Luxury Goods Externalities, Taxation and Endogenous Cycles

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Abstract

This thesis aims to analyze aggregate instability due to volatile expectations in a simple OLG model with money. We assume there are two types of goods, necessity and luxury goods, from which agents take utility, with the particularity of having a consumption externality affecting the consumption of the latter. We also assume government to follow a balanced budget rule with public spendings financed exclusively through consumption taxation. Tax rates for each type of good may be different and may to react to the cycle. We verify that the distinction between necessity and luxury goods is not relevant for the emergence of indeterminacy, if there is no government intervention and if the externality has no influence. Then we show that the fiscal policies considered may create local indeterminacy, in the absence of externalities, if tax rates are strongly pro-cyclical or counter-cyclical. We also show that externalities, *per se*, may create indeterminacy. However, consumption taxation can, in fact, be a stabilizing instrument, by eliminating local indeterminacy, if one of the tax rates is set pro-cyclically (counter-cyclically) for a positive (negative) externality degree.

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Contents

1 Introduction

Whether or not Governments should adopt balanced budget rules is a wide open debate in Europe and also in the U.S.. Many contributions are given for it, coming from politics, ethics, law and some other areas. Economics, of course, is also a very important part of the discussion and one of its more valuable contributions, in our opinion, is the analysis of business cycles when such balanced budget rules are running.

It is well known that rational expectations economic models offer two types of complementary explanations for cycles. One considering exogenous shocks to the fundamentals of the economy (*Real Business Cycles approach* or *RBC*) and the other considering that some endogenous fluctuations may arise, even without those shocks, through self-fulfilling prophecies (*Endogenous Business Cycles approach* or *EBC*). People take current decisions relying on expectations about the future value of variables and the latter depend on agents' current decisions. Under some economic environments, changes in expectations and current decisions may lead to changes in future variables in the direction initially expected. Since rational behavior requires those expectations to be right on average, though they may be subjected to predicting errors, the result is that aggregate variables can exhibit rational expectations stochastic bounded fluctuations around the trend, even if all the fundamentals hold the same — that is why we call them *endogenous.*

Since the late 80's lots of literature concerning endogenous cycles has been developed. Recently, some authors have shown how empirically plausible this explanation is, at least to explain part of the magnitude of business cycles. This particular thesis is inserted in this branch of the study of cycles and tries to analyze, from a theoretical point of view, the emergence of endogenous cycles and if there is a stabilizing role for government, when it follows a balanced budget rule and public spendings are financed by consumption taxation, with different tax rates over luxury and necessity goods.

Since the early 90's lots of literature has come to the light on the impact of income taxation, when no public debt is allowed, for the emergence of endogenous cycles. However, few works were yet published on the effects of government spending financed with taxes over consumption. Giannitsarou (2007) is one of the pioneers in this chapter, arriving to very interesting and robust results. Within a neo-classical model with capital, infinite horizon agents deriving utility from consumption of one unique good and leisure, constant returns to scale technologies, perfectly competitive markets and an exogenous schedule of public expenditures financed by a proportional tax rate over consumption, her main conclusion is that self-fulfilling prophecies are not possible, since the labor supply is independent from consumption tax rate. Moreover, she tested several utility functions and the previous results show up again: though in fact indeterminacy may be allowed for some utility formulations, it would demand empirically implausible values for the fundamentals and so endogenous fluctuations are not likely to occur. Anyway it is clear that indeterminacy is related with the preferences specification. Another very interesting point pinned down by this work is that, when considered side by side with the rest of the literature that relates balanced budget rules and expectations-driven cycles, consumption taxation may be preferred to income taxation, from the point of view of stability, since income taxes affect labor supply and may induce indeterminacy, though Lloyd-Braga *et all.* (2008) already show that when heterogeneity in households is introduced, consumption taxation in deed has destabilizing effects.

Nourry *et all.* (October 2011) are also preparing a very complete analysis of this topic. They go even further generalizing the results of Giannitsarou. They claim, for instance, that, in the context of a standard Ramsey framework, "local indeterminacy is ruled out under constant government spendings, and requires extreme conditions with counter-cyclical government spendings" (Nourry *et all.*, October 2011, p. 6).

There is, however, a common assumption in all these works that actually doesn't fit reality. Actually, according to OECD (2011) just two of the members of this community of countries (Chile and Japan) don't have a scheme of taxation over consumption with multiple rates[1]. This multiplicity, in most of the countries, is related to some categorization of the goods according to their degree of necessity, mainly fruit of social and equity considerations.

In this context, our reasoning to fix a research topic was quite straightforward: if indeterminacy has to do with preferences specification and if consumption taxation has to do with the distinction between luxury and necessity goods (a preferences related issue), why not to study the implications of this distinction in endogenous cycles emergence?

Arriving here, several questions came to hand concerning preferences. In fact distinguishing necessity and luxury goods calls for changes in the way usually used to describe how consumers value the goods. As we know, classically this distinction relies on income sensitivity of demand, an *individual* related attribute. However, we believe there is some *social* component in the individual valorization of luxury goods, that results from social interaction of agents and make them to value consumption of luxury goods relatively to some reference level. And here is where the introduction of consumption externalities can be of help. Though we tend to assume *positive social networking effects* prevail on consumption of luxury goods, which is known in some literature as a *keeping up with the Joneses* behavior (see, for example, Galí, 1994), we also analyze the other case, where those effects are negative, which we call *keeping away from the Joneses*, though some literature refers also to this case with the previously mentioned expression (see, for instance, Wendner, 2009).

The relevance of this type of externalities has been a topic of interest in several ways. Galí (1994), for example, already studied the impact of *keeping up with Joneses* externalities on financial assets acquisition decisions, assuming those decisions to be a *social activity* by setting up preferences depending not only on individual consumption but on the average level of consumption in the economy. And Wendner (2009), to give another example, also shows that this type of externalities may influence optimal income taxation,

[1] Whether or not these taxes are of the type *Value Added Taxes* or of the type *Sales Taxes* is not very relevant, because. the principals of both fiscal schemes are identical, i.e. in both cases the goal is to tax final consumption and as such, consumers are supposed to bear entirely the tax burden. What changes is the collection mechanism, which doesn't affect our results.

since they affect steady state savings and growth rates.

In the context of this thesis, our aim is to study if this type of externalities has influence on local expectations-driven cycles emergence and so if there are stabilizing policies able to smooth those effects. In fact, those policies need to be such that they eliminate local indeterminacy, if the externality is a cause for it, as we shall see it is. Eliminating indeterminacy will, in effect, rule out the volatility of local endogenous cycles, that appear due volatility in agents' expectations (Grandmont *et all.*, 1998). Then, we will study fiscal policies characterized by two attributes of taxes (ciclicity and rates) and define conditions for them to be stabilizing instruments. Meanwhile, we will also see that, even in the absence of externalities, variable fiscal policies may cause indeterminacy, and so, endogenous fluctuations, under certain conditions.

The way this work is organized is the following: in Section 2 we describe the model and derive the equilibrium conditions, focusing specially on those who determine the perfect foresight equilibrium dynamics of the model. In Section 3, we state the conditions for a stationary solution and analyze the local stability properties of such a solution, if it exists, trying to figure out the impact of government policies in the dynamics of the model and to what extent can government have a stabilizing or destabilizing role through fiscal policy. Then, in Section 4, we see two scenarios for fiscal policy, where one of the taxes is constant over time or doesn't even exist, while the other moves with the cycle, and discuss their impact. Finally, in Section 5, we state some conclusion remarks, organizing our main results and pointing out to some future research topics. At the very end we have an Appendix section, where we collect all the proofs of the results we obtained.

2 The model

In this section we set up our model, which is a simple *OLG* (*Overlapping Generations*) model, where money works as an asset to transfer value in time. We assume there are two perfectly competitive sectors producing two types of goods and a government that raises taxes over consumption of both goods and use them to finance public spendings, keeping a balanced budget.

2.1 Firms

This economy has two perfectly competitive sectors: the *necessity* sector and the *luxury* sector. In each period and in each sector, there is a continuum of identical price-takers firms willing to maximize profits, when deciding the amount of labor they want to hire and the output they want to offer in the market at price p_t^c, for the necessity good, or p_t^x, for the luxury good.

Furthermore, we assume the production technology in both sectors to depend linearly on labor as a production factor.

Assumption 1. *The technology used in each production sector is described by:*

$$y_t^i = A^i l_t^i, \quad i = c,\, x \tag{1}$$

where A^c and A^x are the productivity parameters of the necessity and of the luxury sectors, l_t^c is labor employed in the necessity sector and l_t^x has the same meaning for the luxury sector and y_t^c the output produced in the necessity sector and y_t^x the output produced in the luxury sector.

As a result of maximizing profits in each competitive market, using the technologies in (1), we obtain that firms should hire labor units until:

$$\frac{w_t}{p_t^c} = A^c \tag{2}$$

$$\frac{w_t}{p_t^x} = A^x \tag{3}$$

Notice that these two results in equations (3) and (2) imply a constant ratio of contemporaneous prices between the two types of goods:

$$\frac{p_t^x}{p_t^c} = \frac{A^c}{A^x} \tag{4}$$

2.2 Households

We consider an *Overlapping Generations* model (OLG) with money. In this economy each household lives just for two periods, working while young and consuming while old. In each period, a new generation of n identical households is born, substituting the generation who dies.

Assumption 2. *Preferences of any household born at period* t *is described by the following life-time utility function:*

$$U\left(c_{t+1},\, x_{t+1},\, l_t\right) = \beta\left(c_{t+1} - \overline{c}\right)^{\alpha}\left(x_{t+1}\tilde{\chi}_{t+1}^{\rho}\right)^{1-\alpha} - \frac{l_t^z}{B} \tag{5}$$

In equation (5)c_{t+1} represents the quantity of *necessity good* consumed by the household when old, x_{t+1} is the quantity of *luxury good* consumed by the household when old and l_t is the total amount of labor offered in the market by the household when young. We use a Stone-Geary type of preferences (Geary 1953), where $\overline{c}$ represents a minimum amount of the necessity good, below which utility becomes negative in consumption for any positive quantity of any good. Moreover, $0 < \beta < 1$ is a discount factor, which intends to give relatively more value to leisure than to consumption since consumption is one period ahead from leisure. B and \alpha are parameters such that $B > 0$ and $0 < \alpha < 1$[2].

[2] B is a scaling parameter that will ensure the existence of a steady state as we shall see later in Section 3 and α is related to the share of the expenditure in consumption of the necessity good in total consumption

$\tilde{\chi}_{t+1}$ represents a point expectation in period t for the average amount of consumption of the luxury good in the economy in $t+1$ ($\chi_{t+1} \equiv X_{t+1}/n = \sum_{i=1}^{n} x_{t+1}^{i}/n = x_{t+1}$), which is taken as given by families when deciding about their private consumption levels, due to their atomistic size. This aims to take in account that consumption of luxury good has some *social effect*. It is easy to see that this effect crucially depends on the signal of parameter ρ. If $\rho > 0$ the representative household is considered to give more value to an extra unity of luxury goods when the average level of consumption of those goods in the economy is higher, as if there was a positive *social networking effect* in the consumption of those goods. This type of externality is known in the literature as *keeping up with the Joneses* (see, for example, Galí 1994), and, in simple terms, it's like agents in this economy value more to have an iPad when everybody around has one. However, if $\rho < 0$ the effect is exactly the opposite: the more the luxury good is consumed on average, the less each agent values its consumption. Again, in simple terms, it's like agents in this economy value more to have a Rembrandt painting when nobody around has one. Here we must say that literature is a little bit confusing, because some authors (see, for instance, Wendner 2009) also refer to this case as *keeping up with the Joneses*. But, in fact, assuming a strictly negative ρ means that agents' marginal utility is decreasing with the average level of luxury consumption. Thus, *keeping away from the Joneses* would probably be a better description for this particular sort of externality.

It seems both considerations are empirically plausible. Nevertheless, and even if we discuss both cases in our analysis, we tend to assume a strictly positive ρ. For this reason, a statement must be done on our definition of *luxury good*.

First of all, it is important to remember the standard definition of luxury goods in economics.

