Testing the rationality of expectations using aggregate data*

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In this paper it is argued that tests of rationality of expections based on aggregate data, specifically when the aggregate expectations series come from business survey data, are not conclusive.
In fact it is shown that even when individual agents have rational expectations, aggregate expectations series and aggregate prediction errors should not pass the traditional rationality tests.
This can account for the observed persistent correlation in deviations of real GNP from trend in a Lucas-supply function without the need of a lagged output variable. But it also suggests that the macroeconomic implications of rational expectations will be weakened.

1. Introduction

Several studies on the formation of expectations (concerning mainly inflation) used survey data to construct the corresponding aggregate expectations series (see Carlson and Parkin (1975), Knobl (1974) and Batchelor (1982)).

In this paper it is argued that tests of rationality of expectations based on this type of series are not conclusive.

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In fact it is shown that when individual agents expectations are rational, in the sense that each agent uses its available information in the best possible way, so that individual prediction errors are orthogonal to the agent's information set and expectations, and that individual prediction errors follow a white noise stochastic process, we dont get generally the same properties for the aggregate expectations and aggregate prediction errors series. This means that we can not reject the hypothesis that agent's expectations are rational on the basis of tests that use aggregate data.

This problem, that can be considered as an aggregation problem, simply states the fact that as long as agents have different information sets the average (aggregate) prediction and prediction error do not coincide with the prediction and prediction error of a representative agent i.e. of an agent that is not subject to local shocks in our terminology.

Although in this paper we have concentrated on the case where aggregate expectations data are obtained from business survey data, our point is more general. In fact what we want to stress is that aggregate expectation series and aggregate prediction errors should not verify the so called tests of rationality in order to conclude that agents have rational expectations. More specifically aggregate prediction error series will generally show autocorrelation even when individual prediction errors are uncorrelated and orthogonal to expectations. This fact can account for the observed persistent correlation in the unemployment rate and in deviations of real GNP from trend in a Lucas-supply function without the need of an ad.hoc. lagged output variable.[1] But more important this fact suggests that, as in general, aggregate expectations series will not pass the traditional rationality tests, the implications of rational expectations e.g. concerning the efficacity of policy will be weakened.

In the remainder of the paper we will first consider for the sake of clarity a rather simple model concerning the formation of expectations for output growth by firms. The implications for aggregation are discussed mainly in the light of the type of aggregation considered when using business survey data to

1 See also Sargent (1979) p. 330-331 for some comments on this point.

generate aggregate expectations series. Then a more general and plausible model, that generates the first model as a particular case, is considered. Several variants of this model depending on the information set available are discussed. Finally an empirical application using portuguese business survey data is presented.

2. The simple model

Firms are assumed to have rational expectations i.e. they are assumed to from the forecasts of their future production growth by taking conditional[2] mathematical expectations with respect to the stochastic process that actually governs output growth.

Let us assume that each firm's growth rate of production deviates from the average growth rate of production by an error term that is specific to the firm i.e.

$$\Delta \ln q_{tj} = \Delta \ln q_t + e_{tj} \qquad (1)$$

where a subscript j designates a variable at the firm's level. Moreover we also have that

$$(\Delta \ln q_t - \Delta \ln \bar{q}) = a\,(\Delta \ln q_{t-1} - \Delta \ln \bar{q}) + V_t \qquad (2)$$

where $\Delta \ln \bar{q}$ designates the normal (long-run) growth rate of production.

We will also assume that

$e_{tj} \sim N(0, \sigma_e^2)$

$V_t \sim N(0, \theta)$

2 The expectations are conditional on all available information at the time when the prediction is made.

and that e_{tj} and V_t are independent.

To complete the model we have to specify the information set of firm j at the end of period (t-1), [I(t-1)j]. It includes $\Delta \ln q(t-1)j$, that the firm observes by the end of period (t-1) and all previous values of $\Delta \ln q_{tj}$ i.e. dated (t-2) and earlier, together with $\Delta \ln \bar{q}$, [I(t-2)j]. This way of specifying I(t-1)j implies that a firm never observes any average quantity. Later on, this hypothesis will be relaxed and we will see that the specification of I(t-1)j has some important consequences.

The problem of making an estimate of $\Delta \ln q_{tj}$ given the information set, I(t-1)j, can be solved using the Kalman filtering technique. Using this method and noting from (1)

$$E\,[\Delta \ln q_{tj} / I_{(t-1)j}] = E\,[\Delta \ln q_t / I_{(t-1)j}] \qquad (3)$$

we have that

$$E\,[\Delta \ln q_{tj} / I_{(t-1)j}] = E\,[\Delta \ln q_{tj} / I_{(t-2)j}] \qquad (4)$$

$$+ \psi\,[\Delta \ln q_{(t-1)j} - E\,\Delta \ln q_{(t-1)j} / I_{(t-2)j}]$$

where $\psi = \dfrac{a\,\phi}{\phi + \sigma_e^2}$

and ϕ is the steady-state value of

$$\text{var}\,[\Delta \ln q_t - E\,\Delta \ln q_t / I_{(t-1)j}]$$

that in this case is given by:

$$\phi = a^2 \phi + \theta - \psi\, a\, \phi$$

Now as

$$E\,[\Delta \ln q_{tj} / I_{(t-2)j}] = (1-a)\,\Delta \ln \bar{q} + a\,E\,[\Delta \ln q_{(t-1)j} / I_{(t-2)j}] \qquad (5)$$

we can rewrite (4) as

$$E\,[\Delta \ln q_{tj} / I_{(t-1)j}] = (1-a)\,\Delta \ln \bar{q} + (a-\psi)\,E\,[\Delta \ln q_{(t-1)j} / I_{(t-2)j}] + \psi\,\Delta \ln q_{(t-1)j} \qquad (4')$$

This period prediction error of firm j

$$Z_{tj} = \Delta \ln q_{tj} - E\,[\Delta \ln q_{tj} / I_{(t-1)j}]$$

(that is given by expression (6)) is next period's innovation and therefore by the orthogonality principle firm's prediction errors are not correlated in time.[3]

$$Z_{tj} = \Delta \ln q_{tj} - E\,\Delta \ln q_{tj} / I_{(t-1)j}\,] = (a-\psi)\,[\Delta \ln q_{(t-1)} - E\,\Delta \ln q_{(t-1)} / I_{(t-2)j}] - \psi\, e_{j(t-1)} + V_t + e_{jt} \qquad (6)$$

Now let's consider the case of a firm that behaves like the average of the industry i.e. the so called representative firm that is not subject to local shocks, or equivalently that observes $\Delta \ln q_{(t-1)}$ and all other lagged $\Delta \ln q_t$ variables. For the representative firm we have simple that

$$E\,[\Delta \ln q_t / I_{(t-1)}] = (1-a)\,\Delta \ln \bar{q} + a\,\Delta \ln q_{(t-1)} \qquad (7)$$

so that the prediction error of the representative agent is

$$Z_t = \Delta \ln q_t - E\,[\Delta \ln q_t / I_{(t-1)}] = V_t \qquad (8)$$

3 Note however that $[\Delta \ln q_t - E\,\Delta \ln q_t / I_{(t-1)j}]$ shows serial correlation as firms never observe $\Delta \ln q_t$.

