Portfolio Optimization: Does the optimization methodology have a significant impact on portfolio measures, in a context of elevated market volatility?

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This dissertation is dedicated to my parents and brothers.
For their endless love, support and encouragement.

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ABSTRACT

This submitted master's dissertation focuses on the practical application of resampling as a portfolio optimization methodology, in a context of elevated market volatility.

In the process of constructing investment portfolios, the optimization methodology plays a crucial role, since it must output portfolios that are able to withstand unexpected unfavorable market conditions.

In this context, portfolio resampling is a methodology that explicitly considers information uncertainty about assets, and outputs asset portfolios that are, according to the literature, more resilient to volatile environments. This dissertation explores and tries to assess the ex-post performance of hypothetical portfolios of the US and the EU stock markets, constructed using the resampling technique, during the initial stages of the COVID-19 pandemic, in the first semester of 2020. The findings indicate that in general, resampling strategy enhances portfolio performance and reduces the portfolio volatility.

Keywords: Portfolio Optimization, Resampling, Performance, Volatility

JEL Classification: C610, G110, G170

RESUMO

Esta dissertação de mestrado tem como foco a aplicação prática da reamostragem como metodologia de optimização de carteiras, num contexto de elevada volatilidade do mercado.
No processo de construção de carteiras de investimento, a metodologia de optimização desempenha um papel crucial, uma vez que deve produzir carteiras capazes de resistir a condições desfavoráveis inesperadas de mercado.
Neste contexto, a reamostragem de carteiras é uma metodologia que considera explicitamente a incerteza de informação sobre activos, e produz carteiras que, segundo a literatura, são mais resilientes a ambientes voláteis. Esta dissertação explora e tenta avaliar o desempenho *ex-post* de carteiras hipotéticas dos mercados acionistas dos EUA e da UE, construídas utilizando a técnica de reamostragem, durante as fases iniciais da pandemia COVID-19, no primeiro semestre de 2020. Os resultados indicam que, em geral, a estratégia de reamostragem melhora o desempenho das carteiras e reduz a sua volatilidade.

Palavras chave: Otimização de Carteiras, Reamostragem, Desempenho, Volatilidade

JEL Classificação: C610, G110, G170

CONTENTS

1 INTRODUCTION 1

1.1 Motivation and Practical Relevance 1

1.2 Document Structure 2

2 LITERATURE REVIEW 3

2.1 Mean-Variance Framework 3

2.2 Limitations of MV Framework 4

2.3 Resampled Efficient Frontier 6

3 AN OVERVIEW OF PORTFOLIO OPTIMIZATION 7

3.1 Risk and Return 7

3.2 Portfolio Optimization 7

3.3 Portfolio Measures 10

3.4 Portfolio Resampling 12

4 PROBLEM FOUNDATION AND METHODOLOGY 14

4.1 Data Description 14

4.2 Problem Objective 15

5 EMPIRICAL RESULTS AND ANALYSIS 18

6 CONCLUSION 22

BIBLIOGRAPHY 23

APPENDIX 25

A Python Script 26

A.1 Portfolio Optimization Process 26

A.2 Portfolio Resampling Analysis 32

B Data Results 36

B.1 Performance evaluation of S&P 500 stock market index 37

B.2 Performance evaluation of EURO STOXX 50 stock market index 38

LIST OF FIGURES

Figure 3.1 – Markowitz Efficient Frontier 9
Figure 4.1 – Algorithm of Portfolio Optimization Inputs 16
Figure 4.2 – Algorithm of Portfolio Optimization Routine 16
Figure 4.3 – Algorithm of Portfolio Optimization Resampling Step 17

LIST OF TABLES

Table 4.1 – Data Sets Considered in the Analysis 15
Table 5.1 – Portfolio 1/HHI Concentration.. 20
Table 5.2 – Portfolio Turnover Ratio 20
Table 5.3 – Portfolio Risk and Return Measures 21

ACRONYMS

CAPM	Capital Asset Pricing Model
COVID-19	Syndrome Coronavirus 2 (SARS-CoV-2)
E.U.	The European Union
HHI	Herfindahl-Hirschman Index
MDD	Maximum Drawdown
MEF	Markowitz Efficient Frontier
MV	Mean Variance
MVP	Minimum Variance Portfolio
Nikkei 225	Nikkei Stock Average
REF	Resampled Efficient Frontier
S&P 500	Standard & Poor's 500
U.S.	The United States of America
VaR	Value-at-Risk
VIX	CBOE Volatility Index
WHO	World Health Organization

SYMBOLS

$\boldsymbol{\mu}$	Vector of portfolio expected returns
$\boldsymbol{w}$	Vector of portfolio weights
$\boldsymbol{\sigma}$	Standard deviation
$\boldsymbol{\sigma}^2$	Portfolio variance
$\boldsymbol{\rho}$	Correlation coefficient
Σ	Variance-covariance matrix

1

INTRODUCTION

This chapter provides the motivation for the research question, describes its aims and practical relevance, as well as the document structure.

1.1 Motivation and Practical Relevance

Investment decisions are greatly influenced by the frameworks and methodologies that underlie the construction of asset portfolios. The output portfolios must be able to withstand unexpected and unfavorable conditions, and contexts of elevated market volatility. As a result, the study of new optimization techniques, aiming to output the most efficient portfolios, keeps motivating further research and development, due to the shortcomings of previously proposed models. In addition, there is plethora of research regarding the effect of major global events, such as financial crisis, on financial markets.

The portfolio resampling technique, firstly introduced by Michaud and Michaud (1998), is a methodology that explicitly incorporates the information uncertainty about assets, and outputs portfolios that exhibit, according to the literature, a higher degree of diversification, and thus are more robust to volatile market environments, such as the one created by the COVID-19 pandemic, in the first semester of 2020. This period of unforeseeable and extreme market volatility, created new challenges for portfolio managers, and enlightened once again the role of risk management in the investment decision process.

Therefore, this dissertation aims to assess the ex-post performance of hypothetical stock portfolios constructed using the resampling technique, during the beginning stages of the COVID-19 pandemic, and whether this methodology yields superior investment performance, when compared to the standard mean-variance optimization firstly proposed by Markowitz (1952).

1.2 Document Structure

The remainder of this document is organized as follows:

- Chapter 2 provides a literature review of relevant research within the field of portfolio optimization, and further motivates the theoretical framework and methodology employed in this dissertation;
- Chapter 3 introduces the underlying theory used as a framework for portfolio optimization;
- Chapter 4 describes the methodology employed for investigating the research question in this dissertation;
- Chapter 5 presents and analyzes the empirical findings of this study;
- Finally, Chapter 6 concludes the dissertation and provides some avenues for further research.

2

LITERATURE REVIEW

This chapter provides a review of the literature associated with portfolio optimization. Later, it discusses the limitations of the traditional portfolio optimization framework and proposes an alternative.

2.1 Mean-Variance Framework

In his seminal work, Markowitz (1952) introduced the mean-variance (MV) framework for portfolio optimization, in which the optimal portfolio is defined as a set of weights that achieves an acceptable baseline expected rate of return, with a minimal level of volatility, measured as the standard deviation of returns. His work set the foundation for various equilibrium models such as the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), the Black and Litterman model (1992) and the three-factor model of Fama and French's (1993).

In the MV framework, the optimal portfolio is defined as a trade-off between risk and expected return, taking in to account a pre-established risk aversion behavior. This framework enlightened the importance of portfolio diversification and represented a breakthrough in portfolio management theory, which was until then solely focused on maximizing returns. Portfolio diversification refers to the process of combining a set of different asset classes and securities, in a way the exposure to any one type of asset is limited and, accordingly, the overall portfolio risk is minimized. As shown, for example, by Elton, Gruber, Brown, and Goetzmann (2006), in a diversified portfolio, the overall risk is more affected by the interrelationships between the assets rather than by the risk of each asset itself.

Markowitz (1952) split the definition of risk into two factors, systematic risk and unsystematic risk. The first refers to the risk that it is inherent of a certain market or market segment, while the second refers to specific risk that is particular to a specific investment, and therefore can be mitigated through diversification. Despite the fact that portfolio diversification cannot eliminate all risk, there is a rate at which the investor can obtain a larger expected return by taking on risk or, alternatively, giving up expected return by reducing risk. In addition, the MV framework associates portfolio diversification with the notion of efficiency, since it establishes that an optimal diversification is obtained along the Markowitz Efficient Frontier (MEF). This classic paradigm of modern finance, further described in Chapter 3, represents graphically all possible combinations of risky assets with respect to an optimal level of return, given a certain level of risk.

Hence, the optimization framework principal objective is to determine the best trade-off between risk and return, subject to a set of constraints. Under these circumstances, it establishes a spectrum of reasonable expectations that provide influential insight into investment decision making. The MV theoretical framework relies on certain underlying assumptions (described, for example, in Markowitz (1952, 1959) and Fabozzi, Kolm, et al. (2007)) that set the foundation for the application of the theory, namely:

- The markets are considered to be perfectly efficient;
- The prices of the assets are exogenous and given;
- The assets are infinitely divisible;
- Risk-free assets are available to all investors;
- There are neither taxes or other costs of transaction;
- Investors are assumed to be rational and risk averse;
- Investors aim is to maximize their expected utility;
- Investors possess homogeneous investment information and expectations;
- Unlimited long and short positions are permitted;
- Distribution of returns follows a normal distribution.

2.2 Limitations of MV Framework

The MV theory is one of the most referenced formulations in portfolio management literature. Nonetheless, it is important to note that the methodology encompasses certain shortcomings, for which it received considerable criticism. These arise from the fact that the framework assumptions do not hold in reality, and consequently compromise the direct model application of the model.

The most significant limitations of this framework are the following (as described, for example, in Bodie et al. (2009), Elton et al. (2010) and Hult et al. (2012)):

- It implicitly assumes that the asset returns follow a normal distribution. However, empirical evidence shows that asset returns usually follow leptokurtic and skewed distributions;
- Assets are usually not infinitely divisible, i.e., do not have a minimum order size and cannot be traded in fractions;
- Taxes and transactions costs are in fact an unpleasant reality;
- Investors' expectations are not homogenous and each has their own preferences;

- It is a single period model. However, investors usually possess a long-term investment strategy and therefore the impact of decisions arising in subsequent periods has to be considered;
- It is assumed that correlations across assets are fixed, which is not verifiable in reality;
- The model output is highly sensitive to the quality of the underlying parameter inputs (return, variance and covariance), and consequently maximizes estimation error;
- The model tends to identify efficient portfolios that are highly concentrated in certain asset classes.