Definition 1. A good x is considered to be *luxury* if its demand has high sensitiveness to changes in consumer's income m, i.e., if its *income elasticity of marshallian demand*, defined as:

$$\eta \equiv \frac{\partial x}{\partial m}\frac{m}{x}$$

is higher than unity ($\eta > 1$), which means that the percentage change in the quantity consumed of x is proportionally higher than the percentage change observed in income. Similarly, a good x is considered to be a *necessity* good if $0 < \eta < 1$ and an *inferior* good if $\eta < 0$.

Applying the previous definition in the context of our model, with Stone-Geary preferences defined over two goods, the following Proposition is straightforward:

Proposition 1. *For any $\bar{c} > 0$, x is a luxury good and c is a necessity good, under*

expenditures.

Definition 1, since income elasticities of demand of x and c, are such that:

$$\eta^c_{t+1} \equiv \frac{l_{t+1}}{l_{t+1} + \frac{1-\alpha}{\alpha}\frac{\left(1+\tau^c_{t+1}\right)\bar{c}}{A^c}} < 1$$

$$\eta^x_{t+1} \equiv \frac{l_{t+1}}{l_{t+1} - \frac{\left(1+\tau^c_{t+1}\right)\bar{c}}{A^c}} > 1$$

Proof. See appendix D. □

So, *luxury*, before being an hypothesis about *social interaction* of agents, it is an *individual attribute* implicit in preferences, that characterizes *sensitiveness to income*. For this reason, when we call x a luxury good, this is not properly an assumption, but a result. Here, *luxury* is defined relatively, as opposed to *necessity*, and states for *the good that agents don't require a survival consumption in order to draw utility from*, which has the natural consequence of having an income elasticity of demand higher than one. And this fact is independent from the externality (it is not the externality what causes x to be luxury) and from the social effect we want to consider through it.

But even under this clarification, a couple of questions remain: what is, then, the social effect we want to model with $\rho > 0$? Is it observable? Though we actually don't have empirical evidence on it, what we want to describe with the introduction of a *keeping up with the Joneses* effect over the *non-necessity good* (possibly, the best tag) is that agents don't seek for *exclusivity* when they consume it, but for *living standards updating*. They don't suffer from *cupidity*, but from *envy*.

This idea has also some support in the occidental thought for almost three centuries. Kierkegaard (1978), a danish philosopher from the late eighteen century already stated how the life of societies is organized upon *envy*, as a consequence of rationalism:

> "(...) *envy* becomes the *negatively unifying principle* in a passionless and very reflective age [as the one emerged after the French Revolution]. This must not promptly be interpreted ethically, as an accusation; no, reflection's idea, if it may be called that, is envy (...). Envy in the process of establishing itself takes the form of *leveling*."

Of course here is not the place to develop a debate on to what extent Kierkegaard was or not right, but it illustrates how reasonable is our assumption of social *leveling* instead of *outstanding*.

Moreover, notice that when consumption is driven by a social status highlighting desire, consumers increase expenses on luxury goods by choosing more and more rare (and so, expensive) luxury goods. Rich people (or, better, consumers of luxury goods) don't run away from iPads to buy apples, when iPad gets widely used; if they really want to underline their social condition, they possibly run away from iPads to Rembrandt paintings (*another* and more expensive luxury good). In the context of our model, since there is a single luxury good, the way to increase expenditures in luxury goods is increasing the quantity consumed of it. This to say that even if we were to assume *cupidity* instead of *envy* in consumption of the luxury good, a positive ρ would work fine, in the context of our model[3].

Finally, the labor offered by households, as we saw before, will be used by firms of the necessity and of the luxury sectors. For that amount of labor offered, the household gets paid a nominal wage w_t, equal across sectors, since the labor market is assumed to be perfectly competitive and to allow for perfect mobility of workers. In each period t, the labor income can be saved through a stock of money (the unique asset in this economy to transfer value through time), which will be available to spend in consumption in period $t+1$ and, therefore, we denote by m_{t+1}. So, on top of a *budget constraint* there is also a sort of *liquidity constraint* that affects our household's decision.

Assumption 3. *When maximizing its utility, each household faces the following constraints:*

$$w_t l_t = m_{t+1} \tag{6}$$

$$m_{t+1} = \tilde{p}^c_{t+1}\left(1+\tilde{\tau}^c_{t+1}\right)c_{t+1} + \tilde{p}^x_{t+1}\left(1+\tilde{\tau}^x_{t+1}\right)x_{t+1} \tag{7}$$

where $\tilde{p}^c_{t+1}$ *and* $\tilde{p}^x_{t+1}$ *are point expectations in* t *for the prices of each good in* $t+1$*, and* $\tilde{\tau}^c_{t+1}$ *and* $\tilde{\tau}^x_{t+1}$ *are also point expectations in* t *for the value of tax rates in period* $t+1$*, which were not yet announced by government.*

This way, under Assumptions 2 and 3, a household born in t will maximize its life-time utility, described by equation (5), setting the labor supply in t and demand for both goods in $t+1$ according to the following first order conditions:

$$x_{t+1} = (1-\alpha)\frac{w_t l_t - \tilde{p}^c_{t+1}\left(1+\tilde{\tau}^c_{t+1}\right)\bar{c}}{\tilde{p}^x_{t+1}\left(1+\tilde{\tau}^x_{t+1}\right)} \tag{8}$$

$$c_{t+1} = \alpha\frac{w_t l_t}{\tilde{p}^c_{t+1}\left(1+\tilde{\tau}^c_{t+1}\right)} + (1-\alpha)\bar{c} \tag{9}$$

$$l_t = \left[B\alpha\beta\frac{w_t}{\tilde{p}^c_{t+1}\left(1+\tilde{\tau}^c_{t+1}\right)}\left(c_{t+1}-\bar{c}\right)^{\alpha-1}\left(x_{t+1}\tilde{\chi}^{\rho}_{t+1}\right)^{1-\alpha}\right]^{\frac{1}{z-1}} \tag{10}$$

Notice that equation (10), which represents the total labor supply of households in period t, depends on expectations for future prices, taxes and average level of luxury consumption. For future reference, we must also add two more details that emerge from the previous results. The first is that, taking as given the externality and the tax rates, the *private*

[3] Notice that considering several luxury goods would require much more heavy assumptions on the model. It will not only change the shape of preferences, but, perhaps, it would also require other assumptions on the way the luxury production sector works (for example, monopolistic competition).

indirect utility of consumption[4]:

$$v = \beta \left[\alpha \frac{\left(1+\tilde{\tau}^x_{t+1}\right)\tilde{p}^x_{t+1}}{\left(1+\tilde{\tau}^c_{t+1}\right)\tilde{p}^c_{t+1}} \right]^{\alpha} (1-\alpha)^{(1-\alpha)} \frac{w_t l_t - \left(1+\tilde{\tau}^c_{t+1}\right)\tilde{p}^c_{t+1}\bar{c}}{\tilde{p}^x_{t+1}\left(1+\tilde{\tau}^x_{t+1}\right)} \tag{11}$$

is linear in income. And the second is that the private elasticity of labor supply with respect to wage is given by $\frac{1}{z-1}$.

2.3 Government

In this economy, government follows a balanced budget rule, avoiding public debt and money issuing. This way, public spendings are given by revenues collected by issuing taxes on consumption, at a tax rate τ^c_t for the necessity good and at a tax rate τ^x_t for the luxury good. The tax rates are supposed to react to cycle according to the following Assumption.

Assumption 4. *Tax rates over consumption of each type of good are determined by the following policy rules:*

$$\tau^c_t \equiv \theta^c \left(\frac{l_t}{l}\right)^{\varphi^c} \tag{12}$$

$$\tau^x_t \equiv \theta^x \left(\frac{l_t}{l}\right)^{\varphi^x} \tag{13}$$

where $\theta^c > 0$ *and* $\theta^x > 0$ *are the steady state tax rates,* l_t/l *is a ratio that measures deviations of labor supply from its steady state value, here used as a proxy for measuring the cycle, and* φ^c *and* φ^x *are parameters that impose each tax rate to be pro-cyclical (counter-cyclical), if strictly positive (negative).*

The interpretation of parameters θ^i and φ^i, $i = x, c$, is similar to that consider in Lloyd-Braga *et all.* (2008). It is clear, this way, that if $\varphi^i = 0$ then the tax rate τ^i is constant.

We also assume that public spendings are lost, as if they were wasted resources in the economy: neither useful, nor productive. They are also not transferred to households, which could, in fact, be an option. But we preferred not to assume them this way, as if the State is paying back a huge stock of debt accumulated in past periods, like Portugal right now. A careful reader would also notice that we don't establish any assumption on what goods government spends fiscal revenue, assuming only that public spendings adjust endogenously to the revenue collected, since a balanced budget rule is followed. This is possible because, as we shall see later, the way government uses fiscal revenues has no impact in the dynamics of the model with respect to the aggregate level of employment.

[4]Since we have additive separable preferences, this equation is simple obtained isolating the consumption counter-part of equation (5) and plugging in it the marshallian demands for c and x in equations (2) and (3) respectively.

2.4 Equilibrium and Dynamics

In this subsection we obtain the intertemporal perfect foresight equilibrium dynamics, which is an equilibrium path for variables such that households don't incur in predicting errors. This is a particular case of rational expectations: households correctly anticipate in period t future values for the variables in $t+1$, such that, formally, $\tilde{p}^i_{t+1} = p^i_{t+1}$, $\tilde{\tau}^i_{t+1} = \tau^i_{t+1}$, for $i = c$, x, and $\tilde{\chi}_{t+1} = x_{t+1}$. Thus, as the future values of variables depend on present decisions and these, by their turn, depend on expected values for variables, these expected values can become confirmed in future, originating self-fulfilling prophecies phenomena.

As we saw before, in our model, the dynamics are particularly dependent on the labor supply, because current decisions on labor supply depend on expectations for future values of some variables. Using the market clearing condition for the money market, $M = m_t \Rightarrow w_t l_t = w_{t+1} l_{t+1}$, where $M > 0$ is an exogenous constant money supply, and after some simple substitutions, we can conclude that those dynamics can be summarized in a single equation as stated in the next Proposition.

Proposition 2. *Any* intertemporal perfect foresight equilibrium *for variable l obeys to the following motion equation:*

$$\beta\alpha^{\alpha}(1-\alpha)^{(1+\rho)(1-\alpha)}\left(\frac{A^x}{1+\tau^x_{t+1}}\right)^{(1+\rho)(1-\alpha)} \times \\ \times \left[l_{t+1} - \frac{1+\tau^c_{t+1}}{A^c}\bar{c}\right]^{\rho(1-\alpha)} l_{t+1}\left(\frac{A^c}{1+\tau^c_{t+1}}\right)^{\alpha} = z\frac{l^z_t}{B} \tag{14}$$

Proof. See appendix C. □

It is remarkable that the dynamics of the model can be written exclusively in terms of *total* labor supply[5]. It is also of notice that equation (14) defines a *one dimensional dynamic system* in labor l. This variable is non predetermined since its current value depends on expectations for the future.

3 Steady State and Local Dynamics

Once we already described the conditions which determine the the perfect foresight dynamic behavior of the model, we focus now our attention in a stationary solution, if it exists, since we are interested in analyzing the emergence of local expectations-driven cycles. These type of fluctuations are exclusively caused by volatility in agents' expectations and they occur in a neighborhood of the *stationary* solution for the equilibrium system. This will be the case as far as *local indeterminacy* characterizes that stationary solution of the deterministic counter-part of the model (See Grandmont *et all.,* 1998).

[5]Since relative prices are constant, the total output in each sector and the amount of labor affected to each sector will be entirely determined by the demand side, for which will be crucial to know the path for public consumption of both necessity and luxury goods.

3.1 Steady State

Since we are interested in the emergence of local expectations-driven cycles, we will analyze the local dynamics close to a stationary solution, i.e. an equilibrium path for the variables such that $l_{t+1} = l_t = l$, implying $\tau_t^c = \theta^c$, $\tau_t^x = \theta^x$. Hence, we need to establish conditions ensuring the existence of such a steady state l. Using equations (5), (9), (8) and (10) it is straightforward to obtain the following Proposition.

