Ramsey optimal taxation with wage rigidities

by

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Abstract

This thesis aims to assess the impact of downward wage rigidities on optimal taxation. We study a dynamic general equilibrium neoclassical growth model for a one sector, cashless stochastic closed economy with an infinitely lived representative household, a representative firm with a constant returns to scale technology, a government deciding how to finance its exogenous expenditures without access to lump sum taxes, and competitive markets. We conclude that the optimal labor income tax exhibits both a reactive and a precautionary nature. Regarding the reactive nature, when the wage rigidity is binding, labor taxes increase since it is possible to raise revenue without additional distortions. On the precautionary side, the expectation of a future constraint lowers labor taxes, which, in turn, decreases the wage that clears the labor market, thus loosening future constraints. In the nominal small open economy with downwardly rigid nominal wages and exogenous nominal exchange rates, we show that the same conclusions apply. Additionally, it is possible to use consumption taxes in such a way that the optimal capital control tax is zero for a broad family of instantaneous utility functions. Finally, we introduce a consumption tax that discriminates between the good produced in the domestic economy and the good produced abroad to show that downward wage rigidities and exogenous exchange rates are irrelevant if the correct policy is used.

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Resumo

Esta tese pretende avaliar o impacto de rigidezes salariais decrescentes sobre a política fiscal óptima. Para tal, usamos um modelo dinâmico de equilibrío geral, concordante com a corrente neoclásica, para uma economia estocástica fechada, sem moeda, de horizonte temporal infinito com uma família representativa, uma empresa representativa que usa uma tecnologia com rendimentos constantes à escala num único sector, um governo que precisa de financiar a sequência exógena de gastos sem ter acesso a impostos lump sum, e mercados competitivos. É possível concluir que o imposto óptimo sobre o rendimento do trabalho apresenta características reactivas e precautórias. No que respeita à vertente reactiva, quando a restrição salarial é activa, o imposto deve aumentar visto ser possível recolher mais receita sem introduzir distorções adicionais. Na vertente precautória, a expectativa de uma restrição futura diminui o imposto, o que, por sua vez, reduz o salário que equilibra o mercado do trabalho, relaxando, desta forma, as restrições futuras. Para a pequena economia aberta nominal, com rigidez decrescente no salário nominal juntamente com taxas de câmbio nominais exógenas, o mesmo resultado é obtido. Além disso, mostramos que é possível implementar impostos sobre o consumo tais que o imposto óptimo de controlo de capitais seja zero para uma variedade de funções utilidade instantâneas. Finalmente, introduzimos um imposto sobre o consumo que discrimina o bem doméstico do bem produzido no exterior para demonstrar que rigidezes decrescentes no salário nominal, juntamente com taxas de câmbio nominais exógenas, se tornam irrelevantes para uma correcta utilização desta política.

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1 Introduction

Mundell's trillema of monetary policy states that an open economy cannot have an independent monetary policy with fixed exchange rates and free capital mobility. This idea is perfectly illustrated by the covered interest rate parity condition, for it shows that the domestic nominal interest rate is completely determined outside the economy provided there is free capital mobility and fixed exchange rates. Consequently, many countries with fixed exchange rate regimes (or in the particular case of a monetary union), have to bear the costs "resulting from the loss in ability to use policy for stabilization purposes" (Adão, Correia, and Teles 2009). Particularly, when facing nominal rigidities, such countries are negated the possibility of attaining the same allocations as it would be possible under flexible nominal exchange rates. For instance, if the economy needs to reduce the real wage in order to boost competitiveness, as a response to a low aggregate productivity parameter, for example, but nominal wages cannot be lowered, a nominal devaluation would solve this problem. However, if there is a constraint on the exchange rate policy, the real wage will not adjust completely and inefficiencies arise. This is one of the reasons for which Milton Friedman (1953) has argued in favor of flexible exchange rate regimes under circumstances of nominal rigidities.

The recent European crisis provides a good example of these costs. "Membership in the eurozone has been blamed for the inability of countries like Greece, Ireland, Portugal, and Spain to devalue their exchange rates and restore their competitiveness in international markets" (Farhi, Gopinath and Itskhoki 2012). Even though these problems have been extensively studied and solved, the fact remains that they are not isolated, i.e. they tend to be recurrent throughout the course of history, ever since they first appeared with the gold standard. Schmitt-Grohé and Uribe (2016) report other examples, which include the case of Argentina during the second half of the Convertibility Plan, specifically for the period of 1998 to 2001, when the Argentine central bank was pegging the Argentine peso to the U.S. dollar. There, the negative effects of the combination of downward nominal wage rigidity and currency pegs was quite visible, for the period considered was a period of rising unemployment. On the episode of the Great Recession of 2008, in which the main protagonists are the European periphery countries on the euro area, data used by Schmitt--Grohé and Uribe (2016) suggests the same rise in unemployment accompanied by rising nominal wages. And so, given these examples, it is important to shed new light on this topic.

Problems such as the one presented above have led economists to study how to conduct fiscal and monetary policy in order to make fixed exchange rate regimes irrelevant for the equilibrium allocations, or to replicate the effects of nominal devaluations. For this reason, this field came to be known as the field of fiscal devaluations. For instance, Adão, Correia, and Teles (2009) use a Ramsey optimal fiscal and monetary policy approach in a two open economies model to show that there is a minimal set of instruments such that, under the assumptions of no international labor mobility and no tradeability of state contingent private debt, the equilibrium allocations in the flexible producer price, flexible nominal exchange rate model can be implemented with stable producer prices and a fixed exchange rate. The implication of this result is that fixed exchange rate regimes are irrelevant for the allocations, no matter the type and degree of price rigidity that may exist, i.e. even in the presence of some price rigidity such as staggered Calvo prices, fixed exchange rate regimes come with zero welfare costs. In Adão, Correia, and Teles (2010) the irrelevance result is extended to encompass nominal wage rigidities. They show that the minimal set of instruments needs just to be augmented to include payroll taxes, so that, in the flexible prices, flexible nominal wages, and flexible nominal exchange rate economies, the equilibrium allocations can be implemented with fiscal and monetary policy that allows for prices and nominal wages to be stable, even with a constant nominal exchange rate. The use of payroll taxes, particularly decreasing payroll taxes in order to allow the effective real wage to decrease, presents, nevertheless, some problems. In fact, the financing of these lower payroll taxes is through an increase in labor income taxes, which might be perceived as an income transfer from households to capitalists, and so, it may be hard to implement. In short, these contributions suggest that, since prices and wages can be made stable in the economy with flexible prices and nominal wages, then restrictions on these variables, whatever they may be, are completely irrelevant.

Farhi, Gopinath, and Itskhoki (2012) highlight that one of the problems concerning Adão, Correia, and Teles' (2009 and 2010) irrelevance results lies in the fact that the minimal set of instruments includes both domestic and foreign taxes. Consequently, the irrelevance results cannot be implemented unilaterally by one of the economies. With this disadvantage in mind, the authors study a New Keynesian open economy and develop fiscal policies, namely a combination of a value-added tax with payroll taxes and export subsidies with import tariffs, that robustly replicate the effects of a devaluation of domestic currency, i.e. that apply to a significant variety of environments. Indeed, they show that both these policies are applicable across different price setting assumptions and different configurations of the assets market. There are, however, some additional instruments that should be used depending on the circumstances. For instance, when foreign debt is denominated in domestic currency, replicating the effects of a nominal devaluation requires using a tax on the return of foreign investors so as to induce the same loss of wealth that would stem from a nominal devaluation. Furthermore, Farhi, Gopinath, and Itskhoki's (2012) proposed fiscal policies can be implemented with zero net impact on the overall government's budget deficit. In their words, these policies are "revenue-neutral".

Nevertheless, none of these works studies how to conduct fiscal policy in economies characterized by the failure of these results, i.e. in economies where nominal rigidities combined with fixed exchange rates indeed incur in welfare costs. That is the scope of this work.

In a related line of work, Schmitt-Grohé and Uribe (2016) start from a first best environment for a small open economy with a currency peg and introduce a distortion on nominal wages - downward nominal wage rigidity - thus generating a negative externality stemming from overborrowing in expansionary periods, which leads to a rise in the nominal wage and consequent unemployment during recessions. From that point on, the authors' approach is to introduce another distortion, summarized in a capital control tax, which is shown to be prudential in nature, i.e. the optimal capital control is high during economic expansions to prevent the economy from overheating and nominal wages from inflating, and low during recessions to provide stimuli. As it turns out, the second best principle stating that additional distortions can improve welfare is valid in their framework. Other references on optimal capital controls include the work of Farhi and Werning (2012). They study the optimality of capital controls in response to a variety of shocks and show that they are highly effective when employed as a response to risk-premium shocks.

This work strives to understand how the existence of downward nominal wage rigidities in the open economy, combined with an exogenous stochastic sequence of nominal exchange rates, impacts on optimal taxation policy when only labor income, capital income, and consumption taxes are available. With this in mind, we are going to study a dynamic general equilibrium neoclassical growth model for both a real stochastic closed economy and a nominal stochastic small open economy with an infinitely lived representative household and competitive firms, where the single composite good produced domestically and abroad is homogeneous. In contrast with Adão, Correia, and Teles (2009), whose environment is characterized by imperfect competition, we propose a simple competitive environment where all firms in the domestic economy are price takers. The main advantage of this assumption is that we can simplify the economy by abstracting from price setting assumptions, thus allowing us to study only the implications of nominal wage rigidities in economies without control over the nominal exchange rate. Money is not going to be used in transactions, though it is going to serve as mean of value. Additionally, we assume, in contrast to Schmitt-Grohé and Uribe (2016), that the environment is characterized by the absence of lump sum taxes, thus implying the first best solution is not attainable. Ergo, we follow the standard approach in the literature on optimal fiscal and monetary policy after Lucas and Stokey (1983), which takes a Ramsey central planner deciding how to raise distortionary taxes to finance exogenous government expenditures.

In the closed economy framework, where real wages are assumed to be downwardly rigid à la Schmitt-Grohé and Uribe (2016), we conclude that the optimal labor income tax is both reactive and precautionary in nature. Its reactive nature, consisting on increasing the tax rate in states of the current period in which the wage rigidity is binding, is explained by the fact that the level of employment is demand determined, thus implying that the government can raise revenue without introducing any additional distortions. For the precautionary nature, it arises when there are expectations of a future wage constraint. In that case, labor income taxes decrease in that state of the current period in order to lower the labor market clearing wage, thus loosening future constraints. The optimal capital income tax is shown to vary negatively with the expectation of a constraint for the following period, and positively with the expectation of a constraint for two periods from the current one. In the small open economy, we show that the optimal labor income tax exhibits the same reactive and precautionary nature. Additionally, we show that there is a choice of consumption taxes, irrelevant for the allocations, such that the optimal capital control tax is zero. In a second best environment, such choice of consumption taxes also determines an optimal zero capital income tax. Hence, we revisit Correia's (1996) result stating that optimal taxation in a small open economy, taxing income from abroad using the worldwide system, and in a closed economy is quite similar in the sense that Chamley's (1986) and Judd's (1985) result of zero capital income taxation in steady state, which can be interpreted as no intertemporal distortions, holds even in our framework of a stochastic economy, where there is no steady state. Furthermore, we show that consumption taxes can be used to discriminate between the good produced domestically and the good produced abroad, thus allowing us to establish an irrelevance result in the same spirit as that of Adão, Correia, and Teles (2009). Particularly, when used correctly, consumption taxes can replace the role of the nominal exchange rate. thus implying that the combination of downwardly rigid nominal wages and exogenous nominal exchange rates comes with zero welfare costs, i.e.

consumption taxes allow for the implementation of the second best solution.

The outline of this work is as follows. First, we start by analyzing the effects of downward wage rigidities on optimal taxes in a one sector closed economy environment. The purpose is to understand how the central planner incorporates the externality arising from the assumption of downward real wage rigidity on the optimal taxes. Then, in section 3, we open the economy to a foreign sector with which a homogenous good and non state contingent net foreign debt can be traded, and impose downwardly nominal wages and an exogenous stochastic sequence of nominal exchange rates, in order to see how the introduction of the foreign debt dynamics impacts the Ramsey planner's decision of optimal taxes. Section 4 follows with the establishment of our irrelevance result arising from a special use of consumption tax policy. Section 5 concludes.

2 The stochastic representative agent closed economy with downward real wage rigidity

This section presents a stochastic neoclassical growth model for a one sector closed economy. The purpose is to understand how wage rigidities - real wage rigidities in particular as we are going to consider a real economy where the composite good is the numeraire - impact on optimal Ramsey taxation policy. In doing so, we intend to isolate the impact of this friction on optimal taxation policy, which will be helpful in section 4 where the economy is open, for the effects arising from wage rigidities and foreign debt can be separated.

The closed economy has an infinitely lived representative household with preferences over consumption and labor, competitive firms with a constant returns to scale technology, and a government that issues state contingent debt but is not allowed to use lump sum taxes to finance its exogenous sequence of expenditures. In each period $t \geq 0$ the economy experiences one of finitely many events s_t, which are due to uncertainty on productivity and government expenditures. The initial realization s_0 is given. The set of all possible events in period t is denoted by S_t, the history of these events up to and including period t, defined as state at t, $(s_0, s_1, ..., s_t)$ is represented by s^t, and the set of all possible states in period t is denoted by S^t.

Furthermore, real wages are assumed to be downwardly rigid. In particular, we borrow and adapt Schmitt-Grohé and Uribe's (2016) assumption on downward nominal wage rigidity to introduce real wage rigidity in this economy as

$$w_t\left(s^t\right) \geq \gamma w_{t-1}\left(s^{t-1}\right), \tag{1}$$

where $\gamma \in]0,1]$ represents the degree of downward wage rigidity. Higher values of γ are, therefore, associated with a higher degree of rigidity, where $\gamma = 1$ characterizes full wage rigidity. On the other hand, lower values of γ put the economy closer to the flexible wage economy. In the limit, wages are fully flexible when $\gamma = 0$.

This assumption can be thought to be associated with some structural problem of the economy, i.e. some institutional imposition, legislation, or a status quo bias in wage bargaining. With employment being determined by the short side of the labor market, this assumption generates a negative externality, according to which involuntary unemployment may exist in contractionary periods, for the adverse shock may lead to an equilibrium in which the labor market clearing real wage is below the lower bound of period t, γw_{t-1}. The reason is that agents are too small to internalize the fact that their decisions can collectively affect aggregate variables.

