RESOURCE MISALLOCATION IN PORTUGAL:

BETWEEN SECTOR MISALLOCATION AND INTERSECTORAL LINKAGES

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Abstract

Resource misallocation helps to explain cross-country *per capita* income differences as the result of differences in how economies allocate their available resources. In poorer economies additional distortions drive the equilibrium allocation further away from the optimal allocation, thus lowering aggregate output. The present dissertation applies the misallocation hypothesis to Portugal between 2000 and 2014/2017. In particular, it focuses on how distortions affect the allocation of primary inputs over the various industries, in an input-output economy. Portugal's allocative efficiency performance is then compared to Germany's.

The results suggest that if sector primary input distortions were fully eliminated, on average, GDP could have been more than 20 percent above the observed level, in Portugal. In Germany, the average gain was 13 percent. However, allocative efficiency (ratio between GDP and undistorted GDP), in Portugal, improved, from 0.80 in 2000 to 0.86 in 2014. Meanwhile, Germany's allocative efficiency remained broadly unchanged.

The dissertation explores the role of the input-output network for these results: first, keeping the calibration but closing the intermediate goods channel; second, modelling GDP as the sum of sector's value-added, which allows the use of available data up to 2017. The first suggests that the input-output network may have amplified the misallocation losses about 1.5 times. The second finds an equivalence between the input-output and the value-added frameworks, underlying that there is not an unique way to assess the importance of the input-output network and that the appropriate approach depends on the particular question.

Keywords: Resource misallocation, allocative efficiency, primary inputs, sectors, input-output networks.

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Resumo

Disparidades no rendimento *per capita* entre economias podem, em parte, resultar de diferenças na qualidade da afetação de recursos. Em economias mais pobres podem existir distorções adicionais que levam a uma pior afetação dos recursos disponíveis e, consequentemente, um menor rendimento. A presente dissertação explora esta hipótese para Portugal entre 2000 e 2014/2017, utilizando a Alemanha como comparação. O exercício foca o impacto de distorções setoriais que condicionam a distribuição de recursos primários entre setores tendo em conta ligações intersectoriais.

Os resultados sugerem que, se as distorções na afetação de recursos primários entre setores fossem eliminadas, o PIB poderia ser superior em mais de 20 por cento, em Portugal. Na Alemanha, o ganho seria 13 por cento. Contudo, a eficiência na afetação (rácio entre o PIB e o PIB sem distorções) melhorou em Portugal entre 2000 e 2014, de 0.80 para 0.86, enquanto a eficiência na afetação alemã se manteve globalmente estável.

A dissertação explora a importância das ligações intersectoriais para os resultados: primeiro, mantendo a calibração original e fechando o canal de bens intermédios; segundo, representando o PIB como soma do valor acrescentado, permitindo explorar a disponibilidade de dados até 2017. A primeira sugere que as ligações intersectoriais podem ter amplificado as perdas cerca de 1.5 vezes. Na segunda é encontrada uma equivalência entre a especificação com ligações intersectoriais e a valor acrescentado, realçando que não há uma maneira única de analisar o impacto das ligações intersectoriais e que a abordagem apropriada depende da questão particular.

Palavras-chave: Afetação de recursos, eficiência na afetação, recursos primários, setores, relações intersectoriais.

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Table of Contents

List of Figures

List of Tables

1. Introduction

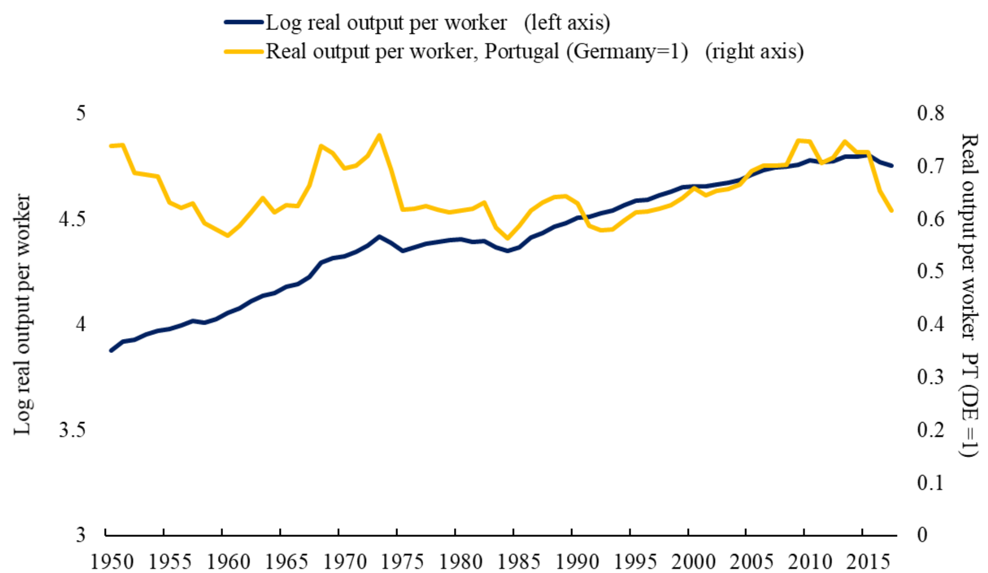
In every period, an economy has a certain amount of resources and multiple valuable uses for them. Therefore, there are various possible splits. To illustrate, imagine an economy where aggregate output comes in the form of ice-cream cones. Ice-cream production requires ice-cream scoops, produced by industry[1] A, and biscuit cones, produced by industry B. Using all available resources there are splits where to increase production in one industry production the production of the other industry must fall. These splits define the efficient frontier – the technological ability to transform ice-cream scoops into biscuit cones. But, overall the optimal ratio of ice-cream scoops to biscuit cones depends also on preferences, whether the best ice-cream is scoop intensive or biscuit intensive. All these elements define the optimal split of resources over the various uses. In reality, we may have too many scoops or too many biscuit cones but fewer than otherwise possible ice-creams. This because if industry A (B) is receiving too many resources and producing too much compared to the optimum, industry B (A) must be receiving too few resources and producing too little compared to the optimum. Compared to the optimum, the available resources are the same but production is smaller. With the available resources, it would have been possible to produce more ice-creams and yet they are not produced as a result of an inefficient resource allocation. The smaller amount of ice-creams compared to the potential amount reflects the misallocation loss.

Why is resource misallocation relevant? Why some countries are rich and others poor is one of the most important issues in economics. The misallocation literature offers a possible explanation. In two identical economies in terms of technology, available resources and preferences, the optimal (if unique) division of resources between various uses is the same. However, the observed allocations may differ and differ from the optimal. According to the misallocation hypothesis, additional distortions in lower-income economies bring them further away from the efficiency frontier thus lowering aggregate output. The aggregate, the number of ice-creams produced in the example above, is insufficient to get the full picture. Looking deeper, to the firm or sector level, it is possible to assess whether resource misallocation is a more severe issue in one country or another.

[1] In this dissertation industry and sector are used interchangeably.

In 1960, of the 15 countries that would become first Member States of the European Union (EU-15[2]) Portugal had the lowest *per capita* GDP (Banco de Portugal, 2019). Given the similarities with the European counterparts, the (absolute) real convergence hypothesis would predict faster growth in Portugal inducing a catch-up process. Portuguese real output per worker is larger today than it was 60 years ago. But the gap between real output per worker in Portugal and in Germany, measured in Purchasing Power Parities (PPPs), was about the same in 2017 as it was in 1986, when Portugal joined the EU (Figure 1). Differences may be the result of many factors, for example technological or in the quantity and quality of resources. But, how important was resource misallocation?

Figure 1: Real output per worker, Portugal (1950-2017)



Source: Penn World Tables (PWT) 9.1. (Feenstra, Inklaar and Timmer, 2015). Author's own computations.

Note: Real output per worker is the ratio between expenditure-side real GDP at current Purchasing Power Parities (PPPs) (in millions of 2011 US$) and the number of persons engaged (in millions).

The objective of this dissertation is to measure the extent of between-sector primary inputs misallocation in Portugal between 2000 and 2017, complementing the within-sector resource misallocation analysis of Dias, Robalo Marques and Richmond (2016a, 2016b) and Gopinath *et al.* (2017). This will be done using Germany as a benchmark economy. For this exercise, a static input-

[2] The EU-15 countries are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, and the United Kingdom.

output model in line with the proposal of Long and Plosser (1983) later augmented by Jones (2013) to include international trade is used. The model's static nature leads to an abstraction from any intertemporal distortions. Furthermore, the exercise also abstracts from distortions affecting the supply of labor, which is exogenously supplied. Instead, distortions shall be introduced as distortions to industries' choice of primary inputs, labor and capital. The chosen distortions are the measurable ones using sector-level data publicly available in EU-KLEMS and World Input-Output Database, thereby avoiding further identification issues and relying on previous estimates non-specific to the Portuguese or German economies. The proposed framework shall be calibrated to match the Portuguese and German data. This calibration allows for the characterization of the optimal split of available resources among producers. The optimal split depends on the economies' structure, and how far the economy is from this optimal split will crucially depend on heterogeneity of implicit factor prices. Allocative efficiency, defined as the ratio between GDP and undistorted GDP, will be the summary statistic capturing the distance to the frontier; the misallocation loss is the percentage increase in output if the economy were to move to the optimal resource split. Besides quantifying the misallocation loss, the analysis looks into the allocation of capital and labor and to the contributions of the various industries.

Misallocation is the outcome of distortions that drive the equilibrium resource split away from the optimal. There is an array of potential distortion sources: legal aspects[3], as well as discretionary provisions by governments or credit institutions, and market imperfections[4] are just a few examples (Restuccia and Rogerson, 2017). These distortions may affect allocations between the multiple producers of a sector (within-sector misallocation) or affect allocations between sectors (between-sector misallocation). In effect, both within- and between-sector distortions play a role on aggregate outcomes. Dias, Robalo Marques and Richmond (2016a, 2016b), and Gopinath *et al.* (2017) studied resource misallocation in Portugal focusing on within-sector misallocation, in line with the proposal of Hsieh and Klenow (2009). Dias, Robalo Marques and Richmond (2016b) report high and increasing misallocation losses over their sample period (1996-2011), estimating, in 2011, value-added gains of 79 percent were within-sector distortions eliminated. Instead, this dissertation looks at between-sector resource misallocation, in a new application, to the best of our knowledge,

[3] Dias, Robalo Marques and Richmond (2016b) list the size-dependent policies passed by the Portuguese government since the early 1990s.
[4] See, for example, Harberger (1954).

to the Portuguese economy. Jones (2013) highlights that in the presence of sector-specific distortions the allocation of resources between sectors may also be sub-optimal and that this less studied channel may provide relevant insights. The within-sector analysis begins with a certain amount of productive resources at the sector level and assesses whether these are properly allocated among the sector's producers. By doing so, it disregards the possibility of between-sector reallocation. The present between-sector analysis takes a step back, going to the other extreme, wondering whether those sector available resources were optimal, with the misallocation loss coming from between-sector resource misallocation.

Working with sector-level data allows for the examination of the role of input-output networks. The multiple producers and industries are connected through direct and indirect supplier-costumer relations (Carvalho and Tahbaz-Salehi, 2019). Acemoglu *et al.* (2012) highlight that these relations may be an important channel through which microeconomic shocks can spread to the macroeconomic level, such that the importance of a sector for the economy as a whole depends on its importance both as a final good producer and as an intermediate goods supplier (directly and indirectly) – its "centrality" (Bonacich, 1987). To illustrate why this may be an important channel, picture again the ice-cream economy but with an additional sector, agriculture. In this example, agriculture is a supplier both to the production of ice-cream scoops (industry A) and to the production of biscuit cones (industry B), but does not directly contribute to the production of ice-creams. If agricultural production is distorted, if for example it receives too few resources compared to the optimum, this will have consequences for both the production of ice-cream scoops and biscuit cones through direct relationships, but also for aggregate production, indirectly, through the impact on industries A and B. Input-output network data is not commonly available at the firm level and as a result it is often disregarded in the traditional (within-sector) misallocation analysis. Working with sector data allows for this rich information to be accounted for and, to the propose of this dissertation, account for its potential impact on misallocation. Bearing this in mind, the input-output network will be explicitly considered. In addition, this dissertation will investigate the importance of the input-output network for the found misallocation loss. First, the original calibration shall be mainted but the intermedite goods channel will be closed: the purpose is to know how would the misallocation loss change, all else constant, by closing the sector interconnections. Second, the economic framework will be simplified, modelling aggregate output as the sum of industries' value-added. Afterwards, the parameters are recalibrated accordingly and the misallocation loss computed. The purpose of this exercise is to assess how would the computed misallocation loss change with this simplification.

The dissertation presents three main results. First, on average over the sample period, the misallocation loss in Portugal was about 20 percent. Yet, through this period, allocative efficiency in Portugal improved such that, by 2017, the misallocation loss was around 15 percent, compared to 25 percent in 2000. In line with Reis (2013) and Dias, Robalo Marques and Richmond (2016b), capital distortions are found of primary importance for this outcome. The allocation of capital is further away from the optimal than the allocation of labor, but its improvements drove the overall improvement in allocative efficiency. Furthermore, there is a remarkable concentration of industries around the efficiency line suggesting that most industries receive just about the "right" among of resources. This concentration, however, comes partly from compensation of capital and labor distortions. There are some outliers to this concentration and they are about the same throughout. Compared to Germany, the misallocation loss is larger in Portugal. On average and compared to its undistorted output, the misallocation loss in Germany was 13 percent, a magnitude that remained stable over the period (2001-2017). This lack of improvement in Germany coupled with the reported improvement in Portugal led to the convergence of the allocative efficiency measure of the two countries. Second, when keeping the original calibration but closing the intermediate goods channel, in a *ceteris paribus* exercise, the network is found to have amplified the misallocation loss, on average in Portugal and Germany, about 1.5 times, albeit fluctuating through the period. Notwithstanding, the third result is the underscore, through the comparison of the *ceteris paribus* exercise with the value-added simplification, that there is not an unique way to assess the importance of the network and that the two approaches provide different answers as they resolve distinct questions.

The sections will proceed as follows: Section 2 summarizes the main results in the literature. Section 3 presents the data and Section 4 introduces the theoretical framework that will be used to calculate the proposed measure of allocative efficiency in both Portugal and Germany. The results will be interpreted and decomposed into factor and sector contributions in Section 5. Sections 6 and 7 explore the role of the input-output structure. In Section 6 a *ceteris paribus* exercise where the original calibration is maintained but the intermediate goods channel is closed is used.

Differently, in Section 7 the model is further simplified as a value-added model and the implications for the allocative efficiency measure assessed. Section 8 concludes.

2. Literature Review

Solow (1957) finds that Total Factor Productivity (TFP) contributed nearly 90 percent to the growth of output *per capita* in the U.S. between 1909 and 1949. Prescott (1998) underscores that cross-country income differences are not due to quantity and quality of resources or usable knowledge, but rather to differences in TFP. The misallocation hypothesis presents a possible explanation - income differences (partly) result from the effectiveness of resource allocation (Restuccia and Rogerson, 2017)[5]. This effectiveness reflects distortions that drive equilibrium allocations away from the optimal. Distortions, in turn, may come from several sources. Thus, a commonly followed approach is to introduce distortions as affecting the decisions of agents without referencing the underlying source of those distortions (Restuccia and Rogerson, 2013), an approach also followed in the present work.

In static frameworks where factor supply is exogenous, like the one used in this dissertation, to maximize GDP is to maximize utility of the representative household. Hence, allocative efficiency can be defined as the ratio between GDP and undistorted GDP. Baqaee and Farhi (2020) call this notion "allocative efficiency relative to the frontier" (Baqaee and Farhi, 2020, p. 143). Alternatively, they also characterize changes in allocative efficiency relative to the previous period's allocation and changes in allocative efficiency due to changes in wedges. In this analysis, the goal is to quantify how production would have risen if there were no distortions, such that the used definition is allocative efficiency relative to the frontier. Overall, the paper by Baqaee and Farhi (2020) takes a very general approach, despite relying mostly on first order approximations, by employing nonparametric formulas to aggregate microeconomic shocks in inefficient economies with input-output linkages.

[5] The Selection channel also highlighted by Restuccia and Rogerson (2017) captures which producers thrive. This channel can be understood as a special form of misallocation. Measuring its impact is, however, difficult as data is available solely for active firms (Restuccia and Rogerson, 2017). In Bartelsman, Haltiwanger and Scarpetta (2013), for example, distortions play a role in the allocation among operating firms but also in selection.

The misallocation hypothesis was put forward and applied to explain cross-country income differences in the seminal contributions of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Restuccia and Rogerson (2008) find that idiosyncratic distortions, particularly when correlated with establishment size, can lead to substantial losses of output and TFP ("(…) in the range of 30 to 50 percent" (Restuccia and Rogerson, 2008, p. 708)). In their application to the U.S., China and India, Hsieh and Klenow (2009) use firm-level data to find larger reallocation gains in China and India's manufacturing than in U.S. manufacturing, suggesting that misallocation may be significant to explain the income gap between these countries[6, 7]. Specifically, they find TFP gains of 30 to 50 percent in China and of 40 to 60 percent in India if the distortion level was the same as in the U.S.. For these analyses, productivity differences within sector's producers play the prominent role. To the extent that it studies the impact of resource misallocation, the present dissertation follows this broad literature. Crucially, however, the focus here is on between-sector resource misallocation.

Jones (2013) highlights that, in the presence of sector-specific distortions, the distribution of resources between sectors may also be sub-optimal, affecting aggregate production. Jones (2013) proposed a model, based on Long and Plosser (1983), with Cobb-Douglas production functions in an open input-output economy. Jones' goal was to have a tractable framework that could be linked to input-output data. This goal is also present in this dissertation and hence the application of Jones' framework. In fact, despite not quantifying the misallocation loss and introducing distortions as output (markup) distortions, Jones (2013) is the main reference for this work. By looking to the misallocation loss from distortions in line with Jones' proposal, the dissertation is close to Leal (2015), and Shao and Tang (2020a, 2020b). Leal (2015) uses the framework to study the Mexican economy. The author introduces both a markup wedge and a labor wedge affecting industry-level decisions[8]. Leal finds, for a case when rents from distortions are not given back to the household,

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[6] The framework of Hsieh and Klenow has been extensively applied. A few examples are: Ziebarth (2013) that also studies India and China but uses as a benchmark second half of the 19th century. Fujii and Nozawa (2013) analyzing the role of misallocation during Japan's two lost decades (1989-2009). De Vries (2014) focusing on the Brazilian retail sector and on the heterogeneity among Brazilian states (1996-2006). Finally, Chen and Irarrazabal (2015) relate the improvements in allocative efficiency in Chile between 1980 and 1996 to the banking reforms of the 1980s.

[7] As an alternative to Hsieh and Klenow's method, Bartelsman, Haltiwanger and Scarpetta (2013) focus on the covariance between size and productivity. By changing the distribution of distortions, the authors are able to match within-sector covariance between size and productivity in the chosen countries.

[8] This choice creates identification issues which are circumvented by assuming that the U.S. economy is undistorted and that, at the sector level, the labor share and that intermediate inputs share is the same in

that aggregate productivity would increase over 60 percent if all markup wedges were eliminated. Different from Leal (2015) and closer to this dissertation, as there is an abstraction from markup wedges, is the application to the U.S. and Canada of Shao and Tang (2020a, 2020b). Likewise, Shao and Tang also make use of KLEMS and World Input-Output Database data. Differently, their main focus was to use this framework to understand the widening productivity gap between Canada and the U.S.. They conclude that a substantial amount can be accounted for by lack of improvements in Canada's allocative efficiency. Osotimehin and Popov (2020), also based on the World Input-Output Database, use a framework similar to Jones' but allow for more flexible elasticities. In their analysis restricted to labor distortions, they find TFP gains of 0.5 percent for the median country in their benchmark calibration. Their analysis critically looks to the use of a Cobb-Douglas specification: TFP gains of 0.8 percent were found for the median country assuming unitary elasticity of substitution.

The between-sector misallocation analysis is closely linked to the literature on the macroeconomic impact of microeconomic distortions through input-output networks for two main reasons. First, resource misallocation can be thought of as a virtual shock: a sector-level deviation from the optimal production. Second, an advantage of working with sector level data is the possibility to explore the role of interconnections between the various sectors, as this information that is not available at the firm-level. Jones (2011) emphasizes that when intermediate goods are accounted for, a substantial multiplier arises. Acemoglu *et al.* (2012) reason how input-output links allow for the spread of microeconomic shocks to the economic aggregate by connecting various players of the economic system. The magnitude of the impact, nevertheless, depends on the input-output network's structure – on whether the various producers play asymmetric roles. Recent contributions can be found in Baqaee and Farhi (2019), Bigio and La'O (2020), and Osotimehin and Popov (2020). Baqaee and Farhi (2019) use second-order approximations to highlight that Hulten's theorem[9] (Hulten, 1978) is a first-order (or Cobb-Douglas) estimate of the impact of

Mexico as in the U.S., a common practice in this literature. Many authors use the U.S. economy as a benchmark when analyzing the extent of misallocation. Other examples of this practice are Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Bartelsman, Haltiwanger and Scarpetta (2013). This hypothesis is not uncontroversial. It can be argued that because countries are in different development stages their technology and factor shares are different (Leal, 2015). In this case, part of the estimated misallocation loss would be the due to poor parameter identification.

[9] The Hulten theorem says that "(…) for efficient economies and under minimal assumptions, the impact on aggregate TFP of a microeconomic TFP shock is equal to the shocked producer's sales as a share of GDP." (Baqaee and Farhi, 2019, p. 1155).

microeconomic shocks on the aggregate and that, to higher orders, the microeconomic structure, which includes the input-output network, is relevant for the macroeconomic effects of micro shocks. In line with Jones (2013), Bigio and La'O (2020) use a Cobb-Douglas specification, but allow for endogenous labor supply, such that distortions impact both TFP and labor supply. They apply such framework to study the network amplification effects of the 2008-09 financial crisis by keeping the original calibration but closing the intermediate goods channel. They find that the input-output network may have doubled the impact of financial distortions during the crisis. A similar analysis to assess the impact of the input-output network is carried in this dissertation. Lastly, Osotimehin and Popov (2020) show that the input-output network does not systematically amplify the effects of distortions. Importantly, departing from unitary elasticity, they find a counterintuitive result: the TFP loss from distortions is smaller with lower input substitutability. Here, despite the focus on primary input resource misallocation rather than on misallocation of intermediate goods, the sector interconnections coming from supplier-customer relationships are accounted for and their impact on misallocation assessed.