In conclusion, when applied to real life scenarios, the MV optimization framework does not, in practice, output portfolios that optimally maximize the expected return subject to a certain given level of diversified risk. Michaud (1989) argues that extreme and unstable portfolio weights are inherent to MV optimizers, thus calling them "estimation-error maximizers". This argument is later reinforced by subsequent studies, such as Chopra and Ziemba (1993). In this article, the authors provide one of the most noticeable empirical examples regarding error maximization and demonstrate the sensitivity of the optimal portfolio composition in the MV framework to estimation errors in problems inputs. Consequently, this sensitivity causes an unstable optimization framework and generates ambiguous optimized portfolios. Likewise, Michaud and Michaud (1998) state that the main problems concerning the practical application of the MV framework were this implicit instability and ambiguity, since small changes in input parameters will often lead to large changes in the optimized portfolios. In consonance, Fabozzi et al. (2014) highlight the considerable sensitivity of portfolio allocation to changes in the input parameters and the unclear relation between the inputs and outputs of MV framework. Best and Grauer (1991) further illustrate the sensitivity of MV optimized portfolios weights to changes in the return input parameters and that it influences the overall portfolio performance. Similarly, Kan and Zhou (2007) confirm that portfolios are not optimal when considering the uncertainty underlying input parameters. In addition, DeMiguel et al. (2009) provide a comprehensive review of the performance of various methods in efforts to reduce the error in the estimation of input parameters and covariances. Hence, and according with Michaud and Michaud (2008), MV optimizers tend to create biased portfolios, due to a considerable reliance on input statistical information. This bias is present in both estimates for risk and return since the inherent uncertainty of these inputs is not taken into consideration, as real markets exhibit complexities with unknown and unobservable distributions of both return and risk. Moreover, as described Maringer (2005), the MV framework is excessively simplified in order to be solvable under unrealistic assumptions and does not reflect the constraints faced by real-world investors. Additionally, the result obtained if the model is constrained, in order to increase its applicability in real market conditions, could mean that it is not clear anymore if the optimizer contributes to portfolio investment value. Accordingly, deploying this biased traditional methodology without any improvements will lead to an undesired poor performance and questionable results. These limitations have consequently motivated further studies in portfolio optimization frameworks, in order to improve its applicability in real life complex scenarios.

2.3 Resampled Efficient Frontier

In view of the previously stated limitations, Michaud and Michaud (2008) proposed a portfolio optimization framework that incorporates the inherent uncertainty in the model input parameters, which, if not accounted for, gives rise to a heavily degraded performance of the resulting portfolios. Conversely, to incorporate this uncertainty, the authors use Monte Carlo simulations to generate a considerable amount of plausible scenarios from the estimated inputs, and create alternative efficient frontiers, that differ from the MEF portfolios. These alternative efficient frontiers and optimized portfolios are then employed to create the Resampled Efficient Frontier (REF). When compared to the MEF portfolios, the REF risk and return input parameters estimates become optimal when these statistically equivalent efficient portfolios are averaged over a substantially high amount of possible scenarios of the unknown true input parameters. As such, the resampling efficiency provides an improvement over traditional methods since it is possible to obtain greater investment performance without the need for added constraints, while simultaneously factoring in uncertainty and using investment information in a more perceptive manner. However, is important to acknowledge the disadvantages of this proposed portfolio optimization framework as stated by Scherer (2002) namely, the lack of a robust theoretical foundation, its degree of complexity, the associated expensive costs and the fact that to some degree it still relies on the accuracy of input data.

3

An Overview of Portfolio Optimization

This chapter presents the conceptual and theoretical framework foundations that support this dissertation.

3.1 Risk and Return

One of the main objectives for any investor is the accurate construction of an optimal portfolio. In order to achieve that, is crucial to analyze the return and risk investment profile of each asset, to help select the most efficient portfolios, and ultimately choose the optimal portfolio tailored to the investor's preferences.

Return is defined as the rate of change in the value of an investment, in a specific time interval. Thus, the expected return of a portfolio provides an estimate of how much return it is expected to yield, and henceforth is denoted as R_p. Additionally, in order to convert historical price data into values that can be integrated in a statistical distribution and help draw statistical inferences for research, it is assumed that returns are stationary.

In finance literature, risk refers to the probability of failing to meet the objectives for a given investment, and consequently represents the possibility that actual investment returns will differ from the expected returns. Standard deviation of returns, denoted as σ_p, is used as a proxy for the portfolio risk.

3.2 Portfolio Optimization

As previously mentioned in Chapter 2, the optimization framework model provides important insights into the investment decision process, by establishing a trade-off between risk and return, subject to a set of constraints.

Consider a portfolio p of i risky assets, with expected returns R_1, R_2, … R_n, composing a $n \times 1$ vector of returns, and portfolio weights w_1, w_2, …w_n, forming a $n \times 1$ vector of weights, according with the following equations,

$$R_p = \mu = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{bmatrix} \tag{3.2.1}$$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{3.2.2}$$

The expected return of a portfolio, denoted as R_p, corresponds to the sum of the expected returns of the individual securities R_n, each multiplied by the corresponding portfolio weights w_n.

$$R_p = \sum_{i=1}^{n} (w_i \times R_i) \tag{3.2.3}$$

The portfolio variance is given by the following equation

$$\sigma_p{}^2 = Var(R_p) = Var\left(\sum_{i=1}^{n} w_i R_i\right) \tag{3.2.4}$$

$$= \sum_{i,j=1}^{n} w_i w_j Cov(R_i, R_j) = \sum_{i=1}^{n} w_i{}^2\, Var(R_i) + \sum_{i,j=1, i \neq j}^{n} w_i w_j Cov(R_i, R_j) \tag{3.2.5}$$

where $Var(R_i)$ is the variance of returns for security i, while $Cov(R_i, R_j)$ is the covariance of returns between securities i and security j, which can be expressed as $Cov(R_i, R_j) = \rho_{i,j}\sigma_i\sigma_j$. A variance-covariance matrix $\sum$ is a square matrix that contains the variances and covariances associated with the portfolio securities. The diagonal elements of the matrix represent the asset variances σ_i^2, while the off-diagonal elements contain all possible pairwise covariances $Cov(R_i, R_j)$. The variance-covariance matrix is defined as follows:

$$\sum = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \cdots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \sigma_{n-1,n} \\ \sigma_{n,1} & \sigma_{n,n-1} & \cdots & \sigma_n^2 \end{bmatrix} \tag{3.2.7}$$

With this framework, the portfolio optimization is performed through the minimization of the portfolio risk, given a fixed level of expected portfolio return and constraints. The MV optimization problem is defined as follows:

minimize

$$\sigma_p{}^2 = \sum_{i,j=1}^{n} w_i w_j Cov(R_i, R_j) \tag{3.2.8}$$

subject to

$$R_p{}^* = \sum_{i=1}^{n}(w_i \times R_i) \tag{3.2.9}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{3.2.10}$$

$$0 \leq w_i \leq 1 \tag{3.2.11}$$

In the above formulation, the first optimization constraint (equation 3.2.9) ensures that the portfolio return achieves a given desired portfolio return $R_p{}^*$, while the second constraint (equation 3.2.10), ensures that the capital is fully invested. Finally, the last constraint (equation 3.2.11) imposes a restriction on short selling.

This optimization problem is solved through standard numerical methods, yielding the Markowitz efficient frontier. This line represents graphically the portfolios with the greatest expected level of return given a certain level of risk. The set of portfolios (feasible set) that lie below the Efficient Frontier are considered sub-optimal. Moreover, there is a diminishing marginal expected return with respect to increases in risk levels.

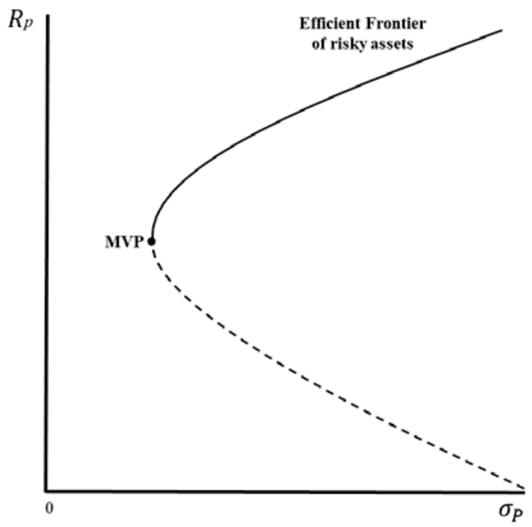

Figure 3.1 – Markowitz Efficient Frontier

Figure 3.1 depicts the global Minimum Variance Portfolio (MVP), the portfolio with the smallest possible variance.

3.3 Portfolio Measures

Portfolio measures are fundamental for assessing investment decisions and strategies. Despite the fact that conducting an ex-post analysis of historical data does not guarantee future performance, these indicators still prevail as critical tools that provide information to assess investment performance. Additionally, risk-adjusted measures, that incorporate the risk-return trade-off, can also be applied to compare mutually exclusive portfolios and support the investment decision. Furthermore, it is impossible to accurately gauge the performance of an investment resorting only to a single measure. As such, to analyze the efficiency of the generated portfolios, measures such as Sharpe ratio, average expected return, maximum drawdown, standard deviation and Herfindahl-Hirschman Index were used. Further bellow is defined each measure and how it is computed.

- **Average Return**

Average return is a simple measure and it is achieved through the average of a series of returns accrued over a specified period of time.

- **Standard Deviation**

It is a measure of the dispersion of a set of data from its mean, which means that the more disperse the data, the higher the volatility of an investment. The standard deviation of a portfolio is given by the following equation:

$$\sigma_p = \sqrt{\sum_{i,j=1}^{n} w_i w_j Cov(R_i, R_j)} \tag{3.3.1}$$

Notwithstanding, standard deviation if used alone as a measurement of risk can have its limitations. Namely, it is not forward looking so it does not guarantee that the outcomes will be consistent in the future. Accordingly, it must be assessed in conjunction with other risk measures.

