# Japan in 70's: A Speculative Run on an Interest Rate Peg?

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**Abstract**

This thesis aims to present the Japanese Great Inflation of 1974 as a case study of the equilibria developed by Bassetto and Phelan (2015), identifying in a set of Japanese macroeconomic variables the mechanisms that characterise a corner equilibrium and its recovery. The theoretical framework of the authors is explained and discussed, a terse account of the Japanese markets and institutions is provided, and, finally, an analysis of the period between 1973-1975 is presented, concluding that there are strong similarities between the Japanese context and a corner equilibrium.

**Keywords:** High Inflation, Corner Equilibrium, Japanese Great Inflation, Official Interest Rate, Shadow Interest Rate, Fiscal Intervention.

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**Resumo**

O objetivo desta tese é apresentar a Grande Inflação Japonesa de 1974 como um estudo de caso dos equilíbrios descritos por Bassetto and Phelan (2015), procurando identificar num conjunto de variáveis macroeconómicas japonesas os mecanismos que caracterizam um equilíbrio de canto e a sua respetiva recuperação. Primeiramente, o modelo destes autores é explicado e discutido, seguindo-se uma apresentação concisa dos mercados e das instituições Japonesas. Por último, uma análise ao período entre 1973-1975 permite concluir que existem fortes semelhanças entre o contexto Japonês e um equilíbrio de canto.

**Palavras-chave:** Inflação Alta, Equilíbrio de Canto, Grande Inflação Japonesa, Taxa de Juro Oficial, Taxa de Juro Sombra, Intervenção Fiscal.

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# 1 Introduction

After the Second World War, Japan, the second-largest economy at the time, was experiencing high growth rates of 10% and an inflation which steadily moved around 5%. By the 1970's, the Bank of Japan and the government were pursuing a policy of reducing nominal interest rates when the country faced, in 1974, a very high average year-over-year inflation rate of 23%. Real interest rates were very negative, real GDP declined by 1.2%, the first recession since WWII, and there was a noteworthy dissociation and departure between the official interest rate set by the Bank of Japan and inter-bank interest rates.

The work set down hereafter proposes to revisit this unusual macroeconomic period through the lens of the model presented by Bassetto and Phelan (2015) in "Speculative Runs on Interest Rate Pegs", as it appears to be a suitable case study for the theoretical framework developed by these authors.

This thesis is divided in two sections.

Chapter 2 provides a thorough presentation and discussion of the model. In general terms, when the central bank sets an interest rate, based on any arbitrary function of available information, at which it is willing to exchange money for public assets under a quantity restriction, and inflation is expected to be high, there is the possibility of having equilibria where households coordinate to borrow money by selling assets until the limit the central bank is willing to accept. The central bank is forced to a corner that is characterised by high inflation, high growth of money and an output recession, as a privately-traded shadow interest rate rises above the policy rate. The severity of the real and nominal consequences of a corner equilibrium, as well as the recovery from it, depend on fiscal policy, interest rate policy being an insufficient tool to contain inflation.

Chapter 3 conveys an outline of the Japanese financial institutions and markets in the period between 1973 and 1975, a context that is then translated into an economic environment based on Bassetto and Phelan (2015). Taking this into consideration, relevant data for the period is analysed descriptively to argue that Japan went through a corner solution in 1974. Throughout the chapter, the policy choices of the Bank of Japan are also analysed, together with the exit and recovery from the corner equilibrium.

# 2 The Model from Bassetto and Phelan (2015)

Bassetto and Phelan (2015) present a stochastic cash-in-advance model with flexible prices and discrete time, with a government, a central bank, and a private sector. In each period $t$, the sunspot variable $s_t$ is uniformly and i.i.d between [0,1].

The government has a fixed real consumption and sets taxes according to a fiscal rule. The central bank follows an interest rate policy, where the interest rate is set for each period as an arbitrary function of the available information as of the beginning of $t$ and collapsed in history $s^t$: the way the interest rate is set is thus immaterial for the analysis.

The private sector consists of a continuum of identical households, from where a representative Household is extracted.

In each period $t$, after the realisation of $s_t$, the government decides taxes $T_t$. The central bank observes the tax plan and sets a nominal interest rate $R_t$, which can coincide with the discount rate of public debt issued in $t$ and payable in $t + 1$, $B_t$.

After that, the asset markets open. The private sector uses its after tax nominal wealth, which consists of the nominal income earned in the previous period and its bond holdings, to save in public and private securities and demand money $M_t$ to consume $C_t$, according to a cash-in-advance constraint $M_t = P_tC_t$. Public securities $B_t$ are traded between the public sector and the Household, while private securities $A_{t+1}$ are traded between households. Private securities can be risk-free, paying an interest rate of $\hat{R}_t$, or state contingent. In the latter case, the probability-weighted present value as of $t$ of a unit of consumption received in a state in $t+1$ is $Q_{t+1}$, and thus the price of a state-contingent security that delivers a unit of consumption considering all realisable states in $t + 1$ is $E_t(Q_{t+1})$.

Since there are no restrictions and all assets consist of promises to deliver one monetary unit tomorrow, the no-arbitrage condition relating the returns of the available assets is given by:

$$\hat{R}_t = \frac{1}{E_t(Q_{t+1})} - 1 = R_t \tag{1}$$

Once the asset markets close, the goods market opens where households use labour to produce output, purchase consumption from other households, and the government purchases public consumption $\bar{G}$. Each household has one unit of time and the technology is labour-only and linear, $F(Y_t) = Y_t$. Labour income paid in $t$ is accumulated and used in the asset markets of $t + 1$. Finally, the market-clearing condition is $C_t + \bar{G} = Y_t$.

## 2.1 Household Problem

The representative Household has preferences over labour and consumption, solving a usual expected utility maximisation problem subject to its intratemporal budget constraint. Given prices $\{P_t, R_t, Q_{t+1}\}_{t=0}^{\infty}$, tax policy $\{T_t\}_{t=0}^{\infty}$ and initial wealth $W_{-1}$, the representative household chooses $\{C_t, Y_t, A_{t+1}, B_t, M_t\}_{t=0}^{\infty}$ such that:

Max $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, y_t)$[1] subject to:

The intratemporal budget constraint:

$$E_t(A_{t+1}Q_{t+1}) + \frac{B_t}{1 + R_t} = P_{t-1}(Y_{t-1} - C_{t-1}) + B_{t-1} + A_t + M_{t-1} - T_t - M_t \tag{2}$$

[1] $u$ is continuously differentiable, and the Inada conditions hold $\lim_{c\to 0} u_c(c, y) = \infty$ and $\lim_{y\to 1} u_y(c, y) = -\infty$.

Where $P_{t-1}Y_{t-1} + B_{t-1} + A_t - T_t$ denotes the net of tax nominal wealth available at the asset markets in $t$.

The cash-in-advance constraint:

$$M_t \geq P_t C_t \tag{3}$$

and the no-arbitrage condition (1)

Furthermore, there is the initial condition:

$$E_0(A_1 Q_0) + \frac{B_0}{1+R_0} = W_{-1} - T_0 - M_0 \tag{4}$$

Where $W_{-1}$ is exogenous initial wealth as of $t = 0$.

The problem is completed with the no-ponzi and transversality conditions for each asset, guaranteeing their stable and bounded accumulation[2].

## 2.2 Government Conditions

For a given sequence of prices $\{P_t, R_t\}_{t=0}^{\infty}$, the government policy is a sequence of taxes, money supply and debt $\{T_t, M_t, B_t\}_{t=0}^{\infty}$ and is feasible for all $t > 0$ if

$$\frac{B_t}{1+R_t} = P_{t-1}\bar{G} + B_{t-1} + M_{t-1} - T_t - M_t \tag{5}$$

Where $\bar{G} \geq 0$, and taxes are levied in $t$ to pay for public consumption purchased in the goods market in $t-1$. $\frac{B_t}{1+R_t}$ is debt issued in the public assets market in $t$, maturing in $t+1$.

With the initial condition for $t = 0$

$$\frac{B_0}{1+R_0} = W_{-1} - M_0 - T_0 \tag{6}$$

## 2.3 Satisfying the No-Ponzi and Transversality Conditions

Any sequence of debt chosen under the Household problem is stable and bounded if the government sets taxes under a Ricardian tax rule, whereby taxes adjust to the accumulation of debt, paying a share of it overtime, or fiscally back currency issued in the past: $T_t = \alpha(B_{t-1} + M_{t-1})$.

For taxes to be Ricardian, the present-value of seigniorage must be finite. This is the case if either the official interest rate set by the central bank, taken as the opportunity cost of money, is upper bounded, $R_t \leq \overline{R}$, or preferences are such that $\lim_{C \to 0} u(c_t, y_t) > -\infty$. In appendix 5.1 the limit condition is explained together with an attempt to show that seigniorage revenues exhibit a "Laffer" curve for preferences that satisfy it. Either one of

[2]See Bassetto and Phelan (2015), pp.102-103

these two conditions impose that seigniorage cannot be used permanently, so taxes have to adjust overtime.

The authors only consider the limit condition, since an upper bound on $R_t$ restricts the set of arbitrary interest rate rules that can fall under the analysis, such as Taylor rules.

## 2.4 Equilibrium Conditions

The equilibrium conditions from the government and Household problems are summarised by:

$$-\frac{u_y(t)}{u_c(t)} = \frac{1}{1+R_t} \tag{7}$$

$$E_t\left(\beta\frac{u_c(t+1)}{u_c(t)}\frac{P_t}{P_{t+1}}\right) = \frac{1}{1+R_t} \tag{8}$$

$$E_t\left(\beta\frac{u_y(t+1)(1+R_{t+1})}{u_y(t)}\frac{P_t}{P_{t+1}}\right) = 1 \tag{9}$$

$$\hat{R}_t = \frac{1}{E_t(Q_{t+1})} - 1 = R_t \tag{10}$$

$$M_t = P_t C_t \tag{11}$$

$$\frac{B_0}{1+R_0} = W_{-1} - M_0 - T_0 \tag{12}$$

$$\frac{B_t}{1+R_t} = P_{t-1}\bar{G} + B_{t-1} + M_{t-1} - T_t - M_t \tag{13}$$

$$C_t + \bar{G} = Y_t \tag{14}$$

$A_{t+1} = 0$, being in net-zero aggregate supply, since private securities are traded within the representative Household: some households are net borrowers, others are net lenders and $\hat{R}_t$ is such that for each borrower there is a lender.

## 2.5 Stochastic Equilibria and Steady State

When the Household faces no borrowing constraints, $B_t$ being unrestricted, an equilibrium consisting of a sequence of prices, allocations and policies $\{P_t, R_t, Q_{t+1}, C_t, Y_t, A_{t+1}, B_t, M_t, T_t\}_{t=0}^{\infty}$ and its steady state are obtained as follows:

In each period, a Ricardian tax rule determines $T_t$ and an arbitrary interest rate rule sets $R_t$. For assumed preferences and replacing $Y_t$ in (7) by the market-clearing condition

(14), the interest rate pins down $C_t$ and then $Y_t$ is recovered. In period 0, as in any other period, real allocations are determined once the interest rate is set.