2.1 The representative household

The representative household has preferences over the consumption of the composite good, c_t, and labor, l_t, described by the expected utility function

$$U = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t \Pr_t\left(s^t\right) u\left[c_t\left(s^t\right), l_t\left(s^t\right)\right] = E_0 \sum_{t=0}^{\infty}\beta^t u\left[c_t\left(s^t\right), l_t\left(s^t\right)\right], \tag{2}$$

where $0 < \beta < 1$ is the subjective discount factor. The instantaneous utility function $u\left[c_t\left(s^t\right), l_t\left(s^t\right)\right]$ satisfies the usual properties that guarantee the existence of an interior solution, i.e. the instantaneous utility function is continuous and twice differentiable, it is increasing in consumption, decreasing in labor, and globally concave, and Inada conditions $\lim_{c\to 0} u_c = \infty$, $\lim_{c\to\infty} u_c = 0$, $\lim_{l\to 0} u_l = 0$, and $\lim_{l\to\bar{l}} u_l = -\infty$ hold, where $\bar{l}$ stands for a maximal feasible level of labor.

In each state s^t of period $t \geq 0$, the representative household receives income from labor $l_t\left(s^t\right)$, which is taxed at the proportional rate $\tau_t^l\left(s^t\right)$, from the predetermined level of capital $k_t\left(s^{t-1}\right)$, taxed at the proportional rate $\tau_t^k\left(s^{t-1}\right)$, and from state contingent real government bonds $b_t\left(s^t\right)$. Profits are not considered, though firms are owned by households, due to the fact that the assumptions of competitive markets and a constant returns to scale production function imply zero profits for firms in equilibrium. This disposable income can be used to consume $c_t(s^t)$, taxed at the proportional rate $\tau_t^c(s^t)$, to invest in capital $k_{t+1}(s^t)$ or in state contingent real government bonds $b_{t+1}(s^{t+1})$ that cost $q_t(s^{t+1})$ units of the good in period t and deliver one unit of the good in the state of period $t+1$, where the event s_{t+1} is realized, and zero otherwise. These state contingent real government bonds ensure the completeness of the assets market. Hence, the representative household's intratemporal budget constraints in each state can be written as

$$\left(1+\tau_t^c\left(s^t\right)\right) c_t\left(s^t\right)+k_{t+1}\left(s^t\right)+\sum_{s^{t+1} \mid s^t} q_t\left(s^{t+1}\right) b_{t+1}\left(s^{t+1}\right)= \tag{3}$$

$$=\left(1-\tau_t^l\left(s^t\right)\right) w_t\left(s^t\right) l_t\left(s^t\right)+\left(1+\left(1-\tau_t^k\left(s^{t-1}\right)\right) r_t^k\left(s^t\right)-\delta\right) k_t\left(s^{t-1}\right)+b_t\left(s^t\right),$$

together with a no Ponzi games condition.

In its essence, the budget constraints (3) are inequality constraints as the representative household is not allowed to spend more than the disposable income. Nevertheless, these constraints are written with equality for simplification purposes, since the assumption of monotonic preferences implies that at the optimal solution the household exhausts all disposable income.

The representative household chooses $\left\{c_t\left(s^t\right), l_t\left(s^t\right), k_{t+1}\left(s^t\right), b_{t+1}\left(s^{t+1}\right)\right\}_{t=0}^{\infty}$, $\forall\ s^t \in S^t$ to maximize lifetime expected utility (2) subject to (3) and the no Ponzi games condition, taking prices, taxes, the initial stock of capital k_0, and the initial stock of government bonds b_0 as given. We take the standard assumption on the literature that capital is chosen in each period after the realization of uncertainty. The Lagrangean function for this problem is

$$\begin{aligned}
\mathcal{L} = {} & E_0 \sum_{t=0}^{\infty} \beta^t\left[u\left[c_t\left(s^t\right), l_t\left(s^t\right)\right]-\lambda_t\left(s^t\right)\left(\begin{array}{c}
\left(1+\tau_t^c\left(s^t\right)\right) c_t\left(s^t\right)+k_{t+1}\left(s^t\right)+ \\
+\sum_{s^{t+1} \mid s^t} q_t\left(s^{t+1}\right) b_{t+1}\left(s^{t+1}\right)
\end{array}\right)\right] \\
& +E_0 \sum_{t=0}^{\infty} \beta^t\left[\lambda_t\left(s^t\right)\left(\begin{array}{c}
\left(1-\tau_t^l\left(s^t\right)\right) w_t\left(s^t\right) l_t\left(s^t\right)+ \\
+\left(1+\left(1-\tau_t^k\left(s^{t-1}\right)\right) r_t^k\left(s^t\right)-\delta\right) k_t\left(s^{t-1}\right)+b_t\left(s^t\right)
\end{array}\right)\right].
\end{aligned}$$

The first order condition with respect to $c_t(s^t)$ is

$$\beta^t\left[u_{c,t}\left(s^t\right)-\lambda_t\left(s^t\right)\left(1+\tau_t^c\left(s^t\right)\right)\right]=0, \quad t \geq 0, \forall\ s^t.$$

Since there might exist involuntary unemployment, i.e. for the market wage rate households may be willing to supply more labor than firms are willing to use in production, then the first order condition with respect to $l_t\left(s^t\right)$ is

$$\beta^t\left[u_{l,t}\left(s^t\right)+\lambda_t\left(s^t\right)\left(1-\tau_t^l\left(s^t\right)\right)w_t\left(s^t\right)\right]\geq 0,\quad t\geq 0,\forall\ s^t.$$

The first order condition with respect to $k_{t+1}\left(s^t\right)$ is

$$-\beta^t\lambda_t\left(s^t\right)+E_t\left[\beta^{t+1}\lambda_{t+1}\left(s^{t+1}\right)\left(1+\left(1-\tau_{t+1}^k\left(s^t\right)\right)r_{t+1}^k\left(s^{t+1}\right)-\delta\right)\right]=0,\quad t\geq 0,\forall\ s^t.$$

The first order condition with respect to $b_{t+1}\left(s^{t+1}\right)$ is

$$-\Pr_t\beta^t\lambda_t\left(s^t\right)q_t\left(s^{t+1}\right)+\beta^{t+1}\Pr_{t+1}\lambda_{t+1}\left(s^{t+1}\right)=0,\quad t\geq 0,\forall\ s^t.$$

The following conditions, together with the budget constraints (3), k_0, b_0 and the imposed transversality conditions

$$\lim_{T\to\infty}Q_T\left(s^T\right)k_{T+1}\left(s^T\right)=0, \tag{4}$$

and

$$\lim_{T\to\infty}\sum_{s^{T+1}|s^T}Q_{T+1}\left(s^{T+1}\right)b_{T+1}\left(s^{T+1}\right)=0, \tag{5}$$

where $Q_{t+1}\left(s^{t+1}\right)=q_t\left(s^{t+1}\right)Q_t\left(s^t\right)$ and $Q_0=1$, characterize the solution to the representative household's problem:

$$\frac{-u_{l,t}\left(s^t\right)}{u_{c,t}\left(s^t\right)}\leq\frac{\left(1-\tau_t^l\left(s^t\right)\right)w_t\left(s^t\right)}{\left(1+\tau_t^c\left(s^t\right)\right)},\quad t\geq 0,\forall\ s^t, \tag{6}$$

$$u_{c,t}=\beta E_t\left[u_{c,t+1}\left(1+\left(1-\tau_{t+1}^k\right)r_{t+1}^k-\delta\right)\frac{\left(1+\tau_t^c\right)}{\left(1+\tau_{t+1}^c\right)}\right],\quad t\geq 0,\forall\ s^t, \tag{7}$$

where the variables are a function of the state.

$$q_t\left(s^{t+1}\right)=\beta\Pr_{t+1}\left(s^{t+1}|s^t\right)\frac{u_{c,t+1}\left(s^{t+1}\right)}{1+\tau_{t+1}^c\left(s^{t+1}\right)}\frac{1+\tau_t^c\left(s^t\right)}{u_{c,t}\left(s^t\right)},\quad t\geq 0,\forall\ s^t. \tag{8}$$

Conditions (6) are going to be verified with inequality whenever the downward wage rigidity condition (1) binds. The sign of the inequality makes sense, for the existence of unemployment is a synonym of less employment and, consequently, less consumption relative to the flexible wage situation. Hence, at such a solution, the marginal disutility of labor, $u_{l,t}\left(s^t\right)$, which is decreasing, is excessively small, in absolute value, while the decreasing marginal utility of consumption, $u_{c,t}\left(s^t\right)$, is excessively high. The remaining conditions can be interpreted as no arbitrage conditions. Conditions (7), which state that the intertemporal marginal rate of substitution is equal to the relative price of current consumption, relate to the alternative ways a unit of consumption can be used. It can either be used to consume in the current period or it can be invested in capital, generating a state contingent return and allowing for consumption in the future period, whose valuation depends on the current expectations of the future state of nature. Essentially, both alternatives must yield the same marginal benefit, i.e. the optimal allocation of consumption over time must leave the representative household indifferent between these alternatives. Conditions (8) represent the equality between the marginal rate on substitution between a state of period t and a given state of period $t+1$ and the relative price of current consumption in those states of different periods.

2.2 The representative firm

Firms in this economy are competitive, i.e. price takers. The production technology uses labor and capital according to the production function

$$Y_t\left(s^t\right) \equiv A\left(s^t\right) F\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right)\right),$$

where $A\left(s^t\right)$ represents the aggregate stochastic productivity parameter. This production function is assumed to verify the standard properties, i.e. it is continuous and twice differentiable, it exhibits constant returns to scale, the marginal productivities of inputs are positive and decreasing ($F_k, F_l > 0$ and $F_{kk}, F_{ll} < 0$), and it verifies the Inada conditions $\lim_{k\to 0} F_k\left(k,l\right) = \lim_{l\to 0} F_l\left(k,l\right) = \infty$ and $\lim_{k\to \infty} F_k\left(k,l\right) = \lim_{l\to \infty} F_l\left(k,l\right) = 0$. Additionally, both inputs are assumed to be essential in the sense that $F\left(0,l\right) = F\left(k,0\right) = 0$.

Taking prices as given, the representative firm is going to choose in each state s^t of period $t \geq 0$ the inputs l_t and k_t in order to maximize profits. These are written as

$$\pi_t\left(s^t\right) = A\left(s^t\right) F\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right)\right) - w_t\left(s^t\right) l_t\left(s^t\right) - r_t^k\left(s^t\right) k_t\left(s^t\right).$$

Profit maximizing behavior implies choosing the level of inputs such that their marginal productivities are equal to their real remuneration. This means that the optimal conditions that characterize the solution to the representative firm's problem are

$$r_t^k\left(s^t\right) = A\left(s^t\right) F_{k,t}\left(s^t\right), \quad t \geq 0, \forall\ s^t. \tag{9}$$

and

$$w_t\left(s^t\right) = A\left(s^t\right) F_{l,t}\left(s^t\right), \quad t \geq 0, \forall\ s^t. \tag{10}$$

Regardless of the existence of unemployment, conditions (10) are always verified. This has to do with the assumption of downward wage rigidity (1), assuring the equilibrium in the labor market during boom episodes, which, together with the dominance of the short side of the market, guarantee that overemployment never exists in this economy.

As mentioned before, the representative firm earns zero profits, for it is price taker and uses a constant returns to scale technology. This can be proved using Euler's theorem.

2.3 The government

The government needs to finance the exogenous sequence of stochastic public consumption $g\left(s^t\right)$. To that end, and since lump sum taxation is not allowed, it will raise distortionary taxes. Specifically, the government is going to use state contingent time varying taxes on consumption $\tau_t^c\left(s^t\right)$ and on labor income $\tau_t^l\left(s^t\right)$, and non state contingent time varying taxes on capital income $\tau_t^k\left(s^{t-1}\right)$. Moreover, the government can also raise revenue by issuing state contingent real government bonds $b_{t+1}\left(s^{t+1}\right)$ that pay one unit of good in period $t+1$, provided event s_{t+1} is realized, which are traded at a real price $q_t\left(s^{t+1}\right)$ in period t. Consequently, the government's state by state intratemporal budget constraints can be written as

$$g\left(s^t\right) + b_t\left(s^t\right) = \sum_{s^{t+1}|s^t} q_t\left(s^{t+1}\right) b_{t+1}\left(s^{t+1}\right) + \tag{11}$$

$$+\tau_t^c\left(s^t\right) c_t\left(s^t\right) + \tau_t^l\left(s^t\right) w_t\left(s^t\right) l_t\left(s^t\right) + \tau_t^k\left(s^{t-1}\right) r_t^k\left(s^t\right) k_t\left(s^{t-1}\right),$$

together with a no Ponzi games condition.

2.4 Market clearing conditions

The presence of downwardly rigid real wages implies that there might exist states of nature with involuntary unemployment. In the event of labor market disequilibrium, which would be the case were condition (1) binding, condition (6) would hold with strict inequality for the reasons discussed above. On the other hand, if real wages were to face no problem adjusting, i.e. condition (1) does not bind, there would be no reason for involuntary unemployment to exist and condition (6) would be verified with equality. To incorporate these features, one needs to introduce the following complementary slackness conditions:

$$\left(\frac{-u_{l,t}\left(s^t\right)}{u_{c,t}\left(s^t\right)}-\frac{\left(1-\tau_t^l\left(s^t\right)\right)w_t\left(s^t\right)}{\left(1+\tau_t^c\left(s^t\right)\right)\left(s^t\right)}\right)\left(w_t\left(s^t\right)-\gamma w_{t-1}\left(s^{t-1}\right)\right)=0,\quad t\geq 0,\forall\ s^t. \tag{12}$$

For the remaining markets (goods, capital, and government bonds), only the goods market clearing conditions should be considered, given that the same variable was used to define demand and supply in the capital and government bonds markets. That being said, the goods market clearing conditions, also called resource constraints, for each state of nature s^t in each period $t \geq 0$ are presented as

$$c_t\left(s^t\right)+g\left(s^t\right)+k_{t+1}\left(s^t\right)-\left(1-\delta\right)k_t\left(s^{t-1}\right)=A\left(s^t\right)F\left(k_t\left(s^{t-1}\right),l_t\left(s^t\right)\right). \tag{13}$$

2.5 Competitive equilibrium

Definition 1 *In this stochastic closed economy, a competitive equilibrium is a set of allocations* $\left\{c_t\left(s^t\right),l_t\left(s^t\right),k_{t+1}\left(s^t\right)\right\}_{t=0}^{\infty}$*, prices* $\left\{w_t\left(s^t\right),r_t^k\left(s^t\right),q_t\left(s^{t+1}\right),Q_t\left(s^t\right)\right\}_{t=0}^{\infty}$*, and policies* $\left\{b_{t+1}\left(s^{t+1}\right),\tau_t^c\left(s^t\right),\tau_t^l\left(s^t\right),\tau_{t+1}^k\left(s^t\right)\right\}_{t=0}^{\infty}$ *for all* $s^t\in S^t$*, such that given the exogenous stochastic government expenditures* $g\left(s^t\right)$ *and aggregate productivity* $A\left(s^t\right)$*, the initial stock of capital* k_0*, the initial stock of government bonds* b_0*, and the historical real wage* w_{-1}*, the following conditions are met:* (*i*) *the representative household maximizes lifetime expected utility subject to the budget constraints, taking prices and taxes as given;* (*ii*) *the representative firm chooses* k_t *and* l_t *in each state in order to maximize profits subject to the technology constraints, taking prices as given;* (*iii*) *the government satisfies its budget constraints; and* (*iv*) *markets clear.*

It follows from Definition (1) that the conditions that characterize the competitive equilibrium in this stochastic closed economy are (3), (6)-(10), (12), and (13), together with the terminal conditions (4) and (5), with k_0, and b_0 given.

