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Risk Management in Bond Markets: A Country-Specific Study of Volatility Scaling Strategies

Leonor Conde Ferreira

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Abstract

This study takes a close look at how to deal with the market's time-variation in risk, with a particular focus on applying volatility-management strategies to bond portfolios, featuring different levels of credit risk and maturities in international fixed-income markets. Notably, the volatility-managed high-yield portfolio, consistently outperforms its original counterpart, both in terms of positive alpha in the spanning regression and significant increase in the Sharpe ratio. However, for high-quality bonds, there is no evidence indicating an enhancement in risk-adjusted returns.

Mixed evidence is found on the performance of volatility-managed government bond portfolios, with longer maturities portfolios being more likely to profit from the dynamic strategy. When analysing the risk-return trade-off, I show that the reported difference across markets mainly arises from differences in return timing. By applying different volatility estimation methods, I highlight the sensitivity of results to distinct methodologies. In addition to the methodology used, the selected sample and prevailing market dynamics have been shown to have a significant influence on the outcomes. Nevertheless, although evidence seems to suggest some potential benefits, universal improvements in fixed-income markets remain elusive.

Keywords: Volatility-management; Market timing; Fixed-income markets; Risk-return trade-off

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Resumo

Este estudo analisa de perto a abordagem da variação temporal do risco de mercado, com especial incidência na aplicação de estratégias de gestão da volatilidade a carteiras de obrigações, que apresentem diferentes níveis de risco de crédito e maturidades em mercados internacionais de rendimento fixo. Em particular, a carteira obrigacionista de alto rendimento quando aplicada a estratégia de gestão de volatilidade apresenta um desempenho superior ao da carteira original, tanto em termos de alfa positivo na *spanning regression* como de aumento significativo do rácio de Sharpe. No entanto, no que respeita às obrigações de alta qualidade, não existem evidências que indiquem uma melhoria significativa dos retornos ajustados ao risco.

Adicionalmente, não são encontradas provas consistentes sobre o desempenho das carteiras de obrigações de governos geridas em função da volatilidade, sendo que as carteiras com prazos mais longos têm maior probabilidade de beneficiar da estratégia dinâmica. Ao analisar o *trade-off* risco-retorno, mostro que a diferença registada entre mercados resulta principalmente de diferenças no *timing* dos retornos. Ao aplicar diferentes métodos para estimar a volatilidade, saliento a sensibilidade dos resultados a metodologias distintas. Para além da metodologia utilizada, o período de observação e a dinâmica de mercado prevalente demonstraram ter uma influência significativa nos resultados. Conclui-se que embora os dados demonstrem a existência de eventuais benefícios, melhorias universais a nível dos mercados obrigacionistas continuam a ser difíceis de alcançar.

Palavras-Chave: Gestão da volatilidade; *Market timing*; Mercados Obrigacionistas; Relação risco-retorno

Título: Gestão de risco nos mercados obrigacionistas: Um estudo das estratégias de gestão de volatilidade em diversos mercados internacionais

Autora: Leonor Conde Ferreira

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1. Introduction

Fixed-income markets allow both governments and corporations to issue bonds, aiming to raise capital. This has a major role in the funding of corporations and governments, as it benefits borrowers by increasing the available funds while also providing investors with rewards.

Investing in the corporate bond market demands a greater level of expertise compared to the stock market. As, while in the latter investors choose the company they want to invest in, to trade corporate bonds investors need to select among hundreds of outstanding issues within the same company. Consequently, the bond market is mainly dominated by professional investors. Among these, insurers have the largest share of institutional ownership (Kojien & Yogo, 2023). In 2022, U.S. households had 23% of their investable assets invested in the stock market, in contrast to a modest 3% invested in bonds (SIFMA, 2023).

Due to security and stability reasons, government bonds are more prone to attract interest from institutional investors, such as central banks and pension funds (OECD, 2019). Conversely, high reward and high-risk characteristics, make high-yield¹ corporate bonds have less interest for institutional investors than investment-grade² corporate bonds (Cao et al., 2021). The latter tend to have a significant institutional ownership ratio, being insurance companies crucial in that market (Kojien & Yogo, 2023). Considering all these divergent factors, it is recommended to study each instrument individually, as well as in international markets with different dynamics.

Moreover, high-quality bonds tend to be used to enhance portfolio diversification as they present low volatility and low or even negative correlation with equity returns. Nevertheless, after decades of low returns, in 2022 due to the culmination of inflationary pressures and consequent abrupt increase in interest rates, bonds saw their returns declining even further and both equity and bond prices moved in the same direction, resulting in historically significant losses (Birdsall & Duensing, 2023). The bond market has also become volatile in recent times, and at times beyond the stock market, thus there is the need for applying risk management strategies to the bond market; furthermore, as a result of rising yields and falling prices brought about by an increase in interest rates, investors are becoming more attracted to the bond market (Sommer, 2023). Anticipating the conclusion of the Federal Reserve's hiking cycle in 2024,

¹ High-yield bonds (non-investment-grade, speculative-grade, or junk bonds) are a designation based on the probability of default associated with the bonds. These bonds are rated below investment-grade by credit rating agencies, like Fitch, S&P, and Moody's.

² Investment-grade bonds are bonds with a credit rating above or equal to BBB by S&P or Fitch, and above or equal to Baaa by Moody's. The ratings aim to reflect the quality of the corporation's credit and the associated probability of default.

high-quality bonds are expected to outperform in case of a recession and gain from an increase in prices once the Federal Reserve is capable of reducing interest rates (Birdsall & Duensing, 2023).

Motivated by these facts, the present study builds on the existing literature and analyses volatility-management strategies applied to government and corporate bonds in an international context, helping to fill a gap in the literature, since previous studies on volatility-management have focused mainly on stock markets.

Volatility scaling is a dynamic strategy that consists of increasing the exposure to the underlying asset following periods of low volatility and reducing the position when volatility has been high, advising investors to take less risk at times of severe uncertainty. Volatility-managed portfolios are constructed by scaling the original portfolio by the inverse of its conditional volatility. In the present study, I analyse diverse methods of estimating conditional volatility and show that a more precise estimation of volatility can substantially affect the performance of volatility-managed portfolios. To analyse the performance of the volatility-managed portfolios, I mainly focus on a direct comparison of the Sharpe ratios and Moreira & Muir's (2017) spanning regressions of the scaled portfolios on the unscaled counterparts, following Cederburg et al. (2020) study of volatility-management strategies on 103 equity strategies. The present study is divided into three separate analyses.

Firstly, I build on Rebelo (2022) study of the effectiveness of volatility management in U.S. investment-grade bonds and I examine the performance of volatility-management in corporate bonds and their respective credit risk factors in international markets. I confirm Rebelo (2022) findings that there is no advantage of timing the volatility for corporate bonds with ratings above investment-grade and for the respective credit spread, in a broader sample including foreign markets. However, for low-rated bonds, there is evidence that adopting volatility scaling can create value for investors.

Interestingly, when studying volatility-management in different segments of the yield curve, despite the results displaying mixed findings across countries, I report a trend across all study markets of increase in the alphas as well as the t-statistics with the maturity. In addition, I extended the study focusing on the term spread, by broadening the timeframe and adopting different methods to estimate the volatility. The benefits of volatility-management can arise due to a positive relationship between lagged volatility and future volatility or a negative relationship between lagged volatility and future expected return. Hence, I explore these relations and discover that the variations observed across international markets and different fixed-income securities emerge from distinct dynamics in the risk-return trade-off.

2. Literature Review

Throughout the history of financial markets investors have been looking for strategies to improve their performance, while reducing their risk exposure. Over the last two decades, volatility-management strategies have been studied and applied to several markets, across multiple geographies and different timespans. These strategies weight the investments in risky assets by the inverse of their past volatility, thus being called volatility-management. Empirical evidence on this, indicates that it not only allows for the prediction of expected returns but also enhances risk-adjusted returns.

Fleming et al. (2003) find that for a mean-variance investor, there are significant economic benefits to applying a volatility timing approach to a portfolio of four assets: stocks, bonds, gold, and cash, by estimating the conditional covariance matrix. The abovementioned strategy on the momentum anomaly has also been a focus of study for Barroso & Santa-Clara (2015) and Daniel & Moskowitz (2016). Barroso & Santa-Clara's (2015) approach was to manage the specific risk of the momentum strategy while Daniel and Moskowitz (2016) focus on systematic risk and report that the dynamic strategy doubles both alphas and the Sharpe ratio when compared to the original momentum strategy, with this result being consistent across multiple equity markets, time periods, and asset classes. Furthermore, the authors find that in bear markets the payoffs of the momentum strategy resemble being short a call option on the market.

More recently volatility-management strategies have been applied to multiple equity factors (Moreira & Muir, 2017). Inspired by the Modern Portfolio Theory, Moreira & Muir (2017) built their analysis on the fact that current volatility is highly correlated with past volatility. Nevertheless, this is not the case when examining the relationship between historical volatility and average returns, suggesting that a mean-variance investor should increase its exposure to risky assets in periods of low volatility and on the other hand, reduce its exposure to risky assets when volatility has been recently high. Hence, indicating a weakened mean-variance trade-off during high-volatility periods. This analysis was eventually extended to a larger sample of 103 equities trading strategies (Cederburg et al., 2020) and international equity risk factors by Grobys & Äijö (2018). The latter authors find that volatility-management, when applied to the Fama & French (2017) local risk factors, enhances performance across the European and Asian markets. Nevertheless, this does not extend to the Japanese market.

In turn, Grobys et al. (2018) apply Barroso & Santa-Clara (2015) risk management strategy to industrial portfolios and observe an increase in risk-managed industrial momentum payoffs

when compared to its unmanaged counterpart, and higher profits when using smaller time windows to estimate the variance forecast. Following Daniel & Moskowitz (2016), the authors also study the optionality effects of plain industry momentum and risk management industry momentum after bear markets. However, in the studied sample the authors do not yield any evidence to support these effects.

The benefits of volatility-management were also discovered in financial distress strategies (Eisdorfer & Misirli, 2017), in mutual funds in the U.S. market (Wang et al., 2021) and betting-against-beta strategy³ (Barroso et al., 2023). The study by Wang et al. (2021) emphasize the positive and significant relationship between funds past volatility and future volatility, as well as the negative relationship between past performance and future returns. Moreover, reducing the exposure in funds with high past volatility, mitigates the risk of increased volatility in the future, while also minimizing the risk of low returns. Hence, the advantages of volatility-management in mutual funds can be explained by this phenomenon. Contrarily to most of the presented studies, Wang & Yan (2021) use downside volatility⁴ to scale portfolios of equity factors and find that volatility scaling using downside volatility outperforms total volatility-managed portfolios, being this a consequence of return timing.