In summary, this dissertation applies the framework of Jones (2013), with Cobb-Douglas production functions in an open input-output economy, to study the extent of resource misallocation as a result of sector-specific distortions to primary inputs, as in Shao and Tang (2020a, 2020b). Furthermore, it assesses the importance of the input-output structure in line with both Bigio and La'O (2020), and Osotimehin and Popov (2020). But, here the focus is different, attempting to understand their results, looking both to the Portuguese case and to their theoretical foundations.

The dissertation is also related to the literature on the experience and performance of the Portuguese economy. In 1960, Portugal had the lowest *per capita* GDP of the countries that would become the EU-15 (Banco de Portugal, 2019) and the economy was expected to catch-up to the richer European counterparts. The Boom (Reis, 2013), 1995-1999, with falling unemployment and rising current account deficit, is associated with the prospect of Portugal joining the euro area (EA) (Blanchard, 2007). In the subsequent period, 2000-2007, despite substantial capital inflows from abroad and low long-term interest rates, Portugal's economic performance was disappointing, with rising unemployment (Blanchard, 2007). Reis (2013) argues "(…) Portugal's financial sector caused the capital inflows to be largely misallocated (…) and thus to a fall in measured productivity" (Reis, 2013, p. 146). The study of misallocation in Portugal, in line with the proposal of Hsieh and Klenow (2009), was carried by Gopinath *et al.* (2017) and Dias, Robalo Marques and Richmond (2016a, 2016b). Gopinath *et al.* (2017), focusing on the manufacturing sector, find a rise in allocative inefficiency in Spain, Italy and Portugal[10] but not in Germany, France or Norway. They associate their findings with financial frictions that are plausibly more severe in the Southern countries. Their findings support misallocation as an explanation for the productivity divergence between Northern and Southern European countries. Following that approach, this dissertation also juxtaposes Portugal's performance to Germany's for a better understanding of the phenomenon. Dias, Robalo Marques and Richmond (2016b) analysis covers all economic activity sectors. The authors find high levels of allocative inefficiency that increased substantially during their sample period: they estimate that "(…) output would have been 48% and 79% above actual GDP levels in 1996 and 2011, respectively" if distortions were eliminated (Dias, Robalo Marques and Richmond, 2016b, p. 48). Capital distortions, in line with Reis (2013), are found to be of primary importance for these results. Their results are not directly comparable to those presented in this dissertation: the former focused on within-sector distortions; the latter focuses on between-sector distortions. Such complementary may contribute to a better understanding of misallocation in the Portuguese economy.

3. The Data

Leal (2015), Shao and Tang (2020a, 2020b) and Osotimehin and Popov (2020) are a few examples of recent analyses that, like the present work, aim at cross-country comparisons. Following their lead, two main datasets were used in this work: World Input-Output Database (WIOD) (2016 Release) and EU-KLEMS (2019 Release). Both are publicly available for a large number of countries, following uniformed procedures thus enabling cross-country comparisons. They provide annual data at the industry level. The industry classification used is ISIC (United Nations International Standard Industrial Classification of All Economic Activities) Rev. 4. The list of the 28 industries considered for the estimation is shown in Table 1[11,12]. For Portugal, in addition to these datasets, Instituto Nacional de Estatística's (INE) per industry capital stock information was

[10] For Portugal, the analysis of Gopinath and co-authors covers the 2006 to 2012 period.
[11] For additional details on ISIC Rev. 4, see United Nations (2008).
[12] The list of industries is the result of data availability in EU-KLEMS, WIOD and INE.

also used. Importantly, these datasets allow for the calibration of the proposed model and for the computation of allocative efficiency and misallocation loss, in Portugal and Germany.

Table 1: Industry list

ISIC Rev. 4 industry code	Sector description
A	Agriculture, Forestry and Fishing
B	Mining and Quarrying
C10-12	Manufacture of food products, beverages and tobacco
C13-15	Manufacture of textiles, wearing apparel, leather and related products
C16-18	Manufacture of wood and paper products; printing and reproduction of recorded media
C19	Manufacture of coke and refined petroleum products
C20-21	Manufacture of chemicals and chemical products
C22-23	Manufacture of rubber and plastics products, and other non-metallic mineral products
C24-25	Manufacture of basic metals and fabricated metal products, except machinery and equipment
C26-27	Manufacture of electrical and optical equipment
C28	Manufacture of machinery and equipment not elsewhere classified (n.e.c.)
C29-30	Manufacture of transport equipment
C31-33	Other manufacturing; repair and installation of machinery and equipment
D-E	Electricity, Gas and Water Supply
F	Construction
G	Wholesale and Retail Trade; Repair of Motor Vehicles and Motorcycles
H	Transportation and Storage
I	Accommodation and Food Service activities
J58-60	Publishing, audiovisual and broadcasting activities
J61	Telecommunications
J62-63	IT and other information services
K	Financial and Insurance activities
L	Real Estate activities
M-N	Professional, Scientific, Technical, Administrative and Support Service activities
O	Public administration and defense; compulsory social security
P	Education
Q	Health and social work
R-S	Arts, entertainment, recreation; other services and service activities, etc.

Input-output matrices for Portugal and Germany were collected from WIOD. The national tables report on the consumption of intermediate inputs by industry, dividing consumption by source industry and origin (domestic, Dom, *versus* imported, Imp). Table 2 shows a simplified example of an input-output matrix. The columns indicate who is using the good and the rows who produced the good. The sources (rows) are also segmented according to origin – domestic versus imported goods. By dividing the elements of each column by the corresponding gross-output (GO), presented in the last row, one obtains the transposed technical coefficient matrix. Value-added at current basic prices is also reported in the input-output tables. These tables are available from 2000 to 2014, thus restricting the analysis of allocative efficiency, taking into account the input-output links, to that period. The exercises in Sections 5 and 6 only cover the 2000-2014 period.

Table 2: Simplified input-output matrix

Origin	Usages of production→ Producing sector ↓	1	2	Other	GO
Dom	1	$p_1 x_{11}$	$p_1 x_{21}$	Other uses of 1's output	$p_1 y_1$
Dom	2	$p_2 x_{12}$	$p_2 x_{22}$	Other uses of 2's output	$p_2 y_2$
Imp		$p_1^m m_1$	$p_2^m m_2$	Other imports	
	GO	$p_1 y_1$	$p_2 y_2$		

Note: Dom and Imp stand for domestic imports, respectively. GO stands for gross-output. In the matrices the "Other" column is divided in six columns: Final consumption expenditure by households, final consumption expenditure by non-profit organizations, final consumption by government, gross fixed capital formation and changes in inventories and valuables. In the input-output matrices the row imports is actually N rows (one for each industry). x_{ij} is the consumption of intermediate goods from domestic industry *j* by industry *i,* and m_i is the consumption of imported intermediate goods by industry *i*. $p_i y_i$ is nominal gross-output of industry *i.* All information is provided in expenditures.

EU-KLEMS (2019 Release)[13] contains information from 1995 to 2017[14]. From this dataset information was collected on value-added (VA), gross-output (GO), intermediate inputs (II), number of persons employed (EMP), and labor and capital compensation (LAB and CAP,

[13] For additional information on the 2019 EU-KLEMS release, please see Stehrer *et al.* (2019).
[14] At the time of the EU-KLEMS 2019 Release, some 2017 figures were still estimates.

respectively). For Germany, data was also collected on the capital stock[15]. This capital stock information is based on the European System of Accounts 2010 (ESA 2010) and includes machinery and equipment, cultivated assets, dwellings, other buildings and structures and intellectual property products (Stehrer *et al.*, 2019). INE provides capital stock information per industry (2000-2017), also in line with ESA 2010, for Portugal, both at current prices and at previous year's prices. This information was used to create a capital stock series. The choice of this wide ranging capital stock measure avoids missing potentially important capital forms other than fixed capital[16]. The exercise in Section 7 only relies on EU-KLEMS and INE's information and therefore covers the entire (2000-2017) period.

Table 3: Databases (EU-KLEMS and INE)

Variable	Description	Data Availability
EU-KLEMS (2019) Growth Accounts File (Portugal and Germany)		
VA	Gross value added at current basic prices[17] (in millions of euros)	1995-2017
GO	Gross output at current basic prices (in millions of euros)	
II	Intermediate inputs at current purchasers prices (in millions of euros)	
EMP	Number of persons employed (thousands) (quantity)	
EU-KLEMS (2019) National Accounts File (Portugal and Germany)		
LAB	Labor compensation (in millions of euros)	1995-2017
CAP	Capital compensation (in millions of euros)	
EU-KLEMS (2019) Capital File (Germany)		
Kq_GFCF	Real fixed capital stock (2010 prices) - All assets (quantity)	1995-2017
INE Capital Stock (Portugal)		
Capital stock by industry	Current prices; annual (in millions of euros)	2000-2017
Capital stock by industry	Previous year's prices; annual (in millions of euros)	

For primary inputs, information is available on both expenditures (CAP and LAB) and quantities (Kq_GFCF and EMP). For intermediate goods, unfortunately, WIOD only reports on expenditures. Information on quantities and expenditures, as available for primary inputs, enables the identification of variations of gross input prices faced by the various producers and misallocation,

[15] Per industry capital stock data was not available for Portugal, a situation circumvented using INE's per industry capital stock information.
[16] This type of capital stock data is also used in Shao and Tang (2020a, 2020b), and Osotimehin and Popov (2020).
[17] Please see Appendix D for the definition of basic prices.

as explained in detail in the next section. If for intermediate goods expenditure data was employed as quantity data it would not be possible to finding dispersion, and hence no misallocation would be found[18]. Therefore, the lack of data led to the subsequent abstraction of misallocation of intermediate inputs. Also, the capital compensation provided in KLEMS is constructed as the difference between industry value-added and industry labor compensation. Thus, there is no distinction between payments to capital and profits (Stehrer *et al.*, 2019). To be consistent with this information, in the theoretical framework there is an abstraction from markups, as in Shao and Tang (2020a, 2020b). Alternatively, Osotimehin and Popov (2020) disregard the capital compensation information, and estimate capital costs based on a 4 percent nominal interest rate, and the Bureau of Economic Analysis (BEA) sector level depreciation rates. This creates a gap between sales and costs that is used as the sector's markup. A possible extension of the present work, still using the presented datasets as the main source of information, would be to infer markups as Osotimehin and Popov. This, however, also implies stringent assumptions. [19]

4. Primary input resource misallocation in an input-output economy with distortions

The main objective of this section is to present the theoretical framework[20,21] supporting the results shown in the next section. It introduces the concept of allocative efficiency as well as its decompositions. It also includes an explanation on how the available statistical information, presented in Section 3, was used to calibrate the model and ultimately quantify the extent of the misallocation loss in Portugal and Germany.

The model economy shall be presented allowing the connection with the data described (Section 3) – to calibrate the economy that imposes the aggregate misallocation. Relying on the previous referred data, the elasticity of substitution in the model will be assumed one, so Cobb-Douglas technologies will be used[22]. The model will be further used to compute the counterfactual. The counterfactual is the economy's characterization if all distortions were eliminated – the first best.

[18] Please see Shao and Tang (2020a) for additional details.

[19] Additional information on the datasets employed can be found in Appendix D.

[20] The full details of the model are presented in Appendix A.

[21] Matrices are identified by their dimensions and in bold.

[22] This specification is employed for example by Jones (2013), Leal (2015, 2016), Bigio and La'O (2020), and Shao and Tang (2020a, 2020b).

The comparison between the first characterization and the first best equilibrium delivers the loss from resource misallocation.

4a. A distorted General Equilibrium

The model economy is defined as a sequence of static economies. In each period, total capital (K) is exogenous. A further simplifying assumption is that the labor supply in this economy (L) is also exogenous. Therefore, the total amounts of primary inputs, capital and labor are part of the economy's fundamentals that characterize its equilibrium, in every period. In addition to the supply of primary inputs, the fundamentals also include the utility function of the representative household, the available technologies that constrain the production of the final good producer and of the N industries. Lastly, the industry and sector-specific taxes imposed by the government (t_{ki} and $t_{li}, \forall_i$) and the prices of imported goods ($p_i^m, \forall_i$) are also fundamentals. Factors are perfectly mobile across sectors. All agents are price takers and markets clears in every period.

This is a small open economy. Hence, international import prices ($p_i^m, \forall_i$) are exogenous and so unaffected by the economy's equilibrium. As the goal is to apply the model to the Portuguese economy this is the best assumption – that sector level prices of imported inputs are exogenous. Furthermore, the trade balance is assumed equal to zero, i.e. exports (Exp) are equal to imports (Imp).

The problem of the representative household is to maximize his/her consumption (C). He/she receives factor payments as well as a lump-sum subsidy (tax) from the government. Therefore the budget constrain is: $C \leq wL + rK + T$. The solution is trivial: the representative household will spend all his/her available income on consumption.

The final good producer purchases inputs from the different industries (f_i) to produce the final good (F). The available technology for this producer is a constant returns to scale (CRS), Cobb-Douglas production function. The goal of the aggregate output producer is to maximize profits ($\Pi = F - (\sum_{i=1}^{N} p_i f_i\ )$) subject to the available technology and prices.

$$F = \left(\prod_{i=1}^{N} f_i^{\beta_i} \right) \tag{1}$$

CRS implies:

$$\sum_{i=1}^{N} \beta_i = 1$$

An industry *i* has CRS, Cobb-Douglas technology available. Production requires primary inputs labor (l_i) and capital (k_i), domestic intermediate goods, and imported intermediate goods. x_{ij} is the consumption of intermediate goods from domestic industry *j* by industry i, and m_i is the consumption of imported intermediate goods by industry *i*. Gross-output of industry *i* (y_i) is given by:

$$y_i = A_i[(k_i)^{\alpha_i}(l_i)^{1-\alpha_i}]^{1-\sigma_i-\gamma_i}\left(\prod_{j=1}^{N} x_{ij}^{\sigma_{ij}}\right)\left(m_i^{\gamma_i}\right) \qquad (2)$$

CRS implies:

$$\left(\sum_{j=1}^{N} \sigma_{ij}\right) = \sigma_i$$

The goal of a producer in industry *i* is to maximize profits subject to the available technology and prices. Absent taxes, industries would pay a wage (w) per unit of labor employed and a rental rate (r) per unit of capital employed. With taxes, the price faced by the producer of industry *i* for labor is $(1+t_{li})w = \tau_{li}w$, and for capital $(1+t_{ki})r = \tau_{ki}r$. Further, for intermediate goods, industry *i* pays p_i^m per unit of imported intermediate good and p_j per unit of intermediate good produced by the domestic industry *j*.

$$Max\ \pi_i = p_i y_i - \tau_{li} w l_i - \tau_{ki} r k_i - \left(\sum_{j=1}^{N} p_j x_{ij}\right) - p_i^m m_i$$

$$\text{subject to } y_i = A_i[(k_i)^{\alpha_i}(l_i)^{1-\alpha_i}]^{1-\sigma_i-\gamma_i}\left(\prod_{j=1}^{N} x_{ij}^{\sigma_{ij}}\right)\left(m_i^{\gamma_i}\right)$$

The government collects taxes (pays subsidies) on capital and labor from the industries and gives a lump-sum rebate (T) of the same amount to the representative household. As a result, the analysis of the effects of these fiscal instruments is limited to substitution effects, not income effect. Without the rebate the cost of this instruments to the aggregate economy would be much larger. This is the sole action of the government.

$$T = \sum_{i=1}^{N} (t_{li} w l_i + t_{ki} r k_i) \quad (3)$$

Definition: the competitive equilibrium is a set of allocations $\{C, F, Exp, Imp, y_i, f_i, l_i, k_i, m_i\}_{i=1}^{N}$ $\{\{x_{ij}\}_{i=1}^{N}\}_{j=1}^{N}$ and prices w, r and $\{p_i\}_{i=1}^{N}$ such that, given the exogenous variables $\{t_{li}, t_{ki}\}_{i=1}^{N}, \{p_i^m\}_{i=1}^{N}$, L and K the following are satisfied:

i. The representative household maximizes consumption (C) taking prices (w, and r), and lump-sum subsidy (T) as given.

ii. The producer of aggregate output maximizes profits (Π) by choosing input quantities $\{f_i\}_{i=1}^{N}$ taking prices $\{p_i\}_{i=1}^{N}$ as given, subject to (1).

iii. The producer of a generic industry i maximizes profits (π_i) by choosing inputs labor, capital, imported intermediate goods and domestic intermediate goods (l_i, k_i, m_i and $\{x_{ij}\}_{j=1}^{N}$) taking prices ($p_i, w, r, \{p_j\}_{j=1}^{N}$ and p_i^m) and taxes (t_{li} and t_{ki}) as given, subject to (2).

iv. The government satisfies (3).

v. The labor market, the capital market, the N industry goods' markets and the final good market clear.

In the description of this economy, prices are expressed in units of the final good (F).

The market clearing conditions are:

$$\left(\sum_{i=1}^{N} l_i\right) = L \quad (4)$$

$$\left(\sum_{i=1}^{N} k_i\right) = K \quad (5)$$

The production of industry i can be used by the final good producer as an input (f_i) or as an input to the remaining industries $(x_{ji}, \forall_j)$. As a result, the market clearing condition for the good produced by industry i is:

$$y_i = f_i + \left(\sum_{j=1}^{N} x_{ji}\right) \tag{6}$$

The final good produced by the aggregate producer can be used for private consumption (C) and exports (Exp).The value of this final good is the economy's value-added (GDP) plus the value of imports (Imp), in units of the domestic final good. As a result, the market clearing condition in the final good market is:

$$F = C + Exp \leftrightarrow GDP + Imp = C + Exp \tag{7}$$

With trade balance equal to zero, $Exp = Imp$ and thus $GDP = C$.

Solving the general equilibrium:

Condition (6) coupled with the industries' first order conditions (FOC) with respect to domestic intermediate goods can be written as:

$$y_i = f_i + \left(\sum_{j=1}^{N} \frac{\sigma_{ji} p_j y_j}{p_i}\right) \tag{8}$$

Define v_i as: $v_i \equiv \frac{\beta_i y_i}{f_i}$. From the FOC of the aggregate producer, in equilibrium, v_i is also the share of gross-output of industry i on aggregate output: $v_i = \frac{p_i y_i}{F}$. With this, condition (8) can be written as:

$$v_i = \beta_i + \left(\sum_{j=1}^{N} \sigma_{ji} v_j\right) \tag{9}$$

v_i is element i in matrix $\boldsymbol{v}_{\mathbf{N\,x\,1}}$.

Where,

$$\boldsymbol{v}_{\mathrm{N\,x\,1}} = \left[\mathbf{I}_{\mathrm{N\,x\,N}}{}^{23} - \boldsymbol{\sigma}'_{\mathrm{N\,x\,N}}{}^{24}\right]^{-\mathbf{1}\,25} \boldsymbol{\beta}_{\mathbf{N\,x\,1}}{}^{26}$$

v_i can be computed separately from both allocations and other equilibrium prices that it captures. From (9) it can be seen that v_i summarizes the importance of industry *i* for aggregate production, directly and indirectly. Directly, it is the immediate contribution for final output as captured by β_i. Indirectly, it reflects how important *i*'s production is as an input for other producers (σ_{ji}) weighted by how important those other producers are for aggregate production (v_j). Noticeably, v_i will always be at least as high as the direct contribution of the sector. In summary, v_i resumes information on the importance of the sector for the economy as a whole, the so-called sector's "centrality" (Bonacich, 1987), and it is only a function of economy's fundamentals, β and the input-output network.

From the industries' FOC with respect to capital and the market clearing condition in the capital market:

$$\frac{k_i}{K} = \frac{\frac{\alpha_i(1-\sigma_i-\gamma_i)p_i y_i}{r\tau_{ki}}}{\sum_{j=1}^{N}\frac{\alpha_j(1-\sigma_j-\gamma_j)p_j y_j}{r\tau_{kj}}} \tag{10}$$

Using the definition of v_i and its equilibrium value, (10) can be written as:

$$\frac{k_i}{K} = \frac{\frac{\alpha_i(1-\sigma_i-\gamma_i)v_i}{\tau_{ki}}}{\sum_{j=1}^{N}\frac{\alpha_j(1-\sigma_j-\gamma_j)v_j}{\tau_{kj}}} = \theta_{ki} \tag{11}$$

θ_{ki} only depends on fundamentals, $\alpha_j(1-\sigma_j-\gamma_j)$, v_j and τ_{kj}, $\forall_j$. Intuitively, the equilibrium share of total capital allocated to industry *i* depends on how important industry *i* is for aggregate output, as captured by v_i and on how much industry *i* relies on capital for its production, as captured by the Cobb-Douglas parameter $\alpha_i(1-\sigma_i-\gamma_i)$. These elements contribute positively to the equilibrium amount of capital received by the industry. On the contrary, the higher the capital

[23] $\mathbf{I}_{\mathbf{N\,x\,N}}$ is the identity matrix.
[24] $\boldsymbol{\sigma}_{\mathbf{N\,x\,N}}$ is the N by N matrix whose element (i, j) is σ_{ij}.
[25] Matrix $[\mathbf{I}_{\mathrm{N\,x\,N}} - \boldsymbol{\sigma}'_{\mathrm{N\,x\,N}}]^{-\mathbf{1}}$ is the Leontief inverse matrix.
[26] $\boldsymbol{\beta}_{\mathbf{N\,x\,1}}$ is a column vector with element *i* equal to β_i.

wedge (τ_{ki}) the lower the equilibrium capital share of industry i, all else constant. All these elements appear in a comparative fashion, i.e. how much the industry relies on capital, how important the industry is and the wedge it faces matter when compared to the remaining sectors. Interestingly, the equilibrium share does not depend on how productive the industry is, i.e. it does not depend on A_i. This feature appears due to the Cobb-Douglas specification employed.

