Stress-Testing the 1/N Rule: Portfolio Strategy Performance Across Financial Crisis Phases

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Abstract

This thesis investigates whether optimised portfolio strategies can consistently outperform naïve equal-weight diversification across distinct phases of financial crises. The analysis extends an unconditional portfolio evaluation framework by introducing explicit crisis-regime conditioning. Crises are identified using a drawdown-based classification methodology that partitions the market timeline into four sequential phases: pre-crisis, crisis, drawdown, and recovery. The empirical analysis spans three geographic regions, the United States, Europe, and Japan, and four asset universes derived from common equity factor models.

Two layers of strategies are evaluated. The first comprises classical mean-variance and minimum-variance optimisation strategies. The second comprises phase-conditioned long-short strategies that construct portfolio signals from the return histories of prior crisis episodes. Results reveal a striking crisis and drawdown-recovery asymmetry. During the sub-phases crisis and drawdown, constrained minimum-variance strategies produce statistically significant outperformance of the equal-weight benchmark across all regions and universes. During the recovery phase, all classical strategies significantly underperform the benchmark, reflecting the cost of backward-looking covariance estimates calibrated on high-stress market environments.

Phase-conditioned long-short strategies generate full-sample Sharpe ratios and certainty-equivalent returns above the equal-weight benchmark, an achievement absent from every phase of every classical strategy, with outperformance concentrated in the sub-phases crisis and drawdown. These findings demonstrate that the aggregate conclusion of the prior literature conceals structurally distinct phase-level behaviour. Regime-aware signal construction, rather than optimisation refinement alone, is the primary source of exploitable outperformance relative to naïve diversification.

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Resumo

Esta tese investiga se as estratégias de carteira otimizadas superam a diversificação ingénua de pesos iguais em diferentes fases de diversas crises financeiras. As crises são identificadas através de uma metodologia baseada na queda de valor máximo acumulado, dividindo a linha temporal de cada crise em quatro fases: pré-crise, crise, queda e recuperação. A análise empírica abrange os Estados Unidos, a Europa e o Japão, em quatro universos de ativos financeiros derivados de modelos de fatores de ações.

São avaliadas duas categorias de estratégias: estratégias clássicas de otimização de média-variância e de variância mínima, assim como estratégias long-short condicionadas à fase, que constroem sinais a partir dos históricos de rendibilidade de crises anteriores.

Os resultados revelam uma assimetria entre a crise e a recuperação. Na subfase de crise e na fase de queda, as estratégias de variância mínima produzem um desempenho estatisticamente significativo superior face ao índice de referência em todas as regiões e universos de ativos. Na fase de recuperação, todas as estratégias clássicas apresentam um desempenho significativamente inferior, refletindo o custo de estimativas de covariância calibradas em condições de stress elevado. As estratégias long-short condicionadas à fase são as únicas a gerar rácios de Sharpe e retornos equivalentes à certeza superiores ao índice de referência na amostra completa, com um desempenho concentrado na crise e queda. Estes resultados demonstram que conclusões agregadas ocultam comportamento distinto entre fases, e que sinais sensíveis ao regime constituem a principal fonte de desempenho superior face à diversificação ingénua.

Título: Avaliação da regra 1/N em condições de crise: desempenho da estratégia de carteira nas diferentes fases da crise financeira

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Palavras-chave: Diversificação Ingénua; Optimização de Carteiras; Crise Financeira; CMAX; Estratégias Condicionadas por Fase

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Abbreviations

Ann. Ret.	Annualized Return
BH	Buy-and-Hold
BM	Book-to-Market
bs	Bayes-Stein portfolio strategy
bs-c	Constrained Bayes-Stein Portfolio Strategy
CAPM	Capital Asset Pricing Model
CEQ	Certainty-Equivalent Return
CMA	Conservative Minus Aggressive
CMAX	Cumulative Maximum (drawdown measure)
EW	Equal-Weighted
ew-min	Equal-Weight / Minimum-Variance Mixture Portfolio
FF3	Fama-French Three-Factor Model
FF5	Fama-French Five-Factor Model
g-min-c	Globally Constrained Minimum-Variance Portfolio
HML	High Minus Low
L	Long-Only (strategy variant)
LS	Long-Short (strategy variant)
M	Estimation Window Length (months)
MDD	Maximum Drawdown
min	Unconstrained Minimum-Variance Portfolio
min-c	Constrained Minimum-Variance Portfolio
MKT	Market Factor
Mom-12	12-1 Month Rolling Momentum Strategy
mv	Unconstrained Mean-Variance Portfolio
mv-c	Constrained Mean-Variance Portfolio
mv-min	Mean-Variance / Minimum-Variance Mixture Portfolio
N	Number of Assets in the Portfolio Universe
NYSE	New York Stock Exchange
OOS	Out-of-Sample
P1F	Past-1 Full: Ranked by Total Return of Most Recent Prior Crisis Episode
P1P	Past-1 Phase: Ranked by Same-Phase Return of Most Recent Prior Episode
PAF	Past-All Full: Ranked by Average Total Return Across All Prior Episodes
PAP	Past-All Phase: Ranked by Average Same-Phase Return Across All Prior Episodes

Phase-Mom	Phase-Momentum: Long Best Performers of Most Recently Completed Phase
Phase-Rev	Phase-Reversal: Long Worst Performers of Most Recently Completed Phase
RMW	Robust Minus Weak
ST-Rev	Short-Term Reversal Strategy
SIC	Standard Industrial Classification
SMB	Small Minus Big
SR	Sharpe Ratio
TO	Portfolio turnover
US	United States
VIX	Volatility Index
VW	Value-Weighted
1/N	Equal-Weight Naïve Diversification Rule

1. Introduction

A central challenge in portfolio management is whether formally optimal diversification rules can improve on simple, parameter-free heuristics when deployed in real markets. Under known parameters, mean-variance optimization dominates any fixed weighting scheme (Markowitz, 1952). The practical challenge is that parameters must be estimated, and estimation errors transform the optimizer's precision into a liability, amplifying noise into concentrated, unstable allocations. In a landmark empirical study across fourteen optimized strategies and seven datasets, none of the optimized strategies is found to consistently outperform the naive equal-weight (1/N) portfolio in terms of out-of-sample Sharpe ratio, certainty-equivalent return, or turnover after transaction costs (DeMiguel et al., 2009). The required estimation window for the sample mean-variance portfolio to dominate 1/N in expectation is shown to be on the order of thousands of months, far exceeding any feasible sample (DeMiguel et al., 2009).

This finding carries a consequential limitation: the entire evaluation is conducted unconditionally, averaging performance across all market environments without distinguishing between different regimes. Whether the 1/N rule's competitive performance is stable across market conditions, and whether crisis periods alter the relative ranking of naive and optimized strategies, is a question that prior work explicitly leaves open (DeMiguel et al., 2009).

Crisis periods are not merely scaled versions of normal markets. The empirical literature documents systematic changes in correlation structure and volatility dynamics during market downturns. Correlations between equity markets increase significantly during bear markets but not during bull markets, a pattern of asymmetric dependence incompatible with the constant-correlation assumption of classical mean-variance theory (Longin and Solnik, 2001). After accounting for heteroscedasticity-induced bias, significant increases in cross-market linkages are documented during major crisis episodes (Forbes and Rigobon, 2002). For optimized strategies that rely on pre-crisis covariance estimates, these findings carry a direct implication: the covariance matrix used at portfolio formation will systematically understate the correlations prevailing during the crisis, potentially undermining the risk-reduction objectives of the strategy at precisely the moment they are most needed.

The identification of crisis periods requires a formal, data-driven classification procedure. This thesis adopts the CMAX methodology, which defines a crisis as a period in which the current market index falls below 80 percent of its historical maximum over a specified window (Patel and Sarkar, 1998). This approach is entirely data-driven, requires no prior specification of crisis dates, and is applicable consistently across different regions and sample periods. Crucially, the CMAX measure implies a phase structure: a crisis phase with the CMAX falling under one before a threshold breach, a drawdown phase spanning from the initial threshold breach to the portfolio trough, and a recovery phase from the trough back to the pre-crisis maximum. This phase decomposition allows the analysis to test whether portfolio strategies perform differently at different stages of the crisis cycle, a dimension absent from prior work.

The present thesis extends the prior framework along three dimensions. First, it introduces explicit crisis-regime conditioning, using the CMAX methodology to partition the evaluation period into distinct phases and assess whether the relative performance of naive and optimized strategies changes systematically across regimes. Second, it evaluates a layer of phase-conditioned long-short strategies that construct portfolio signals specifically from the return history of prior crisis episodes, investigating whether crisis-specific return patterns carry predictive information for subsequent portfolio performance. Third, the analysis is conducted across three geographic regions (United States, Europe, and Japan) and four datasets (FF3, FF5, BM and IND), providing evidence formalized through a pooled cross-regional regression that simultaneously test the asymmetry phase coefficients across all regions and universes.

Three research questions guide the empirical analysis. First, does binary crisis classification alter the relative performance of classical optimised strategies versus the 1/N benchmark, and does the constrained versus unconstrained distinction determine which strategies benefit from crisis conditioning? Second, does the four-phase decomposition of the crisis cycle reveal differential performance patterns for classical optimised strategies relative to 1/N that binary classification cannot identify, and in which phases does the performance differential concentrate? Third, can phase-conditioned long-short strategies generate full-sample out-of-sample outperformance relative to 1/N, and in which phases of the crisis cycle does this outperformance concentrate?

The empirical results reveal a consistent pattern across all three analytical layers and all four datasets. The sub-phases crisis and drawdown produce statistically significant outperformance of the 1/N benchmark for classical optimised and long-short strategies, while the recovery phase produces significant underperformance. This crisis and drawdown-recovery asymmetry is the most robust finding in the thesis. A pooled cross-regional regression across all three regions and five investable asset universes confirms the asymmetric phase coefficients simultaneously as statistically significant results. Under binary classification, the two effects partially offset each other, producing aggregate results broadly consistent with prior evidence (DeMiguel et al., 2009). The four-phase decomposition reveals that this aggregate conclusion conceals structurally distinct behaviour across crisis phases. Among the long-short strategies, phase-conditioned signal construction generates full-sample outperformance not observed in any phase of any classical strategy, establishing that the source of alpha is regime-aware signal design rather than optimization or estimation window choice alone.

The remainder of the thesis proceeds as follows. Section 2 reviews the relevant literature on modern portfolio theory, robust portfolio construction, momentum and reversal strategies, naive diversification, and crisis identification. Section 3 describes the data, methodology, and strategy definitions. Section 4 presents the empirical results across three analytical layers. Section 5 discusses the findings in relation to the research questions and positions the results within the existing literature. Section 6 concludes.

2. Literature Review

2.1 Foundations of Modern Portfolio Theory

The theoretical foundations of this thesis rest on the mean-variance framework, which establishes the conditions under which rational investors should construct portfolios (Markowitz, 1952). The framework demonstrates that the relevant objective for a risk-averse investor is not merely to maximize expected return but to optimize the trade-off between expected return and variance. The resulting efficient frontier characterizes all portfolios offering the highest expected return for a given level of risk. By combining assets whose returns are imperfectly correlated, an investor can reduce portfolio variance without sacrificing expected return, providing the first rigorous justification for diversification as a value-creating activity.

The equilibrium implications of the mean-variance framework are formalized in the Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Lintner, 1965). Under the assumptions of homogeneous expectations and frictionless markets, the CAPM implies that the market portfolio is mean-variance efficient and that all investors should hold a combination of the risk-free asset and the value-weighted market portfolio. This proposition has a direct practical corollary: the value-weighted market portfolio, requiring no parameter estimation beyond observable market capitalizations, constitutes a legitimate portfolio.

The single-factor CAPM was subsequently challenged by accumulating evidence of cross-sectional return patterns that market beta alone could not explain. Firm size and book-to-market equity are identified as two additional dimensions of systematic risk, with factor-mimicking portfolios SMB and HML explaining a large share of cross-sectional return variation alongside the market factor (Fama and French, 1993). The model is extended to five factors by adding profitability (RMW) and investment (CMA) premia, with theoretical and empirical justification for their independent explanatory power (Fama and French, 2015).

2.2 The Limits of Mean-Variance Optimization: Estimation Error

Despite its theoretical elegance, the practical implementation of the mean-variance framework is severely hampered by estimation errors. The sample-based mean-variance portfolio treats sample estimates of expected returns and the covariance matrix as if they were true population parameters. This approach is fragile because the portfolio optimizer amplifies noise: assets with spuriously high sample means receive excessive weight, and the resulting portfolios are highly concentrated and unstable across rebalancing periods. This fragility is documented empirically: the very process of optimization transforms estimation errors into extreme, unintuitive allocations that frequently perform poorly out-of-sample (Michaud, 1989).

The sensitivity of mean-variance portfolios to input perturbations is established analytically: small changes in expected return estimates produce dramatic revisions in optimal portfolio weights, and the efficient frontier is not a stable object but a construct reflecting the idiosyncrasies of the estimation sample (Best and Grauer, 1991). Finite-sample evidence reinforces this: standard errors of estimated efficient portfolio weights are so wide that even directional conclusions about optimal allocations to individual assets are often statistically unreliable (Britten-Jones, 1999). This joint weight of evidence points to a fundamental tension, the strategies theoretically optimal under known parameters are precisely those that perform worst when parameters must be estimated.

A quantification of the relative damage inflicted by errors in different inputs finds that errors in expected returns are over ten times more costly than errors in variances, and over twenty times more costly than errors in covariances (Chopra and Ziemba, 1993). This hierarchy implies that resources devoted to improving covariance estimation yield far less benefit than equivalent effort directed at mean return estimation and explains why strategies that sidestep expected return estimation entirely, such as the minimum-variance portfolio, tend to perform competitively out-of-sample. The result also underscores why the robust portfolio construction literature has increasingly focused on either shrinking or eliminating the reliance on estimated means.

2.3 Robust Portfolio Construction: Shrinkage, Constraints, and Combination Strategies

The recognition that estimation error undermines mean-variance optimization has generated a large literature on robust portfolio construction. These approaches fall broadly into three categories: shrinkage of parameter estimates toward structured targets, imposition of constraints that implicitly limit the influence of extreme estimates, and direct combination of portfolio strategies to spread estimation risk.

Shrinkage estimation, rooted in the decision-theoretic result establishing the inadmissibility of the sample mean as an estimator of the multivariate mean under squared error loss, provides a principled way to reduce estimation error by pulling sample estimates toward a common target (Stein, 1955; James and Stein, 1961). Applied to portfolio selection, the Bayes-Stein estimator shrinks sample mean returns toward the expected return on the global minimum-variance portfolio (Jorion, 1991). The resulting portfolio is more diversified and more stable than the unconstrained mean-variance portfolio and delivers better out-of-sample performance.

Short-sale constraints offer a conceptually simpler route to limiting extreme allocations. Constraining portfolio weights to be non-negative is formally equivalent to shrinking the off-diagonal elements of the sample covariance matrix toward zero, providing theoretical justification for why constraints improve out-of-sample performance even without economic motivation (Jagannathan and Ma, 2003).

A third category combines multiple portfolio strategies at the weight level. An extension of the two-fund separation theorem derives an optimal portfolio that combines the sample mean-variance portfolio and the minimum-variance portfolio with mixing weights chosen to account for parameter uncertainty (Kan and Zhou, 2007). The key insight is that the minimum-variance portfolio provides a natural hedge against the estimation error inherent in the tangency portfolio, so that a convex combination of the two dominates either alone for a mean-variance investor.

2.4 Momentum, Reversal, and Phase-Conditioned Long-Short Strategies

The momentum premium, the tendency for assets that have performed well over the recent past to continue outperforming, is one of the most extensively documented return anomalies in empirical asset pricing. Portfolios formed by buying recent winners and selling recent losers are shown to generate significant positive returns over three- to twelve-month holding periods (Jegadeesh and Titman, 1993). Incorporating a momentum factor is subsequently found to substantially improve cross-sectional explanatory power beyond the three-factor model (Fama and French, 1993), cementing momentum as a systematic return premium (Carhart, 1997). Complementary evidence of longer-horizon reversal shows that portfolios of long-horizon past losers significantly outperform portfolios of past winners over three- to five-year periods, consistent with investor overreaction subsequently correcting (DeBondt and Thaler, 1985).

Long-short portfolio strategies that exploit these return predictability patterns operate by simultaneously holding long positions in assets expected to outperform and short positions in assets expected to underperform. Such strategies have been extensively studied in the cross-sectional return literature.

2.5 Naive Diversification and the Out-of-Sample Performance Puzzle

The extensive literature proposing increasingly sophisticated approaches to portfolio optimization collectively presupposes that the gains from exploiting estimated moments justify the costs of estimation error. In a comprehensive empirical evaluation across fourteen models and seven distinct datasets, none of the optimized strategies consistently outperforms the naïve heuristic of allocating equal weight to all available assets, the 1/N rule, in terms of Sharpe ratio, certainty-equivalent return, or turnover after accounting for transaction costs (DeMiguel et al., 2009). A theoretical explanation for this result shows that the estimation window required for the sample mean-variance strategy to outperform the equally weighted portfolio in expectation is approximately 3,000 months for a portfolio of 25 assets and around 6,000 months for a portfolio of 50 assets (DeMiguel et al., 2009). Critically, this conclusion extends to shrinkage, Bayesian, and constrained variants: even strategies specifically designed to reduce estimation error fail to overcome the fundamental problem consistently enough to beat the naïve benchmark.

2.6 Identifying Market Crises and Asset Behavior under Stress

Assessing portfolio performance conditional on market regimes requires a formal and reproducible procedure for identifying crisis periods. The literature has developed several distinct approaches to this task, each with different conceptual foundations and empirical implications. Regime-switching models identify latent market states by fitting Markov-chain models to return or volatility series, allowing transition probabilities between states to be estimated from the data (Hamilton, 1989). This class of models captures the persistence of regimes and accommodates gradual transitions but requires distributional assumptions and does not necessarily align the identified states with economically meaningful crisis episodes. Volatility-based approaches, exemplified by threshold rules on realised variance or the VIX, define stress periods as months in which market volatility exceeds a specified level. While intuitive, these methods conflate elevated uncertainty with market crises and do not directly capture the magnitude of peak-to-trough price declines that characterise genuine crises from an investor perspective.

Drawdown-based approaches offer a direct connection to the investment losses experienced during crisis episodes. Within this class, the CMAX methodology is particularly well-suited to the present analysis. The CMAX measure is defined as the ratio of the current market index level to its historical maximum over a specified window, so that a value below a specified threshold triggers a crisis classification. Proposed thresholds are 0.80 for developed markets and 0.65 for emerging markets, calibrated to identify economically significant downturns while excluding minor corrections (Patel and Sarkar, 1998). This approach is entirely data-driven, requires no prior specification of crisis dates, and provides a transparent and reproducible classification criterion applicable consistently across different markets and sample periods. Crucially, the CMAX measure implies a phase structure, comprising a drawdown phase from the initial threshold breach to the portfolio trough and a recovery phase from the trough back to the pre-crisis peak.

The broader empirical literature on stock market crises has established several regularities bearing directly on the portfolio comparison in this thesis. The most consequential finding is the systematic increase in cross-asset correlations during market downturns. Applying extreme value theory to international equity returns, correlations between equity markets are shown to increase significantly in bear markets but not in bull markets, a pattern of asymmetric dependence fundamentally at odds with the constant-correlation assumption of classical mean-variance theory (Longin and Solnik, 2001). This finding is corroborated by evidence of significant increases in cross-market linkages during multiple crisis episodes after controlling for the heteroscedasticity-induced bias that can spuriously inflate estimated correlations in high-volatility periods (Forbes and Rigobon, 2002). For strategies relying on pre-crisis covariance estimates, these findings carry a direct implication: such estimates will systematically understate the correlations prevailing during the crisis, potentially undermining risk-reduction objectives precisely when they are most needed.