Despite the popularity of this topic in recent studies, fixed-income markets have received much less attention. Rebelo (2022) follow Wang et al. (2021) analysis to study the benefits of volatility-management in investment-grade portfolios in the U.S. and found that for the analysed sample there was no evidence of potential advantages of applying volatility-managed strategies to investment-grade portfolios and their respective portfolios, when isolated the credit risk.

Throughout the years, various methodologies have been developed to build volatility-managed strategies. In order to deal with the high risk of momentum, Barroso & Santa-Clara (2015) scale the strategy using a forecast of standard deviation. This portfolio is calculated with the realized variance of daily returns, as well as targeting a constant level of volatility which is set to 12% (by targeting a constant level of volatility, the authors eliminate the look-ahead bias). The authors find that the risk of momentum can be easily predicted and therefore managed. The results of this study suggest that the volatility-managed momentum eliminates the crash risk of

³ Betting-against-beta strategy was developed by Frazzini & Pedersen (2014) and is a self-financing strategy that is long the low-beta portfolio and short the high-beta portfolio.

⁴ Downside volatility only takes into consideration for the calculation, periods that have negative returns. This measure of downside risk can be considered more appropriate to evaluate the risk of an investment as investors link risk with losses.

the plain momentum and almost doubles the Sharpe ratio. The study also extends to international markets and different subsamples and similar results of enhanced performance are found.

On the other hand, rather than choosing an ex-ante level of volatility, Moreira & Muir (2017) set it constant so that scaled and unscaled portfolios present the same level of volatility. The authors conclude that for market, value, momentum, profitability, return on equity, investment, and betting-against-beta, volatility timing increases Sharpe ratios and presents positive and significant alphas in the spanning regression of the scaled strategy on the unscaled counterpart. Besides, the implementation of volatility-management strategies is still beneficial for an investor even when transaction costs and leverage constraints are taken into consideration.

According to Liu et al. (2019), Moreira & Muir's (2017) approach features look-ahead bias and is challenging to apply in situations considering the bias. Cederburg et al. (2020) carried out a more in-depth study on the real-time implementation of volatility-management strategies, showing that volatility-management strategies tend to present positive and significant alphas in the spanning regressions, despite the mixed results from the direct comparison of managed and unmanaged counterparts. However, the authors further argue that the spanning regressions cannot be carried out in practice as they rely on ex-post optimal weights and the benefits that the volatility-management strategy present in-sample cannot be extended to out-of-sample.

The present research contributes to the literature by closing the gap in the study of volatility-management strategies in fixed-income markets by applying these strategies on a comprehensive set of bond portfolios, including both corporate and government bonds as well as the respective credit and term factors, in international market dynamics and by using various methods to estimate the conditional volatility, from the simple realized volatility to more complex methods, EWMA and GARCH. Moreover, this study innovates on the analysis of the benefits of volatility-management strategy in fixed-income markets to real-time investors.

3. Data

3.1 Credit Spread using daily data

For the purpose of studying the effect of volatility scaling strategies in bond portfolios with varying degrees of credit risk and matching maturities, I downloaded the Total Return Index from DataStream for an index of corporate bonds classified as Investment-Grade (IG), an index of corporate bonds classified as High-Yield (HY) and an index of Long-Term Government bonds (LTGB). All indexes are composed of securities with comparable maturities, of 10 years or longer in order to isolate the credit risk of these bond portfolios. The presented variables were retrieved for the U.S. bond market in the local currency (USD) with a daily frequency from January 29, 1993, to July 31, 2023 (consisting of 7957 daily observations).

3.2 Term Spread using daily data

To verify if volatility scaling strategies benefits investors when compared to the respective unscaled strategies on government bond portfolios with different maturities, I retrieved the Total Return Index of benchmark indexes of government bonds. This index divides the bonds into 5 brackets depending on its maturities, from DataStream. The first index pertains to government bonds with maturities between 1 and 3 years, the other to bonds with maturities between 3 and 5 years, then between 5 and 7 years, 7 to 10 years, and lastly to government bonds with maturities over 10 years. The different variables represent bond portfolios with the same credit risk and different maturities and were downloaded with a daily frequency, spanning from January 1, 1989, to July 31, 2023 (9022 daily observations) for the U.S. in the local currency (USD).

3.3 Term Spread using monthly data

Furthermore, I repeat the previous analysis using monthly data with the aim of extending the time period under consideration. To achieve this, I must use monthly returns on government bonds. However, this data is not publicly available, and the yields-to-maturity of both the Long-Term Government Bond (with 10 years of maturity) and the Short-Term Government Bond (with 3 months of maturity) are published on the Federal Reserve Economic Data (FRED) database.

The monthly yields for both the Long-Term Government Bond⁵ and the Short-Term Government Bond⁶ were retrieved from the FRED database for the United States of America in USD for the period from June of 1964 to July of 2023 (presenting 710 monthly yield observations).

I follow the Tuckman & Serrat (2012) methodology to transform the yield into monthly investor returns, as further described in Appendix A.

After obtaining the returns, I calculate the term spread as being the difference between the 10-year government bond and the 3-month T-bill. This portfolio is the one considered for the remainder of this analysis.

The U.S. government market is considered the most efficient and liquid fixed-income market in the world (SIFMA, 2023). Hence, I use the one-month U.S. treasury bill rate as a benchmark for the risk-free rate. This data, as well as the monthly Fama and French five risk factors were retrieved from the Kenneth French Data Library.

⁵ OECD Interest Rates: Long-Term Government Bond Yields: 10-Year: Main (Including Benchmark) for United States [IRLTLT01USM156N], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/IRLTLT01USM156N>

⁶ OECD Interest Rates: 3-Month or 90-Day Rates and Yields: Interbank Rates: Total for United States [IR3TIB01USM156N], retrieved from FRED, Federal Reserve Bank of St. Louis; <https://fred.stlouisfed.org/series/IR3TIB01USM156N>

4. Methodology

Volatility-managed portfolios are constructed by scaling the excess returns by the inverse of the past month volatility times a constant c . Following the literature on this topic (Moreira & Muir, 2017; Cederburg et al., 2020; Wang & Yan, 2021), I calculated the constant c so that the volatility-managed portfolio presents the same standard deviation as the buy-and-hold portfolio. Although this value is not known to real-time investors, it has no influence on the strategy's Sharpe ratio. Such use of the same ex-post level of volatility is necessary to ensure the comparability of the strategies throughout the analysis.

As the analysed portfolios represent zero-investment strategies, I scale them without constraints. Therefore, the volatility-managed portfolio excess return (R_t^σ) is given by the following formula:

$$R_t^\sigma = \frac{c}{\hat{\sigma}_{t-1}} \cdot R_t \quad (1)$$

, where R_t is the buy-and-hold portfolio excess return and $\hat{\sigma}_{t-1}$ is the lagged volatility of returns.

4.1 Daily data

In order to calculate the variance of monthly returns using daily data, I use the following Realized Variance formula:

$$RV_t^2 = \sum_{i=1}^{D_t} r_{i,t}^2 \quad (2)$$

, where RV_t^2 is the monthly realized variance at month t , which equals to the sum of the squared observed daily returns ($r_{i,t}^2$) of the respective month. D_t represents the number of trading days in month t .

With the aim of calculating the volatility-managed portfolios, I need to calculate the realized volatility. As in this sample not all months present the same number of trading days, I first create a standardized measure of realized variance, by using the following formula:

$$\text{Standard } RV_t^2 = 21 \cdot \frac{RV_t^2}{D_t} \quad (3)$$

Using the standardized realized variance, I calculate the realized volatility used to scale the portfolios.

$$\hat{\sigma}_t = \sqrt{\text{Standard } RV_t^2} \quad (4)$$

4.2 Monthly data

Ideally, it is preferable to use daily returns to estimate the volatility. However, to be able to conduct the study of volatility-management on government bond portfolios in a broader sample, including more 295 months, I had to adopt a monthly frequency approach.

To calculate the variance of monthly returns using monthly data, I use the Exponentially Weighted Moving Average (EWMA) formula. This statistical method is characterized by giving more importance to more recent data, and decreasing the weights exponentially for older data, without ever reaching zero. Hence, it rapidly responds to changes in prices, essential in volatile times.

For the first observation:

$$\hat{\sigma}_t^2 = r_t^2 \quad (5)$$

, where r_t^2 is the observed monthly return squared at month t .

And the following formula for the remaining observations:

$$\hat{\sigma}_t^2 = (1 - \lambda).r_{t-1}^2 + \lambda.\hat{\sigma}_{t-1}^2 \quad (6)$$

, where $\hat{\sigma}_t^2$ is the variance at month t and $0 < \lambda \leq 1$ is the decay factor. Following, Mina & Xiao (2001), I used 0.97 for the decay factor.

To add up to this analysis, I repeated the same study but by using the GARCH(1,1) model⁷ discovered by Bollerslev (1986) to estimate volatility based on the full sample, as it allows to capture the persistence of volatility, time-varying volatility and volatility clustering, providing a more accurate forecast.

$$\hat{\sigma}_t^2 = \omega + \alpha.r_{t-1}^2 + \beta.\hat{\sigma}_{t-1}^2 \quad (7)$$

, where ω is the GARCH(0) term, r_{t-1}^2 is the previous month observed monthly return squared and $\hat{\sigma}_{t-1}^2$ is the variance in the previous month.

⁷ To estimate conditional volatility using the GARCH model, I used the GARCH volatility model included in the `arch_model` function of the `arch` library in python.

5. Empirical Results

In this section, I discuss the main results of my empirical studies, which are divided into 3 analyses: daily credit spread, daily term spread, and monthly term spread. I analyse the risk and return metrics of the original buy-and-hold and volatility-managed portfolios, by first comparing the descriptive statistics followed by an evaluation of the performance in a set of regressions.

5.1 Daily Credit Spread

I started by studying the impact of volatility scaling strategies in government bonds (LTGB) and corporate bonds with different levels of credit risk (IG and HY). I then isolate the credit risk, creating two more portfolios (Credit-IG and Credit-HY). The latter consist of the default spread, which is the difference between the excess returns of the respective corporate bonds and the LTGB. These portfolios present different levels of credit risk but equal maturities.

Table 1: Descriptive statistics for unscaled and scaled excess returns for the Credit Spread using daily data
This table reports descriptive statistics of the original bond portfolios and the risk-managed bond portfolios. The sample period under consideration is from 29/01/1993 to 31/07/2023. Panel A presents the unscaled results and Panel B the results of the volatility scaling strategy on bond portfolios using the realized variance of the previous month calculated from daily data to scale the portfolio. The reported statistics are the maximum and minimum monthly excess returns, the annualized mean excess return, the annualized standard deviation of excess return, the Skewness, Excess Kurtosis and the annualized Sharpe Ratio. The difference in the Sharpe ratio between the volatility-managed and the original portfolio is reported, along with the statistical significance according to Jobson & Korkie (1981) test. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Excess returns are presented as percentages.

