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**MAINTENANCE INCENTIVES IN HIGHWAY CONCESSION  
CONTRACTS**

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# Maintenance Incentives in Highway Concession Contracts\*

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## Abstract

In most European countries, the private sector has a direct or indirect participation in the construction, overhaul, maintenance or operation of highways, normally through concession contracts with a pre-specified duration. The concession company is frequently remunerated through direct payments by road users (road tolls). In this context, it is important to understand the incentives it has to maintain a highway in proper conditions whilst at the same time it seeks to maximise its profits. We model this profit-maximisation problem in a dynamic setting where demand is partly a function of road quality in each period. We find that concession companies have incentives to “shirk” on their maintenance duties and let road quality degrade early in their concession contract; later on, the concession company invests more heavily in maintenance so as to return the highway to the public authority in good working conditions. We also analyse how these results are affected by changes in the road toll, costs and the duration of the concession contract.

JEL Classification: L2, L5

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# 1 Introduction

The use of concession contracts “by which a public authority grants specific rights to an organization (whether private or semi-public) to construct, overhaul, maintain and operate an infrastructure for a given period” (Bousquet and Fayard (2001); see also Kessides (2004)) is a common practice in most European countries when dealing with road infrastructure and in particular highways. The company which is granted the concession contract is normally remunerated through direct payments by road users (road tolls) or through payments by the concession authority (shadow toll), normally on the basis of traffic observed on the highway.

In many cases, the company granted the concession is charged with making the investments required to create the service at its own cost and operate the service at its own risk, for a limited period (Bousquet and Fayard (2001)). However, there are several possible variations to the infrastructure concession, and it may be possible to have the public authority funding the project and later making the infrastructure available to the concession company, who becomes responsible for managing the operation of the highway. In this case, the concession is a lease contract (Bousquet and Fayard (2001)).

In such lease contracts, the concession company is exclusively responsible for the operation of the service. In this paper, we focus on one issue related to service operation: road maintenance. This issue has received little attention in the literature (see Vickerman (2004)). Dekker *et al* (1997) argue that the timing of the rehabilitation actions is a “fundamental aspect of the road maintenance planning”. “On the one hand one must rehabilitate the roads before the damage is bothering the motorists; on the other hand one must not repair the roads too soon as it is quite expensive to maintain small road-sections” (*idem*). Dekker *et al* (1997) also suggest that road maintenance is characterised by economies of scale: the cost of maintenance per square meter decreases when the repaired area becomes larger, i.e. it is less costly to carry out a one-off larger repair than to spread out the maintenance task over several small repairs.

A popular discussion is one which concentrates on issues such as whether private sector operators will put profits before safety (Vickerman (2004)). This would imply, however, that there would be no revenue implications of operating an unsafe or under-maintained network, or at least that these implications would be smaller than any cost saving. “Since most infrastructure has an expected life greater than the typical franchise granted to an

operator, there might be an incentive to depreciate the asset more rapidly if there is no penalty for the condition at the end of this period” (Vickerman (2004)). This appears to justify lease contracts’ transfer clauses which typically stipulate that the infrastructure has to be transferred back to the public authority, at the end of the concession, in good working order.

Vickerman (2004) analyses maintenance incentives in privatised infrastructures, namely the road and rail networks. In particular, he discusses the definition of the optimal level of maintenance and the incentives which might ensure that the infrastructure operator always maintains the infrastructure in this optimal state. Vickerman (2004) considers that “those maintaining and operating networks will have a better knowledge of their long-term potential to deliver a given level of service than those regulating that provision”.

We construct a model which analyses the incentives of the concession company to maintain a highway in proper conditions whilst at the same time it seeks to maximise profits. We model the profit-maximisation problem faced by the concession company throughout its concession contract in the case where (i) it is remunerated directly by users (road tolls), (ii) demand is partly a function of road quality and (iii) maintenance is costly. We consider that the company has no part in the construction phase of the infrastructure and is only responsible for the operation of the highway (lease contract).

We model this profit-maximisation as an optimal control problem which ultimately determines the optimal path of the intensity of road maintenance efforts by the concession company throughout the concession. There is significant value added associated with this modelling framework: it allows us not only to understand whether the company has incentives to invest in maintenance, but also to analyse how such incentives evolve throughout the concession. In that respect, we depart from standard principal-agent models (Laffont and Tirole (1993) and Vickerman (2004)) which analyse the incentives of the agent (the concession company) to exert maintenance efforts, but regard such effort to be constant and applicable to the concession as a whole. By contrast, an optimal control framework allows us to see how such maintenance effort may vary *within* the concession. In order to do so, we assume the company receives the road, at the beginning of the contract, in an almost-as-new condition and has to return it, at the end of the concession, in the same condition. Road quality in any period depends on the depreciation rate and on the intensity of road maintenance: if the company invests more in maintenance in a given period than what is necessary

to compensate for that period's depreciation, road quality increases and vice-versa.