- **Maximum Drawdown (MDD)**

Measures the largest single drop from peak to bottom in the value of a portfolio, during a time period. This maximum observed loss is used as an indicator of the downside risk in the period under analysis. However, it is noteworthy to mention that it does not take into consideration the frequency of large losses since it only measures the size of the largest drawdown. MDD is calculated as:

$$MDD = \frac{(lowest\ value\ before\ largest\ drop\ -\ peak\ value\ before\ largest\ drop)}{peak\ value\ before\ largest\ drop} \quad (3.3.2)$$

- **Sharpe Ratio**

Sharpe ratio is a measure that measures the desirability of a risky asset or investment strategy, by measuring the risk-adjusted performance or excess return per unit of deviation.
The higher Sharpe ratio, the greater the investment return with reference to the amount of investment risk taken. Note that a higher Sharpe ratio indicates a better risk-adjusted performance and does not necessarily translate to a lower volatility investment.
Additionally, a negative Sharpe ratio can either mean that the risk-free rate R_f is greater that the portfolio return, or that the portfolio return is expected to be negative. The Sharpe ratio is calculated by subtracting the risk-free rate R_f from the rate of return for a portfolio R_p and dividing the result by the standard deviation of the portfolio returns σ_p, as follows:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p} \quad (3.3.3)$$

- **Herfindahl-Hirschman Index (HHI)**

The Herfindahl–Hirschman Index is a standard measure of concentration. The index is computed by adding all squared weights of the individual stocks w_i in the portfolio, as follows:

$$HHI = \sum_{i=1}^{n} {w_i}^2 \quad (3.3.4)$$

Accordingly, it can range from $\frac{1}{n}$ to 1, shifting from an equally-weighted portfolio to a single stock portfolio. Thus, the higher HHI, the higher the portfolio concentration risk. Additionally, the reciprocal of the HHI, the effective number of stocks, has a more intuitive interpretation to analyze the level of concentration in a portfolio and to help understand if certain stocks have a disproportionate effect on the overall portfolio performance. This indicator is calculated as follows:

$$Effective\ number\ of\ stocks = \frac{1}{HHI}$$

- **Value-at-Risk (VaR)**

Value-at-Risk quantifies the maximum potential loss of an investment, at a given confidence level $(1-\alpha)$, over a specific period of time. By assessing the extent of possible losses, Value-at-Risk helps control the level of risk exposure of an investment, and allows for a worst-case scenario analysis. The VaR can be computed directly from past returns, by simply sorting them in ascending order, from worst to best (this indicator is also named the Historical VaR). Although this technique is a simple and fast method to calculate VaR, it is not forward looking, entirely dependent on past market movements.

3.4 Portfolio Resampling

Portfolio resampling, first proposed by Michaud (1998), is a statistically rigorous procedure based on the resampled efficiency concept (the resampling optimization process was introduced in Chapter 2) that aims to minimize the impact of the estimation risk and obtain a more robust optimized portfolios.

The general concept of Michaud's resampled efficiency can be outlined as follows:

- Creating sequences of various returns scenarios, through a Monte Carlo simulations;
- Performing an optimization exercise for each return scenario; and
- Obtaining the Michaud optimal portfolio weights taking into consideration the various returns scenarios.

In this dissertation, the procedure for computing the resampled efficient portfolios was simplified. It was considered three plausible scenarios from the return input parameters namely, a neutral, a positive and a negative scenario that characterize alternative market conditions and try to reflect the variability implicit in historical data, as follows:

- The neutral scenario reflects the unchanged expected value of the portfolios after a given period of time. Thus, $R_{neutral} = \mu$;
- The positive scenario, best-case oriented and a more optimistic approach, reflects the expected value of the portfolios after a given period of time, assuming favorable market conditions. Thus, $R_{positive} = \mu + 1\ \sigma_p$; and
- The negative scenario, worst-case oriented and a more conservative approach, reflects the expected value of the portfolios after a given period of time, assuming unfavorable market conditions. Thus, $R_{negative} = \mu - 1\ \sigma_p$.

Note that in the optimization step no short selling is allowed. In the resampling step, the three return scenarios were incorporated by considering a probability for each scenario (40% for the neutral scenario, and 30% for the positive and the negative scenarios). Additionally, it should be considered the following:

- The expected value of the portfolios μ , is equivalent to the average monthly return over the last 12 months; and
- The portfolios variance-covariance matrix Σ, is computed based on the monthly returns of the last 12 months.

4

PROBLEM FOUNDATION AND METHODOLOGY

This chapter addresses the methodology used to answer the research question in this dissertation. First, the data included in the portfolio optimization approach is presented. This is followed by a description of the methodology adopted to assess portfolio efficiency.

4.1 Data Description

The empirical analysis employs a dataset that consists of monthly closing prices, retrieved from Thomson Reuters Datastream (Refinitiv Datastream), for constituent stocks of the following market indices:

- S&P 500 – Stock market index based on the market capitalization of 500 large companies listed on stock exchanges in the United States of America.
- EURO STOXX 50 – Stock market index based on the market capitalization of 50 companies from eight Eurozone countries (Belgium, Finland, France, Germany, Ireland, Italy, Netherlands and Spain).
- NASDAQ 100 – Stock market index based on the market capitalization of 101 largest non-financial companies listed on the Nasdaq Stock Exchange.
- DAX 40 – Stock market index based on the market capitalization of the 40 leading German companies trading on the Frankfurt Stock Exchange.

The above stock market indexes were selected to represent major worldwide stock markets and allow the observation of the elevated market volatility impact on the wider scale possible. Therefore, it enables a more accurate evaluation of the portfolio resampling strategy efficiency.

The data sets analyzed in this dissertation are listed in the following Table 4.1.

Table 4.1 – Data Sets Considered in the Analysis

Name	N	Time period
(1) S&P 500	96x458	01/01/2012 - 30/04/2020
(2) XX 50	96x50	01/01/2012 - 30/04/2020
(3) NASDAQ 100	96x84	01/01/2012 - 30/04/2020
(4) DAX 40	96x25	01/01/2012 - 30/04/2020

Table 4.1 lists the data and its sources; *N* denotes the total number of monthly observations of each portfolio data set. The last column represents the time line for each data index.

4.2 Problem Objective

As discussed in the previously chapter 2, the MV optimization framework is subjected to considerable criticism due to the generated estimation errors inherent of the uncertainty of input parameter estimates. As an alternative, heuristic methods such the one proposed by Michaud and Michaud (2008) been developed to minimize the impact of estimation risk on the portfolio composition, in order to improve portfolio performance with respect to the MV portfolio optimization framework. Additionally, the portfolio resampling strategy takes into account various plausible scenarios from the estimated inputs, and create more diversified and robust portfolios, that differ from the MV optimized portfolios.

This study attempts to analyze the performance and feasibility of the resampling technique in a context of elevated market volatility, as a result of events such as the COVID-19 global pandemic in 2020. In addition, it is assumed that the agent is risk tolerant and therefore the level of risk aversion had no effect in the study. The following procedure is adopted:

1. Computation of a portfolio optimization exercise on 31/01/2020;
2. Analysis of the performance of the generated portfolios in February;
3. Computation of a portfolio optimization exercise on 29/02/2020;
4. Analysis of the performance of the generated portfolios in March;
5. Computation of a portfolio optimization exercise on 31/03/2020;
6. Analysis of the performance of the generated portfolios in April.

The necessary algorithms were implemented in Jupiter Notebook, a web-based interactive computing platform to create documents (called "notebooks") in which it is possible to execute Python source code, and create text, equations and visualizations. This allowed the creation of an interactive data science environment organized in two notebooks each with its specific purpose.

The first notebook, *Portfolio Optimization Process*, is organized as follows:

1. The algorithm starts by selecting the historical data necessary for performing the optimization exercise for the period under analysis;
2. Next, it computes the monthly returns of the generated data subsets;
3. Based upon these estimates, the following step entails the computation, for the three considered scenarios, of the return and risk input parameters, namely the vector of expected returns and the variance-covariance matrix. The code for this part of the algorithm is presented below:

Figure 4.1 – Algorithm of Portfolio Optimization Inputs

Algorithm Portfolio Optimization Inputs

```
def var_covar(sigma,rho):
    sigma_aux = np.diag(sigma)
    return np.matmul(np.matmul(np.transpose(sigma_aux),rho),sigma_aux)
def portfolio_variance(w,vcv):
    w = np.expand_dims(w, axis=1)
    wt = w.T
    return np.matmul(np.matmul(wt,vcv),w)[0][0]
ret = df_monthly_returns
sigma = ret.std()
rho = ret.corr()
sigma = sigma.values
rho = rho.values
n_port = 50
cons_w = {'type': 'eq', 'fun': lambda x: np.sum(x)-1}
```

4. Next, the portfolio optimization routine takes place. The code for this part of the algorithm is presented below:

Figure 4.2 – Algorithm of Portfolio Optimization Routine

Algorithm Portfolio Optimization Routine

```
bds = ()
for i in range(n):
  bds += ((0, 0.9999),)
opts = {'disp': True, 'maxiter': 1000}
ret_c_min = np.quantile(ret_c , 0.1)
ret_c_max = np.quantile(ret_c , 0.9)
for i in range (1,n_port+1):
  random_n = np.random.rand(n)
  w_guess = random_n/np.sum(random_n)
  cons_ret_c = {'type': 'eq', 'fun': lambda x: np.sum(x*ret_c)-(ret_c_min+(ret_c_max-ret_c_min)
/n_port*i)}
  cons = ([cons_w, cons_ret_c])
  sol = spo.minimize(portfolio_variance, w_guess, args=(vcv_c,), method = 'SLSQP', bounds = bds,
constraints = cons, options = opts)
  opt_w = sol.x
```

5. The Michaud resampling process is conducted. The generated alternative efficient frontiers and optimized portfolios of the three considered scenarios (neutral, positive and negative) are sorted into buckets according to their variance level and then employed to create the resampled efficient frontier with an assign probability of 40%/30%/30% to each scenario respectively. The code for this part of the algorithm is presented below:

Figure 4.3 – Algorithm of Portfolio Optimization Resampling Step

Algorithm Portfolio Resampling Step

```
port_w_c = eff_port_w_c[(eff_port_sd_c >= int_a) & (eff_port_sd_c < int_b),:]
port_w_c = np.mean(port_w_c,0)
port_w_c_final[:,i] = port_w_c
port_w_p = eff_port_w_p[(eff_port_sd_p >= int_a) & (eff_port_sd_p < int_b),:]
port_w_p = np.mean(port_w_p,0)
port_w_p_final[:,i] = port_w_p
port_w_n = eff_port_w_n[(eff_port_sd_n >= int_a) & (eff_port_sd_n < int_b),:]
port_w_n = np.mean(port_w_n,0)
port_w_n_final[:,i] = port_w_n
port_w_res = 0.4*port_w_c + 0.3*port_w_p + 0.3*port_w_n
port_w_res_final[:,i] = port_w_res
```

Secondly, the notebook *Portfolio Resampling Analysis,* takes the aforementioned notebook as input and together with the CSV files of the financial data computes the portfolio performance measures, and finally exports the resulting data frames to CSV files.

5

Empirical Results and Analysis

This chapter presents the empirical results. First, it provides a brief historical description of the market conditions during the period under analysis. Lastly, the portfolio performance measures are presented and interpreted.