	<i>Min</i>	<i>Max</i>	<i>Mean</i>	<i>Std</i>	<i>Skew</i>	<i>Kurt</i>	<i>SR</i>	<i>SR dif.</i>
Panel A: U.S. Unscaled								
GOV	-9.85	11.21	4.05	10.41	0.18	1.01	0.39	-
IG	-12.85	12.80	4.30	9.37	-0.31	3.44	0.46	-
HY	-19.88	12.89	6.99	10.51	-1.44	10.25	0.67	-
Credit-IG	-16.82	9.21	0.25	7.33	-1.61	13.67	0.03	-
Credit-HY	-25.96	17.44	2.94	13.83	-1.20	8.58	0.21	-
Panel B: U.S. Scaled								
GOV	-7.94	14.37	4.48	10.41	0.56	2.49	0.43	0.04
IG	-13.29	13.29	4.86	9.37	0.04	2.98	0.52	0.06
HY	-17.51	9.47	8.69	10.51	-1.14	4.96	0.83	0.16*
Credit-IG	-13.57	6.46	-0.51	7.33	-1.65	8.60	-0.07	-0.10
Credit-HY	-34.60	14.91	2.58	13.83	-2.05	16.49	0.19	-0.02

Firstly, upon contrasting the descriptive statistics of the unscaled and the scaled excess returns in Table 1, it is possible to state that employing a scaling strategy leads to an improvement in the Sharpe ratios of the portfolios. This enhancement in the performance is particularly pronounced for the HY portfolio, a foreseeable outcome, as this portfolio presents the most

equity features, and due to past literature, it is known that this strategy tends to perform well in the equity market (Moreira & Muir, 2017; Cederburg et al., 2020). This reflects a creation in value for investors, as the scaling strategy produces a more appealing risk-to-reward trade-off. However, when comparing the Sharpe ratio for the portfolios where I isolated the credit risk, the Sharpe ratio of these portfolios decreases when compared to the unscaled strategy, even yielding negative results, although the decreases are not statistically significant.

Additionally, I use the Jobson & Korkie (1981) methodology to evaluate if each of these changes are statistically significant. I concluded that only the HY portfolio systematically outperforms its original counterpart. Moreover, for the corporate bond portfolios, the volatility-management strategy reduces high-order risk, as there is a decrease in the kurtosis of these portfolios, and it helps with the negative skewness problem reducing, or even eliminating it for the IG portfolio. This improvement does not consistently extend to the credit factor portfolios, as there is an increase in the left-skewness of both portfolios paired with an increase in the kurtosis for the Credit-HY portfolio.

To test for the presence of positive and higher alphas when comparing scaled excess returns to unscaled excess returns, I run regressions for the different portfolios with different levels of credit risk. In these regressions, I computed the t-statistics based on White (1980) heteroskedasticity robust standard errors, to account for the heteroskedasticity that may be present in the time-series analysis of portfolio's returns.

Firstly, I analyse if the CAPM (Capital Asset Pricing Model) of Sharpe (1964) can explain the bond portfolios' excess returns for both the unscaled and scaled ($R_{i,t}^{(\sigma)}$).

$$R_{i,t}^{(\sigma)} = \alpha_i + \beta_i \cdot MKT_t + \epsilon_{i,t} \quad (8)$$

Fama & French (1993) argued that the stock market factors have little explanation for the returns of both government and corporate bonds, with the only exception being HY corporate bonds. The remaining variation in bond returns is related to maturity and default risk. These factors are called the term factor, which is approximated here by the LTGB and reflects changes in the interest rates, and the default factor, which reflects changes in the probability of default and is represented by both credit portfolios.

Table 2: Scaled and Unscaled Excess Returns on the Market factor for the Credit Spread using daily data

This table reports the results from the time-series regressions of the unscaled excess return (Panel A) and the volatility scaled excess returns (Panel B) on the market factor, as represented by equation (8). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from 29/01/1993 to 31/07/2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>MKT</i>	<i>R-squared</i>
Panel A: U.S. Unscaled			
GOV	0.393** (2.34)	-0.088* (-1.94)	0.02
IG	0.223 (1.52)	0.179*** (3.94)	0.09
HY	0.294** (1.97)	0.395*** (7.20)	0.34
Credit-IG	-0.170 (-1.62)	0.266*** (7.17)	0.31
Credit-HY	-0.099 (-0.49)	0.483*** (7.46)	0.29
Panel B: U.S. Scaled			
GOV	0.446*** (2.71)	-0.102** (-2.48)	0.02
IG	0.297** (2.05)	0.150*** (3.87)	0.06
HY	0.479*** (3.20)	0.341*** (8.06)	0.25
Credit-IG	-0.221** (-2.07)	0.248*** (7.42)	0.27
Credit-HY	-0.092 (-0.43)	0.426*** (5.54)	0.23

Supporting Fama & French (1993), in the studied sample I find a term premium and that the excess return over the market risk premium explains a great proportion of the monthly HY bond unscaled and scaled excess returns. However, I do not find the credit factor to be significant. Moreover, the HY portfolio before applying the scaling strategy presents a beta of 0.395, suggesting that part of its return predictability comes from the market.

When comparing the alphas of the scaled portfolios with their original counterparts, it is possible to observe that there is an increase in the alphas for the LTGB portfolio and both corporate portfolios, with all values being significant in the scaled version. This increase is particularly evident for the HY portfolio, with 0.205 pp increase. Additionally, the market factor has positive and significant coefficients for all portfolios except for the LTGB which is negative and significant. This might be justified by the fact the U.S. government bond market is considered the most efficient in the world (SIFMA, 2023) and therefore its returns might be better explained by factors different from the equity market factors.

Then I test if the Fama-French five-factor model (5FF) can explain the bond portfolio excess returns.

$$R_{i,t}^{(\sigma)} = \alpha_i + \beta_{1i} \cdot MKT_t + \beta_{2i} \cdot SMB_t + \beta_{3i} \cdot MHML_t + \beta_{4i} \cdot RMW_t + \beta_{5i} \cdot CMA_t + \epsilon_{i,t} \quad (9)$$

Table 3: Scaled and Unscaled Excess Returns on the Five Fama and French Factors for the Credit Spread using daily data

This table reports the results from the time-series regressions of the unscaled excess return (Panel A) and the volatility scaled excess returns (Panel B) on the 5 Fama and French Factors, as represented by equation (9). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from 29/01/1993 to 31/07/2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>MKT</i>	<i>SMB</i>	<i>HML</i>	<i>RMW</i>	<i>CMA</i>	<i>R-squared</i>
Panel A: U.S. Unscaled							
GOV	0.343** (1.97)	-0.049 (-1.01)	-0.114** (-2.04)	-0.175* (-1.95)	0.115 (1.46)	0.063 (0.53)	0.06
IG	0.176 (1.13)	0.202*** (4.07)	-0.027 (-0.51)	-0.018 (-0.22)	0.093 (1.29)	0.009 (0.08)	0.10
HY	0.250* (1.73)	0.396*** (7.83)	0.105* (1.78)	0.118 (1.12)	0.061 (0.85)	-0.025 (-0.21)	0.37
Credit-IG	-0.167 (-1.62)	0.251*** (8.23)	0.087** (2.18)	0.157* (1.84)	-0.022 (-0.49)	-0.054 (-0.64)	0.38
Credit-HY	-0.093 (-0.51)	0.445*** (8.80)	0.219*** (3.11)	0.293* (1.85)	-0.054 (-0.61)	-0.088 (-0.57)	0.37
Panel B: U.S. Scaled							
GOV	0.414** (2.36)	-0.062 (-1.36)	-0.156*** (-2.69)	-0.182** (-2.45)	0.046 (0.61)	0.121 (1.20)	0.07
IG	0.284* (1.84)	0.166*** (4.02)	-0.065 (-1.12)	0.002 (0.03)	0.026 (0.39)	0.001 (0.01)	0.07
HY	0.456*** (3.09)	0.344*** (8.73)	0.040 (0.61)	0.085 (0.91)	0.001 (0.01)	0.024 (0.22)	0.26
Credit-IG	-0.187* (-1.70)	0.222*** (7.03)	0.066* (1.73)	0.163** (2.38)	-0.055 (-1.18)	-0.100 (-1.30)	0.32
Credit-HY	-0.079 (-0.37)	0.376*** (5.44)	0.263*** (3.08)	0.227** (2.56)	-0.002 (-0.02)	-0.157 (-1.32)	0.29

Consistent with the previous results, the alphas exhibit an enhancement from the unmanaged to their volatility-managed version, with the results being significant post-scaling (the HY at 1%, the IG at 10%, and the LTGB at 5%). The benefit of this strategy seems particularly interesting for the HY portfolio, as it sees a great increase in both the alpha coefficient and its t-statistic.

The Fama and French five-factor model does not present a higher explanatory power than its previous model (3FF) as both the profitability and investment factor present insignificant coefficients for all the analysed portfolios. However, the equity factors present a higher predictive ability for the credit premium and HY portfolios, being consistent with Bektić et al.'s

(2019) finding that these factors do not translate into fixed-income markets, except for the HY market.

Additionally, I add to equation (8) the LTGB excess return to further analyse the profitability of the corporate bond portfolios.

$$R_{i,t}^{(\sigma)} = \alpha_i + \beta_{1i} \cdot MKT_t + \beta_{2i} \cdot LTGB_t + \epsilon_{i,t} \quad (10)$$

Table 4: Excess returns on the Market factor and Government Bond excess returns for the Credit Spread using daily data

This table reports the results from the time-series regressions of the unscaled excess return (Panel A) and the volatility scaled excess returns (Panel B) on the Market factor and Government Bond excess returns, as represented by equation (10). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from 29/01/1993 to 31/07/2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>MKT</i>	<i>LTGB</i>	<i>R-squared</i>
Panel A: U.S. Unscaled				
IG	-0.053 (-0.59)	0.240*** (7.08)	0.703*** (17.24)	0.69
HY	0.213 (1.42)	0.414*** (7.59)	0.206*** (2.96)	0.38
Credit-IG	-0.053 (-0.59)	0.240*** (7.08)	-0.297*** (-7.28)	0.49
Credit-HY	0.213 (1.42)	0.414*** (7.59)	-0.794*** (-11.40)	0.64
Panel B: U.S. Scaled				
IG	0.035 (0.36)	0.209*** (5.85)	0.668*** (14.43)	0.60
HY	0.386*** (2.66)	0.362*** (8.36)	0.237*** (4.29)	0.31
Credit-IG	-0.094 (-1.10)	0.219*** (7.99)	-0.323*** (-9.57)	0.48
Credit-HY	0.215 (1.31)	0.357*** (5.44)	-0.781*** (-14.15)	0.57

By looking at the R-squares in Table 4, I find that, as opposed to earlier regressions, the market component paired with the LTGB portfolio greatly explains the excess return of the corporate bond portfolios and respective default spreads. However, contrary to the previous results, none of the buy-and-hold portfolios do present a better performance than the benchmark, which in this case is a combination of instruments related to the stock and government bond markets.