The equilibrium share of labor for industry *i* can be written as:

$$\frac{l_i}{L} = \frac{\frac{(1-\alpha_i)(1-\sigma_i-\gamma_i)v_i}{\tau_{li}}}{\sum_{j=1}^{N}\frac{(1-\alpha_j)(1-\sigma_j-\gamma_j)v_j}{\tau_{lj}}} = \theta_{li} \tag{12}$$

Using (11) and (12) as well as the optimal choices of intermediate inputs ($x_{ij} = \sigma_{ij}\frac{v_i}{v_j}y_j$ and $m_i = \frac{\gamma_i v_i F}{p_i^m}$)[27], equilibrium log gross-output of industry *i* can be written as:

$$\mathbf{log}\,\boldsymbol{y}_{\mathrm{N\,x\,1}} = \left[\boldsymbol{I}_{\mathrm{N\,x\,N}} - \boldsymbol{\gamma}_{\mathrm{N\,x\,1}}{}^{28}\boldsymbol{\beta}'_{\mathrm{1\,x\,N}} - \boldsymbol{\sigma}_{\mathrm{N\,x\,N}}\right]^{-1}[\mathbf{log}\,\boldsymbol{A}_{\mathrm{N\,x\,1}} + \mathbf{log}\,\boldsymbol{VA}_{\mathrm{N\,x\,1}} + \boldsymbol{\zeta}_{\mathrm{N\,x\,1}}] \tag{13}$$

Where element *i* of the matrix $\mathbf{log}\,\boldsymbol{VA}_{\mathrm{N\,x\,1}}$ is:

$$\log VA_i = (1-\sigma_i-\gamma_i)[\alpha_i \log(k_i) + (1-\alpha_i)\log(l_i)] \tag{14}$$

From the final good producer FOC and the definition of v_i, the equilibrium log of aggregate production can be written as:

$$\mathbf{log}(\boldsymbol{F})_{\mathbf{1\,x\,1}} = \boldsymbol{\beta}'_{\mathrm{1\,x\,N}}\left[\mathbf{log}\left(\frac{\boldsymbol{\beta}}{\boldsymbol{v}}\right)_{\mathrm{N\,x\,1}} + \mathbf{log}\,\boldsymbol{y}_{\mathrm{N\,x\,1}}\right] \tag{15}$$

Where $\mathbf{log}\,\boldsymbol{y}_{\mathrm{N\,x\,1}}$ is given by (14).

Lastly, from the market clearing condition in the final goods market and the trade balance condition:

$$C = GDP = F - Imp \tag{16}$$

[27] See Appendix A1 for the details.

[28] $\boldsymbol{\gamma}_{\mathrm{N\,x\,1}}$ is a column vector where element *i* is γ_i.

4b. Allocative efficiency

The optimal allocations can be obtained by eliminating the distortions. In the model's notation, undistorted aggregate consumption is (achievable) aggregate consumption if $\tau_{li} = \tau_{ki} = 1, \forall_i$, as all agents behave competitively and there are no externalities. This solution is nested in the previously presented equilibrium conditions[29].

If, in (15), $\tau_{li} = \tau_{ki} = 1, \forall_i$ is imposed, $\log(F)$, now $\log(F^*)$, is:

$$\mathbf{log}(\boldsymbol{F}^*)_{1\,\mathrm{x}\,1} = \boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}}\left[\mathbf{log}\left(\frac{\boldsymbol{\beta}}{\boldsymbol{v}}\right)_{\mathrm{N}\,\mathrm{x}\,1} + \mathbf{log}\,\boldsymbol{y}^*{}_{\mathrm{N}\,\mathrm{x}\,1}\right] \tag{17}$$

Where:

$$\mathbf{log}\,\boldsymbol{y}^*{}_{\mathrm{N}\,\mathrm{x}\,1} = [\boldsymbol{I}_{\mathrm{N}\,\mathrm{x}\,1} - \boldsymbol{\gamma}_{\mathrm{N}\,\mathrm{x}\,1}\boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}} - \boldsymbol{\sigma}_{\mathrm{N}\,\mathrm{x}\,\mathrm{N}}]^{-1}[\mathbf{log}\,\boldsymbol{A}_{\mathrm{N}\,\mathrm{x}\,1} + \mathbf{log}\,\boldsymbol{VA}^*_{\mathrm{N}\,\mathrm{x}\,1} + \boldsymbol{\zeta}_{\mathrm{N}\,\mathrm{x}\,1}] \tag{18}$$

With element *i* in matrix $\mathbf{log}\,\boldsymbol{VA}^*_{\mathrm{N}\,\mathrm{x}\,1}$ being:

$$\log VA_i^* = (1 - \sigma_i - \gamma_i)[\alpha_i \log(k_i^*) + (1 - \alpha_i)\log(l_i^*)] \tag{19}$$

Where

$$\frac{k_i^*}{K} = \frac{\alpha_i(1 - \sigma_i - \gamma_i)v_i}{\sum_{j=1}^{N} \alpha_j(1 - \sigma_j - \gamma_j)v_j} = \theta_{ki}^* \tag{20}$$

And

$$\frac{l_i^*}{L} = \frac{(1 - \alpha_i)(1 - \sigma_i - \gamma_i)v_i}{\sum_{j=1}^{N}(1 - \alpha_j)(1 - \sigma_j - \gamma_j)v_j} = \theta_{li}^* \tag{21}$$

Comparing (20) and (21) with (11) and (12), respectively, shows that differences to the distribution of primary inputs are solely the result of dispersion of τ_ks and τ_ls, since the total stock of capital and of labor is exogenous. In fact, if τ_{ki} is equal to the average τ_k ($\bar{\tau}_k$) for all industries condition (11) can be written as:

[29] Ahead, it will be shown that this is not the only way to recover the undistorted solution.

$$\frac{k_i}{K} = \frac{\frac{\alpha_i(1-\sigma_i-\gamma_i)v_i}{\bar{\tau}_k}}{\sum_{j=1}^{N}\frac{\alpha_j(1-\sigma_j-\gamma_j)v_j}{\bar{\tau}_k}} = \frac{\alpha_i(1-\sigma_i-\gamma_i)v_i}{\sum_{j=1}^{N}\alpha_j(1-\sigma_j-\gamma_j)v_j}$$

Which is exactly condition (20). Imposing $\tau_{ki} = 1, \forall_i$ is a special case of this more general way to recover the undistorted resource distribution. The same holds for labor. Even if there are taxes (subsidies), provided that all industries face the same taxes (subsidies), the distribution of resources will be the same in the two solutions. On the other hand, if industries face different gross primary input prices, the equilibrium distributions will differ.

As mentioned, in the present (static) model, to maximize GDP is to maximize aggregate consumption, and to maximize the utility of the representative household. In this context, allocative efficiency (E) is defined as the ratio between consumption and undistorted consumption. In this model, allocative efficiency coincides with productive efficiency namely due to the inelastic supply of labor. In the model, $\log(E)$ is given by:

$$\log(E) = \log\left(\frac{C}{C^*}\right) =^{30} \log\left(\frac{F}{F^*}\right) = \boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}}[\boldsymbol{I}_{\mathrm{N}\,\mathrm{x}\,\mathrm{N}} - \boldsymbol{\gamma}_{\mathrm{N}\,\mathrm{x}\,1}\boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}} - \boldsymbol{\sigma}_{\mathrm{N}\,\mathrm{x}\,\mathrm{N}}]^{-1}\,\mathbf{log}\left[\frac{\boldsymbol{VA}}{\boldsymbol{VA}^*}\right]_{\mathrm{N}\,\mathrm{x}\,1} \quad (22)$$

Where element *i* in matrix $\mathbf{log}\left[\frac{\boldsymbol{VA}}{\boldsymbol{VA}^*}\right]_{\mathrm{N}\,\mathrm{x}\,1}$ is:

$$\log\left(\frac{VA_i}{VA_i^*}\right) = (1-\sigma_i-\gamma_i)[\alpha_i(\log(k_i)-\log(k_i^*)) + (1-\alpha_i)(\log(l_i)-\log(l_i^*)] =$$

$$= [1-\sigma_i-\gamma_i][\alpha_i(\log(\theta_{ki})-\log(\theta_{ki}^*)) + (1-\alpha_i)(\log(\theta_{li})-\log(\theta_{li}^*)] \quad (23)$$

With (22) and (23), allocative efficiency is the outcome of industry level gaps between the actual share of total resources employed and the optimal share of total resources to be allocated to that industry. These gaps are then weighted by the factor's importance for industry production. Lastly, for the economy as a whole, these industry level gaps are weighted by the industry's importance as captured by φ_i, where φ_i is element *i* in $[\boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}}[\boldsymbol{I}_{\mathrm{N}\,\mathrm{x}\,\mathrm{N}} - \boldsymbol{\gamma}_{\mathrm{N}\,\mathrm{x}\,1}\boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}} - \boldsymbol{\sigma}_{\mathrm{N}\,\mathrm{x}\,\mathrm{N}}]^{\mathbf{-1}}]'$. These various

[30] The proof that $\log\left(\frac{C}{C^*}\right)$ is equal to $\log\left(\frac{F}{F^*}\right)$ is shown in Appendix A4.

elements are separable, suggesting that allocative efficiency can be decomposed in the cross-sector misallocation of labor and capital and in the contributions of each industry[31].

In Table 4, allocative efficiency is decomposed in capital's allocative efficiency, E_k, and labor's allocative efficiency, E_l. The decomposition underscores the separability in that how capital (labor) is allocated does not directly impact the allocation of labor (capital) across industries. Nonetheless, for economy's allocative efficiency they augment each other: provided that either E_k or E_l is less to one, E will be lower than both.

Table 4: Allocative efficiency decompositions – capital and labor decomposition

$$E = \left[\prod_{i=1}^{N}\left(\frac{\theta_{ki}}{\theta_{ki}^{*}}\right)^{\varphi_i\alpha_i(1-\sigma_i-\gamma_i)}\right]\left[\prod_{i=1}^{N}\left(\frac{\theta_{li}}{\theta_{li}^{*}}\right)^{\varphi_i(1-\alpha_i)(1-\sigma_i-\gamma_i)}\right] =$$

$$= E_k \times E_l$$

Where

$$E_k = \prod_{i=1}^{N}\left(\frac{\theta_{ki}}{\theta_{ki}^{*}}\right)^{\varphi_i\alpha_i(1-\sigma_i-\gamma_i)}$$

And

$$E_l = \prod_{i=1}^{N}\left(\frac{\theta_{li}}{\theta_{li}^{*}}\right)^{\varphi_i(1-\alpha_i)(1-\sigma_i-\gamma_i)}$$

Table 5: Allocative efficiency decompositions – industry decomposition

$$E = \prod_{i=1}^{N}\left[\left(\frac{\theta_{ki}}{\theta_{ki}^{*}}\right)^{\alpha_i}\left(\frac{\theta_{li}}{\theta_{li}^{*}}\right)^{1-\alpha_i}\right]^{\varphi_i(1-\sigma_i-\gamma_i)} = \prod_{i=1}^{N} E_i$$

Where

$$E_i = \left[\left(\frac{\theta_{ki}}{\theta_{ki}^{*}}\right)^{\alpha_i}\left(\frac{\theta_{li}}{\theta_{li}^{*}}\right)^{1-\alpha_i}\right]^{\varphi_i(1-\sigma_i-\gamma_i)}$$

As for the contribution of the various industries (Table 5), unlike E_k and E_l, E_i may be higher or lower than one. E_i greater (smaller) than one means that, compared to the optimum, the industry is receiving at least more (less) than the optimal amount of one of the resources. Knowing that E_i is greater than one, for example, is, however, insufficient to know whether the industry is receiving too much of both inputs or just of one of them. Importantly, even if some E_i are greater than one,

[31] The decompositions in Table 4 and Table 5 were proposed by Shao and Tang (2020a, 2020b).

at the aggregate, the larger production in these sectors will always be outweighed by the loss in other sectors.

4c. Bringing the model to the Portuguese and German data

With the series for employment and capital stock, the aggregate stock of these inputs is the sum of the industry level quantities, such that:

$$\left(\sum_{i=1}^{N} l_{i,t}\right) = L_t, \forall_t \tag{24}$$

And,

$$\left(\sum_{i=1}^{N} k_{i,t}\right) = K_t, \forall_t \tag{25}$$

The value of gross-output of industry *i* in year *t* ($p_{i,t}y_{i,t}$) is the sum of its value-added and expenditures on intermediate goods (domestic and imported)[32]. Value-added, in turn, is the sum of labor and capital compensation[33].

$$p_{i,t}y_{i,t} = CAP_{i,t} + LAB_{i,t} + \left(\sum_{j=1}^{N} p_{j,t}x_{ij,t}\right) + p_{i,t}^{m}m_{i,t}, \forall_{i,t} \tag{26}$$

Using the FOC from an industry i, the Cobb-Douglas parameters can be recovered as[34]:

$$\alpha_{i,t}(1-\sigma_{i,t}-\gamma_{i,t}) = \frac{CAP_{i,t}}{p_{i,t}y_{i,t}}, \forall_{i,t} \tag{27}$$

$$(1-\alpha_{i,t})(1-\sigma_{i,t}-\gamma_{i,t}) = \frac{LAB_{i,t}}{p_{i,t}y_{i,t}}, \forall_{i,t} \tag{28}$$

[32] NIOT report on industry gross-output. This value differs from the sum of value-added and expenditures on intermediate inputs. The difference is due to taxes less subsidies on products and international transport margins. The difference is, on average in 2014, less than 4 percent of industry's reported gross-output.
[33] For Germany, in 2000, Agriculture, Forestry and Fishing (industry A) labor compensation exceeded value added and, consequently, capital compensation was negative. For consistency, this year was excluded for Germany's analysis. The exercise covers the 2001-2014 period for Germany and the 2000-2014 period for Portugal.
[34] See Appendix A1.

$$\sigma_{ij,t} = \frac{p_{j,t} x_{ij,t}}{p_{i,t} y_{i,t}}, \forall_{i,j,t} \tag{29}$$

With $\sigma_{i,t} = \sum_{j=1}^{N} \sigma_{ij,t}$.

And

$$\gamma_{i,t} = \frac{p_{i,t}^{m} m_{i,t}}{p_{i,t} y_{i,t}}, \forall_{i,t} \tag{30}$$

Where $p_{i,t}^{m} m_{i,t}$ is the sum of all industry *i*'s expenditures on imported intermediate inputs, in year t.

The information on σ_{ij} for all i and j, in a given year, can be summarized in a matrix ($\boldsymbol{\sigma}_{t_{\mathrm{N\,x\,N}}}$) – the technical coefficient matrix.

$$\boldsymbol{\sigma}_{t_{\mathrm{N\,x\,N}}} = \begin{bmatrix} \sigma_{11,t} & \cdots & \sigma_{1\mathrm{N},t} \\ \vdots & \ddots & \vdots \\ \sigma_{\mathrm{N}1,t} & \cdots & \sigma_{\mathrm{NN},t} \end{bmatrix}, \forall_t$$

In a similar fashion, γ_i for all *i*, in a given year, can be organized in a column vector ($\boldsymbol{\gamma}_{t_{\mathrm{N\,x\,1}}}$).

$$\boldsymbol{\gamma}_{t_{\mathrm{N\,x\,1}}} = \begin{bmatrix} \gamma_{1,t} \\ \vdots \\ \gamma_{\mathrm{N},t} \end{bmatrix}, \forall_t$$

Using the FOC of the final good producer:

$$\beta_{i,t} = \frac{p_{i,t} y_{i,t} - \left(\sum_{j=1}^{N} p_{i,t} x_{ji,t}\right)}{\sum_{j=1}^{N} \left(p_{j,t} y_{j,t} - \left(\sum_{n=1}^{N} p_{j,t} x_{nj,t}\right)\right)} = \frac{p_{i,t} f_{i,t}}{\sum_{j=1}^{N} \left(p_{j,t} f_{j,t}\right)}, \forall_{i,t} \tag{31}$$

Which can also be organized, for any given year, in a column vector ($\boldsymbol{\beta}_{t_{\mathrm{N\,x\,1}}}$):

$$\boldsymbol{\beta}_{t_{\mathrm{N\,x\,1}}} = \begin{bmatrix} \beta_{1,t} \\ \vdots \\ \beta_{\mathrm{N},t} \end{bmatrix}, \forall_t$$

This procedure was applied to every year in the Portuguese (2000-2014) and German (2001-2014) samples. The imposed Cobb-Douglas production functions result in parameter fluctuations even in this small period. Table 6 presents for both Portugal and Germany, the (simple) average industry level Cobb-Douglas parameters, in 2014. Despite some fluctuations, the parameter values are relatively stable through the sample. Further, the two countries are quite similar in terms of average parameter values.

Table 6: (Simple) average parameter values, Portugal (PT) and Germany (DE) (2014)

	$\alpha_i(1-\sigma_i-\gamma_i)$	$(1-\alpha_i)(1-\sigma_i-\gamma_i)$	$1-\sigma_i-\gamma_i$	$\sigma_i+\gamma_i$	σ_i	γ_i
PT	0.18	0.27	0.45	0.55	0.38	0.17
DE	0.15	0.31	0.46	0.54	0.39	0.15

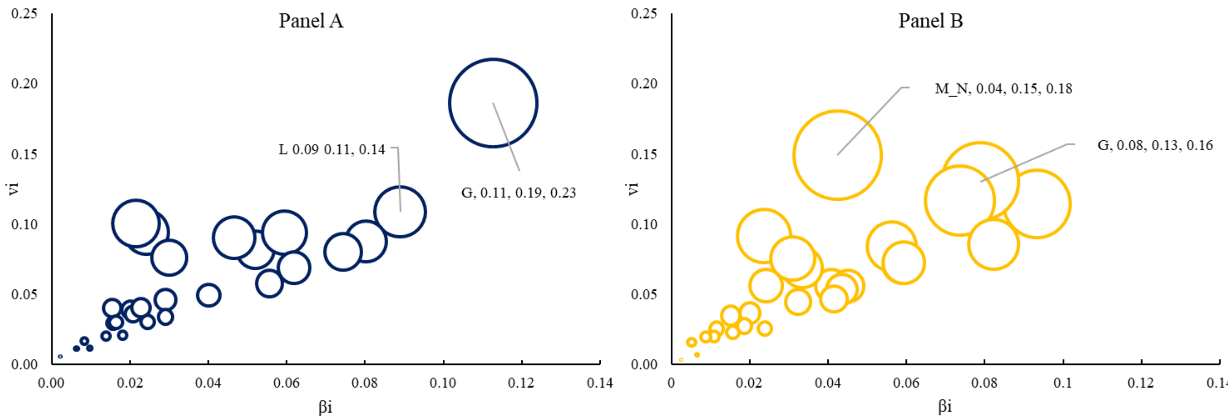
Source: EU-KLEMS, WIOD. Author's own computations.

If the decisions are made as predicted by the model, the calibration and sector employed inputs can be used to derive the implicit prices faced by the sectors. For the eventual price heterogeneity to be understood as misallocation there cannot be adjustment costs. With dynamic decision making, which we abstract from, this could be relevant. Hall (2004), however, finds small adjustment costs on both capital and labor even after considering potential biases coming from neglect of discrete adjustment costs, aggregation over time and aggregation across heterogeneous firms. Hall's analysis is based on yearly two-digit industries. David and Venkateswaran (2017), assuming that capital adjustment costs are convex, also find that although adjustment costs could be important, they play a small role for the observed productivity dispersion. Their analysis was carried using firm-level data for the Chinese manufacturing sector (1998-2009).

As suggested in 4a and 4b, the input-output network appears to be a key element for the analysis of misallocation. Reiterating, v_i captures importance of a given industry for aggregate production, it is a summary of the sectors' network "centrality", and it is this centrality, not just the direct contribution for final production (β_i), that determines the weight of the sector for the economy and is a crucial element in the optimal distribution of resources. Furthermore, in a closed economy, for simplicity, the aggregating weights (φ_i), that amplify the sector level gaps to the aggregate, are also equal to v_i. Given this importance, it is worth to present a few remarks on the Portuguese and German input-output networks. As a whole, intermediate goods account, on average, for over 50 percent of total input costs. This result is not specific to the Portuguese and German economies - Jones (2013) finds an intermediate goods share of gross output of around 50 percent across a large number of countries. Yet, industries differ significantly on how much they rely on intermediate inputs. Manufacturing of coke and refined petroleum products (industry C19) relies the most on intermediate inputs (98 and 93 percent of total input costs in 2014 for Portugal and Germany, respectively). For Portugal, real estate activities (industry L) relied the least on intermediate goods (only 6 percent of total input costs in 2014), whereas for Germany it was Education (industry P) that relied the least on intermediate goods (21 percent of total input costs in 2014). Furthermore, the diagonal of the input-output matrix is quite strong, suggesting that, for most industries, a prominent input is produced by the industry itself – own industry inputs, either domestically produced or imported, represent, on average, around 15 percent of industry's input costs. There is, nevertheless, substantial dispersion of this own-industry reliance within each country and between Portugal and Germany. Figure 2 shows v_i against β_i, with the bubble size proportional to φ_i, in 2014. This figure emphasizes several important elements. First, the two economies have different structures, i.e. the distribution of φ_i, v_i and β_i is not the same in Portugal and in Germany. Second, despite the relation between the parameters, they are noticeably different and their distinction is relevant. Third, v_i and φ_i are always at least as large as β_i, such that their sum will always be larger than one. For Portugal and Germany, the sum of v_i was always above 1.5 over the sample period. Jones (2013) calls this sum the *domestic intermediate goods multiplier* as it measures, ignoring international trade, the impact of a 1 percent increase in productivity in all sectors on final production (Hulten, 1978). Fourth, in both countries industry G (Wholesale and retail trade) stands out as a key sector. In Portugal, in 2014, it was the largest sector in the three dimensions considered. In Germany, it was industry M-N (Professional, Scientific, Technical, Administrative and Support Service activities) that had the highest φ_i and v_i. In Portugal, after G, real estate activities (industry L) are the second most prominent sector[35].

[35] Additional figures and tables on the input-output matrices are provided in Appendix E.