In order to better interpret the empirical results it is fundamental to contextualize them considering the time events of the period of analysis as follows:

- **December 2019:** Wuhan, a central city in China, reported the first COVID-19 case.
- **January 2020:** Wuhan Health Committee reported 44 cases of viral pneumonia of unknown cause. The disease spread silently across the globe and on 30/01, the WHO issued its first global alert regarding COVID-19 outbreak. The S&P 500 and the Euro Stoxx 50 close out the month down 0,16% and 2,78% respectively and VIX up 36,72%.
- **February 2020:** On 19/02, the capital markets reached their pre-COVID-19 high. This first month of the crisis was characterized by historically substantial and rapid declines across all business sectors. In the beginning of the pandemic, all news coverage were portraying a disastrous scenario, uncertainty was extraordinary, and the downside risk seemed unlimited. The S&P 500 and the Euro Stoxx 50 close out the month down 8,41% and 8,55% respectively and VIX up 112,90%.
- **March 2020:** As the number of confirmed cases soared throughout the world, the WHO decided to officially announce it as a pandemic on 11/03 and VIX it skyrocketed to an all-time high of 85,47, surpassing its 2008 record. From mid-March it was observable the substantial differentiation across sectors and governments began responding with record stimulus packages. Sectors such as high tech and some consumer categories staged recoveries since the impact of the pandemic led to tailwinds of growing demand. However, several industries remained down significantly from their pre-pandemic peaks, namely aerospace, travel, banking, insurance, and petroleum (Mazur et al., 2020). The S&P 500 and the Euro Stoxx 50 close out the month down 12,51% and 16,30% respectively and VIX up 33,48%.
- **April 2020:** The decline started to deaccelerate as the health crisis and mitigation measures have become more widespread. Additionally, the implemented policies changed consumer behavior and enabled some companies to surpass their sector peers. Other sectors, such as pharmaceuticals and biotechnology, almost fully regained their

market losses. The S&P 500 and the Euro Stoxx close out the month with an increase of 12,68% and 5,06% respectively and VIX down 36,22%.

Note that the stock market indexes DAX 40 and NASDAQ 100 were considered in the analysis but given their relevance and study contribution were not included in the final analysis.
This dissertation compares the generated portfolios performances and volatility with regard to the resampling strategy implemented as follows:

- **Portfolio Concentration**

Portfolio diversification is an important risk mitigation strategy (the higher the level of portfolio diversification, the lower the correlation coefficient between the different assets of the portfolio) and the results presented in table 5.1 show clearly that the resampled portfolios show a higher degree of diversification and consequently a lower volatility. This is explained because the resampling portfolio strategy takes into account three return scenarios (neutral, positive and negative) and, therefore delivers optimized portfolios that are more diversified and better prepared for all market conditions. In more detail, the results show that the resampled optimized portfolios present consistently a higher degree of diversification not only during the months of lower volatility (February and April) but also during the month of more accentuated volatility (March). Therefore, results show that the portfolio resampling strategy delivers a more robust asset allocation than the MV portfolio optimization framework.

Table 5.1 – Portfolio 1/HHI Concentration

Month	Portfolio	S&P 500				EUROSTOXX 50			
February		*neutral*	*positive*	*negative*	*resampled*	*neutral*	*positive*	*negative*	*resampled*
	0	27,69	30,12	31,10	29,67	14,011	14,112	14,169	14,127
	1	28,71	34,15	30,99	33,26	13,123	14,529	13,463	14,018
	2	31,58	33,43	29,48	36,22	11,840	12,285	13,963	13,275
	3	30,04	31,83	27,38	36,38	10,626	11,199	11,814	12,226
March									
	0	29,31	27,35	26,72	28,15	14,805	15,014	11,919	14,319
	1	27,69	28,33	28,90	30,22	13,492	15,674	10,996	13,906
	2	28,11	27,57	26,40	31,42	12,996	14,073	9,264	12,940
	3	32,98	29,31	27,91	36,69	11,996	13,019	8,666	12,361
April									
	0	29,60	28,17	24,86	28,30	12,962	13,006	12,261	12,915
	1	27,37	29,25	25,27	28,80	13,924	14,701	12,332	13,957
	2	27,51	29,25	27,96	30,97	15,657	15,555	15,491	15,948
	3	27,18	28,38	25,99	31,14	14,916	14,020	11,967	14,627

Note: The HHI concentration index was computed for each portfolio as an indicator of diversification relative to the naive portfolio (i.e., a well-diversified portfolio of n assets should have its HHI index close to $1/n$).

- **Portfolio Turnover**

Portfolio turnover is an important measure that indicates the extent to which the portfolio assets changed during a determined period of time. Additionally, it is important to consider that this indicator is affected by the level of portfolio diversification, with less diversified portfolios having a higher turnover ratio. Moreover, the level of portfolio turnover ratio de-pends on the market conditions. During high volatility market conditions, less diversified portfolios observe more abrupt changes in asset weights, increasing the portfolio turnover. From the results presented in table 5.2, the portfolio resampling strategy tends to lead to a lower portfolio turnover ratio, in the neutral and negative scenarios. On the other hand, the resampling strategy, when compared with the positive scenario, have a higher turnover ratio. Therefore, it reflects that generally and considering all markets conditions, the resampling strategy has a more balanced investment approach, with a portfolio composition that varies less over time.

Table 5.2 – Portfolio Turnover Ratio

Month	Portfolio	S&P 500				EUROSTOXX 50			
February		*neutral*	*positive*	*negative*	*resampled*	*neutral*	*positive*	*negative*	*resampled*
	0	59,72%	57,29%	56,51%	57,46%	24,48%	3,38%	28,93%	23,90%
March									
	0	52,20%	46,64%	53,15%	49,89%	45,85%	51,35%	66,33%	51,00%
April									
	0	29,73%	30,51%	28,47%	28,12%	29,54%	26,63%	38,29%	29,18%

Note: The Portfolio Turnover Ratio was computed for the minimum variance portfolio.

- **Portfolio Risk and Return**

In terms of risk, as measured by the standard deviation of returns, MDD and VaR reported in table 5.3, it is possible to observe that during the total period of analysis, the resampling strategy displayed generally a balanced risk performance. Additionally, even though the resampling strategy was outperformed in a general sense by the negative scenario, it displayed very close results in comparison. This is evidence that the high diversification level of the resampled portfolios result in a resilient investment strategy capable of adapting to the different markets conditions in a consistent manner. As far as the risk-return measures are concerned reported in table 5.3, the resampling strategy during the period of analysis, presents the second best results regarding Sharpe ratio in comparison with the other scenarios. It further demonstrates that the resampling strategy could consistently have an adequate investment return relative to the amount of risk taken. Likewise, regarding the portfolios average daily returns shown in table 5.3, the resampling strategy delivered consistently the second best results in comparison with the neutral and negative scenarios. Thus, the resampling strategy was able to achieve a balanced return over the total period of analysis.

Table 5.3 – Portfolio Risk and Return Measures

Month	**S&P 500**				**EUROSTOXX 50**			
February	***neutral***	***positive***	***negative***	***resampled***	***neutral***	***positive***	***negative***	***resampled***
Avg. Daily Return	-0,18%	-0,19%	-0,19%	-0,19%	-0,25%	-0,30%	-0,23%	-0,26%
MDD	-9,90%	-9,89%	-9,67%	-9,84%	-11,24%	-11,45%	-10,91%	-11,21%
Stand. Deviation	1,34%	1,30%	1,33%	1,32%	1,50%	1,49%	1,45%	1,48%
Sharpe Ratio	-12,63%	-13,81%	-14,38%	-13,65%	-16,98%	-20,30%	-15,74%	-17,64%
VaR	-2,79%	-2,85%	-2,69%	-2,79%	-3,32%	-3,48%	-3,13%	-3,28%
March								
Avg. Daily Return	0,10%	0,04%	0,09%	0,08%	-0,31%	-0,35%	-0,20%	-0,29%
MDD	-21,68%	-22,27%	-21,62%	-21,84%	-24,32%	-24,85%	-22,42%	-23,82%
Stand. Deviation	5,04%	5,03%	5,09%	5,05%	3,92%	3,92%	3,72%	3,83%
Sharpe Ratio	1,94%	0,72%	1,85%	1,56%	-7,85%	-8,79%	-5,48%	-7,46%
VaR	-8,37%	-8,40%	-8,50%	-8,44%	-5,84%	-5,95%	-5,59%	-5,83%
April								
Avg. Daily Return	0,31%	0,28%	0,29%	0,29%	0,38%	0,37%	0,35%	0,37%
MDD	-12,64%	-12,19%	-12,51%	-12,38%	-10,17%	-9,81%	-9,34%	-9,81%
Stand. Deviation	2,06%	2,10%	2,10%	2,08%	1,43%	1,46%	1,35%	1,41%
Sharpe Ratio	15,00%	13,40%	13,91%	13,88%	26,88%	25,09%	25,78%	26,07%
VaR	-2,48%	-2,60%	-2,38%	-2,54%	-1,84%	-1,90%	-1,75%	-1,82%

6

CONCLUSION

In the beginning of 2020, the world faced an unprecedented global outbreak pandemic that triggered stock market crashes. The COVID-19 crisis brought about long-lasting changes that affected not only worldwide economies and businesses but also our daily lives. Additionally, it provided an ideal case study to illustrate market efficiency and test various hypotheses concerning optimal portfolio strategies that adequately hedge against investment risk.

Throughout this elevated volatility period, the expectations concerning the duration of the global pandemic and its impact in global markets were substantial, and more than ever reinforced the relevant impact of a dependable and efficient portfolio optimization framework.

In investment decision making, is crucial to have a precise analysis and interpretation of the portfolio risk, return and performance. The more consistent and efficient the optimization framework, the greater the ability to withstand unexpected unfavorable circumstances.

The aim of this dissertation was to assess if the resampling optimization method developed by Michaud and Michaud (2008), during the beginning stages of the COVID-19 pandemic, provided superior investment performance in comparison with the Markowitz MV optimization framework (1952).

As presented in Chapter 5, devoted to the empirical results, the general outcome result from the resampled optimization is that not only the degree of diversification is higher and the portfolio turnover lower but also displayed results regarding measures like the MDD, the Sharpe ratio, the standard deviation and the VaR, that are very close to the best in comparison. This methodology allowed the construction of efficient frontiers that are in fact more resilient and diversified than traditional MV optimized portfolios. Moreover, the resampling method demonstrated that it has the propensity to reduce turnover and increase financial out-of-sample efficiency. Therefore, the resampled efficiency within the framework of this study has proven to be an interesting heuristic method that influences the portfolio measures of both risk and return, and seems to provide on average, superior investment performance. However, even though resampling demonstrates that it is an effective mechanism for portfolio construction, this methodology does not have a consistent theoretical foundation crucial to facilitate the comprehension of concepts, variables and the relationship between them. Another important remark is that the empirical results presented might only be valid within this framework since they are based on a study with historic data of limited number of stocks over a specific short period of time.

