Healthcare Analytics 7 (2025) 100385

Contents lists available at ScienceDirect

Healthcare Analytics

journal homepage: www.elsevier.com/locate/health

A data-driven approach to pricing models for balanced public–private healthcare systems ☆

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ARTICLE INFO

Keywords:
Healthcare system
Contract mechanisms
Data analytics
Pricing model
Nash equilibrium
Social utility

ABSTRACT

This study focuses on a real-world healthcare system with coexisting public and private hospitals with distinct characteristics. While public hospitals have lower costs, they also suffer from long waiting times and diminishing patients' perceived quality of care. Conversely, despite their higher fees, private hospitals offer shorter waiting times, leading to a more favorable perception of quality. A balanced healthcare system could provide societal benefits. Pricing strategies greatly influence a patient's hospital selection. For instance, reduced fees in private hospitals attract more patients, consequently reducing overcrowding in public facilities and elevating the overall quality of services provided. This study aims to develop pricing models to foster a balanced and socially advantageous healthcare system. This system determines private hospital pricing through contract mechanisms with the government. Thus, we delve into the ramifications of various contract models between the government and private hospitals on social utility. Our findings underscore the communal advantages of contract mechanisms. Furthermore, we generalize the proposed models to apply to similar systems.

1. Introduction

Public and private hospitals are integral components of several healthcare systems, each playing distinct roles in serving diverse patient populations. In this study, we model a real healthcare system in which public and private hospitals coexist, each with distinct features. Private hospitals typically provide high service levels and short waiting times but at significantly higher fees. Conversely, public hospitals offer much lower or even free services, but long waiting times and lower service quality often diminish patient satisfaction. Efficient healthcare systems require a delicate balance between these hospitals to ensure equitable access and patient satisfaction [1]. While existing models often analyze systems with one public and one private hospital, they fail to represent more complex settings involving multiple interacting hospitals [2,3]. To address this limitation, we propose a generalized framework comprising two private hospitals and one public hospital, capturing the dynamics of competition and collaboration. We assume that hospitals interact, and their pricing decisions influence patients' preferences. Additionally, this study incorporates critical factors such as waiting times, service quality, and pricing to model hospital selection more realistically [4].

Building on previous studies [2,3], this study uniquely focuses on how contract mechanisms can enhance social utility within competitive private hospital settings. By examining the dynamics of competition, we contribute to a deeper understanding of how private hospitals interact strategically under government-proposed contracts . Unlike former studies [2,3], in addition to scenarios where no contracts exist, this study introduces new approaches by analyzing cases where the government proposes contracts to private hospitals, exploring outcomes when private hospitals reject these contracts, and independently determining their pricing strategies. Using game theory techniques, we identify Nash equilibrium points for private hospital pricing decisions, providing unique insights into competitive and cooperative behaviors in healthcare management. The models are evaluated based on their impact on social utility, with results detailed in the experimental section. Our framework, which is generalizable to systems with multiple hospitals, enables the analysis of intricate healthcare environments and the role of contract mechanisms in optimizing social outcomes. Additionally, novel heuristic solution methods offer practical approaches for real-world applications.

☆ This work is supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under grant number 115M185.
This work is supported by the Portuguese Foundation for Science and Technology (FCT) in the framework of the project CEECINST/00137/2018.
Financial support from Fundação para a Ciência e Tecnologia (through project UIDB/00731/2020) is gratefully acknowledged.

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https://doi.org/10.1016/j.health.2025.100385
Received 13 August 2024; Received in revised form 22 January 2025; Accepted 9 February 2025
Available online 17 February 2025

The conceptual framework of this work can be summarized as follows:

- We model a real regional healthcare system that contains one public hospital and two private hospitals with different characteristics.
- Distinct from many prior studies, we investigate the influence of contract mechanisms on public expenditures, patient decisions, and the profit of private hospitals within the system.
- In addition to the state where there is no contract between the government and the private hospitals, we analyze the model in which the government proposes a contract to both the private hospitals.
- We explore the Nash equilibrium points for the pricing decisions of the private hospitals using the game theory techniques.
- We demonstrate that it is possible to raise both the social utility and the profits of private hospitals by applying suitable contract mechanisms.
- We generalize the model and the solution methods for the case involving multiple public and private hospitals.

In the subsequent sections of this study, following the literature review, we first describe the system and models in detail. In the results section, we compare the models based on social utility, defined as a multi-objective function incorporating both the utility derived by patients and the government's expenditure.

2. Literature review

Patients' hospital selection decisions significantly impact the overall situation of healthcare systems [2,3]. Patients typically select hospitals based on factors such as income levels and sensitivity to quality, underscoring the significant influence of pricing and service quality on their decisions. Several researchers have investigated the key factors that shape patients' hospital preferences [5]. Qin and Prybutok [6] identify price as one of the most important factors affecting patient satisfaction and hospital choice. Similarly, Andritsos and Tang [7], as well as Andritsos and Aflaki [8], report that incorporating private service providers in healthcare systems can reduce patient waiting times and government spending. However, other studies caution against unrestricted support for private healthcare. For instance, Duckett [9] and March and Shroyen [10] argue that improper support for the private sector can harm the overall healthcare system rather than benefit it. This highlights the importance of carefully balancing public and private hospital interactions to optimize societal outcomes, a challenge that often requires robust collaboration mechanisms.

Collaboration between governments and private hospitals often involves pricing decisions, which are governed by contract mechanisms. These mechanisms play a crucial role in designing efficient and equitable healthcare systems. While contract mechanisms and price competition have been widely studied in management science [11,12], their application in the health sector remains relatively unexplored. Some studies provide insights into specific implementations. For example, Kreis and Schmidt [13] survey the impact of health technology evaluations on public health in France, Germany, and the UK. In China, You and Kobayashi [14] analyze the effects of mandatory health insurance on healthcare expenditures. Chick and Mamani [15] explore cost-sharing contracts in the health sector and highlight their potential to address supply uncertainties, such as those in the flu vaccine supply chain. These contract mechanisms are often operationalized through payment schemes, which directly influence service performance and financial sustainability in healthcare systems.

In service systems, payment schemes significantly influence performance and revenue [16,17]. Consequently, performance-based contracts are becoming increasingly common in healthcare systems [18]. For instance, to reduce preventable readmissions, many systems have adopted reimbursement models such as pay-for-performance or bundled payments instead of traditional fee-for-service structures [19]. So and Tang [20] developed a mathematical model to evaluate the effects of reimbursement policies on drug usage. Similarly, Guo et al. [21] analyzed the impact of these reimbursement schemes on patient welfare, readmission rates, and waiting times in public healthcare systems. Similarly, government subsidies serve as another critical mechanism to influence hospital performance and ensure equitable access, particularly in systems involving public and private facilities.

The subsidization of private institutions by governments is another critical issue in healthcare reform, particularly for ensuring access to safe and effective care [22]. Governments can increase subsidy rates to enable more patients to use private facilities; however, this must align with the system's objectives and available resources. Hoel and Sæther [23] highlight that in mixed healthcare systems, private facilities should receive subsidies, or public services should impose fees, especially when capacity constraints exist. Without proper subsidy allocation, public institutions may face overcrowding while private institutions underutilize capacity. Conversely, excessive subsidies may lead to crowding in private institutions and increased government expenditures. To address this, finding an optimal subsidy ratio is a key area of research. Qian et al. [24] suggest that differentiated pricing policies can benefit healthcare systems, while Qian and Zhuang [25] propose subsidy mechanisms to direct patients with higher sensitivity to waiting times toward private hospitals. These approaches can reduce congestion in public hospitals, create a more balanced system, and improve overall social utility [1–3]. Effective subsidy policies also shape the competitive dynamics within healthcare systems, underscoring the need for strategic models to analyze interactions among stakeholders.

Competition in health systems has been extensively studied, particularly through the application of game theory. Acuna et al. [26] present two quantitative frameworks for negotiating with local and regional actors to reduce waiting lists in two-tier health systems, demonstrating that their game-theoretic model can significantly reduce waiting times. Expanding on this, Acuna et al. [27] introduce a two-level Nash-in-Nash model to analyze interactions between insurers, hospitals, and patients in the healthcare market, modeling eight distinct scenarios to account for hospital mergers and insurance network growth. Similarly, Li and Zou [28] use a game-theoretic queueing model to identify optimal contract decisions and characterize equilibrium points.

While game theory has proven effective in modeling competitive dynamics, mechanism design extends these approaches by focusing on creating strategies that align stakeholders' interests and improve system-wide efficiency. Incorporating mechanism design into healthcare, Sun et al. [29] propose multi-strategy combination plans that align the interests of healthcare systems with patient needs. Yang et al. [30] examine three categories of hospital relationships — autonomous decision-making, regional medical information sharing, and government-led collaboration — and find that the proportion of transferred patients significantly influences decision-making, cost-sharing, and hospital profitability. Moscelli et al. [31] examine the impact of competition on waiting time disparities, highlighting that although competition can improve efficiency in certain scenarios, it may also result in longer and more chaotic situations in specific regions.