We find that concession companies have incentives to “shirk” on their maintenance duties and let road quality degrade early in their concession contract; later on, because at the end of the contract the highway must be in good working conditions, the concession company invests more heavily in maintenance. This result is directly related to the comparison, for the concession company, between the marginal revenue and the marginal cost associated with a strategy of no-road-depreciation, i.e. a strategy whereby maintenance effort is always just sufficient to compensate for road depreciation. When they are not equal, the concession company finds it profit-maximising to always increase (or decrease) its maintenance efforts throughout the concession. This behaviour does not allow it to satisfy the end point constraint which requires the highway to be handed over at the end of the concession in an almost-as-new condition. In order to maximise profits and satisfy such constraints, the concession company chooses to underinvest (relative to the depreciation rate) in road maintenance early in the concession (and road quality deteriorates) and to overinvest (also relative to the depreciation rate) in maintenance later in the concession (and road quality improves until it reaches an almost-as-new condition at the end of the concession).

We also find that increasing the duration of the concession reinforces this behaviour: longer concessions generally result in lower road quality levels in every period, as the concession company has even more incentives to underinvest in road maintenance early in the concession. Increased maintenance costs have the opposite effect: it is profit-maximising to provide better road quality in every period of the concession. Finally, increases in the road toll may contribute to an increase or decrease of road quality in each period and this depends on whether the marginal revenue associated with this price increase is positive (which leads to lower road quality) or negative (which leads to improved road quality). We show that this is associated with an inelastic and elastic demand respectively.

The paper is structured in the following way: section 2 describes the model and contains the main results; in section 3 we analyse some comparative statics; section 4 provides a graphical analysis of the results; finally, section 5 concludes.

## 2 The model

We assume the concession company must maximise its profits over a given planning horizon  $[0, T]$ , the concession period, in continuous time and with no discount rate.

Revenues are given by demand (number of road users) multiplied by the road toll,  $p$  which we assume is fixed throughout the concession period and exogenous to the concession company<sup>1</sup>. Demand in period  $t$  is given by:

$$D_t(p) = K_t(a - bp) \quad (1)$$

where  $K_t$  is road quality at time  $t$ . We assume that  $K_t \in [0, 1]$ , where  $K_t = 1$  means that the highway is in an almost-as-new state and cannot be further improved and  $K_t = 0$  means that the highway is in poor condition and cannot get any worse. Therefore, demand is a decreasing function of the road toll and an increasing function of road quality. The following relationship is assumed to hold:  $a > b$ .

Let  $Q_t \in [0, 1]$  denote the intensity of road maintenance efforts by the concession company in period  $t$ :  $Q_t = 0$  implies that no maintenance is carried out in period  $t$  and  $Q_t = 1$  implies that in a given period the concession company fully restores the road into an almost-as-new condition.

The concession company is assumed to have the following cost function:

$$C(Q_t) = cQ_t - Q_t^2 \quad (2)$$

This cost function conveys the idea that average costs are a decreasing function of maintenance efforts, as suggested by Dekker *et al* (1997). Therefore, it is less costly for the concession company to carry out a given maintenance task  $Q_t$  in one single period than to spread it out over two or more periods. Average cost in period  $t$  is given by  $c - Q_t$ , whereas marginal cost is given by  $c - 2Q_t$ . We impose the restriction that  $c > 2$  so that both average and marginal costs are positive for any  $Q_t \in [0, 1]$ .

We assume that there is a constant depreciation rate of road quality given by  $(1 - \delta)$ . Therefore, if road quality at period  $t$  is  $K_t$ , in the following period it will be given by  $K_{t+1} = \delta K_t$ . We assume  $\delta \in (0, 1)$ .

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<sup>1</sup>In many countries, the applicable road toll is stipulated in the concession contract and any changes to it are also regulated.

The maximisation problem of the concession company consists of choosing a level of  $Q_t$  for every  $t \in [0, T]$  (the concession period) which maximises profits; this is defined in dynamic optimisation problems as the stable or optimal path and it is the solution of:

$$\begin{aligned} & \max_{Q_t} \int_0^T p [K_t (a - bp)] - cQ_t + Q_t^2 \\ & \text{s.t. } \dot{K} = (\delta - 1) K_t + Q_t \\ & K_0 = 1 \\ & K_T = 1 \end{aligned} \tag{3}$$

where  $\dot{K}$  is the instantaneous rate of growth of road quality at any point in time. In this dynamic optimisation problem,  $K_t$  is the state variable and  $Q_t$  is the control variable. There are three restrictions: (i) road quality evolves over time in a way which depends on the depreciation rate  $(1 - \delta)$  and the optimal choice of  $Q_t$  by the concession company; these two factors determine the rate of growth of road quality in each period,  $\dot{K}$ ; (ii) the road is handed over to the concession company at the beginning of the concession ( $t = 0$ ) in an almost-as-new condition ( $K_0 = 1$ ) and (iii) the road must be handed over at the end of the concession ( $t = T$ ) also in an almost-as-new condition ( $K_T = 1$ ). With this setup, we obtain the following results:

**Proposition 1** *The profit-maximising level of maintenance effort for each period  $t$  is given by:*

$$Q_t = \frac{c}{2} - \frac{p(a - bp)}{2(1 - \delta)} - C \frac{e^{(1-\delta)t}}{2} \tag{4}$$

*This effort level results in the following road quality level for each period  $t$ :*

$$K_t = \frac{L}{e^{(1-\delta)t}} + \frac{c}{2(1 - \delta)} - \frac{p(a - bp)}{2(1 - \delta)^2} - \frac{1}{4(1 - \delta)} C e^{(1-\delta)t} \tag{5}$$