Figure 2: βi, vi, and φi, Portugal (Panel A) and Germany (Panel B) (2014)



Source: EU-KLEMS, WIOD and INE. Author's own computations.

Note: The bubble width is proportional to the industry φi. The reported values are βi, vi and φi, in this order.

5. How did the Portuguese misallocation losses evolve between 2000 and 2014?

The misallocation loss measures by how much output could have increased if all distortions were eliminated. In the present exercise, the misallocation loss is just the result of sector level distortions to the choice of primary inputs, labor and capital.

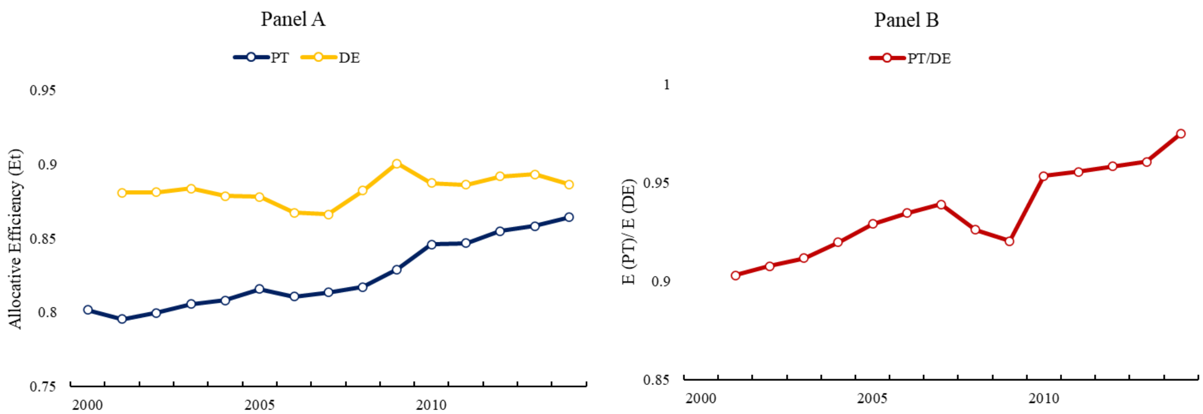
Over the sample period (2000-2014) in Portugal, allocative efficiency ranged between 0.80 (in 2001) and 0.86 (in 2014), for an average allocative efficiency of 0.82 (Figure 3, Panel A). This means a yearly average misallocation loss of 21 percent. In other words, on average, eliminating all distortions would have boosted output by 21 percent (Figure 3, Panel B). Still, allocative efficiency, in Portugal, improved during the period, growing at an annual average of 0.5 percent. The largest annual improvement occurred in 2010 (2 percent), following a period of initial deterioration (2006) and small subsequent improvements (2007-2008).

Figure 3: Allocative efficiency and misallocation loss, Portugal (2000-2014)

Source: EU-KLEMS, WIOD and INE. Author's own computations.

Compared to Germany, the loss in Portugal is substantially bigger. Over the sample period (2001-2014), allocative efficiency in Germany was, on average, 0.88 and the average misallocation loss was 13 percent (21 percent in Portugal). The average annual growth rate of Germany's allocative efficiency was only 0.1 percent: the faster improvement in Portugal contributed to the convergence of the allocative efficiency measure for the two countries. From 2001 to 2005, a combination of improvements in Portugal's allocative efficiency and a deterioration in Germany's was observed. In 2006, the misallocation loss increased in both countries, still, the deterioration was more severe in Germany and so Portugal continued to converge. The initial recovery (2008-2009) was faster in Germany, the only sub-period in the sample when Portugal's allocative efficiency deviated from Germany's. The German recovery was shorter-lived and in 2010, Portugal returned to a convergence trajectory (Figure 4, Panel B). From 2010 onwards, allocative efficiency stayed fairly constant in Germany, while improving in Portugal. By 2014, the gap was substantially smaller than in 2001. In that year, Germany's allocative efficiency was 0.88 (about 15 percent misallocation loss) while Portugal's was 0.80 (25 percent misallocation loss), compared to the respective undistorted outputs. By 2014, allocative efficiency was still 0.89 in Germany and 0.86 in Portugal (Figure 4). Despite the improvements in Portugal, the misallocation loss by 2014 was still greater than Germany's misallocation loss at the beginning of the period (Figure 4, Panel A). It is important to emphasize that this measure looks at allocative efficiency in a static manner, i.e. the allocative efficiency measures, in every period, how far each economy was from the respective undistorted outputs. The analysis does not detail the evolution of the frontier over time, that is, it does not detail the evolution of undistorted output.

Figure 4: Allocative efficiency Portugal (PT) (2000-2014) versus Germany (DE) (2001-2014)

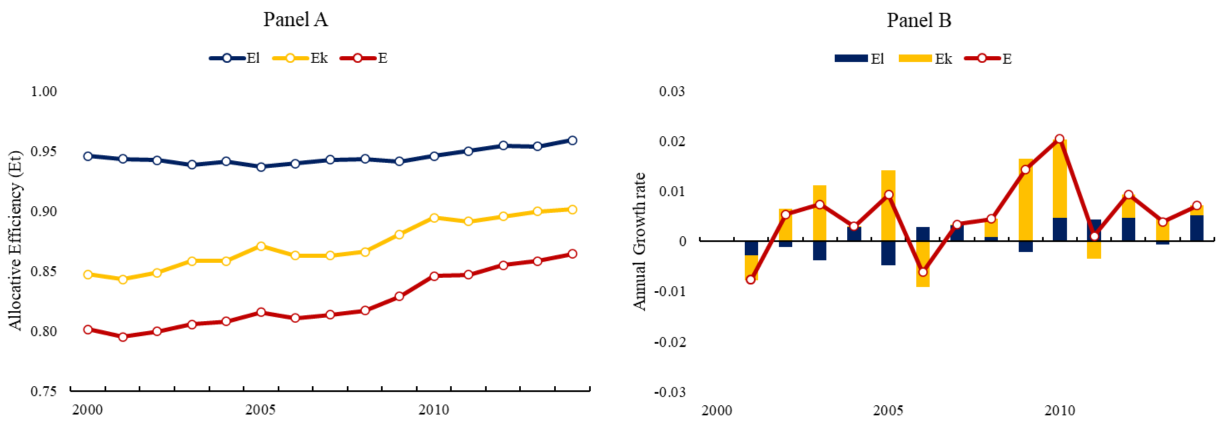


Source: EU-KLEMS, WIOD and INE. Author's own computations.

But how was this result achieved? As previously highlighted, allocative efficiency, in the present context, is the outcome of capital and labor allocation between the different industries. To understand Portugal's performance, then, a key analysis is the measure and evolution of labor and capital's allocative efficiency, E_l and E_k, respectively. This is shown in Figure 5 Panel A: the allocation of labor was much closer to the optimal than the allocation of capital, with E_l averaging at 0.95 and E_k at 0.87. Still, the allocation of capital improved more during the sample period than labor's, with annual growth rates of 0.4 percent and 0.1 percent, respectively. Figure 5 Panel B shows the yearly contributions of growth in labor and capital's allocative efficiency to the overall growth of allocative efficiency. For all years but 2011, capital's allocative efficiency growth and overall allocative efficiency moved in tandem, with capital's allocative efficiency driving most of the evolution of E. The correlation between the annual growth of E_k and E was 0.90. In Germany, both the distribution of capital and labor were closer to the optimal than in Portugal, with E_l averaging at 0.98 and E_k at 0.90[36] (Figure 6, Panel A). Like in Portugal, the movement of overall misallocation loss in Germany was mostly driven by changes in the allocation of capital, while the allocation of labor stayed fairly constant. Arguably, there was little room for improvement of Germany's between-sector labor distribution.

[36] Not finding a substantial capital misallocation, especially in Germany, can be seen as support for the employed measures of capital stock and capital compensation.

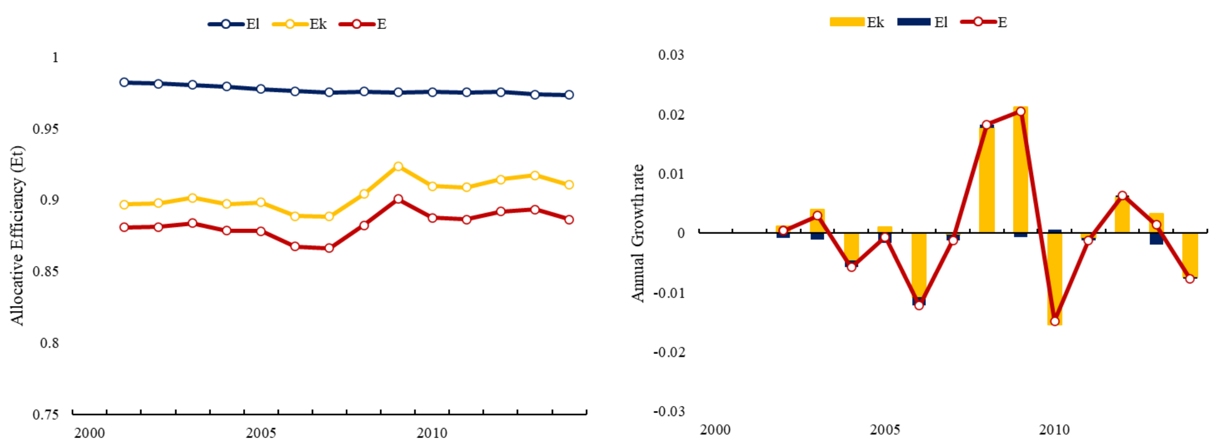
Figure 5: Allocative efficiency decomposition: Labor versus capital's allocative efficiency, Portugal (2000-2014)



Source: EU-KLEMS, WIOD and INE. Author's own computations.

Note: $E_t = E_{l,t} \times E_{k,t}$ (see Table 4).

Figure 6: Allocative efficiency decomposition: Labor versus capital's allocative efficiency, Germany (2001-2014)

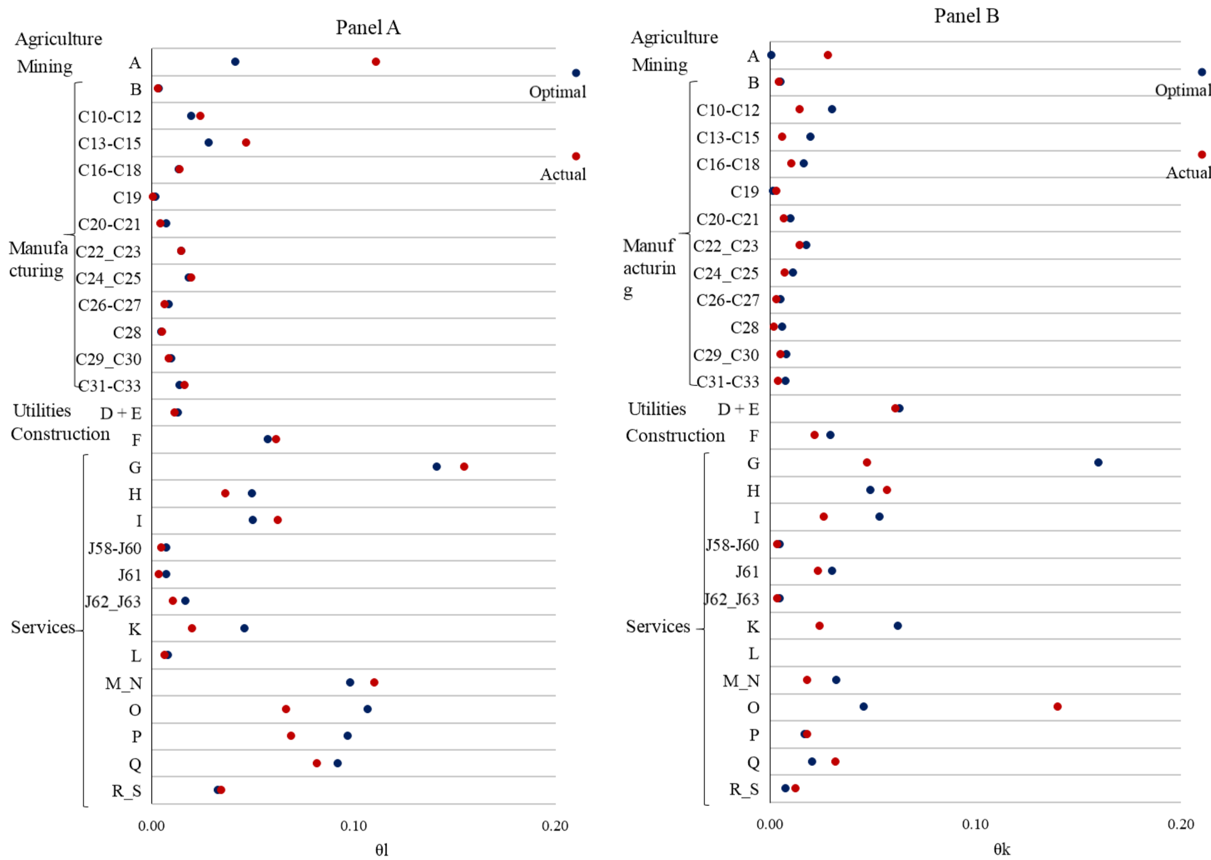


Source: EU-KLEMS, WIOD and INE. Author's own computations.

Note: $E_t = E_{l,t} \times E_{k,t}$ (see Table 4).

Looking further to the Portuguese case, as emphasized previously, the allocative efficiency of capital and labor are the outcome of the sector level gaps between the actual share of resources employed by each industry and the optimal share. As there is a fixed stock of capital (labor), in order for some industries to have more than their optimal share some others will necessarily have less, such that, at this level, gains and losses cancel out. Figure 7 Panel A shows the optimal distribution of labor versus the actual distribution of labor, and Figure 7 Panel B shows the optimal distribution of capital versus the actual distribution of capital. Figure 8 summarizes the information showing the difference between the actual share of resources and the optimal share. Both figures report for 2014. For capital, the sum of absolute deviations from the optimal is 0.54 comparing with just 0.28 for labor, a result consistent with the worse overall allocation of capital (Figure 5). This summary measure does not provide the full details to quantify the loss from misallocation as sector level gaps still need to be weighted by the importance of the factor for industry production and by the industry importance. Nonetheless, it is a good abstraction: through the sample period, the correlation between the absolute deviations of labor and E_l was -0.96 while for capital the correlation between the absolute deviations and E_k was -0.99. Intuitively, a larger dispersion, as measured by the sum of absolute deviations, indicates a worse overall between-sector factor allocation. Agriculture, Forestry and Fishing (industry A), Real Estate activities (industry L) and public administration (industry O) stand out as having a capital share clearly above the optimal share, plus 0.03, plus 0.12 and plus 0.09, respectively. Contrastingly, Wholesale and Retail Trade (industry G) and Financial and Insurance activities (industry K) stand out as receiving less than their optimal share, minus 0.11, and minus 0.04, respectively. As suggested by the previous analysis, the distribution of labor is much closer to the optimum than the distribution of capital such that most industries have about the optimum labor share. The industry that stands out as having a labor share above optimum is again Agriculture, Forestry and Fishing (industry A) with plus 0.07, in 2014.

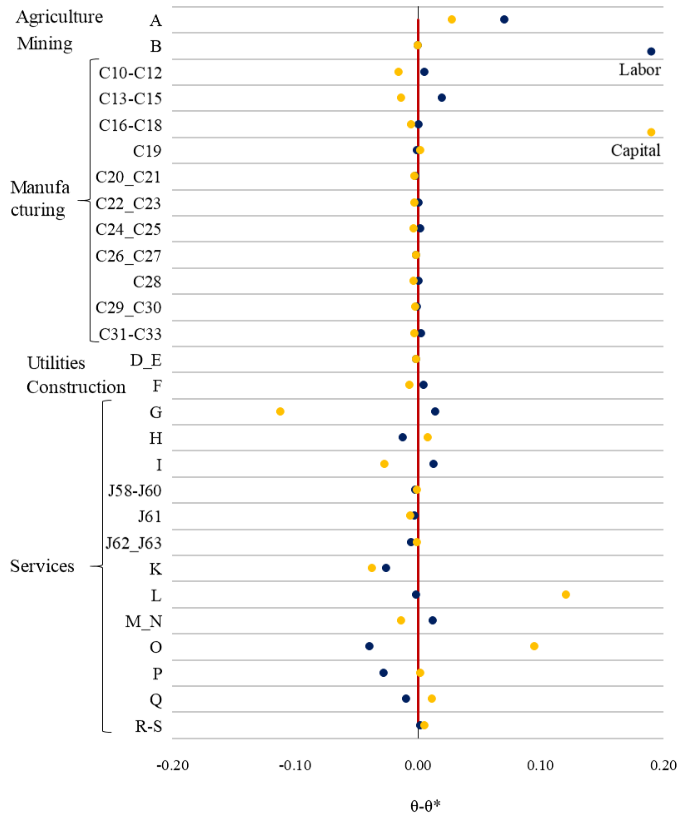
Figure 7: Distribution of labor (Panel A) and capital (Panel B) across industries, Portugal (2014)



Source: EU-KLEMS, WIOD and INE. Author's own computations.

Note: For the industry list, vide Table 1 in Section 3.

Figure 8: Industry-level gap between actual share and optimal share, Portugal (2014)

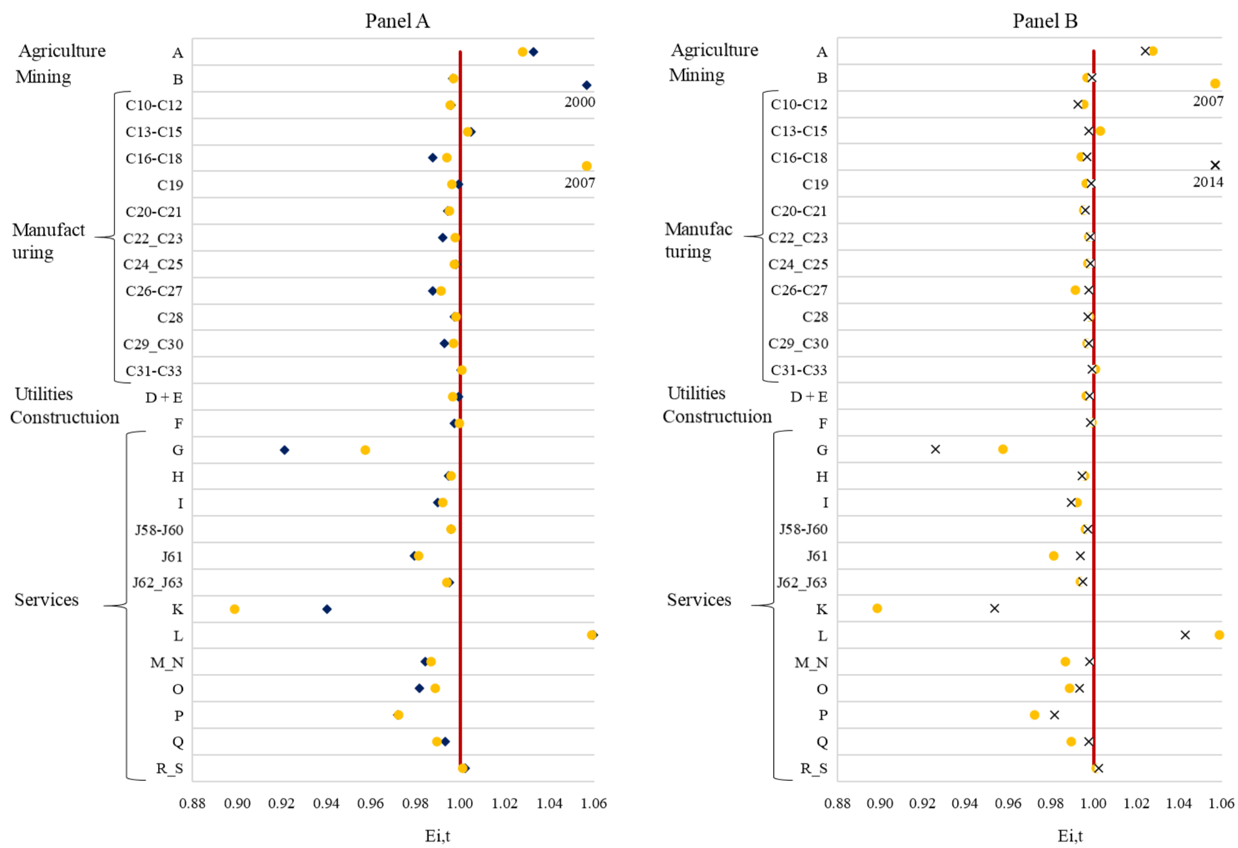


Source: EU-KLEMS, WIOD and INE. Author's own computations.

Note: For the industry list, vide Table 1 in Section 3.

Overall, whether an industry is bigger or smaller than optimal is a combination of the distortions it faces on both factors. While some industries may receive too much of both inputs compared to the optimum this is not necessarily the case and, in some instances, it may be that capital and labor distortions compensate, such that, at the industry level, the industry is about the "right" size. Figure 8 suggests that this might be the case as many industries seem to be receiving more than one factor (to the right of the red line) but less than the optimal amount of the other factor (to the left of the red line). Moreover, the contribution of any industry to aggregate allocative efficiency depends on the gaps on both capital and labor weighted by the respective factor importance and finally, by the industry importance for the economy, as captured by φ_i. Figure 9 shows the contribution of each industry to allocative efficiency in 2000, 2007 and 2014. Industries to the right of the red line are larger than optimal, while industries to the left of the red line are depressed compared to their optimal size. The red line is thus the efficient line. The correlation coefficient between the series in 2000 and in 2014 is 0.97, suggesting that the industries that were too big (small) in 2000 are about the same in 2014. Agriculture, Forestry and Fishing (industry A) and Real Estate activities (industry L) stand out as bigger than optimal. While Wholesale and retail trade; Repair of Motor Vehicles and Motorcycles (industry G) and Financial and Insurance activities (industry K) and Education (industry P) stand out as receiving too little resources compared to the optimum. Using broader industry groups, the average absolute deviations to the efficiency line is largest in Services. Although through a different perspective Dias, Robalo Marques and Richmond (2016a) also find larger reallocation gains in Services than in Manufacturing. This is the case, even if real estate activities are excluded from this group. As for real estate activities the deviation from the efficiency line peaked in 2004, dropped slightly in 2005 and increased again in 2006-07. Since then, the deviation in real estate has been falling.