Proposition 3. *Any* $l > \frac{1+\theta^c}{A^c}\overline{c} \Leftrightarrow lw > p^c(1+\theta^c)\overline{c}$ *is a steady state of (14) if and only if* B *is a unique solution of:*

$$B = \frac{zl^{z-1}}{\beta\alpha^\alpha(1-\alpha)^{(1+\rho)(1-\alpha)}\left(\frac{A^x}{1+\theta^x}\right)^{(1+\rho)(1-\alpha)}\left(\frac{A^c}{1+\theta^c}\right)^\alpha\left[l - \frac{1+\theta^c}{A^c}\overline{c}\right]^{\rho(1-\alpha)}}$$

And this can always be the case, because B is a "free" scaling parameter, whose introduction is a standard procedure in related literature (see, for example, Cazzavillan *et all.* 1998 and Nourry *et all.* 2011).

In our local dynamic analysis we will assume that the steady state l is constant. Proposition 3 allows for keeping the steady state the same, once we change any of the exogenous parameters, as long as B adjusts. Note that, in any case B doesn't depend on parameters φ^c and φ^x, which means we can analyze changes in these parameters without affecting or adjusting the value of B, and also without affecting the underlying steady state.

3.2 Local Dynamics Analysis

Because we are just interested in studying the stability around some steady state, we can limit our study to the local dynamics of the system around it, using a Taylor linear approximation. If a steady state exists, which we already proved, we can analyze if equilibrium paths deviated from it in a small arbitrary neighborhood would converge or, on the contrary, become explosive.

Proposition 4. *Let* $\hat{l}_{t+1}$ *and* $\hat{l}_t$ *represent percentage deviations from the steady state in two consecutive periods and* η^x *be the income elasticity of demand for the luxury good as defined in Proposition 1. Then, the log-linearized equilibrium dynamics for* $\hat{l}$ *is given by:*

$$\hat{l}_{t+1} = \lambda\hat{l}_t, \tag{15}$$

$$\lambda \equiv \frac{z}{1+\rho(1-\alpha)\left[\eta^x - \varphi^x\frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} + \varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)\right]} \tag{16}$$

Proof. See appendix E. □

Notice that λ represents a single eigenvalue because our equilibrium dynamics are characterized by a single equation.

What this relationship tells us is that, in an arbitrary small neighborhood of the steady state, there can be equilibrium paths for labor supply starting with a slightly different value from the stationary solution (since labor supply is not a predetermined variable and depends on future expectations subjected to predicting errors) that would soon converge to it. This will happen as far as the system is locally indeterminate around that specific stationary solution, according to the following Definition.

Definition 2. The dynamic system in equation (14) will be *locally indeterminate* in an arbitrary small neighborhood of any solution l that verifies Proposition 3 for a given B, if and only if the absolute value of the eigenvalue of the system in (16), evaluated at the steady state, is small than unity, i.e., if and only if $|\lambda| < 1$.

Given this definition, analyzing the expression of λ as defined in (16) allows us to infer some immediate reasonings.

First, since the fiscal policy parameters only influence λ through $\varphi^i \frac{\theta i}{1+\theta^i}$, $i = x, c$, we see that if government establishes constant tax rates, setting $\varphi^i = 0$, it will not be able to influence the local dynamics of the model. The idea that it is the variability of the tax rate that is relevant for local dynamic properties was already emphasized in Lloyd-Braga *et all.* (2008).

Second, when government is absent, with $\theta^c = \theta^x = 0$, or when a fiscal policy with constant tax rates is followed, with $\varphi^c = \varphi^x = 0$, whether there is local indeterminacy or not crucially depends on the existence of the externality. Indeed, in such a case, in the absence of externalities ($\rho = 0$) we obtain that $\lambda = z > 1$, and so any deviation from the steady state value would have an underlying linear explosive path for labor supply, i.e., there is no indeterminacy and no bounded expectations-driven fluctuations close to the steady state would exist. In the absence of externalities and government intervention, the log-linearized system is equivalent to the case where there is only one aggregate consumption good Y, with $U(Y_{t+1}, l_t) = \beta Y_{t+1} - \frac{l_t^z}{B}$, Y being produced out of labor according to $Y_{t+1} = l_{t+1}$. This equivalence is due to the linearity of consumption's utility in income in both situations (recall equation (11) in subsection 2.2). In case of a unique good, indeterminacy requires a negatively sloped labor supply, so indeterminacy cannot occur with $1/z - 1 > 0 \Leftrightarrow z > 1$. Due to the equivalence referred, the same happens in our case in absence of externalities and government intervention. The introduction of consumption externalities breaks the linear relation between consumption utility and income and, as shown below in the next subsection, it may allow the occurrence of indeterminacy.

Third, the expression in (16) also tells us that, when $\varphi^i \frac{\theta i}{1+\theta^i} = 0$, for $i = x, c$ (with $\theta^c = \theta^x = 0$ or $\varphi^c = \varphi^x = 0$) , if an externality of the type $\rho > 0$, that we tend to consider is in place, then the distinction between necessity and luxury goods may become relevant for the emergence of endogenous cycles: the income elasticity of demand of the good over whose the externality relies on, empowers the destabilizing effect the externality may have. This leads us to another important reasoning: for the same (perhaps low) degree of influence of the externality, social networking effects ($\rho > 0$) are more likely to be destabilizing the higher the income elasticity of demand of the good which the externality affects. Anyway, it is also remarkable that the indeterminacy that may arise because of the externality can be ruled out by government, through a cycle-indexed fiscal policy, whose effectiveness we analyze in more detail in next section.

Fourth, it comes also from the expression above that a variable (cycle-indexed) fiscal policy is, by itself and in the absence of the externality, a source of indeterminacy. Considering separately the case of a variable tax rate on luxury good from a variable tax rate on the necessity good, we have the following two Propositions.

Proposition 5. *In the absence of externality ($\rho = 0$), but in the presence of a fiscal policy involving only a variable tax rate on luxury goods as in (13) of Assumption 4, i.e., considering $\theta^c = 0$ or $\varphi^c = 0$, local indeterminacy will emerge if and only if $\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{1+z}{1-\alpha} > 0$, i.e., a sufficiently* strong pro-cyclical tax rate *on luxury goods, or if $\varphi^x \frac{\theta^x}{1+\theta^x} < \frac{1-z}{1-\alpha} < 0$, i.e., a sufficiently* strong counter-cyclical tax rate.

Proof. See appendix G. □

Proposition 6. *In the absence of externality ($\rho = 0$), but in the presence of a fiscal policy involving only a variable tax rate on necessity goods as in (12) of Assumption 4, i.e., considering $\theta^x = 0$ or $\varphi^x = 0$, local indeterminacy will emerge if and only if $\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{1+z}{\alpha} > 0$, i.e., a sufficiently* strong pro-cyclical tax rate *on luxury goods, or if $\varphi^c \frac{\theta^c}{1+\theta^c} < \frac{1-z}{\alpha} < 0$, i.e., a sufficiently* strong counter-cyclical tax rate.

Proof. See appendix G. □

We can conclude from the previous Propositions that when one of the taxes holds invariant with cycle, indeterminacy will emerge if the other is *sufficiently pro-cyclical* or *sufficiently counter-cyclical.*

Here additive separable preferences between consumption and labor (or leisure) are considered. Giannitsarou (2007), using a one sector Ramsey model with capital and no externalities, and considering additive separable preferences in consumption and leisure, found that, if government keeps the budget balanced and uses proportional consumption taxation to finance a constant flow of government spendings, indeterminacy cannot occur. This result is also referred in Nourry *et all.* (October 2011) to be obtained at least for additively separable preferences. In contrast to their results, we have seen that, even with additively separable preferences as considered in (5) and no externalities, within an Overlapping Generations model with money as the unique asset, indeterminacy is possible, provided consumption taxes are sufficiently pro-cyclical or counter-cyclical. This might be due to the fact that, under our policy rule, government spending is not constant.

3.3 Indeterminacy in the Absence of Government

Our main aim is to characterize the ciclicity of the tax rates that are able to eliminate indeterminacy caused by consumption externalities. Therefore we establish conditions on the degree of externalities ρ under which indeterminacy prevails in the absence of government intervention, or if tax rates are constant. In the Appendix we prove the following Proposition.

Proposition 7. *If government policy is absent, i.e.,* $\theta^c = \theta^x = 0$*, or characterized by a constant tax rate over time, i.e.,* $\varphi^c = \varphi^x = 0$*, the eigenvalue in equation (16) will take the form:*

$$\lambda = \frac{z}{1 + \rho\,(1-\alpha)\,\eta^x}$$

In such a case there is indeterminacy if and only if, either:

$$\rho > \frac{z-1}{\eta^x\,(1-\alpha)} > 0, \quad \textit{so that}\, \lambda \in (0,1)$$

or:

$$\rho < -\frac{z+1}{\eta^x\,(1-\alpha)} < 0, \quad \textit{so that}\, \lambda \in (-1,0)$$

Proof. See appendix F. □

So, this Proposition tells us is that, in the absence of variable tax rates, a sufficiently strong influence of the externality, i.e., a $|\rho|$ high enough, may indeed cause indeterminacy.

4 Stabilizing Fiscal Policies Analysis

In this section we analyze two types of stabilizing fiscal policies, considering separately the case of a variable tax rate on luxury goods and then the case of a tax rate on the necessity good. The aim of our study will be to define conditions on policy factors $\varphi^i \frac{\theta^i}{1+\theta^i}$, for $i = c\,x$, such that the indeterminacy, that would emerge in the absence of government, is eliminated.

4.1 Variable Tax Rate in the Luxury Good

We will start our study about government policies by the case where it sets a constant tax rate on necessity goods, i.e., $\varphi^c = 0$, but a variable tax rate on luxury goods according to (13), with $\varphi^x \neq 0$. Using (16) it is easy to see that the eigenvalue becomes:

$$\lambda = \lambda^x \equiv \frac{z}{1 + \rho\,(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right]}$$

Following the mission intended for the present section, the next Proposition states conditions on policy parameter $\varphi^x \frac{\theta^x}{1+\theta^x}$ such that indeterminacy is ruled out, with $|\lambda^x| > 1$.

Proposition 8. *Under variable tax rates on luxury goods, and considering* $\rho > -1$*, indeterminacy is ruled out if and only if :*

$$\frac{\rho\,(1-\alpha)\,\eta^x + 1 - z}{(1-\alpha)\,(1+\rho)} < \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho\,(1-\alpha)\,\eta^x + 1 + z}{(1-\alpha)\,(1+\rho)}, \quad \frac{\theta^x}{1+\theta^x}\varphi^x \neq \frac{(1-\alpha)\,\rho\eta^x + 1}{(1-\alpha)\,(1+\rho)}$$

Proof. See appendix H.1. □

Using Proposition 7 we see that when indeterminacy emerges, in the absence of government intervention, with a positive externality degree, then $\rho(1-\alpha)\eta^x + 1 - z > 0$. Therefore using also Proposition 8 we have the following result:

Corollary 1. *If $\rho > 0$ and if indeterminacy prevails in the absence of government, i.e. values of $\rho > \frac{z-1}{\eta^x(1-\alpha)} > 0$, then only a* pro-cyclical tax rate, *with $\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{\rho(1-\alpha)\eta^x+1-z}{(1-\alpha)(1+\rho)} > 0$, is able to eliminate indeterminacy.*

Thus, a pro-cyclical tax rate is required for stabilizing purposes when externalities of the type *keeping up with the Joneses* are sufficiently strong. Notice that the boundary defined this way allows government to achieve the same fiscal result combining a low steady state level for the tax rate, but highly pro-cyclical or, otherwise, a high steady state tax rate over luxury goods, but less reactive to cycle, in order to achieve the same stabilizing goal.

However, if a *keeping away from the Joneses* behavior was to be assumed, the conclusion is exactly the opposite, since indeterminacy emerges in the absence of government intervention when $\rho(1-\alpha)\eta^x + 1 + z < 0$.