For an arbitrary $P_0$ and given initial wealth $W_{-1}$, the cash-in-advance (11) determines $M_0$, and $B_0$ is obtained from (12). For $t > 0$, the price level is not a function of past information but depends on the possible realisations of the sunspot $s_t$. The many possible price levels that can occur in $t$, or simply $P_t(s_t)$, are bounded by:

$$E_{t-1}(\frac{P_{t-1}}{P_t}) = \frac{u_y(t-1)}{\beta u_y(t)(1+R_t)} \tag{15}$$

Given the realised $P_t$, (11) determines $M_t$ and (13) pins down $B_t$.

As such, the necessary and sufficient conditions to have an equilibrium are (7), (9), (11), (12), (13), (14).

The steady state is obtained by setting a constant $R_t$ for all periods. The levels of consumption and labour are constant, independent from the money and price levels, and are determined by (7) and (14). With real allocations constant overtime, the condition that relates the steady state interest rate $R$ and expected inverse inflation is given by:

$$\frac{1}{\beta(1+R)} = E_{t-1}(\frac{P_{t-1}}{P_t}) \tag{16}$$

Condition (16) shows that albeit realised inflation is indeterminate, a constant interest rate policy translates into a constant expected inverse inflation. If the nominal interest rate is set at $\frac{1}{\beta} - 1$, the price level realisations across time are such that the real value of the currency is not expected to depreciate. Therefore, as long as the interest rate is constant, consumption and labour remain at their steady state levels, and the price level, money and debt fluctuate approximately one-for-one[3].

## 2.6 Stochastic Equilibria with Runs

Consider $R_t$ as an interest rate set by the central bank at which it is willing to exchange public bills $B_t$ for money $M_t$ with the private sector. This rate is pegged (i.e. fixed) before markets open, and it can either be set as a constant level for the entire period, as in the author's examples, or one can also think of it as a trajectory known beforehand.

In each period there is the possibility, with probability $\phi$, that, given expectations of high inflation and negative real official interest rates, households want to anticipate consumption and increase borrowing, deciding to not save through government debt, demanding liquidity instead. To this end, households sell their assets in exchange for money at the official interest rate, performing a "run" against the central bank's interest rate peg.

[3] Appendices 5.3 and 5.4, discuss the optimality of the nominal interest rate and the relation between the steady-state level of $B_t$ and the price level, respectively.

When this possibility exists, quantity restrictions, such as $B_t \geq 0$, have to be in place, otherwise households could infinitely borrow from the central bank.

Broadly speaking, the "run" is taken as a theoretical instrument to think of an economy that can face an equilibrium where, given expectations of high inflation, households coordinate to borrow as much as possible by selling their assets until the lower limit of an aggregate quantity restriction is reached, i.e, $B_t = 0$. Further borrowing goes through a private market, so that the private and official interest rates detach, the former rising sufficiently above the latter to have an equilibrium. Money rises fast, consumption decreases, and inflation surpasses money growth. These inflationary and real consequences that can arise when some households start exchanging their assets for money lead other households to do the same, validating a corner equilibrium.

Formally, let the period in which a run realises be $s$, such that $B_s = 0$. To have an equilibrium where $B_t > 0$ for all $t \neq s$ , i.e, outside a corner solution, then:

$$T_0 < W_{-1} \tag{17}$$

$$P_0 < \frac{W_{-1} - T_0}{\hat{C}(R_0)} \tag{18}$$

where $\hat{C}(R_0)$ is the solution of (7) for an initial interest rate $R_0$, and

$$\frac{T_t}{P_{t-1}} < \bar{G} + \frac{B_{t-1}}{P_{t-1}} + \frac{M_{t-1}}{P_{t-1}} - \beta \frac{C(R_t)(1+R_t)u_y(R_t)}{u_y(R_{t-1})} \tag{19}$$

Condition (17) imposes that taxes in the first period cannot exceed the household's initial assets. Condition (18) imposes a positive upper bound on the initial price level, which is indeterminate. Together, they imply $B_0 > 0$. Finally, condition (19), which is extracted by joining (13) with the deterministic version of (15), guarantees that in every period up to $s$ and from $s+1$ onwards, government revenues excluding debt issuance are always insufficient to cover for expenses and thus the revenue needs are met by borrowing from the private sector such that $B_t > 0$.

Additionally, when $B_{t-1} = 0$, it must be that:

$$\frac{T_t}{P_{t-1}} < \bar{G} + \frac{M_{t-1}}{P_{t-1}} \tag{20}$$

Where $\bar{G} + \frac{M_{t-1}}{P_{t-1}} = Y_{t-1}$. (20) imposes that in a period after a corner solution, real taxes never exceed the real wealth available before taxation, otherwise the household would not be able to pay its taxes under the borrowing constraint $B_t \geq 0$.

Condition (19) is used to extract a deterministic Ricardian fiscal rule that satisfies (20): $T_t = \alpha(B_{t-1} + M_{t-1}) - P_{t-1}(C(R) - \bar{G})$, where $C(R)$ is steady state consumption for $R = \frac{1}{\beta} - 1$, and $\bar{G} < C(R)$.

Having said this, $\{P_t, R_t, Q_{t+1}, C_t, Y_t, A_{t+1}, B_t, M_t, T_t\}_{t=0}^{s-1}$ are determined as in the previous section. $P_0$ is arbitrarily set respecting (18), and $T_0$ following (17). The interest rate is set at the optimal level $\frac{1}{\beta} - 1$ so that before a run the economy is at the steady state where currency is not expected to depreciate. Additionally, the official rate $R_t$ coincides with the private market interest rate $\hat{R}_t$.

Then, for any period $s$, there exists $0 < \overline{\phi}_s < 1$ such that there are equilibria in which $B_s = 0$ and $\hat{R}_s > R_s$ with any probability $\phi \in [0, \overline{\phi}_s]$[4].

Thus, from the perspective of $s-1$, there are two possible realisations in $s$. With probability $\phi$, expected high inflation makes the official interest rate too low for the household to save through public debt, as seen in (8). Seeking to anticipate consumption and borrow as much as possible, all public debt is monetised and the private interest rate $\hat{R}_s^H$ rises above the official interest rate $R$ once the quantity restriction becomes active. On the other hand, there is a probability $1-\phi$ that the economy remains at the steady-state with $\hat{R}_s^L = R$, where the realised price level, $P_s^L$, is lower than $P_{s-1}$.

Equilibrium allocations and prices are determined separately for each of the two possible branches.

For the run branch, $B_s = 0$ implies that (13) becomes:

$$\frac{M_{s-1} + B_{s-1}}{P_{s-1}} + \bar{G} = \frac{T_s}{P_{s-1}} + \frac{M_s^H}{P_{s-1}} \tag{21}$$

Given the fiscal rule that determines $T_s$, (21) solves for $M_s^H$ [5]. For an arbitrary private interest rate such that $\hat{R}_s^H > R_s$, the opportunity cost of money is higher relative to the steady state so consumption, $C(\hat{R}_s^H)$, decreases, as implied by (7). By market-clearing, output falls one-for-one with consumption. Lastly, the price level rises by $M_s^H = P_s^H C(\hat{R}_s^H)$, where the fall in consumption and the rise in money mean that inflation, $\frac{P_s^H}{P_{s-1}}$, must be higher than money growth.

Obtaining the money and the price levels in this way reflects an important feature. When the economy is at the steady state, there is a close relation between the official interest rate and expected inflation as depicted by condition (16), but, once the economy is at the corner, the price level is associated with the expansion of the money level. In turn, the rise in money is determined by the amount of public debt up to $s$ that can be presented as collateral or sold in exchange for money, as well as by taxes. In this sense, the parameter $\alpha$ in the Ricardian fiscal rule has a relevant role in determining $P_s^H$. Everything else constant, for a higher $\alpha$, the share of debt from the previous period that is taxed is higher, so there would be a lower stock of public debt in the hands of the

[4] $\phi$ has to be low in order to have an equilibrium where the economy starts in a steady state and stays there even if there is the possibility of a run. See apendix 5.2.2

[5] Having $B_{s-1} > 0$ (positive debt prior to a run) and setting taxes as $T_t = \alpha(B_{t-1} + M_{t-1}) - P_{t-1}(C(R) - \bar{G})$ results in $M_s^H > M_{s-1}$

private sector ready to be monetised. Thus, a higher $\alpha$ would imply a lower increase in the money supply and, consequently, on the price level and inflation.

Alternatively, the authors present a strategy that also reduces the increase in the money level.

Consider that the government has two types of debt, blue bonds and red bonds, both with the same maturity and promising to deliver one dollar tomorrow, denoted by $B_t^R$ and $B_t^B$. The central bank sets an $R_t$ for which it is willing to trade red bonds with the private sector, under $B_t^R \geq 0$. Conversely, $B_t^B$ are auctioned at the prevailing private market interest rate, under $B_t^B \geq 0$. The value auctioned follows a rule similar to how taxes are defined, being a share of past debt and money: $B_t^B = \rho(B_{t-1} + M_{t-1})$. When expected inflation is very high, the official interest rate is too low for the household to save through red bonds. As such, $B_t^R = 0$ and the money level expands. However, the government would now auction $B_t^B$ at a sufficiently high private interest rate, such that the household would apply part of its wealth in these bonds rather than applying all of it in money, whose level would not be given by (21) but by:

$$M_s^H = P_{s-1}\bar{G} - T_s + M_{s-1} + B_{s-1} - \frac{B_s^B}{1 + \hat{R}_s^H} \tag{22}$$

Where $B_s^R = 0$, $B_s = B_s^B$

Everything else constant, the auction of an asset whose return tracks the private interest rate would partially offset the increase in the money level and, therefore, the rise in the price level.

Overall, a corner equilibrium is characterised by a higher money supply and a higher price level together with a decrease in consumption and output, as a shadow private market interest rate, which reflects the opportunity cost of money, rises above the official interest rate to clear the credit market. By the cash-in-advance constraint, the contraction in output implies that money has to grow below inflation. The higher the stock of public debt that can be sold or used as collateral, which depends on the fiscal choice of the parameter $\alpha$, the more severe the effects of moving to a corner solution. This is the new way by which fiscal policy could undermine price stability, according to Bassetto and Phelan (2015).

For the no-run branch, real allocations remain at the steady state level since $\hat{R}_s^L = R$. To determine $\frac{P_s^L}{P_{s-1}}$, the intertemporal labour choice is lagged one period and rearranged to obtain:

$$\beta E_{s-1}[u_y(s)(1 + R)\frac{P_{s-1}}{P_s}] = u_y(s-1) \tag{23}$$

And then the expected value is unfolded to have:

$$\beta[\phi u_y(\hat{R}_s^H)(1 + \hat{R}_s)\frac{P_{s-1}}{P_s^H} + (1-\phi)u_y(R)(1 + R)\frac{P_{s-1}}{P_s^L}] = u_y(s-1) \tag{24}$$

Equation (24) can be solved for $\frac{P_{s-1}}{P_s^L}$, provided that there are non-negative bond holdings. This is a necessary condition since in the no-run branch the economy is outside of the corner solution. $B_s^L > 0$ if:

$$M_{s-1} + B_{s-1} + P_{s-1}\bar{G} \geq \frac{T_s}{P_{s-1}} + C(R)\frac{P_s^L}{P_{s-1}} \tag{25}$$

A sufficient condition for (24) to assume a solution that fulfills (25) is that $\phi$ is small. Note that, as $\phi \to 0$, (24) collapses in the deterministic intertemporal labour condition from where $\frac{P_s^L}{P_{s-1}}$ can be written as a function of past information, and (25) would then be (19), which is satisfied for the specified tax function.

