%	(0,048)	0,133	0,229	54,67%	75,63%	112,44%	34,38%	78,15%	88,22%
95,0%	(37,456)%	233,453%	502,278%	(0,056)	0,128	0,224	57,83%	79,22%	117,97%	35,47%	80,31%	89,79%
100,0%	(43,765)%	236,602%	513,905%	(0,064)	0,123	0,219	61,04%	82,87%	123,68%	36,50%	82,38%	91,21%

Table 9: Key Portfolio Metrics by Bitcoin Allocation (between 10% and 100%, with 5% increments), assuming a 3-year rolling window quarterly rebalanced.

Chapter 5
Conclusion

In this dissertation we aimed to expand the framework used in Hougan and Lawant, 2020 and its extension in Hougan and Lawant, 2021. Not only did we extend the study period under analysis, but also tested the impact of a wider scope of Investment-Time Horizons, Rebalancing Frequencies and Bitcoin positions sizing using a Rolling Analysis Framework, which allowed to capture the most robust and stable portfolio's construction techniques throughout the sample period analyzed.

Firstly, we tested what was the minimum holding period for which it would be attractive to allocate Bitcoin to a 60/40 portfolio. We conclude that for a minimum holding period of 3 years, all portfolios exposed to Bitcoin outperformed the baseline portfolio, both in terms of Cumulative Returns and Sharpe Ratio. Secondly, we tested which is the "ideal" rebalancing frequency, based on the historical performance of the assets considered. The quarterly rebalancing frequency is shown to have consistently outperformed the other rebalancing frequencies under analysis, regardless of the Bitcoin position considered. Thirdly, we tested which Bitcoin percentage allocation has been the most beneficial for the investor. As such, a Bitcoin position around 7.5% seems to be the most favorable in terms of risk-adjusted returns, despite the increase in volatility and maximum drawdown associated. Hence, we believe we have shown that with an intuitive rebalancing strategy, Bitcoin can significantly increase the overall performance of a portfolio and be attractive for market participants.

Moreover, we acknowledge that this study did not account for the impact of some important factors, such as transaction fees and taxes. Although it can be argued that the impact of the former on the overall performance of a portfolio may be significant – since the high volatility of inputs may mean that the portfolio composition changes frequently, implying additional costs of rebalancing –, we believe that applying a flat fee or a trading volume fee would be inaccurate. This is due to the different fees systems applied in different exchanges, OTC market and even the volatility of the fees system within the BTC network itself[16]. As for the latter, we believe that the introduction of taxes – considering the different regulatory frameworks for the different crypto assets[17] around the world – would reduce the scope of applicability of this research.

Finally, additional robustness tests using a larger basket of cryptocurrencies, traditional and alternative assets could have provided better insights, given the dynamics within the different assets.

[16] See https://mempool.space/pt/graphs/mining/block-fees#all for more details.
[17] See (Hammond and Ehret, 2022) for more details.

References

Astrakhantseva, I., Astrakhantsev, R. and Los, A. (2021) 'Cryptocurrency fraud schemes analysis', *SHS Web of Conferences*, 106, p. 02001. doi: 10.1051/shsconf/202110602001.

Back, A. (2002) 'Hashcash - A Denial of Service Counter-Measure', *http://Www.Hashcash.Org/Papers/Hashcash.Pdf*, (August), pp. 1–10.

Baek, C. and Elbeck, M. (2015) 'Bitcoins as an investment or speculative vehicle? A first look', *Applied Economics Letters*, 22(1), pp. 30–34.

Bakry, W., Rashid, A., Al-Mohamad, S., El-Kanj, N. (2021) 'Bitcoin and Portfolio Diversification: A Portfolio Optimization Approach', *Journal of Risk and Financial Management*, 14(7), p. 282.

Baur, D. G., Hong, K. H. and Lee, A. D. (2018) 'Bitcoin: Medium of exchange or speculative assets?', *Journal of International Financial Markets, Institutions and Money*, 54, pp. 177–189.

BIS (2022) *Basel Committee on Banking Supervision Consultative Document Prudential treatment of cryptoasset exposures*. Available at: www.bis.org.