To address these challenges, several studies investigate ways to enhance collaboration and coordination in competitive healthcare environments. Niu et al. [32] examine incentives for sharing health information among competing institutions, showing that such exchanges are feasible when profit margins from health examination fees are sufficiently reduced. Carvalho and Lodi [33] develop a multiplayer kidney replacement program using Nash equilibria, emphasizing the need for further research to assess transplant quality. Alvarado et al. [34] employ a Stackelberg game to model insurer–hospital interactions, identifying optimal policy designs for healthcare systems. Yaya et al. [35] merge k-means clustering and data envelopment analysis with a game-theoretic model to evaluate healthcare efficiency.

Table 1
A comparison between studies in the literature and our work.

	Patient's hospital choice modeling	Contracting/Pricing	Multiple hospitals	Competition in hospitals	Staffing	Capacity sharing health system	Hospital mergers
Our study	√	√	√	√			
Smith et al. [5]	√						
Qin and Prybutok [6]	√						
Jiang et al. [18]		√					
Andritsos and Tang [19]		√					
So and Tang [20]		√					
Guo et al. [21]		√					
Zhou et al. [22]	√	√					
Qian et al. [24]		√					
Kaya et al. [2]	√	√					
Teymourifar et al. [3]	√	√			√		
Acuna et al. [26]	√		√	√		√	
Acuna et al. [27]	√		√	√			√
Li and Zou [28]		√			√		
Sun et al. [29]		√					
Yang et al. [30]			√	√			
Moscelli et al. [31]		√		√	√		
Niu et al. [32]		√		√			
Carvalho and Lodi [33]			√	√			
Alvarado et al. [34]				√			
Yaya et al. [35]			√	√			
Bisceglia et al. [36]		√		√			
Han et al. [37]		√	√	√			
Panayides et al. [38]			√	√			
Kaya et al. [39]	√	√	√		√		
Balan and Brand [40]			√				√

Expanding on these efforts, other studies utilized game-theoretic models to address interdependencies and competition at regional and hospital levels. Bisceglia et al. [36] use a game-theoretic model to investigate the interdependence of regional healthcare regulators, focusing on interactions between price setters across regions. Han et al. [37] evaluate competition between hospitals in an insurer's plan, offering recommendations for selecting fee-for-service or bundled payment plans. Finally, Panayides et al. [38] apply a game-theoretic model involving emergency services, demonstrating the utility of simulation for analyzing interactions between multiple hospitals [39,40].

However, as these models grow in complexity and involve multi-player scenarios, heuristic-based solution methods have emerged as indispensable tools for identifying equilibrium points and optimizing decision-making. Thus, in this study, we design heuristic-based solution methods to detect Nash equilibrium points, building on approaches explored in the broader literature. While these methods are not limited to healthcare applications, they offer significant potential for modeling complex systems. For instance, Porter et al. [41] develop simple search strategies to identify Nash equilibrium points in both two-player and n-player games. Expanding on this, Konak and Kulturel-Konak [42] propose a regret-based fitness assignment strategy for evolutionary algorithms, enabling the detection of Nash equilibria in computationally intractable combinatorial game models. Zaman et al. [43] introduce a co-evolutionary method for identifying multiple Nash equilibria in continuous n-player games, which has been successfully applied to competitive electricity markets.

Following these developments, more advanced techniques have been developed to cope with the increasing complexity of systems and the challenges of multi-player scenarios. Lung and Dumitrescu [44] address the challenge of determining Nash equilibria in multi-player normal-form games, presenting a Nash-based domination relation that enhances convergence to multiple solutions. Similarly, Konak et al. [45] develop the Nash equilibrium sorting genetic algorithm, designed to identify equilibria in competitive maximal covering location problems. Recognizing that exact Nash equilibrium solutions are not always feasible, Wei et al. [46] propose a multi-objective migratory bird optimization algorithm that approximates Nash equilibria using game theory, neighborhood operators, and path relinking techniques. Belabid et al. [47] combine hybrid greedy algorithms with Nash equilibrium concepts and genetic operators, further advancing solution methods.

Given their ability to address complex constraints and multi-player interactions, these algorithms hold significant potential for application in healthcare systems. Their capacity to model competition and decision-making under intricate constraints makes them valuable tools for analyzing interactions in multi-hospital systems.

The similarities and differences between this study and those found in the literature are outlined in Table 1, which also highlights the contributions of this work.

3. Description of the system

The used notations in this section are outlined in Table 2. As mentioned in the introduction, the dealt problem is from a real healthcare system, in which there are one public and two private hospitals in a region i.e. n=2 and m=1. The primary subject of the problem is to investigate the impacts of contract mechanisms to enhance social benefit in this system.

In the current state of the system, there is a contract based on fixed prices between the government and private hospitals, and also the government pays a definite subsidy to the patients that receive service from a private hospital. In this case, as seen in Fig. 1, the service fee, the average waiting time and the perceived quality level in the i-th private hospitals are denoted as b_{o_i}, w_{o_i} and q_{o_i}, $\forall i = 1, 2$, which are b_d, w_d and q_d in the public one. We assume that quality has a favorable impact on the utility while the influence of price and waiting time is adverse. In the literature, a utility function has been defined for similar models [1–3]. In this study, taking advantage of them we define the utility of the patients in the i-th private hospital as $\frac{q_{o_i}}{w_{o_i}} - k{b_{o_i}}^2$, while it is $\frac{q_d}{w_d} - kb_d^2$ in the public hospital. k is the price sensitivity of the patients and we assume $k \geq 0$, otherwise, the negative effect of price in the utility function is omitted.

The literature supports the assumptions regarding the utility function, reflecting findings that shorter waits improve perceived quality. The quadratic price term captures increasing disutility with rising payments [2]. It aligns with observed system behaviors and other models incorporating quality, waiting time, and price, making it a justified choice for healthcare systems with public and private hospitals.

We suppose that a strategic patient makes a choice between the hospitals according to the utility she will receive in the chosen hospital.

Table 2
Used notations.

Notation	Description	Type
n	Number of private hospitals	Parameter
m	Number of public hospitals	Parameter
c_{d_j}	Cost of care for each patient in the j-th public hospital	Parameter
b_{d_j}	Amount paid by the patients to the j-th public hospital	Parameter
c_{o_i}	Cost of care for each patient in the i-th private hospital	Parameter
r_i	Total price of service in the i-th private hospital	Decision variable
b_{o_i}	Amount paid by the patients to the i-th private hospital	Variable
s_{o_i}	Subsidy payment made by the government to the i-th private hospital for each patient	Decision variable
k_{o_i}	Cost of unit capacity in the private hospital	Parameter
k_{d_j}	Cost of unit capacity in the j-th public hospital	Parameter
q_{o_i}	Service quality level in the i-th private hospital	Parameter
q_{d_j}	Service quality level in the j-th public hospital	Parameter
λ	Total arrival rate of all hospitals' patients	Parameter
p_{o_i}	Probability of selecting the i-th private hospital by a patient	Variable
p_{d_j}	Probability of selecting the j-th public hospital by a patient	Variable
ca_{o_i}	Capacity of the i-th private hospital	Parameter
ca_{d_j}	Capacity of the j-th public hospital	Parameter
μ_{o_i}	Service rate per unit capacity in the i-th private hospital	Parameter
μ_{d_j}	Service rate per unit capacity in the j-th public hospital	Parameter
H_d	Amount of expenditure made by the government (public expenditure)	Variable
w_{o_i}	Average waiting time in the i-th private hospital	Variable
w_{d_j}	Average waiting time in the j-th public hospital	Variable
k	Price sensitivity of a patient	Variable
A_n	Thresholds of k	Variable
k^{max}	Upper bound of k	Parameter
$F_k(x)$	Cumulative probability function of k	Definition
I_{min}	Minimum income level of patients	Definition
I_{max}	Maximum income level of patients	Definition
U_1	Total utility received by all patients	Objective function
U_2	Average government expenses per patient	Objective function
U	Total social utility	Objective function
Z_{o_i}	The profit of the i-th private hospital	Objective function
T	Computational time	Output of the solution algorithm

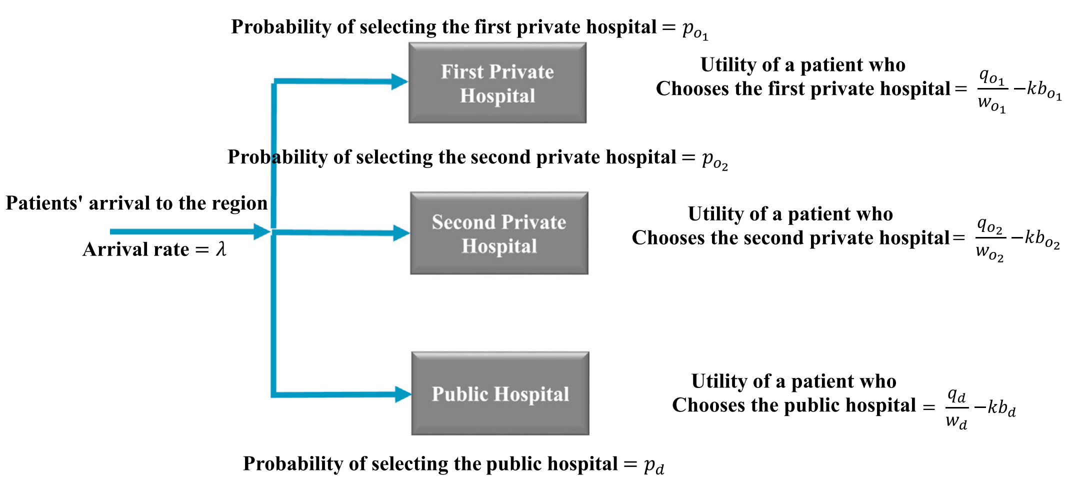

Fig. 1. A strategic patient chooses one of the hospitals based on the utility she gets.

