Endogenous Product Design and Quality When Consumers Have Heterogeneous Limited Attention*

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Abstract

In some markets, consumers do not know the attributes of all the products that are available in the market, or the prices at which they are offered. To reduce this uncertainty consumers may, at a cost, gather and process information about the attributes and prices of the different products. The uncertainty that persists at the time of purchase affects the competition in the market, via product attributes and prices. We examine the consequences of information costs on firms' product multi-attribute and pricing decisions when consumers have heterogeneous information costs and limited attention. We find that consumers that can gather and process information at approximately no cost rationally select to be attentive while consumers that must incur a cost rationally select to be inattentive. We find also that firms have an incentive to respond to lower information costs by increasing differentiation, but solely along the least-costly attribute dimension and if the proportion of attentive consumers in the market is small. This implies that, as information costs decrease, equilibrium prices may increase in some markets and decrease in others. Further, it implies also that when the cost of an attribute dimension in a market changes, there can be radical shifts in product attributes.

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1 Introduction

In some markets, consumers are imperfectly informed. They do not know the attributes of all the products in the market, or the prices at which they are available (Stiglitz, 1989). To reduce this uncertainty consumers may gather and process information about the attributes and prices of the different products (e.g., contact the different sellers, examine the products, ask questions, seek expert advice, read internet sites or forums, among many other). However, to do so, consumers may incur a cost, since gathering and processing information takes money, time and effort. This cost impacts the choice of consumers on what and how much information to gather and process and, as a consequence, the uncertainty that will persist at the time of purchase, which, in turn, affects the competition in the market, via product attributes (in the long-run) and prices (in the short-run).

This article is part of a significant and growing literature that investigates the consequences of consumer information costs on firms' product attribute and pricing decisions (Kuksov, 2004; Bar-Isaac et al., 2012; Larson, 2013; Fishman and Levy, 2015). This problem is particularly relevant given the growth of electronic commerce and internet marketplaces that has contributed to decrease - but not eliminate - the cost of gathering and processing information on product attributes and prices.[1] We examine the same problem. The main innovation is that we consider the case in which firms position their products on *multiple attributes*. Moreover, we allow also consumers to be *heterogeneous* in their information costs and have *limited attention*. This contrasts with the existing literature that has investigated the problem assuming a single endogenous product attribute, homogeneous information costs and a rational expectations framework. There are several reasons why we focus on these three contributing aspects. First, most products embody multiple attributes - both horizontal and vertical - and production requires their joint specification, which yields that they are, as a consequence, jointly endogenous. Second, the money, time and effort required to gather and process information may vary extensively from consumer to consumer.[2] Finally, consumers can not fully process all available information as assumed by the rational expectations

[1] The existing empirical evidence suggests that gathering and processing information on product attributes and prices remains costly for consumers: see, for example, Lach (2002), Lewis (2008), and Dubois and Perrone (2019). Lach (2002) examines the Israeli refrigerator, chicken, coffee, and flour markets. Lewis (2008) examines the San Diego gasoline market. Dubois and Perrone (2019) examine the French food retail market for beer, cola, coffee, and whisky. They all find evidence of price dispersion, even after controlling for observed and unobserved product characteristics, which suggests that even information on price remains not freely and readily available.

[2] Hortaçsu and Syverson (2004) document that information costs in the U.S. mutual fund industry are considerably heterogeneous across consumers, with a highly skewed distribution. Nishida and Remer (2018) document that information costs in the U.S. retail gasoline industry are, in part, driven by consumers opportunity costs and vary considerably both within and across markets. Jolivet and Turon (2019) document that information costs in the French on-line CD trading industry are considerably heterogeneous across consumers (and sometimes substantial).

framework. Their attention is a scarce resource whose allocation must be decided.[3]

In order to examine this problem, we set up a discrete-choice framework in which two firms, manufacturing a single product characterized by two attributes, *design* (which we portray as horizontal differentiation) and *quality* (which we portray as vertical differentiation), compete over a continuum of consumers. On the demand side, consumers are assumed to select an information gathering and processing strategy, i.e., to choose what and how much information to gather and process.[4] We assume also there are two groups of consumers: one that can gather and process information at approximately no cost (who may simply love shopping) and another that must incur a cost to gather and process information. The information strategies selected by consumers generate signals that are then used to update consumers prior beliefs about the attributes and prices of the different products and, in turn, select (and purchase) the product whose perceived attributes and price yield the highest utility. On the supply side, firms are assumed to select the product attributes (design and quality) and pricing that yield the highest own profit. We assume also that there are no costs associated with choosing different product designs, following Bar-Isaac et al. (2012), while there may exist costs associated with choosing different product qualities, following Mussa and Rosen (1978).

The main results are as follows. On the demand side, we find that consumers that can gather and process information at approximately no cost fully eliminate their uncertainty about product attributes and prices. They rationally select to be attentive. Consumers that must incur a cost to gather and process information, on the other hand, do not fully eliminate their uncertainty. They rationally select to be *inattentive* and partially base their decision on prior beliefs. This implies that limited attention has an important impact on decision making, as long suggested by Simon (1959) and Kahneman (1973), and it is consistent with several recent empirical studies documenting that consumers process relatively little information in car insurance (Honka, 2014), S&P 500 index funds (Hortaçsu and Syverson, 2004), and automobiles (Moorthy et al., 1997; Morton et al., 2011). On the supply side, we find that the key result of the existing literature, that lower consumer information costs may in fact lead to higher equilibrium prices since firms may respond to those lower costs by increasing product differentiation (and thus decrease price competition), does not always generalize. Firms do have an incentive to respond to lower information costs by increasing

[3]There is ample evidence of consumer limited attention from laboratory studies in psychology as well as from field experiments, calibration and econometric studies in many domains, including shipping fees, nontransparent taxes, rankings of hospitals and colleges, financial news, outside buy-it-now options, car mileage, bank overdraft fees and psychological pricing. Please see DellaVigna (2009) and Zhong (2022) and the references therein.

[4]Consumers may not consciously decide, in reality, which information and processing strategy they are going to select, but over time they are likely to optimize this behaviour, even if subconsciously.

differentiation, but solely along *the least-costly attribute dimension* and *if the proportion of rationally attentive consumers in the market is small.* This is consistent with (i) the theoretical literature that examines firms' product attribute and pricing decisions under perfect information, which shows that standard differentiation results no longer hold when firms compete in a multi-attribute space ((Neven and Thisse, 1987, 1990); Tabuchi, 1994; Ansari et al., 1998; Irmen and Thisse, 1998; Heeb, 2001); and (ii) the theoretical literature that examines firms' pricing decisions under imperfect information, which shows that the competitiveness of a market depends crucially on the shape of the information cost distribution ((Stahl, 1989, 1996); Moraga-Gonzáles et al., 2017). Further, it implies that, as information costs decrease, equilibrium prices may increase in some markets and decrease in others. This is consistent with mixed empirical evidence on the impact of lower information costs on equilibrium prices (see, for example, Lynch and Ariely, 2000; Brown and Goolsbee, 2002). Finally, it implies also that when the cost of an attribute dimension in a market changes, there can be radical shifts in equilibrium product attributes. This is consistent with, for example, the shift observed in the U.S. coffee market after the introduction by Starbucks of a technology which drastically reduced the cost of quality improvement (Bordalo et al., 2016).

The remainder of the article is organized as follows: Section 2 describes the consumer and firm behaviour, Section 3 addresses the timing and equilibrium of the model, Section 4 offers relevant managerial and policy implications, and Section 5 concludes.

