



Industry investing under Parametric Portfolio Policy

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Abstract

This dissertation investigates the performance of the Parametric Portfolio Policy in an industry level over the 1971 – 2019 period. I find that investors can take advantage of the Parametric Portfolio Policy strategy in an industry level, with lower cost than direct stock investing due to the increasing amount of industry exchanged traded funds in the market. In all strategies proposed the *Naïve Industry + Long-short* and *Long-only* portfolios performed better than the benchmark. However, the out-of-sample portfolio turnover is extremely high compared to the benchmark. I consider three different trading costs to have a more reliable net-of-costs performance. Taking into account different investors' type, this paper also reports performance data varying the risk aversion coefficient.

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Keywords: Parametric Portfolio Policy, Industry Investing, Sector Investing, Asset allocation

Abstrato

Esta dissertação investiga a performance do *Parametric Portfolio Policy* no nível de indústrias durante o período de 1970 a 2019. Eu considero que os investidores conseguem tirar proveito da estratégia do *Parametric Portfolio Policy* a um nível de indústrias com custos mais baixos comparativamente a investimentos diretamente em ações devido ao número crescente de fundos de indústrias listados em bolsa de valores (*Exchanged Traded Funds*) no mercado. Todas as estratégias propostas no *Naïve Industry + Long-short* e *Long-only* portfólios superaram a performance do *benchmark*. Porém, o *turnover* do portfólio *out-of-sample* é significativamente mais elevado comparativamente ao *benchmark*. Esta questão é abordada assumindo três custos de transação diferentes com o objetivo de ter uma performance pós custos mais fidedigna. Tendo em consideração diferentes tipos de investidores, esse artigo também reporta a performance variando o coeficiente de aversão ao risco.

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Palavras-Chave: Parametric Portfolio Policy, Investimento em Indústrias, Investimento em Setores, Alocação de Ativos

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1. Introduction

During the past decades, financial markets have registered a growing number of exchange traded funds (ETFs), supported by an increase in the preference for passive investment. From May 2009 to April 2019, passive investment has registered about 1.5 trillion dollars inflow, while active investment funds – where a portfolio manager actively chooses in which stocks to invest – has registered an outflow of more than 1 trillion dollars, according to Forbes. According to Mirae Asset Management, the amount of assets under management of ETFs globally presented a cumulative annual growth rate of 24% between 2003 and 2016.

In 2019, the amount of asset under management of passive equity funds has surpass for the first time the amount of assets under management of active equity funds, according to Morningstar. Following this trend, investors increasingly allocate wealth not only on stocks but also on industry or country equity portfolios, since the exchange traded funds give the investors the possibility to do it easily and in a cheap manner.

Motivated by this trend, I propose in this work using the parametric portfolio policy, method developed by Brand et al. (2009), to develop an optimal industry portfolio using the 49 Fama-French Industries. To the best of my knowledge, using this model in order to design an industry investment strategy is a novel approach. This differentiates my study from previous works on parametric portfolio policy, such as Brandt, Santa-Clara and Valkanov (2009), DeMiguel, Martin-Utrera, Nogales and Uppal (2017) and Barroso, Menichetti and Reichenecker (2021).

The parametric portfolio policy is a quantitative approach to develop an investment strategy using asset specific characteristics, such as value, momentum and size for stocks, the so-called “greeks” for options and fx volatility and carry for currencies.

In this study, I use different combinations of characteristics and find robust OOS results, achieving significant reductions in the correlation to the naïve industry portfolio, higher Sharpe ratios and positive Jensen-Alpha, when compared to the naïve industry portfolio, used as benchmark.

The base case model presented in this paper uses Book-to-Market, Momentum and Profitability as industry-specific characteristics and a risk aversion coefficient equal to 5. It outperforms the equal-weighted industry portfolio over the sample period and enhances the

Sharpe ratio from 0.53 to 0.67 out-of-sample. The long-only strategy also outperforms the benchmark with a Sharpe ratio of 0.63 out-of-sample.

The structure of the paper is as follows. Section II exposes a brief overview of the related literature. Section III describes the sample of data used on the work. Section IV describes the methodology and empirical framework used to build and analyse the portfolios created, Section V exhibit the results obtained and Section VI the summary and conclusion of the study.

2. Literature Review

Markowitz (1952) proposes the traditional mean-variance approach, which introduces the concept of trade-off between risk and return and is often considered the standard theoretical model of investment behaviour, as mentioned by Brandt (2010). However, many researchers point out Markowitz's approach as computationally difficult, noisy, and unstable. As Michaud (1989) describes, "equal weighting may significantly outperform unconstrained mean-variance optimization in many cases". This idea is also documented by DeMiguel, Garlappi and Uppal (2007).

The problems that many authors document about the Markowitz optimization approach are contrasted by a growing abundance of evidence for predictability in the cross section of stock returns. Even though financial markets are characterized by a component of randomness in assets returns, since the end of 20th century, many studies have documented patterns in the cross section of individual stock returns. According to Lewellen (2014), "the asset-pricing literature finds significant cross-sectional predictability in stocks returns", additionally, "many of the documented patterns are highly significant and seem almost certainly to be real".

Banz (1981) documents the size factor, finding that small-cap stocks on average tend to outperform large-caps stocks over time. Later, De Bondt and Thaler find patterns on short-term reversals, where stocks with relatively low returns over the past month or week earn positive abnormal returns in the following period.

Jegadeesh (1990) and Lehmann (1990) provide empirical evidence of short-term return reversals. Later, Jegadeesh and Titman (1993) find that buying past winners and selling past losers, investors can earn a significant positive abnormal return. This strategy became commonly known as Momentum.

Fama and French (1992) argue that the multifactor model including other factors such as size and B/M ratios can explain excess returns better than just the traditional Capital Asset Pricing Model (CAPM) containing only the market beta factor. They find that CAPM can only explain about 60% of market returns, while, by including the two other factors, the regression can explain over 90% of stock market returns.

More recently, Brandt, Santa-Clara and Valkanov (2009) presents a new approach for portfolio optimization using only a few cross-sectional parameters based on firm characteristics. They propose to parameterize the portfolio weight of each stock by maximizing the investor's utility function. The parameters suggested as firm-specific characteristics were value, size, and momentum. The authors entitle this method Parametric Portfolio Policies (PPPs). As stated by DeMiguel, Garlappi and Uppal (2009), "exploiting information about the cross-section characteristics of assets may be a promising direction to pursue".

The advantage of this approach derives from the easiness to replicate on real data. In order to construct a portfolio with N stocks, the traditional mean-variance approach requires modelling N first and $(N^2 + N)/2$ second moments and even for a small number of stocks implicates an almost unmanageable number of moments to estimate. On the other hand, the PPP model requires designing only N portfolio weights based on the investor's risk averse coefficient and the joint distribution of assets returns. Due to this decreased dimensionality, this method has less imprecise coefficient estimates and less overfitting problem, therefore it can tolerate an exceptionally large number of assets with arbitrary preferences. It also captures the relation between the firm characteristics and those of the expected returns, variances, covariances and even higher order moments of returns through the distribution of the optimized portfolio's returns and thus the investor's expected utility, as state by Brandt et al. (2009).

As Barroso et al. (2017) describes, one advantage of this method is that they bypass the issue of estimating the conditional distribution of returns by designing weights directly as a function of each asset characteristic. Moreover, by using the PPP, it is possible to construct an investment strategy fully adaptable to a specific asset class, using the advantage of the flexibility of the parametric portfolio policy.

Initially, the Parametric Portfolio Policies developed by Brandt, Santa-Clara and Valkanov (2009) applies in the formation of individual stocks portfolios. Further on, DeMiguel, Martin-Utrera, Nogales and Uppal (2017) emphasize on the same area of parametric portfolio formation as Brandt et al. (2009) and obtain similar results with a larger set of characteristics.

Barroso and Santa-Clara, (2015), apply this portfolio formation on currencies and obtain results which are statistically significant. In 2017, a study by Faias and Santa-Clara (2017) applies the similar structure developed by Brandt, Santa-Clara and Valkanov (2009) but uses options as the centre of their research paper. DeMiguel, Martin-Utrera, Nogales and Uppal (2020) extend the Parametric Portfolio Policy methodology in order to study how transaction costs change the number of characteristics that are jointly significant for an investor's portfolio. Barroso, Menichetti and Reichenecker (2021) uses the same method using currency momentum, value, carry and autocorrelations to reduce the cost of hedging and find significant results, with 38% gain in Sharpe ratio. My contribution to this area of research is the application of the parametric portfolios policies for equity industries portfolios.

Besides individual stocks, currencies and options portfolios, industry investing has also been of great interest to both academics and practitioners' researchers in the past years. Fama and French (1997) focus their study trying to estimate the industry cost of equity. According to Moskowitz and Grinblatt (1999), the momentum effect seems to be stronger when applied to industries rather than individual stocks. Similar results are also documented by Menzly and Ozbas (2005). Narayan, Ahmed and Narayan (2017) apply the Markowitz framework in an industry level and found that the performance of dynamic trading strategies outperform passive trading strategies in all sectors studied.