*where  $L$  and  $C$  are constants given by:*

$$C = 4(1 - \delta) \frac{1 - e^{(1-\delta)T}}{1 - e^{2(1-\delta)T}} \left[ \frac{c}{2(1 - \delta)} - \frac{p(a - bp)}{2(1 - \delta)^2} - 1 \right] \tag{6}$$

*and*

$$\begin{aligned} L &= \frac{e^{2(1-\delta)T} - e^{(1-\delta)T}}{1 - e^{2(1-\delta)T}} \left[ \frac{c}{2(1 - \delta)} - \frac{p(a - bp)}{2(1 - \delta)^2} - 1 \right] \\ &= -\frac{e^{(1-\delta)T}}{4(1 - \delta)} C \end{aligned} \tag{7}$$

These equations fully characterise the optimal path of  $Q_t$  and  $K_t$  over the planning horizon  $[0, T]$ .

**Proof.** In order to solve this optimal control problem, where the concession company must decide on a given path of  $Q_t$  over the planning horizon  $t \in [0, T]$ , we set up the Hamiltonian:

$$H = p [K_t (a - bp)] - cQ_t + Q_t^2 + \pi (t) [(\delta - 1) K_t + Q_t] \quad (8)$$

The stable path must satisfy the following first-order conditions

$$\frac{\partial H}{\partial Q_t} = 0 \quad (9)$$

$$\dot{\pi}(t) = -\frac{\partial H}{\partial K_t} \quad (10)$$

$$\dot{K} = \frac{\partial H}{\partial \pi(t)} \quad (11)$$

Applying the first-order conditions to our maximisation problem yields:

$$\frac{\partial H}{\partial Q_t} = -c + 2Q_t + \pi(t) = 0 \quad (12)$$

$$\dot{\pi}(t) = -p(a - bp) + (1 - \delta) \pi(t) \quad (13)$$

$$\dot{K} = (\delta - 1) K_t + Q_t \quad (14)$$

Rearranging the second condition yields:

$$\dot{\pi} + (\delta - 1) \pi = -p(a - bp) \quad (15)$$

This is a linear differential equation. In order to solve it, we multiple both sides by  $e^{(\delta-1)t}$  and integrate:

$$\pi = \frac{p(a - bp)}{1 - \delta} + C e^{(1-\delta)t} \quad (16)$$

where  $C$  is assumed to be the constant of integration. Substituting into the first equation yields:

$$Q_t = \frac{c}{2} - \frac{p(a - bp)}{2(1 - \delta)} - C \frac{e^{(1-\delta)t}}{2} \quad (17)$$

Finally, substituting this equation in the third first-order condition and rearranging yields:

$$\dot{K} + (1 - \delta) K_t = \frac{c}{2} - \frac{p(a - bp)}{2(1 - \delta)} - C \frac{e^{(1-\delta)t}}{2} \quad (18)$$

Again, this is a linear differential equation, but now the right hand side depends on  $t$ .

Define:

$$\begin{aligned} u(t) &= 1 - \delta \\ w(t) &= \frac{c}{2} - \frac{p(a - bp)}{2(1 - \delta)} - C \frac{e^{(1-\delta)t}}{2} \end{aligned} \quad (19)$$

A differential equation of the form:

$$\dot{K} + u(t) K_t = w(t) \quad (20)$$

Has as its solution:

$$K_t = \frac{1}{e^{\int u(t) dt}} \left( A + \int w(t) e^{\int u(t) dt} \right) \quad (21)$$

Where A is an arbitrary constant. Therefore the solution of equation (18) is:

$$K_t = \frac{1}{e^{(1-\delta)t+G}} \left[ A + \int \left( \frac{c}{2} - \frac{p(a - bp)}{2(1 - \delta)} - C \frac{e^{(1-\delta)t}}{2} \right) (e^{(1-\delta)t+G}) dt \right] \quad (22)$$

where A and G are arbitrary constants. After some algebraic manipulation, this expression simplifies to:

$$K_t = \frac{L}{e^{(1-\delta)t}} + \frac{c}{2(1 - \delta)} - \frac{p(a - bp)}{2(1 - \delta)^2} - \frac{1}{4(1 - \delta)} C e^{(1-\delta)t} \quad (23)$$

where  $L$  is an arbitrary constant of integration.

Using the last two restrictions of the maximisation problem, the starting point and the end point of the state variable,  $K_t$ , it is possible to solve for values of  $C$  and  $L$ . In particular, we assume that  $K_0 = 1$  and  $K_T = 1$ . Using the optimal level of  $K_t$  (equation (23)) this implies that the following equations must hold:

$$K_0 = L + \frac{c}{2(1 - \delta)} - \frac{p(a - bp)}{2(1 - \delta)^2} - \frac{1}{4(1 - \delta)} C = 1 \quad (24)$$

$$K_T = \frac{L}{e^{(1-\delta)T}} + \frac{c}{2(1 - \delta)} - \frac{p(a - bp)}{2(1 - \delta)^2} - \frac{1}{4(1 - \delta)} C e^{(1-\delta)T} = 1 \quad (25)$$

