The basis determinants: The European Case

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Abstract

With subprime mortgage crisis, Lehman Brothers Holdings Inc. bankruptcy and European government credit crisis, the CDS market assisted to a generalized turmoil, contributing for a decrease of CDS market in more than 50% in less than 3 years.

This dissertation focuses on testing possible determinants of the basis spread for several European companies, analysing data between June 18 2008 and December 31 2012. All financial information and data used in this thesis was gathered from *Bloomberg*.

Literature on single-name credit modelling and valuing credit derivatives is revised and applied to calculate the basis, with special focus on estimating hazard rates, where we used the optimization method instead of the generally used bootstrap method.

We than, followed Zhu work and analysed the proposed determinants for the basis, updating his work by introducing two new variables as potential determinants of the basis: the CDS Big Bang and the Lehman Brothers bailout.

Finally, we have found some evidence that efforts to standardize and regulate the credit derivative contracts, the CDS Big Bang has contributed to mitigate part of the counterpart risk, and that have also been reflected on the CDS-ASW basis.

Foreword

This dissertation could not have been done without the direct and indirect support of several people.

I would like to express my gratitude to Professor Joaquim Cadete for the unconditional availability, superior knowledge and experience, promoting very high guidance and delivery necessary tools for the dissertation development and conclusion.

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Introduction

Credit default swaps (CDS) have not only become the most widely used credit derivative, but have also turned into a journalistic buzz word due to subprime mortgage crisis in late-2000s and, more recently, due to European sovereign credit crisis.

These richness of events in the last 5 years, start to struggle the credit derivatives desks in major financial institutions in order to price their products. One of the major parameter to price a credit derivative is the spread, which represents the credit risk of a *name* involved in the deal and it had no more logic. Subsequently, most of the products could not be priced any longer, creating some discrepancies in their value and subsequently some arbitrage opportunities.

One of those arbitrage opportunities relied on taking advantage of the difference between the asset-swap spread and the CDS of the respective underline. This difference is called basis. In a market without any arbitrage opportunity the basis is expected to be zero, but recently this basis has moved deeply away from zero.

One of the most interesting papers regarding the basis determinants, where developed by (Zhu, 2006). Zhu analyses data between the years 1999 and 2002 and addresses two major concerns that have significant implications for financial regulators and risk managers. First, is credit risk equally priced between the derivatives market and the cash market? Zhu, refers this question as accuracy of credit risk pricing issue, where low financial transparency and the existence of asymmetric information between protection buyers and sellers leads to potential arbitrage of credit risk across markets. Second, which market reacts more rapidly to changes in credit conditions? This question, reports to price discovery efficiency of both markets, where traders could take potential gains from price differentials.

Using Zhu's work, we have applied the same framework for the between years 2008 and 2012, but with the introduction of two new dummy variables: the Lehman Brothers bankruptcy (September 15 2008) and CDS Big Bang (April 8 2009).

With the Lehman Brothers bailout, it is expected that the basis move away from zero, hence the risk perception in the market players has worsen. On the opposite, with the CDS Big Bang, it is expected that the basis approximate more to zero, because after April 8 2009 where introduced changes in CDS contracts and conventions in order to make CDS more standardised and consequently to help central clearing of CDS trades, mitigating the counterpart risk.

With our investigation, we tried to bring new information for the bank's derivatives desk, in order to contribute for the definition of the liquidity *premia* necessary to face future credit events. Thus, it is expected to answer the following questions: Does the Lehman Brothers bailout augment the risk perception? In which way affected the basis? Does the Big Bang event helped to mitigate the counterpart risk? How it influenced the basis? How does rating events and liquidity influence the basis?

Finally, unlike Zhu did in his paper, hazard rates are taken through the optimization process and not through bootstrapping method. The optimization method has the advantage of dealing with the liquidity of CDS quotes, putting more emphasis in liquid ones and less emphasis in illiquid maturities.

The rest of this thesis is structured as follows. Chapter 2 starts to briefly analyse the dimension of the credit derivative market, explain what is a CDS and introduces a theoretical background regarding the main models of CDS valuation. Chapter 3 explains in detailed the analysis framework used in this dissertation, including the relationship between the credit spreads in the bond market and the derivatives market from a theoretical perspective. Chapter 4 describes the data. Chapter 5 identifies the determinants of the basis. Chapter 6 concludes.

Background Theory

What is a Credit Default Swap?

The most important credit derivative is the credit default swap (CDS). This is a contract between two parties, where one party transfers to another the credit risk of a reference entity, corporate or sovereign, for a specific period of time. A CDS is designed to protect an investor against the loss from par on a bond or loan following the credit event of the reference entity, including bankruptcy, failure-to-pay and restructuring. In return for this, the protection buyer pays a premium to the protection seller.

There are also a number of option-based credit derivatives. These include single-name default swaptions in which the option buyer has the option to enter into a CDS contract on a future date. More recently, we have assisted to the growth of portfolio swaptions, where the holder has the option to enter into a portfolio of CDS. These contracts work by "tranching" up the credit risk of the underlying portfolio. Tranching is a mechanism by which different securities or tranches are structured so that any default losses in the portfolio are incurred in a specific order. The first default losses are incurred by the riskiest equity tranche. If the size of these losses exceeds the face value of the equity tranche then the remaining losses are incurred by the mezzanine tranche. If there are still remaining losses after this, then these are incurred by the senior tranches. The risk of this credit derivatives contract is sensitive to the tendency of the credits in the portfolio to default together. This is known as default correlation and, for this reason, these derivatives are known as correlation products.

An important extension of the CDS is the CDS index. This product allows the investor to enter into a portfolio consisting of 100 or more different CDS in one transaction, exposing the issuer to the default risk of more than one credit or "name", we call this transaction *multi-name* product. The *multi-name* products have several advantages to *the single-name* products, mainly considerable liquidity and diversification.

Lastly, we have the credit Constant Proportion Portfolio Insurance (CPPI) structure and the more recent Constant Proportion Debt Obligation (CPDO) structure. These structures exploit a rule-based dynamic trading strategy typically involving a CDS index. In the case of CPPI, it is designed to provide a leveraged credit exposure while protecting the investor's principal. In the case of CPDO, the strategy is designed to produce a high coupon with low default risk. Due to the complexity involving the valuation of *multi-named* products, we will focus our thesis in *single-named* products.

Single-name credit modelling – Default rate statistics

In order to define a model for pricing credit derivatives, we need to establish a modelling framework which can capture the appropriate risks, which includes default risk, recovery rate risk, spread risk, interest rate risk and credit rating transition risk. Default risk is the risk that a planned payment of interest or principal on a bond or loan is not received. Recovery risk is the risk that following a default, the size of the recovered amount is less than the amount due. Spread risk is the risk that the value of a credit security falls as the market's view regarding the credit quality of the borrower changes, causing us to realise a loss if we sell the credit security. Interest rate risk is the risk that changes in the level of the treasury curve will cause the value of the credit security to fall. Credit rating transition risk is the risk of credit note change of the reference entity or issuance.

Default is a complex event that can occur for several reasons. In some cases it is an entirely idiosyncratic event that strikes just one company. In other cases can be a systemic event in which several companies or sovereigns are all affected by the same factor.

The main sources of default statistics are the credit rating agencies. In order to better assess the issuers' credit quality, they have collected a significant amount of data over their lifetime. In order to measure default, it is important to understand the default definition. According to Moody's, a default is "any missed or delayed payment of interest or principal, bankruptcy or distressed exchange where,

(i) the issuer offered bondholders a new security or package of securities that amount to a diminished financial obligation (such as preferred or common stock or debt with a lower coupon or par amount), or
(ii) the exchange had apparent purpose of helping the borrower avoid default".

If a failure to pay principal or coupon occurs as a result of some omission, which is quickly rectified, then the event is known as *technical default.* Such an occurrence is not usually included in rating agency default statistics.

The rating agency methodology for calculating default statistics has been to construct databases of issuers and to monitor the dating and default behaviour of the senior unsecured bonds of each issuer through time. This is done by identifying a *cohort* – a group of issuers with the same initial rating. The rating agency then keeps track of the *cohort* and records if any of the issuers default. As a result, it is possible to calculate the number of defaults in each *cohort*. Dividing this by the number of issuers in the *cohort* gives the average default rate of issuers with a specific rating. Each year new *cohorts* are defined. Averages can then be taken across *cohorts* with different initial dates but with the same initial rating to give a time-averaged default rate for each rating class.

The next table shows time-averaged cumulative default rates and so has averaged out any time variability in default rate statistics. In practice, market participants will use these historical default rates as proxies for the default probabilities used within their credit risk models. This assumes that time-averaged historical default rates by rating are a good predictor of future default rates, and that all issuers with the same rating have the same probability of default. It therefore ignores the current state of the credit environment and differences in credit quality that exist within a rating category. The averages are global, so that differences in the triggering of default and the workout process which may exist across different legal jurisdictions are not captured. The data is also biased towards US corporate credits since this has traditionally been the dominant market for corporate credit bonds. However, this issue is now being addressed by the rating agencies that have recently begun to produce separate statistics for the European credit market.

Figure 1 Average cumulative default rates of corporate bond issuers by letter rating from 1983 to 2005

	Cumulative default rate (%)									
Years	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Aaa	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.2	0.2	0.2
Aa	0.0	0.0	0.1	0.1	0.2	0.3	0.3	0.3	0.4	0.4
A	0.0	0.1	0.3	0.4	0.6	0.7	0.9	1.0	1.1	1.2
Baa	0.2	0.6	1.1	1.6	2.2	2.8	3.3	3.7	4.1	4.5
Ba	1.3	3.6	6.3	9.0	11.1	12.9	14.5	15.8	16.9	17.8
B	5.7	12.1	17.6	21.8	25.1	27.7	29.6	30.8	31.6	32.1
Caa-C	21.0	30.3	36.1	39.5	41.2	42.1	42.6	42.9	43.1	43.3

Source: Hamilton *et al.* *(2005).*

Historical default data is not used by the market for determining the price of a security. It is primarily used as a way of calibrating risk models. When we come to pricing credit risky assets such as credit derivatives, we need to be in a world in which we can hedge out the risk that these contracts present. For that, we need to be in a risk-neutral framework.

Single-name credit modelling – Recovery rate statistics

Credit risk is also about the risk associated with the amount of the claim that can be recovered after default. In the credit derivatives market, the recovery price is the price of some reference obligation determined within 72 days of the default event. The measure of recovery rate used in the credit markets is the defaulted bond price divided by the face value. There are a number of sources for recovery data. In our study, we will a recovery rate of 40% of the face value. This value, is used as general accepted in most models of CDS valuation, and is based on the study performed by Altman *et al.* (2003b). In this study, it is empirically estimated recovery rates based on prices just after default on loans. The following table resumes the findings:

Figure 2 Empirical estimates of recovery rates

Seniority of debt	Debt type	Number of issues	Median recovery (%)	Mean recovery (%)	Standard deviation (%)
Senior secured	Loans	155	73.00	68.50	24.4
Senior unsecured	Loans	28	50.50	55.00	28.4
Senior secured	Bonds	220	54.49	52.84	23.1
Senior unsecured	Bonds	910	42.27	34.89	26.6
Senior subordinated	Bonds	395	32.35	30.17	25.0
Subordinated	Bonds	248	31.96	29.03	22.5
All bonds and loans		1909	40.05	34.31	24.9

Source: Altman *et al.* *(2003b)*

The pricing of credit derivatives

The pricing of credit derivatives is not an easy task. One of the major reasons is that the market price of the underlying asset is not often easily observable. This is particularly applicable for loans, which are rarely traded in a secondary market. Nevertheless, if the underlying company is rated by an agency, the rating can be used as a proxy to value the respective debt, but published ratings are often outdate, since agencies are not able to analyse the underlying debt on a timely basis, and defaults are rare events. Especially, since a company typically only defaults once, empirical data on the default of a solvent company is typically unavailable.

In addition, default is usually triggered by a combination of factors, like as credit, market and operational risk, whose correlation has to be integrated into the pricing model. Moreover, with credit derivatives, the counterparty risk is an important pricing element, since the default of the underlying debt typically leads to a large settlement payment of the protection selling counterparty. Ideally, the correlation between the default risk of the counterparty and the default risk of the underlying debt should be considered in the pricing process. All this makes pricing credit derivatives complex.

Lastly, there is no pricing model generally accepted as a benchmark as, for example, the Black-Scholes model for standard options. Additionally, incorporating all input variables, summarized in the next table, it is not trivial.

Figure 3 List of variables for valuing a credit derivatives price

Input for deriving the price of a credit derivative	
1)	Default probability and credit deterioration probability of the reference asset
2)	Default probability and credit deterioration probability of the credit derivatives seller
3)	Correlation between 1) and 2)
4)	Volatility of the underlying reference asset
5)	Volatility of the credit derivatives seller
6)	Correlation between 4) and 5)
7)	Maturity of the credit derivative
8)	Expected recovery rate of the reference asset
9)	Expected recovery rate of the credit derivatives seller
10)	Return of the reference asset (e.g. coupon of the reference bond)
11)	Risk-free interest rate term structure used to discount future cash-flows
12)	Default probability of the credit derivatives buyer in case of periodic credit derivative premium
13)	Expected recovery rate of the credit derivatives buyer in case of periodic credit derivative premium
14)	Correlation between de default probability of the credit derivatives buyer and the reference asset in case of periodic credit derivatives premium
15)	Market risks (as interest rate risk, currency risk, commodity risk, and stock price risk) and the correlation between market risk and credit risk
16)	Operational risks (e.g. legal risks, documentation risks, or settlement risks), which might endanger the enforceability of the payoff and the correlation between operational risk and credit risk
17)	Liquidity of the credit derivative

18) Liquidity of the underlying reference asset
19) BIS risk weight on the credit derivatives seller
20) Urgency of protection (e.g. in an immediate credit deterioration expected or does the protection free up credit lines to enable further business with a client)
21) Transaction costs

Source: (Meissner, 2005)

Furthermore, the credit risk models can be divided into two major groups, the *structural models* and *reduced form models* (also known as intensity-based models). The *structural models* were pioneered by (Merton, 1974), in his framework a firm issues two types of assets: equities and bonds. A default occurs if the total asset value falls below a default boundary, this is a level of asset value, sufficiently low so that the firm decides to default on its debt if asset value falls beneath this level. By contrast, *reduced form models* treat default as a random stopping time with stochastic arrival intensity. The credit spread is determined by risk neutral valuation under the absence of arbitrage opportunities. This method has been widely used in the pricing of CDSs, the main literature was developed by (Jarrow & Turnbull, 1995), (Das, 1995), (Duffie, 1999), (Duffie & Singleton, 1999), (Das & Sundaram, 2000), (Madan & Unal, 2000), (Hull & White, 2000, 2001), (Archarya, et al., 2002), (Jarrow & Yildirim, 2002), (Das, et al., 2003) and (Schönbucher, 2003).

Before we discuss *structural* and *reduced-form models* in detail, is important to understand simple pricing features of credit derivatives, namely *the default swap premium derived from asset swaps*, *deriving the default swap premium using arbitrage arguments* and *obtaining the default probability on a binomial model*.

Deriving the default swap premium using arbitrage arguments

An important arbitrage argument, used in trading practice to help determine the price of a default swap can be expressed in the following terms:

Default swap premium = Return on risky bond – Return on risk-free bond. (1)

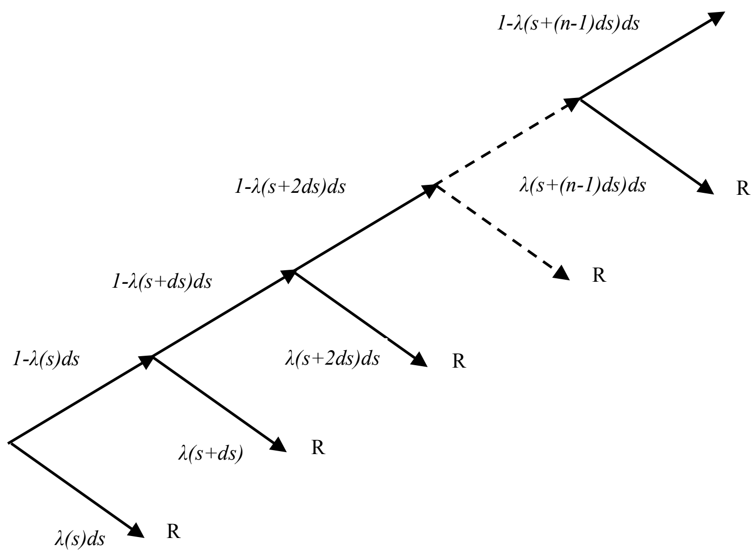
This equation can only serve as an approximation, hence it abstracts from several inputs, already described, which have to be included in the pricing of a default swap. One of the most important points, not included in the previous equation, is the counterparty risk, this is the risk that the default protection seller defaults. In addition, the correlation between counterparty default risk and default risk of the underlying asset assumes also a degree of importance, since the default protection buyer will incur a loss in the amount of his reference asset value plus the default swap premium (minus the recovery rate of the reference asset issuer and the counterparty), if both the protection seller and the underlying asset default.