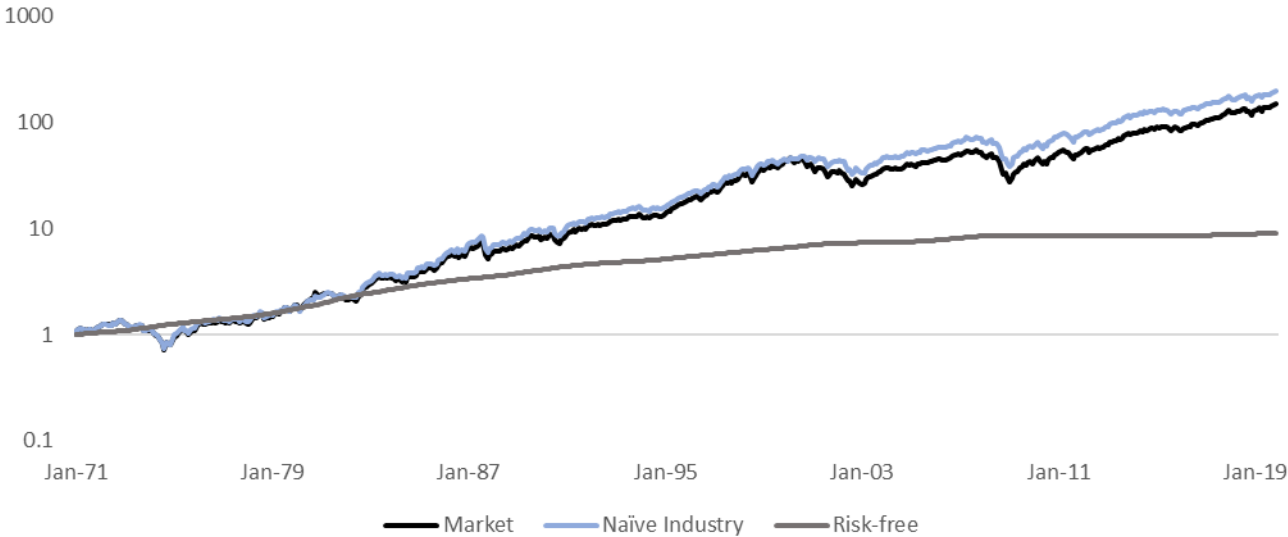
The objective of this paper is to apply the Parametric Portfolio Policy model in industry level by introducing three new characteristics – profitability, volatility and beta – and demonstrate how this model performs with industry investing when applying different industry-specific variables and compare it to the naïve industry portfolio and US risk-free rate.

3. Data

The monthly industry ratios are taken from the Center for Research of Security Prices (CRSP) and Compustat database using all CRSP US common stocks distinguished in the Fama-French 49 Industries using the median as the aggregation method. The value-weighted stock index is taken from Kenneth French Data Library, constructed with all CRSP firms incorporated in the US and listed on the NYSE, AMEX or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t and used as a proxy for the market portfolio. As a proxy for the US risk-free rate, it was used the one-month Treasury bill rate, also from Kenneth French Data Library.

In order to achieve a robust sample for the model’s portfolio, it was collected monthly data ranging from January 1970 to December 2019.

Figure 1
Market, Naïve Industry and Risk-free Portfolio Performance



Performance of the three analyzed portfolios during the sample period of 1970 to 2019.

As one can see in Figure 1, during the sample period, the market experienced at least three periods of bear market: 1973 oil crisis (between January 1973 and December 1974), Dot-com Bubble (between March 2000 and October 2002) and global financial crisis (between October 2007 and March 2009).

Table 1:

Table 1 shows statistics on the performance of the market, naïve industry and risk-free portfolios during the sample period.

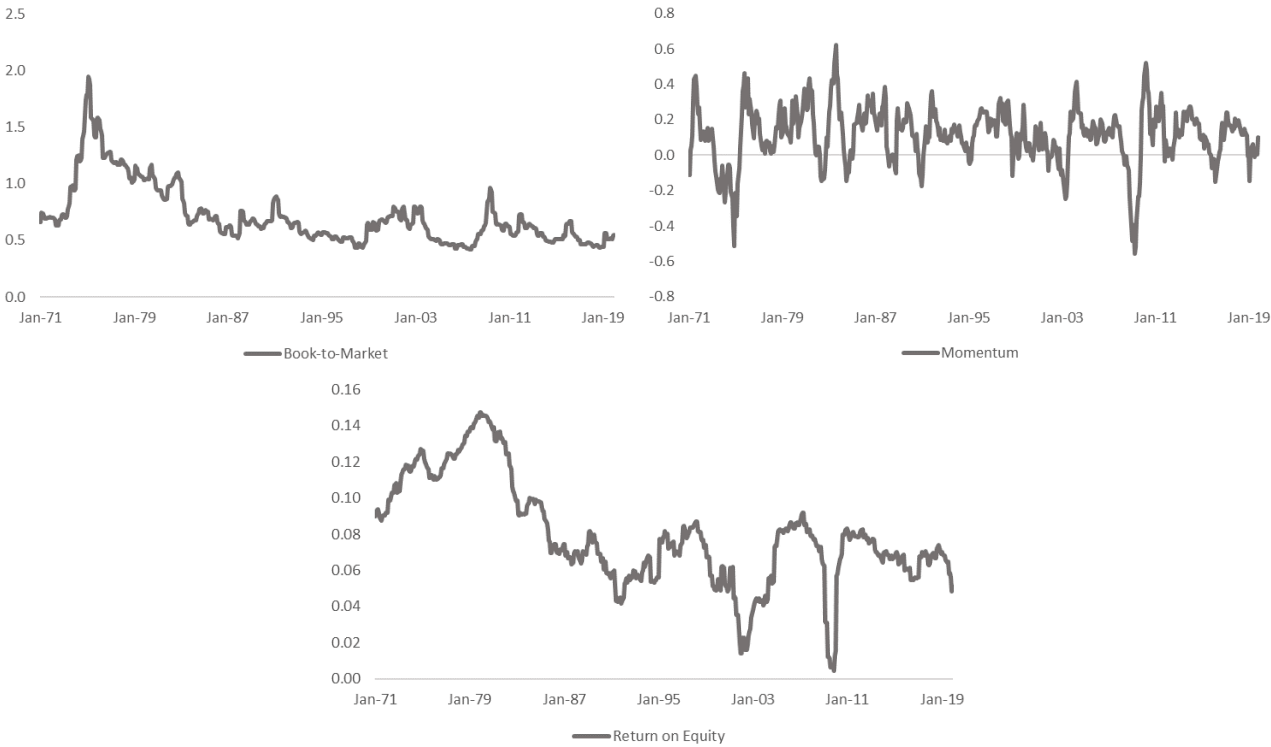
Table 1
Market, Naïve Industry and Risk-free Portfolio Performance

The table provides the average return, the standard deviation and Sharpe ratio from January 1970 to December 2019. The first column represents the market statistics, followed by the naïve industries data and finally, the last column, the risk-free rate information. All values are annualized.

	Market [1]	Naïve industry [2]	Risk-free [3]
Mean Return	11.49%	11.91%	4.51%
Standard Deviation	15.43%	14.55%	0.97%
Sharpe ratio	0.45	0.51	-

The average annualized return for the market portfolio is 11.49% with a standard deviation of 15.43% during the sample period. On the other hand, the equal-weighted industry portfolio presents an average return equal to 11.91% per year with a standard deviation of 14.55%. During the same period, the risk-free rate delivered an annual average return of 4.51%. The market portfolio has a Sharpe ratio equal to 0.45 and the naïve industry portfolio equal to 0.51.

Figure 2
Characteristics Mean During the Sample Period



Monthly average of the three base non-standardized characteristics during the sample period of 1971 to 2019.

The figure above presents cross-sectional non-standardized average of the Book-to-Market, Momentum and Return on Equity characteristics in the industry level across the data sample.

From figure 2, it is possible to identify that during moments of market crashes, the momentum factor decreases drastically, even assuming negative values, in opposite to the value factor that tends to increase during crises. The profitability factor tends to increase during bull markets and decrease during bear market.

4. Methodology

The base portfolio was constructed using industries' book-to-market (book equity divided by market equity), industries' momentum (compounded return between months $t - 12$ and $t - 2$) and return on equity as a measure of profitability. The size factor, from the original Parametric Portfolio Policies described by Brandt, Santa-Clara and Valkanov (2009), was substituted for the profitability factor, in line with Novy-Marx (2012), since size is essentially a firm-level factor and profitability is likely to be a more relevant factor for industries.