But what is the relation between the prediction or the prediction error of the average (representative) firm and the average of predictions or the average of predictiuon errors ($\bar{Z}_{tj}$) across firms?

It is easy to see that

$$\bar{Z}_{tj} = E_j Z_{tj} = \Delta \ln q_t - E_j E [\Delta \ln q_{tj} / I_{(t-1)j}]$$

can be written as:

$$\bar{Z}_{tj} = Z_t + \{ E [\Delta \ln q_t / I_{(t-1)}] - E_j E [\Delta \ln q_{tj} / I_{(t-1)j} \} \qquad (9)$$

i.e. the average prediction error deviates from the prediction error of the representative agent by what can be considered an aggregation error, as long as firms have different information sets.

Now noting that from expressions (4') and (7) we have that

$$E [\Delta \ln q_t / I_{(t-1)}] - E_j E [\Delta \ln q_{tj} / I_{(t-1)j}] =$$

$$(a-\psi) [\Delta \ln q_{(t-1)} - E_j E [\Delta \ln q_{(t-1)j} / I_{(t-2)j}]]$$

we can write (9) as

$$\bar{Z}_{tj} = Z_t + (a-\psi) \bar{Z}_{(t-1)j} \qquad (9')$$

From (9') it is easy to see that average (aggregate) prediction errors are correlated in time, although we have that the prediction error of a representative agent (Zt = Vt) shows no serial correlation. In fact as

$$\bar{Z}_{tj} = [1 + (a-\psi) L + (a-\psi)^2 L^2 + ...] V_t \qquad (10)$$

we have the following results:[4]

$$\text{Var}\ \bar{Z}_{tj} = \frac{\theta}{1-(a-\psi)^2} = \frac{\phi\,[1-a\,(a-\psi)]}{1-(a-\psi)^2} \qquad (11a)$$

$$\text{and } E(\bar{Z}_{tj}\,,\bar{Z}_{(t-1)j}) = \frac{(a-\psi)\theta}{1-(a-\psi)^2} = \frac{[1-a(a-\psi)]\psi\sigma^2 e}{1-(a-\psi)^2} \qquad (11b)$$

Moreover we also have that (see the Appendix) aggregate prediction errors and aggregate prediction are correlated despite the fact that individual prediction errors and prediction are orthogonal.

$$E\,[\,\bar{Z}_{tj}\,,E_j\,E\,[\Delta \ln\ q_{tj} / I_{(t-1)j}]\,] = \frac{(a-\psi)\psi\,\phi}{1-(a-\psi)^2} \qquad (12)$$

$$E\,[\,Z_{tj}\,,E\,\Delta \ln\ q_{tj} / I_{(t-1)j}] = 0$$

$$E\,[\,Z_t\,,E\,\Delta \ln\ q_t / I_{(t-1)}] = 0$$

So far, using a very simple model, we have shown that when agents are subject to local shocks and have different information sets, aggregate prediction series and prediction errors will not pass the rationality tests even when agents have rational expectations. However when formulating the model we used two simplifying assumptions that can be considered somewhat irrealistic. Firstly we assumed that local shocks were not serially correlated and secondly we assumed that individual agents never observed aggregate variables. We will now drop the first hypothesis and later on we will deal with the second one.

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4 Note that $E\,(\bar{Z}_{tj},\bar{Z}_{(t-1)j})$ is zero when $(a = \psi)$ i.e. when $\sigma^2_{\varepsilon} = 0$ so that all firms are identical to the representative firm. In this case we also get that $\text{Var}\ \bar{Z}_{tj} = \theta$.

3. The simple model with local correlated disturbances

The present case can be formulated as:

$$\Delta \ln q_{tj} = \Delta \ln q_t + e_{tj} \quad (1)$$

$$(\Delta \ln q_t - \Delta \ln \bar{q}) = a(\Delta \ln q_{t-1} - \Delta \ln \bar{q}) + V_t \quad (2)$$

$$e_{tj} = \rho\, e_{(t-1)j} + u_{tj} \quad (13)$$

where $u_{tj} \sim N(0, \sigma_u^2)$

$V_t \sim N(0, \theta)$

and where u_{tj} and V_t are independent.

The information set of firm j is specified as before.

Now from (1) and (13) we have that

$$E[\Delta \ln q_{tj} / I_{(t-1)j}] = E[\Delta \ln q_t / I_{(t-1)j}] + E[e_{tj} / I_{(t-1)j}] \quad . \quad (14)$$

For this case the problem of making an estimate of $\Delta \ln q_{tj}$ conditional on the information set, $I_{(t-1)j}$ can be again solved using the Kalman filtering technique but with an augmented state-vector.[5]

For this purpose let

$$m_t = \begin{bmatrix} \Delta \ln q_{tj} \\ e_{tj} \end{bmatrix}, \quad \bar{m} = \begin{bmatrix} (1-a)\, \Delta \ln \bar{q} \\ 0 \end{bmatrix}, \quad g_t = \begin{bmatrix} V_t \\ u_{tj} \end{bmatrix}$$

So that we can write

$$m_t = A\, m_{(t-1)} + \bar{m} + g_t \quad (15)$$

5 See Meditch (1969).

$$\Delta \ln q_{(t-1)j} = I_2 \, m_{(t-1)} \tag{16}$$

where $A = \begin{bmatrix} a & 0 \\ 0 & \rho \end{bmatrix}$ $\qquad I_2 = [\,1 \quad 1\,]$

and where $g_t \sim N(0,G)$ with $G = \begin{bmatrix} \theta & 0 \\ 0 & \sigma_u^2 \end{bmatrix}$