Solving these two equations yields:

$$C = 4(1 - \delta) \frac{1 - e^{(1-\delta)T}}{1 - e^{2(1-\delta)T}} \left[ \frac{c}{2(1 - \delta)} - \frac{p(a - bp)}{2(1 - \delta)^2} - 1 \right] \quad (26)$$

and

$$\begin{aligned}
L &= \frac{e^{2(1-\delta)T} - e^{(1-\delta)T}}{1 - e^{2(1-\delta)T}} \left[ \frac{c}{2(1-\delta)} - \frac{p(a-bp)}{2(1-\delta)^2} - 1 \right] \\
&= -\frac{e^{(1-\delta)T}}{4(1-\delta)} C
\end{aligned} \tag{27}$$

Equations (17) and (23) fully characterise the solution of this dynamic optimisation problem, where the constants  $C$  and  $L$  are the expressions of equations (26) and (27). ■

Proposition 1 identifies the equations for  $K_t$  and  $Q_t$  which contain the optimal path over time of the state ( $K_t$ ) and control ( $Q_t$ ) variables of this dynamic optimisation problem. Naturally, the optimal path depends on various factors, namely (i) the cost function of the concession company (through parameter  $c$ ), (ii) the revenue of the concession company (through parameters  $p$ , the road toll, and  $a$  and  $b$ , from the linear demand specification), (iii) the depreciation rate of the highway ( $1 - \delta$ ) and (iv) time ( $t$ ). It is especially with respect to this latter variable that we are most interested in the profit-maximising choice of the concession company.

**Proposition 2**  $C < 0$  is a necessary and sufficient condition for  $\partial Q_t / \partial t > 0$ , i.e. for maintenance effort to be increasing throughout the concession period.

**Proof.** The profit-maximising maintenance effort (equation (17)) has as its derivative with respect to time:

$$\frac{\partial Q_t}{\partial t} = -\frac{1-\delta}{2} C e^{(1-\delta)t} \tag{28}$$

Given that  $\delta \in (0, 1)$ , provided  $C < 0$  this derivative is always positive. ■

The following result is obtained for the profit-maximising road quality level ( $K_t$ ) throughout the concession:

**Proposition 3** Provided  $C < 0$ ,  $\partial K_t / \partial t < 0$  for  $t < T/2$  and  $\partial K_t / \partial t > 0$  for  $t > T/2$ , i.e. road quality decreases with time in the first half of the concession and increases with time in the second half of the concession.

**Proof.** The profit-maximising road quality level (equation (23)) has as its derivative with respect to time:

$$\frac{\partial K_t}{\partial t} = C \frac{e^{(1-\delta)T} - e^{2(1-\delta)t}}{4e^{(1-\delta)t}} \quad (29)$$

The denominator is always positive; if  $C < 0$ , this derivative will be negative when the numerator is positive, i.e. when:

$$e^{(1-\delta)T} - e^{2(1-\delta)t} > 0 \quad (30)$$

which is equivalent to:

$$\begin{aligned} (1-\delta)T &> 2(1-\delta)t \\ \Leftrightarrow t &< T/2 \end{aligned} \quad (31)$$

Similarly, the derivative is positive when the numerator is negative, which occurs when  $t > T/2$ . ■

These two propositions contain the essence of the profit maximisation problem of the concession company which is neatly captured by the sign of  $C$ , a constant. In order for  $C < 0$ , and rearranging equation (26), the following must hold:

$$c - \frac{p(a-bp)}{1-\delta} - 2(1-\delta) < 0 \quad (32)$$

We can now explain the economic message underlying this expression which is so important for Propositions 2 and 3. Firstly, by rearranging the expression for  $K_t$  obtained in equation (23), we obtain:

$$\begin{aligned} K_t &= \frac{1}{1-\delta} \left[ \frac{c}{2} - \frac{p(a-bp)}{2(1-\delta)} - C \frac{e^{(1-\delta)t}}{2} \right] + C \left( \frac{e^{2(1-\delta)t} - e^{(1-\delta)T}}{4(1-\delta)e^{(1-\delta)t}} \right) \\ &= \frac{1}{1-\delta} Q_t + C \left( \frac{e^{2(1-\delta)t} - e^{(1-\delta)T}}{4(1-\delta)e^{(1-\delta)t}} \right) \end{aligned} \quad (33)$$

Inserting this expression into the revenue of the concession company in period  $t$ ,  $R_t$ , we obtain revenue as a function of maintenance effort,  $Q_t$ :

$$\begin{aligned} R_t &= p [K_t (a-bp)] \\ &= \left[ \frac{1}{1-\delta} Q_t + C \left( \frac{e^{2(1-\delta)t} - e^{(1-\delta)T}}{4(1-\delta)e^{(1-\delta)t}} \right) \right] p (a-bp) \end{aligned} \quad (34)$$

Marginal revenue in period  $t$ ,  $MR_t$ , is constant and given by:

$$MR_t = \frac{\partial R_t}{\partial Q_t} = \frac{p(a - bp)}{(1 - \delta)} \quad (35)$$

