



Investment Strategies in Dynamic Economies:
*Application of the Bakshi Chen Dong Valuation Model on a
Sample of 25 Stocks*

Ayline Sarkis

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Keywords: *stock valuation, BCD model, mispricing, earnings-to-price, size, momentum, return, investment, underpriced, overpriced.*

Abstract: This dissertation studies the effectiveness of the Bakshi-Chen-Dong (2001) valuation model (BCD) in selecting stocks for investment. The BCD model has a closed-form solution where the firm's equilibrium stock price is a function of its (1) current net earnings per share, (2) expected growth of earnings, and (3) current interest rates. A stock is deemed mispriced whenever the percentage difference between the model price and the actual market value of the stock is far from zero. The performance of this mispricing measure in picking profitable stocks is compared to three indirect valuation indicators (earnings-to-price, size, and past return momentum) in the case of 25 stocks in the Dow Jones Industrial Average Index of 2022. First, alike Dong (2000), the mispricing measure is shown to be close to zero, and mean-reverting every ten months, allowing for potentially profitable trades. Second, when taken explicitly, exploiting the model mispricing is proven to be the most profitable strategy in picking stocks for investment over a one-month time horizon. Finally, a strategy that combines high momentum stocks with low mispricing showed the best performance among all studied.

Estratégias de Investimento em Economias Dinâmicas: *Aplicação do Modelo de Avaliação Bakshi Chen Dong em uma Amostra de 25 Ações*

Ayline Sarkis

Palavras-Chave: *avaliação de ações, modelo BCD, erro de avaliação, relação lucro/preço, tamanho, dinâmica, retorno, investimento, subvalorizado sobrevalorizado.*

Resumo: Esta tese estuda a eficácia do modelo de avaliação (BCD) de Bakshi-Chen-Dong (2001) na seleção de ações para investimento. O modelo BCD fornece uma solução fechada na qual o preço de equilíbrio das ações da empresa é uma função de (1) seu lucro líquido atual por ação, (2) crescimento esperado dos lucros e (3) taxas de juros atuais. Uma ação é considerada mal avaliada sempre que a diferença percentual entre o preço do modelo e o valor real de mercado da ação está longe de zero. O desempenho desta medida de avaliação incorreta na seleção de ações lucrativas é comparado com três indicadores de avaliação indiretos (relação lucro/preço, tamanho e dinâmica de retorno passado) no caso de 25 ações do Dow Jones Industrial Index Average 2022. Primeiro, tal como Dong (2000), a medida de mispricing é próxima de zero e reverte para a média a cada dez meses, permitindo negociações potencialmente lucrativas. Em segundo lugar, quando tomada explicitamente, a exploração da avaliação incorrecta do modelo revela-se a estratégia mais lucrativa para seleccionar acções nas quais investir num horizonte de um mês. Uma estratégia que combina ações de alto momentum com baixo mispricing apresentou o melhor desempenho entre todas as estudadas.

Table of Contents

1 	Introduction	1
2 	Literature Review	4
	2.1 Intrinsic and Relative Equity Valuation Methods	4
	2.2 Structural Equity Valuation Methods	6
	2.3 Stock Selection Strategies	9
3 	The Model	11
	3.1 Model Assumptions	11
	3.2 Equilibrium Stock Price	13
4 	Data Description	15
5 	Model Implementation and Mispricing Measure	17
	5.1 Parameter Estimation	17
	5.2 Rolling Process Implementation	19
	5.3 Construction of the Mispricing Measure	20
	5.4 Estimated parameters	20
6 	Overview of Results	22
	6.1 BCD Model Results	22
	6.2 Mispricing Mean-Reverting Property	24
7 	Comparison of Stock Selection Strategies	27
	7.1 Construction of the Other Valuation Measures	27
	7.2 Descriptive Statistics of Each Measure	28
	7.3 Investment Performance of Stock Selection Strategies	32
8 	Conclusion	37
A 	References	39
B 	Appendices	41

List of Tables

Table 1. Data Gathered and Total Number of Observations	15
Table 2. Summary Statistics of the Inputs to the BCD Stock Valuation Model	16
Table 3. Description of the Structural Parameters	18

Table 4. Estimated Parameters of the BCD model	21
Table 5. Summary Statistics on Mispricing	22
Table 6. Average of SSE and absolute MP for Each Stock	23
Table 7. Summary Statistics of Firm Characteristics for the Filtered Data	28
Table 8. Pearson Correlation Matrix for the Filtered Data	29
Table 9. Characteristics of Sorted Tercile Portfolios	30
Table B.1. List of Stocks in the Sample	41
Table B.2. Summary Statistics of Firm Characteristics for the Unfiltered Data	41
Table B.3. Characteristics of Bi-Dimensionally Sorted Portfolios	42

List of Figures

Figure 1. Illustration of the BCD Monthly Average Results for the Entire Sample	24
Figure 3. Mispricing Reversals Across Terciles	25
Figure 4. Autocorrelation Plot for the Most Undervalued Tercile	26
Figure 2. Average Ret+1 by Tercile Portfolios MP and EP	31
Figure 5. Relationship Between Measures and Future Returns	33
Figure 6. Future Returns on Bi-Dimensionally Sorted Portfolios	34

List of Codes

Code 1. R(t) Parameter Estimation under the Objective Probability Measure	44
Code 2. Implementation of the BCD Model Rolling Process	45

1 | Introduction

The quest for optimal stock selection strategies has long captivated the interests of investors and researchers alike. This paper contributes to the ongoing discussion by investigating the structural model of equity valuation proposed by Bakshi and Chen (2005) and generalized by Dong (2000). In addition, it evaluates the performance of its model-based mispricing measure in selecting profitable stocks, relative to established methods.

The mispricing measure posits that a stock is undervalued if its current market price is lower than its equilibrium price as determined by the Bakshi, Chen, and Dong model (Chen & Dong, 2001), hereafter BCD model. Similarly, a stock is considered overvalued if its market price is higher than the BCD model price. Three main premises guide the BCD model of stock valuation. First, shareholders are entitled to continuous dividend payments, which correspond to a fixed fraction of net earnings per share (EPS), with an added noise component. Second, adjusted net earnings per share (i.e., EPS adjusted for negative values) are assumed to follow a stochastic process. Accordingly, adjusted EPS, and thus dividends, are subject to random fluctuations caused by both firm-specific and broader economic factors. The considered stochastic process assumes that the adjusted EPS growth rate follows a mean-reverting stochastic process. This aligns with the idea that growth rates tend to stabilize as a company matures. Finally, the instantaneous interest rate also follows a mean-reverting stochastic process, consistent with the Vasicek (1977) term structure of interest rates. Based on (1) current EPS, (2) expected future EPS, and (3) current interest rates, the BCD model price is calculated.

The BCD model remains highly relevant today, due to its emphasis on earnings rather than effectively paid dividends to determine shareholders' total payoff, and consequently share value. This focus reflects the evolving financial environment, notably in sectors like technology, where traditional dividend-based valuation models, such as the dividend discount model (DDM), fail to deliver. The necessity for a model that accurately values zero-dividend stocks is highlighted by the growing prevalence of reinvestment strategies and stock buybacks, aimed at stimulating growth and innovation (Fama & French, 2001; Grullon & Michaely, 2002). Another feature that underscores the significance of the BCD model is its parametrization of EPS and its growth

processes. By differentiating between long-term and current EPS growth, this approach helps to gain a deeper understanding of a firm's position within its business cycle. The model theorizes that two stocks can be valued fairly but have significantly different E/P ratios (Chen & Dong, 2001). This is influenced by variations in their business nature or stage in the business cycle.

The BCD mispricing measure can vary significantly. A relatively high or low pricing error could indicate that the model is incorrect, or that the market is wrong in its pricing of a stock. For this reason, it is essential to assess the validity of the BCD mispricing measure. Three hypotheses must be studied: (1) mispricing should be close to zero on average, with no significant fluctuations; (2) it should be mean reverting over time, preferably at a fast speed; (3) and finally, the BCD mispricing should be able to predict future returns to help guide investment decisions. Proving the third hypothesis shows that the pricing errors are due to the market rather than modeling errors (Dong, 2000).

This paper examines the usefulness of the BCD model by analyzing a sample of 25 stocks in the Dow Jones Industrial Average Index (DIJA) of 2022, from January 1990 to December 2000. During this full economic cycle, major economic events like the dot-com bubble occurred, offering a reliable testing ground for the model. By selecting well-established stocks in 2022 and valuing them at earlier dates, we aim to assess the effectiveness of the BCD model at estimating value at a time when these companies were experiencing different stages of growth. This method provides a thorough assessment of the model's robustness and applicability in stock valuation by testing the model's predictive abilities in a range of market conditions and corporate development stages. Due to the complexity of the BCD model, the extended period of our analysis, and the time-intensive nature of the model's algorithm, we limited our analysis to 25 stocks for the 11-year period.

Recognizing the limitations and objectives mentioned, our approach consists of three steps. First, the monthly fair value and mispricing measure of each stock in our sample is estimated using the BCD model. Second, the mispricing measure is evaluated by examining its descriptive statistics and mean-reversion property. Third, we assess the mispricing measure's ability to accurately predict a firm's future returns and compare its capability against three traditional measures: size, earnings-to-price (E/P) ratio, and momentum. The predictive power of all measures is studied by

sorting the sample of stocks into quintile portfolios and analyzing each portfolio's future return. The dataset used covers the period between January 1990 to December 2000.

This dissertation is structured as follows. The literature review in section 2 examines equity valuation approaches and common stock selection strategies for investment decisions. Section 3 discusses the BCD model assumptions and introduces the formula for calculating the BCD equilibrium stock price. Section 4 details our data gathering and cleaning process. In section 5, we explain how the BCD model can be used to calculate the monthly mispricing measure of each stock out-of-sample. In section 6, we outline the results of the BCD model and investigate the mean-reversion property of the mispricing measure. In section 7, we show how the other valuation measures are constructed, and we study the return performance of the sorted quintile portfolios. The conclusion of our study is presented in section 8.

2 | Literature Review

In today's financial markets, equity valuation plays a pivotal role in stock selection decisions. Refining stock valuation models is an ongoing research topic. Valuations are becoming ever more complex, offering more nuanced insights, and enhancing conviction in decision-making. There are countless methodologies for valuing a publicly traded company. These approaches fall under three umbrellas: intrinsic valuation methods, relative valuation methods, and structural valuation methods. Analysts often use a combination of these models to develop a comprehensive analysis of a company's worth and outlook. In sections 2.1 and 2.2, we discuss the three types of equity valuation. Next, we turn our attention to documented methods used to select stocks for profitable investments.

2.1 Intrinsic and Relative Equity Valuation Methods

Intrinsic valuation methods use a company's fundamentals to determine its true value. It includes variations of the discounted cash flow (DCF) model, and the residual income model (RIM). As a rule, people prefer to receive a fixed sum of money today rather than in the future. This concept refers to the time value of money. Fisher's (1930) seminal work, *The Theory of Interest*, posits that the decision to receive money immediately or in the future is contingent upon the prevailing interest rate. This rate encompasses the expected returns on current investments and borrowing costs. Later, Williams (1938) developed this idea and provided an equation for valuing assets. According to him, the value of an asset today corresponds to the present value of its projected future cash flows. His pioneering work has become a cornerstone of modern finance and laid the foundation for the discounted cash flow (DCF) model.

The DCF model assumes that a company's value is determined by the present value of all its future cash flows. Cash flows are typically forecast using two or three-stage growth rates, depending on the company's maturity level. A terminal value accounting for cash flows beyond the forecasting period is also estimated. The choice between using the free cash flows to equity holders (FCFE) or the free cash flows to the firm (FCFF) depends on the context. When considering FCFF, the analyst includes the total cash flows generated by the business and determines enterprise value. On the other hand, FCFE give insight into the value available to

common shareholders, after accounting for the company's financial obligations. For decades the DCF approach has been the choice of investors. They have fine-tuned its accuracy by adjusting for factors like tax implications, and risk through sensitivity analysis and discount rate adjustments.

The Dividend Discount Model (DDM) is a specialization of the DCF based on the premise that the intrinsic value of a stock is fundamentally linked to the dividends distributed to its shareholders. It is primarily used for valuing dividend-paying stocks, and generally assumes scenarios with stable and predictable dividend patterns, including no growth, constant growth, or two/three-stage growth. Within the category of DDMs, the Gordon Growth Model (GGM) stands out as it posits that dividends will perpetually increase at a fixed rate (Gordon, 1962). It is critical to note that although these assumptions streamline the model, they do not accurately reflect the discretionary nature of dividends. Dividend distributions are unpredictable by nature. Management decides whether to payout or reinvest earnings continuously. This introduces forecasting challenges and limits the suitability of the model to companies with a well-established track record of consistent dividend payments and minimal financial risks.