Corollary 2. *If $\rho < 0$ and if indeterminacy prevails in the absence of government, i.e. for values of ρ such that $-1 < \rho < -\frac{z+1}{\eta^x(1-\alpha)} < 0$, indeterminacy cannot be ruled out unless government sets a* counter-cyclical tax rate *such that $\varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho(1-\alpha)\eta^x+1+z}{(1-\alpha)(1+\rho)} < 0$.*

4.2 Variable Tax Rate in the Necessity Good

This case is a little bit more complex. Now we pay attention to a fiscal policy characterized by having a constant tax rate on luxury goods, which means $\varphi^x = 0$, and a cycle-indexed tax rate in necessity goods, such that $\varphi^c \neq 0$ in (12). This way, the eigenvalue in (16) around the steady state becomes:

$$\lambda = \lambda^c \equiv \frac{z}{1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x_{ss} - \frac{\alpha}{\rho(1-\alpha)}\right)\right]}$$

Again we need to find the boundaries for government policy factor $\varphi^c \frac{\theta^c}{1+\theta^c}$ between which it will be a useful instrument for eliminating the indeterminacy that may arise in the absence of government intervention. And, again, those boundaries will be different according to the signal of ρ. So, the following Proposition states conditions on $\varphi^c \frac{\theta^c}{1+\theta^c}$ such that $|\lambda^c| > 1$ and indeterminacy is ruled out.

Proposition 9. *Under variable tax rates on necessity goods, and considering $\rho > -\frac{\alpha}{(\eta^x-1)(1-\alpha)}$, indeterminacy is ruled out if and only if :*

$$\frac{\rho(1-\alpha)\eta^x + 1 - z}{\rho(1-\alpha)(\eta^x - 1) + \alpha} < \varphi^c \frac{\theta^c}{1+\theta^c} < \frac{\rho(1-\alpha)\eta^x + 1 + z}{\rho(1-\alpha)(\eta^x - 1) + \alpha}, \quad \varphi^c \frac{\theta^c}{1+\theta^c} \neq \frac{\rho(1-\alpha)\eta^x + 1}{\rho(1-\alpha)(\eta^x - 1) + \alpha}$$

Proof. See appendix H.2. □

Using Proposition 7 we see that when indeterminacy emerges, in the absence of government intervention, with a positive externality degree, then $\rho(1-\alpha)\eta^x + 1 - z > 0$. Therefore, from Proposition 9 above, we can derive the following Corollary:

Corollary 3. *If $\rho > 0$ and if indeterminacy prevails in the absence of government, i.e. for values of $\rho > \frac{z-1}{\eta^x(1-\alpha)} > 0$, then only* a pro-cyclical tax rate *with $\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{\rho(1-\alpha)\eta^x+1-z}{(1-\alpha)(1+\rho)} > 0$ is able to eliminate indeterminacy.*

Hence, we arrived to a similar conclusion to the one we obtained for luxury goods in Corollary 1: a pro-cyclical tax rate is required for stabilizing purposes when externalities of the type *keeping up with the Joneses* are sufficiently strong. It remains also true as before that the same stabilization result can be achieved by government combining a low steady state level for the tax rate, but highly pro-cyclical, or, reversely, a high steady state tax rate, but less reactive to cycle.

Also as previously, if a *keeping away from the Joneses* behavior is in place, the conclusion on the ciclicity of the fiscal policy is exactly the opposite, since indeterminacy emerges in the absence of government intervention when $\rho(1-\alpha)\eta^x_{ss} + 1 + z < 0$. In fact, from Propositions 9 and 7 the following result is clear.

Corollary 4. *If $\rho < 0$ and if indeterminacy prevails in the absence of government, i.e. for values of ρ such that $-\frac{\alpha}{(\eta^x-1)(1-\alpha)} < \rho < -\frac{z+1}{\eta^x(1-\alpha)} < 0$, indeterminacy cannot be ruled out unless government sets a counter-cyclical tax rate such that $\varphi^c \frac{\theta^c}{1+\theta^c} < \frac{\rho(1-\alpha)\eta^x+1+z}{(1-\alpha)(1+\rho)} < 0$.*

From these Propositions we can conclude that, if indeterminacy is caused by luxury consumption externalities ($\rho \neq 0$), then a cycle-indexed tax rate over one of the goods is suitable to eliminate indeterminacy, no matter whether this ciclicity exists on the tax rate over luxury goods or over necessity goods: given a positive value of $\rho > \frac{z-1}{\eta^x(1-\alpha)} > 0$, the tax rate should be *pro-cyclical*, whereas given a negative value for ρ, such that $\max\left\{-1, -\frac{\alpha}{(\eta^x-1)(1-\alpha)}\right\} < \rho < -\frac{z+1}{\eta^x(1-\alpha)} < 0$, the tax rate should be *counter-cyclical.*

Thus, if instability do arise in the absence of government, which will be the case if a sufficiently strong degree of externalities exists, then it is possible to correct those fluctuations with cycle-indexed fiscal policies. If the externality is positive, in a boom people will over react consuming more luxury goods, in order to *keep up with the Joneses.* In such a case, a pro-cyclical tax rate will cause an increase in prices, making people consume less and, this way, the effect of the externality becomes smoothed. On the contrary, if the externality is negative, in a boom people will react avoiding consumption of luxury goods, in order to *run away from the Joneses.* Hence, a counter-cyclical tax rate may correct expectations on prices in a way that attenuates the effect of the externality.

5 Concluding Remarks

This thesis discusses the impact of consumption externalities on luxury goods in the emergence of local expectations-driven cycles, when government is committed with a balanced budget rule and public spendings are financed exclusively through consumption taxation, with different tax rates for each type of good. We confirmed a well known result in the literature: externalities are, indeed, a source of indeterminacy. In our case, this happens because the marginal utility, and, thus, demand, of luxury goods depends on the contemporaneous average level of consumption of those goods in the economy. However, since labor supply is one period lagged in the OLG model, and because households want to level their luxury consumption by the one of their peers, the labor supply decision will be influenced by expectations on the average level of consumption of those goods in the future. And these expectations, which are volatile by nature, even if they are rational as we assume they are, may lead to bounded fluctuations on labor supply around a stationary value and induce aggregate instability. Moreover, when a positive externality is considered, and so *social networking effects* are assumed to affect consumption of luxury goods, this effect is amplified by income-elasticity of demand for those goods, i.e., indeterminacy is more likely to occur, for high levels of sensitiveness of demand to income. This is because labor is the unique income resource of households and so, the more reactive the demand of luxury goods is to income, the less is the change needed in labor supply to *keep up with the Joneses*, according to expectations of households for the average level of consumption of those goods.

We also verified that neither the distinction between luxury and necessity goods itself nor their different taxation is relevant for generating indeterminacy, in the absence of externalities and government intervention. In fact, since the Stone-Geary preferences we use are quasi-homothetic, their related indirect utility function is linear in income, sharing this feature with linear homogenous preferences. Thereby, it is equivalent to one sector Overlapping Generations model with money and, as well known in the literature, indeterminacy can not occur, unless labor supply has a negative slope, which is not our departing point, since we assume elasticity of labor supply to be positive with respect to wage. However, the introduction of consumption externalities removes aggregate utility linearity in income and indeterminacy may indeed show up, even in the absence of externalities and government intervention.

Furthermore, though our work is not directly comparable with the work developed by Giannitsarou (2007) and Nourry *et all.* (October 2011) since they used a standard Ramsey framework, we arrived to a different conclusion from them, with respect to additive separable preferences, as the ones we used. Actually, in the context of our simple two-sectors OLG model with money, consumption taxation, right without the presence of consumption externalities, may have destabilizing effects, as far as sufficiently pro-cyclical or counter-cyclical tax rates are adopted. This may happen because, on the contrary of these authors, that consider constant public spendings, we assume public spendings to adjust to the tax revenues collected. Nevertheless, it would be interesting to analyze this topic in more detail in a future work.

Another interesting conclusion arising from our work is that, in line with what we expected, whatever the rate government chooses to move with cycle, i.e. be it the one over necessity goods or the other over luxury goods, it must set them pro-cyclical, if a positive externality is assumed (*keeping up with the Joneses*), and counter-cyclical, if a negative externality is considered (*keeping away from the Joneses*). However, this conclusion seems not to be that straightforward when both taxes are variable, though we didn't analyze that case.

Finally, we think there are many other topics for future research departing from here, that we didn't have time to cover. Two that we already have thought about was the analysis of other types of fiscal policies. For instance, it would be of value to see a scenario where government uses one of the tax rates to keep a balance budget with exogenous public spendings, and the other one to stabilize expectations-driven fluctuations. On top of that, a welfare analysis, considering the fiscal policies presented in this work, would also be interesting to develop. Lastly, it would also be interesting to consider different technologies and market structures from the ones we assumed to derive our results.

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A Solution for Household Utility Maximization Problem

The original problem:

$$
\begin{aligned}
\max_{c_{t+1},\, x_{t+1},\, l_t} \quad & \beta \left(c_{t+1} - \overline{c}\right)^{\alpha} \left(x_{t+1}\chi^{\rho}\right)^{1-\alpha} - \frac{l_t^z}{B} \\
\text{s.t.} \quad & w_t l_t = m_{t+1} \\
& m_{t+1} = p_{t+1}^c \left(1 + \tau_{t+1}^c\right) c_{t+1} + p_{t+1}^x \left(1 + \tau_{t+1}^x\right) x_{t+1} \\
& c_0 \,\text{and}\, x_0 \,\text{given}
\end{aligned}
$$

Using the restrictions to substitute c_{t+1} is equivalent to:

$$
\max_{x_{t+1},\, l_t} \quad \beta \left(\frac{w_t}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} l_t - \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} x_{t+1} - \overline{c} \right)^{\alpha} \left(x_{t+1}\chi^{\rho}\right)^{1-\alpha} - \frac{l_t^z}{B}
$$

Taking First Order Conditions:

$$
\begin{aligned}
\frac{\partial}{\partial x_{t+1}} = 0 \Leftrightarrow & -\alpha \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} \beta \left(\frac{w_t}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} l_t - \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} x_{t+1} - \overline{c} \right)^{\alpha-1} \\
& \times \left(x_{t+1}\chi^{\rho}\right)^{1-\alpha} + (1-\alpha)\chi^{\rho} \\
& \times \beta \left(\frac{w_t}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} l_t - \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} x_{t+1} - \overline{c} \right)^{\alpha} \\
& \times \left(x_{t+1}\chi^{\rho}\right)^{-\alpha} = 0 \\
\Leftrightarrow & \alpha \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} \beta \left(\frac{w_t}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} l_t - \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} x_{t+1} - \overline{c} \right)^{\alpha-1} \\
& \times \left(x_{t+1}\chi^{\rho}\right)^{1-\alpha} = (1-\alpha)\chi^{\rho} \\
& \times \beta \left(\frac{w_t}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} l_t - \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} x_{t+1} - \overline{c} \right)^{\alpha} \\
& \times \left(x_{t+1}\chi^{\rho}\right)^{-\alpha} \\
\Leftrightarrow & \frac{\alpha}{1-\alpha} \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} x_{t+1} = \frac{w_t}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} l_t - \frac{p_{t+1}^x \left(1 + \tau_{t+1}^x\right)}{p_{t+1}^c \left(1 + \tau_{t+1}^c\right)} x_{t+1} - \overline{c}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial x_{t+1}} \Leftrightarrow & \frac{\alpha}{1-\alpha} x_{t+1} = \frac{w_t}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} l_t - x_{t+1} - \frac{p^c_{t+1}\left(1+\tau^c_{t+1}\right)}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \overline{c} \\
\Leftrightarrow & \frac{\alpha+1-\alpha}{1-\alpha} x_{t+1} = \frac{w_t l_t}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} - \frac{p^c_{t+1}\left(1+\tau^c_{t+1}\right)\overline{c}}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \\
\Leftrightarrow & x_{t+1} = \frac{1-\alpha}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \left[w_t l_t - p^c_{t+1}\left(1+\tau^c_{t+1}\right)\overline{c}\right]
\end{aligned}
$$