After period $s$, equilibrium proceeds deterministically and recursively for each branch. Since debt was positive in the node with an interior equilibrium, a corner solution can now take place in $s+1$ with the exact same dynamics as the ones described. Nevertheless, what is relevant is to check whether remaining at a corner solution for more than one period is part of an equilibrium.

Combining the intratemporal condition, the intertemporal labour condition and the government budget for $t \geq 1$, the following equilibrium condition is extracted:

$$\frac{u_c(C_{s+1}^H, C_{s+1}^H + \bar{G})C_{s+1}^H}{P_s^H\bar{G} + M_s^H - T_{s+1}} = -\frac{C_s^H u_y(C_s^H, C_s^H + \bar{G})}{M_s^H} \tag{26}$$

If (26) admits a solution $C_{s+1}^H$ that is lower than the consumption level that would be observed if $R$ had become again the effective interest rate, i.e., $C_{s+1}(R)$, then the private interest rate continued to rise and the economy remained in the corner.

Conversely, if for all values between $[0, C_{s+1}(R)]$ the left-hand side is smaller than the right-hand side then the economy is out of the corner. This is true because

$$\frac{u_c(C_{s+1}^H, C_{s+1}^H + \bar{G})C_{s+1}^H}{P_s^H\bar{G} + M_s^H - T_{s+1}} < -\frac{C_s^H u_y(C_s^H, C_s^H + \bar{G})}{M_s^H} \tag{27}$$

(27) can be rewritten as

$$\frac{M_{s+1} - M_s^H}{P_s^H} < \bar{G} - \frac{T_{s+1}}{P_s^H} \tag{28}$$

Condition (28) implies that a fiscal policy intervention is required to recover from a corner solution, namely running a deficit in $s+1$ that is sufficiently higher than seigniorage so that enough new nominal liabilities are issued, $B_{s+1} > 0$: Public debt would again be positive and the economy is back to an interior equilibrium. Therefore, a higher primary fiscal deficit contributes to satisfy (27), lowering the left-hand side.

Provided (28) holds, $C_{s+1}^H$ is set to $C_{s+1}(R)$, this $R$ coinciding with the one effective before the run, and $P_{s+1}$ would be determined according to:

$$E_s(\frac{P_s^H}{P_{s+1}}) = \frac{u_y(s)}{\beta u_y(s+1)(1+R)} \tag{29}$$

However, since a run has not continued, there is only one possible state in $s+1$, so $P_{s+1}$ is not a function of the sunspot $s_{t+1}$. Thus, (29) holds deterministically, reflecting that the price level is again associated with the official interest rate.

During recovery, consumption and output go back to their steady state levels. For output to grow between $s$ and $s+1$, the expected real interest rate at which labour income earned in $s$ is saved in $s+1$ must be negative so that the household postpones labour. This requires further inflation, which is nevertheless lower than in the corner. For real balances to recover, money has to continue to increase, growing above inflation. Moreover, and looking at the intertemporal consumption choice (8), the official interest rate is no longer too low to save through public debt, as long as the household looks forward to remain at the steady state after recovery.

Finally, since $B_{s+1} > 0$, a new "run" could take place in $s+2$ with probability $\phi$. The steps described before would again be used to obtain real allocations and nominal variables.

Having in mind the features that characterise a recovery, one can discuss further the fiscal policy intervention, how the deficit is obtained and how taxes are set.

Returning to (28), all others things constant, since government consumption is fixed, a deficit is obtained by sufficiently lowering real taxes after a corner equilibrium. From the perspective of households, if real taxes do not decrease, then there would not be sufficient income available to acquire money to consume the steady state level and to hold newly-issued public debt, making a recovery unfeasible.

To see this, consider the household's budget constraint (4) in real terms, recalling that, in equilibrium, $A_{t+1} = 0$ and $M_t = P_t C_t$:

$$\frac{P_{t-1}Y_{t-1}}{P_t} + \frac{B_{t-1}}{P_t} - \frac{T_t}{P_t} = C_t + \frac{B_t}{(1+R_t)P_t}$$

The left-hand side encompasses real net of tax income available to save and consume. Having $t = s$ as the period where the economy is at the corner and $B_s = 0$, and $t = s+1$ be the recovery period, one has that:

$$\frac{P_{s-1}Y_{s-1}}{P_s^H} + \frac{B_{s-1}}{P_s^H} - \frac{T_s}{P_s^H} = C_s(\hat{R})$$

$$\frac{P_s^H Y_s}{P_{s+1}} - \frac{T_{s+1}}{P_{s+1}} = C_{s+1}(R) + \frac{B_{s+1}}{(1+R)P_{s+1}}$$

Where $C_s(\hat{R}) < C_{s+1}(R)$, as $R < \hat{R}$.

Everything else constant, in $s+1$, real income goes down relative to the period before since the household has not saved through public debt, $B_s = 0$, receiving no debt payments. Additionally, further inflation during recovery reduces the real income available to bring consumption back to its steady state level and to save through public debt.

However, if, in real terms, the household is to consume and save more than when the economy was at the corner, then, during recovery, available real income must be relatively higher. This is only possible if the change in nominal taxes, $\frac{T_{s+1}}{T_s}$, is lower than realised inflation in the corner, $\frac{P_s^H}{P_{s-1}}$[6], which is equivalent to saying that real taxes must go down. Therefore, lower real taxes, for the same government consumption, help to satisfy condition (28) while being associated with this economic interpretation.

A follow-up question is how lower real taxes are obtained. Condition (28) can be rewritten as:

$$\bar{G} - \frac{M_{s+1} - M_s^H}{P_s^H} > \frac{T_{s+1}}{P_s^H}$$

The high realised price level in the previous period already decreases real taxes, so nominal taxes can either decrease or increase by less than the rise in $P_s^H$. Note that as government consumption becomes smaller, or as $\bar{G}$ approaches 0, the left-hand side is negative. In this way, the sign of nominal taxes depends on the parameter $\bar{G}$, or the size of government spending. If the government has a low consumption and the real income from seigniorage generated during the recovery is high, then a lump-sum subsidy might be the only way to reestablish debt so that $B_{s+1} > 0$.

Although seigniorage revenues are not known beforehand when taxes are being set, the deterministic tax rule presented by the authors, $T_t = \alpha(B_{t-1} + M_{t-1}) - P_{t-1}(C(R) - \bar{G})$, is able to anticipate that the money level will continue to rise after a corner. For a higher price level in the previous period, the government takes the price and money levels to be permanently higher in the future, since taxes are set as if real allocations and inflation always remain at their deterministic steady state levels. Everything else constant, this would reduce taxes in the recovery period, and its the main intuition on having a minus sign before $P_{t-1}$ in the tax rule. Moreover, the lower the government spending is relative to steady state consumption, the higher this effect will be.

On the other hand, and abstracting from this specific tax rule, one can also make sense of this by arguing that the seigniorage gains generated in the corner by the government are rebated to the household by lowering lump-sum taxes. In other words, in the corner, the government financed itself with money rather than debt, saving in interest payments, which is then compensated by a reduction in future taxes.

In sum, when corner equilibria arise, real taxes must be unequivocally lower to recover to an interior equilibrium. However, even if the tax rule used by the authors points towards a tax cut in order to reestablish an interior equilibrium, this is not straightforward, as the movement in nominal taxes depends on the size of real government spending.

As a last remark, a particular case arises if the government follows a balanced budget rule in $s + 1$, $T_{s+1} = P_s^H \bar{G}$, whereby taxes would rise and there would not be a deficit

[6]Recall that, from a fiscal standpoint, real taxes in the present are associated with the price paid for government consumption in the previous period.

such that $B_{s+1} > 0$. Hence, $B_{s+1} = 0$ and (13) implies a constant money supply. Real income would be insufficient to recover enough real balances to return to the steady-state. Consumption further decreases and its level is given by (26). Since money is constant, $P^H_{s+1}$ has to rise, accelerating inflation. When tax rules that do not satisfy (28) are followed permanently, a corner equilibrium goes on forever and both trajectories for consumption and the price level would become explosive: the economy would face hyperinflation.

# 3 The Japanese Great Inflation as a Case Study

The main purpose of this section is to discuss if it is possible to identify in the Japanese Great Inflation of 1974 the mechanisms and features of a corner equilibrium as in Bassetto and Phelan (2015), bearing in mind that the model is highly simplified and was not formulated to be directly applied to data. For instance, the model excludes capital, the cash-in-advance constraint does not consider neither velocity nor the substitutability between types of money, or the technology does not include a total factor productivity parameter, which are all examples to the interpretation of data.

To that end, the section is organised in three parts. Firstly, the Japanese financial system and institutions are described. Secondly, the model framework is combined with the Japanese context to extract an economic environment providing an idea of the relevant agents, markets, prices and restrictions for the analysis. Lastly, the case study is presented.

## 3.1 The Japanese Financial System and Institutions

The Japanese financial system during the Great Inflation of 1973-1974 was characterised by ceilings on deposit rates, administrative control on bank lending rates, a close-to-liberalised inter-bank market, undeveloped securities markets, selective lending and quantity controls (Suzuki (1984), Yulek (2005)).

Twelve city banks comprised the most important financial institution in Japan, each one leading a network of large corporate firms (Horiuchi (1984), Schaede (1989)). The benchmark lending rate was associated with the official discount rate (ODR) at which banks could borrow funds from the Bank of Japan. In principal, the ODR should coincide with the effective interest rate offered by city banks to their corporate customers, but the latter deviated from the former through loopholes, fees, or collusion (Horiuchi (1984), Yulek (2005), Suzuki (1984)).

Japanese households applied their savings in deposits with local or regional banks, and invested in long-term fixed-rate corporate bonds, apart from holding physical assets such as housing.

Corporations relied primarily on banks rather than on indirect finance, as securities markets were underdeveloped and restricted. Only firms selected by the Ministry of Finance could issue corporate bonds, which had to be issued overpriced relative to the ODR to prevent secondary markets from developing. Most bonds were acquired by banks, which would not sell them in a secondary market involving non-financial institutions unless they were willing to take a loss. To balance this out, the city banks would raise the effective lending rate (Yulek (2005)). This strong reliance on bank financing meant that changes in the ODR, and thus in the lending rates of banks, had a high impact on business activity (Suzuki (1984)).

Fiscal policy consisted of surpluses and small deficits, resulting in a low stock of public debt by the beginning of the 1970's, as seen in table 1.

| Table 1: Government Debt as % of GDP | |
|---|---|
| Year | Public Debt to GDP |
| 1970 | 7.45 |
| 1971 | 8.39 |
| 1972 | 11.47 |
| 1973 | 10.61 |
| Source: IMF | |

Public debt was mostly long term (Japanese Government Bonds, or JGB), with short-term bills starting to be issued in significant amounts after 1974, where a closed and well-defined group of banks had to compulsory underwrite the public bonds. Just as with corporate securities, Suzuki (1984) points that the rate on public debt was purposefully set below, but not far from, the official discount rate to prevent banks from selling the bonds afterwards in a secondary market. The Bank of Japan would purchase these claims at a premium to provide funds to banks (Yulek (2005)), who could could also obtain liquidity through the central bank's loan window, or through the inter-bank market.

In the loan window, the Bank of Japan provided funds at the ODR on a credit rationing basis, especially for city banks, each bank facing a quota (Suzuki (1984)).