Crisis episodes are also documented to be almost universally associated with regional contagion, with most individual markets within a region entering crisis simultaneously, and stock prices declining for several years following onset (Patel and Sarkar, 1998). These features reinforce the notion that crisis periods represent a structurally distinct market regime rather than a mere intensification of normal conditions, lending weight to the hypothesis that the relative performance of portfolio strategies shifts systematically between crisis and non-crisis regimes.

3. Methodology

3.1 Data Description and Portfolio Universes

This dissertation employs monthly equity portfolio return series obtained from the Kenneth R. French Data Library (Fama and French, 1993).[1] This data source offers several methodological advantages that justify its selection over individual stock data. Factor portfolios keep the cross-sectional dimension N tractable, which is essential for the mean-variance strategies evaluated in this thesis: with individual stocks, N can reach into the hundreds, rendering the sample covariance matrix near-singular and making all optimized strategies except the constrained minimum-variance infeasible without additional regularization. Furthermore, the French Data Library constructs each portfolio by incorporating all eligible firms at each rebalancing date, including those that subsequently delist or fail, so the return series are free from survivorship bias (Fama and French, 1993).

All returns are monthly total returns expressed in U.S. dollars, including both dividends and capital gains. The sample period differs by region and, for the United States, also by asset universe, to exploit the full extent of available data. For the United States, the FF3 portfolios, the 25 Size and BM portfolios, and the 10 Industry portfolios are available from July 1929 to November 2025, spanning nearly a century of monthly observations. The FF5 portfolios are available for the United States from July 1963 to November 2025. For Europe and Japan, all series are available from July 1990 to November 2025, reflecting the later onset of comparable regional factor data. Cross-regional comparisons employ only the common window of July 1990 to November 2025 to ensure full comparability, while the extended US history is additionally exploited in US-specific analyses to maximise the number of identifiable crisis episodes.

The empirical analysis focuses on developed equity markets comprising the United States, Europe, and Japan, with Europe treated as a single aggregated region throughout. Four distinct datasets are employed as investable asset sets, each representing a different dimension of equity market segmentation: (1) FF3 portfolios, (2) FF5 portfolios, (3) 25 Size and BM sorted portfolios, and (4) 10 Industry portfolios.

[1] Kenneth R. French Data Library, available at: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

3.1.1 Fama-French 3-Factor Portfolios

The first asset universe consists of the three factor-mimicking portfolios from the Fama-French three-factor model: MKT, SMB, and HML, treated as investable assets in the portfolio optimization framework (Fama and French, 1993). This universe ($N = 3$) has been extensively studied in portfolio optimisation research and presents non-trivial optimisation challenges because two of the three assets (SMB and HML) are zero-cost portfolios whose means may not be reliably positive across all sample periods, leading to unstable portfolio weights when estimation error is present (DeMiguel et al., 2009; Pástor, 2000).

3.1.2 Fama-French 5-Factor Portfolios

The second asset universe extends the three-factor model by incorporating the profitability factor (RMW) and the investment factor (CMA) from the five-factor model (Fama and French, 2015). Together with MKT, SMB, and HML, these five factors ($N = 5$) provide a richer opportunity set, potentially allowing portfolio strategies to exploit additional return premia beyond size and value. This, however, comes at the cost of increased estimation uncertainty from higher dimensionality.

3.1.3 25 Size and Book-to-Market Sorted Portfolios

The third asset universe consists of portfolios formed by independently sorting firms into five size quintiles and five BM quintiles using NYSE breakpoints, generating 25 intersection portfolios each value-weighted by market capitalization. The five portfolios corresponding to the largest size quintile are excluded, as these are highly correlated with the market index and create severe multicollinearity in the covariance matrix (Wang, 2005; DeMiguel et al., 2009). The resulting investable universe therefore contains $N = 20$ portfolios.

3.1.4 10 Industry Portfolios

The fourth asset universe consists of ten broad industry portfolios constructed using Standard Industrial Classification (SIC) codes, as defined by the data library: Consumer Nondurables, Consumer Durables, Manufacturing, Energy, Chemicals, Business Equipment, Telecommunications, Utilities, Retail, and Other. Each portfolio is value-weighted within its industry. Industry portfolios capture sector-specific risk exposures and exhibit time-varying correlations and sector rotation patterns, particularly during crisis periods, making this universe ($N = 10$) an empirically relevant test case. This universe is available only for the United States.

3.1.5 Investable Asset Universes

The four base datasets serve as building blocks from which a structured set of composite universes is constructed. Six investable universes are used for the United States and five for Europe and Japan (which lack comparable industry return data): FF3 ($N = 3$); FF5 ($N = 5$); BM_MKT ($N = 21$), the 20 retained Size and BM portfolios combined with MKT; BM_FF3 ($N = 23$), augmented by the full FF3 set; BM_FF5 ($N = 25$), augmented by the full FF5 set; and IND_MKT ($N = 11$, United States only), the 10 Industry portfolios combined with MKT.

3.2 Return Construction and Excess Returns

All portfolio's return series are initially obtained as monthly total returns expressed as simple arithmetic returns, inclusive of dividends and capital gains. For portfolio optimization and performance evaluation, excess returns are computed by subtracting the risk-free rate from each asset's total return. The risk-free rate is proxied by the one-month U.S. Treasury bill rate from the data library. Crisis identification employs total returns rather than excess returns to avoid spurious signals from variation in the risk-free component.

All returns are transformed from simple returns into continuously compounded log returns: $R = \ln(1 + r)$. This transformation is applied to both total returns (for crisis identification) and excess returns (for portfolio optimisation). Log returns are time-additive, so multi-period returns decompose naturally into sums of single-period returns, facilitating performance decomposition across crisis phases. They also exhibit improved distributional properties relative to simple returns, with reduced skewness and excess kurtosis, which benefits statistical inference (Campbell et al., 1997). All subsequent analysis employs log excess returns for portfolio weight computation and log total returns for the cumulative wealth indices underlying the CMAX crisis measure, unless explicitly stated otherwise.

3.3 Portfolio Formation Mechanics

This section describes the rolling out-of-sample portfolio construction procedure, which tries to evaluate portfolio strategies under conditions that mimic an investor's real-time decision environment, where future returns are unknown and portfolio weights must be determined using only information available at the time of each rebalancing.

The baseline specification employs an estimation window of $M = 60$ months (5 years) balancing estimation precision against the assumption of parameter stationarity. Robustness analysis using $M = 120$ months (10 years) is also conducted. Portfolios are rebalanced monthly: weights are recalculated at the end of each month and adjusted at the beginning of the subsequent month. In the baseline analysis, weight constraints are not imposed except where explicitly specified by the strategy (e.g., the constrained mean-variance strategy, which imposes $w_i \geq 0$).

Transaction costs are incorporated by assuming proportional costs of 50 basis points per transaction. Portfolio turnover in month t is measured as:

$$TO_t = \sum_{i=1}^{N} \left| w_{i,t} - w_{i,t}^{+} \right|$$

where $w_{i,t}$ is the desired weight in asset i at the end of month $t-1$ (after rebalancing) and $w_{i,t}^{+}$ is the weight immediately before rebalancing in month t (the weight resulting from the previous period's returns without trading). The transaction cost incurred in month t is $0.005 \times TO_t$.

3.4 Portfolio Construction Methodologies

3.4.1 Theoretical Framework

This section presents the portfolio construction methodologies evaluated in this thesis. Let R_t denote the N-vector of excess returns on the N risky assets at date t, with expected value μ_t and variance-covariance matrix Σ_t. Their sample counterparts, estimated over a window of M months, are $\hat{\mu}_t$ and $\hat{\Sigma}_t$. Throughout, $\mathbf{1}_N$ denotes an N-vector of ones.

The portfolio weight vector x_t represents the fraction of wealth invested in the N risky assets at time t, with $1 - \mathbf{1}_N{}' x_t$ invested in the risk-free asset. The vector of relative weights among risky assets is defined as:

$$w_t = \frac{x_t}{|\mathbf{1}_N{}' x_t|}$$

A mean-variance investor should then maximizes the expected utility at each time t by solving:

$$\max_{x_t} \; x_t{}' \mu_t - \frac{\gamma}{2} x_t{}' \Sigma_t x_t$$

where γ denotes the coefficient of risk aversion. The solution yields $x_t = \frac{1}{\gamma}\Sigma_t^{-1}\mu_t$, and the corresponding vector of relative portfolio weights is:

$$w_t = \frac{\Sigma_t^{-1}\mu_t}{\mathbf{1}_N{}'\Sigma_t^{-1}\mu_t}$$

Most portfolio strategies evaluated in this thesis deliver weights expressible in this form, with primary differences arising from how μ_t and Σ_t are estimated or constrained.

3.4.2 Benchmark Strategies

3.4.2.1 Equal-Weighted Portfolio (1/N)

The naïve diversification strategy (EW), denoted $1/N$, serves as the primary benchmark against which all strategies are evaluated. It allocates wealth equally across all available assets:

$$w_t^{ew} = \frac{1}{N}\mathbf{1}_N$$

This heuristic requires no parameter estimation and ignores historical return data entirely. The portfolio is rebalanced monthly to maintain equal weights, generating transaction costs that are accounted for in all net-of-cost comparisons.

3.4.2.2 Value-Weighted Portfolio (VW)

The VW portfolio uses the total return on the value-weighted market index (MKT_total) as its realized return and involves zero active rebalancing, as market-capitalization weights adjust automatically with price changes. The VW benchmark corresponds to the passive market portfolio and serves as the zero-turnover comparison case, isolating the impact of active rebalancing on strategy performance.

3.4.3 Classical Optimization Strategies

For all strategies, a small ridge regularization term proportional to the trace of the sample covariance matrix is added to the diagonal of $\hat{\Sigma}_t$ prior to inversion, preventing numerical instability from near-singular covariance matrices.

The **unconstrained mean-variance portfolio** (mv) sets weights proportional to $\hat{\Sigma}_t^{-1}\hat{\mu}_t$, normalised to sum to unity. This is the tangency portfolio under full reliance on sample estimates and is susceptible to extreme long and short positions when estimation error is present. The **constrained mean-variance portfolio** (mv-c) solves the same optimization but imposes non-negativity constraints ($w_i \geq 0$) via quadratic programming, which implicitly shrinks off-diagonal covariance estimates and improves out-of-sample stability (Jagannathan and Ma, 2003).

The **Bayes-Stein portfolio** (bs) and its **constrained variant** (bs-c) shrink the sample mean vector toward the expected return of the global minimum-variance portfolio, with shrinkage intensity estimated from the data (Jorion, 1991). The **minimum-variance portfolio** (min) and its **constrained variant** (min-c) set $\hat{\mu}_t = \mathbf{1}_N$, so that the weight vector is determined entirely by $\hat{\Sigma}_t^{-1}$ without relying on mean estimates; the constrained version additionally imposes $w_i \geq 0$. The **globally constrained minimum-variance portfolio** (g-min-c) combines the minimum-variance criterion with a global non-negativity constraint, providing an additional layer of regularization against extreme weights.

The **mean-variance/minimum-variance mixture** (mv-min) constructs the portfolio as a convex combination of the mv and min weights with equal mixing coefficients, in the spirit of combining strategies to hedge parameter uncertainty (Kan and Zhou, 2007). The **EW/minimum-variance mixture** (ew-min) combines the EW portfolio with the unconstrained minimum-variance portfolio in equal proportions, exploiting covariance information while avoiding mean estimation entirely (Wang, 2005).

3.4.4 Phase-Conditioned Long-Short Strategies

The phase-conditioned strategies are implemented using the 4-phase CMAX classification (pre-crisis, crisis, drawdown, recovery). During pre-crisis periods, all strategies in this group default to the EW allocation; signal-based positions are formed only during crisis, drawdown, and recovery phases. This design directly targets the research question of phase-specific performance while avoiding look-ahead bias in signal construction.

Eight signal families are implemented, each available in two variants: long-only (L), which invests equally in the top-ranked X of assets; and long-short (LS), which simultaneously holds the top X long and shorts the bottom X, where X depends on the asset universe (e.g., 1, 3 or 5), with positions in each leg summing to unity in absolute value. Long-only variants require no short-selling and incur transaction costs only on rebalancing trades at phase transitions; long-short variants additionally incur costs on short-leg rebalancing. The first episode encountered in the data is excluded for cross-episode strategies (Past-1 and Past-All variants) since no prior episode is available.

Past-1 Full (P1F): assets are ranked by their total return over the full prior crisis episode, with the portfolio formed at the beginning of each new crisis episode and held without rebalancing until the episode ends. **Past-1 Phase** (P1P): assets are ranked by their return in the same phase of the most recent prior crisis episode, with the portfolio rebalancing at each phase transition within the current episode. **Past-All Full** (PAF): assets are ranked by their average total return across all previously observed full crisis episodes, aggregating historical information to reduce sensitivity to any single episode. **Past-All Phase** (PAP): assets are ranked by their average same-phase return across all previously observed episodes.

Phase-Reversal (Phase-Rev): assets are ranked by the return of the most recently completed phase, and the portfolio goes long the worst performers, implementing a contrarian signal (DeBondt and Thaler, 1985). **Phase-Momentum** (Phase-Mom): uses the same signal as Phase-Rev but goes long the best performers, implementing a momentum signal (Jegadeesh and Titman, 1993). **Momentum-12** (Mom-12): a rolling 12-1 month return signal is used to rank assets (Jegadeesh and Titman, 1993). This strategy is always active and is not restricted to crisis phases, serving as an unconditional momentum benchmark.

3.5 Crisis Identification via the CMAX Methodology

Crisis periods are identified using the CMAX methodology, which provides an objective, data-driven procedure for dating market crises based on observed drawdowns in cumulative market returns without requiring ex-ante specification of crisis dates (Patel and Sarkar, 1998).

Let P_t denote the cumulative wealth index of the market portfolio at the end of month t, computed from total log returns as $P_t = \exp\left(\sum_{s=1}^{t} r_{m,s}\right)$, where $r_{m,s}$ is the log total return on the market portfolio in month s and $P_0 = 1$. For each month t, the running historical maximum over the most recent 24 months is defined as $P_t^{max} = \max_{1 \le s \le t} P_s$. The CMAX measure at time t is then:

$$CMAX_t = \frac{P_t}{P_t^{\max}}$$

By construction, $CMAX_t \in (0,1]$, with $CMAX_t = 1$ indicating a new peak and $CMAX_t < 1$ indicating a decline from the historical maximum. A crisis begins in month t if $CMAX_t$ falls below a specified threshold τ for the first time following a period in which CMAX was equal to 1. The developed-market threshold $\tau = 0.80$ is applied throughout, as the dataset comprises developed equity markets (Patel and Sarkar, 1998). The crisis trough is defined as the month within the episode in which CMAX reaches its minimum value. The crisis ends when CMAX returns to 1, indicating full recovery to the pre-crisis peak. If a crisis remains unresolved by the end of the sample period, it is treated as ongoing through the final observation. To prevent double counting, a new crisis can only begin after the previous one has fully ended and CMAX subsequently falls below the threshold again.

The CMAX methodology serves two complementary purposes in this thesis. For the crisis chronology and associated visualizations, a four-phase structure is employed: (i) the **pre-crisis phase** spans the period from the previous peak ($CMAX = 1$) to the last month immediately before the uninterrupted decline that will eventually breach the threshold; (ii) the **crisis phase** covers the interval from the first month in which CMAX falls below 1 (after the final pre-crisis peak) through the last month before CMAX first breaches 0.80; (iii) the **drawdown phase** begins at the first month in which CMAX falls below $\tau = 0.80$ and ends at the month of minimum CMAX; and (iv) the **recovery phase** runs from the month immediately following the trough until the first month in which CMAX returns to 1. For the conditional portfolio performance evaluation, a simpler binary classification is employed, assigning each observation month to either a pre-crisis or crisis phase.

3.6 Performance Evaluation and Statistical Inference

Portfolio performance is evaluated using a set of metrics capturing both traditional mean-variance trade-offs and downside risk characteristics relevant during crisis periods.

The primary metric is the Sharpe ratio (SR) (Sharpe, 1966), defined as:

$$SR = \frac{\hat{\mu}_p}{\hat{\sigma}_p}$$

where $\hat{\mu}_p$ is the mean log excess return and $\hat{\sigma}_p$ is the standard deviation of log excess returns. Out-of-sample SR values are computed from the time series of realized portfolio returns generated by the rolling evaluation procedure.

The certainty-equivalent return (CEQ) represents the risk-free rate that a mean-variance investor would accept in lieu of holding the risky portfolio:

$$CEQ_p(\gamma) = \hat{\mu}_p - \frac{\gamma}{2}\sigma_p^2$$

CEQ values are reported for $\gamma = 5$. This value is well-established in the portfolio literature as a representation of a moderately risk-averse investor (DeMiguel et al., 2009; Campbell and Viceira, 2002).

Portfolio turnover in month t is measured as $TO = \sum_{i=1}^{N} \left| w_{i,t} - w_{i,t}^{+} \right|$. Average turnover is reported both in absolute terms and relative to the 1/N strategy.

To evaluate downside risk, the Sortino ratio (Sortino and Price, 1994) is computed as:

$$Sortino = \frac{\hat{\mu}_p}{\hat{\sigma}_{d,p}}$$

where the downside deviation is $\hat{\sigma}_{d,p} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} \min\left(r_{p,t}, 0\right)^2}$, penalising only negative excess returns. This construction treats positive excess returns as non-penalised outcomes and scales performance by the magnitude of negative outcomes only, making it particularly informative when return distributions are asymmetric during crisis periods.

Drawdown risk is measured via maximum drawdown (MDD). Let $W_t = \exp\left(\sum_{s=1}^{t} r_{p,s}\right)$ with $W_0 = 1$ denote the cumulative wealth index and define the running maximum $W_t^{\max} = \max_{1 \le s \le t} W_s$. MDD is then:

$$MDD_p = 1 - \min_{1 \le t \le T} \frac{W_t}{W_t^{\max}}$$

equaling the largest peak-to-trough proportional loss over the evaluation period (Young, 1991).

The Calmar ratio rewards strategies delivering higher average excess returns while limiting extreme drawdowns:

$$Calmar_p = \frac{\hat{\mu}_p}{MDD_p}$$

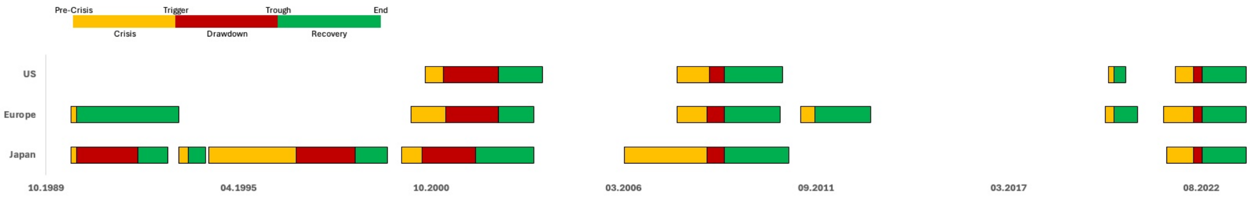
All performance metrics are computed both unconditionally (using all out-of-sample months) and conditionally (using only months classified as pre-crisis, crisis, drawdown, or recovery).

