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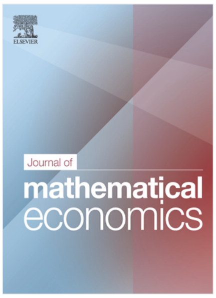

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Revised manuscript (Clean version)

Imperfect competition in the banking sector and economic instability*

Francesco Carli,[†] Teresa Lloyd-Braga[‡] and Leonor Modesto[§]

Abstract

We study the impact of competition in the banking sector on the emergence of endogenous cycles driven by self-fulling volatile expectations. We consider an OLG model with two sectors and two household types: workers, who consume and work when young and save through bank deposits; and entrepreneurs, who seek bank loans to finance current consumption and to invest in a productive technology that transforms the consumption good into capital. When old, entrepreneurs rent this capital to firms, who produce the consumption good using capital and labor. All markets are perfectly competitive, except the loans market where banks compete *à la* Cournot under free entry and exit.

In the absence of externalities in the capital producing technology, more competition in the banking sector promotes the emergence of local indeterminacy and sunspots fluctuations. In contrast, under constant social returns to scale in the capital producing technology, bank market power alone triggers the emergence of local indeterminacy. With increasing social returns to scale, both market power and externalities facilitate the emergence of local indeterminacy. Additionally, when banks have market power, steady state multiplicity may emerge, opening the way to global indeterminacy and fluctuations.

Keywords: Banking Sector, endogenous fluctuations, indeterminacy, imperfect competition.

JEL Codes: D43, E32, E44.

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1 Introduction

The critical roles played by financial intermediaries in the real economy sparked research interest in the macroeconomics of banking. Numerous studies, in particular, emphasized the role of the financial system as a source of endogenous fluctuations driven by self-fulfilling volatile expectations. These studies underscore the role of financial market imperfections for indeterminacy of equilibria and the emergence of endogenous fluctuations. For instance, the literature placed emphasis on costly state verification (Boyd and Smith (1997), Huybens and Smith (1998)), adverse selection (Azariadis and Smith (1998)), and limited commitment (Gu et al. (2023)). This body of research, however, typically operates under the assumption of a perfectly competitive banking sector. In many developed economies, however, a small number of banks exerts substantial control over a significant portion of the country's banking industry. For instance, in 2021, the five largest banks in the US had assets equal to 49.68% of total commercial banking assets. In many developed economies, such measure of banking concentration is even higher.[1] This observation leads to the following question: what is the effect of bank competition on the emergence of indeterminacy and endogenous fluctuations? In this paper, we fill this gap and study how competition in the banking sector affects the emergence of macroeconomic endogenous cycles driven by self-fulfilling volatile expectations.

To answer this question, we consider a simple overlapping generations (OLG) model with some intra-generational heterogeneity and where capital investment is credit financed through intermediaries (banks) operating in Cournotian markets. To be more specific, we consider a two-period overlapping generation economy where some agents are lenders (workers) and others are borrowers (entrepreneurs). The final output is competitively produced using a constant returns to scale technology with capital and labor as inputs. Lenders work when young, earning labor income, and are retired when old. To finance their consumption while retired they save in the form of bank deposits. Borrowers have access to a technology which converts time t consumption goods into time $t+1$ capital goods. This technology exhibits decreasing returns to scale at the private level. However, if there are positive externalities in the production of the capital good, it may exhibit constant or increasing returns to scale at the social level. To finance

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[1] In 2021, such top 5-bank concentration measure in the UK was 59.58%, in Italy 73.78%, in France 78.48%, and in Germany 94.28%. See the *Global Financial Development Report, September 2022 version.*

the capital investments, borrowers need external credit. Banks provide intermediation between borrowers and lenders, writing deposit contracts with the latter and loan contracts with the former. However, while banks are price-takers in the market for deposits, they compete à la Cournot under free entry and exit in the market for loans. Thus, the equilibrium number of banks, obtained under a zero profit condition, depends on labor income. This varies according to the real activity level, introducing a two-way interaction between the real economy and the banking sector.

In this framework, we obtain the following results. Provided that some technical conditions are met to ensure the existence of a *normalized* steady state, necessary conditions for the emergence of endogenous fluctuations associated with local indeterminacy, are that the elasticity of labor supply is sufficiently large, and that the share of current consumption in wage income as well as the distortions in the economy are not too large. Under these conditions, local indeterminacy emerges for empirically plausible values of the elasticity of capital-labor substitution. In particular, there are two concurrent mechanisms at work in the economy: the market power of banks in the market for loans and the externality in the production of the investment good. These two distortions play a pivotal role on the emergence of local indeterminacy. Importantly, the effects of bank market power on the emergence of local indeterminacy, depend on the strength of the externality, i.e., on whether social returns to scale in the production of the capital good, are decreasing, constant or increasing. In the absence of externalities, so that the capital producing technology has decreasing social returns to scale, more competition in the banking sector, measured by the number of banks in the system, promotes macroeconomic instability by facilitating the onset of local indeterminacy and sunspot fluctuations. This is one novel result of the paper. Since entrepreneurs earn competitive rents from operating their technology because of decreasing returns to scale, market power makes banks extract part of these profits from entrepreneurs. Hence, a high market power offsets the effects on local indeterminacy resulting from decreasing social returns to scale in the production of the investment good. Differently, when the externality is large, so that the capital producing technology has increasing social returns to scale, a high market power may facilitate the emergence of local indeterminacy. Finally, with constant social returns to scale, bank market power *per se* can lead to the emergence of local indeterminacy.

We also show that, in the presence of bank market power, steady state multiplicity may naturally emerge, with or without externalities in the production of the capital good. As this opens the way to global indeterminacy and fluctuations, we conclude bank market power constitutes a potentially destabilizing mechanism.

Our paper relates to three strands of literature. The first one studies the role of imperfections on the emergence of indeterminacy. Using also an OLG structure, Dos Santos Ferreira and Lloyd-Braga (2005) and Seegmuller (2005) study the effects of imperfect competition on the output market on endogenous fluctuations. Like us, Dos Santos Ferreira and Lloyd-Braga (2005) consider Cournot competition under free entry and exit, and find that markup variability is responsible for the occurrence of endogenous cycles. Here, with constant social returns to scale in the capital producing technology we obtain a similar result. Seegmuller (2005) considers a fixed number of firms under monopolistic competition, and finds that imperfect competition stabilizes fluctuations. We also find that a higher market power of banks promotes stability in the case of decreasing social returns to scale.

The second strand of literature studies the link between financial intermediation and real activity. Williamson (1987) constructs a real business cycle model with costly state verification and perfect competition in the banking sector. Using an OLG model, Azariadis and Smith (1998) find that adverse selection can result in indeterminacy of equilibrium and excessive fluctuations. Boyd and Smith (1997) consider an open economy OLG model and find that international markets can result in endogenous volatility when borrowing is subject to costly state verification. Finally, in a model of delegated investment, Gu et al. (2023) show that intermediation improves welfare, but renders the economy subject to sunspots and cycles. Our results complement these work by showing that endogenous volatility can also emerge in a closed economy with imperfect competition in the banking sector.

Finally, our paper relates to the strand of literature that studies the consequences for financial stability of the degree of competition in the banking sector. These work include, among others, Allen and Gale (2001), Dos Santos Ferreira and Modesto (2021), Boyd and De Nicolo (2005), and Martinez-Miera and Repullo (2010). We complement these papers as in our model there is no fundamental uncertainty, fluctuations in banking activity, entry and exit, and output are linked to autonomous volatility in self-fulfilling expectations.

2 The model

We consider an OLG model where a continuum of unit mass of households, of which $1-\alpha$ are *entrepreneurs*, and α are *lenders*, lives for two periods, consuming in both of them. Population is constant. There are two goods, a capital and a consumption good. The consumption good is produced in a perfectly competitive sector by *a representative firm* using labor l_t and capital K_t. The capital good is produced by *entrepreneurs*, which are endowed with a technology that transforms the consumption good into next period capital good. As they have no initial endowment, young entrepreneurs must borrow from a bank in order to invest in their technology. When old, they rent the capital produced to firms at the perfectly competitive rental rate. The rentals received are used to repay the loan and to finance old age consumption. Lenders work when young for the representative firm in exchange for the competitive wage. They deposit part of their income with a bank in order to finance consumption when old.

Our model involves therefore five markets: the market for the consumption good, the capital services market, the labor market and the *loan* and the *deposit markets*, and four sets of agents: two types of households, a representative firm and n *banks*.

2.1 The Firm

Each period there is a perfectly competitive sector — or *a representative firm* — that produces the consumption good Y_t using capital K_t and labor l_t in a linearly homogeneous technology, $Y_t = \Theta F(K_t, l_t) = \Theta l_t f(K_t/l_t)$, where $k_t \equiv K_t/l_t$ and Θ is a scaling constant representing total factor productivity. We assume that capital used in production fully depreciates, hence consumption goods can only be produced if new capital is provided each period. The firm maximizes profits, so that from the first orders conditions we get

$$w_t = \Theta[f(k_t) - k_t f'(k_t)] \equiv w(k_t), \quad k_t \equiv K_t/l_t \tag{1}$$

$$\rho_t = \Theta f'(k_t) \equiv \rho(k_t) \tag{2}$$

where w_t denotes the real wage and ρ_t the real rental rate of capital (both measured in terms of the consumption good). Denoting by $s(k)$ the share of capital in total income and by $\sigma(k)$

the elasticity of capital-labor substitution, we can write:

$$s(k_t) = \frac{kf'(k_t)}{f(k_t)} \in (0,1) \quad \text{and} \quad \sigma(k_t) = -\left[1 - s(k_t)\right] \frac{f'(k_t)}{kf''(k_t)} > 0. \tag{3}$$

2.2 Lenders

Lenders work when young and save in the form of bank deposits to finance their consumption while retired. Each lender born at time t (indexed by L^t) wants to maximize his utility, by choosing the amount to deposit in a bank ($D_t^{L^t}$), consumption at young and old age ($c_t^{L^t}$ and $c_{t+1}^{L^t}$ respectively), and the labor supplied ($l_t^{L^t}$). Preferences are represented by the utility function $U(c_t^{L^t}, c_{t+1}^{L^t}, l_t^{L^t}) = (c_t^{L^t})^{s_h}(c_{t+1}^{L^t})^{1-s_h} - \frac{\varphi(l_t^{L^t})^{(1+\frac{1}{\varepsilon})}}{(1+\frac{1}{\varepsilon})}$, where $s_h \epsilon (0,1)$ denotes the share of current consumption in wage income, $\varepsilon > 0$ is the elasticity of labour supply and $\varphi > 0$ is a scaling parameter. So, a lender born at time t will solve

$$\begin{aligned} \max_{\{c_t^{L^t}, c_{t+1}^{L^t}, l_t^{L^t}, d_t^{L^t}\} \in \Re_+^4} \quad & (c_t^{L^t})^{s_h}(c_{t+1}^{L^t})^{1-s_h} - \frac{\varphi(l_t^{L^t})^{(1+\frac{1}{\varepsilon})}}{(1+\frac{1}{\varepsilon})} \\ s.t. \quad & P_t c_t^{L^t} + D_t^{L^t} = W_t l_t^{L^t} \\ & P_{t+1} c_{t+1}^{L^t} = D_t^{L^t} g_{t+1} \end{aligned}$$

where P_t is the price of consumption, W_t is the nominal wage and g_{t+1} is the gross interest factor, received on $t+1$ on each unit deposited in t, which is determined in period t. Denote by $d_t^{L^t} = D_t^{L^t}/P_t$ the real value of deposits, by $w_t = W_t/P_t$ the real wage and by $r_{t+1} = g_{t+1}/\pi_{t+1}$ the real interest factor on deposits, with $\pi_{t+1} = P_{t+1}/P_t$ denoting the inflation rate. Then, the budget constraints can be rewritten

$$\begin{aligned} c_t^{L^t} + d_t^{L^t} &= w_t l_t^{L^t} \\ c_{t+1}^{L^t} &= d_t^{L^t} r_{t+1}. \end{aligned}$$

The solution to lenders' problem is

$$l_t^{L^t} = \left[\frac{s_h^{s_h}(1-s_h)^{1-s_h} r_{t+1}^{1-s_h} w_t}{\varphi}\right]^{\varepsilon} \tag{4}$$

$$
\begin{aligned}
d_t^{L^t} &= (1-s_h)w_t l_t^{L^t} = \left[\frac{s_h^{s_h}(1-s_h)^{(\frac{1}{\varepsilon}+1-s_h)} r_{t+1}^{1-s_h} w_t^{(\frac{1}{\varepsilon}+1)}}{\varphi}\right]^{\varepsilon} && (5)\\
c_t^{L^t} &= s_h w_t l_t^{L^t} && (6)\\
c_{t+1}^{L^t} &= (1-s_h) r_{t+1} w_t l_t^{L^t}. && (7)
\end{aligned}
$$

2.3 Entrepreneurs

A young entrepreneur born at time t (indexed by E^t) asks for a bank loan ($B_t^{E^t}$) to finance consumption in her first period of life ($c_t^{E^t}$) and to invest in her technology, $I_t^{E^t}$. This technology returns $A(\bar{I}_t^{E^t})^{\eta}(I_t^{E^t})^{\theta}$ units of capital in period $t+1$, where $\bar{I}_t^{E^t}$ denotes the average investment level in this technology in the economy. The parameter $\eta \geq 0$ represents the degree of productive externalities in the production of the capital good. We assume that $0 < \theta < 1$, implying that this technology exhibits decreasing returns to scale at the private level and, in the absence of externalities, at the social level as well. However, if there are positive externalities in the production of the capital good, $\eta > 0$, this technology may exhibit constant returns to scale $\eta + \theta = 1$, or increasing returns to scale $\eta + \theta > 1$, at the social level.[2]

The capital produced is then rented to firms. The rentals received will be used to pay for old-age consumption and to repay the loan. Each entrepreneur born at time t maximizes her utility, which is given by $U(c_t, c_{t+1}) = \log(c_t) + \beta \log(c_{t+1})$, where $1 > \beta > 0$. We assume that each borrower takes as given the loan repayment rate, and thus solves

$$
\begin{aligned}
\max_{\{c_t^{E^t}, c_{t+1}^{E^t}, I_t^{E^t}, b_t^{E^t}\} \in \Re_+^4} \quad & \log(c_t^{E^t}) + \beta \log(c_{t+1}^{E^t})\\
s.t. \quad & P_t I_t^{E^t} + P_t c_t^{E^t} = B_t^{E^t}\\
& P_{t+1} c_{t+1}^{E^t} = \Gamma_{t+1} A(\bar{I}_t^{E^t})^{\eta}(I_t^{E^t})^{\theta} - B_t^{E^t} Z_{t+1}
\end{aligned}
$$

where Γ_{t+1} denotes the rental rate of capital at time $t+1$, and Z_{t+1} denotes the credit interest factor, which is determined in t but paid in period $t+1$.