The inter-bank market was composed of a call market, to borrow and lend funds with maturities up to one month, and a bills discount market, for transactions of two to six months (Schaede (1989), Suzuki (1984)). In both markets city banks were net borrowers and regional banks were net lenders, the latter collecting deposits from the entire country and then supplying funds to city banks via the inter-bank markets (Suzuki (1984), Meerschwam (1991)).

In the call market, reserves held at the Bank of Japan were traded by commercial banks at the call rate (Angrick and Yoshino (2018)). This rate was determined by the interaction between the demand of city banks and the supply of regional banks, being the closest to a freely-determined market interest rate in Japan at the time (Horiuchi (1984)).

In the discount bills market, financial institutions transacted corporate bills either with each other or with the Bank of Japan.

When the loan window was more restrictive, banks would borrow funds at the call rate or sell bills in the discount bills market, driving the inter-bank interest rates upwards (Suzuki (1984)). The Bank of Japan influenced the call rate by conducting open-market operations in the discount bills market, maintaining it as close as possible to the official discount rate (Suzuki (1984), Rhodes and Yoshino (2007)). In this way, the Bank of Japan sought to retain control over the quantity of funds lent to city banks, in the sense that if the two rates were close, the city banks would be indifferent between the loan window or the call market.

Furthermore, Horiuchi (1984) presents evidence (figure 1) that there was a consistent average disparity of around 2 p.p between the call rate and the official discount rate from 1953 to 1972.

TABLE I

OFFICIAL DISCOUNT RATES, DEPOSIT RATES, AND MONEY MARKET RATE (ANNUAL AVERAGE)

(%)

| | 1953–57 | 1958–62 | 1963–67 | 1968–72 | 1973–77 | 1978–82 |
|---|---|---|---|---|---|---|
| Japan: | | | | | | |
| Discount rate | 6.9 | 7.1 | 5.9 | 5.4 | 7.1 | 5.6 |
| Call money rate | 8.7 | 9.6 | 7.4 | 7.0 | 8.6 | 7.1 |
| Deposite rate[a] | 5.1 | 5.3 | 5.0 | 5.0 | 6.0 | 5.2 |
| WPI rate of change | 1.1 | −1.0 | 1.4 | 1.3 | 11.4 | 5.1 |
| CPI rate of change | 3.1 | 3.6 | 5.6 | 5.9 | 13.1 | 4.6 |

Figure 1: Facts on Japanese main interest rates. Source: Horiuchi (1984)

Overall, monetary policy was conducted by the Bank of Japan to control the funds lent to corporations by the major banks. To that end, the Bank of Japan would set an official interest rate and target the call rate through open market operations. In periods of monetary tightening, the Bank of Japan would raise the official discount rate to increase the call rate, signalling the banks to cut lending to clients and instead start lending in the inter-bank markets (Suzuki (1984)).

## 3.2 The Economic Environment

From the above institutional description, the main agents, assets and restrictions can be summarised and linked to the model in the following way.

**Agents:** Let the private sector be populated by banks, both city and other commercial banks, as regional banks, and households. As the Bank of Japan was not independent (Ito (2009)), the ministry of finance and the central bank can be spoken of as a unique fiscal/monetary sector.

In each period, the Bank of Japan set an official interest rate $R_t$ according to some interest rate rule. The arbitrariness around this rule in the model has now a practical implication: the existence of a corner equilibrium is independent from the specification of the interest rate rule, this is, from the reasons behind why and how the interest rate level was set as it was.

Throughout the analysis, it can be useful to take the representative household as a pair composed by an household and a bank, as in Alvarez, Lucas and Weber (2001)[7]. After looking at the interest rate set by the Bank of Japan, the bank goes to the asset markets to buy securities and demand money. The household then collects the money to conduct transactions in a goods market.

**Assets and Restrictions:** Let $B_t$ be the short-term assets traded between the Bank of Japan and the banks in the loan window. When $B_t < 0$ the Bank of Japan is lending to the banks at the official discount rate $R_t$. Consider the possibility of $B_t \geq -\delta$, where $\delta$ is the maximum amount of aggregate lending that the Bank of Japan can provide to banks in a moment in time through the loan window. In absolute, the lower the $\delta$ the more stringent the quota. Furthermore, $\delta$ is not the sum of the quotas of each individual bank at the loan window, otherwise all banks could borrow at the same time, each depleting its quota.

Banks could also obtain liquidity by selling government bonds, $B_t^b$, to the Bank of Japan. As explained in the previous section, public debt was issued below the official interest rate to prevent secondary markets from appearing. However, the relevant rate for transactions of this asset with the central bank was the official interest rate, $R_t$.

Let $A_{t+1}$ be the assets traded in the short-term inter-bank call market at the market call rate $\hat{R}_t$. Recall that $A_{t+1}$ is the privately traded asset whose return is the equilibrium rate in a corner solution. As the interpretation of $A_{t+1}$ changes what $\hat{R}_t$ means, a first interpretation associating $\hat{R}_t$ to the call rate does not imply that the latter has to be the shadow equilibrium rate. More broadly, $A_{t+1}$ can denote another type of asset that is transacted inside and across households whose return is an interest rate associated with, but not equal to, the call rate, such as savings transferred between household members to apply in a family-owned small business.

Lastly, $B_t^c$ denotes corporate bills held by the private sector. These were traded at a price that reflected $\hat{R}$, since the call rate was the opportunity cost of trading in the discount bills market. As in the red bond-blue bond example in the model, assume that $B_t^c$ is available according to a function of past information. For what follows, one can think that banks held, at the beginning of each period, a share $\rho$ of a fixed endowment of corporate debt defined exogenously, $B_t^c = \rho \bar{B}^c$, with $\bar{B}^c > 0$ and $\rho \in [0, 1]$.

As corporate bills were transacted close to the call rate, it is irrelevant whether this

[7] This avoids using the model as a thinking tool while having a bank solving an household maximisation problem. Of course, this requires the strong assumption that banks are similar and identical.

asset is quantity restricted or not: what matters for the dissociation between official and private market interest rates is that funds obtained under the official rate are quantity restricted.

### 3.3 Japan 1973-1975

Taking into account, the model, the institutions, the main assets and their restrictions, figure 2 presents the relationship between the official discount rate $R$, the call rate $\hat{R}$, and the inflation rate.

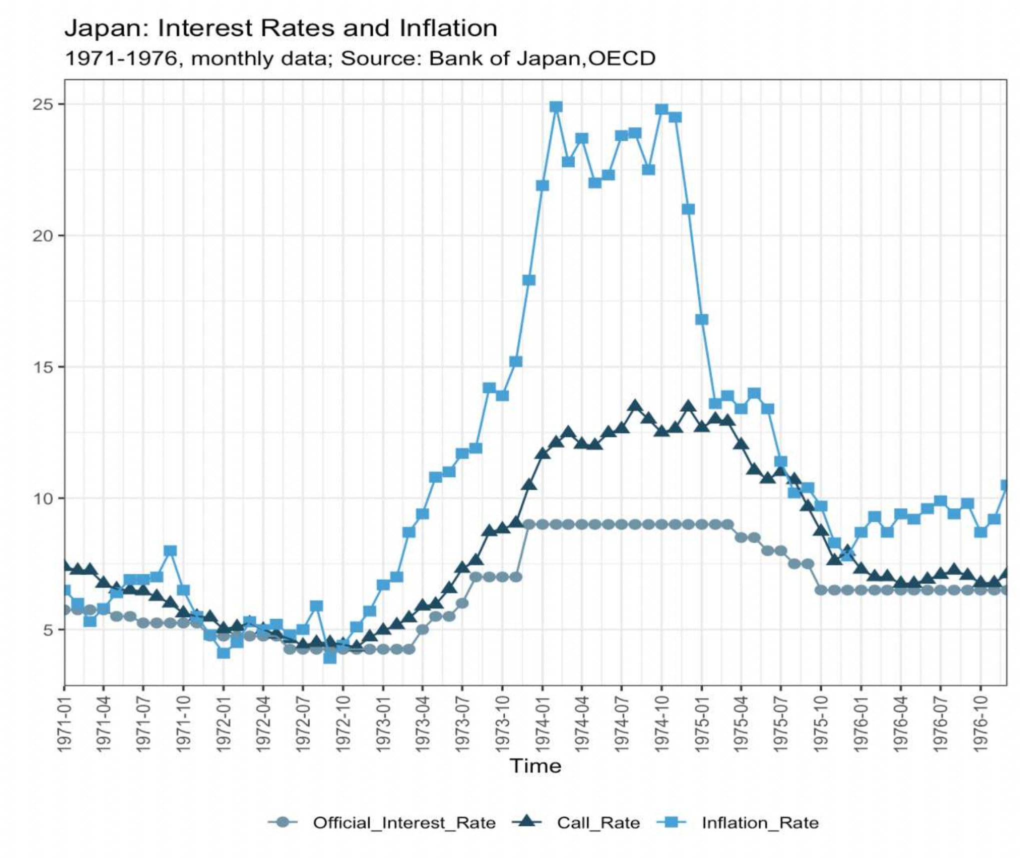



Figure 2: Interest rates and Inflation 1971-1976

In 1972 one sees that there was not only a strong association but also a some moments of coincidence between the call rate and the official discount rate, which was set at 5% but was progressively lowered to 4% until the end of the year. As in the model, the analysis initiates with $R_t = \hat{R}_t$. This coincidence meant that the Bank of Japan was supplying liquidity to the major banks without a stringent quota: the banks would not need to use the call market for funding, putting no upward pressure on the call rate. Hence, $B_t > -\delta$.

Whereas during 1972 inflation tended to fluctuate around the official interest rate, it started to rise fast in the beginning of 1973, and further accelerated after the first oil shock in October, particularly between December and the first quarter of 1974, the Bank of Japan responding through increases in the official discount rate.

Throughout 1973 the two interest rates diverged by an average of 1.5 p.p, lower than the 2 p.p difference that was usually observed in periods of five years (Horiuchi (1984)). Albeit it cannot be strictly said that $R_t = \hat{R}_t$ in 1973, the difference was on average small and one can still observe a close association between the rates.

In 1974, monthly year-over-year inflation was consistently higher than 20%, while the official interest rate remained pegged at 9% for the whole year. It is evident that during the Great Inflation the official rate was no longer able to anchor the call rate, as the two interest rates diverged by a maximum difference of 4 p.p, the average call rate being 12.5%. This resulted in an average negative real official interest rate of -14.2%, and an average real call rate of -10.7%.

This clear separation, above what was historically observed, and dissociation between the official interest rate and the call rate in 1974, in the presence of high inflation, is the key similarity to the Bassetto and Phelan's corner equilibrium.

From the model's perspective, the dissociation between rates is explained by a state where the private sector tries to borrow as much as possible, given expectations of high inflation and negative real interest rates, but faces quantity restrictions with the monetary authority. The economy would move to a corner equilibrium as the private sector transferred assets to the monetary authority until the restrictions on the amount of assets in their possession become active, further borrowing going through a private market.

To what extent can this be true in Japan?

By the end of 1973, inflation was already close to 15% and agents expected that it would keep rising, as attested by Ito (2009), who reports that there was a mood of general panic, with wholesalers buying and storing goods, waiting for further price increases, on the one hand, and consumers buying in bulk, on the other. In face of anticipated negative real rates, there may have been a coordination between households to anticipate consumption and increase borrowing. Banks would have borrowed as much as possible through the loan window and by discounting government bonds at the Bank of Japan. In the Bank for International Settlements report for 1975 one can read that:

"The Bank of Japan substantially increased its purchases of government securities so as to help meet the additional needs of the financial system for central-bank money" (BIS (1975), p.41).

