European Journal of Operational Research 315 (2024) 1–18

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Invited Review

A literature review of economic efficiency assessments using Data Envelopment Analysis

Ana Santos Camanho[a,*], Maria Conceicao Silva[b], Fabio Sartori Piran[c], Daniel Pacheco Lacerda[c]

[a] *Faculdade de Engenharia, Universidade do Porto, Portugal*
[b] *CEGE, Católica Porto Business School, Portugal*
[c] *Universidade do Vale do Rio dos Sinos, UNISINOS, Brasil*

A R T I C L E I N F O

Article history:
Received 7 June 2022
Accepted 19 July 2023
Available online 22 July 2023

Keywords:
Data envelopment analysis
Economic efficiency
Cost efficiency
Revenue efficiency
Profit efficiency

A B S T R A C T

This paper presents a literature review on Data Envelopment Analysis assessments of economic efficiency, covering methodological developments and empirical applications. We review the seminal models for economic efficiency measurement, involving the optimization of cost, revenue, and profit. The applications of the different modelling approaches are also discussed. Based on a content analysis of papers published between 1978 and 2020 in various sectors, the main areas of study are identified, and the pathways of research developments are discussed. Most studies are based on disaggregated quantity and price data. In addition, the use of panel data is prevalent compared to cross-sectional studies. There is a preponderance of input-oriented studies focused on cost efficiency rather than revenue or profit efficiency. Informed by the historical evolution of economic efficiency assessments portrayed in this review, we suggest directions for future developments.

1. Introduction

Since the development of Data Envelopment Analysis (DEA) by Charnes, Cooper, & Rhodes (1978), there has been a considerable growth in academic publications in this scientific field. Interest in the general topic of production frontiers and the measurement of efficiency relative to these frontiers has grown greatly in the last decade. According to Emrouznejad & Yang (2018), there are approximately 10,300 articles from 1978 to 2016 related to DEA. In 2014, 2015 and 2016, the volume of publications exceeded 1000 articles per year (Emrouznejad & Yang, 2018). This path appears to have continued in recent years. A google scholar search in September 2022 reveals above 5100 articles with the complete words ‘Data envelopment Analysis’ in the title from 2017 on wards. It is now possible to find DEA studies in virtually every imaginable production activity. Interest has also extended to policy issues of great significance. This high volume of publications led to several literature reviews on DEA, aiming to summarise the state of the art and propose research agendas for the future.

Many literature reviews address methodological developments associated with the DEA method in general, and seek to identify promising research developments (Cook & Seiford, 2009; Dyson et al., 2001; Emrouznejad, Parker, & Tavares, 2008; Emrouznejad & Yang, 2018; Liu, Lu, & Lu, 2016; Seiford & Thrall, 1990). Other reviews are more specific, focusing on particular theoretical issues, such as weight restrictions and value judgments (Allen, Athanassopoulos, Dyson, & Thanassoulis, 1997), network DEA (Kao, 2014), or fuzzy DEA (Hatami-Marbini, Emrouznejad, & Tavana, 2011). In addition, there are literature reviews focused on specific areas of application of DEA, such as sustainability (Zhou, Yang, Chen, & Zhu, 2018a; Zhou, Ang, & Poh, 2008), human development (Mariano, Sobreiro, & do Nascimento Rebelatto, 2015), health (Pelone et al., 2015), banking and finance (Berger & Humphrey, 1997; Fethi & Pasiouras, 2010), transportation (Cavaignac & Petiot, 2017), insurance industry (Kaffash, Azizi, Huang & Zhu, 2020), and education (Johnes, 2015; Johnes, Portela, & Thanassoulis, 2017). These literature reviews contribute to improving the understanding of the evolution of DEA applications. They guide new researchers and help to broaden the vision of experienced researchers.

Considering specifically the analysis of economic efficiency, there is no literature review addressing theoretical/methodological developments or empirical applications of DEA. Despite the importance of this topic in the performance assessment literature, there

* Corresponding author.
E-mail address: acamanho@fe.up.pt (A.S. Camanho).

https://doi.org/10.1016/j.ejor.2023.07.027

is no critical analysis and synthesis of the knowledge generated in the last decades. Thus, a literature review with this focus can contribute to identify the state of art, and guide future theoretical or empirical studies on economic efficiency using DEA.

The main concern on efficiency analysis is technical efficiency. This concept arises directly from the concept of production function, which is the mathematical expression that yields the maximum quantity of output that is producible from the set of inputs consumed by a firm or an industry (see e.g. Shephard, 2015). When a firm is located below the frontier defining the maximum possibility of production, it is said to be technically inefficient, and its inefficiency can be gauged by the distance to the frontier. At the frontier an infinite number of combinations for producing the same quantity of maximum output are considered technically efficient. That is, it is reasonable to assume that the production of one unit of output using a capital intensive strategy or a labour intensive strategy are equally efficient because they yield exactly the same quantity of output. However, from a value perspective this may not be so. That is, when the value of inputs is considered, a strategy that is more labour intensive may be more cost effective than a strategy that is capital intensive. The measurement of economic efficiency takes into account the monetary value of the factors of production to devise plans of production that are more efficient from a cost (or revenue) perspective. In this paper the term economic efficiency is used to describe this setting where the value of inputs (and/or the value of outputs) is considered in the definition of the efficient production plan.

This review covers both theoretical-methodological developments and empirical applications on economic efficiency measurement through non-parametric techniques (in particular DEA). It is not always easy to distinguish theoretical from empirical applications since most theoretical developments in a DEA context happen as a result of application-driven theory. For the review of applications, we selected papers published in journals indexed in Web of Science and Scopus, and conducted a content analysis of 326 papers published from 1978 to 2020.The list of 326 papers included in this study is available in Supplementary Material.

The review of applications classifies the papers by the type of efficiency estimated (cost, revenue, or profit), the models used, and the application area. The definition of application area tried as much as possible to be consistent with the literature and therefore we divided papers into the most significant application areas (e.g., education, agriculture, banking and finance, energy, water and wastewater utilities, among others). In some cases, this classification in not straightforward. For example, in the case of studies that compare municipalities or regions on certain issues, such as public services performance, environmental performance, or attractiveness of foreign direct investment, the studies could either be classified in the area of municipality applications or in the area of energy applications. For this reason we also consider a sub-area of analysis, which in some cases is obvious (e.g. in the area of education, the sub-areas correspond to schools, universities and university libraries), but in others is less obvious (e.g. in the area of municipalities we considered sub-areas related to foreign direct investment, social well being, public services and cities). The studies using aggregate value data are distinguished from studies that used quantity and price data to estimate allocative efficiency. The period considered in the assessment (i.e., cross-sectional, panel or time series data) is also reported, as well as whether second stage analysis was undertaken to explain the sources of inefficiency.

2. Review of economic efficiency models and their applications

As mentioned previously, production theory assumes that there is a mathematical function that can gauge the relationship between the outputs obtained from the inputs used in a given production process. This relationship is called a production function which defines the maximum output that is producible from a set of inputs (or defined in another way, the minimum inputs that can be consumed for the production of a certain output). In a DEA context, the production function is not an analytical mathematical function, such as the Cobb Douglas, but is determined non-parametrically from a set of firms or units whose transformation process has been observed. The shape of the production function is specified based on axioms imposed on the input and output sets observed in the sample under assessment (Banker, Charnes, & Cooper, 1984; Banker & Thrall, 1992). The estimation of efficiency is then operationalised using linear programming models that enable estimating the distance of observations to a best-practice frontier, corresponding to the boundary of the so called Production Possibility Set (PPS).

Without value/price information, only technical efficiency can be computed. But price information makes estimating economic efficiency (cost, revenue, profit) possible. Economic efficiency implies, therefore, not only that the firm is technically efficient but also that the combination of resources used is the one that minimises the cost, or that the combination of outputs produced is the one that maximises revenue, or both. Using a cost efficient combination of resources or producing a revenue efficient combination of outputs implies that firms are allocative efficient.

The value assigned to inputs and/or outputs does not need to be their price, but rather it can be some sort of preference information establishing that some combinations of inputs (and/or outputs) is better than others. This has been called in the literature Value Efficiency Analysis, in the strand initiated by Halme, Joro, Korhonen, Salo, & Wallenius (1999). Recently, given the raising environmental concerns, environmental efficiency has been measured in the DEA literature by several means. One of these means is the replacement of the prices paid for inputs, by a pollution 'price' of each input, and so obtain a mix of inputs that minimise pollution (see Coelli, Lauwers, & Van Huylenbroeck, 2007 or Welch & Barnum, 2009).

The classical DEA models for economic efficiency use price information and can be cost, revenue, or profit efficiency models. Cost efficiency measures a decision-making unit's (DMU) ability to minimise costs given the output levels produced and the input prices observed (Färe, Grosskopf, & Lovell, 1985; Grosskopf, 1986). Revenue efficiency measures a DMU's ability to maximise revenue, given the input levels consumed and output prices (Färe et al., 1985). Profit efficiency measures a DMU's ability to maximise profit, given the existing input and output prices. Thus, profit efficiency involves both cost minimisation and revenue maximisation, for a certain scale size of operation (Banker & Maindiratta, 1988; Fare, Grosskopf, & Lovell, 1994).

2.1. Classical cost and revenue efficiency models

2.1.1. Theoretical background

The classical approach to the estimation of **Cost efficiency** involves an assessment based on model (1) (Färe et al., 1985). For each DMU j $(j = 1, \ldots, n)$ there is an input vector $\boldsymbol{x_{ij}}$ represented by $(x_{1j}, x_{2j} \ldots, x_{mj})$ that reflects the amount of input i $(i = 1, \ldots, m)$ used to produce a given output vector $\boldsymbol{y_{rj}}$, represented by $(y_{1j}, y_{2j} \ldots, y_{sj})$, that reflects the quantities produced of output r $(r = 1, \ldots, s)$. The classical assumption of cost efficiency assessment is that DMUs are so small in relation to the size of the market that they have no influence on the input prices - they must take the prices as given. This implies that DMUs are said to be perfectly competitive in input markets. In this case, there are two possible situations: one with common unit prices for all DMUs, and the other with different prices from DMU to DMU. The more general case is the latter, with input prices being represented by the vector $\boldsymbol{c_{ij}}$ $(c_{1j}, c_{2j} \ldots, c_{mj})$, as considered in model (1). The linear programming model (1) is solved for estimating the minimum cost at which DMU k under assessment can obtain the observed outputs, given the existing prices.

$$C_k^{\min}(y_{rk}, c_{ik}) = \min_{\lambda_j, x_i} \left\{ \sum_{i=1}^{m} c_{ik} x_i \mid \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, i = 1, \ldots, m, \right. \\ \times \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk}, r = 1, \ldots, s, \\ \left. \lambda_j \ge 0, j = 1 \ldots, n, x_i \ge 0, i = 1 \ldots, m \right\} \quad (1)$$

The decision variables are the optimal input levels for DMU k (x_i) and the intensity variables λ_j, that allow constructing a frontier target point as a linear combination of peer DMUs. The value of y_{rk} corresponds to the observed value of output r for the DMU k under evaluation, and c_{ik} corresponds to the price of input i $(i = 1, \ldots, m)$ for the DMU k under assessment (price data for each DMU's input is fixed and known, although the prices may vary from DMU to DMU). Model (1) represents a formulation with constant returns to scale (CRS). The variable returns to scale (VRS) formulation can be obtained by adding restriction $\sum_{j=1}^{n} \lambda_j = 1$ to model (1).

The cost efficiency (CE) of DMU k is defined as the ratio between the minimum cost with current prices, obtained at the optimal solution to model (1), and the actual cost observed at DMU k, as shown in expression (2) (Färe et al., 1985).

$$CE_k = \frac{C^{\min *}}{\sum_{i=1}^{m} c_{ik} x_{ik}} = \frac{\sum_{i=1}^{m} c_{ik} x_i^*}{\sum_{i=1}^{m} c_{ik} x_{ik}} \quad (2)$$

In expression (2), x_{ik} corresponds to the observed value of input i for DMU k under evaluation. The superscript * signals the value obtained at the optimal solution to the linear programming model. The cost efficiency measure indicates by how much the observed cost could be proportionally reduced while being able to secure the observed output, given the prices observed at each DMU. The excess of cost must logically be either due to excess usage of inputs (i.e., technical inefficiency) and/or because inputs are used in the wrong mix given their prices (i.e., input allocative inefficiency). The relationship between cost efficiency, input technical efficiency and input allocative efficiency is as follows:

Cost efficiency = Input technical efficiency × Input allocative efficiency. (3)

As a result, in the DEA framework the measure of input allocative efficiency can be obtained residually as the ratio of cost efficiency and the input oriented technical efficiency measure.

$$AE_k = \frac{CE_k}{TE_k} \quad (4)$$

The technical efficiency score (TE_k) of the DMU k can be obtained using the original DEA formulation of Charnes et al. (1978). Under CRS, the linear programming model can be formulated in the 'envelopment form' (model (5)) or in the 'weights' form (model (6)), where u_r represent output weights and v_i represent input weights.

$$TE_k(x_{ik}, y_{rk}) = \min_{\lambda_j, \theta} \left\{ \theta \mid \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{ik}, i = 1, \ldots, m, \right. \\ \left. \times \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk}, r = 1, \ldots, s, \lambda_j \ge 0, j = 1 \ldots, n \right\} \quad (5)$$

$$TE_k(x_{ik}, y_{rk}) = \max_{u_r, v_i} \left\{ \sum_{r=1}^{s} u_r y_{rk} \mid \sum_{i=1}^{m} v_i x_{ik} = 1, \sum_{r=1}^{s} u_r y_{rj} \right. \\ - \sum_{i=1}^{m} v_i x_{ij} \le 0, j = 1, \ldots, n, \\ \left. u_r \ge 0, r = 1, \ldots, s, v_i \ge 0, i = 1, \ldots, m \right\} \quad (6)$$

According to Coelli et al. (2007), if we replaced prices by the cost of pollution associated to each input one would obtain an iso-polluting line and a projection to this line would result in an environmental efficiency measure that satisfies the materials balance equation. The resulting environmental efficiency (defined as the ratio between the minimum pollution levels and the current pollution levels) can be decomposed in the same way as the cost efficiency score into a technical efficiency score and an allocative efficiency score.

Revenue efficiency can be seen as a straightforward extension of cost efficiency assessments, that instead of being oriented towards inputs, adopts an output oriented approach. Revenue efficiency can be calculated using model (7) (Färe et al., 1985), where notation is as in (1) and decision variables are the revenue efficient output levels of the DMU k under assessment (y_r for $r = 1, \ldots. s$) and the intensity variables λ_j for $j = 1, \ldots, n$. The prices of the outputs r $(r = 1, \ldots, s)$ are given by the vector $\boldsymbol{p_j}$ $(p_{1j}, p_{2j}, \ldots, p_{sj})$.

$$R^{\max}(x_{ik}, p_{rk}) = \max_{\lambda_j, y_r} \left\{ \sum_{r=1}^{s} p_{rk} y_r \mid \sum_{j=1}^{n} \lambda_j x_{ij} \right. \\ \le x_{ik}, i = 1, \ldots, m, \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_r, r = 1, \ldots, s, \\ \left. \lambda_j \ge 0, j = 1, \ldots, n, y_r \ge 0, r = 1, \ldots, s \right\} \quad (7)$$

Model (7) represents a CRS formulation. The VRS formulation can be obtained by adding restriction $\sum_{j=1}^{n} \lambda_j = 1$ to model (7). The revenue efficiency (RE) of DMU k is the ratio between the observed revenue and the maximum revenue estimated from model (7), given the inputs consumed and the output prices, as shown in (8) (Färe et al., 1985).

$$RE_k = \frac{\sum_{r=1}^{s} p_{rk} x_{rk}}{R^{\max *}} = \frac{\sum_{r=1}^{s} p_{rk} y_{rk}}{\sum_{r=1}^{s} p_{rk} y_r^*} \quad (8)$$

Output Allocative Efficiency is the ratio between revenue efficiency in (8) and the technical efficiency measure computed through a traditional output oriented model (Charnes et al., 1978). Revenue efficiency is therefore the product between output oriented technical efficiency and output allocative efficiency.