Statistical inference is conducted to formally test whether observed performance differences are statistically significant or could plausibly arise from sampling variation. For SR comparisons, an asymptotically standard normal test statistic for the difference in two SRs is constructed under the assumption of identically and independently distributed, approximately normally distributed returns (Jobson and Korkie, 1981; Memmel, 2003). For performance metrics without well-established asymptotic distributions, including MDD, the Sortino ratio, and the Calmar ratio, bootstrap-based inference is employed using the stationary bootstrap with $N = 500$ which resamples blocks of returns with geometrically distributed block lengths to preserve serial dependence (Politis and Romano, 1994). Bootstrap confidence intervals and p-values are constructed from the empirical distribution of the resampled statistics, providing valid inference without distributional assumptions and remaining robust to the heteroscedasticity and serial correlation characteristic of crisis periods. All tests are conducted at significance levels $\alpha \in \{0.01, 0.05, 0.10\}$, and Bonferroni corrections are applied to account for multiple comparisons across strategies and metrics.

4. Empirical Results

4.1 Crisis Identification

Figure 1: CMAX Crisis Episode Timeline | Europe, United States, and Japan (1990–2025)



Notes: Each bar represents one crisis episode identified by the CMAX methodology (Patel and Sarkar, 1998). The colour coding distinguishes the three phases: yellow (crisis lead-in), red (drawdown: $\mathrm{CMAX} < 0.80$ through trough), and green (recovery: trough through $\mathrm{CMAX} = 1$). Episodes are plotted on a common time axis for the overlapping sample period (July 1990 – November 2025). The full US history from July 1926 is presented in Figure A1.

Figure 1 documents the complete CMAX crisis chronology across the three regions over the common sample period from July 1990 to November 2025. The visualization makes transparent both the internal phase structure of each episode and the degree of cross-regional synchronisation. The Global Financial Crisis of 2007–2009 stands out as the most severe and geographically synchronised event: all three regions enter crisis within a few months of each other, with Europe recording a peak-to-trough loss of 59.0%, and the United States and Japan sustaining drawdowns of 50.3% and 44% respectively. The dot-com bust of 2000–2003 is also highly synchronised, though lead-in periods differ: Europe and Japan exhibit longer pre-crisis phases, while the United States breaches the threshold more sharply. Japan exhibits markedly longer and more persistent episodes than the other two regions, most notably the Lost Decade episode beginning in July 1994, spanning 61 months in total. Post-2020 episodes are comparatively short: the COVID-19 shock of March 2020 resolves within six months for the United States, reflecting the speed of policy intervention and market recovery.

Table A1 provides episode-level statistics for all 24 identified episodes across the three regions: 12 for the United States, 7 for Europe, and 5 for Japan, including duration, trough depth, and drawdown severity for each episode. The full historical US chronology extending to July 1926 is presented in Figure A1, which reveals additional episodes not captured in the common post-1990 window, most notably the Depression Relapse of 1937–1939 and the Oil Crisis bear market of 1973–1976, and illustrates the long-run applicability of the CMAX methodology across very different macroeconomic environments.

4.2 Binary Classification: Classical Optimized Strategies

As reported in Table A2, results across the full-sample and pre-crisis phases broadly confirm the central finding that optimised portfolio strategies fail to outperform naive diversification out-of-sample (DeMiguel et al., 2009). Across the majority of strategies, Sharpe ratios are either statistically indistinguishable from 1/N or significantly negative, with unconstrained strategies generating the most severe underperformance due to extreme weight allocations and high estimation error sensitivity. An important qualification applies to the constrained strategy subset and the value-weighted benchmark: MV-C, BS-C, and G-Min-C are not significantly below 1/N on the Sharpe ratio in most universe specifications and, in several cases, record statistically significant positive SR differentials during the pre-crisis period, with VW exhibiting similar results. Min-Var-C is the one constrained strategy that does not reliably share this property. On other metrics, including AnnRet and CEQ, even the better-performing constrained strategies are largely statistically indistinguishable from 1/N rather than outperforming. These results are robust across asset universes; the $M = 120$ estimation window serves as a sensitivity check confirming the same directional pattern, with MV-C, BS-C, and G-Min-C retaining their relatively competitive SR profile under the longer window.

The binary crisis period produces a more mixed but broadly consistent picture. Unconstrained strategies continue to exhibit deeply negative performance across all metrics. Among constrained strategies, the relative disadvantage narrows on both Sharpe ratio and CEQ, indicating that the constrained minimum-variance objective begins to offer competitive risk-adjusted performance under stressed market conditions. These results do not overturn the main prior finding but establish that constraints provide material protection against the most severe underperformance under crisis conditioning, a distinction that the four-phase decomposition in the next section delve deeper into.

4.3 Four-Phase Classification: Classical Optimized Strategies

As shown in Table 1, the four-phase decomposition reveals structural patterns that binary classification obscures. The full-sample and pre-crisis results replicate the binary findings, with the great majority of strategies either insignificant or significantly below 1/N on both metric groups. The novel and empirically central result emerge in the drawdown and recovery phases, which move in sharply opposite directions.

During the drawdown phase, strategies broadly and significantly outperform 1/N on AnnRet, CEQ, MDD, and Calmar. Every strategy with constrained weights shows significantly positive AnnRet and CEQ relative to 1/N, and significantly lower maximum drawdown. The unconstrained strategies, MV and BS, while still negative in absolute terms, also register significantly above the 1/N level during the drawdown phase. The one notable exception within the drawdown phase is the Sharpe ratio: most strategies register a Sharpe ratio significantly below 1/N at drawdown, because total-variance penalisation works against strategies that reduce extreme losses but retain some return volatility in a turbulent environment. The drawdown advantage is therefore concentrated in raw return and maximum loss protection rather than in volatility-adjusted measures.

The crisis phase shows similar outperformance on AnnRet and CEQ for most strategies, with unconstrained MV, BS, and Min-Var significantly above the 1/N level. Constrained strategies such as MV-C and BS-C are indistinguishable from 1/N on most metrics, while G-Min-C shows some of the sharpest improvements relative to benchmark across AnnRet, CEQ, and MDD in this phase.

The recovery phase inverts the picture entirely. Every optimised strategy underperforms 1/N significantly on AnnRet, CEQ, and SR during the recovery phase. Strategies positioned defensively at the trough, specifically underweighting the high-volatility assets that lead the rebound, are structurally penalised. G-Min-C and Min-Var-C show the least severe recovery underperformance: G-Min-C records the least severe recovery underperformance among the classical strategies, significantly below 1/N but considerably closer to it than unconstrained MV and BS. MDD at recovery is largely inconclusive across strategies, consistent with both the strategy and 1/N portfolios experiencing limited drawdowns during a rising market. The two crisis-period phases thus partially cancel in the binary classification, which explains why the binary crisis signal appears weaker than the individual phase results.

Table 1: Four-Phase CMAX Performance Analysis | Optimized Strategies vs. 1/N (Europe, M=60) | Universe: BM_FF3 (N=23 assets)

Strategy	N	Ann. Ret.	SR	CEQ	Sortino	Max DD	Calmar
				Full Sample			
1/N (EW)	**365**	**0.049**	**0.289**	**-0.022**	**0.351**	**-0.595**	**0.082**
VW	365	0.052	0.296***	-0.025	0.379	-0.599	0.087
		(0.77)	*(0.00)*	*(0.80)*	*(0.69)*	*(0.82)*	*(0.85)*
MV	365	-0.036**	-0.603***	-0.045	-1.717	-0.722	-0.049*
		(0.03)	*(0.00)*	*(0.59)*	*(0.15)*	*(0.42)*	*(0.06)*
MV-C	365	0.069	0.478***	0.017	0.609	-0.455	0.152
		(0.30)	*(0.00)*	*(0.11)*	*(0.16)*	*(0.25)*	*(0.23)*
BS	365	-0.039**	-0.878***	-0.044	-2.349**	-0.698	-0.055*
		(0.01)	*(0.00)*	*(0.60)*	*(0.04)*	*(0.51)*	*(0.05)*
BS-C	365	0.069	0.439***	0.007	0.549	-0.459	0.150
		(0.26)	*(0.00)*	*(0.16)*	*(0.22)*	*(0.19)*	*(0.17)*
Min-Var	365	-0.044***	-2.781***	-0.045	-3.632***	-0.735	-0.060**
		(0.00)	*(0.00)*	*(0.56)*	*(0.00)*	*(0.33)*	*(0.05)*
Min-Var-C	365	-0.006**	-0.112***	-0.012	-0.163	-0.311*	-0.019*
		(0.04)	*(0.00)*	*(0.78)*	*(0.12)*	*(0.07)*	*(0.10)*
G-Min-C	365	0.031	0.327***	0.008	0.403	-0.419**	0.075
		(0.31)	*(0.00)*	*(0.16)*	*(0.64)*	*(0.01)*	*(0.82)*
MV-Min	365	-0.041***	-1.561***	-0.043	-3.231***	-0.709	-0.058**
		(0.01)	*(0.00)*	*(0.60)*	*(0.00)*	*(0.43)*	*(0.04)*
EW-Min	365	-0.036***	-1.541***	-0.037	-2.956***	-0.663	-0.054**
		(0.01)	*(0.00)*	*(0.67)*	*(0.00)*	*(0.65)*	*(0.05)*
				Pre-Crisis (CMAX between 1 and 0.80, no threshold breach)			
1/N (EW)	**225**	**0.122**	**1.071**	**0.089**	**1.712**	**-0.216**	**0.563**

VW	225	0.131	1.072	0.093	1.590	-0.202	0.647
		(0.47)	*(0.72)*	*(0.72)*	*(0.46)*	*(0.47)*	*(0.44)*
MV	225	-0.029***	-0.389***	-0.042***	-1.229	-0.602***	-0.048**
		(0.00)	*(0.00)*	*(0.00)*	*(0.17)*	*(0.01)*	*(0.04)*
MV-C	225	0.112	1.139***	0.088	1.822	-0.142**	0.787
		(0.59)	*(0.00)*	*(0.91)*	*(0.67)*	*(0.03)*	*(0.10)*
BS	225	-0.036***	-0.661***	-0.043***	-2.028*	-0.581***	-0.062**
		(0.00)	*(0.00)*	*(0.00)*	*(0.05)*	*(0.01)*	*(0.03)*
BS-C	225	0.121	1.170***	0.094	1.919	-0.146**	0.830*
		(0.99)	*(0.00)*	*(0.78)*	*(0.42)*	*(0.04)*	*(0.06)*
Min-Var	225	-0.051***	-3.262***	-0.052***	-3.990***	-0.614***	-0.083**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.03)*
Min-Var-C	225	-0.001***	-0.020***	-0.006***	-0.034***	-0.235	-0.004**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.85)*	*(0.02)*
G-Min-C	225	0.061***	0.903***	0.050***	1.490	-0.163**	0.373**
		(0.00)	*(0.00)*	*(0.01)*	*(0.12)*	*(0.04)*	*(0.05)*
MV-Min	225	-0.043***	-1.388***	-0.045***	-3.265***	-0.549**	-0.078**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.02)*	*(0.03)*
EW-Min	225	-0.039***	-1.419***	-0.040***	-2.969***	-0.510**	-0.076**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.04)*	*(0.03)*
Crisis (CMAX declining from 1 toward the 0.80 threshold)							
1/N (EW)	**35**	**-0.312**	**-2.304**	**-0.358**	**-3.535**	**-0.575**	**-0.543**
VW	35	-0.316	-2.171***	-0.369	-3.070*	-0.587	-0.539
		(0.85)	*(0.00)*	*(0.66)*	*(0.08)*	*(0.64)*	*(0.83)*
MV	35	-0.052***	-3.916***	-0.052***	-4.822	-0.136***	-0.384***
		(0.00)	*(0.00)*	*(0.00)*	*(0.37)*	*(0.00)*	*(0.00)*
MV-C	35	-0.282	-1.950***	-0.334	-2.440***	-0.529	-0.532

		(0.60)	*(0.00)*	*(0.74)*	*(0.00)*	*(0.53)*	*(0.78)*
BS	35	-0.052***	-4.221***	-0.053***	-4.931	-0.136***	-0.384***
		(0.00)	*(0.00)*	*(0.00)*	*(0.28)*	*(0.00)*	*(0.00)*
BS-C	35	-0.328	-2.146***	-0.386	-2.728**	-0.580	-0.565
		(0.78)	*(0.00)*	*(0.70)*	*(0.02)*	*(0.92)*	*(0.58)*
Min-Var	35	-0.052***	-4.555***	-0.052***	-5.312**	-0.135***	-0.386***
		(0.00)	*(0.00)*	*(0.00)*	*(0.04)*	*(0.00)*	*(0.00)*
Min-Var-C	35	-0.020***	-0.470***	-0.024***	-0.809***	-0.117***	-0.171*
		(0.00)	*(0.00)*	*(0.00)*	*(0.01)*	*(0.00)*	*(0.08)*
G-Min-C	35	-0.124***	-1.650***	-0.139***	-2.483	-0.321***	-0.388***
		(0.00)	*(0.00)*	*(0.00)*	*(0.11)*	*(0.00)*	*(0.00)*
MV-Min	35	-0.052***	-4.533***	-0.052***	-5.273	-0.135***	-0.385***
		(0.00)	*(0.00)*	*(0.00)*	*(0.12)*	*(0.00)*	*(0.00)*
EW-Min	35	-0.052***	-4.560***	-0.053***	-5.320**	-0.135***	-0.386***
		(0.00)	*(0.00)*	*(0.00)*	*(0.04)*	*(0.00)*	*(0.00)*
Drawdown (trigger month through trough)							
1/N (EW)	**40**	**-0.483**	**-1.771**	**-0.670**	**-2.135**	**-0.789**	**-0.613**
VW	40	-0.498	-1.845***	-0.680	-2.481**	-0.805	-0.619
		(0.68)	*(0.00)*	*(0.82)*	*(0.02)*	*(0.41)*	*(0.85)*
MV	40	-0.048***	-2.129***	-0.049***	-2.838	-0.144***	-0.331**
		(0.00)	*(0.00)*	*(0.00)*	*(0.41)*	*(0.00)*	*(0.03)*
MV-C	40	-0.233***	-1.162***	-0.334***	-1.743	-0.580**	-0.402**
		(0.00)	*(0.00)*	*(0.00)*	*(0.41)*	*(0.01)*	*(0.02)*
BS	40	-0.043***	-2.115***	-0.044***	-2.851	-0.130***	-0.329**
		(0.00)	*(0.00)*	*(0.00)*	*(0.42)*	*(0.00)*	*(0.03)*
BS-C	40	-0.277**	-1.193***	-0.411**	-1.635	-0.626**	-0.442**
		(0.01)	*(0.00)*	*(0.01)*	*(0.14)*	*(0.04)*	*(0.03)*

Min-Var	40	-0.028***	-1.858***	-0.028***	-2.612	-0.085***	-0.327**
		(0.00)	*(0.00)*	*(0.00)*	*(0.61)*	*(0.00)*	*(0.03)*
Min-Var-C	40	-0.095***	-1.351***	-0.107***	-1.542	-0.294***	-0.322**
		(0.00)	*(0.00)*	*(0.00)*	*(0.23)*	*(0.00)*	*(0.01)*
G-Min-C	40	-0.241***	-1.550***	-0.302***	-1.797	-0.564***	-0.428***
		(0.00)	*(0.00)*	*(0.00)*	*(0.17)*	*(0.00)*	*(0.00)*
MV-Min	40	-0.035***	-2.135***	-0.035***	-2.948	-0.106***	-0.327**
		(0.00)	*(0.00)*	*(0.00)*	*(0.38)*	*(0.00)*	*(0.03)*
EW-Min	40	-0.027***	-1.794	-0.028***	-2.494	-0.083***	-0.327**
		(0.00)	*(0.16)*	*(0.00)*	*(0.71)*	*(0.00)*	*(0.04)*
Recovery (trough+1 through CMAX returning to 1)							
1/N (EW)	**65**	**0.319**	**1.803**	**0.241**	**3.515**	**-0.147**	**2.167**
VW	65	0.316	1.663***	0.226	3.551	-0.155	2.041
		(0.91)	*(0.00)*	*(0.49)*	*(0.93)*	*(0.78)*	*(0.84)*
MV	65	-0.044***	-2.214***	-0.045***	-3.325**	-0.215	-0.204*
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.20)*	*(0.07)*
MV-C	65	0.295	1.628***	0.213	2.289*	-0.209*	1.415
		(0.31)	*(0.00)*	*(0.33)*	*(0.06)*	*(0.08)*	*(0.23)*
BS	65	-0.038***	-2.123***	-0.039***	-3.205**	-0.192	-0.200*
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.41)*	*(0.07)*
BS-C	65	0.313	1.651***	0.223	2.415*	-0.209*	1.499
		(0.81)	*(0.00)*	*(0.55)*	*(0.09)*	*(0.08)*	*(0.33)*
Min-Var	65	-0.025***	-1.466***	-0.025***	-2.391**	-0.135	-0.181*
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.82)*	*(0.07)*
Min-Var-C	65	0.040***	0.657***	0.031***	1.250	-0.112	0.355*
		(0.00)	*(0.00)*	*(0.00)*	*(0.21)*	*(0.38)*	*(0.08)*
G-Min-C	65	0.181***	1.741***	0.154**	3.600	-0.106*	1.709

		(0.00)	*(0.00)*	*(0.01)*	*(0.83)*	*(0.05)*	*(0.19)*
MV-Min	65	-0.031***	-1.875***	-0.032***	-2.901**	-0.159	-0.196*
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.84)*	*(0.07)*
EW-Min	65	-0.024***	-1.452***	-0.025***	-2.411**	-0.134	-0.181*
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.79)*	*(0.07)*

Notes: **1/N (EW)** = equal weight 1/N, rebalanced monthly. **VW** = value-weighted market return (MKT_total), zero turnover. **MV** = mean-variance. **MV-C** = constrained MV. **BS** = Bayes-Stein. **BS-C** = constrained BS. **Min-Var** = minimum-variance. **Min-Var-C** = constrained Min-Var. **G-Min-C** = global minimum-variance constrained. **MV-Min** = convex combination of MV and Min-Var. **EW-Min** = convex combination of 1/N and Min-Var. All optimized strategies use a rolling M-month estimation window. Metrics on monthly log excess returns $ln(1 + r - rf)$. **N** = monthly observations. Stars vs 1/N (EW): * p<0.10, ** p<0.05, *** p<0.01. P-values in parentheses below each strategy. **SR**: Jobson-Korkie (1981)/Memmel (2003); others: stationary bootstrap (B=500). All significance tests measure the difference of each strategy relative to 1/N (EW); a self-referential test is not defined.

4.4 Pooled Cross-Regional Phase Regression

A concern with this single-region evidence is that the documented patterns could reflect characteristics specific to the European factor universe rather than a structural feature of crisis dynamics. To test whether the phase-level asymmetry holds independently of regional composition and factor universe construction, a pooled OLS regression is estimated with the monthly log excess return differential of each strategy relative to 1/N as the dependent variable and phase dummies as regressors, with standard errors clustered by calendar month to account for within-month cross-sectional correlation. Region dummies (Europe and Japan, with the United States as baseline) and universe dummies (FF3, FF5, BM_FF3, BM_FF5, with BM_MKT as baseline) are included as controls, allowing the phase coefficients to be interpreted as the within-phase performance differential net of any systematic region or factor universe effects. Full regression output including all control coefficients is reported in Table A7 and Table A8.