Obtaining the default probability on a binomial model

One of the most important features when pricing credit derivatives is deriving the probability of default of the underlying debt.

We can model the default in a one-period setting as a binomial tree, which we survive with probability $1- \lambda(s)ds$ or default and receive a recovery value R with probability $\lambda(s)ds$. For a n-period, the risky debt with a notional amount of 1, can be designed as follows:

Figure 4 Binomial model to find the risk-neutral probability of default



Risk-neutrality is an important concept when pricing derivatives. If investors are risk-neutral, they do not require a compensation for taking risk. As a consequence, the expected return on all securities (including derivatives) is the risk-free interest rate. Hence, the present value of any security can be derived by discounting all future cash flows with the risk-free interest rate.

Valuing credit derivatives using *Structural Models*

As we have presented earlier, structural models derive the probability of default by analysing the capital structure of a firm, especially the value of the firm's assets compared to the value of the firm's debt.

The original 1974 Merton model

In 1974 Robert Merton created a firm value model to estimate a company's value of debt and the probability of default (Merton, 1974).

The Merton call

Merton combined the simple equation, shareholders' equity (E) = company's assets (V) – company's liabilities (D), with the Black-Scholes option pricing framework. Merton's model is mathematically identical with the original Black-Scholes equation for valuing a call:

$$E_0 = V_0N(d_1) - De^{-rT}N(d_2) \tag{2}$$

where

$$d_1 = \frac{ln\left[\frac{V_0}{De^{-rT}}\right] + \frac{1}{2}\sigma_V^2 T}{\sigma_V\sqrt{T}} \text{ and } d_2 = d_1 - \delta_V\sqrt{T}$$

where E_0 is the current value of equity, V_0 is the current value of assets, D is the debt to be repaid at time T, N is the cumulative standard normal distribution, r is the risk-free continuously compounded interest rate, δ_V is the expected volatility of the asset, and T is the option maturity, measured in years. The previous equation states that equity holders have a claim on the assets of a company: If the asset value V increases, the equity value E will increase with unlimited upside potential; on the downside, it the debt D exceeds the assets V, the company will go bankrupt. In this case the equity holders will take the remaining assets to repay part of the debt, the equity value being zero.

A well-known property of the Black/Scholes model is that the risk-neutral probability of exercising a call option is $N(d_2)$. Therefore, the probability of not exercising the option is $N(-d_2)$. Not exercising the equity option means that the debt D is bigger than the assets V. This is the case of bankruptcy. Therefore, the probability of default in the Merton framework is $N(-d_2)$.

The Merton put

The value of credit risk and the probability of a company's default in Merton's model can also be found by expressing credit risk with the help of a put option on the assets of the company: The equity holders can hedge the credit risk by buying a put on the assets with strike D, the put seller being the asset holders. In case of default, i.e. $V < D$, the equity holders will deliver the assets to the asset holders, the loss for the asset holders being $D – V$. Thus, the put option can be expressed as in the following equation:

$$P_0 = -V_0N(-d_1) + De^{-rT}N(-d_2) \tag{3}$$

where

$$d_1 = \frac{ln\left[\frac{V_0}{De^{-rT}}\right] + \frac{1}{2}\delta_V^2 T}{\delta_V\sqrt{T}} \text{ and } d_2 = d_1 - \delta_V\sqrt{T}$$

where P_0 is the current value of a put option on the company's assets *V* with strike *D*, σ is the volatility of the underlying asset, and *T* is the option maturity expressed in years.

The equity holders will exercise the put option in the last equation at time *T* if *D* > *V*. In the Merton model, this is the case of bankruptcy. Thus the probability of exercising the put, which is $N(-d_2)$, is again the probability of default.

Rewriting the previous equation as $P_0 = \left(-\frac{N(-d_1)}{N(-d_2)}V_0 + De^{-rT}\right)N(-d_2)$ results in an intuitive interpretation of the default risk, where the term $\frac{N(-d_1)}{N(-d_2)}V_0$ reflects the amount retrieved of the asset value V_0 in case of default, thus the recovery value. The term De^{-rT} is the present value of the debt, thus $\left(-\frac{N(-d_1)}{N(-d_2)}V_0 + De^{-rT}\right)$ is the present value of the loss in the event of default. Multiplying $\left(-\frac{N(-d_1)}{N(-d_2)}V_0 + De^{-rT}\right)$ with the probability of default $N(-d_2)$ gives the present value of the default risk, which equals the put value P_0.

The put option in equation (3) serves as a basis to find a closed form solution for the value of the underlying risky bond B. We can start by expressing B_0 as the debt *D* to be repaid at time T discounted by e^{-rT} minus the value of the credit risk, which is the put in equation (3):

$$B_0 = D_T e^{-rT} - [-V_0 N(-d_1) + De^{-rT} N(-d_2)] \tag{4}$$

Rearranging the equation and assuming 1-N(-d_2)= N(d_2) results in the value of the risky bond of:

$$B_0 = D_T e^{-rT} N(d_2) + VN(-d_1) \tag{5}$$

Where

$$d_1 = \frac{ln\left[\frac{V_0}{De^{-rT}}\right] + \frac{1}{2}\delta_V^2 T}{\sigma_V \sqrt{T}} \text{ and } d_2 = d_1 - \delta_V \sqrt{T}$$

One drawback of Merton's model is that we need the asset value *V* and the asset volatility δ_V as inputs. Both parameters are not easily available in practice. However the equity value *E* and the equity volatility δ_E are observable. Using equation (2) and equation (6) derived from Itô's lemma:

$$E_0 = \frac{N(d_1)V_0\sigma_V}{\sigma_V} \tag{6}$$

We have two equations with two unknowns to solve for, V and δ_V.

As mention earlier, the Merton model serves as a basis for structural and reduced form models that value credit risk. Meanwhile, the model simplifies a number of aspects. It principally only allows default at the maturity of the debt *T* and the debt can only take the form of zero-coupon bonds. Coupons as well as different seniorities cannot be handled. There is only one bankruptcy event, which occurs when the asset value falls below the value of the debt at maturity of the debt. Other bankruptcy event such as illiquidity, restructuring of debt, or a moratorium is not taken into account.

Nevertheless, the Merton model has served as an excellent basis for developing more realistic complex models.

The Black-Cox 1976 model

(Black & Cox, 1976) suggest an exogenous exponential default boundary with two exogenous constants, k and γ. If the asset value drops below the default boundary during a period of time, the asset holders can force the company in to bankruptcy or restructuring. The mandatory bankruptcy or restructuring, expressed as a *safety covenant* of the asset holders, is an important feature of the model. It protects asset holders from further deterioration of the company's assets. In that sense a high value of k and a low value of γ forces early bankruptcy or restructuring and principally protects asset holders.

Besides safety covenants, Black and Cox also investigate subordination arrangements and restrictions for the equity holders to finance interest and dividend payments. All three provisions tend to increase the value of the risky bond.

Black and Cox also find a closed form solution for the risky bonds, which includes (continuous) dividends to the stockholders and the underlying interest rate process and the recovery rate are rather simple. Interest rates do not follow a stochastic process but are assumed constant at a constant rate and the recovery rate is simply set to the asset value at the time of default.

The Kim, Ramaswamy, and Sundaresan 1993 Model

(Kim, et al., 1993) use a simpler default boundary but a more realistic stochastic interest rate process than Black and Cox. Default is triggered if the asset value drops below a exogenous constant variable. The interest rate process follows the risk neutral Cox-Ingersoll-Ross model, where interest rates mean-revert with a defined rate to the long-term average of rates. These rates cannot get negative, because are taken to the square root.

The default boundary in this model takes into account the coupon rate and the cash outflow of the firm. Thus the default boundary is endogenous but not time-dependent as in the Black-Cox model. The recovery rate is the minimum of the asset value and the face value of the debt, if default occurs before the debt maturity, the recovery rate is the minimum of an exogenous recovery rate expressed in percentage of a risk-free bond and the asset value.

According to the authors of the study, this model had better results in deriving realistic default swap premiums than the original Merton model.

The Longstaff-Schwartz 1995 model

(Longstaff & Schwartz, 1995) suggest a first-time passage model with an exogenous and constant default boundary and recovery rate. For the interest rate, Longstaff and Schwartz use the Vasicek[1] model.

With the help of the closed form solution for a zero-coupon bond derived in the Vasicek model, Longstaff and Schwartz find a solution for the price of risky zero-coupon bonds and floating rate bonds.

Key findings of Longstaff and Schwartz imply that credit-spreads decrease when the risk-free Treasury rate increases. This appears counterintuitive but can be explained by the fact that a higher interest rate means a higher growth rate of the asset value. As a consequence of the higher asset value the probability of default is lower, and with it the credit-spreads.

The inverse relationship between long term risk-free interest rates and credit-spread is stronger for firms with lower credit quality. This is intuitive since a strong growth in the asset value can improve the asset-liability relationship of a low rated firm to a significant degree.

Drawbacks of the Longstaff-Schwartz model are the complex parameter calibration of the numerous parameters for the bond equations, and the fact that the underlying Vasicek model for interest rates is generally not arbitrage-free.

The Briys-deVarenne 1997 model

In 1997, (Briys & Varenne, 1997) addressed shortcomings of the Black-Cox, Kim-Ramaswamy-Sundaresan, and Longstaff-Schwartz models. In these models, the payoff to bondholders in case of bankruptcy may be larger than the firm's asset value. In this respect, payoff demands of the equity holders are not taken into account. Consequently, Briys and de Varenne suggest a default boundary and recovery rate, which guarantee that the payoff to bondholders at the time of default is realistic with respect to demands from the equity holders, and cannot be higher than the firm's asset value.

Critical appraisal of *structural models*

The major achievement of the models presented is that unlike in the original Merton model, default before the maturity of the debt at time T is possible. However, several

[1] Vasicek model is a mathematical model describing the evolution of interest rates.

significant drawbacks remain. First, with the exception of the Kim-Ramaswamy-Sundaresan model, the default boundary involves an exogenous constant. Furthermore, the recovery rate of the models, with the exception of the Black-Cox model, also involves an exogenous constant. Consequently the default boundary and recovery rate are difficult to determine for practical purposes.

In addition, the closed form solutions for the risky bond price, equations of the four last models are quite complex and the calibration of the numerous parameters to match market credit-spreads is difficult in trading practice. Other shortcomings of structural models include the fact that some underlying stochastic processes for the asset value (e.g. CIR[2] and Vasicek) are generally not arbitrage-free. Altogether, these drawbacks have so far limited the use *structural models* in credit risk practice.

Valuing credit derivatives using *Reduced form Models*

They are called *reduced form,* since they abstract from the explicit economic reasons for the default, i.e. they do not include the asset-liability structure of the firm to explain the default. Rather, *reduced form models* use debt prices as a main input to model the bankruptcy process. Default is modelled by a stochastic process with an exogenous *default intensity* or *hazard rate*, which multiplied by a certain time frame, results in the risk-neutral default probability, also called pseudo-or martingale[3] default probability. The value of hazard rate is derived by calibration of the variables of the stochastic process. Since *reduced* form models only model the timing of the default not the severity, the recovery rate is usually exogenous.

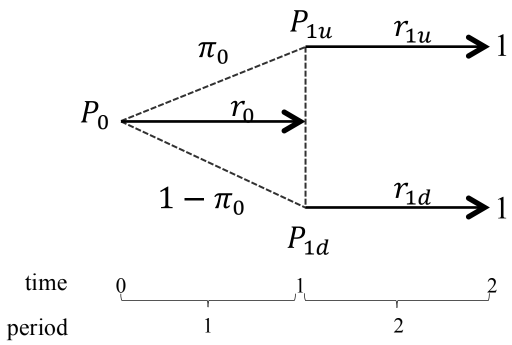
The Jarrow-Turnbull 1995 model

(Jarrow & Turnbull, 1995) were one of the first to derive the value of credit derivative and to price credit derivatives in the arbitrage-free *reduced form* model environment. They combine a process for risk-free interest rates and a bankruptcy process of the risky debt to derive default probabilities and credit derivatives prices. The two processes are assumed to be independent from each other.

Let's define P as the price of the risk-free zero-coupon bond with notional amount 1 and maturity at time 2. π_0 is the risk-neutral probability of an interest rate increase. This brings us to the following interest rate tree:

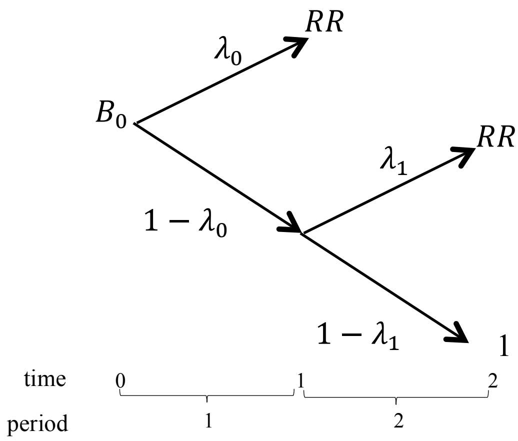
Figure 5 Risk-free interest rate tree in the Jarrow-Turnbull model

[2] Cox–Ingersoll–Ross model (or CIR model) describes the evolution of interest rates.
[3] In probability theory, martingale refers to model, which past events doesn't help to predict future events.



Where r = risk-free interest rate, P = risk-free zero-coupon bond price

Figure 6 Bankruptcy process of risky bond B in the Jarrow-Turnbull model

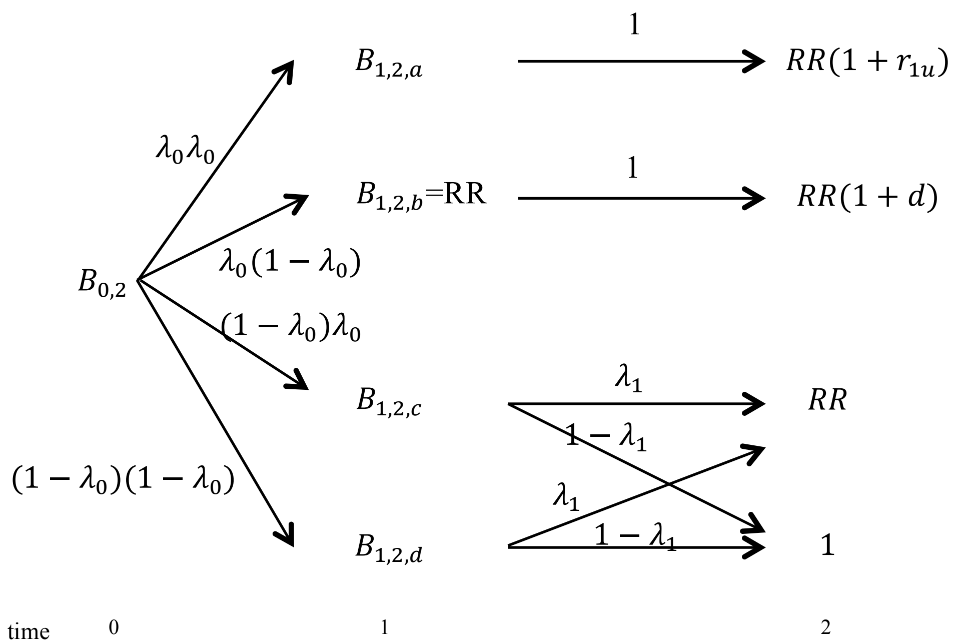


The risk-free bond price at time *t* with maturity *T*, is $P_{t,T} = 1/(1 + r_{t,T})$. Since $r_{1u} > r_{1d}$, it follows that $P_{1u} > P_{1d}$.

Let *B* be the price of a risky zero-coupon bond with a notional amount of 1 and maturity at time2. Let λ be the risk-neutral probability of default, $(1-\lambda)$ the risk-neutral probability of survival, and RR the recovery rate in case of default. Thus, we derive the default process for the risky bond *B*, in the Figure 6

Combining the last two figures, we get the following quadruple tree:

Figure 7 A combined interest rate and bankruptcy process



The unique, risk-neutral or pseudo-probabilities λ and π guarantee that the prices P and B are martingales, this means that past events doesn't help to forecast the future, thus the model is arbitrage-free. Furthermore, the Markov[4] property allows displaying the combined interest rate and bankruptcy tree as a recombining tree. They also use a foreign exchange rate analogy to model the risky bond price B. The risky bond price at any time t with maturity T, $B_{t,T}$, is equal to the risk-free bond price $P_{t,T}$ multiplied with the "exchange rate" e, which is 1 in case of no default and equal to the recovery rate RR in case of default. Thus $B_{t,T} = P_{t,T}\, e_t$. If $E(e_T)$ is the expected payoff at time T, the risky bond price can be expressed as:

$$B_{t,T} = P_{t,T}E(e_T). \tag{7}$$

The previous equation states that the risky bond price is the expected payoff $E(e_T)$ discounted by the risk-free price $P_{t,T}$.