2.1.2. Classical models' applications

Classical cost or revenue models are the most commonly found in the literature. From the total number of papers reviewed in this study, around 51% employed the traditional cost efficiency model outlined in this section and just around 4% employed the traditional revenue model. There is therefore a staring difference between the use of the cost and the revenue model, with the prevalence of the former. One possible reason for this prevalence is that in most business settings costs are easier to control than revenues, which are many times related to the exogenous and uncontrollable demand for the product or service. For example, in the banking and finance context from the 107 studies reviewed 66 are exclusively cost oriented, only 15 are exclusively profit oriented and 2 are exclusively revenue oriented. The remaining mix cost, revenue and profit models. Note that from the 66 studies that are concerned with cost issues in banking, 48 apply the classic cost model. In banking, outputs are most of the times the revenue generating services which are obviously dependent on the market of the branch or the bank. In agriculture cost orientation also prevails (45 out of 52 studies), since outputs, being mostly related to crops, are not only dependent on demand but also on external factors such as the weather. The same happens for other application contexts like health care, energy, telecommunications, transport or hospitality services, where the services provided are mainly dictated by their demand. Consequently, management has more control over cost than over revenues. Interestingly, in for-profit organizations many times the most controllable factor within revenue is the price. However, as we will detail later, traditional cost and revenue models assume that prices are given and dictated by perfectly competitive markets. Dropping this assumption, has resulted in some new revenue models such as the one of Sotiros, Rodrigues, & Silva (2022).

The classic cost model dominates in all areas of application, with the exception of municipalities, water and waste, and supply chain studies. In the first two areas, the aggregate cost models (that we will mention later on) dominate, whereas in the supply chain area, the network economic models dominate. Supply chains are a classical example of a number of processes organized in series, where network models are an obvious choice.

2.2. Weight restrictions and economic efficiency models

2.2.1. Theoretical background

The classical models yield economic efficiency in a direct way. An ‘indirect’ way of getting economic efficiency measures is trough the introduction of weight restrictions, conveying information on prices, in traditional technical efficiency models. One of the mostly used weight restrictions are Assurance Regions (ARs). ARs restrictions, as proposed by Thompson & Thrall (1994) and Thompson, Dharmapala, & Thrall (1995), were originally defined as being related to the economic idea of profit. This notion was further discussed in Thanassoulis (2001, chapter 8), showing that DEA weights convey information on marginal rates of substitution/transformation. Thompson, Dharmapala, Rothenberg, & Thrall (1996) used this economic interpretation of DEA weights by restricting the weights to vary within pre-specified bounds that could represent lower and upper bounds on observed prices. Consequently, when weight restrictions are specified based on the relative values of prices, the efficiency assessment will reflect economic efficiency rather than technical efficiency.

In this section we will first focus on the estimation of the weight restricted input oriented measures of cost efficiency. As proposed in Schaffnit, Rosen, & Paradi (1997), cost efficiency can be obtained by imposing that the relative value of input weights (v_i) underlying the DEA assessment is equal to the relative values of input prices observed at each DMU k under evaluation, as shown in (9).

$$\frac{v_{i^a}}{v_{i^b}} = \frac{c_{i^a k}}{c_{i^b k}} \quad \Leftrightarrow \quad v_{i^a} - \frac{c_{i^a k}}{c_{i^b k}} v_{i^b} = 0, \quad \forall i^a, i^b \in i = 1, \dots s \tag{9}$$

In expression (9) v_{i^a} and v_{i^b} are the input weights underlying the efficiency assessment using model (6), for any two inputs i^a and i^b considered in the assessment. $c_{i^a k}$ and $c_{i^b k}$ are the input prices observed at DMU k under assessment. The total number of restrictions that need to be added to model (6) to estimate CE is given by $\binom{m}{2}$.

Note that using model (6), a technical efficiency model, added of the weight restrictions in (9), implies a change in the frontier against which the DMUs are assessed that corresponds to a ‘value frontier’ and no longer coincides with the frontier of the Production Possibility Set (PPS) defined from the TE assessment. Consequently, the original TE measure of model (6) becomes a cost efficiency estimate with the inclusion of weight restrictions (9). For an output-oriented setting, revenue efficiency can be estimated applying a similar reasoning, and the weight constraints are defined as shown in (10). As a result, an output-oriented technical efficient added with the constraints shown in (10) would result in the computation of a revenue efficiency score (see also Thanassoulis, & Allen, 2004).

$$\frac{u_{r^a}}{u_{r^b}} = \frac{p_{r^a k}}{p_{r^b k}} \quad \Leftrightarrow \quad u_{r^a} - \frac{p_{r^a k}}{p_{r^b k}} u_{r^b} = 0 \tag{10}$$

The restrictions imposing bounds to the relative values of weights, as expressed in Eqs. (9) and (10) are known as ‘Assurance Regions type I’ (ARI) (Thompson, Langemeier, Lee, Lee, & Thrall, 1990). The relationships expressed as ratios between input and output weights are termed ‘Assurance Regions type II’ (ARII) or linked cone Assurance regions (LC-ARs). ARII allow a generalisation of the above notions of ‘value frontiers’ to a profit efficiency context. However, Thompson et al. (1990) recognised the existence of some problems with ARII, some of which relating to infeasibilities (see also Thompson & Thrall, 1994 and Thanassoulis et al., 2004 that discuss the links between ARIIs and profit efficiency, and the limitations of these measures).

To the extent that in most practical applications the weight constraints imposed in (6) are not strict equalities (because of uncertain price information or incomplete price information), it implies that the resulting efficiency score under weight restrictions will lie somewhere between a technical efficiency score (upper bound) and a cost efficiency score (lower bound). In any case an important note is that in order to obtain cost related efficiency scores an input oriented efficiency model should be used together with restrictions on input weights and to obtain a revenue related efficiency score an output oriented model should be used together with restrictions on output weights. If some sort of mix is followed, like in Liu, Tsai, & Wu (2018) where an input oriented model was used together with output weight constraints, the resulting efficiency score will not be a cost efficiency measure. To see this imagine an input oriented model with just two outputs to which the WR $u_1/u_2 = p_1/p_2$ was added. Consider also the dual version of the corresponding multiplier model which is shown in (11) where z is the dual variable associated to the constraint $u_1 p_2 - u_2 p_1 = 0$ (see also Thanassoulis et al., 2004):

$$\begin{aligned} TE_k(x_{ik}, y_{1k}, y_{2k}) = \min_{\lambda_j, \theta} \Bigg\{ \theta \mid \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{ik}, \\ i = 1, \dots, m, \sum_{j=1}^{n} \lambda_j y_{1j} \ge y_{1k} - p_2 z, \\ \times \sum_{j=1}^{n} \lambda_j y_{2j} \ge y_{2k} + p_1 z, \quad \lambda_j \ge 0, j = 1 \dots, n, \quad z \in R \Bigg\} \end{aligned} \tag{11}$$

Following Thanassoulis et al. (2004) we can say that the target output 1 $y_1^* = y_{1k} - p_2 z$ and the target output 2 $y_2^* = y_{2k} + p_1 z$, meaning that $z = \frac{y_{1k} - y_1^*}{p_2} = \frac{y_2^* - y_{2k}}{p_1}$. So the output weight restriction in the TE input model relates somehow the targets of the two outputs but the efficiency score thus obtained is not a cost efficiency measure. If an output model was used the weight restricted envelopment model would be written as:

$$\begin{aligned} TE_k(x_{ik}, y_{1k}, y_{2k}) = \max_{\lambda_j, \beta} \Bigg\{ \beta \mid \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik}, \\ i = 1, \dots, m, \sum_{j=1}^{n} \lambda_j y_{1j} \ge \beta y_{1k} - p_2 z, \\ \times \sum_{j=1}^{n} \lambda_j y_{2j} \ge \beta y_{2k} + p_1 z, \quad \lambda_j \ge 0, j = 1 \dots, n, \quad z \in R \Bigg\} \end{aligned} \tag{12}$$

that means that $z = \frac{\beta y_{1k} - y_{*1}}{p_2} = \frac{y_{*2} - \beta y_{2k}}{p_1}$, and from this equality we get that $\beta = \frac{p_1 y_{*1} + p_2 y_{*2}}{p_1 y_{1k} + p_2 y_{2k}}$. That is, β equals maximum revenue divided by observed revenue and therefore it is the inverse of the revenue efficiency score (Thanassoulis et al., 2004).

2.2.2. Weight restricted models's applications

Applications in the literature with weight restrictions related to price information will be classified in this paper under the same heading of 'Weight restricted models' in spite of the many variants that may occur, such as consideration of price ranges and the consequent determination of efficiency in pessimistic and optimistic ranges of price, as in Camanho & Dyson (2005), Toloo & Ertay (2014), or cost efficiency models under incomplete price information, as in Kuosmanen & Post (2001, 2002, 2003), and also in Mostafaee & Saljooghi (2010), or probabilistic assurance regions as in Olesen & Petersen (2002). Following this literature Shiraz, Hatami-Marbini, Emrouznejad, & Fukuyama (2020) propose a chance-constrained model for dealing with stochastic data in cost efficiency assessments.

Related to the above literature is also the multiple criteria framework called Value Efficiency Analysis as described in Halme et al. (1999), where the Decision Makers preferences are included in a value function estimated through the knowledge of the DMs' most preferred solution. This value function works in a similar way to a revenue or cost function except that it may reveal other preferences than those related with prices (see also Korhonen, Tainio, & Wallenius, 2001).

Although the imposition of weight restrictions in technical efficiency models may be seen as a good way to measure or approximate efficiency to economic efficiency scores, we could not find many cases of applications. From the 327 studies reviewed we could identify only 11 studies applying price related weight restrictions. From such studies one is revenue oriented (the study of Liu et al., 2018 that, as we saw, is not really a revenue model) and one is profit oriented (García-Cestona & Surroca, 2008). In García-Cestona & Surroca (2008) the authors use a technical efficient model with constraints only on the relative importance of outputs meaning that the resulting efficiency score may not indeed be related with profit, as the authors claim. In Thanassoulis et al. (2004) the authors show that the introduction of ARII in a technical efficiency model implies a concept of efficiency that is related, but is not equivalent, to profit efficiency. The remaining papers are cost efficiency models that also dominate in this context.

The reason for the lack of papers within this strand of literature is probably a mis-identification of such papers. Indeed, many papers apply weight restrictions to DEA models without mentioning the link between weight restricted models and economic models, which, by definition, exist when the restrictions are related to prices or some other value of inputs and/or outputs. Note that this distinction can be blurred. For example, suppose we impose constraints on production time. In that case, this could also be considered related to economic efficiency, as outputs that take more time to be produced may be more expensive. Nevertheless, this connection has not been widespread in the literature, which is something researchers should be aware of.

2.3. Price/Market components of economic efficiency

2.3.1. Theoretical background

The traditional cost efficiency model assumes that firms operate in perfectly competitive markets (i.e., all firms sell an identical product and use identical resources, meaning that the inputs and outputs are homogeneous across firms). All firms are small in comparison to the market size, so they are price takers, meaning that they cannot influence the price of their products or the price paid for the inputs. Furthermore, it is assumed that there is complete or 'perfect' information about the prices paid for the inputs and the prices charged for the products sold by each firm. These assumptions imply that comparisons of efficiency on the input/output space are fair, given the homogeneity assumption of resources used and products obtained. Eventual price differences that may exist are outside the firms control, and should not be deemed as inefficiencies that could potentially be removed.

Tone (2002) was among the first to note that the assumption of exogenous prices may hinder the full extent measurement of potential cost savings. For example, the traditional cost efficiency of two DMUs may be the same, even if one faces input prices two times higher than the other (as long as both have the same levels of inputs and outputs). In essence, if prices are truly exogenous this is acceptable, since both DMUs use the same mix of inputs for the prevailing input prices, notwithstanding the fact that one of the DMUs has to pay twice as much for its inputs than the other.

Tone (2002) solve the noted problem by moving from the input and output quantity space to the cost of inputs and outputs space (see also Belas, Kocisova, & Gavurova (2019) or Dong, Hamilton, & Tippett (2014) for two applications of Tone (2002) to the case of banks). When input quantities are replaced by their costs, both quantity and price effects are reflected in the efficiency scores obtained.

Following the work of Tone (2002), Fukuyama & Weber (2004) and Färe & Grosskopf (2006) propose the use of directional distance functions and the computation of differences between costs, rather than ratios. The approach of Färe & Grosskopf (2006) estimates the same TE score for two DMUs employing the same quantities of inputs to produce the same quantities of outputs, but estimates an allocative inefficiency twice as higher for the DMU whose input prices are the double of those of another DMU. That is, the allocative efficiency measure seemed to reflect only price differences and not wrong mixes of inputs given prices.

Therefore, following Tone (2002), Tone & Tsutsui (2007) proposed a decomposition of cost efficiency into technical, allocative and price efficiency. The price efficiency component of Tone & Tsutsui (2007) reflects the scope for savings through price changes, whereas allocative efficiency is defined as "the adjustment to the optimal cost mix, viz., the combination of the optimal input amount and input price mixture"[p. 95]. These two concepts in the Tone & Tsutsui (2007) approach are misspecified, as allocative efficiency is unrelated to its traditional meaning and price efficiency does not capture solely the changes in prices (see also Sahoo & Tone, 2013 for a recent application and Thanassoulis, Portela, & Graveney, 2012 for an adaptation of this model). Camanho & Dyson (2008) propose the computation of the traditional Farrell cost efficiency measure and its decomposition into technical and allocative components, and then identifies a third component called 'market efficiency'. This component is identified by solving the traditional cost efficiency model under different price assumptions (one of which is the assumption that the minimum price observed on each input can be attained by all DMUs, and another is that each DMU can choose from the set of observed price vectors the one that minimises the aggregate cost of inputs for their output bundle). Ray, Chen, & Mukherjee (2008) also proposed a related method for modelling situations where firms can choose their location depending on the input prices offered in each location. The main novelty in the Ray et al. (2008) model is that it considers the possibility of partially producing an output bundle from an input bundle in different locations and at different prices. More recent developments on this issue can be seen in Sotiros et al. (2022), in this case applied to the context of revenue efficiency measurement.