Recalling c_{t+1}:

$$
\begin{aligned}
c_{t+1} =& \frac{w_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} l_t - \frac{p^x_{t+1}\left(1+\tau^x_{t+1}\right)}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} x_{t+1} \Leftrightarrow \\
c_{t+1} =& \frac{w_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} l_t - \frac{p^x_{t+1}\left(1+\tau^x_{t+1}\right)}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} \frac{1-\alpha}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \left[w_t l_t - p^c_{t+1}\left(1+\tau^c_{t+1}\right)\overline{c}\right] \Leftrightarrow \\
c_{t+1} =& \frac{\alpha}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} w_t l_t + (1-\alpha)\overline{c}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial l_t} = 0 \Leftrightarrow & \alpha \frac{w_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} \beta \left(\frac{w_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} l_t - \frac{p^x_{t+1}\left(1+\tau^x_{t+1}\right)}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} x_{t+1} - \overline{c} \right)^{\alpha-1} \\
& \times \left(x_{t+1}\chi^{\rho}\right)^{1-\alpha} - z\frac{l_t^{z-1}}{B} = 0 \\
\Leftrightarrow & \alpha \frac{w_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} \beta \left(\frac{w_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} l_t - \right. \\
& \left. - \frac{p^x_{t+1}\left(1+\tau^x_{t+1}\right)}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} \frac{1-\alpha}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \left[w_t l_t - p^c_{t+1}\left(1+\tau^c_{t+1}\right)\overline{c}\right] - \overline{c} \right)^{\alpha-1} \\
& \times \left(\frac{1-\alpha}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \left[w_t l_t - p^c_{t+1}\left(1+\tau^c_{t+1}\right)\overline{c}\right] \chi^{\rho} \right)^{1-\alpha} = z\frac{l_t^{z-1}}{B} \\
\Leftrightarrow & \beta\alpha\alpha^{\alpha-1}\chi^{\rho(1-\alpha)}(1-\alpha)^{1-\alpha} \frac{w_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} \left(\frac{w_t l_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} - \overline{c} \right)^{\alpha-1} \\
& \times \left(\frac{w_t l_t}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} - \frac{p^c_{t+1}\left(1+\tau^c_{t+1}\right)}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \overline{c} \right)^{1-\alpha} = z\frac{l_t^{z-1}}{B} \\
\Leftrightarrow & \beta\alpha^{\alpha}\chi^{\rho(1-\alpha)}(1-\alpha)^{1-\alpha} \frac{w_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} \left(\frac{w_t l_t}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} - \overline{c} \right)^{\alpha-1} \\
& \times \left(\frac{w_t l_t}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} - \frac{p^c_{t+1}\left(1+\tau^c_{t+1}\right)}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \overline{c} \right)^{1-\alpha} = z\frac{l_t^{z-1}}{B}
\end{aligned}
$$

B Solution for Firms Problem

$$\max_{l_t^i} \quad \Pi_t^i = p_t^i A^i l_t^i - w_t l_t^i$$

Taking FOC:

$$\begin{aligned}\frac{\partial \Pi_t^i}{\partial l_t^i} = 0 \Leftrightarrow & p_t^i A^i - w_t = 0 \\ \Leftrightarrow & \frac{w_t}{p_t^i} = A^i\end{aligned}$$

C Dynamics Simplification

Assuming a stationary exogenous money supply denoted by M, such that $M > 0$, the market clearing condition for the money market requires:

$$\begin{aligned}M =& m_t = m_{t+1} \Rightarrow \\ w_t l_t =& w_{t+1} l_{t+1}\end{aligned}$$

Combining equations (6), (2) and (3) we know that:

$$\begin{aligned}m_t =& m_{t+1} \Leftrightarrow \\ w_t l_t =& w_{t+1} l_{t+1} \Leftrightarrow \\ w_t l_t =& \frac{w_{t+1}}{p_{t+1}^c} p_{t+1}^c l_{t+1} \Leftrightarrow \\ \frac{w_t}{p_{t+1}^c} l_t =& A^c l_{t+1} \\ \frac{w_t}{p_{t+1}^x} l_t =& A^x l_{t+1}\end{aligned}$$

This way, we can re-write equation (10) as:

$$\begin{aligned}\beta \alpha^\alpha \chi^{\rho(1-\alpha)} (1-\alpha)^{1-\alpha} \frac{w_t}{p_{t+1}^c \left(1+\tau_{t+1}^c\right)} \left(\frac{w_t l_t}{p_{t+1}^c \left(1+\tau_{t+1}^c\right)} - \bar{c}\right)^{\alpha-1} \times \\ \times \left(\frac{w_t l_t}{p_{t+1}^x \left(1+\tau_{t+1}^x\right)} - \frac{p_{t+1}^c \left(1+\tau_{t+1}^c\right)}{p_{t+1}^x \left(1+\tau_{t+1}^x\right)} \bar{c}\right)^{1-\alpha} &= z \frac{l_t^{z-1}}{B} \Leftrightarrow \\ \beta \alpha^\alpha \chi^{\rho(1-\alpha)} (1-\alpha)^{1-\alpha} \frac{A^c l_{t+1}}{l_t \left(1+\tau_{t+1}^c\right)} \left(\frac{A^c l_{t+1}}{1+\tau_{t+1}^c} - \bar{c}\right)^{\alpha-1} \times \\ \times \left(\frac{A^x l_{t+1}}{1+\tau_{t+1}^x} - \frac{A^x \left(1+\tau_{t+1}^c\right)}{A^c \left(1+\tau_{t+1}^x\right)} \bar{c}\right)^{1-\alpha} &= z \frac{l_t^{z-1}}{B} \Leftrightarrow\end{aligned}$$

$$\beta\alpha^{\alpha}\chi^{\rho(1-\alpha)}\left(1-\alpha\right)^{1-\alpha}\frac{A^{c}l_{t+1}}{\left(1+\tau^{c}_{t+1}\right)}\left(\frac{A^{c}}{1+\tau^{c}_{t+1}}\right)^{\alpha-1}\left(l_{t+1}-\frac{1+\tau^{c}_{t+1}}{A^{c}}\overline{c}\right)^{\alpha-1}\times$$

$$\times\left(\frac{A^{x}}{1+\tau^{x}_{t+1}}\right)^{1-\alpha}\left(l_{t+1}-\frac{1+\tau^{c}_{t+1}}{A^{c}}\overline{c}\right)^{1-\alpha}=z\frac{l_t^{z-1}l_t}{B}\Leftrightarrow$$

$$\beta\alpha^{\alpha}\chi^{\rho(1-\alpha)}\left(1-\alpha\right)^{1-\alpha}l_{t+1}\left(\frac{A^{c}}{1+\tau^{c}_{t+1}}\right)^{\alpha}\left(\frac{A^{x}}{1+\tau^{x}_{t+1}}\right)^{1-\alpha}=z\frac{l_t^{z}}{B}$$

Since in the general equilibrium $\chi = x_{t+1}$, using equation (8) we get:

$$x_{t+1}=\frac{1-\alpha}{p^{x}_{t+1}\left(1+\tau^{x}_{t+1}\right)}\left[w_t l_t-p^{c}_{t+1}\left(1+\tau^{c}_{t+1}\right)\overline{c}\right]\Leftrightarrow$$

$$x_{t+1}=\frac{1-\alpha}{1+\tau^{x}_{t+1}}\left[\frac{w_t l_t}{p^{x}_{t+1}}-\frac{p^{c}_{t+1}}{p^{x}_{t+1}}\left(1+\tau^{c}_{t+1}\right)\overline{c}\right]\Leftrightarrow$$

$$x_{t+1}=\frac{1-\alpha}{1+\tau^{x}_{t+1}}\left[A^{x}l_{t+1}-\frac{A^{x}}{A^{c}}\left(1+\tau^{c}_{t+1}\right)\overline{c}\right]\Leftrightarrow$$

$$x_{t+1}=\frac{\left(1-\alpha\right)A^{x}}{1+\tau^{x}_{t+1}}\left[l_{t+1}-\frac{1+\tau^{c}_{t+1}}{A^{c}}\overline{c}\right]$$

$$\beta\alpha^{\alpha}\chi^{\rho(1-\alpha)}\left(1-\alpha\right)^{1-\alpha}l_{t+1}\left(\frac{A^{c}}{1+\tau^{c}_{t+1}}\right)^{\alpha}\left(\frac{A^{x}}{1+\tau^{x}_{t+1}}\right)^{1-\alpha}=z\frac{l_t^{z}}{B}\Leftrightarrow$$

$$\beta\alpha^{\alpha}\left(1-\alpha\right)^{\rho(1-\alpha)}\left(\frac{A^{x}}{1+\tau^{x}_{t+1}}\right)^{\rho(1-\alpha)}\left[l_{t+1}-\frac{1+\tau^{c}_{t+1}}{A^{c}}\overline{c}\right]^{\rho(1-\alpha)}\times$$

$$\times\left(1-\alpha\right)^{1-\alpha}l_{t+1}\left(\frac{A^{c}}{1+\tau^{c}_{t+1}}\right)^{\alpha}\left(\frac{A^{x}}{1+\tau^{x}_{t+1}}\right)^{1-\alpha}=z\frac{l_t^{z}}{B}\Leftrightarrow$$

$$\beta\alpha^{\alpha}\left(1-\alpha\right)^{(1+\rho)(1-\alpha)}\left(\frac{A^{x}}{1+\tau^{x}_{t+1}}\right)^{(1+\rho)(1-\alpha)}\left[l_{t+1}-\frac{1+\tau^{c}_{t+1}}{A^{c}}\overline{c}\right]^{\rho(1-\alpha)}\times$$

$$\times l_{t+1}\left(\frac{A^{c}}{1+\tau^{c}_{t+1}}\right)^{\alpha}=z\frac{l_t^{z}}{B}$$

D Computing Income-Elasticities of Demand

As we shall see later, this step is very convenient. Picking up demands:

$$x_{t+1}=\frac{1-\alpha}{p^{x}_{t+1}\left(1+\tau^{x}_{t+1}\right)}\left[w_t l_t-p^{c}_{t+1}\left(1+\tau^{c}_{t+1}\right)\overline{c}\right]\Leftrightarrow$$

$$x_{t+1}=\frac{1-\alpha}{p^{x}_{t+1}\left(1+\tau^{x}_{t+1}\right)}\left[m_{t+1}-p^{c}_{t+1}\left(1+\tau^{c}_{t+1}\right)\overline{c}\right]$$

$$c_{t+1}=\frac{\alpha}{p^{c}_{t+1}\left(1+\tau^{c}_{t+1}\right)}m_{t+1}+\left(1-\alpha\right)\overline{c}$$

$$
\begin{aligned}
\eta^c_{t+1} \equiv & \frac{\partial c_{t+1}}{\partial m_{t+1}} \frac{m_{t+1}}{c_{t+1}} = \\
& \frac{\alpha}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} \frac{m_{t+1}}{\frac{\alpha}{p^c_{t+1}\left(1+\tau^c_{t+1}\right)} m_{t+1} + (1-\alpha)\,\overline{c}} = \\
& \frac{m_{t+1}}{m_{t+1} + \frac{1-\alpha}{\alpha} p^c_{t+1}\left(1+\tau^c_{t+1}\right)\overline{c}} = \\
& \frac{m_{t+1}}{m_{t+1} + \frac{1-\alpha}{\alpha} \frac{m_{t+1}}{A^c l_{t+1}}\left(1+\tau^c_{t+1}\right)\overline{c}} = \\
& \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{\left(1+\tau^c_{t+1}\right)\overline{c}}{A^c l_{t+1}}} = \\
& \frac{l_{t+1}}{l_{t+1} + \frac{1-\alpha}{\alpha} \frac{\left(1+\tau^c_{t+1}\right)\overline{c}}{A^c}}
\end{aligned}
$$

$$
\begin{aligned}
\eta^x_{t+1} \equiv & \frac{\partial x_{t+1}}{\partial m_{t+1}} \frac{m_{t+1}}{x_{t+1}} = \\
& \frac{1-\alpha}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \frac{m_{t+1}}{\frac{1-\alpha}{p^x_{t+1}\left(1+\tau^x_{t+1}\right)} \left[m_{t+1} - p^c_{t+1}\left(1+\tau^c_{t+1}\right)\overline{c}\right]} = \\
& \frac{m_{t+1}}{m_{t+1} - p^c_{t+1}\left(1+\tau^c_{t+1}\right)\overline{c}} = \\
& \frac{1}{1 - \frac{p^c_{t+1}}{m_{t+1}}\left(1+\tau^c_{t+1}\right)\overline{c}} = \\
& \frac{1}{1 - \frac{1}{A^c l_{t+1}}\left(1+\tau^c_{t+1}\right)\overline{c}} = \\
& \frac{l_{t+1}}{l_{t+1} - \frac{\left(1+\tau^c_{t+1}\right)\overline{c}}{A^c}}
\end{aligned}
$$

