Application of an Income-based Structural Model to Measure the Probabilities of Default of Five European Banks

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Abstract

This dissertation analyses whether a modified version of the EBIT-based structural model by (Goldstein, Ju, & Leland, 2001) is able to replicate the default metrics published by major credit rating agencies in the case of banks. This research studies five European banks from 2001 until 2020. As the reference model focus on non-financial institutions, it was adapted to fit the characteristics of banks. In particular, the assumption that firms have fixed financial costs was replaced by the hypothesis that a fraction of banks' non-interest costs are fixed. This share was determined in order to match credit rating agencies average probabilities of default, which equals 1.14% during our 20 years sample. After gathering all data, the model was calibrated following the iterative approach, first proposed by (Vassalou & Xing, 2004). A regression of the mean model probabilities of default and distances to default at each moment in time on the equivalent ratings-implied measures showed an R-squared of 0.27 and 0.40, respectively. Furthermore, this dissertation presents a panel data regression that assesses the fixed effects of each bank. The significance test shows that the coefficients in all regressions are significant at 5% significance levels except the fixed effects associated with three banks. I concluded that the model's credit risk indicators are very comparable to the ratings given by credit rating agencies, though the correlation is far from perfect.

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Resumo

O objetivo desta dissertação é analisar se uma versão modificada do modelo estrutural de (Goldstein, Ju, & Leland, 2001) é capaz de replicar as métricas de risco de crédito publicadas pelas principais agências de classificação de risco no caso de bancos. Esta pesquisa estuda cinco bancos europeus entre 2001 e 2020. Como o modelo de referência foi desenvolvido tendo por base empresas não financeiras, o modelo foi adaptado às características dos bancos. Em particular, a hipótese de que as empresas têm custos financeiros fixos foi substituída pela hipótese de que uma fração dos custos sem juros dos bancos são fixos. Esta parcela foi determinada para corresponder, em média, às probabilidades de incumprimento implícitas nos *ratings* das principais agências de classificação de risco de crédito, o que equivale a 1,14% durante a nossa amostra de 20 anos. Após a coleta de todos os dados, o modelo foi calibrado seguindo uma abordagem iterativa, inicialmente proposta por (Vassalou & Xing, 2004). Uma regressão das probabilidades de incumprimento e distâncias ao incumprimento médias resultantes do modelo em cada ano nas medidas comparáveis implícitas nos *ratings* mostrou um R-quadrado de 27% e 40%, respetivamente. Um modelo de regressão com dados em painel e efeitos fixos de cada banco mostra que os coeficientes de todas as regressões são significativos ao nível de confiança de 5%, os efeitos fixos associados a três bancos. Em suma, concluiu-se que os indicadores de risco de crédito do modelo são comparáveis aos ratings dados pelas agências de classificação de crédito, ainda que a correlação esteja longe de ser perfeita.

Título: Aplicação de um Modelo Estrutural Baseado no Rendimento para Avaliar as Probabilidades de Insolvência de Cinco Bancos Europeus

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Palavras-chave: Modelo estrutural; Classificações de crédito; Previsão de insolvência; Risco de crédito;

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1 Introduction

Assessing the risk of default has always been of extreme importance throughout history, especially for firms operating in the financial sector because they are simultaneously responsible for allocating funds in the economy and guaranteeing that these funds are well-managed from savers point of view. Markets and firms follow continuous changes implying changes in their creditworthiness. Very often, those changes are smooth as they are only a series of tiny shocks. Nevertheless, sometimes firms bear large shocks that cause extreme movements in creditworthiness. For instance, a global financial crisis or a pandemic can completely shake markets and revenues of particular firms. In the banking sector, where institutions are highly leveraged, sudden movements in borrowers' credit risk highly affects banks' risk profile. On a bank's perspective, it would be very risky to lend money to a borrower that does not show a clear ability of solvency. Consequently, it is primordial for firms to have the relevant tools to analyze their credit worthiness and the credit worthiness of their peers so that they can differentiate between "healthy" firms and firms on the verge of default.

Credit risk evaluation also matters for banks' regulators. Regulations' role is to guarantee a safer environment and to avoid a crisis that would affect the whole financial system. To achieve their mission, regulators usually require banks to put aside a certain amount of available cash that restricts them from accepting non-reasonable risks due to leverage. The right risk assessment tools prevent regulators from underestimating or overestimating the required amount of available deposits. Nevertheless, even with the existence of this cash buffer, banks are not fully protected from the bankruptcy risk, but at least, it plays a role in minimizing systemic risk.

Credit risk has been analyzed through many approaches. (Merwin, 1942) suggests an unprecedented model based on accounting data. The model is constructed using a sample of ratios that the author considers as predictors of default. His successor (Altman, 1968) developed the model further and made it famous globally. He uses a discriminant analysis approach on public manufacturing firms and suggests a formula based on liquidity, leverage, profitability, activity and solvency ratios to predict how close a public firm is to bankruptcy. Since these seminal papers, several other approaches have been proposed: logits, probits, decision trees, neural networks, support vector machines are among the most relevant.

All models referred until now are reduced-form models in the sense that they lack a theoretical structure. One notable exception in the literature are structural credit risk models, which were initially proposed by (Black & Scholes, The pricing of options and corporate liabilities. Journal of political, 1973) and by (Merton, 1974). The model proposed by these authors offers a unified setting able to simultaneously price corporate contingent claims like equity and debt and justify the occurrence of default, which they assume to occur when the firm asset value falls below nominal debt in a specific future date. In contrast to accounting-based models, Merton's model was able to use forward looking observable data like stock prices to predict default (in a theoretically consistent way). As a result, the revolutionary (Merton, 1974) became the benchmark of credit risk evaluation for traded firms, especially after the KMV company (thereafter Moody's KMV) adapted the model for commercial use. Later, limitations of Merton's model started being noticed, which led to the emergence of a whole new study area related to structural models. Through the years, they have proved their accuracy for long-terms however they failed to perform better than accounting-based models on short-horizons. Another limitation of structural traditional models is that they fail to model a dynamic default barrier. Luckily, years later, (Kane, McDonald, & Marcus, 1984) proposed a ground shaking dynamic model of the structure of capital.

Credit rating agencies use all types of methods (quantitative and qualitative) to assess firms' creditworthiness. The world accounts for three market leaders in this industry, namely: Moody's, Stand & Poor's and Fitch. Credit rating agencies use different notation systems' from one another. However, they all publish matrices to convert their letters, numbers and symbols into probabilities of default. Those agencies have access to a huge amount of databases and have sharp ways to evaluate the risk of default. They are considered today as a benchmark in this field.

In this dissertation, I will implement a structural model to estimate credit risk for five European Banks and compare it to the credit risk assessment provided by credit rating agencies. The scope of study covers the years between 2001 and 2020. Similar to (Goldstein, Ju, & Leland, 2001), the approach followed in this dissertation associates the project value of a bank to its ability to generate earnings. In other words, the state variable of the GJL model is earnings before interest and taxes (EBIT). However, in my dissertation I will include coupon payments to the state variable which leads to earnings before taxes (EBT) as a state variable. This is more adapted to the banking sector because banks have variable coupon payments as well as variable EBT. The second adaptation of the model to the banking sector concerns fixed costs. Instead of using coupon payments as a proxy for fixed costs, I will use non-interest expenses. As mentioned earlier, in opposition to non-financial firms, banks tend to have fluctuating coupon payments as it is one of the pillars of their business models, so it is not reasonable to assume coupon payments as fixed.

2 Literature Review

2.1 Structural Models in Different Industries

Structural credit risk models were initially applied in (Black & Scholes, The pricing of options and corporate liabilities. Journal of political, 1973) as well as by the pioneer (Merton, 1974). The model (Black & Scholes, The pricing of options and corporate liabilities. Journal of political, 1973) was innovative as it introduced the idea of considering equity of a company as a European call option on assets where the strike price equals its nominal debt. Similarly, the paper suggests the evaluation of debt as the value of a risk-free bond minus a European put option on the assets of a firm. In this framework, the default probability is equal to the probability that the market value of assets ends below nominal debt or that the put matures in the money. To explain this idea further: if the assets' market value ends below the value of debt, shareholders will have the possibility (or the option) to transfer the assets' to debtholders. The assumptions of the (Merton, 1974) model became well-known in the literature. Among the assumptions we cite the constant interest rate, liquidation of the firm at maturity and the tradability of assets.

Even though (Merton, 1974) and (Black & Scholes, The pricing of options and corporate liabilities. Journal of political, 1973) were well received both in the academia and by the industry, they soon started being criticized in face of their strict assumptions and difficulty to cope with the data. One of these limitations was the assumption that default can only occur at maturity. Their successors (Black & Cox, Valuing corporate securities: Some effects of bond indenture provisions, 1976) suggested a model based on the following - and closer to reality - assumption: the firm defaults the first time its assets value hits a defined barrier. Models based on a such assumption are known in the literature as first-time passage models. In the case of (Black & Cox, Valuing corporate securities: Some effects of bond indenture provisions, 1976) the possibility of early default was motivated by creditors covenants. Intrinsically, in their model creditors have the right to call the firm assets to themselves whenever they see things deteriorating below a certain level. This option leads to a higher debt value.

Around twenty years later, (Leland, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, 1994) enhanced the previous model (Black & Cox, Valuing corporate securities: Some effects of bond indenture provisions, 1976). Leland included corporate income taxes and bankruptcy costs in a model designed to find the optimal capital structure in a tradeoff theory setting. He determined the consequences of taxes and bankruptcy costs on the optimized capital structure in two cases: when the default barrier is chosen by the shareholders (endogenous barrier) and when creditors take the firm whenever its net-worth is no more positive. In Leland's model, shareholders pay coupons on debt from their own pocket. Therefore, default happens when keeping the firm alive does not bring any value to shareholders anymore. He showed that the model's inputs guide shareholders to decide whether they should keep the firm alive or abandon it. They should be aware that, on one hand, coupons and the default barrier level evolve in positive correlation. On the other hand, volatility, interest rate, and tax rate have a negative correlation with the default barrier level.

The next year, (Longstaff & Schwartz, 1995) suggested more improvements to the model (Black & Cox, Valuing corporate securities: Some effects of bond indenture provisions, 1976). In particular, they relaxed the rigid constant interest rate assumption. The authors proposed a closed-form solution for the first passage time framework using floating interest rates. (Longstaff & Schwartz, 1995) is a model that takes into consideration interest rates as well as default risk to value corporate bonds. It introduced a clear relationship between default probability, interest rate, and credit spread.

Leland improved his own model by cowriting another paper with Klaus B. Toft. The two researchers focused on the assumption that debt has infinite duration. The original work by Leland assumes that debt is perpetual. However, (Leland & Toft, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, 1996) introduced the assumption of debt rollover. In fact, they discussed that a given company must substitute debt in a continuous manner in order to keep the debt level constant. This concept adds an extra risk bared by shareholders: the rollover risk. In other words, when shareholders are solely able to substitute debt under worse conditions than the precedent ones, they are facing rollover risk, because they are forced to buy more expensive debt. The authors also assumed that the default barrier is determined by shareholders' strategic choices: a certain level of coupon expenses implies that shareholders leave their firm earlier in this framework than in the framework suggested by (Leland, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, 1994). Furthermore, Leland and Toft showed that distress costs are important in the determination of the endogenous default barrier contrarily to Leland's paper. The approach discussed in (Leland & Toft, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, 1996) delivered closer results to historical credit and default data.

Previously, the evolution of a company's value was assessed through a diffusion process. By following this type of processes, the variability around the future value of assets has a positive relation with time, which means that it is impossible that the value of assets changes drastically in a short period. In the early 2000s and for the first time, (Zhou, 2001) used a jump-diffusion processes. The latter processes, as their name suggest, allow sudden jumps to a firm value. Large shocks might happen for example in crisis' times or when a sudden information regarding a company becomes public. As the jump-diffusion process allows firm's value to jump below the default barrier, the value of the company at default can change (Zhou, 2001).

In the same year, Robert Goldstein, Nengjiu Ju and Hayne Leland published a paper that had an important added value in the research field. (Goldstein, Ju, & Leland, 2001), GJL hereafter, compare the company to a project that generates earnings in a continuous way with some agents (shareholders, creditors, government, distress costs) having claims on this project. GJL derive the dynamics of the project value in the case where earnings follow a geometric Brownian motion (GBM) and show that their project value also follows a GBM. The main difference between their model and Leland 1994 paper is that they do not assume that there is a traded security corresponding to the project value. As result, preferences matter to determine the value of contingent claims. They assume nevertheless that the project value is observable and that there is a liquid market for contingent claims, implying that their prices must be related. Later in the paper, the authors consider the case where the company can increase its indebtedness depending on the evolution of the income of the project. Consequently, it is ideal for the firm to issue less debt in its early life. This is closer to what happens usually. Additionally, this idea increases shareholders' willingness to hold the company for a longer period. The two previously stated factors, result in a lower default barrier than the one generated in a frame characterized by a fixed coupon, (Goldstein, Ju, & Leland, 2001). The model is also very interesting as it relaxes the assumption that the firm is liquidated in a predefined date.

(Eisdorfer, Goyal, & Zhdanov, 2019) affirm that firms issue debt and bear fixed operating costs. In this paper, the sum of selling, administrative and general expenses is a proxy for fixed expenses. The authors created a structural model that determines the value of the option to default. Their model assumes that because of the presence of the fixed costs component, equity holders might abandon operations if those costs become too high, even in the case where debt is null. They concluded that generally the value of options to default are not fully embedded in equities' values which means equities are not estimated correctly in the markets.In 2003, (Crosbie & Bohn, 2003) wrote a paper that explained the Moody's KMV approach. . They noticed that (Merton, 1974) did not include steps for calibrating the model and did not discuss its performance for default prediction. Based on Moody's database, they focused their research on this topic. (Merton, 1974) required the estimation of the value, the standard deviation of assets and the default barrier. The original model considered that the default barrier is equal to the book value of debt given that the company has to pay it. (Crosbie & Bohn, 2003) thought that one should give less weight to long-term debt because it would be paid a long time in the future. Similarly, one should give higher debt to short-term debt because it would mature in a short horizon. After fixing the default trigger, they estimated the asset value based on the market capitalization and estimated the volatility of asset return by using an iterative approach. After calibrating the model, they calculated the distance to default (DD) of each firm. The distance to default was defined as the log-difference between the expected value of assets at maturity and the default level expressed in the number of standard deviations of the assets' return. They discuss that DD is a strong assessor of a firm's solvability even though the normal distribution cannot transform it into probabilities of default (PD). Finally, they claim whether a company will default or not in a certain year based on the default ratio of companies in their database characterized by the same DD. As the study is based on confidential information, the results cannot be replicated. However, the Moody's distribution in general results in higher PD than the normal distribution for high DD observations and lower PD than the normal distribution for low DD observations. The method of probability estimation was introduced by KMV and was further explained in the technical paper by Crosbie and Bohn. This approach combined with the Vasieck-Kealhofer model is referred to as Moody's MKMV Expected Default Frequency or MKMV EDF. It is an approach elaborated by Moody's to determine the probability of default in the short-run.

2.2 Structural Models Application' in the Banking Sector

(Pennacchi, 2010) introduced a structural credit risk model that values banks' contingent capital and shareholders' equity. The paper studies the evolution deposits' equilibrium, contingent capital and equity while taking into consideration different parameters that define banks' risk as well the terms of the contingent capital. The model's state variables are the current asset to deposits ratio and the short-term default free interest rate. The researcher assumed that assets' return evolves through a jump-diffusion process in a framework where the bank has short-run deposits, equity, and coupon contingent bonds. He studied the mechanism of pricing contingent capital during crisis and confirmed the evolution of credit spreads using a sample of banks. He also compared a sample of risk-taking banks that issued varied contingent claims to another sample of risk-averse banks that issued only non-convertible subordinated debt to understand the incentives of taking risk. He concluded that banks that took more risk saw in contingent capital an opportunity to deal with financial distress especially when it occurs at the early life of the company.