From the various developments in the strand of economic efficiency that do not consider firms as price takers, there are some clear insights and also issues that need to be clarified. What seems clear is that when prices are not given, there is a component of overall economic inefficiency that is related with inefficiency in managing prices (e.g. firms that do not buy the resources at the lowest possible prices either because they do not buy resources to the right suppliers, they buy at wrong times, or they do not use price quantity discounts). Less clear is how to name this component of inefficiency that is price related, and whether allocative inefficiency is still a component of overall economic inefficiency. Indeed, most existing approaches that compute allocative inefficiency in the non-price taking context, do not compute a traditional measure of allocative inefficiency, as it does not reflect the extent to which the mix of inputs needs to be changed given the prevailing prices. The discussion on whether allocative efficiency is a component of overall cost efficiency when prices are not exogenous has been addressed in Portela & Thanassoulis (2014). These authors propose a measure of cost efficiency that allows for simultaneously changing quantities of inputs and their prices. They propose a decomposition of this measure into a technical efficiency component and a price efficiency component - leaving aside the notion of allocative efficiency. This approach has been developed based on previous works (e.g. Camanho & Dyson, 2008; Kuosmanen, Cherchye, & Sipiläinen, 2006), while at the same time it has also been employed by others as Thanassoulis, Sotiros, Koronakos, & Despotis (2018).

Furthermore, under the assumption that price variations are possible and outside the firms' control, fuzzy approaches to cost efficiency assessment have also been proposed. These approaches estimate a minimum and maximum bound to the cost efficiency estimate for a certain price range (Camanho & Dyson, 2005).

2.3.2. Price/Market efficiency applications

In this review we considered all classes of the above mentioned economic models in the same category 'Non-exogenous prices', in spite of the different approaches used in the empirical applications that considered that the assumption of exogenous prices was not met. Note that only a very reduced number of applications (12 in total) was found in this category. This means that in spite of the fact that exogenous prices are not met frequently in practice, researchers are still computing cost efficiency based on this assumption. As mentioned by Ray et al. (2008, p.208) "Even when input markets are competitive, input prices may vary across locations (like countries or regions within a country) although they are given at any particular location. Such variations in prices may occur due to lack of mobility of inputs. A firm effectively chooses between the input price vectors by producing its output at one location or another", which means that the degree of control over prices is potentially larger than what theoretical economic models imply.

Most applications of non-exogenous price models used the Tone (2002) procedure and all of them assessed cost efficiency. Therefore, there is a lack of applications of economic measures of efficiency when output prices are non-exogenous. The exceptions to the use of Tone (2002) are Camanho & Dyson (2008, 2005), and Thanassoulis et al. (2018) that adapted the model of Portela & Thanassoulis (2014), and Ray et al. (2008). There is therefore a wide range of possibilities for further research in this strand of the economic literature - either by extending the cost efficiency to other economic measures of efficiency and by further applying the existing approaches and comparing them. An unresolved issue is the concept of allocative efficiency and what it means when prices and quantities vary.

2.4. Aggregate value data

2.4.1. Theoretical background

Tone (2002) defined a cost efficiency model in the value space and showed that the cost model considering each input cost disaggregated (in quantity and price data) was equivalent to a model with a single aggregated cost value. As seen in the previous section, the reason for doing this was to solve the problem of traditional cost efficiency measures not reflecting differences in prices. However, the use of aggregate value data can be justified in other circumstances, for example when price and quantity data are not available in a disaggregated way.

Many researchers apply classical models of technical efficiency to value-data (cost or revenue), disregarding the true meaning of the efficiency score thus obtained. In fact, the use of value-data poses some challenges in the interpretation of the resulting efficiency scores, and restricts the choice of applicable models. Since value-data reflect both quantities and prices, it is not evident what is measured by the technical efficiency model when such data are used (Banker, Chang, & Natarajan, 2007). Or, put it another way, "how does the value-based DEA model relate to the quantity based DEA models? If they do not coincide, then what exactly does the value-based DEA model measure and how do we interpret the difference?" (Cross & Fare, 2008).

The answer to the above question depends on whether prices can be assumed similar or different across DMUs. If all DMUs have similar prices, the use of value-data is not a problem, as this does not hinder the computation of cost efficiency nor its decomposition. The way to proceed is to use value-data in models (5) or (6) for the computation of technical efficiency, since under the assumption of equal prices the use of quantities or values yields equivalent results for the TE assessment. The computation of cost efficiency requires collapsing input values (costs) into a single input value variable, representing total cost $\left(C_j = \sum_{i=1}^{m} c_i x_{ij}\right)$. The cost efficiency measure is then obtained by solving the standard TE model shown in (5) or (6) with a single input representing total cost as shown in (13) or its equivalent (14) (where the relationship between the optimal objective function values of the two models is as follows $\theta^* = C^*/C_k$) (see Banker et al., 2007; Banker, Janakiraman, & Natarajan, 2004; Cross & Fare, 2008; Portela, 2014).

$$CE_k(C_k, y_{rk}) = \min_{\lambda_j,\theta} \left\{ \theta \mid \sum_{j=1}^{n} \lambda_j C_j \le \theta C_k, \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk}, \right.$$
$$\left. r = 1, \ldots, s, \lambda_j \ge 0, j = 1 \ldots, n \right\} \tag{13}$$

$$C_k^{\min}(y_{rk}) = \min_{\lambda_j,C} \left\{ C \mid \sum_{j=1}^{n} \lambda_j C_j \le C, \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rk}, \right.$$
$$\left. r = 1, \ldots, s, \lambda_j \ge 0, j = 1 \ldots, n \right\} \tag{14}$$

If revenue efficiency is being computed, the procedure is similar: to use output value data (revenues) in the technical efficiency assessment, and to collapse all output values into a single output value (representing total revenue) to compute revenue efficiency. In the economic efficiency applications reported in this paper, we consider the studies that applied this procedure as the 'aggregate economic model'. We distinguish these models from the classic cost models since when this procedure is adopted, it is not possible to know the cost efficient targets in terms of individual inputs, corresponding to the variables x_i in model (1), which could provide valuable information regarding input mix improvements. Using these aggregate economic models, under equal prices across DMUs, allows for the traditional decomposition of economic efficiency into its technical and allocative components.

If input prices differ among DMUs, the estimates obtained using models (5) or (6) with value-data instead of the quantities of individual inputs, yields an efficiency estimate that is neither TE nor CE (according to Farrell's concepts). This type of evaluation, using value-data (the cost of individual inputs, or the revenues of individual outputs) instead of quantity-data, is appropriate for situations of heterogeneous inputs, where the price can be seen as a way to incorporate other qualitative features, such that evaluations in the cost space (product of inputs with their prices) are a fairer way to benchmark firms. Adopting this course of action implies different assumptions from those of perfect competition, as firms are no longer considered price takers operating with homogeneous inputs (see previous section). The prices reflect the quality of inputs and may be under managerial control (as they depend on the features of the inputs selected). With different assumptions, the efficiency scores reflect different realities, and consequently Farrell's CE and cost efficiency evaluations in the cost space will result in different efficiency metrics. Using the aggregate economic model (14) under the assumption of different prices across DMUs implies that cost efficiency cannot be disentangled into technical and allocative components, except when quantities of inputs are available together with their prices (and this implies the use of the models seen in the previous section).

2.4.2. Aggregate value data applications

The revision of empirical applications undertaken in this paper disregards studies that applied standard technical efficiency models to value data, even when the authors used the denomination of economic efficiency. Given the reasons put forth above, in fact the estimates obtained from technical efficiency models applied to value data are not economic efficiency estimates in Farrell's sense. We did not exclude, however, those studies that used a single input (total cost) or a single output (total revenue) because in these cases a cost or revenue measure was indeed computed. Examples of applications that mention the term 'economic efficiency' but in fact used technical efficiency models based on several inputs and/or outputs can be found in Psaraki-Kalouptsidi & Kalakou (2011), Wang & Feng (2015), Zafra-Gómez, Antonio, & Muñiz (2010), or Jitsuzumi & Nakamura (2003). Some papers, like Byrnes, Crase, Dollery, & Villano (2010) applied a single total cost input but describe it mainly as if a technical efficiency score was being computed decomposing it into scale efficiency too.

Overall we have identified 50 applications that used aggregate data, either acknowledging the challenges posed in the assessment with aggregate data, or employing typical measures of technical efficiency in a total cost input, or a total revenue output model. From these 50 papers 40 use cost models, 7 use revenue models, one uses both, and one uses a profit model (Cherchye & Van Puyenbroeck, 2007) with aggregate data. From the 40 papers using cost models, more than half (28) use the classical aggregate cost model. 5 papers use slack based cost models (Agrell & Bogetoft, 2005; Castellet & Molinos-Senante, 2016; Fukuyama & Weber, 2009; Khan, Kutan, Naz, & Qureshi, 2017; Murphy, Wang, Wang, & Tkacz, 2013), and 3 use cost network models (Seyedboveir, Kordrostami, Daneshian, & Amirteimoori, 2017; Cherchye, Rock, Dierynck, Roodhooft, & Sabbe, 2013; Ray, 2016). The remaining used other methods like Ahn, Bogetoft, & Lopes (2019) who applied a network model that they call potential subunit efficiency together with weight restrictions, or Cherchye & Van Puyenbroeck (2007) who estimate profit efficiency of German farms where some variables were measured in quantity and others in value terms, and Shi, Li, Emrouznejad, Xie, & Liang (2017) who applied a two stage DEA model adapted to the situation of bank mergers.

The applications on revenue and profit efficiency using aggregate data employed mostly the classical approaches, with the only exception being the work of Färe, Grosskopf, & Lee (1990), who employed an indirect profit model.

2.5. Indirect measures of cost and revenue efficiency

Fare et al. (1994) have proposed indirect measures of cost and revenue efficiency, that attempt to simultaneously optimize input and output levels, given the objectives of minimizing the cost required to achieve a pre-specified target revenue, or maximizing the revenue achievable from a pre-specified cost budget.

This section presents the input-oriented case, corresponding to the Revenue Indirect Input Cost Efficiency model, as the generalisation to the output oriented case is trivial.

Consider that the pre-specified revenue target for DMU k under assessment is ϕ_k. The linear programming model to estimate the minimum cost that allows obtaining this revenue level is shown in (15).

$$
\begin{aligned}
RIC^{\min}(c_{ik}, p_{rk}) = \min_{\lambda_j, x_i, y_r} \Bigg\{ & \sum_{i=1}^{m} c_{ik} x_i \;\Big|\; \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, \\
& i = 1, \ldots, m, \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_r, r = 1, \ldots, s \\
& \times \sum_{r=1}^{s} p_{rk} y_r = \phi_k, \quad \lambda_j \ge 0, j = 1 \ldots, n, \quad y_r \ge 0, \\
& r = 1, \ldots, s, \quad x_i \ge 0, i = 1 \ldots, m \Bigg\}
\end{aligned}
\tag{15}
$$

A revenue indirect cost efficiency (RICE) measure can be obtained as the ratio of the minimum cost obtained at the optimal solution to model (15) to the current cost at DMU k, as shown in expression (16).

$$
RICE_k = \frac{RIC^{\min *}}{\sum_{i=1}^{m} c_{ik} x_{ik}} \tag{16}
$$

In the specific case that the pre-specified revenue target equals the observed total revenue of the DMU k under evaluation $\phi_k = \sum_{r=1}^{s} p_{rk} y_{rk}$, the Revenue Indirect Cost Efficiency measure can be estimated using a weights formulation of the DEA model as shown in (17), where the ratios between any two input weights equals the ratio between input costs, and the ratio between any two output weights equals the ratio between output prices. Camanho & Dyson (2005) dubbed this measure 'Cost-effectiveness', i.e. the ability of a DMU to achieve current revenue levels at minimal cost, allowing for changes to input-output levels and mix.

$$
\begin{aligned}
& \text{Cost-effectiveness}(c_{ik}, p_{rk}) = \max_{v_i, u_r} \Bigg\{ \sum_{r=1}^{s} u_r y_{rk} \;\Big|\; \sum_{i=1}^{m} v_i x_{ik} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \; j = 1, \ldots, n, \\
& v_{i^a} - \frac{c_{i^a k}}{c_{i^b k}} v_{i^b} = 0, \forall i^a i^b, \;\; u_{r^a} - \frac{p_{r^a k}}{p_{r^b k}} u_{r^b} = 0, \forall r^a r^b, \\
& u_r \ge 0, r = 1, \ldots, s, v_i \ge 0, i = 1, \ldots, m \Bigg\}
\end{aligned}
\tag{17}
$$

The cost-effectiveness measure can be decomposed into cost efficiency and output mix efficiency. These measures can be defined more precisely as follows (note that all measures assume fixed input and output prices):

Cost efficiency measures the ability to produce current outputs at minimal cost, allowing for changes to input levels only.

Output mix efficiency measures the extent to which a DMU produces the outputs with the right mix to enable the attainment of the current level of total revenue with minimal cost. Output mix efficiency is obtained as the ratio of the minimal cost of DMU, obtained at the optimal solution to model (16), where output levels can be changed, to the minimal cost of DMU obtained at the optimal solution to model (1), where the output levels are fixed.

In spite of the importance of this extension to classical cost and revenue models, the indirect measures of economic efficiency have not been much applied in the literature. This is somehow surprising given that the work of Färe et al. (1990) was among the first economic models proposed and applied in the literature, considering a profit model with expenditure constraints applied to rice farms in California. We could only find two additional applications of indirect economic measures of efficiency, one paper by Fukuyama & Weber (2002) in a cost application to school districts, and Oliveira, Camanho, & Gaspar (2010) in a revenue application to dredge fleet. In the last case the additional constraint to the revenue model was related to the quotas on the outputs and therefore this application employs a different specification of the indirect economic measures of efficiency.

2.6. Profit efficiency

2.6.1. Theoretical background

The maximum profit for DMU k can be estimated given the unit prices of inputs and outputs available for that DMU, and the axioms specifying the shape of the production possibility set based on the input and output levels observed in other DMUs of the sample under assessment.

Model (18) (Fare et al., 1994, p.213) is the model generally used to determine the profit-maximizing input and output levels, where both input quantities and output quantities are decision variables (corresponding to x_i and y_r, respectively, in model (18)), in contrast to oriented models used to estimate cost efficiency and revenue efficiency (e.g., models that focus on reducing inputs or expanding outputs, respectively).

$$\Pi(c_{ik}, p_{rk}) = \max_{\lambda_j, y_r, x_i} \left\{ \sum_{r=1}^{s} p_{rk}\, y_r - \sum_{i=1}^{m} c_{ik}\, x_i \mid \sum_{j=1}^{n} \lambda_j\, y_{rj} \ \geq\ y_r, r = 1, \ldots, s, \right.$$
$$\left. \sum_{j=1}^{n} \lambda_j\, x_{ij} \leq x_i, i = 1, \ldots, m,\ \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0, j = 1, \ldots, n \right\} \quad (18)$$

The maximum profit model (18) needs to be defined with variable returns to scale (VRS), since a definition with constant returns to scale (CRS) presents unlimited or zero solutions (Fare et al., 1994; Varian, 1992) (the CRS assumption implicitly assumes perfect competition where maximum profit is zero). According to Portela & Thanassoulis (2007) another implication of assuming VRS in (18) is that maximum profit units do not need to be the most productive scale size units, that is, maximum profit units do not need to be efficient in scale.