Denote by $b_t^{E^t} = B_t^{E^t}/P_t$ the real value of loans, by $\rho_{t+1} = \Gamma_{t+1}/P_{t+1}$ the real rental rate of capital, and by $R_{t+1} = Z_{t+1}/\pi_{t+1}$ the real interest factor on loans. Then, the budget constraints

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[2] Harrison (2003) provides evidence of externalities in the investment sector.

can be rewritten

$$\begin{aligned} I_t^{E^t} + c_t^{E^t} &= b_t^{E^t} \\ c_{t+1}^{E^t} &= \rho_{t+1} A(\bar{I}_t^{E^t})^{\eta} (I_t^{E^t})^{\theta} - b_t^{E^t} R_{t+1}. \end{aligned}$$

The solution to this problem is

$$b_t^{E^t} = \frac{1+\theta\beta}{\theta(1+\beta)} \left(\frac{A\theta(\bar{I}_t^{E^t})^{\eta}\rho_{t+1}}{R_{t+1}} \right)^{\frac{1}{1-\theta}} \tag{8}$$

$$c_t^{E^t} = \frac{1-\theta}{1+\theta\beta} b_t^{E^t} \tag{9}$$

$$c_{t+1}^{E^t} = \frac{\beta\,(1-\theta)}{1+\theta\beta} R_{t+1} b_t^{E^t} \tag{10}$$

$$I_t^{E^t} = \frac{\theta(1+\beta)}{1+\theta\beta} b_t^{E^t}. \tag{11}$$

2.4 Banks

Banks are Cournot competitors on the market for loans. To solve the problem of a bank $i = 1, ..., n_t$ we first need to obtain the aggregate demand for loans, solved with respect to the loan interest factor. We start by multiplying equation (8) by the number of borrowers $1 - \alpha$. The inverted total demand for loans is then given by

$$R_{t+1} = A\theta(\bar{I}_t^{E^t})^{\eta}\rho_{t+1} \left[\frac{\theta\,(1+\beta)}{(1-\alpha)\,(1+\theta\beta)} \right]^{\theta-1} \overline{B}_t^{\theta-1} \tag{12}$$

where $\overline{B}_t = b_t^i + \sum_{j \neq i} b_t^j$, represents the total amount of loans granted by the n_t banks at the interest factor R_{t+1}. Each bank maximizes profits $R_{t+1} b_t^i - r_{t+1} d_t^i$, which are fully distributed every period. The problem of a bank can therefore be written as

$$\max_{\{b_t^i, d_t^i\} \in \Re_+^2} \quad R_{t+1} b_t^i - r_{t+1} d_t^i \tag{13}$$

$$s.t. \quad b_t^i = (1-\tau) d_t^i - \phi \tag{14}$$

$$R_{t+1} = A\theta(\bar{I}_t^{E^t})^{\eta}\rho_{t+1} \left[\frac{\theta\,(1+\beta)}{(1-\alpha)\,(1+\theta\beta)} \right]^{\theta-1} \left(b_t^i + \sum_{j \neq i} b_t^j \right)^{\theta-1}. \tag{15}$$

Due to a legal reserve requirement, only a fraction $0 < (1-\tau) < 1$ of deposits can be converted into loans. See (14). Moreover we assume that banks buy a license to operate each period t, whereas loans are repaid only at the beginning of $t+1$. Because of this timing mismatch, and because deposits are the only source of financing for banks, it follows that the cost of this license reduces the credit that banks can offer from deposits, corresponding to a fixed cost $\phi \geq 0$, measured in terms of wasted credit.[3] In this respect, we can interpret equation (14) as representing banks' technology for producing loans out of deposits. Note that equation (14) also represents banks' i balance sheet, where the license appears as an asset

The problem of a bank is worked out in the Appendix, and the first order conditions are

$$\frac{R_{t+1}}{r_{r+1}} = \frac{1}{(1-\tau)\left[1 - \frac{(1-\theta)b_t^i}{b_t^i + \sum_{j\neq i} b_t^j}\right]} \tag{16}$$

together with (14) and (15).

We restrict our attention to symmetric equilibria, meaning that $b_t^i = b_t^j$, for $i \neq j$. Then, we can rewrite (16) as

$$\frac{R_{t+1}}{r_{t+1}} = \frac{1}{(1-\tau)\left[1 - \frac{1-\theta}{n_t}\right]} \tag{17}$$

where n_t is the number of banks operating at time t (in this case we neglect the discrete nature of n_t). Denoting by μ_t the markup factor of the loans interest R_{t+1} on marginal costs $r_{t+1}/(1-\tau)$, and using (17), we have that

$$\mu_t \equiv \frac{R_{t+1}}{r_{t+1}}(1-\tau) = \frac{n_t}{n_t - 1 + \theta} \equiv \mu(n_t). \tag{18}$$

Note that μ_t decreases with the number of banks, becoming 1 when n_t is infinitely big. Since in Cournot equilibrium at least two banks should exist, we have that $1 \leqq \mu_t \leqq \frac{2}{1+\theta} < 2$, given $0 < \theta < 1$.

At equilibrium, due to free entry and exit, we have the zero profit condition of banks,

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[3] This is similar in spirit to the bank facing a liquidity constraint. We could have considered instead equity financing the fixed cost. In this circumstance, the fixed cost would enter the profit function. However, this modification would not change the solution to the problem of the bank, which would be characterized by the same first-order condition (16).

$b_t^i R_{t+1} = d_t^i r_{t+1}$, which using (18) we can rewrite as

$$b_t^i = \frac{(1-\tau)d_t^i}{\mu_t}. \tag{19}$$

Using (14) and (19), we have that, under free entry, we can write the markup as:

$$\mu_t = \frac{(1-\tau)d_t^i}{(1-\tau)d_t^i - \phi} \geqq 1, \text{ and } \ \mu_t = 1 \text{ when } \ \phi = 0. \tag{20}$$

3 Perfect Foresight Equilibria

In this environment, an equilibrium is defined as a sequence of quantities $\{c_t^{L^t},\ c_{t+1}^{L^t},\ d_t^{L^t},\ l_t^{L^t}\}_{t=0}^{+\infty}$, $\{c_t^{E^t},\ c_{t+1}^{E^t},\ I_t^{E^t},\ b_t^{E^t}\}_{t=0}^{+\infty}$, $\{d_t^i,\ b_t^i,\ n_t\}_{t=0}^{+\infty}$, and prices $\{R_{t+1},\ r_{t+1},\ \rho_{t+1},\ w_{t+1}\}_{t=0}^{\infty}$ such that lenders maximize their utility subject to their budget constraint, entrepreneurs maximize their utility subject to their budget constraint and the investment project technology, firms maximize profits, banks maximize their profits behaving *à la Cournot* in the loans market with free entry (so that bank´s profits are zero) and markets clear, i.e., the following equations are verified at equilibrium

$$n_t b_t^i = (1-\alpha) b_t^{E^t} \tag{21}$$

$$n_t d_t^i = \alpha d_t^{L^t} \tag{22}$$

$$l_t = \alpha l_t^{L^t}. \tag{23}$$

In the following we obtain the dynamic equilibrium conditions written in terms of labor and capital. From the equilibrium in the deposit market, i.e., using (5) and (22) we have that $n_t d_t^i = \alpha(1-s_h) w_t l_t^{L^t}$. Using the market clearing condition of the labor market (23) we obtain

$$n_t d_t^i = (1-s_h) w_t l_t. \tag{24}$$

Substituting this last equation in (20), and using also (18), we obtain

$$n_t = \sqrt{\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi} w_t l_t} \tag{25}$$

so that

$$\mu_t = \frac{\sqrt{\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi} w_t l_t}}{\sqrt{\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi} w_t l_t} - (1-\theta)}. \tag{26}$$

We can see that the equilibrium number of banks, n_t, and the markup, μ_t, depend on labor income, introducing a two-way interaction between the real economy and the banking sector.

Substituting (1) in (26) we can see that the markup varies endogenously with k and l, $\mu_t = \mu(k_t, l_t)$. Below, using (3), we present the respective elasticities

$$\varepsilon_{\mu,l} = -\left[1 - \frac{s(k)}{\sigma(k)}\right] \frac{(\mu(k,l)-1)}{2} \quad \text{and} \quad \varepsilon_{\mu,k} = -\frac{s(k)}{\sigma(k)} \frac{(\mu(k,l)-1)}{2}. \tag{27}$$

Note that when $\sigma(k) > s(k)$ the markup is decreasing in k and l and that its variability in absolute value is increasing in μ.

We now obtain the capital accumulation equation. Using (11) and (21) we have that $(1-\alpha)\, I_t^{E^t} = \frac{\theta(1+\beta)}{1+\theta\beta} n_t b_t^i$. This expression, recalling that loans per bank depend on the deposits collected, becomes $(1-\alpha)\, I_t^{E^t} = \frac{\theta(1+\beta)}{1+\theta\beta} \frac{(1-\tau) n_t d_t^i}{\mu_t}$ if we use (19). Hence, using (24), we obtain

$$I_t^{E^t} = \zeta \frac{w_t l_t}{\mu_t}, \quad \text{with} \ \ \zeta \equiv \frac{\theta\,(1+\beta)}{1+\theta\beta} \frac{(1-\tau)(1-s_h)}{(1-\alpha)}. \tag{28}$$

At equilibrium, since $\bar{I}_t^{E^t} = I_t^{E^t}$, capital available for production in period t+1 evolves according to

$$K_{t+1} = (1-\alpha) A (I_t^{E^t})^{\theta+\eta}. \tag{29}$$

Substituting (28) in (29) we obtain the following equation for capital accumulation

$$K_{t+1} = (1-\alpha) A \left(\zeta \frac{w_t l_t}{\mu_t}\right)^{\theta+\eta}. \tag{30}$$

Let us now deduce the intertemporal equilibrium condition in the labor market, considering perfect foresight. Since labor supply depends on r_{t+1} , see (4), we first have to obtain r_{t+1} which, at equilibrium, will clear the deposits market. From (5) the supply of deposits is $\alpha d_t^{L^t} = \alpha \left[\frac{s_h^{s_h} (1-s_h)^{(\frac{1}{\varepsilon}+1-s_h)} r_{t+1}^{1-s_h} w_t^{(\frac{1}{\varepsilon}+1)}}{\varphi}\right]^{\varepsilon}$, and using (19) total demand can be written as $n_t d_t^i = \frac{\mu_t n_t b_t^i}{(1-\tau)}$, where from (21) $n_t b_t^i = (1-\alpha) b_t^{E^t}$, with $b_t^{E^t}$ given in (8). Hence, using (28), r_{t+1} at equilibrium satisfies the following equality

$$\alpha\left[\frac{s_h^{s_h}(1-s_h)^{(\frac{1}{\varepsilon}+1-s_h)}r_{t+1}^{1-s_h}w_t^{(\frac{1}{\varepsilon}+1)}}{\varphi}\right]^{\varepsilon}=(1-\alpha)\frac{\mu_t}{(1-\tau)}\frac{1+\theta\beta}{\theta(1+\beta)}\left(\frac{A\theta(\zeta\frac{w_tl_t}{\mu_t})^{\eta}\rho_{t+1}}{R_{t+1}}\right)^{\frac{1}{1-\theta}}$$

where $R_{t+1}=\frac{\mu_t r_{t+1}}{(1-\tau)}$. Solving in r_{t+1} we obtain

$$r_{t+1}=\left\{\left[\frac{(1-\alpha)(1+\theta\beta)}{\alpha\theta(1+\beta)}\right]^{(1-\theta)}\left[\frac{\varphi}{s_h^{s_h}(1-s_h)^{(\frac{1}{\varepsilon}+1-s_h)}}\right]^{\varepsilon(1-\theta)}\frac{A\theta\zeta^{\eta}\rho_{t+1}(1-\tau)^{\theta}l_t^{\eta}}{(\mu_t)^{\eta+\theta}}w_t^{\eta-(1-\theta)(1+\varepsilon)}\right\}^{\frac{1}{1+(1-s_h)\varepsilon(1-\theta)}}. \tag{31}$$