It is also primordial to accurately estimate default probabilities because of the increasing globalization of the economy. Structural models are interesting not only because they allow the estimation of PD but also because they can predict firms that need a bailout and can estimate the costs of the operation. (Correia, Dubiel-Teleszynski, & Javier Población García, 2017) determined the time periods of a high probability of default and the costs related to that. They claimed that injection of equity, insurance deposits and loan guarantees are the major expenses related to a bank bailout. Those conclusions can be used by decision makers to optimize their regulations in the times of crisis.

3 Model

3.1 General Explanation of the Model

I will follow the EBIT-based model developed by (Goldstein, Ju, & Leland, 2001) in my dissertation. Even though in the previously mentioned paper the authors proposed a version with an option to issue further debt in the future and another one without that option, I will only consider the version of the model without the option. The EBIT-based model was initially created to decide on the optimal leverage level, however, with small modifications, it can be useful to estimate the probability of default.

(Goldstein, Ju, & Leland, 2001) consider a firm that has a project that generates a perpetual payout flow (EBIT). The project is perpetual contrarily to the firm that can be closed. A nice metaphor to understand this idea is to compare the project to a water spring. Let's consider a firm that sells water that comes from a water a spring. If at some time *t* the firm closes, the water spring will continue running just like the project.

The EBIT dynamics of the project follow the process below:

Equation 1

$$\frac{d\delta}{\delta} = \mu_p dt + \sigma dz$$

Here, μ_p is a constant that represents the drift of the project while σ is a constant that defines the volatility of the projects' returns. dz is the variation of Wiener Process or Brownian Motion. The Brownian motion is a stochastic process with continuous paths meaning that the process does not show jumps between levels. The variation of the Brownian motion is known to be Normal-distributed with mean 0 and variance equal to the variation of time. The process in Equation 1 is known as the geometric Brownian motion (GBM) because the rates of change (or shocks) to δ follow a normal distribution.

Assuming the non-existence of any arbitrage opportunity and following the risk neutral approach, it is possible to value any asset by discounting the future cash flows using the risk-free interest rate. Therefore, to estimate the value of the project under the risk neutral assumption, we apply the following equation:

Equation 2

$$V(t) = E_t^Q \int_t^{\infty} (ds\delta_s e^{-r}) = \frac{\delta_t}{r - \mu'} = \frac{\delta_t}{r + \theta\sigma - \mu_p}$$

where r is the risk-free rate and is assumed as constant. $\mu_Q = (\mu_p - \theta\sigma)$ is defined as the risk-neutral drift of the project and is assumed constant as well (Girsanov, 1960). In opposition to (Black & Scholes, The pricing of options and corporate liabilities. Journal of political, 1973) model $\mu_p - \theta\sigma$ is not the risk-free rate. We define θ as the market price of risk which is a measure of additional return that investors require to bear risk.

The project value dynamics can be assessed by applying Ito's lemma to project value function as mentioned in Equation 3.

Equation 3

$$\frac{dV}{V} = \mu_Q \mathrm{dt} + \sigma dz^Q$$

The previous equation is written under measure Q which is a probability measure that has the specificity of giving high probabilities to bad events. In contrast with measure Q, measure P is the usual and known probability measure. The previous expression can be written under measure P.

Equation 4

$$\frac{dV}{V} = \mu_p \mathrm{dt} + \sigma dz^P$$

We notice that V and δ follow a GBM under measure P from equations 1 and 4. The same is also valid under measure Q. The difference is only in the drift. Measure Q is used for pricing. Measure P is used to compute real-world probabilities.

We can now define the payout ratio of the project as EBIT over the project value as in Equation 5:

Equation 5

$$k = \frac{\delta_t}{V_t}$$

The substitution of k in Equation 2 proves that the payout ratio is simply the risk-free rate minus the risk-neutral drift of the project and, as result, k must be a constant because all terms in the expression $r + \theta\sigma - \mu_p$ are also constants.

Equation 6

$$k = \frac{\delta_t}{V_t} = r - \mu_Q$$

By rearranging Equation 6 we obtain:

Equation 7

$$\mu_Q = \mathrm{r} - \mathrm{k}$$

We adapt slightly the Equation 3 to obtain the dynamics of the project value:

Equation 8

$$\frac{dV}{V} = (r - k)dt + \sigma dW_t^Q$$

We note that μ_P can be estimated using an interative approach, while θ can be estimated using the CAPM model. However, determining the previously stated two variables is complex. In fact it is difficult to predict the growth rate of the EBIT and the right risk premium, so it is more convenient to substitute $\mu_p - \theta\sigma$ by $r - k$ and determine both r and k.

Numerous papers used the previous risk-neutral dynamics prior to (Goldstein, Ju, & Leland, 2001). For instance we cite (Leland & Toft, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, 1996) , (Leland, Agency Costs, Risk Management, and Capital Structure, 1998) and (Collin-Dufresne & Goldstein, 2001). They were all motivated by the assumption that a firm pays a fraction of its assets as dividends and coupons. Thus, deducting k from μ controls for those payouts and avoids assets' divergence from the barrier at an increased velocity. In contrast to GJL, in neither of these papers the authors consider that the project value is not tradable.

GJL assume that the enterprise issues a perpetual bond that pays a constant coupon C. This coupon must be paid no matter the project payout. More specifically, when the project payout is not enough to cover the totality of the coupon C, shareholders must pay the difference between the coupon C and what can be paid using the project payout. Shareholders have to bear this difference because the model does not have a cash buffer. In addition, shareholders define a firm level that triggers the liquidation of the firm when reached. This level is known in the literature as the endogenous default barrier V_B . It is endogenous because it is set by the shareholders within the model.

All claims to the project earnings can be priced by solving Equation 9, which is a non-homogenous partial differential equation (PDE) of second order. The equation is non-homogenous because of the existence of the constant P .

Equation 9

$$(r-k)VF_v+\frac{\sigma^2}{2}V^2+F_{vv}+F_t+P=rF$$

P represents the payout related to any claim. The claim can be for instance all EBIT or δ before bankruptcy. In this case P is simply equal to δ. In another situation, the claim can be on all coupons before default. Here, P is equal to C.

Because the value of all claims we are interested in are independent of time, the previous equation can be simplified to a second order ordinary differential equation (ODE):

Equation 10

$$0=(r-k)VF_v+\frac{\sigma^2}{2}V^2F_{vv}+P-rF$$

The solution of Equation 10 is the sum of a general solution of the homogeneous equation (without the payout) and a particular solution to the non-homogenous equation. In other words, we want to find the function F that satisfies the condition of the previous equation and the specificities of the security that we want to price.

The general solution to the homogeneous equation (when P is null) is:

Equation 11

$$F_{GS}=A_1V^{-y}+A_2V^{-x}$$

x and y are defined below.

Equation 12

$$x = \frac{1}{\sigma^2}[\left(\mu_Q - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu_Q - \frac{\sigma^2}{2}\right) + 2r\sigma^2}]$$

Equation 13

$$y = \frac{1}{\sigma^2}[\left(\mu_Q - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu_Q - \frac{\sigma^2}{2}\right) + 2r\sigma^2}]$$

The previously defined F_{GS} does not take into account the cash flow payouts. A_1 and A_2 are constants and can be found through the boundary conditions of the particular claim that we want to price. I will start by explaining how one can price the total value of the payout flow until the barrier V_B is reached.

For this, (Goldstein, Ju, & Leland, 2001) define first $P_B(V)$, which is understood as a claim that pays $1 if the company's value reaches the barrier of default V_B. The value of $P_B(V)$ does not have payouts associated and thus can be found as the solution of a homogeneous equation.

Equation 14

$$P_B(V) = A_1 V^{-y} + A_2 V^{-x}$$

From the definition of this claim, it is known that:

Equation 15

$$\lim_{V \to V_B} P_B(V) = 1$$

In words, the closer the value of the firm is to the default barrier, the closer is the value of the claim to one.

Besides, the higher is the value of the company, the safer it is from default, so the value of the claim approaches a null value or:

Equation 16

$$\lim_{V \to \infty} P_B(V) = 0$$

If we take the reasoning further we get:

Equation 17

$$P_B(V) = (\frac{V}{V_B})^{-x}$$

Equation 17 can be interpreted as the distance between the assets and the barrier. We notice also a similarity between this equation and the one in (Leland, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, 1994).

The authors introduce another claim: the claim related to all the intertemporal cashflows of the project. As we know, if the firm produces positive cashflows, shareholders, the government and debtholders are the stakeholders that receive all the payout through dividends, taxes and coupons respectively.

We introduce V_{solv} as the total value of the previously stated claims. We replace P by $(K \times v)$ in Equation 10. V_{solv} will be determined as the following where V is a particular solution to the ODE.

Equation 18

$$V_{solv} = V + A_1 V^{-y} + A_2 V^{-x}$$

A_1 and A_2 can again be determined by imposing boundary condition:

Equation 19

$$\lim_{V \to \infty} V_{solv} = V$$

Equation 20

$$\lim_{V \to V_B} V_{solv} = 0$$

Thus, V_{solv} is:

Equation 21

$$V_{solv} = V - V_B P_B(V)$$

Following the same rationale as for V_{solv} the value of the claim of interest payments is:

Equation 22

$$V_{int} = \frac{C}{r}\,[1 - P_B(V)]$$

The following three equations determine the claims of shareholders, the government, and debtholders respectively:

Equation 23

$$E_{solv}(V) = \left(1 - \tau_{eff}\right)(V_{solv} - V_{int})$$

Equation 24

$$G_{solv}(V) = \tau_{eff}(V_{solv} - V_{int}) + \tau_{int}V_{int}$$

Equation 25

$$D_{solv}(V) = (1 - \tau_i)V_{int}$$

With τ_{eff} represents the effective tax rate [1] and τ_i represents the tax on interest.

It is important to note that the value of the three claims is equal to V_{solv}.

As mentioned previously, default happens when the project hits the barrier V_B. The barrier can be determined by finding the solution to an optimal stopping time problem. We note here the similarity between the choice of shareholders to give up the firm and an investor determining when it is optimal to exercise an American put option. In fact, defaulting can be seen as exercising a put option that at exercise enables the sale of the company. Both the investor and shareholders want to determine what is the optimal time to exercise the option or to sell the company. This type of optimal stopping time problems can be resolved through the application of the smooth pasting condition.

The barrier is determined by the shareholders as:

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[1] Please refer to section 5.2.4 for the details of calculation of the effective tax rate.

Equation 26

$$\left.\frac{\partial E}{\partial V}\right|_{V=V_B} = 0$$

Solving, it leads to:

Equation 27

$$V_B^* = \lambda \frac{C^*}{r'}$$

With

- $\lambda = \frac{x}{x+1}$
- C^* is the optimal leverage, achieved by solving the maximum function of the sum of equity and debt values.

We note here that the default barrier is dependent on taxes through the risk-free rate and that x is affected by the payout ratio k. We also notice that the assets' value is endogenous contrarily to the previous model by (Leland, Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, 1994). Consequently, any change on assets' value would affect the distance to the barrier.

It is mandatory as well to define the expected return on assets, μ_a. This variable will be used to calculate the probability of default and the distance of default later.

Equation 28

$$\mu_a = r + \theta \times \sigma_a$$

Where: σ_a represents the standard deviation of assets.

As the objective of (Goldstein, Ju, & Leland, 2001) is not to determine the probability of default (PD), they didn't provide any formula for that. However, according to (Forte & Lidija, 2012), the physical probability of survival at time t can be calculated as the following:

Equation 29

$$P_{nd}(\sigma, \mu_v) = \Phi\left[\frac{\left(\mu_P - \frac{{\sigma_a}^2}{2}\right)T - \ln\left(\frac{V_B}{V_t}\right)}{\sigma_a\sqrt{T}}\right] - e^{\frac{2}{\sigma^2}\left(\mu_P - \frac{\sigma^2}{2}\right)\ln\left(\frac{V_B}{V_1}\right)}\,\Phi\left[\frac{\left(\mu_P - \frac{{\sigma_a}^2}{2}\right)T + \ln\left(\frac{V_B}{V_t}\right)}{\sigma_a\sqrt{T}}\right]$$

Where: $\mu_P = \mu_a - k$

The second term of Equation 29 represents the probability of hitting the barrier between t_0 and T. Probability of default is known as the probability of survival minus one.

We consider also the distance to default measure in the model, computed as the following:

Equation 30

$$DD = -\frac{\log\left(\frac{V}{V_B}\right) + \left(\mu_P - \frac{\sigma_a}{2}\right)T}{\sigma_a\sqrt{T}}$$

3.2 Adaptation of GJL Model to the Banking Sector

(Goldstein, Ju, & Leland, 2001) consider that the coupon payment is constant. This common simplifying assumption might be acceptable for firms operating in many sectors, however, it is a strong assumption for the banking sector. This being said, assuming some type of fixed costs is essential to determine the endogenous default barrier. In the literature, financial fixed costs are not the only type of fixed costs used to set the default barrier. In (Eisdorfer, Goyal, & Zhdanov, 2019) both financial and operating fixed costs are used to set the default barrier. In the dissertation, constant interest payments are substituted by non-interest expenses, which are considered to be fixed (i.e. mandatory) payments. Those non-interest expenses represent the operational expenses for banks. If a bank keeps the same business model and does not incur unexpected expenses (due to restructuring for example) the assumption of constant non-interest expenses holds. As a result, Equation 22 will be rewritten as the following:

Equation 31

$$V_{non-i} = \frac{Non - interest}{r}\left[1 - P_B(V)\right]$$

Also, Equation 23 will be written as:

Equation 32

$$E(V) = \left(1 - \tau_{eff}\right)(V_{solv} - V_{non-int})$$

Finally, I rewrote Equation 27 as:

Equation 33

$$V_B = \lambda \frac{Non - interest}{r}$$

Another amendment of the initial model in necessary at this step. The state variable δ in this dissertation is the sum of earnings before taxes EBT. In opposition to GJL model who took earnings before interest and taxes EBIT as a state variable, I will include interest costs in the state variable as they are highly variable in the case of banks. GJL model assumes that the state variable δ follows a GBM meaning that variable has a log-normal distribution.

4 Model Calibration

4.1 Parameters Calibrated Through the Iterative Approach

Before applying the GJL model to estimate the probability of default, few steps still need to be done in order to calibrate the model. Several calibrating methods are suggested in the academia. The first approach was suggested by (Jones, Mason, & Rosenfeld, 1984) and is named the proxy approach. They used it to calibrate the Merton model. The idea was to estimate the assets value as the sum of the market value of equity and the book value of debt. Then, the σ_a is computed as the standard deviation of log returns of assets. The drawbacks of this method are the following: it estimates a backward looking σ_a and does not consider the put option on debt. In 1986, (Ronn & Verma, 1986) suggested another calibration method: the system of equation approach. They estimated assets and their volatility by solving a system of equations at each moment in time. The system of equations is composed of an equity value equation and an equity volatility equation where the unknowns are assets value and volatility. In this case, the equity volatility can be calculated in two ways: either by using historical equity observations or by using option implied volatilities. Estimating σ_e through historical observations unfortunately leads to backward looking value. Estimating implied equity volatility using the value of a call option is not perfect either. In fact, all companies do not have traded options. Added to that, the implied volatility of the option varies depending on its maturity and its moneyness. We note also that the system of equations method does not give an interval of confidence for the estimation. Ultimately, resolving the system of equations at each moment in time leads to different σ_a values through moments in time which violates the assumption of constant σ_a through time. Later, in 1994, (Duan, 1994) introduced the maximum likelihood calibrating approach. The rationale behind this method is to find the numeric values of μ_a and σ_a that imply that the observed values of assets are the most likely from all possible values of assets estimated by the model. Here the values of assets are not historical market values, rather they are found by solving a non linear equation of equity pricing conditional on a certain value of asset's volatility. The maximum likelihood approach provides confidence intervals for the first and second moments of the assets' distribution.