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APPENDIX

A

Python Script

The present Appendix A displays the Python Script calculus foundation for this dissertation portfolios optimal optimization framework. The script is organized in two notebooks, Portfolio Optimization Process (appendix A.1) and Portfolio Resampling Analysis (appendix A.2).

A.1 Portfolio Optimization Process

```
#Libraries
import time
import datetime as dt
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.optimize as spo
import scipy.stats

#df_dataset = pd.read_excel('values.xlsx')
#df_EUROSTOXX50 = pd.read_excel('EUROSTOXX50.xlsx')
#df1 = df_EUROSTOXX50.set_index(['Dates'])
#df_SP500 = pd.read_excel('S&P500_100.xlsx')
#df_SP500 = pd.read_excel('S&P500_superclean.xlsx')
#df1 = df_SP500.set_index(['Dates'])
#df_DAX30 = pd.read_excel('DAX30.xlsx')
#df1 = df_DAX30.set_index(['Dates'])
#df_NASDAQ100 = pd.read_excel('NASDAQ100.xlsx')
#df1 = df_NASDAQ100.set_index(['Dates'])

monthly_dates = pd.date_range(start='1/1/2012',end='31/4/2020',freq='BM')

#Daily Returns
df_daily_returns = df1.pct_change()
df_daily_returns = df_daily_returns[1:] #skip first line 'NaN'
cum_return = ((df1.iloc[-1] - df1.iloc[0]) / df1.iloc[0])*100

df_cum_daily_returns = (1 + df_daily_returns).cumprod() - 1
df_cum_daily_returns = df_cum_daily_returns.reset_index()
df_cum_daily_returns

df_daily_returns_mean = df_daily_returns.mean()

#Monthly Returns
df_monthly = df1.loc[monthly_dates]
df_monthly_returns = df_monthly.pct_change()
df_monthly_returns = df_monthly_returns[1:]
df_cum_monthly_returns = (1 + df_monthly_returns).cumprod() - 1
df_cum_monthly_returns = df_cum_monthly_returns.reset_index()
df_cum_monthly_returns

df_monthly_returns_mean = df_monthly_returns.mean()

#Covariance
covariance = df_monthly_returns.cov()*252
#Optimization Process
```

```
def var_covar(sigma,rho):
    sigma_aux = np.diag(sigma)

    return np.matmul(np.matmul(np.transpose(sigma_aux),rho),sigma_aux)

def portfolio_variance(w,vcv):
    w = np.expand_dims(w, axis=1)
    wt = w.T

    return np.matmul(np.matmul(wt,vcv),w)[0][0]

#Calculation of return matrix, standard deviation matrix and correlation
matrix
ret = df_monthly_returns

sigma = ret.std()
rho = ret.corr()

sigma = sigma.values
rho = rho.values

n_port = 50

cons_w = {'type': 'eq', 'fun': lambda x: np.sum(x)-1}

#MICHAUD - Neutral/Central scenario
#Convert pandas to numpy, to minimize matrix calculation error
ret_c = ret.tail(12).mean().values
stdv_c = ret.tail(12).std()

#Calculus var-covar matrix
vcv_c = var_covar(sigma,rho)

n = np.shape(ret_c)[0]

eff_port_w_c = np.zeros((n_port,n))
eff_port_ret_c = np.zeros(n_port)
eff_port_var_c = np.zeros(n_port)

bds = ()
for i in range(n):
    bds += ((0, 0.9999),)

#opts = {'ftol': 1e-12, 'disp': True, 'eps': 1e-12}
opts = {'disp': True, 'maxiter': 1000}

ret_c_min = np.quantile(ret_c , 0.1)
ret_c_max = np.quantile(ret_c , 0.9)

for i in range(1,n_port+1):

    print(i)
    random_n = np.random.rand(n)
    w_guess = random_n/np.sum(random_n)

    #Constraints setup ()
    cons_ret_c = {'type': 'eq', 'fun': lambda x: np.sum(x*ret_c)-
(ret_c_min+(ret_c_max-ret_c_min)/n_port*i)}
    cons = ([cons_w, cons_ret_c])
```

```
    sol = spo.minimize(portfolio_variance, w_guess, args=(vcv_c,), method =
'SLSQP', bounds = bds, constraints = cons, \
                       options = opts)
     opt_w = sol.x

    #Save the optimal portfolio weights
    eff_port_w_c[i-1,:] = opt_w
    eff_port_ret_c[i-1] = np.sum(opt_w*ret_c)

    #Save the optimal portfolio returns and variances
    opt_w = np.expand_dims(opt_w, axis=1)
    opt_wt = opt_w.T

    eff_port_var_c[i-1] = np.matmul(np.matmul(opt_wt,vcv_c),opt_w)[0][0]

print(np.sqrt(eff_port_var_c))
print(eff_port_ret_c)

plot = plt.scatter(np.sqrt(eff_port_var_c), eff_port_ret_c)
plt.xlabel("Standard Deviation")
plt.ylabel("Expected Return")
plt.show()

#MICHAUD - Positive scenario
stdv_p = ret.tail(12).std()
ret_p = (ret.tail(12).mean()+1*stdv_p).values

#Calculus var-covar matrix
vcv_p = var_covar(sigma,rho)

n = np.shape(ret_p)[0]

#print(np.shape(ret))
eff_port_w_p = np.zeros((n_port,n))
eff_port_ret_p = np.zeros(n_port)
eff_port_var_p = np.zeros(n_port)

bds = ()
for i in range(n):
    bds += ((0, 0.9999),)

#opts = {'ftol': 1e-12, 'disp': True, 'eps': 1e-12}
opts = {'disp': True, 'maxiter': 1000}
ret_p_min = np.quantile(ret_p , 0.1)
ret_p_max = np.quantile(ret_p , 0.9)

for i in range(1,n_port+1):
    print(i)
    random_n = np.random.rand(n)
    w_guess = random_n/np.sum(random_n)

    #Constraints setup ()
    cons_ret_p = {'type': 'eq', 'fun': lambda x: np.sum(x*ret_p)-
(ret_p_min+(ret_p_max-ret_p_min)/n_port*i)}
    cons = ([cons_w, cons_ret_p])

    sol = spo.minimize(portfolio_variance, w_guess, args=(vcv_p,), method =
'SLSQP', bounds = bds, constraints = cons, \
                       options = opts)
    opt_w = sol.x
```

```
    #Save the optimal portfolio weights
    eff_port_w_p[i-1,:] = opt_w
    eff_port_ret_p[i-1] = np.sum(opt_w*ret_p)

    #Save the optimal portfolio returns and variances
    opt_w = np.expand_dims(opt_w, axis=1)
    opt_wt = opt_w.T

    eff_port_var_p[i-1] = np.matmul(np.matmul(opt_wt,vcv_p),opt_w)[0][0]

print(np.sqrt(eff_port_var_p))
print(eff_port_ret_p)

plot = plt.scatter(np.sqrt(eff_port_var_p), eff_port_ret_p)
plt.xlabel("Standard Deviation")
plt.ylabel("Expected Return")
plt.show()

#MICHAUD - Negative scenario
stdv_n = ret.tail(12).std()
ret_n = (ret.tail(12).mean()-1*stdv_n).values

print(ret_n)
print(np.shape(ret_n))

#Calculus var-covar matrix
vcv_n = var_covar(sigma,rho)

n = np.shape(ret_n)[0]

#print(np.shape(ret_n))
eff_port_w_n = np.zeros((n_port,n))
eff_port_ret_n = np.zeros(n_port)
eff_port_var_n = np.zeros(n_port)

bds = ()
for i in range(n):
    bds += ((0, 0.9999),)

#opts = {'ftol': 1e-12, 'disp': True, 'eps': 1e-12}
opts = {'disp': True, 'maxiter': 1000}

ret_n_min = np.quantile(ret_n , 0.1)
ret_n_max = np.quantile(ret_n , 0.9)

for i in range(1,n_port+1):
    print(i)
    random_n = np.random.rand(n)
    w_guess = random_n/np.sum(random_n)

    #Constraints setup ()
    cons_ret_n = {'type': 'eq', 'fun': lambda x: np.sum(x*ret_n)-
(ret_n_min+(ret_n_max-ret_n_min)/n_port*i)}
    cons = ([cons_w, cons_ret_n])

    sol = spo.minimize(portfolio_variance, w_guess, args=(vcv_n,), method =
'SLSQP', bounds = bds, constraints = cons, \
                       options = opts)
    opt_w = sol.x

    #Save the optimal portfolio weights
```

```
    eff_port_w_n[i-1,:] = opt_w
    eff_port_ret_n[i-1] = np.sum(opt_w*ret_n)

    #Save the optimal portfolio returns and variances
    opt_w = np.expand_dims(opt_w, axis=1)
    opt_wt = opt_w.T

    eff_port_var_n[i-1] = np.matmul(np.matmul(opt_wt,vcv_n),opt_w)[0][0]

print(np.sqrt(eff_port_var_n))
print(eff_port_ret_n)

plot = plt.scatter(np.sqrt(eff_port_var_n), eff_port_ret_n)
plt.xlabel("Standard Deviation")
plt.ylabel("Expected Return")
plt.show()

#MICHAUD - Resampling
print(np.sqrt(eff_port_var_n))
print(np.argmin((np.sqrt(eff_port_var_c))))
print(np.argmin((np.sqrt(eff_port_var_p))))
print(np.argmin((np.sqrt(eff_port_var_n))))
print(np.sqrt(eff_port_var_n[3:]))

#filter inefficient portfolios
idx_c = np.argmin(eff_port_var_c)
eff_port_sd_c = np.sqrt(eff_port_var_c[idx_c:])
eff_port_ret_c = eff_port_ret_c[idx_c:]
eff_port_w_c = eff_port_w_c[idx_c:,:]
print(eff_port_sd_c)

idx_p = np.argmin(eff_port_var_p)
eff_port_sd_p = np.sqrt(eff_port_var_p[idx_p:])
eff_port_ret_p = eff_port_ret_p[idx_p:]
eff_port_w_p = eff_port_w_p[idx_p:,:]
print(eff_port_sd_p)

idx_n = np.argmin(eff_port_var_n)
eff_port_sd_n = np.sqrt(eff_port_var_n[idx_n:])
eff_port_ret_n = eff_port_ret_n[idx_n:]
eff_port_w_n = eff_port_w_n[idx_n:,:]
print(eff_port_sd_n)

#Level of risk aversion considering standard deviation
buckets = np.arange(0.028, 0.036, 0.001)
print(buckets)
port_w_c_final = np.zeros((n,7))
port_w_p_final = np.zeros((n,7))
port_w_n_final = np.zeros((n,7))
port_w_res_final = np.zeros((n,7))

for i in range(0,np.shape(buckets)[0]-1):

    print(i)

    int_a = buckets[i]
    int_b = buckets[i+1]

    #print(eff_port_sd_c[(eff_port_sd_c >= int_a) & (eff_port_sd_c <
int_b)])
```

```
    #print(eff_port_sd_p[(eff_port_sd_p >= int_a) & (eff_port_sd_p <
int_b)])
    #print(eff_port_sd_n[(eff_port_sd_n >= int_a) & (eff_port_sd_n <
int_b)])

        port_w_c = eff_port_w_c[(eff_port_sd_c >= int_a) & (eff_port_sd_c <
int_b),:]
    port_w_c = np.mean(port_w_c,0)
    port_w_c_final[:,i] = port_w_c

    port_w_p = eff_port_w_p[(eff_port_sd_p >= int_a) & (eff_port_sd_p <
int_b),:]
    port_w_p = np.mean(port_w_p,0)
    port_w_p_final[:,i] = port_w_p

    port_w_n = eff_port_w_n[(eff_port_sd_n >= int_a) & (eff_port_sd_n <
int_b),:]
    port_w_n = np.mean(port_w_n,0)
    port_w_n_final[:,i] = port_w_n

    port_w_res = 0.4*port_w_c + 0.3*port_w_p + 0.3*port_w_n
    port_w_res_final[:,i] = port_w_res

#print(port_w_res_final)
#print(port_w_c_final)

#Exporting to Excel
with pd.ExcelWriter('Dados - NASDAQ100.xlsx') as writer:
        ret.to_excel(writer, sheet_name='ret')
    pd.DataFrame(ret_c).to_excel(writer, sheet_name='ret_c')
    pd.DataFrame(ret_p).to_excel(writer, sheet_name='ret_p')
    pd.DataFrame(ret_n).to_excel(writer, sheet_name='ret_n')
        pd.DataFrame(eff_port_var_c).to_excel(writer,
sheet_name='eff_port_var_c')
    pd.DataFrame(eff_port_var_p).to_excel(writer,
sheet_name='eff_port_var_p')
    pd.DataFrame(eff_port_var_n).to_excel(writer,
sheet_name='eff_port_var_n')
    pd.DataFrame(eff_port_ret_c).to_excel(writer,
sheet_name='eff_port_ret_c')
    pd.DataFrame(eff_port_ret_p).to_excel(writer,
sheet_name='eff_port_ret_p')
    pd.DataFrame(eff_port_ret_n).to_excel(writer,
sheet_name='eff_port_ret_n')
    pd.DataFrame(eff_port_w_c).to_excel(writer, sheet_name='eff_port_w_c')
    pd.DataFrame(eff_port_w_p).to_excel(writer, sheet_name='eff_port_w_p')
    pd.DataFrame(eff_port_w_n).to_excel(writer, sheet_name='eff_port_w_n')
    pd.DataFrame(port_w_c_final).to_excel(writer, sheet_name='port_w_c_fi-
nal')
    pd.DataFrame(port_w_p_final).to_excel(writer, sheet_name='port_w_p_fi-
nal')
    pd.DataFrame(port_w_n_final).to_excel(writer, sheet_name='port_w_n_fi-
nal')
    pd.DataFrame(port_w_res_final).to_excel(writer,
sheet_name='port_w_res_final')
```