Hence, substituting (31) in the labor supply (4), and using the labor market clearing condition (23), we obtain

$$l_t = w_t^{\frac{[1-(1-s_h)(1-\theta-\eta)]\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}}\left[\frac{\rho_{t+1}}{(\mu_t)^{\eta+\theta}}\right]^{\frac{(1-s_h)\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}}Z_l \qquad \textit{where} \tag{32}$$

$$Z_l \equiv \left\{\alpha\left(\frac{s_h^{s_h}}{\varphi}\right)^{\varepsilon}\left[A\theta(1-\tau)^{(\eta+\theta)}(1-s_h)^{(\theta+\eta)}\right]^{(1-s_h)\varepsilon}\left[\frac{(1-\alpha)(1+\theta\beta)}{\theta(1+\beta)}\right]^{(1-\theta-\eta)(1-s_h)\varepsilon}\right\}^{\frac{1}{1+(1-s_h)\varepsilon(1-\theta-\eta)}} \tag{33}$$

Finally, considering (1), (2) and (26), the equilibrium dynamic equations (30) and (32) can be written as

$$K_{t+1} = (1-\alpha)A\left(\zeta\frac{w(k_t)l_t}{\mu(k_t,l_t)}\right)^{\theta+\eta} \tag{34}$$

$$l_t = w(k_t)^{\frac{[1-(1-s_h)(1-\theta-\eta)]\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}}\left[\frac{\rho(k_{t+1})}{\mu(k_t,l_t)^{\eta+\theta}}\right]^{\frac{(1-s_h)\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}}Z_l \tag{35}$$

$$\textit{where} \quad \mu(k_t,l_t) = \frac{\sqrt{\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}w(k_t)l_t}}{\sqrt{\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}w(k_t)l_t}-(1-\theta)}. \tag{36}$$

We then define:

Definition 1. *An intertemporal equilibrium with perfect foresight is a sequence* $\{K_t,l_t\}_{t=0}^{\infty}$ *which, given the initial capital stock* $K_0>0$*, satisfies the capital accumulation equation (34) and the labour market equilibrium condition (35) where* $k_t\equiv K_t/l_t$ *and* $w(k_t)$*,* $\rho(k_{t+1})$ *and* $\mu(k_t,l_t)$ *are given respectively by (1), (2) and (36), and with* ζ *and* Z_l *respectively defined in (28) and (33).*

Equations (34) and (35) rule the dynamics of our economy, and define a two-dimensional dynamic system with one predetermined variable, the aggregate capital stock, which is given by past decisions of entrepreneurs. In contrast, employment in t is affected by expectations about the future real rental rate of capital, opening the way for expectations driven fluctuations.

4 Steady state

A steady state solution $(K, l) \in \Re^2_{++}$ of the dynamic system (34) and (35) is a stationary solution $K_{t+1} = K_t = K > 0$ and $l_{t+1} = l_t = l > 0$ of that system that, using (1), (2) and (36) satisfies the following equations

$$\begin{aligned} kl &= (1-\alpha)A\left(\frac{\theta\,(1+\beta)}{1+\theta\beta}\frac{(1-\tau)(1-s_h)}{(1-\alpha)}\right)^{\theta+\eta}\left(\frac{\Theta[f(k)-kf'(k)]l}{\mu(k,l)}\right)^{\theta+\eta} && (37) \\ l &= \Theta^{\frac{[1+(1-s_h)(\theta+\eta)]\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}}\left[f(k)-kf'(k)\right]^{\frac{[1-(1-s_h)(1-\theta-\eta)]\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}}\left[\frac{f'(k)}{\mu(k,l)^{\eta+\theta}}\right]^{\frac{(1-s_h)\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}} Z_l && (38) \end{aligned}$$

where $k \equiv K/l$, and

$$\mu(k,l) \equiv \frac{\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta[f(k)-kf'(k)]l\right)^{\frac{1}{2}}}{\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta[f(k)-kf'(k)]l\right)^{\frac{1}{2}}-(1-\theta)} \qquad (39)$$

with Z_l given in (33).

4.1 Normalized Steady State

We ensure existence of a steady state, namely the normalized steady state $K_{nss} = 1$, $l_{nss} = 1$ so that $k_{nss} \equiv K_{nss}/l_{nss} = 1$, by following the usual procedure of fixing the parameters A and φ at the appropriate levels, A^* and φ^*.

Proposition 2. ***Normalized Steady State:*** *Define*

$$\begin{aligned} K_{nss} &= 1 = (1-\alpha)A\left(\frac{\theta\,(1+\beta)}{1+\theta\beta}\frac{(1-\tau)(1-s_h)}{(1-\alpha)}\right)^{\theta+\eta}\left(\frac{\Theta[f(1)-f'(1)]}{\mu(1,1)}\right)^{\theta+\eta} && (40) \\ l_{nss} &= 1 = \Theta^{\frac{[1+(1-s_h)(\theta+\eta)]\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}}\left[f(1)-f'(1)\right]^{\frac{[1-(1-s_h)(1-\theta-\eta)]\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}}\left[\frac{f'(1)}{\mu(1,1)^{\eta+\theta}}\right]^{\frac{(1-s_h)\varepsilon}{1+(1-s_h)\varepsilon(1-\theta-\eta)}} Z_l && (41) \end{aligned}$$

where Z_l *is given in (33) and* $\mu(1,1)$ *satisfies the following equation:*

$$\mu(1,1) = \frac{\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta[f(1)-f'(1)]\right)^{\frac{1}{2}}}{\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta[f(1)-f'(1)]\right)^{\frac{1}{2}}-(1-\theta)}. \tag{42}$$

Then, $(K_{nss} = 1, l_{nss} = 1)$, *with the corresponding normalized capital-labor ratio* $k_{nss} = K_{nss}/l_{nss} = 1$ *is the normalized steady state of the dynamic system (34) and (35) if and only if* $A = A^*$ *and* $\varphi = \varphi^*$ *with*

$$\begin{aligned}
A^* &\equiv \frac{\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta[f(1)-f'(1)]\right)^{\frac{\theta+\eta}{2}}}{(1-\alpha)\left\{\frac{\theta(1+\beta)}{1+\theta\beta}\frac{(1-\tau)(1-s_h)}{(1-\alpha)}\Theta[f(1)-f'(1)]\left[\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta[f(1)-f'(1)]\right)^{\frac{1}{2}}-(1-\theta)\right]\right\}^{\theta+\eta}} \\
\varphi^* &\equiv A^{*(1-s_h)}\Psi[f(1)-f'(1)]^{[1-(1-s_h)(1-\theta-\eta)]}f'(1)\left[\frac{\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta[f(1)-f'(1)]\right)^{\frac{1}{2}}-(1-\theta)}{\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta[f(1)-f'(1)]\right)^{\frac{1}{2}}}\right]^{(\eta+\theta)} \\
\textit{where } \Psi &= \theta^{(1-s_h)}\Theta^{[1+(1-s_h)(\theta+\eta)]}(1-\tau)^{(\eta+\theta)(1-s_h)}\alpha s_h^{s_h}(1-s_h)^{(1-s_h)[\theta+\eta]}\left[\frac{(1-\alpha)(1+\theta\beta)}{\theta(1+\beta)}\right]^{(1-\theta-\eta)(1-s_h)}.
\end{aligned}$$

From now on we assume that $A = A^*$ and $\varphi = \varphi^*$, so that the normalized steady state exists. However, we still have to ensure that at the normalized steady state $n(1,1) \geq 2$. Using (25) we can see that by restricting by assumption the parameter ϕ to the interval $\phi < \phi^* \equiv \frac{(1-\theta)(1-\tau)(1-s_h)[f(1)-f'(1)]}{4}$ we guarantee that $n(1,1) \geq 2$, and consequently, that at the normalized steady state $1 \leq \mu(1,1) \leq \frac{2}{1+\theta}$. However, nothing guarantees steady state uniqueness. In the following subsection we discuss the issue of steady state multiplicity.

4.2 Steady State Multiplicity

Using (37) and (38) we are able to express l as a function of k:[4]

$$l = \lambda(k) \equiv c_l[f(k)-kf'(k)]^{\varepsilon s_h}\left[f'(k)\right]^{(1-s_h)\varepsilon}(k)^{(1-s_h)\varepsilon} \tag{43}$$

where

$$c_l \equiv \frac{\Theta^{\varepsilon}Z_l^{1+(1-s_h)\varepsilon(1-\theta-\eta)}}{[(1-\alpha)A]^{(1-s_h)\varepsilon}\left(\frac{\theta(1+\beta)}{1+\theta\beta}\frac{(1-\tau)(1-s_h)}{(1-\alpha)}\right)^{(1-s_h)\varepsilon(\eta+\theta)}}.$$

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[4] See Appendix A.2.1.

Substituting now (39) and (43) in (37) we obtain

$$h(k) \equiv B_1 g_1(k) + B_2 g_2(k) = 1 \tag{44}$$

where

$$\begin{aligned}
g_1(k) &\equiv [f(k) - kf'(k)]^{-\frac{1+\varepsilon s_h}{2}} [kf'(k)]^{\frac{-(1-s_h)\varepsilon}{2}} \\
g_2(k) &\equiv [f(k) - kf'(k)]^{\frac{\varepsilon s_h(1-\eta-\theta)-(\eta+\theta)}{(\eta+\theta)}} [f'(k)]^{\frac{(1-s_h)\varepsilon(1-\eta-\theta)}{\eta+\theta}} (k)^{\frac{1+(1-\eta-\theta)(1-s_h)\varepsilon}{\eta+\theta}} \\
B_1 &\equiv \left[\frac{(1-\theta)\,\phi}{\Theta c_l(1-\tau)(1-s_h)}\right]^{\frac{1}{2}} \\
B_2 &\equiv \frac{c_l^{\frac{1-\eta-\theta}{(\eta+\theta)}}(1+\theta\beta)(1-\alpha)}{\Theta\left[(1-\alpha)A\right]^{\frac{1}{\eta+\theta}}\theta\,(1+\beta)\,(1-\tau)(1-s_h)}.
\end{aligned}$$

A steady state solution is a value $k > 0$ satisfying equation (44) and the corresponding $l = \lambda(k)$ satisfying (43). The number of steady states is determined by the number of intersections of the curve $h(k)$ with 1. Of course, if the elasticity of the function $h(k)$ is positive (negative) for all $k > 0$, so that $h(k)$ is always increasing (decreasing), there is at most one steady state. It follows that a necessary condition for steady state multiplicity is that $h(k)$ is non monotonic. In Appendix A.2.2 we show that the elasticity of $h(k)$, $\varepsilon_{h(k)}$, is a function of k. Therefore its sign may change, so that steady state multiplicity can naturally emerge in our model. To further discuss this issue, in the following we consider the simple case of a Cobb-Douglas production function for the consumption good where $Y = \Theta l f(k) = \Theta l k^s$, $\sigma(k) = 1$ and $s(k) = s$, obtaining in this case that $\varepsilon_{h(k)} \geq 0$ when[5]

$$k \geq k^* \equiv \left[\frac{B_1(\theta+\eta)(1+\varepsilon)s}{2B_2[1+\varepsilon s - s(1+\varepsilon)(\theta+\eta)](1-s)^{\frac{2\varepsilon s_h-(\eta+\theta)(1+\varepsilon s_h)}{2(\eta+\theta)}} s^{\frac{(1-s_h)\varepsilon(2-\eta-\theta)}{2(\eta+\theta)}}}\right]^{\frac{2(\eta+\theta)}{2(1+\varepsilon s)-(\theta+\eta)(1+\varepsilon)s}}. \tag{45}$$

Hence, in the Cobb-Douglas case, provided $\phi > 0$, ensuring that $B_1 > 0$, and $\theta + \eta < \frac{1+\varepsilon s}{s(1+\varepsilon)}$, there is a unique $k = k^* > 0$ such that $\varepsilon_{h(k)} = 0$. So, under these conditions, $\varepsilon_{h(k)}$ changes sign only once. For $0 < k < k^*$ the function $h(k)$ is decreasing, reaching its minimum when $k = k^*$, becoming increasing for $k > k^*$. Note also that in the Cobb-Douglas case, with $k^* > 0$,

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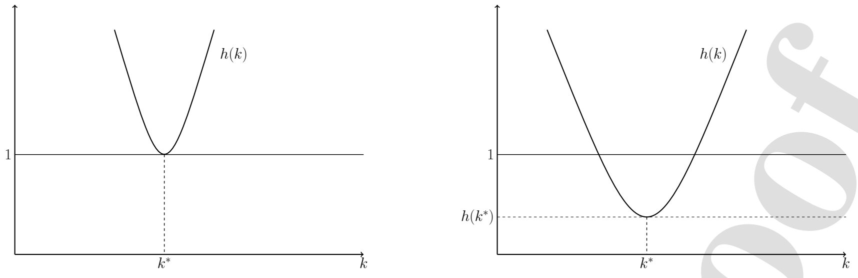
[5] See Appendix A.2.2.