2 Theoretical Model

We contribute to the literature of product positioning under consumer information frictions (Kuksov, 2004; Bar-Isaac et al., 2012; Larson, 2013; Fishman and Levy, 2015) in three dimensions. First, we examine product positioning on a *multi-attribute space.* Second, we model consumers to be *heterogeneous* in information costs. Third, we model the process of gathering and processing information according to the *rational inattention* framework. This section details our consumer and firm behavioral assumptions to do so.

2.1 The Setup

We consider a continuum of heterogeneous consumers of measure 1, indexed by i, each of which, following the discrete-choice framework, is assumed to choose one of the $j = 1, 2$ products available in the market. Each product j is characterized by its position in a two-dimensional attribute space, a setting similar to Neven and Thisse (1987, 1990)'s seminal article. The first attribute, which we denote by x_j, represents the design characteristics of the product. The range of potential designs is, without loss of generality, represented by the $[0, 1]$ interval. The second attribute, which we denote by δ_j, represents the level of quality of the product. The range of potential qualities is represented by the interval $\left[\delta^{low}, \delta^{upp}\right]$. The lower bound level of quality can be interpreted as a minimum standard legal requirement or as being inherent to the production process, following Motta (1993). Without loss of generality, we define $\delta^{low} = 1$. The upper bound level of quality can be interpreted as the maximum quality level that is sustained by a market with a finite measure, following Berry and Waldfogel (2010). Without loss of generality, and solely for comparison purposes, we define $\delta^{upp} = 4$, such that $\delta^{upp} - \delta^{low}$ falls inside the nondegenerate segment in which the two product positioning equilibria (the *max-min* equilibrium and the *min-max* equilibrium), established by Neven and Thisse (1987, 1990), coexist.

2.2 Consumer Behaviour

We model consumer preferences using a characteristics approach in the lines of Lancaster (1966) and model consumer information frictions using the rational inattention framework in the lines of Matějka and McKay (2015).

2.2.1 Consumer Preferences

The preferences of each consumer are, in a characteristics-based approach (Lancaster, 1966), defined directly over the attribute dimensions of the available products. We consider that consumers do not rank designs in the same way, which portrays horizontal differentiation, following Hotelling (1929) and d'Aspremont et al. (1979). However, we consider that all consumers prefer a high quality to a low quality, which portrays vertical differentiation, following Spence (1975) and Mussa and Rosen (1978).

We allow consumer preferences over the two attribute dimensions to be heterogeneous. First, each consumer i has a most preferred design, denoted by $v_i \in [0, 1]$, and incurs in an utility loss whenever purchasing a product with a design that differs from her ideal preference point. The utility loss is quadratic with respect to the distance between the two points. This implies that the flow utility loss derived by this consumer from the design attribute of product j is given by $-\left(v_i - x_j\right)^2$. Second, each consumer i has a specific valuation per unit of quality, which we denote by $\theta_i \in [0, 1]$. This implies that the flow utility derived by this consumer from the quality dimension of product j is given by $\theta_i \delta_j$.[5]

[5] The quadratic utility loss assumption above avoids the discontinuities in the firms profit functions that may cause a problem for the existence of a pure-strategy price equilibrium. However, it introduces a functional form distinction between the marginal

The conditional indirect utility derived by each consumer i from purchasing a unit of product j (without considering the costs of gathering and processing information) aggregates the flow utilities associated to the product's attributes with the flow utilities associated to the consumption of goods from other markets. We assume a linear functional form for this aggregation, as follows:

$$u_{ij} = (y_i - p_j) - (v_i - x_j)^2 + \theta_i \delta_j, \tag{1}$$

where y_i denotes the income of consumer i, p_j denotes the price of product j and $(y_i - p_j)$ denotes the flow utility from consuming all other goods, which we treat as a composite commodity. We follow Neven and Thisse (1987, 1990) in assuming that y_i is large enough for all consumers to find a product that generates a positive utility in equilibrium.

The conditional indirect utility function above makes use of the common assumption in the discrete-choice framework literature that income and prices are additive separable, i.e., that income effects from price changes are negligible (see, e.g., Martin, 2017). This implies that income can be omitted from the indirect utility specification, since it does not vary across products:

$$u_{ij} = -p_j - (v_i - x_j)^2 + \theta_i \delta_j. \tag{2}$$

Exploring the implications of relaxing the additive separability assumption seems a very interesting area of future research.

2.2.2 Consumer Information Frictions

We consider information frictions to be an important part of the consumers' product choice environment. We do so by assuming that consumers have imperfect information in the following lines. Before entering the choice situation, consumers know the number of available products, but lack specific knowledge about their attributes and prices. However, they do hold a prior belief about the probability distribution of the unknown attributes and prices, which we denote by $G(\mathbf{x}, \delta, \mathbf{p})$, with $\mathbf{x} = (x_1, x_2)'$, $\delta = (\delta_1, \delta_2)'$ and $\mathbf{p} = (p_1, p_2)'$ representing the vector of designs, qualities and prices, respectively, of the different products.

In order to counteract the lack of specific knowledge, each consumer i can engage in an information gathering and processing strategy that refines (updates) her knowledge. For example, she can contact the firms, examine the products, ask questions, seek expert advice, read internet sites or forums, among many other strategies.[6] Such strategies generate signals

flow utility associated to the two attribute dimensions. The marginal flow utility of design is given by $2(v_i - x_j)$, which is product-specific and decreases with the design position, whereas the marginal flow utility of quality for consumer i is given by θ_i, which is constant with respect to the identity of the product and the level of quality. This functional form distinction has implications (although very slight) for the equilibrium designs and qualities, an issue we address in Section 3.

[6] Throughout the paper, the consumer is referred to as "she".

that consumers can use to update their beliefs about the attributes and prices of the different products. Let $\mathbf{s}_i = (\mathbf{s}_{i1}, \mathbf{s}_{i2})'$ denote the vector of signals (about the attributes and prices of all the products in the market) obtained from consumer i's information gathering and processing strategy, where $\mathbf{s}_{ij} = \left(x_j^{s_i}, \delta_j^{s_i}, p_j^{s_i}\right)'$ represents the subvector of signals associated to the design, quality and price of product j: $x_j^{s_i}$, $\delta_j^{s_i}$ and $p_j^{s_i}$, respectively.

We follow Matějka and McKay (2015) and allow consumers to choose any information gathering and processing strategy. They are completely free in deciding *what* and *how much* information to gather and process, i.e., in deciding, for example, what and how many questions to ask or posts to read. However, since different information gathering and processing strategies generate different signals (asking questions to a shop assistant is inherently different from reading internet forums, reading five forum posts is inherently different from reading fifty), the choice of an information gathering and processing strategy is implicitly a choice of the (distribution of) signals that are generated. As a consequence, and for simplicity, we model consumer i's information strategy choice as a decision involving the joint distribution of signals, attributes, and prices, i.e., $\mathbf{s}_i$ and $(\mathbf{x}, \delta, \mathbf{p})$, that are implicitly generated (in the lines of Caplin and Dean, 2013, and Matějka and McKay, 2015). Let $F(\mathbf{s}_i, \mathbf{x}, \delta, \mathbf{p})$ denote this joint distribution. Having chosen an information strategy (or equivalently, a joint distribution of signals, attributes, and prices), consumers use the signals received to update their beliefs. Let $F(\mathbf{x}, \delta, \mathbf{p}|\mathbf{s}_i)$ denote the updated beliefs of consumer i.