Following the rationale of revenue and cost efficiency, the profit efficiency score (PE) of DMU k can be obtained by the ratio between the observed profit and maximum profit that can be achieved, as shown in expression (19) (Banker & Maindiratta, 1988; Fare et al., 1994). Note that y_{rk} is the observed value of the output r and x_{ik} is the observed value of the input i for the DMU k under evaluation, while the asterisk stands for optimal values from model (18).

$$PE_k = \frac{\Pi(c_{ik}, p_{rk})}{\Pi^*(c_{ik}, p_{rk})} = \frac{\sum_{r=1}^{s} p_{rk} y_{rk} - \sum_{i=1}^{m} c_{ik} x_{rk}}{\sum_{r=1}^{s} p_{rk} y_r^* - \sum_{i=1}^{m} c_{ik} x_i^*} \quad (19)$$

Such measure of profit efficiency is going to be considered in this paper as the 'classic profit efficiency', because it mimics the classic cost and revenue settings.

Model (18) is well established and widely used for finding maximum profit (in spite of some developments like the one in Maudos, Pastor, Perez, & Quesada, 2002 where output quantities are replaced by revenue values in model (18)). However, the actual measurement of profit efficiency, based on the optimal solution of (18), and the decomposition of profit efficiency into its components of technical and allocative efficiency can be performed through a variety of alternatives.

Banker & Maindiratta (1988) suggest the computation of profit efficiency using (19), and finding technical efficient targets through a traditional DEA methods (e.g. the standard VRS model of Banker et al., 1984). With technical efficient targets on the denominator of (19) the resulting measure is a technical efficiency measure expressed as a profit ratio. By defining both profit and technical efficiency as profit ratios, then a decomposition of profit efficiency into allocative and technical efficiency can be performed, where allocative efficiency is the ratio between profit and technical efficiency.

The above procedure for computing profit efficiency and its corresponding decomposition has some problems, as Portela & Thanassoulis (2007) highlighted. The most important of such problems is the fact that observed profit can be negative (implying negative efficiency scores with a difficult interpretation) and the fact that profit ratios do not have the same proportional interpretability as cost or revenue ratios.

As profits can be negative, Banker & Maindiratta (1988) suggested solving the maximum profit and the technical efficient models with an additional constraint: that maximum profit and technical efficient profit could not be negative, and models would be solved only for units showing non-negative observed profit. This is an obvious limitation of the approach, which lead some authors to solve the problem of negative profits by using revenue/cost ratios, which can never be negative (see e.g Cooper, Seiford, & Tone, 2000b; Kuosmanen, 1999), or using differences between profits to avoid negative efficiency measures (e.g. Berger, Hancock, & Humphrey, 1993; Coelli, Grifell-Tatjé, & Perelman, 2002; Banker & Maindiratta, 1988 also suggest this possibility), or even using other sort of approaches like a different objective function as in Asmild, Paradi, Reese, & Tam (2007).

Another issue in the computation of profit efficiency is that the technical efficiency measures should be non-oriented. That is, under cost minimization we assume that inputs are discretionary and should be reduced, under revenue maximization we assume that outputs are discretionary and should be increased, and under profit maximisation the assumption should be that both inputs and outputs (if not all at least some) are discretionary. This has led a variety of authors to propose new measures of technical efficiency that are specifically appropriate to the context of profit maximisation. For example, the hyperbolic model of Färe et al. (1985) (see also Fare et al., 1994) or the geometric distance function of Portela & Thanassoulis (2007), both rely on multiplicative decompositions of profit efficiency. Alternatively, the directional model of Chambers, Chung, & Färe (1996, 1998), and the additive model of Cooper, Park, & Pastor (1999), Cooper, Seiford, & Tone (2000a), rely on additive decompositions of profit efficiency.

The **hyperbolic model** of Färe et al. (1985) (see also Fare et al., 1994) also requires the estimation of the maximum attainable profit $\Pi^*(y_r, x_i)$ using model (18), given the price vector observed for unit k. In accordance with an hyperbolic path, the overall profit efficiency (ϕ_k^h) of unit k, is derived by solving (20), where $\Pi^*(c_{ik}, p_{rk})$ is the maximum profit of k estimated using model (18). That is, overall profit efficiency (ϕ_k^h) represents the amount by which inputs and outputs should be hyperbolicly adjusted, so that they are projected on the profit boundary.

$$\Pi^*(c_{ik}, p_{rk}) = \sum_{r=1}^{s} p_{rk}(y_{rk}/\phi_k^h) - \sum_{i=1}^{m} c_{ik}(x_{rk}\phi_k^h) \quad (20)$$

In this case, the decomposition of profit efficiency (ϕ_k^h) is based on the computation of a measure of technical efficiency that defines an hyperbolic path towards the production frontier, as shown in (21).

$$TE^h = \min_{\lambda_j,\theta^h} \left\{ \theta^h \mid \sum_{j=1}^{n} \lambda_j\, y_{rj} \ \geq \ \frac{y_{rk}}{\theta^h} \quad r = 1,\ldots,s \quad \sum_{j=1}^{n} \lambda_j\, x_{ij} \leq x_{ik}\theta_k^h \quad i = 1,\ldots,m \ \sum_{j=1}^{n} \lambda_j = 1 \ \ \lambda_j \geq 0 \ j = 1,\ldots,n \right\} \quad (21)$$

where θ^h defines the technical efficiency of unit k. The overall profit efficiency can then be decomposed as: $\phi^h = \theta^h \times \gamma^h$, where γ^h is the allocative profit efficiency.

The non-linear nature of the hyperbolic model is easily handled as shown in Färe et al. (1985), Fare et al. (1994), through a linearization of the model.

A generalised version of the hyperbolic model can be found in Chavas & Cox (1999), who calculate the technical efficiency of unit k by replacing in model (21) the θ^h by $\delta^{1-\alpha}$ when associated to inputs and δ^{α} when associated to outputs. So when alpha equals 1 clearly the Chavas and Cox model reduces to the hyperbolic model (alpha can be set a priori or can be optimised). Chavas & Cox (1999) also provide a decomposition of profit efficiency with a multiplicative nature that considers profit ratios.

The geometric distance function of Portela & Thanassoulis (2007) is related to the above hyperbolic model, but is more flexible than the above measures in the sense that it allows different expansion factors (β_r) and contraction factors (θ_i) for each output and input. This implies that all sources of inefficiency can be captured by the resulting efficiency measure. Technical efficiency using the geometric distance function notion is defined in (22).

$$\min_{\beta_r,\theta_i} \left\{ \frac{(\prod_{i=1}^{m} \theta_i)^{\frac{1}{m}}}{(\prod_{r=1}^{s} \beta_r)^{\frac{1}{s}}} \mid \sum_{j=1}^{n} \lambda_j\, y_{rj} = \beta_r y_{rk}, \ \sum_{j=1}^{n} \lambda_j\, x_{ij} = \theta_i x_{ik}, \times \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, \forall j, 0 \leq \theta_i \leq 1, \forall i, \ \beta_r \geq 1, \forall r \right\} \quad (22)$$

Model (22) has linear constraints but a non-linear objective function. However, it becomes linear for the special cases of traditional input-oriented or output-oriented models that consider equiproportional changes to all inputs or outputs, respectively. For example, input-oriented measures can assume that $\beta_r = \beta = 1\ \forall\ r$, and also that $\theta_i = \theta\ \ \forall\ i$, and thus the final efficiency score in (22) reduces to θ, which corresponds to the Farrell input efficiency measure. A similar reasoning applies for output-oriented efficiency measures. If β_r is assumed to be identical for all outputs, and θ_i is assumed to be equal for all inputs, and if we further impose that $\beta = \frac{1}{\theta}$, then (22) reduces to the hyperbolic model of Färe et al. (1985), with the only difference being that the resulting measure of efficiency is θ^2 and not θ.

If one wishes to solve the model (22) for a non-oriented case where a different expansion factor and contraction factor is associated to each output and each input, then either a non-linear solver needs to be used or the model can be approximated to a linear function as shown in Portela & Thanassoulis (2007). Portela & Thanassoulis (2007) proposed a decomposition of the profit efficiency score into technical and allocative components that is very similar to the approaches outlined above, except that in their case 'distances' between observed and target points are measured through a geometric mean.

Therefore if a point of maximum profit (x_i^*, y_r^*) is found through model (18), then expression (23) is applied to yield profit efficiency.

$$\mathrm{PE}^{GDF} = \frac{(\prod_{i=1}^{m} \frac{x_i^*}{x_{ik}})^{\frac{1}{m}}}{(\prod_{r=1}^{s} \frac{y_r^*}{y_{rk}})^{\frac{1}{s}}} \quad (23)$$

This measure can be decomposed into technical and allocative components by introducing the technically efficient point of unit k $(\theta_i x_{ik}, \beta_r y_{rk})$ as computed through model (22) as shown in (24).

$$\frac{(\prod_{i=1}^{m} \frac{x_i^*}{x_{ik}})^{\frac{1}{m}}}{(\prod_{r=1}^{s} \frac{y_r^*}{y_{rk}})^{\frac{1}{s}}} = \frac{(\prod_{i=1}^{m} \frac{\theta_i x_{ik}}{x_{ik}})^{\frac{1}{m}}}{(\prod_{r=1}^{s} \frac{\beta_r y_{rk}}{y_{rk}})^{\frac{1}{s}}} \times \frac{(\prod_{i=1}^{m} \frac{x_i^*}{\theta_i x_{ik}})^{\frac{1}{m}}}{(\prod_{r=1}^{s} \frac{y_r^*}{\beta_r y_{rk}})^{\frac{1}{s}}} \quad (24)$$

That is, adopting an approach based on geometric distances, we obtain a multiplicative decomposition of profit efficiency (i.e., Profit Eff^{GDF} = Technical Eff^{GDF} × Allocative Eff^{GDF}).

The **directional model** of Chambers et al. (1996, 1998) follows a procedure that is similar to that of the hyperbolic model, except that the overall profit inefficiency (ϕ^d), would decompose as: $\phi^d = \beta^d + \gamma^d$, where β^d represents technical inefficiency and γ^d represents allocative profit inefficiency (for details Chambers, Chung, & Färe, 1998). This directional distance function provides an additive decomposition that avoids negative efficiency measures and has been much used for profit efficiency decompositions. In this case, two models must be solved: one is the maximum profit model (18) and the other is the technical efficiency directional distance function model shown in (25).

$$\max_{\lambda_j,\beta^d} \left\{ \beta^d \mid \sum_{j=1}^{n} \lambda_j\, y_{rj} \ \geq \ y_{rk} + \beta^d g_{rk}, \ r = 1,\ldots,s, \ \sum_{j=1}^{n} \lambda_j\, x_{ij} \leq x_{ik} - \beta^d g_{ik}, \ i = 1,\ldots,m, \sum_{j=1}^{n} \lambda_j = 1 \ \ \lambda_j \geq 0, j = 1,\ldots,n \right\} \quad (25)$$

In model (25), $(\mathbf{g_x}, \mathbf{g_y}) = (g_{ik}, g_{rk})$ is the directional vector and β^d is an inefficiency score. The directional vector can be defined as being equal to the observed input and output levels of DMU k under assessment $(g_{ik}, g_{rk}) = (x_{ik}, y_{rk})$. In this case, the value of β^d represents a proportional adjustment to both inputs and outputs. The maximum profit derived from (18), is higher than the profit at the technical efficiency point derived from the optimal solution of (25), $(\mathbf{x_k} - \beta^{d*}\mathbf{g_x}, \mathbf{y_k} + \beta^{d*}\mathbf{g_y})$. That is, $\Pi^*(c_{ik}, p_{rk}) < \mathbf{p_k}(\mathbf{y_k} + \beta^{d*}\mathbf{g_y}) - \mathbf{c_k}(\mathbf{x_k} - \beta^{d*}\mathbf{g_x})$ and therefore,

$$\beta_k^d < \frac{\Pi^*(c_{ik}, p_{rk}) - \Pi(c_{ik}, p_{rk})}{\sum_{r=1}^{s} p_{rk} g_{rk} - \sum_{i=1}^{m} c_{ik} g_{ik}} \quad (26)$$

The right-hand side of (26) is a measure of profit efficiency, Φ_k^d, (the difference between maximum profit and observed profit is normalised using the directional vector), and the difference between $\Phi_k^d - \beta_k^d$ is the allocative efficiency. Φ_k^d is called the Nerlovian profit efficiency by Chambers et al. (1998). This measure will be equal to 0 if maximum profit is equal to observed profit and greater than zero otherwise. This means that (26) is indeed a shortage function and therefore measures inefficiency rather than efficiency.

The allocative component of the Nerlovian profit efficiency can be estimated by the difference between profit and technical efficiency, so this decomposition has an additive nature (see also Zofio, Pastor, & Aparicio, 2013, for recent references and applications.). Aparicio, Pastor, & Ray (2013b) introduced a modified DDF model where a different decision variable β was considered for inputs and outputs. The objective function of the directional model was specified as $max\ \beta^y + \beta^x$. The resulting decomposition is similar to the Nerlovian profit decomposition described above, except that the normalisation is done by the average cost (see also Ray, 2007).

The **additive model** of Charnes, Cooper, Golany, Seiford, & Stutz (1985) can be used to identify a technically efficient target (y_{rk}^T, x_{ik}^T) obtained as $(y_{rk}^T = y_{rk} + s_{rk}^{+*}, x_{ik}^T = x_{ik} - s_{ik}^{-*})$ where s_{rk}^{+*} and s_{ik}^{-*} are the values obtained at the optimal solution of model (27) solved for each unit k.

$$\max_{s^+_{rk}, s^-_{ik}} \left\{ s^+_{rk} + s^-_{ik} \mid \sum_{j=1}^{n} \lambda_j \, y_{rj} \ \geq \ y_{rk} + s^+_{rk}, \ r = 1, \ldots, s, \right.$$
$$\times \sum_{j=1}^{n} \lambda_j \, x_{ij} \leq x_{ik} - s^-_{ik}, \ i = 1, \ldots, m,$$
$$\left. \times \sum_{j=1}^{n} \lambda_j = 1 \ \ \lambda_j \geq 0, j = 1, \ldots, n \right\} \tag{27}$$

Note that there are many variants of the additive model, and for all of them the approach to decompose **profit efficiency** is the same, irrespectively of the model used to achieve the technical efficient targets. Among the most used variants of the additive model are the Slacks Based Measures (SBM) that weight the sum of slacks in such a way that makes the model units invariant (see for example Tone, 2001).

Under the additive model decomposition the profit at the maximum profit point ($\Pi^*(y_{rk}, x_{rk})$ identified through model (18)) and the profit at the technical efficient point ($\Pi(y^T_{rk}, x^T_{ik})$), are used in the following decomposition (Cooper et al., 1999; Cooper et al., 2000b): profit lost due to overall profit inefficiency equals the sum of the profit lost due to technical inefficiency and the profit lost due to allocative inefficiency as in (28).

$$\Pi^*(y_{rk}, x_{ik}) - \Pi(y_{rk}, x_{ik})) = [\Pi(y^T_{rk}, x^T_{ik}) - \Pi(y_{rk}, x_{ik})] + [\Pi^*(y_{rk}, x_{ik}) - \Pi(y^T_{rk}, x^T_{ik})] \tag{28}$$

This relationship is not, however, expressed in efficiency terms but in absolute profit values (see Berger et al., 1993; Coelli et al., 2002, who also used profit differences, though not using the additive model). The above decomposition can be applied whatever the slack based approach used for achieving technical efficient targets. In fact, one could even use oriented models, where only inputs or outputs were allowed to change towards the technical efficient frontier, but that would not be consistent with the underlying value measure of profit efficiency - where implicitly one assumes that at least some inputs and some outputs are discretionary (otherwise profit maximisation would reduce to cost or revenue optimization).