(a) (b)

Figure 1: Conditions for steady state existence

we have $\underset{k\longrightarrow 0}{lim}\, h(k) = \underset{k\longrightarrow +\infty}{lim} h(k) = +\infty$. Hence, a condition for steady state existence is that $h(k^*) \leq 1$. If $h(k^*) = 1$, k^* is the unique steady state, whereas if $h(k^*) < 1$, two steady states exist, each on a different side of k^*. See figure 1.

Note that, since when $\phi = 0$ we have $k^* = 0$, the existence of bank power in the loans market ($\phi > 0$) is necessary for the existence of two steady states.[6]

As in Proposition 2 we have ensured existence of the normalized steady state $k_{nss} = 1$, so that $h(k_{nss}) = 1$, if $k^* = k_{nss} = 1$, k_{nss} is the unique steady state.[7] If $k^* \neq k_{nss}$, the function $h(k)$ will cross the value 1 a second time, so that another steady state exists in the Cobb-Douglas case. See Figure 2, where the case where $k^* > k_{nss} = 1$ is illustrated by curve h_1 and the case where $k^* < k_{nss} = 1$ is illustrated by curve h_2. Of course, for this second steady state to be admissible we still have to ensure that the number of banks at this other steady state is equal or higher than two.

5 Local dynamics

In this section we discuss the existence of local endogenous fluctuations due to autonomous changes in selffullfilling expectations, by analyzing the conditions under which local indeterminacy emerges. We start by loglinearizing the dynamic system (34)-(35) around the steady state. Denoting percentage deviations from the steady state respectively by $\hat{K}_t \equiv \frac{dK_t}{K_t}$ and

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[6] Recall that a positive ϕ is necessary to ensure that $\mu > 1$. See (20).

[7] In this case σ_T given in Proposition 3 equals 1 as shown in Appendix A.2.3.

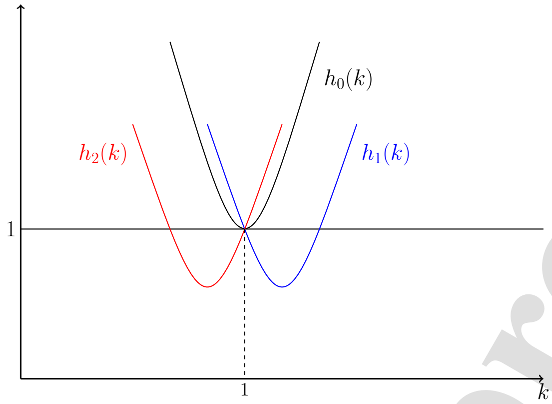

Figure 2: Steady state multiplicity

$\hat{l}_t \equiv \frac{dl_t}{l_t}$, we show in Appendix A.3 that we can write the linearized system as follows

$$\hat{K}_{t+1} = \frac{(\theta+\eta)(1+\mu)}{2}\frac{s}{\sigma}\hat{K}_t + \frac{(\theta+\eta)(1+\mu)}{2}\left[1-\frac{s}{\sigma}\right]\hat{l}_t \tag{46}$$

$$\begin{aligned}\hat{l}_{t+1} &= -\frac{s}{1-s}\left[\frac{(\theta+\eta)(1+\mu)}{2}\left(1-\frac{1-s}{\sigma}\right)+\frac{s_h}{(1-s_h)}\right]\hat{K}_t + \\ &+\left\{\frac{1}{(1-s_h)\varepsilon^*}\frac{\sigma}{1-s}-\left[\frac{\sigma-s}{1-s}\right]\left[\frac{(\theta+\eta)(1+\mu)}{2}\left(1-\frac{1-s}{\sigma}\right)+\frac{s_h}{(1-s_h)}\right]\right\}\hat{l}_t\end{aligned} \tag{47}$$

where σ denotes the elasticity of substitution in production between capital and labor and s represents the capital share in production, both evaluated at the steady state. Moreover, we define $\varepsilon^* \equiv \frac{\varepsilon}{1+\varepsilon}\epsilon(0,1)$ where $\varepsilon > 0$ is the elasticity of labour supply. See (4).

The trace, $\mathcal{T}$, and determinant, $\mathcal{D}$ of the jacobian matrix J of our dynamic system, correspond respectively to the sum and product of the two roots (eigenvalues) of the associated characteristic polynomial $P(\lambda) \equiv \lambda^2 - \lambda\mathcal{T} + \mathcal{D}$ and, defining

$$\nu \equiv \frac{(\theta+\eta)(1+\mu)}{2} \tag{48}$$

can be written as:

$$\mathcal{D} = \frac{s\nu}{(1-s)(1-s_h)\varepsilon^*} > 0 \tag{49}$$

$$\mathcal{T} = \frac{[s_h s + (1-s_h)\nu]\,\varepsilon^*}{(1-s)(1-s_h)\varepsilon^*} + \frac{(1-s_h\varepsilon^*) - \nu(1-s_h)\varepsilon^*}{(1-s)(1-s_h)\varepsilon^*}\sigma. \tag{50}$$

Note that when $\nu = 1$ these are the usual expressions for the trace and the determinant in the case of a Cobb-Douglas utility function in one sector models with constant returns to scale technologies and no distortions, where indeterminacy can only emerge for $\sigma < s$.[8] Indeed the parameter ν summarizes the new mechanisms considered in this paper: the market power of banks in the market for loans, μ, externalities in the production of the investment good, η, and the existence of a profitable entrepreneurial activity (the transformation of the consumption into the investment good) which finances current and future consumption of entrepreneurs. See (9) and (10). Note that the lower θ the more profitable this activity is. Furthermore, in our model these parameters satisfy the following assumption:

Assumption 1. $0 < \theta < 1$, $\eta \geq 0$, *and* $1 \leq \mu \leq \frac{2}{1+\theta}$ *as* $n \geq 2$.

Note also that without externalities ($\eta = 0$) we always have $\nu < 1$. Indeed, substituting (18) in (48) with $\eta = 0$, the inequality $\nu < 1$ can be rewritten as $n > 1 - \frac{\theta}{2}$, which is always satisfied under Assumption 1.

Following Grandmont et al. (1998), we study the local stability properties of our model, which are determined by the eigenvalues of the characteristic polynomial $P(\lambda)$, by referring to the diagram represented in the next Figure.

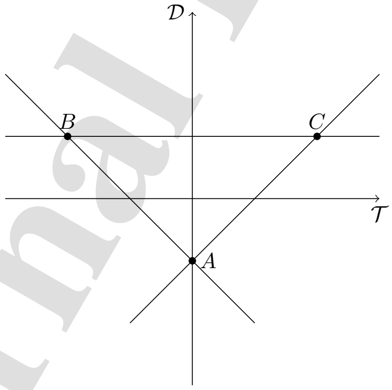

Figure 3: The ABC triangle

One eigenvalue is equal to 1 on the line AC ($\mathcal{D} = \mathcal{T} - 1$). On the line AB ($\mathcal{D} = -\mathcal{T} - 1$) one eigenvalue is equal to -1. On the segment BC the two eigenvalues are complex conjugates

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[8] See Lloyd-Braga et al. (2007) without externalities, or Seegmuller (2005) and Dos Santos Ferreira and Lloyd-Braga (2005) when the markup factor in the product market is one with $s_h = 0$.

with modulus equal to 1. Therefore the steady state is a sink (both eigenvalues with modulus lower than one) when $(\mathcal{T}, \mathcal{D})$ is inside the triangle ABC. Since only capital is a predetermined variable, when the steady state is a sink, it is locally indeterminate[9] and there are infinitely many stochastic endogenous fluctuations (sunspots) arbitrarily close to the steady state. The steady state is a source (both eigenvalues with modulus higher than one) if $(\mathcal{T}, \mathcal{D})$ is above AB, AC and BC or below AB and AC. It is saddle stable (one eigenvalue with modulus higher than one and one eigenvalue with modulus lower than one) in the remaining cases.

We can also use the same diagram to study local bifurcations. When, by slightly changing a (bifurcation) parameter, the values of $\mathcal{T}$ and $\mathcal{D}$ cross the AB line, a flip bifurcation generically occurs and deterministic cycles of period two appear. When the values of $\mathcal{T}$ and $\mathcal{D}$ cross the AC line, a transcritical bifurcation generically occurs.[10] In this case, if $(\mathcal{T}, \mathcal{D})$ is close enough to the AC line, two infinitely close steady states coexist, which exchange stability properties as $(\mathcal{T}, \mathcal{D})$ crosses line AC.[11]

In our analysis we take s_h, s, ε^*, ν and σ as parameters characterizing our economy. In the following we consider fixed values for $s < 1/2$ since empirical values for the share of capital in production are typically between 0.25 and 0.5. We further assume that $0 \leq s_h < \frac{1-2s}{1-s}$, a necessary condition for the emergence of local indeterminacy when $\nu > 1$, ensuring $\mathcal{D} < 1$. Note that the case $s_h = 0$ has been extensively considered in the literature. See for instance Reichlin (1986), Seegmuller (2005), and Dos Santos Ferreira and Lloyd-Braga (2005).[12] We organize our discussion in terms of σ, ν and ε^*. We consider σ as the bifurcation parameter, by varying its value continuously in its domain for given values of ν and ε^*. Furthermore we restrict our analysis to empirically plausible values of the parameters. According to the survey by Knoblach and Stöckl (2020), the majority of empirical estimates for the elasticity of substitution are between 0.25 and 1.2. However, other values of σ below 2 can not be excluded.[13] Therefore, in the following, we will consider values of σ above s. Note that for this range of

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[9] Indeterminacy occurs when the number of eigenvalues strictly lower than one in absolute value is larger than the number of predetermined variables.

[10] As we have ensured existence of the normalized steady state in Proposition 2, and it is likely that at most two steady exist we rule out saddle node and pitchfork bifurcations.

[11] When $(\mathcal{T}, \mathcal{D})$ is on line AC the two steady states collapse into only one.

[12] Remark that the equilibrium conditions considered in the Woodford (1986) model, where infinitely lived workers face cash in advance constraints and capitalists do not work, are similar to ours in the case where $s_h = 0$. See Grandmont et al. (1998).

[13] See their figure 2.

values for σ the wage bill increases with l. We do not restrict a priory the set of possible values for the labor supply elasticity ε. Accordingly we consider values for ε^* between zero and one. Note that, as $\varepsilon = \frac{\varepsilon^*}{1-\varepsilon^*}$, the limit case $\varepsilon^* = 1$ corresponds to the case of an infinitely elastic labor supply, whereas the labor supply does not respond to the wage for $\varepsilon^* = 0$. In the next Assumption we summarize the restrictions considered on the remaining parameters' values.

Assumption 2. $s < 1/2$, $s_h < \frac{1-2s}{1-s}$, $\sigma > s$, $0 < \varepsilon^* < 1$.

We present our results on the local stability properties of the model in the Proposition below.

Proposition 3. *Considering that Assumptions 1 and 2 are satisfied, define* $\nu_1 \equiv -\frac{s_h}{2(1-s_h)} + \sqrt{\frac{s_h^2}{4(1-s_h)^2} + \frac{(1-s)}{s}}$, $\nu_2 \equiv \frac{1-s-s_h}{s(1-s_h)}$, $\nu_3 \equiv \frac{(1-s)(1-s_h)}{s}$, $\varepsilon_1^* \equiv \frac{s}{(1-s)(1-s_h)}$, $\varepsilon_2^* \equiv \frac{s\nu}{(1-s)(1-s_h)}$, $\varepsilon_3^* \equiv \frac{1}{\nu(1-s_h)+s_h}$, $\sigma_T \equiv \frac{(1-s-s_h)\varepsilon^*+\nu[s-(1-s_h)\varepsilon^*]}{(1-s_h\varepsilon^*)-\nu(1-s_h)\varepsilon^*}$ *and* $\sigma_F \equiv \frac{[(1-s_h)(1-s)+s_h s]\varepsilon^*+\nu[s+(1-s_h)\varepsilon^*]}{\nu(1-s_h)\varepsilon^*-(1-s_h\varepsilon^*)} > 1$. *Then the following generically holds:*

(i) *For* $\nu < 1$.