The reliance on dividends as a valuation metric is not without its critics. Modigliani and Miller's (1961) theory of dividend irrelevance challenges the foundation of the DDM. They argue that in a perfect capital market (free from transaction costs, taxes, and information asymmetry), dividend policy does not affect equity valuation. They posit that a firm's value is anchored on its earnings power and investment decisions rather than dividends. This theory suggests that whether a company distributes earnings or reinvests them internally, shareholder wealth remains unchanged. Conversely, Litzenberger and Ramaswamy (1982) contend that dividends negatively affect shareholder wealth since they are taxed less favorably than capital gains. They argue that retained earnings, unlike dividends, have the potential to increase stock value. Several scholars have also argued that dividends fail to reflect a firm's true value, since they dismiss factors such as brand loyalty, customer retention, or the effects of share buybacks (Grullon & Michaely, 2002).

In the cashflow models discussed above, a considerable portion of the value is derived from the terminal value. This terminal value is highly sensitive to the growth rate estimate; even small

adjustments to this rate have a significant impact on the result. The Residual Income Model (RIM) addresses this sensitivity by placing more weight on short-term residual earnings forecasts (Courteau, Kao, & Richardson, 2001). Using this model, a firm's intrinsic value is computed by summing its current book value to the present value of expected future residual income. Residual income is defined as net income less an equity charge. Deducting equity costs accounts for investors' opportunity costs and investment risks. Frankel and Lee (1998) used this model to estimate the value-to-price ratio (V/P) of companies and test its efficiency in predicting stock returns. Ang and Liu (2001) developed a generalization of the RIM, incorporating stochastic measures that reflect dynamic environments.

Relative valuation methods suggest that companies in the same industry, with similar operations and risk profiles should have similar valuations. Companies' market value is evaluated by comparing it with its peers' market value. Choosing the right metric is crucial and depends on the sector and the nature of the business. The ratio between enterprise value and earnings before interest, taxes, depreciation, and amortization (EV/EBITDA) is a commonly used metric. It focuses on a company's operational profitability, independent of its financing decisions, making it ideal for assessing companies with different capital structures. The price-to-book (P/B) ratio compares a company's current market price to the book value of its shares, which is particularly relevant when valuing asset-heavy businesses, such as manufacturing companies. It is less effective, for instance, when valuing technology companies that primarily own intangible assets. In contrast, the price-to-earnings (P/E) ratio, which depicts a stock's current market price relative to its earnings, can be applied to a wide variety of industries, including technology. It provides insights into market sentiment towards a company.

2.2 Structural Equity Valuation Methods

Traditional valuation models rely on a static set of conditions, which do not account for the dynamic nature of financial markets and stakeholders' embedded options. In practice, a firm's leverage, cost of capital, and macroeconomic conditions, are subject to fluctuations over time and shareholders have to take decisions depending on these dynamics. All these materially impact the risk profile of the company and, as a result, its valuation. To address these limitations, dynamic financial models, known as structural models, have been developed.

Merton (1974), Black and Cox (1976), and Vasicek (1977), are seen as the origins to the structural approach as they established the foundational framework for evaluating credit risk and pricing derivative securities. Merton (1974) introduced the idea of modeling a company's equity as a call option on its assets. This strategy was novel since it integrated the ideas of option theory, acknowledging stockholders' limited liability. It states that, like a call option, shareholders essentially hold an "option" on the company's assets, with the right, but not the obligation, to fulfill the firm's obligations beyond their investment. This notion reflects the asymmetric risk profile of equity investments, capturing both the limited downside risk and the limitless upside potential. Using this perspective, provides a more complete framework to be used for equity valuation. Black and Cox (1976) extended Merton's (1974) model by allowing for the possibility of default before the debt's maturity. Their model added more nuance and depth to our understanding of a company's risk profile. Vasicek (1977) further contributed to the field creating a model for the term structure of interest rates, crucial for valuing bonds and other fixed-income securities. These pioneering models, instrumental in advancing derivative securities and bond valuation, set a precedent for incorporating option theory and structural components into financial valuation.

Subsequent structural models built on the earlier models, moving to more dynamic settings with no predetermined liquidation dates. Recognizing that a company can default at any time offered a more realistic and adaptable representation of a firm's financial lifecycle. Leland (1994) led the new wave, introducing a dynamic trade-off theory that considered how businesses strategically manage their debt and equity levels over time. His paper explored the optimal capital structure by balancing the tax benefits of debt against bankruptcy costs. This methodology provided a more sophisticated comprehension of the default process. It expanded the use of structural models in financial valuations by highlighting the significance of strategic debt management and its implications. Additional models, such as the Goldstein-Ju-Leland (GJL) model (2001), advanced the field by focusing on firm fundamentals. By integrating the firm's capital structure decisions with its endogenous default barrier, the GJL model tied the firm's credit risk to its financial and operational strategies. Though it had great potential, the model was not directly used for equity valuation; instead, it was used to analyze the firm's debt structure and default risk. This discrepancy highlighted the challenges in modifying structural models—which were

initially created mostly for bond pricing and credit risk assessment—to meet the complex requirements of equity valuation.

Despite the groundbreaking work, these structural models pose certain challenges when applied directly to equity valuation. Merton (1974), for example views a company's equity as a call option on its assets, but the idea is hard to translate into an actual equity valuation. Practical limitations arise from the model's assumptions, mainly related to the market's capacity to monitor, and respond to a firm's asset value. However, structural models still provide insightful information about equity pricing, particularly when it comes to knowing how leverage, default risk, changes in market conditions and earnings, and strategic debt management affect equity value.

Recent developments in the field have seen the emergence of models tailored for equity valuation, but models with closed-form solutions are still scarce. Eisdorfer, Goyal, & Zhdanov model (2019) – hereafter EGZ model – is one of the few structural models that have been proposed for equity valuation. It is similar to the GJL model, also focusing on firm fundamentals, and incorporating endogenous default, in line with Leland (1994). This model introduces a two-stage valuation process, but only a partial closed-form solution in its second stage. It is unique in that it considers gross margin instead of after-tax earnings (typically used in these models, like in the GJL) as its state variable. The geometric Brownian motion (GBM), used to model the state variable effectively captures the underlying uncertainty and fluctuations within the financial variable. A limitation to the GJL is the GBM's rejection of negative state variables. The EGZ addresses this limitation in scenarios where EBIT values are negative, allowing for broader applicability. Leveraging this approach, Eisdorfer, Goyal, & Zhdanov (2019) applied their model across the entire cross-section of U.S. stocks to demonstrate that investors frequently misprice the default option of stocks. Their findings suggest a potential strategy for financial gain: going long on underpriced shares and short on overpriced ones.

A significant development in structural equity valuation techniques is the model developed by Bakshi and Chen (2005). It is unique in that it integrates several factors, including dynamic capital structure, firm-specific risks, and stochastic interest rates. This model can be viewed as a more realistic evolution of Gordon's (1962) model, with two primary modifications: a dynamic

discount factor and a dynamic expected earnings per share (EPS) growth rate. It assumes that dividends are linked to the firm's EPS, the discount factor follows the stochastic Vasicek (1977) model, and the expected growth rate follows a mean-reverting stochastic process. By using market-implied expectations to determine a firm's future stock price, the Bakshi-Chen (BC) model claims that the market more accurately forecasts a firm's future than analysts do.

Bakshi and Chen (2005) applied the model to firms in the S&P 500 index, the 30 stocks in the Dow Jones Industrial Average Index, and 20 technology stocks. The model prices across these stocks deviated from the actual prices by 8.17% to 23.87%. The pricing errors were also persistent over several months and correlated across stocks, indicating the potential presence of systematic factors not captured by the model. The paper shows that the model had more difficulties pricing high-tech growth stocks, such as Intel, compared to blue-chip stocks like General Electric and Exxon. The significant variation in pricing errors for technology stocks suggests the need for a more complex model to capture the unique factors influencing the growth and pricing of these stocks.

A notable limitation of the Bakshi and Chen model (2005) is its inability to accommodate negative EPS values. Dong (2000) resolved this issue by providing a generalized version of the model that further improved the model's robustness and applicability. A more sophisticated examination of equity valuation was made possible by Dong's adaptation, which added new levels of complexity and realism. This generalization stands as a testament to the evolving nature of structural models, continually improving to provide more accurate and comprehensive tools for equity valuation. This dissertation discusses the Bakshi-Chen-Dong model (2001) in more detail and implements it on a sample of blue-chip stocks.

2.3 Stock Selection Strategies

Investors utilize techniques and analytical tools known as stock selection strategies to find stocks that have the potential to be profitable. These strategies use a variety of metrics and models, including exploiting well-known priced risk factors such as the market risk factor, size, the earnings-to-price (E/P) ratio and momentum.

The Capital Asset Pricing Model (CAPM), introduced by Sharpe (1964) and developed by Lintner (1965), is central in finance, as it translates the trade-off between risk and return. It posits that the expected return of an investment is equal to the risk-free rate plus a multiple of the market risk premium, known as the beta of the investment. The market risk premium represents the additional return that investors anticipate in exchange for assuming the risk that is inherent in the market and thus non-diversifiable. The beta measures the sensitivity of an investment's returns to changes in the overall market. Investing in stocks with higher beta is expected to yield higher returns as a compensation for the increased market risk. Investors can use the CAPM model to identify potentially over/undervalued stocks.

In the seventies, Basu (1977) found that the price-to-earnings (P/E) ratio is correlated to risk-adjusted returns, challenging the Efficient Market Hypothesis (EMH). Stocks with low P/E ratios often outperform those with high P/E ratios due to investors' tendency to overestimate potential returns. Shortly after, Banz (1981) found that smaller firms tend to generate higher risk-adjusted returns than larger ones, suggesting that firm size has an impact on return performance. These findings paved the way for the seminal work of Fama and French (1992), who presented a three-factor model that included size (small minus big or SMB), book-to-market value (high minus low or HMB), and market risk (beta) as factors influencing stock returns.

In a 1993 study, Jegadeesh, and Titman demonstrated the profitability of momentum strategies over three to twelve month holding periods. By forming portfolios based on previous six to twelve-month returns, and buying past winners while selling past losers, investors can achieve higher returns. On a previous work, De Bondt and Thaler (1985) observed that over longer periods of time (three to five years), the trend reverses, and past losers tend to outperform past winners.

While traditional metrics for stock selection - such as the CAPM model, the price/earnings ratio, size, and momentum - do not serve alone to determine a firm's equity value, they are helpful tools for identifying and exploiting investment opportunities.

3 | The Model

The Bakshi-Chen-Dong (2001) stock valuation model is a dynamic model of equity valuation that explicitly accounts for the random nature of earnings and interest rates within a continuous-time, infinite horizon economy. The purpose of the model is to develop a closed-form stock valuation formula for a generic publicly traded company that offers a direct relationship between the stock's fair value and fundamental variables that can be readily observed.

3.1 Model Assumptions

To determine the per-share value at time t , denoted by $S(t)$, the model posits three core assumptions. *First*, the instantaneous interest rate, $R(t)$, follows an Ornstein-Uhlenbeck mean-reverting process:

$$dR(t) = \kappa_r[\mu_r^0 - R(t)]dt + \sigma_r d\omega_r(t) \quad (1)$$

where the long-run mean μ_r^0 , the speed at which $R(t)$ adjusts to its long-run mean κ_r (under the objective probability measure), and the interest rate volatility σ_r are constant. This pricing kernel process, which was adopted from Vasicek (1977) single-factor model for the term structure of interest rates, serves as the economy-wide valuation standard for future payoffs.

Second, a share of a company's stock entitles its holder to a continuous stochastic dividend stream $D(t)$. The firm's dividend policy dictates that at each time t , the dividend is given by:

$$D(t) = \delta Y(t) + \epsilon(t) \quad (2)$$

where δ is the target dividend payout ratio, $Y(t)$ the earnings per share flow at time t (net of all interests, taxes, and expenses), and $\epsilon(t)$ a mean-zero random deviation from the target dividend policy. The payout ratio δ , can vary at each time t , making this parametrization more flexible. $\epsilon(t)$ is assumed to be uncorrelated with any other stochastic variable in the economy, ensuring that the dividend stream, while variable, does not directly depend on other economic factors within the model's framework. The firm's dividend policy is considered exogenous, meaning it is determined outside the model's framework. This includes the firm's investment policy, growth

opportunities, production plan, operating revenues, expenses, and target payout ratio. As a result, the net earnings process, $Y(t)$, indirectly incorporates these aspects.

In recent years, a growing number of companies, have shifted away from paying cash dividends. Instead, they often reinvest earnings or repurchase shares. Equation (2) ensures that even in the case of zero-dividend paying firms, the BCD model can tackle the valuation by linking dividends to current EPS. This approach acknowledges that net earnings are often more indicative of a firm's financial and operating conditions than dividends, supporting Lintner's (1956) groundbreaking work.