In the review of empirical applications dedicated to the assessment of profit efficiency, we have classified as 'slack based models' all models that have computed the distance to the frontier based on slacks, including the additive model presented in (27). The reader is directed to Aparicio, Ortiz, & Pastor (2017) for an approach that decomposes profit efficiency based on the SBM model. Such decomposition uses measures of profit efficiency, technical and allocative efficiency that vary between 0 and 1 and are relatively more complex than the former approach based on targets.

The above models and decompositions can also be used in approaches that look just for cost or for revenue efficiency. The adaptation of the above general approach on profit efficiency to the situation where only cost or revenue is being optimised is straightforward - Cost or revenue targets are obtained from the corresponding cost or revenue model and technical efficiency targets are obtained from the corresponding oriented technical efficiency model.

2.6.2. Profit efficiency applications

Overall, our revision of empirical applications identified 25 applications of profit efficiency measures alone and 24 that applied profit efficiency together with other measures of economic efficiency. From these we found 9 applications of the DDF approach, and 31 of the classical economic models. Slack based models (or additive decomposition) have also been applied in the profit setting by Susaeta, Adams, Carter, Gonzalez-Benecke, & Dwivedi (2016) and by Kamarudin, Sufian, & Nassir (2016) and the GDF measure has been applied in Portela & Thanassoulis (2007, 2005).

Despite the variety of models that exist for assessing profit efficiency, empirical applications resort just to a small subset of them - in concrete the classical approach, the DDF, and the Slack based models. Models such as the hyperbolic, the GDF, or the Chavas and Cox model have not been applied empirically except by the authors that developed the approaches. Some approaches do not fit in any of the models that we just mentioned (see also next section), like Cherchye, Rock, & Hennebel (2017) or Aghayi & Raayatpanah (2019), which address issues of limited price information and data uncertainty in the context of profit efficiency assessments, or Haghighatpisheh, Kordrostami, Amirteimoori, & Lotfi (2022) who estimate average economic efficiency models and consider optimization of profitability (ratio between revenues and costs) rather than profit to obtain optimal scale size models.

2.7. Other models

There are many variations found in the literature of economic efficiency models that for practical reasons could not be detailed in this paper. Here we tried to focus on the most relevant and/or the most used models. For example, in certain studies, developments are proposed for interpreting technical efficiency scores on economic terms, since shadow prices allow for an economic interpretation of such measures. We excluded from this revision approaches that are based on shadow prices such as Kuosmanen, Post, & Sipiläinen (2004) or Singbo, Lansink, & Emvalomatis (2015).

We mentioned before that when there is uncertainty about prices, weight restrictions can be used to reflect that uncertainty and that weight restricted models are linked with economic efficiency models. Another variant in the DEA literature is the one that assumes uncertainty on input and output values. Important models on this strand that is called 'Chance Constrained DEA' can be found in Cooper, Deng, Huang, & Li (2002). The economic models in stochastic DEA have been recently developed by Shiraz et al. (2020) for the case of uncertain values of inputs and outputs and fixed values of prices.

We also found some applications applying Network DEA (NDEA) models where one or various stages of the modelled process were related to economic measures of efficiency. NDEA has been originally developed to handle technical efficiency in multiple stages or processes within a DMU, but there have been some extensions of NDEA to account for economic measures of efficiency. For example, Wu (2010) developed a cost model for a series network structure with two stages (also called bi-level decision network structure). This model or similar models have been applied in the literature (e.g. in Shafiee, Lotfi, Saleh, & Ghaderi, 2016; Zhou, Luo, Tu, Lev, & Pedrycz, 2018b) with some adaptations. We call these models generically 'Economic network models' in our review. These include not only the traditional series or parallel structures but also industry models that in essence are equivalent to network parallel models (e.g. Cesaroni, 2018; Ray, 2016) as well as multi-output settings with output specific technologies (e.g. Cherchye et al., 2013).

In some cases the bootstrapping procedure of Simar & Wilson (2000) is applied to provide robust estimates of efficiency. The bootstrapping is considered in this revision as an additional technique for estimating confidence intervals and distributions of efficiency scores, and therefore we include papers using bootstrap classifying them on the basis of the original model used for the estimation of efficiency. In most cases, these applications use a two stage DEA analysis, where in the second stage contextual variables are used to explain the efficiency scores.

Non-parametric measures of dynamic efficiency in the context of an adjustment-cost technology and inter-temporal cost minimization, are not considered in this review because the type of models used is very different from traditional DEA models. The reader is directed to Nemoto & Goto (1999) and Silva & Stefanou (2003) for theoretical developments and to Rungsuriyawiboon & Hockmann (2015) for examples of application.

All economic efficiency measures presented in this review assume the convexity of the production possibility set. Convexity, is not however a necessary assumption for the computation of economic efficiency. Briec, Kerstens, & Eeckaut (2004) were among the first to analyse issues of convexity in cost functions. Recently Kerstens & Van de Woestyne (2021) review previous applications of cost functions that address convexity issues, and call the attention to the fact that convex cost functions may yield downwardly biased cost estimates in the order between 0.00% and 38.08%. The authors consider therefore that care is needed regarding the assumptions of the production technology implicit in economic efficiency measurement.

3. Review of economic efficiency applications

3.1. Search methodology

In order to find economic efficiency applications in the literature, we searched papers by accessing the Web of Science and Scopus databases. We considered papers published from 1978 (year of publication of the seminal paper on DEA by Charnes et al. (1978) to 2020. We used the terms "Data Envelopment Analysis" AND "Economic Efficiency", "Allocative Efficiency", "Cost Efficiency", "Revenue Efficiency", "Profit Efficiency". We excluded from the sample the pure methodology-oriented papers, since most of these are mentioned in the description of the theoretical models. The final scope of the research considered 326 papers.

In many cases, empirical studies compute a traditional technical efficiency score with input or output variables defined in monetary values, and call the resulting measure a cost or revenue efficiency score. As it became clear in the methodological section, such studies are not in fact computing economic efficiency measures. These measures correspond to a technical efficiency score in the case of similar prices for all DMUs, and are neither TE nor CE when the prices differ among DMUs. Monetary values can be used with the objective of incorporating qualitative differences in inputs among DMUs, and thus the assessment is closer to a technical efficiency evaluation, in the sense that it allows the quantification of proportional adjustments to input or output variables that lead to production on the frontier of the technology. For the above reasons, the studies that applied technical efficiency models to measure efficiency with several input or output value variables (without using a single cost or revenue indicator) were not included in our review.

We start the analysis of empirical applications by an analysis of the evolution of the number of publications over time. This is followed by a broad analysis of the main application areas where economic efficiency was employed and the main empirical challenges that researchers have been facing. The type of data collected was also a matter of concern, so we distinguish the number of studies that used aggregate data versus disaggregate data (separate measures of quantities and prices), and panel vs cross-sectional data. Finally, we identified whereas the approach used was a single-stage DEA analysis or a two-stage DEA analysis. In the two-stage DEA analysis, the first stage calculates efficiency using DEA, and the second stage aims to explore the impact of contextual factors on the measured efficiency (Liu et al., 2016).

3.2. Overview of empirical assessments of economic efficiency

The literature on empirical analyses of economic efficiency can be traced back to 1990, when Färe et al. (1990) analyzed the economic efficiency of 82 rice farms in California. In the same year Ferrier & Lovell (1990) also applied a cost model to banking and compared the DEA approach to the stochastic frontier model. Subsequently, other authors explored the US company AT&T (Sueyoshi, 1991), US farmers (Chavas & Aliber, 1993), the US steel industry (Ray & Kim, 1995), and private sectors of Chinese industry (Ka-Yiu Fung & Kai-Hong Wan, 1996). After 1998, the analysis of economic efficiency using DEA further developed with analyses of the iron and steel industry in China (Ray, Seiford, & Zhu, 1998), of Japanese and American electric utilities (Goto & Tsutsui, 1998), and of the Finnish health sector (Linna, 1998). In the subsequent years (1999–2020), there was an increasing trend in the number of papers published per year. The range of applications was wide, considering several areas such as manufacturing, food, agriculture, health, energy, transport, telecommunications, education, among others.

Table 1
Number of papers by area of application.

Application area (activities)	Number of papers	percentage of papers
Finance	130	39.88%
Agriculture and Forest	53	16.26%
Health Care	25	7.67%
Energy	23	7.06%
Transport	19	5.83%
Aquaculture	15	4.60%
Education	13	3.99%
Manufacturing	12	3.68%
Water and waste	9	2.76%
Municipalities	5	1.53%
Retail Industry	5	1.53%
Telecommunications Industry	4	1.23%
Hotels Industry	4	1.23%
Supply Chain	4	1.23%
Postal Services	3	0.92%
Civil Construction	1	0.31%
Public accounting industry	1	0.31%

Table 1 details the papers by application area. Finance (where we included banking and insurance) is largely the dominating sector in economic efficiency analysis. Agriculture and Forest, an application area with much tradition in efficiency analysis, is the second most frequent area of application of economic efficiency models. Agriculture covers a wide range of applications, such as wine (Aparicio, Borras, Pastor, & Vidal, 2013a), wheat farms (Asghar, Sasaki, Jourdain, & Tsusaka, 2018), poultry farms (Begum, Alam, Buysse, Frija, & Van Huylenbroeck, 2012), dairy farms (Huysveld et al., 2017), cotton (Zulfiqar, Datta, & Thapa, 2017), and others (see Supplementary Material 1). To a lesser extent, analyses in the area of aquaculture are concentrated on the production of different fish cultures (Sharma, Leung, Chen, & Peterson, 1999) and we included them in an autonomous area, which covers 4.6% of the overall number of applications reviewed.

There are sectors where one would expect a low level of application of economic efficiency models because of the difficulty of using prices of inputs and/or outputs. This is the case of education, the field where originally the DEA model originated from Charnes et al. (1978), and where the first applications of DEA (named as such) were originated. Indeed, educational inputs and outputs are often specified with a value-added perspective, considering the prior ability of students, their socio-economic-cultural background, or grades in standardized tests, whose prices are either unavailable or are meaningless. In this area, applications of economic models have been undertaken at various levels of analysis, such as public school districts (Banker et al., 2004; Haelermans & Ruggiero, 2013, universities Casu & Thanassoulis, 2006, and research centers Kuosmanen et al., 2006). In some other sectors, it is less clear why economic models have seldom been employed. This is the case of the manufacturing sector, the retail industry or the water and waste regulated sector.

Table 2
DEA Models.

Model	Total Papers	% of Total
Cost Model	**286**	**76,27%**
- Classic Cost	203	54,13%
- Aggregate Cost Model	30	8,00%
- Non Exogenous Prices	12	3,20%
- Weight Restricted Models	10	2,67%
- Economic Network Models	9	2,40%
- Slack Based Models	7	1,87%
- Directional Distance Function	3	0,80%
- Indirect Economic Measures	1	0,27%
- Other	11	2,93%
Revenue Model	**40**	**10,67%**
- Classic Revenue	29	7,73%
- Aggregate Revenue Model	7	1,87%
- Slack Based Models	1	0,27%
- Directional Distance Function	1	0,27%
- Indirect Economic Measures	1	0,27%
- Other	1	0,27%
Profit Model	**49**	**13,07%**
- Classic Profit	31	8,27%
- Directional Distance Functions	8	2,13%
- Slack Based Models	2	0,53%
- Geometric Distance Function	2	0,53%
- Indirect Economic Measures	1	0.27%
- Other	5	1,33%
Total	375	100,00%

Table 3
Type of data used in the analyses.

Data	Model		
	Cost	Revenue	Profit
Panel data	158	29	35
Cross-sectional	125	11	14
Time series	3		
Total	286	40	49

The reasons for a reduced number of applications in manufacturing may be the difficulty in obtaining data of comparable units for benchmarking. Piran, Lacerda, Camanho, & Silva (2021) explored the idea of internal benchmarking in a manufacturing setting and called precisely the attention to the reduced number of papers addressing economic efficiency in this area. In the case of water and waste, several papers assessed economic efficiency. But many did that using parametric techniques (according to the review of Berg & Marques, 2011) 58% of the studies in the water and wastewater sector used parametric methods), others do not use the terms cost, revenue or profit efficiency in spite of computing them, and others use technical efficiency models in a setting where various costs are used on the input side - typically CAPEX and OPEX (e.g. Carvalho & Marques, 2011). Indeed, the water sector is underrepresented in our review because of the inadequate use of technical efficiency models using aggregate cost data on the input side. When a single input, total expenditures (TOTEX) or operating expenditures (OPEX), is used, the technical efficiency models in fact are equivalent to cost efficiency models. For example, Thanassoulis (2000) show that DEA models that use OPEX as a single input can be interpreted in terms of cost savings and are related to models that regress opex on a set of output variables. The fact that many of these papers do not use the term cost or economic efficiency in their title or abstract made these papers unnoticed in our revision.

Table 2 summarizes the type of efficiency models used. In the previous sections, we already addressed many issues regarding the type of economic models, and as a result, here we will just summarise the main statistics regarding the models so far employed in economic efficiency measurement. There are a few studies (e.g., Cummins, Tennyson, & Weiss, 1999; Cummins, Weiss, Xie, & Zi, 2010) that analyzed more than one type of economic efficiency (e.g., cost, revenue and profit). Therefore, the total number of studies reported (375) exceeds the number of papers reviewed (326). Cost efficiency analyses are predominant among the applications reviewed (76,27% or 286 out of 375). Profit efficiency models have been used about the same as revenue models.

In general, most studies (90% or 338 out of 375) use disaggregate data on quantity and prices, which allow the analysis of allocative efficiency. The remaining 10% of the works use price and quantity data aggregated in total cost or revenue values. So, in spite of the fact that there is difficulty in obtaining the prices of inputs and outputs, the literature on economic efficiency has clearly found ways of getting price data. In some cases, prices are estimated to overcome such difficulties, or studies consider approximate values (e.g., Puig-Junoy, 2000). This can be problematic as it can compromise the validity of allocative efficiency and economic efficiency estimates.

Table 2 reveals that revenue and profit efficiency have been neglected in economic analyses using DEA. This confirms what has been previously pointed out by the literature (Kuosmanen, Kortelainen, Sipiläinen, & Cherchye, 2010; Portela & Thanassoulis, 2005). Some aspects may explain the reduced number of studies dedicated to profit efficiency. First, profit is computed as a difference between revenue and cost, not as a ratio. This feature imposes mathematical limitations, such as the mandatory use of DEA models assuming VRS. Secondly, the analysis requires that the costs of inputs and output prices be known, making 'price requirements' more stringent and difficult. In addition, in many sectors, a not-for-profit logic predominates (e.g., health and education). Consequently, output prices are not available, making it impossible to analyze profit and revenue efficiency. The measurement of revenue efficiency is methodologically similar to that of cost efficiency, and therefore there is no methodological reason for such a reduced use of revenue models. The justification in this case lies most likely in the empirical context, where typically input production factors are easier to control than outputs, which are in many cases dependent on random and uncontrollable demand.