If $\varepsilon^* < \varepsilon_1^*$, *the steady state is always a saddle. If* $\varepsilon^* > \varepsilon_1^*$ *the steady state is a sink for* $\sigma \in (s, \sigma_T)$ *and a saddle for* $\sigma > \sigma_T$.

(ii) *For* $1 < \nu < \nu_1$.

If $\varepsilon^* < \varepsilon_1^*$, *the steady state is a source for* $\sigma \in (s, \sigma_T)$ *and a saddle for* $\sigma > \sigma_T$. *If* $\varepsilon_1^* < \varepsilon^* < \varepsilon_3^*$ *the steady state is always a saddle. If* $\varepsilon_3^* < \varepsilon^* < 1$, *the steady state is a saddle for* $\sigma \in (s, \sigma_T)$, *a sink for* $\sigma \in (\sigma_T, \sigma_F)$,*and a saddle for* $\sigma > \sigma_F$.

(iii) *For* $\nu_1 < \nu < \nu_2$

If $\varepsilon^* < \varepsilon_1^*$, *the steady state is a source for* $\sigma \in (s, \sigma_T)$ *and a saddle for* $\sigma > \sigma_T$. *If* $\varepsilon_1^* < \varepsilon^* < \varepsilon_3^*$ *the steady state is always a saddle. If* $\varepsilon_3^* < \varepsilon^* < \varepsilon_2^*$, *the steady state is a saddle for* $\sigma \in (s, \sigma_T)$, *a source for* $\sigma \in (\sigma_T, \sigma_F)$,*and a saddle for* $\sigma > \sigma_F$. *If* $\varepsilon_2^* < \varepsilon^* < 1$ *the steady state is a saddle for* $\sigma \in (s, \sigma_T)$, *a sink for* $\sigma \in (\sigma_T, \sigma_F)$,*and a saddle for* $\sigma > \sigma_F$.

(iv) *For* $\nu_2 < \nu < \nu_3$

If $\varepsilon^* < \varepsilon_3^*$, *the steady state is a source for* $\sigma \in (s, \sigma_T)$ *and a saddle for* $\sigma > \sigma_T$. *If* $\varepsilon_3^* < \varepsilon^* < \varepsilon_1^*$, *the steady state is a source for* $\sigma \in (s, \sigma_F)$ *and a saddle for* $\sigma > \sigma_F$. *If* $\varepsilon_1^* < \varepsilon^* < \varepsilon_2^*$ *the steady state is a saddle for* $\sigma \in (s, \sigma_T)$, *a source for* $\sigma \in (\sigma_T, \sigma_F)$,*and a saddle for* $\sigma > \sigma_F$. *If* $\varepsilon_2^* < \varepsilon^* < 1$ *and* $s_h < s_{h_2}$, *the steady state is a saddle for* $\sigma \in (s, \sigma_T)$, *a sink for* $\sigma \in (\sigma_T, \sigma_F)$,*and a saddle for* $\sigma > \sigma_F$.

(v) *For* $\nu > \nu_3$

If $\varepsilon^* < \varepsilon_3^*$, *the steady state is a source for* $\sigma \in (s, \sigma_T)$ *and a saddle for* $\sigma > \sigma_T$. *If* $\varepsilon_3^* < \varepsilon^* < \varepsilon_1^*$, *the steady state is a source for* $\sigma \in (s, \sigma_F)$ *and a saddle for* $\sigma > \sigma_F$. *If* $\varepsilon_1^* < \varepsilon^* < 1$ *the steady state is a saddle for* $\sigma \in (s, \sigma_T)$, *a source for* $\sigma \in (\sigma_T, \sigma_F)$, *and a saddle for* $\sigma > \sigma_F$.

In all the cases considered above, when σ *crosses the value* σ_T *a transcritical bifurcation generically occurs and when* σ *crosses the value* σ_F *a flip bifurcation generically occurs.*

Proof: See Appendix A.3.

6 Discussion of the results

From Proposition 3 we can see that indeterminacy requires a sufficiently high labour supply elasticity, and is possible for intermediate values of the elasticity of substitution in production, provided v is not too high ($\nu < \nu_3$) and s_h is sufficiently low ($s_h < \frac{1-2s}{1-s}$) as considered in Assumption 1. Indeed, for the first four different cases for ν considered in Proposition 3, where $\nu < \nu_3$, indeterminacy only emerges when ε^* exceeds a lower limit, which is always above $\varepsilon_1^* \equiv \frac{s}{(1-s)(1-s_h)} < 1$. See Figure 4, where we have represented the indeterminacy regions in the space (v, ε^*). These results are in accordance with the ones found in one sector models in the absence of externalities by Cazzavillan and Pintus (2004), Nourry (2001), and Nourry and Venditti (2006) who also find that indeterminacy requires a sufficiently elastic labour supply and a small share of current consumption. Lloyd-Braga et al. (2007) find that local indeterminacy is compatible with a sufficiently high s_h provided labor externalities are sufficiently higher than capital externalities. Otherwise, indeterminacy requires a sufficiently small s_h like in our case, without labor externalities in the production of the final output, and where all distortions are related with the production of the capital good.

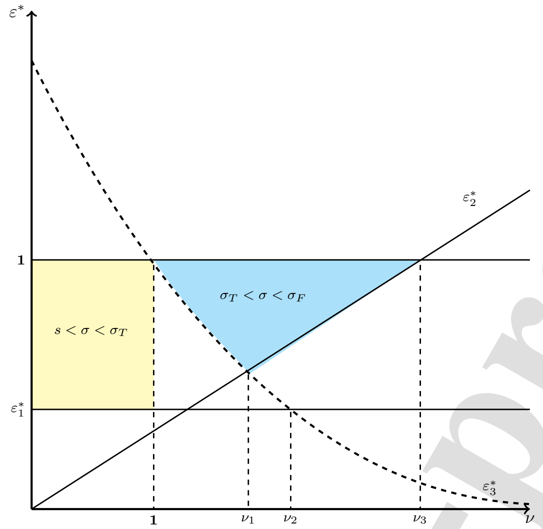

Figure 4: Indeterminacy regions in the (ν, ε^*) space.

From Figure 4 we can see that whenever ε^* is sufficiently high there is always an interval for σ ensuring the occurrence of local indeterminacy, provided $\nu < \nu_3$. So, considering a given value for ε^* sufficiently high, we study the influence of v on the set of values for the elasticity of substitution in production compatible with local indeterminacy. Below we present the derivatives of the critical values σ_T and σ_F with respect to v:

$$\begin{aligned} \frac{\partial \sigma_T}{\partial \nu} &= \frac{[s - (1-s)(1-s_h)\varepsilon^*](1-\varepsilon^*)}{\left[(1 - s_h\varepsilon^*) - \nu(1-s_h)\varepsilon^*\right]^2} \\ \frac{\partial \sigma_F}{\partial \nu} &= -\frac{s + (1-s_h)\varepsilon^*[1 + (1-s)(1-2s_h)\varepsilon^*]}{\left[\nu(1-s_h)\varepsilon^* - (1 - s_h\varepsilon^*)\right]^2} < 0. \end{aligned}$$

Note that when $\varepsilon^* > \varepsilon_1^*$ we have $\frac{\partial \sigma_T}{\partial v} < 0$, implying that σ_T decreases with ν, whenever the steady state is indeterminate (a sink). We also always have that $\frac{\partial \sigma_F}{\partial v} < 0$. From Figure 4 we can also see that the range of values for σ for which indeterminacy emerges is limited by different critical values, depending on whether v is below or above one. In the following we will consider these cases separately, allowing v to change in each case as μ is made to vary in its domain $\mu \,\epsilon [1, \frac{2}{1+\theta}]$.[14] We start with the case where $\nu < 1$, individualizing the benchmark case

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[14] We do this by changing ϕ. See (20).

of no externalities to better highlight the mechanisms involved. Recall that in the absence of externalities, $\eta = 0$, we have $\nu < 1$. See (48).

6.1 The case where $\nu < 1$

6.1.1 The case with no externalities ($\eta = 0$)

When $\eta = 0$, as we fall into configuration *(i)* of Proposition 3, where $\nu < 1$, local indeterminacy emerges for values of the elasticity of substitution between s and σ_T. Hence, as $\frac{\partial \sigma_T}{\partial v} < 0$, in this case, local indeterminacy becomes easier with a smaller ν, which implies a smaller μ, i.e., with a larger number of banks, as μ decreases with n.[15] So, in this case, more competition, measured by the number of banks in the system, promotes macroeconomic instability by facilitating the appearance of local indeterminacy and sunspot fluctuations. A similar result is present in Seegmuller (2005), where the existence of profits due to monopolistic competition in the product market, renders the emergence of local indeterminacy more difficult than with perfect competition. However, in Seegmuller (2005) local indeterminacy only occurs when $\sigma < s$. In our case, as σ_T decreases with v, the highest possible value for σ_T is obtained when $v \to 0$, i.e., $\sigma_T = \frac{(1-s-s_h)\varepsilon^*}{(1-s_h\varepsilon^*)}$, which attains its maximal value of $\frac{(1-s-s_h)}{(1-s_h)} < 1 - s$ when $\varepsilon^* = 1$. We conclude that without externalities in the production of the capital good, i.e. when $\eta = 0$, local indeterminacy may occur for values of the elasticity of substitution in production between s and $\frac{(1-s-s_h)}{(1-s_h)}$. This is an important and new result, as it contrasts with what happens under perfect competition and constant returns to scale in one sector models where profits are zero,[16] and local indeterminacy only occurs for values of σ below s, which are not empirically plausible. However in our framework, because $\theta < 1$, entrepreneurs earn positive profits from operating their technology. When banks have a higher market power, they are able to extract a larger part of these profits from entrepreneurs. As, due to the free entry condition, they do not retain the surplus that they extract from borrowers, a higher market power of banks, i.e. a higher μ, countervails the effects on local indeterminacy of decreasing returns to scale in the capital production technology.

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[15] Note that the markup μ only influences local dynamics because it varies endogenously with K and l. See (27), (57) and (58).

[16] Note that in our model this corresponds to the limit case with $\theta + \eta = 1$ (constant social returns to scale) and $\mu = 1$ (perfect competition in the market for loans), where $\nu = 1$.

6.1.2 The case with externalities

If externalities are not too high, so that $\nu < 1$, implying decreasing social returns to scale, $(\theta + \eta < 1)$, we fall again into case (i) of Proposition 3, and the results presented above still apply.[17]

6.2 The case where $\nu > 1$

In contrast, whenever $\theta + \eta \geq 1$ we obtain $\nu > 1$,[18] falling in the remaining configurations of Proposition (3). Local indeterminacy is only possible in configurations *(ii)-(iv)* of Proposition (3), where $1 < \nu < \nu_3$, occuring for values of the elasticity of substitution between σ_T and σ_F, provided ε^* is sufficiently high. As both σ_T and σ_F decrease when v increases, i.e., as both decrease with n, it is difficult to evaluate the effect of ν on the likelihood of local indeterminacy when $1 < \nu < \nu_3$. However, in the case of constant social returns to scale, where $\theta + \eta = 1$, bank market power in the loans market *per se* is responsible for the emergence of local indeterminacy. In this case, for $\mu = 1$, i.e., in the absence of market power in the market for loans, we would have $\nu = 1$ and, as referred above, local indeterminacy could not occur with $\sigma > s$. In contrast, when $\mu > 1$ so that $\nu > 1$, local indeterminacy becomes possible for values of σ around one, which, as discussed above, are empirically plausible. Indeed, we know that $\sigma_F > 1$, and it can be easily shown that $\sigma_T < 1$ when ε^* is sufficiently high, i.e. when $\varepsilon^* > \varepsilon_4^* \equiv \frac{1-s\nu}{1-s}$.[19] Similarly, in the absence of market power in the banking sector ($\mu = 1$), local indeterminacy becomes possible for values of σ around one, whenever positive externalities are such that increasing social returns to scale $(\theta + \eta > 1)$ are obtained. As departing from $\nu = 1$, both externalities and bank power in the credit market are able to increase v above one, we conclude that both facilitate the emergence of local indeterminacy[20] for plausible values of the elasticity of substitution in production around one.[21] Note that, with increasing social returns to scale,

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[17] Note that, since μ does not take values higher than 2 (see Assumption 1), ν is always lower than 1 for all admissible values of μ when $\eta < 2/3 - \theta$.

[18] It is sufficient to have $\theta + \eta > 1$ to obtain $\nu > 1$. However, we may have $\theta + \eta = 1$ provided $\mu > 1$.

[19] Note that when $1 < v < v_1$ we have $\varepsilon_4^* > \varepsilon_3^*$.

[20] We are considering that $\nu < \nu_3$. Since $1 \leqq \mu < 2$, $\nu < \nu_3$ is ensured for any admissible value of the bank power in the loans market if $s < 0.4$, $s_h < \frac{1-2.5s}{1-s}$ and $\eta < \frac{2}{3}\frac{(1-s)(1-s_h)}{s} - \theta$.