Consumers have, as discussed above, complete freedom to choose their information gathering and processing strategy. Nevertheless, they must consider that all such strategies are costly. For example, examining the products, asking questions or reading internet forums takes money, time and effort. We follow Caplin and Dean (2013), de Oliveira (2014), and Matějka and McKay (2015) and assume the cost of an information gathering and processing strategy to be proportional to the *amount* of information gathered and processed. We capture the latter by the reduction in the expected uncertainty involving the attributes and prices of the different products, where uncertainty (following Shannon, 1948) is measured by entropy. This reduction (even in cases associated a multivariate distributions like ours) is summarized in a single number, the mutual information between the prior and the updated (posterior) beliefs of consumers about $(\mathbf{x}, \delta, \mathbf{p})$. The cost of any information gathering and processing strategy $F(\mathbf{s}_i, \mathbf{x}, \delta, \mathbf{p})$ chosen by consumer i can then be expressed as:

$$c\left(F\left(\mathbf{s}_i, \mathbf{x}, \delta, \mathbf{p}\right); \gamma_i\right) \equiv \gamma_i \left(H\left(G\left(\mathbf{x}, \delta, \mathbf{p}\right)\right) - \int_{\mathbf{s}_i} H\left(F\left(\mathbf{x}, \delta, \mathbf{p}|\mathbf{s}_i\right)\right) F\left(d\mathbf{s}_i, \mathbf{x}, \delta, \mathbf{p}\right) \right), \quad (3)$$

where $c\left(F\left(\mathbf{s}_i, \mathbf{x}, \delta, \mathbf{p}\right); \gamma_i\right)$ denotes the cost of strategy $F(\mathbf{s}_i, \mathbf{x}, \delta, \mathbf{p})$, $\gamma_i > 0$ denotes consumer i's unit cost of gathering and processing information, $H(\cdot)$ denotes Shannon (1948)'s entropy function, $H(G(\mathbf{x},\delta,\mathbf{p}))$ denotes the uncertainty associated with the prior belief and, finally, $\int_{\mathbf{s}_i} H(F(\mathbf{x},\delta,\mathbf{p}|\mathbf{s}_i))\, F(d\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p})$ denotes the expected uncertainty associated with the posterior belief. We allow the unit cost of gathering and processing information to be consumer-specific, since the money, time and effort required to, for example, examine the products, ask questions or read internet forums may vary extensively from consumer to consumer.

To sum up, consumers face a trade-off. Strategies that gather and process more information are more informative, in the sense that generate more precise signals about $(\mathbf{x},\delta,\mathbf{p})$, but are also more costly. Due to this trade-off, it may happen that strategies that could generate signals precise enough to fully eliminate the uncertainty about $(\mathbf{x},\delta,\mathbf{p})$ are, from the consumer perspective, too costly. This implies that some uncertainty about the attributes of the different products may *rationally* persist when consumers make a purchase decision, leading consumers to select a product that may not be the one that yields the highest conditional indirect utility (inattention). In other words, incorporating consumer information frictions into the model introduces errors, and therefore, randomness, in the purchase decisions of consumers.

2.3 Firm Behaviour

We consider that there are two single-product risk-neutral firms in the industry, each of which producing one of the $j = 1, 2$ products available in the market. We assume that there are no costs associated with choosing different product designs, following Bar-Isaac et al. (2012), while there may exist costs associated with choosing different product qualities, following Mussa and Rosen (1978) and Motta (1993), as firms may engage in more skilled labour or more expensive raw materials and inputs to improve quality. Further, we assume that the production technology of each firm is characterized by the following marginal cost function:

$$mc(\delta_j) = \frac{\varphi}{2}\delta_j^2, \tag{4}$$

with $\varphi \geq 0$. This production technology is assumed identical for both firms so to rule out the trivial case in which product differentiation arises from technological differences between firms (Moorthy, 1988). The parameter φ, which drives the production technology, is exogenously drawn from probability distribution $T(\varphi)$.

3 Game, Timing and Equilibrium

We consider that consumers and firms play the following game, timed as depicted in Figure 1. At the beginning of the game, nature draws (i) the probability distribution of the production technology parameters $T(\varphi)$, from which the prior belief of consumers about the probability distribution of product attributes and prices, $G(\mathbf{x}, \delta, \mathbf{p})$ is constructed, (ii) the probability distribution of consumer types (associated with consumers' unit costs of gathering and processing information and preferences regarding product attributes), which we denote $P(\gamma_i, v_i, \theta_i)$ and (iii) the realization of the production technology of the firms φ. We assume that the probability distribution $T(\varphi)$ - and, consequently, $G(\mathbf{x}, \delta, \mathbf{p})$ - and $P(\gamma_i, v_i, \theta_i)$ are common knowledge among firms and consumers while the realization φ is known only to firms. We also make the following assumptions regarding $G(\delta, \mathbf{x}, \mathbf{p})$ and $P(\gamma_i, v_i, \theta_i)$.

Assumption 1. *Consumers have no prior knowledge about the attributes of the different products before entering the choice situation.*

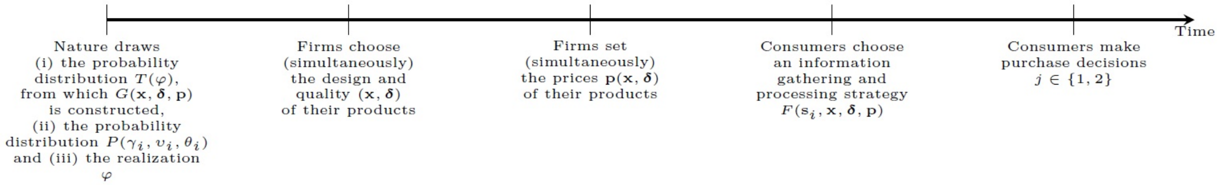
Assumption 2. *Consumer types over the unit cost of gathering and processing information and the different product attributes are independently distributed:* $P(\gamma_i, v_i, \theta_i,) = P_\gamma(\gamma_i) P_v(v_i) P_\theta(\theta_i)$.

Assumption 3. *Consumer types for each product attribute are uniformly distributed.*

Assumption 1 implies that the products are exchangeable in the prior $G(\delta, \mathbf{x}, \mathbf{p})$ and therefore, from a consumer perspective, *a priori* homogeneous. As a result, $\Pr^0_{i1} = \Pr^0_{i2} = 1/2$ for all i, where $\Pr^0_{ij}$ denotes the unconditional probability (i.e. before engaging in any information gathering and processing strategy) with which consumer i chooses to purchase product $j \in \{1, 2\}$. This constitutes a plausible benchmark case - in line with the search literature (see, e.g., Bakos, 1997). Further, it is consistent with a rational expectations assumption since, as we show below, the two products are exchangeable in equilibrium. Exploring the implications of more general consumers' prior beliefs seems a very interesting potential area for future research.

Assumption 2 allows us to rule out the trivial case in which product differentiation arises from correlation between consumer types, whereas Assumption 3 allows us to eliminate the effect of nonuniformity of preferences over attributes as a possible explanation of equilibrium product positioning (Moorthy, 1988). Both regularities, correlation between consumer types and non-uniform preference distribution (e.g., unimodal or bimodal), may lead to trivial standardization or differentiation (Neven, 1986), and confounds the effect of information frictions, which is what we wish to analyze.

Figure 1: *Timing of the Game*



Next, firms address a two-stage decision problem so to maximize own-profit. In the first stage, each firm (simultaneously) chooses the design and the quality of its single product.[7] In the second-stage, each firm (simultaneously) sets prices. The intuition behind the firms' two-stage structure assumption is borrowed from Hotelling (1929) and lies on the fact that prices are more flexible than design or quality in the short run. Thus, as discussed above, the second stage can be interpreted as the short-run where only prices are flexible, while the first stage can be viewed as the long-term when strategic decisions to determine the positions in the attribute space are taken. We model the decisions about design and quality as being simultaneous because production will often require the joint specification of these attributes. Finally, consumers also address a two-stage decision problem so to maximize their expected utility. In the first stage, each consumer chooses an information gathering and processing strategy, which generates signals that the consumer uses to refine her prior beliefs about the probability distribution of the unknown product attributes and prices. In the second stage, each consumer selects the product that provides the highest expected conditional indirect utility, given her updated beliefs.