Figure 9: Industry contributions to allocative efficiency, Portugal (2000, 2007, 2014)

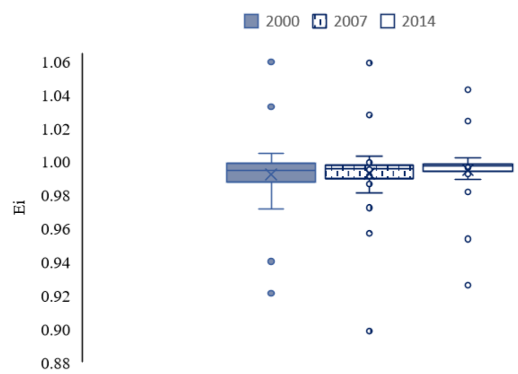


Source: EU-KLEMS, WIOD and INE. Author's own computations

Note: $E_t = \prod_{i=1}^{N} E_{i,t}$ (see Table 5). For the industry list, vide Table 1 in Section 3.

Still, there is a remarkable concentration of industries around the efficient line and this concentration increased with the passage of time (Figure 10) resulting in an improvement in allocative efficiency (Figure 3).

Figure 10: Box and whiskers plot of industry contributions (Ei,t), Portugal (2000, 2007, 2014)



Source: EU-KLEMS, WIOD and INE. Author's own computations

All in all, allocative efficiency in Portugal improved considerably from 2000 to 2014. However, compared to Germany, our benchmark, the larger misallocation loss was both the outcome of a worse allocation of available capital and of available labor. The allocation of capital between industries was further away from the optimal allocation than that of labor. It however improved more during the considered period driving the decline of the Portuguese misallocation loss. At the industry level, distortions in labor maybe compensated by distortions in capital, such that the industry is near the efficiency line. In Portugal, despite some four to five outliers, there is a considerable concentration of industries around the efficiency line and this concentration increased from 2000 to 2014 in accordance with the overall decline of the misallocation loss.

6. What is the role of the input-output structure on primary input misallocation loss?

The input-output structure was at the center of the representation chosen in this work, and the results found and discussed in previous section should be strongly determined by the specific network found in Portugal. In the analysis, the importance of any sector, used to determine the optimal amount of total resources it should employ, was determined by the sector's centrality (v_i) which was a function of the importance of the sector for final good production, directly, but also indirectly as a supplier to others that would then contribute to final production. Furthermore,

φ_i also depends on the input-output network. The purpose of this section is then to investigate the importance of the input-output network for the misallocation loss results (Section 5) for Portugal and Germany. To do so, a *ceteris paribus* exercise was designed whereby the original calibration is maintained but the multipliers associated with the intermediate goods network are closed. Essentially, this exercises investigates the misallocation loss in laboratory economy where the contribution of each sector for final production and the importance of capital and labor at the industry level are the same as in Portugal but there is no input-output transmission channel. Afterwards, the same exercise is carried for Germany.

To close the intermediate goods channel $\sigma_{ij} = \gamma_i = 0, \forall_{i,j}$ is imposed on the multipliers implying:

$$\boldsymbol{v}^{WIO}_{\mathrm{N\,x\,1}} = [\mathrm{I}_{\mathrm{N\,x\,N}} - \boldsymbol{\sigma}'_{\mathrm{N\,x\,N}}]^{-1}\boldsymbol{\beta}_{\mathrm{N\,x\,1}} = \boldsymbol{\beta}_{\mathrm{N\,x\,1}} \qquad (32)$$

And

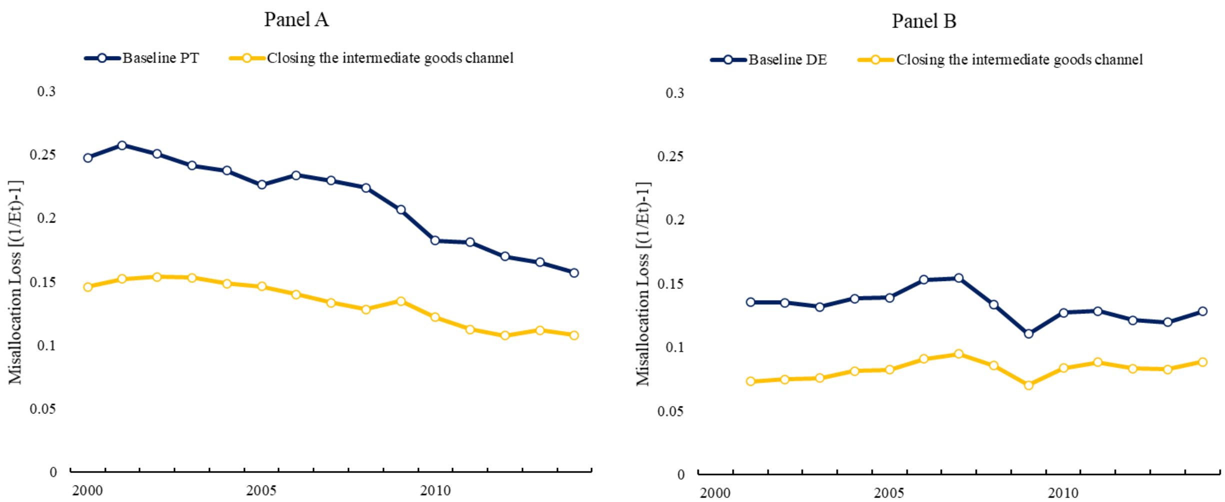
$$\boldsymbol{\varphi}^{WIO}_{\mathrm{1\,x\,N}} = \boldsymbol{\beta}'_{\mathrm{1\,x\,N}}\left[\mathbf{I}_{\mathrm{N\,x\,N}} - \boldsymbol{\gamma}_{\mathrm{N\,x\,1}}\boldsymbol{\beta}'_{\ \mathrm{1\,x\,N}} - \boldsymbol{\sigma}_{\mathrm{N\,x\,N}}\right]^{-1} = \boldsymbol{\beta}'_{\mathrm{1\,x\,N}} \qquad (33)$$

Now there is no multiplier effect, the elements in matrices $\boldsymbol{v}^{WIO}_{\mathrm{N\,x\,1}}$ and $\boldsymbol{\varphi}^{WIO}_{\mathrm{1\,x\,N}}$ sum to one. This brief analysis would suggest that the input-output network serves as an amplifier of sector-level distortions and thus that it amplifies the misallocation loss. This is also what the extensive literature on the matter points to. Yet, by changing the $\boldsymbol{v}_{\mathrm{N\,x\,1}}$ matrix also changes in the relative importance of sectors thereby changing the optimal resource division. Thus, sectors may be closer or further away from the optimal division such that *a priori* the outcome is not obvious. Figure 11 shows the result of the exercise for Portugal (Panel A) and Germany (Panel B). The dark blue line (baseline PT and DE) refer to the baseline estimations for the two countries presented in Section 5 and the yellow line is the computed misallocation loss when the intermediate goods channel is closed. For both countries and throughout the period, the blue line is always above the yellow line indicating an amplification of the misallocation loss from the input-output network. On average, over the sample period, the misallocation loss is over 1.5 times higher in the baseline estimation than in the laboratory version without input-output links. For Portugal, the maximum amplification occurs in 2007 and 2008, when the misallocation loss was about 1.75 times higher in the baseline scenario. In that same period, the amplification coming from the network was about 1.6 in Germany. For both countries, the amplification was the smallest by the end of the sample period (2013-2014).

Still, the amplification was 1.45. This result is in line with the literature and specifically with the empirical analysis of Bigio and La'O (2020).

Figure 11: Misallocation loss, baseline and closing the input-output network.

Panel A: Portugal (2000-2014). Panel B: Germany (2001-2014).



Source: EU-KLEMS, WIOD and INE. Author's own computations.

Note: The Baseline PT and DE refer to the misallocation loss estimates presented in Section 5, for Portugal and Germany, respectively.

7. Adjusting to the value-added economy

The importance of the complex input-output network was addressed, in a particular way, in the previous section. But, economic models are a simplification, a way to make sense of the complex economic system. The within-sector misallocation analysis typically disregards the role of the input-output structure as input-output matrices are not commonly available at the firm level. Consequently, the analysis usually represents the economy as a value-added economy, where capital and labor are the sole inputs to production. In this line, a slightly different question from the one in Section 6 is designed: how would the misallocation loss change if a value-added economy (where industry value-added is the same as its gross-output) was used? To answer, an alternative model economy is built, recalibrated using the data previously described, and the misallocation loss computed. This exercise covers the 2000-2017 period. Comparing the misallocation loss with the one obtained in Section 5 gives the importance of the input-output network.

7a. A distorted General Equilibrium

The value-added economy is nested within the richer input-output economy model previously presented. In particular, it is the special case when $\sigma_{ij} = \gamma_i = 0, \forall_{i,j}$ [37,38]. In this case, the previous market clearing conditions are:

$$\left(\sum_{i=1}^{N} l_i\right) = L \tag{34}$$

$$\left(\sum_{i=1}^{N} k_i\right) = K \tag{35}$$

The production of industry *i* can now be used by the final good producer as an input (f_i).

$$y_i = f_i \tag{36}$$

The good produced by the aggregate producer can be used for private consumption (C) such that:

$$F = C \tag{37}$$

Previously, v_i was defined as $v_i \equiv \frac{\beta_i y_i}{f_i}$. From (36), in the value-added economy $v_i \equiv \beta_i$. Adjusting, the equilibrium shares of capital and labor for any industry *i* are now:

$$\frac{k_i}{K} = \frac{\frac{\alpha_i \beta_i}{\tau_{ki}}}{\sum_{j=1}^{N} \frac{\alpha_j \beta_j}{\tau_{kj}}} = \theta_{ki} \tag{38}$$

$$\frac{l_i}{L} = \frac{\frac{(1-\alpha_i)\beta_i}{\tau_{li}}}{\sum_{j=1}^{N} \frac{(1-\alpha_j)\beta_j}{\tau_{lj}}} = \theta_{li} \tag{39}$$

Log industry's gross-output is, in equilibrium:

$$\log \boldsymbol{y}_{\mathrm{N\,x\,1}} = [\log \boldsymbol{A}_{\mathrm{N\,x\,1}} + \log \boldsymbol{VA}_{\mathrm{N\,x\,1}}] \tag{40}$$

---

[37] As in Section 4, matrices are identified by their dimensions and in bold.
[38] Additional details of this framework are provided in Appendix B.

Where element *i* of the matrix $\mathbf{log}\,\boldsymbol{VA}_{\mathrm{N\,x\,1}}$ is:

$$\log VA_i = [\alpha_i \log(k_i) + (1-\alpha_i)\log(l_i)]$$

And log aggregate consumption $\log(C)$ is:

$$\log(C)_{1\,\mathrm{x}\,1} = \log(F)_{1\,\mathrm{x}\,1} = \boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}}[\mathbf{log}\,\boldsymbol{y}_{\mathrm{N\,x\,1}}] \tag{41}$$

Allocative efficiency is still defined as the ratio between aggregate consumption and undistorted aggregate consumption. Here undistorted aggregate consumption, C^*, is equilibrium C when all distortions are eliminated.

From (38) and (39) if $\tau_{kj} = \tau_{lj} = 1, \forall_j$, (38) and (39) become:

$$\frac{k_i^*}{K} = \frac{\alpha_i\beta_i}{\sum_{j=1}^{N}\alpha_j\beta_j} = \theta_{ki}^* \tag{42}$$

$$\frac{l_i^*}{L} = \frac{(1-\alpha_i)\beta_i}{\sum_{j=1}^{N}(1-\alpha_j)\beta_j} = \theta_{li}^* \tag{43}$$

Log industry's gross-output is, in equilibrium:

$$\mathbf{log}\,\boldsymbol{y}^*_{\mathrm{N\,x\,1}} = [\mathbf{log}\,\boldsymbol{A}_{\mathrm{N\,x\,1}} + \mathbf{log}\,\boldsymbol{VA}^*_{\mathrm{N\,x\,1}}] \tag{44}$$

Where element *i* of the matrix $\mathbf{log}\,\boldsymbol{VA}^*_{\mathrm{N\,x\,1}}$ is:

$$\log VA_i^* = [\alpha_i \log(k_i^*) + (1-\alpha_i)\log(l_i^*)]$$

And log (undistorted) aggregate consumption $\log(C^*)$ is:

$$\log(C^*)_{1\,\mathrm{x}\,1} = \log(F^*)_{1\,\mathrm{x}\,1} = \boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}}[\mathbf{log}\,\boldsymbol{y}^*_{\mathrm{N\,x\,1}}] \tag{45}$$

Using (41) and (45) the expression for log allocative efficiency ($\log(E)$) is given by:

$$\log(E) = \log\left[\frac{C}{C^*}\right] = \log\left[\frac{F}{F^*}\right] = \boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}}\left[\mathbf{log}\left[\frac{\boldsymbol{VA}}{\boldsymbol{VA}^*}\right]_{\mathrm{N\,x\,1}}\right] \tag{47}$$

Where element *i* in matrix $\mathbf{log}\left[\frac{\boldsymbol{VA}}{\boldsymbol{VA}^*}\right]_{\mathrm{N\,x\,1}}$ is:

$$\log\left[\frac{VA_i}{VA_i^*}\right] = \left[\alpha_i[\log(\theta_{ki}) - \log(\theta_{ki}^*)] + (1-\alpha_i)[\log(\theta_{li}) - \log(\theta_{li}^*)]\right]$$

7b. Bringing the model to the Portuguese and German data

The previous subsection suggests that the model's parameters need to be recalibrated. For the sake of clarity, the parameters appear with the superscript 'VA'.

Gross-output of industry $i\,(p_{i,t}y_{i,t})^{VA}$, which is the same as its value-added, is now the sum of capital ($CAP_{i,t}$) and labor compensation ($LAB_{i,t}$).

$$(p_{i,t}y_{i,t})^{VA} = CAP_{i,t} + LAB_{i,t} \tag{48}$$

As such, the Cobb-Douglas parameters of industry *i*'s technology in a given year *t* can now be recovered as:

$$\alpha_{i,t}^{VA} = \frac{CAP_{i,t}}{(p_{i,t}y_{i,t})^{VA}}, \forall_{i,t} \tag{49}$$

$$(1-\alpha_{i,t}^{VA}) = \frac{LAB_{i,t}}{(p_{i,t}y_{i,t})^{VA}}, \forall_{i,t} \tag{50}$$

And, $\beta_{i,t}^{VA}$ is now the share of industry *i*'s value-added on total value-added, for a given year *t*:

$$\beta_{i,t}^{VA} = \frac{(p_{i,t}y_{i,t})^{VA}}{\sum_{j=1}^{N}(p_{j,t}y_{j,t})^{VA}} = \frac{(p_{i,t}f_{i,t})^{VA}}{\sum_{j=1}^{N}(p_{j,t}f_{j,t})^{VA}}, \forall_{i,t} \tag{51}$$

As before, this can be organized, for any given year, in matrix form:

$$\boldsymbol{\beta}_{t\ \mathrm{N\,x\,1}}^{VA} = \begin{bmatrix}\beta_{1,t}^{VA}\\ \vdots \\ \beta_{\mathrm{N},t}^{VA}\end{bmatrix}, \forall_t$$

The procedure was applied for Portugal (2000-2017) and Germany (2001-2017).

7c. How did the Portuguese misallocation losses evolve between 2000 and 2017?

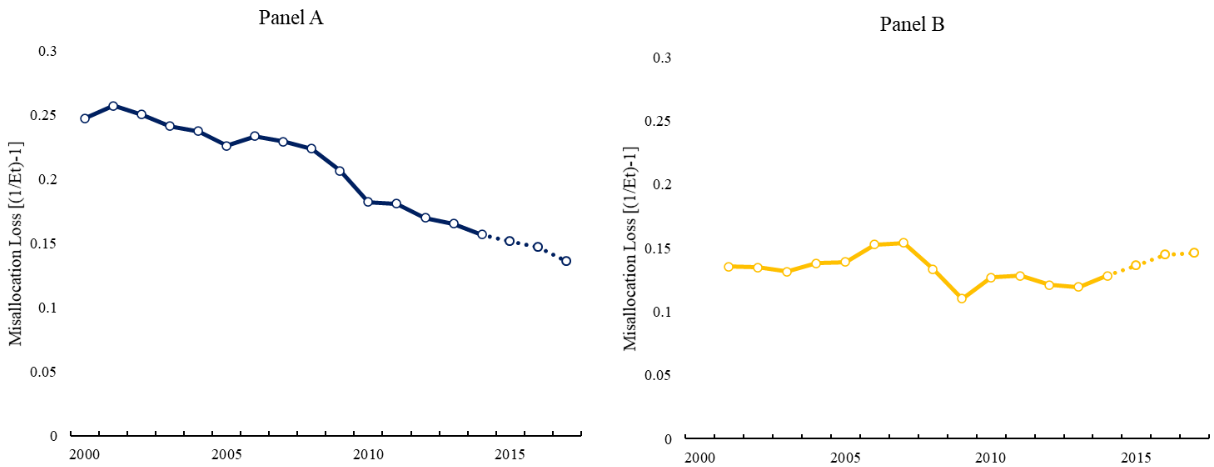
The recalibrated model can now be employed to compute allocative efficiency in the context of the value-added economy. The analysis began by computing allocative efficiency in Portugal (2000-

2017). Over the common period (2000-2014), the misallocation loss measure is found to be exactly the same as the one computed taking into account the full input-output structure (Section 5). This is true not only at the aggregate level but also at the industry level: the optimal share of capital and labor to be employed in each industry were the same as in the counterfactual exercise for the input-output economy. The results are shown in Figure 12 Panel A. This seemed puzzling in light of the results presented in Shao and Tang (2020a, 2020b), which, in a framework very similar to the one presented here, show an amplification effect of the input-output network on the misallocation loss.

In light of the puzzling results, the same exercise was carried for Germany whereby, solely using data from EU-KLEMS, the model's parameters were recalibrated accordingly and the calibrated model was used to compute the corresponding misallocation loss. This additional exercise served two main purposes. First, the German economy is maintained as a benchmark to compare the evolution of the misallocation loss in Portugal, extending the comparison until 2017. Second, it would serve as an indication as to whether the result was due to a particular feature of the Portuguese economy or was of a more general nature. The results are shown in Figure 12 Panel B: again, over the common period (2001-2014), the computed misallocation loss is the same as presented in Section 5. The results suggest that the misallocation loss continued to fall in Portugal until 2017, a downward trajectory that started in 2010. Contrastingly, in Germany, the misallocation loss started to increase in 2013 and only decreased again in 2017. Unlike Portugal, in Germany the misallocation loss was greater by 2017 than in the beginning of the period. In 2017, the two countries had about the same misallocation loss – if they were to eliminate all distortions, output would increase around 15 percent in both.

Figure 12: Allocative efficiency and misallocation loss, Portugal (2000-2017), the value-added economy

Panel A: Portugal (2000-2017). Panel B: Germany (2001-2017).



Source: EU-KLEMS, WIOD and INE. Author's own computations.

Note: The solid line is the full estimation with input-output data (shown in Section 5). The dotted line is the extension to cover the 2015-2017 period through the value-added economy estimation.

7d. One equivalence result

This subsection presents the intuition, supported by Appendix C, for the result found above showing that it is not due to a feature of the data but rather due to a theoretical equivalence between the value-added and the input-output economies.

Result: *In an economy with Cobb-Douglas production functions where distortions solely affect primary inputs, allocative efficiency will be the same in the input-output and in the value-added specifications*.

This result was already suggested in Leal (2017) and Osotimehin and Popov (2020), although both argue it in a closed economy setting and neither apply it empirically. Leal (2017) shows the conditions under which the value-added economy is equivalent to the input-output economy. Osotimehin and Popov (2020), from a characterization of the effects of distortions on TFP, show that when distortions only affect labor (in their analysis labor was the sole primary input) the Cobb-Douglas is the threshold for whether the input-output network has an amplifier or a dampening effect. This final subsection will attempt to give the intuition for this result. The full details are provided in Appendix C for a 2 by 2 economy. Table 7 places the value-added optimal capital share of industry i (${\theta_{ki}^*}^{VA}$) and the allocative efficiency (E) measure next to the same measure for the input-output economy, in the case of a closed economy, for simplicity.

Table 7: Value-added versus input-output specification (closed economy)

Value added closed economy	Input-output closed economy
${\theta_{ki}^*}^{VA} = \frac{\alpha_i^{VA}\beta_i^{VA}}{\sum_{j=1}^N \alpha_j^{VA}\beta_j^{VA}}$	$\theta_{ki}^* = \frac{\alpha_i(1-\sigma_i)v_i}{\sum_{j=1}^N \alpha_j(1-\sigma_j)v_j}$
$E = \prod_{i=1}^N \left[\left(\frac{\theta_{ki}}{\theta_{ki}^*}\right)^{\alpha_i^{VA}} \left(\frac{\theta_{li}}{\theta_{li}^*}\right)^{1-\alpha_i^{VA}}\right]^{\beta_i^{VA}}$	$E = \prod_{i=1}^N \left[\left(\frac{\theta_{ki}}{\theta_{ki}^*}\right)^{\alpha_i} \left(\frac{\theta_{li}}{\theta_{li}^*}\right)^{1-\alpha_i}\right]^{v_i(1-\sigma_i)}$

Here the intuition shall be built based on a simple closed economy with two industries but only industry 1 uses intermediate inputs. First, notice that $\alpha_i^{VA} = \alpha_i$ as this share is equal in the two frameworks. The value-added of industry 1 will be its gross-output minus the intermediate goods it purchases from industry 2:

$$(p_1y_1)^{VA} = p_1y_1 - p_2x_{12} \tag{52}$$

Using the first order condition with respect to x_{12}

$$(p_1y_1)^{VA} = p_1y_1 - p_2x_{12} = (p_1y_1)(1-\sigma_1) \tag{53}$$

For industry 2, the value-added in this simple example is just its gross output

$$(p_2y_2)^{VA} = p_2y_2 \tag{54}$$

The value-added share of industry 1 is:

$$\beta_1^{VA} = \frac{(p_1y_1)^{VA}}{(p_1y_1)^{VA} + (p_2y_2)^{VA}} = \frac{(p_1y_1)(1-\sigma_1)}{p_1y_1 - p_2x_{12} + (p_2y_2)^{VA}} \tag{55}$$

From the market clearing condition, the gross-output of industry 1 will only be used for the production of the final good and the gross-output of industry 2 will be used for production of final good and as an intermediate good for industry 1, such that

$$\beta_1^{VA} = \frac{(p_1y_1)(1-\sigma_1)}{p_1f_1 + p_2f_2} \tag{56}$$

Dividing by F ($F = p_1f_1 + p_2f_2$):

$$\beta_1^{VA} = v_1(1 - \sigma_1) \tag{57}$$

The value-added share resumes the entire information on the industries centrality – an industry's centrality is measured by its contribution as a supplier to the remaining industries, weighted by those industries' importance, and the direct contribution it makes for final consumption. Intuitively, this centrality requires that the industry produce an amount which will be captured by its value-added. The intuition would hold in a more complex example. For an open economy the presentation is slightly more complex as the centrality matrix and the final aggregating matrix are not the same. Nonetheless the equivalence still holds.