[21] A similar equivalence between imperfect competition in the output market and increasing returns to scale due to externalities in the production of output is referred in Cazzavillan et al. (1998) and Lloyd-Braga et al. (2014).

entrepreneurs still make positive profits as $\theta < 1$. However, in this case a higher market power of banks, which increases R, leads to a higher demand for loans at the general equilibrium level.[22] This increases investment in the capital production technology (see (11)) which, in turn, increases the profits of entrepreneurs. Hence, in the presence of increasing social returns to scale, a higher μ, exacerbates the effects on local indeterminacy of decreasing returns to scale in the capital production technology.

7 The local indeterminacy mechanism

Without externalities, so that we have decreasing returns to scale, we fall in case *(i)* of Proposition 3 where $v < 1$, and even in the absence of bank market power, indeterminacy is possible for $\sigma > s$, becoming more likely as v decreases, moving away from $v = 1$. In this case, the introduction of bank market power increases v towards one, hindering therefore the emergence of indeterminacy, as discussed above. In the presence of increasing social returns to scale ($\theta + \eta > 1$), v is higher than one and, again, indeterminacy is possible even in the absence of market power. However, with constant social returns to scale ($\theta + \eta = 1$), bank market power *per se* may lead to the emergence of local indeterminacy for empirically plausible values of σ. We conclude that the indeterminacy mechanisms operating in our framework are different for $v < 1$ and $v > 1$, i.e. without externalities and when externalities are sufficiently important. In the next subsection we discuss them separately.

7.1 Without externalities

Our local indeterminacy mechanism when $\eta = 0$ works as follows. Suppose that, departing from the steady state, at time t agents expect that the future rental rate of capital, ρ_{t+1}, will increase. As $\theta < 1$ entrepreneurs want to invest more, (see (11)). Accordingly, they demand more loans today and, thereby, banks end up demanding more deposits, which leads to an increase in r_{t+1}. Hence, l_t increases (see (4)) and so does the wage bill. Therefore, for a given predetermined value of K_t, μ_t decreases (see (26)) because as banks become more profitable, new banks enter the market. This confirms the increase in investment in the capital production

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[22] Indeed, substituting (11) in (8) with $\bar{I}_t^{E^t} = I_t^{E^t}$, we have that $\varepsilon_{b^{E^t},R} = \frac{-1}{1-\theta-\eta} > 0$.

technology, see (28), leading to an increase in future capital, K_{t+1}, as shown in (30). Using (46), we can easily see that the smaller is μ, (the smaller is ν) the lower will be the observed increase in future capital. In the absence of further changes in expectations, so that future labor supply does not shift, the increase in K_{t+1} implies an increase in the future marginal productivity of labor, which results in an increase in l_{t+1}. This increase will be higher, the higher is the elasticity of labor supply and the smaller is σ.[23] For the expectation to become self fulfilled the ratio K_{t+1}/l_{t+1} must decrease, i.e., K_{t+1} must increase less than l_{t+1}. As explained above, this requires a sufficiently small $v < 1$, an ε^* sufficiently high and a small $\sigma < \sigma_T$.

7.2 With externalities

When externalities are sufficiently small so that $v < 1$ the mechanism presented above still applies. In contrast when $\eta + \theta > 1$, so that $\nu > 1$ for any $\mu \geq 1$, our indeterminacy mechanism works as follows. Suppose that, departing from the steady state, at time t agents expect that the future rental rate of capital, ρ_{t+1}, will increase. As $\theta + \eta > 1$ investment in the capital production technology at the general equilibrium level decreases, so that entrepreneurs demand less loans today and, thereby, banks end up demanding less deposits, which leads to a decrease in r_{t+1}. Hence, l_t decreases (see (4)) and so does the wage bill. Note that the decrease in l_t is higher the higher is $(1 - s_h)\varepsilon$, i.e., the higher is the labor supply elasticity. Therefore, for a given predetermined value of K_t, μ_t increases, see (26), as banks exit the market. This confirms the decrease in investment in the capital production technology, see (28), leading to a decrease in future capital, K_{t+1}, as shown in (30). We can easily see, using (46), that the higher is v, the higher will be the observed decrease in future capital. In the absence of further changes in expectations, so that future labor supply does not shift, the decrease in K_{t+1} implies a decrease in the future marginal productivity of labor, which results in an decrease in l_{t+1}. For the expectation to become self fulfilled, the ratio K_{t+1}/l_{t+1} must decrease, i.e., K_{t+1} must decrease more than l_{t+1}. As this decrease will be smaller the higher is σ, a sufficiently high $\sigma > \sigma_T$ is required. However, we still have to guarantee that the economy will return to the steady state in the absence of further shocks on expectations. From our previous analysis we know that this future reversal of the trajectory requires an upper bound on the elasticity of

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[23] Note that a smaller σ corresponds to a larger shift of the labor demand curve. See (47).

substitution between capital and labor in production, $\sigma < \sigma_F$.

8 Some considerations on global indeterminacy for $\sigma = 1$

Without externalities, so that $v < 1$, we know that $\sigma_T < 1$, so that from Proposition 3 we conclude that when $\sigma = 1$ the normalized steady state is a saddle. However, in subsection 4.2 we have seen that another steady state, which we denote by k_2, may exist. See figure 1b. In Appendix A.2.3 we prove that as $\sigma_T < 1$ we have $k^* < 1$.[24] Therefore, we have $k_2 < k^* < k_{nss} = 1$, as in the curve h_2 depicted in figure 2. This implies that, although the normalized steady state is locally stable, another steady state with a lower level of capital and employment (see (43)) may exist, and global indeterminacy can not be ruled out. This means that, for the same initial given value of the capital stock, there may be several different equilibrium trajectories, pinned by expectations, converging to different steady states.

When externalities are sufficiently high so that $\theta + \eta > 1$, $v > 1$. As shown above $\sigma_T \leq 1$ when $\varepsilon^* \geq \varepsilon_4^* \equiv \frac{1-s\nu}{1-s}$. Hence, for the Cobb-Douglas production function, a transcritical bifurcation occurs when ε^* crosses the value ε_4^*, two steady states which are infinitely close to each other exchanging stability properties. Recall that $\varepsilon_4^* > \varepsilon_3^*$ when $1 < v < v_1$.[25] So, from Proposition 3, we conclude that in configuration (ii), where $1 < v < v_1$, the two steady states exchange stability properties from a sink to a saddle or vice versa as ε^* crosses the value ε_4^*. For example, if $\varepsilon_4^* < \varepsilon^* < 1$ the normalized steady state is a sink, as $\sigma_T < 1$. This, as shown in Appendix A.2.3, implies that $k^* > 1$. Therefore, we have $k^* > k_{nss} = 1$, as in the curve h_1 depicted in figure 2, the other steady state being a saddle. If $\varepsilon_3^* < \varepsilon^* < \varepsilon_4^*$ the normalized steady state is a saddle since $\sigma_T > 1$. We conclude that in this case $k^* < 1$. See Appendix A.2.3. Therefore, we have $k^* < k_{nss} = 1$, as in the curve h_2 depicted in figure 2. This means that, although the normalized steady state is locally saddle-stable, there is a nearby steady state k_2, which is a sink (indeterminate). Hence, equilibria exhibiting sunspot fluctuations around the second steady state exist. Moreover, global indeterminacy and coordination failures become possible.

In subsection 6.1 we have seen that, without externalities, bank market power reduces the

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[24] Note that in configuration *(i)* $\varepsilon_3^* > 1$.

[25] See footnote 19.

likelihood of local indeterminacy, locally stabilizing the economy. In contrast, in the presence of constant or increasing social returns to scale, bank market power promotes local indeterminacy, constituting a locally destabilizing force. Moreover, as shown in subsection 4.2, in the presence of bank market power, steady state multiplicity may emerge when $\sigma = 1$, opening the door to global indeterminacy, as discussed above. Therefore, we conclude that in the Cobb-Douglas case, market power of banks constitutes a potentially destabilizing mechanism, as it may promote economic fluctuations driven by volatile expectations, even when externalities are absent.

9 Concluding remarks

The literature identifies good and bad economic effects of bank competition, usually linked to the lower/higher degree of 'implicit' credit rationing or higher/lower risk of bank failure. In this work we highlight a different economic effect of (im)perfect competition in the banking sector, namely its influence on fluctuations in banking activity and output, linked to autonomous volatility in self-fulfilling expectations.

We consider an OLG model with two sectors and heterogenous agents: workers who work when young, saving in the form of deposits; and entrepreneurs who operate a technology that transforms the consumption good into capital. Capital production is financed by banks, who compete à la Cournot in the credit market.

We find that the effects of bank market power on the emergence of local indeterminacy and fluctuations, crucially depend on the strength of the externalities in the production of the capital good, i.e. on whether social returns to scale in the capital producing technology are decreasing, constant or increasing. Without externalities, implying decreasing social returns to scale, a high market power reduces the scope of local indeterminacy, whereas, with constant social returns to scale, bank market power alone is responsible for the emergence of local indeterminacy. When social returns to scale are increasing, both high markups in the loans market and high externalities in the capital production technology play a similar role in facilitating the emergence of local indeterminacy. This suggests that competition policy/regulation in the credit market, with stabilization goals, should take into consideration the existing degree of social returns to scale characterizing the technology that is financed with bank loans.

However, we also show that, in the presence of bank market power, steady state multiplicity can naturally emerge, opening the door to coordination failures, global indeterminacy and fluctuations, even in the absence of externalities in the production of the capital good. This means that, independently of the degree of social returns to scale, market power in the banking sector is potentially a destabilizing factor. Still, in the presence of decreasing social returns to scale, regulators may be unable to prevent fluctuations by fighting concentration in the banking sector. Indeed, in this case promoting competition increases the likelihood of local cycles.

Finally, in this work we have considered exclusively endogenous fluctuations associated with changes in expectations. It would be interesting to consider also the possibility of exogenous shocks on fundamentals, and investigate whether their joint consideration can yield cycles in credit, investment and real output that match those observed in the data.

A Appendix

A.1 Banks' problem

Replacing the two constraints in the problem of a bank we obtain

$$\max_{\{b_t^i\}} : \pi_{t+1}^i = \left[A\theta(\bar{I}_t^{E^t})^\eta \rho_{t+1}\left(\frac{1+\beta}{(1-\alpha)\left(\frac{1}{\theta}+\beta\right)}\right)^{\theta-1}\left(b_t^i+\sum_{j\neq i} b_t^j\right)^{\theta-1}\right] b_t^i - r_{t+1}\left(\frac{b_t^i}{1-\tau}+\frac{\phi}{1-\tau}\right).$$

The first order condition is bank i's reaction function:

$$A\theta(\bar{I}_t^{E^t})^\eta \rho_{t+1}\left[\left(\frac{1+\beta}{(1-\alpha)\left(\frac{1}{\theta}+\beta\right)}\right)^{\theta-1}\left(b_t^i+\sum_{j\neq i} b_t^j\right)^{\theta-1} + (\theta-1)b_t^i\left(\frac{1+\beta}{(1-\alpha)\left(\frac{1}{\theta}+\beta\right)}\right)^{\theta-1}\left(b_t^i+\sum_{j\neq i} b_t^j\right)^{\theta-2}\right] = \frac{r_{t+1}}{1-\tau}.$$

Using the inverted total demand for loans, $R_{t+1} = A\theta(\bar{I}_t^{E^t})^\eta \rho_{t+1}\left(\frac{1+\beta}{(1-\alpha)\left(\frac{1}{\theta}+\beta\right)}\right)^{\theta-1}\left(b_t^i+\sum_{j\neq i} b_t^j\right)^{\theta-1}$, we can rewrite the previous expression as

$$R_{t+1}\left[1+(\theta-1)b_t^i\left(b_t^i+\sum_{j\neq i} b_t^j\right)^{-1}\right] = \frac{r_{t+1}}{1-\tau}$$

that finally gives us

$$\frac{R_{t+1}}{r_{t+1}} = \frac{1}{(1-\tau)\left[1 - \frac{(1-\theta)b_t^i}{b_t^i + \sum_{j\neq i} b_t^j}\right]}$$

which corresponds to the solution in equation (16).

A.2 Steady State Multiplicity

A.2.1 Expressing l as a function of k

Solving (37) for μ we obtain

$$\mu = [(1-\alpha)A]^{\frac{1}{\eta+\theta}} \left(\frac{\theta\,(1+\beta)}{1+\theta\beta}\frac{(1-\tau)(1-s_h)}{(1-\alpha)}\right) \frac{(\Theta[f(k) - kf'(k)])}{(k)^{\frac{1}{\eta+\theta}}} l^{\frac{\eta+\theta-1}{\eta+\theta}}. \tag{51}$$

Similarly, solving (38) for μ we obtain

$$\mu = \Theta^{\frac{[1+(1-s_h)(\theta+\eta)]\varepsilon}{(1-s_h)\varepsilon(\eta+\theta)}} [f(k) - kf'(k)]^{\frac{[1-(1-s_h)(1-\theta-\eta)]\varepsilon}{(1-s_h)\varepsilon(\eta+\theta)}} [f'(k)]^{\frac{(1-s_h)\varepsilon}{(1-s_h)\varepsilon(\eta+\theta)}} \left(\frac{Z_l}{l}\right)^{\frac{1+(1-s_h)\varepsilon(1-\theta-\eta)}{(1-s_h)\varepsilon(\eta+\theta)}}. \tag{52}$$

Equating now (51) with (52) we obtain (43).