Note that using this notation (14) becomes

$$E\,[\Delta \ln q_{tj} / I_{(t-1)j}] = I_2 \, E\,[m_t / I_{(t-1)j}] \tag{14'}$$

Now

$$E\,[m_t / I_{(t-1)j}] = E\,[m_t / I_{(t-2)j}] + \psi_{(t-1)} \, [\Delta \ln q_{(t-1)j} \tag{14'}$$

$$- E\,[\ln q_{(t-1)j} / I_{(t-2)j}]\,] \tag{17}$$

Where $\psi_{(t-1)} = A \; \phi_{(t-1)} \, I'_2 \; (I_2 \, \phi_{(t-1)} \, I'_2)^{-1}$

and $\phi_{(t-1)} = E\,[m_{t-1} - E\,m_{(t-1)} / I_{(t-2)}] \; [m_t - E\,m_{(t-1)} / I_{(t-2)}]'$

is given by the following recursion

$$\phi_t = A\phi_{(t-1)} A' + G - (A\,\phi_{(t-1)} I'_2)\;(I_2\,\phi_{(t-1)} I'_2)^{-1}\,(A\;\phi_{(t-1)} I'_2)'$$

that we will approximate by

$$\phi_t = A\phi\, A' + G - (A\,\phi\, I'_2)\,(I_2\,\phi\, I'_2)^{-1}\,(A\phi\, I'_2)' \tag{18}$$

so that $\psi_{(t-1)} = \psi = A\,\phi\, I'_2\,(I_2\,\phi\, I'_2)^{-1}$ for all t (19)

and therefore (17) becomes

$$E\,[m_t / I_{(t-1)j}] = \bar{m} + A\,E\,[m_{(t-1)j} / I_{(t-2)j}] \tag{17'}$$

$$+ \psi\,[\Delta \ln q_{(t-1)j} - E\,[\Delta \ln q_{(t-1)j} / I_{(t-2)j}]\,]$$

where we have used the following relation:

$$E\,[m_t / I_{(t-2)j}] = \bar{m} + A\,E\,[m_{(t-1)} / I_{(t-2)\,j}\,]\,.$$

Now noting that from (16) we have that

$$E\,[\Delta \ln q_{(t-1)j} / I_{(t-2)\,j}\,] = I_2\,E\,[m_{(t-1)} / I_{(t-2)j}\,]$$

we can rewrite (17') as

$$E\,[m_t / I_{(t-1)j}\,] = \bar{m} + [A - \psi\, I_2]\,E\,[m_{(t-1)} / I_{(t-2)j}] + \psi\,\Delta \ln q_{(t-1)j} \qquad (17'')$$

This period prediction error of firm j that we will denote again by Z_{tj} is, as before, next period's innovation so that we will have that, by the orthogonality principle firm's prediction errors are not correlated in time.

$$Z_{tj} = I_2\,[A - \psi\, I_2]\,[m_{(t-1)} - E\,m_{(t-1)} / I_{(t-2)j}\,] + I_2\,g_t \qquad (20)$$

Now let's consider the case of the representative firm. We will have again that

$$E\,[\Delta \ln q_t / I_{(t-1)}] = (1\text{-}a)\,\Delta \ln \bar{q} + a\,\Delta \ln q_{(t-1)}$$

so that $Z_t = \Delta \ln q_t - E\,[\Delta \ln q_t / I_{(t-1)}] = V_t$

We get again the same relation between Z_{tj} and $\bar{Z}_t$

i.e. $Z_{tj} = Z_t + \{E\,[\Delta \ln q_t / I_{(t-1)}] - E_j\,E\,[\Delta \ln q_{tj} / I_{(t-1)j}]\,\}$

that we can in this case rewrite as

$$Z_{tj} = [\,a - (\psi_1 + \psi_2\,)\,]\,\bar{Z}_{(t-1)j} + \varepsilon_t \qquad (21)$$

where $\varepsilon_t = V_t + (a - \rho)\,E_j\,E\,[e_{j(t-1)} / I_{(t-2)j}\,]$

and where we have used the following notation $\psi' = [\psi_1 \ \psi_2]$.

Again we have the result (see the Appendix for detailed computations) that aggregate prediction errors are correlated in time despite the fact that individual prediction errors show no serial correlation. Moreover we also have that aggregate prediction errors and aggregate predictions are correlated despite the fact that this does not happen at the firm's level (see also the Appendix).

i.e. $E\,[\,\bar{Z}_{tj}\,,\,E_j\,E\,[\Delta \ln q_{tj} / I_{(t-1)j}]\,] \neq 0$

and $E\,[\,Z_{tj}\,,\,E\,[\Delta \ln q_{tj} / I_{(t-1)j}]\,] = 0$

Remark that when $\rho = 0$, as $\psi_2 = 0$ [6], we obtain the results of the previous model. Another interesting case is the case where $a = \rho$. As we have that $(\psi_1 + \psi_2) = a$ [7] using (20) we get $Z_{tj} = V_t + u_{tj}$ so that $\bar{Z}_{tj} = V_t$ and therefore shows no serial correlation. In fact when the autogressive coefficient is the same for the aggregate and the local components we have that

$$\Delta \ln q_{tj} = (1-a)\,\Delta \ln \bar{q} + a\,\Delta \ln q_{(t-1)} + a\,e_{(t-1)j} + V_t + u_{tj} \qquad (22)$$

i.e.

$$\Delta \ln q_{tj} = (1-a)\,\Delta \ln \bar{q} + a\,\Delta \ln q_{(t-1)j} + V_t + u_{tj} \qquad (22')$$

so that the signal extraction problem disappears i.e. last period's observed individual growth rate of production is much more informative.

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6 See the Appendix for the proof.

7 See also the Appendix for the proof.

Having dealt with the first restrictive hypothesis we will now take care of the second one. This means that now we will consider that individual agents have access to some aggregate infomation.

4. The case where individual agents have access to noisy aggregate information.

We will now admit that each agent has access to some information conceming average quantities i.e. we will assume that each agent observes by the end of period (t-1) a common variable from which he can extract information on q (t-1), e.g. a government prediction for $\Delta \ln q_{(t-1)}$.

i.e.

$$\Delta \ln i_{(t-1)} = b\, \Delta \ln q_{(t-1)} + f_{(t-1)} \qquad (23)$$

where $f_{(t-1)} \sim N(0,\sigma_f^2)$ and is uncorrelated with u_{tj} and V_t .