The fact that there is no need to have the government budget constraints (11) to characterize the competitive equilibrium is worth mentioning. Indeed, the irrelevance of these conditions is associated with Walras' law, i.e. the idea that if all the other equilibrium conditions are met, so must be conditions (11), for they are a linear combination of the others.

2.6 Ramsey optimal taxation

2.6.1 The set of attainable allocations

Following Lucas and Stokey (1983), it is possible to write the representative household's budget constraints (3) as the following implementability condition[1]:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{c,t}\left(s^t\right) c_t\left(s^t\right) + u_{l,t}\left(s^t\right) l_t\left(s^t\right) \right] \geq V_0, \tag{14}$$

where $V_0 \equiv \frac{u_{c,0}}{1+\tau_0^c}\left[\left(1+\left(1-\tau_0^k\right) A_0 F_{k,0} - \delta\right) + b_0\right]$ is the exogenous level of initial wealth in units of utility as in Chari, Nicolini, and Teles (2016), thus allowing us to abstract from initial taxation considerations[2].

Furthermore, one can use the representative firm's optimal conditions (10) to rewrite the wage rigidity constraints (1) as

$$\begin{gathered} A_0 F_{l,0} \geq \gamma w_{-1} \\ A\left(s^t\right) F_{l,t}\left(s^t\right) \geq \gamma A\left(s^{t-1}\right) F_{l,t-1}\left(s^{t-1}\right), \quad t \geq 1, \forall\, s^t. \end{gathered} \tag{15}$$

Lemma 1 *If a set of allocations* $\left\{c_t\left(s^t\right), l_t\left(s^t\right), k_{t+1}\left(s^t\right)\right\}_{t=0}^{\infty}$, $\forall\, s^t \in S^t$ *meets conditions (13), (14) with equality, and (15), then it can be implemented as a competitive equilibrium.*

Proof. To prove Lemma (1), one needs to show that conditions (13), (14) with equality, and (15) are necessary and sufficient to characterize the competitive equilibrium allocations.

To that effect, we take an arbitrary allocation $\left\{c_t\left(s^t\right), l_t\left(s^t\right), k_{t+1}\left(s^t\right)\right\}_{t=0}^{\infty}$, verifying (13), (14) with equality, and (15) as given, and show that the competitive equilibrium conditions (3), (6)-(10), (12), and (13) are met.

The household's budget constraints (3) are satisfied, for the allocation considered meets the implementability condition (14) with equality, which is just an alternative representation of the household's budget constraints. Furthermore, the completeness of the assets market guarantees that the implementability condition implies that the state by state budget constraints are met, for there are as many instruments as there are states of nature. Hence, we need only to choose $b_{t+1}\left(s^{t+1}\right)$ in every state. The intratemporal marginal conditions (6) determine the state contingent labor income tax $\tau_t^l\left(s^t\right)$ in every state. The intertemporal

[1]The proof can be found in Appendix 6.A.

[2]References for initial taxation include the work of Straub and Werning (2015) and Chari, Nicolini, and Teles (2016).

marginal conditions (7) and (8) are satisfied choosing, respectively, the capital income tax $\tau_{t+1}^{k}\left(s^{t-1}\right)$ and the price of state contingent real government bonds $q_t\left(s^{t+1}\right)$ for every Euler equation. The capital rent $r_t^k\left(s^t\right)$ and the real wage $w_t\left(s^t\right)$ in every state can be used to recover the firm's conditions (9) and (10). And the complementary slackness conditions (12) are met since the allocation complies with the implementability condition (14) verified with equality. This idea comes from the fact that even in states in which the wage constraint is binding, the central planner is always going to conduct policy such that condition (6) is verified with equality, which, in turn, implies that the implementability condition (14) is holds with equality. Finally, the resource constraints (13), which are one of the sufficient conditions, are, by definition of the allocation considered, satisfied. ■

The proof of Lemma (1) highlights the fact that the competitive equilibrium can be implemented for any choice of the consumption tax, $\tau_t^c\left(s^t\right)$, for this instrument was not used to satisfy any of the competitive equilibrium conditions, i.e. any arbitrary choice for the sequence of state contingent consumption taxes, either a constant pattern or a varying one, has no implications for the equilibrium allocations. The only adjustments would occur in the appropriate choices of $\tau_t^l\left(s^t\right)$, $\tau_{t+1}^k\left(s^{t-1}\right)$, and $q_t\left(s^{t+1}\right)$.

2.6.2 Optimal taxes

The purpose of this part is to determine the sequence of taxes $\left\{\tau_t^c\left(s^t\right), \tau_t^l\left(s^t\right), \tau_{t+1}^k\left(s^t\right)\right\}_{t=0}^{\infty}$, $\forall\ s^t \in S^t$ a benevolent central planner should choose in this economy with wage rigidities. Although it was pointed out that consumption taxes are irrelevant in this economy, we are going to abstract from the solution $\left\{\tau_t^c\left(s^t\right)\right\}_{t=0}^{\infty} = 0$, $\forall\ s^t \in S^t$, for in doing so, it will be possible to cancel out some intertemporal distortions for a more general instantaneous utility function.

In consideration of the foregoing, we follow Lucas and Stokey (1983) and define the benevolent social planner's problem, who follows a Ramsey approach, as the choice of the allocations $\left\{c_t\left(s^t\right), l_t\left(s^t\right), k_{t+1}\left(s^t\right)\right\}_{t=0}^{\infty}$, $\forall\ s^t \in S^t$ that maximizes lifetime expected utility (2) subject to the resource constraints (13), the implementability condition (14) with equality, and the wage rigidity constraints (15), with k_0, b_0, and w_{-1} given. The

Lagrangean function of this problem is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left[c_t\left(s^t\right), l_t\left(s^t\right) \right] + \varphi\left(u_{c,t}\left(s^t\right) c_t\left(s^t\right) + u_{l,t}\left(s^t\right) l_t\left(s^t\right) \right) \right] -$$

$$-E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_t\left(s^t\right) \left(c_t\left(s^t\right) + g\left(s^t\right) + k_{t+1}\left(s^t\right) - (1-\delta) k_t\left(s^{t-1}\right) - A\left(s^t\right) F\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right)\right) \right) \right] +$$

$$+E_0 \sum_{t=1}^{\infty} \beta^t \mu_t\left(s^t\right) \left(A\left(s^t\right) F_{l,t}\left(s^t\right) - \gamma A\left(s^{t-1}\right) F_{l,t-1}\left(s^{t-1}\right) \right) + \mu_0\left(A_0 F_{l,0} - \gamma w_{-1}\right) - \varphi V_0.$$

The first order condition with respect to $c_t\left(s^t\right)$ is

$$\beta^t \left[u_{c,t}\left(s^t\right) + \varphi\left(u_{cc,t}\left(s^t\right) c_t\left(s^t\right) + u_{c,t}\left(s^t\right) + u_{lc,t}\left(s^t\right) l_t\left(s^t\right) \right) - \lambda_t\left(s^t\right) \right] = 0, \quad t \geq 0, \forall\ s^t.$$

The first order condition with respect to $l_t\left(s^t\right)$ is

$$\beta^t \left[u_{l,t}\left(s^t\right) + \varphi\left(u_{cl,t}\left(s^t\right) c_t\left(s^t\right) + u_{ll,t}\left(s^t\right) l_t\left(s^t\right) + u_{l,t}\left(s^t\right) \right) + \lambda_t\left(s^t\right) A\left(s^t\right) F_{l,t}\left(s^t\right) \right] +$$
$$+\beta^t \mu_t\left(s^t\right) A\left(s^t\right) F_{ll,t}\left(s^t\right) - E_t\left[\beta^{t+1} \mu_{t+1}\left(s^{t+1}\right) \gamma A\left(s^t\right) F_{ll,t}\left(s^t\right) \right] = 0, \quad t \geq 0, \forall\ s^t.$$

The first order condition with respect to $k_{t+1}\left(s^t\right)$ is

$$-\beta^t \lambda_t + \beta^{t+1} E_t\left[\lambda_{t+1}\left(1 - \delta + A\left(s^{t+1}\right) F_{k,t+1}\right) + \mu_{t+1} A\left(s^{t+1}\right) F_{lk,t+1} \right] -$$
$$-E_{t+1}\left[\beta^{t+2} \mu_{t+2} \gamma A\left(s^{t+1}\right) F_{lk,t+1} \right] = 0, \quad t \geq 0, \forall\ s^t,$$

where the variables are a function of the state.

In states of nature where restriction (15) does not bind, it must be that $\mu_t\left(s^t\right) = 0$. On the other hand, once the restriction binds, then $\mu_t\left(s^t\right) > 0$. All in all, the solution to the central planner's problem is also characterized by the complementary slackness conditions

$$\begin{gathered} \mu_0\left(A_0 F_{l,0} - \gamma w_{-1}\right) = 0 \\ \mu_t\left(s^t\right) \left(A\left(s^t\right) F_{l,t}\left(s^t\right) - \gamma A\left(s^{t-1}\right) F_{l,t-1}\left(s^{t-1}\right) \right) = 0, \quad t \geq 1, \forall\ s^t. \end{gathered} \tag{16}$$

The conditions which, together with the resource constraints (13), the implementability constraint (14) with equality, the complementary slackness conditions (16), and initial values of capital and government bonds, k_0 and b_0, and w_{-1}, characterize the solution to the social planner's problem are

$$\frac{-u_{l,t}}{u_{c,t}}\frac{\left[1+\varphi\left(1+\sigma_t^l+\sigma_t^{lc}\right)\right]}{\left[1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)\right]}=A\left(s^t\right)F_{l,t}+ \tag{17}$$

$$+\frac{A\left(s^t\right)F_{ll,t}}{u_{c,t}\left[1+\phi\left(1-\sigma_t^c+\sigma_t^{cl}\right)\right]}\left[\mu_t-\beta\gamma E_t\mu_{t+1}\right],\quad t\geq 0,\ \forall\ s^t,$$

and

$$u_{c,t}=\beta E_t\left[u_{c,t+1}\frac{1+\varphi\left(1-\sigma_{t+1}^c+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}1+A\left(s^{t+1}\right)F_{k,t+1}-\delta\right]+$$

$$+\beta E_t\left[\frac{A\left(s^{t+1}\right)F_{lk,t+1}}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}\left[\mu_{t+1}-\beta\gamma E_{t+1}\mu_{t+2}\right]\right],\quad t\geq 0,\ \forall\ s^t, \tag{18}$$

where the variables are a function of the state, and $\sigma_t^c=\frac{u_{cc,t}}{u_{c,t}}c_t$ is the inverse of the intertemporal elasticity of substitution in consumption, $\sigma_t^l=\frac{u_{ll,t}}{u_{l,t}}l_t$ denotes the inverse of the intertemporal elasticity of substitution in labor, $\sigma_t^{cl}=\frac{u_{cl,t}}{u_{c,t}}l_t$ stands for the elasticity of the marginal utility of consumption with respect to labor, and $\sigma_t^{lc}=\frac{u_{lc,t}}{u_{l,t}}c_t$ represents the elasticity of the marginal disutility of labor with respect to consumption.

For the analysis that follows, one needs to find the appropriate competitive equilibrium conditions that are comparable with conditions (17) and (18) above. With that purpose, we put conditions (6) and (10) together in order to obtain

$$\frac{-u_{l,t}\left(s^t\right)}{u_{c,t}\left(s^t\right)}=\frac{1-\tau_t^l\left(s^t\right)}{1+\tau_t^c\left(s^t\right)}A\left(s^t\right)F_{l,t}\left(s^t\right), \tag{19}$$

where conditions (19) are written with equality instead of the mathematically implied inequality, since the planner is going to set τ_t^l so that the equality holds, as discussed throughout the proof of Lemma (1). The reason is that whenever conditions (19) are not verified with equality, the planner can increase τ_t^l, thus relaxing the budget constraint of the government (11), without introducing any additional distortion, for in such a case, labor income taxes are lump sum. Also, putting conditions (7) and (9) together yields

$$u_{c,t}\left(s^t\right)=\beta E_t\left[u_{c,t+1}\left(s^{t+1}\right)\left[1+\left(1-\tau_{t+1}^k\left(s^t\right)\right)A\left(s^{t+1}\right)F_{k,t+1}\left(s^{t+1}\right)-\delta\right]\frac{1+\tau_t^c\left(s^t\right)}{1+\tau_{t+1}^c\left(s^{t+1}\right)}\right]. \tag{20}$$

The negative externality generated by the downward real wage rigidity assumption (1) is observable once conditions (17) and (19) are confronted, as well as conditions (18) and (20). Indeed, when deciding on their own, agents do not incorporate the fact that the gross growth rate of return of wages cannot exceed γ in their decisions, i.e. the current real wage might influence the future one, even though they are aware of that fact. As a result, the economy may experience involuntary unemployment, as well as excessively high or excessively small levels of capital. It depends on the values $\mu_t\left(s^t\right)$, $\mu_{t+1}\left(s^{t+1}\right)$, and $\mu_{t+2}\left(s^{t+2}\right)$.

Although the present conditions provide all the information that is necessary to take conclusions, one can, nonetheless, work on them in order to obtain the optimal wedges, thus allowing for a more clear interpretation of the results.