A.2.2 The elasticity of $h(k)$

Using (44) we obtain

$$\begin{aligned}\varepsilon_{h(k)} \quad = \quad & \frac{-B_1 g_1(k)(\theta+\eta)\left[(1+\varepsilon s_h)s(k) + (1-s_h)\varepsilon(\sigma(k) - 1 + s(k))\right]}{2\sigma(k)(\theta+\eta)h(k)} \\ & + \frac{2B_2 g_2(k)[\sigma(k)(1 + (1-\theta-\eta)(1-s_h)\varepsilon) - \varepsilon(1-\theta-\eta)(1-s(k)-s_h) - s(k)(\theta+\eta)]}{2\sigma(k)(\theta+\eta)h(k)}.\end{aligned}$$

As $\varepsilon_{h(k)}$ is a function of k, its sign may change, so that steady state multiplicity can naturally emerge in our model.

In the case of a Cobb-Douglas production function for the consumption good where $Y = \Theta l f(k) = \Theta l k^s$, $\sigma(k) = 1$ and $s(k) = s$, we have that

$$\varepsilon_{h(k)} \quad = \quad \frac{-B_1(\theta+\eta)(1+\varepsilon)s(1-s)^{\frac{-(1+\varepsilon s_h)}{2}} s^{\frac{-(1-s_h)\varepsilon}{2}} k^{\frac{-s(1+\varepsilon)}{2}}}{2(\theta+\eta)h(k)}$$

$$+\frac{2B_2(1-s)^{\frac{\varepsilon s_h(1-\theta-\eta)-(\eta+\theta)}{(\eta+\theta)}}s^{\frac{(1-s_h)\varepsilon(1-\eta-\theta)}{\eta+\theta}}[1+\varepsilon s-s(\theta+\eta)(1+\varepsilon)]k^{\frac{(1+\varepsilon s)-(\theta+\eta)(1+\varepsilon)s}{(\eta+\theta)}}}{2(\theta+\eta)h(k)}$$

so that $\varepsilon_{h(k)} \geq 0$ when $k \geq k^*$ given in (45).

A.2.3 Showing that $k^* = k_{nss} = 1 \Leftrightarrow \sigma_T = 1$ in the Cobb-Douglas case

Substituting B_1 and B_2 in (45) we obtain

$$k^* \equiv \left[\frac{[\Theta(1-\tau)(1-s_h)]^{\frac{1}{2}}\,[(1-\alpha)A]^{\frac{1}{\eta+\theta}}\,\theta\,(1+\beta)\,[(1-\theta)\,\phi]^{\frac{1}{2}}\,(\theta+\eta)(1+\varepsilon)s}{2c_l^{\frac{2-\eta-\theta}{2(\eta+\theta)}}(1+\theta\beta)(1-\alpha)[1+\varepsilon s-s(1+\varepsilon)(\theta+\eta)](1-s)^{\frac{2\varepsilon s_h-(\eta+\theta)(1+\varepsilon s_h)}{2(\eta+\theta)}}s^{\frac{(1-s_h)\varepsilon(2-\eta-\theta)}{2(\eta+\theta)}}}\right]^{\frac{2(\eta+\theta)}{2(1+\varepsilon s)-(\theta+\eta)(1+\varepsilon)s}}. \tag{53}$$

As we have ensured existence of the normalized steady state $(k_{nss} = 1, l_{nss} = 1)$ we know that $A = A^*$ and from (43) we have that in the Cobb-Douglas case $c_l = (1-s)^{-\varepsilon s_h}s^{-(1-s_h)\varepsilon}$. So, substituting these expressions in the equation above and reorganizing terms, we obtain

$$k^* \equiv \frac{(1-\theta)\,(\theta+\eta)(1+\varepsilon)s}{2[1+\varepsilon s-s(1+\varepsilon)(\theta+\eta)]\left[\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta(1-s)]\right)^{\frac{1}{2}}-(1-\theta)\right]}.$$

As we have assumed that $1+\varepsilon s-s(1+\varepsilon)(\theta+\eta) > 0$ we conclude that $k^* \geq 1$ when

$$2[1+\varepsilon s-s(1+\varepsilon)(\theta+\eta)]\left(\frac{(1-\theta)(1-\tau)(1-s_h)}{\phi}\Theta(1-s)]\right)^{\frac{1}{2}} < (1-\theta)\,[2(1+\varepsilon s)-s(\theta+\eta)(1+\varepsilon)]. \tag{54}$$

Substituting $\varepsilon^* \equiv \frac{\varepsilon}{1+\varepsilon}$, and ν given in (48) in the expression for σ_T given in Proposition 3, and $\mu(1,1)$ in ν, we can rewrite σ_T as

$$\sigma_T = \frac{2[(1-s-s_h)\varepsilon+[s(1+\varepsilon)-(1-s_h)\varepsilon](\theta+\eta)]\left(\frac{(1-\theta)(1-\tau)(1-s_h)(1-s)\Theta}{\phi}\right)^{\frac{1}{2}}-(1-\theta)\,[2(1-s-s_h)\varepsilon+s(1+\varepsilon)(\theta+\eta)-(1-s_h)\varepsilon(\theta+\eta)]}{2\,[1+\varepsilon(1-s_h)-(1-s_h)\varepsilon(\theta+\eta)]\left(\frac{(1-\theta)(1-\tau)(1-s_h)(1-s)\Theta}{\phi}\right)^{\frac{1}{2}}-(1-\theta)\,[2(1+\varepsilon(1-s_h))-(1-s_h)\varepsilon(\theta+\eta)]}.$$

This means that, if the denominator of σ_T is positive, we have that $\sigma_T \geq 1$ if condition (54) is satisfied. Note that the denominator of σ_T, that we can rewrite as $(1-s_h\varepsilon^*)-\nu(1-s_h)\varepsilon^*$, is positive for $\varepsilon^* < \varepsilon_3^*$.

We conclude that $k^* = k_{nss} = 1 \Leftrightarrow \sigma_T = 1$.

A.3 Local Dynamics

Consider the dynamic system (34)-(35), with $w(k_t)$, $\rho(k_{t+1})$ and $\mu(k_t, l_t)$ given in (1), (2) and (36). Here we log-linearize this system around the normalized steady state. We consider that $\hat{K}$, $\hat{l}$ denote the percentage deviations from the steady state of K and l respectively and that $\mu \equiv \mu\left(k_{nss}, l_{nss}\right)$, $\sigma \equiv \sigma(k_{nss})$ and $s \equiv s(k_{nss})$.

Using (1), (2) and (3) we first obtain

$$\varepsilon_{w,K} = \frac{s}{\sigma};\ \varepsilon_{w,l} = -\frac{s}{\sigma} \tag{55}$$

$$\varepsilon_{\rho,K} = -\frac{1-s}{\sigma};\quad \varepsilon_{\rho,l} = \frac{1-s}{\sigma}. \tag{56}$$

Log-linearization of the capital accumulation equation leads to

$$\hat{K}_{t+1} = (\theta+\eta)\varepsilon_{w,K}\hat{K}_t + (\theta+\eta)\left[1+\varepsilon_{w,l}\right]\hat{l}_t - (\theta+\eta)\varepsilon_{\mu,K}\hat{K}_t - (\theta+\eta)\varepsilon_{\mu,l}\hat{l}_t, \tag{57}$$

so that, using (27) and (55) we finally obtain:

$$\hat{K}_{t+1} = \frac{(\theta+\eta)(1+\mu)}{2}\frac{s}{\sigma}\hat{K}_t + \frac{(\theta+\eta)(1+\mu)}{2}\left[1-\frac{s}{\sigma}\right]\hat{l}_t.$$

Log-linearizing now the dynamic equation for labor, and defining $\varepsilon^* \equiv \frac{\varepsilon}{1+\varepsilon}$ we have:

$$\begin{aligned}\hat{l}_{t+1} + \frac{\varepsilon_{\rho,K}}{\varepsilon_{\rho,l}}\hat{K}_{t+1} \quad = \quad & -\frac{\{\varepsilon^*[s_h+(1-s_h)(\theta+\eta)]\varepsilon_{w,K} - \varepsilon^*(1-s_h)(\theta+\eta)\varepsilon_{\mu,K}\}}{\varepsilon^*(1-s_h)\varepsilon_{\rho,l}}\hat{K}_t \\ & + \frac{\{1-\varepsilon^* s_h - \varepsilon^*(1-s_h)(\theta+\eta) - \varepsilon^*[s_h+(1-s_h)(\theta+\eta)]\varepsilon_{w,l} + \varepsilon^*(1-s_h)(\theta+\eta)\varepsilon_{\mu,l}\}}{\varepsilon^*(1-s_h)\varepsilon_{\rho,l}}\hat{l}_t.\end{aligned} \tag{58}$$

Substituting now (27) and (55)-(56) in the previous expression we obtain

$$\begin{aligned}\hat{l}_{t+1} - \hat{K}_{t+1} \quad = \quad & -\frac{s}{\varepsilon^*(1-s_h)(1-s)}\left\{\varepsilon^* s_h + \varepsilon^*(1-s_h)(\theta+\eta) - \varepsilon^*(1-s_h)(\theta+\eta)\frac{(1-\mu)}{2}\right\}\hat{K}_t + \\ + \frac{\sigma}{\varepsilon^*(1-s_h)(1-s)} & \left\{1-\varepsilon^* s_h - \varepsilon^*(1-s_h)(\theta+\eta) + [\varepsilon^* s_h + \varepsilon^*(1-s_h)(\theta+\eta)]\frac{s}{\sigma} + \varepsilon^*(1-s_h)(\theta+\eta)\left[1-\frac{s}{\sigma}\right]\frac{(1-\mu)}{2}\right\}\hat{l}_t.\end{aligned}$$

Finally, substituting $\hat{K}_{t+1}$ we obtain

$$\hat{l}_{t+1} \quad = \quad -\frac{s}{(1-s)}\left[\frac{(\theta+\eta)\left(1+\mu\right)}{2}\left[1-\frac{1-s}{\sigma}\right] + \frac{sh}{(1-s_h)}\right]\hat{K}_t + \tag{59}$$

$$+\frac{\sigma}{1-s}\left\{\frac{1}{\varepsilon^*(1-s_h)}-\left[1-\frac{s}{\sigma}\right]\left[\frac{sh}{(1-s_h)}+\frac{(\theta+\eta)(1+\mu)}{2}\left(1-\frac{1-s}{\sigma}\right)\right]\right\}\hat{l}_t.$$

Defining $\nu \equiv \frac{(\theta+\eta)(1+\mu)}{2}$ we can write

$$\begin{bmatrix} \hat{K}_{t+1} \\ \hat{l}_{t+1} \end{bmatrix} = [J] \begin{bmatrix} \hat{K}_t \\ \hat{l}_t \end{bmatrix}$$

where $[J]$ is the Jacobian matrix associated with the log-linearized system, given below

$$J \equiv \begin{bmatrix} \nu\frac{s}{\sigma} & \nu\left[1-\frac{s}{\sigma}\right] \\ -\frac{s}{1-s}\left[\nu\left(1-\frac{1-s}{\sigma}\right)+\frac{s_h}{(1-s_h)}\right] & \left\{\frac{1}{(1-s_h)\varepsilon^*}\frac{\sigma}{1-s}-\left[\frac{\sigma-s}{1-s}\right]\left[\nu\left(1-\frac{1-s}{\sigma}\right)+\frac{s_h}{(1-s_h)}\right]\right\} \end{bmatrix}.$$

We use now the geometrical method considering σ as the bifurcation parameter.

A.3.1 The Δ line

Choosing σ as our bifurcation parameter, we can immediately see from (49) that the Δ line is a horizontal line that only exists for positive values of the determinant. Moreover, using (50), we have that $\frac{\partial \mathcal{T}}{\partial \sigma} > 0$ if $\varepsilon^* < \varepsilon_3^* \equiv \frac{1}{\nu(1-s_h)+s_h}$. Therefore, for $\varepsilon^* < \varepsilon_3^*$ the trace is always positive. We then conclude that in this case the Δ line is a horizontal line, that points to the right as σ increases from s to ∞, and only exists for positive values of $\mathcal{T}$ and $\mathcal{D}$. In contrast, for $\varepsilon^* > \varepsilon_3^*$ the Δ line is a horizontal line, that points to the left as σ increases from s to ∞, again only existing for positive values of $\mathcal{D}$. Note also that $\varepsilon_3^* > 1$ when $\nu < 1$.