We follow Bakos (1997) and Kuksov (2004) in assuming that the game is played in a single period setting. This assumption is illustrative and is presented for simplicity. It can be relaxed by incorporating into consumers' prior beliefs the eventual reputation effects that could result from the repeated interaction of consumers in the market. This extension to the analysis seems a very interesting area of future research.

We focus on the sub-game perfect Nash equilibrium of the game. We begin by addressing the consumers decision problem.

3.1 Consumers Decision Problem

We model, as discussed above, the decision problem of each consumer i in two stages. In the second stage, each consumer i is assumed to select the product which provides the highest

[7] Having firms choose quality is entirely equivalent to having firms choose vertical innovation rates, given identical initial qualities (Heeb, 2001).

expected conditional indirect utility, given her posterior belief $F(\mathbf{x},\delta,\mathbf{p}|\mathbf{s}_i)$:

$$U(F(\mathbf{x},\delta,\mathbf{p}|\mathbf{s}_i)) \equiv \max_{j\in\{1,2\}} \int_{(\delta,\mathbf{x},\mathbf{p})} u_{ij} F(d\mathbf{x},d\delta,d\mathbf{p}|\mathbf{s}_i), \tag{5}$$

where $U(F(\mathbf{x},\delta,\mathbf{p}|\mathbf{s}_i))$ denotes the highest expected utility induced by the information gathering and processing strategy $F(\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p})$ chosen in the first stage.

We assume that the choice of strategy $F(\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p})$, in the first stage, is driven by the desire to maximize the *ex-ante* expectation over $U(F(\mathbf{x},\delta,\mathbf{p}|\mathbf{s}_i))$ deducted of the cost of engaging in such strategy:

$$\max_{F(\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p})} \int_{\mathbf{s}_i}\int_{(\mathbf{x},\delta,\mathbf{p})} U(F(\mathbf{x},\delta,\mathbf{p}|\mathbf{s}_i))\, F(d\mathbf{s}_i|\mathbf{x},\delta,\mathbf{p})\, G(d\mathbf{x},d\delta,d\mathbf{p}) - c(F(\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p});\gamma_i) \tag{6}$$
$$\text{such that } \int_{\mathbf{s}_i} F(d\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p}) = G(\mathbf{x},\delta,\mathbf{p}),$$

where $F(\mathbf{s}_i|\mathbf{x},\delta,\mathbf{p})\,G(\mathbf{x},\delta,\mathbf{p}) = F(\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p})$ and $\int_{\mathbf{s}_i} F(d\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p}) = G(\mathbf{x},\delta,\mathbf{p})$ ensures that the posterior beliefs about $(\mathbf{x},\delta,\mathbf{p})$ are consistent with the prior.

Lemma 1. *The solution to consumer i's decision problem involves a first stage choice of information gathering and processing strategy, $F(\mathbf{s}_i,\mathbf{x},\delta,\mathbf{p})$, that implies a second stage purchase of product j, conditional on the realization $(\mathbf{x},\delta,\mathbf{p})$, with probability:*

$$\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i) = \frac{e^{\left(-p_j-(v_i-x_j)^2+\theta_i\delta_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}} e^{\left(-p_k-(v_i-x_k)^2+\theta_i\delta_k\right)/\gamma_i}}. \tag{7}$$

Proof. See Appendix. □

Lemma 1 establishes, following Matějka and McKay (2015), a multinomial logit formula for consumer i's conditional probability of purchasing product j. In particular, it establishes that this conditional probability, under Assumption 1, has two drivers. First, consumer i's indirect utilities u_{ik} for $k \in \{1,2\}$, whose impact follows the lines of the discrete-choice literature: the probability that the consumer purchases product j increases with the utility derived from product j and decreases with the utility derived by the competing product $k \neq j$. Second, consumer i's unit cost of gathering and processing information γ_i, which weights the importance of the above two drivers: when γ_i is high, the consumer rationally gathers and processes less information and so a higher degree of uncertainty about the attributes (and therefore about the induced indirect utilities) of the different products will persist at the time she makes the purchase decision. In such case, the conditional probability approaches

1/2 and so consumer bases her decision more on prior beliefs. This result is consistent with several recent empirical studies documenting that consumers process relatively little information in car insurance (Honka, 2014), S&P 500 index funds (Hortaçsu and Syverson, 2004), and automobiles (Moorthy et al., 1997; Morton et al., 2011), industries associated (for different reasons) with high unit costs of gathering and processing information.

Having computed the conditional purchase probabilities of each consumer i for the two products, we derive the aggregate demand for each product by integrating the corresponding consumer-specific probabilities over the probability distribution of consumer types $P(\gamma_i, \theta_i, v_i)$. Under Assumptions 2 and 3, the aggregate demand, $D_j(\mathbf{x}, \delta, \mathbf{p})$, for each product j is thereby given by:

$$D_j(\mathbf{x}, \delta, \mathbf{p}) = \int_{\gamma_i} \int_0^1 \int_0^1 \frac{e^{\left(-p_j - (v_i - x_j)^2 + \theta_i \delta_j\right)/\gamma_i}}{\sum_{k \in \{1,2\}} e^{\left(-p_k - (v_i - x_k)^2 + \theta_i \delta_k\right)/\gamma_i}} P_\gamma(d\gamma_i)\, P_v(dv_i)\, P_\theta(d\theta_i). \tag{8}$$

3.2 Firms Decision Problem

We model, as discussed above, the decision problem of each single-product firm j in two stages. The sub-game perfect Nash equilibrium of the game involving the decision problems of the two firms is obtained by backward induction. In the second stage, each firm is assumed to (simultaneously) set the prices that provide the highest expected profit, taking as fixed the set of first stage product designs and qualities, $(\bar{\mathbf{x}}, \bar{\delta})$:[8]

$$\max_{p_j} \Pi_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right) = \left(p_j - mc\left(\bar{\delta}_j\right)\right) D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right) = \left(p_j - \frac{\varphi}{2}\bar{\delta}_j^2\right) D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right). \tag{9}$$

A Nash equilibrium $\mathbf{p}^*$ in the second stage sub-game is a pair of prices p_j^* and p_{-j}^* such that, for any pair of product designs, $\bar{\mathbf{x}} = (\bar{x}_j, \bar{x}_{-j})'$, and qualities, $\bar{\delta} = \left(\bar{\delta}_j, \bar{\delta}_{-j}\right)'$, we have that:

$$\Pi_j\left(\bar{\mathbf{x}}, \bar{\delta}, p_j^*, p_{-j}^*\right) \geq \Pi_j\left(\bar{\mathbf{x}}, \bar{\delta}, p_j, p_{-j}^*\right), \ \forall p_j \geq 0, \ j = 1, 2. \tag{10}$$

The following results characterize the price equilibrium $\mathbf{p}^*$ in pure strategies.

Lemma 2. *If $P_\gamma(\gamma_i)$ is a log concave function, there exists a unique Nash equilibrium $\mathbf{p}^*$ in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\delta}$.*

Proof. See Appendix. □

[8] Firms can not strategically use prices to signal the products' attributes since consumers, in our framework, lack specific knowledge about the products' prices. Exploring the implications of using price to signal the products' attributes seems a very interesting area of future research.