Regarding the optimal state contingent labor wedge, we derive the following proposition:

Proposition 1 *In this closed economy, the optimal state contingent labor wedge is*

$$\frac{1-\tau_t^l\left(s^t\right)}{1+\tau_t^c\left(s^t\right)}=\frac{1+\varphi\left(1-\sigma_t^c\left(s^t\right)+\sigma_t^{cl}\left(s^t\right)\right)}{1+\varphi\left(1+\sigma_t^l\left(s^t\right)+\sigma_t^{lc}\left(s^t\right)\right)}+\frac{F_{ll,t}\left(s^t\right)}{F_{l,t}\left(s^t\right)}\frac{\mu_t\left(s^t\right)-\beta\gamma E_t\mu_{t+1}\left(s^{t+1}\right)}{u_{c,t}\left(s^t\right)\left[1+\varphi\left(1+\sigma_t^l\left(s^t\right)+\sigma_t^{lc}\left(s^t\right)\right)\right]}. \tag{21}$$

Proof. The proof of Proposition (1) is straightforward. Indeed, we can use conditions (17) and (19) to write the optimal labor wedge as presented in conditions (21). ■

It follows from Proposition (1) that the optimal labor income tax (a) increases with the current lower bound on real wages and (b) decreases with the expectation of a future wage constraint, for the marginal productivity of labor is assumed to be decreasing in the level of labor ($F_{ll}<0$). We refer to the optimal labor income tax for a given consumption tax, since the consumption tax is indeterminate. The intuition is presented for each case separately. For (a), notice that whenever the wage constraint is binding in a given state of the current period ($\mu_t\left(s^t\right)>0$, where the value of μ_t is higher the higher the current lower bound on the real wage, $\gamma w_{t-1}\left(s^{t-1}\right)$, is), which can, for instance, be due to a negative productivity shock, there is involuntary unemployment, as the real wage cannot fully adjust, i.e. the real wage consistent with flexible wages is below the current lower bound. Consequently, taxing labor income is equivalent to lump sum taxation, for the level of employment is completely determined by the short side of the market, i.e. labor demand. The central planner is, therefore, going to increase $\tau_t^l\left(s^t\right)$ to reduce labor supply up to the point where the labor market clears with $w_t=\gamma w_{t-1}$. As a result, the central planner is able to reduce $\tau_t^l\left(s^t\right)$ in other states of nature. Regarding hypothesis (b), the intuition is that whenever expectations in the current period regarding the period that follows are such that the wage constraint may be binding $\left(E_t\mu_{t+1}\left(s^{t+1}\right)>0\right)$, which might be associated to the expectations of a future negative productivity shock, the central planner should decrease $\tau_t^l\left(s^t\right)$ in that state, for in doing so the current real wage exclusive of taxes, i.e. the lower bound of the following period, decreases through an increase of labor supply, hence loosening future constraints. These lower revenues in this particular state of nature have to be compensated with higher revenues in other states. In short, labor income taxation policy has two distinguishable purposes. First, in (a) it assumes a reactive role, since it is just reacting to the current inefficiency in the labor market. And in (b), it assumes a precautionary role, for it is used in the current period to prevent the negative impacts of an expected future inefficiency.

If we take the second best optimal labor wedge as a reference ($\mu_t=0$, for all $t\geq 0$ and $s^t\in S^t$), we can conclude that the optimal state contingent labor income tax is higher in the economy with wage rigidities whenever the wage constraint is binding $\left(\mu_t\left(s^t\right)>0\right)$. The reason follows directly from the intuition presented above. Whenever the wage constraint is binding, the level of labor is demand determined, thus implying that $\tau_t^l\left(s^t\right)$ is a lump sum tax. Consequently, the central planner chooses to set a labor income tax above the second best level. For expectations of future wage constraints $\left(E_t\mu_{t+1}\left(s^{t+1}\right)>0\right)$, the central planner chooses levels for the labor income tax below the second best solution, as doing so reduces the current real wage, thus relaxing future constraints.

Before the considerations on the optimal capital income tax, we introduce the following assumption:

Assumption 1 *Consumption taxes are chosen in such a way that*

$$\frac{1+\tau_t^c\left(s^t\right)}{1+\tau_{t+1}^c\left(s^{t+1}\right)}=\frac{1+\varphi\left(1-\sigma_{t+1}^c\left(s^{t+1}\right)+\sigma_{t+1}^{cl}\left(s^{t+1}\right)\right)}{1+\varphi\left(1-\sigma_t^c\left(s^t\right)+\sigma_t^{cl}\left(s^t\right)\right)}.$$

Assumption (1) is justified by the fact that the optimal consumption taxes are indeterminate, for, as Lemma (1) suggests, these are irrelevant for the equilibrium allocations. Consequently, one can use consumption taxes to allow for a broader family of instantaneous utility functions that are consistent with a zero non state contingent capital income tax in a second best environment. Without consumption taxes this argument could not be made, unless the instantaneous utility function was assumed to exhibit separability in consumption and labor, as well as a constant coefficient of relative risk aversion in consumption $\left(\sigma_t^{cl}=0\text{ and }\sigma_t^c=\sigma_{t+1}^c,\text{ for all }t\geq 0\text{ and }s^t\in S^t\right)$.

Let us now move on to the optimal capital income tax, for which the following proposition arises:

Proposition 2 *In this closed economy, with Assumption (1), the optimal non state contingent capital income tax is*

$$\tau_{t+1}^{k}\left(s^{t}\right)=-\frac{E_{t}\left[\frac{A\left(s^{t+1}\right)F_{lk,t+1}\left(s^{t+1}\right)}{1+\varphi\left(1-\sigma_{t}^{c}\left(s^{t}\right)+\sigma_{t}^{cl}\left(s^{t}\right)\right)}\left[\mu_{t+1}\left(s^{t+1}\right)-\beta\gamma E_{t+1}\mu_{t+2}\left(s^{t+2}\right)\right]\right]}{E_{t}\left[u_{c,t+1}\left(s^{t+1}\right)A\left(s^{t+1}\right)F_{k,t+1}\left(s^{t+1}\right)\frac{1+\tau_{t}^{c}\left(s^{t}\right)}{1+\tau_{t+1}^{c}\left(s^{t+1}\right)}\right]}. \qquad (22)$$

Proof. From conditions (18) and (20), it is possible to write the optimal non state contingent capital income tax as

$$\tau_{t+1}^{k}\left(s^{t}\right)=\frac{E_{t}\left[\begin{array}{c} u_{c,t+1}\left(1+A\left(s^{t+1}\right)F_{k,t+1}-\delta\right)\left(\frac{1+\tau_{t}^{c}}{1+\tau_{t+1}^{c}}-\frac{1+\varphi\left(1-\sigma_{t+1}^{c}+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_{t}^{c}+\sigma_{t}^{cl}\right)}\right)- \\ -\frac{A\left(s^{t+1}\right)F_{lk,t+1}}{1+\varphi\left(1-\sigma_{t}^{c}+\sigma_{t}^{cl}\right)}\left[\mu_{t+1}-\beta\gamma E_{t+1}\mu_{t+2}\right] \end{array}\right]}{E_{t}\left[u_{c,t+1}A\left(s^{t+1}\right)F_{k,t+1}\frac{1+\tau_{t}^{c}}{1+\tau_{t+1}^{c}}\right]},$$

where the variables are a function of the state. Notice that after imposing Assumption (1) the optimal non state contingent capital income tax reduces to conditions (22). ■

Proposition (2) shows that the optimal non state contingent capital income tax (a) decreases with the expected lower bound on the real wage for the next period and (b) increases with the expected lower bound on the real wage for two periods from the current one. One can observe this result by noticing that the production function assumes both inputs to be complementaries in production ($F_{lk}=F_{kl}>0$)[3], which, in turn, implies that the first derivatives of the optimal capital income tax with respect to $\mu_{t+1}\left(s^{t+1}\right)$ and $\mu_{t+2}\left(s^{t+2}\right)$ are, respectively, negative and positive. We present the intuition for each case separately. For (a), the interpretation relates to the fact that, expecting real wages to hit the lower bound in the following period, the central planner can, by decreasing the capital income tax, i.e. give a subsidy to capital, make this asset more attractive, in expected value terms, relative to government bonds, thus incentivizing the household to accumulate more capital in the current period. As a result, labor productivity in any state of the following period will be higher, which will pressure labor demand and, consequently, the real wage, upwards, hence compensating for the expected negative shock. In what concerns (b), the idea is the opposite: if the wage constraint is expected to be binding two periods from the current one, then, by increasing the non state contingent capital income tax, the central planner is preventing capital accumulation in the current period. As a result, labor productivity in

[3]This result comes as a direct consequence of the assumed constant returns to scale tecnhology with positive and decreasing marginal productivities, as one can show using Euler's theorem.

the next period is lower in any state of nature, thus contributing towards the reduction of that period's real wage, i.e. the lower bound for two periods from the current one.

This concludes the closed economy section. In the section that follows, we open the economy by introducing a foreign sector. In that open economy, we will ask the same questions, i.e., the implications for optimal fiscal policy that stem from the downward wage rigidity assumption.

3 The stochastic representative agent small open economy with downward nominal wage rigidity and fixed exchange rates

In this section, we study a stochastic neoclassical growth model for the open version of the economy studied in section 2. In particular, the set up is exactly the same as that described in the previous section, but for a one sector small open economy, which allows us to abstract from the strategic interactions that may arise when different territories are able to implement their specific tax policies. There, the additional agent that needs to be introduced is a foreign sector, with which the domestic economy trades consumption goods and non state contingent debt denominated in foreign currency. We assume the tradeable good produced domestically is identical in everything to the good produced abroad, i.e. the representative household draws the same marginal utility out of the domestic and the foreign goods. The implication is that the representative household buys the consumption good wherever it finds it cheaper. As a result, the law of one price, which states that the a good must cost the same independently of the placed from which it is purchased, provided that cost is measured in the same currency, must hold, i.e.

$$P_t\left(s^t\right) = \varepsilon_t\left(s^t\right) P_t^*, \quad t \geq 0, \forall\ s^t. \tag{23}$$

We further assume that physical capital is produced domestically, and that labor is immobile.

The small open economy takes stochastic international nominal interest rates $i_t^*\left(s^{t-1}\right)$, the nominal exchange rates $\varepsilon_t\left(s^t\right)$, and international prices P_t^* as given.[4], where $i_t^*\left(s^{t-1}\right)$

[4]The way to think about this assumption is as follows: by pegging to a certain currency, the small open economy loses all control over the exchange rate with all the other currencies. For instance, after joining the euro, Portugal lost control over the exchange rate between the escudo and the U.S. dollar.

Additionally, the process for the international price is not that relevant, for we are considering stochastic is the international nominal interest rate set in period $t-1$ and due in period t. The economy is a nominal economy, for money is used as mean of value, though cashless, in the sense that money is not needed for transactions[5]. Consequently, there are only markets for goods, labor, capital, state contingent real government bonds, and non state contingent foreign debt.

With these assumptions, together with an additional instrument, which we loosely denominate a capital control tax, within a worldwide system of taxation, it is possible to show that optimal taxation in the small open economy is quite similar to optimal taxation in a closed economy, as demonstrated by Correia (1996).

Aside from the fact that the small open economy has no control over the nominal exchange rate, we further impose that nominal wages are downwardly rigid à la Schmitt-Grohé and Uribe (2016), i.e.

$$W_t\left(s^t\right) \geq \gamma W_{t-1}\left(s^{t-1}\right), \tag{24}$$

where $\gamma \in]0,1]$ has the same meaning as in conditions (1), though now it stands for the gross growth rate of nominal wages.

It is no longer necessary to assume downward real wage rigidity (1), for using the law of one price (23) and defining the real wage as $w_t \equiv \frac{W_t}{P_t}$, one can manipulate conditions (24) to obtain a real wage rigidity in small open economy, i.e.

$$w_t\left(s^t\right) = \gamma w_{t-1}\left(s^{t-1}\right) \frac{\varepsilon_{t-1}\left(s^{t-1}\right)}{\varepsilon_t\left(s^t\right)} \frac{P^*_{t-1}}{P^*_t},$$

where it is clear that without control over the nominal exchange rate or without flexible nominal wages, there might exist periods of involuntary unemployment. Nevertheless, if at least one of these variables is flexible, there will be no real wage rigidities, thus implying the second best solution is attainable.

nominal exchange rates. The domestic price responds in the same way to a nominal appreciation or a lower international price. All in all, what matters is that the international price measured in domestic currency is stochastic.

[5] This can be thought of as the limit case of a cash-in-advance economy, in which the transaction technology is

$$P_t c_t \leq v M_t,$$

where the velocity tends to infinite, $v \to \infty$.

3.1 The representative household

The representative household has preferences over the consumption of the composite good $c_t(s^t)$, which now includes domestic and foreign goods $\left(c_t = c_t^d + c_t^f\right)$, and labor $l_t(s^t)$ described by the expected utility function (2).

Apart from the sources of income and expenditures considered in section 2, the representative household can, in this open version of the economy, borrow, in each state s^t of period $t \geq 0$, from the Rest of the World using non state contingent one period nominal securities denominated in foreign currency $D_{t+1}(s^t)$, which yield a stochastic nominal payment $i_{t+1}^*(s^t)$, in the following period, subject to the non state contingent capital control tax $\tau_{t+1}^D(s^t)$[6]. This marginal tax rate can be thought of as a capital control tax through prices, since, by increasing the cost of international financing, it reduces the household's incentives to borrow from abroad. The assumption of external debt being denominated in foreign currency is important, as it excludes the possibility of the small open economy incurring in default on its foreign debt through a depreciation of the domestic currency, were nominal exchange rates flexible, which would decrease the value of the return of the Rest of the World. Farhi, Gopinath and Itskhoki (2012) relax this assumption and show that under certain circumstances, it is possible to replicate the effects of a nominal devaluation only by incurring in partial default on external debt, namely, when foreign debt is denominated in domestic currency. The nominal exchange rate $\varepsilon_t(s^t)$ represents the price of foreign currency in units of domestic currency.

The household's intratemporal budget constraints measured in units of money in each state can be written as

$$\left(1+\tau_t^c(s^t)\right) P_t(s^t) c_t(s^t) + P_t(s^t) k_{t+1}(s^t) + P_t(s^t) \sum_{s^{t+1}|s^t} q_t(s^{t+1}) b_{t+1}(s^{t+1}) =$$

$$= \left(1-\tau_t^l(s^t)\right) W_t(s^t) l_t(s^t) + P_t(s^t)\left[1+\left(1-\tau_t^k(s^{t-1})\right) r_t^k(s^t) - \delta\right] k_t(s^{t-1}) +$$

$$+\varepsilon_t(s^t) D_{t+1}(s^t) - \left[1+\left(1+\tau_t^D(s^{t-1})\right) i_t^*(s^{t-1})\right] \varepsilon_t(s^t) D_t(s^{t-1}) + P_t(s^t) b_t(s^t), \quad (25)$$

together with a no Ponzi games condition.