The pooled estimates in Table 2 confirm the crisis and drawdown-recovery asymmetry as a structural feature that holds across regions and universes. For the constrained minimum-variance family, crisis and drawdown coefficients are uniformly positive and highly significant, establishing that the outperformance documented is not a Europe-specific finding but replicates consistently across the United States and Japan. Region dummies for these strategies are small and statistically insignificant, directly ruling out a regional composition explanation. In the recovery phase, coefficients for the same strategies are uniformly negative and significant, confirming the reversal that constitutes the second half of the asymmetry.

Table 3 reports the pooled regression estimates under $M = 120$, serving as a robustness check on the main results. For the constrained minimum-variance family, the finding is unchanged: crisis and drawdown coefficients retain their sign and significance, and recovery coefficients remain negative and significant. The asymmetry for this strategy group is therefore not sensitive to the estimation window choice. A notable difference under $M = 120$ concerns the unconstrained strategies MV, BS, and MV-Min: while their recovery coefficients are negative under $M = 60$, they turn positive under $M = 120$. This sign reversal under the longer window precludes asserting the asymmetry for unconstrained strategies as a pooled structural result, and further reinforces that the documented pattern is a property of the constrained minimum-variance family specifically.

Table 2: Pooled Phase Regression | Strategy Return Differential vs. 1/N (All Regions, M=60)

Variable	VW	MV	MV-C	BS	BS-C	Min-Var	Min-Var-C	G-Min-C	MV-Min	EW-Min
Constant	0.0030**	-0.6029***	0.0006	-0.3957***	0.0018*	-0.0011	-0.0017*	-0.0012*	-0.2341***	-0.0013
	(2.34)	*(-5.40)*	*(0.65)*	*(-4.35)*	*(1.93)*	*(-0.67)*	*(-1.74)*	*(-1.89)*	*(-3.53)*	*(-0.93)*
Crisis	-0.0124***	0.1650***	-0.0016	0.1308***	-0.0036*	0.0137***	0.0128***	0.0083***	0.1092***	0.0118***
	(-4.52)	*(4.34)*	*(-0.74)*	*(4.11)*	*(-1.66)*	*(4.73)*	*(5.31)*	*(5.31)*	*(4.02)*	*(4.46)*
Drawdown	-0.0183***	0.0145	0.0071**	0.0679	0.0020	0.0237***	0.0209***	0.0135***	0.0763*	0.0211***
	(-5.08)	*(0.18)*	*(2.10)*	*(1.12)*	*(0.66)*	*(4.73)*	*(5.17)*	*(5.26)*	*(1.69)*	*(4.59)*
Recovery	0.0017	-0.2553***	-0.0024	-0.1347	-0.0013	-0.0095***	-0.0066***	-0.0038***	-0.1138	-0.0083***
	(0.74)	*(-2.64)*	*(-1.20)*	*(-1.57)*	*(-0.73)*	*(-3.03)*	*(-2.75)*	*(-2.69)*	*(-1.45)*	*(-2.99)*
N	5475	5475	5475	5475	5475	5475	5475	5475	5475	5475
R^2	0.0561	0.0163	0.0112	0.0087	0.0060	0.0734	0.0829	0.0797	0.0056	0.0676

Notes: See notes below.

Table 3: Pooled Phase Regression | Strategy Return Differential vs. 1/N (All Regions, M=120)

Variable	VW	MV	MV-C	BS	BS-C	Min-Var	Min-Var-C	G-Min-C	MV-Min	EW-Min
Constant	0.0023*	-0.3952***	0.0021*	-0.1792***	0.0026**	-0.0017	-0.0025**	-0.0015**	-0.2508***	-0.0013
	(1.70)	*(-4.20)*	*(1.77)*	*(-2.78)*	*(2.37)*	*(-0.96)*	*(-2.27)*	*(-2.28)*	*(-3.25)*	*(-0.81)*
Crisis	-0.0112***	0.0840	0.0042	0.0312	0.0021	0.0141***	0.0137***	0.0086***	0.0398	0.0129***
	(-3.54)	*(0.94)*	*(1.63)*	*(0.54)*	*(0.93)*	*(4.51)*	*(5.21)*	*(4.98)*	*(0.48)*	*(4.50)*
Drawdown	-0.0199***	-0.0114	0.0106***	0.0169	0.0061**	0.0231***	0.0197***	0.0128***	0.0324	0.0205***
	(-5.03)	*(-0.11)*	*(3.00)*	*(0.24)*	*(2.01)*	*(4.16)*	*(4.74)*	*(4.77)*	*(0.38)*	*(4.17)*
Recovery	0.0025	0.1267**	-0.0029	0.0674***	-0.0026	-0.0068**	-0.0047**	-0.0030**	0.1018**	-0.0059**
	(1.09)	*(2.25)*	*(-1.46)*	*(2.63)*	*(-1.44)*	*(-2.22)*	*(-2.03)*	*(-2.12)*	*(2.32)*	*(-2.17)*
N	4575	4575	4575	4575	4575	4575	4575	4575	4575	4575
R^2	0.0677	0.0105	0.0260	0.0053	0.0135	0.0732	0.0792	0.0794	0.0063	0.0663

Notes: Region dummies (Europe, Japan; baseline US) and universe dummies (FF3, FF5, BM_FF3, BM_FF5; baseline BM_MKT) included as controls but not reported; full coefficients in Table A7 and Table A8. Sample pools three regions (United States, Europe, Japan; 1990–2025) across five universes (FF3, FF5, BM_MKT, BM_FF3, BM_FF5). Crisis identification follows CMAX (Patel and Sarkar, 1998; $\theta = 0.80$, 24-month rolling window). N and R^2 reflect the complete specification including unreported controls. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

4.5 Four-Phase Classification: Long-Short Strategies

The preceding chapters establish a consistent structural pattern that constrained portfolio strategies generate significant outperformance during the sub-phases crisis and drawdown through their covariance-targeting objective, while the recovery phase systematically reverses this advantage. This asymmetry raises a further question: whether the predictable phase structure of the crisis cycle can itself serve as a basis for active portfolio construction. Specifically, if cross-sectional return patterns within crisis episodes are sufficiently persistent to carry information about subsequent phase-level performance, strategies designed to exploit this structure directly may generate outperformance that the classical optimised strategies, which remain agnostic to crisis phase dynamics, cannot replicate.

As reported in Table 4, the long-short strategy layer presents a markedly more favourable picture than the classical optimised strategies across all crisis phases. Full-sample Sharpe ratios for most long-short variants are significantly above 1/N, with Past-1 Phase and Past-All Phase long-short strategies also recording significant outperformance on CEQ and Sortino, a result absent from every phase of every classical strategy and attributable to phase-conditioned signal construction rather than to optimisation or estimation window choice. Among long-only variants, Past-1 Phase and Past-All Phase also show significant outperformance on AnnRet and downside metrics. ST-Rev is the clear underperformer in both variants, recording significant negative results on almost every metric, consistent across the other regional datasets.

During the drawdown phase, the transformation from long-only to long-short produces a categorical improvement. Every long-only strategy underperforms 1/N significantly on AnnRet and CEQ at drawdown, while virtually every long-short strategy significantly outperforms. Phase-Mom L/S and Mom-12 L/S produce the strongest drawdown results, with significantly positive annualised returns and MDD significantly lower than the benchmark. Past-1 Phase L/S and Past-All Phase L/S also record significant positive AnnRet, CEQ, and MDD improvements at drawdown. The only long-short strategies that do not fully recover at drawdown are Phase-Rev L/S and ST-Rev L/S, which remain negative on return metrics despite partial offset from the short leg. This pattern is consistent with the other regional tables, where phase-conditioned long-short strategies show broadly significant drawdown outperformance across all datasets.

The sub-phase crisis produces a differentiated result across strategy types. Long-only strategies largely fail to outperform at crisis, often significantly underperforming. Among long-short variants, Past-All Full L/S is the standout, posting significantly positive AnnRet and Sharpe ratio against a deeply negative 1/N, a result unique to strategies using the full episode history rather than a phase-conditioned signal. Past-1 Full L/S and Past-1 Phase L/S also outperform at crisis on most metrics. Phase-Reversal and Phase-Momentum strategies show no observations at the crisis sub-phase by construction, as they require a completed prior phase to form their signal.

The recovery phase reintroduces underperformance for most long-short strategies, consistent with the mechanism described in Section 4.3. Phase-Mom L/S records the most severe recovery underperformance, as the assets shorted during drawdown are precisely those that lead the rebound. Mom-12 L/S shows a similar but less extreme pattern. In contrast, Phase-Rev L/S and ST-Rev L/S are approximately neutral at recovery, with insignificant differences from 1/N on most metrics. Past-1 Phase and Past-All Phase long-only variants show positive and, in some cases, significantly above-1/N performance during recovery, largely because the long-only construction avoids the short-leg liability that drives recovery losses for L/S strategies. This recovery asymmetry between strategy types is consistent across Europe, Japan, and the United States datasets.

Table 4: Four-Phase CMAX Performance Analysis | Long-Only and Long-Short Strategies vs. 1/N | Universe: BM_FF3 (N=23 assets)

Strategy	N	Ann. Ret.	SR	CEQ	Sortino	Max DD	Calmar
			Panel A: Full Sample				
1/N (EW)	**425**	**0.039**	**0.237**	**-0.029**	**0.292**	**-0.595**	**0.066**
VW	425	0.047	0.271***	-0.028	0.349	-0.599	0.078
		(0.36)	*(0.00)*	*(0.87)*	*(0.38)*	*(0.94)*	*(0.56)*
Past-1 Full | Long	388	0.061	0.395***	0.001	0.488	-0.534	0.114
		(0.39)	*(0.00)*	*(0.23)*	*(0.27)*	*(0.39)*	*(0.42)*
Past-1 Full | L/S	388	0.074	0.782***	0.052*	1.186**	-0.331*	0.223
		(0.42)	*(0.00)*	*(0.06)*	*(0.03)*	*(0.06)*	*(0.33)*
Past-1 Phase | Long	388	0.071**	0.478***	0.016**	0.692***	-0.474**	0.150**
		(0.02)	*(0.00)*	*(0.01)*	*(0.00)*	*(0.04)*	*(0.05)*
Past-1 Phase | L/S	388	0.084	0.881***	0.061**	1.368**	-0.216***	0.390*
		(0.23)	*(0.00)*	*(0.04)*	*(0.02)*	*(0.00)*	*(0.06)*
Past-All Full | Long	388	0.065	0.433***	0.009	0.525	-0.534	0.121
		(0.30)	*(0.00)*	*(0.10)*	*(0.25)*	*(0.38)*	*(0.39)*
Past-All Full | L/S	388	0.076	0.806***	0.054*	1.237**	-0.225***	0.338*
		(0.39)	*(0.00)*	*(0.06)*	*(0.03)*	*(0.00)*	*(0.08)*
Past-All Phase | Long	388	0.075**	0.490***	0.017**	0.705***	-0.474*	0.158*
		(0.02)	*(0.00)*	*(0.02)*	*(0.01)*	*(0.07)*	*(0.07)*
Past-All Phase | L/S	388	0.089	0.919***	0.066**	1.448**	-0.216***	0.413**
		(0.17)	*(0.00)*	*(0.04)*	*(0.01)*	*(0.00)*	*(0.05)*
Phase-Rev | Long	389	0.055**	0.293***	-0.032***	0.338***	-0.562	0.097
		(0.03)	*(0.00)*	*(0.01)*	*(0.00)*	*(0.14)*	*(0.12)*
Phase-Rev | L/S	389	0.059	0.582***	0.033	0.980	-0.335	0.176
		(0.43)	*(0.00)*	*(0.31)*	*(0.15)*	*(0.24)*	*(0.82)*

Phase-Mom | Long	389	0.087	0.667***	0.044***	0.884***	-0.331**	0.264**
		(0.16)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.02)*	*(0.03)*
Phase-Mom | L/S	389	0.073	0.730***	0.048	1.188	-0.216*	0.338
		(0.96)	*(0.00)*	*(0.26)*	*(0.10)*	*(0.06)*	*(0.25)*
Mom-12 | Long	413	0.084***	0.519***	0.019**	0.675***	-0.502	0.166*
		(0.00)	*(0.00)*	*(0.01)*	*(0.01)*	*(0.23)*	*(0.09)*
Mom-12 | L/S	413	0.016	0.308***	0.009	0.434	-0.239**	0.066
		(0.33)	*(0.00)*	*(0.46)*	*(0.86)*	*(0.02)*	*(0.90)*
ST-Rev | Long	424	-0.004***	-0.021***	-0.079***	-0.023***	-0.722*	-0.005**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.08)*	*(0.04)*
ST-Rev | L/S	424	-0.062***	-1.223***	-0.068	-1.545***	-0.888**	-0.070*
		(0.00)	*(0.00)*	*(0.30)*	*(0.00)*	*(0.03)*	*(0.06)*
Panel B: Crisis (CMAX declining from 1 toward the 0.80 threshold)							
1/N (EW)	**36**	**-0.341**	**-2.363**	**-0.393**	**-3.449**	**-0.600**	**-0.568**
VW	36	-0.346	-2.240***	-0.405	-3.063	-0.608	-0.569
		(0.85)	*(0.00)*	*(0.68)*	*(0.14)*	*(0.76)*	*(0.96)*
Past-1 Full | Long	35	-0.264	-1.832***	-0.316	-2.390*	-0.541	-0.488
		(0.37)	*(0.00)*	*(0.47)*	*(0.06)*	*(0.58)*	*(0.17)*
Past-1 Full | L/S	35	-0.018***	-0.350***	-0.025***	-0.321**	-0.136***	-0.135
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.00)*	*(0.23)*
Past-1 Phase | Long	35	-0.240***	-2.162***	-0.271***	-2.912	-0.475***	-0.505*
		(0.00)	*(0.00)*	*(0.00)*	*(0.18)*	*(0.00)*	*(0.07)*
Past-1 Phase | L/S	35	-0.023***	-0.608***	-0.027***	-0.714*	-0.108***	-0.215
		(0.00)	*(0.00)*	*(0.00)*	*(0.06)*	*(0.00)*	*(0.21)*
Past-All Full | Long	35	-0.175***	-1.443***	-0.212***	-2.006**	-0.429*	-0.408*
		(0.00)	*(0.00)*	*(0.00)*	*(0.04)*	*(0.05)*	*(0.05)*
Past-All Full | L/S	35	0.056***	1.222***	0.051***	2.890**	-0.041***	1.355

		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.00)*	*(0.27)*
Past-All Phase | Long	35	-0.209***	-1.834***	-0.242***	-2.391**	-0.437**	-0.479**
		(0.01)	*(0.00)*	*(0.00)*	*(0.04)*	*(0.01)*	*(0.05)*
Past-All Phase | L/S	35	0.008***	0.135***	-0.000***	0.235**	-0.108***	0.069
		(0.00)	*(0.00)*	*(0.00)*	*(0.03)*	*(0.00)*	*(0.28)*
Mom-12 | Long	35	-0.350	-2.150***	-0.416	-2.978	-0.591	-0.592
		(0.39)	*(0.00)*	*(0.26)*	*(0.40)*	*(0.74)*	*(0.16)*
Mom-12 | L/S	35	-0.070***	-1.004***	-0.082***	-1.129**	-0.179***	-0.390
		(0.00)	*(0.00)*	*(0.00)*	*(0.04)*	*(0.00)*	*(0.21)*
ST-Rev | Long	36	-0.430*	-2.539***	-0.501*	-3.263	-0.701**	-0.613
		(0.05)	*(0.00)*	*(0.05)*	*(0.59)*	*(0.03)*	*(0.21)*
ST-Rev | L/S	36	-0.113**	-1.622***	-0.125***	-1.592*	-0.301**	-0.374**
		(0.01)	*(0.00)*	*(0.00)*	*(0.05)*	*(0.01)*	*(0.02)*
Panel C: Drawdown (trigger month through trough)							
1/N (EW)	**41**	**-0.503**	**-1.828**	**-0.693**	**-2.216**	**-0.800**	**-0.629**
VW	41	-0.523	-1.911***	-0.711	-2.582*	-0.810	-0.646
		(0.58)	*(0.00)*	*(0.68)*	*(0.08)*	*(0.67)*	*(0.53)*
Past-1 Full | Long	40	-0.348***	-1.481***	-0.486***	-1.743	-0.708*	-0.491**
		(0.01)	*(0.00)*	*(0.00)*	*(0.14)*	*(0.08)*	*(0.01)*
Past-1 Full | L/S	40	0.030***	0.901***	0.027***	1.332**	-0.037***	0.805
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.00)*	*(0.11)*
Past-1 Phase | Long	40	-0.334**	-1.819***	-0.418***	-2.158	-0.651***	-0.513*
		(0.02)	*(0.00)*	*(0.00)*	*(0.71)*	*(0.00)*	*(0.06)*
Past-1 Phase | L/S	40	0.068***	1.302***	0.062***	2.219	-0.048***	1.416
		(0.00)	*(0.00)*	*(0.00)*	*(0.12)*	*(0.00)*	*(0.17)*
Past-All Full | Long	40	-0.366**	-1.539***	-0.508**	-1.663	-0.725	-0.505**
		(0.05)	*(0.00)*	*(0.02)*	*(0.15)*	*(0.25)*	*(0.04)*

Past-All Full | L/S	40	0.001***	0.021***	-0.002***	0.032*	-0.090***	0.007*
		(0.00)	*(0.00)*	*(0.00)*	*(0.08)*	*(0.00)*	*(0.09)*
Past-All Phase | Long	40	-0.359**	-1.942***	-0.445***	-2.192	-0.679***	-0.529
		(0.04)	*(0.00)*	*(0.00)*	*(0.64)*	*(0.00)*	*(0.13)*
Past-All Phase | L/S	40	0.038***	0.711***	0.031***	1.226*	-0.068***	0.558
		(0.01)	*(0.00)*	*(0.00)*	*(0.06)*	*(0.00)*	*(0.11)*
Phase-Rev | Long	41	-0.706***	-2.159***	-0.973***	-2.795**	-0.897*	-0.787***
		(0.00)	*(0.00)*	*(0.00)*	*(0.03)*	*(0.05)*	*(0.00)*
Phase-Rev | L/S	41	-0.193**	-2.646***	-0.207***	-3.819*	-0.462**	-0.419*
		(0.02)	*(0.00)*	*(0.00)*	*(0.09)*	*(0.01)*	*(0.06)*
Phase-Mom | Long	41	-0.258***	-1.350***	-0.349***	-1.617**	-0.603***	-0.428**
		(0.00)	*(0.00)*	*(0.00)*	*(0.03)*	*(0.00)*	*(0.01)*
Phase-Mom | L/S	41	0.133***	2.184***	0.124***	4.659**	-0.033***	3.988**
		(0.00)	*(0.00)*	*(0.00)*	*(0.04)*	*(0.00)*	*(0.04)*
Mom-12 | Long	40	-0.283***	-1.287***	-0.404***	-1.462**	-0.649**	-0.436***
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.02)*	*(0.01)*
Mom-12 | L/S	40	0.153***	2.037***	0.139***	3.014**	-0.073***	2.089
		(0.00)	*(0.00)*	*(0.00)*	*(0.02)*	*(0.00)*	*(0.15)*
ST-Rev | Long	41	-0.619***	-1.986***	-0.861***	-2.401	-0.862*	-0.718***
		(0.00)	*(0.00)*	*(0.00)*	*(0.43)*	*(0.08)*	*(0.00)*
ST-Rev | L/S	41	-0.146***	-1.707***	-0.164***	-2.742	-0.386***	-0.378**
		(0.00)	*(0.00)*	*(0.00)*	*(0.59)*	*(0.00)*	*(0.02)*
Panel D: Recovery (trough+1 through CMAX returning to 1)							
1/N (EW)	**100**	**0.201**	**1.174**	**0.128**	**2.349**	**-0.270**	**0.743**
VW	100	0.221	1.236***	0.141	2.470	-0.163**	1.351*
		(0.28)	*(0.00)*	*(0.47)*	*(0.70)*	*(0.01)*	*(0.06)*
Past-1 Full | Long	65	0.267*	1.475***	0.185**	2.510	-0.219**	1.220*