The shortcomings of the Jarrow-Turnbull 1995 model lie in the basic approach of the model: the direct economic reasons for default, i.e. the company's specific asset-liability structure or the company's liquidity are not part of the analysis. Rather, bond prices are the major input, assuming that bond prices can serve to reflect the credit risk of the debtor and to derive default probabilities. However, it has been shown that bond prices overestimate a company's probability of default quite substantially (Altman, 1989). In addition, bond prices are often illiquid, resulting in difficulties in determining a fair mid-market price.

[4] A stochastic process has the Markov property if the conditional probability distribution of future states of the process depends only upon the present state, not on the sequence of events that preceded it.

Additionally, it is assumed that the interest rate process and the default process are independent. Also, the default intensity is assumed constant, thus default is equally likely over the life of the debt. Last, the recovery rate of the model does not depend on the model variables, but is exogenous.

These shortcomings were addressed in extensions of the model, as in the Jarrow-Lando-Turnbull 1997 model.

The Jarrow-Lando-Turnbull 1997 model

(Jarrow, et al., 1997) derive default probabilities and valuation methods for credit derivatives not from rather illiquid bond prices, but on basis of historical transition probabilities. The analysis is done within the arbitrage-free martingale framework. However, Markov properties are not mandatory since the martingale transition probabilities, also termed risk-neutral, may depend on historical data up to the present. Let's first look at a historical default matrix, as shown in the next figure:

Figure 8 Average global cumulative historical default rates with respect to time

	Cumulative default rate (%)									
Years	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Aaa	0.0	0.0	0.0	0.04	0.12	0.21	0.3	0.4	0.52	0.64
Aa	0.02	0.03	0.07	0.16	0.26	0.36	0.46	0.57	0.65	0.73
A	0.02	0.09	0.22	0.36	0.51	0.68	0.86	1.07	1.31	1.56
Baa	0.22	0.61	1.08	1.69	2.25	2.81	3.38	3.94	4.58	5.26
Ba	1.28	3.51	6.09	8.76	11.36	13.74	15.66	17.6	19.46	21.29
B	6.51	14.16	21.03	27.04	32.31	36.73	40.97	44.33	47.17	50.01
Caa-C	23.83	37.12	47.43	55.05	60.09	65.22	69.26	73.88	76.50	78.54

Source: Moody's Investor Service, April 2003

We can get the annual default probability from the previous table, through taking the difference in the cumulative default probability for each entry. Doing so, we get the following table:

Figure 9 Average global annual default rates with respect to time

	Default rate (%)									
Years	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Aaa	0.0	0.0	0.0	0.04	0.08	0.09	0.09	0.10	0.12	0.12
Aa	0.02	0.01	0.04	0.09	0.10	0.10	0.10	0.11	0.08	0.08
A	0.02	0.07	0.13	0.14	0.15	0.17	0.18	0.21	0.24	0.25
Baa	0.22	0.39	0.47	0.61	0.56	0.56	0.57	0.56	0.64	0.68
Ba	1.28	2.23	2.58	2.67	2.60	2.38	1.92	1.94	1.86	1.83
B	6.51	7.65	6.87	6.01	5.27	4.42	4.24	3.36	2.84	2.84
Caa-C	23.83	13.29	10.31	7.62	5.04	5.13	4.04	4.62	2.62	2.04

Source: Moody's Investor Service, April 2003

From the previous table we can see that the historical default probability stays constant or increases slightly in time for highly rated credit. However, for low credits such as Caa, the probability of a default decreases with increasing time. This seems quite intuitive, since for a company with a bad rating, the coming years are the most crucial ones. Once they have passed, it can be assumed that the probability of default declines.

The last two tables only express the probability of a certain credit to move to default, i.e. to move to credit state, Jarrow, Lando and Turnbull use a *transition matrix* in their analysis. A transition matrix Λ shows the historical transition probability of a credit in state *i* to move to a credit in state *j,* within a certain time frame, thus

$$\lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1D} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2D} \\ \vdots & & & \\ \lambda_{D-1,1} & \lambda_{D-1,2} & \cdots & \lambda_{D-1,D} \\ 0 & 0 & & 1 \end{bmatrix}$$

Where the transition probabilities $\lambda_{ij} \geq 0$ for all i,j. The probability of default for a certain credit state i, $\lambda_{i,D}$, is in the last column of Λ. The probability of survival for a bond in rating class i, $Q_i = \sum_{j \neq D} q_{i,j} = 1 - \lambda_{i,D}$. The probability of remaining in the same credit state is on the diagonal and is $\lambda_{i,j} = 1 - \sum_{\substack{j=1 \\ i \neq j}} \lambda_{i,j}$.

The last row in Λ expresses that a credit that has defaulted stays in default. Hence, the transition probability 0, and the probability to stay in default is 1. In the following table, 82.83 reflects the probability of a credit, for instance a bond, which is currently rated A to stay in A; 0.47 reflects the probability of a bond that is currently rate A to migrate to B; 0.14 is the probability of a bond currently rated B to move to A.

Figure 10 One-year historical transition matrix of year 2002 (numbers in %)

		Rating at year-end								
		Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa**	**Default**	**WR**
Initial Rating	Aaa	86.82	7.75	0	0	0	0	0	0	5.43
	Aa	1.38	82.23	12.12	0.14	0	0	0	0	4.13
	A	0	2.18	**82.83**	8.86	1.01	0.47	0.08	0.16	4.43
	Baa	0.17	0.17	2.46	79.47	7.55	2.04	1.87	1.19	5.09
	Ba	0	0.18	0.18	2.39	72.38	13.26	2.03	1.47	8.10
	B	0	0	0.14	0.41	2.71	72.9	9.76	4.88	9.21
	Caa	0	0	0	0	0.34	3.42	56.85	27.74	11.64

Source: Moody's Investor Service, April 2003. WR represents companies that had been rated initially but are not rated at year-end

The next step is to transform historical default probabilities, derived from a transition matrix, into risk-neutral martingale probabilities in order to satisfy no-arbitrage conditions. This can be explained easier with an example.

Let's assume we have four rating classes, A, B, C and default D. Let S_{01A}, S_{01B}, and S_{01C} be the credit-spread, this is the difference between the yield of the risky bond and the yield of the risk-free bond, from time 0 to time 1 for a risky bond currently in rating class A, B, and C, respectively. Let's assume S_{01A}=0.01, S_{01B}=0.015, and S_{01C}=0.02, that can be rewrite in matrix form:

$$S_{01} = \begin{bmatrix} 0.01 \\ 0.015 \\ 0.02 \end{bmatrix}.$$

Let S_{02A}, S_{02B}, and S_{02C} be the credit-spread from time 0 to time 2 for a bond currently in rating A, B, and C, respectively. Let's assume S_{02A}=0.02, S_{02B}=0.025, and S_{02C}=0.03, hence:

$$S_{02} = \begin{bmatrix} 0.02 \\ 0.025 \\ 0.03 \end{bmatrix}.$$

Let's further assume the one-year historical transformation matrix:

$$S = \begin{bmatrix} & A & B & C & D \\ A & 0.70 & 0.15 & 0.10 & 0.05 \\ B & 0.10 & 0.60 & 0.20 & 0.20 \\ C & 0.05 & 0.15 & 0.65 & 0.15 \\ D & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix}$$

Though 0.7 is the probability of a bond currently rated A to stay in A; 0.2 is the probability of a bond currently rated B to be downgraded to C; 0.05 in the 2nd column and 4th row is the probability of a bond currently rated C to move to A. Let's assume the risk-free continuously compounded interest rate from time 0 to time 1, r_{01} = 5% and the risk-free continuously compounded interest rate from time 0 to time 2, r_{02} = 6%. The recovery rate *RR* is assumed to be 40%.

In the risk-neutral environment, we can express the risky zero-coupon bond price *B* at time *t* with maturity *T* and notional of €1 as the value of the discounted expected future cash-flow of 1. We discount with the risk-free interest rate *r* plus the swap spread *s*:

$$B_{t,T} = E_t\left[e^{-(r_{t,T}+s_{t,T})T}\right] \tag{8}$$

where E_t is the risk-neutral expectation value at time *t*, and *s* is the excess yield of the risky asset.

For a bond with a notional of € 1 that matures a time 1, the payoff at time 1 will be € 1 if the bond finishes in rating A, B, or C. The payoff will be the recovery rate *RR*, if the bond defaults. Including the historical default probabilities from the transition matrix, we can express the bond price *B* at time 0 with maturity 1, which is rated *A*, B_{01A} as:

$$B_{01A} = e^{-(r_{t,T}+s_{t,T})T} \equiv e^{-r_{01}}\begin{bmatrix} 1 & 1 & 1 & RR \end{bmatrix}\begin{bmatrix} A \rightarrow A \\ A \rightarrow B \\ A \rightarrow C \\ A \rightarrow D \end{bmatrix} \tag{9}$$

where A → A is the historical probability of a bond in rating class A to stay in A; A → B is the historical probability of a bond currently in class A to move to B; and so on. Continuing with our example, we can state:

$$B_{01A} = e^{-(0.05+0.01)\,1} \neq e^{-r_{01}}\begin{bmatrix}1 & 1 & 1 & 0.4\end{bmatrix}\begin{bmatrix}0.70\\0.15\\0.10\\0.05\end{bmatrix}$$

or

$$e^{-(0.05+0.01)\,1} = 0.9418 \neq e^{-0.05}\begin{bmatrix}1 & 1 & 1 & 0.4\end{bmatrix}\begin{bmatrix}0.70\\0.15\\0.10\\0.05\end{bmatrix}$$

$$= e^{-0.05} \times(1\times0.70 + 1\times0.15 + 1\times0.10 + 0.5\times0.05) = 0.9227$$

As we can state, this result in an inequality, it is important to know that the equation (9) is not usually satisfied in reality.

Now, in order to satisfy the no-arbitrage condition (9), we have to transform the historical transition probabilities into risk-neutral martingale probabilities, which satisfy the condition (9).

In order to find the martingale probabilities λ_m, we have to adjust the historical probabilities λ with a factor η. η can be interpreted as a risk premium or risk adjustment. We can rewrite equation (9) for a bond currently rated in class A as:

$$B_{01A} = e^{-(r_{t,T}+s_{t,T})T} \equiv e^{-r_{01}}\begin{bmatrix}1 & 1 & 1 & RR\end{bmatrix}\begin{bmatrix}1-(1-(A\to A))\lambda_A\\(A\to B)\lambda_A\\(A\to C)\lambda_A\\(A\to D)\lambda_A\end{bmatrix} \tag{10}$$

Generalizing the right side of the previous equation for a bond at time t with maturity T and solving for the risk adjustment of that bond in rating class i, η_i (we assume $i = \{A, B, C, D\}$), we get:

$$\lambda_i = \left\{1-\left(\frac{e^{r_{t,T}}}{e^{(r_{t,T}+s_{t,T})}}\right)^T\right\}\frac{1}{(1-RR)\lambda_{iD}} \tag{11}$$

where $\lambda_{i,D}$ is the probability of default of a bond in rating class i.

One specific shortcoming of the model is that the default probability $\lambda_{i,D}$ can become bigger than 1. This is especially the case for longer maturities *T*. Equation (11) can be reduced to:

$$\lambda_{iD} = \left\{1 - \frac{1}{e^{s_{t,T}T}}\right\} \frac{1}{(1-RR)\lambda_i}. \tag{12}$$

For this equation to be smaller than 1, we require that $\frac{1}{e^{s_{t,T}T}} > 1 + \lambda_i(RR - 1)$. This condition may not be satisfied for large s, *T*, η, and RR.

General shortcomings of the model lie again in the fact that the ultimate reason of default, the asset-liability structure or the liquidity of a company, is not part of the analysis. Also, as in the 1995 model, the interest rate process and the bankruptcy process are assumed independent. Furthermore, the recovery rate RR is exogenously given.

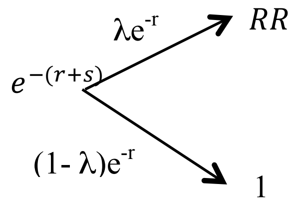
Naturally, the nature of the transition matrix also bears problems. Jarrow, Lando and Turnbull assume that bonds in the same credit class have the same yield spread. This is not necessarily the case as pointed out by (Longstaff & Schwartz, 1995). Rather, the rating-yield relationship is similar within sectors, which suggests conducting sector analysis, rather than aggregating data generally among counterparties.

An additional problem is that ratings are often done infrequently and may not be recent enough to reflect current counterparty risk. In addition, Standard & Poors currently only rates about 1% of all companies worldwide. Nevertheless, the number of rated companies should increase in the future, allowing a widespread usage of the model and its extensions.

Duffie and Singleton (1999)

(Duffie & Singleton, 1999) express the risky bond price *B* at time *t* with maturity *T* based on equation $B_{t,T} = E_t\left[e^{-(r_{t,T}+S_{t,T})T}\right]$. In the Duffie-Singleton model, the swap spread $S_{t,T}$ equals approximately $\lambda_i(1 - RR)$. This result can be derived by a simple binomial tree for a zero-coupon bond with maturity at time 1 and a notional amount of € 1, as shown in the next figure:

Figure 11 Deriving the swap spread s



In the last figure, r is the risk-free interest rate, s is the swap spread, λ is the *hazard rate*, which multiplied by time periods for default of 1 equals the risk-neutral probability of default. *RR* is the recovery rate. Then, we can derive:

$$e^{-(r+s)} = \lambda e^{-r} RR + (1-\lambda)e^{-r}. \tag{13}$$

Solving the last equation for s, using $e^x \approx 1 + x$, we get $s \approx \lambda(1-RR) + \lambda r(1-RR)$. Duffie and Singleton prove than the term $\lambda r(1-RR)$ can be dropped for a continuous time setting. Hence, the interest rate process drops out and we can write for a default swap spread from time *t* to time *T*, $S_{t,T}$:

$$S_{t,T} \approx \lambda_{t,T}(1-RR) \tag{14}$$

Where all variable are viewed at time *t*.

The last equation shows the intuitive approximate relationship between the swap spread s and a hazard rate λ. If the recovery rate *RR* is zero, $s_{t,T} \approx \lambda_{t,T}$. Hence the spread s approximately compensates the investor for the default risk λ. The relationship in the equation $S_{t,T} \approx \lambda_{t,T}(1-RR)$ is often termed *credit triangle*, since two of the three variables are sufficient to generate the third.

The model may include a liquidity premium *l* for the risky asset. In this case the swap spread is simply:

$$S_{t,T} \approx \lambda_{t,T}(1-RR) + l \tag{15}$$

where *l* is a fractional value of the risky bond.

Duffie and Singleton show that any risky claim *B* with an notional amount *N*, for different interest rates r and swap spreads s at various times *j*, and time units of 1, with maturity t + τ, can be expressed as:

$$B_{t,t+\tau} = E_t\left[e^{-\sum_{j=0}^{\tau-1}(r_{t+j}+s_{t+j})} N_{t+\tau}\right] \tag{16}$$

Hence, one crucial finding of the Duffie-Singleton model is that any risky claim *B* can be priced by discounting the notional amount *N* with the default-adjusted process r+s.

The equation $B_{t,t+\tau} = E_t\left[e^{-\sum_{j=0}^{\tau-1}(r_{t+j}+s_{t+j})}N_{t+\tau}\right]$ is an extension of the equation $B_{t,T} = E_t\left[e^{-(r_{t,T}+s_{t,T})T}\right]$.

In the last equations, the recovery rate RR is applied to the expected market value of the risky bond at the time of default, termed *recovery of market value RMV*, hence $E_d(RMV_{d+1}) = RR_d E_d(B_{d+1})$, where d+1 is the time of default. In contrast, in the Jarrow-Turnbull 1995 and Jarrow-Lando-Turnbull 1997 model, the recovery value is a fraction of the risk-free bond price at the time of default. Brennan-Schwartz (1980), Longstaff-Schwartz (1995), and Duffie (1998) apply a simpler assumption with respect to the payoff in default. They assume that creditors at the time of default receive the recovery rate multiplied with the notional amount of the risky bond.

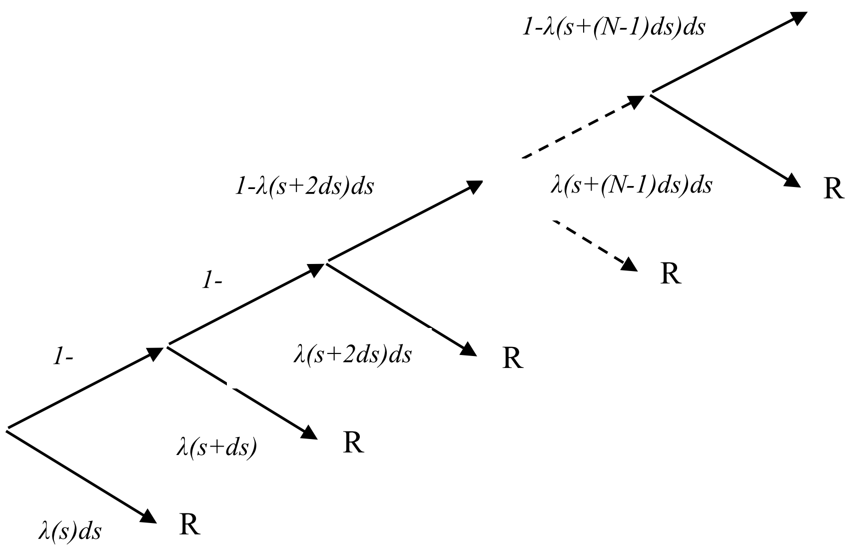
O'Kane and Turnbull (2003)

(O' Kane & Turnbull, 2003) presents in their paper a market standard pricing model. Their approach is based on the work of Jarrow and Turnbull (1995), who characterized a credit event as the first event of a Poisson counting process which occurs at some time τ with a probability defined as:

$$Pr[\tau < t + dt | \tau \geq t = \lambda(t)dt] \tag{17}$$

ie, the probability of a default occurring within the time interval $[t, t + dt]$ conditional on surviving to time *t*, is proportional to sometime dependent function *λ(t)*, known as the hazard rate, and the length of the time interval *dt.* We can therefore think of modelling default in a one-period setting as a simple binomial tree in which we survive with probability *1-λ(t)dt*, or default and receive a recovery value R with probability *λ(t)dt*. O' Kane and Turnbull make the simplifying assumption that the hazard rate process is deterministic. By extension, their assumption also implies that the hazard rate is independent of interest rates and recovery rates.