Regarding the economic models that have been employed more often, we can conclude that the classic cost, revenue, and profit efficiency models (presented in Section 2) are prevalent in empirical applications. However, classic models are not always used, and several theoretical-methodological developments have been proposed and applied to real-world cases (see Table 2). For example, Slack Based Models, Weight Restrictions, Economic Network Models, models for non-exogeneous prices, and Directional Distance Functions, among others, have been used in a few empirical studies.

'Other models' in Table 2 refers to those developed to respond to specific case studies, such as the profit efficiency analysis with incomplete information on prices reported in Cherchye & Van Puyenbroeck (2007), or uncertain prices as in Aghayi & Raayatpanah (2019) and the DEA-based materials balance principle in Xian, Yang, Wang, Wei, & Huang (2019). Regarding the type of data used, Table 3 shows that in general terms, the use of panel data is prevalent (59.2% of the studies), followed by cross-sectional data (40%).

An outstanding aspect is that three studies used time series (longitudinal data) (Kang, 2010; Sueyoshi, 1991; Tone & Sahoo, 2005). In these cases, the analyses are performed over time considering a single Decision Making Unit. Thus, the DMU is compared to itself in different time periods (e.g., annually). This procedure has been named 'internal longitudinal benchmarking' in Piran et al. (2021).

Table 4
Methodological approaches adopted for the two-stage analysis.

		Economic measure of efficiency			
	Approach	Cost	Profit	Revenue	Total
Regression	Tobit regression	65	3	4	72
	Ordinary Least Squares (OLS)	27	10	10	47
	Bootstrap Regression	10	0	1	11
	Log- Regression Model	9	0	0	9
	Truncated Regression (S&W)	5	2	1	8
	Generalized Method of Moments (GMM)	5	1	2	8
	Generalized Least Squares (GLS)	2	1	0	3
	Maximum Likelihood Estimator	2	0	0	2
	Fractional Regression Model (FRM)	1	0	0	1
	Log-Likelihood Distance	0	0	1	1
Hyp. Testing	Wilcoxon Mann-Withney	14	10	7	31
	Kruskal-Wallis	10	9	6	25
	T test	7	1	0	8
	Anova	4	1	1	6
	Kolmogorov-Smirnov test	3	0	0	3
	Granger Causality test	2	0	0	2
	Panel Unit Root test, Panel Co-Integration test	1	1	0	2
	Banker's Sum Ratio test	1	0	0	1
	Simar-Zelenyuk-Adapted Li test	0	1	0	1
Correlation	Spearman's Rank Coefficients	5	1	0	6
	Pearson correlation	3	0	1	4
Other	Other approaches	12	2	3	17

3.3. Two-Stage analysis for explaining economic efficiency scores

Regarding the number of studies conducted with single versus two stage approaches, Table 4 shows the techniques used for two-stage DEA analyses (when performed). Note that although several papers performed post-DEA analysis, we only report in this section the analysis conducted with the purpose of explaining the differences in the efficiency scores that can be attributable to contextual conditions. Most papers in our review undertake a second stage analysis (45%, of studies do not apply second stage techniques) to relate the economic efficiency calculated by DEA with exogenous variables. Among the techniques used to conduct two-stage DEA analyses, Tobit Regression and Ordinary Least Squares (OLS) are predominant. These techniques were used in more than 100 studies, representing 45% of the cases in Table 4.

In general, two-stage DEA analyses seek to capture institutional, demographic, and management factors that affect efficiency. The Supplementary Material 1 details the other approaches used for the second-stage analysis These approaches consist of kernel distribution analysis, Monte Carlo simulations or clustering algorithms.

3.4. Main challenges

Most empirical applications aimed at the measurement of economic efficiency and its decomposition, and many times the analysis of its drivers. There can be found, however, in the literature some specific challenges that have been addressed by some papers. One of such challenges is the relationship between economic efficiency and size (scale economies) and the relationship between economic efficiency and variety (scope economies). Chavas & Aliber (1993) where among the first applications addressing simultaneously issues of scale and scope in Wisconsin farms. Findings show that there are economies of scale for small farms and that the cost of producing livestock and crops separately is higher than the cost of producing them together. This implies significant economies of scope in the sampled farms.

Regarding scale efficiency, we found two ways in the literature of computing it in economic settings. One is proposed by Chavas & Aliber (1993) where two cost models (see (1)) are solved: one under CRS and another under VRS (that is model (1) added of the convexity constraint), and the ratio between the VRS cost efficiency and the CRS cost efficiency is the scale efficiency. On the other hand, authors like Jha, Chitkara, & Gupta (2000), or Kaliba & Engle (2006) used the ratio between technical efficiency scores under VRS and CRS to compute scale efficiency. In Jha et al. (2000) the authors computed cost efficiency under CRS (CE_c) and decomposed cost efficiency into technical and allocative efficiency (AE), and technical efficiency was further decomposed into pure technical (PTE) and scale efficiency (SE) as in (29).

$$CE_c = PTE \times SE \times AE \tag{29}$$

If cost efficiency is computed under VRS (see e.g. Kaliba & Engle, 2006) the decomposition does not incorporate scale effects and is as in (30).

$$CE_v = PTE \times AE_v \tag{30}$$

This implies that defining a ratio between cost efficiency under VRS and CRS is equivalent to the traditional scale efficiency concept, only if the allocative efficiency is the same.

$$\frac{CE_c}{CE_v} = \frac{PTE \times SE \times AE}{PTE \times AE_v} \tag{31}$$

However since the allocative efficiency defined when the cost model follows VRS is not the same as when the cost model follows a CRS technology, it happens that the two concepts are not equivalent. One question therefore remains answered. If one wants to compute scale efficiency in a economic setting how should scale efficiency be measured? in relation to the technological frontier or in relation to the economic frontier? Note that this problem is analogous for revenue efficiency models, whereas for profit models the profit efficiency is normally computed under VRS (since maximum profit is zero or infinite under CRS) meaning that scale effects are not normally computed (see e.g. Färe, He, Li, & Zelenyuk, 2019).

Regarding scope efficiency there are not many papers addressing this important topic. The above mentioned Chavas & Aliber (1993) and Cummins et al. (2010) and Pokharel & Featherstone (2019) are some examples. The estimation of scope economies in the context of the economic efficiency applications has followed varied routes. Chavas & Aliber (1993) estimate scope efficiency by comparing the minimum cost of producing a bundle of outputs with the minimum aggregate cost of producing each output individually (by specifying separable technologies for producing each output). Pokharel & Featherstone (2019) followed a similar approach in a four-output case, estimating the total cost and the cost of producing three outputs jointly. They discuss two approaches to consider partial output sets - setting an appropriate output to zero or dropping an appropriate output constraint. Chavas & Aliber (1993) use the approach where minimum cost is estimated by dropping one or more of the output constraints, whereas Pokharel & Featherstone (2019) constrained one or more outputs to zero, noting that the two approaches yield similar results.

Cummins et al. (2010) used a different approach computing the efficiency of diversified and focused insurance companies and then they estimated a regression on the efficiency scores using a dummy for the type of firm together with other control variables. The coefficient associated to the dummy entailed information regarding scope economies, as it allowed to understand differences in efficiency between diversified and non-diversified firms.

Economies of scope imply that joint production is cheaper than separate production, and this issue is clearly linked with the estimation of economic gains from mergers. Most of the papers addressing this issue in an economic efficiency setting are in the finance area with applications mainly in banks. Some examples of papers that addressed this challenge are Shi et al. (2017) who evaluated the merger cost efficiency of hypothetical DMUs (which have the aggregate inputs and outputs of the merged firms) and decomposed it in technical efficiency, harmony efficiency and scale efficiency following the approach of Bogetoft & Wang (2005). Related to this literature is the analysis over time of pre and post Merger and acquisition periods (as in Chronopoulos, Girardone, & Nankervis, 2013, or Cummins et al., 1999 and Cummins & Xie, 2008 where strong evidence is found that acquired insurance firms achieve greater gains in technical, cost, and revenue efficiency than non-M&A firms) typically through second stage regression type models.

Another recent trend in the literature is related to environmental issues. Technical efficiency implies the specification of the inputs and outputs of the production process, and in a context of great environmental awareness the consideration of pollution, energy consumption or climate change factors is usually required. Traditional ways of computing environmental or energy efficiency is through the use of standard models with variables that address these concerns (see e.g. Agrell & Bogetoft, 2005). In these cases, a common discussion regards the treatment of undesirable inputs or outputs.

Other alternative for estimating environmental efficiency, the most used in economic efficiency applications, is the material balance framework which is an alternative to other methods of treating the undesirable output as an input, applying some sort of transformation to the undesirable output, or modelling undesirable factors as weakly disposable (see e.g. Färe, Grosskopf, Lovell, & Pasurka, 1989). In Coelli et al. (2007) the authors introduce the material balance framework that implies the definition of what is called an 'iso-nutrient' line similar to the isocost in cost efficiency models (Welch & Barnum, 2009, called a similar line as isocarbon line, and Aldanondo-Ochoa, Casasnovas-Oliva, & Almansa-Sáez (2017) called it isomaterial). Efficiency computed in relation to this iso-nutrient line is denoted by environmental efficiency, and it is decomposed into technical and environmental allocative efficiency (see also Xian et al. (2019) or Hai & Speelman (2020) for applications of this approach). The computation of environmental efficiency in the above papers implied the separate computation of the maximum cost reduction, irrespective of environmental outcomes or the maximum reduction in material input, irrespective of economic outcomes. However, in Aldanondo-Ochoa et al. (2017) the two concepts are linked, and models for finding joint environmental and cost allocative efficiency gains through constrained minimization models are implemented.

Blancard & Martin (2014) also used the above framework but in the context of energy efficiency replacing the iso-material line by an iso-energy line (combination of inputs that consume the same amounts of energy), and defined the distance to this line as the energy efficiency considering imprecise information on the energy content (the equivalent to input prices in a cost function). Løvold Rødseth (2017) adapted the material balance framework to a profit setting, computing emission restricted and unrestricted Nerlovian profit efficiencies.

3.5. Discussion and future directions

In summary, this review called the attention to several issues in the literature. Some are related to the (i) inaccurate application or naming of the theoretical concepts of economic efficiency, others are related to the (ii) scarce application of some models, and others are related to (iii) open issues that the literature did not resolve yet. Regarding the first, we noted the widespread application of technical efficiency models to contexts where value-data variables are used and the corresponding use of the terms cost efficiency or revenue efficiency. This common mistake implied the exclusion of many papers that were initially selected for our analysis. Care is therefore needed in clearly putting forward the concepts applied and clarifying the assumptions implicit in economic efficiency models. The use of technical efficiency weight restricted models in fact relate to cost, revenue or profit models when the weight restrictions relate to the relative value of the production factors. However, this link has not always been acknowledged by researchers, and we also could find some imprecise applications of the concept. This is therefore another area in need of more rigor. We hope this review opens the way for this rigor of empirical applications.

We also identified that some models have been scarcely applied. For example, indirect measures of economic efficiency have not been much applied, and we believe that many times they were applied without this name being used. For example, in the context of environmental efficiency recent models tend to impose constraints on the pollution levels on the cost model, and if we regard indirect measures of efficiency as constrained measures we can include this new developments within these models. However, this has not been done in the literature. In the profit efficiency setting we observed a disproportion between available models and existing applications. This is probably the setting where more models for assessing economic efficiency have been developed, but in the empirical applications we observe a prevalence of traditional models. Other scarcely applied models are network DEA models in the economic efficiency setting. In spite of network DEA being a growing field of research in the general efficiency literature, in economic settings there is not yet expression of applications employing such models. Another area with scarcity of applications regards second stage models for explaining efficiency. We were surprised to observe that most empirical application employ standard techniques like regression or Tobit regression for this purpose. This prevalence happens irrespective of the existing discussion around these methods, with Banker & Natarajan (2008) arguing that OLS methods can be applied as second stage tools (and showing that Tobit regression is not significantly better than simple OLS) and Simar & Wilson (2011) arguing that OLS has limited applicability given the peculiar and unusual assumptions on the data-generating process that need to be satisfied for its application.

In addition, models that do not assume firms as price takers are also scarcely employed in practice. This assumption is implicit in traditional cost models, but rarely is it acknowledged in empirical applications. Whereas this happens because the authors believe there is indeed no control over prices or because they are unaware of the assumption is an undisclosed matter. In spite of that, this is an area where there are still some open issues: (i) what sort of models perform better when one wants to change factor's quantities and prices, and (ii) what is the role of allocative efficiency in such a setting?

Other open issues in the literature regard the measurement of scale efficiency and scope efficiency. Regarding the first, we identified two ways that have been used to compute scale efficiency and there are no clear guidelines in the literature as to which of the approaches is preferable. Scope economies have also been analysed in various forms and this is an area where a comparison of methods could serve good purposes.

4. Conclusions

As DEA has its roots in the seminal publication of Charnes et al. (1978) in the Operational Research/Management Science field, the emphasis on the measurement of technical efficiency could be expected. Technical efficiency assessments are primarily concerned with the physical relationships between inputs and outputs. Nevertheless, linear programming techniques can solve all sorts of problems, including price-dependent optimisations aligned with behavioural assumptions of cost, revenue, or profit efficiency. In these cases, the assessments look at how firms decide on the mix of inputs they wish to use, or on the mix of outputs they should produce.

The underlying reason for the relative under-use of economic efficiency models compared to technical efficiency models can be traced back to the concerns expressed by the seminal paper of Farrell (1957), which expressed apprehension about the ability to measure prices accurately enough to make good use of allocative efficiency measurement. Thus, one of the challenges to address in this field, lies with the availability and reliability of price data, not with the analytical and empirical methods used to measure economic efficiency.

This study presented a literature review of the theoretical-methodological developments and empirical applications of DEA to assess economic efficiency (cost, revenue, and profit). We conducted a content analysis of 326 papers published from 1978 to 2020 in journals indexed in the Web of Science and Scopus. The analysis of economic efficiency using DEA began with Färe et al. (1990) and found its way into the literature mostly in operational research and economics journals. There is a focus of the academic community on research analyzing cost efficiency, while revenue efficiency and profit efficiency are relatively understudied.

A limitation of this work is that although we used appropriate keywords for this study and searched papers in the most relevant scientific databases, using other keywords in other databases may yield different results. Thus, some studies may not have been identified in the search we performed. It is our conviction, however, that a very good sample of existing economic DEA-based efficiency studies has been used in this literature review and that the main highlights, challenges and open issues for further research have been shown in this paper.

Acknowledgments

This work is financed by National Funds through the FCT - Fundação para a Ciência e a Tecnologia, I.P. (Portuguese Foundation for Science and Technology) within the project eduBEST - Education Systems Benchmarking with Frontier Techniques, with reference 2022.08686.PTDC.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2023.07.027.