A strategic patient selects the first private hospital if Eq. (3.1) is valid.

$$\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_d}{w_d} - kb_d^2 \quad and \quad \frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2 \tag{3.1}$$

We assume that $b_{o_1} > b_{o_2} > b_d$. Based on our assumption, to select the first one among the private hospitals by a patient, her utility in this hospital would be more than the other, means $\frac{q_{o_1}}{w_{o_1}} - kb_{o_1}^2 \geq \frac{q_{o_2}}{w_{o_2}} - kb_{o_2}^2$ and accordingly $\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}} \geq k(b_{o_1}^2 - b_{o_2}^2)$. Since the right-hand side of the inequality is non-negative, we conclude that $\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}}$, which can be generalized as $\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}} \geq \frac{q_d}{w_d}$.

We define $\frac{q_{o_i}}{w_{o_i}}$ and $\frac{q_d}{w_d}$ as the satisfaction levels in the ith private and public hospital. Thus the previous paragraph means that if the price of service in a hospital is higher than the other, it has to provide more satisfaction, otherwise it is not preferred.

k has an important effect on the decision of patients. It is a non-negative random variable with an upper bound of k_{max}. We suppose that if it is low for a patient, she gives more importance to quality than

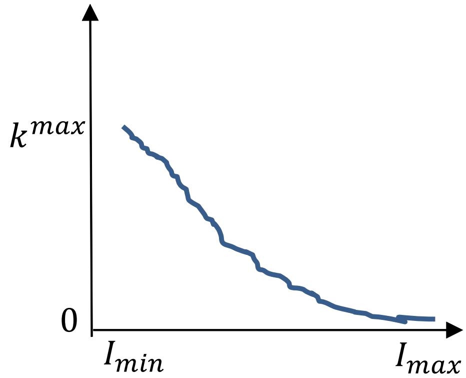

Fig. 2. It is supposed that there is an inverse relationship between k and the income level of patients.

price and vice versa. In addition, we assume that there is an inverse relationship between k and the income level of patients, as seen in Fig. 2.

In Fig. 2, I_{min} and I_{max} are the minimum and maximum income levels of patients, while k^{max} is the upper bound of the price sensitivity of patients.

Each patient has a specific value of k. We assume that there is a threshold shown as A in Fig. 3 and if for a patient $k \leq A$ she chooses the private hospitals.

In Fig. 3, A is the threshold of the price sensitivity of patients to choose private hospitals. k^{max} is the upper bound of the price sensitivity of patients.

We suppose that if for a patient $k \leq A$, then $\frac{q_{o_i}}{w_{o_i}} - kb^2_{o_i} \geq \frac{q_d}{w_d} - kb^2_d$, $\forall i = 1, 2$. Therefore, from Eq. (3.1), if k for a patient is as in Eq. (3.2), she chooses the first private hospital.

$$k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{b^2_{o_1} - b^2_{o_2}} \tag{3.2}$$

It is assumed that the average times in the hospitals are according to the M/M/1 queueing model, then they can be calculated as in Eqs. (3.3) and (3.4) [1,2,39].

$$w_{o_1} = \frac{1}{ca_{o_1}\mu_{o_1} - \lambda p_{o_1}} \tag{3.3}$$

$$w_{o_2} = \frac{1}{ca_{o_2}\mu_{o_2} - \lambda p_{o_2}} \tag{3.4}$$

where μ_{o_i} and ca_{o_i} are the service rate and the capacity of the i-th private hospital, respectively. We suppose that a value of k is attributed to each patient, which is a random variable with a cumulative distribution function, $F_k(x)$ [1,2,39].

So, p_{o_1} is as in Eq. (3.5).

$$p_{o_1} = F_k\left(\frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{b^2_{o_1} - b^2_{o_2}}\right) \tag{3.5}$$

Using Eqs. (3.3) and (3.4), p_{o_1} can be written more clearly as in Eq. (3.6).

$$p_{o_1} = F_k\left(\frac{q_{o_1}(ca_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(ca_{o_2}\mu_{o_2} - \lambda p_{o_2})}{b^2_{o_1} - b^2_{o_2}}\right) \tag{3.6}$$

In a similar way, p_{o_2} is calculated as in Eq. (3.7).

$$p_{o_2} = F_k\left(\frac{q_{o_2}(ca_{o_2}\mu_{o_2} - \lambda p_{o_2}) - q_d(ca_d\mu_d - \lambda p_d)}{b^2_{o_2} - b^2_d}\right) - p_{o_1} \tag{3.7}$$

U_1, the total utility received by all patients is calculated as in Eq. (3.8).

$$U_1 = \int_0^{A_1} \left(\frac{q_{o_1}}{w_{o_1}} - kb^2_{o_1}\right) f(k)dk + \int_{A_1}^{A_2} \left(\frac{q_{o_2}}{w_{o_2}} - kb^2_{o_2}\right) f(k)dk + \int_{A_2}^{k^{max}} \left(\frac{q_d}{w_d} - kb^2_d\right) f(k)dk \tag{3.8}$$

We supposed k is uniformly distributed between 0 and k^{max}. As seen in Fig. 4, A_1 and A_2 are the critical values of k for the patients to select the hospitals, which are calculated as $A_1 = k^{max} p_{o_1}$ and $A_2 = k^{max}(p_{o_1} + p_{o_2}) = k^{max} p_o$.

In Fig. 4, p_{o_1}, p_{o_2} and p_d are the probabilities of selecting the first and second private and public hospitals, respectively. A_1 and A_2 are the critical values of the price sensitivity of patients to select hospitals. k^{max} is the upper bound of the price sensitivity of patients.

H_d, the public expenditure is as in Eq. (3.9).

$$H_d = \lambda p_d(c_d - b_d) + \lambda(p_{o_1}s_{o_1} + p_{o_2}s_{o_2}) + k_d ca^2_d \tag{3.9}$$

Also, U_2, the average public expenditure is defined as in Eq. (3.10).

$$U_2 = \frac{H_d}{\lambda} \tag{3.10}$$

The social utility, which is defined as in Eq. (3.11) consists of U_1 and U_2.

$$U = \alpha_1 U_1 - \alpha_2 U_2 \tag{3.11}$$

The profit functions of the private hospitals are as in Eqs. (3.12) and (3.13).

$$Z_{o_1} = (r_1 - c_{o_1})p_{o_1}\lambda - k_{o_1} ca^2_{o_1} \tag{3.12}$$

$$Z_{o_2} = (r_2 - c_{o_2})p_{o_2}\lambda - k_{o_2} ca^2_{o_2} \tag{3.13}$$

Generalization of the model for *n* private and *m* public hospitals

The described model can be easily generalized to the case where the system contains n private and m public hospitals. We suppose that $b_{d_j} = b_d$, $w_{d_j} = w_d$ and $q_{d_j} = q_d$, $\forall j = 1, 2, \ldots, m$. Therefore, there is no competition among public hospitals and it does not matter which one is chosen. Thus, all public hospitals can be unified under one and as seen in Fig. 5, the system can be modeled as n private hospitals and one public hospital. We also assume that $b_{o_1} > b_{o_2} > \cdots > b_{o_n} > b_{d_j} = b_d$, then it can be concluded that $\frac{q_{o_1}}{w_{o_1}} \geq \frac{q_{o_2}}{w_{o_2}} \geq \cdots \geq \frac{q_{o_n}}{w_{o_n}} \geq \frac{q_{d_j}}{w_{d_j}} = \frac{q_d}{w_d}$, $\forall j = 1, 2, \ldots, m$.