Bitstamp USA, I. (2022) 'Bitstamp'. Available at: https://www.bitstamp.net/.

Bouchey, P., Nemtchinov, V., Paulsen, A., Stein, D. (2012) 'Volatility harvesting: Why does diversifying and rebalancing create portfolio growth?', *Journal of Wealth Management*, 15(2), pp. 26–35.

Brauneis, A. and Mestel, R. (2019) 'Cryptocurrency-portfolios in a mean-variance framework', *Finance Research Letters*, 28(May 2018), pp. 259–264.

Brière, M., Oosterlinck, K. and Szafarz, A. (2015) 'Virtual currency, tangible return: Portfolio diversification with bitcoin', *Journal of Asset Management*, 16(6), pp. 365–373.

Cheah, E. T. and Fry, J. (2015) 'Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin', *Economics Letters*, 130, pp. 32–36.

Chu, J., Nadarajah, S. and Chan, S. (2015) 'Statistical analysis of the exchange rate

of bitcoin', *PLoS ONE*, 10(7), pp. 1–27. doi: 10.1371/journal.pone.0133678.

Colombo, J. A. Cruz, F., Paese, L., Cortes, R. (2021) 'The diversification benefits of cryptocurrencies in multi-asset portfolios: cross-country evidence'. Available at: https://ssrn.com/abstract=3776260.

Corbet, S., Meegan, A., Larkin, C., Lucey, B., Yarovaya, L. (2018) 'Exploring the dynamic relationships between cryptocurrencies and other financial assets', *Economics Letters*, 165, pp. 28–34.

D. Chaum (1982) 'Blind Signatures for Untraceable Payments', *Advances in Cryptology Proceedings of Crypto*, Chaum, D., pp. 199–203. Available at: http://www.chaum.com/publications/publications.html.

Dangi, V. (2020) 'Volatility Dynamics of Cryptocurrencies' Returns: An Econometric Study.', *IUP Journal of Applied Finance*, 26(1), pp. 5–30. Available at: http://search.ebscohost.com/login.aspx?direct=true&db=buh&AN=142016760&site=ehost-live.

DeMiguel, V. Garlappi, L., Nogales, F., Uppal, R. (2009) 'A generalized approach to portfolio optimization: Improving performance by constraining portfolio norms', *Management Science*, 55(5), pp. 798–812.

DeMiguel, V., Garlappi, L. and Uppal, R. (2009) 'Optimal versus Naive Diversification: How Inefficient Is the 1/N Portfolio Strategy?', *The Review of Financial Studies*, 22(5), pp. 1915–1953.

Demiralay, S. and Bayracı, S. (2021) 'Should stock investors include cryptocurrencies in their portfolios after all? Evidence from a conditional diversification benefits measure', *International Journal of Finance and Economics*, 26(4), pp. 6188–6204.

Dwork, C. and Naor, M. (1993) 'Pricing via processing or combatting junk mail', *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, 740 LNCS, pp. 139–147.

Fama, E. F. (1970) 'Efficient Capital Markets: A Review of Theory and Empirical Work', *The Journal of Finance*, 25(2). Available at: http://www.jstor.org/stable/2325486.

Fang, F. Ventre, C., Basios, M., Kanthan, L., Martinez-Rego, D., Wu, F., Li, L. (2022) 'Cryptocurrency trading: a comprehensive survey', *Financial Innovation*, 8(1).

FATF (2022) 'Targeted Update on Implementation of the Fatf Standards on Virtual Assets and Virtual Asset Service Providers', (June). Available at: www.fatf-gafi.org.

FSB (2022) 'Assessment of risks to financial stability from crypto-assets', *Financial Stability Board*, (February), p. 30. Available at: www.fsb.org/emailalert.

García-Corral, F. J. Cordero-García, J., de Pablo-Valenciano, J., Uribe-Toril, J. (2022) 'A bibliometric review of cryptocurrencies: how have they grown?', *Financial Innovation*, 8(1).

Ghabri, Y., Guesmi, K. and Zantour, A. (2020) 'Bitcoin and liquidity risk diversification', *Finance Research Letters*, 40(July 2020), p. 101679.