(Vassalou & Xing, 2004) used another approach to calibrate the Merton model: the iterative approach. I will follow the same steps to calibrate the GJL model by estimating the volatility of assets and the payout ratio for every bank. This method leads to a constant volatility of assets which is in line the Merton model's assumption. However, it does not allow for the estimation of a confidence interval. The following section explains the steps of this method.

4.1.1 Determination of Assets' Volatility σ_a

The iterative approach's steps are:

1. Determining the initial value for σ_a. In our case, the initial σ_a will be equal to the historical volatility of equity .
2. Find the asset value time series by solving a non-linear equation given by Equation 31 (Equity valuation equation), conditional on the initial σ_a estimate, and observed equity value and.
3. Replace the previously estimated σ_a by the standard deviation of the log variation of asset value time series found in step 2.
4. Repeat steps 2 and 3 until the difference in σ_a estimates between two consecutive iterations is inferior to the error tolerance level.

As in (Vassalou & Xing, 2004), the error tolerance level for σ_a will be set equal to 0.0001.

After running the iterative approach and finding the final σ_a, I run it again few more times but with arbitrary different starting values for σ_a. The final σ_a was the same in all iterations. However, setting a relatively close starting value helps the algorithm to be more efficient and to find the final σ_a faster.

4.1.2 Determination of the Payout Ratio k

The calibration's steps of k are the following:

1. Set the initial value of k equal to 5%.
2. Apply the iterative approach presented in (Vassalou & Xing, 2004) to estimate the project value and the volatility of assets.
3. Calculate k as the mean of EBT over the asset values estimated in step 2.
4. Repeat steps 2 and 3 until the difference in k estimates between two consecutive iterations is inferior to the error tolerance. As for σ_a , the error tolerance level was set at 0.0001.

I checked again the sensitivity of the final payout ratio to different starting values of k. After running the algorithm for different starting arbitrary values, I confirmed the final payout ratio was the same no matter the value set in the first step.

In line with the model assumptions, the abovementioned calibration process leads to constant levels of k and σ_a which I will assume valid for the whole period of study. Condition on these estimates, one can determine asset values using Equation 31.

4.2 Determination of the Market Price of Risk θ

The probability of default of each firm can be calculated either under measure Q or under measure P. Measure Q is used for pricing. Measure P is the true probability measure, and thus the only one relevant to compare with credit ratings provided by rating agencies. The computation of probabilities under the physical measure requires however the determination of the market price of risk. Previous researchers suggest many approaches to calculate this parameter. For instance, (Wu, Zhou, & Wang, 2018) suggest a method that relies on joint data on the option and on the underlying asset price to estimate the volatility of the market as well as market return under the G.A.R.C.H diffusion model assumptions. Besides, (Ronn & Kolos, 2008) studied the market price of risk in the energy sector and suggest determining MRP through the spot and future prices of energy. In this dissertation I will follow the capital asset model (CAPM) because it is the most straightforward. The CAPM establishes that the expected return on assets is solely determined by its level of systematic risk. The model is commonly applied to stocks. Equation 33 represents the CAPM model.

Equation 34

$$r_i = rf + \beta_i \times (r_m - rf).$$

In Equation 33, r_i is the expected rate of return on investment, rf is the risk-free rate and β_i is beta of the investment. β_i is also interpreted as a proxy for the systematic risk of a firm. In our case the investment is a stock.

Previous papers showed that under (Goldstein, Ju, & Leland, 2001) model, the application of Ito's lemma to the equity valuation expression results in the below formula for the expected return on equity, μ_e:

Equation 35

$$\mu_e = rf + \theta \times \sigma_e.$$

In Equation 34, θ represents the market price of risk and σ_e represents the volatility of equity. A quick comparison between the expressions of r_i and μ_e depicts that $r_i - rf$ and $r_m - rf$ are the same. As a result, $\beta_i \times (r_m - rf)$ and $\theta \times \sigma_e$ are also the same. Finally, substituting $\mu_e - rf$ with $\beta_i \times EQRP$ leads to the market price of risk formula:

Equation 36

$$\theta = \frac{\beta_i \times EQRP}{\sigma_e}$$

The market price of risk is more known as Sharpe ratio which is the risk-adjusted extra mean return earned compared to the risk-free rate for each unit of volatility.

From a theoretical standpoint, it is important to note that σ_e in Equation 34 varies according to the project's value and according to the volatility of the project's returns. We will nevertheless ignore this theoretical result and estimate σ_e as the standard deviation of equity returns.

5 Data

5.1 Period of Study and Banks' Selection

The period of study chosen starts in 2001 and lasts until 2020. Furthermore, the reference model will be applied on 5 banks in Europe, namely:

Table 1 : Banks & Bloomberg Tickers

Banks	Bloomberg Tickers
Banco Santander	E:SAN
Deutsche Bank	D:DBK
Banco Bilbao Vizcaya Argentaria	E:BBVA
Commerzbank	D:CBK
Banco De Sabadell	E:BSAB

In the step of choosing my sample of banks, I considered banks that satisfy the assumptions of the model. For instance, I chose banks that have on average a positive EBT during the twenty years of study. I also chose banks with stable non-interest payments. It was very challenging to find banks that satisfy the latter condition. In fact, during twenty years, because of inflation and growth, it was frequent for banks to have growing non-interest expenses. Special events like mergers and acquisition also have a direct impact on non-interest expenses and result in sharp movements. An alternative solution would be to apply the model on reduced time windows like 2 or 3 years. This would imply estimating the payout ratio and the volatility of assets for every time window. It would most probably result in different assets' volatilities and violate the constant assets' volatility assumption. Consequently, I chose to exclude the latter alternative. Additionally, I excluded non-European banks for the simplicity of using the same risk-free rate.

5.2 Parameters Calibrated Through Historical Market and Accounting

5.2.1 Historical Earnings Before Interest & Taxes

I downloaded EBT values for each firm yearly from DataStream as it is a reliable source and simple to use. As EBT is an accounting variable, I considered yearly observations from 31/12/2000 to 31/12/2020. However, as I needed weekly observations, I used the linear interpolation method to fill the missing values between each two consecutive yearly observations.

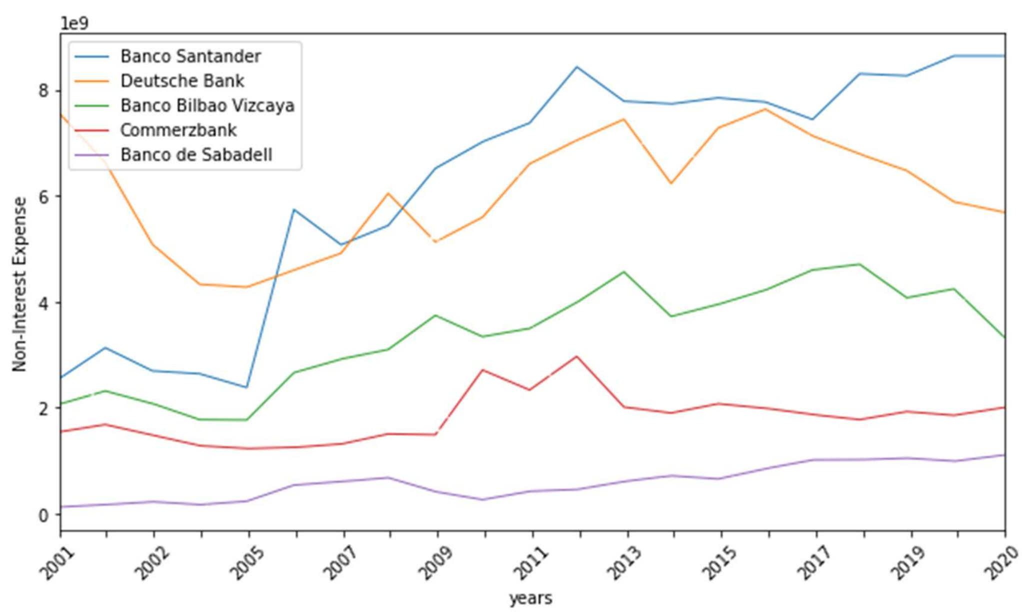
5.2.2 Historical Fixed Costs

In the literature, the default-barrier level is either set endogenously or exogenously. Endogenous barriers are chosen within the model as a result of shareholders or debtholders behavior within the model. In most of the endogenous barrier models, the barrier is chosen by shareholders who decide whether they are willing to inject capital in the firm in case needed. In exogenous barrier models, the barrier is set outside the model and must be calibrated. This can be set as a fraction of nominal debt as proposed by Moodys' or using CDS data. I chose to set it endogenously as it is the approach followed by GJL.

As I am applying the model to firms belonging to the banking sector, I considered using non-interest expenses as a proxy for fixed costs. Non-interest costs in the case of banks represent operational expenses. In particular, they correspond to labor-related expenses, depreciation and amortization, litigation expenses, restructuring expenses, etc… They clearly differ from one bank to another however, they are assumed constant through time for an individual bank. The latter assumption complexifies the calibration of those costs because it is in opposition to what is observed. Notably, inflation and growth of banks result in growing fixed costs in the long run. Furthermore, the literature does not precise what sub-non-interest expenses should be added to the state variable of the model.

Steps that led to weekly observations of non-interest expenses are similar to those that led to weekly EBT values. The graph below depicts this series:

Figure 1 : Evolution of Non-Interest Expenses Between 2001 and 2020

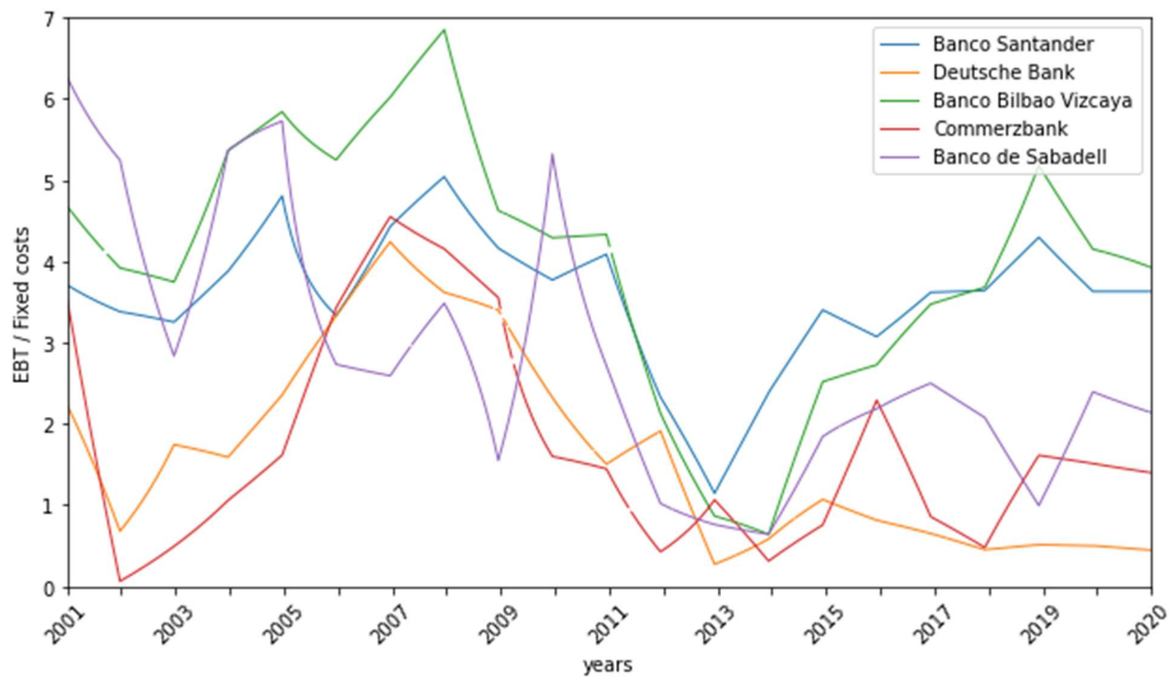


My initial estimates of the model's probabilities of default using non-interest expenses as fixed costs were relatively high compared to probabilities of default provided by credit rating agencies. In particular, the average PD obtained by the model was 2.77% compared to 1.05% by credit rating agencies. In an attempt to remove the gap between the average of PD estimated by credit rating agencies and the one obtained through the model, I decided to multiply non-interest costs by a constant γ and find the level of γ that matches the probabilities of default implied by credit ratings. For consistency reasons, any decrease in non-interest expenses must be accounted by an equal decrease in the state variable. Hence, I run the model several times while decreasing the constant at each iteration until the average PD values matched the average PD values by credit rating agencies. This technique cancelled the gap with γ of 10%.

The optimized γ of 10% allows the model probabilities of default to match the ones implied by credit rating agencies while keeping the mean DD (computed using the symmetric of the inverse normal of the probability of default) also close to the one implied by credit rating agencies.

In the next step, calculated the ratio EBT over fixed costs. Figure 2 shows the evolution of this ratio during the period of study for each bank. The ratio depicts, for each bank, how many times EBT can cover operational costs of the bank. The ratio follows the same movement overall with different magnitudes for each bank during the 20 years. Banco Bilabo Vizcaya Argentaria records the highest EBT/fixed costs value of 6.85 in 2008. The minimum value of 0.06 was achieved in 2002 by Commerzbank.

Figure 2 : Evolution of EBT over Fixed Costs per Bank Between 2001 and 2020

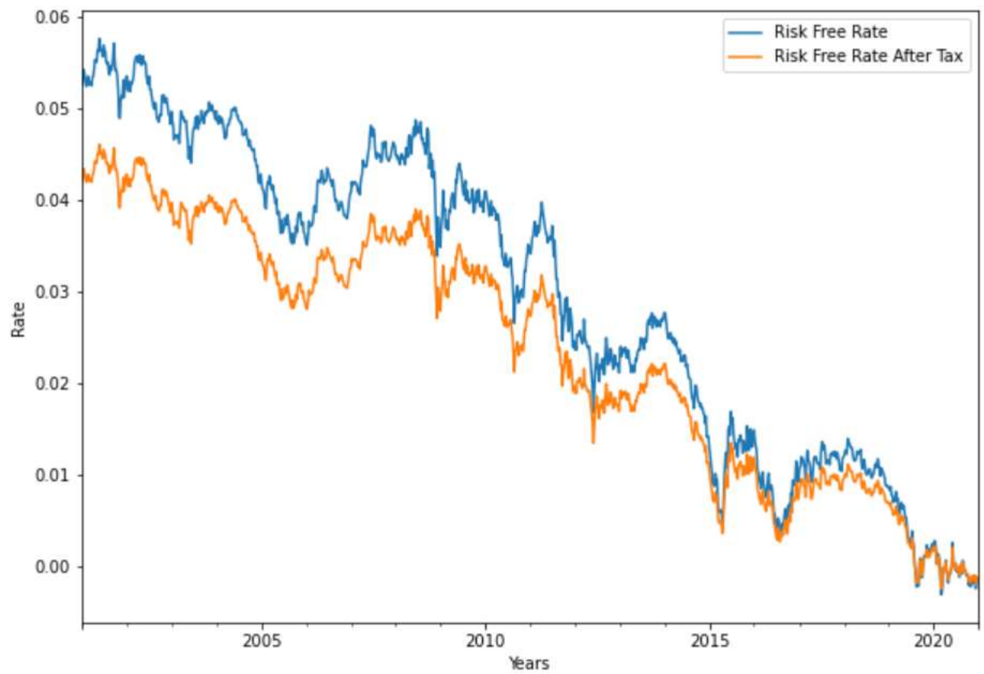


5.2.3 Historical Interest Rate

The 30Y German bond yield was used as a proxy for the constant interest rate assumed by the model. Therefore, weekly observations of this variable were downloaded from DataStream. As risk-free rate in the model is calculated after taxes, it was necessary to apply a tax deduction on the values observed. After researching in the previous published papers, I found that 20% is a frequently used tax rate so I chose to use it as well.