Similarly, going back to the cost function (equation (2)), marginal cost in period  $t$ ,  $MC_t$ , is given by:

$$MC_t = \frac{\partial C(Q_t)}{\partial Q_t} = c - 2Q_t \quad (36)$$

One option the concession company has is to maintain constant road quality, i.e. in every period it invests just enough to prevent the road from depreciating:  $Q'_t = (1 - \delta)$ . Such a strategy would allow the concession company to have constant road quality:  $K_t = 1, \forall t$ . The marginal revenue associated with such a strategy,  $MR'_t$ , would be the same as in equation (35) because marginal revenue is constant. For this strategy, the relationship between marginal revenue and marginal cost would be:

$$\begin{aligned} MR'_t - MC'_t &= \frac{p(a - bp)}{(1 - \delta)} - c + 2(1 - \delta) \\ &= - \left[ c - \frac{p(a - bp)}{(1 - \delta)} - 2(1 - \delta) \right] \end{aligned} \quad (37)$$

When  $C < 0$  (see equation (32) and the term in square brackets in the equation above), the marginal revenue ( $MR'_t$ ) associated with the strategy of maintaining constant road quality ( $Q'_t = (1 - \delta)$ ) is larger than marginal cost ( $MC'_t$ ). In that case, the equation above is positive. This suggests that maintaining constant road quality is not a feasible strategy: because marginal revenue is larger than marginal cost, the concession company would find it profitable to increase maintenance efforts; however, in doing so, it will find it impossible to satisfy the end point constraint ( $K_T = 1$ ). It will always have incentives to invest more and more in maintenance and road quality will increase, never being able to return to the level  $K_T = 1$ , the end point constraint (this will be shown graphically in section 4). Therefore, when such an imbalance exists between marginal revenue and marginal cost, it is profit-maximising (subject to the starting and end point constraints) for the concession company to choose a path for  $Q_t$  which depends on time. In particular, if  $C < 0$  the imbalance between marginal revenue and marginal cost is such that  $MR'_t > MC'_t$  and it is profit-maximising for the concession company to increase its maintenance efforts over time.

### 3 Comparative statics

The optimal path for  $Q_t$  chosen by the concession company depends on various variables and it is important to understand how they affect maintenance incentives.

#### 3.1 Duration of the concession ( $T$ )

**Corollary 1** *When  $C < 0$ , both maintenance effort ( $Q_t$ ) and road quality ( $K_t$ ) are decreasing with the duration of the concession ( $T$ ), for every period  $t$ .*

**Proof.** The derivative of  $K_t$  (equation (23)) with respect to  $T$  is given by:

$$\frac{\partial K_t}{\partial T} = -\frac{1}{4(1-\delta)e^{(1-\delta)t}} \left[ \frac{\partial C}{\partial T} (e^{(1-\delta)T} + e^{2(1-\delta)t}) + C(1-\delta)e^{(1-\delta)T} \right] \quad (38)$$

From equation (26) we obtain:

$$\frac{\partial C}{\partial T} = \frac{C(1-\delta)e^{(1-\delta)T}(e^{(1-\delta)T} - 1)}{1 - e^{2(1-\delta)T}} \quad (39)$$

Substituting in equation (38) we obtain:

$$\frac{\partial K_t}{\partial T} = -\frac{Ce^{(1-\delta)T}}{4e^{(1-\delta)t}} \left[ \frac{(1 - e^{2(1-\delta)t})(1 - e^{(1-\delta)T})}{1 - e^{2(1-\delta)T}} \right] \quad (40)$$

The expression between square brackets is always negative, which implies that  $\partial K_t/\partial T$  is negative provided  $C < 0$ . Similarly, the derivative of  $Q_t$  (equation (17)) with respect to  $T$  is given by:

$$\begin{aligned} \frac{\partial Q_t}{\partial T} &= -\frac{e^{(1-\delta)t}}{2} \frac{\partial C}{\partial T} \\ &= \frac{Ce^{(1-\delta)t}e^{(1-\delta)T}(1-\delta)}{2} \cdot \frac{1 - e^{(1-\delta)T}}{1 - e^{2(1-\delta)T}} \end{aligned} \quad (41)$$

The expression on the right is always positive, which implies that  $\partial Q_t/\partial T$  is negative provided  $C < 0$ . ■

When  $C < 0$ , the optimal strategy for the concession company is to underinvest in road maintenance early in the concession (road quality decreases) and to increase that effort later in the concession (see Propositions 2 and 3). In that case, allowing the concession company to have a longer contract duration (a larger value of  $T$ ) reinforces that behaviour: maintenance effort in each period will be lower, as will road quality.

### 3.2 Costs ( $c$ )

**Corollary 2** *An increase in costs of road maintenance ( $c$ ) induces an overall increase in road quality,  $K_t$ , in each period  $t \in (0, T)$ ; this is accomplished by a relative increase in road maintenance early in the concession and a relative decrease later in the concession.*

**Proof.** The derivative of road quality,  $K_t$  (equation (23)), with respect to  $c$  is given by:

$$\frac{\partial K_t}{\partial c} = \frac{(e^{(1-\delta)t} - 1)(e^{(1-\delta)T} - e^{(1-\delta)t})(1 - e^{(1-\delta)T})}{2(1 - \delta)e^{(1-\delta)t}(1 - e^{2(1-\delta)T})} \quad (42)$$

The denominator is always negative; the numerator is always negative except for  $t = 0$  and  $t = T$ , the starting and end points of the concession, where it is equal to 0. Therefore,  $\partial K_t / \partial c \geq 0, \forall t$  and  $\partial K_t / \partial c > 0, \forall t \in (0, T)$ .