Lemma 3. *If $P_\gamma(\gamma_i)$ is a log concave function, the price vector $\mathbf{p}^* = \left(p_j^*, p_{-j}^*\right)$ that supports the unique Nash equilibrium in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\delta}$, is strictly positive.*

Proof. See Appendix. □

Lemmas 2 and 3 imply that the unique Nash equilibrium $\mathbf{p}^* = \left(p_j^*, p_{-j}^*\right)$ in pure strategies in the second stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\delta}$, must satisfy the following system of first-order equations for all $j \in \{1, 2\}$:

$$\frac{\partial \Pi_j\left(\bar{\mathbf{x}}, \bar{\delta}, p_j^*, p_{-j}^*\right)}{\partial p_j} = D_j\left(\bar{\mathbf{x}}, \bar{\delta}, p_j^*, p_{-j}^*\right) + \left(p_j^* - \frac{\varphi}{2}\bar{\delta}_j^2\right)\frac{\partial D_j\left(\bar{\mathbf{x}}, \bar{\delta}, p_j^*, p_{-j}^*\right)}{\partial p_j} = 0, \tag{11}$$

which must have a unique solution $\mathbf{p}^*$, since any solution $\mathbf{p}^*$ must be a Nash equilibrium in pure strategies, and Lemma 2 establishes that $\mathbf{p}^*$ is unique. This unique Nash equilibrium defines prices to be functions of the pair of product designs and qualities in the market: $p_j^*(\mathbf{x}, \delta)$ and $p_{-j}^*(\mathbf{x}, \delta)$, which establish an equilibrium mapping from the vector of product attributes $(\mathbf{x}, \delta)$ to the vector of prices chosen by firms $\mathbf{p}$.

Having established that, if $P_\gamma^*(\gamma_i)$ is a log concave function, there exists a unique Nash price equilibrium in pure strategies in the second-stage sub-game, for any pair of product designs $\bar{\mathbf{x}}$ and qualities $\bar{\delta}$, we now address the first stage sub-game. If we substitute $p_j^*(\mathbf{x}, \delta)$ and $p_{-j}^*(\mathbf{x}, \delta)$ in firm $j \in \{1, 2\}$'s profits, we have:

$$\Pi_j\left(\mathbf{x}, \delta, p_j^*(\mathbf{x}, \delta), p_{-j}^*(\mathbf{x}, \delta)\right) = \Pi_j^*(\mathbf{x}, \delta). \tag{12}$$

The first stage Nash equilibrium in designs and qualities $(\mathbf{x}^*, \delta^*)$ is a pair of designs x_j^* and x_{-j}^*, and a pair of qualities δ_j^* and δ_{-j}^*, such that, for all $j \in \{1, 2\}$:

$$\Pi_j^*\left(x_j^*, x_{-j}^*, \delta_j^*, \delta_{-j}^*\right) \geq \Pi_j^*\left(x_j, x_{-j}^*, \delta_j, \delta_{-j}^*\right), \ \forall x_j \in [0, 1], \delta_j \in \left[\delta^{low}, \delta^{upp}\right]. \tag{13}$$

The complexity of this problem makes it difficult to find an analytical solution, since although we can show the existence of a unique solution $\mathbf{p}^*$, we cannot characterize the solution analytically. This has been the main obstacle preventing wider applications of the rational inattention framework (Tutino, 2011; Boyaci and Akçay, 2018). In order to address this issue, we make use of a numerical procedure (in line with Rhee et al., 1992; Heeb, 2001; Matějka and McKay, 2012; Boyaci and Akçay, 2018) and focus on particular realizations of the production technology (in line with Matějka and McKay, 2012).[9] We acknowledge this

[9] Dubé et al. (2009) follow a similar procedure to examine whether switching costs make markets less competitive.

concentrates the analysis on very special cases. Hopefully, however, our contribution can be seen as a stepping stone in the direction of a more complete and general analysis.

The numerical procedure incorporates a grid of product designs (x_j, x_{-j}) and qualities (δ_j, δ_{-j}) to cope with eventual multiple equilibria.[10,11] The details are as follows. *For each pair* of product designs and qualities $(\mathbf{x}, \delta)$ in the grid, we derive the unique Nash equilibrium $\mathbf{p}^*$ in pure strategies in the second stage sub-game using the structural system of first-order equations (11). We then use $(\mathbf{x}, \delta, \mathbf{p}^*)$ to compute the corresponding profits for the two firms. Next, we use these profits to find the best response function of each firm $j \in \{1, 2\}$ in terms of product design and quality. Finally, we identify the intersection (intersections) that characterize the Nash equilibrium (equilibria) in designs and qualities $(\mathbf{x}^*, \delta^*)$. The vectors of product designs and qualities $(\mathbf{x}^*, \delta^*)$ and prices $\mathbf{p}^*$ constitute a sub-game perfect Nash equilibrium. We examine this equilibrium considering that information costs can be homogeneous or heterogeneous across consumers. We do so for a realization of the production technology in which (i) $\varphi = 0$, corresponding to the case in which there are no costs associated with choosing different product qualities and (ii) $\varphi = 1$, corresponding to the case in which there are such costs.

3.2.1 Homogeneous Information Costs

Given the second stage Nash equilibrium in prices, we begin by examining the first stage Nash equilibrium in designs and qualities for the case in which consumers are homogeneous in their unit costs of gathering and processing information. This analysis mimics the existing literature that investigates the consequences of consumer information costs on firms' product attribute and pricing decisions (Kuksov, 2004; Bar-Isaac et al., 2012; Larson, 2013; Fishman and Levy, 2015), which assumes that consumers are *homogeneous* in information costs. We do so to examine the separate impact of considering a *multi*-endogenous product attributes framework.

Assumption 4. $\gamma_i = \gamma$ *for all* i.

This implies that the probability distribution of the unit cost of information across consumers is a $0-1$ indicator function over a convex set, as follows:

$$P^*_\gamma(\gamma_i) = \begin{cases} 1 & \text{if } \gamma_i = \gamma > 0 \text{ for all } i \\ 0 & \text{otherwise} \end{cases}, \tag{14}$$

which constitutes a classical example of a log concave function, as required by Lemma 2.

[10]We define the grid with an initial size of 5×10^{-2}, which we decrease whenever necessary to narrow our results.

[11]Assumption 1 is instrumental in significantly reducing the computational burden of this numerical procedure.

No Quality Costs

We first examine the implications of Assumption 4 for a realization of the production technology costs in which there are no costs associated with choosing different product qualities, following Shaked and Sutton (1982) and Neven and Thisse (1987, 1990). Such case corresponds to the following assumption.

Assumption 5. *The realization of the production technology costs is* $\varphi = 0$*.*

The following result establishes the first stage Nash equilibria in designs and qualities for the setting described above.

Proposition 1. *Under Assumptions 4 and 5:*

(a) If $\gamma \geq 0.51$, there exists a single Nash equilibrium in designs and qualities: a min-min equilibrium, in which firms minimize design and quality differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 4$.

(b) If $0.43 \leq \gamma < 0.51$, there exist two Nash equilibria in designs and qualities: (1) an intermediate-min equilibrium, in which firms select an intermediate level of design differentiation (that gradual and symmetrically increases as γ decreases), and minimize quality differentiation, given by: $x_j < 1$, $x_{-j} = 1 - x_j > 0$ and $\delta_j = \delta_{-j} = 4$, and (2) a min-min equilibrium, in which firms minimize design and quality differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 4$.

(c) If $0.39 \leq \gamma < 0.43$, there exist two Nash equilibria in designs and qualities: (1) a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by: $x_j = 1$, $x_{-j} = 0$ and $\delta_j = \delta_{-j} = 4$, and (2) a min-min equilibrium, in which firms minimize design and quality differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 4$.