This means that now the information set of firm j at the end of period (t-1), I(t-1)j is specified as:

$$I_{(t-1)j} = \{I_{(t-2)j}\, ,\, S_{((t-1)j}\}$$

Where $S_{((t-1)j} = \begin{bmatrix} \Delta \ln q_{(t-1)j} \\ \Delta \ln i_{(t-1)} \end{bmatrix}$

Noting that

$$S_{((t-1)j} = B\, m_{(t-1)} + h_{(t-1)} \qquad (24)$$

Where $B = \begin{bmatrix} 1 & 1 \\ b & 0 \end{bmatrix}$ and $h_{(t-1)} = \begin{bmatrix} 0 \\ f_{(t-1)} \end{bmatrix} \sim N(0,H)$

we can write the full model as:

$$m_t = A\, m_{(t-1)} + \bar{m} + g_t$$

$$S_{(t-1)j} = B\, m_{(t-1)} + h_{(t-1)}$$

Using again the Kalman filtering technique and approximating $\phi(t)$ by ϕ we will have:

$$E[\, m_t / I_{(t-1)j}\,] = E[m_t / I_{(t-2)j}\,] + \psi\,[S_{(t-1)j} - E[S_{(t-1)j} / I_{(t-2)}]\,] \quad (25$$

that we can rewrite as (25')

$$E[\, m_t / I_{(t-1)j}\,] = \bar{m} + [A-\psi B]\, E[m_{(t-1)} / I_{(t-2)j}\,] + \psi\, S_{(t-1)j} \quad (25')$$

where $\psi = A\, \phi\, B'\, (B\, \phi\, B' + H)^{-1}$ (26)

and $\phi = A\, \phi\, A' + G - (A\, \phi\, B')\,(B\, \phi\, B' + H)^{-1}\, (A\, \phi\, B')'$ (27)

Again the prediction error of firm j, (Z_{tj}) (that is given by expressi (28)) is, by the orthogonality principle, not serially correlated and orthogor to firm's j prediction of its future rate of output.

$$Z_{tj} = I_2\,[\, m_t - E[m_t / I_{(t-1)j}\,]\,] = \quad (28)$$

$$I_2\, [A-\psi B]\, [m_{(t-1)} - E[m_{(t-1)} / I_{(t-2)j}\,]\,] - I_2\, \psi h_{(t-1)} + I_2\, g_t$$

From (28) we obtain easily the average prediction error ($\bar{Z}_{tj}$)

$$\bar{Z}_{tj} = I_2 [A-\psi B] E_j [m_{(t-1)} - E [m_{(t-1)} / I_{(t-2)j}]] - I_2 \psi h_{(t-1)} + V_t \qquad (29)$$

and we have also that

$$Var (\bar{Z}_{tj}) = I_2 \ \omega I'_2 = I_2 [A-\psi B] \psi [A-\psi B]' I'_2 + I_2 \psi H \psi' I'_2 + \theta \qquad (30)$$

where ω the variance-covariance matrix of $E_j [m_t - E [m_t / I_{(t-1)j}]]$

is given by the following expression:

$$\omega = [A-\psi B] \omega [A-\psi B]' + \psi H \psi' + K \qquad (31)$$

where $K = E [[E_j g_t] \ [E_j g_t]'] = \begin{bmatrix} \theta & 0 \\ 0 & 0 \end{bmatrix}$

Moreover from (29) we also have that (see the Appendix)

$$E [\bar{Z}_{tj} , \bar{Z}'_{(t-1)j}] = I_2 [A-\psi B] \ \omega I'_2 \neq 0 \qquad (32)$$

so that average prediction errors are in general correlated in time.

We also get the result that the average prediction error is generally correlated with the average prediction (see also the Appendix).

Let us now consider the case where $\Delta \ln i_{(t-1)} = \Delta \ln q_{(t-1)}$. (This mean that b=1 and $\sigma_f^2 = 0$) . In this case we have that

$$E \ [\Delta \ln q_{tj} / I_{(t-1)j}] = (1-a) \Delta \ln \bar{q} + a \Delta \ln q_{(t-1)} + \rho e_{j(t-1)} \qquad (33)$$

and $Z_{tj} = \Delta \ln q_{tj} - E \; [\Delta \ln q_{tj} \; / \; I_{(t-1)j} \,] = V_t + u_{tj}$ (34)

So, in this case the average of the firms behaves like a representative firm and therefore $\bar{Z}_{tj}$ is not serially correlated and is orthogonal to the average prediction.

However admiting that each firm observes at the same time, its own local information and the corresponding aggregate variable is very unrealistic, as we all know that there is a certain lag in the publication of statistical series. In our framework it is sufficient to admit that aggregate series are published with one period lag in order to get the result that average prediction errors show serial correlation and that they are correlated with average predictions.

In order to make this point clearer we will now consider again, for the sake of simplicity, the case where local shocks are not serially correlated and we will introduce the idea that individual agents can observe ($\Delta \ln q_t$) but with a certain lag.

5. The case where agents observe the aggregate series with one period lag.

At the end of period (t-1), firm j predicts $\Delta \ln q_{tj}$ using all its available information, $I_{(t-1)j}$. Now we will specify $I_{(t-1)j}$ in the following way: $I_{(t-1)j}$ contains $\Delta \ln q_{(t-1)j}$ that the firm observes by the end of period (t-1), $\Delta \ln q_{(t-2)}$ that the firm observes by the middle of period (t-1) and all previous values of these variables.

This means that

$$I_{(t-1)j} = \{\Delta \ln q_{(t-1)j} \, , \Omega_{(t-1)} \}$$

where $\Omega_{(t-1)} = \{\Delta \ln q_{(t-2)} \, ,\Delta \ln q_{(t-3)} \, , \ldots, \Delta \ln q_{(t-2)j} \, , \ldots\}$

i.e. $\Omega_{(t-1)} = \{\Delta \ln q_{(t-2)} \, , I_{(t-2)j}\}$

Again using the Kalman filtering technique and noting that[8]

$$E[\Delta \ln q_{tj} / I_{(t-1)j}] = E[\Delta \ln q_t / I_{(t-1)j}] \tag{35}$$

$$E[\Delta \ln q_t / I_{(t-1)j}] = a\, E[\Delta \ln q_{(t-1)} / I_{(t-1)j}] \tag{36}$$

we have that

$$E[\Delta \ln q_t / I_{(t-1)j}] = [a - \psi]\, E[\Delta \ln q_{(t-1)} / \Omega_{(t-1)}] + \psi\, \Delta \ln q_{(t-1)j} \tag{37}$$

$$\text{where } \psi = \frac{a \operatorname{Var}[\Delta \ln q_{(t-1)} - E[\Delta \ln q_{(t-1)} \mid \Omega_{(t-1)}]\,]}{\operatorname{Var}[\Delta \ln q_{(t-1)} - E[\Delta \ln q_{(t-1)} \mid \Omega_{(t-1)}]\,] + \sigma_e^2}$$