In this small open economy, the representative household's utility maximization problem consists of choosing $\left\{c_t(s^t), l_t(s^t), k_{t+1}(s^t), b_{t+1}(s^{t+1}), D_{t+1}(s^t)\right\}_{t=0}^{\infty}$ to maximize

[6] Regardless of the non state contingent nature of net foreign debt, the completeness of the assets market is still assured, for we have state contingent real government bonds.

lifetime expected utility (2) subject to the budget constraints (25) and the no Ponzi games condition, with prices, taxes, international nominal interest rates, nominal exchange rates, the initial stock of capital k_0, the initial stock of government bonds b_0, and the initial stock of net foreign debt D_0 taken as given.

The Lagrangean function is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left[c_t\left(s^t\right), l_t\left(s^t\right)\right] - \lambda_t\left(s^t\right) \left(\begin{array}{c} \left(1+\tau_t^c\left(s^t\right)\right) P_t\left(s^t\right) c_t\left(s^t\right) + P_t\left(s^t\right) k_{t+1}\left(s^t\right) + \\ +P_t\left(s^t\right) \sum_{s^{t+1}|s^t} q_t\left(s^{t+1}\right) b_{t+1}\left(s^{t+1}\right) \end{array} \right) \right] +$$

$$+E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t\left(s^t\right) \left[\left(1-\tau_t^l\left(s^t\right)\right) W_t\left(s^t\right) l_t\left(s^t\right) + P_t\left(s^t\right) \left[1+\left(1-\tau_t^k\left(s^{t-1}\right)\right) r_t^k\left(s^t\right) - \delta\right] k_t\left(s^{t-1}\right)\right] +$$

$$+E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t\left(s^t\right) \left[\varepsilon_t\left(s^t\right) D_{t+1}\left(s^t\right) - \left[1+\left(1+\tau_t^D\left(s^{t-1}\right)\right) i_t^*\left(s^{t-1}\right)\right] \varepsilon_t\left(s^t\right) D_t\left(s^{t-1}\right) + P_t\left(s^t\right) b_t\left(s^t\right)\right].$$

Defining the real wage as $w_t \equiv \frac{W_t}{P_t}$, one can conclude that the conditions which, together with the budget constraints (25), k_0, b_0, D_0, and the imposed transversality conditions (4), (5), and

$$\lim_{T\to\infty} Q_T\left(s^T\right) \frac{\varepsilon_T}{P_T} D_{T+1} = 0 \tag{26}$$

characterize the solution to the representative household's problem are still conditions (6)-(8). However, the introduction of net foreign debt as an alternative way of allocating consumption over time implies the existence of an additional marginal condition, which is the Euler equation that follows:

$$u_{c,t} = \beta E_t \left[u_{c,t+1} \left[1+\left(1+\tau_{t+1}^D\right) i_{t+1}^*\right] \frac{\varepsilon_{t+1}\left(s^{t+1}\right)}{\varepsilon_t\left(s^t\right)} \frac{P_t}{P_{t+1}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right], \quad t \geq 0, \forall\ s^t, \tag{27}$$

where the variables are a function of the state.

3.2 The representative firm

The price taking representative firm in the small open economy has a problem identical to that of the representative firm in the closed economy, though written in nominal terms. In every state of every period, it has to choose k_t and l_t in order to maximize profits, which are now written as

$$\pi_t\left(s^t\right) = P_t A\left(s^t\right) F\left(k_t\left(s^t\right), l_t\left(s^t\right)\right) - W_t\left(s^t\right) l_t\left(s^t\right) - P_t\left(s^t\right) r_t^k\left(s^t\right) k_t\left(s^t\right).$$

The conditions that characterize the solution to this problem are still (9) and (10), where $w_t \equiv \frac{W_t}{P_t}$.

3.3 Government

The government in the small open economy needs to finance exogenous stochastic public consumption $g\left(s^t\right)$. It can resort to all of the taxes considered in the closed version of the economy (state contingent labor income taxes, state contingent consumption taxes, and non state contingent capital income taxes), though now it can also raise revenues or incur in higher expenditures through, respectively, the tax on the return of international inflows $\tau_t^D > 0$, or the subsidy to international capital outflows $\tau_t^D < 0$, assuming $D_{t+1} > 0$[7]. Consequently, the state by state intratemporal budget constraints in nominal terms of the government are

$$P_t\left(s^t\right) g\left(s^t\right) + P_t\left(s^t\right) b_t\left(s^t\right) = \tau_t^c\left(s^t\right) P_t\left(s^t\right) c_t\left(s^t\right) + \tag{28}$$

$$+\tau_t^l\left(s^t\right) W_t\left(s^t\right) l_t\left(s^t\right) + P_t\left(s^t\right) \tau_t^k\left(s^{t-1}\right) r_t^k\left(s^t\right) k_t\left(s^{t-1}\right) +$$

$$+\tau_t^D\left(s^{t-1}\right) i_t^*\left(s^{t-1}\right) \varepsilon_t\left(s^t\right) D_t\left(s^{t-1}\right) + P_t\left(s^t\right) \sum_{s^{t+1}|s^t} q_t\left(s^{t+1}\right) b_{t+1}\left(s^{t+1}\right),$$

together with a no Ponzi games condition.

3.4 Foreign sector

The foreign sector is the agent with which the small open economy makes transactions of goods and net foreign debt. For this reason, the external accounts must be balanced, i.e. the budget constraints with the foreign sector, which are simply a representation of the balance of payments, must be verified. These state by state intratemporal budget constraints with the foreign sector are represented by the following condition that can be interpreted as the law of motion of net foreign debt:

$$\varepsilon_t\left(s^t\right) D_{t+1}\left(s^t\right) = -P_t\left(s^t\right) TB_t\left(s^t\right) + \left(1 + i_t^*\left(s^{t-1}\right)\right) \varepsilon_t\left(s^t\right) D_t\left(s^{t-1}\right), \tag{29}$$

[7] If the representative household chooses to get a positive net asset position towards the foreign sector ($D_{t+1} < 0$), the reasoning would be the opposite, i.e. $\tau_t^D > 0$ would represent an expenditure for the government, whereas $\tau_t^D < 0$ would consist of a revenue.

together with a no Ponzi games condition, where TB represents the trade balance of the domestic small open economy. We cannot individually distinguish exports from imports, for there is only one single good in the model.

3.5 Market clearing conditions

The presence of downward nominal wage rigidity combined with the absence of control over the nominal exchange rate determines the possible existence of states of nature where the market clearing real wage is below its implied lower bound, i.e. there may exist involuntary unemployment. Such a situation is to arise whenever condition (24) for a certain state is binding, thus implying the level of employment is demand determined. As a result, condition (6) is verified with inequality in that state. The opposite could also be said. If the market clearing real wage is consistent flexible wages (condition (6) holds with equality), that is a consequence of having the nominal wage fully adjusting to the current circumstances (condition (24) does not bind). All in all, the complementary slackness condition (12) needs to be updated to the open economy version. Specifically, it needs to be consistent with the assumption of downwardly rigid nominal wages, which implies

$$\left(\frac{-u_{l,t}\left(s^{t}\right)}{u_{c,t}\left(s^{t}\right)}-\frac{\left(1-\tau_{t}^{l}\left(s^{t}\right)\right)w_{t}\left(s^{t}\right)}{\left(1+\tau_{t}^{c}\left(s^{t}\right)\right)}\right)\left(W_{t}\left(s^{t}\right)-\gamma W_{t-1}\left(s^{t-1}\right)\right)=0,\quad t\geq 0,\forall\ s^{t}. \tag{30}$$

In the closed economy of section 2, if wage rigidity was nominal, as in conditions (24), conditions (30) would not exist, for it would be possible to use the price level to overcome the nominal wage rigidity. To illustrate this point, recall that $w_t \equiv \frac{W_t}{P_t}$ and rewrite the firm's optimal marginal conditions for labor (10) as

$$P_{t}\left(s^{t}\right)=\frac{W_{t}\left(s^{t}\right)}{A_{t}\left(s^{t}\right)F_{l,t}\left(s^{t}\right)}.$$

It becomes clear that if prices are flexible, they can be used to overcome the nominal wage rigidity. The problem, however, is that in this small open economy environment, the domestic price level is, in equilibrium, determined outside the economy according to the law of one price (23), thus invalidating the use of this variable to overcome the wage rigidity when nominal exchange rates are taken as given.

Since the variable representing net foreign debt stands for both its demand and supply, the other necessary market clearing conditions are those concerning the goods market. This condition needs to incorporate the fact that, now, the domestic economy can trade goods with the foreign sector. That being said, the goods market clearing conditions for each state of nature s^t in each period $t \geq 0$ are

$$c_t\left(s^t\right)+g\left(s^t\right)+k_{t+1}\left(s^t\right)-(1-\delta)\,k_t\left(s^{t-1}\right)+TB_t\left(s^t\right)=A\left(s^t\right)F\left(k_t\left(s^{t-1}\right),l_t\left(s^t\right)\right). \quad (31)$$

3.6 Competitive equilibrium

Definition 2 *In this stochastic small open economy, a competitive equilibrium is a set of allocations* $\left\{c_t\left(s^t\right),l_t\left(s^t\right),k_{t+1}\left(s^t\right)\right\}_{t=0}^{\infty}$*, net foreign debt* $\left\{D_{t+1}\left(s^t\right)\right\}_{t=0}^{\infty}$*, trade balances* $\left\{TB_t\left(s^t\right)\right\}_{t=0}^{\infty}$*, prices* $\left\{P_t\left(s^t\right),W_t\left(s^t\right),r_t^k\left(s^t\right),q_t\left(s^{t+1}\right),Q_t\left(s^t\right)\right\}_{t=0}^{\infty}$*, and policies* $\left\{\tau_t^c\left(s^t\right),\tau_t^l\left(s^t\right),\tau_{t+1}^k\left(s^t\right),\tau_{t+1}^D\left(s^t\right),b_{t+1}\left(s^{t+1}\right)\right\}_{t=0}^{\infty}$ *for all* $s^t \in S^t$*, such that given the exogenous stochastic government expenditures* $g\left(s^t\right)$*, productivity* $A\left(s^t\right)$*, nominal exchange rates* $\varepsilon_t\left(s^t\right)$*, international nominal interest rates* $i_t^*\left(s^{t-1}\right)$*, the international price* P_t^**, the initial stock of capital* k_0*, government bonds* b_0*, external debt* D_0*, and the historical nominal wage* W_{-1}*, the following conditions are met: (i) the representative household maximizes lifetime expected utility subject to the budget constraints and taking prices, taxes, international nominal interest rates, and nominal exchange rates as given; (ii) firms maximize state by state profits subject to their technological constraints and taking prices as given; (iii) the government satisfies its budget constraints; (iv) the budget constraints with the foreign sector are satisfied; and (v) markets clear.*

Based on Definition (2), the competitive equilibrium in this economy is characterized by conditions (25), (6)-(8), (27), (9), (10), (29), (30), (31), and (23), together with the terminal conditions (4), (5), and (26), with k_0, b_0, D_0, and W_{-1} given. Once again, the budget constraints of the government (28) are a linear combination of the other equilibrium conditions, i.e. they are met by Walras' law.

3.7 Ramsey optimal taxation

3.7.1 The set of attainable allocations

Following Lucas and Stokey (1983), the implementability condition for this stochastic small open economy is[8]

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{c,t}\left(s^t\right) c_t\left(s^t\right) + u_{l,t}\left(s^t\right) l_t\left(s^t\right) \right] \geq V_0, \tag{32}$$

where $V_0 \equiv \frac{u_{c,0}}{1+\tau_0^c} \left[\left(1 + \left(1 - \tau_0^k\right) A_0 F_{k,0} - \delta\right) k_0 + b_0 - \left(1 + \left(1 + \tau_0^D\right) i_0^*\right) \frac{\varepsilon_0}{P_0} D_0 \right]$ is the exogenous level of initial wealth in units of utility as in Chari, Nicolini, and Teles (2016) Once again, this assumption allows us to abstract from initial taxation considerations.

Moreover, the resource constraints can be written as depending on the level of net foreign debt in real units. One needs only to replace conditions (23) and (29) in (31) to obtain

$$\begin{gathered} c_t\left(s^t\right) + g\left(s^t\right) + k_{t+1}\left(s^t\right) - (1-\delta) k_t\left(s^{t-1}\right) - \frac{D_{t+1}\left(s^t\right)}{P_t^*} + \\ + \left(1 + i_t^*\left(s^{t-1}\right)\right) \frac{D_t\left(s^{t-1}\right)}{P_t^*} = A\left(s^t\right) F\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right)\right), \quad t \geq 0, \forall\ s^t. \end{gathered} \tag{33}$$

Finally, using the representative firm's optimal conditions (10), the law of one price (23), and the nominal wage rigidity assumption (24), we get the nominal wage constraint as a function of the allocations:

$$\varepsilon_0 P_0^* A_0 F_{l,0} \geq \gamma W_{-1} \tag{34}$$

$$\varepsilon_t\left(s^t\right), P_t^* A\left(s^t\right) F_{l,t}\left(s^t\right) \geq \gamma \varepsilon_{t-1}\left(s^{t-1}\right) P_{t-1}^* A\left(s^{t-1}\right) F_{l,t-1}\left(s^{t-1}\right), \quad t \geq 1, \forall\ s^t.$$

Lemma 2 *A sequence of allocations* $\left\{c_t\left(s^t\right), l_t\left(s^t\right), k_{t+1}\left(s^t\right)\right\}_{t=0}^{\infty}$ *and net foreign debt* $\left\{D_{t+1}\left(s^t\right)\right\}_{t=0}^{\infty}$*, for all* $s^t \in S^t$*, that meets conditions (32) with equality, (33), and (34) can be implemented as a competitive equilibrium.*

Proof. The proof of Lemma (2) is similar to that of Lemma (1). We have to show that conditions (32) with equality, (33), and (34) are necessary and sufficient for the competitive

[8] The proof is similar to that found in Appendix 6.A, which concerns the model for the closed economy, though with an additional no arbitrage condition between state contingent government bonds and non state contingent international bonds.

equilibrium allocations.