(d) If $\gamma < 0.39$, there exist two Nash equilibria in designs and qualities: (1) a min-max equilibrium, in which firms minimize design differentiation and maximize quality differentiation, given by: $x_j = x_{-j} = 1/2$, and $\delta_j = 4$, $\delta_{-j} = 1$, and (2) a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by: $x_j = 1$, $x_{-j} = 0$ and $\delta_j = \delta_{-j} = 4$.[12]

[12] Proposition 1 implies that Assumption 1 is consistent with a rational expectations assumption, since the two products are - for all possible values of the unit cost of gathering and processing information - exchangeable.

Proposition 1 implies that when the unit cost of gathering and processing information is sufficiently high, i.e., $\gamma \geq 0.51$, there exists a single equilibrium in which firms do not differentiate the attributes of their products at all, neither in design nor quality, which yields a symmetric outcome in terms of price, aggregate demand, and consequently, profit. In this equilibrium, firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, and the top quality, $\delta_j = \delta_{-j} = \delta^{upp} = 4$. The reason for this *min-min* differentiation equilibrium is that given the high costs of gathering and processing information, consumers rationally choose to gather and process a low level of information. As a result, a high degree of uncertainty about the attributes of the products in the market will rationally persist at the time consumers make a purchase decision. As a consequence, consumers base the purchase decision mostly on prior beliefs. This implies that they are not too sensitive to actual prices and, thus, attribute differentiation is not required to relax price competition.

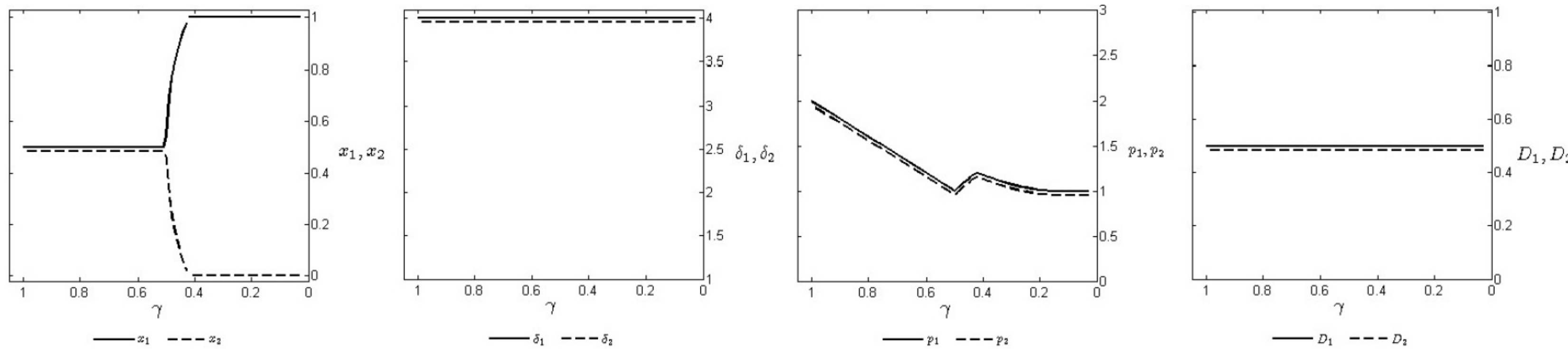
Proposition 1 implies also that, as the unit cost of gathering and processing information decreases, product attributes become instrumental in relaxing price competition. In order to see why, note that, as that cost decreases, consumers rationally gather and process relatively more information, which generates more precise signals about the attributes of the products in the market. As a result, the degree of uncertainty that rationally persists about those attributes at the time consumers make a purchase decision, decreases, increasing price competition between the two firms and decreasing the equilibrium price level. Firms respond, so to relax the increasing price competition, by engaging in attribute differentiation strategies. *Three* equilibrium strategy paths (depicted in Figure 2) emerge from Proposition 1, as the unit cost of gathering and processing information decreases to levels below $\gamma = 0.51$.[13]

A *min-min >>> intermediate-min >>> max-min path*, characterized by a continuous, gradual convergence, starting at $\gamma = 0.51$, from the *min-min* equilibrium towards the *max-min* equilibrium, achieved at $\gamma = 0.43$, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$). This convergence occurs through a series of

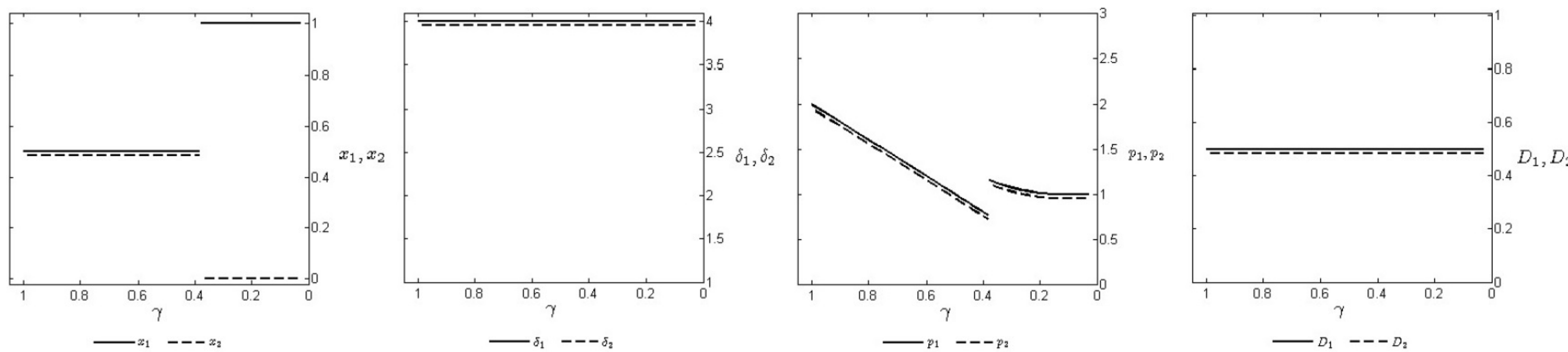
[13] The difference among the three equilibrium strategy paths presented is due to the functional form distinction between the marginal flow utility of design and quality, discussed in Section 2.2. First, the primary attribute dimension to be differentiated, as the unit cost of gathering and processing information decreases, is design. The reason being that as that cost decreases to levels below $\gamma = 0.51$, the incentives to deviate from the min-min equilibrium (in which firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, and the top quality, $\delta_j = \delta_{-j} = \delta^{upp} = 4$) by differentiating the design attribute are higher than the incentives to deviate by differentiating the quality attribute. In order to see why this is the case, note that the expectation of the marginal flow utility due to a decrease in the quality of a given product across consumers is given by $-E(\theta_i) = -0.5$, whereas the expectation of the marginal utility due to an increase in the design of a given product across consumers is given by $E[2(v_i - 0.5)] = 0$. Second, differentiation in quality always exhibits a discrete path (in γ), in contrast with differentiation in design, which also exhibits a continuous and gradual path (in γ). The reason being that the marginal utility for design is product-specific and decreases with the level of design, as discussed in Section 2.2, which penalizes high magnitude deviations in the design level.