Ghiro, L. Restuccia, F., D'Oro, S., Basagni, S., Melodia, T., Maccari, L., Cigno, R. (2021) 'What is a Blockchain? A Definition to Clarify the Role of the Blockchain in the Internet of Things'. Available at: http://arxiv.org/abs/2102.03750.

Hammond, S. and Ehret, T. (2022) 'Cryptos on the rise 2022 - A complex regulatory future emerges'.

Harry Markowitz (1952) 'Portfolio Selection', *The Journal of Finance*, 7(1), pp. 77–91. Available at: https://www.jstor.org/stable/2975974.

Hougan, M. and Lawant, D. (2020) 'The Case for Crypto In An Institutional Portfolio', (May).

Hougan, M. and Lawant, D. (2021) *Cryptoassets: The Guide to Bitcoin, Blockchain, and Cryptocurrency for Investment Professionals*, *SSRN Electronic Journal*.

Hughes, E. (1993) 'A Cypherpunk's Manifesto', 22(March), pp. 1–2. Available at: https://www.activism.net/cypherpunk/manifesto.html.

Kakinaka, S. and Umeno, K. (2021) 'Exploring asymmetric multifractal cross-correlations of price–volatility and asymmetric volatility dynamics in cryptocurrency markets', *Physica A: Statistical Mechanics and its Applications*, 581, p. 126237.

Karaa, R., Slim, S., Goodell, J. W., Goyal, A., Kallinterakis, V. (2021) 'Do investors feedback trade in the Bitcoin—and why?', *European Journal of Finance*.

Karame, G. O. Androulaki, E., Roeschlin, M., Gervais, A., Čapkun, S. (2015) 'Misbehavior in Bitcoin: A study of double-spending and accountability', *ACM Transactions on Information and System Security*, 18(1), pp. 1–32.

Katsiampa, P. (2019) 'An empirical investigation of volatility dynamics in the cryptocurrency market', *Research in International Business and Finance*, 50(May), pp. 322–335.

Klein, T., Pham Thu, H. and Walther, T. (2018) 'Bitcoin is not the New Gold – A comparison of volatility, correlation, and portfolio performance', *International Review of Financial Analysis*, 59, pp. 105–116.

Kristoufek, L. (2015) 'What are the main drivers of the bitcoin price? Evidence from wavelet coherence analysis', *PLoS ONE*, 10(4), pp. 1–15. doi: 10.1371/journal.pone.0123923.

Lamport, L. (1983) 'The Weak Byzantine Generals Problem', *Journal of the ACM (JACM)*, 30(3), pp. 668–676. doi: 10.1145/2402.322398.

Lee, D. K. C., Guo, L. and Wang, Y. (2018) 'Cryptocurrency: A new investment opportunity?', *Journal of Alternative Investments*, 20(3), pp. 16–40.

Letho, L., Chelwa, G. and Alhassan, A. L. (2022) 'Cryptocurrencies and portfolio diversification in an emerging market', *China Finance Review International*, 12(1), pp. 20–50.

Merton, R. C. (1980) 'On estimating the expected return on the market: An exploratory investigation', *Topics in Catalysis*, 8(4), pp. 323–361. doi: 10.1016/0304-405X(80)90007-0.

Nakamoto, S. (2008) 'Bitcoin: A Peer-to-Peer Electronic Cash System'. Available at: https://bitcoin.org/bitcoin.pdf.

Pérez-Solà, C. Delgado-Segura, S., Navarro-Arribas, G., Herrera-Joancomartí, J. (2019) 'Double-spending prevention for Bitcoin zero-confirmation transactions', *International Journal of Information Security*, 18(4), pp. 451–463.

Petukhina, A. Trimborn, S., Karl, W., Elendner, H. (2020) 'Investing with Cryptocurrencies – evaluating their potential for portfolio allocation', 49(0), pp. 1–67.

Phillip, A., Chan, J. and Peiris, S. (2018) 'A new look at Cryptocurrencies', *Economics Letters*, 163, pp. 6–9.

Platanakis, E. and Urquhart, A. (2019) 'Portfolio management with cryptocurrencies: The role of estimation risk', *Economics Letters*, 177, pp. 76–80.