Data showed that the tax rate varied slightly through the period of study. Yet, for consistency reasons and in order to be in line with the model's assumptions, I chose to consider a constant rate. Figure 3 illustrates the downward overall trend of the risk free between 2001 and 2021. One of the reasons behind this trend is the increase in savings (Dossche, Boeckx, & Cordemans, 2013).

Figure 3 : The Evolution of the Risk-Free Rate in Europe Between 2001 and 2021.



5.2.4 Effective Tax Rate

The reference model requires the determination of an effective tax rate on corporate profit in addition to the tax rate on interest. Looking at historical tax data, I noticed that tax on corporate income and on dividends fluctuate for each bank during the period of study. However, for simplicity, I will keep the two rates constant through the considered period. I will assume that each rate is equal to 20%. This assumption is similar to the one made by (Goldstein, Ju, & Leland, 2001) and confirmed by (Asen, 2021). Thus, the following expression shows the calculation of the effective tax rate:

Equation 37

$$1 - Eff_{tax} = (1 - Corp_{tax}) \times (1 - Div_{tax})$$

With:

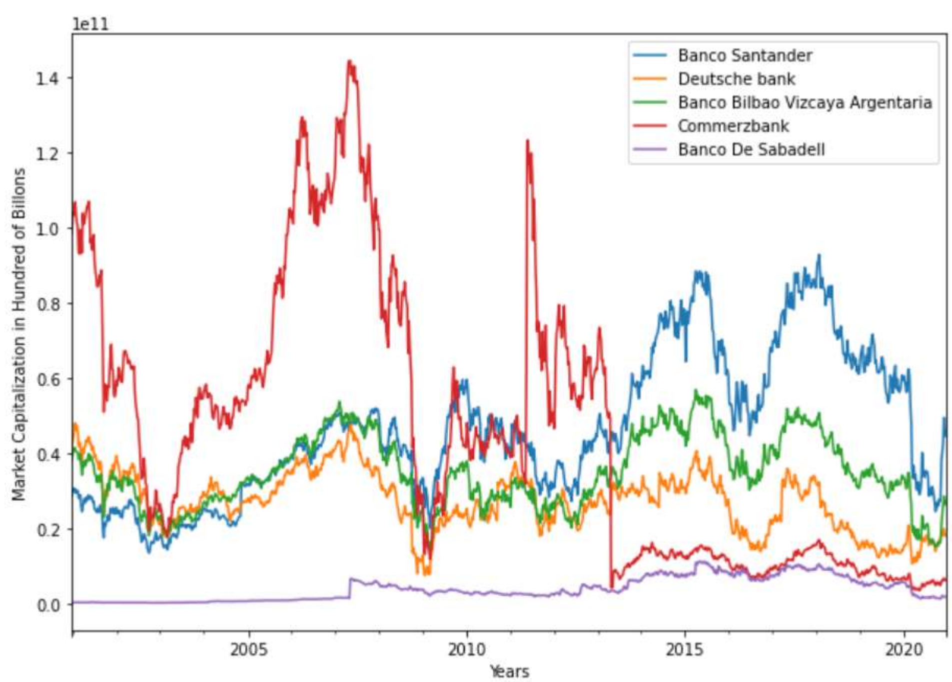
- $Corp_{tax}$: tax paid by shareholders and equal to 20%
- Div_{tax} : tax paid by shareholders and equal to 20%

Consequently $Eff_{tax} = 36\%$

5.2.5 Historical Equity

The value of equity was calculated by multiplying the market price of one share issued by a specific bank by the number of shares outstanding. Again, weekly observations were retrieved from DataStream. Figure 4 compares the evolution of equity of banks in the sample. It is known that banks do not buyback or issue shares frequently which means that oscillations of each line are mainly due to the change of closing prices of shares. Looking at the graph, it can be seen that the crisis of 2008-2009 was marked by a collapse of the market capitalization of all the banks. Furthermore, Commerzbank records the highest volatility. In fact, its market capitalization severely fluctuated until 2012. The bank also suffered the sharpest drop of market capitalization in the financial crisis of 2008-2009. Indeed, despite the efforts of Commerzbank, it endured high income losses during 2009 which directly decreased the share price. On the other hand, Banco de Sabadell has the lowest and the most stable market capitalization.

Figure 4 : Equity Evolution of the Banks Between 2001 and 2021



5.2.6 Historical Equity Volatility

Historical equity volatility is necessary for the determination of the market price of risk (MRP). It will also be used to set the starting guess for the volatility of assets. This was computed as the annualized standard deviation of weekly equity log returns. I controlled for outliers in this process by winsorizing the time series of log equity changes before computing the standard deviation. This was done so that all the data falls in the range between minus three standard deviations and plus three standard deviations. Annualization was done multiplying the obtained standard deviation by the square root of 52. Table 2 summarizes the values obtained. Besides, Appendix 1 represents in histograms the log equity returns of each bank. We note that the high volatility of Commerzbank is in line with the evolution of equity depicted in Figure 4.

Table 2 : Average Equity Volatility per Bank

Banco Santander	Deutsche Bank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell	Commerzbank
0.3082	0.3560	0.3110	0.3488	0.4509

5.2.7 Historical Beta

I extracted historical quarterly beta values from DataStream. Figure 5 below shows the evolution of historical beta of each bank while Table 3 depicts the mean beta per bank. On average, all betas follow the same trend. In fact, there are two major periods of rise of beta: years following the financial crisis of 2008 and the covid-19 crisis in 2019. In the dissertation, I chose to assume a constant beta value for each bank to avoid unwanted variation of probabilities of default due to the fluctuation of beta. Hence, the average quarterly beta will be considered for each bank. The assumption of constant beta is also justified by the fact that the banks of the sample kept the same major business model pillars during the period of study. As expected, Commerzbank has the highest average beta while Banco de Sabadell has the lowest average.

Figure 5 : Quarterly Beta Evolution of Each Bank Between 2001 and 2020

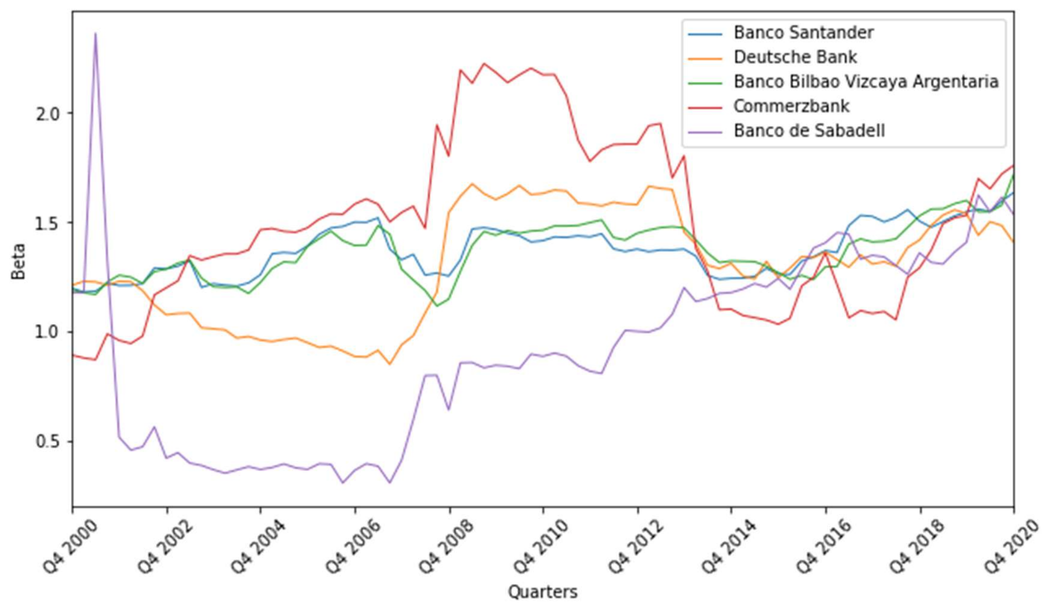


Table 3 : Average Historical Beta per Bank

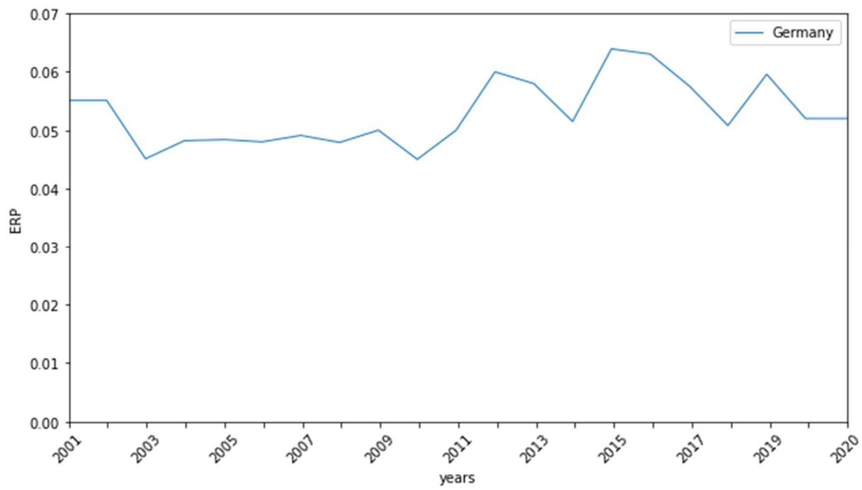
Banco Santander	Deutsche Bank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell	Commerzbank
1.374	1.301	1.371	0.921	1.491

5.2.8 Equity Risk Premium

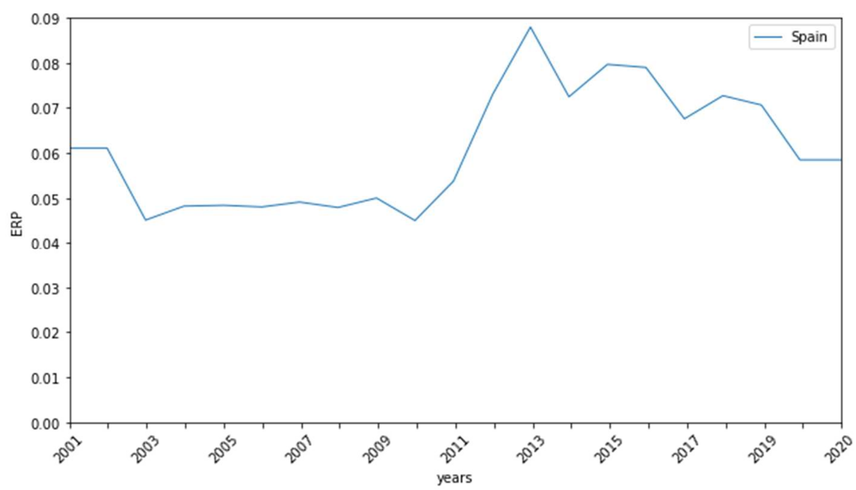
I extracted the yearly equity risk premium (ERP) data for the period of study from professor's Damodaran website. Professor Damodaran's estimates are widely used among practitioners. Panel A and B of Figure 6 compare the evolution of the ERP in the two countries where the banks studied are based:

Figure 6 : Evolution of ERP Between 2001 and 2020 in Germany and Spain

Panel A: Germany



Panel B: Spain



5.2.9 Market Price of Risk

I calculated the market price of risk (MRP) following Equation 35. As previously stated, β and equity volatility σ_e are constant during the period of study so the variation of the market price of risk is only caused by the variation of ERP. Figure 7 describes the MRP of each bank during the study period while Table 4 gives an idea about the average MRP of each bank.

Figure 7 : Evolution of Market Price of Risk Between 2001 and 2021

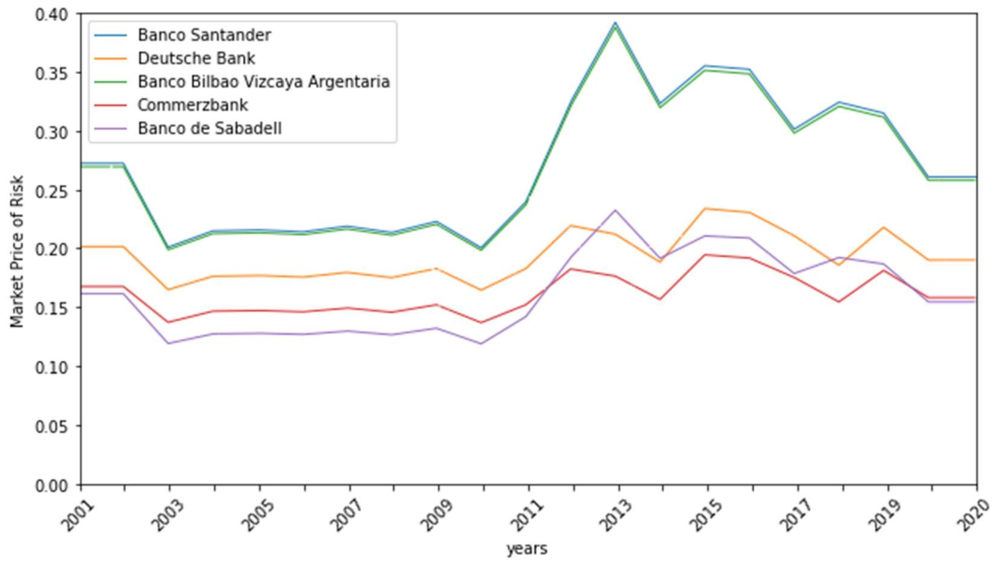


Table 4 : Average Historical Market Price of Risk

Banco Santander	Deutsche Bank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell	Commerzbank
0.2718	0.1933	0.2690	0.1610	0.1608

5.3 Parameters Calibrated Through the Iterative Approach

5.3.1 Assets' Volatility

As mentioned in section 4.1.1, the iterative approach is used to determine the asset volatility. Results are presented in the table below:

Table 5 : Average Assets' Volatility

Banco Santander	Deutsche Bank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell	Commerzbank
0.2686	0.3017	0.2801	0.4283	0.7400

A comparison between asset and equity volatilities shows that the two mean values are almost equal. The average of asset volatility is higher by 0.05 than the \average of equity volatility. This result is nevertheless opposite to my expectations. Due to leverage effect created by fixed operating costs, I expected assets' volatility to be higher than equity volatility. I furthermore aimed to assess a correlation between the volatilities of assets and equity though a regression that compares the two variables. I found a strong positive correlation with a coefficient of determination equal to 0.91. In other words, 91% of the variation of the volatility of equity is explained by the variation the change in asset volatility. In our sample, the higher the business risk, the higher the equity risk.

5.3.2 Payout Ratio

The iterative approach is used again for the determination of the payout ratio k. Steps of this method were explained in detail in section 4.1.2. Furthermore, Table 6 shows the average payout ratio estimated for each of the five banks. We notice that, except for Commerzbank, all the banks have payout ratios close to each other's. A possible explanation is that Commerzbank profits in this period were relatively lower and thus most of its expected return comes from expected asset appreciation rather than payouts. At the same time, the rest of the spectrum had a more stable business, so the higher payout ratio is not surprising.