E Log-linearized Model

Taking logs of the dynamic equation (14):

$$
\log\left[\frac{B}{z}\beta\alpha^\alpha (1-\alpha)^{(1+\rho)(1-\alpha)} \left(\frac{A^x}{1+\tau^x_{t+1}}\right)^{(1+\rho)(1-\alpha)} \left(\frac{A^c}{1+\tau^c_{t+1}}\right)^{\alpha}\right] + \\
+\rho(1-\alpha)\log\left[l_{t+1} - \frac{1+\tau^c_{t+1}}{A^c}\overline{c}\right] + \log l_{t+1} = z\log l_t
$$

Differentiating this equation:

$$
\begin{aligned}
\rho\left(1-\alpha\right)\frac{\mathrm{d}l_{t+1}}{l_{t+1}-\frac{1+\tau^c_{t+1}}{A^c}\overline{c}}+\frac{\mathrm{d}l_{t+1}}{l_{t+1}} =& z\frac{\mathrm{d}l_t}{l_t}\Leftrightarrow\\
\rho\left(1-\alpha\right)\frac{\mathrm{d}l_{t+1}}{l_{t+1}-\frac{1+\tau^c_{t+1}}{A^c}\overline{c}}+\hat{l}_{t+1} =& z\hat{l}_t\Leftrightarrow\\
\rho\left(1-\alpha\right)\frac{\frac{\mathrm{d}l_{t+1}}{l_{t+1}}}{\frac{1}{l_{t+1}}\left[l_{t+1}-\frac{1+\tau^c_{t+1}}{A^c}\overline{c}\right]}+\hat{l}_{t+1} =& z\hat{l}_t\Leftrightarrow\\
\hat{l}_{t+1}\left[\rho\left(1-\alpha\right)\frac{1}{1-\frac{1+\tau^c_{t+1}}{A^c l_{t+1}}\overline{c}}+1\right] =& z\hat{l}_t\Leftrightarrow\\
\hat{l}_{t+1}\left[\rho\left(1-\alpha\right)\frac{l_{t+1}}{l_{t+1}-\frac{1+\tau^c_{t+1}}{A^c}\overline{c}}+1\right] =& z\hat{l}_t\Leftrightarrow\\
\hat{l}_{t+1}\left[\rho\left(1-\alpha\right)\eta^x_{t+1}+1\right] =& z\hat{l}_t\Leftrightarrow\\
\hat{l}_{t+1} =& \frac{z}{1+\rho\left(1-\alpha\right)\eta^x_{t+1}}\hat{l}_t
\end{aligned}
$$

F Constant Tax Rate Policy or Absent Government

$$
\lambda\equiv\frac{z}{1+\rho\left(1-\alpha\right)\left[\eta^x-\varphi^x\frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}+\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)\right]}
$$

If there was a stationary government intervention or absent government, our eigenvalue would become:

$$
\lambda=\frac{z}{1+\rho\left(1-\alpha\right)\eta^x}
$$

which corresponds to the case where $\varphi^c=\varphi^x=0$ or $\theta^c=\theta^x=0$.

There will be indeterminacy if $|\lambda|<1$, which means that the conditions which determine it to emerge depends on the signal of ρ.

1. If $\rho>0\Rightarrow\lambda>0$. So $\lambda<1$ iff $1+\rho\left(1-\alpha\right)\eta^x>z$:

$$
\begin{aligned}
1+\rho\left(1-\alpha\right)\eta^x>z\Leftrightarrow\\
\rho>\frac{z-1}{\left(1-\alpha\right)\eta^x}
\end{aligned}
$$

2. If $\rho<0$ and $\rho\neq-\frac{1}{(1-\alpha)\eta^x}$:

   (a) $0<\lambda<1$

- $\lambda > 0$if:

$$1 + \rho (1 - \alpha) \eta^x > 0 \Leftrightarrow$$
$$\rho > -\frac{1}{(1 - \alpha) \eta^x}$$

- $\lambda < 1$if:

$$1 + \rho (1 - \alpha) \eta^x > z \Leftrightarrow$$
$$\rho > \frac{z - 1}{(1 - \alpha) \eta^x}$$

- So indeterminacy will not occur, because $(z - 1) / (1 - \alpha) \eta^x > 0$ and $\rho < 0$.

(b) $-1 < \lambda < 0$

- $\lambda < 0$if:

$$1 + \rho (1 - \alpha) \eta^x < 0 \Leftrightarrow$$
$$\rho < -\frac{1}{(1 - \alpha) \eta^x}$$

- $\lambda > -1$if:

$$1 + \rho (1 - \alpha) \eta^x < -z \Leftrightarrow$$
$$\rho < -\frac{z + 1}{(1 - \alpha) \eta^x}$$

- So indeterminacy will occur when:

$$\rho < -\frac{1}{(1 - \alpha) \eta^x} \quad \wedge \quad \rho < -\frac{z + 1}{(1 - \alpha) \eta^x} \Rightarrow$$
$$\rho < -\frac{z + 1}{(1 - \alpha) \eta^x}$$
$$\rho \neq -\frac{1}{(1 - \alpha)\eta^x}$$

G Variable Fiscal Policy in the Absence of Externalities

$$\lambda \equiv \frac{z}{1 + \rho (1 - \alpha) \left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x} \frac{1+\rho}{\rho} + \varphi^c \frac{\theta^c}{1+\theta^c} \left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right]}$$

If there was no externality, $\rho = 0$. Thus, the eigenvalue would turn into:

$$\lambda = \frac{z}{1 - (1 - \alpha) \varphi^x \frac{\theta^x}{1+\theta^x} - \alpha \varphi^c \frac{\theta^c}{1+\theta^c}}$$

We now show under which conditions variable fiscal policy can induce indeterminacy.

There will be indeterminacy if $|\lambda| < 1$. We will study the case of variable tax rate on luxury goods separately from variable tax rate on the necessity good.

G.1 The case of $\varphi^c = 0$ and $\varphi^x \neq 0$

1. $0 < \lambda < 1$

   - Assuming $z > 1$, $\lambda > 0$ iff $1 - (1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} > 0$:

$$\begin{aligned} 1 - (1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} &> 0 \Leftrightarrow \\ -(1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} &> -1 \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{1}{1-\alpha} \end{aligned}$$

   - And $\lambda < 1$ just iff $1 - (1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} > z$:

$$\begin{aligned} 1 - (1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} &> z \Leftrightarrow \\ -(1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} &> z - 1 \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{1-z}{1-\alpha} < 0 \end{aligned}$$

   - Thus, indeterminacy will require:

$$\begin{aligned} \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{1}{1-\alpha} \quad \wedge \quad \varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{1-z}{1-\alpha} \Rightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{1-z}{1-\alpha} < 0 \end{aligned}$$

2. $-1 < \lambda < 0$

   - Assuming $z > 1$, $\lambda < 0$ iff $1 - (1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} < 0$:

$$\begin{aligned} 1 - (1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} &< 0 \Leftrightarrow \\ -(1-\alpha)\,\varphi^x \frac{\theta^x}{1+\theta^x} &< -1 \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{1}{1-\alpha} \end{aligned}$$

- And $\lambda > -1$ just iff $1 - (1-\alpha)\varphi^x \frac{\theta^x}{1+\theta^x} - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} < -z$:

$$\begin{aligned} 1 - (1-\alpha)\varphi^x \frac{\theta^x}{1+\theta^x} &< -z \Leftrightarrow \\ -(1-\alpha)\varphi^x \frac{\theta^x}{1+\theta^x} &< -z - 1 \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{1+z}{1-\alpha} > 0 \end{aligned}$$

- Thus, indeterminacy will require:

$$\begin{aligned} \varphi^x \frac{\theta^x}{1+\theta^x} > \frac{1}{1-\alpha} \quad \wedge \quad \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{1+z}{1-\alpha} \Rightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{1+z}{1-\alpha} > 0 \end{aligned}$$

Getting together these results, indeterminacy, i.e., $|\lambda| < 1$, requires: $\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{1+z}{1-\alpha} > 0$, i.e., a sufficiently *strong pro-cyclical tax rate* on luxury goods, or $\varphi^x \frac{\theta^x}{1+\theta^x} < \frac{1-z}{1-\alpha} < 0$, i.e., a sufficiently *strong counter-cyclical tax rate.*

G.2 The case of $\varphi^c = 0$ and $\varphi^x \neq 0$

1. $0 < \lambda < 1$

   - Assuming $z > 1$, $\lambda > 0$ iff $1 - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} > 0$:

   $$\begin{aligned} 1 - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} &> 0 \Leftrightarrow \\ \varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{1}{\alpha} \end{aligned}$$

   - And $\lambda < 1$ just iff $1 - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} > z$:

   $$\begin{aligned} 1 - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} &> z \Leftrightarrow \\ 0 &> z - 1 + \alpha\varphi^c \frac{\theta^c}{1+\theta^c} \Leftrightarrow \\ \varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{1-z}{\alpha} < 0 \end{aligned}$$

   - Thus, indeterminacy will require:

   $$\begin{aligned} \varphi^c \frac{\theta^c}{1+\theta^c} < \frac{1}{\alpha} \quad \wedge \quad \varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{1-z}{\alpha} < 0 \Rightarrow \\ \varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{1-z}{\alpha} < 0 \end{aligned}$$

2. $-1 < \lambda < 0$

- Assuming $z > 1$, $\lambda < 0$ iff $1 - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} < 0$:

$$1 - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} < 0 \Leftrightarrow$$
$$\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{1}{\alpha}$$

- And $\lambda > -1$ just iff $1 - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} < -z$:

$$1 - \alpha\varphi^c \frac{\theta^c}{1+\theta^c} < -z \Leftrightarrow$$
$$0 < -z - 1 + \alpha\varphi^c \frac{\theta^c}{1+\theta^c} \Leftrightarrow$$
$$\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{1+z}{\alpha} > 0$$

- Thus, indeterminacy will require:

$$\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{1}{\alpha} \quad \wedge \quad \varphi^c \frac{\theta^c}{1+\theta^c} > \frac{1+z}{\alpha} \Rightarrow$$
$$\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{1+z}{\alpha} > 0$$

Getting together these results, indeterminacy, i.e., $|\lambda| < 1$, requires: $\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{1+z}{\alpha} > 0$, i.e., a sufficiently *strong pro-cyclical tax rate* on luxury goods, or$\varphi^c \frac{\theta^c}{1+\theta^c} < \frac{1-z}{\alpha} < 0$, i.e., a sufficiently *strong counter-cyclical tax rate.*

H Variable tax rates policy

$$\lambda = \lambda^x \equiv \frac{z}{1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x} \frac{1+\rho}{\rho} + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right]}$$

H.1 The case of $\varphi^c = 0$ and $\varphi^x \neq 0$

$$\lambda = \lambda^x \equiv \frac{z}{1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x} \frac{1+\rho}{\rho}\right]}$$

Indeterminacy requires $|\lambda^x| < 1$.