References

Aghayi, N., & Raayatpanah, M. A. (2019). A robust optimization approach to overall profit efficiency with data uncertainty: Application on bank industry. *Journal of the Chinese Institute of Engineers, 42*(2), 160–168.
Agrell, P. J., & Bogetoft, P. (2005). Economic and environmental efficiency of district heating plants. *Energy Policy, 33*(10), 1351–1362.
Ahn, H., Bogetoft, P., & Lopes, A. (2019). Measuring potential sub-unit efficiency to counter the aggregation bias in benchmarking. *Journal of Business Economics, 89*(1), 53–77.
Aldanondo-Ochoa, A. M., Casasnovas-Oliva, V. L., & Almansa-Sáez, M. C. (2017). Cross-constrained measuring the cost-environment efficiency in material balance based frontier models. *Ecological Economics, 142*, 46–55.
Allen, R., Athanassopoulos, A., Dyson, R. G., & Thanassoulis, E. (1997). Weights restrictions and value judgements in data envelopment analysis: Evolution, development and future directions. *Annals of Operations Research, 73*, 13–34.
Aparicio, J., Borras, F., Pastor, J. T., & Vidal, F. (2013a). Accounting for slacks to measure and decompose revenue efficiency in the Spanish designation of origin wines with DEA. *European Journal of Operational Research, 231*(2), 443–451.
Aparicio, J., Ortiz, L., & Pastor, J. T. (2017). Measuring and decomposing profit inefficiency through the slacks-based measure. *European Journal of Operational Research, 260*(2), 650–654.
Aparicio, J., Pastor, J. T., & Ray, S. C. (2013b). An overall measure of technical inefficiency at the firm and at the industry level: The 'lost profit on outlay'. *European Journal of Operational Research, 226*(1), 154–162.
Asghar, S., Sasaki, N., Jourdain, D., & Tsusaka, T. W. (2018). Levels of technical, allocative, and groundwater use efficiency and the factors affecting the allocative efficiency of wheat farmers in Pakistan. *Sustainability, 10*(5), 1619.
Asmild, M., Paradi, J. C., Reese, D. N., & Tam, F. (2007). Measuring overall efficiency and effectiveness using DEA. *European Journal of Operational Research, 178*(1), 305–321.
Banker, R., Chang, H., & Natarajan, R. (2007). Estimating DEA technical and allocative inefficiency using aggregate cost or revenue data. *Journal of Productivity Analysis, 27*, 115–121.
Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*, 1078–1092.
Banker, R. D., Janakiraman, S., & Natarajan, R. (2004). Analysis of trends in technical and allocative efficiency: An application to Texas public school districts. *European Journal of Operational Research, 154*(2), 477–491.
Banker, R. D., & Maindiratta, A. (1988). Nonparametric analysis of technical and allocative efficiencies in production. *Econometrica: Journal of the Econometric Society*, 1315–1332.
Banker, R. D., & Natarajan, R. (2008). Evaluating contextual variables affecting productivity using data envelopment analysis. *Operations Research, 56*(1), 48–58.
Banker, R. D., & Thrall, R. M. (1992). Estimation of returns to scale using data envelopment analysis. *European Journal of Operational Research, 62*(1), 74–84.
Begum, I. A., Alam, M. J., Buysse, J., Frija, A., & Van Huylenbroeck, G. (2012). Contract farmer and poultry farm efficiency in Bangladesh: A data envelopment analysis. *Applied Economics, 44*(28), 3737–3747.
Belas, J., Kocisova, K., & Gavurova, B. (2019). Determinants of cost efficiency: Evidence from banking sectors in eu countries. *Acta Polytechnica Hungarica, 16*(5), 101–123.
Berg, S., & Marques, R. (2011). Quantitative studies of water and sanitation utilities: A benchmarking literature survey. *Water Policy, 13*(5), 591–606.
Berger, A. N., Hancock, D., & Humphrey, D. B. (1993). Bank efficiency derived from the profit function. *Journal of Banking and Finance, 17*, 314–347.
Berger, A. N., & Humphrey, D. B. (1997). Efficiency of financial institutions: International survey and directions for future research. *European Journal of Operational Research, 98*(2), 175–212.
Blancard, S., & Martin, E. (2014). Energy efficiency measurement in agriculture with imprecise energy content information. *Energy Policy, 66*, 198–208.
Bogetoft, P., & Wang, D. (2005). Estimating the potential gains from mergers. *Journal of Productivity Analysis, 23*(2), 145–171.
Briec, W., Kerstens, K., & Eeckaut, P. V. (2004). Non-convex technologies and cost functions: Definitions, duality and nonparametric tests of convexity. *Journal of Economics, 81*(2), 155–192.
Byrnes, J., Crase, L., Dollery, B., & Villano, R. (2010). The relative economic efficiency of urban water utilities in regional New South Wales and Victoria. *Resource and Energy Economics, 32*(3), 439–455.
Camanho, A., & Dyson, R. (2008). A generalisation of the Farrell cost efficiency measure applicable to non-fully competitive settings. *Omega, 36*(1), 147–162.
Camanho, A. S., & Dyson, R. (2005). Cost efficiency, production and value-added models in the analysis of bank branch performance. *Journal of the Operational Research Society, 56*(5), 483–494.
Carvalho, P., & Marques, R. C. (2011). The influence of the operational environment on the efficiency of water utilities. *Journal of Environmental Management, 92*(10), 2698–2707.
Castellet, L., & Molinos-Senante, M. (2016). Efficiency assessment of wastewater treatment plants: A data envelopment analysis approach integrating technical, economic, and environmental issues. *Journal of Environmental Management, 167*, 160–166.
Casu, B., & Thanassoulis, E. (2006). Evaluating cost efficiency in central administrative services in UK universities. *Omega, 34*(5), 417–426.
Cavaignac, L., & Petiot, R. (2017). A quarter century of data envelopment analysis

applied to the transport sector: A bibliometric analysis. *Socio-Economic Planning Sciences, 57*, 84–96.
Cesaroni, G. (2018). Industry cost efficiency in data envelopment analysis. *Socio-Economic Planning Sciences, 61*, 37–43.
Chambers, R. G., Chung, Y., & Färe, R. (1996). Benefit and distance functions. *Journal of economic theory, 70*(2), 407–419.
Chambers, R. G., Chung, Y., & Färe, R. (1998). Profit, directional distance functions, and Nerlovian efficiency. *Journal of Optimization Theory and Applications, 98*(2), 351–364.
Charnes, A., Cooper, W. W., Golany, B., Seiford, L., & Stutz, J. (1985). Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions. *Journal of Econometrics, 30*, 91–107.
Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429–444. https://doi.org/10.1016/0377-2217(78)90138-8.
Chavas, J.-P., & Aliber, M. (1993). An analysis of economic efficiency in agriculture: A nonparametric approach. *Journal of Agricultural and Resource Economics*, 1–16.
Chavas, J.-P., & Cox, T. L. (1999). A generalized distance function and the analysis of production efficiency. *Southern Economic Journal, 66*(2), 294–348.
Cherchye, L., Rock, B. D., Dierynck, B., Roodhooft, F., & Sabbe, J. (2013). Opening the "black box" of efficiency measurement: Input allocation in multioutput settings. *Operations Research, 61*(5), 1148–1165.
Cherchye, L., Rock, B. D., & Hennebel, V. (2017). Coordination efficiency in multi--output settings: A DEA approach. *Annals of Operations Research, 250*, 205–233.
Cherchye, L., & Van Puyenbroeck, T. (2007). Profit efficiency analysis under limited information with an application to german farm types. *Omega, 35*(3), 335–349.
Chronopoulos, D. K., Girardone, C., & Nankervis, J. C. (2013). How do stock markets in the US and Europe price efficiency gains from bank M&As? *Journal of Financial Services Research, 43*, 243–263.
Coelli, T., Grifell-Tatjé, & Perelman, S. (2002). Capacity utilisation and profitability: A decomposition of short run profit efficiency. *International Journal of Production Economics, 79*, 261–278.
Coelli, T., Lauwers, L., & Van Huylenbroeck, G. (2007). Environmental efficiency measurement and the materials balance condition. *Journal of Productivity Analysis, 28*(1), 3–12.
Cook, W. D., & Seiford, L. M. (2009). Data envelopment analysis (DEA)–thirty years on. *European Journal of Operational Research, 192*(1), 1–17.
Cooper, W. W., Deng, H., Huang, Z., & Li, S. X. (2002). Chance constrained programming approaches to technical efficiencies and inefficiencies in stochastic data envelopment analysis. *Journal of the Operational Research Society, 53*(12), 1347–1356.
Cooper, W. W., Park, K. S., & Pastor, J. T. (1999). RAM: A range adjusted measure of inefficiency for use with additive models, and relations to other models and measures in DEA. *Journal of Productivity Analysis, 11*(1), 5–42.
Cooper, W. W., Seiford, L. M., & Tone, K. (2000a). Data envelopment analysis. In *Handbook on data envelopment analysis* (pp. 1–40).
Cooper, W. W., Seiford, L. M., & Tone, K. (2000b). *Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software*. Boston: Kluwer Academic Publishers.
Cross, R., & Fare, R. (2008). Farrell efficiency under value and quantity data. *Journal of Productivity Analysis, 29*, 193–199.
Cummins, J. D., Tennyson, S., & Weiss, M. A. (1999). Consolidation and efficiency in the us life insurance industry. *Journal of Banking & Finance, 23*(2–4), 325–357.
Cummins, J. D., Weiss, M. A., Xie, X., & Zi, H. (2010). Economies of scope in financial services: A DEA efficiency analysis of the us insurance industry. *Journal of Banking & Finance, 34*(7), 1525–1539.
Cummins, J. D., & Xie, X. (2008). Mergers and acquisitions in the us property-liability insurance industry: Productivity and efficiency effects. *Journal of Banking & Finance, 32*(1), 30–55.
Dong, Y., Hamilton, R., & Tippett, M. (2014). Cost efficiency of the chinese banking sector: A comparison of stochastic frontier analysis and data envelopment analysis. *Economic Modelling, 36*, 298–308.
Dyson, R., Allen, R., Camanho, A., Podinovski, V., Sarrico, C., & Shale, E. (2001). Pitfalls and protocols in DEA. *European Journal of Operational Research, 132*(2), 245–259. https://doi.org/10.1016/s0377-2217(00)00149-1.
Emrouznejad, A., Parker, B. R., & Tavares, G. (2008). Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA. *Socio-Economic Planning Sciences, 42*(3), 151–157.
Emrouznejad, A., & Yang, G.-l. (2018). A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016. *Socio-Economic Planning Sciences, 61*, 4–8.
Färe, R., & Grosskopf, S. (2006). Resolving a strange case of efficiency. *Journal of the Operational Research Society, 57*(11), 1366–1368.
Färe, R., Grosskopf, S., & Lee, H. (1990). A nonparametric approach to expenditure-constrained profit maximization. *American Journal of Agricultural Economics, 72*(3), 574–581.
Färe, R., Grosskopf, S., & Lovell, C. A. K. (1985). *The measurement of efficiency of production*. Dordrecht, The Netherlands: Springer Netherlands. https://doi.org/10.1007/978-94-015-7721-2.
Fare, R., Grosskopf, S., & Lovell, C. K. (1994). *Production frontiers*. Cambridge University Press.
Färe, R., Grosskopf, S., Lovell, C. K., & Pasurka, C. (1989). Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. *The Review of Economics and Statistics*, 90–98.
Färe, R., He, X., Li, S., & Zelenyuk, V. (2019). A unifying framework for Farrell profit efficiency measurement. *Operations Research, 67*(1), 183–197.
Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society. Series A (General), 120*(3), 253. https://doi.org/10.2307/2343100.
Ferrier, G. D., & Lovell, C. K. (1990). Measuring cost efficiency in banking: Econometric and linear programming evidence. *Journal of Econometrics, 46*(1–2), 229–245.
Fethi, M. D., & Pasiouras, F. (2010). Assessing bank efficiency and performance with operational research and artificial intelligence techniques: A survey. *European Journal of Operational Research, 204*(2), 189–198.
Fukuyama, H., & Weber, W. L. (2002). Evaluating public school district performance via DEA gain functions. *Journal of the Operational Research Society, 53*(9), 992–1003.
Fukuyama, H., & Weber, W. L. (2004). Economic inefficiency measurement of input spending when decision-making units face different input prices. *Journal of the Operational Research Society, 55*(10), 1102–1110.
Fukuyama, H., & Weber, W. L. (2009). Output slacks-adjusted cost efficiency and value-based technical efficiency in DEA models (<special issue> operations research for performance evaluation). *Journal of the Operations Research Society of Japan, 52*(2), 86–104.
García-Cestona, M., & Surroca, J. (2008). Multiple goals and ownership structure: Effects on the performance of spanish savings banks. *European Journal of Operational Research, 187*(2), 582–599.
Goto, M., & Tsutsui, M. (1998). Comparison of productive and cost efficiencies among Japanese and US electric utilities. *Omega, 26*(2), 177–194.
Grosskopf, S. (1986). The role of the reference technology in measuring productive efficiency. *Economic Journal, 96*(Jun), 499–513.
Haelermans, C., & Ruggiero, J. (2013). Estimating technical and allocative efficiency in the public sector: A nonparametric analysis of Dutch schools. *European Journal of Operational Research, 227*(1), 174–181.
Haghighatpisheh, H., Kordrostami, S., Amirteimoori, A., & Lotfi, F. H. (2022). Optimal scale sizes in input–output allocative data envelopment analysis models. *Annals of Operations Research, 315*(2), 1455–1476.
Hai, A. T. N., & Speelman, S. (2020). Economic-environmental trade-offs in marine aquaculture: The case of lobster farming in Vietnam. *Aquaculture, 516*, 734593.
Halme, M., Joro, T., Korhonen, P., Salo, S., & Wallenius, J. (1999). A value efficiency approach to incorporating preference information in data envelopment analysis. *Management Science, 45*(1), 103–115.
Hatami-Marbini, A., Emrouznejad, A., & Tavana, M. (2011). A taxonomy and review of the fuzzy data envelopment analysis literature: Two decades in the making. *European Journal of Operational Research, 214*(3), 457–472.
Huysveld, S., Van Meensel, J., De Meester, S., Peiren, N., Muylle, H., Dewulf, J., Lauwers, L., et al., (2017). Communicative farm-specific diagnosis of potential simultaneous savings in costs and natural resource demand of feed on dairy farms. *Agricultural Systems, 150*, 34–45.
Jha, R., Chitkara, P., & Gupta, S. (2000). Productivity, technical and allocative efficiency and farm size in wheat farming in India: A DEA approach. *Applied Economics Letters, 7*(1), 1–5.
Jitsuzumi, T., & Nakamura, A. (2003). Measuring DEA efficiency in cable television network facilities: What are appropriate criteria for determining the amounts of governmental subsidies? *Socio-Economic Planning Sciences, 37*(1), 29–43.
Johnes, J. (2015). Operational research in education. *European Journal of Operational Research, 243*(3), 683–696.
Johnes, J., Portela, M., & Thanassoulis, E. (2017). Efficiency in education. *Journal of the Operational Research Society, 68*(4), 331–338.
Ka-Yiu Fung, M., & Kai-Hong Wan, K. (1996). Ownership and efficiency differentials in chinese industry: Further evidence from data envelopment analysis. *Applied Economics Letters, 3*(7), 475–482.
Kaffash, S., Azizi, R., Huang, Y., & Zhu, J. (2020). A survey of data envelopment analysis applications in the insurance industry 1993–2018. *European Journal of Operational Research, 284*(3), 801–813. https://doi.org/10.1016/j.ejor.2019.07.034.
Kaliba, A. R., & Engle, C. R. (2006). Productive efficiency of Catfish farms in Chicot county, arkansas. *Aquaculture Economics & Management, 10*(3), 223–243.
Kamarudin, F., Sufian, F., & Nassir, A. M. (2016). Global financial crisis, ownership and bank profit efficiency in the Bangladesh's state owned and private commercial banks. *Contaduría y administración, 61*(4), 705–745.
Kang, C.-C. (2010). Liberalization policy, production and cost efficiency in Taiwan's telecommunications industry. *Telematics and Informatics, 27*(1), 79–89.
Kao, C. (2014). Network data envelopment analysis: A review. *European Journal of Operational Research, 239*(1), 1–16.
Kerstens, K., & Van de Woestyne, I. (2021). Cost functions are nonconvex in the outputs when the technology is nonconvex: Convexification is not harmless. *Annals of Operations Research, 305*(1), 81–106.
Khan, H. H., Kutan, A. M., Naz, I., & Qureshi, F. (2017). Efficiency, growth and market power in the banking industry: New approach to efficient structure hypothesis. *The North American Journal of Economics and Finance, 42*, 531–545.
Korhonen, P., Tainio, R., & Wallenius, J. (2001). Value efficiency analysis of academic research. *European Journal of Operational Research, 130*(1), 121–132.
Kuosmanen, T. (1999). Some remarks on scale efficiency and returns to scale in DEA. Helsinki School of economics and business administration.
Kuosmanen, T., Cherchye, L., & Sipiläinen, T. (2006). The law of one price in data envelopment analysis: Restricting weight flexibility across firms. *European Journal of Operational Research, 170*(3), 735–757.
Kuosmanen, T., Kortelainen, M., Sipiläinen, T., & Cherchye, L. (2010). Firm and industry level profit efficiency analysis using absolute and uniform shadow prices. *European Journal of Operational Research, 202*(2), 584–594.
Kuosmanen, T., & Post, T. (2001). Measuring economic efficiency with incomplete