Figure 12 The equivalent of a binomial tree in the modelling of default in which the tree terminates and makes a payment K at default



Extending this model to continuous time survival probability to time T conditional on surviving to valuation time, t_v, by considering the limit ds→0.

Survival probability can be shown as

$$Q(t_v, T) = \exp\left(-\int_{t_v}^{T} \lambda(s)ds\right). \tag{18}$$

This model is used to value both the premium and protection legs, and then the breakeven spread of a default swap. With this model, we can get the implied term structure of arbitrage-free survival probabilities from market spreads.

In order to value the *premium leg*, this is the series of payments of the default swap spread made to maturity or to the time of the credit event, which occurs first, and ignoring the accrued premium payment from the previous premium payment date until the time of the credit event, the present value of the premium leg can be written as:

$$Premium\ Leg\ PV(t_v, t_N) = S(t_0, t_n) \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B) Z(t_v, t_n)\, Q(t_v, t_n). \tag{19}$$

where there are $n=1,\dots,N$ contractual payment dates $t_1,\dots,t_N$ and t_N is the maturity date of the default swap, with the spread $S(t_0, t_n)$ between today and the maturity date. $\Delta(t_{n-1}, t_n, B)$ is the day count fraction between premium dates t_{n-1} and t_n in the appropriate basis convention denoted by B. $Q(t_v, t_n)$ is the arbitrage-free survival probability of the reference entity from the valuation time t_v to premium payment time t_n. $Z(t_v, t_n)$ is the Libor discount factor from valuation date to premium payment date n.

In order to include the effect of premium accrued, we have to work out the expected accrued premium payment by considering the probability of defaulting at each time between two premium dates, and calculating the probability weighted accrued premium payment, resulting in the following expression for the premium leg:

$$S(t_0, t_n) \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \Delta(t_{n-1}, s, B) Z(t_v, s)\, Q(t_v, s) \lambda(s) ds. \tag{20}$$

This integral makes complicated expression to evaluate exactly. Though, as demonstrated by O' Kane and Turnbull, it is possible to approximate this equation with

$$\frac{S(t_0,t_n)}{2} \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B) Z(t_v, t_n) \left(Q(t_v, t_{n-1}) - Q(t_v, t_n)\right) \tag{21}$$

by noting that if a default does occur between two premium dates, the average accrued premium is half the full premium due to be paid at the end of the premium period. The full value of the premium leg is then given by

$$S(t_0, t_N) \times RPV01 \tag{22}$$

where RPV01 is the risky PV01 defined as

$$RPV01 = \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B) Z(t_v, t_n) \left[Q(t_v, t_n) + \frac{1_{PA}}{2}\left(Q(t_v, t_{n-1}) - Q(t_v, t_n)\right)\right] \tag{23}$$

where 1_{PA}=1 if the contract specifies premium accrued (PA) and 0 otherwise.

The effect of premium accrued on the spread can be very well approximated by

$$\frac{S^2}{2(1-R)f}$$

The protection leg is the contingent payment of *(100%-R)* on the face value of the protection made following the credit event. *R* is the expected recovery rate. In pricing the protection leg, it is important to take into account the timing of the credit event, because this can have a significant effect on the present value of the protection leg – especially for longer maturity default swaps. Within the hazard rate approach we can solve this timing problem by conditioning on each small time interval $[s, s + ds]$ between time t_v and t_N at which the credit event can occur. The expected present value of the recovery payment is:

$$(1-R) \int_{t_v}^{t_N} Z(t_v, s) Q(t_v, s) \lambda(s) ds \tag{24}$$

where, *R* is the expected recovery price of the *cheapest to deliver* asset at the time of the credit event, $Q(t_v, s)$ is the probability of surviving to some future time *s*. $Z(t_v, s)$ is the risk free rate between today and the future time *s*, and $\lambda(s)ds$ is the probability of a credit event in the next small time increment *ds*.

From now on, it is possible to get the survival probabilities from the market quoted default swap spread, through

PV of Premium Leg = PV of the Protection Leg

Other significant reduced form models that have received recognition are Brennan-Schwartz (1980); Iben-Litterman (1991); Longstaff-Schwartz (1995); Das-Tufano (1996); Duffee (1996); Schoenbucher (1997); Henn (1997); Brooks-Yan (1998); Madan-Unal (1998); Duffee (1998); Das-Sundaram (2000); Hull and White (2000); Hull and White (2001); Wei (2001); Duffie-Lando (2001); and Jarrow-Yildirim (2002); Martin, Thompson and Browne (2003)

Analysis Framework

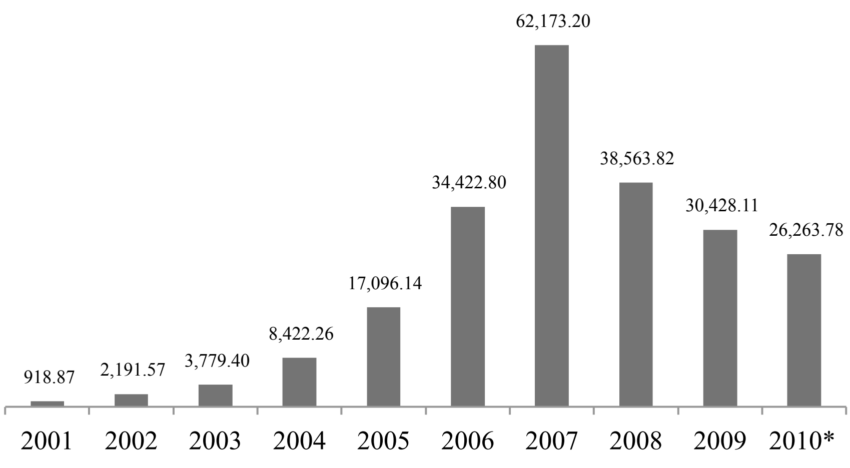
Credit Default Swaps in numbers

In 1998, International Swaps and Derivatives Association (ISDA®) facilitated the CDS trading, through standard documentation and procedures, allowing credit risks to be traded and managed in much the same way as market risks. In 2009, the market saw a "Big Bang" for CDS contracts and the way in which they are traded, including important convention changes in order to make CDS more standardised to help support efforts for central clearing of CDS trades.

According to the ISDA® market survey, the notional amount outstanding of CDS had an outstanding expansion in the last decade, rising from US$920 billion at 2001 to US$62.2 trillion at 2007. This breakthrough growth was interrupted by a financial turmoil starting in 2007, caused by a subprime mortgage crisis, and then by the Lehman and Brothers, AIG and Bern and Stearns bankruptcy, downing to US$26.3 trillion at mid-year 2010, a decrease of 57.8% from year-end 2007.

As in past surveys held by ISDA®, the US$26.3 trillion notional amount was approximately evenly divided between bought and sold protection: bought protection notional amount was approximately US$13.3 trillion and sold protection was about US$13.0 trillion, with a net bought notional amount of US$359.0 billion, representing 5.6% of the total derivatives reported to the ISDA® Market Survey.

Figure 13 Outstanding Credit Default Swaps. Notional amounts in billions of US dollars, adjusted for double counting



Source: ISDA® Market Survey

According to ISDA® CDS Marketplace™, in May 5th 2012 the total par amount of credit protection bought or sold was around US$14.9 trillion, 81% was related to corporates and the rest to sovereigns. More than 91% of this amount, mentioned as gross notional value in ISDA® studies was concentrated in Europe and America.

It is important to refer the overall amount of credit risk in the financial system does not increase with this significant size of the credit derivatives market, because every credit derivative contract has a buyer and a seller of the credit risk and so there is no net increase of credit risk. In some situations credit derivatives can increase the amount of credit risk in the *capital markets*, due to the counterparty credit risk associated with each contract. This is the risk that the protection seller does not pay the compensation in case of default to the protection buyer.

The equivalence relation between CDS and bond yields

In our study, we will use the reduced-form hence it offers a suitable framework to connect bond spreads with CDS *premia*. Using the risk neutral default probability and no-arbitrage conditions, it is direct to establish the parity link between the two spreads, which will be used as the testable hypothesis in the empirical part of the thesis.

This framework, initially developed by (Duffie, Credit swap valuation, 1999), is simplified by assuming that the risk-free rate (r) is constant over time. The protection buyer of a CDS must pay a constant premium (s) until the contract matures or a credit event occurs to the protection seller. If a credit event does occur, the protection buyer receives the difference between the cheapest-do-deliver bond and the face value, and must pay accrued CDS premium upon default. For simplicity, we will assume the recovery value as the difference between the par value and the market value and there is no accrued premium after default.

Assuming no-arbitrage conditions, we can replicate synthetically the acquisition of a CDS through shorting a par fixed coupon bond, with a coupon rate of (c) on the same reference entity with the same maturity date, and investing the proceeds in a par fixed coupon risk-free bond. Therefore, the CDS premium should be equal to the credit spread of the par fixed coupon bond. That is,

$$s = c - r \tag{25}$$

If this parity relationship is violated, the arbitrageur can take profits. This is, if $s > c - r$, an investor can sell the CDS in the derivatives market, buy a risk-free bond and sell the bond of the reference company in the cash market, resulting in arbitrage returns.

If $s < c - r$, the investor can implement the reverse strategy in order to collect profits.

Meanwhile, this equilibrium may not hold, because some of the key assumptions may not be satisfied in practice. First, the protection buyer normally needs to pay the accrued premium when the credit event occurs, making the CDS premium to be smaller after taking into the account of this accrued payment. Second, the existence of the cheapest-to-deliver option, hence the majority of the CDS contracts are settle via physical delivery, resulting in an increase of the CDS *premia*. Third, the definition of credit event is not unanimous, making harder the valuation of a credit event. Fourth, there is no initial exchange of cash-flows in a CDS transaction, in contrast to the bond market. This difference could cause the CDS market to respond faster than the bond market to changes in the underlying credit risk, generating price discrepancies in the short run. Fifth, short-sale of corporate bonds is practically not allowed, making difficult to gain from the price difference when the CDS premium is higher than the bond spread. This asymmetry may have important implications for the credit spreads adjustment dynamics. Sixth, transaction costs will reduce the number of arbitrage opportunities between the two markets. Lastly, the two spreads may be influenced differently by other factors than credit risk, such as liquidity and/or counterparty risk.

Consequently, in our study we will assume that:

1. Market participants can short single name corporate bonds. This means that bond holders can sell bonds, buy riskless bonds and sell default protection when $s > c - r$;
2. Market participants can short riskless bonds. This is equivalent to assuming that market participants can borrow at the risk free rate;
3. We ignore the *cheapest-to-deliver bond* option in a CDS;
4. There is no counterparty default risk in a CDS;
5. The circumstances under which the CDS pays off are those defined by ISDA®;
6. We ignore that there may be tax and liquidity reasons that cause investors to prefer a riskless bond to a corporate bond plus a CDS or vice versa;
7. CDS gives the holder the right to sell a bond for its face value;
8. We ignore transaction costs.

Estimation of risk-neutral default probabilities from CDS spreads

As we state previously, we will use the *reduced form* to modelling credit events and estimate our risk-neutral default probabilities from CDS spreads through optimization algorithm, based on the work of (Martin, Thompson, & Browne, 2001). Thus, we will assume the default process to follow an non-homogeneous Poisson process and as such for any $0 \leq \tau \leq T$ the default time t and default intensity $\lambda(t)$ satisfy

$$Q_S(t) = \mathbb{P}(\tau > t) = exp\left(-\int_0^t \lambda(u)\mathrm{d}u\right) \quad (26)$$

where $\mathbb{P}$ is the risk-neutral probability and T is the final maturity. The single name survival probabilities $\mathbb{P}(\tau > t)$ are usually implied from the CDS market.

The fair CDS spread balances the present value of the contingent leg C, given by

$$C = N(1-R)\sum_{i=1}^{n} d(t_i)(Q_s(t_{i-1}) - Q_s(t_i)) \quad (27)$$

and the present values of the fee leg F is given by

$$F = NS(\sum_{i=1}^{n} Q_s(t_i)d(t_i)\Delta t_i + A_D)) \tag{28}$$

where, N is the CDS notional and AD is the accrual on default, defined by

$$A_D = \frac{1}{2}\sum_{i=1}^{n} Nd(t_i)(Q_s(t_{i-1}) - Q_s(t_i))\Delta t_i. \tag{29}$$

In these equations the summations run over the payment dates, N is the notional, S is the spread premium on a yearly basis, paid quarterly, $Q_s(t_i)$ is the survival probability at time t_i, R is the recovery rate, $d(t)$ is the risk-free discount factor and Δt is the fraction of the year, corresponding to $\Delta t_i = t_i - t_{i-1}$ an Actual/360 day counter. The standard maturity dates on corporate CDS contract are the 20th of March, June, September and December. The fee leg can be written as

$$F = NSB, \tag{30}$$

where $B = BVP(0, T)$ is the present value of 1 basis point (bp) paid from time zero until maturity, T. It is given by

$$BVP(0,T) = \sum_{i=1}^{n} Q_s(t_i)d(t_i)\Delta t_i + \frac{1}{2}\sum_{i=1}^{n} N^{CDS}d(t_i)(Q_s(t_{i-1}) - Q_s(t_i))\Delta t_i. \tag{31}$$

At this time we can determine the survival probability in (5) from the observed CDS market quotes, through the optimization method to determine all the probabilities of default simultaneously. At a start of a CDS in non-arbitrage equivalence, both sides, the fee and the contingent legs, must be equal and from (6) and (7) we have that the spread at time zero is assumed by

$$S = \frac{(1-R)\sum_{i=1}^{n} d(t_i)(Q_s(t_{i-1}) - Q_s(t_i))}{\sum_{i=1}^{n} Q_s(t_i)d(t_i)\Delta t_i + A_D} \tag{32}$$

At beginning all CDS quotes follows (11) for all maturities, therefore in an optimization process the set of survival probabilities at each point in time is bound such that the error in recovering the observed CDS quotes is minimized and probabilities are constrained between zero and one. Assuming $Y\ [Q_S(t_1), Q_S(t_2), \ldots, Q_S(t_n)]$ is a possible set of survival probabilities, S_i are the market CDS spreads at time zero with maturity T_i, Z_i is the value of spread using Y. The objective function $T\ [Q_S(t_1), Q_S(t_2), \ldots, Q_S(t_n)]$ to be minimized is given by

$$T\ [QS(t1), QS(t2), \ldots, QS(tn)] = \frac{1}{2}\sum_{i=1}^{m}\left(\frac{S_i - Z_i}{\sigma_i}\right)^2 + \mu\sum_{i=0}^{n-1} d^2(q_{i+1}, q_i), \tag{33}$$

where $d(q_i, q_{i+1})$ is a probability distance measure given by

$$d(q_{i+1}, q_i) = \sqrt{(q_{i+1} - q_i)ln\left(\frac{q_{i+1}}{q_i}\right) - (q_i - q_{i+1})ln\left(\frac{1-q_{i+1}}{1-q_i}\right)}. \tag{34}$$

The weights σ_i adjusts the importance given to the different quotes when implying the probabilities and the parameter μ controls the importance of the continuity part of the error function. In our study, we will follow the (Martin, Thompson, & Browne, 2001) suggestion and use $\mu = 10$, which gives a very good fit of the market data, and fix the five and then year contracts to σ_i to 10^{-2} and the three and seven year contracts we will fix σ_i to 10^{-4}, hence the five and ten year contracts have usually been more liquid than the three and seven year contracts. The $d(q_{i+1}, q_i)$ is an *entropic distance measure* that assures smoothness between two adjacent points in time of the probability distribution. In practice, when a probability moves close to zero or one, the parameter d will increase guaranteeing that the probabilities stay constrained.

Building a hazard rate term structure

The standard modelling assumption used in the credit default swap market is to assume that the hazard rate is a piecewise flat function of maturity time. This is an entirely reasonable assumption because, given only one data point, it is not possible to extract more than one piece of information about the term structure of hazard rates.