The important takeaway from this exercise is that compared to the input-output framework (Section 4) the value-added framework used here is an equally good abstraction to compute the misallocation loss from between-sector primary inputs misallocation.

8. Conclusion

This dissertation aimed to measure the impact and evolution of between-sector primary input resource misallocation as a contribution to understand the weak economic performance of the Portuguese economy in the early 21st century.

The findings suggest that, over the 2000-2017 period, the misallocation loss in Portugal was substantial – on average, were all between-sector distortions to primary inputs eliminated, output would have been about 20 percent greater than actual output. The misallocation loss in Portugal is found to be mostly driven by capital misallocation, both in level and in evolution. Compared to Germany, the allocation of available labor and of available capital were worse in Portugal. Nonetheless, there is a substantial decline of the Portuguese misallocation loss between 2000 and 2017, from about 25 percent to about 15 percent. Contrastingly, the misallocation loss in Germany stayed fairly constant, around 13 percent.

This work looked at between-sector misallocation. The recent literature on input-output networks has emphasized that the network, by connecting the sectors, may be an important propagation channel of microeconomic shocks to the aggregate and thus suggests that the network is a key element of this analysis. To assess this, two different questions were posed. First, how does the computed misallocation loss change, all else constant, by closing the input-output network?

Second, how does the computed misallocation loss change using a simpler value-added framework? For the first, the analysis found the misallocation loss to be about 1.5 times larger in the baseline estimation than when the input-output multipliers were closed, in both Portugal and Germany. For the second, the analysis found an equivalence between the sectors' value-added shares and their "centrality", such that to measure the misallocation loss from primary input misallocation the full network information was not required. So, yes and no. The full network information was not necessary for the found misallocation loss but yes, the information gives relevant insights to the underlying mechanism.

Substantial recent research addresses between-sector resource misallocation and the macroeconomic impact of microeconomic shocks. As highlighted by Baqaee and Farhi (2020), the Cobb-Douglas production function is very tractable but also very special. Ultimately, future work owes to understand how the results are affected by the assumptions made, in particular in regards to the available technology, exploring more general specifications.

References

Acemoglu, D. *et al.* (2012) 'THE NETWORK ORIGINS OF AGGREGATE FLUCTUATIONS', *Econometrica*, 80(5), pp. 1977–2016. Available at: http://www.jstor.org/stable/23271439.

Banco de Portugal (2019) *Real convergence in the European Union and the relative performance of the Portuguese economy*, *Economic Bulletin*. Lisbon, Portugal: Bank of Portugal.

Baqaee, D. R. and Farhi, E. (2019) 'The macroeconomic impact of microeconomic shocks: beyond Hulten's Theorem', *Econometrica*, 87(4), pp. 1155–1203.

Baqaee, D. R. and Farhi, E. (2020) 'Productivity and misallocation in general equilibrium', *The Quarterly Journal of Economics*, 135(1), pp. 105–163.

Bartelsman, E., Haltiwanger, J. and Scarpetta, S. (2013) 'Cross-country differences in productivity: The role of allocation and selection', *American Economic Review*, 103(1), pp. 305–334. doi: 10.1257/aer.103.1.305.

Bigio, S. and La'O, J. (2020) 'Distortions in Production Networks*', *The Quarterly Journal of Economics*, 135(4), pp. 2187–2253. doi: 10.1093/qje/qjaa018.

Blanchard, O. (2007) 'Adjustment within the euro. The difficult case of Portugal', *Portuguese Economic Journal*, 6(1), pp. 1–21. doi: 10.1007/s10258-006-0015-4.

Bonacich, P. (1987) 'Power and centrality: A family of measures', *American journal of sociology*, 92(5), pp. 1170–1182.

Carvalho, V. M. and Tahbaz-Salehi, A. (2019) 'Production networks: A primer', *Annual Review of Economics*, 11, pp. 635–663.

Chen, K. and Irarrazabal, A. (2015) 'The Role of Allocative Efficiency in a Decade of Recovery', *Review of Economic Dynamics*, 18(3), pp. 523–550. Available at: https://econpapers.repec.org/RePEc:red:issued:13-61.

David, J. M. and Venkateswaran, V. (2017) *Capital Misallocation: Frictions or Distortions?* National Bureau of Economic Research.

Dias, D., Robalo Marques, C. and Richmond, C. (2016a) 'Comparing misallocation between sectors in Portugal'. Lisbon, Portugal: Bank of Portugal. Available at: https://www.bportugal.pt/paper/afetacao-de-recursos-em-portugal-comparacao-entre-setores.

Dias, D., Robalo Marques, C. and Richmond, C. (2016b) 'Misallocation and productivity in the lead up to the Eurozone crisis', *Journal of Macroeconomics*, 49, pp. 46–70.

European Comission *et al.* (2009) *System of National Accounts 2008*. New York.

Feenstra, R. C., Inklaar, R. and Timmer, M. P. (2015) 'The next generation of the Penn World Table', *American economic review*, 105(10), pp. 3150–3182.

Fujii, D. and Nozawa, Y. (2013) 'Misallocation of Capital during Japan's Lost Two Decades', *Development Bank of Japan Working Paper*, 1304.

Gopinath, G. , Kalemli-Özcan, Ş., Karabarbounis, L., and Villegas-Sanchez, C. (2017) 'Capital allocation and productivity in south europe', *Quarterly Journal of Economics*, 132(4), pp. 1915–1967. doi: 10.1093/qje/qjx024.

Hall, R. E. (2004) 'Measuring factor adjustment costs', *The Quarterly Journal of Economics*, 119(3), pp. 899–927.

Harberger, A. C. (1954) 'Monopoly and Resource Allocation', *The American Economic Review*, 44(2), pp. 77–87. Available at: http://www.jstor.org/stable/1818325.

Hsieh, C.-T. and Klenow, P. J. (2009) 'Misallocation and Manufacturing TFP in China and India*', *The Quarterly Journal of Economics*, 124(4), pp. 1403–1448. doi: 10.1162/qjec.2009.124.4.1403.

Hulten, C. R. (1978) 'Growth Accounting with Intermediate Inputs', *The Review of Economic Studies*, 45(3), pp. 511–518. doi: 10.2307/2297252.

Jones, C. I. (2011) 'Intermediate Goods and Weak Links in the Theory of Economic Development', *American Economic Journal: Macroeconomics*, 3(2), pp. 1–28. Available at: http://www.jstor.org/stable/41237141.

Jones, C. I. (2013) 'Misallocation, Economic Growth, and Input–Output Economics', in Acemoglu, D., Arellano, M., and Dekel, E. (eds) *Advances in Economics and Econometrics: Tenth World Congress*. Cambridge University Press (Econometric Society Monographs), pp. 419–456. doi: 10.1017/CBO9781139060028.011.

Leal, J. (2015) *Key sectors in economic development: A perspective from input-output linkages and cross-sector misallocation*. Ciudad de México: Banco de México. Available at: http://hdl.handle.net/10419/129963.

Leal, J. (2016) 'Cross-Sector Misallocation with Sector-Specific Distortions', *The World Bank Economic Review*, 30(Supplement_1), pp. S42–S56. doi: 10.1093/wber/lhw017.

Leal, J. (2017) *Equivalence between input-output and value-added economies*. Ciudad de México: Banco de México. Available at: http://hdl.handle.net/10419/174466.

Long Jr, J. B. and Plosser, C. I. (1983) 'Real business cycles', *Journal of political Economy*, 91(1), pp. 39–69.

Osotimehin, S. and Popov, L. (2020) 'Misallocation and Intersectoral Linkages'.

Prescott, E. C. (1998) 'Lawrence R. Klein Lecture 1997: Needed: A Theory of Total Factor Productivity', *International Economic Review*, 39(3), pp. 525–551. Available at: http://www.jstor.org/stable/2527389.

Reis, R. (2013) 'The Portuguese Slump-Crash and the Euro Crisis', *Brookings Papers on Economic Activity*, 2013, pp. 143–210. doi: 10.1353/eca.2013.0005.

Restuccia, D. and Rogerson, R. (2008) 'Policy Distortions and Aggregate Productivity with Heterogeneous Plants', *Review of Economic Dynamics*, 11(4), pp. 707–720. Available at: https://econpapers.repec.org/RePEc:red:issued:07-48.

Restuccia, D. and Rogerson, R. (2013) 'Misallocation and productivity', *Review of Economic Dynamics*, 16(1), pp. 1–10. doi: https://doi.org/10.1016/j.red.2012.11.003.

Restuccia, D. and Rogerson, R. (2017) 'The causes and costs of misallocation', *Journal of Economic Perspectives*, 31(3), pp. 151–174. doi: 10.1257/jep.31.3.151.

Shao, L. and Tang, R. (2020a) *Allocative Efficiency and Aggregate Productivity Growth in Canada and the US*. 25 August. Available at: http://www.lin-shao.com/files/AEAPGCU.pdf.

Shao, L. and Tang, R. (2020b) *Optimal Allocation, Input-output Linkages, and Aggregate Productivity Growth*. February 15.

Solow, R. M. (1957) ‘Technical change and the aggregate production function’, *The review of Economics and Statistics*, pp. 312–320.

Stehrer, R. *et al.* (2019) *Industry level growth and productivity data with special focus on intangible assets*, *Vienna Institute for International Economic Studies Statistical Report*.

Timmer, M. P. *et al.* (2015) ‘An illustrated user guide to the world input--output database: the case of global automotive production’, *Review of International Economics*, 23(3), pp. 575–605.

United Nations (2008) ‘International Standard Industrial Classification of All Economic Activities (ISIC), Rev. 4’, *United Nations Statistical Papers*, 4.

De Vries, G. J. (2014) ‘Productivity in a Distorted Market: The Case of B razil’s Retail Sector’, *Review of Income and Wealth*, 60(3), pp. 499–524.

Ziebarth, N. L. (2013) ‘Are China and India backward? Evidence from the 19th century U.S. Census of Manufactures’, *Review of Economic Dynamics*, 16(1), pp. 86–99. doi: https://doi.org/10.1016/j.red.2012.09.003.

Appendix A: Input output economy[39]

In the framework employed in the main text, there is a representative household, an aggregate output producer, industries and a government. All agents behave competitively. The fundamentals are the stocks of primary inputs, labor (L) and capital (K), the technologies available for the industry producers as well as for the aggregate output producer. The taxes paid by the industries to the government, t_{li} and t_{ki}, $\forall_i$ for labor and capital, respectively are also exogenously determined. The representative household receives a lump-sum rebate of the amount collected from producers by the government. This economy is a small open economy and the trade balance is zero.

In this economy, all prices are expressed in units of the final good (F).

A1. Competitive equilibrium with taxes

The problem of the representative household:

$$Max\ C \text{ subject to } C = wL + rK + T$$

The solution to this problem is trivial: the representative household spends all his available income in consumption. There is no need to take first order conditions.

Problem of the final good producer:

$$Max\ \Pi = F - \left(\sum_{i=1}^{N} p_i f_i\right) \text{ subject to } F = \prod_{i=1}^{N} f_i^{\beta_i}$$

First order condition with respect to f_i:

$$(f_i)\ \ \frac{\beta_i F}{f_i} = p_i$$

The problem of the producer in industry *i:*

$$Max\ \pi_i = p_i y_i - \tau_{li} w l_i - \tau_{ki} r k_i - \left(\sum_{j=1}^{N} p_j x_{i,j}\right) - p_i^m m_i$$

[39] As in the main text, matrices are identified by their dimensions and in bold.

$$\text{subject to } y_i = A_i\left(k_i^{\alpha_i} l_i^{1-\alpha_i}\right)^{1-\sigma_i-\gamma_i} (m_i)^{\gamma_i} \prod_{j=1}^{N} \left(x_{ij}\right)^{\sigma_{ij}}$$

$$\text{with } \sum_{j=1}^{N} \sigma_{ij} = \sigma_i$$

Taking first order conditions with respect to labor (l_i), capital (k_i) and intermediate goods (domestic and imported):

$$\begin{cases} (l_i)\ \ p_i y_i (1-\alpha_i)(1-\sigma_i-\gamma_i) = w\tau_{li} l_i \\ (k_i)\ \ p_i y_i (\alpha_i)(1-\sigma_i-\gamma_i) = r\tau_{ki} k_i \\ (x_{ij})\ \ p_i y_i \sigma_{ij} = p_j x_{ij} \\ (m_i)\ p_i y_i \gamma_i = p_i^m m_i \\ y_i = A_i[(k_i)^{\alpha_i}(l_i)^{1-\alpha_i}]^{1-\sigma_i-\gamma_i} \left(\prod_{j=1}^{N} x_{ij}^{\sigma_{ij}}\right) m_i^{\gamma_i} \end{cases}$$

The government satisfies its budget constraint:

$$T = \sum_{i=1}^{N} (t_{li} w l_i + t_{ki} r k_i)$$

The market clearing conditions in the labor, capital, industry goods market and final good market are:

$$\sum_{i=1}^{N} l_i = L$$

$$\sum_{i=1}^{N} k_i = K$$

$$y_i = f_i + \left(\sum_{j=1}^{N} x_{ji}\right)$$

$$F = C + Exp \ \leftrightarrow GDP + Imp = C + Exp$$

Since the trade balance is equal to zero, $Exp = Imp$ and thus $GDP = C$.

Solving the competitive equilibrium:

Replacing the First Order Condition with respect to $x_{ji}, \forall_j$ in the market clearing condition for y_i:

$$y_i = f_i + \left(\sum_{j=1}^{N} x_{ji}\right) \leftrightarrow$$

$$\leftrightarrow\ y_i = f_i + \left(\sum_{j=1}^{N} \frac{p_j y_j \sigma_{ji}}{p_i}\right)$$

From the optimality condition of the final good producer $\frac{\beta_j f_i}{\beta_i f_j} = \frac{p_j}{p_i}$.

Replacing this condition in the previous equation:

$$y_i = f_i + \left(\sum_{j=1}^{N} \frac{\beta_j f_i}{\beta_i f_j} y_j \sigma_{ji}\right)$$

Defining v_i as $v_i \equiv \frac{\beta_i y_i}{f_i}$, the previous condition can be written as:

$$v_i = \beta_i + \left(\sum_{j=1}^{N} \sigma_{ji} v_j\right)$$

There are N equations like this one. Writing them in matrix notation:

$$\boldsymbol{v}_{\mathrm{N\,x\,1}} = \boldsymbol{\beta}_{\mathrm{N\,x\,1}} + \boldsymbol{\sigma}'_{\mathrm{N\,x\,N}} \boldsymbol{v}_{\mathrm{N\,x\,1}}$$

And we can solve for $\boldsymbol{v}_{\mathrm{N\,x\,1}}$:

$$\boldsymbol{v}_{\mathrm{N\,x\,1}} = (\boldsymbol{I}_{\mathrm{N\,x\,N}} - \boldsymbol{\sigma}'_{\mathrm{N\,x\,N}})_{\mathrm{N\,x\,N}}{}^{-1} \boldsymbol{\beta}_{\mathrm{N\,x\,1}}$$

Where $\boldsymbol{v}_{\mathrm{N\,x\,1}} = \begin{bmatrix} v_1 \\ \vdots \\ v_{\mathrm{N}} \end{bmatrix}$, $\boldsymbol{I}_{\mathrm{N\,x\,N}} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$, $\boldsymbol{\sigma}'_{\mathrm{N\,x\,N}} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{\mathrm{N}1} \\ \vdots & \ddots & \vdots \\ \sigma_{1\mathrm{N}} & \cdots & \sigma_{\mathrm{NN}} \end{bmatrix}$, and $\boldsymbol{\beta}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_{\mathrm{N}} \end{bmatrix}$.

Joining the first order conditions with respect to capital $(k_i, \forall_i)$ with the market clearing condition in the capital market $(\sum_{i=1}^{N} k_i = K)$:

$$\frac{k_i}{K} = \frac{\frac{\alpha_i(1-\sigma_i-\gamma_i)p_i y_i}{r\tau_{ki}}}{\sum_{\mathrm{j}=1}^{N} \frac{\alpha_j(1-\sigma_j-\gamma_j)p_j y_j}{r\tau_{kj}}} = \frac{\frac{\alpha_i(1-\sigma_i-\gamma_i)p_i y_i}{\tau_{ki}}}{\sum_{j=1}^{N} \frac{\alpha_j(1-\sigma_j-\gamma_j)p_j y_j}{\tau_{kj}}}$$

Replacing $p_i = \frac{\beta_i F}{f_i}$ from the problem of the aggregate producer and using $v_i \equiv \frac{\beta_i y_i}{f_i}$:

$$\frac{k_i}{K} = \frac{\frac{\alpha_i(1-\sigma_i-\gamma_i)\frac{\beta_i F}{f_i}\,y_i}{\tau_{ki}}}{\sum_{j=1}^{N}\frac{\alpha_j(1-\sigma_j-\gamma_j)\frac{\beta_j F}{f_j}\,y_j}{\tau_{kj}}} =$$

$$= \frac{\frac{\alpha_i(1-\sigma_i-\gamma_i)\frac{\beta_i}{f_i}\,y_i}{\tau_{ki}}}{\sum_{j=1}^{N}\frac{\alpha_j(1-\sigma_j-\gamma_j)\frac{\beta_j}{f_j}\,y_j}{\tau_{kj}}} = \frac{\frac{\alpha_i(1-\sigma_i-\gamma_i)v_i}{\tau_{ki}}}{\sum_{j=1}^{N}\frac{\alpha_j(1-\sigma_j-\gamma_j)v_j}{\tau_{kj}}} = \theta_{ki}$$

In a similar fashion for labor:

$$\frac{l_i}{L} = \frac{\frac{(1-\alpha_i)(1-\sigma_i-\gamma_i)v_i}{\tau_{li}}}{\sum_{j=1}^{N}\frac{(1-\alpha_j)(1-\sigma_j-\gamma_j)v_j}{\tau_{lj}}} = \theta_{li}$$

Replacing the previous conditions as well as the optimal conditions for intermediate goods (domestic and imported) in the production function of industry *i*:

$$y_i = A_i[(k_i)^{\alpha_i}(l_i)^{1-\alpha_i}]^{1-\sigma_i-\gamma_i}\prod_{j=1}^{N}\left[\frac{\sigma_{i,j}p_i y_i}{p_j}\right]^{\sigma_{i,j}}\left[\frac{\gamma_i p_i y_i}{p_i^m}\right]^{\gamma_i} \leftrightarrow$$

$$\leftrightarrow\; y_i = A_i[(\theta_{ki}K)^{\alpha_i}(\theta_{li}L)^{1-\alpha_i}]^{1-\sigma_i-\gamma_i}\prod_{j=1}^{N}\left[\frac{\sigma_{i,j}\beta_i f_j y_i}{\beta_j f_i}\right]^{\sigma_{i,j}}\left[\frac{\gamma_i p_i y_i}{p_i^m}\right]^{\gamma_i}$$

$$\leftrightarrow\; y_i = A_i[(\theta_{ki}K)^{\alpha_i}(\theta_{li}L)^{1-\alpha_i}]^{1-\sigma_i-\gamma_i}\prod_{j=1}^{N}\left[\frac{\sigma_{i,j}\beta_i f_j y_i}{\beta_j f_i}\right]^{\sigma_{i,j}}\left[\frac{\gamma_i v_i F}{p_i^m}\right]^{\gamma_i}$$

Taking logs and using the relationship between $\log(F)$ and $\log y_i$, i.e. $\log F = \sum_{i=1}^{N}\beta_i \log(f_i)$ and $\log(f_i) = \log\left(\frac{\beta_i}{v_i}\right) + \log(y_i)$, it is possible to write the N equations similar to the previous one in matrix form:

$$\mathbf{log\,} \boldsymbol{y}_{\mathrm{N\,x\,1}} = \mathbf{log\,} \boldsymbol{A}_{\mathrm{N\,x\,1}} + \mathbf{log\,} \boldsymbol{VA}_{\mathrm{N\,x\,1}} + \boldsymbol{\sigma}_{\mathrm{N\,x\,N}} \mathbf{log\,} \boldsymbol{y}_{\mathrm{N\,x\,1}} + \boldsymbol{\gamma}_{\mathrm{N\,x\,1}} \boldsymbol{\beta}'_{\mathrm{1\,x\,N}} \mathbf{log\,} \boldsymbol{y}_{\mathrm{N\,x\,1}} + \boldsymbol{\zeta}_{\mathrm{N\,x\,1}} \leftrightarrow$$

$$\leftrightarrow \mathbf{log\,} \boldsymbol{y}_{\mathrm{N\,x\,1}} = [\boldsymbol{I}_{\mathrm{N\,x\,1}} - \boldsymbol{\gamma}_{\mathrm{N\,x\,1}} \boldsymbol{\beta}'_{\mathrm{1\,x\,N}} - \boldsymbol{\sigma}_{\mathrm{N\,x\,N}}]^{-1} [\mathbf{log\,} \boldsymbol{A}_{\mathrm{N\,x\,1}} + \mathbf{log\,} \boldsymbol{VA}_{\mathrm{N\,x\,1}} + \boldsymbol{\zeta}_{\mathrm{N\,x\,1}}]$$

Where $\mathbf{log\,} \boldsymbol{y}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \log y_1 \\ \vdots \\ \log y_N \end{bmatrix}, \boldsymbol{\gamma}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_N \end{bmatrix}, \boldsymbol{\beta}'_{\mathrm{1\,x\,N}} = [\beta_1 \quad \dots \quad \beta_N], \boldsymbol{\sigma}_{\mathrm{N\,x\,N}} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_{NN} \end{bmatrix}$,

$\mathbf{log\,} \boldsymbol{A}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \log A_1 \\ \vdots \\ \log A_N \end{bmatrix}$,

$$\mathbf{log\,} \boldsymbol{VA}_{\mathrm{N\,x\,1}} = \begin{bmatrix} (1-\sigma_1-\gamma_1)(\alpha_1 \log[(\theta_{k1}K)] + (1-\alpha_1)\log[(\theta_{l1}L)]) \\ \vdots \\ (1-\sigma_N-\gamma_N)(\alpha_N \log[(\theta_{kN}K)] + (1-\alpha_N)\log[(\theta_{lN}L)]) \end{bmatrix}$$

And

$$\boldsymbol{\zeta}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \left(\sum_{j=1}^{N} \sigma_{1j} \log\left(\frac{v_1}{v_j}\sigma_{1j}\right)\right) + \gamma_1 \left[\log\left(\frac{v_1\gamma_1}{p_1^m}\right) + \sum_{i=1}^{N} \beta_1 \log\left(\frac{\beta_1}{v_1}\right)\right] \\ \vdots \\ \left(\sum_{j=1}^{N} \sigma_{Nj} \log\left(\frac{v_N}{v_j}\sigma_{Nj}\right)\right) + \gamma_N \left[\log\left(\frac{v_N\gamma_N}{p_N^m}\right) + \sum_{i=N}^{N} \beta_N \log\left(\frac{\beta_N}{v_N}\right)\right] \end{bmatrix}$$

Using, again, the relationship between $\log(F)$ and $\log y_i$:

$$\mathbf{log}(\boldsymbol{F})_{\mathrm{1\,x\,1}} = \boldsymbol{\beta}'_{\mathrm{1\,x\,N}} \left[\mathbf{log}\left(\frac{\boldsymbol{\beta}}{\boldsymbol{v}}\right)_{\mathrm{N\,x\,1}} + \mathbf{log\,} \boldsymbol{y}_{\mathrm{N\,x\,1}}\right]$$

Where $\mathbf{log}\left(\frac{\boldsymbol{\beta}}{\boldsymbol{v}}\right)_{\mathrm{N\,x\,1}} = \begin{bmatrix} \log\left(\frac{\beta_1}{v_1}\right) \\ \vdots \\ \log\left(\frac{\beta_N}{v_N}\right) \end{bmatrix}$.