In other words, we have to show that a given arbitrary set of allocations $\{c_t(s^t), l_t(s^t), k_{t+1}(s^t)\}_{t=0}^{\infty}$ and net foreign debt $\{D_{t+1}(s^t)\}_{t=0}^{\infty}$ verifying (32) with equality, (33), and (34) meets the competitive equilibrium conditions (25), (6)-(8), (27), (9), (10), (29), (30), (31), and (23).

The household's budget constraints (25) are met, for the implementability condition is an alternative representation of those conditions. Furthermore, the completeness of the assets market guarantees that the implementability condition implies that the state by state budget constraints are met, for there are as many instruments as there are states of nature. Hence, we need only to choose $b_{t+1}(s^{t+1})$ in every state. The intratemporal marginal conditions (6) determine the state contingent labor income tax $\tau_t^l(s^t)$ for all the states, while the intertemporal marginal conditions (7), (8), and (27) are satisfied with the appropriate choice of the non state contingent capital income tax $\tau_{t+1}^k(s^t)$, the price of state contingent real government bonds $q_t(s^{t+1})$, and the non state contingent capital control tax $\tau_{t+1}^D(s^t)$, respectively. The firm's conditions (9) and (10) are recovered choosing the return on capital $r_t^k(s^t)$ and the nominal wage $W_t(s^t)$ in every state, and the law of one price (23) pins down $P_t(s^t)$. Finally, the law of motion of net foreign debt (29) determines the trade balance $TB_t(s^t)$. The complementary slackness conditions (30) are satisfied, for the central planner is always going to conduct policy in such a way that condition (6) is verified with equality, which, in turn, implies that the implementability condition (14) holds with equality. Finally, the resource constraints (31) are satisfied by definition of the given allocation. ■

Once again, it can be concluded that the state contingent consumption tax is an irrelevant instrument in this economy, for it is possible to attain any allocation with any path for $\tau_t^c(s^t)$.

3.7.2 Optimal taxes

As in section 2, we follow a Ramsey approach in order to determine the optimal tax policy in this stochastic small open economy. There, we have imposed Assumption (1). This feature also applies for the present section.

In consideration of the foregoing, we follow Lucas and Stokey (1983) and define the social planner's problem as the choice of $\{c_t(s^t), l_t(s^t), k_{t+1}(s^t), D_{t+1}(s^t)\}_{t=0}^{\infty}\ \forall\ s^t \in S^t$ that maximizes lifetime expected utility (2) subject to the implementability condition (32) with equality, the resource constraints (33), and the nominal wage constraints (34), with

k_0, b_0, D_0, and W_{-1} given. The Lagrangean function is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left[c_t\left(s^t\right), l_t\left(s^t\right)\right] + \varphi\left(u_{c,t}\left(s^t\right) c_t\left(s^t\right) + u_{l,t}\left(s^t\right) l_t\left(s^t\right)\right)\right] -$$

$$-E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t\left(s^t\right) \left[\begin{array}{c} c_t\left(s^t\right) + g\left(s^t\right) + k_{t+1}\left(s^t\right) - (1-\delta) k_t\left(s^{t-1}\right) - \frac{D_{t+1}\left(s^t\right)}{P_t^*} + \\ + \left(1 + i_t^*\left(s^{t-1}\right)\right) \frac{D_t\left(s^{t-1}\right)}{P_t^*} - A\left(s^t\right) F\left(k_t\left(s^{t-1}\right), l_t\left(s^t\right)\right) \end{array} \right] +$$

$$+E_0 \sum_{t=1}^{\infty} \beta^t \mu_t\left(s^t\right) \left(\varepsilon_t\left(s^t\right) P_t^* A\left(s^t\right) F_{l,t}\left(s^t\right) - \gamma \varepsilon_{t-1}\left(s^{t-1}\right) P_{t-1}^* A\left(s^{t-1}\right) F_{l,t-1}\left(s^{t-1}\right)\right) +$$

$$+\mu_0\left(\varepsilon_0 P_0^* A_0 F_{l,0} - \gamma W_{-1}\right) - \varphi \omega_0.$$

The first order condition with respect to $c_t\left(s^t\right)$ is

$$\beta^t \left[u_{c,t}\left(s^t\right) + \varphi\left(u_{cc,t}\left(s^t\right) c_t\left(s^t\right) + u_{c,t}\left(s^t\right) + u_{lc,t}\left(s^t\right) l_t\left(s^t\right)\right) - \lambda_t\left(s^t\right)\right] = 0, \quad t \geq 0, \forall\ s^t.$$

The first order condition with respect to $l_t\left(s^t\right)$ is

$$\beta^t \left[u_{l,t}\left(s^t\right) + \varphi\left(u_{cl,t}\left(s^t\right) c_t\left(s^t\right) + u_{ll,t}\left(s^t\right) l_t\left(s^t\right) + u_{l,t}\left(s^t\right)\right) + \lambda_t\left(s^t\right) A\left(s^t\right) F_{l,t}\left(s^t\right)\right] +$$

$$+\beta^t \mu_t\left(s^t\right) \varepsilon_t\left(s^t\right) P_t^* F_{ll,t}\left(s^t\right) - E_t\left[\beta^{t+1} \mu_{t+1}\left(s^{t+1}\right) \gamma \varepsilon_t\left(s^t\right) P_t^* A\left(s^t\right) F_{ll,t}\left(s^t\right)\right] = 0, \quad t \geq 0, \forall\ s^t.$$

The first order condition with respect to $k_{t+1}\left(s^t\right)$ is

$$-\beta^t \lambda_t\left(s^t\right) + \beta^{t+1} E_t \left[\begin{array}{c} \lambda_{t+1}\left(s^{t+1}\right)\left[1 + A\left(s^{t+1}\right) F_{k,t+1}\left(s^{t+1}\right) - \delta\right] + \\ +\mu_{t+1}\left(s^{t+1}\right) \varepsilon_{t+1}\left(s^{t+1}\right) P_{t+1}^* A\left(s^{t+1}\right) F_{lk,t+1}\left(s^{t+1}\right) \end{array} \right] -$$

$$-\beta^{t+2} E_{t+1}\left[\beta^{t+2} \mu_{t+2}\left(s^{t+2}\right) \gamma \varepsilon_{t+1}\left(s^{t+1}\right) P_{t+1}^* A\left(s^{t+1}\right) F_{lk,t+1}\left(s^{t+1}\right)\right] = 0, \quad t \geq 0, \forall\ s^t.$$

The first order condition with respect to $D_{t+1}\left(s^t\right)$ is

$$\beta^t \lambda_t\left(s^t\right) \frac{1}{P_t^*} - E_t\left[\beta^{t+1} \lambda_{t+1}\left(s^{t+1}\right)\left(1 + i_{t+1}^*\left(s^t\right)\right) \frac{1}{P_{t+1}^*}\right] = 0, \quad t \geq 0, \forall\ s^t.$$

The solution to the central planner's problem also requires a complementary slackness condition, for whenever $\mu_t\left(s^t\right) = 0$ condition (34) is verified with strict inequality, whereas

$\mu_t\left(s^t\right) > 0$ implies that condition (34) binds. Hence, we need the following condition:

$$\mu_0\left(\varepsilon_0 P_0^* A_0 F_{l,0} - \gamma W_{-1}\right) = 0 \tag{35}$$

$$\mu_t\left(s^t\right)\left(\varepsilon_t\left(s^t\right) P_t^* A\left(s^t\right) F_{l,t}\left(s^t\right) - \gamma\varepsilon_{t-1}\left(s^{t-1}\right) P_{t-1}^* A\left(s^{t-1}\right) F_{l,t-1}\left(s^{t-1}\right)\right) = 0, \quad t \geq 1, \forall\ s^t.$$

In conclusion, the conditions which, together with the implementability condition (32) with equality, the resource constraints (33), the complementary slackness conditions (35), and with k_0, b_0, D_0, and W_{-1} given, characterize the solution to the social planner's problem are

$$\begin{aligned} \frac{-u_{l,t}}{u_{c,t}}\frac{1+\varphi\left(1+\sigma_t^l+\sigma_t^{lc}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)} &= A\left(s^t\right) F_{l,t}+ \\ +\frac{\varepsilon_t\left(s^t\right) P_t^* A\left(s^t\right) F_{ll,t}}{u_{c,t}\left[1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)\right]}&\left[\mu_t-\beta\gamma E_t\mu_{t+1}\right], \quad t \geq 0, \forall\ s^t, \end{aligned} \tag{36}$$

$$\begin{aligned} u_{c,t} = \beta E_t&\left[u_{c,t+1}\frac{1+\varphi\left(1-\sigma_{t+1}^c+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}\left(1+A\left(s^{t+1}\right) F_{k,t+1}-\delta\right)\right]+ \\ +\beta E_t&\left[\frac{\varepsilon_{t+1}\left(s^{t+1}\right) P_{t+1}^* A\left(s^{t+1}\right) F_{lk,t+1}}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}\left[\mu_{t+1}-\beta\gamma E_{t+1}\mu_{t+2}\right]\right], \quad t \geq 0, \forall\ s^t, \end{aligned} \tag{37}$$

and

$$u_{c,t} = \beta E_t\left[u_{c,t+1}\frac{1+\varphi\left(1-\sigma_{t+1}^c+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}\left(1+i_{t+1}^*\right)\frac{P_t^*}{P_{t+1}^*}\right], \quad t \geq 0, \forall\ s^t, \tag{38}$$

where the variables are a function of the state and σ_t^c, σ_t^l, σ_t^{cl}, and σ_t^{lc} are defined as before.

The competitive equilibrium conditions (19) and (20) still apply in this small open economy and from them, respectively, it is possible to determine the optimal state contingent labor tax and the non state contingent capital income tax. Nonetheless, in order to determine the optimal non state contingent capital control tax, we need first to use conditions (27), which together with the law of one price (23) become

$$u_{c,t}\left(s^t\right) = \beta E_t\left[u_{c,t+1}\left(s^{t+1}\right)\left[1+\left(1+\tau_{t+1}^D\left(s^t\right)\right) i_{t+1}^*\left(s^t\right)\right]\frac{P_t^*}{P_{t+1}^*}\frac{1+\tau_t^c\left(s^t\right)}{1+\tau_{t+1}^c\left(s^{t+1}\right)}\right]. \tag{39}$$

Regarding the optimal state contingent labor wedge and the optimal non state contingent capital income tax, we derive the following proposition:

Proposition 3 *In this small open economy, the optimal state contingent labor wedge is*

$$\frac{1-\tau_t^l\left(s^t\right)}{1+\tau_t^c\left(s^t\right)}=\frac{1+\varphi\left(1-\sigma_t^c\left(s^t\right)+\sigma_t^{cl}\left(s^t\right)\right)}{1+\varphi\left(1+\sigma_t^l\left(s^t\right)+\sigma_t^{lc}\left(s^t\right)\right)}+ \\ +\frac{\varepsilon_t\left(s^t\right)P_t^* F_{ll,t}\left(s^t\right)}{F_{l,t}\left(s^t\right)}\frac{\mu_t\left(s^t\right)-\beta\gamma E_t\mu_{t+1}\left(s^{t+1}\right)}{u_{c,t}\left(s^t\right)\left[1+\varphi\left(1-\sigma_t^c\left(s^t\right)+\sigma_t^{cl}\left(s^t\right)\right)\right]}, \tag{40}$$

and the optimal non state contingent capital income tax is

$$\tau_{t+1}^k\left(s^t\right)=-\frac{E_t\left[\frac{\varepsilon_{t+1}\left(s^{t+1}\right)P_{t+1}^* A\left(s^{t+1}\right)F_{lk,t+1}\left(s^{t+1}\right)}{1+\varphi\left(1-\sigma_t^c\left(s^t\right)+\sigma_t^{cl}\left(s^t\right)\right)}\left[\mu_{t+1}\left(s^{t+1}\right)-\beta\gamma E_{t+1}\mu_{t+2}\left(s^{t+2}\right)\right]\right]}{E_t\left[u_{c,t+1}\left(s^{t+1}\right)A\left(s^{t+1}\right)F_{k,t+1}\left(s^{t+1}\right)\frac{1+\tau_t^c\left(s^t\right)}{1+\tau_{t+1}^c\left(s^{t+1}\right)}\right]}. \tag{41}$$

The proof of Proposition (3) can be found in Appendix 6.B.

Conditions (40) and (41) only differ from (21) and (22), respectively, in that in the closed economy the source of inefficiency was a downward real wage rigidity, while in the small open economy it is a combination of downward nominal wage rigidity and exogenous exchange rates. Hence the nominal terms $\varepsilon_t\left(s^t\right)P_t^*$ and $\varepsilon_{t+1}\left(s^{t+1}\right)P_{t+1}^*$ in conditions (40) and (41).

Consequently, it should be noticed that both the optimal state contingent labor income tax and the optimal non state contingent capital income tax exhibit, in this small open economy, the same behavior as in the closed economy. This conclusion follows from the fact that the marginal productivity of labor is decreasing in the level of labor ($F_{ll}<0$) and that labor and capital are complementaries in production ($F_{lk}=F_{kl}>0$), thus determining the aforementioned behavior for these variables. As a result, the intuitions provided in section 2 still apply in this small open economy. In particular, we highlight the dual role of the optimal state contingent labor income tax: if the wage constraint is binding in the current period, then the optimal labor income tax exhibits a reactive behavior, for it increases to raise revenue without introducing any additional distortion; and if there is the expectation that the wage constraint is going to be binding in the next period, then the optimal labor income tax exhibits a precautionary behavior, since it decreases in order to relax future constraints.