Given 1Y, 3Y, 5Y, 7Y and 10Y default swap spread values, we would assume that we have a hazard rate term structure with five sections $\lambda_{0,1}$, $\lambda_{1,3}$, $\lambda_{3,5}$, $\lambda_{5,7}$ and $\lambda_{7,10}$. The process of constructing the term structure of hazard rates is the *bootstrapping method*. It starts with taking the shortest maturity contract and using it to calculate the first survival probability. In this case, the 1Y default swap has to be used to calculate the value $\lambda_{0,1}$. Assuming a quarterly premium payment frequency, using a value of M=12, and assuming that premium accrued is not paid, this is achieved by solving

$$\frac{S(t_v,t_v+1Y)}{1-R}\sum_{m=3,6,9,12}\Delta(t_{n-3},t_n,B)Z(t_v,t_n)e^{-\lambda_{0,1}t_n} = \sum_{m=1}^{12} Z(t_v,t_m)\left(e^{-\lambda_{0,1}t_{m-1}} - e^{-\lambda_{0,1}t_m}\right) \quad (35)$$

This equation can be solved using a one-dimensional root-searching algorithm[5]. This procedure is then repeated to solve for $\lambda_{1,3}$ and so on until the final maturity default swap is reached. Beyond this, it is often assumed that the hazard rate is flat. Defining τ=T-t_v, we have:

$$Q(t_v - T) = \begin{cases} \exp(-\lambda_{0,1}\tau) & if\ 0 < \tau \le 1 \\ \exp(-\lambda_{0,1}\tau - \lambda_{1,3}(\tau-1)) & if\ 1 < \tau \le 3 \\ \exp(-\lambda_{0,1}\tau - 2\lambda_{1,3} - \lambda_{3,5}(\tau-3)) & if\ 3 < \tau \le 5 \\ \exp\left(-\lambda_{0,1}\tau - 2\lambda_{1,3} - 2\lambda_{3,5} - \lambda_{5,7}(\tau-5)\right) & if\ 5 < \tau \le 7 \\ \exp(-\lambda_{0,1}\tau - 2\lambda_{1,3} - 2\lambda_{3,5} - 2\lambda_{5,7} - \lambda_{7,10}(\tau-7)) & \tau > 7 \end{cases}$$

The second approach uses an optimization algorithm to determine all the probabilities at the same time. (Martin, Thompson, & Browne, 2001) present an algorithm for the

[5] Bisection or gradient-based methods such as Newton-Raphson. This method is used for finding successively better approximations to the roots.

optimization, that assumes the absence of arbitrage at the start of a CDS, which means that both sides, the premium leg and protection leg must be equal, this means, that the spread at time zero is given by:

$$S(t_v, t_v + 1Y) = (1 - R)\frac{\sum_{i=1}^{12} Z(t_v,t_m)(e^{-\lambda_{0,1}t_{m-1}} - e^{-\lambda_{0,1}t_m})}{\sum_{m=3,6,9,12} \Delta(t_{n-3},t_n,B)Z(t_v,t_n)e^{-\lambda_{0,1}t_n}} \tag{36}$$

We consider there are CDS quotes associated with several maturities, with several maturities, for instance 3, 5, 7 and 10-year maturities.

In an optimization algorithm the set of survival probabilities at each point in time is determined such that the error in recovering the observed CDS quoted is minimized, with probabilities constrained between 0 and 1.

With our example, for 1Y, 3Y, 5Y, 7Y and 10Y default swap spread values, we would assume that we have a possible set of hazard rates Y [$\lambda_{0,1}$, $\lambda_{1,3}$, $\lambda_{3,5}$, $\lambda_{5,7}$ and $\lambda_{7,10}$]. S_i are the market CDS rates at time zero with maturity t_N. Z_i is the value of the spread using Y. The objective function T [$\lambda_{0,1}$, $\lambda_{1,3}$, $\lambda_{3,5}$, $\lambda_{5,7}$ and $\lambda_{7,10}$] will be minimized with:

$$\mathrm{T}\,[\lambda_{0,1}, \lambda_{1,3}, \lambda_{3,5}, \lambda_{5,7}, \lambda_{7,10}] = \frac{1}{2}\sum_{i=1}^{m}\left(\frac{S_i - Z_i}{\sigma_i}\right)^2 + \mu \sum_{i=0}^{n-1} d^2(p_{i+1}, p_i)\,. \tag{37}$$

where $d(p_{i+1}, p_i)$ is a probability distance measure given by:

$$d(p_{i+1}, p_i) = \sqrt{(p_{i+1} - p_i)ln\left(\frac{p_{i+1}}{p_i}\right) + (p_i - p_{i+1})ln\left(\frac{1-p_{i+1}}{1-p_i}\right)} \tag{38}$$

The weights σ_i serve to adjust the importance given to the diferente quotes when implying the probabilities, because some contracts are more liquid than others. The $d(p_{i+1}, p_i)$ serves to guarantee the smoothness betweem two adjacent points in time of the probability distribution. The importance of the continuity part of the error function is controlled via the parameter μ.

Equating both methods we can say that the bootstrap is easier to implement when comparing with the optimization method, meanwhile it has their drawbacks. In the bootstrap method, if there is any unreliable quote due to illiquidity, any mistakes on the determination of the short-term probabilities are propagated to the subsequent ones. Additionally, depending on the form of the CDS curve, one may come up with negative probabilities.

Regarding the optimization method, the algorithm searches for the distribution of probabilities that minimize CDS errors and observe some constrains, for example that probabilities remain positive and smooth. It can also control which quotes are reliable and which are not. In our study, we use the optimization method.

Estimation of the risk-free rates using German Treasuries

Choosing the risk-free rate may not be a simple task. Bond traders tend to regard the Treasury zero curve as the risk-free zero curve. By contrast, derivatives traders working for large financial institutions tend to use the swap zero curve (sometimes also called the LIBOR zero curve) as the risk-free zero curve in their pricing models because they consider LIBOR/swap rates to correspond closely to their opportunity cost of capital.

The choice of the Treasury zero curve as the risk-free zero curve is based on the argument that a bond issued by a government in its own currency has no credit risk so that its yield should equal the risk-free rate of interest. However, there are many other factors such as liquidity, taxation and regulation that can affect the yield on a bond. For example, the yields on German Treasury bonds tend to be much lower than the yields on other instruments that have very low credit risk. One reason for this is that Treasury bonds have to be used by financial institutions to fulfil a variety of regulatory requirements. A second reason is that the amount of capital a financial institution is required to hold to support an investment in Treasury bonds is substantially smaller than the capital required supporting a similar investment in low risk corporate bonds. A third reason is that the interest on Treasury bonds is not taxed at the state level whereas the interest on other fixed income investments is taxed at this level. For all of these non-credit-risk reasons, the yields on German Treasury bonds tend to be depressed relative to the yields on other low risk bonds.

Thus, since zero-coupon rates are rarely directly observable, they have to be estimated from market data. In this thesis we used the parametric term structure estimation method, the Nelson-Siegel-Svensson model.

Calculating the basis

The CDS basis is the difference between the CDS spread (derivative market) and the asset swap spread (cash market) of the same reference asset and in theory should be zero.

$$\text{Basis}_{i,n} = \text{CDS spread}_{i,n} - \text{ASW spread}_{i,\,n} \tag{39}$$

where *i* is the reference underline and *n* is the period.

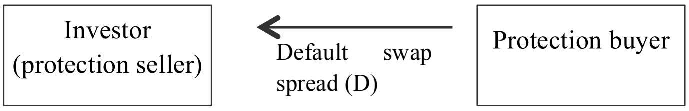
The basis is positive when CDS is higher than ASW spread, and negative when ASW spread is higher than CDS. The difference between both spreads happens because of funding and optionality[6].

Usually, the basis is positive, but after the subprime crisis, we have assisted to a generally negative basis for the major *names* due to this funding crisis.

[6] Cadete, Joaquim, 2012. Master in Finance Lectures, Session 2 slides.

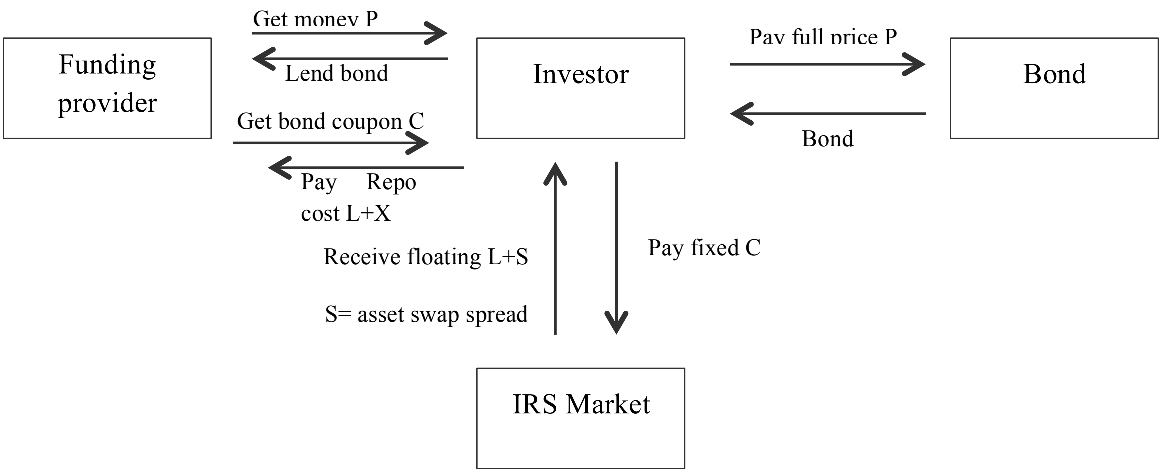
Imagine that one investor takes on credit risk and gets paid the default swap spread "D":

Figure 14 Price of credit risk in the *default swap market*



We can replicate the CDS in the *asset swap market*.

Figure 15 Replication of the CDS in the *asset swap market*



Netting out the cash-flows, shows that the investor is paid (S-X) bps for taking on the bonds credit risk. If Libor flat funding is assumed (i.e. X=0), then the *asset swap spread* is a close proxy for the *default swap spread*.

Most corporate bonds fund below Libor, X is negative. The breakeven default swap spread then becomes:

$$\text{Default Swap Spread} \approx \text{Asset Swap Spread} + \text{Funding Cost} \qquad (40)$$

In addition, the protection seller has sold a number of options to the protection buyer and will need to be compensated for it with a wider default swap spread:

$$\text{Default Swap Spread} = \text{Asset Swap Spread} + \text{Funding Cost} + \text{Optionality Cost} \qquad (41)$$

Several factors can increase or decrease the basis, namely:

Factors that increase the basis:

- *Natural market*: The default swap market has grown into the natural market to hedge credit risk, especially for 3- to 5-year maturities many credit hedgers use the liquid default swap market rather than the asset swap market, driving default premiums up;

- *Convertible bond arbitrage:* Hedge funds and other financial institutions strip the credit risk from the convertible bond and hedge it with default swaps to concentrate on managing the equity option;
- *The delivery option:* In a default swap the delivery option allows the protection buyer to choose delivery from a pre-defined pool of assets, which increases de value of default swaps compared to asset swaps;
- *Default criteria:* Default criteria are clearly defined by ISDA®, which facilitate trading. Also, default swap payments may be triggered by events, which do not constitute default in the cash market.

Factors that decrease the basis:

- *Counterparty risk:* The default swap buyer is exposed to higher counterparty credit risk than the asset swap payer, since in an asset swap two cash-flows of similar value are exchanged on a regular basis;
- *Marking-to-market in default:* In case of default, it is typically quite difficult to mark-to-market an asset swap. Default swaps are designed to function in a default, so their marking-to-market is typically easier to achieve. This might drive asset swap spreads up, since the asset swap fixed rate payer, who will suffer a financial loss in the event of default, might want to be compensated for the higher uncertainty with receiving a higher spread.

In order to determine the ASW in our study, whenever possible, we have selected bonds with expected life between 4 and 6 years, hence the basis tend to zero when maturity approximates.

In order to match life of both CDS and ASW spread, the ASW spread of each *single-name* was found through the cubic splines interpolation.

Finally, when quotes where not available, we used the last available price, because we assumed that there was no new information that could influence the price.

Determinants of basis spreads

Besides (Zhu, 2006), the most relevant literature along this matter are (Blanco, Brennan, & Marsh, 2005) and (Longstaff, Mithal, & Neis, 2005). (Blanco, Brennan, & Marsh, 2005) show that for most entities, the parity relationship between the two credit spreads holds on average over time, but extensive differences can arise in the short run either, because of inadequacies in contract terms or due to a clear lead for CDS *premia* over bond spreads. (Longstaff, Mithal, & Neis, 2005) uses weekly data and find that price discrepancies between bond spreads and CDS prices can be largely explained by measures of individual corporate bond illiquidity. (Cossin & Hricko, 2001) shows that the determinants of CDS *premia* are quite similar to those of bond spreads, including ratings, yield curves, stock prices and leverage ratios. (Hull, Predescu, & White, 2004) and (Houwelling & Vorst, 2005) compare the pricing of credit risk between the CDS market and the bond market, they also suggest that, when using swap rates as benchmark risk-free rates, the price differences between bond spreads and CDS *premia* are quite small, around 10 basis points. (Hull, Predescu, & White, 2004) and (Norden & Weber, 2004) find robust evidence that the CDS market anticipates credit rating announcements, particularly negative rating events.

According to (Zhu, 2006), price differences between the money market and the derivatives market are very small on average over time, there is substantial variation over time and cross entities, that could be explained by the reasons described previously in the section "The equivalence relation between CDS and bond yields". We will use the regression technique to investigate the determinants of basis spread movements: The explanatory variables include:

- *Ratings and rating events.* (Houwelling & Vorst, 2005) suggest that the price discrepancy could be different for high-grade and low-grade bond issues. Intuitively, a same level of absolute price differential is proportionally less important for low-grade bond issuers, because the credit spread is higher. In this study we include the time series of the *Standard & Poors* (*S&P*) rating for each single name. The rating categories AAA, AA+, AA,…,CCC+ are transformed into the numbers 1,2,3,…,17. Another issue of interest is whether the bond market and the CDS market have different predicting power over future rating events. (Hull, et al., 2004) and (Norden & Weber, 2004) pointed out that the derivatives market tends to anticipate future rating events, especially between the 90th and the 60th day before of a rating downgrade or upgrade event, and during the 10 days after the credit event, bond spreads increase or decrease more substantially, offsetting the price discrepancies accumulated before (Zhu, 2006). This implies that the derivatives market does a better job in incorporating future rating events into the price. To examine this issue, we include five dummy variables, following (Hull, et al., 2004), that can capture the impact of rating actions: *DUMB6190*, *DUMB3160* and *DUMB0130* represent a rating event occurring on future days [t + 61, t + 90], [t + 31, t + 60] and [t + 61, t + 90], respectively. *DUMA0110* and *DUMA1130* represent a past rating event during [t

– 1, *t* - 10] and [*t* – 11, *t* - 30]. In each of the dummy variables a value of 1 refers to a downgrade of the rating, -1 to an upgrading and 0 to no action;

- *The CDS Big Bang event*. In April 08 2009, we assisted to the introduction of new forms of CDS contracts, expressed as upfront payments plus fixed coupon. In order to capture and analyse the impact of this event we create a dummy variable: *DUMBIG*. The value of 1 refers to after the event and 0 to before.
- *The Lehman & Brothers Bailout*. In September 15 2008, we assisted to the Lehman & Brothers bailout. This event had impact in all names. In order to capture and analyse the impact of this event we create a dummy variable: *DUMLEHMAN*. The value of 1 refers to after the event and 0 to before.
- *Liquidity.* Both CDS *premia* and bond spreads may be influenced by factors unrelated to the underlying credit risk. We use the bid-ask spread of CDS-5 year contract differential to represent the relative liquidity between the two instruments. To be more specific, this measure is defined as the average bid-ask spread in the CDS market in the past 20 business days.
- *Macroeconomic conditions.* To test the pricing accuracy, we will include one macro-financial variable. Regional stock market index (*EURO STOXX 50* in Europe), because this variable reflects the performance of the macroeconomy and thus have an impact on the pricing of credit risk. However, if both markets are equally efficient in pricing the changes in macro-financial conditions, their impact on basis spreads should be zero.

Our study differs slightly from Zhu work hence we do not test the *lagged basis spreads* and *changes in credit spreads* as determinants. Meanwhile, we introduce two new variables, the *Big Bang Event* and *Lehman and Brothers Event*, as possible determinants of basis spreads.

Data set

The period of analysis is from 1 January 2007 to 30 November 2012. We first group the quotes by the characteristics of reference entities, including company names, currency of denomination, maturity and seniority, treating different currency denominated bonds as two distinct entities. The following two filtering criteria are then used: (i) the entity is a bank, a corporate or a sovereign; (ii) there are at least 150 days with valid quotes for the contract during the period of analysis; (iii) we choose only denominations in US dollars (USD) or euros (Eur). The filtering leaves 96 entities to start with. We then construct the time series of daily CDS quotes for those entities, which are defined as the middle point of average bid and average offer on each day.