Finally, from the market clearing condition in the final good's market and the trade balance equal to zero condition:

$$C = F - Imp$$

A2. Competitive equilibrium without taxes

The competitive equilibrium without taxes is nested in the previously described equilibrium conditions. If, in the previous solution, we impose $\tau_{ki} = \tau_{li} = 1, \forall_i$ then the equilibrium allocations become:

$$k_i^* = \frac{\alpha_i(1-\sigma_i-\gamma_i)v_i}{\sum_{j=1}^{N}\alpha_j(1-\sigma_j-\gamma_j)v_j} * K = \theta_{ki}^* K$$

$$l_i^* = \frac{(1-\alpha_i)(1-\sigma_i-\gamma_i)v_i}{\sum_{j=1}^{N}(1-\alpha_j)(1-\sigma_j-\gamma_j)v_j} * L = \theta_{li}^* L$$

And

$$\mathbf{log}\,\boldsymbol{y}^*{}_{\mathrm{N\,x\,1}} = [\boldsymbol{I}_{\mathrm{N\,x\,1}} - \boldsymbol{\gamma}_{\mathrm{N\,x\,1}}\boldsymbol{\beta}'_{\mathrm{1\,x\,N}} - \boldsymbol{\sigma}_{\mathrm{N\,x\,N}}]^{-1}[\mathbf{log}\,\boldsymbol{A}_{\mathrm{N\,x\,1}} + \mathbf{log}\,\boldsymbol{VA}^*_{\mathrm{N\,x\,1}} + \boldsymbol{\zeta}_{\mathrm{N\,x\,1}}]$$

Where $\mathbf{log}\,\boldsymbol{y}^*{}_{\mathrm{N\,x\,1}} = \begin{bmatrix}\log y_1^* \\ \vdots \\ \log y_N^*\end{bmatrix}, \boldsymbol{\gamma}_{\mathrm{N\,x\,1}} = \begin{bmatrix}\gamma_1 \\ \vdots \\ \gamma_N\end{bmatrix}, \boldsymbol{\beta}'_{\mathrm{1\,x\,N}} = [\beta_1 \quad \dots \quad \beta_N],$

$$\boldsymbol{\sigma}_{\mathrm{N\,x\,N}} = \begin{bmatrix}\sigma_{11} & \cdots & \sigma_{1\mathrm{N}} \\ \vdots & \ddots & \vdots \\ \sigma_{\mathrm{N}1} & \cdots & \sigma_{\mathrm{NN}}\end{bmatrix},$$

$$\mathbf{log}\,\boldsymbol{A}_{\mathrm{N\,x\,1}} = \begin{bmatrix}\log A_1 \\ \vdots \\ \log A_N\end{bmatrix},$$

$$\mathbf{log}\,\boldsymbol{VA}^*_{\mathrm{N\,x\,1}} = \begin{bmatrix}(1-\sigma_1-\gamma_1)(\alpha_1\log[(\theta_{k1}^*K)] + (1-\alpha_1)\log[(\theta_{l1}^*L)]) \\ \vdots \\ (1-\sigma_N-\gamma_N)(\alpha_N\log[(\theta_{kN}^*K)] + (1-\alpha_N)\log[(\theta_{lN}^*L)])\end{bmatrix}$$

And $\boldsymbol{\zeta}_{\mathrm{N\,x\,1}} = \begin{bmatrix}\left(\sum_{j=1}^{N}\sigma_{1j}\log\left(\frac{v_1}{v_j}\sigma_{1j}\right)\right) + \gamma_1\left[\log\left(\frac{v_1\gamma_1}{p_1^m}\right) + \sum_{i=1}^{N}\beta_1\log\left(\frac{\beta_1}{v_1}\right)\right] \\ \vdots \\ \left(\sum_{j=1}^{N}\sigma_{Nj}\log\left(\frac{v_N}{v_j}\sigma_{Nj}\right)\right) + \gamma_N\left[\log\left(\frac{v_N\gamma_N}{p_N^m}\right) + \sum_{i=N}^{N}\beta_N\log\left(\frac{\beta_N}{v_N}\right)\right]\end{bmatrix}$

Such that $\mathbf{log}(\boldsymbol{F}^*)_{\mathrm{1\,x\,1}}$

$$\mathbf{log}(\boldsymbol{F}^*)_{\mathrm{1\,x\,1}} = \boldsymbol{\beta}'_{\mathrm{1\,x\,N}}\left[\mathbf{log}\left(\frac{\boldsymbol{\beta}}{\boldsymbol{v}}\right)_{\mathrm{N\,x\,1}} + \mathbf{log}\,\boldsymbol{y}^*{}_{\mathrm{N\,x\,1}}\right]$$

With $\mathbf{log}\left(\frac{\boldsymbol{\beta}}{\boldsymbol{v}}\right)_{\mathrm{N\,x\,1}} = \begin{bmatrix} \log\left(\frac{\beta_1}{v_1}\right) \\ \vdots \\ \log\left(\frac{\beta_N}{v_N}\right) \end{bmatrix}$.

A3. Allocative efficiency

The allocative efficiency measure is defined as the ratio between aggregate consumption in the competitive equilibrium with taxes and aggregate consumption in the equilibrium without taxes. Comparing the solutions in the two problems we can compute log of allocative efficiency $(\log E)$ as:

$$\log E = \log\left(\frac{C}{C^*}\right) = \log\left(\frac{F}{F^*}\right) = \boldsymbol{\beta}'_{\mathrm{1\,x\,N}}[\boldsymbol{I}_{\mathrm{N\,x\,1}} - \boldsymbol{\gamma}_{\mathrm{N\,x\,1}}\boldsymbol{\beta}'_{\mathrm{1\,x\,N}} - \boldsymbol{\sigma}_{\mathrm{N\,x\,N}}]^{-1}\left[\mathbf{log}\left(\frac{\boldsymbol{VA}}{\boldsymbol{VA}^*}\right)_{\mathrm{N\,x\,1}}\right]$$

Where

$$\mathbf{log}\left(\frac{\boldsymbol{VA}}{\boldsymbol{VA}^*}\right)_{\mathrm{N\,x\,1}} =$$
$$= \begin{bmatrix} (1-\sigma_1-\gamma_1)(\alpha_1(\log(\theta_{k1}) - \log(\theta^*_{k1})) + (1-\alpha_1)(\log(\theta_{l1}) - \log(\theta^*_{l1}))) \\ \vdots \\ (1-\sigma_{\mathrm{N}}-\gamma_{\mathrm{N}})(\alpha_N(\log(\theta_{kN}) - \log(\theta^*_{kN})) + (1-\alpha_{\mathrm{N}})(\log(\theta_{lN}) - \log(\theta^*_{lN}))) \end{bmatrix}$$

A4. Proof that $\frac{\mathbf{C}}{\boldsymbol{C}^*}$ is equal to $\frac{\mathbf{F}}{\boldsymbol{F}^*}$

In the previous subsection, it was used that $\log\left(\frac{C}{C^*}\right)$ is equal to $\log\left(\frac{F}{F^*}\right)$. The proof of this result is presented here.

From the market clearing condition in the final goods market:

$$F = C + Exp$$

From the trade balance equal to zero condition:

$$Exp = Imp$$

Furthermore,

$$Imp = \sum_{i=1}^{N} p_i^m m_i$$

Using the industries' first order conditions with respect to m_i ($p_i y_i \gamma_i = p_i^m m_i$), the previous condition can be re-written as:

$$Exp = Imp = \left(\sum_{i=1}^{N} p_i y_i \gamma_i\right)$$

Replacing $p_i y_i = F v_i$

$$Exp = Imp = \left(\sum_{i=1}^{N} F v_i \gamma_i\right) = F\left(\sum_{i=1}^{N} v_i \gamma_i\right)$$

Returning to the market clearing condition in the final goods market:

$$F - Exp = C \leftrightarrow$$

$$\leftrightarrow F - F\left(\sum_{i=1}^{N} v_i \gamma_i\right) = C \leftrightarrow$$

$$\leftrightarrow C = F\left(1 - \sum_{i=1}^{N} v_i \gamma_i\right)$$

Similarly for F^* and C^*:

$$C^* = F^*\left(1 - \sum_{i=1}^{N} v_i \gamma_i\right)$$

Therefore

$$\frac{C}{C^*} = \frac{F(1 - \sum_{i=1}^{N} v_i \gamma_i)}{F^*(1 - \sum_{i=1}^{N} v_i \gamma_i)} = \frac{F}{F^*} \rightarrow Q.E.D.$$

Appendix B: Value-added economy

In the framework employed in the main text, there is a representative household, an aggregate output producer, industries and a government. All agents behave competitively. The fundamentals are the stocks of primary inputs, labor (L) and capital (K), the technologies available for the industry producers as well as for the aggregate output producer. The taxes paid by the industries to the government, t_{li} and t_{ki}, $\forall_i$ for labor and capital, respectively are also exogenously determined. The representative household receives a lump-sum rebate of the amount collected from producers by the government.

In this economy, all prices are expressed in units of the final good (F).

B1. Competitive equilibrium with taxes

The value-added economy is also special case of the input-output economy, described in detail in Appendix A1. In this special case, there is no consumption of intermediate goods, such that $\sigma_{ij} = \gamma_i = 0, \forall_{i,j}$.

The market clearing condition in the good market of industry *i* is now:

$$y_i = f_i$$

And $v_i \equiv \frac{\beta_i y_i}{f_i} = \beta_i$.

In this particular case, the equilibrium allocations become:

$$\frac{k_i}{K} = \frac{\frac{\alpha_i \beta_i}{\tau_{ki}}}{\sum_{j=1}^{N} \frac{\alpha_j \beta_j}{\tau_{kj}}} = \theta_{ki}$$

$$\frac{l_i}{L} = \frac{\frac{(1-\alpha_i)\beta_i}{\tau_{li}}}{\sum_{j=1}^{N} \frac{(1-\alpha_j)\beta_j}{\tau_{lj}}} = \theta_{li}$$

And equilibrium gross-output becomes:

$$\mathbf{log}\, \boldsymbol{y}_{\mathrm{N\,x\,1}} = \mathbf{log}\, \boldsymbol{A}_{\mathrm{N\,x\,1}} + \mathbf{log}\, \boldsymbol{VA}_{\mathrm{N\,x\,1}} \leftrightarrow$$

$$\leftrightarrow \mathbf{log}\, \boldsymbol{y}_{\mathrm{N\,x\,1}} = [\mathbf{log}\, \boldsymbol{A}_{\mathrm{N\,x\,1}} + \mathbf{log}\, \boldsymbol{VA}_{\mathrm{N\,x\,1}}]$$

Where $\mathbf{log}\, \boldsymbol{y}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \log y_1 \\ \vdots \\ \log y_N \end{bmatrix}, \boldsymbol{\beta}'_{\mathrm{1\,x\,N}} = [\beta_1 \quad \dots \quad \beta_N]$

$$\mathbf{log}\,\boldsymbol{A}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \log A_1 \\ \vdots \\ \log A_N \end{bmatrix},$$

$$\mathbf{log}\,\boldsymbol{VA}_{\mathrm{N\,x\,1}} = \begin{bmatrix} (\alpha_1 \log[(\theta_{k1}K)] + (1-\alpha_1)\log[(\theta_{l1}L)]) \\ \vdots \\ (\alpha_N \log[(\theta_{kN}K)] + (1-\alpha_N)\log[(\theta_{lN}L)]) \end{bmatrix},$$

Finally, $\log(F)$ is given by:

$$\mathbf{log}(\boldsymbol{F})_{1\,\mathrm{x}\,1} = \boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}}[\mathbf{log}\,\boldsymbol{y}_{\mathrm{N\,x\,1}}]$$

B2. Competitive equilibrium without taxes

The competitive equilibrium without taxes is nested in the previously described equilibrium conditions. If, in the previous solution, we impose $\tau_{ki} = \tau_{li} = 1, \forall_i$ then the equilibrium allocations become:

$$\frac{k_i^*}{K} = \frac{(\alpha_i)\beta_i}{\sum_{j=1}^{N}(\alpha_j)\beta_j} = \theta_{ki}^*$$

$$\frac{l_i^*}{L} = \frac{(1-\alpha_i)\beta_i}{\sum_{j=1}^{N}(1-\alpha_j)\beta_j} = \theta_{li}^*$$

Solving for

$$\mathbf{log}\,\boldsymbol{y}^*{}_{\mathrm{N\,x\,1}} = [\mathbf{log}\,\boldsymbol{A}_{\mathrm{N\,x\,1}} + \mathbf{log}\,\boldsymbol{VA}^*_{\mathrm{N\,x\,1}}]$$

Where $\mathbf{log}\,\boldsymbol{y}^*{}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \log y_1^* \\ \vdots \\ \log y_N^* \end{bmatrix}$, $\boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}} = [\beta_1 \quad \dots \quad \beta_N]$,

$$\mathbf{log}\,\boldsymbol{A}_{\mathrm{N\,x\,1}} = \begin{bmatrix} \log A_1 \\ \vdots \\ \log A_N \end{bmatrix},$$

$$\mathbf{log}\,\boldsymbol{VA}^*_{\mathrm{N\,x\,1}} = \begin{bmatrix} (\alpha_1 \log[(\theta_{k1}^*K)] + (1-\alpha_1)\log[(\theta_{l1}^*L)]) \\ \vdots \\ (\alpha_N \log[(\theta_{kN}^*K)] + (1-\alpha_N)\log[(\theta_{lN}^*L)]) \end{bmatrix}$$

And $\mathbf{log}(\boldsymbol{F}^*)_{1\,\mathrm{x}\,1}$

$$\mathbf{log}(\boldsymbol{F}^*)_{1\,\mathrm{x}\,1} = \boldsymbol{\beta}'_{1\,\mathrm{x}\,\mathrm{N}}[\mathbf{log}\,\boldsymbol{y}^*{}_{\mathrm{N\,x\,1}}]$$

B3. Allocative efficiency

The allocative efficiency measure is defined as the ratio between aggregate consumption in the competitive equilibrium with taxes and aggregate consumption in the equilibrium without taxes. Comparing the solutions in the two problems we can compute log of allocative efficiency ($\log E$) as:

$$\log E = \log\left(\frac{C}{C^*}\right) = \log\left(\frac{F}{F^*}\right) = \boldsymbol{\beta'}_{1 \times \mathrm{N}} \left[\mathbf{log}\left(\frac{\boldsymbol{VA}}{\boldsymbol{VA}^*}\right)_{\mathrm{N} \times 1}\right]$$

Where

$$\mathbf{log}\left(\frac{\boldsymbol{VA}}{\boldsymbol{VA^*}}\right)_{\mathrm{N} \times 1} = \begin{bmatrix} \alpha_1(\log(\theta_{k1}) - \log(\theta_{k1}^*)) + \ (1-\alpha_1)(\log(\theta_{l1}) - \log(\theta_{l1}^*)) \\ \vdots \\ \alpha_N(\log(\theta_{kN}) - \log(\theta_{kN}^*)) + \ (1-\alpha_{\mathrm{N}})(\log(\theta_{lN}) - \log(\theta_{lN}^*)) \end{bmatrix}$$

In this case, because there is no intermediate goods consumption $\log\left(\frac{C}{C^*}\right)$ is trivially equal to $\log\left(\frac{F}{F^*}\right)$ as $C = F$ and $C^* = F^*$.

Appendix C: Equivalence results

Let us think of the economy represented by either the abstraction described in Appendix A or in Appendix B. This appendix shows that when we calibrate these two abstractions, using the same data equivalent results, are obtained. Without loss of generality, the proof is done in a simplified economy with only two sectors (a 2 x 2 economy).

In this section, parameters without any superscript refer to elements from the input-output specification while elements with the superscript "VA" refer to elements from the value-added specification.

Input-output economy

In equilibrium, a producer from industry *i* makes zero profits such that:

$$p_i y_i = w_i{}^{40} l_i + r_i k_i + \left(\sum_{j=1}^{2} p_j x_{i,j} \right) + p_i^m m_i$$

The v matrix in this 2 x 2 setting is given by:

$$\mathbf{v}_{2\text{x}\,1} = (\boldsymbol{I}_{2\,\text{x}\,2} - \boldsymbol{\sigma}'_{2\,\text{x}\,2})_{2\,\text{x}\,2}{}^{-1} \boldsymbol{\beta}_{2\,\text{x}\,1} = \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \right]^{-1} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} =$$

$$= \frac{1}{(1-\sigma_{11})(1-\sigma_{22}) - \sigma_{12}\sigma_{21}} \begin{bmatrix} (1-\sigma_{22})\beta_1 + \sigma_{21}\beta_2 \\ (1-\sigma_{11})\beta_2 + \sigma_{12}\beta_1 \end{bmatrix} = \frac{1}{DET} \begin{bmatrix} (1-\sigma_{22})\beta_1 + \sigma_{21}\beta_2 \\ (1-\sigma_{11})\beta_2 + \sigma_{12}\beta_1 \end{bmatrix}$$

Where $DET = (1-\sigma_{11})(1-\sigma_{22}) - \sigma_{12}\sigma_{21}$.

The final aggregation matrix is given by:

$$\boldsymbol{\beta}'_{1\,\text{x}\,2}[\mathbf{I} - \boldsymbol{\gamma}\boldsymbol{\beta}' - \boldsymbol{\sigma}]^{-\mathbf{1}}_{2\,\text{x}\,2} = [\beta_1 \quad \beta_2] \begin{bmatrix} 1 - \gamma_1\beta_1 - \sigma_{11} & -\gamma_1\beta_2 - \sigma_{12} \\ -\gamma_2\beta_1 - \sigma_{21} & 1 - \gamma_2\beta_2 - \sigma_{22} \end{bmatrix}^{-1} =$$

$$\frac{1}{DET_2} [\beta_1 - \beta_1\sigma_{22} + \beta_2\sigma_{21} \quad \beta_2 - \beta_2\sigma_{11} + \beta_1\sigma_{12}]$$

With $DET_2 = (1 - \gamma_1\beta_1 - \sigma_{11})(1 - \gamma_2\beta_2 - \sigma_{22}) - (\gamma_1\beta_2 + \sigma_{12})(\gamma_2\beta_1 + \sigma_{21})$.

Value-added economy

In equilibrium, a producer from industry *i* makes zero profits such that:

$$(p_i y_i)^{VA} = w_i l_i + r_i k_i$$

The v matrix in this 2 x 2 setting is given by:

$$\mathbf{v}_{2\text{x}\,1} = \boldsymbol{\beta}_{2\,\text{x}\,1} = \begin{bmatrix} \beta_1^{VA} \\ \beta_2^{VA} \end{bmatrix}$$

The final aggregation matrix is given by:

$$\boldsymbol{\beta}'_{1\,\text{x}\,2} = [\beta_1^{VA} \quad \beta_2^{VA}]$$

Relationship between the parameters

[40] For simplicity, $w_i = w\tau_{li}$ and $r_i = r\tau_{ki}$. This notation is carried throughout this subsection.

$$(p_i y_i)^{VA} = p_i y_i - \left[\left(\sum_{j=1}^{2} p_j x_{i,j}\right) + p_i^m m_i\right]$$

Using the first order conditions with respect to $x_{i,j}$ and m_i, the previous condition can be written as:

$$(p_i y_i)^{VA} = p_i y_i - \sigma_i p_i y_i - \gamma_i p_i y_i \leftrightarrow$$

$$\leftrightarrow \ (p_i y_i)^{VA} = p_i y_i (1 - \sigma_i - \gamma_i)$$

The Cobb-Douglas parameters in the value-added economy are:

$$\alpha_i^{VA} = \frac{r_i k_i}{(p_i y_i)^{VA}} = \frac{r_i k_i}{p_i y_i (1 - \sigma_i - \gamma_i)} = \frac{\alpha_i (1 - \sigma_i - \gamma_i)}{(1 - \sigma_i - \gamma_i)} = \alpha_i$$

α_i^{VA} is the same as α_i. Equally, $1 - \alpha_i^{VA} = 1 - \alpha_i$.