The case of $\rho > 0$

1. $0 < \lambda^x < 1$

- Assuming $z > 1$, $\lambda^x > 0$ iff $1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] > 0$:

$$\begin{aligned} 1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] > 0 &\Leftrightarrow \\ \eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} > -\frac{1}{\rho(1-\alpha)} &\Leftrightarrow \\ -\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} > -\frac{1}{\rho(1-\alpha)} - \eta^x &\Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} < \frac{1}{\rho(1-\alpha)} + \eta^x &\Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho}{1+\rho}\left[\eta^x + \frac{1}{\rho(1-\alpha)}\right] &\Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)} & \end{aligned}$$

- Assuming $z > 1$, $\lambda^x < 1$ iff $1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] > z$:

$$\begin{aligned} 1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] > z &\Leftrightarrow \\ \eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} > \frac{z-1}{\rho(1-\alpha)} &\Leftrightarrow \\ -\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} > \frac{z-1}{\rho(1-\alpha)} - \eta^x &\Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} < \eta^x - \frac{z-1}{\rho(1-\alpha)} &\Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho}{1+\rho}\left[\eta^x - \frac{z-1}{\rho(1-\alpha)}\right] &\Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} & \end{aligned}$$

- So for indeterminacy to emerge:

$$\begin{aligned} \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} \quad \wedge & \\ \wedge \quad \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)} &\Rightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} & \end{aligned}$$

2. $-1 < \lambda^x < 0$

- Assuming $z > 1$, $\lambda^x < 0$ iff $1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] < 0$:

$$\begin{aligned}
1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] < 0 &\Leftrightarrow \\
\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} < -\frac{1}{\rho(1-\alpha)} &\Leftrightarrow \\
-\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} < -\frac{1}{\rho(1-\alpha)} - \eta^x &\Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} > \frac{1}{\rho(1-\alpha)} + \eta^x &\Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{\rho}{1+\rho}\left[\eta^x + \frac{1}{\rho(1-\alpha)}\right] &\Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)} &
\end{aligned}$$

- Assuming $z > 1$, $\lambda^x > -1 \Leftrightarrow -\lambda < 1$ iff $1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] < -z$:

$$\begin{aligned}
1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] < -z &\Leftrightarrow \\
\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} < -\frac{z+1}{\rho(1-\alpha)} &\Leftrightarrow \\
-\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} < -\frac{z+1}{\rho(1-\alpha)} - \eta^x &\Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} > \eta^x + \frac{z+1}{\rho(1-\alpha)} &\Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{\rho}{1+\rho}\left[\eta^x + \frac{z+1}{\rho(1-\alpha)}\right] &\Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)} &
\end{aligned}$$

- So indeterminacy requires:

$$\begin{aligned}
\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)} \quad &\wedge \\
\wedge \quad \varphi^x \frac{\theta^x}{1+\theta^x} > \frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)} &\Rightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x} > \frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)} &
\end{aligned}$$

Getting together these results, eliminating indeterminacy, i.e., $|\lambda^x| > 1$, requires:

$$\begin{aligned}
\frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} < \varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)} \\
\varphi^x \frac{\theta^x}{1+\theta^x} &\neq \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)}
\end{aligned}$$

Notice that $\frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)} > 0$ under our assumptions. However the signal of $\frac{\rho}{1+\rho}\eta^x -$

$\frac{z-1}{(1+\rho)(1-\alpha)}$ is not clear. But if we take in account that, if there was no policy, indeterminacy will just arise if $\rho > \frac{z-1}{\eta^x(1-\alpha)}$ this implies $\frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} > 0$ and so a *pro-cyclical tax rate* over luxury goods, i.e. $\varphi^x > 0$ is needed to rule out indeterminacy.

The case of $\rho < 0$ In fact, this case makes it necessary to consider three more detailed cases: $\rho = -1$, $\rho < -1$ and $\rho > -1$, but we will limit our analysis to $\rho > -1$.

1. $0 < \lambda^x < 1$

   - Assuming $z > 1$, $\lambda > 0$ iff $1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] > 0$:

$$\begin{aligned}
1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] &> 0 \Leftrightarrow \\
\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &< -\frac{1}{\rho(1-\alpha)} \Leftrightarrow \\
-\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &< -\frac{1}{\rho(1-\alpha)} - \eta^x \Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &> \frac{1}{\rho(1-\alpha)} + \eta^x \Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x}(1+\rho) &< \frac{1}{(1-\alpha)} + \rho\eta^x
\end{aligned}$$

   Now, if $\rho > -1$:

$$\begin{aligned}
\varphi^x \frac{\theta^x}{1+\theta^x}(1+\rho) &< \frac{1}{(1-\alpha)} + \rho\eta^x \Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{1}{(1+\rho)(1-\alpha)} + \frac{\rho}{1+\rho}\eta^x
\end{aligned}$$

   - Assuming $z > 1$, $\lambda < 1$ iff $1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] > z$:

$$\begin{aligned}
1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] &> z \Leftrightarrow \\
\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &< \frac{z-1}{\rho(1-\alpha)} \Leftrightarrow \\
-\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &< \frac{z-1}{\rho(1-\alpha)} - \eta^x \Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &> \eta^x - \frac{z-1}{\rho(1-\alpha)}
\end{aligned}$$

   If $\rho > -1$:

$$\begin{aligned}
\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &> \eta^x - \frac{z-1}{\rho(1-\alpha)} \Leftrightarrow \\
\varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)}
\end{aligned}$$

- So, if $\rho > -1$, for indeterminacy to emerge we have:

$$\begin{aligned} \varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)} \quad \wedge \\ \wedge \quad \varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} \Rightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &< \frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} < 0 \end{aligned}$$

2. $-1 < \lambda < 0$

- Assuming $z > 1$, $\lambda < 0$ iff $1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] < 0$:

$$\begin{aligned} 1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] &< 0 \Leftrightarrow \\ \eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &> -\frac{1}{\rho(1-\alpha)} \Leftrightarrow \\ -\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &> -\frac{1}{\rho(1-\alpha)} - \eta^x \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &< \frac{1}{\rho(1-\alpha)} + \eta^x \end{aligned}$$

If $\rho > -1$:

$$\begin{aligned} \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &< \frac{1}{\rho(1-\alpha)} + \eta^x \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{1}{(1+\rho)(1-\alpha)} + \frac{\rho}{1+\rho}\eta^x \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)} \end{aligned}$$

- Assuming $z > 1$, $\lambda > -1 \Leftrightarrow -\lambda < 1$ iff $1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] < -z$:

$$\begin{aligned} 1 + \rho(1-\alpha)\left[\eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho}\right] &< -z \Leftrightarrow \\ \eta^x - \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &> -\frac{z+1}{\rho(1-\alpha)} \Leftrightarrow \\ -\varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &> -\frac{z+1}{\rho(1-\alpha)} - \eta^x \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &< \eta^x + \frac{z+1}{\rho(1-\alpha)} \end{aligned}$$

If $\rho > -1$:

$$\begin{aligned} \varphi^x \frac{\theta^x}{1+\theta^x}\frac{1+\rho}{\rho} &< \eta^x + \frac{z+1}{\rho(1-\alpha)} \Leftrightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)} \end{aligned}$$

- So, if $\rho > -1$, indeterminacy requires:

$$\begin{aligned} \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)} \quad \wedge \\ \wedge \quad \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)} \Rightarrow \\ \varphi^x \frac{\theta^x}{1+\theta^x} &> \frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)} \end{aligned}$$

So combining these results, if $\rho > -1$, eliminating indeterminacy, i.e., $|\lambda^x| > 1$, requires:

$$\frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} < \varphi^x \frac{\theta^x}{1+\theta^x} < \frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)}$$
$$\varphi^x \frac{\theta^x}{1+\theta^x} \neq \frac{\rho}{1+\rho}\eta^x + \frac{1}{(1+\rho)(1-\alpha)}$$

In this case $\frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} < 0$. However, the signal of $\frac{\rho}{1+\rho}\eta^x + \frac{z+1}{(1+\rho)(1-\alpha)}$ is not clear. Since indeterminacy will just arise if $\rho < -\frac{z+1}{\eta^x(1-\alpha)}$, in the absence of government intervention, this implies $\frac{\rho}{1+\rho}\eta^x - \frac{z-1}{(1+\rho)(1-\alpha)} < 0$ and so a *counter-cyclical tax rate* over luxury goods, i.e. $\varphi^x < 0$ is needed for stabilizing purposes.

H.2 The case of $\varphi^x = 0$ and $\varphi^c \neq 0$

$$\lambda = \lambda^c \equiv \frac{z}{1+\rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1-\eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right]}$$

Indeterminacy requires $|\lambda^c| < 1$. Since $\eta^x > 1$, then this analysis will again depend on the signal of ρ.

The case of $\rho > 0$

1. $0 < \lambda^c < 1$

   - Assuming $z > 1$, $\lambda^c > 0$ iff

$1+\rho(1-\alpha)\left[\eta^x+\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)\right]>0$:

$$\begin{aligned}
1+\rho(1-\alpha)\left[\eta^x+\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)\right]&>0 \Leftrightarrow\\
\eta^x+\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)&>-\frac{1}{\rho(1-\alpha)} \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)&>-\frac{1}{\rho(1-\alpha)}-\eta^x \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}&<-\frac{\frac{1}{\rho(1-\alpha)}+\eta^x}{1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}} \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\frac{1}{\rho(1-\alpha)}+\eta^x}{\eta^x+\frac{\alpha}{\rho(1-\alpha)}-1} \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\frac{1+\eta^x\rho(1-\alpha)}{\rho(1-\alpha)}}{\frac{\eta^x\rho(1-\alpha)+\alpha-\rho(1-\alpha)}{\rho(1-\alpha)}} \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\rho(1-\alpha)\eta^x+1}{\rho(1-\alpha)(\eta^x-1)+\alpha}
\end{aligned}$$

- Assuming $z>1$, $\lambda^c<1$ iff
$1+\rho(1-\alpha)\left[\eta^x+\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)\right]>z$:

$$\begin{aligned}
1+\rho(1-\alpha)\left[\eta^x+\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)\right]&>z \Leftrightarrow\\
\eta^x+\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)&>\frac{z-1}{\rho(1-\alpha)} \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)&>\frac{z-1}{\rho(1-\alpha)}-\eta^x \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\frac{z-1}{\rho(1-\alpha)}-\eta^x}{1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}} \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\frac{z-1-\eta^x\rho(1-\alpha)}{\rho(1-\alpha)}}{\frac{(1-\eta^x)\rho(1-\alpha)-\alpha}{\rho(1-\alpha)}} \Leftrightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\rho(1-\alpha)\eta^x+1-z}{(\eta^x-1)\rho(1-\alpha)+\alpha}
\end{aligned}$$

- So for indeterminacy to emerge:

$$\begin{aligned}
\varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\rho(1-\alpha)\eta^x+1}{\rho(1-\alpha)(\eta^x-1)+\alpha} \quad \wedge\\
\wedge \quad \varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\rho(1-\alpha)\eta^x+1-z}{(\eta^x-1)\rho(1-\alpha)+\alpha} \Rightarrow\\
\varphi^c\frac{\theta^c}{1+\theta^c}&<\frac{\rho(1-\alpha)\eta^x+1-z}{(\eta^x-1)\rho(1-\alpha)+\alpha}
\end{aligned}$$

2. $-1<\lambda^x<0$

- Assuming $z > 1$, $\lambda^x < 0$ iff $1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right] < 0$:

$$
\begin{aligned}
1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right] < 0 &\Leftrightarrow \\
\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < -\frac{1}{\rho(1-\alpha)} &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < -\frac{1}{\rho(1-\alpha)} - \eta^x &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{-\frac{1}{\rho(1-\alpha)} - \eta^x}{1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}} &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} > -\frac{\frac{1}{\rho(1-\alpha)} + \eta^x}{1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}} &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} > -\frac{\frac{1+\rho(1-\alpha)\eta^x}{\rho(1-\alpha)}}{\frac{\rho(1-\alpha)(1-\eta^x)-\alpha}{\rho(1-\alpha)}} &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{\rho(1-\alpha)\eta^x + 1}{\rho(1-\alpha)(\eta^x - 1) + \alpha}
\end{aligned}
$$