Before proceeding to the portfolio optimization problem, all characteristics were cross-sectionally standardized to have mean equal zero and standard deviations equal to one, using the formula below:

$$C_{i,t}^S = \frac{(C_{i,t} - \mu_t)}{\sigma_t}$$

where $C_{i,t}$ is the original characteristic for industry i at month t , μ_t is the characteristic mean for the cross-section of industries at month t , σ_t is the characteristic standard deviation for the cross-section of industries at month t . The characteristics are standardized due to the cross-sectional distribution of the standardized $C_{i,t}^S$ being stationary over time, although the raw $C_{i,t}$ might be non-stationary. Furthermore, the standardization state that the cross-sectional average of deviations of the optimal portfolio weights from the benchmark weights sum to zero. The results in the long-short portfolio are comparable to those described by Brandt, Santa-Clara and Valkanov (2009).

The linear specification for the portfolio weight function is similar to the one used by Brandt et al. (2009). The exception in relation to the original model is the distinction between the original weight on the equal-weighted industry portfolio to the deviation that the model is implying. This study presents this small change to assess the benefit that those deviations impact on the investor's portfolio and it is represented by the following formula:

$$\omega_{i,t} = \frac{1}{N_t} \theta^T C_{i,t}^S$$

Where θ^T is the transposed vector of coefficients to be estimated for each characteristic and $C_{i,t}^S$ is the cross-sectionally standardized characteristic for industry i at month t , N_t is the number of sectors at month t .

The term $1 / N_t$ is a normalization ensures that the portfolio weights function to be used to an arbitrary and time-varying number of industries. Without this modification, if investor doubles the assets universe it will result in twice as aggressive wealth allocation, without any

fundamental change in the investment opportunities. As mentioned by Brandt, Santa-Clara and Valkanov (2009), rather than estimating one weight for each industry, the model estimates weights as a single function of characteristics that applies to all industries each month.

The most crucial condition of the parametrization is that the vector of coefficients θ are constant over assets and time. Constant coefficients across assets ensures that two assets that are similar to each other in characteristics related with expected returns and risk should have close weights in the portfolio, even though their sample returns differ from each other. Constant coefficients over time express that the coefficients that maximize the investor's conditional expected utility at a specific month are the same for all months and consequently maximize the investor's unconditional expected utility. In ensuring these conditions, the chances of in-sample overfitting are considerably reduced, since only a $k \times 1$ vector of characteristics is predicted.

After these two conditions being explained, it is possible to rewrite the conditional optimization relative to the portfolio weights $\omega_{i,t}$ as the unconditional optimization regarding to the coefficients θ as follows:

$$\max_{\theta} E[u(r_{p,t+1})] = E[u(\sum_{i=1}^{N_t} (\frac{1}{N_t} \theta^T C_{i,t}^S) r_{i,t+1})]$$

It is possible to estimate the vector of coefficients θ by maximizing the corresponding sample analogue:

$$\max_{\theta} \frac{1}{T} \sum_{t=0}^{T-1} u(r_{p,t+1}) = \frac{1}{T} \sum_{t=0}^{T-1} u(\sum_{i=1}^{N_t} (\frac{1}{N_t} \theta^T C_{i,t}^S) r_{i,t+1})$$

for any pre-specified utility function, here is the constant relative risk aversion (CRRA) preferences over wealth, in line with Brandt, Santa-Clara and Valkanov (2009) and described below:

$$u(r_{p,t+1}) = \frac{(1 + r_{p,t+1})^{1-\gamma}}{1 - \gamma}$$

where γ represents the level of the investor's risk-aversion. The benefit of using a CRRA utility function is that it includes preferences concerning higher-order moments without the need to introduce any additional preference parameters. In other words, this utility function takes into account kurtosis and skewness, in contrast to mean-variance utility which concentrate only on the first two moments of the returns' distribution.

The base portfolio construct under Parametric Portfolio Policies is a Long-short Portfolio, where each month the sum of weights of long positions equals the sum of weights of short positions. After this portfolio is developed, two others portfolios are created: A long only portfolio, where only the long positions are used and the net position equals 100% long every month; And a Naïve Industry + Long-Short portfolio, similar to the base model develop by Brandt, Santa-Clara and Valkanov (2009), where the intuition here is that the investor is buying the Naïve Industry portfolio with more weights in the long positions and less weights in the short positions of the PPP model. In other words, the long-short portfolio captures the deviations of the optimal portfolio weights from the benchmark portfolio weights.

In order to construct the long only portfolio some adjustments should be done to ensure that all months will have a 100% long exposure. The simplest way to establish this constraint cross the portfolio policy is to limit the portfolio weights computed in the long-short portfolio at zero and renormalize the portfolio weights as expressed in the following formula:

$$\omega_{i,t}^+ = \frac{\max[0, \omega_{i,t}]}{\sum_{j=1}^{N_t} \max[0, \omega_{j,t}]}$$

By doing so, it is guaranteed the positivity of the portfolio weights and that every month it will sum to one.

In spite of analysing the differences in performance on the three portfolios strategies that I propose on this paper and the benchmark, it is used the following metrics: (I) Sharpe ratio (SR), (II) Jensen-Alpha, (III) Information ratio and (IV) turnover. Each metric is briefly explained bellow:

(I) Sharpe ratio (SR):

Developed by Nobel laureate William F. Sharpe (1966), it is most famous measure to assess the trade-off between risk, measured by standard deviation of returns (σ), and return of an investment, measured by the portfolio excess return ($\mu - rf$):

$$SR = \frac{\mu - rf}{\sigma}$$

(II) Jensen-Alpha:

Alpha is used in order to determine the abnormal return of an asset or a portfolio of assets over the theoretical expected return using the Capital Asset Pricing

Model (CAPM). The Jensen's alpha could be easily obtain applying the formula below:

$$\alpha_i = (R_i - R_f) - \beta_{iM}(R_m - R_f)$$

Where R_i is the expected return of the asset or portfolio, R_m is the return of the benchmark portfolio, R_f is the risk-free rate and β_{iM} is the beta of the asset or portfolio in relation to the benchmark portfolio.

(III) Information ratio:

The information ratio is a measurement of excess return (Jensen-alpha) compared to the volatility of the residuals and can be computed applying the formula below:

$$IR = \left(\frac{\alpha_i}{\sigma_r} \right)$$

Where α_i is the Jensen-Alpha and σ_r is the volatility of the residuals.

(IV) Turnover:

Turnover is an essential component of an investment strategy. In a world without transaction costs this measure would be unnecessary. However, as it is a relevant concern to portfolio managers, it was considered in the analysis. A high turnover expresses that a large proportion of capital gains were lost through transaction costs, affecting the net return of the strategy. In order to take into account all these concerns, it is provided a turnover measure in line with the one defined by DeMiguel, Garlappi, and Uppal (2007):

$$Turnover = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^N (|\omega_{i,t+1} - \omega_{i,t}|)$$

Where $\omega_{i,t}$ is the inertia weight, that is, the weight at the end of the period before the rebalancing, $\omega_{i,t+1}$ is the new weight given by the rebalancing, T represents the number of periods in the sample and N is the number of assets that are invested in each period. This coefficient averages the absolute change in weights from one month to the following.

5. Results

This section presents the analysis of both “naïve industry + long-short” and “long-only” portfolios, varying the characteristics used. I compute the Sharpe ratio, Jensen-Alpha, information ratio and portfolio’s turnover in order to compare the different strategies proposed with the benchmark. It is also computed a significance test of the characteristics and models through a bootstrap method and the results of the test is used to propose a final portfolio strategy developed in the end of this section varying the risk aversion coefficient.

5.1 Base case

Table 2:

Table 2 reports descriptive information regarding the unconstrained monthly rebalanced Naïve Industry + Long-short and Long-only portfolios constructed using book-to-market, momentum and profitability as characteristics and a risk aversion coefficient of 5 in line with the standard model presented by Brandt, Santa-Clara and Valkanov (2009).

The table displays the parameter estimates for each of the characteristics, followed by performance measures in order to compare different portfolio strategies.

The coefficients estimated through the optimization process can be directly compared to each other as all the characteristics used are standardized in the cross-section.

Table 2
In-sample Performance of the Base Case Strategy

The table provides descriptive information for the in-sample base case strategy from January 1971 to December 2019. The first row presents the value parameter, followed by the momentum and profitability parameters. The fourth row presents the average return, followed by the standard deviation and Sharpe ratio. The seventh row presents the correlation to the benchmark. All values are annualized.

	IS Naïve Industry + Long-short	IS Long-only	Naïve Industry
Value	1.087	1.087	-
Momentum	2.309	2.309	-
Profitability	0.075	0.075	-
Mean Return	19.2%	15.7%	12.2%
Standard Deviation	20.1%	18.9%	16.8%
Sharpe Ratio	0.73	0.59	0.46
Correlation to the benchmark	82.1%	88.8%	100.0%

As a result of the in-sample analysis (“IS Naïve Industry + Long-short” and “IS Long-only”), it can be said that all three characteristics proposed for the base case model presents a

positive coefficient. In other words, the parametric portfolio's optimal weights deviate positively with both value, momentum and profitability. However, it is easy to observe that the momentum characteristic is the one with the highest coefficient and profitability characteristics is the one with the lowest coefficient. This means that a higher momentum provokes a larger overweight of an industry in the final portfolio and a smaller momentum provokes a larger underweight of an industry in the final portfolio. In other words, the model deviates more from the benchmark with respect to a higher or lower momentum than to a higher or lower profitability.