A.2 Portfolio Resampling Analysis

```
#Libraries
import time
import calendar
import datetime as dt
from datetime import timedelta
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import scipy.optimize as spo
import scipy.stats
from scipy.stats import norm
import math
from pandas.tseries.holiday import USFederalHolidayCalendar
from pandas.tseries.offsets import CustomBusinessDay

df_NASDAQ100 = pd.read_excel('NASDAQ100.xlsx')
#df_DAX30 = pd.read_excel('DAX30.xlsx')
#df_SP500 = pd.read_excel('S&P500_superclean.xlsx')
#df_EUROSTOXX50 = df_SP500 = pd.read_excel('EUROSTOXX50.xlsx')

port_w_c = pd.read_excel('Dados - NASDAQ100.xlsx', sheet_name='port_w_c_fi-
nal', index_col = 0)
port_w_p = pd.read_excel('Dados - NASDAQ100.xlsx', sheet_name='port_w_p_fi-
nal', index_col = 0)
port_w_n = pd.read_excel('Dados - NASDAQ100.xlsx', sheet_name='port_w_n_fi-
nal', index_col = 0)
port_w_res = pd.read_excel('Dados - NASDAQ100.xlsx',
sheet_name='port_w_res_final', index_col = 0)
port_w_c = port_w_c.values
port_w_p = port_w_p.values
port_w_n = port_w_n.values
port_w_res = port_w_res.values

us_bd = CustomBusinessDay(calendar=USFederalHolidayCalendar())

df1 = df_NASDAQ100.set_index(['Dates'])
#df1 = df_SP500.set_index(['Dates'])
#df1 = df_SP500.set_index(df_SP500.columns[0])
#df1 = df_EUROSTOXX50.set_index(['Dates'])
#df1 = df_DAX30.set_index(['Dates'])

annual = pd.date_range(start='31/12/2019',end='31/12/2020',freq=us_bd)
trim_1 = pd.date_range(start='31/12/2019',end='31/03/2020',freq=us_bd)
jan = pd.date_range(start='31/12/2019',end='31/1/2020',freq=us_bd)
feb = pd.date_range(start='31/1/2020',end='28/2/2020',freq=us_bd)
mar = pd.date_range(start='28/2/2020',end='31/3/2020',freq=us_bd)
apr = pd.date_range(start='31/3/2020',end='30/4/2020',freq=us_bd)

df_annual = df1.loc[annual]
df_trim_1 = df1.loc[trim_1]
#df_jan = df1.loc[jan]
df_feb = df1.loc[feb]
df_mar = df1.loc[mar]
df_apr = df1.loc[apr]

ret_annual = df_annual.pct_change()[1:].values
ret_trim_1 = df_trim_1.pct_change()[1:].values
#ret_jan = df_jan.pct_change()[1:].values
ret_feb = df_feb.pct_change()[1:].values
```

```
ret_mar = df_mar.pct_change()[1:].values
ret_apr = df_apr.pct_change()[1:].values

#Expected returns - CHECK MONTH OF ANALYSIS BELOW
ret_dataframe = ret_apr
df_dataframe = df_apr
ret_port_c = np.matmul(ret_dataframe, port_w_c)
ret_port_p = np.matmul(ret_dataframe, port_w_p)
ret_port_n = np.matmul(ret_dataframe, port_w_n)
ret_port_c_annual = np.matmul(ret_annual, port_w_c)
ret_port_p_annual = np.matmul(ret_annual, port_w_p)
ret_port_n_annual = np.matmul(ret_annual, port_w_n)
ret_port_res = np.matmul(ret_dataframe, port_w_res)
ret_port_res_annual = np.matmul(ret_annual, port_w_res)

#Volatility
port_sd_c = np.std(ret_port_c,axis=0)
port_sd_p = np.std(ret_port_p,axis=0)
port_sd_n = np.std(ret_port_n,axis=0)
port_sd_res = np.std(ret_port_res,axis=0)

#Average Daily Return
avg_returns_c = np.mean(ret_port_c, axis=0)
avg_returns_p = np.mean(ret_port_p, axis=0)
avg_returns_n = np.mean(ret_port_n, axis=0)
avg_returns_res = np.mean(ret_port_res, axis=0)

#max
max_ret_c = np.amax(ret_port_c, axis = 0)
max_ret_p = np.amax(ret_port_p, axis = 0)
max_ret_n = np.amax(ret_port_n, axis = 0)
max_ret_res = np.amax(ret_port_res, axis = 0)

#min
min_ret_c = np.amin(ret_port_c, axis = 0)
min_ret_p = np.amin(ret_port_p, axis = 0)
min_ret_n = np.amin(ret_port_n, axis = 0)
min_ret_res = np.amin(ret_port_res, axis = 0)

#Maximum Drawdown (MDD)
valor_port_0 = 100000
valor_port_c_0 = port_w_c*valor_port_0
valor_port_p_0 = port_w_p*valor_port_0
valor_port_n_0 = port_w_n*valor_port_0
valor_port_res_0 = port_w_res*valor_port_0

unidades_port_0_c = valor_port_c_0/np.reshape(df_dataframe.values[0], (-
1,1))
unidades_port_0_p = valor_port_p_0/np.reshape(df_dataframe.values[0], (-
1,1))
unidades_port_0_n = valor_port_n_0/np.reshape(df_dataframe.values[0], (-
1,1))
unidades_port_0_res = valor_port_res_0/np.reshape(df_dataframe.values[0],
(-1,1))

valor_port_c = np.matmul(df_dataframe.values[1:], unidades_port_0_c)
valor_port_p = np.matmul(df_dataframe.values[1:], unidades_port_0_p)
valor_port_n = np.matmul(df_dataframe.values[1:], unidades_port_0_n)
valor_port_res = np.matmul(df_dataframe.values[1:], unida-des_port_0_res)
```

```
mdd_c = (np.min(valor_port_c, axis = 0) - np.max(valor_port_c, axis =
0))/np.max(valor_port_c, axis = 0)
mdd_p = (np.min(valor_port_p, axis = 0) - np.max(valor_port_p, axis =
0))/np.max(valor_port_p, axis = 0)
mdd_n = (np.min(valor_port_n, axis = 0) - np.max(valor_port_n, axis =
0))/np.max(valor_port_n, axis = 0)
mdd_res = (np.min(valor_port_res, axis = 0) - np.max(valor_port_res, axis =
0))/np.max(valor_port_res, axis = 0)
#Sharpe ratio
rf = 0.0
sharpe_ratio_c = (avg_returns_c-rf)/port_sd_c
sharpe_ratio_p = (avg_returns_p-rf)/port_sd_p
sharpe_ratio_n = (avg_returns_n-rf)/port_sd_n
annual_sharpe_ratio_c = (252**0.5) * sharpe_ratio_c
annual_sharpe_ratio_p = (252**0.5) * sharpe_ratio_p
annual_sharpe_ratio_n = (252**0.5) * sharpe_ratio_n
sharpe_ratio_res = (avg_returns_res-rf)/port_sd_res
annual_sharpe_ratio_res = (252**0.5) * sharpe_ratio_res
#Sortino ratio
rf = 0.0
sd_neg_c = ret_port_c[ret_port_c < 0].std() #desvio padrão do down-side
sd_neg_p = ret_port_p[ret_port_p < 0].std()
sd_neg_n = ret_port_n[ret_port_n < 0].std()

sortino_ratio_c = (avg_returns_c-rf)/sd_neg_c
sortino_ratio_p = (avg_returns_p-rf)/sd_neg_c
sortino_ratio_n = (avg_returns_n-rf)/sd_neg_c
annual_sortino_ratio_c = (252**0.5) * sortino_ratio_c
annual_sortino_ratio_p = (252**0.5) * sortino_ratio_p
annual_sortino_ratio_n = (252**0.5) * sortino_ratio_n
sd_neg_res = ret_port_res[ret_port_res < 0].std()
sortino_ratio_res = (avg_returns_res-rf)/sd_neg_res
annual_sortino_ratio_res = (252**0.5) * sortino_ratio_res

#Herfindahl-Hirschman Index (HHI)
hhi_c = np.sum(port_w_c**2, axis=0)
hhi_p = np.sum(port_w_p**2, axis=0)
hhi_n = np.sum(port_w_n**2, axis=0)
hhi_res = np.sum(port_w_res**2, axis=0)
hhindex_c = 1/hhi_c
hhindex_p = 1/hhi_p
hhindex_n = 1/hhi_n
hhindex_res = 1/hhi_res

#Historic Value-at-Risk
hist_var_95_c = np.percentile(ret_port_c,5,axis=0)
hist_var_95_p = np.percentile(ret_port_p,5,axis=0)
hist_var_95_n = np.percentile(ret_port_n,5,axis=0)
hist_var_95_res = np.percentile(ret_port_res,5,axis=0)
hist_var_99_c = np.percentile(ret_port_c,1,axis=0)
hist_var_99_res = np.percentile(ret_port_res,1,axis=0)

#Exporting to Excel
with pd.ExcelWriter('Resultadoscpn2 - NASDAQ100_apr.xlsx') as writer:
    pd.DataFrame(ret_dataframe).to_excel(writer, sheet_name='ret')
    pd.DataFrame(df_dataframe).to_excel(writer, sheet_name='df')

    #Descriptive Statistics (Risk and Return Analysis)
    pd.DataFrame(ret_port_c).to_excel(writer, sheet_name='ret_port_c')
    pd.DataFrame(ret_port_p).to_excel(writer, sheet_name='ret_port_p')
    pd.DataFrame(ret_port_n).to_excel(writer, sheet_name='ret_port_n')
```

```
    pd.DataFrame(ret_port_res).to_excel(writer, sheet_name='ret_port_res')
    pd.DataFrame(port_sd_c).to_excel(writer, sheet_name='port_sd_c')
    pd.DataFrame(port_sd_p).to_excel(writer, sheet_name='port_sd_p')
    pd.DataFrame(port_sd_n).to_excel(writer, sheet_name='port_sd_n')
    pd.DataFrame(port_sd_res).to_excel(writer, sheet_name='port_sd_res')
    pd.DataFrame(avg_returns_c).to_excel(writer, sheet_name='avg_re-
turns_c')
    pd.DataFrame(avg_returns_p).to_excel(writer, sheet_name='avg_re-
turns_p')
    pd.DataFrame(avg_returns_n).to_excel(writer, sheet_name='avg_re-
turns_n')
    pd.DataFrame(avg_returns_res).to_excel(writer, sheet_name='avg_re-
turns_res')
    pd.DataFrame(max_ret_c).to_excel(writer, sheet_name='max_ret_c')
    pd.DataFrame(max_ret_p).to_excel(writer, sheet_name='max_ret_p')
    pd.DataFrame(max_ret_n).to_excel(writer, sheet_name='max_ret_n')
    pd.DataFrame(max_ret_res).to_excel(writer, sheet_name='max_ret_res')
    pd.DataFrame(min_ret_c).to_excel(writer, sheet_name='min_ret_c')
    pd.DataFrame(min_ret_p).to_excel(writer, sheet_name='min_ret_p')
    pd.DataFrame(min_ret_n).to_excel(writer, sheet_name='min_ret_n')
    pd.DataFrame(min_ret_res).to_excel(writer, sheet_name='min_ret_res')
    pd.DataFrame(mdd_c).to_excel(writer, sheet_name='mdd_c')
    pd.DataFrame(mdd_p).to_excel(writer, sheet_name='mdd_p')
    pd.DataFrame(mdd_n).to_excel(writer, sheet_name='mdd_n')
    pd.DataFrame(mdd_res).to_excel(writer, sheet_name='mdd_res')

    #Additional Statistics
    pd.DataFrame(sharpe_ratio_c).to_excel(writer, sheet_name='sharpe_ra-
tio_c')
    pd.DataFrame(sharpe_ratio_p).to_excel(writer, sheet_name='sharpe_ra-
tio_p')
    pd.DataFrame(sharpe_ratio_n).to_excel(writer, sheet_name='sharpe_ra-
tio_n')
    pd.DataFrame(sharpe_ratio_res).to_excel(writer, sheet_name='sharpe_ra-
tio_res')
    pd.DataFrame(sortino_ratio_c).to_excel(writer, sheet_name='sortino_ra-
tio_c')
    pd.DataFrame(sortino_ratio_p).to_excel(writer, sheet_name='sortino_ra-
tio_p')
    pd.DataFrame(sortino_ratio_n).to_excel(writer, sheet_name='sortino_ra-
tio_n')
    pd.DataFrame(sortino_ratio_res).to_excel(writer,
sheet_name='sortino_ratio_res')
    pd.DataFrame(hhi_c).to_excel(writer, sheet_name='hhi_c')
    pd.DataFrame(hhi_p).to_excel(writer, sheet_name='hhi_p')
    pd.DataFrame(hhi_n).to_excel(writer, sheet_name='hhi_n')
    pd.DataFrame(hhi_res).to_excel(writer, sheet_name='hhi_res')
    pd.DataFrame(hhindex_c).to_excel(writer, sheet_name='hhindex_c')
    pd.DataFrame(hhindex_p).to_excel(writer, sheet_name='hhindex_p')
    pd.DataFrame(hhindex_n).to_excel(writer, sheet_name='hhindex_n')
    pd.DataFrame(hhindex_res).to_excel(writer, sheet_name='hhindex_res')
    pd.DataFrame(hist_var_95_c).to_excel(writer,sheet_name='hist_var_95_c')
    pd.DataFrame(hist_var_95_p).to_excel(writer,sheet_name='hist_var_95_p')
    pd.DataFrame(hist_var_95_n).to_excel(writer,sheet_name='hist_var_95_n')
    pd.DataFrame(hist_var_95_res).to_excel(writer,
sheet_name='hist_var_95_res')
```