Given the alphas of the volatility-managed portfolios, all portfolios show an increase in the coefficient when compared to the unmanaged counterparts (except the credit IG which decreases by 0.041pp). This increase is only significant for the HY portfolio, which presents a substantial increase in both the alpha and the t-stat, presenting an alpha for the scaled portfolio of 0.386% which is significant at 1%. Moreover, even though in the first regression the HY

portfolio had the highest proportion of its excess returns explained by the market factor, when the LTGB is also added to the regression, this portfolio is the one that is least explained. This reflects the differences in the credit risks present in the different portfolios, being corporate bond portfolios a combination of risk-free assets (in the present analysis I assume it to be the LTGB, neglecting the term spread) and an equity component (Merton, 1973). The IG has lower probability of default and is more similar to a risk-free asset than to an equity instrument, reflected by a high LTGB coefficient (0.703) and t-stat (17.24) and a smaller proportion of its return being explained by the market component. On the other hand, the HY portfolio presents more equity features and less similarities to a risk-free asset.

Following the work of Moreira & Muir (2017), to evaluate the performance of the volatility-managed portfolios I run a series of time-series regressions that regress the volatility-managed portfolio excess returns ($R_{i,t}^\sigma$) on the buy-and-hold portfolio excess returns ($R_{i,t}$).

$$R_{i,t}^\sigma = \alpha_i + \beta_i \cdot R_{i,t} + \epsilon_{i,t} \quad (11)$$

This regression allows to verify if positive alphas exist, implying that the volatility scaling expands the mean-variance frontier, and increase the Sharpe ratios when compared to the buy-and-hold strategy. However, I refrain from conducting the spanning regression controlling for the three Fama and French factors, as in the present thesis I am studying bond portfolios, which are not well explained by these factors, as previously discussed.

Considering the superior performance of the volatility-management strategy for the HY portfolio, a positive alpha is foreseen in the spanning regression. The results for this as well as the remaining portfolios are presented in Table 5.

Table 5: Spanning regressions for the bond portfolios for the Credit Spread using daily data

This table reports results from spanning regressions of the volatility scaled portfolios excess returns on their unscaled version, as represented by equation (11). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from 29/01/1993 to 31/07/2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>Unscaled excess returns</i>	<i>R-squared</i>
United States of America			
GOV	0.070 (1.21)	0.919*** (21.14)	0.84
IG	0.082 (1.52)	0.918*** (17.49)	0.84
HY	0.229** (2.38)	0.856*** (11.79)	0.73
Credit-IG	-0.062 (-1.26)	0.896*** (16.40)	0.80
Credit-HY	0.020 (0.15)	0.783*** (10.34)	0.61

Despite the observable improvement in the Sharpe ratios and an increase in the alphas from the unscaled to the scaled portfolios, combining the volatility-managed portfolios with their unmanaged counterparts do not offer an advantage for most of the portfolios, as the alpha is only significant for the portfolios constituted with HY corporate bonds at a 5% significance level. Hence, the volatility scaling strategies can increase the Sharpe ratio and expand the mean-variance frontier for the HY portfolio. Results from a similar analysis, excluding the HY portfolio due to data limitations, for a broader set of countries are in section 6.1. These broadly confirm this pattern in international data.

Implementing volatility scaling strategies in HY bonds, using the underlying assets, can be challenging due to the low liquidity and high transaction costs that are negatively correlated with the size of the trade and the credit rating inherent to the corporate bond (Edwards et al. 2007), which could eliminate the positive performance after accounting for it. However, more recently with the use of ETF specialized in HY bonds there is likely enough liquidity on the market to implement volatility-management strategies.

5.2 Daily Term Spread

From the previous analysis, there seems to be a term premium with potential for volatility scaling. Therefore, I decide to further study the effect of volatility scaling strategies on government bonds with different maturities.

Table 6: Descriptive statistics for unscaled and scaled excess returns for the Term Spread using daily data
This table reports descriptive statistics of the original bond portfolios and of the risk-managed bond portfolios. The sample period under consideration is from 01/01/1989 to 31/07/2023. Panel A presents the unscaled results and Panel B the results of the volatility scaling strategy on bond portfolios using the realized variance of the previous month calculated from daily data to scale the portfolio. The reported statistics are the maximum and minimum monthly excess returns, the annualized mean excess return, the annualized standard deviation of excess return, the Skewness, Excess Kurtosis and the annualized Sharpe Ratio. The difference in the Sharpe ratio between the volatility-managed and the original portfolio is reported, along with the statistical significance according to Jobson & Korkie (1981) test. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Excess returns are presented in percentages.

	<i>Min</i>	<i>Max</i>	<i>Mean</i>	<i>Std</i>	<i>Skew</i>	<i>Kurt</i>	<i>SR</i>	<i>SR dif.</i>
Panel A: U.S. Unscaled								
1-3Y	-1.46	1.43	1.09	1.47	0.05	0.98	0.74	-
3-5Y	-2.96	3.69	2.14	3.41	-0.03	0.40	0.63	-
5-7Y	-3.90	5.57	2.79	4.81	-0.01	0.41	0.58	-
7-10Y	-5.31	7.73	3.18	6.30	0.03	0.89	0.50	-
10Y+	-10.51	11.45	4.39	10.33	0.18	1.36	0.43	-
Panel B: U.S. Scaled								
1-3Y	-1.28	1.70	0.93	1.47	0.06	0.52	0.64	-0.10
3-5Y	-3.18	4.77	1.81	3.41	0.18	1.41	0.53	-0.10
5-7Y	-4.77	5.88	2.62	4.81	0.14	1.25	0.54	-0.04
7-10Y	-6.08	9.32	3.25	6.30	0.37	2.15	0.52	0.02
10Y+	-10.76	18.40	5.26	10.33	0.57	3.42	0.51	0.08

In line with the findings of Frazzini & Pedersen (2014), in Panel A I document the yield curve for the U.S. and observed that this risk is more rewarded for lower maturities than for higher maturities. Hence, Sharpe ratios decline gradually from 0.74 in the short-maturity bonds to 0.43 for the long-maturity bonds.

The differences in the Sharpe ratios of the unscaled and the scaled portfolios are negative and more evident for the lower maturity portfolios (with maturities lower than 5 years), being the reduction of 0.1 p.p. These differences decline with the increase in the maturity, becoming positive for maturities over 7 years. The improvement in the Sharpe ratios is more substantial for portfolios with maturities over 10 years, suggesting that for the same volatility, the scaling strategy demonstrate greater average excess returns than their unmanaged counterpart. Additionally, none of these differences are significant, suggesting that volatility-scaling portfolios do not systematically outperform or underperform the original portfolios.

The scaling strategy deals with the negative skewness presented in this sample, indicating that, after the scaling, all 5 portfolios present a potential for some gains. I record an increase in the kurtosis, in exception of the 1-3y portfolio, suggesting that these bond portfolios have a higher probability of extreme events, which can be concerning for investors as it suggests that there is a greater risk of large losses or gains than one might expect.

The results of the regressions (8) and (9) for the Term Spread using daily data are reported in the Table B1 and Table B2. An investor who buys only the unscaled portfolios as well as the scaled version can still benefit from risk-adjusted returns as all alphas are positive and significant when regressing the excess return on both the market factor and the Fama & French (1993) five risk factors. The difference in the alphas deliver the same conclusions as the direct comparison of the Sharpe ratios and similar results to the ones observed on the LTGB in the credit spread analysis were found observing the regression estimated parameters.

To evaluate the performance of government bond portfolios derived from volatility-management strategies in greater depth, I perform a set of time-series regressions of the volatility-managed portfolios excess returns on their original excess returns.

Table 7: Spanning regressions for the bond portfolios for the Term Spread using daily data

This table reports results from spanning regressions of the volatility scaled portfolios excess returns on their unscaled version, as represented by equation (11). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from 01/01/1989 to 31/07/2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>Unscaled excess returns</i>	<i>R-squared</i>
United States of America			
1-3Y	-0.003 (-0.30)	0.890*** (28.35)	0.79
3-5Y	-0.011 (-0.55)	0.909*** (26.70)	0.83
5-7Y	0.004 (0.16)	0.925*** (26.01)	0.86
7-10Y	0.029 (0.88)	0.919*** (22.13)	0.85
10Y+	0.106* (1.92)	0.915*** (22.68)	0.84

Despite the null alphas found for the LTGB in section 5.1, when using a sample from 1989 to 2023, the volatility-management strategy generated positive and significant alphas (at a 10% significance level) for the portfolio constituted by U.S. government bonds with maturity longer than 10 years. Although, this portfolio presents a positive and significant alpha, from the direct comparison of the Sharpe ratios it is known that the difference in the Sharpe ratios is not significant, suggesting that the volatility-managed portfolio is more lucrative when combined with their original portfolio rather than as a stand-alone investment.

For the remaining portfolios, it is possible to observe that all alphas are null and therefore found no evidence that the volatility scaling strategies increase the Sharpe ratios and expand the mean-variance frontier, for the study of U.S. government bond with maturities lower than 10 years. When I compare how effective the volatility management strategy is, for government bond portfolios with different maturities, it becomes evident that longer-term bonds perform better while shorter-term bonds underperform.

5.3 Monthly Term Spread

The final empirical study in my research reveals how volatility management behaves in a broader sample that includes taking a long position in a 10-year government bond and shorting a 3-month T-bill. To extend it to contain data from 1964, monthly data is used to estimate volatility with two techniques, EWMA and GARCH as shown in 4.2.

Table 8: Descriptive statistics for unscaled and scaled excess returns for the Term Spread using monthly data

This table reports descriptive statistics of the original bond portfolios and of the risk-managed bond portfolios. The sample period under consideration is from June of 1964 to July of 2023. Panel A presents the unscaled results, Panel B the results of the volatility scaling strategy on bond portfolios using the GARCH model to estimate the volatility, and Panel C the results of the volatility scaling strategy on bond portfolios using the EWMA model to estimate the volatility. The reported statistics are the maximum and minimum monthly excess returns, the annualized mean excess return, the annualized standard deviation of excess return, the Skewness, Excess Kurtosis and the annualized Sharpe Ratio. The difference in the Sharpe ratio between the volatility-managed and the original portfolio is reported, along with the statistical significance according to Jobson & Korkie (1981) test. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Excess returns are presented as percentages.