Using (2) and (37) we also have that

$$\operatorname{Var}[\Delta \ln q_t - E\, \Delta \ln q_t \mid I_{(t-1)}] = a^2 \operatorname{Var}[\Delta \ln q_t - E\, \Delta \ln q_{(t-1)} \mid \Omega_{(t-1)}] - a\, \psi \operatorname{Var}[\Delta \ln q_{(t-1)} - E\, \Delta \ln q_{(t-1)} \mid \Omega_{(t-1)}] + \theta \tag{38}$$

Now note that using (36) we get

$$\begin{aligned} E[\Delta \ln q_{(t-1)} / \Omega_{(t-1)}] &= E[\Delta \ln q_{(t-1)} / I_{(t-2)j}] \\ &\quad + a[\Delta \ln q_{(t-2)} - E\, \Delta \ln q_{(t-2)} / I_{(t-2)j}] \\ &= a\, \Delta \ln q_{(t-2)} \end{aligned} \tag{39}$$

This implies that

$$[\Delta \ln q_{(t-1)} - E\, \Delta \ln q_{(t-1)} / \Omega_{(t-1)}] = V_{(t-1)} \tag{40}$$

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8 For simplicity, we will from now on set $\Delta \ln \bar{q} = 0$.

so that $\text{Var}\,[\Delta \ln q_{(t-1)} - E\,\Delta \ln q_{(t-1)} / \Omega_{(t-1)}] = \theta$ (41)

Substituting now (39) in (37) we obtain

$$E\,[\Delta \ln q_t / I_{(t-1)j}] = a^2\,\Delta \ln q_{(t-2)} + \psi\,[\Delta \ln q_{(t-1)j} - a\,\Delta \ln q_{(t-2)}] \quad (42)$$

$$= a^2\,\Delta \ln q_{(t-2)} + \psi\,[V_{(t-1)} + e_{(t-1)j}]$$

so that

$$\Delta \ln q_t - E\,[\Delta \ln q_t / I_{(t-1)j}] = (a-\psi)\,V_{(t-1)} + V_t - \psi\,e_{(t-1)j} \quad (43)$$

From expression (43) we get the result that prediction errors when predicting $\Delta \ln q_t$ using the information contained in $I_{(t-1)j}$ are correlated in time as

$$E\,[\,[\Delta \ln q_t - E\,[\Delta \ln q_t / I_{(t-1)j}]\,],\,[\Delta \ln q_{(t-1)} - E\,[\Delta \ln q_{(t-1)} / I_{(t-2)j}]\,]$$

$$= (a-\psi)\,\theta \neq 0$$

Note however that $Z_{tj} = \Delta \ln q_{tj} - E[\Delta \ln q_{tj} / I_{(t-1)j}]$

does not show serial correlation as Z_{tj} is next's period innovation.

$$E\,[Z_{tj}, Z_{(t-1)j}] = (a-\psi)\,\theta - \psi\,\sigma_e^2 = 0$$

Looking now at $\bar{Z}_{tj} = E_j\,Z_{tj}$, we obtain again the result that this series show serial correlation. In fact

$$E\,[\bar{Z}_{tj}, \bar{Z}_{(t-1)j}] = (a-\psi)\,\theta$$

Moreover

$$E\,[\,\bar{Z}_{tj}, E_j\,E\,[\Delta \ln q_{tj} / I_{(t-1)j}]\,] = \psi\,(a-\psi)\,\theta$$

and $E[Z_{tj}, E \Delta \ln q_{tj} / I_{(t-1)j}] = 0$

This means that again although individual prediction errors and individual predictions are orthogonal we don't get the same properties for the aggregate series.

6. Some empirical results

The data used in this study come from the quarterly business survey conducted by the I.N.E. (Lisbon) among manufacturing firms, on recent performances, observed and expected trends and appraisals of levels for variables like production, demand and inventories. For example, firms are asked whether their own production is expected to fall, rise or remain the same over the following quarter, and they are also asked tio report the same for their own present evolution of production. The percentage of responses falling in each category can then be used to construct the corresponding aggregate series i.e. the aggregate expected growth rate of production and aggregate observed growth rate of production. The basic idea behind this quantification of business survey data is that business survey variables are generated by an underlying theoretical distribution across firms for the variable on which the question is being answered.

For example that

$$\Delta \ln q_{tj} = \Delta \ln q_t + e_{tj} \qquad e_{tj} \sim N(0,\sigma_e^2)$$

and then we regard the fraction of firms giving a particular answer as the probability that a firm will report this type of answer. Moreover it is assumed that firms will only report increase or decrease for significative changes of the variable. For simplicity it is also considered that this indifference interval is the same for all firms, for the same variable.

The quantification tecnhique used in this paper differs from the one developed by Carlson and Parkin (1975), and Knobl (1974) mainly by the fact that we approximate the normal by the logistic distribution[9] in order to obtain more tractable expressions for the business survey percentages and also by the fact we do not impose that the indifference interval is symmetric[10].

Using our technique the expression used to construct the aggregate variables from the business survey percentages is the following:

$$\Delta \ln y_t = \frac{1}{c} \left[\omega \ln \left(\frac{x_1}{1-x_1}\right)_t + (1 - \omega) \ln \left(\frac{1-x_3}{x_3}\right)_t \right] \qquad (44)$$

$$c = \frac{\Pi}{\sqrt{3}} \frac{15}{16} \frac{1}{\sigma}$$

where y_t is the aggregate variable (e.g. q_t), $x_1(3)$ is the percentage of firms reporting increase (decrease) and σ is the standard-error of the distribution of $\Delta \ln y_{tj}$ across firms. The weights ω , depend on whether the indifference interval is symmetric (ω=0.5) or not.

Having constructed in this way $\Delta \ln q_t$ and $E_j\ E\ [\Delta \ln q_{tj} / (t-1)]$ we run the following regressions

$$\bar{Z}^*_{tj} = \Pi_1 \bar{Z}^*_{(t-1)j} + e_{1t} \qquad (45)$$

$$\bar{Z}^*_{tj} = a_0 + a_1 E_j\ E\ [\Delta \ln q_{tj} / t-1]^* + e_{2t} \qquad (46)$$

that are the ones that are normally used when testing for the rationality of expectations. We introduced the star superscript to denote that we don't have precisely the variables $\bar{Z}_{tj}$ and $E_j\ E\ [\Delta \ln q_{tj} / (t-1)]$ but their values up to a constant.

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9 The corrected logistic distribution is known to be a good approximation to the standard normal cumulative distribution (see Johnson and Kotz (1972)).

10 For more details on this approach see Modesto (1987) and also Lambert (1987) for a similar procedure.

The results obtained using a sample from 1977.2 to 1986.4 and OLS are presented below.