The in-sample naïve industry + long-short portfolio ("IS Naïve Industry + Long-short") has an average return of 19.2%, a standard deviation of the returns of 20.1% and a correlation with the equal weighted industry portfolio of 82.1% during the sample period studied.

As one can see, the in-sample Sharpe ratio from the equal weighted + long-short portfolio ("IS Naïve Industry + Long-short") and from the long-only portfolio ("IS Long-only") are equal to 0.73 and 0.59 respectively, while the equal weighted industry portfolio presents a Sharpe ratio equal to 0.46 for the sample period studied.

It should be recognized that in this analysis. Using an in-sample optimization, it is expected that the policy portfolio outperforms the benchmark. Thus, I conduct an out of sample analysis in order to check the robustness of the model in terms of return, standard deviation and Sharpe ratio and the results are shown in the table below.

Although our sample period started in January 1970, the optimization needs an initial period of 10 years of data in order to estimate the coefficients for the out-of-sample strategy. Thus, the results shown in the table below are only for the period between January 1981 to December 2019.

Table 3:

Table 3 reports descriptive information regarding the unconstrained monthly rebalanced Naïve Industry + Long-short and Long-only portfolios constructed using book-to-market, momentum and profitability as characteristics and a risk aversion coefficient of 5 in line with the standard model presented by Brandt, Santa-Clara and Valkanov (2009).

The table displays the parameter estimates for each of the characteristics, followed by performance measures in order to compare different portfolio strategies.

The coefficients estimated through the optimization process can be directly compared to each other as all the characteristics used are standardized in the cross-section.

Table 3
Out-of-sample Performance of the Base Case Strategy

The table provides descriptive information for the base case strategy from January 1981 to December 2019. The first row presents the average value parameter, followed by the average momentum and profitability parameters. The fourth row presents the average return, followed by the standard deviation and Sharpe ratio. The seventh row presents the correlation to the benchmark, followed by the Jensen-Alpha, Jensen-Alpha t-stat, Information ratio and finally, the last row, presents the portfolio's turnover. All values are annualized.

	OOS Naïve Industry + Long-short	OOS Long-only	Naïve Industry
Value	0.642	0.642	-
Momentum	2.117	2.117	-
Profitability	0.124	0.124	-
Mean Return	20.6%	14.5%	12.5%
Standard Deviation	24.7%	16.7%	16.0%
Sharpe Ratio	0.67	0.63	0.53
Correlation to the benchmark	53.7%	92.8%	100.0%
Jensen-Alpha	9.6%	2.3%	-
Jensen-Alpha t-stat	2.82	2.23	-
Information Ratio	0.46	0.36	-
Portfolio turnover	1058.9%	403.5%	60.1%

The out-of-sample naïve industry + long-short portfolio (“OOS Naïve Industry + Long-short”) has an average return of 20.6%, a standard deviation of the returns of 24.7% and a correlation with the equal weighted industry portfolio of 53.7% during the sample period studied.

As one can see, the out-of-sample Sharpe ratio from the equal weighted + long-short portfolio (“OOS Naïve Industry + Long-short”) and from the long-only portfolio (“OOS Long-only”) are equal to 0.67 and 0.63 respectively, while the Naïve Industry portfolio presents a Sharpe ratio are equal to 0.53 for the sample period studied.

The Naïve Industry + Long-short portfolio (“OOS Naïve Industry + Long-short”) presents an annualized Jensen-Alpha of 9.6% and Information ratio of 0.46 for the sample period studies.

An important component regarding the out-of-sample Naïve Industry + Long-short portfolio is that, compared to the in-sample analysis, the correlation of the strategy to the benchmark decreased from 82.1% to 53.7%. This confirms that the performance of the strategy is more associated to characteristics coefficients and therefore to the weights imposed to the

characteristics on each month than to the overall market performance in comparison to the in-sample and benchmark portfolios.

As described by Lesmond, Schill and Zhou (2004) when analysing momentum strategies, “the evidence for positive trading profit net of transaction costs appears weak”. The authors find that momentum strategies are not profitable after transaction costs. In line with Kirby and Ostdiek (2012), most of optimized portfolios involve too much rebalancing trades, resulting on a disappointing OOS net-of-cost performance. Barroso and Santa-Clara (2015) exploits the transaction costs issue on momentum strategies. These studies find that trading costs are important deterrents of profitability for otherwise puzzling strategies.

The out-of-sample Naïve Industry + Long-short portfolio has an average annual turnover of nearly 1058.9%. At this level of turnover, if transaction costs are taken into account, the strategy is practically unable to be implemented. This occurs because the high transaction costs can largely decrease the policy’s performance for the investor. When compared to the benchmark average annual turnover of nearly 60%, we conclude that the portfolio’s turnover is extremely high.

There are significant diverges on the literature addressing transaction costs. As Barroso and Saxena (2020) describe, “these likely depend on trade size, order types (limit or market), the trading algorithm, and market conditions (volatility and liquidity)”. Hasbrouck (2009) considers an effective one-way transaction cost around 50 to 60 basis points (bps) for the median stock by market capitalization. Novy-Marx and Velikov (2016) predict roundtrip costs of value-weighted strategies around 50bps (one-way transaction costs of 25bps). Frazzini, Israel and Moskowitz (2018), use 1.7 billion dollars of live trade execution data from a large institutional investor and includes price impact, which they mention on the study as the largest component of costs for large investors, to estimate one-way costs for larger stocks of just 9.93bps.

Bearing in mind these differences in estimates, it is reported in this study the after-cost performance with one-way costs of 10, 30 and 50bps, in line with Barroso and Saxena (2020). A likely advantage of trading an industry ETF is that the cost should be lower than those for median individual stock which is in practice just a small-cap. Thus, the 50bps transaction costs is probably too conservative.

Table 4:

Table 4 reports descriptive information regarding the after-cost performance with one-way costs of 10, 30 and 50bps in the base case Naïve Industry + Long-short portfolio.

Table 4
Out-of-sample Performance of the Base Case Strategy Assuming Transaction Costs

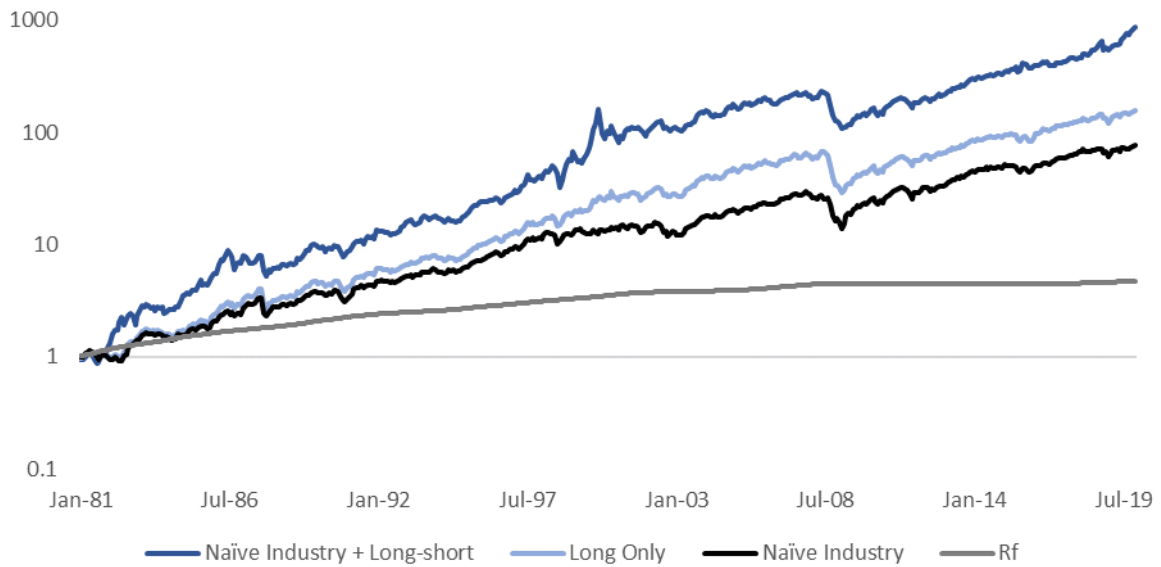
The table provides descriptive information for the base case strategy from January 1981 to December 2019 assuming transaction costs. The column assumes one-way costs of 10bps, following by one-way costs of 30bps and finally, the last column assumes one-way costs of 50bps. All values are annualized.

	OOS Naïve Industry + Long-short (Transaction costs = 10bps)	OOS Naïve Industry + Long-short (Transaction costs = 30bps)	OOS Naïve Industry + Long-short (Transaction costs = 50bps)
Mean Return	19.5%	17.4%	15.3%
Standard Deviation	24.7%	24.7%	24.7%
Sharpe Ratio	0.63	0.54	0.46
Jensen-Alpha	8.5%	6.4%	4.3%

When taking transaction costs of 10, 30 and 50bps into account the net-of-cost annual return of the out-of-sample Naïve Industry + Long-short portfolio decreases to 19.5%, 17.4% and 15.3% respectively, while the standard deviation remains constant at 24.7% in all cases. The impact on the portfolio return decreases the Sharpe ratio to 0.63, 0.54 and 0.46 and the annualized Jensen-Alpha to 8.5%, 6.4% and 4.3% respectively.