If the part of this "central-bank money" that was supplied at the official interest rate was constrained, then the banks had to turn to the inter-bank market for funds, increasing the call rate. In this sense, the sharp rise in the call rate between December 1973 and January 1974, increasing from 9% to 12% as represented in figure 2, is evidence that the Bank of Japan's liquidity supply at the official rate fell short of the amounts demanded, because quantity restrictions were not only present but were also active.

The detachment between the call and the official rate that followed, as observed in the same figure, reflects that during 1974 the quantity restrictions in both the loan window and the implied ones associated with the Bank of Japan's purchases of public bonds were active. Had the banks been able to borrow boundlessly through the Bank of Japan's loan window or by exchanging government bonds for money, then the inter-bank market would have not been used and the two interest rates would still be similar both in their levels and trajectories.

In figure 3, which presents the outright purchases of government bonds (JGB's) and market operations of the Bank of Japan, one can have an idea of the magnitude of the funds borrowed in the discount-bills market, as further borrowing went through the inter-bank market. Banks discounted or presented as collateral the other significant asset in their possession, corporate bills $B_t^c$. It is possible to see that, in December 1973, the Bank of Japan purchased 1.5 trillion yens in corporate bills.

Banks would only be willing to sell corporate bills at a price that reflected a rate below the opportunity cost of borrowing in the discount bills market, this is, the call rate. By providing this amount of liquidity at such a price, the Bank of Japan may have been attempting to prevent the banks from borrowing in the call market, and further raising this rate above the official discount rate, which would increase the funding cost of banks and, by extension, of corporations and households, in line with the transmission mechanisms described by Suzuki (1984).

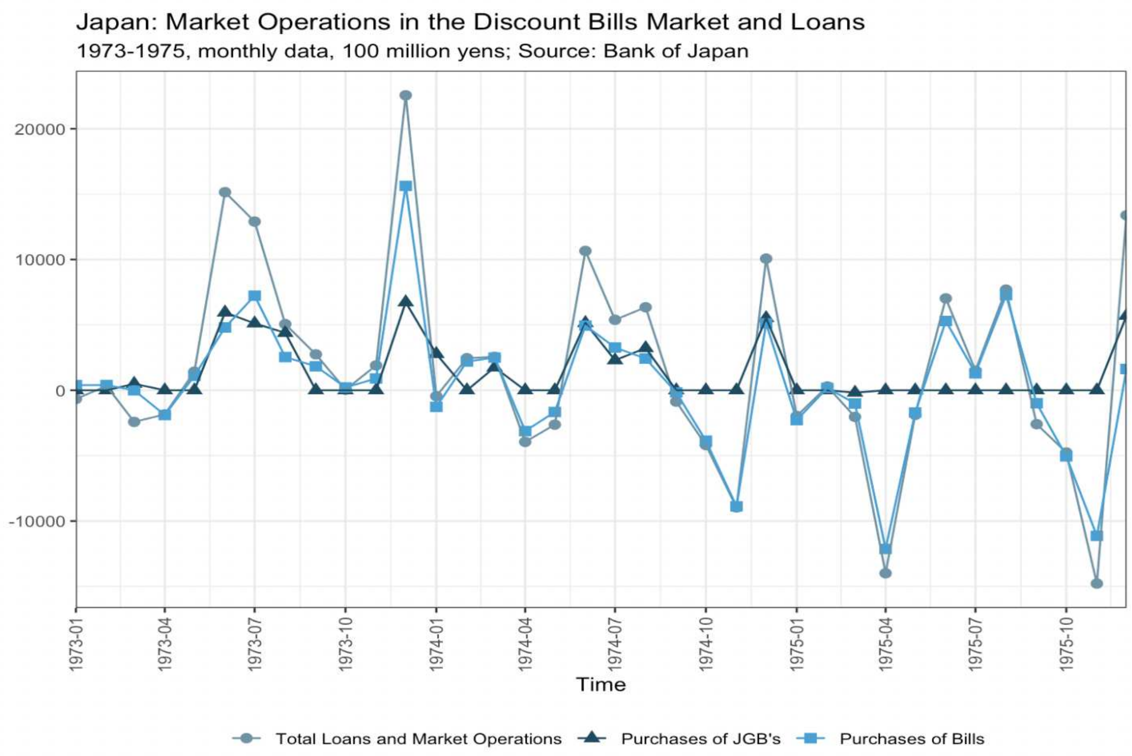



Figure 3: Bank of Japan outright purchases of assets between 1973 and 1975

The corporate bills, $B_t^c$, which here represent the blue bond from the model, i.e. an asset that can be bought or sold by the monetary authority at the market interest rate, performed a different role relative to the model, where, everything else constant, auctioning blue bonds at the private market interest rate offsets part of the increase in the money and price levels, containing inflation.

The public sector budget constraint allows to discuss what may have been the effects on the money level in December 1973 from the loan window borrowings, the selling of government bonds and the purchases of corporate bills. The condition is given by:

$$\frac{B_t}{1+R_t} + \frac{B_t^b}{1+R_t} + \frac{B_t^c}{1+\hat{R}_t} = P_{t-1}\bar{G} + B_{t-1} + B_{t-1}^c + M_{t-1} - T_t - M_t \tag{30}$$

where $t$ is December 1973 and $t-1$ denotes the period before, $B_{t-1} > -\delta$. The interest rate on the loan window, $R_t$, remained constant. For simplification purposes, $P_{t-1}\bar{G} + B_{t-1} + B_{t-1}^c + M_{t-1} - T_t$ is summarised in a function $F(\bar{G}, B_{t-1}, B_{t-1}^c, M_{t-1}, T_t)$, which represents a highly simplified approximation to aggregate wealth available in December 1973 to apply in assets, particularly because, in the model, $M_{t-1} = P_{t-1}C_{t-1}$ and so $P_{t-1}\bar{G} + M_{t-1}$ would be equal to nominal output by market clearing. Then one has that:

$$M_t = F(\bar{G}, B_{t-1}, B_{t-1}^c, M_{t-1}, T_t) + \frac{B_t}{1+R_t} - \frac{B_t^b}{1+R_t} - \frac{B_t^c}{1+\hat{R}_t} \tag{31}$$

Where there is a plus sign before the assets in the loan window in 1973, since $B_t < 0$, banks being net borrowers in the loan window.

Everything else constant, the money level would have increased as banks borrowed as much as possible from the loan window, $B_t$ approaching $\delta$, and government bonds were bought until a certain limit the Bank of Japan was willing to accept. The open market operation conducted by the Bank of Japan decreased $B_t^c$, increasing $M_t$ together with the effect of the loan window.

From figure 2, this implied increase in the money level would have coincided with the sharp rise in inflation at onset of 1974: year-over-year inflation was around 15% before December 1973, and rapidly reached 22% by January. As seen in figure 4, the M1 and M2 growth rates stayed close to 20% and 15%, respectively. Output decreased by 1.7% relative to the previous year, and 3.4% relative to the previous quarter[8].

From the model's perspective, money growing below inflation while output contracts, together with the described dissociation between rates, which mirrors the existence of restrictions when borrowing at the official rate, are all features of an economy that moves from a state with an interior equilibrium into a corner solution, where all signs point towards Japan being at the first in 1972/73 and entering the latter in late 1973/beginning of 1974.

[8]Ito (2009),p.37

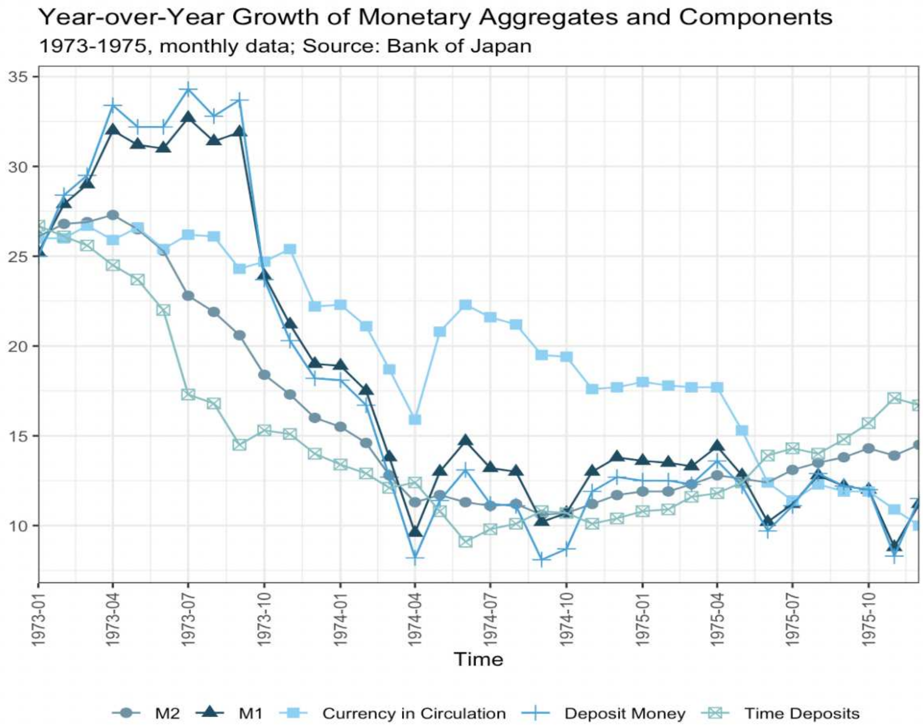



Figure 4: Year-over-year growth of M1, M2 and components.

By the end of 1974 and in 1975, there was a slow down in the call rate trajectory, with this rate progressively decreasing and converging towards the official interest rate. Furthermore, the growth of M1 and M2 stabilised around 12%, as seen in figure 4, while inflation sharply decelerated in late 1974, from 25% to 20%, and fell below 10% in the second semester of 1975, as depicted in figure 2. These features point towards the year of 1975 as being a period of transition towards a different equilibrium relative to the corner solution.

As discussed in section 2.6, and as represented by conditions (27) and (28), a real government deficit is required to have a recovery period a sufficiently high deficit provides the private sector with enough aggregate income to demand money to recover consumption and to accumulate assets once again, these assets being associated with the official rate.

Recall that, in the model, the real deficit, captured in $\bar{G} - \frac{T_t}{P_{t-1}}$, is obtained by lowering real taxes via a reduction in nominal taxes, being ambiguous whether nominal lump-sum taxes decrease sufficiently to become a subsidy, as this would depend on the size of government consumption. Furthermore, the deficit must be sufficiently higher than the seigniorage gains obtained during recovery, so that the government issues new nominal liabilities.

In reality, taxation is more complex than lump-sum taxes, and a higher fiscal deficit can be obtained if, for example, tax revenues collected were lower or government spending increased sufficiently, the government assuming an expansionary stance, where this latter scenario could not be captured by the model.