B

DATA RESULTS

The present Appendix B exhibits the performance evaluation data concerning the stock markets indexes target of the study empirical analysis, S&P 500 and EURO STOXX 50.

B.1 Performance evaluation of S&P 500 stock market index

Month	Portfolio	Return				Standard Deviation				Average Return				Maximum Return				Minimum Return			
		c	p	n	res	c	p	n	res	c	p	n	res	c	p	n	res	c	p	n	res
February	0	-3,96%	-3,86%	-4,10%	-3,97%	1,30%	1,29%	1,30%	1,30%	-0,20%	-0,20%	-0,21%	-0,20%	1,53%	1,50%	1,38%	1,48%	-4,47%	-4,49%	-4,38%	-4,45%
	1	-3,60%	-3,76%	-3,79%	-3,70%	1,33%	1,29%	1,32%	1,31%	-0,18%	-0,19%	-0,19%	-0,19%	1,68%	1,42%	1,22%	1,46%	-4,43%	-4,52%	-4,29%	-4,41%
	2	-3,40%	-3,73%	-3,67%	-3,58%	1,34%	1,31%	1,32%	1,31%	-0,17%	-0,19%	-0,19%	-0,18%	1,63%	1,37%	1,00%	1,36%	-4,39%	-4,55%	-4,30%	-4,41%
	3	-3,32%	-3,83%	-3,67%	-3,57%	1,36%	1,31%	1,34%	1,33%	-0,17%	-0,19%	-0,19%	-0,18%	1,69%	1,32%	0,85%	1,30%	-4,42%	-4,58%	-4,29%	-4,43%
March	0	-1,55%	-1,64%	-1,55%	-1,58%	5,00%	5,01%	5,00%	5,00%	0,05%	0,05%	0,05%	0,05%	9,42%	9,41%	9,41%	9,41%	-8,68%	-8,70%	-8,73%	-8,70%
	1	-0,73%	-1,86%	-1,08%	-1,17%	5,04%	5,04%	5,06%	5,04%	0,09%	0,04%	0,08%	0,07%	9,34%	9,28%	9,32%	9,31%	-8,63%	-8,69%	-8,79%	-8,70%
	2	-0,36%	-2,17%	-0,37%	-0,90%	5,06%	5,05%	5,13%	5,07%	0,11%	0,03%	0,11%	0,09%	9,37%	9,05%	9,61%	9,35%	-8,64%	-8,73%	-8,82%	-8,69%
	3	0,15%	-2,08%	-0,01%	-0,56%	5,07%	5,03%	5,16%	5,07%	0,13%	0,03%	0,13%	0,10%	9,44%	9,03%	9,71%	9,40%	-8,49%	-8,74%	-8,83%	-8,59%
April	0	5,17%	5,10%	5,72%	5,31%	2,10%	2,11%	2,11%	2,11%	0,25%	0,25%	0,28%	0,26%	5,17%	5,24%	5,10%	5,17%	-2,61%	-2,68%	-2,76%	-2,65%
	1	6,13%	5,35%	6,12%	5,90%	2,08%	2,10%	2,10%	2,09%	0,29%	0,26%	0,29%	0,28%	5,09%	5,21%	4,99%	5,10%	-2,58%	-2,73%	-2,83%	-2,66%
	2	6,79%	5,85%	6,25%	6,35%	2,06%	2,09%	2,10%	2,07%	0,32%	0,28%	0,30%	0,30%	5,04%	5,22%	4,94%	5,06%	-2,56%	-2,81%	-2,88%	-2,65%
	3	7,13%	6,30%	6,38%	6,66%	2,05%	2,10%	2,09%	2,07%	0,33%	0,30%	0,30%	0,31%	4,99%	5,25%	4,88%	5,03%	-2,55%	-2,91%	-2,92%	-2,67%