In Fig. 5, p_{o_i} and p_d are the probabilities of selecting the i-th private hospital and all public hospitals. A_n and k^{max} are the thresholds and the upper bound of the price sensitivity of patients.

p_{o_1} is as in Eq. (3.6). p_{o_i}, $\forall i = 2, \ldots, n-1$, and p_{o_n} are as in Eqs. (3.14) and (3.15), respectively.

$$p_{o_i} = F_k\left(\frac{q_{o_i}(ca_{o_i}\mu_{o_i} - \lambda p_{o_i}) - q_{i-1}(ca_{i-1}\mu_{i-1} - \lambda p_{i-1})}{b^2_{o_i} - b^2_{i-1}}\right) - \sum_{ii=1}^{i-1} p_{o_{ii}}, \quad \forall i = 2, \ldots, n-1 \tag{3.14}$$

$$p_{o_n} = F_k\left(\frac{q_{o_n}(ca_{o_n}\mu_{o_n} - \lambda p_{o_n}) - q_d(ca_d\mu_d - \lambda p_d)}{b^2_{o_n} - b^2_d}\right) - \sum_{i=1}^{n-1} p_{o_i} \tag{3.15}$$

U_1, H_d, and Z_{o_i} are as in Eqs. (3.16), (3.17), and (3.18), respectively.

$$U_1 = \int_0^{A_1} \left(\frac{q_{o_1}}{w_{o_1}} - kb^2_{o_1}\right) f(k)dk + \int_{A_1}^{A_2} \left(\frac{q_{o_2}}{w_{o_2}} - kb^2_{o_2}\right) f(k)dk + \cdots + \int_{A_{n-1}}^{A_n} \left(\frac{q_{o_{n-1}}}{w_{o_{n-1}}} - kb^2_{o_{n-1}}\right) f(k)dk + \int_{A_n}^{k^{max}} \left(\frac{q_d}{w_d} - kb^2_d\right) f(k)dk \tag{3.16}$$

$$H_d = \lambda p_d(c_d - b_d) + \lambda\left(\sum_{i=1}^{n} p_{o_i}s_{o_i}\right) + k_d ca^2_d \tag{3.17}$$

$$Z_{o_i} = (r_i - c_{o_i})p_{o_i}\lambda - k_{o_i} ca^2_{o_i} \tag{3.18}$$

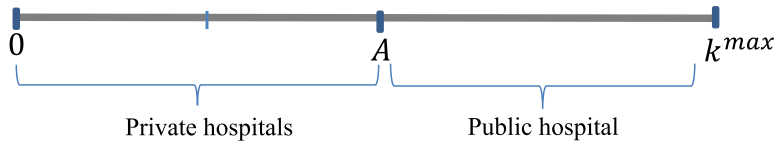

Fig. 3. The price sensitivity threshold of patients influences hospital selection decisions.

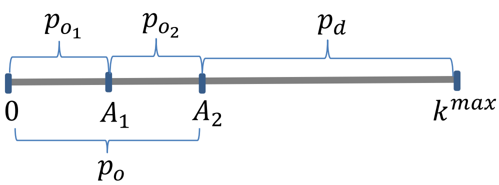

Fig. 4. The threshold of patients' price sensitivity when there are two private hospitals.

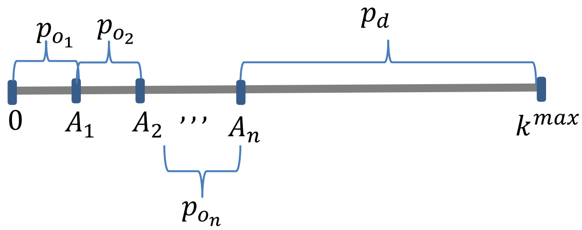

Fig. 5. A_i, $\forall i = 1, 2, \dots, n$ are the thresholds of patients' price sensitivity to select the ith private hospitals.

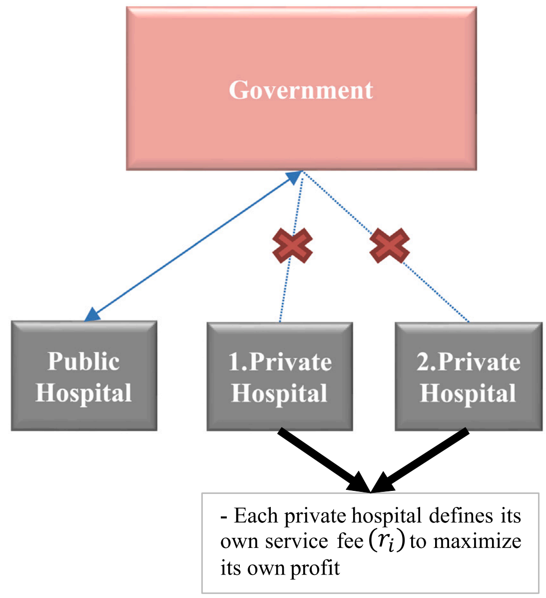

Fig. 6. Model NC.

4. Model NC: No contract between the government and the private hospital

In this model, which is summarized in Fig. 6, as in the base case problem, there are two private hospitals and one public hospital. Each private hospital determines its own examination fees to maximize its profit. The examination fee, the average waiting time and the perceived quality level are r_i, w_{o_i} and q_{o_i} in the i-th private hospital, while in the public hospital they are b_d, w_d and q_d. Thus, the utility of the patients is $\frac{q_{o_i}}{w_{o_i}} - kr_i^2$ in the i-th private hospital and $\frac{q_d}{w_d} - kb_d^2$ in the public hospital. As it is described in Eq. (4.1), a strategic patient selects the first private hospital if she earns more utility there.

$$\frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_d}{w_d} - kb_d^2 \quad and \quad \frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_{o_2}}{w_{o_2}} - kr_2^2 \tag{4.1}$$

As illustrated in Fig. 3, we presume that if for a patient $k \leq A$, then $\frac{q_{o_i}}{w_{o_i}} - kb_{o_i}^2 \geq \frac{q_d}{w_d} - kb_d^2$, $\forall i = 1, 2$. Therefore, for a strategic patient to choose the first private hospital, Eq. (4.2) is sufficient.

$$\frac{q_{o_1}}{w_{o_1}} - kr_1^2 \geq \frac{q_{o_2}}{w_{o_2}} - kr_2^2 \tag{4.2}$$

So, the patient chooses the first private hospital, if the value of k is as in Eq. (4.3).

$$k \leq \frac{\frac{q_{o_1}}{w_{o_1}} - \frac{q_{o_2}}{w_{o_2}}}{r_1^2 - r_2^2} \tag{4.3}$$

p_{o_1} is as in Eq. (4.4), where $F_k(x)$ is the cumulative probability function of k.

$$p_{o_1} = F_k\left(\frac{q_{o_1}(ca_{o_1}\mu_{o_1} - \lambda p_{o_1}) - q_{o_2}(ca_{o_2}\mu_{o_2} - \lambda p_{o_2})}{r_1^2 - r_2^2}\right) \tag{4.4}$$

In a similar way, p_{o_2} is written as in Eq. (4.5).

$$p_{o_2} = F_k\left(\frac{q_{o_2}(ca_{o_2}\mu_{o_2} - \lambda p_{o_2}) - q_d(ca_d\mu_d - \lambda p_d)}{r_2^2 - b_d^2}\right) - p_{o_1} \tag{4.5}$$

It is clear that the probability of choosing the public hospital by a patient is $p_d = 1 - p_{o_1} - p_{o_2}$.

$$H_d = \lambda p_d(c_d - b_d) + k_d ca_d^2 \tag{4.6}$$

In this model, U_1 is defined in Eq. (4.7).

$$U_1 = \int_0^{A_1} \left(\frac{q_{o_1}}{w_{o_1}} - kr_1^2\right) f(k)dk + \int_{A_1}^{A_2} \left(\frac{q_{o_2}}{w_{o_2}} - kr_2^2\right) f(k)dk + \int_{A_2}^{k^{max}} \left(\frac{q_d}{w_d} - kb_d^2\right) f(k)dk \tag{4.7}$$

U_2 and U are as in Eqs. (3.10) and (3.11) and also we assume that k is uniformly distributed between 0 and k^{max}.

In this model, private hospitals attempt to maximize their profits by defining appropriate examination fees. So this model consists of two problems; in the first problem, the profit of the first hospital, and in the second one the profit of the second hospital are to be maximized. The problem for the first hospital is defined as in Eq. (4.8).

$$Max_{r_1} \quad Z_{o_{1_{nc}}} = \lambda p_{o_1}(r_1 - c_{o_1}) - k_{o_1} ca_{o_1}^2 \tag{4.8}$$

The problem of the second hospital is defined in Eq. (4.9).

$$Max_{r_2} \quad Z_{o_{2_{nc}}} = \lambda p_{o_2}(r_2 - c_{o_2}) - k_{o_2} ca_{o_2}^2 \tag{4.9}$$

The constraints of the model are defined in Eq. (4.10) to (4.19).

$$p_{o_1} \leq 1 \tag{4.10}$$

$$p_{o_1} \geq 0 \tag{4.11}$$

$$\lambda p_{o_1} \leq ca_{o_1}\mu_{o_1} \tag{4.12}$$

$$p_{o_2} \leq 1 \tag{4.13}$$

$$p_{o_2} \geq 0 \tag{4.14}$$

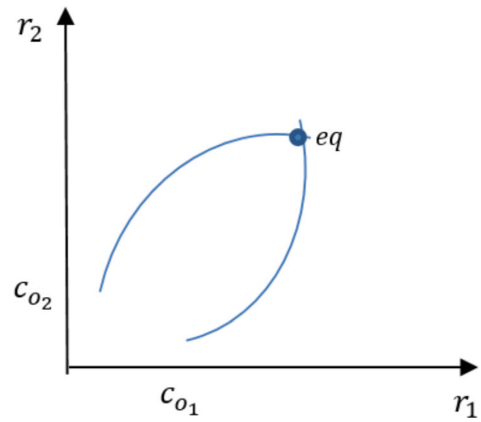

Fig. 7. Equilibrium point of r_1 and r_2.

$$\lambda p_{o_2} \le ca_{o_2}\mu_{o_2} \tag{4.15}$$

$$\lambda p_d \le ca_d \mu_d \tag{4.16}$$

$$r_1, r_2 \ge 0 \tag{4.17}$$

$$r_1 \ge r_2 \tag{4.18}$$

$$\frac{q_{o_1}}{w_{o_1}} \ge \frac{q_{o_2}}{w_{o_2}} \tag{4.19}$$

In Fig. 7, r_i, c_{o_i}, and eq represent the total service price, the cost of care per patient at the i-th private hospital, and the equilibrium point, respectively. As illustrated, the service fees for private hospitals start at their minimum levels, which correspond to the cost of care per patient. When one private hospital increases its service fee, the other hospital responds by raising its own fee as well, reflecting the competitive dynamics between them. As seen in the figure, the equilibrium is achieved at the point where the service fees of the two private hospitals intersect and stabilize, indicating that neither hospital finds it advantageous to further adjust its fee.