Third, earnings (adjusted for negative values) grow at a variable rate, and the growth process is mean reverting. Bakshi and Chen (2005) initially posited that the EPS, denoted as $Y(t)$, adheres to a Geometric Brownian Motion (GBM). This approach prevents the occurrence of negative EPS values. To address this issue, Dong (2000) added a fixed buffer to $Y(t)$, resulting in an adjusted EPS, $X(t)$, defined as:

$$X(t) \equiv Y(t) + y_0 \quad (3)$$

Next, Dong (2000) proposed that $X(t)$ and the expected adjusted-EPS growth, $G(t)$, follow:

$$\frac{dX(t)}{X(t)} = G(t)dt + \sigma_x d\omega_x(t) \quad (4)$$

$$dG(t) = \kappa_g [\mu_g^0 - G(t)]dt + \sigma_g d\omega_g(t) \quad (5)$$

where $G(t)$ is the conditionally expected rate of growth in adjusted EPS, $X(t)$. The volatility of expected EPS σ_x , and the volatility for changes in expected adjusted-EPS growth σ_g , are both time-invariant. The long-run mean for $G(t)$ μ_g^0 , and the speed at which $G(t)$ adjusts to its long-run mean κ_g , are also constant. Furthermore, the term $\frac{1}{\kappa_g}$ is interpreted as the duration of the firm's business growth cycle, providing insight into the length and stages of the firm's growth. This parametrization is crucial as it differentiates between long-run growth and transitory growth

patterns and quantifies the firm's growth cycle. The correlations of $\omega_x(t)$ with $\omega_g(t)$ and $\omega_r(t)$ are respectively denoted by $\rho_{g,x}$ and $\rho_{r,x}$.

3.2 Equilibrium Stock Price

Under the three model assumptions in Section 3.1, the equilibrium stock price $S(t)$ is given by:

$$S(t) = \delta \int_0^{\infty} \{X(t) \exp[\varphi(\tau) - \varrho(\tau)R(t) + \vartheta(\tau)G(t)] - y_0 \exp[\phi_0(\tau) - \varrho(\tau)R(t)]\} d\tau \quad (6)$$

with:

$$\begin{aligned} \varphi(\tau) = & -\lambda_x \tau + \frac{1}{2} \frac{\sigma_r^2}{\kappa_r^2} \left[\tau + \frac{1 - e^{-2\kappa_r \tau}}{2\kappa_r} - \frac{2(1 - e^{-\kappa_r \tau})}{\kappa_r} \right] - \frac{\kappa_r \mu_r + \sigma_x \sigma_r \rho_{r,x}}{\kappa_r} \left[\tau - \frac{1 - e^{-\kappa_r \tau}}{\kappa_r} \right] \\ & + \frac{1}{2} \frac{\sigma_g^2}{\kappa_g^2} \left[\tau + \frac{1 - e^{-2\kappa_g \tau}}{2\kappa_g} - \frac{2}{\kappa_g} (1 - e^{-\kappa_g \tau}) \right] + \frac{\kappa_g \mu_g + \sigma_x \sigma_g \rho_{g,x}}{\kappa_g} \left[\tau - \frac{1 - e^{-\kappa_g \tau}}{\kappa_g} \right] - \\ & \frac{\sigma_r \sigma_g \rho_{g,r}}{\kappa_r \kappa_g} \left\{ \tau - \frac{1}{\kappa_r} (1 - e^{-\kappa_r \tau}) - \frac{1}{\kappa_g} (1 - e^{-\kappa_g \tau}) + \frac{1 - e^{-(\kappa_r + \kappa_g) \tau}}{\kappa_r + \kappa_g} \right\} \end{aligned} \quad (7)$$

$$\varrho(\tau) = \frac{1 - e^{-\kappa_r \tau}}{\kappa_r} \quad (8)$$

$$\vartheta(\tau) = \frac{1 - e^{-\kappa_g \tau}}{\kappa_g} \quad (9)$$

$$\phi_0(\tau) = \frac{1}{2} \frac{\sigma_r^2}{\kappa_r^2} \left[\tau + \frac{1 - e^{-2\kappa_r \tau}}{2\kappa_r} - \frac{2(1 - e^{-\kappa_r \tau})}{\kappa_r} \right] \quad (10)$$

$S(t)$ is subject to the two transversality conditions:

$$\mu_r > \frac{1}{2} \frac{\sigma_r^2}{\kappa_r^2} \quad (11)$$

$$\mu_r - \mu_g > \frac{\sigma_r^2}{2\kappa_r^2} - \frac{\sigma_r \sigma_x \rho_{r,x}}{\kappa_r} + \frac{\sigma_g^2}{2\kappa_g^2} + \frac{\sigma_g \sigma_y \rho_{g,x}}{\kappa_g} - \frac{\sigma_g \sigma_r \rho_{g,r}}{\kappa_g \kappa_r} - \lambda_x \quad (12)$$

Where λ_x is the risk premium for the systematic risk of earnings shocks, μ_g and μ_r are the respective risk-neutralized long-run means of $G(t)$ and $R(t)$. For the detailed derivation of the equilibrium stock price refer to Dong (2000). Formula (6) represents a closed-form solution to

the equity valuation problem. The equilibrium stock price, $S(t)$, is a function of interest rate, current EPS, expected future EPS, the firm's risk premium, and the thirteen structural parameters governing the EPS and interest rate processes.

4 | Data Description

In this dissertation, we evaluate stock selection methods using a sample of blue-chip companies listed on the Dow Jones Industrial Average Index in 2022. This index is composed by 30 firms, but we disregard stocks with missing data (CRM, CSCO, DOW, GS, and V). This results in a portfolio of 25 stocks (see list in Table B.1 in the appendix). This study spans a period of 11 years from January 1990 to December 2000 inclusive. Our final dataset consists of 18,906 observations corresponding to 25 stocks with 750 observations each plus 156 observations on the current interest rate.

Our analysis requires comprehensive monthly data on six financial metrics: current and expected future EPS, current market prices, current interest rates, and the stock's returns and number of shares outstanding. The first four metrics, current and expected 1-year future EPS, $Y(t)$ and $E[Y(t + 1)]$ respectively, along with the current market prices $S_m(t)$ and interest rates $R(t)$ are used as inputs for the BCD model. The data is needed starting in January 1988. This is to achieve the two-year prior data requirement for the model's implementation (see Section 5.2). Table 1 summarizes the data gathered.

Table 1. Data Gathered and Total Number of Observations

Data	Used to Construct	from	No. of Obs.
Current EPS, $Y(t)$	BCD and E/P	1/88 to 12/00	156
Expected future EPS, $E[Y(t + 1)]$	BCD	1/88 to 12/00	156
Current market price, $S_m(t)$	BCD and E/P	1/88 to 12/00	156
Current interest rate, $R(t)$	BCD	1/88 to 12/00	156
Returns	Ret-12, Ret-6, Ret+1, Ret+6	1/89 to 06/01	150
Number of shares outstanding	Size	1/90 to 12/00	132
<i>Total No. of Obs. per stock</i>			906
<i>Total No. of Obs. for the entire sample of 25 stocks*</i>			18,906

*The interest rate data is counted once for the entire sample, because it is the same for each stock.

The stock's fundamental data is gathered from I/B/E/S International. The current EPS $Y(t)$, corresponds to the total EPS over the trailing 12 months. The expected future EPS $E[Y(t + 1)]$ - used to determine the adjusted EPS growth rate $G(t)$ in the model - is taken as the analyst consensus estimate for the firm's total EPS over the upcoming 12 months. We use the 30-year

Treasury yield as a proxy for the interest rate, $R(t)$, since Bakshi and Chen (2005) have demonstrated that it is the best fit for the BCD model. The monthly data for the stock's contemporaneous price $S_m(t)$, as well as $R(t)$, is collected from DataStream International, Inc. The summary statistics of the BCD model inputs are found in Table 2. It should be highlighted that all input data used for the BCD model pricing, current and analyst-consensus expected EPS as well as interest rates data, is available to actual investors during the time of valuation.

Monthly stock returns (dividend-inclusive) and the number of shares outstanding are obtained from the CRSP database, with returns data ranging from January 1989 to June 2000. This data allows us to construct the traditional measures of E/P, size and momentum used for comparison with the model mispricing measure (see Section 7.1). It is also used to construct measures of future returns. All monthly stock prices and returns are from end-of-month to end-of-month. The stock price in January for example, is the one recorded at the end of January, and the returns are from end of December to end of January.

Table 2. Summary Statistics of the Inputs to the BCD Stock Valuation Model

		$S_m(t)$	$Y(t)$	$E[Y(t + 1)]$	
Sample Period 1/1988 - 12/2000 (<i>No. of months</i> \times <i>stocks</i> = Obs. , $156 \times 25 = 3900$)					
<i>Entire Sample</i> (25 stock)	Mean	18.68	0.96	1.12	0.07
	Std. Dev.	16.66	0.87	0.92	30-Year Yield
	Min	0.11	-5.97	-0.01	R(t)
	Max	127.63	4.37	4.98	0.09

		$S_m(t)$	$Y(t)$	$E[Y(t + 1)]$
Sample Period* 1/1980 - 12/2000 ($132 \times 25 = 3300$)				
<i>Entire Sample</i> (25 <i>stock</i>)	Mean	20.76	1.03	1.19
	Std. Dev.	17.09	0.85	0.94
	Min	0.12	-2.7	-0.01
	Max	127.63	4.37	4.98

*Only the out of sample data are shown (this table does not include the initial two years data).

5 | Model Implementation and Mispricing Measure

5.1 Parameter Estimation

$S(t)$ depends on thirteen parameters: $\{y_0, \mu_r, \kappa_r, \sigma_r, \mu_g, \kappa_g, \sigma_g, \sigma_x, \lambda_x, \rho_{g,x}, \rho_{g,r}, \rho_{r,x}, \delta\}$. To implement the BCD model in (6) and find the equilibrium stock price $S(t)$ we need to estimate these structural parameters for each share S at time t . Like in Bakshi and Chen (2005), we preset the correlation of $\omega_x(t)$ with $\omega_g(t)$ to $\rho_{g,x} = 1$, and the correlations $\rho_{g,r} = \rho_{r,x} \equiv \rho$. This was done to reduce the number of parameter estimations. It implies that the actual and expected adjusted-EPS growth rates are subject to the same shocks.

In addition, for each monthly stock estimation, we set the parameters $\kappa_r = 0.185$, $\mu_r = 0.058$, and $\sigma_r = 0.007$. These values are uniformly applied across all stocks for any given time t within our study period. This approach, based on the findings by Bakshi and Chen (2005), recognizes that the impact of interest-rate parameters on the pricing performance of individual stocks is generally less pronounced compared to firm-specific parameters. To get to these parameters, we solve for the maximum-likelihood criterion function detailed in the appendix (see Code 1). Using this maximum likelihood procedure, we estimate the interest-rate parameters from the 30-year yield monthly time series, covering the period from January 1988 to December 2000.

Eight firm-specific parameters remain to be estimated: $\Phi = \{y_0, \mu_g, \kappa_g, \sigma_g, \sigma_x, \lambda_x, \rho, \delta\}$ (see Table 3). For each stock at month t , and a give sample size of T prior observations, we search for estimates of the eight parameters Φ , to solve the least-squares formula:

$$\text{Min}_{\Phi} \sum_{t=1}^T [S_m(t) - S(t)]^2 \quad (13)$$

subject to the transversality conditions (11) and (12). Here, $S(t)$ is given by equation (6), and $S_m(t)$ is the observed market price at date t . We choose the optimal values of Φ so that each past model price closely matches to the observed market price. The estimation is applied separately for each stock S , and at each date t , that make each past model price. In predicting our

model's parameters, we selected a time horizon of $T = 24$ months, due to its enhanced predictive power for the model's prices (Chen & Dong, 2001).

Table 3 summarizes all parameters in the model and the imposed calibration constraints. For a given stock, some parameters are constant across time, and others vary from one month to the next. We fix the constant parameters denoted with an asterisk* in the table and set bounds for the remaining variable parameters. While in practice, the target dividend payout ratio can be negative when firms are raising capital, in this model we assume that firms are continuously distributing a percentage of the firm's net earnings, and δ is non-negative and non-zero. μ_g can be negative implying a decline in long-term adjusted EPS growth. A μ_g of -1 materializes in an extreme scenario where the firm is facing an existential threat. A μ_g of 1 signals a long-term doubling of adjusted EPS, typical of companies in their growth phase. Adjusted EPS $X(t)$ can change unexpectedly due to external factors (e.g., industry wide shift), or internal factors (e.g., change in management strategy). These sudden changes are called earnings shocks. λ_x can be seen as the compensation investors expect on top of the risk-free rate, specifically for bearing the risk associated with the undiversifiable risk of earning shocks. This risk premium is typically positive. For further details on our parameter estimation approach, refer to Code 2 in the appendix.