price information with an application to European commercial banks. *European Journal of Operational Research*. forthcoming
Kuosmanen, T., & Post, T. (2002). Nonparametric efficiency analysis under price uncertainty: A first-order stochastic dominance approach. *Journal of Productivity Analysis, 17*(3), 183–200.
Kuosmanen, T., & Post, T. (2003). Measuring economic efficiency with incomplete price information. *European Journal of Operational Research, 144*(2), 454–457.
Kuosmanen, T., Post, T., & Sipiläinen, T. (2004). Shadow price approach to total factor productivity measurement: With an application to finnish grass-silage production. *Journal of Productivity Analysis, 22*(1), 95–121.
Linna, M. (1998). Measuring hospital cost efficiency with panel data models. *Health Economics, 7*(5), 415–427.
Liu, H., Tsai, H., & Wu, J. (2018). Regional hotel performance and benchmarking in the pearl river delta: An input and output efficiency analysis. *International Journal of Contemporary Hospitality Management*.
Liu, J. S., Lu, L. Y., & Lu, W.-M. (2016). Research fronts in data envelopment analysis. *Omega, 58*, 33–45.
Løvold Rødseth, K. (2017). Environmental regulations and allocative efficiency: Application to coal-to-gas substitution in the us electricity sector. *Journal of Productivity Analysis, 47*(2), 129–142.
Mariano, E. B., Sobreiro, V. A., & do Nascimento Rebelatto, D. A. (2015). Human development and data envelopment analysis: A structured literature review. *Omega, 54*, 33–49.
Maudos, J., Pastor, J. M., Perez, F., & Quesada, J. (2002). Cost and profit efficiency in European banks. *Journal of International Financial Markets, Institutions and Money, 12*(1), 33–58.
Mostafaee, A., & Saljooghi, F. (2010). Cost efficiency measures in data envelopment analysis with data uncertainty. *European Journal of Operational Research, 202*(2), 595–603.
Murphy, O. A., Wang, P., Wang, S. X., & Tkacz, G. (2013). An economic efficiency study on different regions of Ghana via slack-based data envelopment analysis and regression analysis. *Applied Economics, 45*(34), 4773–4780.
Nemoto, J., & Goto, M. (1999). Dynamic data envelopment analysis: Modeling intertemporal behavior of a firm in the presence of productive inefficiencies. *Economics Letters, 64*(1), 51–56.
Olesen, O. B., & Petersen, N. C. (2002). The use of data envelopment analysis with probabilistic assurance regions for measuring hospital efficiency. *Journal of Productivity Analysis, 17*(1), 83–109.
Oliveira, M. M., Camanho, A. S., & Gaspar, M. B. (2010). Technical and economic efficiency analysis of the portuguese artisanal dredge fleet. *ICES Journal of Marine Science, 67*(8), 1811–1821.
Pelone, F., Kringos, D. S., Romaniello, A., Archibugi, M., Salsiri, C., & Ricciardi, W. (2015). Primary care efficiency measurement using data envelopment analysis: A systematic review. *Journal of Medical Systems, 39*(1), 1–14.
Piran, F. S., Lacerda, D. P., Camanho, A. S., & Silva, M. C. (2021). Internal benchmarking to assess the cost efficiency of a broiler production system combining data envelopment analysis and throughput accounting. *International Journal of Production Economics, 238*, 108173.
Pokharel, K. P., & Featherstone, A. M. (2019). Estimating multiproduct and product-specific scale economies for agricultural cooperatives. *Agricultural Economics, 50*(3), 279–289.
Portela, M. C., & Thanassoulis, E. (2007). Developing a decomposable measure of profit efficiency using DEA. *Journal of the Operational Research Society, 58*(4), 481–490.
Portela, M. C. A. S. (2014). Value and quantity data in economic and technical efficiency measurement. *Economics Letters, 124*(1), 108–112.
Portela, M. C. A. S., & Thanassoulis, E. (2005). Profitability of a sample of portuguese bank branches and its decomposition into technical and allocative components. *European Journal of Operational Research, 162*(3), 850–866.
Portela, M. C. A. S., & Thanassoulis, E. (2014). Economic efficiency when prices are not fixed: Disentangling quantity and price efficiency. *Omega, 47*, 36–44.
Psaraki-Kalouptsidi, V., & Kalakou, S. (2011). Assessment of efficiency of greek airports. *Journal of Airport Management, 5*(2), 170–186.
Puig-Junoy, J. (2000). Partitioning input cost efficiency into its allocative and technical components: An empirical DEA application to hospitals. *Socio-Economic Planning Sciences, 34*(3), 199–218.
Ray, S. (2016). Cost efficiency in an indian bank branch network: A centralized resource allocation model. *Omega, 65*, 69–81.
Ray, S. C. (2007). Shadow profit maximization and a measure of overall inefficiency. *Journal of Productivity Analysis, 27*(3), 231–236.
Ray, S. C., Chen, L., & Mukherjee, K. (2008). Input price variation across locations and a generalized measure of cost efficiency. *International Journal of Production Economics, 116*(2), 208–218.
Ray, S. C., & Kim, H. J. (1995). Cost efficiency in the us steel industry: A nonparametric analysis using data envelopment analysis. *European Journal of Operational Research, 80*(3), 654–671.
Ray, S. C., Seiford, L. M., & Zhu, J. (1998). Market entity behavior of chinese state-owned enterprises. *Omega, 26*(2), 263–278.
Rungsuriyawiboon, S., & Hockmann, H. (2015). Adjustment costs and efficiency in polish agriculture: A dynamic efficiency approach. *Journal of Productivity Analysis, 44*(1), 51–68.
Sahoo, B. K., & Tone, K. (2013). Non-parametric measurement of economies of scale and scope in non-competitive environment with price uncertainty. *Omega, 41*(1), 97–111.
Schaffnit, C., Rosen, D., & Paradi, J. C. (1997). Best practice analysis of bank branches: An application of DEA in a large canadian bank. *European Journal of Operational Research, 98*(2), 269–289.
Seiford, L. M., & Thrall, R. M. (1990). Recent developments in DEA: The mathematical programming approach to frontier analysis. *Journal of Econometrics, 46*(1-2), 7–38.
Seyedboveir, S., Kordrostami, S., Daneshian, B., & Amirteimoori, A. (2017). Cost efficiency measurement in data envelopment analysis with dynamic network structures: A relational model. *Asia-Pacific Journal of Operational Research, 34*(05), 1750023.
Shafiee, M., Lotfi, F. H., Saleh, H., & Ghaderi, M. (2016). A mixed integer bi-level DEA model for bank branch performance evaluation by Stackelberg approach. *Journal of Industrial Engineering International, 12*(1), 81–91.
Sharma, K. R., Leung, P., Chen, H., & Peterson, A. (1999). Economic efficiency and optimum stocking densities in fish polyculture: An application of data envelopment analysis (DEA) to Chinese fish farms. *Aquaculture, 180*(3-4), 207–221.
Shephard, R. W. (2015). *Theory of cost and production functions*. Princeton University Press.
Shi, X., Li, Y., Emrouznejad, A., Xie, J., & Liang, L. (2017). Estimation of potential gains from bank mergers: A novel two-stage cost efficiency DEA model. *Journal of the Operational Research Society, 68*(9), 1045–1055.
Shiraz, R. K., Hatami-Marbini, A., Emrouznejad, A., & Fukuyama, H. (2020). Chance–constrained cost efficiency in data envelopment analysis model with random inputs and outputs. *Operational Research, 20*(3), 1863–1898.
Silva, E., & Stefanou, S. E. (2003). Nonparametric dynamic production analysis and the theory of cost. *Journal of Productivity Analysis, 19*(1), 5–32.
Simar, L., & Wilson, P. W. (2000). A general methodology for bootstrapping in non–parametric frontier models. *Journal of Applied Statistics, 27*(6), 779–802.
Simar, L., & Wilson, P. W. (2011). Two-stage DEA: Caveat emptor. *Journal of Productivity Analysis, 36*(2), 205–218.
Singbo, A. G., Lansink, A. O., & Emvalomatis, G. (2015). Estimating shadow prices and efficiency analysis of productive inputs and pesticide use of vegetable production. *European Journal of Operational Research, 245*(1), 265–272.
Sotiros, D., Rodrigues, V., & Silva, M. C. (2022). Analysing the export potentials of the portuguese footwear industry by data envelopment analysis. *Omega, 108*, 102560.
Sueyoshi, T. (1991). Estimation of stochastic frontier cost function using data envelopment analysis: An application to the AT&T divestiture. *Journal of the Operational Research Society, 42*(6), 463–477.
Susaeta, A., Adams, D. C., Carter, D. R., Gonzalez-Benecke, C., & Dwivedi, P. (2016). Technical, allocative, and total profit efficiency of loblolly pine forests under changing climatic conditions. *Forest Policy and Economics, 72*, 106–114.
Thanassoulis, E., M. C. S, & Allen, R. (2004). Incorporating value judgments in DEA. In W.W. Cooper, L.W. Seiford, & J. Zhu (Eds.), Handbook on data envelopment analysis (pp. 99–138). Kluwer Academic Publishers.
Thanassoulis, E. (2000). The use of data envelopment analysis in the regulation of UK water utilities: Water distribution. *European Journal of Operational Research, 126*(2), 436–453.
Thanassoulis, E. (2001). *Introduction to the theory and application of data envelopment analysis: A foundation text with intefrated software*. Kluwer Academic Publishers.
Thanassoulis, E., Portela, M. C. S., & Graveney, M. (2012). Estimating the scope for savings in referrals and drug prescription costs in the general practice units of a UK primary care trust. *European Journal of Operational Research, 221*(2), 432–444.
Thanassoulis, E., Sotiros, D., Koronakos, G., & Despotis, D. (2018). Assessing the cost–effectiveness of university academic recruitment and promotion policies. *European Journal of Operational Research, 264*(2), 742–755.
Thompson, R., Langemeier, L., Lee, C., Lee, E., & Thrall, R. M. (1990). The role of multiplier bounds in efficiency analysis with application to kansas farming. *Journal of Econometrics, 46*, 93–108.
Thompson, R., & Thrall, R. (1994). Polyhedral assurance regions with linked constraints. In W. Cooper, & A. Whinston (Eds.), *New directions in computational economics* (pp. 121–133). Kluwer Academic Publishers.
Thompson, R. G., Dharmapala, P. S., Rothenberg, L. J., & Thrall, R. M. (1996). DEA/AR efficiency and profitability of 14 major oil companies in US exploration and production. *Computers and Operations Research, 23*(4), 357–373.
Thompson, R. G., Dharmapala, P. S., & Thrall, R. M. (1995). Linked-cone DEA profit ratios and technical efficiency with application to illinois coal mines. *International Journal of Production Economics, 39*, 99–115.
Toloo, M., & Ertay, T. (2014). The most cost efficient automotive vendor with price uncertainty: A new DEA approach. *Measurement, 52*, 135–144.
Tone, K. (2001). A slacks-based measure of efficiency in data envelopment analysis. *European Journal of Operational Research, 130*(3), 498–509.
Tone, K. (2002). A strange case of the cost and allocative efficiencies in DEA. *Journal of the Operational Research Society, 53*(11), 1225–1231.
Tone, K., & Sahoo, B. K. (2005). Evaluating cost efficiency and returns to scale in the life insurance corporation of India using data envelopment analysis. *Socio-Economic Planning Sciences, 39*(4), 261–285.
Tone, K., & Tsutsui, M. (2007). Decomposition of cost efficiency and its application to Japanese-US electric utility comparisons. *Socio-Economic Planning Sciences, 41*(2), 91–106.
Varian, H. R. (1992). *Microeconomic analysis* (3rd ed.). W.W. Norton and Company.
Wang, Z., & Feng, C. (2015). A performance evaluation of the energy, environmental, and economic efficiency and productivity in China: An application of global data envelopment analysis. *Applied Energy, 147*, 617–626.
Welch, E., & Barnum, D. (2009). Joint environmental and cost efficiency analysis of electricity generation. *Ecological Economics, 68*(8-9), 2336–2343.

Wu, D. D. (2010). Bilevel programming data envelopment analysis with constrained resource. *European Journal of Operational Research, 207*(2), 856–864.
Xian, Y., Yang, K., Wang, K., Wei, Y.-M., & Huang, Z. (2019). Cost-environment efficiency analysis of construction industry in China: A materials balance approach. *Journal of Cleaner Production, 221*, 457–468.
Zafra-Gómez, J. L., Antonio, M., & Muñiz, P. (2010). Overcoming cost-inefficiencies within small municipalities: Improve financial condition or reduce the quality of public services? *Environment and Planning C: Government and Policy, 28*(4), 609–629.
Zhou, H., Yang, Y., Chen, Y., & Zhu, J. (2018a). Data envelopment analysis application in sustainability: The origins, development and future directions. *European Journal of Operational Research, 264*(1), 1–16.
Zhou, P., Ang, B. W., & Poh, K.-L. (2008). A survey of data envelopment analysis in energy and environmental studies. *European Journal of Operational Research, 189*(1), 1–18.
Zhou, X., Luo, R., Tu, Y., Lev, B., & Pedrycz, W. (2018b). Data envelopment analysis for bi-level systems with multiple followers. *Omega, 77*, 180–188.
Zofio, J. L., Pastor, J. T., & Aparicio, J. (2013). The directional profit efficiency measure: On why profit inefficiency is either technical or allocative. *Journal of Productivity Analysis, 40*(3), 257–266.
Zulfiqar, F., Datta, A., & Thapa, G. B. (2017). Determinants and resource use efficiency of "better cotton": An innovative cleaner production alternative. *Journal of Cleaner Production, 166*, 1372–1380.