Generalization of the Model NC for *n* private and *m* public hospitals

In this generalization, we assume that $r_{o_1} > r_{o_2} > \cdots > r_{o_n} > b_{d_j} = b_d$, then it can be concluded that $\frac{q_{o_1}}{w_{o_1}} \ge \frac{q_{o_2}}{w_{o_2}} \ge \cdots \ge \frac{q_{o_n}}{w_{o_n}} \ge \frac{q_{d_j}}{w_{d_j}} = \frac{q_d}{w_d}$, $\forall j = 1, 2, \ldots, m$. Hence, as shown in Fig. 5 all public hospitals can be unified under one. Thus, the system can be modeled as n private hospitals and one public hospital.

p_{o_1} is as in Eq. (3.6). p_{o_i}, $\forall i = 2, \ldots, n-1$, and p_{o_n} are as in Eqs. (4.20) and (4.21), respectively.

$$p_{o_i} = F_k\left(\frac{q_{o_i}(ca_{o_i}\mu_{o_i} - \lambda p_{o_i}) - q_{i-1}(ca_{i-1}\mu_{i-1} - \lambda p_{i-1})}{r_{o_i}^2 - r_{i-1}^2}\right) - \sum_{ii=1}^{i-1} p_{o_{ii}}, \ \forall i = 2, \ldots, n-1 \tag{4.20}$$

$$p_{o_n} = F_k\left(\frac{q_{o_n}(ca_{o_n}\mu_{o_n} - \lambda p_{o_n}) - q_d(ca_d\mu_d - \lambda p_d)}{r_{o_n}^2 - b_d^2}\right) - \sum_{i=1}^{n-1} p_{o_i} \tag{4.21}$$

U_1 and H_d are respectively as in Eqs. (4.22) and (4.23).

$$\begin{aligned} U_1 = &\int_0^{A_1} \left(\frac{q_{o_1}}{w_{o_1}} - kr_{o_1}^2\right) f(k)dk + \int_{A_1}^{A_2} \left(\frac{q_{o_2}}{w_{o_2}} - kr_{o_2}^2\right) f(k)dk + \cdots \\ &+ \int_{A_{n-1}}^{A_n} \left(\frac{q_{o_{n-1}}}{w_{o_{n-1}}} - kr_{o_{n-1}}^2\right) f(k)dk + \\ &\int_{A_n}^{k^{max}} \left(\frac{q_d}{w_d} - kb_d^2\right) f(k)dk \end{aligned} \tag{4.22}$$

$$H_d = \lambda p_d(c_d - b_d) + \lambda\left(\sum_{i=1}^{n} p_{o_i} s_{o_i}\right) + k_d ca_d^2 \tag{4.23}$$

As defined in Eq. (4.24), the goal of private hospitals is to maximize their own profits and the decision variable is r_i.

$$Max_{r_i} \quad Z_{o_i} = (r_i - c_{o_i})p_{o_i}\lambda - k_{o_i} ca_{o_i}^2 \tag{4.24}$$

The constraints of the model are defined in Eqs. (4.25) to (4.30).

$$p_{o_i} \le 1 \tag{4.25}$$

$$p_{o_i} \ge 0 \tag{4.26}$$

$$\lambda p_{o_i} \le ca_{o_i}\mu_{o_i} \tag{4.27}$$

$$r_i \ge 0 \tag{4.28}$$

$$r_1 \ge r_2 \ge \cdots \ge r_n \tag{4.29}$$

$$\frac{q_{o_1}}{w_{o_1}} \ge \frac{q_{o_2}}{w_{o_2}} \ge \cdots \ge \frac{q_{o_n}}{w_{o_n}} \tag{4.30}$$

Solution method

The designed solution method is summarized in Algorithm 1.

Algorithm 1: Solution method for Model NC

1 **Initialize** Best r_2 and r_2
2 **while** *Best* $r_2 \ne r_2$ **do**
3     $r_2 \leftarrow$ Best r_2;
4   **Search** on r_1;
5   **Find Best** r_1 that maximizes $Z_{o_1}(r_1, r_2)$;
6   **Search** on r_2;
7   **Find Best** r_2 that maximizes $Z_{o_2}(Best\ r_1, r_2)$;
8 **Calculate** $Z_{o_1}(Best\ r_1, Best\ r_2)$;
9 **Calculate** $Z_{o_2}(Best\ r_1, Best\ r_2)$;

Algorithm 1 begins by initializing the search at line 1. Subsequently, each hospital iteratively adjusts its pricing strategy to maximize its profit, factoring in the pricing decision of the other hospital. This iterative process continues until an equilibrium point is reached, where neither hospital can improve its profit by unilaterally changing its price. This stable point is achieved when the condition outlined in line 2 is satisfied, signaling convergence.

The search range for prices, specified between 0 and 500 with an increment value of 1 (as defined in lines 4 and 6), is selected based on expert recommendations to ensure computational efficiency. However, tests with a higher upper limit for the price range yield the same results.

It is important to note that this is a heuristic method. While it provides near-optimal solutions efficiently, it does not guarantee a fully optimal solution. Despite this limitation, the method reliably identifies equilibrium points suitable for practical applications.

While this study does not account for capacity decisions, they can be easily incorporated into Algorithm 1 by adding constraints to the searches at lines 4 and 6.

We consider a region with one public hospital and two private hospitals. However, it should be noted that the system is not isolated, and patients may seek services in neighboring regions. Therefore, the solution methods should be applicable to a greater number of hospitals. Algorithm 1 is easily generalizable to n private hospitals. Algorithm 2 is a generalization for three private hospitals.

Within Algorithm 2, constraints related to capacity decisions can be incorporated into the searches at lines 4, 6, and 8.

5. Model SC: Contract mechanism based on subsidy payments

In this model, which is summarized in Fig. 8, the private hospitals decide to accept or reject the contract proposed by the government

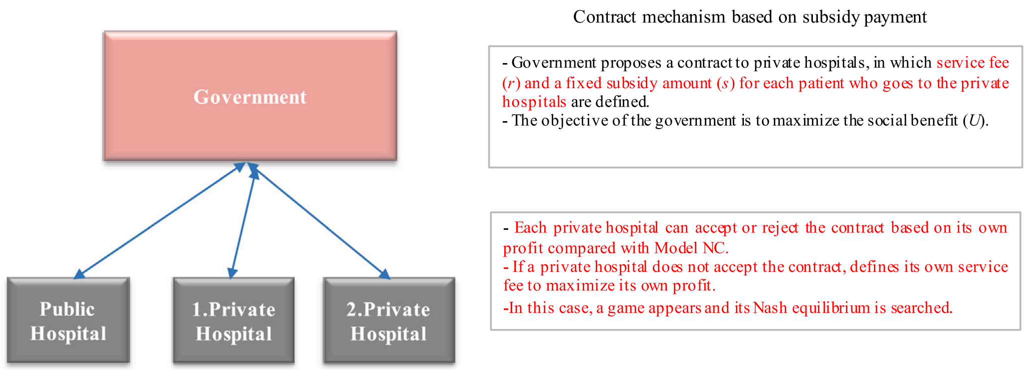

Fig. 8. Model SC.

Algorithm 2: Solution method for Model NC with three private hospitals

Initialize Best r_3, r_3 and r_2
while *Best* $r_3 \neq r_3$ **do**
  $r_3 \leftarrow$ Best r_3;
  Search on r_1;
  Find Best r_1 that maximizes $Z_{o_1}(r_1, r_2, r_3)$;
  Search on r_2;
  Find Best r_2 that maximizes $Z_{o_2}(Best\ r_1, r_2, r_3)$;
  Search on r_3;
  Find Best r_3 that maximizes $Z_{o_3}(Best\ r_1, Best\ r_2, r_3)$;
Calculate $Z_{o_1}(Best\ r_1, Best\ r_2, Best\ r_3)$;
Calculate $Z_{o_2}(Best\ r_1, Best\ r_2, Best\ r_3)$;
Calculate $Z_{o_3}(Best\ r_1, Best\ r_2, Best\ r_3)$;

Table 3
Profit of private hospitals and social utility in different regions.