	<i>Min</i>	<i>Max</i>	<i>Mean</i>	<i>Std</i>	<i>Skew</i>	<i>Kurt</i>	<i>SR</i>	<i>SR dif.</i>
Panel A: U.S. Unscaled	-32.17	13.07	1.03	9.94	-2.30	25.76	0.10	-
Panel B: U.S. Scaled (GARCH)	-11.31	13.22	3.72	9.94	-0.13	1.82	0.37	0.27***
Panel C: U.S. Scaled (EWMA)	-18.76	25.95	2.87	9.94	1.40	20.83	0.29	0.19**

After analysing the summary statistics, I discover that both methods greatly improve the Sharpe ratios and help reduce high-order risk. Moreover, it seems that GARCH is more effective when it comes to minimizing extreme outcomes and improving risk-adjusted performance. This is evident, from the higher Sharpe ratio and the substantial reduction in kurtosis. The statistically significant differences in the Sharpe ratios further support the superiority of the GARCH model in this context, since it is significant at 1% using GARCH, as opposed to 5% when using the EWMA model to estimate volatility.

Subsequent analysis of the performance of both the unscaled portfolio and scaled portfolios while accounting for the CAPM (equation (8)) and the Fama & French (1993) five factor model (equation (9)) are presented in Table B3 and Table B4, respectively. The analysed portfolios only present both significant positive alphas after the implementation of volatility managing strategies and the estimated coefficient are in line with the previous analysis. Additionally, when contrasting the different volatility estimation methods, GARCH outperforms in terms of alpha and t-statistics. Moreover, the market and the 5FF present higher explanatory power on the portfolio's excess returns according to this autoregressive model, which can be attributed to GARCH's better ability to capture clusters of volatility as well as make the linkage between past and future volatility. It hence gives more precise picture of the volatility dynamics in the analysed sample.

To confirm the existence of utility gains for mean-variance investors who hold volatility-managed bond portfolios, I ran Moreira & Muir's (2017) spanning regressions, as illustrated in equation (11).

Table 9: Spanning regressions for the bond portfolios for the Term Spread using monthly data

This table reports results from spanning regressions of the volatility scaled portfolios excess returns on their unscaled version, as represented by equation (11). Panel A presents the results of the univariate spanning regressions of the volatility scaled portfolios excess returns using the GARCH model to estimate volatility, and Panel B reports the results of the univariate spanning regressions of the volatility scaled portfolios excess returns using the EWMA model to estimate volatility. T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from June of 1964 to July of 2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are in percentage whereas, betas, t-statistics values and R-squared are in decimals.

	<i>Alpha</i>	<i>Unscaled excess returns</i>	<i>R-squared</i>
Panel A: U.S. Scaled (GARCH)	0.237*** (3.99)	0.855*** (8.97)	0.73
Panel B: U.S. Scaled (EWMA)	0.174** (2.44)	0.749*** (18.32)	0.56

The results in Table 9 grant strong empirical support for the benefits of applying volatility-management strategies to bond portfolios. The positive and statistically significant intercepts, at 1% using the GARCH to estimate volatility and at 5% using the EWMA, suggest that by applying volatility-management strategies a mean-variance investor can expand the mean-variance frontier relatively to the original buy-and-hold portfolios and increase Sharpe ratios. This improvement is more evident when using the GARCH model, reflected by a higher coefficient (0.237) and associated t-statistic (3.99). The superiority of the strategy when using the GARCH model holds internationally and in different timeframes, for further analysis on this topics refer to section 6.3 and section 7.1, respectively.

6. International Evidence

The differences in the development of corporate bond markets across the globe are more explicit than for government bond markets. One example that highlights this difference is the reliance on debt securities in the United States, for funding diverse types of businesses, whereas in Europe and Japan is mainly used by financial institutions (Schinasi & Smith, 1998).

In their research, Pérignon et al. (2007) analysed the common factor structure in international government bonds with different maturities, taking into consideration country-specific factors. The authors discovered that only one common factor is shared between U.S. bond returns and German and Japanese bond returns, being this factor related to changes in the domestic term structures.

Due to the differences in fixed-income market across the world, I decided to extend my analysis to international fixed-income markets and check if my results hold or if they are particular to the U.S. bond market due to its unique aspects.

Therefore, I repeat the previous analysis on a sample of 6 countries (Japan (JP), the United Kingdom (U.K.), Canada (CAN), Australia (AU), France (FR) and Germany (BD)), when studying the effect of risk management in government bond with different maturities and respective term spread. For the broader period, Japan was excluded for the sample as it did not satisfy the minimum data requirements.

Nevertheless, a sample of three bond markets (Japan, the U.K., and Canada) is taken into consideration in order to assess the benefits of volatility scaling strategies in bond portfolios with different levels of credit risk. However, in this analysis, all portfolios are within the investment grade rating. I restrict the current analysis to a smaller sample of international markets and disregarded corporate bonds with ratings below investment-grade to ensure the validation of my results, as data on bond returns is not widely available and only these portfolios met the minimum data criteria.

I retrieve the required data from the sources explained in section 3, however, in this section I collected a sample of the several portfolios returns in the local currency. To ensure comparability of the results, I convert⁸ all the returns to U.S. dollars, eliminating the impact of currency fluctuations.

⁸ Exchange Rates were retrieved from DataStream covering the entire time period and for the different currencies (Japanese Yen, British Pound, Canadian Dollar, Australian Dollar, and Euro) to U.S. dollars.

Furthermore, in the present analysis, I follow Cederburg et al. (2020) methodology and focus on the direct comparison of the Sharpe ratios of the scaled and unscaled portfolios, in addition to the results of the univariate spanning regressions to ensure the brevity of my research.

6.1 Daily Credit Spread

I begin this analysis by presenting the results of the direct comparison of the 3 portfolios in Table 10. To check the significance of these differences, I follow the method proposed by Jobson & Korkie (1981) and report the number of countries that present positive or negative differences for the government portfolio, corporate portfolio, and the portfolio in which I isolated the credit risk.

Table 10: Differences in the Sharpe ratios of unscaled and scaled excess returns for the Credit Spread using daily data

This table reports the results for the direct comparisons of the original bond portfolios and of the risk-managed bond portfolios. For each type of portfolio, I report the number of Sharpe ratio differences which are positive, in squared brackets the number of positive differences significant at the 5% level and the number of differences that are negative, in squared brackets the number of negative differences significant at the 5% level. To assess the statistically significant of the differences in the Sharpe ratios I used Jobson & Korkie (1981) method. The sample period under consideration is from 01/02/1998 to 31/07/2023.

	<i>Sharpe ratio difference</i>	
	$\Delta SR > 0$ [sig.]	$\Delta SR \leq 0$ [sig.]
GOV	2 [0]	1 [0]
CORP	2 [0]	1 [0]
Credit	1 [0]	2 [0]

Across all 9 portfolios, the findings indicate a lack of systematic benefit from employing volatility-scaling strategies to a sample of bonds with different levels of credit risk. Most of the results, which reflect the case of the U.K. and Canada, confirm the evidence found in the U.S., in that volatility-management enhances Sharpe ratios for the government and corporate bonds but not for the credit component; conversely, the Japanese market exhibits an opposing trend. However, none of the previous results is statistically significant.

Despite this unfavourable evidence of the benefits of volatility-management in this sample, I decided to apply the spanning regressions, computed as in equation (11). Although there is no benefit in invest solely on these volatility-managed portfolios, benefits can arise when complementing this investment with its original counterpart.

The result of this analysis, which can be found in Table C1, shows that there is no combination of the volatility-managed and original portfolios that provide an enhancement in performance as the alphas are not significant. The findings are consistent with the inexistent benefits of

volatility-managed IG corporate bond portfolios demonstrated by Rebelo (2022) and may arise from the elevated institutional ownership of these instruments, where these professional investors tend to have a long-term perspective and follow a buy-and-hold strategy (Bai et al., 2019). Moreira & Muir (2019) analysed the relationship between investor horizon and volatility-management and also found that long-term investors are less susceptible to changes in volatility due to the mean reversion in stock returns.

6.2 Daily Term Spread

In this subsection, I discuss the results of the direct comparisons of the Sharpe ratios, presented in Table 11, and the spanning regressions, showed in Table C2, for 6 fixed-income markets across the globe. In each market I study 5 government bond portfolios each presenting different maturities.

Table 11: Differences in the Sharpe ratios of unscaled and scaled excess returns for the Term Spread using daily data

This table reports the results for the direct comparisons of the original bond portfolios and of the risk-managed bond portfolios. For each type of portfolio, I report the number of Sharpe ratio differences which are positive, in squared brackets the number of positive differences significant at the 5% level and the number of differences that are negative, in squared brackets the number of negative differences significant at the 5% level. To assess the statistically significant of the differences in the Sharpe ratios I used Jobson & Korkie (1981) method. The sample period under consideration is from 01/01/1989 to 31/07/2023.

	<i>Sharpe ratio difference</i>	
	$\Delta SR > 0$ [sig.]	$\Delta SR \leq 0$ [sig.]
1-3Y	3 [1]	3 [0]
3-5Y	3 [1]	3 [0]
5-7Y	4 [1]	2 [0]
7-10Y	5 [1]	1 [0]
10Y+	5[2]	1 [0]

Despite the unfavourable results in the previous analysis for international markets, the present results suggest that volatility-managed portfolios tend to improve performance more frequently than worsen it. The majority of differences are positive for the portfolio that presents higher maturity, which confirms the evidence found that for longer maturities the strategy's benefits are larger, despite not significant for any maturities in the U.S.

For a sample of international markets, volatility-managed portfolios seem to systematically outperform the buy-and-hold portfolios for longer maturities. Notably, in Australia, there is strong evidence that by using volatility timing strategy it is possible to constantly outperform the corresponding unmanaged portfolio for all segments of the yield curve, as the differences in Sharpe ratios are positive and significant at 5%.

According to the evidence in section 6.1, similar alphas were found for Canada and the U.K., despite that, in the ongoing study the alphas are significant for Canada at 10% for lower maturities reaching a significance level of 1% for maturities over 10 years. Additionally, consistent with the work of Grobys & Äijö (2018) that found no evidence of enhanced performance of volatility-management when applied to Fama and French in the Japanese market, I found that the benefits of volatility scaling that are observed in some bond portfolios do not extend to the Japanese market.

France and Germany present results that bear a striking resemblance, a resemblance that may be explained by the fact that changes in interest rates are a significant factor in explaining the returns of government bonds (Longstaff & Schwartz, 1993) and the monetary policies of both countries are influenced by decisions of the European Central Bank (ECB, 2021).

6.3 Monthly Term Spread

Building upon the previous results (in section 5.3), which showed that for the U.S. applying volatility-management strategies to the term spread⁹ provides higher and significant Sharpe ratios when compared to the unscaled portfolio, with the outperformance being particularly significant when using the GARCH model to estimate volatility, I now extend this analysis to investigate whether this holds for a sample of five international markets.