The derivative of  $Q_t$  (equation (17)) with respect to  $c$  is given by:

$$\frac{\partial Q_t}{\partial c} = \frac{1 - e^{2(1-\delta)T} - 2e^{(1-\delta)t} + 2e^{(1-\delta)t}e^{(1-\delta)T}}{2(1 - e^{2(1-\delta)T})} \quad (43)$$

The sign of this derivative depends on the value of  $t$ . When  $t = 0$  this derivative is positive:

$$\frac{\partial Q_0}{\partial c} = \frac{(1 - e^{(1-\delta)T})(e^{(1-\delta)T} - 1)}{2(1 - e^{2(1-\delta)T})} \quad (44)$$

Whereas when  $t = T$  this derivative is negative:

$$\frac{\partial Q_T}{\partial c} = -\frac{(1 - e^{(1-\delta)T})(e^{(1-\delta)T} - 1)}{2(1 - e^{2(1-\delta)T})} \quad (45)$$

The expression in equation (43) is continuous in the interval  $[0, T]$ ; therefore, by the intermediate value theorem, there exists a  $t^*$  such that  $\partial Q_{t^*} / \partial c = 0$ . ■

An increase in  $c$  is equivalent to an increase in both marginal as well as average costs. Therefore, as a reaction to those increased costs the concession company chooses not to let road quality deteriorate as much as it did before. It accomplishes this by investing more than it did before in each period early in the concession, which leads to better road quality, and to invest less than it did before later in the concession (because road quality did not deteriorate as much, it is not necessary to invest as much as before in road maintenance).

### 3.3 Price or road toll ( $p$ )

**Corollary 3** *If  $\partial R_t/\partial p > 0$ , an increase in the road toll ( $p$ ) induces an overall decrease in road quality,  $K_t$ , in each period  $t \in (0, T)$ ; this is accomplished by a relative decrease in road maintenance early in the concession and a relative increase later in the concession. If  $\partial R_t/\partial p < 0$ , the opposite happens: an increase in the road toll ( $p$ ) induces an overall increase in road quality,  $K_t$ , in each period  $t \in (0, T)$ ; this is accomplished by a relative increase in road maintenance early in the concession and a relative decrease later in the concession.*

**Proof.** The derivative of road quality,  $K_t$  (equation (23)), with respect to  $p$  is given by:

$$\frac{\partial K_t}{\partial p} = \frac{(a - 2bp) (1 - e^{(1-\delta)t}) (e^{(1-\delta)T} - e^{(1-\delta)t}) (1 - e^{(1-\delta)T})}{2(1 - \delta)^2 e^{(1-\delta)t} (1 - e^{2(1-\delta)T})} \quad (46)$$

Note that marginal revenue evaluated with respect to  $p$  is given by:

$$\begin{aligned} MR_t|_p &= \frac{\partial R}{\partial p} = \frac{\partial}{\partial p} [K_t (p(a - bp))] \\ &= K_t (a - 2bp) \end{aligned} \quad (47)$$

which is positive when  $(a - 2bp) > 0$ . The denominator of equation (46) is always negative whereas the numerator is always positive when  $(a - 2bp) > 0$ , i.e. when marginal revenue is positive. The only two exceptions occur when  $t = 0$  and  $t = T$ , the starting and end points of the concession, where the numerator is equal to 0. Therefore,  $\partial K_t/\partial p \leq 0$ ,  $\forall t$  if  $(a - 2bp) > 0$ .

The derivative of  $Q_t$  (equation (17)) with respect to  $p$  is given by:

$$\frac{\partial Q_t}{\partial p} = -\frac{(a - 2bp) (1 - e^{2(1-\delta)T} - 2e^{(1-\delta)t} + 2e^{(1-\delta)t}e^{(1-\delta)T})}{2(1 - \delta) (1 - e^{2(1-\delta)T})} \quad (48)$$

Again, the sign of the derivative depends on the value of  $t$ . When  $t = 0$  this derivative is negative provided  $(a - 2bp) > 0$ :

$$\frac{\partial Q_0}{\partial p} = \frac{(a - 2bp) (1 - e^{(1-\delta)T})^2}{2(1 - \delta) (1 - e^{2(1-\delta)T})} \quad (49)$$

Whereas when  $t = T$  this derivative is positive:

$$\frac{\partial Q_T}{\partial p} = \frac{(a - 2bp) (1 - e^{(1-\delta)T}) (e^{(1-\delta)T} - 1)}{2(1 - \delta) (1 - e^{2(1-\delta)T})} \quad (50)$$

The expression in equation (48) is continuous in the interval  $[0, T]$ ; therefore, by the intermediate value theorem, there exists a  $t^*$  such that  $\partial Q_{t^*}/\partial p = 0$ .