		(0.07)	*(0.00)*	*(0.04)*	*(0.19)*	*(0.02)*	*(0.07)*
Past-1 Full | L/S	65	-0.021***	-0.568***	-0.024***	-0.864**	-0.171	-0.121*
		(0.00)	*(0.00)*	*(0.00)*	*(0.03)*	*(0.71)*	*(0.08)*
Past-1 Phase | Long	65	0.306	1.535***	0.206*	2.703	-0.219***	1.393*
		(0.44)	*(0.00)*	*(0.05)*	*(0.12)*	*(0.01)*	*(0.07)*
Past-1 Phase | L/S	65	0.020***	0.364***	0.012***	0.566*	-0.139	0.143*
		(0.00)	*(0.00)*	*(0.00)*	*(0.05)*	*(0.86)*	*(0.07)*
Past-All Full | Long	65	0.253**	1.524***	0.184*	2.922	-0.164	1.540
		(0.03)	*(0.00)*	*(0.06)*	*(0.44)*	*(0.63)*	*(0.36)*
Past-All Full | L/S	65	-0.030***	-0.749***	-0.034***	-0.834**	-0.197	-0.152*
		(0.00)	*(0.00)*	*(0.00)*	*(0.05)*	*(0.58)*	*(0.08)*
Past-All Phase | Long	65	0.327	1.545***	0.215	2.887*	-0.173	1.888
		(0.62)	*(0.00)*	*(0.24)*	*(0.08)*	*(0.14)*	*(0.35)*
Past-All Phase | L/S	65	0.051***	0.807***	0.041***	1.298**	-0.084*	0.610*
		(0.00)	*(0.00)*	*(0.00)*	*(0.04)*	*(0.06)*	*(0.07)*
Phase-Rev | Long	100	0.207	1.055***	0.111	2.237	-0.265	0.782
		(0.65)	*(0.00)*	*(0.21)*	*(0.62)*	*(0.87)*	*(0.82)*
Phase-Rev | L/S	100	0.014***	0.238***	0.005**	0.447**	-0.148*	0.097
		(0.00)	*(0.00)*	*(0.03)*	*(0.04)*	*(0.06)*	*(0.18)*
Phase-Mom | Long	100	0.151**	1.178	0.110	2.216	-0.254	0.593
		(0.02)	*(0.35)*	*(0.35)*	*(0.59)*	*(0.50)*	*(0.35)*
Phase-Mom | L/S	100	-0.065***	-1.013***	-0.076**	-1.365**	-0.398	-0.164
		(0.00)	*(0.00)*	*(0.02)*	*(0.01)*	*(0.30)*	*(0.13)*
Mom-12 | Long	91	0.218	1.322***	0.150	2.079	-0.232	0.941
		(0.50)	*(0.00)*	*(0.57)*	*(0.11)*	*(0.57)*	*(0.58)*
Mom-12 | L/S	91	-0.019***	-0.382***	-0.026**	-0.563**	-0.198	-0.097
		(0.00)	*(0.00)*	*(0.02)*	*(0.04)*	*(0.82)*	*(0.13)*

ST-Rev | Long	100	0.177	1.054***	0.107	1.918	-0.268	0.662
		(0.18)	*(0.00)*	*(0.17)*	*(0.12)*	*(0.94)*	*(0.65)*
ST-Rev | L/S	100	-0.035***	-0.742***	-0.041**	-1.290**	-0.271	-0.130
		(0.00)	*(0.00)*	*(0.02)*	*(0.01)*	*(0.99)*	*(0.13)*

Notes: **1/N (EW)** = equal weight 1/N, rebalanced monthly. **VW** = buy-and-hold from equal initial weights; passive drift, never reset (zero turnover). **Past-1 Full** / **Past-All Full** = rank assets by prior full episode return; **Long** = long top X, **L/S** = long top X (+0.5) short bottom-X (-0.5). **Past-1 Phase** / **Past-All Phase** = same-phase rank in prior episode(s); rebalance at phase transitions. **Phase-Rev** = long worst performers of just-completed phase (reversal). **Phase-Mom** = long best performers (momentum). **Mom-12** = rolling 12-1-month momentum. **ST-Rev** = prior-month reversal. X=1 (FF3, FF5), 5 (BM_*), 3 (IND_MKT). Pre-crisis: all strategies hold 1/N (EW). Metrics on monthly log excess returns $ln(1 + r - rf)$. **N** = monthly observations. Stars vs 1/N (EW): * p<0.10, ** p<0.05, *** p<0.01. P-values in parentheses below each strategy. **SR**: Jobson-Korkie (1981)/Memmel (2003); others: stationary bootstrap (B=500). All significance tests measure the difference of each strategy relative to 1/N (EW); a self-referential test is not defined.

4.6 Transaction Cost Analysis

The performance differentials documented in previous sections are measured gross of transaction costs. Whether the phase-specific outperformance for constrained and phase-conditioned strategies survives realistic implementation costs is a natural and practically important question, since strategies with high turnover may appear attractive on a gross basis while delivering inferior net returns.

Among classical optimised strategies, transaction costs reinforce rather than reverse the performance hierarchy (Tables A4 and A5). Constrained strategies incur turnover close to the 1/N benchmark across all phases, leaving their performance differentials materially unchanged after costs. Unconstrained MV and BS generate substantially higher turnover, compounding their already-negative gross performance and deepening the underperformance gap after costs. Under binary classification, turnover is elevated across all strategies during crisis periods relative to non-crisis periods. In the four-phase decomposition, the turnover pattern reinforces the documented asymmetry: crisis and drawdown phases are associated with the lowest turnover levels, while the recovery phase generates the highest. Since the recovery phase is precisely where gross performance is most severely negative, the additional transaction cost burden at recovery compounds rather than offsets the performance deficit, amplifying the asymmetry in net return terms.

For the phase-conditioned long-short strategies, the transaction cost picture is equally or more favourable (Table A6). The majority of strategies generate cost differentials that are negligible relative to their gross return advantages, with several recording slightly negative net cost contributions.

5. Discussion

5.1 Research Questions Revisited

RQ1*: Does binary crisis classification alter the relative performance of classical optimised strategies versus the 1/N benchmark, and does the constrained versus unconstrained distinction determine which strategies benefit from crisis conditioning?*

Binary crisis conditioning reveals a partial and strategy-dependent shift in relative performance. Unconstrained strategies remain deeply negative across all metrics during crisis periods, showing no benefit from regime conditioning. Among constrained strategies, the underperformance gap narrows considerably. The constrained versus unconstrained distinction is therefore the decisive factor in determining which strategies respond to binary crisis conditioning, though no strategy family achieves broad and consistent outperformance under this binary framework.

RQ2*: Does the four-phase decomposition of the crisis cycle reveal differential performance patterns for classical optimised strategies relative to 1/N that binary classification cannot identify, and in which phases does the performance differential concentrate?*

The four-phase decomposition reveals a structural asymmetry that binary classification obscures by aggregating opposing phase dynamics. During the sub-phases crisis and drawdown, some constrained strategies consistently outperform 1/N on annualised return, CEQ, and downside metrics, with the drawdown phase producing the broadest and most significant outperformance. During the recovery phase, the same strategies systematically and significantly underperform 1/N across all metrics. Unconstrained strategies exhibit more volatile and less consistent patterns across phases.

RQ3*: Can phase-conditioned long-short strategies generate full-sample out-of-sample outperformance relative to 1/N, and in which phases of the crisis cycle does this outperformance concentrate?*

The long-short strategy layer generates full-sample Sharpe ratios and CEQ returns significantly above 1/N, an achievement absent from every phase of every classical strategy. Outperformance concentrates again in the sub-phases crisis and drawdown, where episode-specific return signals correctly identify assets that continue to underperform. The recovery phase introduces a structural liability: assets held short during drawdown lead the rebound, partially reversing drawdown gains for long-short implementations. Long-only variants exhibit more balanced phase profiles by avoiding this short-leg cost. These results hold across all regions and universes, attributing the alpha to regime-aware signal construction.

5.2 Interpreting the Crisis and Drawdown-Recovery Asymmetry

The crisis and drawdown-recovery asymmetry is the most robust finding of this thesis and merits interpretation beyond the quantitative results alone. The mechanism operates through the changing covariance structure during crisis cycles and understanding it clarifies why constrained strategies outperform 1/N during both sub-phases crisis and drawdown, and why the same strategies reliably underperform during recovery. The asymmetry encompasses not only the drawdown phase but also the preceding crisis sub-phase, during which constrained minimum-variance strategies similarly outperform 1/N as correlations begin to rise and the variance-minimising objective begins to gain traction against the equal-weight benchmark.

During both sub-phases crisis and drawdown, the conditions that normally work against covariance-based optimization are temporarily reversed. In normal markets, the dominant source of estimation error lies in expected returns rather than covariances, which explains why strategies that eliminate expected return estimation tend to perform best out-of-sample (Chopra and Ziemba, 1993). However, in the sub-phases crisis and drawdown the relevant covariance matrix changes rapidly and in a predictable direction: correlations rise toward one and volatilities increase, concentrating the covariance signal in a way that makes minimum-variance optimization more tractable. The constrained minimum-variance strategy, which eliminates expected return estimation entirely and focuses exclusively on variance reduction, is therefore better positioned to benefit from the drawdown regime. The equal-weighted portfolio, by construction, ignores covariance information entirely and cannot adapt to the changing correlation structure, leaving it at a systematic disadvantage during this phase.

During the recovery phase, this logic reverses with equal force. The recovery is characterized by a rapid unwinding of crisis-era correlations and a differentiated rebound in which assets most severely discounted during the drawdown outperform disproportionately. The constrained minimum-variance strategy, conditioned on drawdown covariances, holds a portfolio weighted toward low-volatility defensive assets that fail to participate fully in the recovery rally. The equal-weighted portfolio maintains proportional exposure to all assets including those poised to rebound most strongly, and this diversification becomes an advantage precisely because the recovery return profile is concentrated in prior losers. The long-short strategies exhibit an analogous but more extreme version of this pattern: the drawdown momentum signal correctly identifies which assets will continue to underperform, but the recovery involves a mean-reversion that reverses prior rankings, converting the short leg of the drawdown-trained strategy into a substantial liability.

5.3 Contribution to the Literature

5.3.1 Regime-Conditioning Reveals Phase-Specific Heterogeneity

The existing findings of the literature are extended to a crisis-conditioned setting, demonstrating that the aggregate conclusion, that no optimised strategy consistently outperforms 1/N, conceals systematic phase-specific patterns. The sub-phases crisis and drawdown constitute a structural exception in which constrained covariance-based strategies add demonstrable value over 1/N, while the recovery phase represents a regime in which all tested strategies are actively and significantly worse than the naive benchmark. This phase decomposition provides a new lens on the naive versus optimised diversification debate and establishes that aggregating all sub-phases into a single unconditional evaluation is precisely what obscures these patterns. Critically, this heterogeneity is not a property of optimisation in general but of the constrained versus unconstrained distinction specifically: constrained strategies narrow the performance gap during stress periods while unconstrained strategies deepen it, identifying portfolio weight constraints as the operative mechanism through which regime conditioning affects performance.

5.3.2 Phase-Conditioned Long-Short Strategies Generate Full-Sample Outperformance

Phase-conditioned long-short strategies are the only approach in the evaluation generating full-sample Sharpe ratio and CEQ outperformance relative to 1/N, a result absent from every classical optimised strategy across every phase and every region. Outperformance is the dominant finding across strategies, metrics, and samples but is not universal, and the conclusion should be understood as reflecting the central tendency of the evidence rather than unconditional dominance across every metric and region. The finding contributes to the crisis-regime literature by demonstrating that the predictable structure of crisis phases carries exploitable cross-sectional information that fixed-window estimation cannot replicate (Jegadeesh and Titman, 1993).

5.3.3 The Crisis and Drawdown-Recovery Asymmetry is Universal

Replication across the United States, Europe, and Japan, and across four asset classifications, strengthens the interpretation that the documented asymmetry reflects a structural feature of crisis dynamics rather than a data-specific artifact, consistent with evidence that crisis-period correlation dynamics are not confined to a single market (Longin and Solnik, 2001; Forbes and Rigobon, 2002). The pooled cross-regional regression provides the most direct statistical test of this claim: pooling all three regions and five universes with region and universe controls, crisis and drawdown coefficients for Min-Var-C, G-Min-C, and EW-Min retain their sign and significance, and the region dummies for these strategies are small and statistically insignificant, confirming cross-regional consistency as a formal regression result (Table 2 and Table 3; Table A7 and Table A8).

5.4 Practical Implications

The empirical findings carry actionable implications for portfolio construction during market stress. The key qualification throughout is whether an investor can identify crisis phases in real time: no single strategy dominates across all phases of the crisis cycle, and the practical value of the documented patterns depends heavily on this implementation constraint.

For investors who wish to employ mean-variance-type optimization, the evidence strongly favors constrained variants over unconstrained ones. Constrained strategies, particularly g-min-c and min-c, consistently deliver less severe underperformance than their unconstrained counterparts during non-crisis periods and show the strongest outperformance of 1/N during the sub-phases crisis and drawdown. The unconstrained mean-variance and Bayes-Stein strategies offer no compensating advantage and introduce substantially higher turnover and estimation sensitivity. Investors considering optimization should therefore adopt constrained formulations as a baseline regardless of market conditions.

For investors willing to use long-short or phase-conditioned strategies, the phase-conditioned variants generate the strongest outperformance, but this comes with a structural condition: the recovery phase, which follows every drawdown, produces significant underperformance. An investor implementing these strategies must therefore be willing to accept this recovery cost or be able to switch strategies as the market transitions from drawdown to recovery. Since the trough is only observable ex post, any real-time implementation will face classification uncertainty that the retrospective analysis abstracts from. The long-only variants of the phase-conditioned signals provide a more balanced profile by avoiding the short-leg liability during recovery and may represent a more practical compromise. Ultimately, for investors who cannot reliably identify crisis phases in real time, the 1/N equal-weight portfolio or the value-weighted passive benchmark remain difficult to improve upon in expectation.

5.5 Limitations

5.5.1 Ex-post-only phase classification

The CMAX methodology identifies crisis phases only after the fact: the trough separating drawdown from recovery is observable only ex post, meaning any investor attempting real-time implementation faces a classification problem the retrospective analysis abstract from. The performance advantage documented during the drawdown phase represents an upper bound on what a real-time investor could achieve.

5.5.2 Restricted empirical scope

The asset universes are drawn exclusively from the Kenneth R. French Data Library and consist of factor-mimicking and size/value-sorted equity portfolios. This choice limits the conclusions in two ways. First, the long-short strategies cannot be directly implemented without mapping factor signals to investable instruments, and the flat 50-basis-point transaction cost assumption may understate true costs in crisis periods when spreads widen and liquidity deteriorates. Second, restricting the analysis to equity portfolios excludes asset classes, government bonds, corporate bonds, real estate, and commodities, that may exhibit different crisis-phase dynamics and potentially provide natural hedges for equity drawdown risk.

5.5.3 Limited crisis episode counts and episode-length heterogeneity

Phase-specific statistics are computed over a small number of episode observations, reducing statistical power. The bootstrap inference procedure accounts for serial correlation from crisis-phase clustering but cannot fully address the small-sample problem, and significance levels in the four-phase analysis should be interpreted with appropriate caution.

Beyond episode counts, the identified crises vary substantially in length, from the six-month COVID shock to the 61-month Japanese Lost Decade. Longer episodes contribute disproportionately to phase-level statistics, so the documented patterns may reflect the structure of a small number of dominant episodes rather than a general regularity, and significance levels in the phase-conditioned results should be interpreted with this in mind.

5.5.4 Sensitivity of results to parameter choices

The analysis fixes several parameters whose perturbation could alter findings. The CEQ metric is computed at a single risk-aversion coefficient of gamma equal to 5; strategy rankings would differ under alternative assumptions for strategies with substantially different variance profiles. Transaction costs are held at a constant 50 basis points, abstracting from time-variation in market liquidity. The CMAX crisis threshold is fixed at 0.80 for all developed markets; modest perturbations of this parameter (for instance, to 0.75 or 0.85) could alter episode counts, phase boundaries, and the number of observations available in each phase. The conclusions should therefore be understood as conditional on these calibration choices rather than invariant to them.

5.6 Further Research

5.6.1 Emerging market application

Applying the phase-conditioned framework to emerging market portfolios is the most direct extension. The CMAX methodology is valid in both developed and emerging contexts, using a lower threshold of 0.65 for emerging markets (Patel and Sarkar, 1998). Emerging markets experience more frequent and severe crises, which would increase the number of phase observations and allow more powerful inference on phase-specific performance. Whether the drawdown-recovery asymmetry holds where recovery dynamics differ due to currency risk and capital flow reversals remains an open empirical question.

5.6.2 Multi-region phase conditioning

The present analysis treats each region independently. A multi-region framework could exploit differential timing of crisis phases across markets to construct globally diversified phase-conditioned portfolios, potentially moderating the asymmetry through diversification across phase timing. This approach would also allow a direct test of whether the cross-regional correlation increases documented during crises affect the relative performance of constrained and unconstrained strategies at the global level (Longin and Solnik, 2001).

5.6.3 Multi-asset extension beyond equities

Extending to multi-asset portfolios incorporating government bonds, corporate bonds, real estate investment trusts, and commodity futures would allow assessment of whether phase-conditioned outperformance extends beyond equities and whether cross-asset diversification during crisis periods modifies the relative performance of naive and optimised strategies. The drawdown-recovery asymmetry may be attenuated or reversed in a multi-asset context given differing crisis-period dynamics across asset classes.

5.6.4 Crisis heterogeneity by trigger type

The present analysis treats all crisis episodes symmetrically by their price-drawdown characteristics regardless of underlying cause. Whether episodes driven by financial sector stress, commodity shocks, or geopolitical disruption produce systematically different phase-specific patterns is an open question. If the mechanism driving the crisis affects cross-sectional dispersion and correlation structure differently across episode types, phase-conditioned strategies effective during financial crises may not generalise to commodity-driven or geopolitical episodes.

5.6.5 Real-time phase classification via probabilistic regime weighting

A forward-looking variant using real-time regime probability estimates, along the lines of probabilistic state weighting, would address the concerns about the practical ability to implement the strategies, and allow direct assessment of how much real-time classification uncertainty costs relative to the retrospectively measured performance advantage.

6. Conclusion

This thesis extends the portfolio performance evaluation framework of prior work by incorporating an explicit crisis-regime dimension. Using the CMAX methodology to classify monthly observations into pre-crisis, crisis, drawdown, and recovery phases, the analysis evaluates three layers of portfolio strategies, classical optimized, passive benchmark, and phase-conditioned long-short, against the naive equal-weight 1/N benchmark across three geographic regions and four asset universes (Patel and Sarkar, 1998).