		Second private hospital	
		Reject	Accept
First private hospital	Reject	$Z_{o_{1RR}}, Z_{o_{2RR}}, U_{RR}$	$Z_{o_{1RA}}, Z_{o_{2RA}}, U_{RA}$
	Accept	$Z_{o_{1AR}}, Z_{o_{2AR}}, U_{AR}$	$Z_{o_{1AA}}, Z_{o_{2AA}}, U_{AA}$

according to their own profits. The contract includes the price of the examination and a subsidy. In this model, the choice of hospital for a strategic patient is made based on Eqs. (3.1), (3.2), (3.5) and (3.7). Therefore, H_d, U_1, U_2, U, Z_{o1} and Z_{o2} are defined as in Eqs. (3.8) – (3.13). The generalization of p_{o_i} for the case where there are n private and m public hospitals in the system is as in Eqs. (3.14) and (3.15). U_1, H_d, and Z_{o_i} are as in Eqs. (3.16), (3.17), and (3.18), respectively.

In this model, first of all, the profits of the private hospitals are calculated for the case that none of them accepts the contract, which is indicated as Reject–Reject (RR) in Table 3. The other states in the same table are as follows: AA signifies that both private hospitals accept the contract, RA shows that the first private hospital rejects it while the second accepts it, and AR reflects that the first private hospital accepts the contract while the second rejects it. The profits of the private hospitals are calculated according to the price and subsidy proposed by the government. Each private hospital compares its own profit with RR state; if the contract provides more profit, the hospital accepts it. Otherwise, by rejecting the contract, the hospital defines its own service fee.

Solution method

In the algorithm implemented in the MATLAB software, $Z_{o_{1RR}}$ and $Z_{o_{2RR}}$, which are the profits that the two private hospitals obtain when they reject the contract are calculated. Then, the profits are computed for the cases summarized below, and subsequently, the Nash equilibrium point is searched.

The RR state: if $Z_{o_{1RR}} > Z_{o_{1RA}}$ and $Z_{o_{2RR}} > Z_{o_{2AR}}$ then both hospitals reject the contract.

The AA state: if $Z_{o_{1RR}} > Z_{o_{1AR}}$ and $Z_{o_{2RR}} > Z_{o_{2RA}}$ then both hospitals accept the contract.

The RA state: if $Z_{o_{1RA}} > Z_{o_{1AA}}$ and $Z_{o_{2RA}} > Z_{o_{2RR}}$ then the first hospital rejects and the second one accepts the contract.

The AR state: if $Z_{o_{1AR}} > Z_{o_{1RR}}$ and $Z_{o_{2AR}} > Z_{o_{2AA}}$ then the second hospital rejects and the first one accepts the contract.

After determining the Nash equilibrium point, the relevant social utility is also calculated. The solution method is summarized in Algorithm 3.

Similar to the process described in Algorithm 1, the solution approach outlined in Algorithm 3 is a heuristic method designed to provide near-optimal solutions rather than exact optimal ones. The lower and upper bounds, along with the increment values used in the search (lines 2, 4, and 9), remain consistent with those specified in the description following Algorithm 1.

Algorithm 3 is highly adaptable and can be generalized to accommodate a system with n private and m public hospitals, as further elaborated in Algorithm 4. In this generalized scenario, it is assumed that there is no competition among public hospitals, simplifying the dynamics and focusing the analysis on interactions between private and public entities.

In Algorithm 3, constraints tied to capacity decisions can be integrated into the searches on lines 5 and 10.

In Algorithm 4, AAA represents a situation where all three hospitals accept the contract, while RRR signifies all three hospitals declining the contract. Partial situations occur in patterns like ARR, where only the first hospital accepts while the second and third reject, or RAR, where the second hospital accepts while the first and third reject. Similarly, RRA reflects acceptance by the third hospital and rejection by the first and second, while AAR shows acceptance by the first and second hospitals but rejection by the third. In ARA, the first and third hospitals accept while the second rejects and RAA highlights acceptance by the second and third hospitals with rejection by the first.

Capacity decisions can be added to Algorithm 4 as constraints on the searches in lines 8 and 12.

Algorithms 1–4 primarily consist of basic ‘if’ conditions, ‘for’ loops, and ‘while’ loops, making them straightforward to implement. However, computational complexity can increase with the size of the system, particularly in large-scale scenarios with numerous hospitals. The nested iterations over parameters and evaluations of various states could significantly raise computational requirements, especially when combined with optimization steps. In such situations, strategies like

Algorithm 3: Solution method for Model SC

1 **Search** on r and s, and **Calculate** U and Z_{o_i} in states of RA, AR, AA and RR as:
2 Run Algorithm 1;
3 **foreach** r *and* s **do**
4 RA (1. private hospital rejects, while 2. private hospital accepts);
5 **Find** Best r_1 that maximizes $Z_{o_1}(r_1, r, s)$;
6 **Calculate** $Z_{o_{1_{RA}}}(Best\ r_1, r, s)$;
7 **Calculate** $Z_{o_{2_{RA}}}(Best\ r_1, r, s)$;
8 **Calculate** $U_{RA}(Best\ r_1, r, s)$;
9 AR (1. private hospital accepts, while 2. private hospital rejects);
10 **Find** Best r_2 that maximizes $Z_{o_2}(r, s, r_2)$;
11 **Calculate** $Z_{o_{1_{AR}}}(Best\ r, s, r_2)$;
12 **Calculate** $Z_{o_{2_{AR}}}(Best\ r, s, r_2)$;
13 **Calculate** $U_{AR}(Best\ r, s, r_2)$;
14 AA (both private hospitals accept);
15 **Calculate** $Z_{o_{1_{AA}}}(Best\ r, s)$;
16 **Calculate** $Z_{o_{2_{AA}}}(Best\ r, s)$;
17 **Calculate** $U_{AA}(Best\ r, s)$;
18 **if** $Z_{o_{1_{AA}}} > Z_{o_{1_{RA}}}$ *and* $Z_{o_{2_{AA}}} > Z_{o_{2_{AR}}}$ **then**
19 Equilibrium point is AA;
20 **Calculate** U_{AA};
21 **else if** $Z_{o_{1_{RR}}} > Z_{o_{1_{AR}}}$ *and* $Z_{o_{2_{RR}}} > Z_{o_{2_{RA}}}$ **then**
22 Equilibrium point is RR;
23 **Calculate** U_{RR};
24 **else if** $Z_{o_{1_{RA}}} > Z_{o_{1_{AA}}}$ *and* $Z_{o_{2_{RA}}} > Z_{o_{2_{RR}}}$ **then**
25 Equilibrium point is RA;
26 **Calculate** U_{RA};
27 **else if** $Z_{o_{1_{AR}}} > Z_{o_{1_{RR}}}$ *and* $Z_{o_{2_{AR}}} > Z_{o_{2_{AA}}}$ **then**
28 Equilibrium point is AR;
29 **Calculate** U_{AR};
30 **Find** the Best U, the corresponding fees and related Equilibrium point;
31 **Calculated** Z_{o_1} and Z_{o_2};

parallel computing or heuristic methods can be valuable in mitigating complexity.

6. Experimental results

In this section, the numerical results of the models are presented. The used parameters are summarized in Table 4, which are taken from a district of Eskişehir province in Turkey. As mentioned earlier, there are two private hospitals and one public hospital in the district. The data was collected between 2015–2019 from the system.

Service quality levels were obtained through a questionnaire filled out by 250 respondents in the hospitals. In the questionnaire, which is designed by the Turkish Ministry of Health, there are questions about the staff (nurses, doctors, etc.), facilities, and cleanliness in hospitals. More details about the procedure of data collection and the English translation of the questionnaire are available in [39].

The procedures specified in Algorithms are implemented in MATLAB. A system with an Intel Core i5 processor, 2.4 GHz and 12 GB of RAM is utilized. The obtained results are presented in Table 5, in which T is in seconds.