In the input-output economy

$$\theta_{ki}^* = \frac{\alpha_i (1 - \sigma_i - \gamma_i) v_i}{\sum_{j=1}^{N} \alpha_j \left(1 - \sigma_j - \gamma_j\right) v_j}$$

In the value-added economy

$${\theta_{ki}^*}^{VA} = \frac{\alpha_i \beta_i^{VA}}{\sum_{j=1}^{N} \alpha_j \beta_j^{VA}}$$

With only 2 industries:

$$\beta_1^{VA} = \frac{(p_1 y_1)^{VA}}{(p_1 y_1)^{VA} + \ (p_2 y_2)^{VA}} = \frac{p_1 y_1 (1 - \sigma_1 - \gamma_1)}{p_1 y_1 (1 - \sigma_1 - \gamma_1) + \ p_2 y_2 (1 - \sigma_2 - \gamma_2)} =$$

$$= \frac{v_1 (1 - \sigma_1 - \gamma_1)}{v_1 (1 - \sigma_1 - \gamma_1) + \ v_2 (1 - \sigma_2 - \gamma_2)}$$

Replacing in ${\theta_{ki}^*}^{VA}$

$${\theta_{ki}^*}^{VA} = \frac{\alpha_i (1 - \sigma_i - \gamma_i) v_i}{\sum_{j=1}^{N} \alpha_j \ (1 - \sigma_j - \gamma_j) v_j} = \theta_{ki}^*$$

So, the optimal capital share is the same in the value-added economy as in the input-output economy.

In a similar fashion for labor, the optimal labor share is the same in the value-added economy as in the input-output economy.

$$\theta_{li}^{*VA} = \theta_{li}^{*}$$

In the input-output economy

$$\log \boldsymbol{VA}_{\mathrm{N\,x\,1}} = \begin{bmatrix} (1-\sigma_1-\gamma_1)(\alpha_1 \log[(\theta_{k1}K)] + (1-\alpha_1)\log[(\theta_{l1}L)]) \\ \vdots \\ (1-\sigma_N-\gamma_N)(\alpha_N \log[(\theta_{kN}K)] + (1-\alpha_N)\log[(\theta_{lN}L)]) \end{bmatrix}$$

While in the value-added economy

$$\log \boldsymbol{VA}_{\mathrm{N\,x\,1}}^{VA} = \begin{bmatrix} (\alpha_1 \log[(\theta_{k1}K)] + (1-\alpha_1)\log[(\theta_{l1}L)]) \\ \vdots \\ (\alpha_N \log[(\theta_{kN}K)] + (1-\alpha_N)\log[(\theta_{lN}L)]) \end{bmatrix}$$

At the industry level, $\log \boldsymbol{VA}_{\mathrm{N\,x\,1}}$ differs from $\log \boldsymbol{VA}_{\mathrm{N\,x\,1}}^{VA}$ by a factor of $(1-\sigma_i-\gamma_i)$.

Allocative efficiency in the input-output economy is

$$E = \boldsymbol{\beta}'_{\mathrm{1\,x\,N}}[\boldsymbol{I}_{\mathrm{N\,x\,1}} - \boldsymbol{\gamma}_{\mathrm{N\,x\,1}}\boldsymbol{\beta}'_{\mathrm{1\,x\,N}} - \boldsymbol{\sigma}_{\mathrm{N\,x\,N}}]^{-1}\left[\log\left(\frac{\boldsymbol{VA}}{\boldsymbol{VA}^{*}}\right)_{\mathrm{N\,x\,1}}\right]$$

And in the value-added economy

$$E^{VA} = \boldsymbol{\beta}^{VA'}{}_{\mathrm{1\,x\,N}}\left[\log\left(\frac{\boldsymbol{VA}^{VA}}{\boldsymbol{VA}^{VA*}}\right)_{\mathrm{N\,x\,1}}\right]$$

Using the relationship between β_i^{VA} and the parameters of the input-output economy found above, the final matrix in the value-added framework can thus be re-written as:

$$\boldsymbol{\beta}^{VA'}{}_{\mathrm{1\,x\,2}} = \frac{1}{v_1(1-\sigma_1-\gamma_1) + v_2(1-\sigma_2-\gamma_2)}[v_1(1-\sigma_1-\gamma_1) \quad v_2(1-\sigma_2-\gamma_2)]$$

Replacing the conditions of v_1 and v_2 found previously:

$$\boldsymbol{\beta}^{\boldsymbol{VA}'}{}_{1\,\mathrm{x}\,2} =$$

$$= \frac{\frac{1}{DET}}{\frac{1}{DET}[((1-\sigma_{22})\beta_1+\sigma_{21}\beta_2)(1-\sigma_1-\gamma_1)+\ (\beta_2(1-\sigma_{11})+\sigma_{12}\beta_1)(1-\sigma_2-\gamma_2)]} \times$$

$$\times [((1-\sigma_{22})\beta_1+\sigma_{21})\beta_2(1-\sigma_1-\gamma_1) \quad (\beta_2(1-\sigma_{11})+\sigma_{12}\beta_1)(1-\sigma_2-\gamma_2)]$$

Apart from $(1-\sigma_1-\gamma_1)$ and $(1-\sigma_2-\gamma_2)$ the elements inside the matrix are equal in the input-output economy and in the value-added economy. $(1-\sigma_1-\gamma_1)$ and $(1-\sigma_2-\gamma_2)$ compensate the previously mentioned difference.

So, allocative efficiency is the same in the input-output economy and in the value-added economy if $((1-\sigma_{22})\beta_1+\sigma_{21}\beta_2)(1-\sigma_1-\gamma_1)+\ (\beta_2(1-\sigma_{11})+\sigma_{12}\beta_1)(1-\sigma_2-\gamma_2) = DET_2$. Where $DET_2 =\ (1-\gamma_1\beta_1-\sigma_{11})(1-\gamma_2\beta_2-\sigma_{22})-(\gamma_1\beta_2+\sigma_{12})(\gamma_2\beta_1+\sigma_{21})$.

DET_2 can be written as:

$$DET_2 = 1-\gamma_2\beta_2-\sigma_{22}-\gamma_1\beta_1+\gamma_1\beta_1\sigma_{22}-\sigma_{11}+\sigma_{11}\gamma_2\beta_2+\sigma_{11}\sigma_{22}-\gamma_1\beta_2\sigma_{21}-\ \sigma_{12}\gamma_2\beta_1 -\ \sigma_{12}\sigma_{21}$$

And $((1-\sigma_{22})\beta_1+\sigma_{21}\beta_2)(1-\sigma_1-\gamma_1)+\ (\beta_2(1-\sigma_{11})+\sigma_{12}\beta_1)(1-\sigma_2-\gamma_2)$ can be written as:

$$((1-\sigma_{22})\beta_1+\sigma_{21}\beta_2)(1-\sigma_1-\gamma_1)+\ (\beta_2(1-\sigma_{11})+\sigma_{12}\beta_1)(1-\sigma_2-\gamma_2) =$$

$$= (\beta_1-\sigma_{22}\beta_1+\ \sigma_{21}\beta_2)(1-\sigma_1-\gamma_1)+(\beta_2-\beta_2\sigma_{11}+\sigma_{12}\beta_1)(1-\sigma_2-\gamma_2) =$$

$$= \beta_1-\beta_1\sigma_1-\beta_1\gamma_1-\sigma_{2,2}\beta_1+\sigma_{2,2}\beta_1\sigma_1+\sigma_{2,2}\beta_1\gamma_1+\sigma_{2,1}\beta_2-\sigma_{2,1}\beta_2\sigma_1-\sigma_{2,1}\beta_2\gamma_1+\ \beta_2 -\beta_2\sigma_2-\beta_2\gamma_2-\sigma_{1,1}\beta_2+\sigma_{1,1}\beta_2\sigma_2+\sigma_{1,1}\beta_2\gamma_2+\sigma_{1,2}\beta_1-\sigma_{1,2}\beta_1\sigma_2-\sigma_{1,2}\beta_1\gamma_2$$

Taking the difference with respect to DET_2 and recalling that $\beta_1+\beta_2=1$

$$DET_2-\big(\beta_1-\beta_1\sigma_1-\beta_1\gamma_1-\sigma_{2,2}\beta_1+\sigma_{2,2}\beta_1\sigma_1+\sigma_{2,2}\beta_1\gamma_1+\sigma_{2,1}\beta_2-\sigma_{2,1}\beta_2\sigma_1-\sigma_{2,1}\beta_2\gamma_1 +\ \beta_2-\beta_2\sigma_2-\beta_2\gamma_2-\sigma_{1,1}\beta_2+\sigma_{1,1}\beta_2\sigma_2+\sigma_{1,1}\beta_2\gamma_2+\sigma_{1,2}\beta_1-\sigma_{1,2}\beta_1\sigma_2 -\sigma_{1,2}\beta_1\gamma_2\big) =$$

$$= -\sigma_{22} - \sigma_{11} + \sigma_{11}\sigma_{22} + \sigma_{12}\sigma_{21}$$
$$- (-\beta_1\sigma_1 - \sigma_{22}\beta_1 + \sigma_{22}\beta_1\sigma_1 + \sigma_{21}\beta_2 - \sigma_{21}\beta_2\sigma_1 - \beta_2\sigma_2 - \sigma_{11}\beta_2 + \sigma_{11}\beta_2\sigma_2$$
$$+ \sigma_{12}\beta_1 - \sigma_{12}\beta_1\sigma_2)$$

Replacing β_2 by $(1 - \beta_1)$

$$= (-\sigma_{22} - \sigma_{11} + \sigma_{11}\sigma_{22} + \sigma_{12}\sigma_{21})$$
$$- (-\beta_1\sigma_1 - \sigma_{22}\beta_1 + \sigma_{22}\beta_1\sigma_1 + \sigma_{21} - \sigma_{21}\beta_1 - \sigma_{21}\sigma_1 + \sigma_{21}\sigma_1\beta_1 - \sigma_2 + \sigma_2\beta_1$$
$$- \sigma_{11} + \sigma_{11}\beta_1 + \sigma_{11}\sigma_2 - \sigma_{11}\sigma_2\beta_1 + \sigma_{12}\beta_1 - \sigma_{12}\beta_1\sigma_2) =$$

$$= (-\sigma_{22} - \sigma_{11} + \sigma_{11}\sigma_{22} + \sigma_{12}\sigma_{21})$$
$$- (\beta_1(-\sigma_1 - \sigma_{22} + \sigma_{22}\sigma_1 - \sigma_{21} + \sigma_{21}\sigma_1 + \sigma_2 + \sigma_{11} - \sigma_{11}\sigma_2 + \sigma_{12} - \sigma_{12}\sigma_2)$$
$$+ \sigma_{21} - \sigma_{21}\sigma_1 - \sigma_2 - \sigma_{11} + \sigma_{11}\sigma_2) =$$

$$= \left(\beta_1(-\sigma_1 - \sigma_{22} + \sigma_{22}\sigma_1 - \sigma_{21} + \sigma_{21}\sigma_1 + \sigma_2 + \sigma_{11} - \sigma_{11}\sigma_2 + \sigma_{12} - \sigma_{12}\sigma_2)\right) =$$

$$= \beta_1\left((-\sigma_{11} - \sigma_{12} - \sigma_{22} + \sigma_{22}\sigma_1 - \sigma_{21} + \sigma_{21}\sigma_1 + \sigma_{22} + \sigma_{21} + \sigma_{11} - \sigma_{11}\sigma_2 + \sigma_{12} - \sigma_{12}\sigma_2)\right) =$$

$$= \beta_1\left((\sigma_{22}\sigma_1 + \sigma_{21}\sigma_1 - \sigma_{11}\sigma_2 - \sigma_{12}\sigma_2)\right) =$$

$$\beta_1 \times 0 = 0 \rightarrow Q.E.D.$$

Allocative efficiency is the same in the input-output framework and in the value-added framework

Appendix D: Datasets

The main sources of data were the World Input-Output Database (WIOD) and EU-KLEMS. These datasets are publicly available at http://www.wiod.org/database/wiots16 and https://euklems.eu. Both datasets use the industry classification ISIC Rev.4. For Portugal, in addition to these datasets, INE's per industry capital stock was also used. This final dataset can be found in Statistics Portugal - Web Portal (ine.pt).

From the WIOD, 2016 Release, data was gathered from the Portuguese and German National input-output tables (NIOT)[41]. The 2016 Release of the World Input-Output Tables (WIOT) offers National Input-Output Tables (NIOT) for the 28 European Union member states as well as for 15

[41] For the details on the World Input-Output Tables see Timmer *et al.* (2015).

other countries (Australia, Brazil, Canada, China, India, Indonesia, Japan, Korea, Mexico, Norway, Russia, Switzerland, Taiwan, Turkey and the United States). This release covers the 2000 to 2014 period. The NIOT contains detailed information on the input-output network, including the sources of inputs and uses of outputs, value-added and gross-output. Value-added and gross-output are reported at basic prices. Basic prices are "the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any tax payable, and plus any subsidy receivable, by the producer as a consequence of its production or sale" (European Comission *et al.*, 2009, p. 101). The industry classification of the NIOT is for some industries more desegregated than in the other datasets used. The aggregation procedure followed the detailed structure for ISIC Rev. 4 detailed structured[42].

The EU-KLEMS is ran by the Vienna Institute for International Economic Studies (wiiw)[43]. The latest EU-KLEMS release, the 2019 Release, has information from 1995 to 2017[44] for the 28 European Union member states, and for Japan and the United States. For Portugal and Germany, information was collected from the Growth Accounts and National Accounts files on value-added (VA), gross-output (GO), intermediate inputs (II), number of persons employed (EMP), and labor and capital compensation (LAB and CAP, respectively). For Germany, in addition to the Growth Accounts and the National Accounts files' variables, the Capital file was also used, gathering data on the capital stock (at 2010 prices). This capital stock information is based on the European System of Accounts 2010 (ESA 2010). For Portugal, the EU-KLEMS capital file did not provide information on per industry capital stock. To circumvent this, INE's per industry capital stock information was used. INE provides capital stock information per industry, also in line with ESA 2010, for Portugal, both at current prices and at previous year's prices. This information was used to create a capital stock series spanning the 2000-2017 period.

[42] For the details and decompositions of the ISIC Rev. 4 industry classification, please see United Nations (2008).

[43] For additional information on the 2019 EU-KLEMS release, please see Stehrer *et al.* (2019).

[44] At the time of the EU-KLEMS 2019 Release, some 2017 some figures were still estimates.

Appendix E: Additional Figures

Figure 13: Heat Map - Input-output table, Portugal (2014)

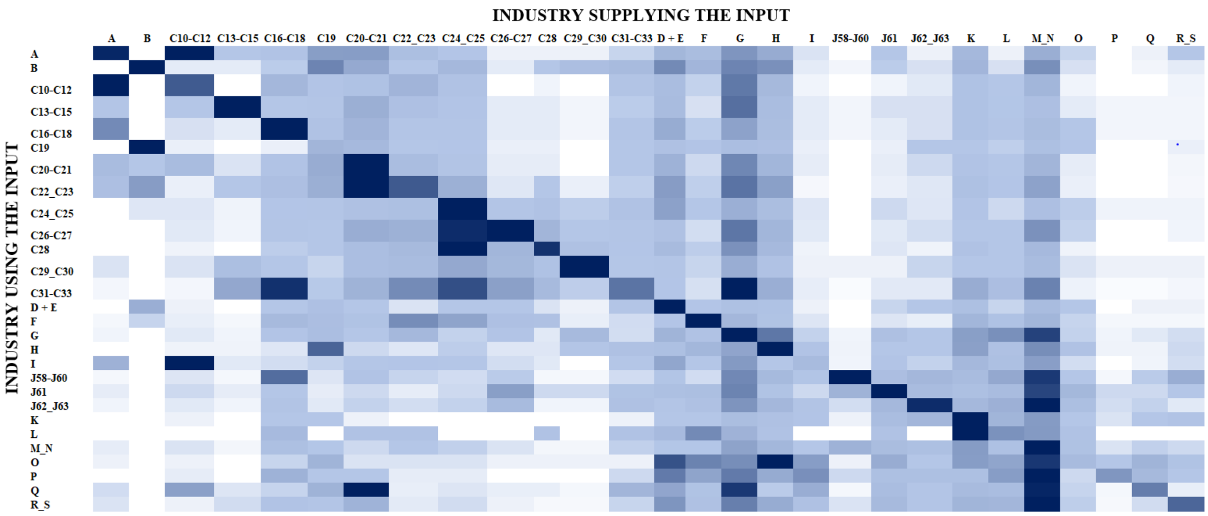


Figure 14: Heat Map - Input-output table, Germany (2014)

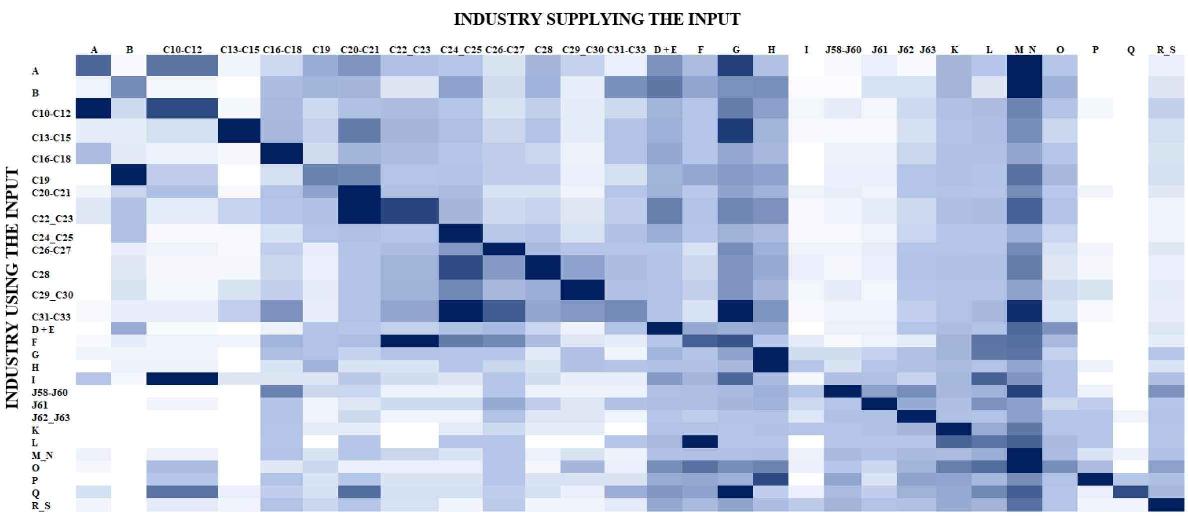
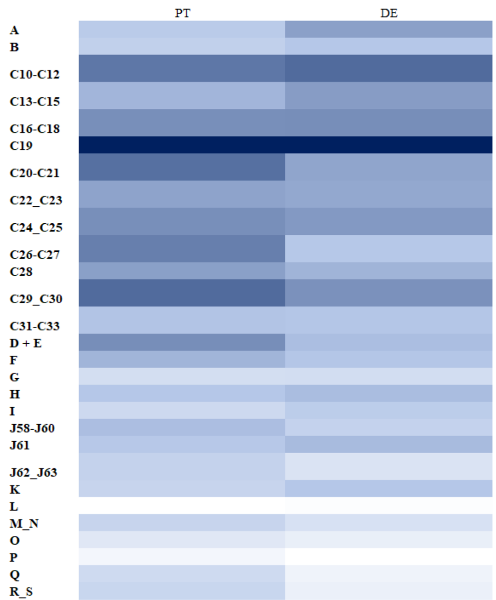


Figure 15: Intensity panel, input-output table, Portugal (PT) and Germany (DE) (2014)



Source: WIOD. Author’s own computations.

Color scale for figures 13, 14 and 15:

Highest value Percentile 50 Lowest value

Note to figures 13, 14 and 15: The scale is based on $\sigma_i + \gamma_i$. For the industry list vide Table 1 in Section 3.

Table 8: Summary statistics input-output tables, Portugal (PT) and Germany (DE) (2000, 2007, 2014)

	PT			DE		
	2000	2007	2014	2000	2007	2014
$\sigma_i + \gamma_i$						
Mean of $\sigma_i + \gamma_i$	0.55	0.56	0.55	0.51	0.54	0.54
75th percentile	0.68	0.70	0.71	0.60	0.63	0.64
50th percentile	0.58	0.59	0.57	0.55	0.57	0.56
25th percentile	0.44	0.46	0.44	0.40	0.41	0.42
Properties of the diagonal elements						
Mean	0.17	0.18	0.17	0.14	0.14	0.14
75th percentile	0.28	0.28	0.24	0.22	0.21	0.21
50th percentile	0.16	0.15	0.15	0.12	0.13	0.13
25th percentile	0.07	0.07	0.07	0.05	0.06	0.05
Percentage of all elements that are:						
equal to zero	6.76	8.42	8.16	10.08	9.31	10.59
below 1 percent	66.96	65.94	65.43	63.90	62.76	60.46
above 10 percent	4.59	4.21	4.34	3.70	3.70	3.70
above 50 percent	0.13	0.13	0.13	0.00	0.00	0.00
v_i						
Mean	0.06	0.06	0.06	0.06	0.06	0.06
Sum	1.65	1.66	1.59	1.71	1.68	1.61

Source: WIOD. Author's own computations.

Notes: Following Jones (2013), the statistics report to the overall input-output matrix i.e. $\sigma_{i,j} + \gamma_{i,j}$.