Table 12: Differences in the Sharpe ratios of unscaled and scaled excess returns for the Term Spread using monthly data

This table reports the results for the direct comparisons of the original bond portfolios and of the risk-managed bond portfolios. For each type of portfolio, I report the number of Sharpe ratio differences which are positive, in squared brackets the number of positive differences significant at the 5% level and the number of differences that are negative, in squared brackets the number of negative differences significant at the 5% level. To assess the statistical significance of the differences in the Sharpe ratios I used Jobson & Korkie (1981) method. The sample period under consideration is from February of 1986 to July of 2023 (United Kingdom), February of 1960 to July of 2023 (Canada), August of 1969 to July of 2023 (Australia), February of 1970 to July of 2023 (France), and from February of 1960 to July of 2023 (Germany).

	<i>Sharpe ratio difference</i>	
	$\Delta SR > 0$ [sig.]	$\Delta SR \leq 0$ [sig.]
GARCH	3(1)	2(0)
EWMA	1(0)	4(2)

Despite the previous evidence of the exceptional benefits of this method, the result of the direct comparisons Sharpe ratios (in Table 12) presents mixed evidence, as it improves and harms performance nearly as frequently. When using the EWMA method to estimate volatility, the

⁹ In the present analysis, as stated in section 3.3, term spread is the difference between a 10-year Government bond with a 3-month Treasury bill.

volatility-management strategy clearly underperforms its original counterpart, with half of the negative differences being significant at 5%. This confirms the advantage of using the GARCH model over the EWMA, although in the broader sample, only one of the differences in the Sharpe ratios is positive.

In addition, from the results reported in Table C3, I find that when using the EWMA to estimate the volatility by combining the scaled with the unscaled portfolio, there are negative and significant at 10% alphas for all markets in the sample. Except for the U.K. in which there is no evidence of under or overperformance. Moreover, I find evidence that using the GARCH leads to a positive and significant alpha at 1% in Germany and at 10% in the U.K., however, these alphas are economically insignificant (0.038% and 0.041%, respectively).

Contrarily to previous findings, where Australia showed evidence of systematically outperformance using volatility-management strategies, in the current analysis it appears to significantly hurt the performance regardless of the approach used, being the alphas negative and significant at 10%.

7. Robustness tests

7.1 Monthly term spread, smaller sample

The two preceding analyses in the study of the term spread diverge in term of method used and temporal scope. I decided to isolate the effect of the different methods to estimate the volatility and check the robustness of the previous results. To address this matter, I reapply the procedure outlined in section 4.2 covering the period used in the daily term spread analyse, from January of 1989 to July of 2023¹⁰. It is important to note that some of the disparities may occur because the assets under analysis are not exactly the same.

Across the spectrum of countries examined, the volatility timing strategies using the GARCH generate a higher alpha with a higher associated t-statistic than the ones using the EWMA, confirming the superior performance of the GARCH model. The new sample offers stronger evidence on the benefits of using volatility-managed strategy to improve bond portfolios performance with the GARCH method, being the differences in the Sharpe ratios all positive with the majority being significant at 5%. The combination of strategies yields positive and significant alphas for the U.S., the U.K., France and Germany, reflecting possible utility gains for a mean-variance investor. However, while these alphas are statistically significant, their economic significance is marginal. On the other hand, the results for the combination of the strategies using the EWMA indicate either negative or null alphas and thus do not offer investors any further advantages over the unscaled portfolios.

Comparing the different methods, I prove that countries that achieve great improvements in performance due to volatility scaling with the daily method, like Canada and Australia, do not achieve the same results when using monthly data, even when controlling for the timeframe. This may be in part explained by the differences in the composition of the portfolios. Moreover, there is no clear evidence on which method consistently leads to a better performance, despite the alphas tending to be larger when using the realized volatility method.

7.2 Risk-Return Trade-Off

In order to explain the differences found among multiple countries and the different methods to estimate the volatility, I studied the risk-return trade-off of each portfolio. The analysis of the risk-return trade-off is consistent in the volatility-management literature, as the performance of this strategy is motivated by the relation between lagged volatility and current volatility and by

¹⁰ I report the detailed results on the performance of volatility-managed portfolios in the Appendix D.

the relation between lagged volatility and current average returns (Cederburg et al., 2020). Wang & Yan (2021) attributed the positive alphas in the spanning regression to be a consequence of volatility timing and return timing.

Hence, I first examine the relationship between current excess returns and the previous month volatility (return timing), with the following univariate regression:

$$R_{i,t} = c_i + \theta_i \cdot RV_{i,t-1} + \epsilon_{i,t} \quad (12)$$

Mixed evidence is found in past literature on the relation between lagged volatility and future excess returns, with the results being affected by the sample period and methodology to estimate the volatility. The results reported in Appendix E, show evidence of a weak risk-return relationship, with the majority of the reported coefficients not being significant. Surprisingly, I found that in Australia past volatility is strongly and negatively associated with current expected returns across all segments of the yield curve, with coefficients averaging -0.340 and significant at 5%. This can explain the systematic outperformance found in these volatility-managed portfolios, in Australia. Since a negative relationship between lagged volatility and average returns makes volatility-management more effective, as there is a negative relation between the conditional Sharpe ratio and its lagged volatility and investors can benefit from this by taking more aggressive investment decisions after low-volatility periods (Cederburg et al., 2020).

Lastly, I estimate the following autoregressive regression of the realized volatility, to study the existence of a positive relation between past and future volatility (volatility timing). I refrained from carrying out the regression for the study of the monthly term spread because the volatility estimation methods used depend intrinsically on past values, according to their definition.

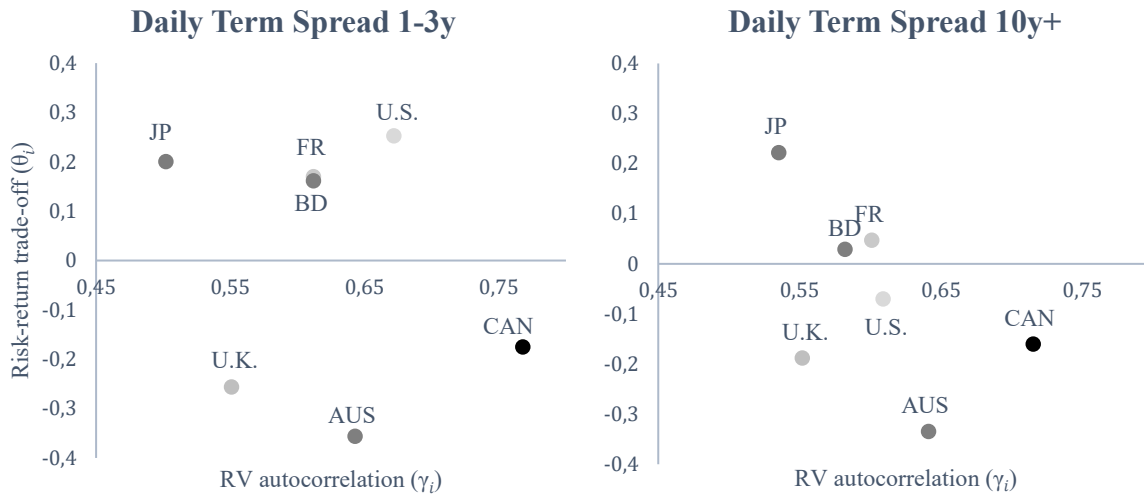
$$RV_{i,t} = c_i + \gamma_i \cdot RV_{i,t-1} + \epsilon_{i,t} \quad (13)$$

The coefficients for this regression, that can be found in Appendix E, show strong support for a positive relation between present volatility and its lag, as the betas range from 0.336 in the credit spread of Canada to 0.768 in Canada's term spread, being all coefficients significant at 1%. These results are in line with the expectation due to volatility clustering phenomenon in the financial market, where periods of high volatility tend to cluster together (Engle, 2004).

For a better understanding of these relationships, I plot, in Figure 1, the coefficients (θ_i) of equation (12) on the coefficients (γ_i) of equation (13).

Figure 1: Risk-Return trade-off

This figure shows the relationship between past volatility and current returns, as well as the relationship between past volatility and current volatility for the shorter maturity portfolio (1-3y) and the longer maturity one (10y+). The detailed results can be found in Table E2. The sample period under consideration is from 01/01/1989 to 31/07/2023, for 7 international government bond markets.



It is clear from the figures above, that the main difference across countries as well as the differences between the different securities arise from the relationship between past volatility and present returns. In Figure 1, it is possible to analyse this relation and conclude that the portfolios presenting negative y-value, suggesting a negative relation between past volatility and present return, and are more distant to the origin tend to show a better performance when volatility-management strategies are applied.

Moreover, I find that the poor performance of the volatility scaling strategy in Japan can be both attributed to a positive relation between past volatility and present returns and a weak correlation of volatility with its lagged counterpart. On the opposite side, the impressive benefits that scaling the volatility bring in Australia are a result of the strong negative risk-return trade-off. Considering this, I conclude that the differences found in the result of applying volatility scaling strategies in different countries are mainly owed to relation of past volatility and current returns.

8. Conclusion

Adding to the growing literature on volatility-management strategies, this paper studies the potential benefits of managing the volatility in bond portfolios. It encompasses a range of portfolios, looking at varying levels of credit risk and maturities, in markets around the world. The empirical findings offer insights, on the effectiveness of these strategies in various market conditions highlighting the need to carefully consider factors such as credit risk, maturity and the dynamics of international markets.

In the study of bonds with varying levels of default probability, I conclude that the benefits of volatility-managed portfolios tend to increase with the credit risk. In particular, volatility-managed HY portfolios, in contrast to the remaining portfolios, show improvements in performance both as stand-alone investments and in combination with their original counterparts but this performance may not be easily implemented, due to high transaction costs and low liquidity of this type of bonds. Moreover, due to limited data on HY corporate bonds, this study is unable to extend the previous conclusion to international markets, leaving a gap in the study of volatility-management on HY portfolios.

Additionally, the study uncovers mixed results on the performance of volatility-managed government bond portfolios with different maturities. I conclude that longer-term portfolios show potential benefits from scaling, being the enhancement in performance more promising when used as a combination of strategies rather than stand-alone investments, being these conclusions robust to international data. As far as government bonds with maturities of less than 10 years are concerned, there is little evidence of any improvement in Sharpe ratios. Australia stands out as an exception, with consistently superior performance in all segments of the yield curve.

The efficacy of volatility scaling strategies in bonds portfolios is conditioned by the methodology used, the time period and the market specific dynamics at play. In particular, the GARCH model consistently improves performance while considerably improving higher-order moments. In the study of the term spread, using the EWMA method shows evidence of underperformance when compared to the original portfolios, which demonstrates the sensitivity of the results to different methodologies.

Essential to understanding the observable differences across countries is the risk-return trade-off, I found that most of these discrepancies result from differences in return timing. To further

explore the sources of the performance of volatility-managed portfolios, I suggest following Wang & Yan's (2021) methodology and decompose the alpha from the spanning regressions.