For each of the chosen reference entities, we retrieve the information for all bonds outstanding during the analysis period. In order to avoid measurement errors caused by various options in corporate bonds, we choose only bond issues that satisfy the following restrictions: (i) bonds must not be puttable, callable, convertible or reverse convertible; (ii) bonds must be denominated in the same currency as the CDS contract; (iii) bonds must not be subordinated, structured or company guaranteed; (iv) the coupon payments must be fixed-term.

Then, we selected one company per activity sector (10), mainly German, which led to the following subset of companies:

Figure 16 Analysed companies

Company name	Sector
Bayer AG	Basic Materials
Deutsche Telekom AG	Communications
Deutsche Lufthansa AG	Consumer Cyclical
Unilever NV	Consumer Non-cyclical
LVMH Moet Hennessy Louis Vuitton SA	Diversified
Total SA	Energy
Deutsche Bank AG	Senior
HeidelbergCement AG	Industrial
STMicroelectronics NV	Technology
E.ON AG	Utilities

Next for each *name*, we got the bond issues and respective information in order to obtain the ASW spread.
For the companies analysed, we have selected the following tickers (corporate bond issuances):

Figure 17 Bond issues analysed

Company name	Period of analysis	Ticker	Issue Date	Maturity
Bayer AG	2007..2012	EF433718 Corp	23-05-2006	23-05-2013
Deutsche Telekom AG	2010..2012	EC918126 Corp	01-04-2003	29-03-2018
Deutsche Lufthansa AG	2007..2009	EF385422 Corp	04-05-2006	06-05-2013
Deutsche Lufthansa AG	2010..2012	EH890127 Corp	07-07-2009	07-07-2016
Unilever NV	2007..2012	EF101824 Corp	29-09-2005	29-09-2015
LVMH Moet Hennessy Louis Vuitton SA	2007..2008	ED9733749 Corp	22-06-2005	22-06-2012
LVMH Moet Hennessy Louis Vuitton SA	2009..2012	EH881133 Corp	29-06-2009	29-06-2017
Total SA	2007..2012	EG476311 Corp	06-06-2007	06-06-2017
Total SA	2007..2012	EC2983939 Corp	24-10-2000	24-10-2010
Allianz SE	2007..2012	ED256184 Corp	11-12-2003	11-12-2013
Allianz SE	2007..2012	EH675203 Corp	16-12-2008	17-12-2018
HeidelbergCement AG	2007..2012	EG911261 Corp	22-10-2007	04-01-2018
HeidelbergCement AG	2007..2012	EC116528 Corp	09-04-1999	09-04-2009
STMicroelectronics NV	2007..2012	ED083495 Corp	05-08-2003	05-07-2013
E.ON AG	2007..2012	EC570562 Corp	29-05-2002	07-06-2032

Lastly, we only analysed companies that had more than 150 daily quotes, in order to compute the ASW. The resulted companies were:

- Bayer AG
- Deutsche Telekom AG
- Deutsche Lufthansa AG
- Unilever NV
- LVMH Moet Hennessy Louis Vuitton SA
- Total SA
- E.ON. AG.

Finally, we decided to focus on the period between June, 18th 2008 till December, 31st 2012, because during this period, we had CDS quotes for all maturities, this is 6 months, 1-year, 2-years, 3-years, 4-years, 5-years, 7-years and 10-years. This is essential to retrieve the hazard rates from daily CDS curves, through optimization process described early. All information regarding CDS spreads, bond information and ASW spread were obtained from Bloomberg.

Empirical analysis: The determinants of the basis between CDS and ASW spread

As we explain earlier, we will use 9 variables as determinants of the basis. The expected sign of each variable is resumed as follows:

Figure 18 Expected sign of variables

Dependent variables	Expected sign
DUMBIGBANG	(+)
DUMLEHMAN	(-)
EUROSTOXX50	(-)
SPREADS5Y	(-)
DUMA3160 [t+61,90]	(-)
DUMA3160 [t+31,60]	(-)
DUMA030 [t+1,30]	(-)
DUMB0110 [t-1,t-10]	(-)
DUMB1130 [t-1,t-30]	(-)

All above variables are compiled into the following regression model:

$$\begin{aligned} Basis_{i,n} = {} & \beta_{0,i,n} + \beta_{1,i} DUMBIGBANG + \beta_{2,i} DUMLEHMAN + \beta_{3,i} EUROSTOXX_n \\ & + \beta_{4,i} \mathrm{SPREADS5Y}_{i,n} + \beta_{5,i} \mathrm{DUMA3160}\ [\mathrm{t}+61{,}90]_{i,n} \\ & + \beta_{6,i} \mathrm{DUMA3160}\ [\mathrm{t}+31{,}60]_{i,n} + \beta_{7,i} \mathrm{DUMA030}\ [\mathrm{t}+1{,}30]_{i,n} \\ & + \beta_{8,i} \mathrm{DUMB0110}\ [\mathrm{t}-1, \mathrm{t}-10]_{i,n} + \beta_{9,i} \mathrm{DUMB1130}\ [\mathrm{t}-1, \mathrm{t}-30]_{i,n} \\ & + \varepsilon_{i,n} \end{aligned}$$

where, *i* is the *company* and *n* is the daily quote.

Note that we can only compare liquidity, rating and macroeconomic factors with Zhu's work. And as we can see, we cannot conclude that credit factors plays an important role in determining basis spreads, like Zhu paper did, hence for some of the analysed companies, we did not observed credit events (e.g. Bayer AG, Deutsche Telekom AG, Unilever NV and Total SA) and for the rest of the companies we got consistently *P-values* higher than 1%, meaning that we will fail to reject the null hypothesis, this is that dependent variable can contribute for the determination of the basis.

All companies in the sample for period the period between June 18 2008 and December 31 2012 have 1658 observations. The F-statistic, shown in appendix 1.1 for all companies, shows that the coefficients are jointly significant. No problems with endogeneity have been detected. The results for each company are shown in the following tables:

Figure 19 The determinants of the basis between CDS spread and ASW spread – Bayer AG

Dependent variables	Coef	*P value*
DUMBIGBANG	93.51	0.00%
DUMLEHMAN	(78.59)	0.00%
EUROSTOXX50	(1.96)	0.00%
SPREADS5Y	(10.85)	0.00%
DUMA3160 [t+61,90]	0.00	n.a.
DUMA3160 [t+31,60]	0.00	n.a.
DUMA030 [t+1,30]	0.00	n.a.
DUMB0110 [t-1,t-10]	0.00	n.a.
DUMB1130 [t-1,t-30]	0.00	n.a.

Figure 20 The determinants of the basis between CDS spread and ASW spread – Deutsche Telekom AG

Dependent variables	Coef	*P value*
DUMBIGBANG	(35.82)	0.00%
DUMLEHMAN	(30.93)	0.00%
EUROSTOXX50	(0.11)	29.56%
SPREADS5Y	(12.76)	0.00%
DUMA3160 [t+61,90]	0	n.a.
DUMA3160 [t+31,60]	0	n.a.
DUMA030 [t+1,30]	0	n.a.
DUMB0110 [t-1,t-10]	0	n.a.
DUMB1130 [t-1,t-30]	0	n.a.

Figure 21 The determinants of the basis between CDS spread and ASW spread – Deutsche Lufthansa AG

Dependent variables	Coef	*P value*
DUMBIGBANG	132.80	0.00%
DUMLEHMAN	(126.22)	0.00%
EUROSTOXX50	(1.15)	0.00%
SPREADS5Y	(2.88)	0.00%
DUMA3160 [t+61,90]	(84.89)	0.00%
DUMA3160 [t+31,60]	5.34	62.52%
DUMA030 [t+1,30]	(3.89)	72.03%
DUMB0110 [t-1,t-10]	2.61	87.37%
DUMB1130 [t-1,t-30]	88.09	0.00%

Figure 22 The determinants of the basis between CDS spread and ASW spread – Unilever N.V.

Dependent variables	Coef	*P value*
DUMBIGBANG	(67.98)	0.00%
DUMLEHMAN	(60.74)	0.00%
EUROSTOXX50	(0.37)	0.00%
SPREADS5Y	(2.19)	0.01%
DUMA3160 [t+61,90]	0	n.a.
DUMA3160 [t+31,60]	0	n.a.
DUMA030 [t+1,30]	0	n.a.
DUMB0110 [t-1,t-10]	0	n.a.
DUMB1130 [t-1,t-30]	0	n.a.

Figure 23 The determinants of the basis between CDS spread and ASW spread – LVMH Moet Hennessy Louis Vuitton SA

Dependent variables	Coef	*P value*
DUMBIGBANG	86.31	0.0%
DUMLEHMAN	(92.77)	0.0%
EUROSTOXX50	(0.99)	0.0%
SPREADS5Y	(4.69)	0.0%
DUMA3160 [t+61,90]	1.40	83.3%
DUMA3160 [t+31,60]	1.02	90.9%
DUMA030 [t+1,30]	(0.83)	92.5%
DUMB0110 [t-1,t-10]	2.85	83.1%
DUMB1130 [t-1,t-30]	(16.02)	3.9%

Figure 24 The determinants of the basis between CDS spread and ASW spread – Total SA

Dependent variables	Coef	*P value*
DUMBIGBANG	(6.56)	1.39%
DUMLEHMAN	(6.90)	12.89%
EUROSTOXX50	(0.07)	46.18%
SPREADS5Y	(10.93)	0.00%
DUMA3160 [t+61,90]	0	n.a.
DUMA3160 [t+31,60]	0	n.a.
DUMA030 [t+1,30]	0	n.a.
DUMB0110 [t-1,t-10]	0	n.a.
DUMB1130 [t-1,t-30]	0	n.a.

Figure 25 The determinants of the basis between CDS spread and ASW spread – E.ON AG

Dependent variables	Coef	*P value*
DUMBIGBANG	0.10	0.00%
DUMLEHMAN	39.49	44.25%
EUROSTOXX50	(0.11)	0.00%
SPREADS5Y	(7,65)	0.00%
DUMA3160 [t+61,90]	(19,97)	2.33%
DUMA3160 [t+31,60]	(7,68)	11.67%
DUMA030 [t+1,30]	7,01	17.52%
DUMB0110 [t-1,t-10]	5.13	48.21%
DUMB1130 [t-1,t-30]	15.02	31.32%

The average R^2 for the period is 49,77% being lowest for Total (27.94%) and highest for Bayer AG (67.69%). This average R^2 is also reflected in part for having good coefficients for DIMBIGBANG and DUMLEHMAN variables.

Of the global variables DUMBIGBANG is significant for all companies, even for Total SA, that have a *P-value* slightly higher than 1%. Meanwhile, not all companies in the sample had a positive coefficient as expected, Deutsche Telekom AG, Unilever NV and Total SA had negative coefficients. For the rest of the companies that had positive coefficients, this reflects the expected, this is the Big Bang event contribute to risk mitigation (counterparty risk), hence after the event the basis had averagely get narrower. The DUMLEHMAN variable was significant and had negative coefficient for Bayer AG, Deutsche Telekom AG, Deutsche Lufthansa AG, Unilever NV and LVMH

SA, as predicted. This means that after Lehman and Brothers event, the basis for these companies had wider, this was the result of the credit crisis confidence after the respective bailout. For Total SA and E.ON AG we cannot conclude, because we have reject the variables hence the *P-value* is quite larger than 1%.

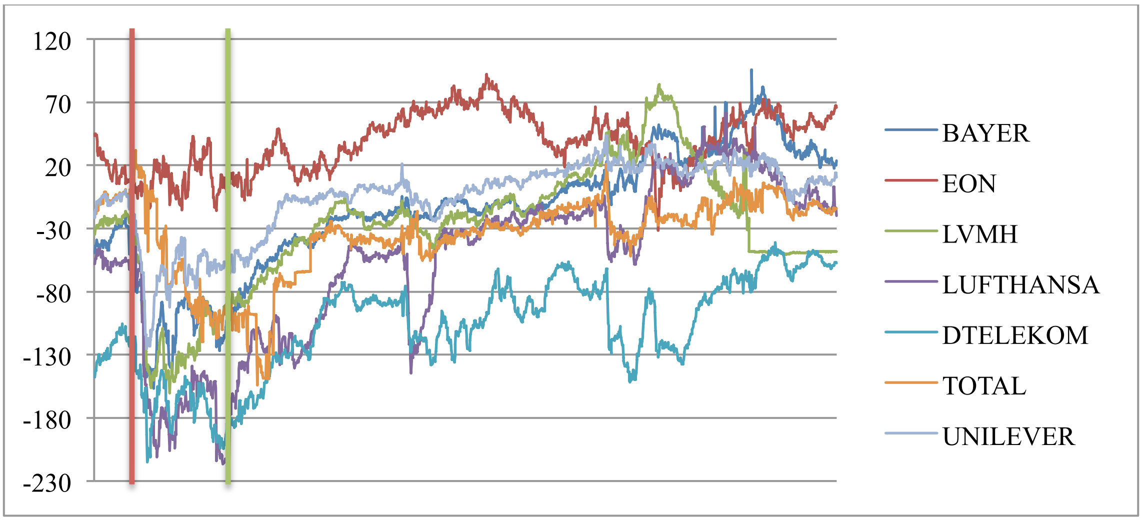
Like Zhu work, we have also confirm that both markets are *quasi*-equally efficient in pricing the changes in macro-financial conditions, hence their impact on basis spreads are near zero. This conclusion reflects the fact that EUROSTOXX50 is significant for the Bayer AG, Deutsche Lufthansa AG, Unilever NV, LVMH SA and E.ON AG, because the *P-value* is near 0% and the respective coefficient varies between (1.99) for Bayer AG and (0.11) for Deutsche Telekom AG and E.ON AG.

The variable SPREAD5Y is significant for all companies and negative as expected, changing from a lower of (12.76) for Deutsche Telekom AG to a higher of (2.19) for Unilever NV. In the case of Deutsche Telekom AG, this means that a decrease of 1% of the average bid-ask spread of the last 20 days for the CDS-5 year reflects in a decrease of 12.76 basis (CDS – ASW spread). This conclusion is also consistent with the findings of (Zhu 2006).

Finally, regarding rating events related variables, this is for DUMA3160 [t+61,90], DUMA3160 [t+31,60], DUMA030 [t+1,30], DUMB0110 [t-1,t-10] and DUMB1130 [t-1,t-30], we didn't have any finding, hence for Bayer AG, Deutsche Telekom AG, Unilever NV and Total SA, we had no credit rating change. For Deutsche Lufthansa AG, LVMH SA and E.ON AG, we cannot accept the variables because they are not significant (*P-values* higher than 1%).

The next figure shows the basis variation of each company. The first line denotes the Lehman Brothers bailout and the second refers the CDS Big Bang event.

Figure 26 Sector analysis: Basis between June 18 2008 and December 31 2012



Assuming that each *name* is representative of its sector, we denote that companies that are in less leveraged sectors are those that basis changes are less severe and negative, like *Utilities* (E.ON) and *Consumer Non-cyclical* (UNILEVER). On the contrary, on those more leveraged sectors like *Communications* (DEUTSCHE TELEKOM), we have assisted quite severe changes on basis and strong negative basis, when compared with less leveraged sectors.

Meanwhile, in order to perform a deeper analysis, it would be necessary to have a sample with more sectorial observations. Though, those sectors represented in the prior figure can denote some evidence on potential correlation between basis and sectors.

Conclusion

With subprime mortgage crisis, Lehman Brothers Holdings Inc. bankruptcy and European government credit crisis, the CDS market assisted to a generalized turmoil, contributing for a decrease of CDS market in more than 50% in less than 3 years.

From a research-related perspective, the primary contributions of the thesis can be divided into two categories: theoretical contribution and empirical contributions.

Regarding theoretical contribution, we revisited the literature on single-name credit modelling and valuing credit derivatives, with special focus on estimating hazard rates, where we introduced the optimization method used by *Martin et al*.

In relation to empirical contribution, we have actualized (Zhu 2006) work, relative to basis determinants analysis for the period between June 18 2008 and December 31 2012. In respect to liquidity and macro-financial variables, we assisted to similar results when comparing with Zhu findings, this means that the market information for both derivatives and cash markets are quasi-similar and liquidity has not the same explanatory power as other variables to determine the basis. In fact, our findings are that the Lehman and Brothers bailout event and the CDS Big Bang have mainly contributed to explain and determine the basis. As expected the Lehman and Brothers bailout contribute to a wider basis, because of a credit risk perception growth in the market and the Big Bang event contribute to a narrowing of the basis spread, because of the introduction of new procedures and standardization of CDS contracts. In fact those efforts on regulating the credit derivative market are working in the basis reduction, through the mitigation of the counterpart risk.

Furthermore, there are certain potential limitations related to the empirical study that need to be addressed. First, consistent data is a prerequisite for obtaining reliable results. It has not been possible to obtain CDS spreads from a single data–supplier that is why most of the data is from Bloomberg, and the daily blanks are filled with last quoted price. This problem was partially mitigated by using the optimization process.