Binary crisis classification reveals that the constrained versus unconstrained distinction is the decisive factor in determining which strategies respond to regime conditioning: constrained strategies narrow the underperformance gap during crisis periods on both Sharpe ratio and CEQ, while unconstrained strategies deepen it. The aggregate binary result remains broadly consistent with the prior unconditional finding (DeMiguel et al., 2009).

The central empirical finding is the crisis and drawdown-recovery asymmetry. During the sub-phases crisis and drawdown, constrained strategies, specifically G-Min-C, Min-Var-C, and EW-Min, generate consistent and significant outperformance relative to 1/N on annualised return, CEQ, and maximum drawdown, while the recovery phase produces a systematic and significant reversal across the same strategies and metrics. This asymmetry is precisely what binary classification hides; by combining drawdown outperformance and recovery underperformance within a single crisis label, the binary framework produces aggregate results that mask structurally distinct phase-level behaviour. A pooled OLS regression across all three regions and five investable asset universes confirms that the phase coefficients cannot be attributed to regional composition or factor universe structure, with region dummies uniformly small and statistically insignificant for the constrained family, establishing the asymmetry as a cross-regional structural feature of crisis dynamics rather than a sample-specific artifact.

Phase-conditioned long-short strategies extend the asymmetry finding by demonstrating that the predictable phase structure of the crisis cycle carries exploitable information for active portfolio construction. The Phase-Mom and Past-phase variants are the only strategies in the evaluation achieving full-sample Sharpe ratio and CEQ outperformance relative to 1/N, with performance concentrating in the drawdown phase and competitive parity maintained during the pre-crisis period through the default to 1/N weights.

Transaction cost analysis confirms that the documented performance patterns reflect genuine investment value. Constrained strategy performance differentials are materially unchanged after a uniform 50-basis-point per-trade cost, and phase-conditioned long-short strategies produce cost differentials negligible relative to their gross return advantages. For unconstrained strategies, costs compound an already-negative performance picture, and the recovery-phase turnover pattern amplifies rather than offsets the gross underperformance at precisely the phase where the deficit is most severe.

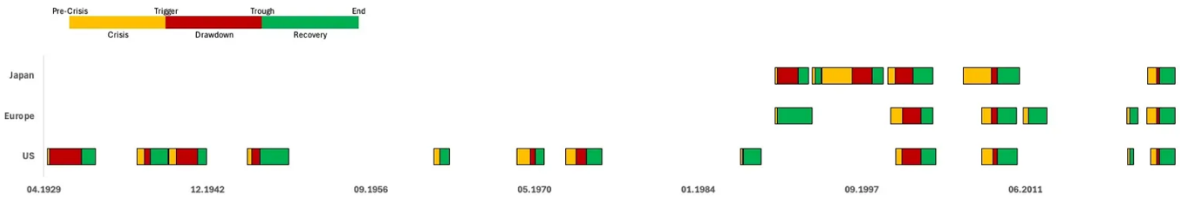
The practical implication is clear but conditional. During the sub-phases crisis and drawdown, constrained minimum-variance strategies and phase-conditioned long-short strategies offer demonstrable advantages over the naive equal-weight rule. During the recovery phase, equal weighting dominates. For investors who cannot reliably identify these phases in real time, or who are unwilling to accept the recovery underperformance that accompanies phase-conditioned long-short strategies, the 1/N equal-weight portfolio remains difficult to improve upon in expectation, a conclusion that reinforces and qualifies the landmark prior finding (DeMiguel et al., 2009).

These findings open several avenues for further research, including the application of the phase-conditioned framework to emerging markets, the extension to multi-asset portfolios, the development of real-time phase classification rules using regime probability estimates, the examination of simultaneous multi-region phase conditioning, and the investigation of whether different crisis origins, such as financial sector stress, commodity shocks, or geopolitical events, produce systematically different phase-specific return patterns.

Appendix

Note: Extended results tables are not included in this appendix due to page constraints are available in the Online Supplementary Material at: link. All materials are view-only.

Figure A1: CMAX Crisis Episode Timeline | United States, Full Sample (1926–2025)



Note: Figure A1 extends the crisis timeline to the full US sample (July 1926 – November 2025), covering an additional 14 episodes not visible in the common post-1990 window of Figure 1. Europe and Japan are included for reference over their available sample (July 1990 – November 2025). Colour coding and phase definitions follow Figure 1. See Section 4.1 for a full description of the CMAX methodology.

Table A1: CMAX Crisis Episode Identification

#	Episode	Crisis Start	Threshold Breached	Trough	Recovery End	CMAX at Trough	Market Drawdown	Lead-in	To Trough	Recovery	Total
					US						
1	Great Depression	Sep 1929	Oct 1929	Jun 1932	Aug 1933	0.2417	75.8%	1	33	14	48
2	Depression Relapse	Mar 1937	Sep 1937	Mar 1938	Sep 1939	0.5054	49.5%	6	7	18	31
3	WWII Onset	Oct 1939	May 1940	Mar 1942	Dec 1942	0.7531	24.7%	7	23	9	39
4	Post-War Correction	Jun 1946	Sep 1946	May 1947	Oct 1949	0.7574	24.3%	3	9	29	41
5	Early 1960s Correction	Jan 1962	Jun 1962	Jun 1962	Apr 1963	0.7693	23.1%	5	1	10	16
6	Late 1960s Bear Market	Dec 1968	Jan 1970	Jun 1970	Mar 1971	0.6645	33.6%	13	6	9	28
7	Oil Crisis Bear Market	Jan 1973	Nov 1973	Sep 1974	Jan 1976	0.5349	46.5%	10	11	16	37
8	Black Monday	Sep 1987	Oct 1987	Nov 1987	May 1989	0.7015	29.9%	1	2	18	21
9	Dot-Com Bust	Sep 2000	Feb 2001	Sep 2002	Dec 2003	0.5917	40.8%	5	20	15	40
10	Global Financial Crisis	Nov 2007	Sep 2008	Feb 2009	Oct 2010	0.4969	50.3%	10	6	20	36
11	COVID-19 Shock	Feb 2020	Mar 2020	Mar 2020	Jul 2020	0.7978	20.2%	1	1	4	6
12	2022 Rate-Hike Bear Market	Jan 2022	Jun 2022	Sep 2022	Dec 2023	0.7516	24.8%	5	4	15	24
					Europe						
1	1990 Market Stress	Aug 1990	Sep 1990	Sep 1990	Aug 1993	0.7937	20.6%	1	1	35	37
2	Dot-Com Bust	Apr 2000	Mar 2001	Sep 2002	Sep 2003	0.6121	38.8%	11	19	12	42
3	Global Financial Crisis	Nov 2007	Aug 2008	Feb 2009	Sep 2010	0.4103	59.0%	9	7	19	35
4	Euro Debt Crisis	May 2011	Sep 2011	May 2012	Apr 2013	0.7265	27.4%	4	9	11	24
5	COVID-19 Shock	Jan 2020	Mar 2020	Mar 2020	Nov 2020	0.7531	24.7%	2	1	8	11
6	COVID-19 Shock	Sep 2021	Jun 2022	Sep 2022	Dec 2023	0.6887	31.1%	9	4	15	28
					Japan						
1	Asset Bubble Burst	Aug 1990	Sep 1990	Jun 1992	Apr 1993	0.6327	36.7%	1	22	10	33
2	1993 Market Stress	Sep 1993	Nov 1993	Nov 1993	May 1994	0.7801	22.0%	2	1	6	9
3	1994 Market Stress	Jul 1994	Dec 1996	Aug 1998	Jul 1999	0.5547	44.5%	29	21	11	61
4	Dot-Com Bust	Jan 2000	Jul 2000	Jan 2002	Sep 2003	0.4404	56.0%	6	19	20	45
5	Global Financial Crisis	May 2006	Aug 2008	Feb 2009	Dec 2010	0.5600	44.0%	27	7	22	56
6	COVID-19 Shock	Oct 2021	Jun 2022	Sep 2022	Dec 2023	0.7107	28.9%	8	4	15	27

Notes. CMAX computed following Patel & Sarkar (1998): $CMAX_t = W_t / \max(W_{t-23}, ..., W_t)$, where W_t is the cumulative wealth index of the regional total-return market index and the rolling window spans 24 months. A crisis episode is triggered when CMAX drops below 0.80. **Crisis Start** = first month CMAX departs from its rolling maximum (CMAX < 1). **Threshold Breached** = first month CMAX < 0.80 (formal crisis trigger). **Trough** = month of minimum CMAX within the episode. **Recovery End** = first month CMAX returns to 1. **Market Drawdown** = (1 - CMAX at Trough) x 100%. **Lead-in** = months from Crisis Start to Threshold Breach. **To Trough** = months from Threshold Breach to Trough (inclusive). **Recovery** = months from Trough to full recovery. (*) Episode had not fully recovered by end of sample (Nov 2025). US sample: Jul 1926 - Nov 2025. Europe and Japan samples: Jul 1990 - Nov 2025.

Table A2: Binary CMAX Performance Analysis | Optimized Strategies vs. 1/N (Europe, M=60) | Universe: BM_FF3 (N=23 assets)

Strategy	N	Ann. Ret.	SR	CEQ	Sortino	Max DD	Calmar
				Full Sample			
1/N (EW)	**365**	**0.049**	**0.289**	**-0.022**	**0.351**	**-0.595**	**0.082**
VW	365	0.052	0.296***	-0.025	0.379	-0.599	0.087
		(0.77)	*(0.00)*	*(0.80)*	*(0.69)*	*(0.82)*	*(0.85)*
MV	365	-0.036**	-0.603***	-0.045	-1.717	-0.722	-0.049*
		(0.03)	*(0.00)*	*(0.59)*	*(0.15)*	*(0.42)*	*(0.06)*
MV-C	365	0.069	0.478***	0.017	0.609	-0.455	0.152
		(0.30)	*(0.00)*	*(0.11)*	*(0.16)*	*(0.25)*	*(0.23)*
BS	365	-0.039**	-0.878***	-0.044	-2.349**	-0.698	-0.055*
		(0.01)	*(0.00)*	*(0.60)*	*(0.04)*	*(0.51)*	*(0.05)*
BS-C	365	0.069	0.439***	0.007	0.549	-0.459	0.150
		(0.26)	*(0.00)*	*(0.16)*	*(0.22)*	*(0.19)*	*(0.17)*
Min-Var	365	-0.044***	-2.781***	-0.045	-3.632***	-0.735	-0.060**
		(0.00)	*(0.00)*	*(0.56)*	*(0.00)*	*(0.33)*	*(0.05)*
Min-Var-C	365	-0.006**	-0.112***	-0.012	-0.163	-0.311*	-0.019*
		(0.04)	*(0.00)*	*(0.78)*	*(0.12)*	*(0.07)*	*(0.10)*
G-Min-C	365	0.031	0.327***	0.008	0.403	-0.419**	0.075
		(0.31)	*(0.00)*	*(0.16)*	*(0.64)*	*(0.01)*	*(0.82)*
MV-Min	365	-0.041***	-1.561***	-0.043	-3.231***	-0.709	-0.058**
		(0.01)	*(0.00)*	*(0.60)*	*(0.00)*	*(0.43)*	*(0.04)*
EW-Min	365	-0.036***	-1.541***	-0.037	-2.956***	-0.663	-0.054**
		(0.01)	*(0.00)*	*(0.67)*	*(0.00)*	*(0.65)*	*(0.05)*
			Pre-Crisis (CMAX between 1 and 0.80, no threshold breach)				
1/N (EW)	**225**	**0.122**	**1.071**	**0.089**	**1.712**	**-0.216**	**0.563**

VW	225	0.131	1.072	0.093	1.590	-0.202	0.647
		(0.47)	*(0.72)*	*(0.72)*	*(0.46)*	*(0.47)*	*(0.44)*
MV	225	-0.029***	-0.389***	-0.042***	-1.229	-0.602***	-0.048**
		(0.00)	*(0.00)*	*(0.00)*	*(0.17)*	*(0.01)*	*(0.04)*
MV-C	225	0.112	1.139***	0.088	1.822	-0.142**	0.787
		(0.59)	*(0.00)*	*(0.91)*	*(0.67)*	*(0.03)*	*(0.10)*
BS	225	-0.036***	-0.661***	-0.043***	-2.028*	-0.581***	-0.062**
		(0.00)	*(0.00)*	*(0.00)*	*(0.05)*	*(0.01)*	*(0.03)*
BS-C	225	0.121	1.170***	0.094	1.919	-0.146**	0.830*
		(0.99)	*(0.00)*	*(0.78)*	*(0.42)*	*(0.04)*	*(0.06)*
Min-Var	225	-0.051***	-3.262***	-0.052***	-3.990***	-0.614***	-0.083**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.03)*
Min-Var-C	225	-0.001***	-0.020***	-0.006***	-0.034***	-0.235	-0.004**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.85)*	*(0.02)*
G-Min-C	225	0.061***	0.903***	0.050***	1.490	-0.163**	0.373**
		(0.00)	*(0.00)*	*(0.01)*	*(0.12)*	*(0.04)*	*(0.05)*
MV-Min	225	-0.043***	-1.388***	-0.045***	-3.265***	-0.549**	-0.078**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.02)*	*(0.03)*
EW-Min	225	-0.039***	-1.419***	-0.040***	-2.969***	-0.510**	-0.076**
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.04)*	*(0.03)*
Crisis (all non-pre-crisis months)							
1/N (EW)	**140**	**-0.068**	**-0.301**	**-0.197**	**-0.396**	**-0.650**	**-0.105**
VW	140	-0.075	-0.320***	-0.210	-0.462	-0.720*	-0.104
		(0.67)	*(0.00)*	*(0.48)*	*(0.49)*	*(0.10)*	*(0.95)*
MV	140	-0.047	-2.442***	-0.048**	-3.375***	-0.418	-0.112
		(0.70)	*(0.00)*	*(0.03)*	*(0.00)*	*(0.13)*	*(0.90)*
MV-C	140	0.000	0.000***	-0.096	0.001	-0.455	0.000

		(0.11)	*(0.00)*	*(0.10)*	*(0.13)*	*(0.19)*	*(0.17)*
BS	140	-0.043	-2.450***	-0.044**	-3.405***	-0.392*	-0.110
		(0.64)	*(0.00)*	*(0.02)*	*(0.00)*	*(0.08)*	*(0.93)*
BS-C	140	-0.016	-0.073***	-0.130	-0.104	-0.459	-0.034
		(0.16)	*(0.00)*	*(0.24)*	*(0.15)*	*(0.11)*	*(0.23)*
Min-Var	140	-0.032	-2.098***	-0.033**	-3.109***	-0.310**	-0.105
		(0.52)	*(0.00)*	*(0.02)*	*(0.00)*	*(0.01)*	*(1.00)*
Min-Var-C	140	-0.014	-0.221***	-0.023**	-0.305	-0.315**	-0.043
		(0.32)	*(0.00)*	*(0.01)*	*(0.85)*	*(0.02)*	*(0.42)*
G-Min-C	140	-0.016	-0.126***	-0.057***	-0.160	-0.400***	-0.040*
		(0.11)	*(0.00)*	*(0.00)*	*(0.14)*	*(0.01)*	*(0.09)*
MV-Min	140	-0.037	-2.401***	-0.038**	-3.447***	-0.349**	-0.107
		(0.58)	*(0.00)*	*(0.02)*	*(0.00)*	*(0.03)*	*(0.98)*
EW-Min	140	-0.032	-2.074***	-0.033**	-3.088***	-0.307**	-0.104
		(0.51)	*(0.00)*	*(0.02)*	*(0.00)*	*(0.01)*	*(1.00)*

Notes: **1/N (EW)** = equal weight 1/N, rebalanced monthly. **VW** = value-weighted market return (MKT_total), zero turnover. **MV** = mean-variance. **MV-C** = constrained MV. **BS** = Bayes-Stein. **BS-C** = constrained BS. **Min-Var** = minimum-variance. **Min-Var-C** = constrained Min-Var. **G-Min-C** = global minimum-variance constrained. **MV-Min** = convex combination of MV and Min-Var. **EW-Min** = convex combination of 1/N and Min-Var. All optimized strategies use a rolling M-month estimation window. Metrics on monthly log excess returns $ln(1 + r - rf)$. **N** = monthly observations. Stars vs 1/N (EW): * p<0.10, ** p<0.05, *** p<0.01. P-values in parentheses below each strategy. **SR**: Jobson-Korkie (1981)/Memmel (2003); others: stationary bootstrap (B=500). All significance tests measure the difference of each strategy relative to 1/N (EW); a self-referential test is not defined.