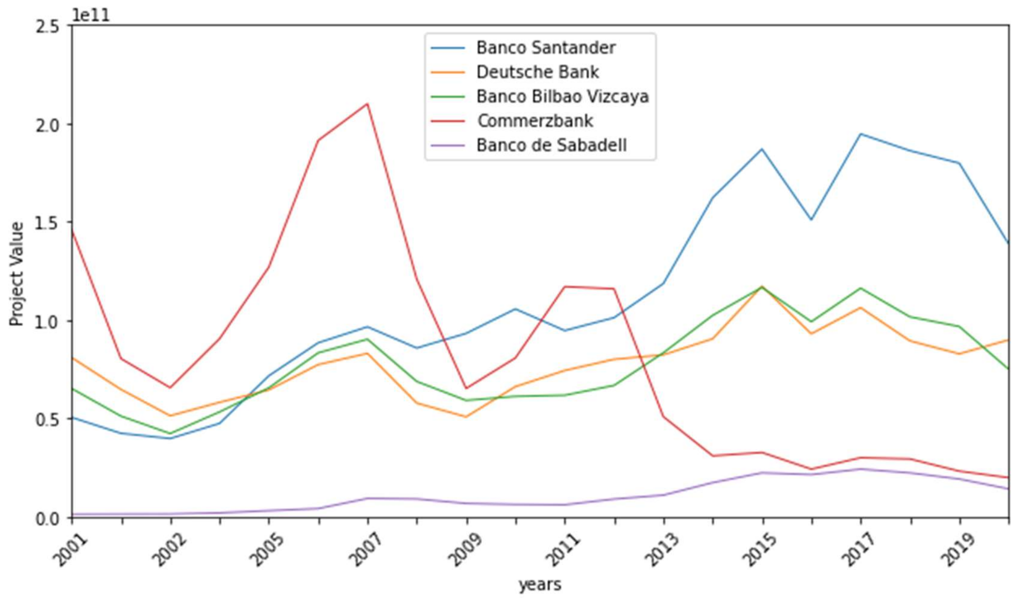
Table 6 : Average Payout Ratio per Bank

Banco Santander	Deutsche Bank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell	Commerzbank
0.0243	0.0154	0.0207	0.0171	0.003

5.3.3 Project Value (or Estimated Assets)

After calibrating the model, it is finally possible to estimate the project value. This variable integrates the market capitalization conditional on the calibrated and observed inputs of the model. Figure 8 shows the evolution of the project value of the five banks. It is noticeable that prior to 2006 there is a high co-movement among all banks. Starting from 2006, all banks follow the same trend except Commerzbank. Additionally, we notice a drop in the lines during the two crisis of 2008 and 2019. It is remarkable that Commerzbank was the most affected by the 2008 crisis.

Figure 8 : Evolution of the Project Value per Bank Between 2001 and 2020



6 Results

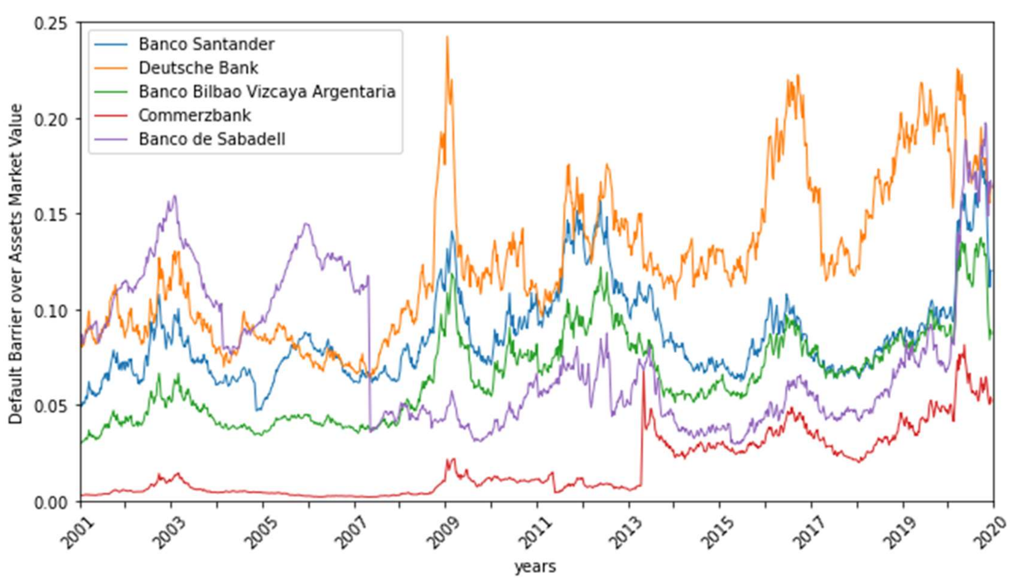
This chapter is split in 2 sections. In the first section, I analyze the probabilities of default that resulted from the application of the structural model. I do this by decomposing them in subcomponents, notably, the ratio between the endogenous barrier and the market value of assets, the drift of the process, and, finally, the distance-to-default. The latter tells us how far the market value of assets is expected to be from the barrier in 5 years, standardized by each bank asset volatility. In the second section, I compare those results to credit ratings given by rating agencies.

6.1 GJL Model Results

6.1.1 Default Barrier over Assets' Market Value

The ratio between the default barrier and the market value of assets measures the distance between the value of assets and the barrier of default without taking into consideration the volatility of assets. Some researchers consider this ratio as a market-based measure of leverage. This ratio clearly ranges between 0 and 1 as a bank would default if the assets fall to a level below the barrier. A value above 1 is not possible because we estimate the asset value using equity data, which is always positive. This ratio has a positive relation with the probability of default and a negative correlation with the distance to default. The graph below compares the evolution of the barrier to assets ratio for all banks. Even though the lines representing all banks do not follow exactly the same movements, we can clearly identify many periods where the trend is the same. For example, the 2009 crisis and the Covid-19 crisis are characterized by a peak of the ratios of the five banks. On one hand, Commerzbank has the lowest Barrier to Assets values through all the period of study. On the other hand, Deutsche bank achieved the highest default Barrier to Assets values during most of the period, especially after 2007. From 2002 to 2007, Banco de Sabadell had the highest Barrier to Assets ratios. During the almost whole period of study, Banco Santander had the second highest values of the ratio followed by Banco Bilbao Vizcaya in the ranking.

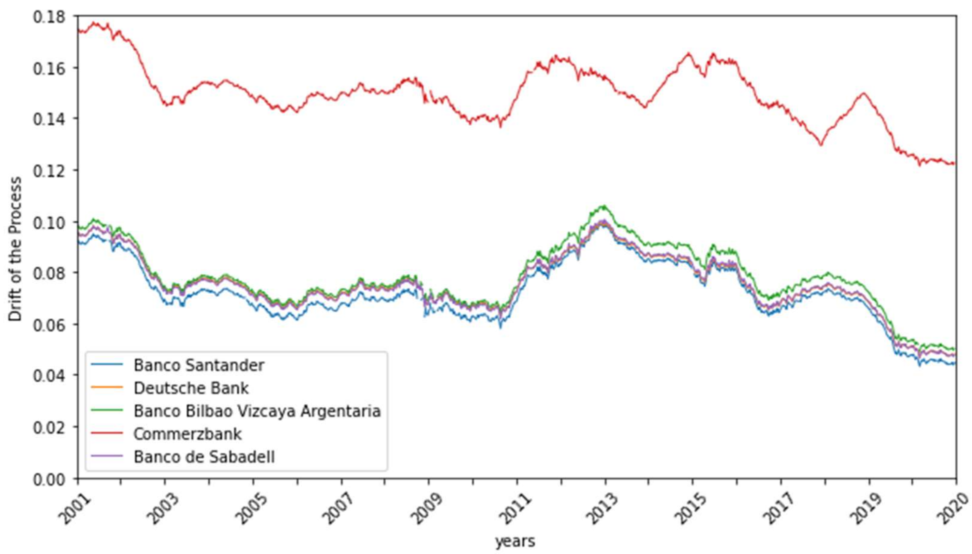
Figure 9 : Evolution of Default Barrier to Assets Market Value Ratio per Bank



6.1.2 Drift of the Process

Figure 10 shows my estimates for the process drifts under measure P (Equation 4). Equivalently, the drift of the process can be seen as the difference between the project discount rate, μ_A, and the payout ratio, k.We notice a big difference between the drift of the process of Commerzbank and all the other banks who have relatively close drifts to each other. We recall that earlier we found that Commerzbank has the lowest payout ratio k.

Figure 10 : Evolution of Drift of the Process per Bank Between 2001 and 2020

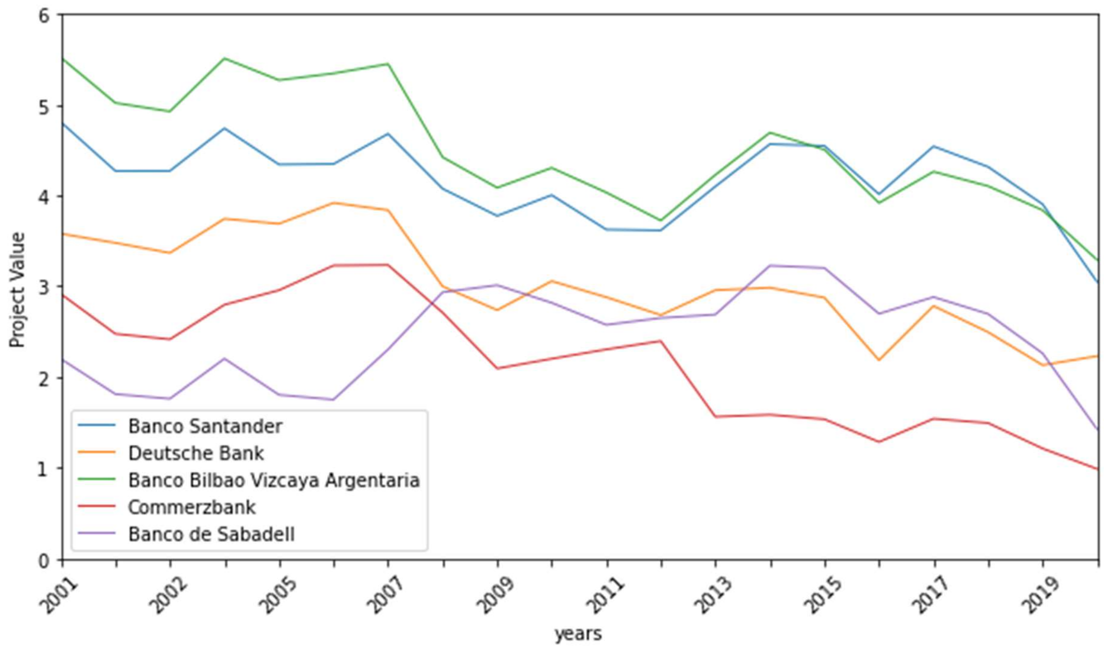


6.1.3 Estimated Distance to Default by the Model

The distance to default metric has won a lot of popularity since the model of Merton was introduced in 1974. It measures the expected distance between the market value of assets and a predefined default barrier, standardized by the asset risk within a fixed number of years. It is interesting because it incorporates in one variable the major characteristics of a firm: the expected volatility of assets as well as market leverage.

I chose to run the DD as well as the PD estimations on a 5 years' time frame. Figure 11 shows my DD estimates for each bank.

Figure 11 : Evolution of Estimated 5-year DD per Bank Between 2001 and 2020



We can see a downward trend for each of the five lines. It is remarkable that all banks reached their minimum in terms of distance to default in 2020. This result shows the importance of the impact of the tight market conditions on banks. In fact, crisis times are generally characterized by low interest rates, which affect negatively the profitability of banks. In simple words, banks make profits by lending to customers and by charging a lending interest rate. Those same banks pay customers a deposit interest rate to encourage them to keep their money in the bank account. Clearly, in this framework the banks' profits are directly linked to the spread between the two rates. In a low interest rate environment, the spread decreases considerably and might even go negative, meaning that the cost of keeping deposits is higher than the income generated through lending. In this case, profits of the banks are negatively impacted by the drop of interest rates.

Subsequently, I continued by calculating the average distance to default to each bank. The results are shown in Table 7.

Table 7 : Average Distance to Default by Bank

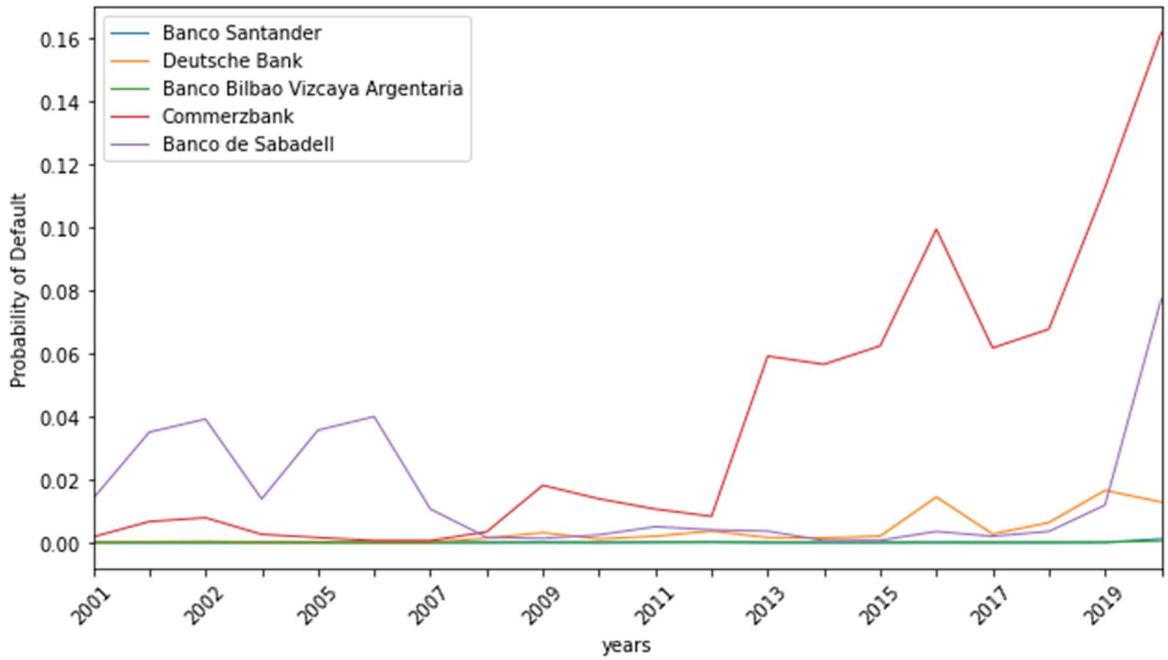
Banco Santander	Deutsche Bank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell	Commerzbank
4.1801	3.0307	4.5232	2.4450	2.1463

The average DDs range between 2.1463 and 4.5232. We notice here that the banks with the lowest distance to default values - namely Banco De Sabadell and Commerzbank - have unexpectedly the lowest barrier over assets ratio values. However, one should not forget that a high asset volatility leads to a low DD. We recall from (Table 5: Average Assets' Volatility) that Banco de Sabadell and Commerzbank are the ones with the highest asset volatility. Consequently, we can say that the low DD values are justified by the high assets' volatility.

6.1.4 Probability of Default

The probability of default is the most common measure of credit risk because it is simple and very intuitive. Same as for DD, I considered 5-years' time horizon when I calculated PD values. The yearly probability of default graph (Figure 12) depicts the general trend for each bank from 2001 until 2020. As it is a yearly graph, it does not show daily shocks to PD (for instance shocks related to the movement of markets) . In other words, the graph is less prone to market noise. It is noticeable that most of the banks recorded yearly PD observations close to zero through the study period. Besides, the average 5-year cumulative PD of the five banks during the 20 years study period is 1.14%. In particular, the yearly means are between 0.12% and 5.07%. It is interesting to notice that the well-known dotcom bubble that hit the financial sectors in the early 2000s had almost no effect on the majority of the banks. Only Banco De Sabadell might have been affected by that bubble. Additionally, the graph highlights sharp increase of PDs in 2016. During that year, the Chinese stock market negative shock as well as the OPEC cut of crude oil production affected the stock markets. Those two events might be indirectly the reason behind the rise of PD in 2016. We note that by far Commerzbank recorded the most significant rise of PD in 2016 to around 9.95%. Historical articles show that the bank went also through deep crisis in that year. During that period, it announced its plan to cut 7300 full time positions until 2020 (Marino, 2015). Moreover, the bank was obliged to enforce its cash position by selling 113.85M stocks that had a value of 1.4B euros in total. Finally, by those times, the bank was fined by 1.33B euros after it was discovered handling transactions for sanctioned countries (Hard Times For Commerzbank, s.d.).