One of the biggest challenges in fixed-income markets is to execute dynamic strategies that need frequent trading. This arises from low liquidity of some bonds and high transaction costs due to wide bid-ask spread. I therefore suggest that future research takes these factors into account, as they could compromise the benefits associated with volatility scaling. Another path could be followed to eliminate ex-post bias from this analysis, such as by targeting a constant level of volatility, a methodology used by Barroso & Santa-Clara (2015). Or even follow Wang & Yan (2021) methodology and use downside volatility instead of total volatility to scale the portfolios, as bondholders are more susceptible to downside risk, which can be explained by the fact that they expect to receive a fixed amount, hence do not benefit from positive developments. On the other hand, due to unexpected defaults investors can see their expected cash flows disappear (Bai et al., 2019).

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Appendix

Appendix A: Calculation of returns from yields-to-maturity

The following formulas are approximations that can be done for par bonds, meaning these bonds coupon rate equals the bond's yield. Besides, this method is only correct for government bond, as it does not take into consideration defaults, and it would overestimate the returns on corporate bonds.

Firstly, I estimate the interest rate sensitivity by calculating the modified duration (D_t), given by the following formula:

$$D_t = \frac{1}{y_t} \cdot \left[1 - \frac{1}{(1 + 0.5 \cdot y_t)^{2 \cdot M_t}} \right] \quad (A1)$$

, where y_t is the yield-to-maturity at month t and M_t the maturity of the bond at month t .

Then, I study the relation between the bond's price and its yields by calculating the convexity (C_t):

$$C_t = \frac{2}{y_t^2} \cdot \left[1 - \frac{1}{(1 + 0.5 \cdot y_t)^{2 \cdot M_t}} \right] - \frac{2 \cdot M_t}{y_t \cdot (1 + 0.5 y_t)^{2 \cdot M_t + 1}} \quad (A2)$$

, where y_t is the yield-to-maturity at month t and M_t the maturity of the bond at month t .

After calculating both the modified duration and the convexity, I can calculate the return in month t (r_t):

$$r_t = [(1 + y_{t-1})^{1/12} - 1] - D_t \cdot \Delta y_t + 0.5 C_t \cdot \Delta y_t^2 \quad (A3)$$

, where the first term is the coupon and the second is the percentage change in price.

For the Long-Term Government Bond, the remaining maturity is constant and $M_t = 10$, whereas for the Short-Term Government Bond the remaining maturity is constant and $M_t = 0.25$.

Appendix B: Additional Regressions

Table B1: Scaled and Unscaled Excess Returns on the Market factor for the Term Spread using daily data

This table reports the results from the time-series regressions of the unscaled excess return (Panel A) and the volatility scaled excess returns (Panel B) on the market factor, as represented by equation (8). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from 01/01/1989 to 31/07/2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>MKT</i>	<i>R-squared</i>
Panel A: U.S. Unscaled			
1-3Y	0.097*** (4.37)	-0.009 (-1.57)	0.01
3-5Y	0.194*** (3.80)	-0.022* (-1.67)	0.01
5-7Y	0.250*** (3.47)	-0.026 (-1.39)	0.01
7-10Y	0.284*** (2.99)	-0.030 (-1.14)	0.01
10Y+	0.405*** (2.58)	-0.059 (-1.36)	0.01
Panel B: U.S. Scaled			
1-3Y	0.084*** (3.82)	-0.008 (-1.42)	0.01
3-5Y	0.166*** (3.25)	-0.020 (-1.39)	0.01
5-7Y	0.236*** (3.31)	-0.026 (-1.28)	0.01
7-10Y	0.290*** (3.10)	-0.027 (-1.02)	0.00
10Y+	0.462*** (3.01)	-0.033 (-0.84)	0.00

Table B2: Scaled and Unscaled Excess Returns on the Five Fama and French Factors for the Term Spread using daily data

This table reports the results from the time-series regressions of the unscaled excess return (Panel A) and the volatility scaled excess returns (Panel B) on the 5 Fama and French Factors, as represented by equation (9). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from 01/01/1989 to 31/07/2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>MKT</i>	<i>SMB</i>	<i>HML</i>	<i>RMW</i>	<i>CMA</i>	<i>R-squared</i>
Panel A: U.S. Unscaled							
1-3Y	0.088*** (3.78)	-0.003 (-0.52)	-0.017** (-2.08)	-0.012 (-1.17)	0.019* (1.80)	0.005 (0.30)	0.05
3-5Y	0.169*** (3.21)	-0.008 (-0.55)	-0.044** (-2.38)	-0.036 (-1.47)	0.058** (2.29)	0.004 (0.10)	0.07
5-7Y	0.213*** (2.87)	-0.005 (-0.27)	-0.061** (-2.34)	-0.063* (-1.74)	0.091** (2.56)	0.005 (0.09)	0.07
7-10Y	0.240** (2.45)	-0.004 (-0.16)	-0.079** (-2.31)	-0.088* (-1.72)	0.118** (2.50)	0.001 (0.01)	0.07
10Y+	0.352** (2.18)	-0.024 (-0.52)	-0.130** (-2.45)	-0.194** (-2.19)	0.143* (1.91)	0.042 (0.38)	0.07

Panel B: U.S. Scaled							
1-3Y	0.075*** (3.29)	-0.003 (-0.42)	-0.018** (-2.27)	-0.016* (-1.85)	0.016* (1.68)	0.009 (0.72)	0.05
3-5Y	0.139*** (2.64)	-0.004 (-0.25)	-0.050*** (-2.69)	-0.053** (-2.52)	0.050** (2.17)	0.028 (0.92)	0.07
5-7Y	0.199*** (2.69)	-0.001 (-0.07)	-0.075*** (-2.80)	-0.079*** (-2.66)	0.075** (2.25)	0.037 (0.87)	0.08
7-10Y	0.247** (2.54)	0.002 (0.08)	-0.098*** (-2.81)	-0.101** (-2.57)	0.093** (2.04)	0.038 (0.69)	0.07
10Y+	0.419** (2.55)	0.004 (0.09)	-0.151*** (-2.72)	-0.143** (-2.08)	0.096 (1.29)	0.055 (0.60)	0.05

Table B3: Scaled and Unscaled Excess Returns on the Market factor for the Term Spread using monthly data

This table reports the results from the time-series regressions of the unscaled excess return (Panel A), the volatility scaled excess returns using the GARCH model (Panel B) and the volatility scaled excess returns using the EWMA model (Panel C) on the market factor, as represented by equation (8). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from June of 1964 to July of 2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>MKT</i>	<i>R-squared</i>
Panel A: U.S. Unscaled	0.133 (1.24)	-0.085** (-2.52)	0.02
Panel B: U.S. Scaled (GARCH)	0.356*** (3.22)	-0.082** (-2.52)	0.02
Panel C: U.S. Scaled (EWMA)	0.269** (2.46)	-0.054** (-2.01)	0.01

Table B4: Scaled and Unscaled Excess Returns on the Five Fama and French Factors for the Term Spread using monthly data

This table reports the results from the time-series regressions of the unscaled excess return (Panel A), the volatility scaled excess returns using the GARCH model (Panel B) and the volatility scaled excess returns using the EWMA model (Panel C) on the 5 Fama and French Factors, as represented by equation (9). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from June of 1964 to July of 2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>MKT</i>	<i>SMB</i>	<i>HML</i>	<i>RMW</i>	<i>CMA</i>	<i>R-squared</i>
Panel A: U.S. Unscaled	0.164 (1.51)	-0.090*** (-2.62)	-0.061 (-1.41)	-0.031 (-0.42)	0.096* (1.71)	-0.121 (-1.32)	0.04
Panel B: U.S. Scaled (GARCH)	0.399*** (3.43)	-0.082** (-2.47)	-0.096** (-2.23)	-0.071 (-1.22)	0.087 (1.56)	-0.096 (-1.09)	0.05
Panel C: U.S. Scaled (EWMA)	0.303*** (2.66)	-0.051* (-1.93)	-0.082** (-2.21)	-0.023 (-0.38)	0.037 (0.71)	-0.076 (-0.86)	0.02

Appendix C: International Evidence

Table C1: Spanning regressions for the bond portfolios for the Credit Spread using daily data

This table reports the results from the univariate spanning regressions of the volatility scaled portfolios excess returns on their unscaled version for Japan, the United Kingdom, and Canada, as represented by equation (11). The sample period under consideration is from 01/02/1998 to 31/07/2023. T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>Unscaled excess returns</i>	<i>R-squared</i>
Japan			
GOV	-0.051 (-0.49)	0.913*** (18.06)	0.83
CORP	-0.075 (-0.72)	0.910*** (18.55)	0.83
Credit	0.027 (1.60)	0.876*** (16.67)	0.77
United Kingdom			
GOV	0.097 (0.97)	0.922*** (20.85)	0.85
CORP	0.054 (0.57)	0.914*** (19.30)	0.84
Credit	-0.079 (-1.27)	0.837*** (14.76)	0.70
Canada			
GOV	0.111 (1.26)	0.937*** (23.88)	0.88
CORP	0.067 (0.74)	0.925*** (18.20)	0.86
Credit	-0.005 (-0.10)	0.866*** (10.78)	0.75

Table C2: Spanning regressions for the bond portfolios for the Term Spread using daily data

This table reports the results from the univariate spanning regressions of the volatility scaled portfolios excess returns on their unscaled version for Japan, the United Kingdom, Canada, Australia, France, and Germany, as represented by equation (11). The sample period under consideration is from 01/01/1989 to 31/07/2023. T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>Unscaled excess returns</i>	<i>R-squared</i>
Japan			
1-3Y	-0.159* (-1.86)	0.910*** (19.82)	0.83
3-5Y	-0.153* (-1.79)	0.912*** (20.10)	0.83
5-7Y	-0.148* (-1.73)	0.913*** (20.66)	0.84
7-10Y	-0.136 (-1.59)	0.916*** (21.41)	0.84
10Y+	-0.116 (-1.31)	0.919*** (22.05)	0.85

United Kingdom			
1-3Y	0.099 (1.60)	0.945*** (33.26)	0.89
3-5Y	0.086 (1.39)	0.948*** (32.98)	0.90
5-7Y	0.077 (1.25)	0.951*** (34.05)	0.91
7-10Y	0.069 (1.09)	0.954*** (36.90)	0.91
10Y+	0.114 (1.53)	0.949*** (35.76)	0.90
Canada			
1-3Y	0.117* (1.58)	0.891*** (18.91)	0.80
3-5Y	0.117* (1.59)	0.902*** (19.59)	0.81
5-7Y	0.126* (1.76)	0.911*** (20.77)	0.83
7-10Y	0.149** (2.11)	0.920*** (22.57)	0.85
10Y+	0.185*** (2.48)	0.927*** (25.45)	0.86
Australia			
1-3Y	0.323*** (3.38)	0.897*** (22.36)	0.80
3-5Y	0.324*** (3.42)	0.903*** (21.74)	0.81
5-7Y	0.333*** (3.47)	0.899*** (21.23)	0.81
7-10Y	0.332*** (3.46)	0.905*** (21.73)	0.82
10Y+	0.347*** (3.65)	0.914*** (23.00)	0.83
France			
1-3Y	-0.031 (-0.43)	0.925*** (23.83)	0.86
3-5Y	-0.015 (-0.20)	0.927*** (25.83)	0.86
5-7Y	0.002 (0.03)	0.930*** (29.08)	0.87
7-10Y	0.023 (0.32)	0.936*** (32.08)	0.88
10Y+	0.079* (1.03)	0.946*** (37.83)	0.90
Germany			
1-3Y	-0.031 (-0.43)	0.925*** (24.08)	0.86
3-5Y	-0.015 (-0.21)	0.927*** (25.92)	0.86
5-7Y	-0.004 (-0.06)	0.931*** (29.29)	0.87
7-10Y	0.022 (0.30)	0.934*** (31.82)	0.87
10Y+	0.066* (0.87)	0.948*** (38.20)	0.90