Table 3. Description of the Structural Parameters

Structural Parameters	Description	Estimation Bounds
γ_0	fixed buffer to keep adjusted EPS, $X(t)$ non-negative	(for pos. $X(t)$, 15)
μ_r^*	risk-neutralized long-run mean of interest rate $R(t)$	0.058
κ_r^*	speed at which $R(t)$ adjusts to its long-run mean	0.185
σ_r^*	volatility of $R(t)$	0.007
μ_g	risk-neutralized long-run mean for adjusted EPS expected growth $G(t)$	(-1, 1)
σ_g	volatility of $G(t)$	(1e-4, 1)
κ_g	speed at which $G(t)$ adjusts to its long-run mean	(1e-4, 10)
λ_x	risk premium for the systematic risk of adjusted earnings $X(t)$ shocks	(1e-4, 1)
σ_x	volatility of $X(t)$	(1e-4, 1)
δ	target dividend payout ratio	(1e-4, 1)
$\rho_{g,x}^*$	correlation of $\omega_x(t)$ with $\omega_g(t)$	1
ρ	$\rho_{g,r} = \rho_{r,x} \equiv \rho$, correlation of $\omega_r(t)$ with $\omega_g(t)$ and $\omega_x(t)$	(-1, 1)

It is important to recognize that the parameters estimated through equation (12) correspond to the risk-neutralized values. These estimates, derived from historical data, inherently consider the standards by which the market values stocks. This approach offers a significant advantage: it allows the parameters to encapsulate a range of factors, including the firm's business model, potential for future growth, and the quality of its management. These aspects are often overlooked by estimation methods that do not factor in past stock prices. Additionally, these parameters provide insight into the supply-demand dynamics specifically associated with the stock in question.

5.2 Rolling Process Implementation

Suppose that we want to value a specific stock in month t . We use the 24 monthly data on the stock and the interest rates prior to month t (i.e., from month $t - 24$ to month $t - 1$) and apply equation (13) to estimate the parameters Φ for that stock in month t . Next, we substitute the estimated Φ parameters, the current $Y(t)$ and $G(t)$ values of that stock, as well as the current $R(t)$ into equation (6) to determine the BCD model price for the stock in month t . The growth component $G(t)$, is given by:

$$G(t) = \frac{X(t+1) - X(t)}{X(t)} = \frac{E[Y(t+1)] - Y(t)}{Y(t) + y_0} \quad (14)$$

As an example, let us suppose we want to estimate the BCD model price at month $t =$ January 1990. We use data from month $t - 24 =$ January 1988, to month $t - 1 =$ December 1989 to find the Φ parameters. We use these Φ parameters estimated as well as the $Y(t)$, $E[Y(t+1)]$, and $R(t)$ at $t =$ January 1990 to find the BCD model price at that date. When we are done with $t =$ January 1990, we repeat these steps and estimate Φ and the BCD price for the stock in month $t + 1 =$ February 1990, month $t + 2 =$ March 1990, and so on. We apply this rolling process between January 1990 and December 2000 meaning that 132 optimization exercises were solved for each stock. The parameters are estimated using a random search algorithm to circumvent the occurrence of local minima values as in Dong (2000). For a more detailed description of the model implementation, refer to Code 2 in the appendix.

We apply the BCD model on our sample of 25 stocks and proceed to optimize the estimation by testing different bounds in Code 2 and increasing the number of trial estimations. The higher the number of trials, the longer it takes for the estimation to run. We settle for 20,000 trials per optimization, meaning 20,000 different combinations of the eight parameters in Φ for each month t . Each stock goes through 132 optimizations (one for every month) with 20,000 trials per optimization, which takes around 45 minutes per stock. Because of our relative lack of computing power, we run the estimation process on the entire sample around 15 times instead of increasing the number of trials. That way we get results every day or two, compare the value of equation (13) to the previous trial, and choose the parameters with the smallest minimized value. Given the independent and separate implementation of this process for each stock and for each month in the sample, all the model prices are determined out of sample.

5.3 Construction of the Mispricing Measure

The BCD Model Mispricing (MP) is the percentage difference of the current market price $S_m(t)$, relative to the estimated BCD model price $S(t)$, at month t . If MP is positive, the stock is considered overvalued by the market. Conversely, if MP is negative, the stock is undervalued by the market.

$$MP (\%) = \frac{S_m(t) - S(t)}{S(t)} \times 100 \quad (15)$$

5.4 Estimated parameters

Table 4 shows some summary statistics on the estimated parameters. The difference between panel A and B is that panel B excludes some outliers based on the mispricing measure. SSE represents the square root of the minimized value computed by equation (13) divided by the number of observations $T = 24$. We reach an optimized average SSE of 0.43 for the filtered sample (Panel B), down from 0.45 (Panel A). The standard deviation is less than 0.5. For comparison, Dong (2000) applied the BCD model on five stocks for varying periods between 1979 and 1999 (depending on data availability) and found that Exxon had the smallest average SSE at 1.51, with a standard deviation of 1.26. Now looking at the parameters Φ : first, the EPS buffer y_0 , averages 4.5 and reaches a maximum of 14.9. Second, the parameters for the growth

of adjusted EPS suggest that on average adjusted EPS growth rate reaches 13.8% in the long run, at a speed of 3.9. The higher the speed, the faster the mean-reversion to its long-run mean. Third, the implied risk premium required by investors to offset the risk of earnings shocks averages 1.3%. Also, average adjusted EPS volatility is almost 0.5. Finally, the 25 firms are expected to pay out an average of 71.1% of their earnings as dividends.

Table 4. Estimated Parameters of the BCD model

This table shows the mean and standard deviation of the Φ parameters estimated for the entire sample of 25 stocks from January 1990 to December 2000. Panel A shows the summary statistics for the parameters for the entire unfiltered data. Panel B shows the summary statistics for the parameters excluding observations with $MP \geq 50\%$, or $MP \leq -50\%$. SSE is given by:

$$SSE = \text{Min}_{\Phi} \sqrt{\frac{1}{24} \sum_{t=1}^{24} [S_m(t) - S(t)]^2} \quad (16)$$

Panel A		SSE	y_0	μ_g	κ_g	σ_g	σ_x	λ_x	ρ	δ
<i>Entire Sample (25 stock)</i>	Mean	0.445	4.455	0.138	3.858	0.470	0.480	0.129	0.216	0.719
	Std. Dev.	0.503	3.205	0.107	2.127	0.255	0.250	0.078	0.598	0.270
	Min	0.006	0.019	-0.288	0.653	0.101	0.102	0.004	-0.983	0.019
	Max	4.716	14.863	0.348	7.976	0.996	0.994	0.296	0.999	1.000

Sample Period 1/1990 - 12/2000 (*No. of Obs., 3300*)

Panel B		SSE	y_0	μ_g	κ_g	σ_g	σ_x	λ_x	ρ	δ
<i>Entire Sample (25 stock)</i>	Mean	0.429	4.462	0.138	3.857	0.470	0.479	0.129	0.213	0.717
	Std. Dev.	0.475	3.212	0.107	2.132	0.254	0.251	0.078	0.599	0.270
	Min	0.006	0.019	-0.288	0.653	0.101	0.102	0.004	-0.983	0.019
	Max	4.716	14.863	0.348	7.976	0.996	0.994	0.296	0.999	1.000

Sample Period 1/1990 - 12/2000 (*No. of Obs., 3195*)

6 | Overview of Results

In this section we analyze the results of the BCD Model and its mispricing measure. We address the first two hypotheses outlined in the introduction to assess the effectiveness of the mispricing measure. In section 6.1 we show that the mispricing average for our entire sample is near zero and exhibits acceptable standard deviations (hypothesis 1). In section 6.2 we demonstrate the mean reversion property of our mispricing results (hypothesis 2).

6.1 BCD Model Results

Table 5 shows some summary statistics for the mispricing measure for of our sample of 25 stocks. The mispricing average is 8.28%. The median MP is -4.65%, showing that half of the studied sample has a market price at least 4.65% smaller than its equilibrium price. These disparities in signs between mean and median could be due to outliers - significantly over/under valued stocks - skewing the distribution. For that reason, we exclude all MP values bigger than 50% and smaller than -50%, which constitute only 3% of the entire sample. The remainder of our study is done using the sample filtered for MP. The resulting average mispricing is 6.54%, 1.74% lower than that of the unfiltered data, and the median is 6.08%.

Table 5. Summary Statistics on Mispricing

		MP (%) Unfiltered	MP (%) Filtered
	Mean	8.28%	6.54%
	Std. Dev.	20.14%	17.21%
<i>Entire Sample (25 stock)</i>	Min	-66.22%	-49.57%
	25th Percentile	-4.65%	-4.90%
	Median	6.60%	6.08%
	75th Percentile	19.44%	17.77%
	Max	128.79%	49.71%

AAPL, CAT and JPM are the stocks with the lowest mispricing averages for the period from 1990 to 2000 inclusive, at -1.4%, 0.1% and 1.5%, respectively: on average, AAPL is 1.4% underpriced by the market in the studied period, CAT is fairly priced by the market according to the BCD pricing, and JPM is 1.5% overpriced by the market. An average MP of 0% would refer

to a stock that is undervalued half of the time and overvalued the other half. To compare SSE results to MP results, we find the mean absolute MP values of each stock in the sample in Table 6. Note that a smaller SSE average does not necessarily imply a smaller absolute mispricing average. Travelers Companies (TRV) for example has the highest average absolute mispricing at 24.4% but has the eighteenth lowest SSE average of the sample (i.e., there are seven firms with a higher SSE). Nike (NKE) on the other hand, has an absolute mispricing average slightly lower at 22.3%, but the second lowest SSE average relative to the other stocks in the sample. JP Morgan has the lowest absolute mispricing average at 8.4%, and one of the highest SSE averages at 0.73 (see Table 6). As seen in Figure 1, SSE tends to be higher for higher market prices as expected from the nature of the squared difference in equation (13). SSE cannot be standardized, and therefore is not comparable among firms.

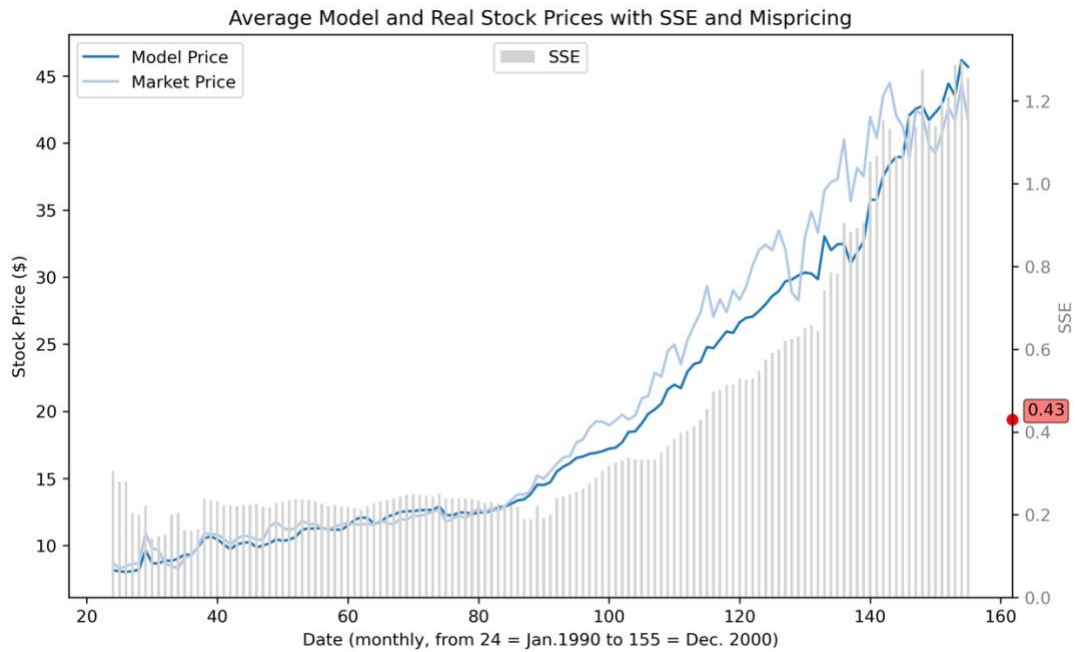
Table 6. Average of SSE and absolute MP for Each Stock

The table presents the average SSE and absolute MP values of each stock in our sample, sorted in ascending order of SSE.