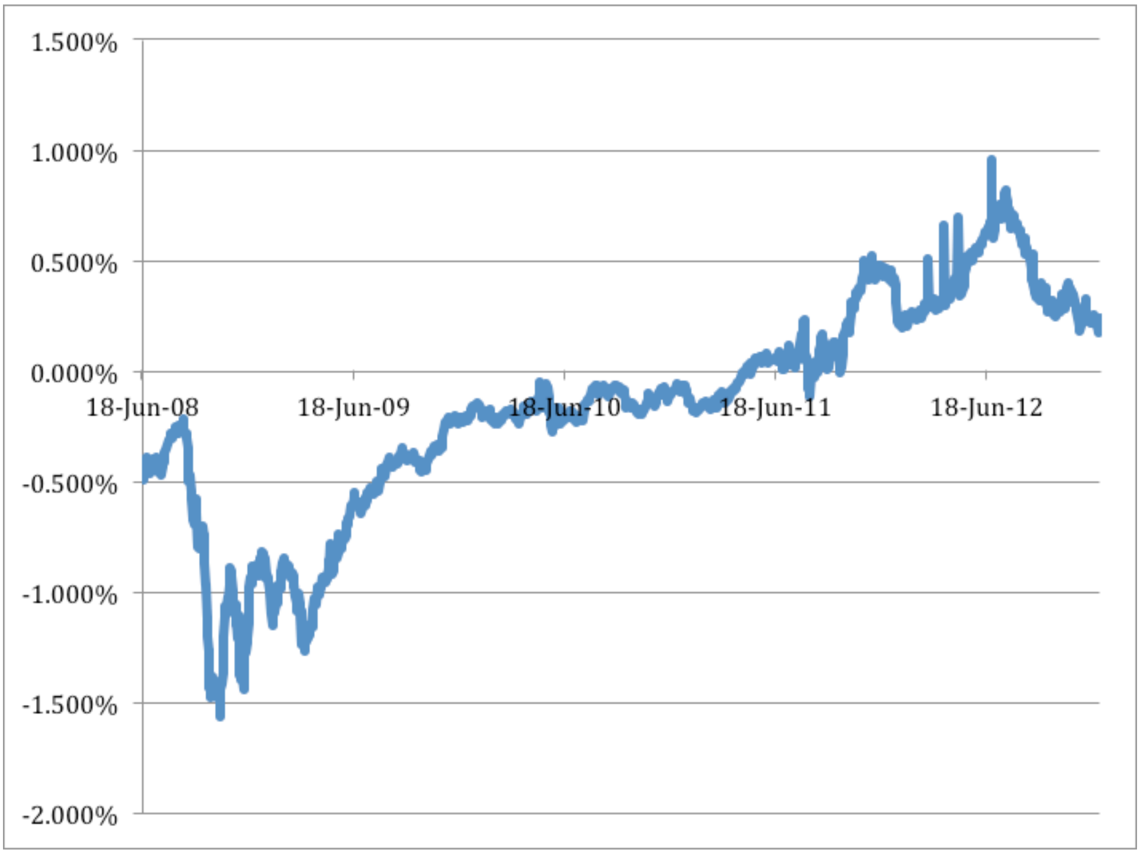
Even tough, this research could be interesting for derivative desks hence it can contribute for the establishment of a *premia* spread in order to face liquidity risk on future events.

Finally, future research could focus on analysing the impact of ECB liquidity strategies on the liquidity in the market and its impact in the diminishment of the spread (bid-offer). It is also important to understand if the diminishment of the spreads is the reflection of a higher turnover of the transactions. Additionally, it would be interesting to introduce the sector analysis and study the impact of basis changes in defensive and less defensive sectors.

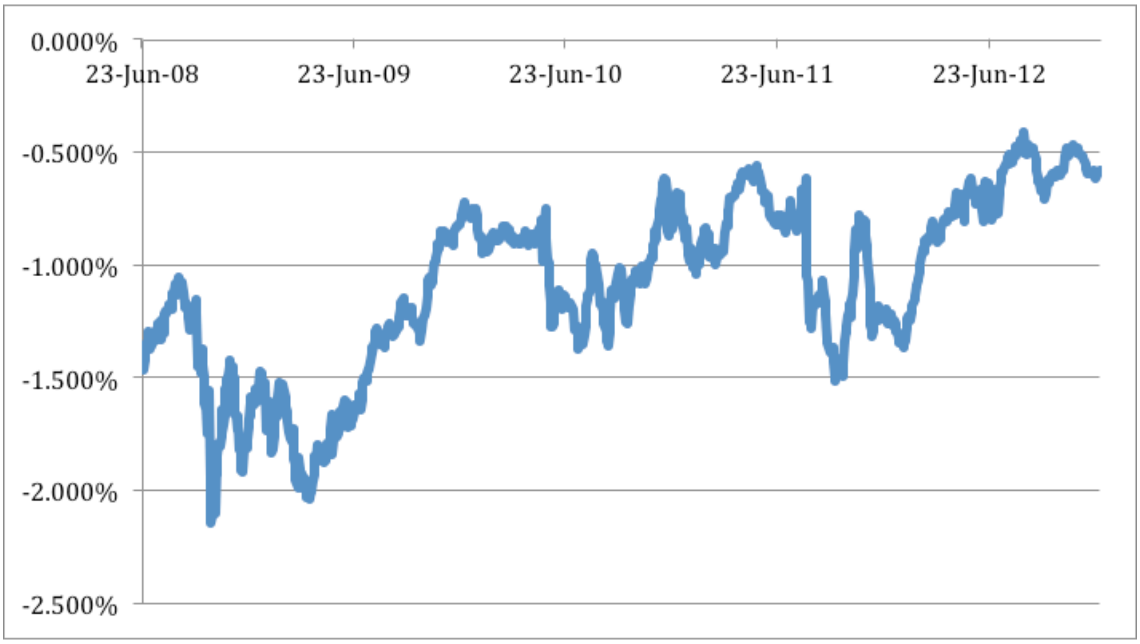
1.1 Appendix – Data analysis

Analysed companies basis

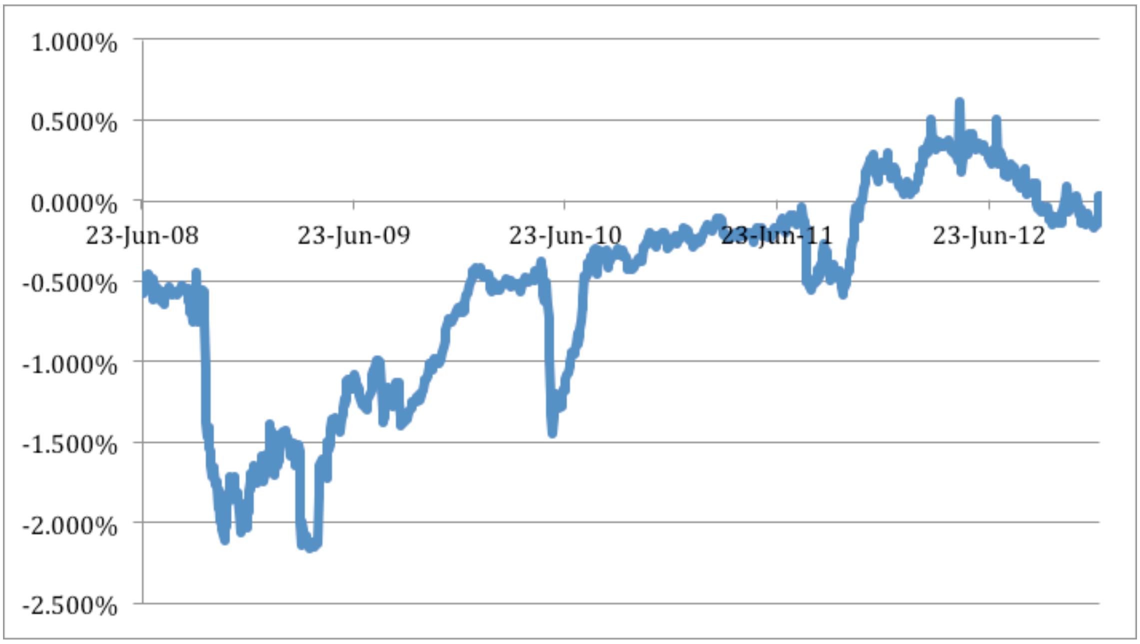
Bayer AG



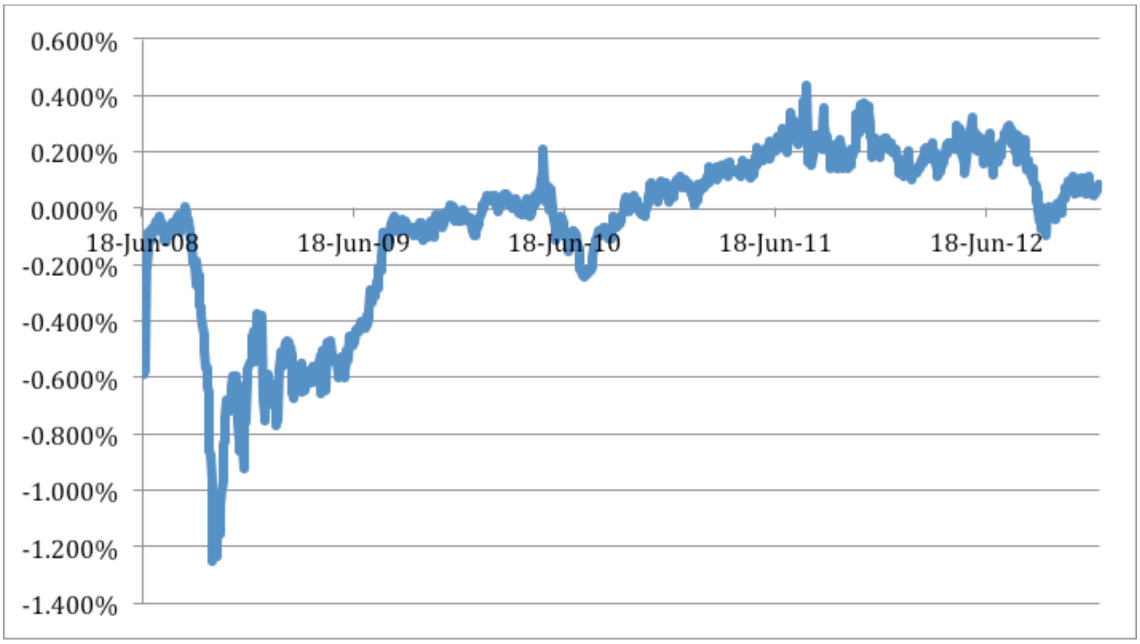
Deutsche Telekom



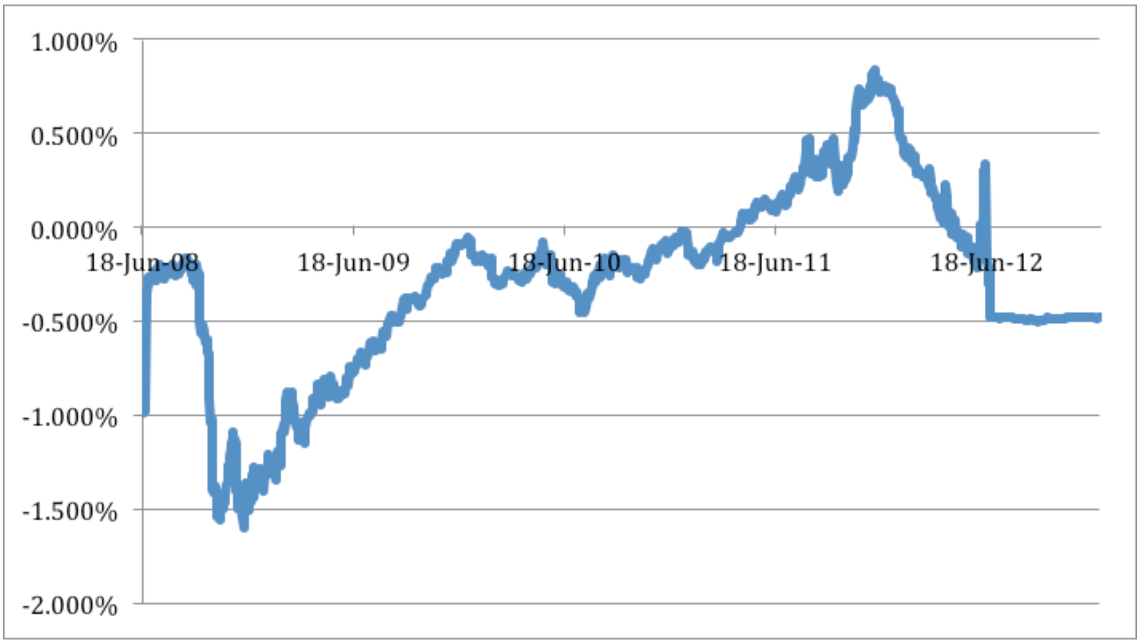
Lufthansa



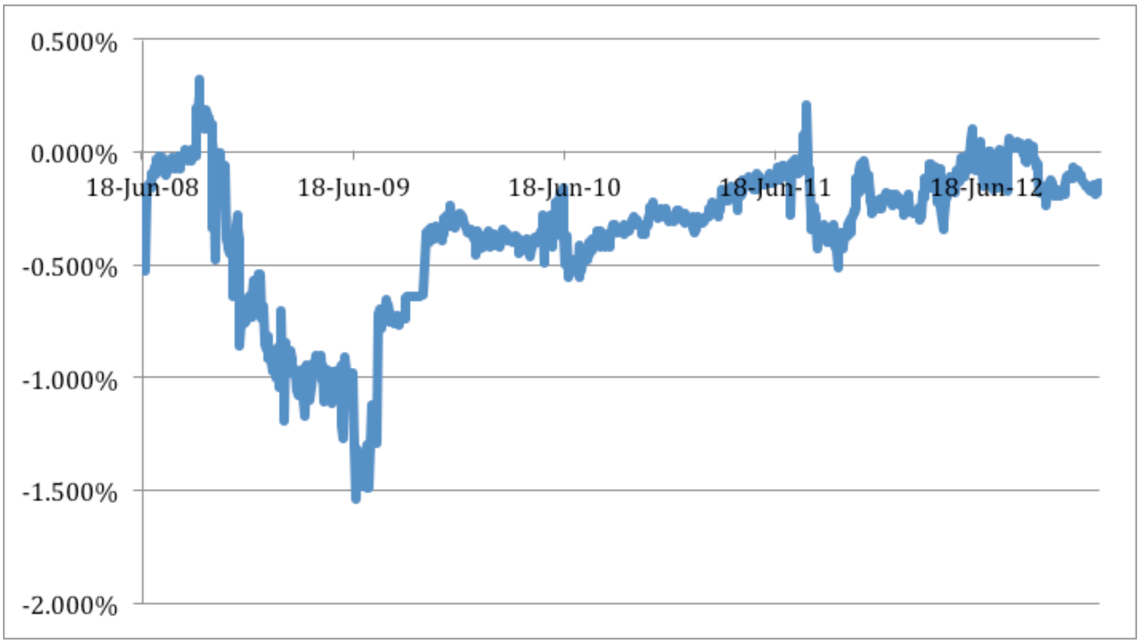
Unilever



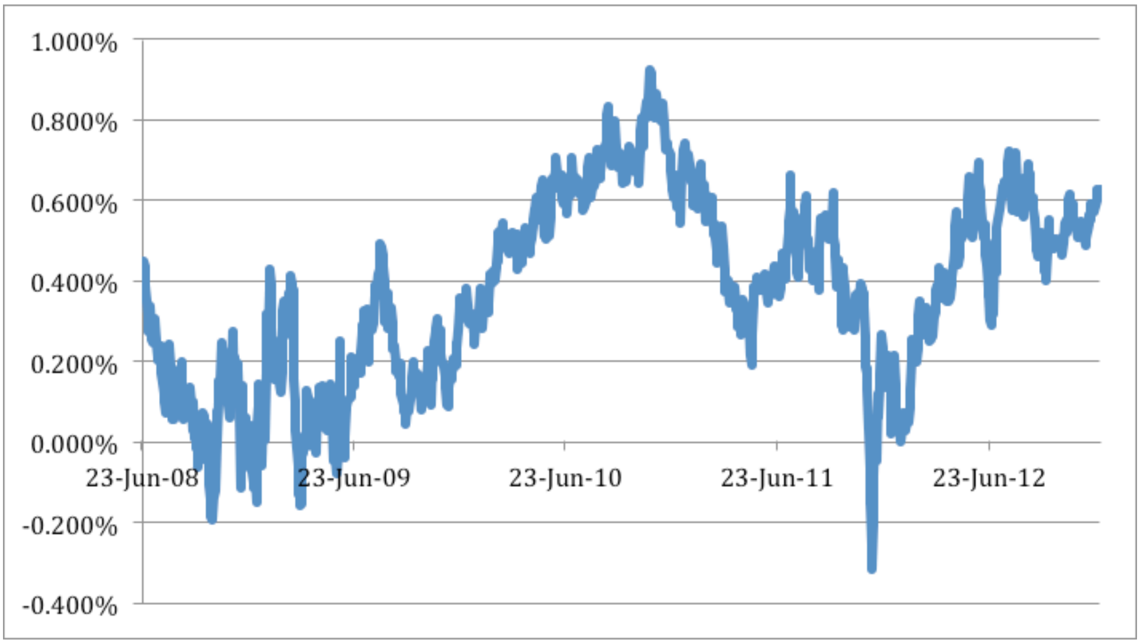
LVMH


1.000%
0.500%
0.000%
-0.500%
-1.000%
-1.500%
-2.000%
18-Jun-08
18-Jun-09
18-Jun-10
18-Jun-11
18-Jun-12

Total


0.500%
0.000%
-0.500%
-1.000%
-1.500%
-2.000%
18-Jun-08
18-Jun-09
18-Jun-10
18-Jun-11
18-Jun-12

E.ON AG


1.000%
0.800%
0.600%
0.400%
0.200%
0.000%
-0.200%
-0.400%
23-Jun-08
23-Jun-09
23-Jun-10
23-Jun-11
23-Jun-12

Analysed companies regressions and correlations

Bayer AG

Regression Statistics	
Multiple R	82.28%
Square R	67.69%
Adjusted Square R	67.52%
Observations	1658

ANOVA

	gl	*SQ*	*MQ*	*F*	*F of Significance*
Regression	9	2637605.343	293067.2604	383.6814614	0.00
Residual	1648	1258791.195	763.8296083		
Total	1657	3896396.538			

	Coef	*Standard Error*	*Stat t*	*P-value*
DUMBIGBANG	93.51	2.33	40.15	0.00
DUMLEHMAN	-78.59	4.48	-17.55	0.00
EUROSTOXX50	-1.96	0.09	-21.16	0.00
BAYER_SPREADS5Y	-10.85	0.46	-23.43	0.00
DUMA3160 [t+61,90]BAYER	0.00	0.00	65535.00	n.a.
DUMA3160 [t+31,60]BAYER	0.00	0.00	65535.00	n.a.
DUMA030 [t+1,30]BAYER	0.00	0.00	65535.00	n.a.
DUMB0110 [t-1,t-10]BAYER	0.00	0.00	65535.00	n.a.
DUMB1130 [t-1,t-30]BAYER	0.00	0.00	65535.00	n.a.