Table C3: Spanning regressions for the bond portfolios for the Term Spread using monthly data

This table reports the results from the univariate spanning regressions of the volatility scaled portfolios excess returns on their unscaled version, using the volatility calculated with both the GARCH and EWMA model to scale the portfolio for the United Kingdom, Canada, Australia, France, and Germany, as represented by equation (11). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from February of 1986 to July of 2023(United Kingdom), February of 1960 to July of 2023(Canada), August of 1969 to July of 2023(Australia), February of 1970 to July of 2023(France), and from February of 1960 to July of 2023(Germany). The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>Unscaled excess returns</i>	<i>R-squared</i>
United Kingdom (GARCH)	0.041* (1.94)	0.964*** (31.50)	0.93
United Kingdom (EWMA)	0.008 (0.42)	0.975*** (45.74)	0.95
Canada (GARCH)	-0.026 (-1.18)	0.932*** (29.58)	0.87
Canada (EWMA)	-0.039* (-1.72)	0.928*** (35.46)	0.86
Australia (GARCH)	-0.021* (-1.24)	0.975*** (51.26)	0.95
Australia (EWMA)	-0.098* (-1.71)	0.723*** (9.04)	0.52
France (GARCH)	0.001 (0.16)	0.997*** (162.61)	1.00
France (EWMA)	-0.031* (-0.92)	0.971*** (103.65)	0.94
Germany (GARCH)	0.038*** (2.96)	0.969*** (32.76)	0.94
Germany (EWMA)	-0.011* (-0.92)	0.976*** (60.09)	0.95

Appendix D: Robustness Test-Smaller Sample

Table D1: Difference in Sharpe ratios of unscaled and scaled excess returns for the Term Spread using monthly data

This table reports the results for the direct comparisons of the original bond portfolios and of the risk-managed bond portfolios for the United States of America, United Kingdom, Canada, Australia, France and Germany. For each type of portfolio, I report the number of Sharpe ratio differences which are positive, in squared brackets the number of positive differences significant at the 5% level and the number of differences that are negative, in squared brackets the number of negative differences significant at the 5% level. To assess the statistical significance of the differences in the Sharpe ratios I used Jobson & Korkie (1981) method. The sample period under consideration is from January of 1989 to July of 2023.

	<i>Sharpe ratio difference</i>	
	$\Delta SR > 0$ [sig.]	$\Delta SR \leq 0$ [sig.]
GARCH	6 [4]	0 [0]
EWMA	2[0]	4[0]

Table D2: Spanning regressions for the bond portfolios for the Term Spread using monthly data

This table reports results from spanning regressions of the volatility scaled portfolios excess returns on their unscaled version, as represented by equation (11) for the United States of America, United Kingdom, Canada, Australia, France and Germany. T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The sample period under consideration is from January of 1989 to July of 2023. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Alphas are presented in percentages whereas, betas, t-statistics values and R-squared are presented in decimals.

	<i>Alpha</i>	<i>Unscaled excess returns</i>	<i>R-squared</i>
United States of America (GARCH)	0.086** (2.25)	0.951*** (16.63)	0.91
United States of America (EWMA)	0.035 (0.61)	0.835*** (18.30)	0.70
United Kingdom (GARCH)	0.047** (2.35)	0.970*** (29.17)	0.94
United Kingdom (EWMA)	-0.024* (-1.66)	0.984*** (53.73)	0.97
Canada (GARCH)	0.010 (0.99)	0.992*** (100.71)	0.99
Canada (EWMA)	-0.052 (-0.89)	0.735*** (18.52)	0.54
Australia (GARCH)	0.006 (0.34)	0.984*** (63.94)	0.97
Australia (EWMA)	-0.009 (-0.52)	0.982*** (58.51)	0.96
France (GARCH)	0.050*** (3.06)	0.982*** (40.92)	0.97
France (EWMA)	-0.009 (-0.74)	0.990*** (69.24)	0.98
Germany (GARCH)	0.048*** (3.09)	0.984*** (36.34)	0.97
Germany (EWMA)	0.000 (0.02)	0.994*** (72.91)	0.99

Appendix E: Robustness Test-Risk-Return Trade-off

Table E1: Risk-Return Trade-off for the Credit Spread using daily data

This table reports the results from the regression of the previous month realized volatility on the present excess return of the buy-and-hold strategy, as represented by equation (12) and the results from the regression of the previous month realized volatility, on the current realized volatility, as represented by equation (13). The sample period under consideration is from 29/01/1993 to 31/07/2023(US) and from 01/02/1998 to 31/07/2023 for the remaining countries. T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Betas and t-statistics values are presented in decimals.

	(12)	(13)
	RV _{t-1}	RV _{t-1}
United States of America		
GOV	-0.037 (-0.12)	0.607*** (8.54)
IG	0.169 (0.64)	0.575*** (6.62)
HY	0.127 (0.28)	0.526*** (9.84)
Credit-IG	0.317 (1.37)	0.570*** (6.12)
Credit-HY	-0.064 (-0.15)	0.640*** (9.14)
Japan		
GOV	0.072 (0.35)	0.566*** (11.97)
CORP	0.101 (0.49)	0.557*** (10.98)
Credit	-0.074 (-0.60)	0.478*** (4.52)
United Kingdom		
GOV	-0.007 (-0.04)	0.563*** (7.76)
CORP	0.183 (0.73)	0.582*** (8.34)
Credit	0.292* (1.81)	0.512*** (9.15)
Canada		
GOV	-0.110 (-0.53)	0.691*** (13.20)
CORP	-0.077 (-0.34)	0.707*** (9.88)
Credit	-0.108 (-0.31)	0.336*** (5.11)

Table E2: Risk-Return Trade-off for the Term Spread using daily data

This table reports the results from the regression of the previous month realized volatility on the present excess return of the buy-and-hold strategy, as represented by equation (12) and the results from the regression of the previous month realized volatility, on the current realized volatility, as represented by equation (13). The sample period under consideration is from 01/01/1989 to 31/07/2023. T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Betas and t-statistics values are presented in decimals.

	(12)	(13)
	RV _{t-1}	RV _{t-1}
United States of America		
1-3Y	0.252** (2.24)	0.672*** (12.37)
3-5Y	0.202 (1.41)	0.670*** (15.82)
5-7Y	0.087 (0.50)	0.623*** (12.55)
7-10Y	0.006 (0.03)	0.614*** (10.22)
10Y+	-0.070 (-0.43)	0.609*** (5.19)
Japan		
1-3Y	0.201 (1.09)	0.502*** (10.48)
3-5Y	0.220 (1.18)	0.507*** (10.69)
5-7Y	0.228 (1.22)	0.511*** (10.93)
7-10Y	0.225 (1.21)	0.518*** (11.39)
10Y+	0.222 (1.21)	0.535*** (12.89)
United Kingdom		
1-3Y	-0.256* (-1.89)	0.551*** (8.75)
3-5Y	-0.221 (-1.59)	0.548*** (8.46)
5-7Y	-0.189 (-1.34)	0.544*** (8.31)
7-10Y	-0.165 (-1.17)	0.547*** (8.40)
10Y+	-0.188* (-1.72)	0.552*** (8.67)
Canada		
1-3Y	-0.175 (-0.95)	0.768*** (19.62)
3-5Y	-0.152 (-0.81)	0.762*** (18.83)
5-7Y	-0.148 (-0.78)	0.755*** (17.80)
7-10Y	-0.163 (-0.86)	0.743*** (16.64)
10Y+	-0.160 (-0.87)	0.715*** (15.52)

Australia		
1-3Y	-0.356** (-2.18)	0.643*** (7.44)
3-5Y	-0.350** (-2.15)	0.644*** (7.42)
5-7Y	-0.330** (-2.11)	0.654*** (6.88)
7-10Y	-0.330** (-2.03)	0.644*** (7.54)
10Y+	-0.334** (-2.09)	0.641*** (7.88)
France		
1-3Y	0.170 (0.80)	0.612*** (14.85)
3-5Y	0.168 (0.81)	0.612*** (14.71)
5-7Y	0.144 (0.72)	0.617*** (14.55)
7-10Y	0.097 (0.50)	0.609*** (14.23)
10Y+	-0.047 (-0.25)	0.601*** (12.75)
Germany		
1-3Y	0.162 (0.77)	0.612*** (14.81)
3-5Y	0.163 (0.79)	0.615*** (14.73)
5-7Y	0.168 (0.86)	0.615*** (14.54)
7-10Y	0.124 (0.67)	0.609*** (14.09)
10Y+	0.029 (0.16)	0.582*** (12.06)

Table E3: Risk-Return Trade-off for the Term Spread using monthly data

This table report the results from the regression of the previous month volatility, calculated with both the GARCH and EWMA model volatility, on the present excess return of the buy-and-hold strategy, as represented by equation (12). The sample period under consideration is from June of 1964 to July of 2023(United States of America), February of 1986 to July of 2023(United Kingdom), February of 1960 to July of 2023(Canada), August of 1969 to July of 2023(Australia), February of 1970 to July of 2023(France), and from February of 1960 to July of 2023(Germany). T-statistics in parentheses are based on White (1980) heteroskedasticity robust standard errors. The *, ** and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively. Betas and t-statistics values are presented in decimals.

	<i>(12)</i>
	RV_{t-1}
United States of America (GARCH)	-0.259 (-1.60)
United States of America (EWMA)	-0.133 (-1.11)
United Kingdom (GARCH)	-0.081 (-0.34)
United Kingdom (EWMA)	-0.046 (-0.20)
Canada (GARCH)	0.223 (1.48)
Canada (EWMA)	0.263** (2.17)
Australia (GARCH)	0.222 (1.28)
Australia (EWMA)	0.380** (2.57)
France (GARCH)	0.299** (2.16)
France (EWMA)	0.012 (0.29)
Germany (GARCH)	-0.284 (-1.33)
Germany (EWMA)	0.159 (0.97)