	SSE rank	SSE		MP rank	MP	
		Mean	Std. Dev.		Mean	Std. Dev.
AAPL	1	0.012	0.008	19	17.0%	12.7%
NKE	2	0.115	0.079	24	22.3%	14.4%
UNH	3	0.135	0.073	12	13.9%	8.5%
WBA	4	0.241	0.285	18	16.7%	11.5%
INTC	5	0.281	0.479	8	11.6%	7.7%
CAT	6	0.353	0.248	3	9.9%	8.0%
AXP	7	0.354	0.324	9	12.4%	9.9%
MCD	8	0.374	0.341	23	19.3%	14.6%
JNJ	9	0.393	0.261	7	11.4%	10.0%
KO	10	0.393	0.282	16	15.6%	12.8%
DIS	11	0.399	0.263	17	16.0%	11.8%
MSFT	12	0.401	0.558	10	13.3%	8.1%
AMGN	13	0.408	0.585	13	14.5%	10.3%
PG	14	0.418	0.408	15	14.8%	10.3%
VZ	15	0.437	0.222	5	11.3%	8.9%
CVX	16	0.440	0.325	2	8.6%	7.0%
HD	17	0.448	0.530	14	14.8%	9.7%
TRV	18	0.464	0.238	25	24.4%	14.2%
MMM	19	0.474	0.338	20	17.1%	11.3%

HON	20	0.495	0.504	6	11.3%	9.6%
WMT	21	0.503	0.511	4	10.8%	9.5%
MRK	22	0.646	0.344	21	17.1%	11.8%
JPM	23	0.730	0.709	1	8.4%	5.7%
BA	24	0.746	0.268	11	13.9%	11.6%
IBM	25	1.033	1.111	22	19.2%	13.4%
Average		0.428			14.6%	

Figure 1. Illustration of the BCD Monthly Average Results for the Entire Sample



6.2 Mispricing Mean-Reverting Property

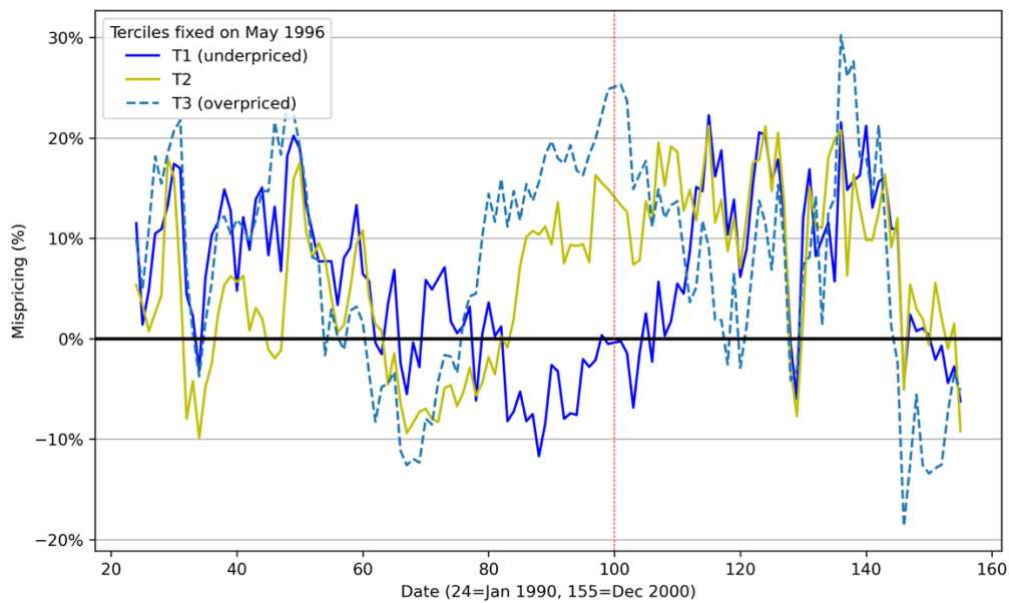
The second hypothesis to assess the viability of the mispricing measure, as stated in the introduction is the following: it should be mean reverting over time, preferably at a fast speed. These are two key ideas in stock valuation from an investment perspective. First, it is assumed that a stock will eventually revert back to a standard level, or a “norm”. Anticipating this correction allows investors to make strategic investment decisions. Second, a “reasonably fast” reversion allows the investment to materialize in a realistic time horizon (Chen & Dong, 2001).

To study the mean-reversion property of the mispricing measure over time, we start by sorting all stocks into terciles (T1 to T3), based on the BCD mispricing values in May 1996. The date was

chosen arbitrarily, and other dates yielded similar results. T1 refers to the most underpriced sample of stocks at this date, and T3 to the most overpriced. The sample of stocks in each tercile portfolio is unchanged for the remainder of the years before and after May 1996 (regardless of MP values). Next, we find the monthly average mispricing of each tercile from January 1990 to December 2000. Refer to Figure 3 for the plot of the monthly average mispricing across time.

Figure 3. Mispricing Reversals Across Terciles

Date = 100 refers to May 1996.



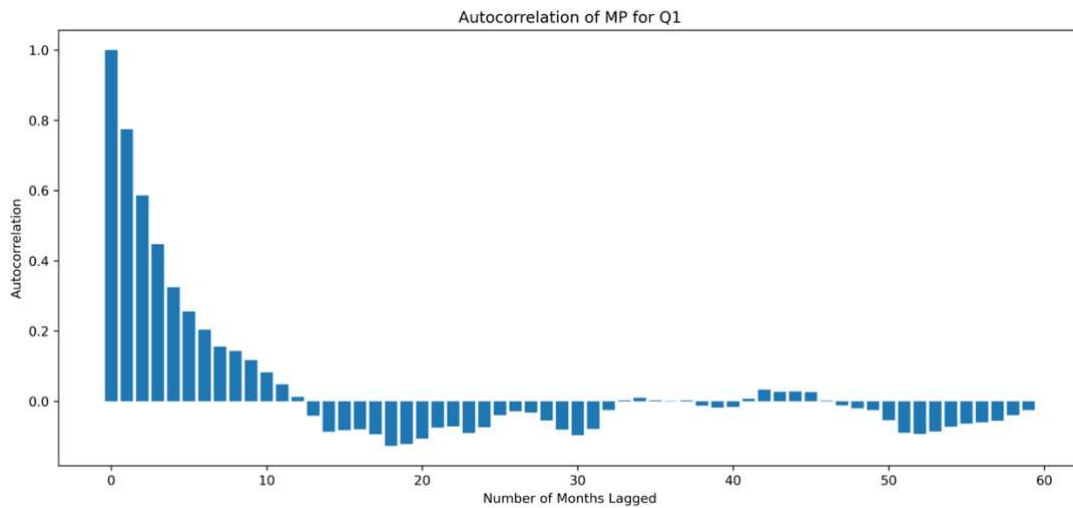
The underlying supposition according to Chen and Dong (2001) is that if the mispricing measure reverts to the mean at a fast speed, then we would expect to see the average mispricing across the terciles moving towards each other, both prior to and following May 1996 marked by the red line. This pattern where the mispricing moves towards the zero line and crosses it from undervalued to overvalued and vice versa would indicate a "correction" or a return to fair value.

Figure 3 clearly shows that T1, T2 and T3 converge towards the 0% mispricing level before and after May 1996 indicating a “reasonably fast” reversion for an investor with a moderate-term investment window. T1 and T3 appear to cross each other multiple times, meaning that the relative order of the terciles shifts: T3 is the most underpriced portfolio at some point, T1 the

most overpriced at a different date etc. Although we can see signs of mean-reverting behavior, the volatility of mispricing doesn't allow for a clear pattern recognition in Figure 3.

Another way to evaluate mean reversion of mispricing is to analyze the persistence property of the mispricing measure using the autocorrelation function. Figure 4 plots the autocorrelation of mispricing of tercile T1 vs. the number of months lagged. The longer it takes for the autocorrelation to move close to zero, the more the mispricing measure is persistent over time. A change in the autocorrelation from negative to positive implies that the portfolio switched from underpriced to overpriced. In Figure 4, it takes around ten months for the autocorrelation to die out. This means the mispricing measure is persistent for ten months.

Figure 4. Autocorrelation Plot for the Most Undervalued Tercile



7 | Comparison of Stock Selection Strategies

In this section, we tackle the third hypothesis: the BCD mispricing should be able to predict future returns to help guide investment decisions. To test this hypothesis, we study how the mispricing measure performs in making investment decisions. We also compare its performance to traditional measures. Section 7.1 shows how we constructed the other traditional measures. In section 7.2 we discuss the characteristics of the other measures of stock selection. Finally, section 7.3 compares the performance of the traditional measures to that of the BCD mispricing measure.

7.1 Construction of the Other Valuation Measures

As mentioned in the literature review, there is an extensive list of indirect measures of valuation, which are useful in predicting future stock returns, and selecting stocks for profitable investments (see Section 2.3). After having determined our monthly BCD stock prices, we now compare the predictive power of its mispricing measure to three commonly used conventional measures: E/P, size, and momentum of returns, as seen in Chen and Dong (2001). This is done by sorting stocks into portfolios based on the mispricing measure (MP), and/or the other traditional measures and comparing the average forward 1-month and 6-month returns (Ret+1) per portfolio (more on this in Section 7.3). The stock-selection measures explored in this study are constructed for every month as follows:

- **Earnings/Price (E/P) Ratio:** the total EPS over the trailing 12 months (equivalent to the monthly current EPS in the BCD model) divided by the market price at month t .
- **Size:** the market capitalization of the firm at month t , equal to the total number of shares outstanding multiplied by the current market price per share. In the regressions presented in Section 7, the logarithm of the dollar market value is used.
- **Momentum (Ret-6):** this measure is based on the stock's past 6-month return.

We don't include the CAPM beta in our analysis because Chen and Dong (2001) demonstrated that it had no statistical significance when regressed against mispricing.

7.2 Descriptive Statistics of Each Measure

Table 7 presents some summary statistics on mispricing and other valuation measures. The average E/P ratio is at 0.053 with a high standard deviation of 0.032. The average firm size varies between a maximum of \$433 million and a minimum of \$10,000, with a mean of \$20 million. All values are in nominal terms. The 6-month return of the sample averages at 10.6% and its 12-month return at 22.4%. Refer to Table B.2 in the appendix for the summary statistics of the unfiltered data.

Table 7. Summary Statistics of Firm Characteristics for the Filtered Data

	MP (%)	E/P	Size (\$ Million)	Ret-6	Ret-12
	Sample Period 1/1990 - 12/2000, (No. of Obs., 3195)				
Mean	6.54%	0.053	20.24	10.59%	22.44%
Std. Dev.	17.21%	0.032	36.95	19.81%	27.74%
Min	-49.57%	-0.271	0.01	-72.42%	-92.65%
Entire Sample (25 stock) 25th Percentile	-4.90%	0.036	1.66	-1.34%	5.47%
Median	6.08%	0.051	7.30	10.63%	22.48%
75th Percentile	17.77%	0.068	19.54	22.65%	38.92%
Max	49.71%	0.150	432.86	97.77%	142.08%

Table 8 presents the Pearson correlation matrix of all the studied measures. MP shows significant correlation with the 6-month and 12-month momentum measures at 62% and 63% respectively. The higher a stock's past returns over 6 or 12 months, the more likely it is to be overpriced by the market according to the BCD model mispricing measure. This can be explained by investors' optimistic outlooks for the stock, known as investor sentiment. E/P on the other hand, is negatively correlated with MP, meaning that on average, the lower a firm's E/P ratio, the more likely it is to be overpriced. At -28%, this relationship is not as significant as MP and momentum. Finally, size is positively correlated with MP at 10%. A high firm size is therefore linked to overpricing by the market.

Table 8. Pearson Correlation Matrix for the Filtered Data

In the Pearson correlation, the logarithm of size is used.

	MP (%)	E/P	Size	Ret-6	Ret-12
MP (%)	1.00				
E/P	-0.28	1.00			
Size	0.10	-0.08	1.00		
Ret-6	0.62	-0.09	-0.02	1.00	
Ret-12	0.63	-0.10	-0.02	0.70	1.00

For further analysis of the characteristics, we sort stocks into terciles based on one of the measures: percentage mispricing (MP), size (in \$ million), earnings/price (E/P), and past 6-month momentum (Ret-6). The portfolios are sorted in ascending order i.e., MP1 are stocks with the lowest mispricing value, and MP3 are those with the highest mispricing value. The same goes for size (S1 to S3), earnings/price (EP1 to EP3), and past 6-month momentum (MO1 to MO3). Table 9 presents the time-series average characteristics of each tercile portfolio. These sorted portfolios corroborate the findings of the correlation matrix. First, smaller mispricing portfolio MP1 has an average past 6-month return of -3.25%, while higher mispricing portfolio MP3 averages 23.79% Ret-6. The same link can be made for Ret-12. As for E/P, the inverse applies. EP1, averages 15.72% mispricing, significantly higher than the -1.35% MP average for EP3.

To study the effectiveness of the measures in predicting future stock returns, we include the 1-month and 6-month forward return averages of our tercile portfolios, denoted Ret+1 and Ret+6 respectively. We find that Ret+1 is the highest at 2.57% for the underpriced portfolio MP1, and lower at 2.04% for the overpriced portfolio MP3. This implies that the more underpriced a firm is the higher its future return. However, the relationship between MP and Ret+1 is not clear seeing as MP2 averages the lowest Ret+1, at 1.39%. This could be due to the small nature of the sample or other factors affecting results. Further analysis is needed (see Section 7.3).