Table A3: Binary CMAX Performance Analysis | Optimized Strategies vs. 1/N (Europe, M=120) | Universe: BM_FF3 (N=23 assets)

Strategy	N	Ann. Ret.	SR	CEQ	Sortino	Max DD	Calmar
				Full Sample			
1/N (EW)	**305**	**0.047**	**0.265**	**-0.032**	**0.328**	**-0.595**	**0.079**
VW	305	0.040	0.219***	-0.044	0.289	-0.599	0.067
		(0.47)	*(0.00)*	*(0.32)*	*(0.57)*	*(0.82)*	*(0.55)*
MV	305	-0.046**	-2.604***	-0.046	-3.602***	-0.684	-0.067*
		(0.01)	*(0.00)*	*(0.79)*	*(0.00)*	*(0.58)*	*(0.09)*
MV-C	305	0.053	0.388***	0.006	0.515	-0.466	0.114
		(0.79)	*(0.00)*	*(0.12)*	*(0.34)*	*(0.20)*	*(0.49)*
BS	305	-0.043**	-2.709***	-0.043	-3.594***	-0.660	-0.065
		(0.01)	*(0.00)*	*(0.83)*	*(0.00)*	*(0.67)*	*(0.10)*
BS-C	305	0.053	0.347***	-0.005	0.443	-0.507	0.104
		(0.80)	*(0.00)*	*(0.27)*	*(0.52)*	*(0.44)*	*(0.62)*
Min-Var	305	-0.035**	-2.552***	-0.036	-3.258***	-0.589	-0.060
		(0.03)	*(0.00)*	*(0.93)*	*(0.00)*	*(0.98)*	*(0.11)*
Min-Var-C	305	0.004	0.074***	-0.003	0.104	-0.351**	0.012
		(0.15)	*(0.00)*	*(0.45)*	*(0.47)*	*(0.05)*	*(0.31)*
G-Min-C	305	0.037	0.368***	0.012*	0.462	-0.413***	0.090
		(0.62)	*(0.00)*	*(0.07)*	*(0.22)*	*(0.00)*	*(0.72)*
MV-Min	305	-0.043**	-2.694***	-0.044	-3.575***	-0.663	-0.065*
		(0.01)	*(0.00)*	*(0.83)*	*(0.00)*	*(0.65)*	*(0.10)*
EW-Min	305	-0.035**	-2.552***	-0.036	-3.258***	-0.589	-0.060
		(0.03)	*(0.00)*	*(0.93)*	*(0.00)*	*(0.98)*	*(0.11)*
				Pre-Crisis (CMAX between 1 and 0.80, no threshold breach)			
1/N (EW)	**168**	**0.138**	**1.176**	**0.104**	**2.053**	**-0.216**	**0.642**

VW	168	0.131	1.096***	0.095	1.865	-0.202	0.650
		(0.39)	*(0.00)*	*(0.35)*	*(0.27)*	*(0.56)*	*(0.95)*
MV	168	-0.050***	-2.746***	-0.051***	-3.892***	-0.504***	-0.100*
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.00)*	*(0.07)*
MV-C	168	0.116	1.114***	0.089	1.852	-0.126**	0.922
		(0.22)	*(0.00)*	*(0.40)*	*(0.53)*	*(0.01)*	*(0.15)*
BS	168	-0.047***	-2.861***	-0.048***	-3.878***	-0.481***	-0.098*
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.01)*	*(0.07)*
BS-C	168	0.124	1.131***	0.094	1.898	-0.131**	0.944
		(0.44)	*(0.00)*	*(0.55)*	*(0.63)*	*(0.01)*	*(0.14)*
Min-Var	168	-0.038***	-2.722***	-0.038***	-3.435***	-0.412**	-0.092*
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.04)*	*(0.07)*
Min-Var-C	168	0.017***	0.400***	0.013***	0.727***	-0.162	0.107*
		(0.00)	*(0.00)*	*(0.00)*	*(0.01)*	*(0.26)*	*(0.08)*
G-Min-C	168	0.081***	1.147***	0.069**	1.942	-0.159**	0.508
		(0.00)	*(0.00)*	*(0.02)*	*(0.41)*	*(0.01)*	*(0.18)*
MV-Min	168	-0.047***	-2.851***	-0.048***	-3.855***	-0.483***	-0.098*
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.01)*	*(0.07)*
EW-Min	168	-0.038***	-2.722***	-0.038***	-3.435***	-0.412**	-0.092*
		(0.00)	*(0.00)*	*(0.00)*	*(0.00)*	*(0.04)*	*(0.07)*
				Crisis (all non-pre-crisis months)			
1/N (EW)	**137**	**-0.065**	**-0.283**	**-0.195**	**-0.372**	**-0.645**	**-0.100**
VW	137	-0.072	-0.305***	-0.210	-0.443	-0.715*	-0.100
		(0.64)	*(0.00)*	*(0.41)*	*(0.47)*	*(0.09)*	*(0.99)*
MV	137	-0.040	-2.442***	-0.041**	-3.294***	-0.361*	-0.111
		(0.67)	*(0.00)*	*(0.03)*	*(0.00)*	*(0.08)*	*(0.88)*
MV-C	137	-0.024	-0.141***	-0.093**	-0.194	-0.542	-0.043

		(0.30)	*(0.00)*	*(0.02)*	*(0.47)*	*(0.35)*	*(0.30)*
BS	137	-0.038	-2.533***	-0.038**	-3.285***	-0.345*	-0.109
		(0.64)	*(0.00)*	*(0.03)*	*(0.00)*	*(0.05)*	*(0.90)*
BS-C	137	-0.035	-0.184***	-0.123	-0.245	-0.603	-0.057
		(0.45)	*(0.00)*	*(0.18)*	*(0.62)*	*(0.73)*	*(0.45)*
Min-Var	137	-0.032	-2.349***	-0.033**	-3.045***	-0.304**	-0.106
		(0.56)	*(0.00)*	*(0.02)*	*(0.00)*	*(0.01)*	*(0.93)*
Min-Var-C	137	-0.012	-0.186***	-0.023***	-0.247	-0.329***	-0.037
		(0.29)	*(0.00)*	*(0.00)*	*(0.71)*	*(0.01)*	*(0.36)*
G-Min-C	137	-0.016	-0.127***	-0.057***	-0.165	-0.397***	-0.041
		(0.15)	*(0.00)*	*(0.00)*	*(0.18)*	*(0.00)*	*(0.12)*
MV-Min	137	-0.038	-2.512***	-0.039**	-3.259***	-0.348*	-0.110
		(0.65)	*(0.00)*	*(0.03)*	*(0.00)*	*(0.05)*	*(0.90)*
EW-Min	137	-0.032	-2.349***	-0.033**	-3.045***	-0.304**	-0.106
		(0.56)	*(0.00)*	*(0.02)*	*(0.00)*	*(0.01)*	*(0.93)*

Notes: **1/N (EW)** = equal weight 1/N, rebalanced monthly. **VW** = value-weighted market return (MKT_total), zero turnover. **MV** = mean-variance. **MV-C** = constrained MV. **BS** = Bayes-Stein. **BS-C** = constrained BS. **Min-Var** = minimum-variance. **Min-Var-C** = constrained Min-Var. **G-Min-C** = global minimum-variance constrained. **MV-Min** = convex combination of MV and Min-Var. **EW-Min** = convex combination of 1/N and Min-Var. All optimized strategies use a rolling M-month estimation window. Metrics on monthly log excess returns $ln(1 + r - rf)$. **N** = monthly observations. Stars vs 1/N (EW): * p<0.10, ** p<0.05, *** p<0.01. P-values in parentheses below each strategy. **SR**: Jobson-Korkie (1981)/Memmel (2003); others: stationary bootstrap (B=500). All significance tests measure the difference of each strategy relative to 1/N (EW); a self-referential test is not defined.

Table A4: Binary CMAX Transaction Cost Analysis | Optimized Strategies vs. 1/N (Europe, M=60)

Strategy	FF3 *(N=3)*	FF5 *(N=5)*	BM_MKT *(N=21)*	BM_FF3 *(N=23)*	BM_FF5 *(N=25)*
			Full Sample		
1/N (EW)	**0.0212**	**0.0193**	**0.0121**	**0.0148**	**0.0176**
			Panel A: Relative Turnover (strategy / 1/N)		
VW	0.00	0.00	0.00	0.00	0.00
MV	360.79	210.29	41253.72	15.52	74.52
MV-C	4.38	3.76	11.71	10.07	4.53
BS	150.83	161.28	20486.93	13.90	60.37
BS-C	4.03	4.05	9.63	11.42	5.42
Min-Var	1.05	1.47	85.33	11.35	9.77
Min-Var-C	1.00	1.49	8.55	2.02	2.35
G-Min-C	0.96	1.27	5.33	1.97	1.92
MV-Min	369.04	202.77	35720.19	13.97	56.50
EW-Min	3.21	1.94	119.55	11.35	9.77
			Panel B: Return-Loss Relative to 1/N (per month)		
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
MV	0.0381	0.0202	2.4966	0.0011	0.0065
MV-C	0.0004	0.0003	0.0006	0.0007	0.0003
BS	0.0159	0.0154	1.2398	0.0010	0.0052
BS-C	0.0003	0.0003	0.0005	0.0008	0.0004
Min-Var	0.0000	0.0000	0.0051	0.0008	0.0008
Min-Var-C	-0.0000	0.0000	0.0005	0.0001	0.0001
G-Min-C	-0.0000	0.0000	0.0003	0.0001	0.0001
MV-Min	0.0390	0.0194	2.1617	0.0010	0.0049
EW-Min	0.0002	0.0001	0.0072	0.0008	0.0008

		Pre-Crisis (CMAX between 1 and 0.80, no threshold breach)			
1/N (EW)	0.0156	0.0144	0.0098	0.0117	0.0136
		Panel A: Relative Turnover (strategy / 1/N)			
VW	0.00	0.00	0.00	0.00	0.00
MV	493.44	58.07	72064.11	17.39	151.38
MV-C	3.91	3.62	7.64	8.07	4.00
BS	253.23	31.16	32038.19	15.03	120.65
BS-C	2.82	2.83	6.31	9.35	4.25
Min-Var	1.04	1.29	88.86	11.63	9.93
Min-Var-C	0.96	1.45	7.61	1.41	2.13
G-Min-C	0.86	1.31	4.84	1.59	1.75
MV-Min	501.81	55.31	64913.13	15.13	112.83
EW-Min	3.12	1.51	131.39	11.63	9.93
		Panel B: Return-Loss Relative to 1/N (per month)			
VW	-0.0001	-0.0001	-0.0000	-0.0001	-0.0001
MV	0.0384	0.0041	3.5337	0.0010	0.0102
MV-C	0.0002	0.0002	0.0003	0.0004	0.0002
BS	0.0196	0.0022	1.5710	0.0008	0.0082
BS-C	0.0001	0.0001	0.0003	0.0005	0.0002
Min-Var	0.0000	0.0000	0.0043	0.0006	0.0006
Min-Var-C	-0.0000	0.0000	0.0003	0.0000	0.0001
G-Min-C	-0.0000	0.0000	0.0002	0.0000	0.0001
MV-Min	0.0390	0.0039	3.1830	0.0008	0.0076
EW-Min	0.0002	0.0000	0.0064	0.0006	0.0006
		Crisis (all non-pre-crisis months)			
1/N (EW)	0.0280	0.0252	0.0149	0.0186	0.0225

Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00
MV	270.42	316.74	16418.89	14.07	17.52
MV-C	4.69	3.86	14.98	11.62	4.92
BS	81.07	252.27	11175.99	13.04	15.67
BS-C	4.86	4.90	12.31	13.01	6.28
Min-Var	1.06	1.59	82.49	11.13	9.65
Min-Var-C	1.02	1.52	9.31	2.50	2.51
G-Min-C	1.03	1.24	5.73	2.25	2.05
MV-Min	278.59	305.90	12189.11	13.09	14.73
EW-Min	3.27	2.25	110.02	11.13	9.65
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
MV	0.0378	0.0399	1.2248	0.0012	0.0019
MV-C	0.0005	0.0004	0.0010	0.0010	0.0004
BS	0.0112	0.0317	0.8337	0.0011	0.0017
BS-C	0.0005	0.0005	0.0008	0.0011	0.0006
Min-Var	0.0000	0.0001	0.0061	0.0009	0.0010
Min-Var-C	0.0000	0.0001	0.0006	0.0001	0.0002
G-Min-C	0.0000	0.0000	0.0004	0.0001	0.0001
MV-Min	0.0389	0.0385	0.9092	0.0011	0.0015
EW-Min	0.0003	0.0002	0.0081	0.0009	0.0010

Notes: **Turnover** = average monthly absolute weight changes. Row below header: absolute monthly **1/N** turnover per universe. **Panel A**: Relative turnover = strategy / 1/N. VW = 0.00 by construction. **Panel B**: Return-loss vs 1/N per month $= TC \times (turnover_{strategy} - turnover_{1/N})$, TC=50bp. Positive = strategy costs more than 1/N per month; negative = cheaper.

Table A5: Four-Phase CMAX Transaction Cost Analysis | Optimized Strategies vs. 1/N (Europe, M=60)

Strategy	FF3 *(N=3)*	FF5 *(N=5)*	BM_MKT *(N=21)*	BM_FF3 *(N=23)*	BM_FF5 *(N=25)*
Full Sample					
1/N (EW)	**0.0211**	**0.0192**	**0.0124**	**0.0149**	**0.0175**
Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00
MV	212.82	491.98	41806.70	218.28	1775.49
MV-C	4.37	7.74	21.67	17.57	9.80
BS	92.22	211.53	31790.08	154.61	585.80
BS-C	5.75	8.26	19.62	17.13	11.24
Min-Var	1.80	2.48	185.23	17.44	16.90
Min-Var-C	1.65	2.41	10.72	3.03	3.23
G-Min-C	1.48	1.75	5.98	2.83	2.62
MV-Min	197.16	540.32	18960.54	81.19	171.25
EW-Min	3.65	3.65	209.31	21.12	19.15
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
MV	0.0224	0.0471	2.5961	0.0162	0.1552
MV-C	0.0004	0.0006	0.0013	0.0012	0.0008
BS	0.0096	0.0202	1.9741	0.0114	0.0511
BS-C	0.0005	0.0007	0.0012	0.0012	0.0009
Min-Var	0.0001	0.0001	0.0114	0.0012	0.0014
Min-Var-C	0.0001	0.0001	0.0006	0.0002	0.0002
G-Min-C	0.0001	0.0001	0.0003	0.0001	0.0001
MV-Min	0.0207	0.0518	1.1774	0.0060	0.0149
EW-Min	0.0003	0.0003	0.0129	0.0015	0.0016

Pre-Crisis (CMAX between 1 and 0.80, no threshold breach)					
1/N (EW)	**0.0168**	**0.0154**	**0.0107**	**0.0125**	**0.0143**
Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00
MV	335.40	246.05	5638.16	403.18	3496.88
MV-C	6.23	9.47	28.54	22.25	12.48
BS	151.61	134.24	4797.17	281.70	1137.44
BS-C	6.45	7.46	25.66	19.40	13.40
Min-Var	2.22	3.19	203.49	19.79	19.97
Min-Var-C	2.11	2.95	13.72	3.82	4.04
G-Min-C	1.65	2.08	8.53	3.17	2.93
MV-Min	308.50	224.57	3108.26	142.29	325.00
EW-Min	4.92	4.42	248.48	26.82	24.44
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
MV	0.0281	0.0189	0.3027	0.0252	0.2497
MV-C	0.0004	0.0007	0.0015	0.0013	0.0008
BS	0.0126	0.0103	0.2576	0.0176	0.0812
BS-C	0.0005	0.0005	0.0013	0.0012	0.0009
Min-Var	0.0001	0.0002	0.0109	0.0012	0.0014
Min-Var-C	0.0001	0.0002	0.0007	0.0002	0.0002
G-Min-C	0.0001	0.0001	0.0004	0.0001	0.0001
MV-Min	0.0258	0.0172	0.1669	0.0088	0.0231
EW-Min	0.0003	0.0003	0.0133	0.0016	0.0017
Crisis (CMAX declining from 1 toward the 0.80 threshold)					
1/N (EW)	**0.0262**	**0.0230**	**0.0159**	**0.0187**	**0.0212**

Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00
MV	119.75	33.37	1186.03	15.92	20.85
MV-C	2.26	5.08	12.33	11.67	10.68
BS	55.99	19.21	719.14	14.75	19.09
BS-C	1.83	6.45	11.91	14.38	12.58
Min-Var	1.55	2.59	116.93	12.43	11.59
Min-Var-C	1.32	2.41	9.17	2.23	3.42
G-Min-C	1.29	1.91	4.28	1.88	2.30
MV-Min	114.14	30.28	383.59	12.46	11.88
EW-Min	2.50	3.33	102.59	12.43	11.59
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
MV	0.0156	0.0037	0.0940	0.0014	0.0021
MV-C	0.0002	0.0005	0.0009	0.0010	0.0010
BS	0.0072	0.0021	0.0570	0.0013	0.0019
BS-C	0.0001	0.0006	0.0009	0.0013	0.0012
Min-Var	0.0001	0.0002	0.0092	0.0011	0.0011
Min-Var-C	0.0000	0.0002	0.0006	0.0001	0.0003
G-Min-C	0.0000	0.0001	0.0003	0.0001	0.0001
MV-Min	0.0148	0.0034	0.0304	0.0011	0.0012
EW-Min	0.0002	0.0003	0.0081	0.0011	0.0011
Drawdown (trigger month through trough)					
1/N (EW)	**0.0341**	**0.0309**	**0.0175**	**0.0228**	**0.0290**
Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00

MV	39.36	1106.57	2703.13	17.79	23.20
MV-C	2.40	5.97	13.11	12.44	5.96
BS	14.94	564.76	2169.37	16.65	21.32
BS-C	3.31	7.84	9.66	14.87	8.29
Min-Var	1.25	1.34	137.49	13.37	11.71
Min-Var-C	1.13	1.35	5.11	2.03	2.00
G-Min-C	1.10	1.24	1.19	1.74	2.05
MV-Min	37.23	1060.32	1167.75	14.23	13.32
EW-Min	2.29	2.61	140.13	13.42	11.71
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0002	-0.0002	-0.0001	-0.0001	-0.0001
MV	0.0065	0.1707	0.2367	0.0019	0.0032
MV-C	0.0002	0.0008	0.0011	0.0013	0.0007
BS	0.0024	0.0871	0.1899	0.0018	0.0029
BS-C	0.0004	0.0011	0.0008	0.0016	0.0011
Min-Var	0.0000	0.0001	0.0120	0.0014	0.0016
Min-Var-C	0.0000	0.0001	0.0004	0.0001	0.0001
G-Min-C	0.0000	0.0000	0.0000	0.0001	0.0002
MV-Min	0.0062	0.1636	0.1022	0.0015	0.0018
EW-Min	0.0002	0.0002	0.0122	0.0014	0.0016
Recovery (trough+1 through CMAX returning to 1)					
1/N (EW)	**0.0254**	**0.0231**	**0.0132**	**0.0162**	**0.0195**
Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00
MV	127.69	800.45	201384.37	23.16	39.97
MV-C	2.92	6.61	15.37	13.16	5.98

BS	40.40	202.54	151727.69	21.12	34.87
BS-C	8.37	11.44	15.73	14.73	7.69
Min-Var	1.45	1.74	216.88	17.79	16.99
Min-Var-C	1.22	2.04	7.84	2.30	2.22
G-Min-C	1.51	1.35	3.80	3.46	2.52
MV-Min	120.92	1115.59	89933.13	18.46	19.08
EW-Min	2.53	2.88	224.49	17.92	16.99
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
MV	0.0161	0.0923	13.3340	0.0018	0.0038
MV-C	0.0002	0.0006	0.0010	0.0010	0.0005
BS	0.0050	0.0233	10.0461	0.0016	0.0033
BS-C	0.0009	0.0012	0.0010	0.0011	0.0007
Min-Var	0.0001	0.0001	0.0143	0.0014	0.0016
Min-Var-C	0.0000	0.0001	0.0005	0.0001	0.0001
G-Min-C	0.0001	0.0000	0.0002	0.0002	0.0001
MV-Min	0.0152	0.1287	5.9546	0.0014	0.0018
EW-Min	0.0002	0.0002	0.0148	0.0014	0.0016

Notes: **Turnover** = average monthly absolute weight changes. Row below header: absolute monthly **1/N** turnover per universe. **Panel A**: Relative turnover = strategy / 1/N. VW = 0.00 by construction. **Panel B**: Return-loss vs 1/N per month $= TC \times (turnover_{strategy} - turnover_{1/N})$, TC =50bp. Positive = strategy costs more than 1/N per month; negative = cheaper.