Algorithm 4: Solution method for Model SC with three private hospitals

1 **Search** on r and s, and **Calculate** U and Z_{o_i} in states of AAA, RRR, ARR, RAR, RRA, AAR, ARA, RAA:
2 Run Algorithm 2;
3 **foreach** r *and* s **do**
4 AAA (all private hospitals accept);
5 **Calculate** $Z_{o_{1_{AA}}}(Best\ r, s)$, $Z_{o_{2_{AA}}}(Best\ r, s)$, $Z_{o_{3_{AA}}}(Best\ r, s)$, $U_{AAA}(Best\ r, s)$;
6 ARR (1. private hospital accepts, while 2. and 3. private hospitals reject);
7 Run Algorithm 1 for 2. and 3. private hospitals;
8 **Find** Best r_2 and r_3 that maximize $Z_{o_2}(r, s, r_2, r_3)$ and $Z_{o_3}(r, s, r_2, r_3)$;
9 **Calculate** $Z_{o_{1_{ARR}}}(Best\ r, s, r_2, r_3)$, $Z_{o_{2_{ARR}}}(Best\ r, s, r_2, r_3)$, $Z_{o_{3_{ARR}}}(Best\ r, s, r_2, r_3)$, $U_{ARR}(Best\ r, s, r_2, r_3)$;
10 **Do** similar steps between lines 6 and 9 for the RAR and RRA states;
11 AAR (1. and 2. private hospitals accept, while 3. private hospital rejects);
12 **Find** Best r_3 that maximizes $Z_{o_3}(r, s, r_3)$;
13 **Calculate** $Z_{o_{1_{AAR}}}(Best\ r, s, r_3)$, $Z_{o_{2_{AAR}}}(Best\ r, s, r_3)$, $Z_{o_{3_{AAR}}}(Best\ r, s, r_3)$, $U_{AAR}(Best\ r, s, r_3)$;
14 **Do** similar steps between lines 11 and 13 for the ARA and RAA;
15 **Find** the state in which equilibrium occurs;
16 **Find** the Best U, the corresponding fees and related Equilibrium point;
17 **Calculated** Z_{o_1}, Z_{o_2} and Z_{o_3};

As seen in Table 5, in Model NC, where no contract exists between the government and private hospitals, the examination prices at the hospitals are set at 230 TL and 176 TL. Under Model SC, when the government offers a contract to private hospitals, the first private hospital rejects the offer, while the second private hospital accepts it. This establishes the equilibrium state in the RA region, with the corresponding result highlighted in bold in Table 5.

Compared to Model NC, Model SC demonstrates a remarkable improvement in social utility, achieving an increase of 786%. This improvement is attributed to the second private hospital offering a discount of approximately 33%, leading to a 47% reduction in patient payments at that hospital. As a result, the number of patients choosing the second private hospital rises by about 6%. Additionally, Model SC significantly reduces the average waiting time at the public hospital, leading to an improvement of 383% in U_1, the associated social utility. While government expenditures increase under Model SC, the benefits to society are substantial. Notably, both private hospitals experience an increase in profits, further emphasizing the effectiveness of the proposed mechanism.

As depicted in Fig. 9, in the RA region where the equilibrium is achieved, the profits of both private hospitals are higher compared to other regions. These results clearly indicate that it is possible to enhance both social utility and the profits of private hospitals by implementing well-designed contract mechanisms, making Model SC a mutually beneficial solution for all stakeholders.

To demonstrate the generalizability of the models and solution methods, we assume that there is a third private hospital in the region. We presume the following parameters for the third hospital: $c_{o_3} = 17$,

Table 4
Values of base case parameters.

$\lambda = 70,000$	$\alpha_1 = 0.01$	$\alpha_2 = 1$	$b_d = 5$	$k^{max} = 1$
$c_{o_1} = 20, c_{o_2} = 15, c_d = 10$	$k_{o_1} = 50,000, k_{o_2} = 40,000, k_d = 10,000$	$q_{o_1} = 0.7, q_{o_2} = 0.6, c_d = 0.5$	$t_{o_1} = 8, t_{o_2} = 8, t_d = 3$	$ca_{o_1} = 3, ca_{o_2} = 3, ca_d = 4$

Table 5
Results obtained by the models with the base case parameters.

Model		Hospital	r	s_o	p_o	w_o	Z	w_d	H_d	U_1	U_2	U	T
	Model NC	First private hospital Second private hospital	230 176	0 0	0.081 0.098	0.07 0.08	739 281.16 744 996.25	5.69	447367.13	516.82	6.39	−1.22	< 1
Model SC	RR region	First private hospital Second private hospital	230 176	0 0	0.081 0.098	0.07 0.08	739 281.16 744 996.25	5.69	447367.13	516.82	6.39	−1.22	6.57
	RA region	First private hospital Second private hospital	**272** **117**	**0** **34**	**0.070** **0.155**	**0.06** **0.13**	**784 625.27** **745 935.01**	**0.22**	**799935.97**	**1979.49**	**11.43**	**8.37**	
	AR region	First private hospital Second private hospital	117 134	34 0	0.141 0.089	0.11 0.07	508 820.00 381 080.00	0.19	765520.00	2029.50	10.94	9.36	
	AA region	First private hospital Second private hospital	117 117	34 34	0.137 0.121	0.11 0.09	480 970.00 506 710.00	0.13	1034700.00	2999.70	14.78	15.21	

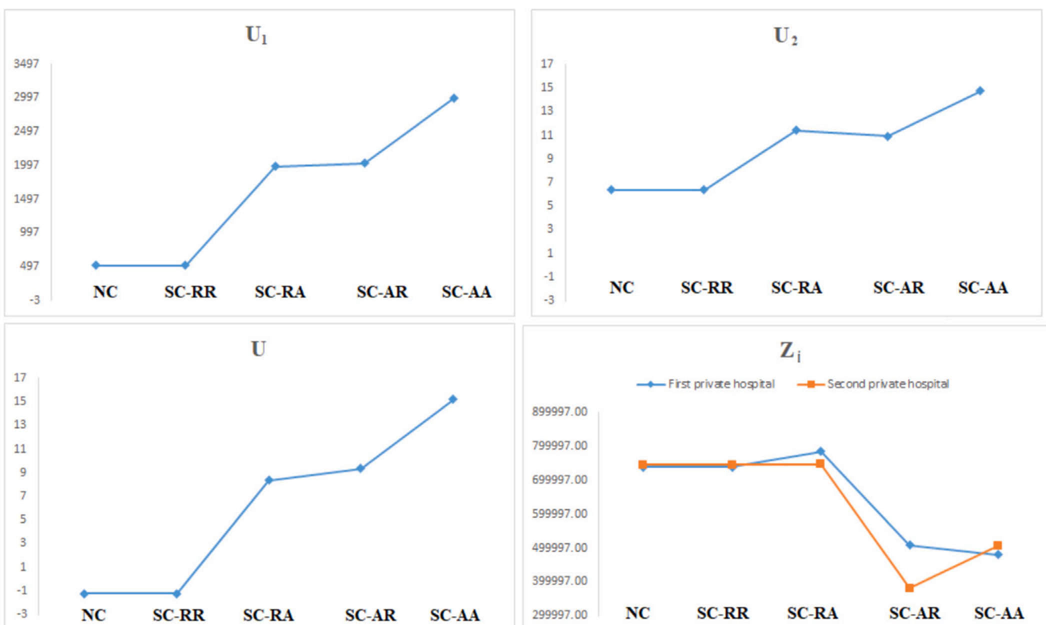

Fig. 9. Values of U_1, U_2, U and Z_i in the models.

Table 6
Results for Model NC with three private hospitals.

Model	Hospital	r	s_o	p_o	w_o	Z	w_d	H_d	U_1	U_2	U	T
Model NC	First private hospital Second private hospital Third private hospital	229 150 136	0 0 0	0.061 0.076 0.090	0.06 0.07 0.07	445 642.39 362 865.26 341 554.11	0.20	430432.54	2132.97	6.15	15.18	< 1

$k_{o_3} = 45000$, $q_{o_3} = 0.6$, $t_{o_3} = 8$ and $ca_{o_3} = 3$. Other parameters are the same as in Table 4. The results of applying Algorithm 2 to solve this problem based on Model NC are shown in Table 6.

As shown in Table 6, the results can be interpreted in a manner similar to those of Model NC in Table 5. However, since the total probability of patients choosing private hospitals is higher in this case compared to the scenario with only two private hospitals, both U_1 and the overall utility U are significantly higher.

It is worth emphasizing that the computation time remains under one second, demonstrating that the solution methods are not computationally complex. This suggests that the methods can be applied to scenarios involving a greater number of hospitals.

6.1. Sensitivity analysis

To show that similar consequences are obtained according to different parameters, the results of sensitivity analysis are provided in Table 7.

If $\lambda = 65000$, in the equilibrium point, the social utility increases but the profit of private hospitals decreases. When $\lambda = 75000$, the social utility decreases, and in this case, while the profit of the first private hospital increases, the profit of the second private hospital reduces. In both states of $\lambda = 65000$ and 75000, the equilibrium point for Model SC is in the RA area, i.e. the first hospital rejects the contract and the second accepts it. The same equilibrium point (RA) is also obtained with parameters $c_{o_1} = 25$, $c_{o_2} = 10$ and $ca_{o_1} = 2.5$. However, if

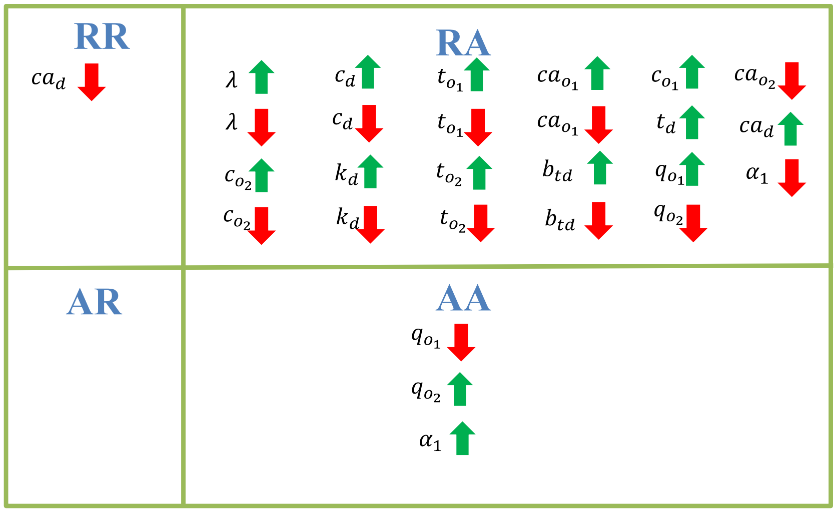

Fig. 10. Graphical summary of the sensitivity analysis.