Table 9. Characteristics of Sorted Tercile Portfolios

Panel A	MP1	MP2	MP3	All Stocks
MP (%)	-12.00%	5.77%	25.30%	6.54%
E/P	0.063	0.055	0.042	0.053
Size (\$Mn)	16.42	17.61	26.50	20.24
Ret-6	-3.25%	10.84%	23.79%	10.59%
Ret-12	3.57%	21.27%	41.91%	22.44%
Ret+1	2.57%	1.39%	2.04%	2.00%
Ret+6	11.70%	9.65%	12.39%	11.26%

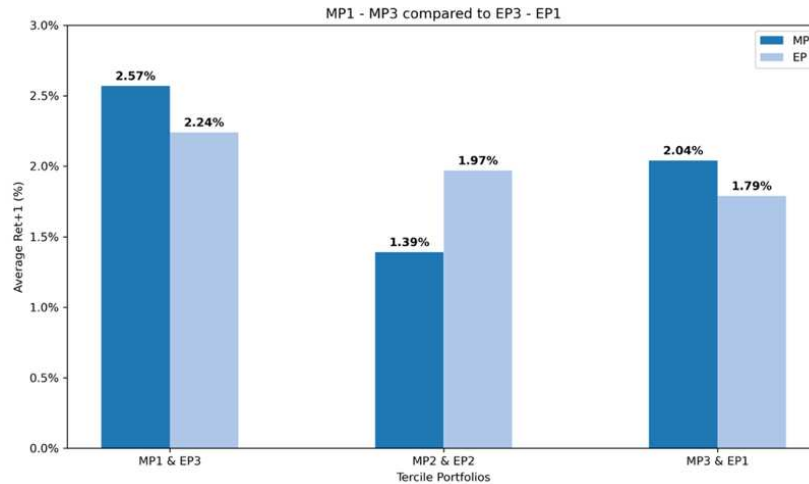
Panel B	EP1	EP2	EP3	All Stocks
E/P	0.025	0.051	0.083	0.053
MP (%)	15.72%	5.47%	-1.35%	6.54%
Size (\$ Mn)	38.01	15.20	7.85	20.24
Ret-6	14.81%	11.05%	6.04%	10.59%
Ret-12	30.19%	21.52%	15.80%	22.44%
Ret+1	1.79%	1.97%	2.24%	2.00%
Ret+6	10.33%	11.52%	11.91%	11.26%

Panel C	S1	S2	S3	All Stocks
Size (\$ Mn)	1.07	7.45	51.27	20.24
MP (%)	4.48%	5.91%	9.15%	6.54%
E/P	0.056	0.061	0.042	0.053
Ret-6	11.16%	9.95%	10.66%	10.59%
Ret-12	23.96%	19.70%	23.62%	22.44%
Ret+1	2.59%	1.76%	1.67%	2.00%
Ret+6	13.50%	10.73%	9.59%	11.26%

Panel D	MO1	MO2	MO3	All Stocks
Ret-6	-10.18%	10.27%	31.07%	10.59%
MP (%)	-5.88%	6.01%	19.12%	6.54%
E/P	0.057	0.056	0.047	0.053
Size (\$ Mn)	21.65	17.70	21.32	20.24
Ret-12	1.69%	22.44%	42.59%	22.44%
Ret+1	2.02%	2.31%	1.69%	2.00%
Ret+6	9.76%	10.32%	13.62%	11.26%

From the tercile portfolios EP1 to EP3 in panel B, we can draw clear connections: forward 1-month returns increase gradually from 1.79% for EP1 to 2.24% for EP3, a difference of 0.45%. Similarly, Ret+6 goes from 10.33% to 11.91%. MP averages decrease gradually from 15.72% to -1.35%.

Figure 2. Average Ret+1 by Tercile Portfolios MP and EP



To show the comparison of MP and EP portfolios, we draw the bar graphs representing the average 1-month forward return values in Figure 2. Because MP and E/P have opposite relationships to Ret+1, and for clarity, MP bars are presented in ascending order, and EP bars in descending order. The standard deviation of Ret+1 by portfolio ranges from 7.7% to 10%. Although tercile portfolios based on mispricing MP do not gradually decline in forward returns, the difference in their Ret+1 averages of 0.53% surpasses that of the EP portfolios (Figure 2). This could potentially indicate a higher effectiveness of MP in predicting future returns.

From Table 9, we can see that the bottom tercile based on mispricing (MP1) and the top tercile based on E/P (EP3), have the lowest Ret-6, i.e., they gather past losers. Still, they have the highest Ret+1 among the corresponding terciles, and in the case of EP3, the highest Ret+6. In other words, panel A and B suggest that past losers, if underpriced or have high E/P, will likely become future winners. To study the effects further, we sort portfolios bi-dimensionally in section 7.3. From panel C, the bigger a firm's size, the smaller its future return. Ret+1 and Ret+6 go from 2.59% and 13.5% to 1.67% and 9.59% respectively. Panel D shows that high stock momentum leads to lower future 1-month returns.

7.3 Investment Performance of Stock Selection Strategies

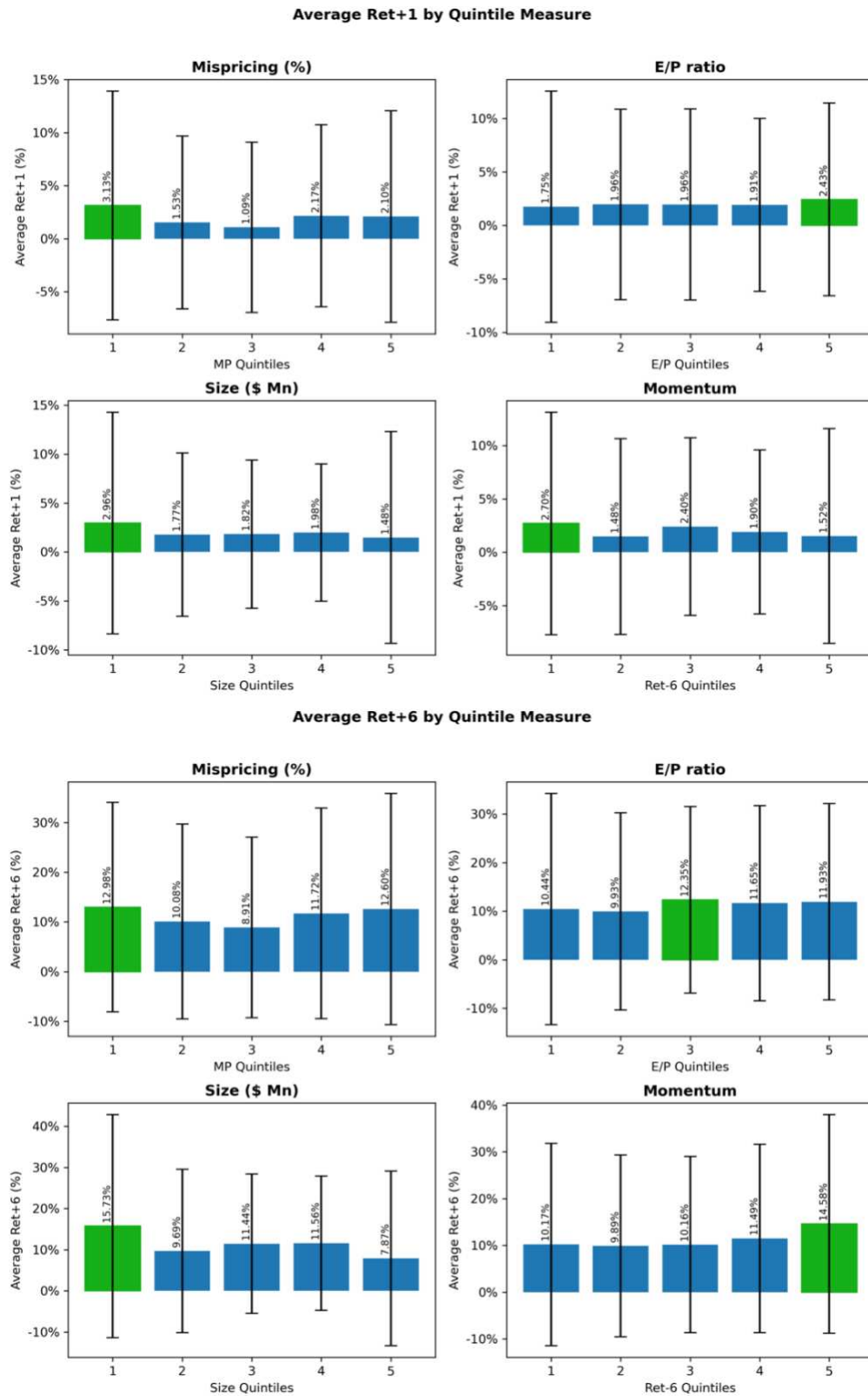
First, we construct sorted quintile portfolios based on the four measures: MP, E/P, size, and momentum. We follow the same procedure used for Table 9. This time we construct five portfolios instead of three to get a better illustration of the behavior of these measures. We then find and plot the average 1-month forward and 6-month forward returns of these portfolios in Figure 5. Quintile 1 represents the portfolio with the lowest value of the measure, and quintile 5 the highest. The portfolios exhibiting the highest average forward returns are highlighted in green.

The relationship between mispricing and forward returns is clearer in Figure 5. The portfolio of stocks with the smallest mispricing in the sample delivers the highest returns at an average 3.13% Ret+1 and 12.98% Ret+6. Investing in the most underpriced stocks provides the highest financial gain. As for size, investing in smaller sized firms yield better returns with S1 (quintile 1 based on size) averaging 2.96% and 15.73% Ret+1 and Ret+6 respectively. On the other hand, investing in the highest E/P portfolio for a month, yields the highest average return at 2.43%. For a six-month investing horizon however, the graph shows that stocks around the median outperform those at the extremes. Finally, the relationship between momentum and future returns is reversed when looking at six months investing horizon as opposed to one month. Here, the bottom quintile outperforms for a month, with an average Ret+1 of 2.70%. For six months however, the opposite is true with the top quintile yielding the highest average return at 14.58%.

Our analysis so far shows that the mispricing strategy is most beneficial for near-term results, delivering superior returns relative to the other strategies within a one-month timeframe. But for a longer six-month period, this strategy is not as effective when compared with size and momentum strategies.

Figure 5. Relationship Between Measures and Future Returns

The portfolios exhibiting the highest average forward returns are highlighted in green.



To further reinforce our findings, we create bi-dimensional quintile portfolios from the intersection of every two groups of quintiles created in Figure 5. Figure 6 reports the average 1-month forward and 6-month forward returns of each of these 25 stocks using a heatmap. By nature, these bi-dimensionally sorted portfolios are not equally weighted, with an average number of stocks ranging from 1.2 to 3.5 stocks per portfolio, as detailed in Table B.3 in the appendix. Although the size of these portfolios is relatively small, they are derived from a comprehensive monthly sorting process over an extensive eleven-year period. This ensures a thorough analysis under various market conditions and economic cycles. We acknowledge the limitations posed by the small portfolio size, but the two-tier criteria used to derive these stocks over an extended period can test the consistency and stability of these relationships.