Month	Portfolio	Maximum Drawdown				Sharpe Ratio				Sortino Ratio				HHI				Historic VaR (95% confidence)			
		c	p	n	res	c	p	n	res	c	p	n	res	c	p	n	res	c	p	n	res
February	0	-9,66%	-9,61%	-9,65%	-9,64%	-14,57%	-14,18%	-15,42%	-14,71%	-13,50%	-13,14%	-14,05%	-13,55%	27,69	30,12	31,10	29,67	-2,90%	-2,86%	-2,86%	-2,88%
	1	-9,73%	-9,73%	-9,61%	-9,70%	-13,06%	-13,73%	-14,27%	-13,64%	-12,19%	-12,78%	-12,91%	-12,58%	28,71	34,15	30,99	33,26	-2,83%	-2,84%	-2,79%	-2,82%
	2	-9,80%	-9,91%	-9,55%	-9,72%	-12,33%	-13,49%	-13,87%	-13,17%	-11,47%	-12,67%	-12,48%	-12,13%	31,58	33,43	29,48	36,22	-2,78%	-2,86%	-2,65%	-2,76%
	3	-10,08%	-10,06%	-9,48%	-9,85%	-11,82%	-13,76%	-13,98%	-13,10%	-11,15%	-13,00%	-12,49%	-12,10%	30,04	31,83	27,38	36,38	-2,73%	-2,86%	-2,56%	-2,72%
March	0	-21,94%	-21,93%	-21,75%	-21,88%	1,07%	1,00%	1,07%	1,05%	2,10%	1,96%	2,09%	2,04%	29,31	27,35	26,72	28,15	-8,47%	-8,46%	-8,49%	-8,47%
	1	-21,78%	-22,26%	-21,85%	-21,95%	1,85%	0,82%	1,54%	1,45%	3,65%	1,62%	3,05%	2,84%	27,69	28,33	28,90	30,22	-8,41%	-8,49%	-8,46%	-8,45%
	2	-21,58%	-22,53%	-21,48%	-21,84%	2,20%	0,54%	2,22%	1,71%	4,35%	1,06%	4,46%	3,37%	28,11	27,57	26,40	31,42	-8,36%	-8,45%	-8,56%	-8,47%
	3	-21,41%	-22,34%	-21,40%	-21,68%	2,65%	0,61%	2,56%	2,02%	5,27%	1,20%	5,18%	3,98%	32,98	29,31	27,91	36,69	-8,23%	-8,32%	-8,50%	-8,41%
April	0	-11,85%	-11,77%	-12,26%	-11,95%	11,94%	11,79%	13,06%	12,23%	39,09%	38,63%	42,83%	37,65%	29,60	28,17	24,86	28,30	-2,56%	-2,57%	-2,50%	-2,57%
	1	-12,49%	-11,90%	-12,54%	-12,33%	14,07%	12,35%	13,90%	13,52%	45,48%	40,34%	45,50%	41,29%	27,37	29,25	25,27	28,80	-2,53%	-2,57%	-2,40%	-2,54%
	2	-12,78%	-12,16%	-12,60%	-12,54%	15,56%	13,42%	14,20%	14,53%	49,78%	43,60%	46,34%	44,06%	27,51	29,25	27,96	30,97	-2,46%	-2,58%	-2,34%	-2,54%
	3	-12,94%	-12,47%	-12,65%	-12,72%	16,32%	14,31%	14,50%	15,22%	52,05%	46,67%	47,16%	46,01%	27,18	28,38	25,99	31,14	-2,44%	-2,63%	-2,26%	-2,54%

B.2 Performance evaluation of EURO STOXX 50 stock market index

Month	Portfolio	Return				Standard Deviation				Average Return				Maximum Return				Minimum Return			
		c	p	n	res	c	p	n	res	c	p	n	res	c	p	n	res	c	p	n	res
February	0	-6,07%	-6,08%	-5,78%	-5,98%	1,42%	1,42%	1,42%	1,42%	-0,32%	-0,32%	-0,30%	-0,31%	1,46%	1,45%	1,49%	1,47%	-3,58%	-3,56%	-3,51%	-3,55%
	1	-5,23%	-5,93%	-4,68%	-5,28%	1,46%	1,45%	1,45%	1,45%	-0,27%	-0,31%	-0,24%	-0,27%	1,66%	1,53%	1,64%	1,61%	-3,61%	-3,57%	-3,48%	-3,56%
	2	-4,70%	-5,60%	-4,74%	-4,98%	1,51%	1,50%	1,45%	1,49%	-0,24%	-0,29%	-0,24%	-0,26%	1,71%	1,66%	1,63%	1,67%	-3,68%	-3,66%	-3,51%	-3,62%
	3	-4,33%	-5,66%	-3,75%	-4,56%	1,54%	1,52%	1,48%	1,51%	-0,22%	-0,29%	-0,19%	-0,23%	1,79%	1,76%	1,67%	1,74%	-3,74%	-3,70%	-3,47%	-3,65%
March	0	-8,27%	-8,15%	-6,69%	-7,76%	3,83%	3,82%	3,78%	3,81%	-0,32%	-0,31%	-0,24%	-0,29%	6,79%	6,38%	6,69%	6,64%	-10,87%	-10,85%	-10,51%	-10,75%
	1	-8,05%	-8,61%	-6,15%	-7,65%	3,86%	3,86%	3,67%	3,79%	-0,30%	-0,33%	-0,22%	-0,29%	6,92%	6,62%	6,65%	6,75%	-10,88%	-10,95%	-10,13%	-10,68%
	2	-8,05%	-8,91%	-5,59%	-7,57%	3,93%	3,88%	3,72%	3,84%	-0,30%	-0,35%	-0,19%	-0,28%	7,27%	7,05%	7,00%	7,12%	-11,03%	-11,01%	-10,05%	-10,73%
	3	-8,20%	-9,20%	-5,03%	-7,55%	3,93%	3,96%	3,70%	3,86%	-0,31%	-0,36%	-0,16%	-0,28%	7,44%	7,19%	7,22%	7,30%	-10,99%	-11,18%	-9,74%	-10,67%
April	0	7,98%	7,30%	6,62%	7,37%	1,42%	1,41%	1,37%	1,40%	0,36%	0,33%	0,30%	0,33%	3,51%	3,49%	3,02%	3,36%	-2,87%	-2,90%	-2,93%	-2,90%
	1	8,31%	8,19%	7,63%	8,07%	1,39%	1,43%	1,30%	1,37%	0,37%	0,37%	0,34%	0,36%	3,35%	3,54%	2,72%	3,22%	-2,83%	-2,84%	-2,76%	-2,81%
	2	8,76%	8,33%	8,30%	8,49%	1,47%	1,48%	1,43%	1,46%	0,39%	0,38%	0,37%	0,38%	3,33%	3,67%	3,16%	3,38%	-2,88%	-2,95%	-2,95%	-2,92%
	3	9,11%	8,81%	8,35%	8,80%	1,43%	1,54%	1,30%	1,42%	0,41%	0,40%	0,37%	0,39%	3,25%	3,85%	2,60%	3,23%	-2,82%	-2,87%	-2,76%	-2,82%

Month	Portfolio	Maximum Drawdown				Sharpe Ratio				Sortino Ratio				HHI				Historic VaR (95% confidence)			
		c	p	n	res	c	p	n	res	c	p	n	res	c	p	n	res	c	p	n	res
February	0	-11,14%	-11,15%	-11,08%	-11,13%	-22,51%	-22,47%	-21,25%	-22,12%	-25,16%	-25,20%	-23,86%	-24,90%	14,01	14,11	14,17	14,13	-3,24%	-3,26%	-3,22%	-3,24%
	1	-11,15%	-11,33%	-11,00%	-11,16%	-18,54%	-21,43%	-16,67%	-18,86%	-21,43%	-24,52%	-19,06%	-21,75%	13,12	14,53	13,46	14,02	-3,32%	-3,39%	-3,05%	-3,26%
	2	-11,24%	-11,43%	-11,05%	-11,24%	-15,96%	-19,43%	-16,89%	-17,30%	-19,04%	-23,00%	-19,29%	-20,40%	11,84	12,29	13,96	13,28	-3,35%	-3,51%	-3,08%	-3,32%
	3	-11,26%	-11,60%	-10,77%	-11,21%	-14,37%	-19,33%	-12,89%	-15,47%	-17,44%	-23,22%	-15,00%	-18,53%	10,63	11,20	11,81	12,23	-3,33%	-3,59%	-3,17%	-3,28%
March	0	-24,44%	-24,39%	-23,29%	-24,08%	-8,25%	-8,12%	-6,39%	-7,66%	-11,14%	-10,93%	-8,49%	-10,41%	14,81	15,01	11,92	14,32	-6,16%	-6,38%	-5,80%	-6,12%
	1	-24,06%	-24,60%	-22,30%	-23,69%	-7,89%	-8,61%	-5,99%	-7,57%	-10,72%	-11,69%	-7,73%	-10,24%	13,49	15,67	11,00	13,91	-5,81%	-6,10%	-5,48%	-5,80%
	2	-24,30%	-24,73%	-22,31%	-23,83%	-7,69%	-8,91%	-5,11%	-7,33%	-10,63%	-12,18%	-6,70%	-10,04%	13,00	14,07	9,26	12,94	-5,82%	-5,82%	-5,51%	-5,72%
	3	-24,15%	-24,91%	-21,80%	-23,67%	-7,87%	-9,04%	-4,44%	-7,27%	-10,88%	-12,59%	-5,79%	-9,99%	12,00	13,02	8,67	12,36	-5,71%	-5,75%	-5,57%	-5,68%
April	0	-9,51%	-8,70%	-8,22%	-8,86%	25,40%	23,54%	21,95%	23,86%	39,83%	36,64%	33,37%	37,14%	12,96	13,01	12,26	12,92	-1,84%	-1,86%	-1,94%	-1,88%
	1	-9,82%	-9,72%	-9,24%	-9,62%	26,84%	25,82%	26,40%	26,48%	41,36%	40,83%	38,02%	40,42%	13,92	14,70	12,33	13,96	-1,77%	-1,83%	-1,66%	-1,75%
	2	-10,56%	-9,96%	-9,91%	-10,18%	26,76%	25,29%	26,03%	26,13%	43,55%	41,57%	41,35%	42,53%	15,66	15,56	15,49	15,95	-1,93%	-1,93%	-1,84%	-1,87%
	3	-10,79%	-10,87%	-9,98%	-10,57%	28,50%	25,72%	28,73%	27,83%	45,11%	43,90%	41,42%	43,88%	14,92	14,02	11,97	14,63	-1,83%	-1,98%	-1,56%	-1,79%

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