Table 7
Results obtained by the models according to different parameters.

	Model NC	Model SC	
	U	U	Equilibrium Region
$\lambda = 75000$	−1.44	−0.79	RA
$\lambda = 65000$	13.18	21.12	RA
$c_{o_1} = 25$	−1.64	7.97	RA
$c_{o_2} = 20$	−1.58	8.12	RA
$c_{o_2} = 10$	−0.87	8.63	RA
$c_d = 15$	−5.33	4.49	RA
$c_d = 5$	2.88	12.24	RA
$k_d = 15000$	−2.37	7.22	RA
$k_d = 5000$	−0.08	9.51	RA
$q_{o_1} = 0.8$	0.01	9.72	RA
$q_{o_1} = 0.6$	−2.07	7.32	AA
$q_{o_2} = 0.7$	−1.34	8.97	AA
$q_{o_2} = 0.5$	−1.38	7.85	RA
$q_d = 0.6$	−1.65	9.50	RA
$t_{o_1} = 9$	−2.05	5.31	RA
$t_{o_1} = 7$	3.05	12.01	RA
$t_{o_2} = 9$	−2.21	5.45	RA
$t_{o_2} = 7$	0.74	11.93	RA
$t_d = 4$	−2.50	−3.40	RA
$ca_{o_1} = 3.5$	3.85	12.62	RA
$ca_{o_1} = 2.5$	−2.20	3.96	RA
$ca_{o_2} = 2.5$	−2.24	4.00	RA
$ca_d = 4.5$	18.90	26.44	RA
$ca_d = 3.5$	−1.08	−1.08	RR
$b_{td} = 10$	2.56	4.58	RA
$b_{td} = 0$	−5.22	4.59	RA
$k_{max} = 5$	−26.69	10.68	RA
$\alpha_1 = 0.02$	3.95	41.33	AA
$\alpha_1 = 0.005$	−3.81	−0.56	RA

$ca_d = 3.5$ and $q_{o_1} = 0.6$, the equilibrium point is in the RR and AA areas, respectively.

Fig. 10 offers a visual summary of the sensitivity analysis results presented in Table 7, showcasing how changes in various parameters affect the equilibrium regions. The green up arrow and red down arrow, respectively, indicate the boost and decline in the corresponding parameter. Equilibrium points are noted in blue.

From Fig. 10, it is evident that parameters impact the equilibrium outcomes. For instance, a decrease in ca_d and q_{o_1} can move the system into equilibrium regions, such as RR or AA, reflecting strategic responses from the hospitals. In general, the highest social utility with the parameters summarized in Table 4 is provided by Model SC. The profit of both private hospitals in this model is higher than in Model NC.

Managerial Discussion

The findings highlight beneficial strategies for policymakers and hospital managers to enhance healthcare systems through contract mechanisms. For policymakers, designing suitable subsidies and monitoring their impact on competition between hospitals can improve social utility while reducing pressure on public hospitals. Hospital managers can strategically define prices, adjust capacity, and enhance quality to balance profitability with patient satisfaction. Challenges such as resistance to contracts and resource constraints can be addressed through capacity-sharing agreements. These policies should always account for the competitive dynamics between hospitals. These insights provide a practical foundation for implementing effective policies to create more balanced public–private healthcare systems.

7. Conclusion and future works

In this study, we cope with a problem from a real healthcare system and we develop models to solve it. The main topic of the problem is to analyze the effects of contract mechanisms on improving social utility. Although the impacts of many contract mechanisms are analyzed in the literature on supply chain management, there is not enough work on this topic in healthcare management studies. In the system, the pricing decisions are based on the contract mechanisms between the government and private hospitals, which affect the general state of the healthcare system owing to that they have significant effects on the preference of patients to choose a hospital.

According to the results obtained, contract mechanisms between the government and private hospitals benefit society. In the model where the government offers a contract to the private hospitals, in most of the obtained results according to different parameters, the first private hospital, i.e., the hospital providing higher quality service, does not accept the contract and determines its own price. In this case, although the number of patients in the first private hospital, which defines a higher price than in the absence of a contract, decreases slightly, its profit increases. In this state, the second private hospital, which provides a quality of service between the public and the first private hospital, significantly reduces the price, supplies service for more patients, and increases its own profit.

The problem is from a regional healthcare system in the Eskişehir Province of Turkey, which contains two private hospitals and one public hospital. However, the proposed models are generalized for multiple hospitals. In addition, the presented sensitivity analysis shows that the results are also valid for different parameters. The described healthcare system may exist in different countries. From managerial insight, the results can be summarized as follows: Governments can provide a more balanced healthcare system with appropriate contract mechanisms. In this case, with the enhancement in social utility, the profit of private hospitals that accept the contract also increases. Contract acceptance may require certain managerial decisions. Capacity levels may need to be redefined. In the solution methods, we explain where capacity decisions can be made. However, private hospitals may not always have adequate personnel or equipment to raise capacity, in which case capacity-sharing policies can be considered.

The designed heuristics effectively determine Nash equilibrium points using logical strategies based on pricing, waiting times, and service quality, providing practical results. However, this study has limitations stemming from theoretical assumptions made to simplify the analysis of patient decision-making and hospital dynamics. For instance, hospitals' capacity decisions are not considered, and service quality is treated as an exogenous, static variable despite the dynamic correlation between patients' perceptions of quality and waiting times. The inclusion of hospital capacity decisions and service quality as endogenous variables warrants further investigation. Furthermore, the model assumes a uniform distribution of patient price sensitivity and a direct inverse correlation between income and sensitivity, which may not universally reflect real-world patterns. Additionally, patient utility is modeled solely as a function of price, waiting time, and service quality, excluding other influential factors such as proximity or hospital reputation. Country-specific elements, such as public policies and financial incentives for public hospitals based on patient volume, also play a critical role in shaping hospital competition and patient demand, but these are not fully captured.

Furthermore, the analysis of contract mechanisms addresses various scenarios but does not explicitly consider all real-world challenges like compliance and negotiation dynamics. While the models are generalizable to systems with multiple hospitals, regional differences in healthcare policies may limit their applicability globally. The data from Eskişehir, Turkey, provides a foundation for the models, but its global representativeness is limited; broader datasets from several regions are needed to enhance robustness. The heuristic methods used to find equilibrium points leave the total number of potential equilibria unknown. Consequently, the competition between hospitals and patients' decision-making processes in the modeled system may not be fully represented. If policymakers implement this model in practice, the data collected during its application could provide valuable insights into whether it accurately reflects patient decision-making and system dynamics. Such empirical validation could refine the model, enhance its applicability, and address its current limitations, making it a promising avenue for future research. Future studies should aim to design more comprehensive models that account for these complexities and improve the current framework.

Simulation can serve as a valuable tool for analyzing models involving more hospitals. In future studies, hybrid models combining simulation and game theory will be developed. Additionally, as highlighted in Table 1, potential research directions include models that examine competition between multiple hospitals while considering staffing and districting factors. Future research will also focus on analyzing the long-term impacts of contract mechanisms and parameter changes on service quality and patient satisfaction. To address the limitations of current tools for modeling long-term dynamics, system dynamics techniques will be employed to better capture time-dependent changes and feedback effects within the healthcare system.

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

Human and animal rights

The research does not involve any data collected from human participants or experiments with animals.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors would like to thank the editors and the anonymous referees for their valuable comments, which significantly improved the manuscript.

ChatGPT was employed to enhance the English writing of this paper, focusing on improvements in grammar, style, and overall clarity.

Appendix. Acronyms

Models:

- **NC:** No contract between the government and the private hospital.
- **SC:** Contract mechanism based on subsidy payments.

Hospital Decision Patterns (Two Private Hospitals):

- **RR:** A state where neither of the private hospitals accepts the contract.
- **AA:** A state where both private hospitals accept the contract.
- **RA:** A state where the first private hospital rejects the contract, while the second accepts it.
- **AR:** A state where the first private hospital accepts the contract, while the second rejects it.

Hospital Decision Patterns (Three Private Hospitals):

- **AAA:** A situation where all three private hospitals accept the contract.
- **RRR:** A state where all three private hospitals decline the contract.
- **ARR:** A state where the first private hospital accepts the contract, while the second and third reject it.
- **RAR:** A state where the second private hospital accepts the contract, while the first and third reject it.
- **RRA:** A state where the third private hospital accepts the contract, while the first and second reject it.
- **AAR:** A state where the first and second private hospitals accept the contract, while the third rejects it.
- **ARA:** A state where the first and third private hospitals accept the contract, while the second rejects it.
- **RAA:** A state where the second and third private hospitals accept the contract, while the first rejects it.

Data availability

Data will be made available on request.

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