Figure 12 : Evolution of Probabilities of Default Between 2001 and 2020 per Bank



6.2 Credit Rating Results Compared to GJL Model Results

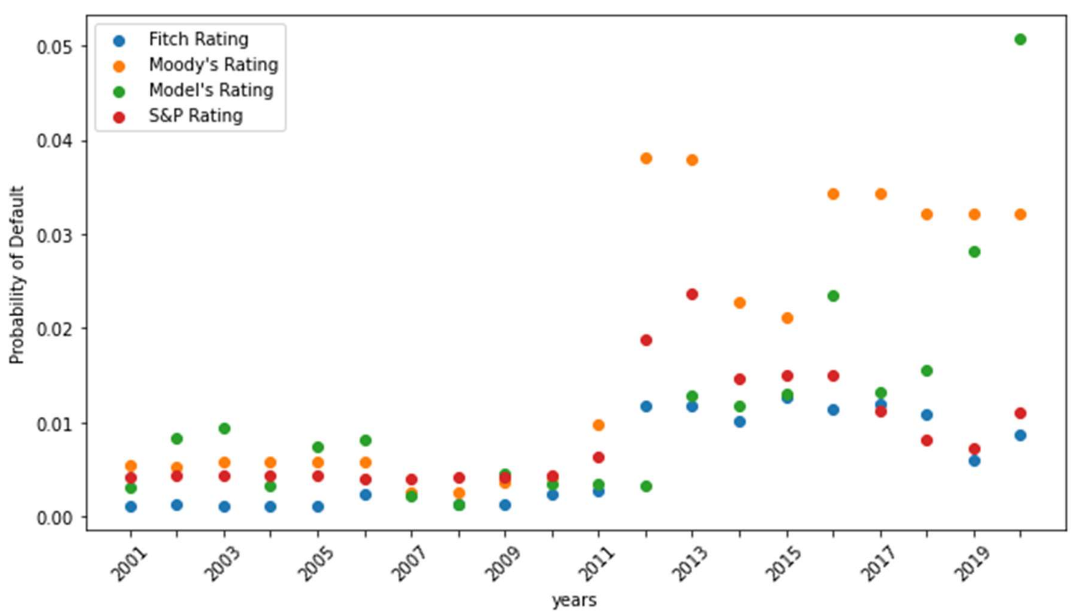
The comparison of my results with those of credit rating agencies is crucial to understand the reliability of the model. To accomplish this task, I will consider the ratings of Moody's, S&P, and Fitch Group who are the three leaders in the credit rating industry. Credit rating agencies have extremely developed methods and have access to a wide range of information that make their estimations taken as benchmarks in the field. However, the fact that they are benchmarks does not mean that they are correct and without any subjectivity. In fact, every credit rating agency might assign a different rating to the same company at the same moment. The model proposed by (Goldstein, Ju, & Leland, 2001) is of course very simple which might not lead to similar results as those suggested by credit rating analysts.

6.2.1 A Comparison Between GJL Model's Probabilities of Default and Credit Rating Agencies Probabilities of Default

As a first step, I retrieved from Eikon Reuters database and from annual reports of the banks their credit ratings according to Moody's, S&P and Fitch. Then, I transformed each credit rating letter and symbol (or number) to a PD using the adequate matrices provided by each agency. Appendices 3, 4, and 5 show the credit rating by bank for each year. It is noticeable that all banks during the study period issued investment grade bonds. In the following step, I averaged the PDs of the five banks combined to finally achieve one PD value per year and per credit rating agency.

On average the model gives a higher PD (1.14%) than Fitch (0.57%) and S&P (0.87%) but lower than Moody's (1.71%). However, as mentioned earlier, by construction, the average PD by the model matches the average PD by credit rating agencies. We can see through the chart that PDs were more or less stable during the first 10 years, while they increased a lot during the last 10 years. This trend was depicted by the credit rating agencies as well as by the GJL model.

Figure 13 : Average Probability of Default per Year by the Model and by Credit Rating Agencies



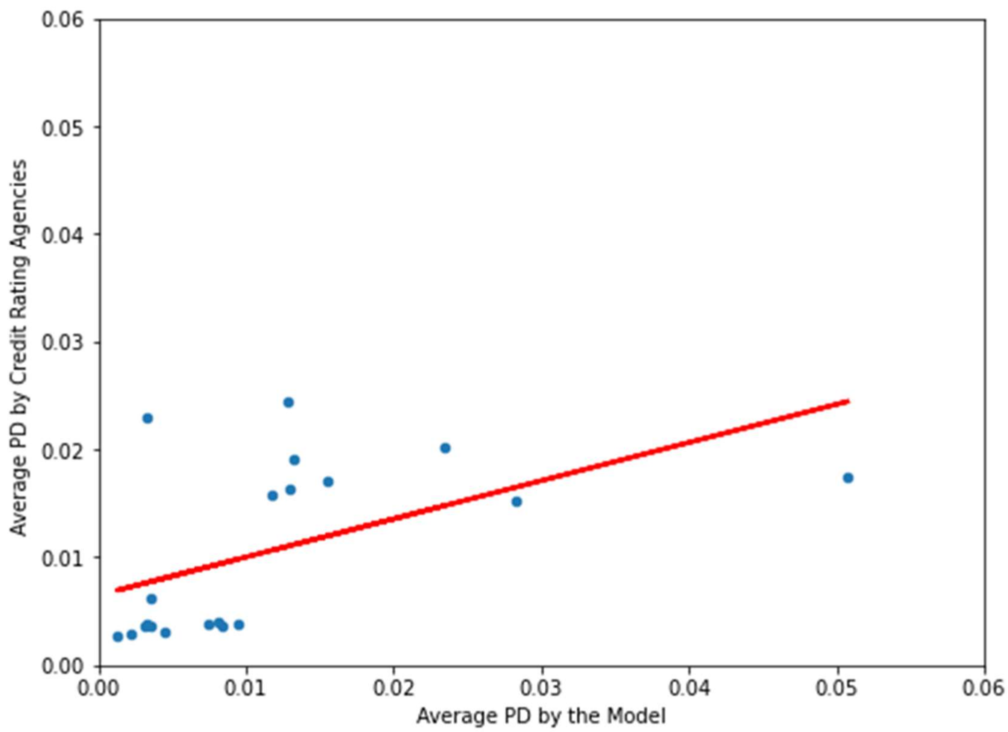
I also run a regression that presents the yearly average PD by agencies compared to the yearly average of the model's PD.

Equation 38 Regression of Average PD by Credit Rating Agencies on Average PD by the Model

$$Average\ PD\ by\ Credit\ Rating\ Agencies_t = 0.35\ Average\ PD\ by\ the\ model_t + 0.006 + \epsilon_t$$

Figure 14 is a graphical representation of Equation 38. The y-axis represents the averages of all credit ratings by professional agencies during each of the 20 years of study and the x-axis represents the same averages but found through our model. The regression suggests a positive relation between the model PDs and the PDs estimated by credit rating agencies. In particular, I found a $R^2$of 0.27 and the p-value of the coefficient in Equation 38 is equal to 0.018, which means that this coefficient is significant at 5% significance level. The graph also highlights that the average PDs by the model range from 0.13% to 5.07%. Credit rating agencies PDs values belong to a much smaller interval between 0.27% and 2.45%.

Figure 14 : Regression of Average Probability of Default by Credit Rating Agencies on the Average Probability of Default by the Model

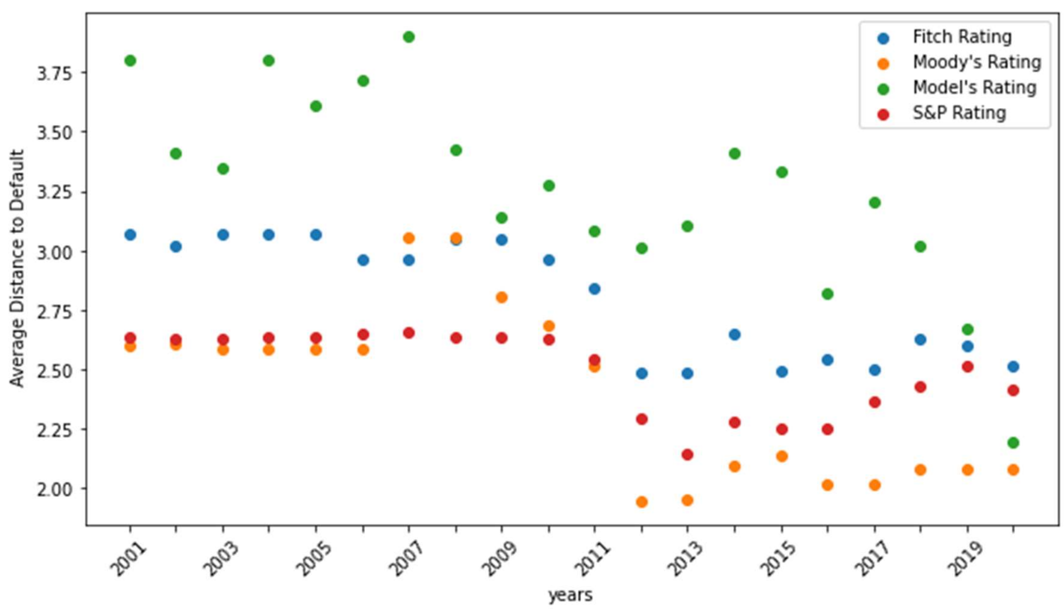


6.2.2 A Comparison Between GJL Model's Distances to Default and Credit Rating Agencies Distances to Default

The probability of default is known to have a non-linear relationship with its fundamentals. This makes complex any econometric analysis usually based on linear models. As a solution, it is frequent to run the analysis using transformed variables. This approach is observed in Logit or Probit credit risk models. Merton model uses a similar concept as well. In the example of Merton model, the DD is calculated and then transformed into a probability of default value through the normal distribution function. Therefore, in the Merton model framework, the DD is nothing but the symmetric of the inverse normal of the probability of default. In the GJL model, the inverse normal of PD is not exactly the distance between the default barrier and the assets value adjusted for risk as it was previously defined because the barrier can be reached before maturity. As ignoring the first passage time may raise problems, I opted for using the symmetric of the inverse normal of our 5-year cumulative PD to calculate the DD values by the model. This can be interpreted as a "Merton-model equivalent" DD. For that reason, I will continue to call it DD, though it is not computed as referred by Equation 30. I summarize the results in the Figure 15 below.

The yearly bank-average DD implied by the model reached its minimum in 2020 (2.19) and its maximum in 2007 (3.90). The all-period average of the model's DD is 3.27 compared to 2.50 average DD estimated by S&P, 2.80 estimated by Fitch and 2.40 estimated by Moody's. We also note that in general, the distance to default is known to be less volatile than the probability of default.

Figure 15 : Average Distance to Default Between 2001 and 2020

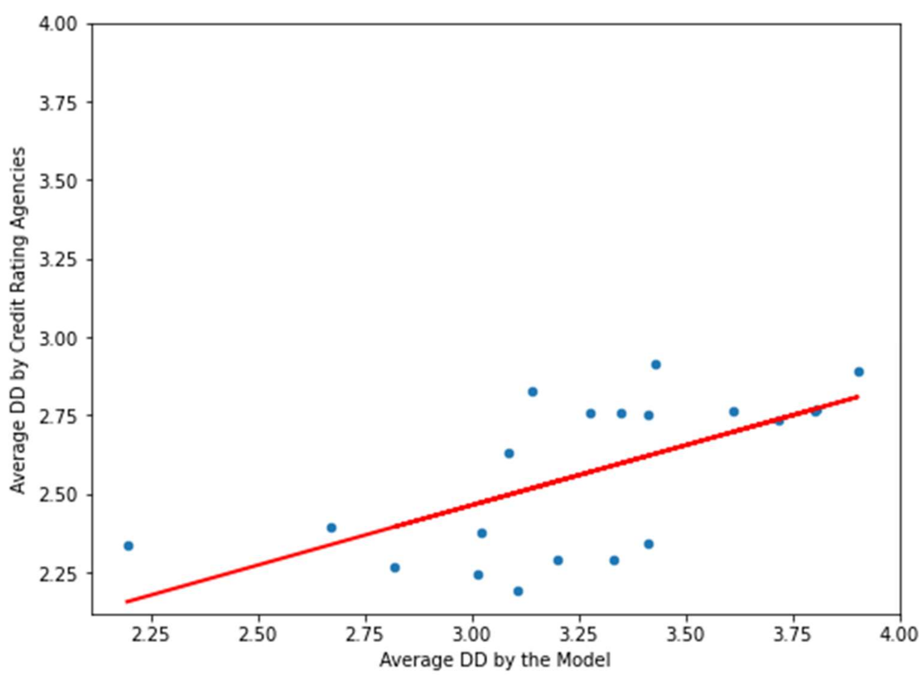


I complemented the averages' analysis by running regressions of distances to default to analyze in an advanced way this metric. Equation 39 and Figure 16 disclose the slope and intercept of a regression of the yearly average DD values estimated by the model compared to the ones estimated by the credit rating agencies. The regression shows an R-square equal to 0.40 which is a relatively high value. The p-value is 0.003 which implies that the coefficient is significant at 5% significance level. Once again, the comparison of intervals shows that the model gives a higher range than credit rating agencies. In other words, while the average DDs values by the model are between 2.20 and 3.90, the average DDs by credit rating agencies range from 2.19 to 2.91.

Equation 39 : Regression of Average DD by the Model by Average DD by Credit Rating Agencies

$$
\begin{aligned}
&Average\ DD\ by\ Credit\ Rating\ Agencies_t \\
&\qquad = 0.38\ Average\ DD\ by\ the\ model_t + 1.32 + \epsilon_t
\end{aligned}
$$

Figure 16 : Regression of Average Distance of Default by Credit Rating Agencies on the Average Distance of Default by the Model



Moreover, I run another regression (Equation 40) that does not average the banks when calculating the distance to default. Instead, it compares credit rating agencies DD per bank (average of the 3 rating agencies) in a certain year to the DD by the model in that year and for that same bank. This latter regression showed a weaker R-square than the previous regressions. In this case the R-square is equal to 0.16. Here again the coefficient is significant at 5% significance level.