Moreover, the Δ line crosses the AC line, where $\mathcal{D} = \mathcal{T} - 1$, when $\sigma = \sigma_T \equiv \frac{(1-s-s_h)\varepsilon^*+\nu[s-(1-s_h)\varepsilon^*]}{(1-s_h\varepsilon^*)-\nu(1-s_h)\varepsilon^*}$, and crosses the AB line, where $\mathcal{D} = -\mathcal{T} - 1$, when $\sigma = \sigma_F \equiv \frac{[(1-s_h)(1-s)+s_h s]\varepsilon^*+\nu[s+(1-s_h)\varepsilon^*]}{\nu(1-s_h)\varepsilon^*-(1-s_h\varepsilon^*)}$.

A.3.2 The Δ_1 line

Letting $\sigma = s$, and solving in ε^*, we obtain

$$\mathcal{D} = \mathcal{D}_1 = -\nu^2 + \nu\mathcal{T}_1 \equiv \Delta_1(\mathcal{T}_1).$$

Note, that when $\mathcal{T}_1 = 2$ we always have $\mathcal{D}_1 < 1$. Also,

$$\frac{\partial \mathcal{D}}{\partial \varepsilon^*} = -\frac{\nu s}{(1-s)(1-s_h)\varepsilon^{*2}} < 0.$$

Then, as ε^* increases from 0 to 1, the Δ_1 line points downwards with a positive slope that can be higher or smaller than 1. Also, the Δ_1 line crosses the $\mathcal{D} = 1$ line for $\varepsilon^* = \frac{s\nu}{(1-s)(1-s_h)} \equiv \varepsilon_2^*$.

In addition note that

$$\mathcal{D}_1 + 1 - \mathcal{T}_1 = \frac{[(1-s)(1-s_h)\varepsilon^* - s][1-\nu]}{(1-s)(1-s_h)\varepsilon^*}$$

so that the Δ_1 line coincides with the AC line when $\nu = 1$. Moreover, it is easy to see that the Δ_1 line crosses the AC line when $\varepsilon^* = \frac{s}{(1-s)(1-s_h)} \equiv \varepsilon_1^*$.

We now define the following critical values for ν: $\nu_1 = -\frac{s_h}{2(1-s_h)} + \sqrt{\frac{s_h^2}{4(1-s_h)^2} + \frac{(1-s)}{s}}$ which is the value of ν for which $\varepsilon_2^* = \varepsilon_3^*$, $\nu_2 = \frac{1-s-s_h}{s(1-s)}$ which is the value of ν for which $\varepsilon_1^* = \varepsilon_3^*$ and $\nu_3 = \frac{(1-s)(1-s_h)}{s}$ which is the value of ν for which $\varepsilon_2^* = 1$. In the following we will consider separately 5 cases:

(i) The case where $\nu < 1$

In this case the $\Delta_1(\mathcal{T}_1)$ line always crosses the AC line below point C. See figure 5a. Indeed, as Δ_1 coincides with the AC line when $\nu = 1$, as ν decreases from 1, its slope also decreases from 1, crossing the AC line below point C, since when $\mathcal{T}_1 = 2$ we always have $\mathcal{D}_1 < 1$. In addition, as $1 > \nu$, we have that $\mathcal{D}_1 \gtreqless -1 + \mathcal{T}_1$ if $\varepsilon^* \gtreqless \frac{s}{(1-s)(1-s_h)} \equiv \varepsilon_1^*$, and that for $\varepsilon^* = \varepsilon_1^*$ the Δ_1 line crosses the AC line. So, when $\varepsilon^* < \frac{s}{(1-s)(1-s_h)}$, as $0 < v < 1$, the $\Delta 1$ line lies below the AC line. As in this case $\varepsilon_3^* > 1$, the Δ line which departs from the Δ_1 line for $\sigma = s$, points to the right as σ grows from s to ∞, so that the steady state is always a saddle. For $\varepsilon^* = \frac{s}{(1-s)(1-s_h)}$ the Δ_1 line crosses the AC line and for $\varepsilon^* > \frac{s}{(1-s)(1-s_h)}$ the Δ_1 line is always inside the ABC triangle, lying above the AC line. Then, the steady state is a a sink for $\sigma \in (s, \sigma_T)$, and a saddle for $\sigma > \sigma_T$.

(ii) The case where $1 < \nu < \nu_1$

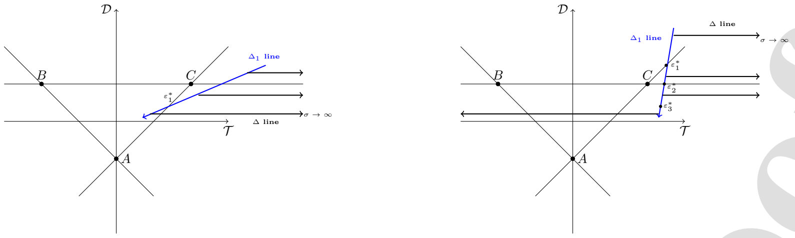
In this case $\varepsilon_1^* < \varepsilon_2^* < \varepsilon_3^* < 1$. For $\varepsilon^* < \varepsilon_3^*$, as $\frac{\partial T}{\partial \sigma} > 0$, the Δ line is a horizontal line, that points to the right as σ increases from s to ∞, while for $\varepsilon^* > \varepsilon_3^*$ we have $\frac{\partial T}{\partial \sigma} < 0$, so that the Δ line is a horizontal line, that points to the left as σ increases from s to ∞. Moreover now the slope of the Δ_1 line is higher than 1, and we know that for $\mathcal{T}_1 = 2$ we always have $\mathcal{D}_1 < 1$. Hence the Δ_1 line crosses the AC line above point C for $\varepsilon^* = \varepsilon_1^* \equiv \frac{s}{(1-s)(1-s_h)}$. See figure 5b. For $\varepsilon^* < \varepsilon_1^*$ the Δ_1 line lies above the AC line. As the Δ line departs from the Δ_1 line for $\sigma = s$, pointing to the right as σ grows from s to ∞, the steady state is a source for $\sigma \in (s, \sigma_T)$, and a saddle for $\sigma > \sigma_T$. For $\varepsilon_1^* < \varepsilon^* < \varepsilon_3^*$ the Δ_1 line lies below the AC line. As the Δ line departs from the $\Delta 1$ line for $\sigma = s$, pointing to the right as σ grows from s to ∞, the steady state is always a saddle. For $\varepsilon_3^* < \varepsilon^* < 1$ we have $\frac{\partial T}{\partial \sigma} < 0$ so that now the Δ line is a horizontal line, that points to the left as σ increases from s to ∞. As the Δ line departs from the Δ_1 line for $\sigma = s$, pointing to the left as σ grows from s to ∞, the steady state is a saddle for $\sigma \in (s, \sigma_T)$, a sink for $\sigma \in (\sigma_T, \sigma_F)$ and a saddle for $\sigma > \sigma_F$.

(iii) The case where $\nu_1 < \nu < \nu_2$

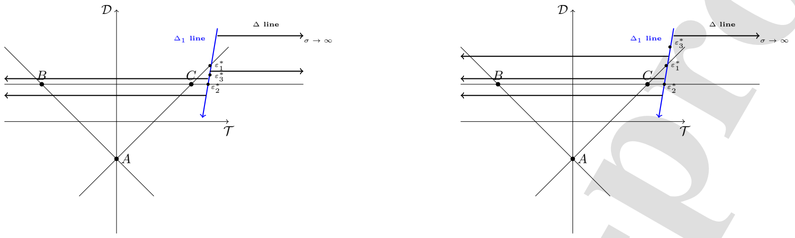
In this case we have $\varepsilon_1^* < \varepsilon_3^* < \varepsilon_2^* < 1$. As in the previous case the slope of the Δ_1 line is higher than 1, and we know that for $\mathcal{T}_1 = 2$ we always have $\mathcal{D}_1 < 1$. Hence the Δ_1 line crosses the AC line above point C for $\varepsilon^* = \varepsilon_1^* \equiv \frac{s}{(1-s)(1-s_h)}$. See figure 5c. For $\varepsilon^* < \varepsilon_1^*$ the Δ_1 line lies above the AC line and as $\varepsilon_1^* < \varepsilon_3^*$ the Δ line departs from the Δ_1 line for $\sigma = s$, pointing to the right as σ grows from s to ∞, the steady state is a source for $\sigma \in (s, \sigma_T)$, and a saddle for $\sigma > \sigma_T$. For $\varepsilon_1^* < \varepsilon^* < \varepsilon_3^*$ the Δ_1 line lies below the AC line, and the Δ line departs from the $\Delta 1$ line for $\sigma = s$, pointing to the right as σ grows from s to ∞. So, the steady state is always a saddle. For $\varepsilon_3^* < \varepsilon^* < \varepsilon_2^*$ the Δ_1 line lies below the AC line, but the Δ line departs from the Δ_1 line for $\sigma = s$, pointing to the left as σ grows from s to ∞. So, the steady state is a saddle for $\sigma \in (s, \sigma_T)$, a source for $\sigma \in (\sigma_T, \sigma_F)$, and a saddle for $\sigma > \sigma_{F.}$ For $\varepsilon_2^* < \varepsilon^* < 1$ the Δ line is a horizontal line, that points to the left as σ increases from s to ∞. As the Δ line departs from the Δ_1 line for $\sigma = s$, pointing to the left as σ grows from s to ∞, the steady state is a saddle for $\sigma \in (s, \sigma_T)$, a sink for $\sigma \in (\sigma_T, \sigma_F)$ and a saddle for $\sigma > \sigma_F$.

(iv) The case where $\nu_2 < \nu < \nu_3$

In this case $\varepsilon_3^* < \varepsilon_1^* < \varepsilon_2^* < 1$. As in the previous case, the slope of the Δ_1 line is higher than 1, and we know that for $\mathcal{T}_1 = 2$ we always have $\mathcal{D}_1 < 1$. Hence, the Δ_1 line crosses the AC line above point C for $\varepsilon^* = \varepsilon_1^* \equiv \frac{s}{(1-s)(1-s_h)}$. See figure 5d. For $\varepsilon^* < \varepsilon_3^*$ we have $\frac{\partial T}{\partial \sigma} > 0$, and the Δ line is a horizontal line, that points to the right as σ increases from s to ∞. Also, for $\varepsilon^* < \varepsilon_3^* < \varepsilon_1^*$ the Δ_1 line lies above the AC line. So, as the Δ line departs from the Δ_1



(a) Case (i) (b) Case (ii)



(c) Case (iii) (d) Case (iv)

Figure 5: The Δ and Δ_1 lines

line for $\sigma = s$, pointing to the right as σ grows from s to ∞, the steady state is a source for $\sigma \in (s, \sigma_T)$, and a saddle for $\sigma > \sigma_T$. For $\varepsilon_3^* < \varepsilon^* < \varepsilon_1^*$ we have $\frac{\partial T}{\partial \sigma} < 0$ so the Δ line is a horizontal line, that points to the left as σ increases from s to ∞. Also, for $\varepsilon^* < \varepsilon_1^*$ the Δ_1 line lies above the AC line. As the Δ line departs from the $\Delta 1$ line for $\sigma = s$, pointing to the left as σ grows from s to ∞, the steady state is a source for $\sigma \in (s, \sigma_F)$, and a saddle for $\sigma > \sigma_F$. For $\varepsilon_1^* < \varepsilon^* < \varepsilon_2^*$ the Δ_1 line lies below the AC line and above the $D = 1$ line. As the Δ line departs from the Δ_1 line for $\sigma = s$, pointing to the left as σ grows from s to ∞, the steady state is a saddle for $\sigma \in (s, \sigma_T)$, a source for $\sigma \in (\sigma_T, \sigma_F)$ and a saddle for $\sigma > \sigma_F$. For $\varepsilon^* > \varepsilon_2^*$ the Δ_1 line lies below the AC line and below the $D = 1$ line. As the Δ line departs from the Δ_1 line for $\sigma = s$, pointing to the left as σ grows from s to ∞, the steady state is a saddle for $\sigma \in (s, \sigma_T)$, a sink for $\sigma \in (\sigma_T, \sigma_F)$. and a saddle for $\sigma > \sigma_F$.

(iv) The case where $\nu > \nu_3$

In this case $\varepsilon_3^* < \varepsilon_1^* < 1 < \varepsilon_2^*$. Everything is as in the previous case, with one exception. Now ε_2^* is above one, so that the case where $\varepsilon^* > \varepsilon_2^*$ does not exist. So, as in the previous case, for $\varepsilon^* < \varepsilon_3^*$ the steady state is a source for $\sigma \in (s, \sigma_T)$, and a saddle for $\sigma > \sigma_T$. For $\varepsilon_3^* < \varepsilon^* < \varepsilon_1^*$ the steady state is a source for $\sigma \in (s, \sigma_F)$, and a saddle for $\sigma > \sigma_F$. Finally, for $\varepsilon_1^* < \varepsilon^* < 1$ the steady state is a saddle for $\sigma \in (s, \sigma_T)$, a source for $\sigma \in (\sigma_T, \sigma_F)$ and a saddle for $\sigma > \sigma_F$.

We summarize these results in Proposition 2 in the main text.

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