Figure 6. Future Returns on Bi-Dimensionally Sorted Portfolios

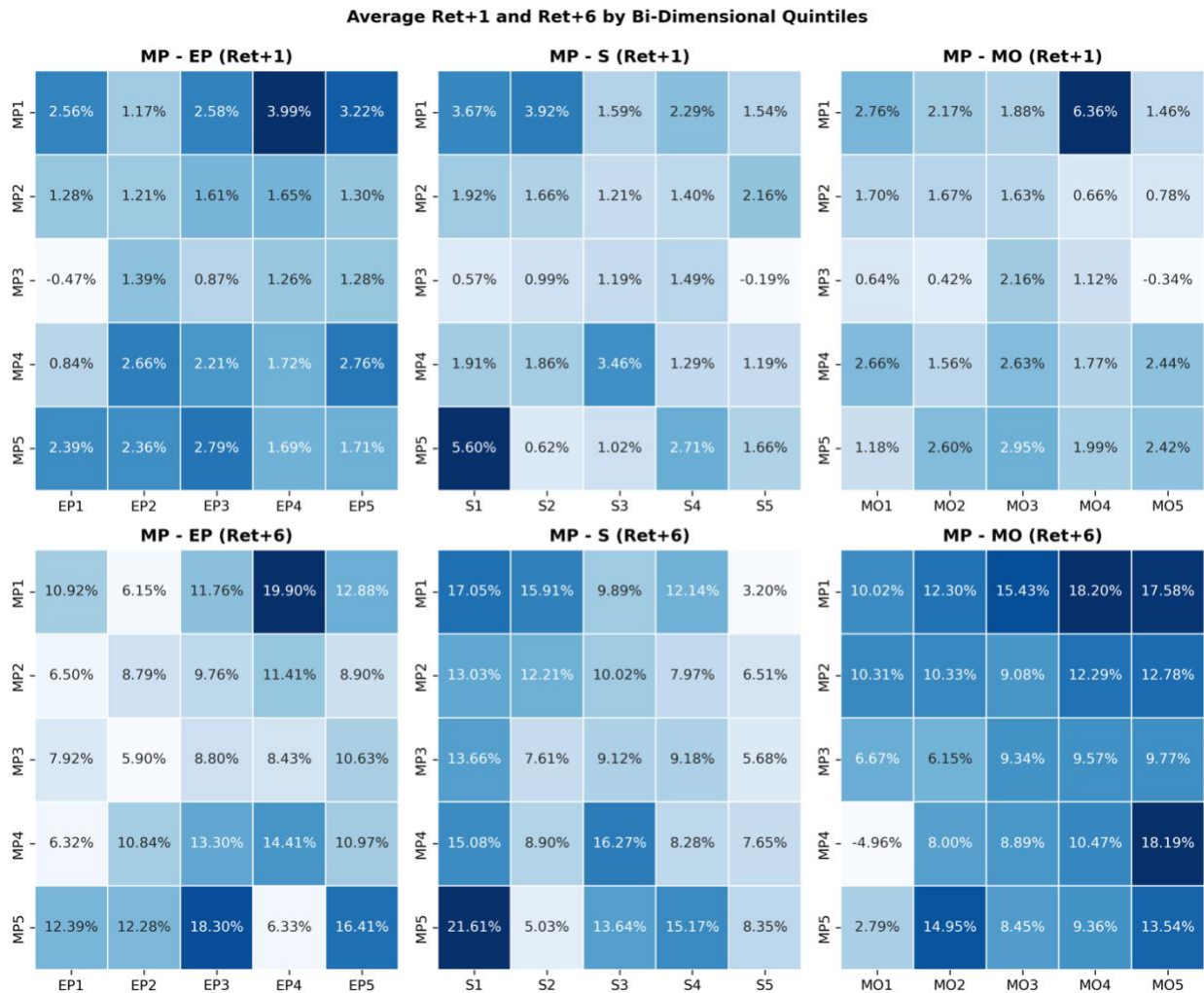


Figure 6 supports our conclusion: MP is inversely proportional to future returns. On the short-term, MP1-EP4 has the best performance among the sorted MP-EP portfolios. With an average Ret+1 of 3.99%, this strategy of picking stocks with low mispricing (MP1) but also high E/P (EP4) bests the one focusing only on low mispricing (MP1, see Figure 5) by 0.86 ppts. MP1-EP5 is not the best performer, this can be due to the small nature of the portfolios and the presence of outliers. However, it is the second best at 3.22%, still outperforming MP1 by 0.09 ppts. For the long-term, the same is true with a MP1-EP4 Ret+6 average of 19.90%, 6.92ppts above MP1 Ret+6 average.

For the portfolios sorted based on mispricing and size, the results are unclear. When adding the size component, Ret+1 and Ret+6 averages are highest for the most overvalued stocks based on mispricing (MP5-S1). Note that the number of small and overpriced firms falling in MP5-S1 is higher than that of small and underpriced firms in MP1-S1 (see Table B.3 in the appendix). The second highest returns occur in the case of low mispricing, small size portfolios: MP1-S2 for Ret+1 and MP1-S1 for Ret+6. This disparity can be due to the small firms that are overvalued having exceptionally high forward returns for some months and skewing the results in favor of size above mispricing.

The portfolio with the best performance among the strategies (MP-EP, MP-S, and MP-MO), for the short-term, is MP1-MO4. Stocks with high momentum and low mispricing average 6.36% in 'Ret+1', more than double the return from MP1 or MO5 strategies alone. There is a clear reversal when considering momentum with mispricing, instead of momentum only. It is more favorable to pick stocks with high momentum instead of low momentum if these stocks are undervalued. On the longer term, the winning stocks fall at the extremes in MP1-MO4/MO5.

From the Pearson correlation matrix, momentum is highly correlated with mispricing at 62% (see Table 8). This implies that high-momentum stocks are 62% likely to be deemed overpriced by the BCD model. Inversely, low momentum stocks are likely to be underpriced. Also, overpriced stocks underperform compared to underpriced stocks. Therefore, the reason why picking low momentum stocks is best when the momentum measure is exclusively considered, can be due to these stocks being underpriced (see Figure 5). Yet, the 38% uncorrelated picks of high

momentum stocks with low mispricing achieve top performance. These stocks collect the premium from low mispricing on top of the premium for high momentum. Meaning, if we pick stocks with high momentum and filter out the overpriced stocks using the BCD model, we achieve the best investment strategy.

However informative these results are, they do not paint an accurate picture of a realistic market environment. Results can be skewed in favor of undervalued stocks that are bad or average throughout the analysis period but performed unusually well for a month. Similarly, overpriced stocks can have one very bad month that could make their performance look worse than usual. To address this issue, we would have to use a much larger sample of stocks that would drown out any extreme returns from a small number of stocks and proceed with an investment algorithm to test out our theories.

8 | Conclusion

This dissertation aims to use the Bakshi Chen Dong (2001) model to perform equity valuation on a sample of 25 blue chip stocks and test its validity. The model can be used by any investor with access to data on current earnings, forecasted future earnings, and interest rates. The mispricing measure we calculate based on the BCD model has an average close to zero, proving promise in this valuation strategy. The mispricing for the entire sample is 8.28% with extremes, and 6.54% in the final filtered sample. Also, mispricing is proven to be mean reverting, a property crucial for the measure to be valid in guiding investment decisions.

Additionally, we test the validity of the mispricing strategy by comparing it to three tried-and-true measures: E/P, size, and momentum. Although our sample of stocks is relatively small, our analysis yields substantial results and ideas open to additional research. We find that using the BCD mispricing strategy is best at picking stocks for short-term returns (one-month forward returns). However, for the long-term (six-month forward returns), using the size strategy yields the best performance. On the other hand, picking stocks based on two measures shows inconclusive results still. The mispricing-momentum strategy looks to be the most profitable on the short-term, but further analysis and a bigger sample of stocks is needed to produce stronger results. While we acknowledge the limitations posed by the small portfolio size, our results are credible due to the focused nature of our criteria and the long-term nature of our eleven-year study, which suggests significant insights despite the small number of stocks.

Our analysis was hampered by the fact that we were unable to apply the BCD model on a larger sample due to time and computational power constraints. A larger sample size would have allowed for the testing of the mispricing measure's predictive power against the other measures using Fama-MacBeth regressions, by regressing one-month future returns against these measures to evaluate their forecasting accuracy.

Additional areas of study are endless. The model can be modified to focus on revenues and costs instead of earnings. By replacing the adjusted earnings with revenues, and the buffer earnings with negative costs, we can derive similar equations and follow the same implementation using data on revenues and costs to find the equity value. The stock price formula could be slightly

altered to include additional parameters and inputs. An analysis can then be performed to see if it yields better empirical results than the earnings approach. The mispricing valuation measure can also be applied to study several concepts such as the market seasonality effect. It can also be considered as the standard fair value of a firm to gain insights into corporate events. For example, we can use the BCD model valuation as a benchmark to compare the prices of the bidding and target firms before and after a merger considering that it relies on earnings and earnings forecasts to determine value.

To conclude, this valuation model reflects the nuances of financial valuations, but potential improvements can still advance its accuracy.

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B | Appendices

Table B.1. List of Stocks in the Sample

	Ticker	Company Name		Ticker	Company Name
1	AAPL	Apple Inc	14	KO	Coca-Cola
2	AMGN	Amgen Inc	15	MCD	McDonalds Corp
3	AXP	American Express	16	MMM	3M
4	BA	Boeing	17	MRK	Merck & Co Inc
5	CAT	Caterpillar Inc	18	MSFT	Microsoft Corp
6	CVX	Chevron Corp	19	NKE	Nike
7	DIS	Walt Disney	20	PG	Procter & Gamble
8	HD	Home Depot Inc	21	TRV	Travelers Companies Inc.
9	HON	Honeywell International Inc.	22	UNH	UnitedHealth Group Inc.
10	IBM	International Business Machines Co.	23	VZ	Verizon Communications Inc.
11	INTC	Intel Corporation Corp	24	WBA	Walgreen Boots Alliance Inc.
12	JNJ	Johnson & Johnson	25	WMT	Walmart Stores Inc
13	JPM	JP Morgan Chase & Co.			

Table B.2. Summary Statistics of Firm Characteristics for the Unfiltered Data

Table B.2 presents summary statistics for BCD model mispricing (MP), size (in \$ million), earnings/price (E/P), and past 6-month and 12-month momentum (Ret-6 and Ret-12) for the entire data sample.

	MP (%)	E/P	Size (\$ Million)	Ret-6	Ret-12	
Sample Period* 1/1990 - 12/2000 (<i>No. of months × stocks = Obs., 132 × 25 = 3300</i>)						
	Mean	8.28%	0.052	21.36	11.49%	23.79%
	Min	20.14%	0.033	40.57	20.89%	29.40%
<i>Entire Sample (25 stock)</i>	25th Percentile	-66.22%	-0.271	0.01	-72.42%	-92.65%
	Median	-4.65%	0.035	1.56	-1.14%	5.98%
	75th Percentile	6.60%	0.050	7.32	11.18%	23.38%
	Max	19.44%	0.067	20.20	23.74%	40.33%

Table B.3. Characteristics of Bi-Dimensionally Sorted Portfolios

At each month in the dataset, we sort stocks into quintiles based on one of the measures: percentage mispricing (MP), size (in \$ million), earnings/price (E/P), and momentum (Ret-6). The 25 portfolios created by the intersection between every two groups of quintile portfolios are given with the average 1-month forward returns of each portfolio in **bold**, followed by the standard deviation of Ret+1, and the number of stocks per portfolio *{in brackets}*.

Panel A	MP1	MP2	MP3	MP4	MP5
EP1	2.56%	1.28%	-0.47%	0.84%	2.39%
<i>Std. dev</i>	13.02%	6.55%	8.28%	6.91%	8.05%
<i>{Nb. Stock}</i>	{1.3}	{1.3}	{1.4}	{1.5}	{2.7}
EP2	1.17%	1.21%	1.39%	2.66%	2.36%
	8.02%	5.94%	6.67%	5.18%	5.77%
	{1.3}	{1.6}	{1.4}	{2.0}	{2.0}
EP3	2.58%	1.61%	0.87%	2.21%	2.79%
	8.41%	6.68%	4.68%	5.03%	4.52%
	{1.8}	{1.5}	{1.7}	{1.6}	{1.3}
EP4	3.99%	1.65%	1.26%	1.72%	1.69%
	6.05%	4.63%	5.13%	4.49%	5.85%
	{1.8}	{1.8}	{1.8}	{1.6}	{1.3}
EP5	3.22%	1.30%	1.28%	2.76%	1.71%
	7.66%	4.59%	4.55%	5.03%	6.76%
	{2.7}	{1.9}	{1.6}	{1.3}	{1.2}

Panel B	MP1	MP2	MP3	MP4	MP5
S1	3.67%	1.92%	0.57%	1.91%	5.60%
<i>Std. Dev.</i>	7.70%	6.91%	5.86%	7.63%	7.53%
<i>{No. Stocks}</i>	{1.9}	{1.7}	{2.0}	{1.5}	{2.2}
S2	3.92%	1.66%	0.99%	1.86%	0.62%
	7.17%	5.47%	5.16%	5.01%	5.42%
	{1.8}	{1.6}	{1.6}	{1.5}	{1.5}
S3	1.59%	1.21%	1.19%	3.46%	1.02%
	6.01%	4.12%	4.31%	5.03%	4.35%
	{1.6}	{1.7}	{1.8}	{1.7}	{1.3}
S4	2.29%	1.40%	1.49%	1.29%	2.71%
	7.27%	4.67%	4.73%	3.60%	6.46%
	{1.3}	{1.5}	{1.8}	{1.9}	{1.9}
S5	1.54%	2.16%	-0.19%	1.19%	1.66%
	11.71%	8.96%	7.27%	7.79%	8.56%

{2.7} {2.1} {2.1} {2.7} {3.1}

Panel C	MP1	MP2	MP3	MP4	MP5
MO1	2.76%	1.70%	0.64%	2.66%	1.18%
<i>Std. Dev.</i>	7.14%	5.38%	7.12%	7.81%	8.92%
<i>{No. Stocks}</i>	{3.4}	{1.9}	{1.5}	{1.2}	{1.2}
MO2	2.17%	1.67%	0.42%	1.56%	2.60%
	7.53%	5.79%	4.67%	6.91%	11.96%
	{1.8}	{2.2}	{1.9}	{1.5}	{1.2}
MO3	1.88%	1.63%	2.16%	2.63%	2.95%
	5.90%	3.95%	5.67%	5.97%	8.44%
	{1.3}	{1.8}	{2.0}	{1.8}	{1.3}
MO4	6.36%	0.66%	1.12%	1.77%	1.99%
	4.79%	3.78%	4.89%	4.46%	4.48%
	{1.3}	{1.4}	{1.8}	{2.2}	{1.8}
MO5	1.46%	0.78%	-0.34%	2.44%	2.42%
	9.10%	3.69%	7.25%	6.91%	7.43%
	{1.0}	{1.1}	{1.4}	{1.8}	{3.5}