Interestingly enough, our model presents a result that is different from that of Schmitt-Grohé and Uribe's (2016), which we summarize as

Proposition 4 *In this small open economy, the optimal non state contingent capital control tax is zero, i.e.*

$$\tau_{t+1}^{D}\left(s^{t}\right)=0.$$

Proof. The proof of Proposition (4) follows straightforwardly from conditions (38) and (39). Indeed, these conditions can be manipulated in order to obtain

$$\tau_{t+1}^{D}\left(s^{t}\right)=\frac{E_{t}\left[u_{c,t+1}\left(s^{t+1}\right)\left(1+i_{t+1}^{*}\left(s^{t}\right)\right)\frac{P_{t}^{*}}{P_{t+1}^{*}}\left(\frac{1+\varphi\left(1-\sigma_{t+1}^{c}\left(s^{t+1}\right)+\sigma_{t+1}^{cl}\left(s^{t+1}\right)\right)}{1+\varphi\left(1-\sigma_{t}^{c}(s^{t})+\sigma_{t}^{cl}(s^{t})\right)}-\frac{1+\tau_{t}^{c}\left(s^{t}\right)}{1+\tau_{t+1}^{c}(s^{t+1})}\right)\right]}{E_{t}\left[u_{c,t+1}\left(s^{t+1}\right)i_{t+1}^{*}\left(s^{t}\right)\frac{P_{t}^{*}}{P_{t+1}^{*}}\frac{1+\tau_{t}^{c}(s^{t})}{1+\tau_{t+1}^{c}(s^{t+1})}\right]}. \tag{42}$$

The optimality of zero capital controls then follows from the imposition of Assumption (1) on conditions (42). ■

In Schmitt-Grohé and Uribe (2016), the optimal capital control tax, prudential in nature, is used to mitigate to some extent the negative externality arising from downward nominal wage rigidities in an economy with a currency peg and free capital mobility, for it should be high during booms, thus preventing overborrowing and an excessive increase in the consumption of the nontradeable good, which in turn inflates nominal wages, and low during recessions. In our model, without any harm for the allocations, since, according to Lemma (2), consumption taxes are irrelevant for the competitive equilibrium allocations, the optimal capital control tax can be set to zero for a large family of instantaneous utility functions. Ergo, all we need is a state contingent labor income tax with both a reactive and a precautionary role, as well as a capital income tax. In other words, the role played by their prudential capital control tax is replaced, in our model, by a labor income tax, which is both reactive and precautionary.

The results obtained thus far are in line with Correia's (1996) finding on capital income taxation in a deterministic small open economy that cannot resort to lump sum taxes to finance the exogenous sequence of government consumption and taxes capital income earned abroad according to the worldwide system. Indeed, with Assumption (1), the planner chooses not to tax capital income earned abroad, while domestic capital income is taxed, but only due to the existence of a restriction that drives the economy away from the second best allocation. In a second best environment $\left(\mu_{t}\left(s^{t}\right)=0, t\geq 0\ \forall\ s^{t}\in S^{t}\right)$, Assumption (1) guarantees the optimality of zero capital income taxation $\left(\tau_{t+1}^{k}=0\right)$. What is, however, interesting is that we were able to replicate this result, which is a generalization of Judd's (1985) and Chamley's (1986) result of zero capital income taxation in the steady state of a deterministic closed economy, in a stochastic small open economy.

4 Extension : a special consumption tax

In section 3 we have argued that the real wage rigidity arises due to the combination of downwardly rigid nominal wages and exogenous nominal exchange rates. Consequently, if nominal wages were fully flexible ($\gamma = 0$ in conditions (24)) or if nominal exchange rates were flexible, the real wage rigidity would not exist, thus implying that the second best solution would be attainable.

In this section, in contrast with Adão, Correia, and Teles' (2010) payroll taxes that implement the competitive equilibrium allocations with stable nominal wages and a constant nominal exchange rate, hence eliminating the costs of fixed exchange rate regimes[9], we use consumption taxes to eliminate these costs. With this instrument, we can discriminate between the good produced domestically and the good produced abroad in such a way that the costs of a combination of downward nominal wage rigidity and exogenous nominal exchange rates are eliminated, i.e. the second best solution can be implemented. The assumption follows:.

Assumption 2 *The incidence of the consumption tax falls only on the final good produced domestically.*

The motivation for this fiscal regime is associated with the irrelevance of consumption taxes for the allocations, i.e. the fact that consumption taxes are a free variable in the small open economy of section 3, as suggested by Lemma (2).

It follows from Assumption (2) that the relevant price for the domestic good is the price charged by domestic firms gross of consumption taxes, whereas the relevant price for the international good is the international price. As a result, the law of one price for this economy must change accordingly. It becomes

$$\left(1 + \tau_t^c\left(s^t\right)\right) P_t\left(s^t\right) = \varepsilon_t\left(s^t\right) P_t^*, \quad t \geq 0, \forall\ s^t. \tag{43}$$

Given that P_t^* is exogenous to the small open economy, it can be thought of as including

[9] However, the necessary decrease in payroll taxes should be accompanied by higher labor income taxes, which might be hard to implement since it represents a transfer of wealth from households to capitalists.

both the case in which the domestic household has to pay consumption taxes abroad for the imported good and the case in which it does not.

The suitability of this assumption relates to the idea that consumption taxes can be used to replicate a nominal devaluation, as the real wage in the small open economy can now be written as

$$w_t\left(s^t\right) \equiv \frac{W_t\left(s^t\right)}{P_t\left(s^t\right)} = \frac{\left(1+\tau_t^c\left(s^t\right)\right) W_t\left(s^t\right)}{\varepsilon_t\left(s^t\right) P_t^*}. \tag{44}$$

From conditions (44), it is possible to conclude that subsidizing exports, or, alternatively, taxing imports, by means of lowering the consumption tax rate works in the same way as a nominal devaluation. The basic intuition is that both these policies contribute to an increase in the relative price of imported goods, exclusive of consumption taxes. We have, thus, gained the freedom to control real wages in the domestic economy in an environment of exogenous exchange rates and downward nominal wage rigidities, for the exogenous nominal exchange rate and international price pin down the domestic price gross of consumption taxes, but not each individual component (τ_t^c and P_t).[10]

4.1 Implications for the allocations

Since consumption taxes can be used to replace the role of flexible nominal exchange rates, then, whenever the wage constraint is binding, decreasing the consumption tax rate, i.e. subsidizing exports or taxing imports, allows the real wage to adjust to its level consistent with flexible wages, for the price of the domestic good exclusive of taxes has to increase to meet the law of one price (43). This, in turn, means that the second best solution can be implemented, i.e. the set of attainable allocations is fully characterized by the implementability condition (32) with equality and the resource constraints (33), even though nominal wages are downwardly rigid and nominal exchange rates taken as given.

The proposition follows:

Proposition 5 *Let the nominal wage be downwardly rigid, as in conditions (24), and the nominal exchange rates be exogenous. Then, the second best solution can still be implemented in this small open economy provided Assumption (2) is met.*

Proof. To prove Proposition (5), one has to show that the set of attainable allocations is fully characterized by the implementability condition (32) with equality and the re-

[10] A limitation that could be presented to this solution is the possible necessity of setting $\tau_t^c < -1$, as that would imply that the consumer was being paid to consume. However, from the nonnegativity of the real wage, $\tau_t^c \geq -1$ is guaranteed.

source constraints (33), even with a combination of downward nominal wage rigidity and exogenous nominal exchange rates. Hence, consider a given arbitrary set of allocations $\{c_t(s^t), l_t(s^t), k_{t+1}(s^t)\}_{t=0}^{\infty}$ and net foreign debt $\{D_{t+1}(s^t)\}_{t=0}^{\infty}$ for all $s^t \in S^t$ verifying (32) with equality and (33).

The household's budget constraints (25) are met, for the implementability condition is an alternative representation of those conditions. Furthermore, the completeness of the assets market guarantees that the implementability condition implies that the state by state budget constraints are met, for there are as many instruments as there are states of nature. Hence, we need only to choose $b_{t+1}(s^{t+1})$ in every state. The intratemporal marginal conditions (6) determine the state contingent labor income tax $\tau_t^l(s^t)$, while the intertemporal marginal conditions (7), (8), and (27) are satisfied choosing, respectively, the non state contingent capital income tax $\tau_{t+1}^k(s^t)$, the price of state contingent real government bonds $q_t(s^{t+1})$, and the non state contingent capital control tax $\tau_{t+1}^D(s^t)$. The firm's optimal conditions (9) and (10) can be recovered with the appropriate choice of the capital rent $r_t^k(s^t)$ and the price of the domestic good $P_t(s^t)$. The budget constraints with the foreign sector (29) determine the trade balance $TB_t(s^t)$. And the law of one price (43) determines the consumption tax $\tau_t^c(s^t)$. The resource constraints (31) are met by definition of the sequence of allocations. ■

It is indeed the case that the second best solution is achievable, as consumption taxes, when used in the right way, replace a nominal devaluation of the domestic currency, thus allowing the economy to have no involuntary unemployment in every state of nature. Consequently, the downward nominal wage rigidity assumption (24) is irrelevant, i.e. $\mu_t(s^t) = 0$, $t \geq 0 \; \forall \; s^t$, even in the presence of exogenous nominal exchange rates[11].

Moreover, Proposition (5) can be extended to other constraints on nominal wages, as argued in Adão, Correia, and Teles (2009) for the case of price rigidities and Adão, Correia, and Teles (2010) for the case of wage rigidities. The proof of Proposition (5) justifies this extrapolation. Notice that the implementation of the second best allocations did not require the use of the nominal wage to meet any competitive equilibrium condition, which means that $W_t(s^t)$ is a free variable. Ergo, whatever nominal wage rigidity exists, it will not be relevant.

[11] Albeit interesting from a theoretical point of view, this result is also hard to implement, for it stems from subsidizing exports and taxing imports. This is, however, one of the usual policies recommended to substitute exchange rate policy (see Farhi, Gopinath, and Itskhoki, 2012), although it goes against free trade. On top of that, our result is only possible since the price level P_t is assumed to be flexible.

4.2 Implications on optimal taxes

The implication of the irrelevance result established is that downward nominal wage rigidities combined with exogenous nominal exchange rates are irrelevant for the equilibrium allocations. As a result, the second best solution, determined from the maximization of lifetime expected utility (2) subject to the implementability condition (32) with equality and the resource constraints (33), with k_0, b_0, and D_0 given, can be implemented.

From the solution to the benevolent central planner's problem we derive the following proposition:

Proposition 6 *In the small open economy with downward nominal wage rigidities and exogenous nominal exchange rates where there is discrimination between the good produced domestically and the good produced abroad, the optimal state contingent labor wedge is*

$$\frac{1-\tau_t^l\left(s^t\right)}{1+\tau_t^c\left(s^t\right)}=\frac{1+\varphi\left(1-\sigma_t^c\left(s^t\right)+\sigma_t^{cl}\left(s^t\right)\right)}{1+\varphi\left(1+\sigma_t^l\left(s^t\right)+\sigma_t^{lc}\left(s^t\right)\right)},$$

the optimal non state contingent capital income tax is

$$\tau_{t+1}^k\left(s^t\right)=\frac{E_t\left[u_{c,t+1}\left(1+A\left(s^{t+1}\right)F_{k,t+1}-\delta\right)\left(\frac{1+\tau_t^c\left(s^t\right)}{1+\tau_{t+1}^c\left(s^{t+1}\right)}-\frac{1+\varphi\left(1-\sigma_{t+1}^c+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}\right)\right]}{E_t\left[u_{c,t+1}A\left(s^{t+1}\right)F_{k,t+1}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right]},$$

and the optimal non state contingent capital control tax is

$$\tau_{t+1}^D\left(s^t\right)=\frac{E_t\left[u_{c,t+1}\left(1+i_{t+1}^*\right)\frac{P_t^*}{P_{t+1}^*}\left(\frac{1+\varphi\left(1-\sigma_{t+1}^c+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}-\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\right]}{E_t\left[u_{c,t+1}i_{t+1}^*\frac{P_t^*}{P_{t+1}^*}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right]},$$

where the optimal consumption tax is determined according to conditions (44) whenever nominal wages hit their lower bound and indeterminate otherwise.

The proof can be found in Appendix 6.C.

The implications of the special consumption tax policy come straightforwardly from Proposition (6), for the wedges presented are the second best optimal wedges. However, it is no longer the case that the second best optimal non state contingent capital income and capital control taxes are zero, since the use of consumption taxes in this specific way implies that Assumption (1) no longer holds in states in which the lower bound on nominal wages is reached. This does not mean that the central planner is distorting intertemporally. In fact, in expected value terms, there is no intertemporal distortion.

5 Concluding remarks

Throughout this thesis, the implications on optimal fiscal policy arising from the consideration of downward wage rigidities were studied. Starting with a closed economy environment, where real wages were assumed to be downwardly rigid, we have shown that the benevolent central planner, who chooses the sequence of optimal taxes according to a Ramsey approach, is able to incorporate the existence of downwardly rigid wages (something that individual agents do not do, though they are aware of that fact) through the use of labor and capital income taxes. Indeed, it was proven that the optimal state contingent labor income tax has a dual role. On the one hand, it is used as a reactive tool, since it behaves as a labor supply shifter to make the excessively high real wage consistent with labor market clearing (though at a level of employment below the second best solution). However, the optimal labor income tax also exhibits properties of a precautionary tool, for, anticipating a future negative shock, it can be used to establish a smaller lower bound on real wages in the period that follows. Relative to the second best solution, the optimal labor income tax, when used as a reactive tool, is higher, since it is a lump sum tax, whereas, when used as a precautionary tool, it becomes lower than its second best counterpart, as there is a need to reduce the current labor market clearing wage.

Once the closed economy was opened to a foreign sector, which was assumed to be taken as given, and non state contingent foreign nominal bonds denominated in foreign currency were introduced as another source of revenue for the domestic representative household, our results have not changed. Indeed, the consideration of downward nominal wage rigidities, combined with an exogenous stochastic sequence of nominal exchange rates, which created a real wage rigidity, produced exactly the same results in terms of the optimal state contingent labor income tax, i.e. it still exhibited both a reactive and a precautionary nature. Additionally, and this is, perhaps, the most interesting part of our findings for the small open economy, there is a consumption tax policy such that the optimal non state contingent capital control tax can be set to zero for a broad family of instantaneous utility functions, thus contradicting Schmitt-Grohé and Uribe's (2016) result regarding the optimality of a capital control tax, prudential in nature.

David Hume's intuition in *Of Money* and *Of Interest* that monetary policy shocks are irrelevant for the allocations is shown to hold in the small open economy (though not in the conventional way), but only under the consideration of a tax system such that the incidence of the consumption tax falls only on the final good produced domestically. Indeed, the central planner can discriminate between the domestic and the foreign goods by using the state contingent consumption taxes to implement the second best solution, thus implying that downward nominal wage rigidities, combined with exogenous nominal exchange rates, are irrelevant for the allocations. Without this irrelevance result, the equilibrium wage rate could move with a nominal appreciation shock, which is something that is not in accordance with the quantity theory of money. Furthermore, this result is interesting, for it diverges from the use of payroll taxes proposed in Adão, Correia, and Teles (2010), though the proposed consumption tax policy is suitable only in this environment in which the nominal price of the domestic good is flexible, although determined according to the law of one price.