Figure 2: *Equilibria under Assumptions 4 and 5*

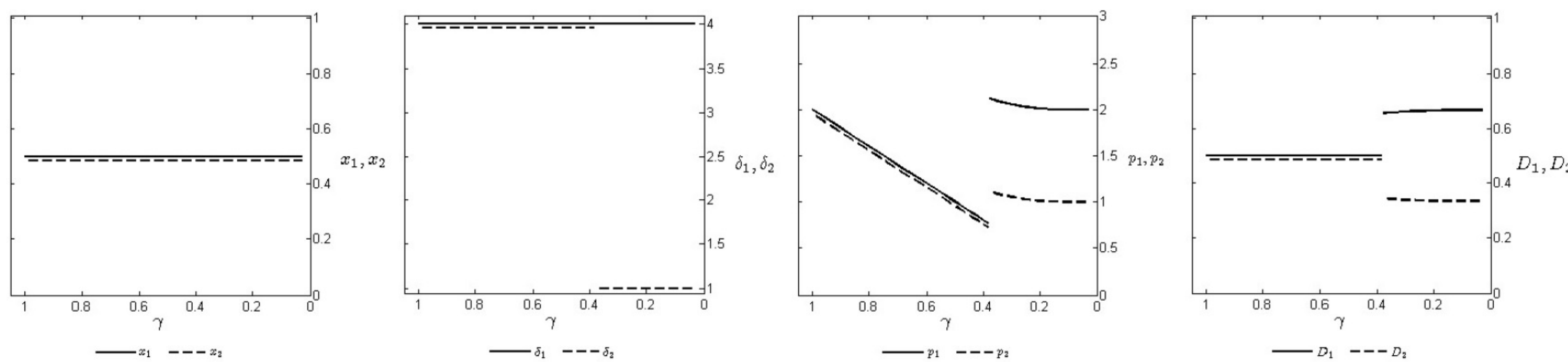
min-min >>> intermediate-min >>> max-min path:



min-min >>> max-min path:



min-min >>> min-max path:



intermediate-min equilibria, in which firms symmetric and gradually decrease differentiation along the design attribute dimension, $x_j < 1$ and $x_{-j} = 1 - x_j > 0$, as γ decreases. Both equilibria (the *intermediate-min* and the *max-min*) segment the market according to the ideal preference point of consumers for design: consumers with low ideal preference points for design are targeted by the low-design product, whereas consumers with high ideal preference points for design are targeted by the high-design product. This yields a symmetric outcome in terms of price, aggregate demand, and profit.

A *min-min* >>> *max-min path*, characterized by a discrete shift, that occurs at $\gamma = 0.43$, from the *min-min* equilibrium towards the *max-min* equilibrium, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$), which, as discussed above, yields a symmetric outcome in terms of price, aggregate demand, and profit.

A *min-min* >>> *min-max path*, characterized by a discrete shift, that occurs at $\gamma = 0.39$, from the *min-min* equilibrium towards the *min-max* equilibrium, in which firms maximize differentiation along the quality attribute dimension, $\delta_j = 4$ and $\delta_{-j} = 1$ (while maintaining no differentiation along the design dimension, $x_j = x_{-j} = 1/2$). The *min-max* equilibrium segments the market according to the valuation of consumers for quality: high-valuation consumers are targeted by the high-quality (hence, high-price) product, whereas low-valuation consumers are targeted by the low-quality (hence, low-price) product. This yields an asymmetric outcome in terms of price, aggregate demand, and profit, which favours the high-quality product.

The three equilibrium strategy paths above imply that when the unit cost of gathering and processing information is negligible, i.e., it decreases to levels below $\gamma = 0.39$, no *min-min* equilibrium is sustainable, establishing a *differentiation principle*. In such situations, the *min-max* and *max-min* equilibria coexist, in the lines of Neven and Thisse (1987, 1990),[14] establishing that the Nash equilibrium is robust to small deviations in the unit cost of information (as it is continuous in the degree of information frictions). Interestingly, firms are not indifferent between the two equilibria. The asymmetric outcome of the *min-max* strategy is favoured by the high-quality firm (but not by the low-quality firm) when compared to the symmetric outcome of the *max-min* strategy.

[14] This implies that the equilibria of our rational inattention model converges to the equilibria established in Neven and Thisse (1987, 1990)'s information frictionless model, as the unit cost of gathering and processing information becomes negligenciable. In other words, the introduction of information frictions does not change per se the nature of the attribute differentiation equilibria, which remains valid as long as that unit cost is negligenciable.

Quality Improvement Costs

We now examine a (more realistic) realization of the production technology costs in which there are costs associated with choosing different product qualities, following Moorthy (1988). In order to do so, we make the following assumption.

Assumption 6. *The realization of the production technology costs is* $\varphi = 1$.

The following result establishes the first stage Nash equilibria in designs and qualities for the setting described by Assumptions 4 and 6.

Proposition 2. *Under Assumptions 4 and 6:*

(a) If $\gamma \geq 0.51$, there exists a single Nash equilibrium in designs and qualities: a min-min equilibrium, in which firms minimize design and quality differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 1$.

(b) If $0.43 \leq \gamma < 0.51$, there exist two Nash equilibria in designs and qualities: (1) an intermediate-min equilibrium, in which firms select an intermediate level of design differentiation (that gradual and symmetrically increases as γ decreases), and minimize quality differentiation, given by: $x_j < 1$, $x_{-j} = 1 - x_j > 0$ and $\delta_j = \delta_{-j} = 1$, and (2) a min-min equilibrium, in which firms minimize design and quality differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 1$.

(c) If $0.27 \leq \gamma < 0.43$, there exist two Nash equilibria in designs and qualities: (1) a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by: $x_j = 1$, $x_{-j} = 0$ and $\delta_j = \delta_{-j} = 1$, and (2) a min-min equilibrium, in which firms minimize quality and design differentiation, given by: $x_j = x_{-j} = 1/2$ and $\delta_j = \delta_{-j} = 1$.

(d) If $\gamma < 0.26$, there exists a single Nash equilibrium in designs and qualities: a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by: $x_j = 1$, $x_{-j} = 0$ and $\delta_j = \delta_{-j} = 1$.[15]

Proposition 2 implies that when the unit cost of gathering and processing information is sufficiently high, i.e., $\gamma \geq 0.51$, there exists, as in the costless quality case, a single equilibrium in which firms do not differentiate the attributes of their products at all, neither in design nor quality, which yields a symmetric outcome in terms of price, aggregate demand, and

[15] Proposition 2 implies that Assumption 1 is consistent with a rational expectations assumption, since the two products are - for all possible values of the unit cost of gathering and processing information - exchangeable.

consequently, profit. In this equilibrium, firms select the design near the "center" of the market, $x_j = x_{-j} = 1/2$, as in the costless quality case, but select, instead, the bottom (and not the top) quality, $\delta_j = \delta_{-j} = \delta^{low} = 1$. The reason is as follows. In face of high information costs, consumers are highly uncertain about the attributes of the products at the time they make a purchase decision (since they rationally gather and process a low level of information and base their purchase decision mostly on prior beliefs), giving firms an incentive to deviate from specifications that incorporate costly attributes on the products.

Proposition 2 implies also, as in the costless quality case, that, as the unit cost of gathering and processing information decreases, attribute differentiation strategies become instrumental in relaxing price competition. *Two* equilibrium strategy paths (depicted in Figure 3) emerge from Proposition 2, as the unit cost of gathering and processing information decreases to levels below $\gamma = 0.51$.

A *min-min >>> intermediate-min >>> max-min path*, characterized by a continuous, gradual convergence, starting at $\gamma = 0.51$, from the *min-min* equilibrium towards the *max-min* equilibrium, achieved at $\gamma = 0.43$, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$). This convergence occurs through a series of *intermediate-min* equilibria, in which firms symmetric and gradually decrease differentiation along the design attribute dimension, $x_j < 1$ and $x_{-j} = 1 - x_j > 0$, as γ decreases (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$). As in the costless quality case, both equilibria (the *intermediate-min* and the *max-min*) segment the market according to the ideal preference point of consumers for design: consumers with low ideal preference points for design are targeted by the low-design product, whereas consumers with high ideal preference points for design are targeted by the high-design product. This yields a symmetric outcome in terms of price, aggregate demand, and profit.