Ram, A. J. (2019) 'Bitcoin as a new asset class', *Meditari Accountancy Research*, 27(1), pp. 147–168. doi: 10.1108/MEDAR-11-2017-0241.

Sharpe, W. F. (1964) 'Capital Asset Prices: A Theory of Market Equilibrium under conditions of risk', *The Journal of Finance*, XIX, pp. 425–442.

Solutions, B. (2022) *Bitcoin User Adoption*. Available at: https://static1.squarespace.com/static/5de588aa3e9c044c1ad8cb59/t/62a0cee064cb4b79ded25894/1654705890141/Bitcoin+User+Adoption+Report.pdf.

Tiwari, A. K., Jana, R. K., Das, D., Roubaud, D. (2018) 'Informational efficiency of Bitcoin—An extension', *Economics Letters*, 163, pp. 106–109.

Urquhart, Andrew ; Zhang, H. (2019) 'Is Bitcoin a hedge or safe haven for currencies? An intraday analysis', pp. 0–25. doi: https://doi.org/10.1016/j.irfa.2019.02.009.

Urquhart, A. (2016) 'The inefficiency of Bitcoin', *Economics Letters*, 148, pp. 80–82.

Urquhart, A. (2017) 'Price clustering in Bitcoin', *Economics Letters*, 159, pp. 145–148.

Yahoo (2022) 'Yahoo! Finance'. Available at: https://finance.yahoo.com/.

Yarovaya, L., Matkovskyy, R. and Jalan, A. (2021) 'The effects of a "black swan" event (COVID-19) on herding behavior in cryptocurrency markets', *Journal of International Financial Markets, Institutions and Money*, 75, p. 101321..

Zeng, T., Yang, M. and Shen, Y. (2020) 'Fancy Bitcoin and conventional financial assets: Measuring market integration based on connectedness networks', *Economic Modelling*, 90(April), pp. 209–220.

Zivot, E. and Wang, J. (2003) 'Rolling Analysis of Time Series', *Modeling Financial Time Series with S-Plus®*, pp. 299–346. doi: 10.1007/978-0-387-21763-5_9.

Appendix I: Performance Metrics

Here we show how the main performance metrics employed in this dissertation were computed:

- **Cumulative return** allows us to capture the aggregate return of the investment (results in percentage).

$$Cumulative\ return_i = \log\left(\frac{P_{iT}}{P_{i0}}\right) * 100 \quad (1)$$

P_{IT} represents the price of portfolio *i* for period *t=T* (last period), P_{I0} represents the price of portfolio *i* for period *t=0* (first period).

- **Annualized returns** (Compound Annual Growth Rate (CAGR)) allows us to represent the average rate required to obtain the cumulative return over the number of years invested, assuming that all profits are reinvested at the end of each period (results in percentage).

$$CAGR_i = [\ (1 + \log\left(\frac{P_{iT}}{P_{i0}}\right))^{\frac{1}{n}} - 1] * 100 \quad (2)$$

n represents the number of years of the investment.

- **Annualized standard deviation (volatility)** allows us to capture the dispersion of a set of returns (results in percentage).

$$S_i = \sqrt{252} * \sqrt{\frac{1}{n-1}\sum_{i=0}^{n-1}(x_i - X)^2} \quad (3)$$

S_i represents the sample standard deviation, x_i represents individual observations, X is the mean of the sample and n is the number of values in the dataset. This formula

considers the Bessel's correction to compute the best unbiased estimator for the population variance.

- **Sharpe ratio**, developed by Sharpe, 1964, following the work of Harry Markowitz, 1952, represents an average return in excess of the risk-free rate per unity of risk (volatility). Assuming a risk-free interest rate of 0%, we have that:

$$Sharpe\ ratio_i = \frac{CAGR_i}{S_i} \tag{4}$$

- **Maximum Drawdown** allows us to capture a measure of downside risk over a specified time (results in percentage).

$$Maximum\ Drawdown_i = \frac{Through\ Value_i - Peak\ Value_i}{Peak\ Value_i} * 100 \tag{5}$$

In which the $Through\ Value_i$ represents a maximum observed loss for portfolio i after reaching its *Peak Value* before a new peak is attained. $Peak\ Value_i$ represents the maximum value of a portfolio for a given period.