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6 Appendix

6.A Proof of the implementability condition in section 2

To obtain the implementability condition (14), we start by defining the no arbitrage conditions between capital and state contingent real government bonds. These follow directly from the representative household's Euler equations (7) and (8). Together, they can be written as

$$1=\sum_{s^{t+1}|s^{t}} q_{t}\left(s^{t+1}\right)\left[1+\left(1-\tau_{t+1}^{k}\left(s^{t}\right) r_{t+1}^{k}\left(s^{t+1}\right)\right)-\delta\right], \quad t \geq 0, \forall\, s^{t}. \tag{45}$$

Next, we derive the representative household's intertemporal budget constraint.

The time $t=0$, $t=1$, and $t=2$ versions of the intratemporal budget constraints (3) are, respectively,

$$\left(1+\tau_{0}^{c}\right) c_{0}+k_{1}\left(s^{0}\right)+\sum_{s^{1}|s^{0}} q_{0}\left(s^{1}\right) b_{1}\left(s^{1}\right)=\left(1-\tau_{0}^{l}\right) w_{0} l_{0}+\left(1+\left(1-\tau_{0}^{k}\right) r_{0}^{k}-\delta\right) k_{0}+b_{0}, \tag{46}$$

$$\begin{aligned}
&\left(1+\tau_{1}^{c}\left(s^{1}\right)\right) c_{1}\left(s^{1}\right)+k_{2}\left(s^{1}\right)+\sum_{s^{2}|s^{1}} q_{1}\left(s^{2}\right) b_{2}\left(s^{2}\right)= \\
&=\left(1-\tau_{1}^{l}\left(s^{1}\right)\right) w_{1}\left(s^{1}\right) l_{1}\left(s^{1}\right)+\left(1+\left(1-\tau_{1}^{k}\left(s^{0}\right)\right) r_{1}^{k}\left(s^{1}\right)-\delta\right) k_{1}\left(s^{0}\right)+b_{1}\left(s^{1}\right),
\end{aligned} \tag{47}$$

and

$$\begin{aligned}
&\left(1+\tau_{2}^{c}\left(s^{2}\right)\right) c_{2}\left(s^{2}\right)+k_{3}\left(s^{2}\right)+\sum_{s^{3}|s^{2}} q_{2}\left(s^{3}\right) b_{3}\left(s^{3}\right)= \\
&=\left(1-\tau_{2}^{l}\left(s^{2}\right)\right) w_{2}\left(s^{2}\right) l_{2}\left(s^{2}\right)+\left(1+\left(1-\tau_{2}^{k}\left(s^{1}\right)\right) r_{2}^{k}\left(s^{2}\right)-\delta\right) k_{2}\left(s^{1}\right)+b_{2}\left(s^{2}\right).
\end{aligned} \tag{48}$$

Multiplying condition (47) by $q_{0}\left(s^{1}\right)$ and adding over the states of period $t=1$, multiplying condition (48) by $q_{0}\left(s^{1}\right) q_{1}\left(s^{2}\right)$ and adding over the states of period $t=2$, and summing the resulting expressions and (46), we notice, from the no arbitrage conditions

(45), that the following expression is obtained:

$$\sum_{t=0}^{2}\sum_{s^t} Q_t\left(s^t\right)\left[1+\tau_t^c\left(s^t\right)c_t\left(s^t\right)-\left(1-\tau_t^l\left(s^t\right)\right)w_t\left(s^t\right)l_t\left(s^t\right)\right]+$$
$$+\sum_{s^2|s^0} Q_2\left(s^2\right)k_3\left(s^2\right)+\sum_{s^2|s^0}\sum_{s^3|s^2} Q_3\left(s^3\right)b_3\left(s^3\right)=\left(1+\left(1-\tau_0^k\right)r_0^k-\delta\right)k_0+b_0,$$

where $Q_{t+1}\left(s^{t+1}\right)=q_t\left(s^{t+1}\right)Q_t\left(s^t\right)$ and $Q_0=1$.

By repeated substitution until a generic period $t=T$ we get:

$$\sum_{t=0}^{T}\sum_{s^t} Q_t\left(s^t\right)\left[1+\tau_t^c\left(s^t\right)c_t\left(s^t\right)-\left(1-\tau_t^l\left(s^t\right)\right)w_t\left(s^t\right)l_t\left(s^t\right)\right]+$$
$$+\sum_{s^T|s^0} Q_T\left(s^T\right)k_{T+1}\left(s^T\right)+\sum_{s^T|s^0}\sum_{s^{T+1}|s^T} Q_{T+1}\left(s^{T+1}\right)b_{T+1}\left(s^{T+1}\right)=\left(1+\left(1-\tau_0^k\right)r_0^k-\delta\right)k_0+b_0.$$

As $T\rightarrow\infty$, the imposed transversality conditions (4) and (5) imply that the representative household's present-value budget constraint is

$$\sum_{t=0}^{\infty}\sum_{s^t} Q_t\left(s^t\right)\left[1+\tau_t^c\left(s^t\right)c_t\left(s^t\right)-\left(1-\tau_t^l\left(s^t\right)\right)w_t\left(s^t\right)l_t\left(s^t\right)\right]=\left(1+\left(1-\tau_0^k\right)r_0^k-\delta\right)k_0+b_0. \tag{49}$$

The next step is to use the household's first order conditions in order to eliminate the prices and taxes from the budget constraint (49). Particularly, by replacing the intratemporal marginal conditions (6) in (49), we obtain:

$$\sum_{t=0}^{\infty}\sum_{s^t} Q_t\left(s^t\right)\left[\left(1+\tau_t^c\left(s^t\right)\right)c_t\left(s^t\right)+\frac{\left(1+\tau_t^c\left(s^t\right)\right)u_{l,t}\left(s^t\right)}{u_{c,t}\left(s^t\right)}l_t\left(s^t\right)\right]\geq\left(1+\left(1-\tau_0^k\right)r_0^k-\delta\right)k_0+b_0,$$

which can be manipulated in such a way that it becomes

$$\sum_{t=0}^{\infty}\sum_{s^t} Q_t\left(s^t\right)\frac{\left(1+\tau_t^c\left(s^t\right)\right)}{u_{c,t}\left(s^t\right)}\left[u_{c,t}\left(s^t\right)c_t\left(s^t\right)+u_{l,t}\left(s^t\right)l_t\left(s^t\right)\right]\geq\left(1+\left(1-\tau_0^k\right)r_0^k-\delta\right)k_0+b_0.$$

Using the Euler equations (8), which, with $Q_{t+1}\left(s^{t+1}\right)=q_t\left(s^{t+1}\right)Q_t\left(s^t\right)$, can be rewritten as

$$\frac{1+\tau_t^c\left(s^t\right)}{u_{c,t}\left(s^t\right)}=\beta\frac{\Pr_t\left(s^t\right)}{\Pr_{t-1}\left(s^{t-1}\right)}\frac{Q_{t-1}\left(s^{t-1}\right)}{Q_t\left(s^t\right)}\frac{1+\tau_{t-1}^c\left(s^{t-1}\right)}{u_{c,t-1}\left(s^{t-1}\right)},$$

we can iterate backwards until we have the implementability condition for this stochastic closed economy, i.e.

$$\sum_{t=0}^{\infty}\sum_{s^t}\Pr_t\left(s^t\right)\beta^t\left[u_{c,t}\left(s^t\right)c_t\left(s^t\right)+u_{l,t}\left(s^t\right)l_t\left(s^t\right)\right]\geq\frac{u_{c,0}}{1+\tau_0^c}\left[\left(1+\left(1-\tau_0^k\right)r_0^k-\delta\right)k_0+b_0\right],$$

which, denoting $V_0\equiv\frac{u_{c,0}}{1+\tau_0^c}\left[\left(1+\left(1-\tau_0^k\right)r_0^k-\delta\right)k_0+b_0\right]$ reduces to condition (14).

6.B Proof of Proposition (3)

The proof of Proposition (3) is rather similar to that of Propositions (1) and (2). In fact, we just need to use conditions (19) and (36) to obtain conditions (40).

Regarding the optimal non state contingent capital income tax, one must use conditions (37) and (20), which together yield

$$\tau_{t+1}^k\left(s^t\right)=\frac{E_t\left[\begin{array}{c}u_{c,t+1}\left(1+A\left(s^{t+1}\right)F_{k,t+1}-\delta\right)\left[\frac{1+\tau_t^c}{1+\tau_{t+1}^c}-\frac{1+\varphi\left(1-\sigma_{t+1}^c+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}\right]-\\-\frac{\varepsilon_{t+1}P_{t+1}^*A\left(s^{t+1}\right)F_{lk,t+1}}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}\left[\mu_{t+1}-\beta\gamma E_{t+1}\mu_{t+2}\right]\end{array}\right]}{E_t\left[u_{c,t+1}A\left(s^{t+1}\right)F_{k,t+1}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right]},$$

where the variables are a function of the state. These expressions reduce to conditions (41) once we impose Assumption (1).

6.C Proof of Proposition (6)

To prove Proposition (6), one needs to solve the second best problem, which is presented as follows:

$$\max_{\{c_t(s^t),l_t(s^t),k_{t+1}(s^t),D_{t+1}(s^t)\}}\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\Pr_t\left(s^t\right)u\left[c_t\left(s^t\right),l_t\left(s^t\right)\right]\qquad s.t.$$

$$E_0\sum_{t=0}^{\infty}\beta^t\left[u_{c,t}\left(s^t\right)c_t\left(s^t\right)+u_{l,t}\left(s^t\right)l_t\left(s^t\right)\right]=V_0$$

$$c_t\left(s^t\right)+g\left(s^t\right)+k_{t+1}\left(s^t\right)-\left(1-\delta\right)k_t\left(s^{t-1}\right)-\frac{D_{t+1}\left(s^t\right)}{P_t^*}+\left(1+i_t^*\left(s^{t-1}\right)\right)\frac{D_t\left(s^{t-1}\right)}{P_t^*}=$$

$$=A\left(s^t\right)F\left(k_t\left(s^{t-1}\right),l_t\left(s^t\right)\right)$$

where $V_0 \equiv \frac{u_{c,0}}{1+\tau_0^c}\left[\left(1+\left(1-\tau_0^k\right)A_0F_{k,0}-\delta\right)k_0+b_0-\left(1+\left(1+\tau_0^D\right)i_0^*\right)\frac{\varepsilon_0}{P_0}D_0\right]$ as in Chari, Nicolini, and Teles (2016).

Let φ be the multiplier associated with the implementability condition and $\beta^t\lambda_t\left(s^t\right)$ the present value multiplier associated with the resource constraints.

The first order condition with respect to $c_t\left(s^t\right)$ is

$$\beta^t\left[u_{c,t}\left(s^t\right)+\varphi\left(u_{cc,t}\left(s^t\right)c_t\left(s^t\right)+u_{c,t}\left(s^t\right)+u_{lc,t}\left(s^t\right)l_t\left(s^t\right)\right)-\lambda_t\left(s^t\right)\right]=0,\quad t\geq 1,\forall\ s^t.$$

The first order condition with respect to $l_t\left(s^t\right)$ is

$$\begin{aligned}\beta^t\left[u_{l,t}\left(s^t\right)+\varphi\left(u_{cl,t}\left(s^t\right)c_t\left(s^t\right)+u_{ll,t}\left(s^t\right)l_t\left(s^t\right)+u_{l,t}\left(s^t\right)\right)\right]+\\+\beta^t\lambda_t\left(s^t\right)A\left(s^t\right)F_{l,t}\left(s^t\right)=0,\quad t\geq 1,\forall\ s^t.\end{aligned}$$

The first order condition with respect to $k_{t+1}\left(s^t\right)$ is

$$-\beta^t\lambda_t\left(s^t\right)+\beta^{t+1}E_t\left[\lambda_{t+1}\left(s^{t+1}\right)\left(1+A\left(s^{t+1}\right)F_{k,t+1}\left(s^{t+1}\right)-\delta\right)\right]=0,\quad t\geq 1,\forall\ s^t.$$

The first order condition with respect to $D_{t+1}\left(s^t\right)$ is

$$\beta^t\lambda_t\left(s^t\right)\frac{1}{P_t^*}-E_t\left[\beta^{t+1}\lambda_{t+1}\left(s^{t+1}\right)\left(1+i_{t+1}^*\right)\frac{1}{P_{t+1}^*}\right]=0,\quad t\geq 1,\forall\ s^t.$$

The conditions, which together with the implementability condition (32) with equality and the resource constraints (33), with k_0, b_0, D_0 given, characterize the solution to this problem are

$$\frac{-u_{l,t}}{u_{c,t}}\frac{1+\varphi\left(1+\sigma_t^l+\sigma_t^{lc}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}=A\left(s^t\right)F_{l,t}, \tag{50}$$

$$u_{c,t}=\beta E_t\left[u_{c,t+1}\frac{1+\varphi\left(1-\sigma_{t+1}^c+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_t^c+\sigma_t^{cl}\right)}\left(1+A\left(s^{t+1}\right)F_{k,t+1}-\delta\right)\right], \tag{51}$$

and (38), where the variables are a function of the state and σ_t^c, σ_t^l, σ_t^{cl}, and σ_t^{lc} are defined as before.

Joint manipulation of conditions (19) and (50) yields

$$\frac{1-\tau_t^l\left(s^t\right)}{1+\tau_t^c\left(s^t\right)}=\frac{1+\varphi\left(1-\sigma_t^c\left(s^t\right)+\sigma_t^{cl}\left(s^t\right)\right)}{1+\varphi\left(1+\sigma_t^l\left(s^t\right)+\sigma_t^{lc}\left(s^t\right)\right)},$$

whereas putting together conditions (20) and (51) yields

$$\tau_{t+1}^{k}\left(s^{t}\right)=\frac{E_{t}\left[u_{c,t+1}\left(1+A\left(s^{t+1}\right)F_{k,t+1}-\delta\right)\left(\frac{1+\tau_{t}^{c}}{1+\tau_{t+1}^{c}}-\frac{1+\varphi\left(1-\sigma_{t+1}^{c}+\sigma_{t+1}^{cl}\right)}{1+\varphi\left(1-\sigma_{t}^{c}+\sigma_{t}^{cl}\right)}\right)\right]}{E_{t}\left[u_{c,t+1}A\left(s^{t+1}\right)F_{k,t+1}\frac{1+\tau_{t}^{c}}{1+\tau_{t+1}^{c}}\right]},$$

where the variables are a function of the state.

Finally, the proof of the optimal non state contingent capital control tax (42) is part of the proof of Proposition (5).