The chart below presents the out-of-sample Naïve Industry + Long-short and long-only portfolios as well as the Naïve Industry portfolio, our benchmark for the strategy, and the risk-free rate.

Figure 3
Out-of-sample Base Case Strategy Performance



Performance of the base case portfolios in comparison to the benchmark and risk-free portfolios during the sample period of 1981 to 2019 before transaction costs.

5.2 Five characteristics strategy

The unconstrained five characteristics model is constructed using the same three characteristics used in the base case developed above and including the volatility and beta as two new characteristics. The volatility factor used was the annualized twelve months standard deviation of monthly returns and the beta factor used was a one-year beta computed using monthly past data. The addition of volatility and beta factors are in line with Ang, Hodrick, Xing and Zhang (2006) and Frazzini and Pedersen (2014) respectively.

Table 5:

Table 5 reports descriptive information regarding the unconstrained monthly rebalanced Naïve Industry + Long-short and Long-only portfolio constructed using a risk aversion coefficient of 5 in line with the standard model presented by Brandt, Santa-Clara and Valkanov (2009) and the base case presented previously.

The table displays the parameter estimates for each of the characteristics, followed by the allocation of assets and performance measures in order to compare different portfolio strategies.

The coefficients estimated through the optimization process could be directly compared to each other as all the characteristics used are standardized in the cross-section.

Table 5
In-sample Performance of the Five Characteristics Strategy

The table provides descriptive information for the in-sample five characteristics strategy from January 1971 to December 2019. The first row presents the value parameter, followed by the momentum, profitability, beta and volatility parameters. The sixth row presents the average return, followed by the standard deviation and Sharpe ratio. The ninth row presents the correlation to the benchmark. All values are annualized.

	IS Naïve Industry + Long-short	IS Long-only	Naïve Industry
Value	1.075	1.075	-
Momentum	2.327	2.327	-
Profitability	0.153	0.153	-
Beta	0.055	0.055	-
Volatility	0.134	0.134	-
Mean Return	19.2%	15.8%	12.2%
Standard Deviation	20.5%	19.2%	16.8%
Sharpe Ratio	0.72	0.59	0.46
Correlation to the benchmark	82.6%	88.5%	100.0%

As a result of the in-sample analysis (“IS Naïve Industry + Long-short”), it can be found that all five characteristics proposed for the model presents a positive coefficient. Once again, the momentum factor has the largest coefficient, and the beta has now the smallest one. In other words, the model deviates more from the benchmark with respect to a higher or lower momentum than to a higher or lower beta.

The in-sample Naïve Industry + Long-short portfolio (“IS Naïve Industry + Long-short”) has an average return of 19.2%, a standard deviation of the returns of 20.5% and a correlation with the equal weighted industry portfolio of 82.6% for the sample period studied.

The resultant in-sample Sharpe ratio from the equal weighted + long-short portfolio (“IS Naïve Industry + Long-short”) and Long-only (“IS Long-only”) are equals to 0.72 and 0.59 respectively, while the equal weighted industry portfolio presents a Sharpe ratio equals to 0.46 for the sample period studied.

In order to check the robustness of the strategy, it is conducted an out of sample analysis in order to check the robustness of the five characteristics strategy in terms of return, standard deviation and Sharpe ratio and the results are shown in the table below.

Table 6:

Table 6 reports descriptive information regarding the out-of-sample unconstrained monthly rebalanced Naïve Industry + Long-short and Long-only portfolios constructed using book-to-market, momentum, profitability, beta and volatility as characteristics and a risk aversion coefficient of 5 in line with the standard model presented by Brandt, Santa-Clara and Valkanov (2009).

The table displays the parameter estimates for each of the characteristics, followed performance measures in order to compare different portfolio strategies.

The coefficients estimated through the optimization process can be directly compared to each other as all the characteristics used are standardized in the cross-section.

Table 6
Out-of-sample Performance of the Five Characteristics Strategy

The table provides descriptive information for the five characteristics strategy from January 1981 to December 2019. The first row presents the average value parameter, followed by the average momentum, profitability, beta and profitability parameters. The sixth row presents the average return, followed by the standard deviation and Sharpe ratio. The ninth row presents the correlation to the benchmark, followed by the Jensen-Alpha, Jensen-Alpha t-stat, Information ratio and finally, the last row, presents the portfolio's turnover. All values are annualized.

	OOS Naïve Industry + Long-short	OOS Long-only	Naïve Industry
Value	0.634	0.634	-
Momentum	2.109	2.109	-
Profitability	0.040	0.040	-
Beta	0.040	0.040	-
Volatility	0.040	0.040	-
Mean Return	19.7%	14.8%	12.5%
Standard Deviation	23.4%	17.3%	16.0%
Sharpe Ratio	0.67	0.63	0.53
Correlation to the benchmark	58.5%	92.4%	100.0%
Jensen-Alpha	8.4%	2.3%	-
Jensen-Alpha t-stat	2.73	2.14	-
Information Ratio	0.44	0.35	-
Portfolio turnover	1049.5%	415.8%	60.1%

As one can see, the out-of-sample Sharpe ratio from the Naïve Industry + Long-short (“OOS Naïve Industry + Long-short”) and Long only (“OOS Long-only”) portfolios for the five characteristics strategy are equal to 0.67 and 0.63 respectively, while the equal weighted industry portfolio presents a Sharpe ratio equals to 0.53 for the sample period studied.

The Naïve Industry + Long-short portfolio (“OOS Naïve Industry + Long-short”) presents an annualized Jensen-Alpha of 8.4% and an Information ratio of 0.44 for the sample period studied.

An important component regarding the out-of-sample Naïve Industry + Long-short portfolio is that, compared to the in-sample analysis, the correlation of the strategy to the benchmark decreased from 82.6% to 58.6%, slightly higher than the base case strategy correlation. This confirms that the performance of the strategy is more associated to

characteristics coefficients and therefore to the weights on the characteristics on each month than to the overall market performance.

The out-of-sample Naïve Industry + Long-short portfolio for the five characteristics strategy has an average annual turnover of nearly 1049.5%, similar to the average annual turnover of the base case strategy. At this level of turnover, if transaction costs are taken into account, the strategy is once again practically unable to be implemented. This occurs because the high transaction costs can largely decrease the policy’s performance for the investor. When compared to the benchmark average annual turnover of nearly 60%, we conclude that the portfolio’s turnover is extremely high.

As on the base case portfolio, on the table below is reported the after-cost performance with one-way costs of 10, 30 and 50bps, in line with Barroso and Saxena (2020).

Table 7:

Table 7 reports descriptive information regarding the after-cost performance with one-way costs of 10, 30 and 50bps in the five characteristics strategy Naïve Industry + Long-short portfolio.

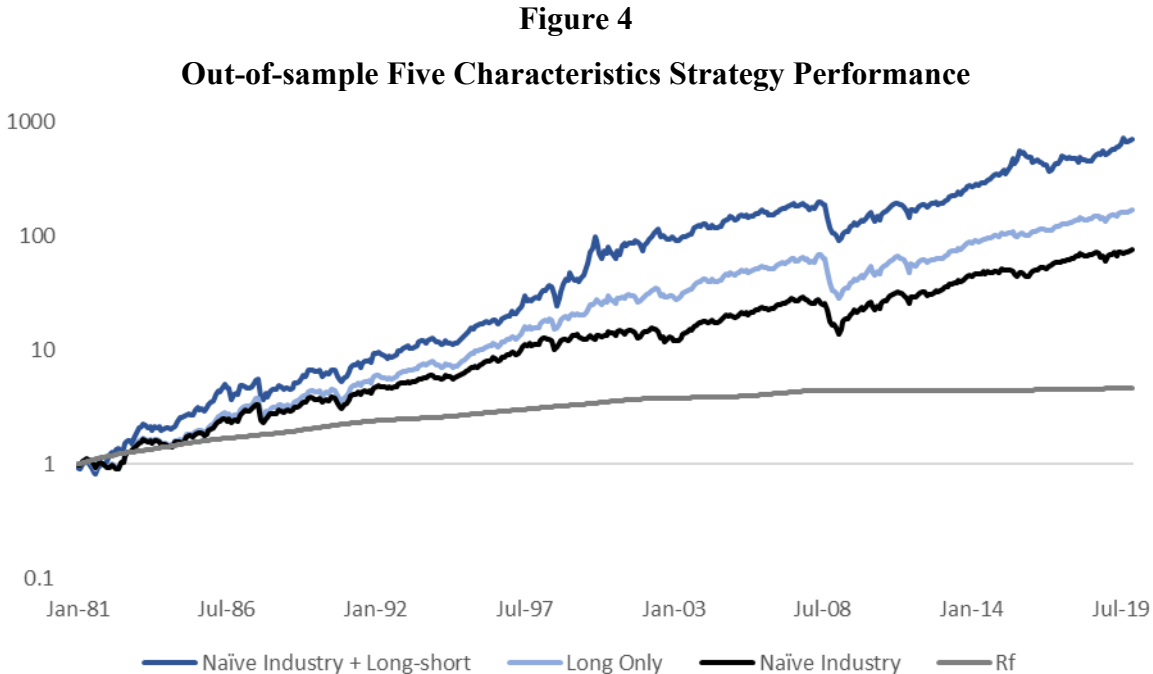
Table 7
Out-of-sample performance of the five characteristics strategy assuming transaction costs

The table provides descriptive information for the five characteristics strategy from January 1981 to December 2019 assuming transaction costs. The column assumes one-way costs of 10bps, following by one-way costs of 30bps and finally, the last column assumes one-way costs of 50bps. All values are annualized.