Table 1[a)]

Ordinary least squares estimates of eq.(45) and eq.(46)

Coefficient Estimates			Statistics		
Eq. (45)		Π_1	$\bar{R}^2$	DW	SEE
		.569	.115	2.22	.082
		(4.31)			
Eq. (46)	a_0	a_1	$\bar{R}^2$	DW	SEE
	-0.64	.189	.083	1.03	.096
	(-2.88)	(2.10)			

a) Figures in parenthesis are t-statistics.

One can see that average prediction errors show serial correlation and that they are also correlated to the average forecast. However in the light of what was presented before we can not conclude that expectations are not rational.

7. Concluding Remarks

If aggregate expectation series will not pass the so called rationality tests even when individual agents have rational expectations how can one test if agents form their expectations rationally? The obvious answer is that these tests should be conducted at the micro-level (see Zimmermann (1984) for this type of exercise). However if one is interested in testing if expectations are rational because of its implications concerning macro-economic policy this paper suggests that as aggregate series will not in general pass the traditional rationality tests, the strenght of these implications will be weakened.

Appendix

1. The simple model

- Showing that aggregate prediction errors and aggregate prediction are correlated.

As $$E_j\,[E\,[\Delta \ln\; q_{tj} / I_{(t-1)j}] = (1\text{-}a)\,\Delta \ln\; \bar{q} +$$

$$(a\text{-}\psi)\,E_j\;\;[E\,[\Delta \ln\;\; q_{(t-1)j} / I_{(t-2)j}]\;] + \psi\,\Delta \ln\;\; q_{(t-1)}$$

and $$\bar{Z}_{tj} = V_t + (a\text{-}\psi)\,\bar{Z}_{(t-1)j}$$

we have that

$$E\,[\;\bar{Z}_{tj}\;,\,E_j\,[E\,[\Delta \ln\;\; q_{tj} / I_{(t-1)j}]\;] =$$

$$(a\text{-}\psi)^2\,E\,[\bar{Z}_{(t-1)j}\;, E_j\,E\,[\Delta \ln\;\; q_{(t-1)j} / I_{(t-2)j}]\;]$$

$$+\,(a\text{-}\psi)\;\psi\,E\,[\bar{Z}_{(t-1)j}\;,\,\Delta \ln\;\; q_{(t-1)}]\;. \qquad (1.A)$$

Now as we can write

$$\Delta \ln\;\; q_{(t-1)} = \bar{Z}_{(t-1)j} + E_j\,E\,[\Delta \ln\;\; q_{(t-1)j} / I_{(t-2)j}]$$

we have that

$$E[\bar{Z}_{(t-1)j}, \Delta \ln q_{(t-1)}] = \text{var}\, \bar{Z}_{(t-1)j}$$

$$+ E[\bar{Z}_{(t-1)j}, E_j E[\Delta \ln q_{(t-1)j} / I_{(t-2)j}]]$$

so that (1.A) becomes

$$E[\bar{Z}_{tj}, E_j E[\Delta \ln q_{tj} / I_{(t-1)j}]] = (a-\psi)\, \psi\, \text{var}\, \bar{Z}_{(t-1)j}$$

$$+ a\,(a-\psi)\, E[\bar{Z}_{(t-1)j}, E_j E[\Delta \ln q_{(t-1)j} / I_{(t-2)j}]] .$$

Using now the steady-state solution we obtain

$$E[\bar{Z}_{tj}, E_j E[\Delta \ln q_{tj} / I_{(t-1)j}]] = \frac{(a-\psi)\,\psi}{[1-a(a-\psi)]} \text{var}\, \bar{Z}_{tj}$$

$$= \frac{(a-\psi)\,\psi}{[1-a(a-\psi)]} \cdot \frac{[1-a(a-\psi)]\,\phi}{1-a(a-\psi)^2}$$

$$= \frac{(a-\psi)\,\psi\,\phi}{1-(a-\psi)^2}$$

- <u>Showing that individual prediction errors and prediction are orthogonal</u>

As $Z_{tj} = (a-\psi)\,[\Delta \ln q_{(t-1)j} - E\,\Delta \ln q_{(t-1)j} / I_{(t-2)j}]$

$$- a\, e_{j(t-1)} + e_{jt} + V_t$$

$$= (a-\psi)\, Z_{(t-1)j} - a\, e_{j(t-1)} + e_{jt} + V_t$$

and $E[\Delta \ln q_{tj} / I_{(t-1)j}] =$

$$(1-a)\,\Delta \ln \bar{q} + (a-\psi)\, E[\Delta \ln q_{(t-1)j} / I_{(t-2)j}]$$

$$+ \psi\, \Delta \ln q_{(t-1)j}$$

we have that

$$E\,[\,Z_{tj}\,,E\,\Delta\ln\,q_{tj}\,/\,I_{(t-1)j}] =$$

$$= (a-\psi)^2\,E\,[Z_{(t-1)j}\,,E\,[\Delta\ln\,q_{(t-1)j}\,/\,I_{(t-2)j}]\,]$$

$$+ (a-\psi)\,\psi\,E\,[Z_{(t-1)j}\,,\Delta\ln\,q_{(t-1)j}] - a\,\psi\,\sigma_e^2\,. \qquad (2.A)$$

Now as we can write

$$\Delta\ln\,q_{(t-1)j} = Z_{(t-1)j} + E\,[\Delta\ln\,q_{(t-1)j}\,/\,I_{(t-2)j}]$$

we have that

$$E\,[Z_{(t-1)j}\,,\Delta\ln\,q_{(t-1)j}\,] = \text{var}\,Z_{(t-1)j}$$

$$+ E\,[Z_{(t-1)j}\,,E\,[\Delta\ln\,q_{(t-1)j}\,/\,I_{(t-2)j}]\,]$$

so that (2.A) becomes:

$$E\,[Z_{tj}\,,E\,\Delta\ln\,q_{tj}\,/\,I_{(t-1)j}]$$

$$= a(a-\psi)\,E\,[Z_{(t-1)j}\,,E\,[\Delta\ln\,q_{(t-1)j}\,/\,I_{(t-2)j}]\,]$$

$$+ (a-\psi)\,\psi\,\text{var}\,Z_{(t-1)j} - a\,\psi\,\sigma_e^2\,.$$

Using now the steady-state solution one obtains

$$E\,[Z_{tj}\,,E\,[\Delta\ln q_{tj}\,/\,I_{(t-1)j}] = \frac{(a-\psi)\,\psi\,\text{var}\,Z_{tj} - a\,\psi\,\sigma_e^2}{1-a(a-\psi)} = 0$$

as $\quad \text{var}\,Z_{tj} = \phi + \sigma_e^2$

so that $\quad (a-\psi)\,\psi\,(\phi + \sigma_e^2) - a\,\psi\,\sigma_e^2 = 0$

because $(a-\psi)\,\phi = \psi\,\sigma_e^2$.