Equation 40 : Regression of Distance to Default of Credit Rating Agencies on Distance to Default by the Model

$$DD\ Credit\ Rating\ Agency_{it} = 0.13\ \ DD\ Model_{it} + 2.13 + \ \epsilon_{it}$$

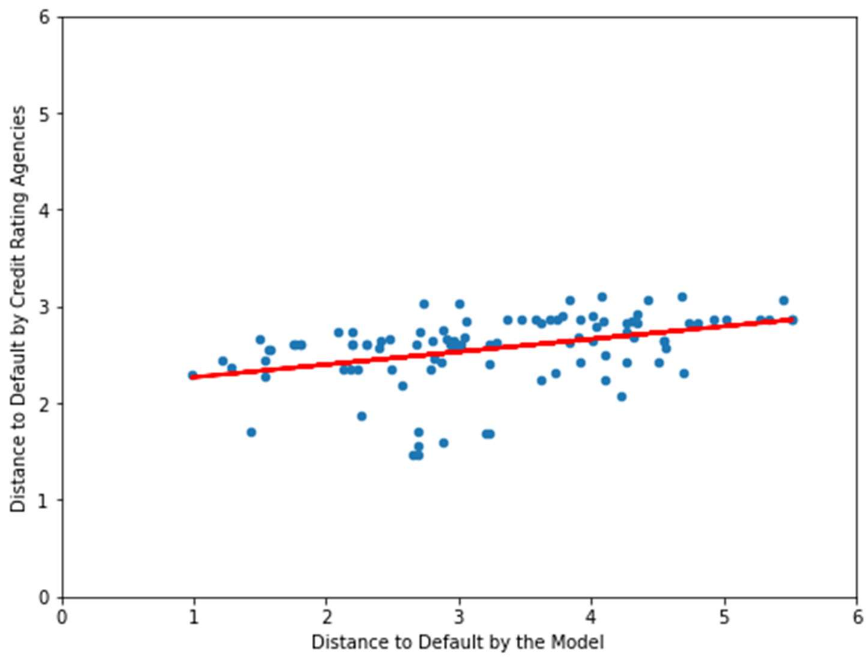
In the next step, I run an ultimate regression that controls for the fixed effects of each bank. I found the following model:

Equation 41: Fixed Effects Regression of Distance to Default of Credit Rating Agencies on Distance to Default by the Model

$$\begin{aligned} DD\ Credit\ Agency_{it} & \\ &= 2.23 + \ 0.10\, DD\ Model_{it} + 0.09\, San_t + 0.16\, Dbk_t + 0.11\, Cbk_t \\ &- 0.02\, Bbva_t - \ 0.34\, Bsab_t + \ \epsilon_{it} \end{aligned}$$

Fixed effects coefficients range from – 0.34 to 0.16. Banco de Sabadell and Banco Bilbao Vizcaya Argentaria had negative fixed effects, (–0.34) and (–0.02) respectively. We also notice that Banco Santander, Deutsche Bank and Commerzbank had positive and close fixed effects to each other (0.09, 0.16 and 0.11 respectively). A positive fixed effect of a bank means that on average credit rating agencies perceive a higher DD (and a lower credit rating) to the bank than the one attributed by this bank. Conversely, when a bank has a negative fixed effect, it means that on average, credit rating agencies recognize a lower DD (and a higher credit rating) to the bank than the one attributed by this same bank. In order to test for significance, I calculated p-values using clustered standard errors, as those are considered as robust results. The model DD is again significant at the 5% level. Regarding the fixed effects, they were found to be significant at the 5% level only in the case of Deutsche Bank and Banco de Sabadell. Taking rating agencies DDs as better measured than the ones provided by the structural model, this suggests that these banks have some characteristics that the model is not taking into account and that ratings agencies are. Besides, as expected, the introduction of the fixed effects increased the model R-square compared to the regression represented by equation 40. The R^2 value found here is equal to 0.39.

Figure 17 Regression of Distance to Default of Credit Rating Agencies on Distance to Default by the Model



7 Conclusion

This dissertation studies whether earnings-based structural models can be applied to estimate probabilities of default in the banking sector. I address this question by comparing how close are these estimates from the ones implied by credit ratings given by major credit rating agencies for a sample of five European banks between 2001 and 2020.

The direct answer to my questions is: the GJL model is able to produce probabilities of default (and distances to default) that are highly correlated with the ones implied by credit rating agencies, especially when we focus on the average distance to default at each moment in time. Econometric analysis at the micro level suggests however the existence of some significant differences among banks. In particular, Banco de Sabadell and Deutsche Bank have positive fixed effects in the DD estimation by credit rating agencies. Notice that the GJL model is based on fundamentals and implies several assumptions. Instead, credit rating leaders take into consideration other factors that cannot be considered by our model. For instance: banks' age. Old banks have a longer history of data that is useful for the assessment of credit risk. In opposition, the evaluation of credit risk is harder for new banks because of the lack of data available.

Through this study, I was able to assess the limitations of the model. The model does not allow for banks that have a negative EBT. Luckily, our calibration approach can overcome this constraint in cases where the average EBT is positive. In fact, in the step of choosing my sample, I had to eliminate banks that have a negative average EBT. Additionally, the geometric Brownian motion assumed in our model does not allow for jumps. This a very strong assumption, especially in times of crisis and market crashes. In the period study between the year 2000 and 2020 there were at least two major market crashes. Another unrealistic assumption imposed by the model is perpetual debt. In fact, debt rollover violates the model's assumption. This is unfortunate because this practice is widely used in the banking sector. Ultimately, the most challenging and binding assumption of the model is the fixed costs. It is impossible to find a bank that keeps the same non-interest level through large time windows like 10 or 20 years due to inflation, markets movements and many other factors. At this level, an interesting topic of study would address the following question: Does applying the model on shorter periods of study like 2 or 3 years leads closer results to reality despite the swings in DD that could be generated between time windows?

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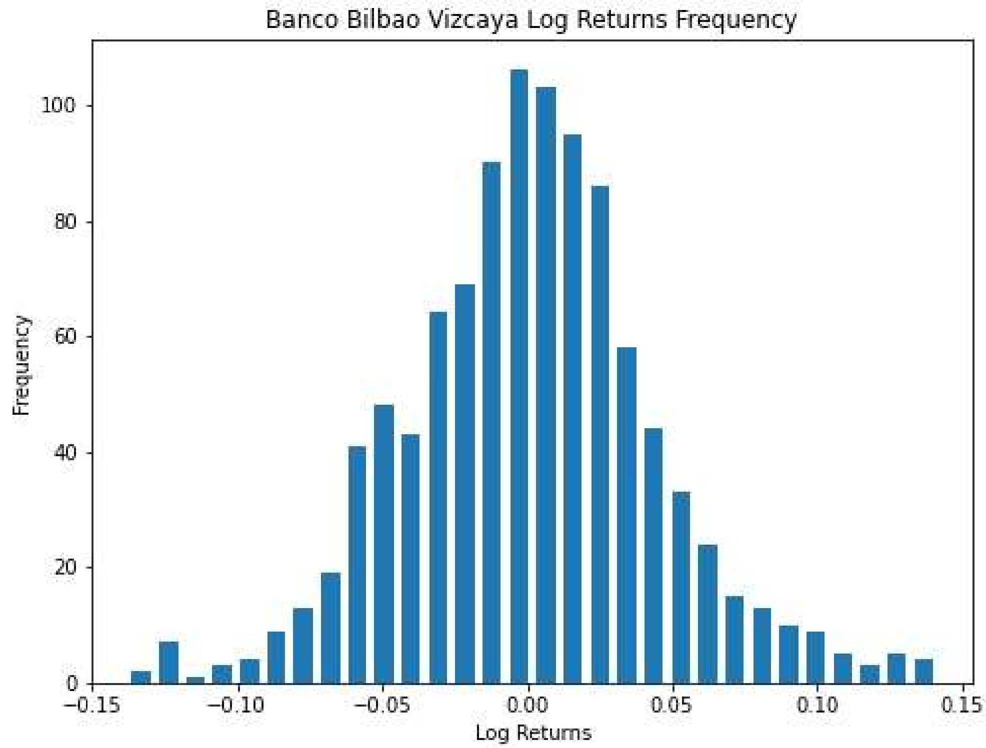
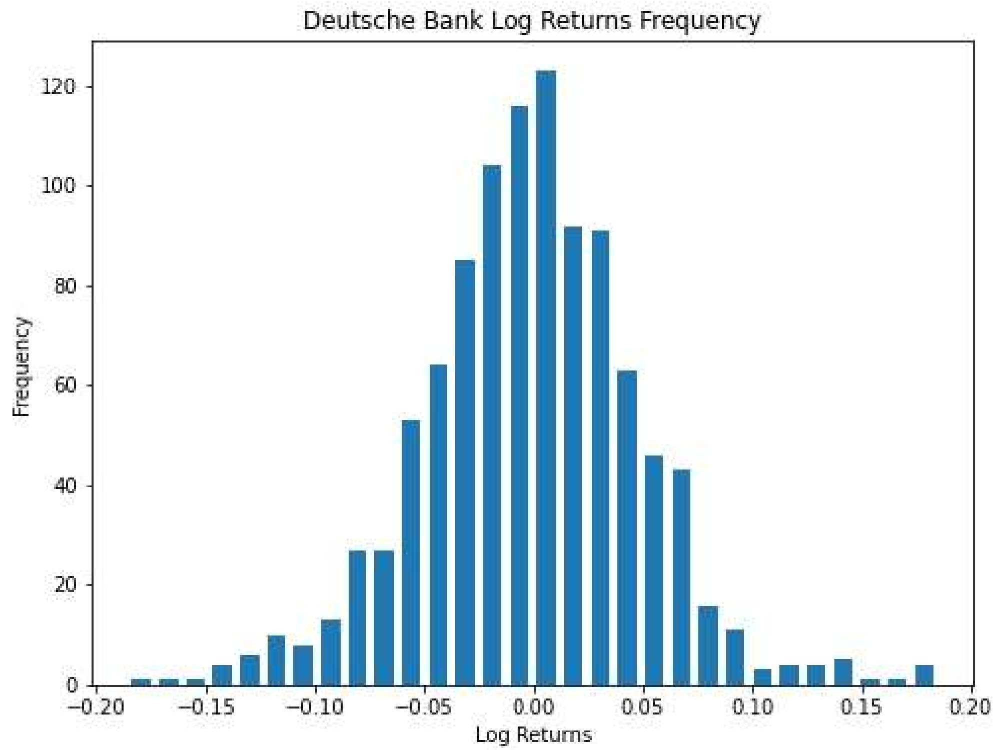
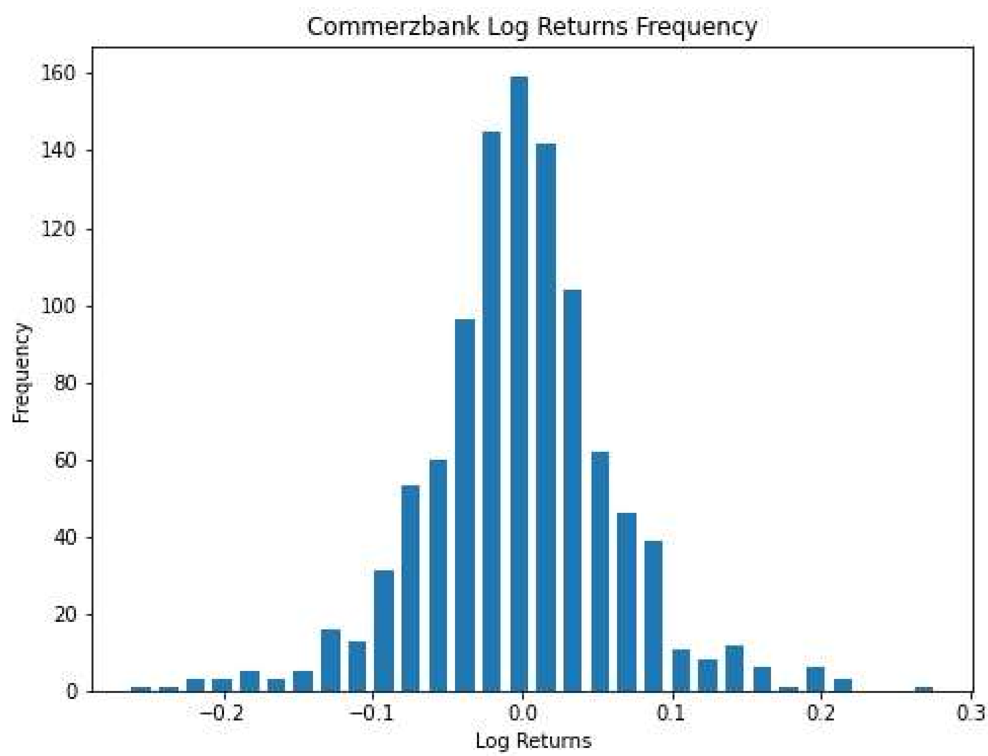
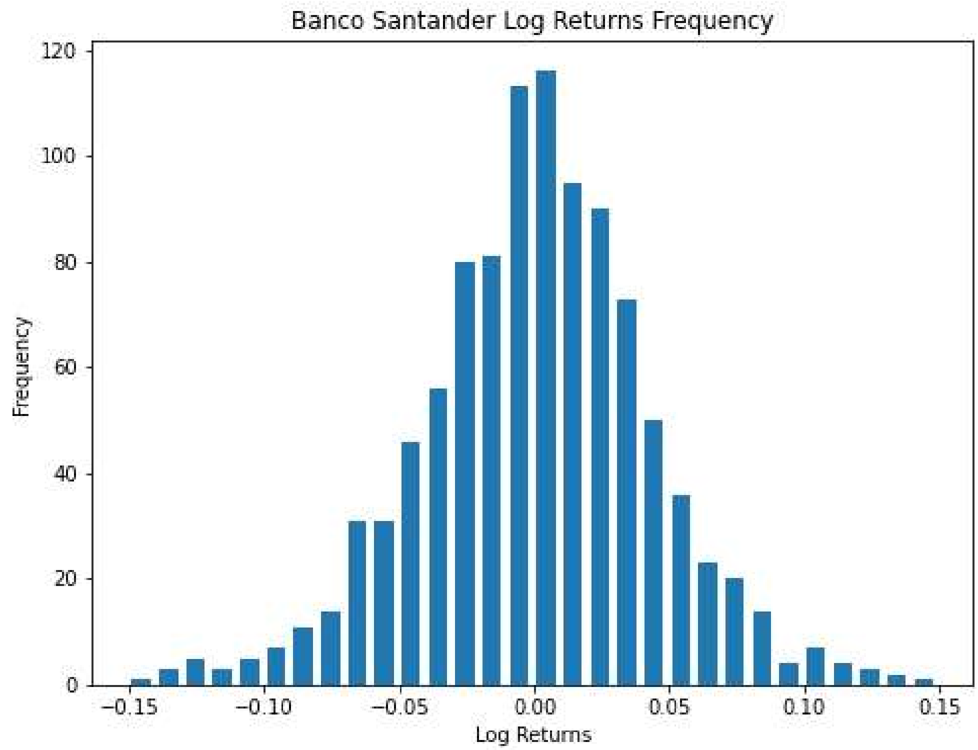
Appendix

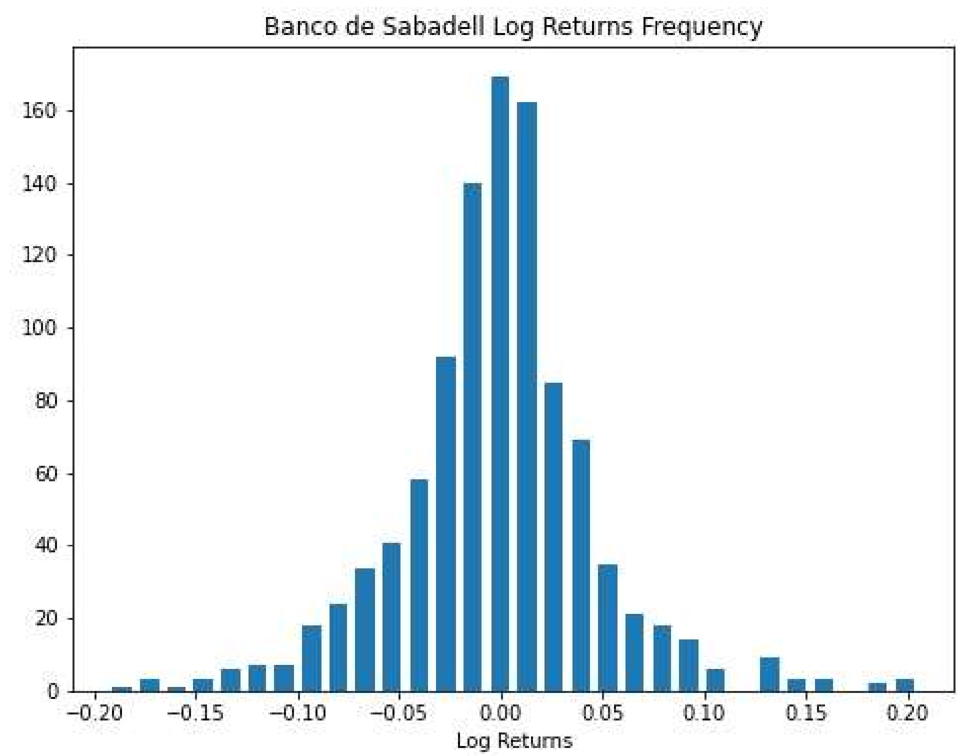
Appendix 1 : Log Returns' Frequency Histograms