	OOS Naïve Industry + Long-short (Transaction costs = 10bps)	OOS Naïve Industry + Long-short (Transaction costs = 30bps)	OOS Naïve Industry + Long-short (Transaction costs = 50bps)
Mean Return	18.6%	16.5%	14.4%
Standard Deviation	23.4%	23.4%	23.4%
Sharpe Ratio	0.63	0.54	0.45
Jensen-Alpha	7.3%	5.2%	3.2%

When taking transaction costs of 10, 30 and 50bps into account the net-of-cost annual return of the out-of-sample Naïve Industry + long-short portfolio decreases to 18.6%, 16.5% and 14.4% respectively, while the standard deviation remains constant at 23.4% in all cases. The impact on the portfolio return decreases the Sharpe ratio to 0.63, 0.54 and 0.45 and the annualized Jensen-Alpha to 7.3%, 5.2% and 3.2% respectively.

The figure below presents the out-of-sample Naïve Industry + Long-short and Long-only for the five characteristics strategy performance, as well as the Naïve Industry portfolio, consider as our benchmark for the strategy, and the risk-free rate for the sample period studied.



Performance of the base case portfolios in comparison to the benchmark and risk-free portfolios during the sample period of 1981 to 2019.

5.3 Characteristics significance test

In order to estimate the t-stat and consequently assess the statistical significance of each characteristic coefficient, it is performed a bootstrap analysis. For this purpose, I generate a large number of samples of returns and characteristics by randomly picking monthly observations from the original data set with replacement. The replacement on the bootstrap method means that some monthly observations can appear twice or more on the random sample generated and it is also possible that some observations do not appear. For each of the bootstrapped samples created, I estimate the coefficients of the in-sample optimal portfolio policy. The bootstrapped standard errors method, in comparison to others statistical significance test methods, are useful in the multi-period investment horizon method implemented on this study. This method of estimating standard errors on Parametric Portfolio Policy optimization are in line with Brandt, Santa-Clara and Valkanov (2009) and Barroso and Santa-Clara (2015).

The resulting estimate of the bootstrapped characteristics coefficients can be used to test individual and joint hypotheses about them. The statistical significance approaches the issue of

whether the characteristic is cross-sectionally associated to the moments of returns in a way that investors can take benefit of it and deviate from the benchmark portfolio weights in order to achieve the optimal portfolio given the characteristics of each asset. The t-stat was computed in this study using 100 bootstrapped samples and the results can be seen on the table below.

Table 8:

Table 8 reports results regarding the statistically significance test for the individual characteristics coefficient and the coefficients on the base case and five characteristics strategies.

Table 8
Characteristics Coefficient Significance Test

The table provides the t-stat for individual characteristics coefficient and jointly test for the base case and five characteristics strategy coefficients. The first column presents the t-stat for value coefficient, followed by momentum, profitability, beta and volatility.

	Value	Momentum	Profitability	Beta	Volatility
Individual test	-5.013	23.228	5.133	-2.773	-2.808
Base case test	7.310	24.528	-0.108	-	-
Five characteristics test	3.473	14.141	1.179	-0.068	0.585

When testing each coefficient individually, it is possible to conclude that all five characteristics seems to be statistically significant at 1% of confidence level. This means that the investor can take advantage of the value (book-to-market), momentum, profitability (return on equity), beta, and volatility characteristics of each security in order to deviate from the benchmark portfolio. The statistically significance of the beta factor is in line with Frazzini and Pedersen (2014) and Barroso, Detzel and Maio (2020) and the volatility factor is in line with Ang, Hodrick, Xing and Zhang (2006) and Barroso, Detzel and Maio (2020).

However, as one can see from the table above, the momentum and book-to-market characteristics are statistically significant in the base case model proposed previously, although the profitability characteristics does not indicate to be statistically significant when jointly testing the three characteristics at 10% of confidence level. Moreover, when jointly testing the five characteristics strategy coefficients, it is possible to conclude that once again only the momentum and book-to-market coefficients are statistically significant on this model, while the profitability, beta and volatility does not indicate to be statistically significant at 10% of confidence level.

Previous asset pricing studies show that these characteristics matter at individual stock level. The jointly bootstrap test results indicates that they do not jointly matter at industry level, suggesting that differences in these characteristics between stocks in the same industry are more relevant than differences across industries.

Taking the results from the joint testing into consideration, it is proposed a final portfolio strategy constructed with value and momentum, the two statistically significant characteristics on the jointly bootstrap tests and described below.

5.4 Final case strategy

As explained above, a final portfolio strategy is constructed using only value and momentum as the characteristics, as a result of the statistical significance of both characteristics together.

Table 9:

Table 9 reports descriptive information regarding the unconstrained monthly rebalanced naïve industry + long-short and long-only portfolios constructed using value and momentum as characteristics and a risk aversion coefficient of 5 in line with the standard model presented by Brandt, Santa-Clara and Valkanov (2009).

The table displays the parameter estimates for each of the characteristics, followed by performance measures in order to compare different portfolio strategies.

The coefficients estimated through the optimization process can be directly compared to each other as all the characteristics used are standardized in the cross-section.

Table 9
In-sample Performance of the Final Case Strategy

The table provides descriptive information for the in-sample final case strategy from January 1971 to December 2019. The first row presents the value parameter, followed by the momentum parameter. The third row presents the average return, followed by the standard deviation and Sharpe ratio. The sixth row presents the correlation to the benchmark. All values are annualized.

	IS Naïve Industry + Long-short	IS Long-only	Naïve Industry
Value	1.073	1.073	-
Momentum	2.316	2.316	-
Mean Return	19.2%	15.7%	12.2%
Standard Deviation	20.1%	18.9%	16.8%
Sharpe Ratio	0.73	0.59	0.46
Correlation to the benchmark	82.1%	88.7%	100.0%

As a result of the in-sample analysis, “IS Naïve Industry + Long-short” and “IS Long-only”, it can be found that all two characteristics proposed for the final portfolio case model presents a positive coefficient. In other words, the parametric portfolio’s optimal weights deviate positively with both value and momentum. However, it is easy to observe that the momentum characteristic is the one with the highest coefficient. This means that a higher momentum provokes a larger overweight of an asset in the final portfolio and a smaller momentum provokes a larger underweight of an asset in the final portfolio in comparison to the value characteristic.

The in-sample Naïve Industry + Long-short portfolio (“IS Naïve Industry + Long-short”) has an average return of 20.1%, a standard deviation of the returns of 19.2% and a correlation with the Naïve industry portfolio of 82.1% for the sample period studied.

As one can see, the in-sample Sharpe ratio from the Naïve Industry + Long-short portfolio (“IS Naïve Industry + Long-short”) and is equal to 0.73, in line with the base case strategy, while the Naïve industry portfolio presented a Sharpe ratio equals to 0.46 for the sample period studied.

It is easy to conclude that the in-sample Sharpe ratio of the base case strategy, 5 characteristics strategy and final portfolio case now proposed for the equal weighted + long-short portfolio is the same and thus, it seems to have no need of using the other three characteristics rather than value and momentum.

It should be recognized that this analysis using an in-sample optimization is expected that the policy portfolio outperforms the benchmark. Thus, it is conducted an out of sample analysis in order to check the robustness of the strategy in terms of return, standard deviation and Sharpe ratio and the results are shown in the table below.

As on the base case portfolio, the results shown in the table below are only for the period between January 1981 to December 2019.

Table 10:

Table 10 reports descriptive information regarding the out-of-sample unconstrained monthly rebalanced naïve industry + long-short and long-only portfolios constructed using book-to-market and momentum as characteristics and a risk aversion coefficient of 5 in line with the standard model presented by Brandt, Santa-Clara and Valkanov (2009).

The table displays the parameter estimates for each of the characteristics, followed by performance measures in order to compare different portfolio strategies.

The coefficients estimated through the optimization process can be directly compared to each other as all the characteristics used are standardized in the cross-section.

Table 10
Out-of-sample Performance of the Final Case Strategy

The table provides descriptive information for the final case strategy from January 1981 to December 2019. The first row presents the average value parameter, followed by the average momentum parameter. The third row presents the average return, followed by the standard deviation and Sharpe ratio. The sixth row presents the correlation to the benchmark, followed by the Jensen-Alpha, Jensen-Alpha t-stat, Information ratio and finally, the last row, presents the portfolio's turnover. All values are annualized.