Most fiscal variables for the period are available as a percentage of nominal GDP. Using the information in table 2, which presents nominal GDP at current prices and gross inflation from 1973 until 1975, one can not only recover the level of the nominal fiscal variables, but also present these variables in real terms, adjusting them to 1972 prices, the year in which this analysis started.

| Table 2: Nominal GDP and Gross Inflation (Billion Yen) | | |
|---|---|---|
| Year | Nominal GDP | Inflation |
| 1973 | 119,950 | 1.05 |
| 1974 | 143,130 | 1.116 |
| 1975 | 158,150 | 1.232 |
| Source: OECD, FRED | | |

For example, to write a nominal variable in 1974, $X_{74}$, in real terms, $x_{74}$, then one would multiply the nominal variable by the the base year's consumer price index, 1972, and divide by the CPI in the current year: $x_{74} = \frac{X_{74}CPI_{72}}{CPI_{74}}$. This is equivalent to divide by the product of year-over-year inflation rates until the period of the variable, $x_{74} = \frac{X_{74}}{(1+\pi_{73})(1+\pi_{74})}$.

Table 3 presents the nominal and real tax revenues, without social security contributions, while table 4 has the government spending, without interest rate payments.

| Table 3: Real Tax Revenues at 1972 prices (Billion Yen) | | |
|---|---|---|
| Year | Nominal Tax Revenues | Real Tax Revenues |
| 1973 | 14,049 | 13,380 |
| 1974 | 15,755 | 13,445 |
| 1975 | 14,506 | 10,048 |
| Source: OECD | | |

| Table 4: Government Primary Spending (Billion Yen) | | | |
|---|---|---|---|
| Year | Spending as % GDP | Nominal Spending | Real Spending |
| 1973 | 22.46 | 26,941 | 25,658 |
| 1974 | 24.6 | 35,210 | 30,048 |
| 1975 | 27.36 | 43,270 | 29,972 |
| Source: IMF | | | |

In Japan, nominal taxes went down by 8% in 1975 relative to 1974, real tax revenues decreased 25%, and real government spending decreased by 0.25%. Taking the difference between real government spending and real taxes as a proxy to the real government deficit, the latter was of 16,603 units in 1974, increasing by 20% to 19,923 units in 1975. Similar to the model, the real deficit is almost totally generated through a reduction in taxes, as real government consumption is close to constant.

Table 5 reports the stock of public debt owed to the private sector at the end of the period.

| Table 5: Stock of Public Debt (Billion Yen) | | | |
|---|---|---|---|
| Year | Debt as % GDP | Nominal Debt | Real Debt |
| 1973 | 10.61 | 12,277 | 11,645 |
| 1974 | 10.96 | 15,687 | 13,387 |
| 1975 | 14.02 | 22,173 | 15,359 |
| Source: IMF | | | |

In the corner, the model tells that public debt should fall relative to the period before, as the official rate would be too low to save through treasuries when considering expected and then realised inflation. As explained before, a considerable restriction on the Japanese context was that a specific set of banks had to compulsory underwrite newly-issued government bonds, provided that the Bank of Japan committed to purchase the securities at a premium in the near future. As such, the interpretation of the change in the stock of debt, from 1973 to 1974, and then relative to 1975, is limited by this fact, which is a friction unparalleled in the model.

Still, according to the model, the stock of real debt would have only increased by 14.7%, if the size of the real deficit surpassed the gains from seigniorage, making the real deficit observed sufficiently high. This "fiscal intervention", whereby nominal and real taxes decreased to generate a real deficit that reestablished debt, would have been a sufficient condition to satisfy the inequality (28) and observe a coincidence between the official and the call rate by the end of 1975. This would imply that the economy was again at an interior equilibrium where the effectiveness of the interest rate set by the monetary authority over inflation was recovered.

From figure 2, even though the two rates were on average separated by Horiuchi's difference of 2 p.p in 1975, it is noteworthy that the call rate appears to be very responsive to the downward changes in the official interest rate, suggesting that the association between rates, which characterises an equilibrium outside of the corner solution, was present once again. A full coincidence between the two rates is only observed in 1976.

Additionally, in figure 5, which provides an annual outline of the main macroeconomic aggregates over the period in question, notice that output recovered in 1975, and then started to grow at a steady growth rate from 1976 onwards, while inflation slowed down and was surpassed by the growth rate of money, in line with what would be observable in the model's recovery period. As a side note, figure 5 also reflects the features in terms of inflation, output and money characteristic of an equilibrium at the corner in 1974, specifically money growing below inflation and a recession in output.

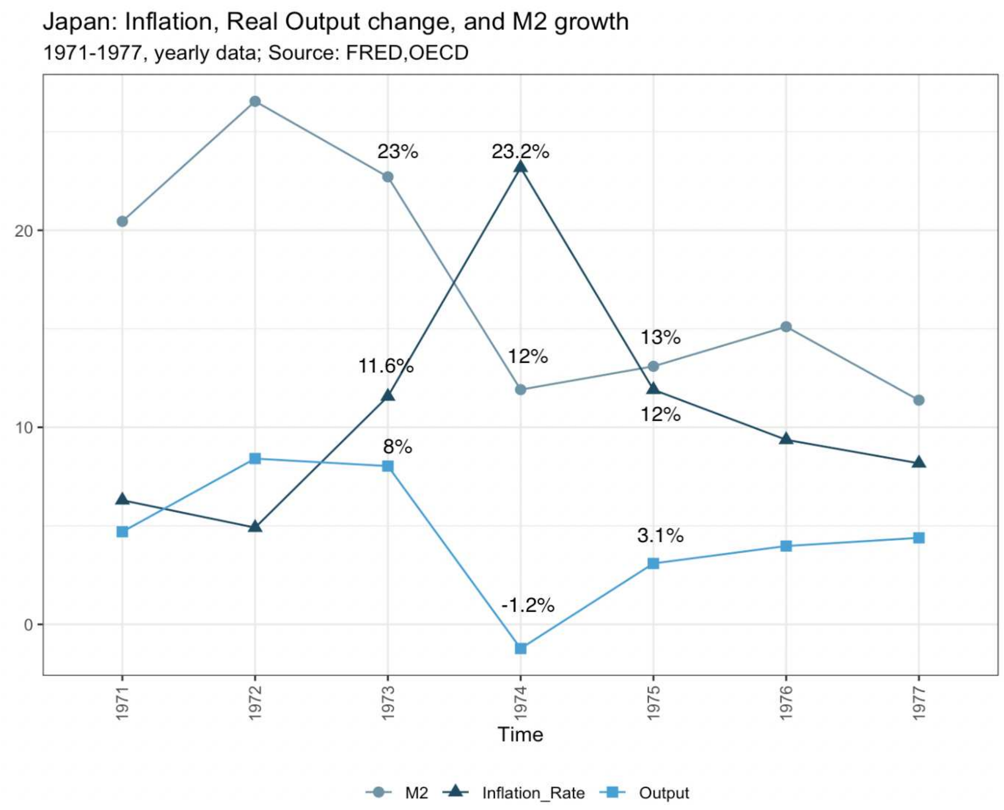



Figure 5: Inflation, Output Growth, and M2 Growth

Nevertheless, the observed fluctuations in output must be approached with care since the model omits two important drivers of the business cycle: government consumption and total factor productivity shocks. Besides, investment is not considered. From the model standpoint, these fluctuations in output would be interpreted by comparing output levels in states characterised by different interest rates.

In this regard, these features would point towards the existence of an equilibrium shadow interest rate in 1974, taken as the opportunity cost of money from the perspective of households, that raised sufficiently above the official interest rate, contributing to reduce consumption and output. However, the call rate does not need to be this shadow interest rate.

This question cannot be answered empirically, since the model cannot be calibrated to solve an exercise of the type “given the data available, what could have been a sufficiently high opportunity cost of money in 1974?”, making the following a theoretical discussion.

Recall the model’s Euler condition for consumption, which is common to more advanced models that include a physical asset whose marginal productivity defines the real interest rate:

$$E_t\left(\beta\frac{u_c(t+1)}{u_c(t)}\frac{P_t}{P_{t+1}}\right)=\frac{1}{1+R_t} \tag{32}$$

Until 1973, the Japanese economy was consistently growing at rates around 7%, reflecting positive real interest rates, consumption being postponed and savings increasing.

From the Euler condition, high expected inflation between 1973 and 1974 would have led the household to anticipate consumption and reduce savings.

Considering that households were not aware in 1974 of what realised inflation would come to be, whatever was taken by them as the opportunity cost of money in this period must have been sufficiently high such that, given the expectation for inflation, the expected real interest rate was positive, consumption being postponed to generate savings and enabling a recovery in 1975.

The dissociation of the call rate from the official interest rate is indicative that other returns in the Japanese economy diverged from the official interest rate, as the call rate became the relevant benchmark for the funding cost of banks, but it is in no way indicative that the call rate was the equilibrium interest rate for which the intertemporal choices of consumption were satisfied.

If households expected inflation to remain high then the call rate would have been by all means too low to allow for this. However, Sargent (1984) showed how, in high-inflation contexts, expectations over inflation are not formed based on its past history, but rather on the present and prospective monetary and fiscal policy.

Households could have looked forward to a high government deficit in 1975, which warranted expectations of a recovery with lower inflation, as the official interest rate would again anchor the inflation rate. In this scenario, the call rate may have been sufficiently high, as households would expect for 1975 inflation rates consistent with a lower official interest rate.

In either case, given the data in figure 5, there is no reason to think that a sufficiently high shadow interest rate was not present in 1974, whether it was the call rate or not, or whether that rate was associated with a financial or non-financial asset, such as the return on a small business or firm, or an investment project that is family-owned, or even the return on land and property, which is difficult to capture in a model without capital or a physical asset.

# 4 Conclusion

The model from Bassetto and Phelan (2015) and the described Japanese experience tell the following. Regardless of why an economy maintains nominal interest rates at a relatively low level, which was the case in Japan in 1972 and 1973, there is the risk of facing an equilibrium where, given expectations of high inflation, agents coordinate a strong borrowing of funds using their assets, both financial or non-financial. Since the borrowing is subject to quantity restrictions, a shadow nominal interest rate that reflects the opportunity cost of money rises above the official interest rate set by a monetary authority, while output and consumption fall, and the price level rises fast with money growing below inflation. This shadow interest can be associated with the inter-bank lending rate but it can also be some implicit rate of return in a private business or a physical asset.

The dissociation and separation above what was historically normal between the official and inter-bank rates, the transfer of assets from the private sector to the central bank, and the behaviour of output, inflation and money, are indicative that Japan faced one of these corner equilibria in 1974. A recovery from the corner equilibrium appears to have started in the end of 1974 and was underway in the following year.

This recovery is observed in the reestablished association between official and inter-bank interest rates, which coincided with a sufficiently high real deficit, associated with lower nominal and real taxes, that supported an increase in public debt, an example of the fiscal intervention described by Bassetto and Phelan (2015) as a necessary condition to move away from a corner equilibrium. Throughout this period, a recovery in output was observed, together with a sharp deceleration of inflation, as Japan converged once again to a balanced growth path.

# 5 Appendices

## 5.1 Laffer Curve for the Inflation Tax

In Bassetto and Phelan (2015) p.109, the authors state that for preferences that satisfy

$$\lim_{c \to 0} u(c_t, y_t) > -\infty \tag{33}$$

"a Laffer curve seems to be present for the inflation tax", which is in line with "The experience of countries that witness hyperinflations". Therefore, one can attempt to extract this Laffer curve for the inflation tax.

Firstly, to understand the limit condition, it is useful to think what would occur if $\lim_{c \to 0} u(c_t, y_t) = -\infty$. In that case, as the interest rate, or the opportunity cost of money, was infinitely raised to generate inflation, seigniorage revenues would still exist because money would still be demanded to consume very small amounts, where further decreases would still provide infinite value. However, if there is a finite lower bound for the value of consumption as it becomes smaller, then the opportunity cost of money cannot be raise infinitely, otherwise the tax base of seigniorage, i.e. the real balances, would disappear converging to 0, the level of consumption for which, having labour constant, this lower bound would be reached.