2. The simple model with local correlated disturbances

- Showing that aggregate prediction errors are correlated in time despite the fact that individual prediction errors (being next period's innovations) show no serial correlation.

As
$$Z_{tj} = I_2\, m_t - E\,[m_t / I_{(t-1)j}] = [\Delta \ln\ q_t - E\,\Delta \ln\ q_t / I_{(t-1)j}]$$
$$+\,[e_{tj} - E\, e_{tj} / I_{(t-1)j}]$$
$$= X_{tj} + Y_{tj}$$

we have that

$$E\,[Z_{tj}\,, Z_{(t-1)j}\,] = E\,[X_{tj}\,, X_{(t-1)j}\,] + E\,[X_{tj}\,, Y_{(t-1)j}\,]$$
$$+\,E\,[Y_{tj}\,, X_{(t-1)j}\,] + E\,[Y_{tj}\,, Y_{(t-1)j}\,]$$

Now as

$$X_{tj} = (a-\psi_1)\, X_{(t-1)j} - \psi_i\, Y_{(t-1)j} + V_t$$
$$Y_{tj} = (\rho-\psi_2)\, Y_{(t-1)j} - \psi_2\, X_{(t-1)j} + u_{tj}$$

we have that

$$E\,[X_{tj}\,, X_{(t-1)j}\,] = (a-\psi_1)\,\phi_{11} - \psi_1\,\phi_{12}$$
$$E\,[X_{tj}\,, Y_{(t-1)j}\,] = (a-\psi_1)\,\phi_{12} - \psi_1\,\phi_{22}$$
$$E\,[Y_{tj}\,, X_{(t-1)j}\,] = (\rho-\psi_2)\,\phi_{12} - \psi_2\,\phi_{11}$$
$$E\,[Y_{tj}\,, Y_{(t-1)j}\,] = (\rho-\psi_2)\,\phi_{22} - \psi_2\,\phi_{12}$$

where we have used the following notation:

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{bmatrix}$$

is the variance-covariance matrix of $[\, m_t - E\, m_t / I_{(t-1)j}]$

Substituting above we obtain

$$E\,[Z_{tj}\ Z_{(t-1)j}\,] = [\, a - (\psi_1+\psi_2)]\,(\phi_{11}+\phi_{12})$$
$$+ [\rho - (\psi_1+\psi_2)]\,(\phi_{12}+\phi_{22}) = 0$$

because $$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} \dfrac{a(\phi_{11}+\phi_{12})}{\phi_{11}+\phi_{22}+2\phi_{12}} \\ \\ \dfrac{\rho - (\phi_{12}+\phi_{22})}{\phi_{11}+\phi_{22}+2\phi_{12}} \end{bmatrix}$$

Looking now at the aggregate prediction error, noting that we can rewrite

$$\bar{Z}_{tj} = I_2\, E_j\, [\, m_t - E\, m_t / I_{(t-1)j}]$$ as:

$$\bar{Z}_{tj} = a\,\bar{X}_{(t-1)j} + \rho\,\bar{Y}_{(t-1)j} - (\psi_1+\psi_2)\,\bar{Z}_{(t-1)j}$$

where $\bar{X}_{tj} = E_j\, X_{tj}$ and $\bar{Y}_{tj} = E_j\, Y_{tj}$

and denoting by $\omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{bmatrix}$

the variance-covariance matriz of $E_j [m_t - E m_t / I_{(t-1)j}]$

we have that

$$E [\bar{Z}_{tj} , \bar{Z}_{(t-1)j}] = a (\omega_{11}+ \omega_{12}) + \rho (\omega_{22}+ \omega_{12})$$

$$- (\psi_1+\psi_2) (\omega_{11}+ \omega_{22} + 2\omega_{12})$$

$$= [a - (\psi_1+\psi_2)] (\omega_{11}+\omega_{12}) + [\rho - (\psi_1+\psi_2)] (\omega_{11}+\omega_{22}) \neq 0$$

because $\omega \neq \phi$.

- Showing that aggregate prediction errors and aggregate prediction are correlated despite the fact that this does not happen at the firm's level.

At the firm's level we have that

$$E [Z_{tj} , E [\Delta \ln q_{tj} / I_{(t-1)j}]] = 0$$

because we have that

$$E [\Delta \ln q_{tj} / I_{(t-1)j}] = I_2 E [m_t / I_{(t-1)j}] =$$

$$= I_2 [I - A]^{-1} \bar{m} + I_2 [I + AL + A^2 L^2 + ..] \psi Z_{(t-1)j}$$

and we also know that Z_{tj} is not correlated in time.

Turning now to the aggregate level we have that

$$E_j E [\Delta \ln q_{tj} / I_{(t-1)j}] = I_2 [I - A]^{-1} \bar{m} +$$

$$I_2 [I + AL + A^2 L^2 + ...] \psi \bar{Z}_{(t-1)j} \qquad (3.A)$$

and that

$$\bar{Z}_{tj} = I_2 \, E_j \, [m_t - E \, m_t / I_{(t-1)j}] = \tag{4.A}$$

$$I_2 \, [A - \psi \, I_2] \, E_j \, [m_{(t-1)} - E \, m_{(t-1)} / I_{(t-2)j}] + V_t.$$

Therefore, using (3.A), we have that:

$$E \, [\bar{Z}_{tj} \, , E_j \, E \, [\Delta \ln \, q_{tj} / I_{(t-1)j}] \,] = E \, [\bar{Z}_{tj} \, , \bar{Z}_{(t-1)j} \,] \, \psi' \, I'_2 \tag{5.A}$$

$$+ E \, [\bar{Z}_{tj} \, , \bar{Z}_{(t-2)j} \,] \, \psi' \, A' \, I'_2 + E \, [\bar{Z}_{tj} \, , \bar{Z}_{(t-3)j} \,] \, \psi' \, A^{2'} \, I'_2 + \dots$$

As from (4.A) one gets that:

$$E \, [\bar{Z}_{tj} \, , \bar{Z}_{(t-1)j} \,] = I_2 \, [A - \psi \, I_2] \, \omega \, I'_2$$

$$E \, [\bar{Z}_{tj} \, , \bar{Z}_{(t-2)j} \,] = I_2 \, [A - \psi \, I_2]^2 \, \omega \, I'_2$$

i.e. that in general

$$E \, [\bar{Z}_{tj} \, , \bar{Z}_{(t-i)j} \,] = I_2 \, [A - \psi \, I_2]^i \, \omega \, I'_2$$

and substituting this last result in (5.A) one gets:

$$E \, [\bar{Z}_{tj} \, E_j \, E \, [\Delta \ln \, q_{tj} / I_{(t-1)j}] \,] = I_2 \, [A - \psi \, I_2] \, \omega \, I'_2 \, \psi' \, I'_2 \tag{5.A'}$$

$$+ I_2 \, [A - \psi \, I_2]^2 \, \omega \, I'_2 \, \psi' \, A' I'_2 + I_2 \, [A - \psi \, I_2]^3 \, \omega \, I'_2 \, \psi' \, A^{2'} I'_2$$

$$+ \dots \neq 0.$$

because $\omega \neq \phi$.