	OOS Naïve Industry + Long-short	OOS Long-only	Naïve Industry
Value	0.574	0.574	-
Momentum	2.290	2.290	-
Mean Return	19.0%	14.4%	12.5%
Standard Deviation	22.2%	17.1%	16.0%
Sharpe Ratio	0.65	0.61	0.53
Correlation to the benchmark	62.9%	91.6%	100.0%
Jensen-Alpha	7.6%	2.1%	-
Jensen-Alpha t-stat	2.70	1.87	-
Information Ratio	0.44	0.30	-
Portfolio turnover	942.4%	455.3%	60.1%

The out-of-sample Naïve Industry + Long-short portfolio (“OOS Naïve Industry + Long-Short”) has an average return of 19.0%, a standard deviation of the returns of 22.2% and a correlation with the Naïve Industry portfolio of 62.9% during the sample period studied.

As one can see, the out-of-sample Sharpe ratio from the Naïve Industry + Long-short (“OOS Naïve Industry + Long-Short”) and Long-only (“OOS Long-only”) portfolios are equals to 0.65 and 0.61 respectively, while the Naïve Industry portfolio presents a Sharpe ratio equals to 0.53 for the sample period studied.

The equal-weighted industry + long-short portfolio (“OOS 1/N + Long-Short”) presents an annualized Jensen-Alpha of 7.6% and an Information ratio of 0.44 for the sample period studied.

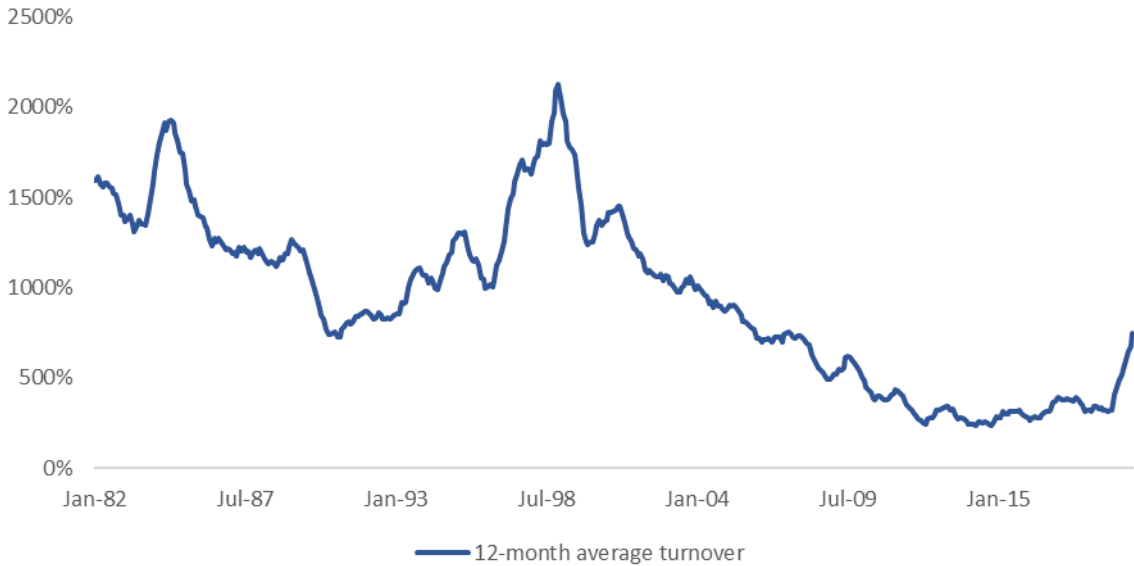
Asness, Moskowitz and Pedersen (2013) find that a blended strategy of value and momentum performs well due to the negative correlation of the returns. Similar conclusions are found on the final case portfolio. The changes on the value and momentum coefficients are

uncorrelated, meaning that a higher (lower) exposure to the momentum factor from one month to the other is not related to a higher (lower) exposure to value factor on the following month.

The out-of-sample Naïve Industry + Long-short portfolio has an average annual turnover of nearly 942.4%. Despite the fact that, compared to the two portfolio strategies proposed with more characteristics, the annual portfolio turnover has decreased, at this level of turnover, if transaction costs are taken into account, the strategy is likely hard to implement profitably. This occurs because the high transaction costs can largely decrease the policy’s performance for the investor. When compared to the benchmark average annual turnover of nearly 60%, we conclude that the portfolio’s turnover is extremely high.

When assessing the 12-month average annual turnover, it is possible to observe in the figure below that the annual turnover has been decreasing over the time, except the spike occurred during the tech bubble in the late 90s and the rapidly increase in 2019 from less than 350% to more than 700%.

Figure 5
12-month Average Annual Turnover



12-month average annual turnover for the final case portfolio during the sample period of 1982 to 2019.

As on the previous cases, on the table below is reported the after-cost performance with one-way costs of 10, 30 and 50bps, in line with Barroso and Saxena (2020).

Table 11**Out-of-sample Performance of the Final Case Strategy Assuming Transaction Costs**

The table provides descriptive information for the final case strategy from January 1981 to December 2019 assuming transaction costs. The column assumes one-way costs of 10bps, following by one-way costs of 30bps and finally, the last column assumes one-way costs of 50bps. All values are annualized.

	OOS Naïve Industry + Long-short (Transaction costs = 10bps)	OOS Naïve Industry + Long-short (Transaction costs = 30bps)	OOS Naïve Industry + Long-short (Transaction costs = 50bps)
Mean Return	18.0%	16.1%	14.2%
Standard Deviation	22.2%	22.2%	22.1%
Sharpe Ratio	0.63	0.55	0.46
Jensen-Alpha	6.6%	4.7%	2.8%

When taking transaction costs of 10, 30 and 50bps into account the net-of-cost annual return of the out-of-sample Naïve Industry + Long-short portfolio decreases to 18.0%, 16.1% and 14.2% respectively, while the standard deviation remains constant at 22.2% in all cases. The impact on the portfolio return decreases the Sharpe ratio to 0.63, 0.55 and 0.46 and the annualized Jensen-Alpha to 6.6%, 4.7% and 2.8% respectively.

An important component regarding the out-of-sample Naïve Industry + Long-short portfolio is that, compared to the in-sample analysis, the correlation of the strategy to the benchmark decreased from 82.1% to 62.9%, confirming that the performance of the strategy is more associated to characteristics coefficients and therefore to the weights on the characteristics on each month than to the overall market performance.

It is important to notice that, by removing the other characteristics proposed previously in the base case and five characteristics strategies, the correlation of the portfolio to the benchmark increased. This states that even though the other characteristics does not seem to be significant on the bootstrap analysis, they are helping to decrease the correlation to the naïve industry portfolio.

In order to check the statistically significance of the two characteristics model, it is performed a jointly test through the same bootstrap standard errors' method, using 100 bootstrapped samples with replacement, as applied previously for the base case and the five characteristics strategies.

Table 12

Characteristics coefficient significance test for the final case strategy

The table provides the t-stat for individual characteristics coefficient and jointly test for the base case and five characteristics strategy coefficients. The first column presents the t-stat for value coefficient, followed by momentum.

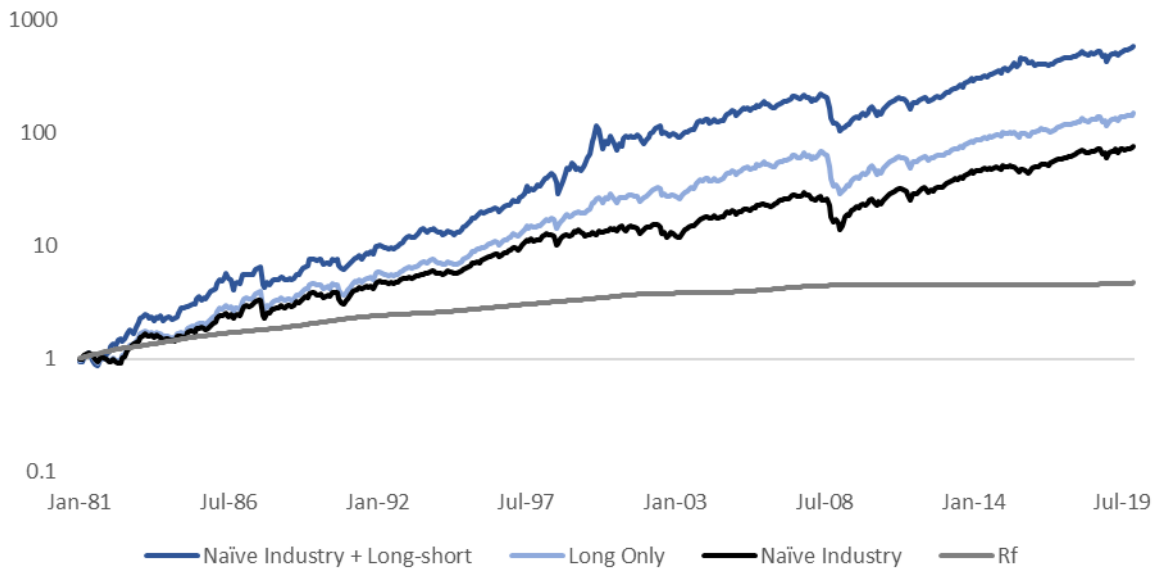
	Value	Momentum
Individual test	-5.013	23.228
Final case test	7.725	25.126

As one can see from the table above, the momentum and book-to-market characteristics is statistically significant in the final case portfolio at 1% of significance level when performed a jointly statistically significance test.