	BASIS	*DUMBIGBANG*	*DUMLEHMAN*	*EUROSTOXX50*	*SPREADS5Y*	*DUMA3160 [t+61,90]*	*DUMA3160 [t+31,60]*	*DUMA030 [t+1,30]*	*DUMB0110 [t-1,t-10]*	*DUMB1130 [t-1,t-30]*
BASIS	1									
DUMBIGBANG	0.661404953	1								
DUMLEHMAN	0.107964942	0.51299963	1							
EUROSTOXX50	-0.093918693	-0.072677357	-0.525983636	1						
SPREADS5Y	-0.510315264	-0.22405211	0.256390064	-0.353251471	1					
DUMA3160 [t+61,90]	n.a.	n.a.	n.a.	n.a.	n.a.	1				
DUMA3160 [t+31,60]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1			
DUMA030 [t+1,30]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1		
DUMB0110 [t-1,t-10]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1	
DUMB1130 [t-1,t-30]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1

E.ON AG

Regression Statistics	
Multiple R	61.97%
Square R	38.40%
Adjusted Square R	38.06%
Observations	1658

ANOVA

	gl	*SQ*	*MQ*	*F*	*F of Significance*
Regression	9	344309.0329	38256.55921	114.1319759	0.00
Residual	1648	552402.6819	335.1958021		
Total	1657	896711.7148			

	Coef	*Standard Error*	*Stat t*	*P-value*
DUMBIGBANG	20.90	1.60	13.04	0.00
DUMLEHMAN	2.21	2.87	0.77	0.44
EUROSTOXX50	-0.34	0.08	-4.31	0.00
EON_SPREADS5Y	-7.03	0.45	-15.64	0.00
DUMA3160 [t+61,90]EON	-7.91	3.49	-2.27	0.02
DUMA3160 [t+31,60]EON	-7.44	4.74	-1.57	0.12
DUMA030 [t+1,30]EON	6.36	4.69	1.36	0.18
DUMB0110 [t-1,t-10]EON	4.99	7.09	0.70	0.48
DUMB1130 [t-1,t-30]EON	4.18	4.14	1.01	0.31

	EON_BASIS	*DUMBIGBANG*	*DUMLEHMAN*	*EUROSTOXX50*	*SPREADS5Y*	*DUMA3160 [t+61,90]*	*DUMA3160 [t+31,60]*	*DUMA030 [t+1,30]*	*DUMB0110 [t-1,t-10]*	*DUMB1130 [t-1,t-30]*
EON_BASIS	1									
DUMBIGBANG	0.514825676	1								
DUMLEHMAN	0.198866712	0.51299963	1							
EUROSTOXX50	0.086215932	-0.072677357	-0.525983636	1						
SPREADS5Y	-0.470109411	-0.364028818	0.135602678	-0.619536131	1					
DUMA3160 [t+61,90]	-0.007185573	0.111228148	0.057059999	0.211093163	-0.189739709	1				
DUMA3160 [t+31,60]	-0.02667504	0.090735856	0.04654746	0.178108823	-0.136306894	0.815763433	1			
DUMA030 [t+1,30]	-0.000449602	0.063023121	0.032330838	0.105709288	-0.087629113	0.566611261	0.694577911	1		
DUMB0110 [t-1,t-10]	-0.004080322	-0.036164951	-0.018552606	-0.034406084	0.053475237	0.018662491	0.015224178	0.010574378	1	
DUMB1130 [t-1,t-30]	-0.018158229	-0.063023121	-0.032330838	-0.061615996	0.0752188	0.03252233	0.026530528	0.018427518	0.573836236	1

LVMH

Regression Statistics	
Multiple R	70.09%
Square R	49.12%
Adjusted Square R	48.84%
Observations	1658

ANOVA

	gl	*SQ*	*MQ*	*F*	*F of Significance*
Regression	9	1887545.612	209727.2902	176.7731901	0.00
Residual	1648	1955220.551	1186.420238		
Total	1657	3842766.163			

	Coef	*Standard Error*	*Stat t*	*P-value*
DUMBIGBANG	86.31	3.80	22.69	0.00
DUMLEHMAN	-92.77	5.55	-16.72	0.00
EUROSTOXX50	-0.99	0.15	-6.52	0.00
SPREADS5Y	-4.69	0.72	-6.49	0.00
DUMA3160 [t+61,90]	1.40	6.64	0.21	0.83
DUMA3160 [t+31,60]	1.02	8.92	0.11	0.91
DUMA030 [t+1,30]	-0.83	8.90	-0.09	0.93
DUMB0110 [t-1,t-10]	2.85	13.34	0.21	0.83
DUMB1130 [t-1,t-30]	-16.02	7.77	-2.06	0.04

	LVMH_BASIS	*DUMBIGBANG*	*DUMLEHMAN*	*EUROSTOXX50*	*SPREADS5Y*	*DUMA3160 [t+61,90]*	*DUMA3160 [t+31,60]*	*DUMA030 [t+1,30]*	*DUMB0110 [t-1,t-10]*	*DUMB1130 [t-1,t-30]*
LVMH_BASIS	1									
DUMBIGBANG	0.57883911	1								
DUMLEHMAN	-0.02065575	0.51299963	1							
EUROSTOXX50	0.11745378	-0.07267736	-0.525983636	1						
SPREADS5Y	-0.51242893	-0.50667705	0.162044693	-0.60272002	1					
DUMA3160 [t+61,90]	-0.02651785	-0.11122815	-0.057059999	-0.17003499	0.06702138	1				
DUMA3160 [t+31,60]	-0.02848131	-0.09073586	-0.04654746	-0.14380325	0.09542403	0.81576343	1			
DUMA030 [t+1,30]	-0.03305649	-0.06302312	-0.032330838	-0.05598603	0.08460593	0.56661126	0.69457791	1		
DUMB0110 [t-1,t-10]	0.00637806	0.03616495	0.018552606	-0.01244578	-0.03818214	0.01866249	0.01522418	0.01057438	1	
DUMB1130 [t-1,t-30]	0.00165359	0.06302312	0.032330838	-0.00751266	-0.05968282	0.03252233	0.02653053	0.01842752	0.57383624	1

Lufthansa AG

Regression Statistics	
Multiple R	75.12%
Square R	56.44%
Adjusted Square R	56.20%
Observations	1658

ANOVA

	gl	*SQ*	*MQ*	*F*	*F of Significance*
Regression	9	3813132.449	423681.3833	237.2087	0.00
Residual	1648	2943513.116	1786.112328		
Total	1657	6756645.565			

	Coef	*Standard Error*	*Stat t*	*P-value*
DUMBIGBANG	132.80	3.37	39.44	0.00
DUMLEHMAN	-126.22	6.57	-19.22	0.00
EUROSTOXX50	-1.15	0.15	-7.64	0.00
SPREADS5Y	-2.88	0.38	-7.48	0.00
DUMA3160 [t+61,90]	-84.89	7.99	-10.62	0.00
DUMA3160 [t+31,60]	5.34	10.92	0.49	0.63
DUMA030 [t+1,30]	-3.89	10.86	-0.36	0.72
DUMB0110 [t-1,t-10]	2.61	16.38	0.16	0.87
DUMB1130 [t-1,t-30]	88.09	9.60	9.17	0.00

	LUFTHANSA_BASIS	*DUMBIGBANG*	*DUMLEHMAN*	*EUROSTOXX50*	*SPREADS5Y*	*DUMA3160 [t+61,90]*	*DUMA3160 [t+31,60]*	*DUMA030 [t+1,30]*	*DUMB0110 [t-1,t-10]*	*DUMB1130 [t-1,t-30]*
LUFTHANSA_BASIS	1									
DUMBIGBANG	0.553106092	1								
DUMLEHMAN	0.007171746	0.51299963	1							
EUROSTOXX50	0.067118043	-0.072677357	-0.525983636	1						
SPREADS5Y	-0.295968814	-0.142886877	0.132439558	-0.388044462	1					
DUMA3160 [t+61,90]	-0.23294146	0.111228148	0.057059999	-0.059899257	0.136500166	1				
DUMA3160 [t+31,60]	-0.191533038	0.090735856	0.04654746	-0.018352648	0.1105638	0.815763433	1			
DUMA030 [t+1,30]	-0.14255071	0.063023121	0.032330838	0.037910735	0.055050054	0.566611261	0.694577911	1		
DUMB0110 [t-1,t-10]	0.097868927	-0.036164951	-0.018552606	-0.03844073	-0.064323468	0.018662491	0.015224178	0.010574378	1	
DUMB1130 [t-1,t-30]	0.162542721	-0.063023121	-0.032330838	-0.08902879	-0.054224866	0.03252233	0.026530528	0.018427518	0.573836236	1

Deutsche Telekom

Regression Statistics	
Multiple R	70.76%
Square R	50.07%
Adjusted Square R	49.80%
Observations	1658

ANOVA

	gl	*SQ*	*MQ*	*F*	*F of Significance*
Regression	9	1219604.408	135511.6009	183.6338207	0.00
Residual	1648	1216132.832	737.9446793		
Total	1657	2435737.24			

	Coef	*Standard Error*	*Stat t*	*P-value*
DUMBIGBANG	35.82	2.81	12.74	0.00
DUMLEHMAN	- 30.93	4.24	- 7.30	0.00
EUROSTOXX50	- 0.11	0.11	- 1.05	0.30
SPREADS5Y	- 12.76	0.78	- 16.42	0.00
DUMA3160 [t+61,90]	0	0	65535	n.a.
DUMA3160 [t+31,60]	0	0	65535	n.a.
DUMA030 [t+1,30]	0	0	65535	n.a.
DUMB0110 [t-1,t-10]	0	0	65535	n.a.
DUMB1130 [t-1,t-30]	0	0	65535	n.a.

	DTELEKOM_BASIS	*DUMBIGBANG*	*DUMLEHMAN*	*EUROSTOXX50*	*SPREADS5Y*	*DUMA3160 [t+61,90]*	*DUMA3160 [t+31,60]*	*DUMA030 [t+1,30]*	*DUMB0110 [t-1,t-10]*	*DUMB1130 [t-1,t-30]*
DTELEKOM_BASIS	1									
DUMBIGBANG	0.57982031	1								
DUMLEHMAN	0.1004051	0.51299963	1							
EUROSTOXX50	0.23065791	-0.07267736	-0.52598364	1						
SPREADS5Y	-0.66851919	-0.66012951	-0.17858024	-0.39469407	1					
DUMA3160 [t+61,90]	n.a.	n.a.	n.a.	n.a.	n.a.	1				
DUMA3160 [t+31,60]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1			
DUMA030 [t+1,30]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1		
DUMB0110 [t-1,t-10]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1	
DUMB1130 [t-1,t-30]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1

Total

Regression Statistics	
Multiple R	52.85%
Square R	27.94%
Adjusted Square R	27.54%
Observations	1658

ANOVA

	gl	*SQ*	*MQ*	*F*	*F of Significance*
Regression	9	478869.4662	53207.71847	70.98409026	0.00
Residual	1648	1235295.398	749.5724502		
Total	1657	1714164.864			

	Coef	*Standard Error*	*Stat t*	*P-value*
DUMBIGBANG	- 6.56	2.66	- 2.46	0.01
DUMLEHMAN	- 6.90	4.54	- 1.52	0.13
EUROSTOXX50	0.07	0.10	0.74	0.46
SPREADS5Y	- 10.93	0.62	- 17.68	0.00
DUMA3160 [t+61,90]TOTAL	0	0	65535	n.a
DUMA3160 [t+31,60]TOTAL	0	0	65535	n.a
DUMA030 [t+1,30]TOTAL	0	0	65535	n.a
DUMB0110 [t-1,t-10]TOTAL	0	0	65535	n.a
DUMB1130 [t-1,t-30]TOTAL	0	0	65535	n.a

	TOTAL_BASIS	*DUMBIGBANG*	*DUMLEHMAN*	*EUROSTOXX50*	*SPREADS5Y*	*DUMA3160 [t+61,90]*	*DUMA3160 [t+31,60]*	*DUMA030 [t+1,30]*	*DUMB0110 [t-1,t-10]*	*DUMB1130 [t-1,t-30]*
TOTAL_BASIS	1									
DUMBIGBANG	0.114877058	1								
DUMLEHMAN	-0.219848156	0.51299963	1							
EUROSTOXX50	0.303211773	-0.072677357	-0.525983636	1						
SPREADS5Y	-0.515317637	-0.415715372	0.229457309	-0.47822455	1					
DUMA3160 [t+61,90]	n.a	n.a	n.a	n.a	n.a	1				
DUMA3160 [t+31,60]	n.a	n.a	n.a	n.a	n.a	n.a	1			
DUMA030 [t+1,30]	n.a	n.a	n.a	n.a	n.a	n.a	n.a	1		
DUMB0110 [t-1,t-10]	n.a	n.a	n.a	n.a	n.a	n.a	n.a	n.a	1	
DUMB1130 [t-1,t-30]	n.a	n.a	n.a	n.a	n.a	n.a	n.a	n.a	n.a	1

Unilever

Regression Statistics	
Multiple R	76.63%
Square R	58.72%
Adjusted Square R	58.49%
Observations	1658

ANOVA

	gl	*SQ*	*MQ*	*F*	*F of Significance*
Regression	9	864051.7277	96005.74752	260.4531899	0
Residual	1648	607469.8951	368.6103732		
Total	1657	1471521.623			

	Coef	*Standard Error*	*Stat t*	*P-value*
DUMBIGBANG	67.98	1.67	40.69	0.00
DUMLEHMAN	-60.74	3.09	- 19.64	0.00
EUROSTOXX50	- 0.37	0.07	- 5.51	0.00
SPREADS5Y	- 2.19	0.57	- 3.85	0.00
DUMA3160 [t+61,90]	0	0	65535	n.a.
DUMA3160 [t+31,60]	0	0	65535	n.a.
DUMA030 [t+1,30]	0	0	65535	n.a.
DUMB0110 [t-1,t-10]	0	0	65535	n.a.
DUMB1130 [t-1,t-30]	0	0	65535	n.a.

	UNILEVER_BASIS	*DUMBIGBANG*	*DUMLEHMAN*	*EUROSTOXX50*	*SPREADS5Y*	*DUMA3160 [t+61,90]*	*DUMA3160 [t+31,60]*	*DUMA030 [t+1,30]*	*DUMB0110 [t-1,t-10]*	*DUMB1130 [t-1,t-30]*
UNILEVER_BASIS	1									
DUMBIGBANG	0.666107848	1								
DUMLEHMAN	0.026572011	0.51299963	1							
EUROSTOXX50	0.102863835	-0.07267736	-0.52598364	1						
SPREADS5Y	-0.39052236	-0.28198039	0.265397567	-0.48706193	1					
DUMA3160 [t+61,90]	n.a.	n.a.	n.a.	n.a.	n.a.	1				
DUMA3160 [t+31,60]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1			
DUMA030 [t+1,30]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1		
DUMB0110 [t-1,t-10]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1	
DUMB1130 [t-1,t-30]	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	1

1.2 References

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1.3 List of figures