Table A6: Four-Phase CMAX Transaction Cost Analysis | Long-Only and Long-Short Strategies vs. 1/N (Europe)

Strategy	FF3 *(N=3)*	FF5 *(N=5)*	BM_MKT *(N=21)*	BM_FF3 *(N=23)*	BM_FF5 *(N=25)*
			Full Sample		
1/N (EW)	**0.0206**	**0.0186**	**0.0121**	**0.0145**	**0.0170**
			Panel A: Relative Turnover (strategy / 1/N)		
VW	0.00	0.00	0.00	0.00	0.00
Past-1 Full | Long	2.17	2.73	3.80	3.31	2.95
Past-1 Full | L/S	2.18	2.76	3.69	3.24	2.91
Past-1 Phase | Long	3.68	4.67	6.61	5.53	4.69
Past-1 Phase | L/S	3.64	4.50	6.91	5.68	4.84
Past-All Full | Long	2.18	2.73	3.80	3.31	2.95
Past-All Full | L/S	2.14	2.71	3.70	3.24	2.89
Past-All Phase | Long	3.44	5.22	5.94	5.41	4.69
Past-All Phase | L/S	3.41	4.63	6.24	5.34	4.63
Phase-Rev | Long	2.77	3.46	5.53	4.77	4.20
Phase-Rev | L/S	2.95	3.95	6.25	5.04	4.24
Phase-Mom | Long	2.73	3.97	5.79	4.67	3.90
Phase-Mom | L/S	3.14	4.17	6.15	5.18	4.49
Mom-12 | Long	16.16	19.74	29.34	25.93	22.95
Mom-12 | L/S	16.24	21.90	32.67	27.00	23.53
ST-Rev | Long	63.42	79.35	117.96	97.40	81.65
ST-Rev | L/S	61.10	79.22	119.89	99.51	84.40
			Panel B: Return-Loss Relative to 1/N (per month)		
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
Past-1 Full | Long	0.0001	0.0002	0.0002	0.0002	0.0002
Past-1 Full | L/S	0.0001	0.0002	0.0002	0.0002	0.0002

Past-1 Phase | Long	0.0003	0.0003	0.0003	0.0003	0.0003
Past-1 Phase | L/S	0.0003	0.0003	0.0004	0.0003	0.0003
Past-All Full | Long	0.0001	0.0002	0.0002	0.0002	0.0002
Past-All Full | L/S	0.0001	0.0002	0.0002	0.0002	0.0002
Past-All Phase | Long	0.0003	0.0004	0.0003	0.0003	0.0003
Past-All Phase | L/S	0.0002	0.0003	0.0003	0.0003	0.0003
Phase-Rev | Long	0.0002	0.0002	0.0003	0.0003	0.0003
Phase-Rev | L/S	0.0002	0.0003	0.0003	0.0003	0.0003
Phase-Mom | Long	0.0002	0.0003	0.0003	0.0003	0.0002
Phase-Mom | L/S	0.0002	0.0003	0.0003	0.0003	0.0003
Mom-12 | Long	0.0016	0.0017	0.0017	0.0018	0.0019
Mom-12 | L/S	0.0016	0.0019	0.0019	0.0019	0.0019
ST-Rev | Long	0.0064	0.0073	0.0071	0.0070	0.0068
ST-Rev | L/S	0.0062	0.0073	0.0072	0.0071	0.0071
Crisis (CMAX declining from 1 toward the 0.80 threshold)					
1/N (EW)	**0.0241**	**0.0212**	**0.0155**	**0.0180**	**0.0202**
Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00
Past-1 Full | Long	7.89	10.81	14.06	12.45	11.34
Past-1 Full | L/S	7.89	10.81	14.06	12.45	11.34
Past-1 Phase | Long	7.97	10.77	14.08	12.49	11.38
Past-1 Phase | L/S	7.97	10.77	14.08	12.49	11.38
Past-All Full | Long	7.94	10.83	14.06	12.46	11.36
Past-All Full | L/S	7.94	10.83	14.06	12.46	11.36
Past-All Phase | Long	8.01	10.79	14.09	12.49	11.38
Past-All Phase | L/S	8.01	10.79	14.09	12.49	11.38
Mom-12 | Long	18.96	18.87	27.08	24.02	21.96

Mom-12 | L/S	19.95	22.73	28.76	24.06	21.91
ST-Rev | Long	54.56	72.89	88.21	76.92	69.07
ST-Rev | L/S	56.18	72.02	89.90	77.26	70.41
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
Past-1 Full | Long	0.0008	0.0010	0.0010	0.0010	0.0010
Past-1 Full | L/S	0.0008	0.0010	0.0010	0.0010	0.0010
Past-1 Phase | Long	0.0008	0.0010	0.0010	0.0010	0.0010
Past-1 Phase | L/S	0.0008	0.0010	0.0010	0.0010	0.0010
Past-All Full | Long	0.0008	0.0010	0.0010	0.0010	0.0010
Past-All Full | L/S	0.0008	0.0010	0.0010	0.0010	0.0010
Past-All Phase | Long	0.0008	0.0010	0.0010	0.0010	0.0010
Past-All Phase | L/S	0.0008	0.0010	0.0010	0.0010	0.0010
Mom-12 | Long	0.0022	0.0019	0.0020	0.0021	0.0021
Mom-12 | L/S	0.0023	0.0023	0.0021	0.0021	0.0021
ST-Rev | Long	0.0065	0.0076	0.0067	0.0068	0.0069
ST-Rev | L/S	0.0067	0.0075	0.0069	0.0068	0.0070
Drawdown (trigger month through trough)					
1/N (EW)	**0.0330**	**0.0291**	**0.0166**	**0.0216**	**0.0271**
Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00
Past-1 Full | Long	0.00	0.00	0.00	0.00	0.00
Past-1 Full | L/S	0.00	0.00	0.00	0.00	0.00
Past-1 Phase | Long	1.52	3.44	8.85	5.16	2.76
Past-1 Phase | L/S	1.89	3.05	7.94	5.19	3.47
Past-All Full | Long	0.00	0.00	0.00	0.00	0.00

Past-All Full | L/S	0.00	0.00	0.00	0.00	0.00
Past-All Phase | Long	0.00	6.87	6.40	3.77	2.09
Past-All Phase | L/S	0.62	3.99	5.42	3.48	2.35
Phase-Rev | Long	6.08	8.17	13.48	10.67	8.71
Phase-Rev | L/S	6.08	8.17	13.48	10.67	8.71
Phase-Mom | Long	5.79	7.96	13.33	10.50	8.53
Phase-Mom | L/S	5.79	7.96	13.33	10.50	8.53
Mom-12 | Long	7.58	5.16	17.23	12.90	10.05
Mom-12 | L/S	7.49	6.84	20.01	15.24	12.30
ST-Rev | Long	32.55	45.26	75.74	61.59	50.89
ST-Rev | L/S	32.95	45.89	77.46	61.64	48.48
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001
Past-1 Full | Long	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001
Past-1 Full | L/S	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001
Past-1 Phase | Long	0.0001	0.0004	0.0007	0.0004	0.0002
Past-1 Phase | L/S	0.0001	0.0003	0.0006	0.0005	0.0003
Past-All Full | Long	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001
Past-All Full | L/S	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001
Past-All Phase | Long	-0.0002	0.0009	0.0004	0.0003	0.0001
Past-All Phase | L/S	-0.0001	0.0004	0.0004	0.0003	0.0002
Phase-Rev | Long	0.0008	0.0010	0.0010	0.0010	0.0010
Phase-Rev | L/S	0.0008	0.0010	0.0010	0.0010	0.0010
Phase-Mom | Long	0.0008	0.0010	0.0010	0.0010	0.0010
Phase-Mom | L/S	0.0008	0.0010	0.0010	0.0010	0.0010
Mom-12 | Long	0.0011	0.0006	0.0014	0.0013	0.0012
Mom-12 | L/S	0.0011	0.0008	0.0016	0.0015	0.0015

ST-Rev | Long	0.0052	0.0064	0.0062	0.0065	0.0068
ST-Rev | L/S	0.0053	0.0065	0.0064	0.0066	0.0064
Recovery (trough+1 through CMAX returning to 1)					
1/N (EW)	**0.0234**	**0.0212**	**0.0126**	**0.0155**	**0.0186**
Panel A: Relative Turnover (strategy / 1/N)					
VW	0.00	0.00	0.00	0.00	0.00
Past-1 Full | Long	0.00	0.00	0.00	0.00	0.00
Past-1 Full | L/S	0.00	0.00	0.00	0.00	0.00
Past-1 Phase | Long	6.56	7.26	8.86	7.93	6.98
Past-1 Phase | L/S	5.82	6.45	7.92	7.09	6.54
Past-All Full | Long	0.00	0.00	0.00	0.00	0.00
Past-All Full | L/S	0.00	0.00	0.00	0.00	0.00
Past-All Phase | Long	6.56	7.26	7.02	8.44	7.60
Past-All Phase | L/S	5.79	6.42	6.63	6.87	6.46
Phase-Rev | Long	0.85	0.94	4.05	3.29	2.75
Phase-Rev | L/S	1.39	2.68	4.54	3.24	2.52
Phase-Mom | Long	0.85	2.83	5.07	3.02	1.77
Phase-Mom | L/S	1.22	2.25	4.60	3.21	2.32
Mom-12 | Long	10.93	23.65	32.71	28.61	25.38
Mom-12 | L/S	14.27	21.98	34.80	30.43	27.99
ST-Rev | Long	53.76	62.32	114.59	91.90	74.47
ST-Rev | L/S	51.67	68.12	115.75	92.71	75.75
Panel B: Return-Loss Relative to 1/N (per month)					
VW	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
Past-1 Full | Long	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
Past-1 Full | L/S	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001

Past-1 Phase | Long	0.0007	0.0007	0.0005	0.0005	0.0006
Past-1 Phase | L/S	0.0006	0.0006	0.0004	0.0005	0.0005
Past-All Full | Long	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
Past-All Full | L/S	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
Past-All Phase | Long	0.0007	0.0007	0.0004	0.0006	0.0006
Past-All Phase | L/S	0.0006	0.0006	0.0004	0.0005	0.0005
Phase-Rev | Long	-0.0000	-0.0000	0.0002	0.0002	0.0002
Phase-Rev | L/S	0.0000	0.0002	0.0002	0.0002	0.0001
Phase-Mom | Long	-0.0000	0.0002	0.0003	0.0002	0.0001
Phase-Mom | L/S	0.0000	0.0001	0.0002	0.0002	0.0001
Mom-12 | Long	0.0012	0.0024	0.0020	0.0021	0.0023
Mom-12 | L/S	0.0016	0.0022	0.0021	0.0023	0.0025
ST-Rev | Long	0.0062	0.0065	0.0072	0.0071	0.0068
ST-Rev | L/S	0.0059	0.0071	0.0072	0.0071	0.0070

Notes: **Turnover** = average monthly absolute weight changes. Row below header: absolute monthly **1/N** turnover per universe. **Panel A**: Relative turnover = strategy / 1/N. VW = 0.00 by construction. **Panel B**: Return-loss vs 1/N per month $= TC \times (turnover_{strategy} - turnover_{1/N})$, TC =50bp. Positive = strategy costs more than 1/N per month; negative = cheaper.

Table A7: Pooled Phase Regression | Strategy Return Differential vs. 1/N (All Regions, M=60)

Variable	VW	MV	MV-C	BS	BS-C	Min-Var	Min-Var-C	G-Min-C	MV-Min	EW-Min
Constant	0.0030**	-0.6029***	0.0006	-0.3957***	0.0018*	-0.0011	-0.0017*	-0.0012*	-0.2341***	-0.0013
	(2.34)	*(-5.40)*	*(0.65)*	*(-4.35)*	*(1.93)*	*(-0.67)*	*(-1.74)*	*(-1.89)*	*(-3.53)*	*(-0.93)*
Crisis	-0.0124***	0.1650***	-0.0016	0.1308***	-0.0036*	0.0137***	0.0128***	0.0083***	0.1092***	0.0118***
	(-4.52)	*(4.34)*	*(-0.74)*	*(4.11)*	*(-1.66)*	*(4.73)*	*(5.31)*	*(5.31)*	*(4.02)*	*(4.46)*
Drawdown	-0.0183***	0.0145	0.0071**	0.0679	0.0020	0.0237***	0.0209***	0.0135***	0.0763*	0.0211***
	(-5.08)	*(0.18)*	*(2.10)*	*(1.12)*	*(0.66)*	*(4.73)*	*(5.17)*	*(5.26)*	*(1.69)*	*(4.59)*
Recovery	0.0017	-0.2553***	-0.0024	-0.1347	-0.0013	-0.0095***	-0.0066***	-0.0038***	-0.1138	-0.0083***
	(0.74)	*(-2.64)*	*(-1.20)*	*(-1.57)*	*(-0.73)*	*(-3.03)*	*(-2.75)*	*(-2.69)*	*(-1.45)*	*(-2.99)*
Europe	-0.0001	0.1299**	0.0009	0.0911*	0.0011	0.0001	0.0003	0.0002	0.0777*	0.0005
	(-0.10)	*(2.15)*	*(0.95)*	*(1.75)*	*(1.19)*	*(0.08)*	*(0.36)*	*(0.26)*	*(1.79)*	*(0.49)*
Japan	-0.0000	-0.0286	-0.0006	0.0002	-0.0013	-0.0004	-0.0004	-0.0001	-0.0123	0.0006
	(-0.02)	*(-0.38)*	*(-0.46)*	*(0.00)*	*(-1.00)*	*(-0.21)*	*(-0.34)*	*(-0.18)*	*(-0.23)*	*(0.39)*
FF3	0.0035**	0.4355***	0.0004	0.2380**	-0.0002	-0.0026*	-0.0020***	-0.0012***	0.0546	-0.0022*
	(2.16)	*(3.27)*	*(0.40)*	*(2.18)*	*(-0.17)*	*(-1.91)*	*(-3.15)*	*(-2.96)*	*(0.58)*	*(-1.80)*
FF5	0.0037*	0.5021***	-0.0001	0.2821***	-0.0006	-0.0019	-0.0014**	-0.0008**	0.1230	-0.0016
	(1.80)	*(3.94)*	*(-0.08)*	*(2.70)*	*(-0.52)*	*(-1.39)*	*(-2.47)*	*(-2.24)*	*(1.41)*	*(-1.40)*
BM_FF3	0.0003	0.5867***	-0.0007	0.3598***	-0.0015**	-0.0084***	-0.0052***	-0.0020*	0.2047***	-0.0079***
	(1.45)	*(4.94)*	*(-0.96)*	*(3.80)*	*(-2.13)*	*(-4.69)*	*(-3.10)*	*(-1.93)*	*(2.70)*	*(-4.02)*
BM_FF5	0.0005	0.5857***	-0.0019	0.3593***	-0.0030**	-0.0080***	-0.0045***	-0.0016	0.2050***	-0.0076***
	(1.21)	*(4.93)*	*(-1.39)*	*(3.79)*	*(-2.40)*	*(-4.82)*	*(-2.73)*	*(-1.53)*	*(2.71)*	*(-4.19)*
N	5475	5475	5475	5475	5475	5475	5475	5475	5475	5475
R^2	0.0561	0.0163	0.0112	0.0087	0.0060	0.0734	0.0829	0.0797	0.0056	0.0676

Notes: Dependent variable is the monthly log excess return differential (strategy minus 1/N). OLS with standard errors clustered by calendar month; t-statistics in parentheses. Sample pools three regions (United States, Europe, Japan; 1990–2025) across five universes (FF3, FF5, BM_MKT, BM_FF3, BM_FF5). Baseline: pre-crisis phase, United States, BM_MKT. Phase dummies measure deviations from the pre-crisis baseline. Region dummies (Europe, Japan; baseline US) and universe dummies (FF3, FF5, BM_FF3, BM_FF5; baseline BM_MKT) control for cross-sectional heterogeneity. Crisis identification follows CMAX (Patel and Sarkar, 1998; $\theta = 0.80$, 24-month rolling window). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A8: Pooled Phase Regression | Strategy Return Differential vs. 1/N (All Regions, M=120)

Variable	VW	MV	MV-C	BS	BS-C	Min-Var	Min-Var-C	G-Min-C	MV-Min	EW-Min
Constant	0.0023*	-0.3952***	0.0021*	-0.1792***	0.0026**	-0.0017	-0.0025**	-0.0015**	-0.2508***	-0.0013
	(1.70)	*(-4.20)*	*(1.77)*	*(-2.78)*	*(2.37)*	*(-0.96)*	*(-2.27)*	*(-2.28)*	*(-3.25)*	*(-0.81)*
Crisis	-0.0112***	0.0840	0.0042	0.0312	0.0021	0.0141***	0.0137***	0.0086***	0.0398	0.0129***
	(-3.54)	*(0.94)*	*(1.63)*	*(0.54)*	*(0.93)*	*(4.51)*	*(5.21)*	*(4.98)*	*(0.48)*	*(4.50)*
Drawdown	-0.0199***	-0.0114	0.0106***	0.0169	0.0061**	0.0231***	0.0197***	0.0128***	0.0324	0.0205***
	(-5.03)	*(-0.11)*	*(3.00)*	*(0.24)*	*(2.01)*	*(4.16)*	*(4.74)*	*(4.77)*	*(0.38)*	*(4.17)*
Recovery	0.0025	0.1267**	-0.0029	0.0674***	-0.0026	-0.0068**	-0.0047**	-0.0030**	0.1018**	-0.0059**
	(1.09)	*(2.25)*	*(-1.46)*	*(2.63)*	*(-1.44)*	*(-2.22)*	*(-2.03)*	*(-2.12)*	*(2.32)*	*(-2.17)*
Europe	-0.0000	-0.1750**	-0.0012	-0.0758	-0.0011	-0.0006	0.0002	0.0001	-0.0942	-0.0005
	(-0.02)	*(-2.32)*	*(-1.32)*	*(-1.62)*	*(-1.25)*	*(-0.51)*	*(0.28)*	*(0.15)*	*(-1.53)*	*(-0.49)*
Japan	-0.0014	-0.0251	-0.0011	0.0012	-0.0007	0.0006	0.0010	0.0003	-0.0151	0.0007
	(-1.08)	*(-0.45)*	*(-0.81)*	*(0.04)*	*(-0.62)*	*(0.36)*	*(0.85)*	*(0.35)*	*(-0.30)*	*(0.51)*
FF3	0.0029	0.1378	-0.0022*	0.0923	-0.0029**	-0.0020	-0.0015**	-0.0007	0.0078	-0.0021*
	(1.63)	*(1.02)*	*(-1.83)*	*(1.10)*	*(-2.46)*	*(-1.45)*	*(-2.21)*	*(-1.62)*	*(0.07)*	*(-1.84)*
FF5	0.0031	0.3742***	-0.0027***	0.1803**	-0.0035***	-0.0017	-0.0012**	-0.0007**	0.2045**	-0.0019*
	(1.35)	*(3.37)*	*(-2.82)*	*(2.52)*	*(-3.14)*	*(-1.25)*	*(-2.32)*	*(-2.19)*	*(2.25)*	*(-1.89)*
BM_FF3	0.0002	0.4230***	-0.0019*	0.1788**	-0.0014**	-0.0077***	-0.0042**	-0.0013	0.2527***	-0.0080***
	(0.91)	*(4.04)*	*(-1.94)*	*(2.50)*	*(-1.98)*	*(-4.18)*	*(-2.20)*	*(-1.08)*	*(3.04)*	*(-3.94)*
BM_FF5	0.0004	0.4221***	-0.0037**	0.1782**	-0.0026**	-0.0072***	-0.0038**	-0.0014	0.2523***	-0.0075***
	(0.79)	*(4.03)*	*(-2.39)*	*(2.50)*	*(-2.05)*	*(-4.29)*	*(-2.03)*	*(-1.21)*	*(3.04)*	*(-4.06)*
N	4575	4575	4575	4575	4575	4575	4575	4575	4575	4575
R^2	0.0677	0.0105	0.0260	0.0053	0.0135	0.0732	0.0792	0.0794	0.0063	0.0663

Notes: Dependent variable is the monthly log excess return differential (strategy minus 1/N). OLS with standard errors clustered by calendar month; t-statistics in parentheses. Sample pools three regions (United States, Europe, Japan; 1990–2025) across five universes (FF3, FF5, BM_MKT, BM_FF3, BM_FF5). Baseline: pre-crisis phase, United States, BM_MKT. Phase dummies measure deviations from the pre-crisis baseline. Region dummies (Europe, Japan; baseline US) and universe dummies (FF3, FF5, BM_FF3, BM_FF5; baseline BM_MKT) control for cross-sectional heterogeneity. Crisis identification follows CMAX (Patel and Sarkar, 1998; $\theta = 0.80$, 24-month rolling window). * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

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Declaration on the Use of AI Tools

In this thesis, I used AI tools solely to refine clarity, improve language, and check grammar and spelling. All content, analyses, and conclusions are my own.