The chart below presents the out-of-sample Naïve Industry + Long-short and Long-only portfolios performance as well as the Naïve Industry portfolio, consider as our benchmark for the strategy, and the risk-free rate for the sample period studied.

Figure 6

Out-of-sample Final Case Strategy Performance



Performance of the final case portfolios in comparison to the benchmark and risk-free portfolios during the sample period of 1981 to 2019.

5.5 Risk aversion coefficient variation

When modelling the utility function and thus the weight of the asset on the optimal portfolio, the risk aversion coefficient performs a big role in the allocation of wealth. The objective in this section is to evaluate how the change of this hypothesis affects the portfolio results in relation to performance metrics and coefficients.

In order to assess this issue, it is recomputed the coefficients for the momentum and book-to-market characteristics for each month varying the risk averse coefficient from 5, as used on all the previous models presented above, to 3, 7 and 9. The risk averse coefficient equals to 3 represents the investor with the biggest appetite for risk taking, while the risk aversion coefficient equals to 9 represents the investor with the lowest appetite for risk taking.

On the table below, it is presented the characteristics coefficients and performance of the final portfolio strategies in terms of average return, standard deviation, Sharpe ratio, correlation to the benchmark, Jensen-alpha, Information ratio and turnover for different risk appetite investors.

Table 13
Out-of-sample Performance of the Final Case Strategy with Risk Aversion Coefficient Variation

The table provides descriptive information for the final case strategy with risk aversion variation from January 1981 to December 2019. The first row presents the average value parameter, followed by the average momentum parameter. The third row presents the average return, followed by the standard deviation and Sharpe ratio. The sixth row presents the correlation to the benchmark, followed by the Jensen-Alpha, Jensen-Alpha t-stat, Information ratio and finally, the last row, presents the portfolio's turnover. The first column presents the All values are annualized.

	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 9$
Value	0.941	0.574	0.412	0.321
Momentum	3.772	2.290	1.644	1.281
Mean Return	23.1%	20.6%	17.1%	16.1%
Standard Deviation	31.1%	24.7%	19.1%	17.7%
Sharpe Ratio	0.61	0.67	0.69	0.69
Correlation to the benchmark	40.7%	62.9%	76.1%	83.9%
Jensen-Alpha	12.4%	7.6%	5.4%	4.2%
Jensen-Alpha t-stat	2.69	2.70	2.71	2.71
Information Ratio	0.44	0.44	0.44	0.44
Portfolio turnover	1552.6%	942.4%	677.7%	529.5%

The sign of the average coefficients stays the same for all levels of risk aversion. Thus, one can affirm that the investors prefer past winners and value firms regardless of their risk preferences. The absolute value of the characteristic coefficient is inversely proportional to the

level of risk aversion. In other words, the characteristics coefficients decrease with the increase in the risk aversion coefficient, meaning that the less the risk averse the investor is, the more the weights will deviate from the benchmark portfolio, developing more drastic allocations. These extreme allocations lead to higher average return. However, it also increases the standard deviation of the optimal portfolio.

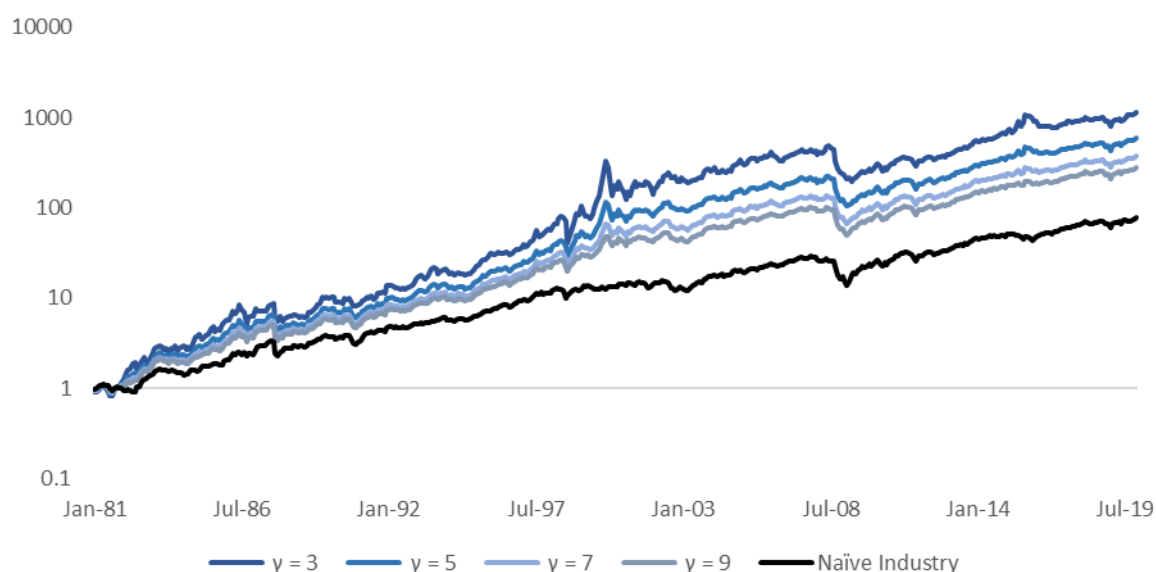
The portfolio with risk aversion is equal to 9 has an average annualized return is equals to 17.7% with a standard deviation equals to 16.1%, while the portfolio with risk aversion coefficient equals to 3 has an average annualized return equals to 23.1% with a standard deviation equals to 31.1%.

The table above show that, due to a larger range of the characteristics coefficients on smaller risk aversion coefficient portfolios, those portfolios present a lower correlation to the Naïve Industry than the ones with higher risk aversion coefficient, reaching a correlation to the benchmark equals to 40.7% on the portfolio with risk aversion coefficient equals to 3. However, the lower risk aversion portfolios also present a higher turnover. The average annual turnover of the portfolio with risk aversion coefficient equals to 3 is 1552%, while the average annual turnover of the portfolio with risk aversion is equal to 9 is 529%.

The chart below presents the evolution of the performance of the final portfolio strategy for the 5 risk aversion coefficients variation presented above, as well as the benchmark portfolio for the sample period studied.

Figure 6

Out-of-sample Performance of the Final Case Strategy with Risk Aversion Variation



Performance of the final case Naïve Industry + Long-short portfolio with risk aversion variation in comparison to the benchmark during the sample period of 1981 to 2019.

6. Conclusion

Through this study, it is applied the Brandt, Santa-Clara and Valkanov (2009) framework, known as Parametric Portfolio Policy, to the 49 Fama-French Industry portfolios. The aim is to assess how the model performs on an industry level context, proposing new characteristics that could address a better performance in this environment and checking if the characteristics possess a statistically significance or not for the optimal portfolio. The base case strategy models the weights allocated to each industry in line with three characteristics: value, momentum and profitability. Secondly, the model is extended including the beta and volatility as two new characteristics and after testing the individual and joint statistically significance of the five characteristics, a final portfolio strategy is design using only value and momentum factors.

Over the entire out of sample period, from 1981 to 2019, the parametric portfolio policy in the final case setup generates a positive Jensen-alpha and a beta lower than 1 in respect to the naïve industry portfolio.

The results for industries under Parametric Portfolio Policy are mostly in line with those documented with the universe of stocks. This suggests that PPP could possibly be performed with industry ETFs with cheaper costs than trading a large universe of individual stocks. The study also finds that some characteristics have seemingly less relevance exploiting across industries than differences across stocks, as profitability factor.

However, the three investment cases that this work develops using 49 industries portfolios present a high average annual turnover and thus are practically unable to be implemented in a real-world setup, depending on the transaction costs considered.

To address this issue, it is tested the performance of the model using 10 industries. Even though the annual turnover decreases with 10 industries optimization, it is still high in comparison to the benchmark and the portfolio performance is disappointing.

The extreme turnover of the optimized portfolio reflects a setting where investors are implicitly assuming they can trade for free. This points out to the importance of adapting portfolio optimizations to incorporate fractions. Alternatively, the Parametric Portfolio Policy could be adapted to incorporate trading costs in the optimization algorithm, as present Brandt, Santa-Clara and Valkanov (2009) and Barroso, Reichenecker and Reichenecker (2019). However, that implies incorporating complex non-linearities in the optimization, as Barroso, Menichetti and Reichenecker (2021) explain, which would be a topic beyond the scope of the present work.

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