The isoelastic preferences used by the authors, $u(c_t, y_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - y_t^{\psi}$, satisfy the limit condition if the curvature of the marginal utility of consumption is lower than 1 ($\sigma < 1$). If $1 - \sigma$ is negative, and so $\sigma > 1$, then as consumption would approach 0, utility would approach $-\infty$, and so the condition would not be met.

The government budget at the beginning of a period $t$ in real terms is given by:

$$\bar{G} - \frac{T_t}{P_{t-1}} + \frac{B_{t-1}}{P_{t-1}} = \frac{B_t}{P_{t-1}(1 + R_t)} + \frac{M_t - M_{t-1}}{P_{t-1}} \tag{34}$$

Where:

$\frac{M_t - M_{t-1}}{P_{t-1}}$ are real seigniorage revenues. Using the cash-in-advance (12) and the definition $\frac{P_t}{P_{t-1}} = (1 + \pi_t)$ they can be rewritten as $\frac{M_t}{P_t}\frac{P_t}{P_{t-1}} - \frac{M_{t-1}}{P_{t-1}} = C(R_t)(1 + \pi_t) - C(R_{t-1})$, consumption in period $t$ being the tax base of the distortionary inflation tax $\pi_t$.

Using the deterministic (trend) version of (9), in a steady state where $R_t$ is constant through time, consumption is constant, and inverse inflation is constant as well. For example, if $R_t = \frac{1}{\beta} - 1$, the currency does not depreciate. Seigniorage revenues become $C(R)\pi$. $C(R)$, which is implicitly defined by the intratemporal condition (8), can be written in terms of inflation, using the steady-state Fischer equation, obtaining $C(\pi)\pi$. Since one can have different steady states with different interest rates and inflation, one can write $C(\pi_t)\pi_t$.

To extract infinite seigniorage the government can move to a steady state with a very high interest rate to generate arbitrarily high inflation (the "tax rate") provided that this increase in inflation surpasses the decrease in real balances (the "tax base"). This will depend on the intratemporal elasticity (the responsiveness of consumption to changes in the nominal interest rate) and hence on the curvature of the marginal utility of consumption, $\sigma$.

The objective is to understand the shape $C(\pi_t)\pi_t$ assumes when the curvature is higher than 1, and the limit condition is not met, and when $\sigma < 1$ and the limit condition is met.

On trend, joining the intratemporal condition (8) and the Fischer condition $\beta(1+R_t) = 1+\pi_t$ gives:

$$\psi(C_t+\bar{G})^{\psi-1}C_t^{\sigma} = \frac{\beta}{1+\pi_t} \tag{35}$$

Which can be rewritten as:

$$F(\pi_t, C_t) = \frac{\psi(C_t+\bar{G})^{\psi-1}C_t^{\sigma}(1+\pi_t)}{\beta} - 1 = 0 \tag{36}$$

To approximate $C(\pi_t)$ we need an initial point $(\pi_0, C_0)$, and the first derivative that can be obtained through the implicit function theorem. After obtaining the first derivative, derivatives of higher order can be extracted and a Taylor rule can then be used.

$\bar{G} = 0.1, \beta = \frac{1}{1.01}$. For $\psi = 1.1$ and $\sigma = 3$, $(\pi_0, C_0) = (0, 0.964)$, and for $\psi = 1.1$ and $\sigma = 0.5$, $(\pi_0, C_0) = (0, 0.823)$. (The values for the parameters follow the ones used by the authors and the results for initial consumption were obtained using the software Mathematica).

For $C_t = F(\pi_t)$, the implicit function theorem can only be used if and only if:

$$\frac{\partial F(\pi_t, C_t)}{\partial C}|_{(\pi_0, C_0)} \neq 0 \tag{37}$$

$\frac{\partial F(\pi_t, C_t)}{\partial C}$ is equal to :

$$\frac{\partial F(\pi_t, C_t)}{\partial C} = \frac{\psi(1+\pi_t)}{\beta}\left[(\psi-1)C_t^{\sigma}(C_t+\bar{G})^{\psi-2} + \sigma C_t^{\sigma-1}(C_t+\bar{G})^{\psi-1}\right] \tag{38}$$

For both initial points, the partial derivative is different 0: For $(\pi_0, C_0) = (0, 0.964)$, the partial derivative is equal to 3.21 .For $(\pi_0, C_0) = (0, 0.823)$ it is equal to 0.716.

The implicit function theorem states that, if the above condition is met, then we can write:

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial x} = 0 \tag{39}$$

or

$$\frac{\partial F}{\partial \pi} + \frac{\partial F}{\partial C}\frac{\partial C}{\partial \pi} = 0 \tag{40}$$

From where:

$$\frac{\partial C}{\partial \pi} = -\frac{F_{\pi}}{F_c} \tag{41}$$

The partial derivative of $F(\pi_t, C_t)$ to inflation is given by:

$$F_{\pi} = \frac{\psi}{\beta}(C_t + \bar{G})^{\psi-1}C_t^{\sigma} \tag{42}$$

And with respect to consumption we have:

$$F_C = \frac{\psi(1+\pi_t)(C_t + \bar{G})^{\psi-1}C_t^{\sigma}}{\beta}\left[(\psi-1)(C_t+\bar{G})^{-1} + \sigma C_t^{-1}\right] \tag{43}$$

Thus, the first derivative $\frac{\partial C}{\partial \pi}$ is given by:

$$\frac{\partial C}{\partial \pi} = -\frac{1}{(1+\pi_t)\left[(\psi-1)(C_t+\bar{G})^{-1} + \sigma C_t^{-1}\right]} \tag{44}$$

Notice that, when taking derivatives of a higher order, $\left[(\psi-1)(C_t+\bar{G})^{-1} + \sigma C_t^{-1}\right]$, is always present and does not change, so we call it $\rho$. For $(\pi_0, C_0) = (0, 0.964)$, $\rho = 3.206$, and for $(\pi_0, C_0) = (0, 0.823)$ $\rho = 0.716$. For example:

$$\frac{\partial^2 C}{\partial^2 \pi} = \frac{1}{(1+\pi_t)^2\left[(\psi-1)(C_t+\bar{G})^{-1} + \sigma C_t^{-1}\right]} = \frac{1}{(1+\pi_t)^2\rho} \tag{45}$$

$$\frac{\partial^3 C}{\partial^3 \pi} = -\frac{2}{(1+\pi_t)^3\rho} \tag{46}$$

$$\frac{\partial^4 C}{\partial^4 \pi} = \frac{6}{(1+\pi_t)^4\rho} \tag{47}$$

A Taylor expansion up to the fourth degree is given by:

$$C(\pi_t) = C(\pi_0) + \sum_{k=1}^{4}\frac{\partial^k C}{\partial^k \pi}|_{(\pi_0, C_0)}\frac{(\pi_t - \pi_0)^k}{k!} \tag{48}$$

For $\sigma = 3$ and $(\pi_0, C_0) = (0, 0.964)$, the Taylor approximation for consumption as a function of inflation is:

$$C(\pi_t) = 0.964 - 0.312\pi_t + 0.156\pi_t^2 - 0.104\pi_t^3 + 0.078\pi_t^4 \tag{49}$$

For $\sigma = 0.5$ and $(\pi_0, C_0) = (0, 0.823)$, the Taylor approximation is:

$$C(\pi_t) = 0.823 - 1.397\pi_t + 0.698\pi_t^2 - 0.466\pi_t^3 + 0.349\pi_t^4 \tag{50}$$

Using a graphing software to plot the intratemporal condition and then the approximations, it is possible to see that (51) is a considerably good approximation for $\pi_t \in [0, 1]$, while (52) could be improved, most likely by choosing a different set of initial values.

Therefore, consider $(\pi_0, C_0) = (0.4, 0.464)$ (Again, the value for consumption was obtained from Mathematica). For this set of initial values, $\rho = 1.255$ and the Taylor approximation becomes:

$$C(\pi_t) = 0.464 - 0.766(\pi_t - 0.4) + 0.368(\pi_t - 0.4)^2 - 0.236(\pi_t - 0.4)^3 + 0.17(\pi_t - 0.4)^4 \quad (51)$$

Which is an improvement on the one obtained using $(\pi_0, C_0) = (0, 0.823)$.

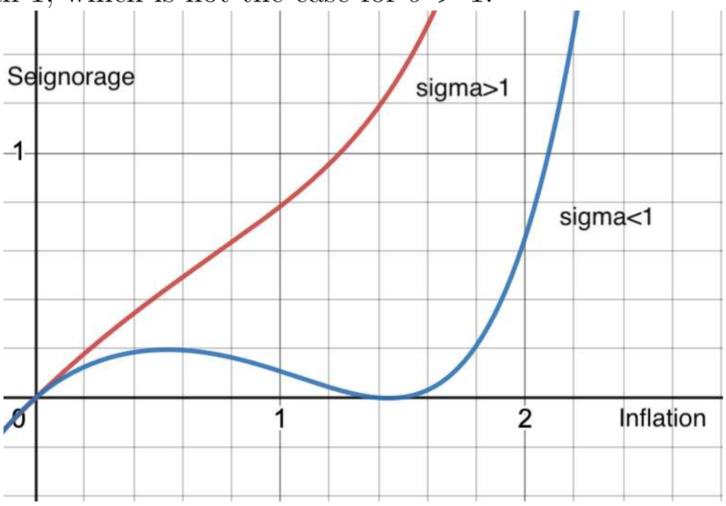
Thus, seignorage revenues for $\sigma = 3$ are approximated as:

$$C(\pi_t)\pi_t = 0.964\pi_t - 0.312\pi_t^2 + 0.156\pi_t^3 - 0.104\pi_t^4 + 0.078\pi_t^5 \quad (52)$$

And for $\sigma = 0.5$:

$$C(\pi_t)\pi_t = 0.464\pi_t - 0.766\pi_t(\pi_t - 0.4) + 0.368\pi_t(\pi_t - 0.4)^2 - 0.236\pi_t(\pi_t - 0.4)^3 + 0.17\pi_t(\pi_t - 0.4)^4 \quad (53)$$

By using the same graphing software and restricting results to $\pi_t$ belonging to $[0, 1]$, it is possible to observe that indeed a "Laffer" curve is present when the curvature is lower than 1, which is not the case for $\sigma > 1$.




## 5.2 Elasticities

Both the intertemporal and the intratemporal elasticities of substitution help to better understand the impact of the values assigned to $\sigma$ and $\psi$, or the curvature of the marginal utility of consumption and the marginal disutility of labour, respectively.