In fact we have that:

$$[A - \psi I_2]\, \phi\, I'_2 = 0$$

Proof: $[A - \psi I_2]\, \phi\, I'_2 = A\, \phi\, I'_2 - \psi\, I_2\, \phi\, I'_2$

now as $\psi = A\phi\, I'_2\, (I_2\, \phi\, I'_2)^{-1}$

we get $A\, \phi\, I'_2 - A\, \phi\, I'_2\, (I_2\, \phi\, I'_2)^{-1}\, I_2\, \phi\, I'_2$

$= A\, \phi\, I'_2 - A\, \phi\, I'_2 = 0.$

but we also have that

$[A - \psi I_2]\, \omega\, I'_2 \neq 0$ because $\omega \neq \phi$

- <u>The case where $\rho = 0$</u>

As

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} \dfrac{a(\phi_{11}+\phi_{12})}{\phi_{11}+\phi_{22}+2\phi_{12}} \\[2ex] \dfrac{\rho\,(\phi_{12}+\phi_{22})}{\phi_{11}+\phi_{22}+2\phi_{12}} \end{bmatrix}$$

one immediately sees that when $\rho = 0$ we get $\psi_2=0$.

- <u>The case where $a = \rho$</u>

In this case it is immediate to see that one gets that

$(\psi_1 + \psi_2) = a$ so that $I_2\ [A - \psi I_2] = 0$

3. The case where individual agents have access to noisy aggregate information.

- Showing that average prediction errors are in general correlated in time.

As $\bar{Z}_{tj} = I_2 [A - \psi B] E_j [m_{(t-1)} - E [m_{(t-1)} I_{(t-2)j}]]$

$$- I_2 \psi h_{(t-1)} + V_t$$

we have that $E [\bar{Z}_{tj} , \bar{Z}_{(t-1)j}] = I_2 [A - \psi B] \omega I'_2 \neq 0$

because $\omega = \phi$.

Note that $I_2 [A - \psi B] \phi I'_2 = 0$.

- Showing that the average prediction error is correlated with the average prediction despite the fact that the prediction error of firm j is orthogonal to firm's j prediction of its future rate of output.

For firm j we have that:

$$E [Z_{tj} , I_2 E [m_t / I_{(t-1)j}]]= 0$$

because as

$$Z_{tj} = I_2 [m_t - E m_t / I_{(t-1)j}] =$$

$$I_2 [A - \psi B] [m_{(t-1)} - E [m_{(t-1)} I_{(t-2)j}]]$$

$$- I_2 \psi h_{(t-1)} + I_2 g_t$$

and $I_2 [E m_t / I_{(t-1)j}] = I_2 [I - A]^{-1} \bar{m}$

$$+ I_2 [I + AL + A^2 L^2 + ...] \psi h_{(t-1)}$$

$$+ \ I_2 [I + AL + A^2 L^2 + ...] \psi B [m_{(t-1)} - E [m_{(t-1)} / I_{(t-2)j}]$$

we get

$$E [Z_{tj} , I_2 E [m_t / I_{(t-1)j}]] = - I_2 [A-\psi B] \psi H \psi' A' I'_2$$

$$- I_2 [A-\psi B]^2 \psi H \psi' A^{2'} I'_2 - I_2 [A-\psi B]^3 \psi H \psi' A^{3'} I'_2 + ...$$

$$+ I_2 [A-\psi B] \phi B' \psi' I'_2 + I_2 [A-\psi B]^2 \phi B' \psi' A' I'_2 \qquad (6.A)$$

$$+ I_2 [A-\psi B]^3 \phi B' \psi' A^{2'} I'_2 + ... - I_2 \psi H \psi' I'_2$$

because we can write:

$$[m_t - E [m_t / I_{(t-1)j}] = - [I + (A - \psi B) L + (A - \psi B)^2 L^2 + ...] \psi h_{(t-1)}$$

$$+ [I + (A - \psi B) L + (A - \psi B)^2 L^2 + ...] \ g_t$$

and we also have that

$$E [[m_t - E m_t / I_{(t-1)j}] , [m_{(t-i)} - E m_{(t-i)} / I_{(t-i-1)j}]'] =$$

$$[A - \psi B)^i]\phi.$$

Organizing terms in (6.A) we have that:

$$E [Z_{tj} , I_2 E [m_t / I_{(t-1)j}]] =$$

$$I_2 [(A-\psi B) \phi B' \psi' - \psi H \psi'] I'_2 \qquad (6A')$$

$$+ I_2 \ [A-\psi B] [(A-\psi B) \phi B' \psi' - \psi H \psi'] A' I'_2$$

$$+ I_2 [A-\psi B]^2 [(A-\psi B) \phi B' \psi' - \psi H \psi'] A^{2'} I'_2 + ... \quad .$$

Now as

$$[(A-\psi B) \ \phi B' \psi' - \psi' H \psi'] = A \phi B' \psi' - \psi (B \phi B' + H) \psi' =$$

$$A \phi B' \psi' - A \phi B' (B \phi B' + H)^{-1} (B \phi B' + H) \psi' = 0$$

we have indeed that $E[Z_{tj}, I_2 E[m_t / I_{(t-1)j}]] = 0$.

For the average values we obtain:

$$E[\bar{Z}_{tj}, I_2 E_j E[m_t / I_{(t-1)j}]] = I_2 [(A-\psi B)\omega B'\psi' - \psi H\psi'] I'_2$$
$$+ I_2 [A-\psi B][(A-\psi B)\omega B'\psi' - \psi H\psi'] A'I'_2 \qquad (7.A)$$
$$+ I_2 [A-\psi B]^2 [(A-\psi B)\omega B'\psi' - \psi H\psi'] A^{2'} I'_2 + \ldots \neq 0$$

because

$$[(A-\psi B)\omega B'\psi' - \psi H\psi'] \neq 0$$

as $\omega \neq \phi$.

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