- Assuming $z > 1$, $\lambda^x > -1 \Leftrightarrow -\lambda < 1$ iff $1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right] < -z$:

$$
\begin{aligned}
1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right] < -z &\Leftrightarrow \\
\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < -\frac{z+1}{\rho(1-\alpha)} &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < -\frac{z+1}{\rho(1-\alpha)} - \eta^x &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} > -\frac{\frac{z+1}{\rho(1-\alpha)} + \eta^x}{1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}} &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} > -\frac{\frac{z+1+\rho(1-\alpha)\eta^x}{\rho(1-\alpha)}}{\frac{\rho(1-\alpha)(1-\eta^x)-\alpha}{\rho(1-\alpha)}} &\Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{\rho(1-\alpha)\eta^x + 1 + z}{\rho(1-\alpha)(\eta^x - 1) + \alpha}
\end{aligned}
$$

- So indeterminacy requires:

$$
\begin{aligned}
\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{\rho(1-\alpha)\eta^x + 1}{\rho(1-\alpha)(\eta^x - 1) + \alpha} \quad &\wedge \\
\wedge \quad \varphi^c \frac{\theta^c}{1+\theta^c} > \frac{\rho(1-\alpha)\eta^x + 1 + z}{\rho(1-\alpha)(\eta^x - 1) + \alpha} &\Rightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} > \frac{\rho(1-\alpha)\eta^x + 1 + z}{\rho(1-\alpha)(\eta^x - 1) + \alpha}
\end{aligned}
$$

Getting together these results, eliminating indeterminacy, i.e., $|\lambda^c| > 1$, requires:

$$\frac{\rho(1-\alpha)\eta^x + 1 - z}{(\eta^x - 1)\rho(1-\alpha) + \alpha} < \varphi^c \frac{\theta^c}{1+\theta^c} < \frac{\rho(1-\alpha)\eta^x + 1 + z}{\rho(1-\alpha)(\eta^x - 1) + \alpha}$$
$$\varphi^c \frac{\theta^c}{1+\theta^c} \neq \frac{\rho(1-\alpha)\eta^x + 1}{\rho(1-\alpha)(\eta^x - 1) + \alpha}$$

Notice that $\frac{\rho(1-\alpha)\eta^x+1+z}{\rho(1-\alpha)(\eta^x-1)+\alpha} > 0$ under our assumptions. However $\frac{\rho(1-\alpha)\eta^x+1-z}{(\eta^x-1)\rho(1-\alpha)+\alpha} \gtrless 0$. But if we take in account that, if there was no policy, indeterminacy will just arise if $\rho > \frac{z-1}{\eta^x(1-\alpha)}$ this implies $\frac{\rho(1-\alpha)\eta^x+1-z}{(\eta^x-1)\rho(1-\alpha)+\alpha} > 0$ and so a *pro-cyclical tax rate* over necessity goods, i.e. $\varphi^c > 0$ is needed to rule out indeterminacy.

The case of $\rho < 0$ We shall consider only the case of $\rho > \frac{\alpha}{(1-\eta^x)(1-\alpha)}$.

1. $0 < \lambda^c < 1$

   - Assuming $z > 1$, $\lambda^c > 0$ iff $1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right] > 0$:

$$1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right] > 0 \Leftrightarrow$$
$$\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < -\frac{1}{\rho(1-\alpha)} \Leftrightarrow$$
$$\varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < -\frac{1}{\rho(1-\alpha)} - \eta^x$$

   If $\rho > \frac{\alpha}{(1-\eta^x)(1-\alpha)} \Rightarrow 1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)} > 0$:

$$\varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < -\frac{1}{\rho(1-\alpha)} - \eta^x \Leftrightarrow$$
$$\varphi^c \frac{\theta^c}{1+\theta^c} < -\frac{\frac{1}{\rho(1-\alpha)} + \eta^x}{1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}} \Leftrightarrow$$
$$\varphi^c \frac{\theta^c}{1+\theta^c} < -\frac{\frac{1+\rho(1-\alpha)\eta^x}{\rho(1-\alpha)}}{\frac{\rho(1-\alpha)(1-\eta^x)-\alpha}{\rho(1-\alpha)}} \Leftrightarrow$$
$$\varphi^c \frac{\theta^c}{1+\theta^c} < \frac{\rho(1-\alpha)\eta^x + 1}{\rho(1-\alpha)(\eta^x - 1) + \alpha}$$

   - Assuming $z > 1$, $\lambda < 1$ iff $1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right] > z$:

$$1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right)\right] > z \Leftrightarrow$$
$$\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < \frac{z-1}{\rho(1-\alpha)} \Leftrightarrow$$
$$\varphi^c \frac{\theta^c}{1+\theta^c}\left(1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)}\right) < \frac{z-1}{\rho(1-\alpha)} - \eta^x$$

If $\rho > \frac{\alpha}{(1-\eta^x)(1-\alpha)} \Rightarrow 1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)} > 0$:

$$\begin{aligned}
\varphi^c \frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right) &< \frac{z-1}{\rho(1-\alpha)} - \eta^x \Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{\frac{z-1}{\rho(1-\alpha)} - \eta^x}{1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}} \Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{\frac{z-1-\rho(1-\alpha)\eta^x}{\rho(1-\alpha)}}{\frac{\rho(1-\alpha)(1-\eta^x)-\alpha}{\rho(1-\alpha)}} \Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{\rho(1-\alpha)\eta^x + 1 - z}{\rho(1-\alpha)(\eta^x-1)+\alpha}
\end{aligned}$$

- So, f $\rho > \frac{\alpha}{(1-\eta^x)(1-\alpha)} \Rightarrow \rho(1-\alpha)(\eta^x-1)+\alpha > 0$, for indeterminacy to emerge:

$$\begin{aligned}
\varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{\rho(1-\alpha)\eta^x + 1}{\rho(1-\alpha)(\eta^x-1)+\alpha} \quad \wedge \\
\wedge \quad \varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{\rho(1-\alpha)\eta^x + 1 - z}{\rho(1-\alpha)(\eta^x-1)+\alpha} \Rightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} &< \frac{\rho(1-\alpha)\eta^x + 1 - z}{\rho(1-\alpha)(\eta^x-1)+\alpha}
\end{aligned}$$

2. $-1 < \lambda < 0$

- Assuming $z > 1$, $\lambda^c < 0$ iff
  $1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)\right] < 0$:

$$\begin{aligned}
1 + \rho(1-\alpha)\left[\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right)\right] &< 0 \Leftrightarrow \\
\eta^x + \varphi^c \frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right) &> -\frac{1}{\rho(1-\alpha)} \Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right) &> -\frac{1}{\rho(1-\alpha)} - \eta^x
\end{aligned}$$

If $\rho > \frac{\alpha}{(1-\eta^x)(1-\alpha)} \Rightarrow 1 - \eta^x - \frac{\alpha}{\rho(1-\alpha)} > 0$:

$$\begin{aligned}
\varphi^c \frac{\theta^c}{1+\theta^c}\left(1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}\right) &> -\frac{1}{\rho(1-\alpha)} - \eta^x \Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} &> -\frac{\frac{1}{\rho(1-\alpha)} + \eta^x}{1-\eta^x-\frac{\alpha}{\rho(1-\alpha)}} \Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} &> -\frac{\frac{1+\rho(1-\alpha)\eta^x}{\rho(1-\alpha)}}{\frac{\rho(1-\alpha)(1-\eta^x)-\alpha}{\rho(1-\alpha)}} \Leftrightarrow \\
\varphi^c \frac{\theta^c}{1+\theta^c} &> \frac{\rho(1-\alpha)\eta^x + 1}{\rho(1-\alpha)(\eta^x-1)+\alpha}
\end{aligned}$$

- Assuming $z > 1$, $\lambda > -1 \Leftrightarrow -\lambda < 1$ iff

$1+\rho\left(1-\alpha\right)\left[\eta^{x}+\varphi^{c}\frac{\theta^{c}}{1+\theta^{c}}\left(1-\eta^{x}-\frac{\alpha}{\rho(1-\alpha)}\right)\right]<-z$:

$$\begin{aligned}1+\rho\left(1-\alpha\right)\left[\eta^{x}+\varphi^{c}\frac{\theta^{c}}{1+\theta^{c}}\left(1-\eta^{x}-\frac{\alpha}{\rho\left(1-\alpha\right)}\right)\right] &< -z \Leftrightarrow\\ \eta^{x}+\varphi^{c}\frac{\theta^{c}}{1+\theta^{c}}\left(1-\eta^{x}-\frac{\alpha}{\rho\left(1-\alpha\right)}\right) &> -\frac{z+1}{\rho\left(1-\alpha\right)} \Leftrightarrow\\ \varphi^{c}\frac{\theta^{c}}{1+\theta^{c}}\left(1-\eta^{x}-\frac{\alpha}{\rho\left(1-\alpha\right)}\right) &> -\frac{z+1}{\rho\left(1-\alpha\right)}-\eta^{x}\end{aligned}$$

If $\rho>\frac{\alpha}{(1-\eta^{x})(1-\alpha)}\Rightarrow 1-\eta^{x}-\frac{\alpha}{\rho(1-\alpha)}>0$:

$$\begin{aligned}\varphi^{c}\frac{\theta^{c}}{1+\theta^{c}}\left(1-\eta^{x}-\frac{\alpha}{\rho\left(1-\alpha\right)}\right) &> -\frac{z+1}{\rho\left(1-\alpha\right)}-\eta^{x} \Leftrightarrow\\ \varphi^{c}\frac{\theta^{c}}{1+\theta^{c}} &> -\frac{\frac{z+1}{\rho(1-\alpha)}+\eta^{x}}{1-\eta^{x}-\frac{\alpha}{\rho(1-\alpha)}} \Leftrightarrow\\ \varphi^{c}\frac{\theta^{c}}{1+\theta^{c}} &> -\frac{\frac{z+1+\rho(1-\alpha)\eta^{x}}{\rho(1-\alpha)}}{\frac{\rho(1-\alpha)(1-\eta^{x})-\alpha}{\rho(1-\alpha)}} \Leftrightarrow\\ \varphi^{c}\frac{\theta^{c}}{1+\theta^{c}} &> \frac{\rho\left(1-\alpha\right)\eta^{x}+1+z}{\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha}\end{aligned}$$

- So, if $\rho>\frac{\alpha}{(1-\eta^{x})(1-\alpha)}\Rightarrow\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha>0$, indeterminacy requires:

$$\begin{aligned}\varphi^{c}\frac{\theta^{c}}{1+\theta^{c}} &> \frac{\rho\left(1-\alpha\right)\eta^{x}+1}{\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha} \quad\wedge\\ \wedge\quad \varphi^{c}\frac{\theta^{c}}{1+\theta^{c}} &> \frac{\rho\left(1-\alpha\right)\eta^{x}+1+z}{\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha} \Rightarrow\\ \varphi^{c}\frac{\theta^{c}}{1+\theta^{c}} &> \frac{\rho\left(1-\alpha\right)\eta^{x}+1+z}{\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha}\end{aligned}$$

Combining these results, if $\rho>\frac{\alpha}{(1-\eta^{x})(1-\alpha)}\Rightarrow\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha>0$, eliminating indeterminacy, i.e., $|\lambda^{c}|>1$, requires:

$$\frac{\rho\left(1-\alpha\right)\eta^{x}+1-z}{\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha}<\varphi^{c}\frac{\theta^{c}}{1+\theta^{c}}<\frac{\rho\left(1-\alpha\right)\eta^{x}+1+z}{\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha}$$
$$\varphi^{c}\frac{\theta^{c}}{1+\theta^{c}}\neq\frac{\rho\left(1-\alpha\right)\eta^{x}+1}{\rho\left(1-\alpha\right)\left(\eta^{x}-1\right)+\alpha}$$

In this case $\frac{\rho(1-\alpha)\eta^{x}+1-z}{\rho(1-\alpha)(\eta^{x}-1)+\alpha}<0$. However, the signal of $\frac{\rho(1-\alpha)\eta^{x}+1+z}{\rho(1-\alpha)(\eta^{x}-1)+\alpha}$ is not clear. Since indeterminacy will just arise if $\rho<-\frac{z+1}{\eta^{x}(1-\alpha)}$, in the absence of government intervention, this implies $\frac{\rho(1-\alpha)\eta^{x}+1+z}{\rho(1-\alpha)(\eta^{x}-1)+\alpha}<0$ and so a *counter-cyclical tax rate* over necessity goods, i.e. $\varphi^{c}<0$ is needed for stabilizing purposes.