When  $(a - 2bp) < 0$ , i.e. when marginal revenue when evaluated with respect to  $p$  is negative, the opposite results hold (see equations (46) and (48)):  $\partial K_t/\partial p \geq 0, \forall t$ ;  $\partial Q_t/\partial p > 0$  early in the concession and  $\partial Q_t/\partial p < 0$  later in the concession. ■

When marginal revenue evaluated with respect to price is positive, an increase in  $p$  increases the revenue of the concession company for any road quality level  $K_t$ . Note that because demand is linear, positive marginal revenue is associated with a demand elasticity lower than 1 (in absolute value)<sup>2</sup>. Therefore, as the road toll increases, demand falls less than proportionally thus yielding higher revenues for the concession company. In turn, this reinforces our previous result: the concession company now prefers to let road quality deteriorate more than it did before. It accomplishes this by investing less than before early in the concession, which leads to a worsening of road quality, and to invest more than it did before later in the concession (because road quality deteriorated more than before the road toll increase, it is necessary to invest more in road maintenance to have the highway in good condition at the end of the concession).

By contrast, when marginal revenue is negative, this is equivalent to a demand elasticity larger than 1 (in absolute value)<sup>3</sup>, which implies that an increase in  $p$  induces a decrease in revenue for the concession company for any road quality level  $K_t$ . This generates incentives for the concession company not to let the road deteriorate as much as it did before, hence recovering some of the lost revenue through an increase in demand (because  $K_t$  increases). Interestingly, this result implies that a concession company operating with sufficiently high road tolls (such that demand elasticity is larger than 1) will typically find it profit-maximising not to let the road deteriorate too much in the concession. Therefore, *ceteris paribus* higher road tolls are associated with incentives for the concession company to offer better road quality throughout the concession.

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<sup>2</sup>Demand elasticity is given by  $\varepsilon = \frac{-bp}{a-bp}$ . The absolute value of demand elasticity is lower than 1 when  $p < a/2b$ , and this results in positive marginal revenue.

<sup>3</sup>The absolute value of demand elasticity is larger than 1 when  $p > a/2b$ , and this results in negative marginal revenue.

## 4 Graphical analysis of results

In order to understand the dynamics of the system and the underlying solution for the maximisation problem, we can draw a phase diagram. Using the first first-order condition (equation (12)), we know that:

$$Q_t = \frac{c}{2} - \frac{\pi(t)}{2} \quad (51)$$

Differentiating with respect to time yields:

$$\dot{Q} = -\frac{\dot{\pi}(t)}{2} \quad (52)$$

Substituting the second first-order condition (equation (13)) into this equation yields:

$$\dot{Q} = \frac{p(a - bp) - (1 - \delta)c}{2} + (1 - \delta)Q_t \quad (53)$$

If we set  $\dot{Q} = 0$ , we will find the values of  $Q_t$  such that no growth over time is observed. This yields:

$$Q_t = \frac{c}{2} - \frac{p(a - bp)}{2(1 - \delta)} \quad (54)$$

This is the first element we need in order to draw the phase diagram. The second element comes from one of the restrictions (equation (14)):

$$\dot{K} = (\delta - 1)K_t + Q_t \quad (55)$$

Therefore, if we set  $\dot{K} = 0$ , we will obtain the values of  $Q_t$  (as a function of  $K_t$ ) such that no growth in road quality is observed through time. This yields:

$$Q_t = (1 - \delta)K_t \quad (56)$$

We can plot these two equations into the  $(Q_t, K_t)$  space in order to understand the dynamics of the system. This is done in Figure 1.