Code 1. R(t) Parameter Estimation under the Objective Probability Measure

The structural parameters for the interest rate R(t) process follow Bakshi and Chen's (2005) maximum-likelihood estimation of the transition density function:

$$\mathcal{L} \equiv \max_{\kappa_r, \mu_r, \sigma_r} \frac{1}{T} \sum_{t=1}^T \log \left(\frac{1}{\pi} \int_0^{\infty} \operatorname{Re} [F(t, \tau, R(t); u) \times \exp(-iuR(t + \tau))] du \right),$$

where $i = \sqrt{-1}$, $T = 156$, (number of observations in the entire sample), $\tau = \frac{1}{12}$ (for monthly sampled observations) and the characteristic function for the density is:

$$F(t, \tau, G(t); u) = \exp \left[iu\mu_r^0(1 - e^{-\kappa_r\tau}) - \frac{u^2\sigma_r^2}{4\kappa_r}(1 - e^{-2\kappa_r\tau}) + iue^{-\kappa_r\tau}R(t) \right]$$

The code aims to minimize the negative expression of the maximum-likelihood function.

```
# Characteristic Function for the Density
def F(t, tau, R_t, u, mu_r, kappa_r, sigma_r):
    ...
    return np.exp(term1 + term2 + term3)

# Maximum Likelihood Criterion Function
# 'data' is the array of the 30-Year Treasury Bond Yields
def likelihood_func(parameters, data):
    kappa_r, mu_r, sigma_r = parameters
    T = len(data)
    tau = 1 / 12
    likelihood = 0
    for t in range(T - 1):
        R_t = data[t]
        R_t_tau = data[t + 1]
        integrand = lambda u: np.real(F(t, tau, R_t, u, mu_r, kappa_r,
            sigma_r) * np.exp(-1j * u * R_t_tau))
        integral, error = quad(integrand, 0, np.inf)
        if integral > 0:
            likelihood += np.log(integral / np.pi)
    return -likelihood / T

# Initial guesses for kappa_g, mu_g, sigma_g
init_params = [0.1, 0.1, 0.1]

# Perform the optimization
result = minimize(likelihood_func, init_params, args=(data))

# Store results
kappa_r, mu_r, sigma_r = result.x
```

Code 2. Implementation of the BCD Model Rolling Process

In Implementing the BCD Model, we follow the process used by Dong (2000). The minimized value in equation (13) is calculated using parameter combinations sampled at random following the bounds in Table 1. The parameter combination $\Phi = \{y_0, \mu_g, \kappa_g, \sigma_g, \sigma_x, \lambda_x, \rho, \delta\}$ with the smallest value of equation (13) is selected as the optimal one.

$$\text{Min}_{\Phi} \sum_{t=1}^T [S_m(t) - S(t)]^2 \quad (13)$$

We use an Optuna hyperparameter optimization framework, and we parallelize the estimation process over the months to make the code faster. To show the logic of our code clearly here, we include the code without parallelization.

We define all the functions that equation (6) of $S(t)$ relies on.

```
# equation (7)
def phi_fn(tau, sigma_x, lambda_x, mu_r, sigma_r, kappa_r, mu_g, sigma_g,
kappa_g, rho):
    term1 = - lambda_x * tau
    term2 = 0.5 * ((sigma_r ** 2) / (kappa_r ** 2)) * (tau + (1 - np.exp(-2 *
kappa_r * tau)) / (2 * kappa_r) - 2 * (1 - np.exp(-kappa_r *
tau)) / kappa_r)
    term3 = - ((kappa_r * mu_r + sigma_x * sigma_r * rho) / kappa_r) * (tau -
(1 - np.exp(-kappa_r * tau)) / kappa_r)
    term4 = 0.5 * ((sigma_g ** 2) / (kappa_g ** 2)) * (tau + (1 - np.exp(-2 *
kappa_g * tau)) / (2 * kappa_g) - 2 * (1 - np.exp(-kappa_g *
tau)) / kappa_g)
    term5 = ((kappa_g * mu_g + sigma_x * sigma_g * rho) / kappa_g) * (tau -
(1 - np.exp(- kappa_g * tau)) / kappa_g)
    term6 = - (sigma_r * sigma_g * rho) / (kappa_r * kappa_g) * (tau - (1 /
kappa_r) * (1 - np.exp(- kappa_r * tau)) - (1 / kappa_g) * (1 -
np.exp(- kappa_g * tau)) + (1 - np.exp(-(kappa_r + kappa_g) *
tau)) / (kappa_r + kappa_g))
    return term1 + term2 + term3 + term4 + term5 + term6

# equation (8)
def rho_fn(tau, kappa_r):
    with warnings.catch_warnings():
        warnings.simplefilter('ignore')
        return (1 - np.exp(- kappa_r * tau)) / kappa_r

# equation (9)
def theta_fn(tau, kappa_g):
    with warnings.catch_warnings():
        warnings.simplefilter('ignore')
        return (1 - np.exp(- kappa_g * tau)) / kappa_g
```

```

# equation (10)
def phi_0_fn(tau, sigma_r, kappa_r):
    term1 = 0.5 * (sigma_r ** 2) / (kappa_r ** 2)
    term2 = tau + (1 - np.exp(-2 * kappa_r * tau)) / (2 * kappa_r)
    term3 = 2 * (1 - np.exp(-kappa_r * tau)) / kappa_r
    return term1 * (term2 - term3)

```

We define the transversality conditions of $S(t)$.

```

# equation (11)
def check_first_condition(mu_r, sigma_r, kappa_r):
    # note that this is not an optimization constraint
    # the interest rate parameters are kept fixed during the optimization
    return mu_r > 0.5 * ((sigma_r ** 2) / (kappa_r ** 2))

```

```

# equation (12)
def check_second_condition(sigma_x, lambda_x, mu_r, sigma_r, kappa_r, mu_g,
sigma_g, kappa_g, rho):
    return mu_r - mu_g > (sigma_r ** 2) / (2 * kappa_r ** 2) - (sigma_x *
sigma_r * rho) / kappa_r + (sigma_g ** 2) / (2 *
kappa_g ** 2) + (sigma_x * sigma_g * rho) / kappa_g
- (sigma_r * sigma_g * rho) / (kappa_r * kappa_g) -
lambda_x

```

We define the equation of the BCD model price $S(t)$.

```

# integrand of equation (6)
# we replace  $X(t) = Y(t) + y_0$ 
# we replace  $G(t) = (E[Y(t+1)] - Y(t)) / (Y(t) + y_0)$ 
def integrand(tau, R, Y, E_Y, y0, sigma_x, lambda_x, mu_r, sigma_r, kappa_r,
mu_g, sigma_g, kappa_g, rho):
    with warnings.catch_warnings():
        warnings.simplefilter('ignore')
        term1 = (Y + y0) * np.exp(phi_fn(tau, sigma_x, lambda_x, mu_r,
sigma_r, kappa_r, mu_g, sigma_g, kappa_g, rho) - rho_fn(tau,
kappa_r) * R + theta_fn(tau, kappa_g) * (E_Y - Y) / (Y + y0))
        term2 = - y0 * np.exp(phi_0_fn(tau, sigma_r, kappa_r) - rho_fn(tau,
kappa_r) * R)
    return term1 + term2

```

```

# equation (6)
# bcd model price  $S(t)$ 
def integral(R, Y, E_Y, y0, sigma_x, lambda_x, mu_r, sigma_r, kappa_r, mu_g,
sigma_g, kappa_g, rho):
    return quad(func=integrand, a=0, b=24, args=(R, Y, E_Y, y0, sigma_x,
lambda_x, mu_r, sigma_r, kappa_r, mu_g, sigma_g, kappa_g,
rho))[0]

```

We fix our constants before running the optimization loop.

```
# constants mu_r, sigma_r, kappa_r and a small value eps
mu_r = 0.057863026104920535
sigma_r = 0.006920974783848631
kappa_r = 0.18526508006653786
eps = 1e-4
```

We define the optimization loop on a stock from dates 0 (Jan. 1990) to 155 (Dec. 2000).

```
# optimization loop function
def run(filepath, T=24):
# when we run this function, we pass the filepath of the stock inputs

# load the data
df = pd.read_csv(os.path.join(filepath))

# add the columns for storing the results
df['S_model(t)'] = np.nan
df['min_value'] = np.nan
df['mispricing(%)'] = np.nan
df['y_0'] = np.nan
df['sigma_x'] = np.nan
df['lambda_x'] = np.nan
df['mu_g'] = np.nan
df['sigma_g'] = np.nan
df['kappa_g'] = np.nan
df['rho'] = np.nan
df['delta'] = np.nan

# loop across the months starting at T = 24
for t in range(T, len(df)):

# extract the previous T values
# .iloc[t - T : t + 1] gives previous T values up to t
# i.e., at the 1st loop, t = 24, we take values from 0 to 24
R = df['R(t)'].iloc[t - T : t + 1]
Y = df['Y(t)'].iloc[t - T : t + 1]
E_Y = df['E_Y(t)'].iloc[t - T : t + 1]
S = df['S_real(t)'].iloc[t - T : t + 1]
```

```

# objective function for equation (13)
def objective(trial):

    # choose random parameters inside the bounds
    params = {
        'y0': trial.suggest_float('y0', eps, 30),
        'sigma_x': trial.suggest_float('sigma_x', eps, 1),
        'lambda_x': trial.suggest_float('lambda_x', -1, 1),
        'mu_g': trial.suggest_float('mu_g', -1, 1),
        'sigma_g': trial.suggest_float('sigma_g', eps, 1),
        'kappa_g': trial.suggest_float('kappa_g', eps, 10),
        'rho': trial.suggest_float('rho', -1, 1),
        'delta': trial.suggest_float('delta', eps, 1)
    }

    # check if the parameters satisfy the transversality condition
    if check_second_condition(params['sigma_x'], params['lambda_x'],
        mu_r, sigma_r, kappa_r, params['mu_g'], params['sigma_g'],
        params['kappa_g'], params['rho']):

        # if they do, use these parameters chosen
        # and calculate the bcd prices for the previous T dates
        prices = []
        for i in range(T):
            price = params['delta'] * integral(R.iloc[i], Y.iloc[i],
                E_Y.iloc[i], params['y0'], params['sigma_x'],
                params['lambda_x'], mu_r, sigma_r, kappa_r,
                params['mu_g'], params['sigma_g'],
                params['kappa_g'], params['rho'])

            # following Dong (2000), if the model price is such that
            #  $0.4 * S(\text{market}) < \text{price} < 5 * S(\text{market})$ , store price
            # else, discard price
            if price > 0 and np.isfinite(price) and (0.4 * S.iloc[i]
                <= price <= 5 * S.iloc[i]):
                prices.append(price)
            else:
                return np.inf

        # calculate the least-square value of equation (13)
        return np.sum((S.iloc[:-1] - np.array(prices)) ** 2)

    else:
        # if the parameters don't satisfy the transversality condition
        # discard them
        return np.inf

```

```

# run the optimization using Optuna
# higher trials lead to better results
optuna.logging.set_verbosity(optuna.logging.ERROR)
study = optuna.create_study(direction='minimize',
                             sampler=optuna.samplers.RandomSampler(seed=42))
study.optimize(objective, n_trials=20000)

# extract the best parameters
y0, sigma_x, lambda_x, mu_g, sigma_g, kappa_g, rho, delta =
study.best_params.values()

# extract the minimum value of least-squares
min_value = study.best_value

# calculate the model price using current values of R, Y and E_Y
# and optimized parameters
S_model = delta * integral(R.iloc[-1], Y.iloc[-1], E_Y.iloc[-1], y0,
                             sigma_x, lambda_x, mu_r, sigma_r, kappa_r, mu_g, sigma_g,
                             kappa_g, rho)

# calculate the model mispricing
mispricing = (S.iloc[-1] - S_model) / S_model

# save the results
with warnings.catch_warnings():
    warnings.simplefilter('ignore')
    df['S_model(t)'].iloc[t] = S_model
    df['min_value'].iloc[t] = min_value
    df['mispricing(%)'].iloc[t] = mispricing
    df['y_0'].iloc[t] = y0
    df['sigma_x'].iloc[t] = sigma_x
    df['lambda_x'].iloc[t] = lambda_x
    df['mu_g'].iloc[t] = mu_g
    df['sigma_g'].iloc[t] = sigma_g
    df['kappa_g'].iloc[t] = kappa_g
    df['rho'].iloc[t] = rho
    df['delta'].iloc[t] = delta

# export the results
# we export after each iteration
# we have a backup in case the code stops for some reason
df.to_csv(filepath.replace('.csv', '_RESULTS.csv'), index=False)

```

We run the loop on stock 'DIS' for example.

```

# run the model
run(filepath='data/DIS.csv')

```