Deutsche Bank Log Returns Frequency
Frequency
Log Returns
0
20
40
60
80
100
120
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20


Commerzbank Log Returns Frequency
Frequency
Log Returns
0
20
40
60
80
100
120
140
160
−0.2
−0.1
0.0
0.1
0.2
0.3


Banco Santander Log Returns Frequency
Frequency
Log Returns
0
20
40
60
80
100
120
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15


Banco de Sabadell Log Returns Frequency
Log Returns
0
20
40
60
80
100
120
140
160
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20

Appendix 2 : Implied Probabilities of Default per Year Between 2001 and 2020 for Each Bank by Standard & Poor's

Date	Banco Santander	Deutsche Bank	Commerzbank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell
2001	0.0045 (A+)	0.0036 (AA)	0.0049 (A)	0.0033 (AA-)	0.0049 (A)
2002	0.0049 (A)	0.0033 (AA-)	0.0057 (A-)	0.0033 (AA-)	0.0049 (A)
2003	0.0049 (A)	0.0033 (AA-)	0.0057 (A-)	0.0033 (AA-)	0.0049 (A)
2004	0.0045 (A+)	0.0033 (AA-)	0.0057 (A-)	0.0033 (AA-)	0.0049 (A)
2005	0.0045 (A+)	0.0033 (AA-)	0.0057 (A-)	0.0033 (AA-)	0.0049 (A)
2006	0.0033 (AA-)	0.0033 (AA-)	0.0057 (A-)	0.0033 (AA-)	0.0049 (A)
2007	0.0036 (AA)	0.0036 (AA)	0.0049 (A)	0.0033 (AA-)	0.0045 (A+)
2008	0.0036 (AA)	0.0045 (A+)	0.0049 (A)	0.0036 (AA)	0.0045 (A+)
2009	0.0033 (AA)	0.0045 (A+)	0.0049 (A)	0.0036 (AA)	0.0049 (A)
2010	0.0036 (AA)	0.0045 (A+)	0.0049 (A)	0.0036 (AA)	0.0049 (A)
2011	0.0045 (AA-)	0.0045 (A+)	0.0049 (A)	0.0033 (AA-)	0.0142 (BBB)
2012	0.0142 (BBB)	0.0045 (A+)	0.0049 (A)	0.0049 (A)	0.0652 (BB)
2013	0.0142 (BBB)	0.0049 (A)	0.0057 (A-)	0.0284 (BBB-)	0.0652 (BB)
2014	0.0103 (BBB+)	0.0049 (A)	0.0057 (A-)	0.0142 (BBB)	0.0386 (BB+)
2015	0.0057 (A-)	0.0103 (BBB+)	0.0103 (BBB+)	0.0103 (BBB+)	0.0386 (BB+)
2016	0.0057 (A-)	0.0103 (BBB+)	0.0103 (BBB+)	0.0103 (BBB+)	0.0386 (BB+)
2017	0.0057 (A-)	0.0057 (A-)	0.0057 (A-)	0.0103 (BBB+)	0.0284 (BBB-)
2018	0.0049 (A)	0.0103 (BBB+)	0.0057 (A-)	0.0057 (A-)	0.0142 (BBB)
2019	0.0049 (A)	0.0103 (BBB+)	0.0057 (A-)	0.0016 (AA+)	0.0142 (BBB)
2020	0.0049 (A)	0.0103 (BBB+)	0.0103 (BBB+)	0.0016 (AA+)	0.0284 (BBB-)

Appendix 3 : Implied Probabilities of Default per Year Beween 2001 and 2020 for Each Bank by Fitch

Date	Banco Santander	Deutsche Bank	Commerzbank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell
2001	0.00092 (AA-)	0.0009 (AA-)	0.00092 (AA-)	0.0009 (AA-)	0.0021 (A+)
2002	0.002068 (A+)	0.0009 (AA-)	0.00092 (AA-)	0.0009 (AA-)	0.0021 (A+)
2003	0.00092 (AA-)	0.0009 (AA-)	0.00092 (AA-)	0.0009 (AA-)	0.0021 (A+)
2004	0.00092 (AA-)	0.0009 (AA-)	0.00092 (AA-)	0.0009 (AA-)	0.0021 (A+)
2005	0.00092 (AA-)	0.0009 (AA-)	0.00092 (AA-)	0.0009 (AA-)	0.0021 (A+)
2006	0.00055 (AA)	0.0009 (AA-)	0.007177 (A)	0.0009 (AA-)	0.0021 (A+)
2007	0.00055 (AA)	0.0009 (AA-)	0.007177 (A)	0.0009 (AA-)	0.0021 (A+)
2008	0.00055 (AA)	0.0009 (AA-)	0.002068 (A+)	0.0009 (AA-)	0.0021 (A+)
2009	0.00055 (AA)	0.0009 (AA-)	0.002068 (A+)	0.0009 (AA-)	0.0021 (A+)
2010	0.00055 (AA)	0.0009 (AA-)	0.002068 (A+)	0.0009 (AA-)	0.0072 (A)
2011	0.00092 (AA-)	0.0021 (A+)	0.002068 (A+)	0.0021 (A+)	0.0065 (BBB+)
2012	0.006489 (BBB+)	0.0021 (A+)	0.002068 (A+)	0.0065 (BBB+)	0.0421 (BB+)
2013	0.006489 (BBB+)	0.0021 (A+)	0.002068 (A+)	0.0065 (BBB+)	0.0421 (BB+)
2014	0.00092 (A-)	0.0021 (A+)	0.002068 (A+)	0.0036 (A-)	0.0421 (BB+)
2015	0.00092 (A-)	0.0036 (A-)	0.012944 (BBB)	0.0036 (A-)	0.0421 (BB+)
2016	0.00092 (A-)	0.0036 (A-)	0.006489 (BBB+)	0.0036 (A-)	0.0421 (BB+)
2017	0.00092 (A-)	0.0065 (BBB+)	0.006489 (BBB+)	0.0036 (A-)	0.0421 (BB+)
2018	0.00092 (A-)	0.0065 (BBB+)	0.00092 (A-)	0.0036 (A-)	0.0421 (BB+)
2019	0.00092 (A-)	0.0065 (BBB+)	0.006489 (BBB+)	0.0036 (A-)	0.0129 (BBB)
2020	0.00092 (A-)	0.0065 (BBB+)	0.012944 (BBB)	0.0036 (A-)	0.0198 (BBB-)

Appendix 4: Implied Probabilities of Default per year between 2001 and 2020 for Each Bank by Moody's

Date	Banco Santander	Deutsche Bank	Commerzbank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell
2001	0.00268 (Aa3)	0.00268 (Aa3)	0.0102 (A1)	0.00304 (Aa2)	0.00917 (A2)
2002	0.00268 (Aa3)	0.00268 (Aa3)	0.00917 (A2)	0.00304 (Aa2)	0.00917 (A2)
2003	0.00268 (Aa3)	0.00268 (Aa3)	0.01173 (A3)	0.00304 (Aa2)	0.00917 (A2)
2004	0.00268 (Aa3)	0.00268 (Aa3)	0.01173 (A3)	0.00304 (Aa2)	0.00917 (A2)
2005	0.00268 (Aa3)	0.00268 (Aa3)	0.01173 (A3)	0.00304 (Aa2)	0.00917 (A2)
2006	0.00268 (Aa3)	0.00268 (Aa3)	0.01173 (A3)	0.00304 (Aa2)	0.00917 (A2)
2007	0.00036 (Aa1)	0.00036 (Aa1)	0.00268 (Aa3)	0.00036 (Aa1)	0.00917 (A2)
2008	0.00036 (Aa1)	0.00036 (Aa1)	0.00268 (Aa3)	0.00036 (Aa1)	0.00917 (A2)
2009	0.00304 (Aa2)	0.00036 (Aa1)	0.00268 (Aa3)	0.00304 (Aa2)	0.00917 (A2)
2010	0.00304 (Aa2)	0.00268 (Aa3)	0.00268 (Aa3)	0.00304 (Aa2)	0.00917 (A2)
2011	0.00268 (Aa3)	0.00268 (Aa3)	0.00917 (A2)	0.00268 (Aa3)	0.0317 (Baa3)
2012	0.0198 (Baa2)	0.00917 (A2)	0.01173 (A3)	0.03171 (Baa3)	0.1183 (Ba3)
2013	0.0198 (Baa2)	0.00917 (A2)	0.01102 (Baa1)	0.03171 (Baa3)	0.1183 (Ba3)
2014	0.01102 (Baa1)	0.01173 (A3)	0.01102 (Baa1)	0.0198 (Baa2)	0.05992 (B1)
2015	0.01173 (A3)	0.01173 (A3)	0.01102 (Baa1)	0.01102 (Baa1)	0.05992 (B1)
2016	0.01173 (A3)	0.0198 (Baa2)	0.01102 (Baa1)	0.01102 (Baa1)	0.1183 (Ba3)
2017	0.01173 (A3)	0.0198 (Baa2)	0.01102 (Baa1)	0.01102 (Baa1)	0.1183 (Ba3)
2018	0.00917 (A2)	0.01173 (A3)	0.0102 (A1)	0.01173 (A3)	0.1183 (Ba3)
2019	0.00917 (A2)	0.01173 (A3)	0.0102 (A1)	0.01173 (A3)	0.1183 (Ba3)
2020	0.00917 (A2)	0.01173 (A3)	0.0102 (A1)	0.01173 (A3)	0.1183 (Ba3)

Appendix 5 : Implied Distance to Default per Year between 2001 and 2020 for Each Bank by Standard & Poor's

Date	Banco Santander	Deutsche Bank	Commerzbank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell
2001	2.612054141	2.687449447	2.582807452	2.716381	2.582807452
2002	2.582807452	2.716380583	2.530192384	2.716381	2.582807452
2003	2.582807452	2.716380583	2.530192384	2.716381	2.582807452
2004	2.612054141	2.716380583	2.530192384	2.716381	2.582807452
2005	2.612054141	2.716380583	2.530192384	2.716381	2.582807452
2006	2.716380583	2.716380583	2.530192384	2.716381	2.582807452
2007	2.687449447	2.687449447	2.582807452	2.716381	2.612054141
2008	2.687449447	2.612054141	2.582807452	2.687449	2.612054141
2009	2.716380583	2.612054141	2.582807452	2.687449	2.582807452
2010	2.687449447	2.612054141	2.582807452	2.687449	2.582807452
2011	2.612054141	2.612054141	2.582807452	2.716381	2.191716273
2012	2.191716273	2.612054141	2.582807452	2.582807	1.512526407
2013	2.191716273	2.582807452	2.530192384	1.904847	1.512526407
2014	2.315236367	2.582807452	2.530192384	2.191716	1.76716858
2015	2.530192384	2.315236367	2.315236367	2.315236	1.76716858
2016	2.530192384	2.315236367	2.315236367	2.315236	1.76716858
2017	2.530192384	2.530192384	2.530192384	2.315236	1.90484671
2018	2.582807452	2.315236367	2.530192384	2.530192	2.191716273
2019	2.582807452	2.315236367	2.530192384	2.947843	2.191716273
2020	2.582807452	2.315236367	2.315236367	2.947843	1.90484671

Appendix 6 : Implied Distance to Default per year Between 2001 and 2020 for Each Bank by Fitch

Date	Banco Santander	Deutsche Bank	Commerzbank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell
2001	3.11501391	3.121389149	3.11501391	3.121389149	2.862736264
2002	2.867596277	3.121389149	3.11501391	3.121389149	2.862736264
2003	3.11501391	3.121389149	3.11501391	3.121389149	2.862736264
2004	3.11501391	3.121389149	3.11501391	3.121389149	2.862736264
2005	3.11501391	3.121389149	3.11501391	3.121389149	2.862736264
2006	3.263536074	3.121389149	2.448278467	3.121389149	2.862736264
2007	3.263536074	3.121389149	2.448278467	3.121389149	2.862736264
2008	3.263536074	3.121389149	2.867596277	3.121389149	2.862736264
2009	3.263536074	3.121389149	2.867596277	3.121389149	2.862736264
2010	3.263536074	3.121389149	2.867596277	3.121389149	2.447127222
2011	3.11501391	2.862736264	2.867596277	2.862736264	2.483769293
2012	2.48438204	2.862736264	2.867596277	2.483769293	1.726819975
2013	2.48438204	2.862736264	2.867596277	2.483769293	1.726819975
2014	3.11501391	2.862736264	2.867596277	2.687449447	1.726819975
2015	3.11501391	2.687449447	2.227890476	2.687449447	1.726819975
2016	3.11501391	2.687449447	2.48438204	2.687449447	1.726819975
2017	3.11501391	2.483769293	2.48438204	2.687449447	1.726819975
2018	3.11501391	2.483769293	3.11501391	2.687449447	1.726819975
2019	3.11501391	2.483769293	2.48438204	2.687449447	2.22920903
2020	3.11501391	2.483769293	2.227890476	2.687449447	2.057897227

Appendix 7 : Implied Distance to Default per Year Between 2001 and 2020 for Each Bank by Moody's

Date	Banco Santander	Deutsche Bank	Commerzbank	Banco Bilbao Vizcaya Argentaria	Banco de Sabadell
2001	2.78456244	2.78456244	2.318908466	2.743435453	2.358681275
2002	2.78456244	2.78456244	2.358681275	2.743435453	2.358681275
2003	2.78456244	2.78456244	2.265859288	2.743435453	2.358681275
2004	2.78456244	2.78456244	2.265859288	2.743435453	2.358681275
2005	2.78456244	2.78456244	2.265859288	2.743435453	2.358681275
2006	2.78456244	2.78456244	2.265859288	2.743435453	2.358681275
2007	3.381847893	3.381847893	2.78456244	3.381847893	2.358681275
2008	3.381847893	3.381847893	2.78456244	3.381847893	2.358681275
2009	2.743435453	3.381847893	2.78456244	2.743435453	2.358681275
2010	2.743435453	2.78456244	2.78456244	2.743435453	2.358681275
2011	2.78456244	2.78456244	2.358681275	2.78456244	1.856375882
2012	2.057897227	2.358681275	2.265859288	1.856235487	1.18352789
2013	2.057897227	2.358681275	2.289677822	1.856235487	1.18352789
2014	2.289677822	2.265859288	2.289677822	2.057897227	1.55544552
2015	2.265859288	2.265859288	2.289677822	2.289677822	1.55544552
2016	2.265859288	2.057897227	2.289677822	2.289677822	1.18352789
2017	2.265859288	2.057897227	2.289677822	2.289677822	1.18352789
2018	2.358681275	2.265859288	2.318908466	2.265859288	1.18352789
2019	2.358681275	2.265859288	2.318908466	2.265859288	1.18352789
2020	2.358681275	2.265859288	2.318908466	2.265859288	1.18352789