For the author's isoelastic preferences:

$$U(C_t, Y_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - Y_t^{\psi} \quad (54)$$

The marginal utility of consumption is equal to: $u_c(t) = C_t^{-\sigma}$, and the second order derivative is equal to: $u_{cc}(t) = -\sigma C_t^{-(1+\sigma)}$

The marginal disutility of labour is given by: $u_y(t) = -\psi Y_t^{\psi-1}$, and the second order derivative is equal to: $u_{yy}(t) = -\psi(\psi-1)Y_t^{\psi-2}$

### 5.2.1 Intratemporal Elasticity of Substitution

The intratemporal elasticity of substitution is derived from the intratemporal condition:

$$-\frac{u_y(t)}{u_c(t)} = \frac{1}{1+R_t} \tag{55}$$

By substituting the partial derivatives the condition becomes:

$$\psi(Y_t)^{\psi-1}C_t^{\sigma} = \frac{1}{1+R_t} \tag{56}$$

The intratemporal elasticity of substitution is thus the percentual change in consumption when the interest rate changes by 1%, or $\varepsilon_{C,1+R}$. To obtain this elasticity both the intratemporal and market clearing conditions are log-linearised, and then their are joined together. Recall that $ln(\frac{C_t}{C})$ represents an approximation to the relative change from $C$ to $C_t$, which can also be represented as $\frac{\mathrm{d}C_t}{C}$, where $\mathrm{d}C_t$ is the absolute difference between $C_t$ and $C$, which is then divided by the starting value to obtain the relative difference. Thus, $ln(\frac{C_t}{C}) \approx \frac{\mathrm{d}C_t}{C}$.

By applying the logarithm to both sides of the intratemporal condition and subtracting its lagged version, the log-linearised intratemporal condition is given by:

$$(\psi-1)\hat{Y} + \sigma\hat{C} = -\tilde{R} \tag{57}$$

Where, $\tilde{R}$ is the relative difference in the gross nominal interest rate $(1+R_t)$.

The log-linearised market clearing condition is given by:

$$C\hat{C} = Y\hat{Y} \tag{58}$$

From here it is straightforward that $\hat{Y} = \frac{C}{Y}\hat{C}$, which can be used in the intratemporal condition to write:

$$(\psi-1)\frac{C}{Y}\hat{C} + \sigma\hat{C} = -\tilde{R} \tag{59}$$

That can be simplified to:

$$\hat{C} = -\frac{\tilde{R}}{\frac{(\psi-1)C}{Y} + \sigma} \tag{60}$$

Therefore, the intratemporal elasticity of substitution is given by:

$$\varepsilon_{C,1+R} = \frac{\frac{\mathrm{d}C}{C}}{\frac{\mathrm{d}(1+R)}{(1+R)}} = \frac{\hat{C}}{\tilde{R}} = -\frac{1}{\frac{(\psi-1)C}{Y} + \sigma} \tag{61}$$

Where $C$ and $Y$ are the starting values for consumption. The important feature to take from this elasticity is that a higher curvature both in the marginal utility of consumption and in the marginal disutility of labour decreases the intratemporal elasticity. For example, everything else constant, for a higher $\sigma$, the consumption fall in response to an increase in the interest rate in the private markets during a run would be lower.

### 5.2.2 Intertemporal Elasticity of Substitution

The intertemporal elasticity of substitution is derived from the intertemporal consumption condition, which is given by:

$$\frac{u_c(t+1)}{u_c(t)} = \frac{Q_{t+1}}{\beta}\frac{P_{t+1}}{P_t} \tag{62}$$

In the deterministic case, $Q_{t+1}$ is simply $\frac{1}{1+R_t}$. Considering the preferences stated above, the Euler condition is given by:

$$\frac{C_{t+1}}{C_t} = \left(\beta(1+R_t)\frac{P_t}{P_{t+1}}\right)^{\frac{1}{\sigma}} \tag{63}$$

Where $R_t$ is the opportunity cost of money in the asset markets of period $t$. It is straightforward that the elasticity of the consumption growth rate with respect to changes in the real interest rate, $1+r_t = (1+R_t)\frac{P_t}{P_{t+1}}$ is equal to:

$$\varepsilon_{\frac{C_{t+1}}{C_t},1+r_t} = \frac{1}{\sigma} \tag{64}$$

Therefore, the higher the curvature in the marginal utility, the lower is the intertemporal change in consumption, or the lower the effects of a change in the real interest rate on the willingness to save in a period $t$, which corresponds to the choice of applying the real wealth in $t$, $\frac{P_{t-1}Y_{t-1}}{P_t} - \frac{T_t}{P_t}$, between either more consumption $\frac{M_t}{P_t}$, or more savings $\frac{B_t}{P_t}$ so to increase wealth in $t+1$.

When considering stochastic equilibria, $\sigma < 1$. Recall that this upper bound on the curvature is imposed by the need to satisfy $\lim_{C\to\infty} u(c_t, y_t) > -\infty$. This condition ensures that the present value of seigniorage remains finite allowing to establish a Ricardian policy that guarantees stable debt sequences.

With uncertainty, the willingness to save is not only affected by the curvature of the marginal utility but also by the probability of a run occurring, which, being small enough, partly offsets the borrowing incentives coming from a potential run next period.

This can be seen by log-linearising the consumption Euler equation with uncertainty, which is given by:

$$\beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{P_t}{P_{t+1}}\right] = \frac{1}{(1+R_t)} \tag{65}$$

The log-linearised version being:

$$E_t(\hat{C}) = \frac{1}{\sigma}\left[\tilde{R} - E_t(\hat{\pi})\right] \tag{66}$$

Where, $\hat{C}$ is the consumption change and $\tilde{R}$ and $\hat{\pi}$ are the deviations of the gross nominal interest rate $(1 + R_t)$ and gross inflation $(1 + \pi_{t+1})$ from their deterministic steady state levels, respectively. In the deterministic steady state, for a constant interest rate in all periods, the consumption change is 0.

Furthermore, notice that there is no expectation formed around the deviations of the gross nominal interest rate: in the markets of period t where the consumption/saving choice is decided, the interest rate for the period is known beforehand.

The expectations for the inflation deviations depend on the probability of a run occurring in the next period. Thus, they are written as:

$$E_t(\hat{\pi}) = \phi\left(\frac{\frac{P^H_{t+1}}{P_t}}{1+\pi} - 1\right) + (1-\phi)\left(\frac{\frac{P^L_{t+1}}{P_t}}{1+\pi} - 1\right) \tag{67}$$

Where $1 + \pi$ is the deterministic steady state inflation, which is equal to 0 when the interest rate $1 + R$ is optimally set to $\frac{1}{\beta}$. The expression can be simplified to:

$$E_t(\hat{\pi}) = \phi\left(\frac{P^H_{t+1}}{P_t} - 1\right) + (1-\phi)\left(\frac{P^L_{t+1}}{P_t} - 1\right) = \phi\left[\left(\frac{P^H_{t+1}}{P_t} - 1\right) + \frac{1-\phi}{\phi}\left(\frac{P^L_{t+1}}{P_t} - 1\right)\right] \tag{68}$$

Replacing this expression in the linearised Euler condition stated above and putting $\phi$ in evidence gives:

$$E_t(\hat{C}) = \frac{\phi}{\sigma}\left[\frac{\tilde{R}}{\phi} - \left[\left(\frac{P^H_{t+1}}{P_t} - 1\right) + \frac{1-\phi}{\phi}\left(\frac{P^L_{t+1}}{P_t} - 1\right)\right]\right] \tag{69}$$

Where $\left[\frac{\tilde{R}}{\phi} - \left[\left(\frac{P^H_{t+1}}{P_t} - 1\right) + \frac{1-\phi}{\phi}\left(\frac{P^L_{t+1}}{P_t} - 1\right)\right]\right]$ is the expected real interest rate as of $t$. Therefore, the intertemporal change in consumption for a change in the expected real interest rate is given by:

$$\varepsilon_{E_t(\hat{C}),E_t(r)} = \frac{\phi}{\sigma} \tag{70}$$

For stochastic equilibria, the requirement that the curvature has to be lower than 1 would imply a high intertemporal effect over the household, the saving decision being highly responsive to expected changes in the real interest rate. The possibility of a run could affect the willingness to save in such a way that the household would move away from the steady state in the period before a run. However, that effect is partially absorbed by a low probability of occurrence, and a small $\phi$ guarantees that the intertemporal elasticity of substitution is sufficiently low. This is in line with the statement of the authors that:

"(...) the potentially negative effect of a run on the households' willingness to save between periods 1 and 2 is tempered by the lower probability of the occurrence".

Besides, notice that for $\phi$ equal to 1, (69) is the linearisation of the deterministic (63)[9].

## 5.3 Planner Problem and First Best

The Pareto Optimum in the model is the solution to a planner problem without markets, prices or money where labour is chosen and a consumption good is allocated by a benevolent ruler so to maximise the expected utility of a representative Household constrained by the available resources and technology. Formally, one has that:

Max $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, y_t)$ subject to $C_t + \bar{G} = Y_t$, which already considers $F(Y_t) = Y_t$.

The conditions that determine $\{C_t, Y_t\}_{t=0}^{\infty}$ are given by:

$$-\frac{u_y(t)}{u_c(t)} = 1 \tag{71}$$

and

$$C_t + \bar{G} = Y_t \tag{72}$$

This real-only economy lacks an intertemporal dimension, this is, a dynamic condition relating consumption in the present and the future, since there is no physical asset with which the Household could move production over time.

By comparing the first-best marginal condition (71) with the equilibrium marginal condition (7), the interest rate should optimally be set at $R_t = 0$, which is otherwise known as the Friedman Rule, eliminating the inflationary tax that distorts the decision to work and consume at the margin.

However, and going back to the model, setting $R_t = \frac{1}{\beta} - 1$, for a discount factor close to 1, makes $R_t$ close to 0 while allowing the cash-in-advance constraint to hold with equality, which is not the case for when $R_t = 0$ and money has no opportunity cost, the household holding money above what is needed for transactions.

## 5.4 Ricardian Coefficient and the Steady State Level of Debt

The government budget at the beginning of a period $t$ in real terms is given by:

$$\bar{G} - \frac{T_t}{P_{t-1}} + \frac{B_{t-1}}{P_{t-1}} = \frac{B_t}{P_{t-1}(1+R_t)} + \frac{M_t - M_{t-1}}{P_{t-1}} \tag{73}$$

Taxes $T_t$ are set according to the Ricardian tax rule that satisfies (20): $T_t = \alpha(B_{t-1} + M_{t-1}) + P_{t-1}\bar{G} - \beta(1+R_t)C(R_t)P_{t-1}\frac{u_y(t)}{u_y(t-1)}$, $\alpha$ being the share of debt received by the household in $t$ that is taxable. Replacing $\frac{T_t}{P_t}$ in the government budget constraint by the

[9]the linearisation of (63) is simply $\hat{C} = \frac{1}{\sigma}\left[\tilde{R} - \pi_{t+1}\right]$, where $\pi_{t+1} = \frac{P_{t+1}}{P_t} - 1$

tax rule and evaluating it at the steady state[10] provides the steady state level of nominal debt, $B$, which is:

$$B = P\frac{C(R)(1-\alpha)}{\beta - 1 + \alpha} \tag{74}$$

Real debt remains constant as long as the interest rate is constant, while nominal debt fluctuates with the price level one-for-one.

By taking a derivative of $B$ with respect to $\alpha$, it becomes clear that the higher the $\alpha$ the lower the steady state level of debt, as one would suppose. Also, a higher $\alpha$ implies that, after a run takes place, debt accumulates at a slower pace, taxes being higher overtime.

[10]To solve for the steady state, $R_t = R$, $C(R_t) = C(R)$ and $Y(R_t) = Y(R)$ for all $t$. Thus, $(1+R) = \frac{1+\pi}{\beta}$, $P_t = P(1+\pi)^t$, $M_t = M(1+\mu)^t$, $B_t = B(1+\eta)^t$. The cash-in-advance imposes that $\pi = \mu$. One can guess that $\eta = \pi$, which is then verified when solving.

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