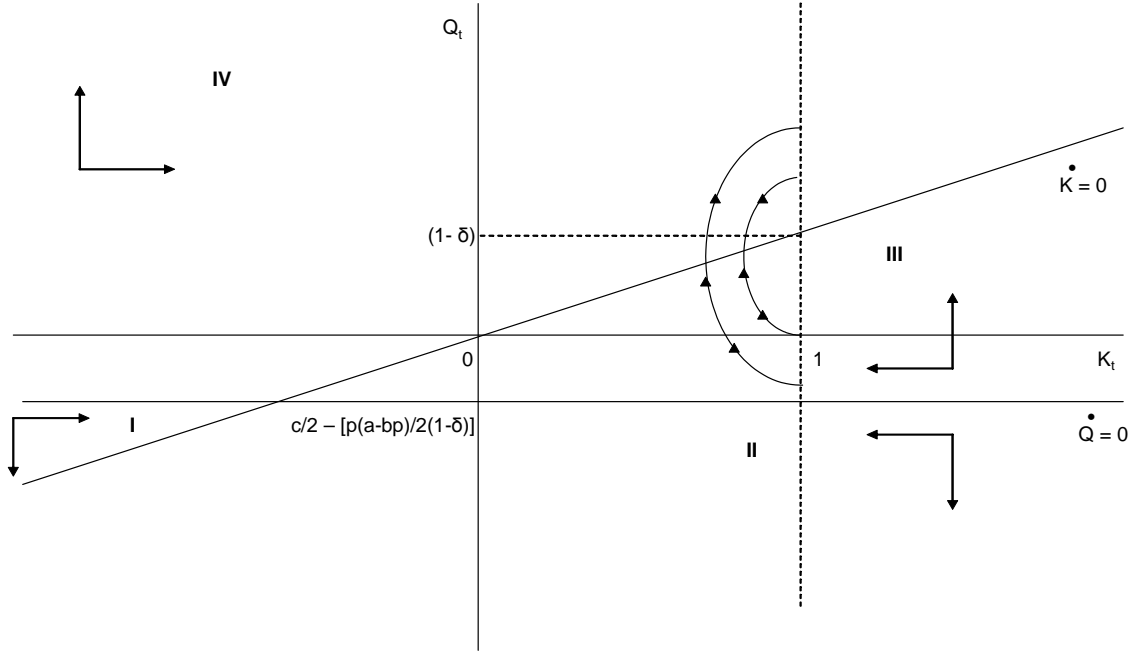


Figure 1: Phase diagram of the optimisation problem

These two equations partition the space into 4 regions. In region I, the dynamics of the system are such that  $\dot{K} > 0$  and  $\dot{Q} < 0$ . Therefore, if the starting point value of  $K_t$  and  $Q_t$  was in this region, the optimal path would follow a rightwards and downwards trajectory, crossing the  $\dot{K} = 0$  line at some point in time. If the starting point values were in region IV, the dynamics of the system would show that in that region  $\dot{K} > 0$  and  $\dot{Q} > 0$ ; therefore, the optimal trajectory would be rightwards and upwards. If the starting point values were in region II, the dynamics of the system show that  $\dot{K} < 0$  and  $\dot{Q} < 0$ ; therefore, the optimal trajectory would lead us in a downwards and leftwards direction. Neither satisfies the starting and end point constraints defined earlier ( $K_0 = K_T = 1$ ).

The interesting cases are those in region III, given that the starting point value of  $K_t$  is 1. Here we see that depending on the equation which describes  $\dot{Q} = 0$ , and in particular the point where it intersects the vertical axis, we obtain a stable path. In particular, it appears to be the case that for some parameter values that intersect would be negative. Therefore, the starting point value of  $Q_t$  could be negative, which is non-sensical in our context (it would be equivalent to negative maintenance effort in any given period). This can easily be solved by introducing the constraint  $Q_t \geq 0, \forall t$  in the maximisation problem (equation (3)).

This restriction would not change the essence of our results, although it would naturally affect the profit levels of the concession company.

The result of Propositions 2 and 3 can be seen assuming the intersect of the  $\dot{Q} = 0$  line is negative. The optimal path could be described by the path in Figure 1 which is more to the left: road quality deteriorates early in the concession ( $\dot{K} < 0$ ) and maintenance efforts increase with time ( $\dot{Q} > 0$ ), i.e. the concession company invests in maintenance but not enough to restore road quality to its previous period's condition. When the path crosses the  $\dot{K} = 0$  line, it must have an infinite slope. It then enters region IV, and follows a rightward and upward direction. In this region, maintenance effort continues to grow but now such growth is more than sufficient to offset the natural road depreciation given by the parameter  $(1 - \delta)$ . Therefore,  $\dot{K} > 0$  and the concession company continues to invest in road maintenance so as to reach the point  $K_T = 1$  when the concession contract expires and the highway must be in perfect working conditions (almost-as-new).

This is true even if for some periods the optimal level of  $Q_t$  is negative because of the parameter values. Indeed, if one were to impose the restriction that  $Q_t \geq 0$ , then the optimal path would be given by the optimal path more to the right in the phase diagram. This would affect profit levels (which would be lower) but not the concession company's behaviour regarding maintenance efforts throughout the concession.

## 5 Conclusion

This paper addresses a pertinent issue in highway concession contracts: are maintenance incentives in concession contracts such that highways are always in good working conditions? As Vickerman (2004) notes, popular discussion concentrates on issues such as whether private sector infrastructure operators put profits before safety. Naturally, such bold statements assume implicitly that the infrastructure operator is unaffected by such decisions, i.e. its profits are not significantly reduced as a consequence of reduced maintenance efforts. This would indeed be the case if the possible loss of revenue was more than offset by potential cost savings.

In order to analyse this issue, we have set up a model where such a trade-off emerges. On the one hand, maintaining highways in good working quality increases the number of users who may decide to use it; however, such maintenance efforts have a cost even though

economies of scale are assumed to exist. In this setup, and imposing the restriction that the concession company must hand over the highway in an almost-as-new condition at the end of the concession, we find that the concession company has incentives to underinvest in maintenance (relative to the road's depreciation rate) early in the concession but will invest increasingly more as time evolves. This maintenance effort is not sufficient to maintain road quality early in the concession and therefore road quality degradation is observed. Later on, as maintenance efforts increase, these will more than offset road depreciation and road quality inverts its trend and starts increasing in such a way as to be in an almost-as-new condition at the very end of the concession. This behaviour is explained by differences between marginal revenue and marginal cost associated with maintenance efforts, which make it infeasible (i.e. not profit-maximising) for the concession company to adopt a policy of constant maintenance effort throughout the concession.

We also find that longer concessions reinforce these incentives, thus contributing to lower road quality levels in every period. Increased costs of maintenance have the opposite effect, thus leading to improved road quality. Increased road tolls have an effect on road quality which depends on whether marginal revenue associated with such a toll increase is positive (leading to lower road quality) or negative (leading to improved road quality). This depends on whether the road toll is set at a level where demand is inelastic or elastic respectively.

This type of model allows for a richer analysis of many issues surrounding concession contracts, particularly allowing for an analysis of incentives within the concession. Starting from this basic model, it is also possible to analyse how these maintenance incentives may be affected by the choice of toll regime (tolls paid directly by users vs shadow tolls paid by the awarding authority) or the existence of competing road concession companies. This analysis will be left for future research.

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