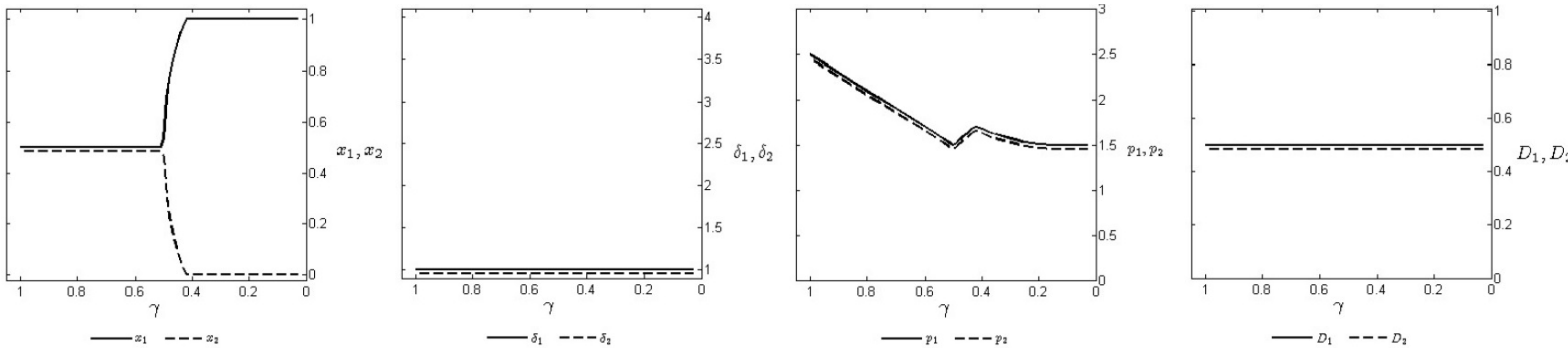
A *min-min >>> max-min path*, characterized by a discrete shift, that occurs at $\gamma = 0.27$, from the *min-min* equilibrium towards the *max-min* equilibrium, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 1$), which, as discussed above, yields a symmetric outcome in terms of price, aggregate demand, and profit.

The two equilibrium strategy paths above imply that when the unit cost of gathering and processing information is negligible, i.e., it decreases to levels below $\gamma = 0.27$, no *min-min* equilibrium is sustainable, establishing, as in the costless quality case, a *differentiation principle*. However, in the costly quality case, in contrast with the costless quality case, a single *max-min* differentiation equilibrium exists.[16] The reason is as follows. Differentiation along

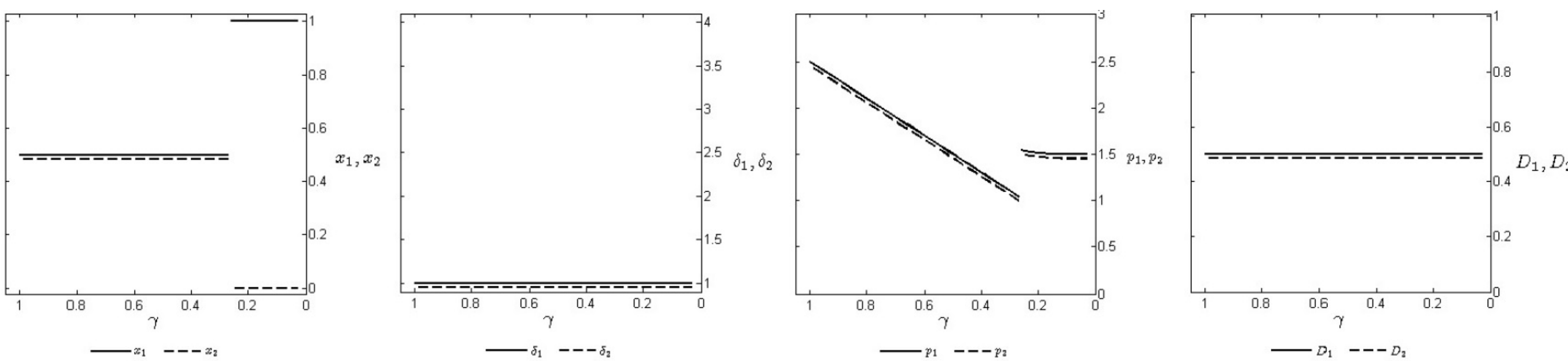
[16] This implies that although the introduction of information frictions does not change per se the nature of Neven and Thisse

Figure 3: *Equilibria under Assumptions* 4 *and* 6

min-min >>> intermediate-min >>> max-min path:



min-min >>> max-min path:



one attribute dimension is, as demonstrated in the costless quality case, more than enough to relax price competition. This implies that differentiation strategies along the quality attribute dimension are substitutes of differentiation strategies along the design attribute dimension. Since the latter are now costly, firms in equilibrium pursue the former.

3.2.2 Heterogeneous Information Costs

Given the second stage Nash equilibrium in prices, we now re-examine the first stage Nash equilibria in designs and qualities for the case in which consumers are heterogeneous in their costs of gathering and processing information, to examine (in comparison with the results above) the separate impact of considering *heterogeneous* information costs. We introduce this heterogeneity in the lines of Salop and Stiglitz (1977) and Boyaci and Akçay (2018), who assume that there are only two groups of consumers: one that can gather and process information at approximately no cost and another that must incur a cost to gather and process information. Finally, in order to illustrate the differential impact towards the homogeneous information costs' clearer, we make the simplest assumption that the proportion of consumers that can gather and process information at approximately no cost is of a sizeable dimension, as in Assumption 7 below. This is consistent with Stahl (1989), who argues that casual empiricism suggests that there is a non-negligible measure of who may simply love shopping. The equilibrium for cases in which the proportion of those consumers is smaller converges gradually from the ones established in the previous section towards the ones established in this section.

Assumption 7. *There are two equally-sized groups of consumers. A group Γ_a with $\gamma_i \to 0$ for all $i \in \Gamma_a$ and a group Γ_b with $\gamma_i = \gamma > 0$ for all $i \in \Gamma_b$.*

This implies that the probability distribution of the unit cost of information across consumers, within each group, is a $0-1$ indicator function over a convex set, as follows:

$$P^*_\gamma(\gamma_i) = \begin{cases} 1 & \text{if } \gamma_i \to 0 \text{ for all } i \in \Gamma_a \\ 1 & \text{if } \gamma_i = \gamma > 0 \text{ for all } i \in \Gamma_b \\ 0 & \text{otherwise} \end{cases}, \tag{15}$$

which constitutes a classical example of a log concave function, as required by Lemma 2. According to Lemma 1, consumers that can gather and process information at approximately no cost fully eliminate their uncertainty about product attributes and prices. They rationally

(1987, 1990)'s attribute differentiation equilibria, the introduction of costs of quality improvement does change it.

select to be attentive. Consumers that must incur a cost to gather and process information, on the other hand, do not fully eliminate their uncertainty. They rationally select to be inattentive and partially base their decision on prior beliefs. As such, Assumption 7 implies the coexistence of two groups of consumers: one attentive and another inattentive.

No Quality Costs

We first examine the implications of Assumption 7 for a realization of the production technology costs in which there are no costs associated with choosing different product qualities, i.e., under Assumption 5. The following result establishes the corresponding first stage Nash equilibrium in designs and qualities.

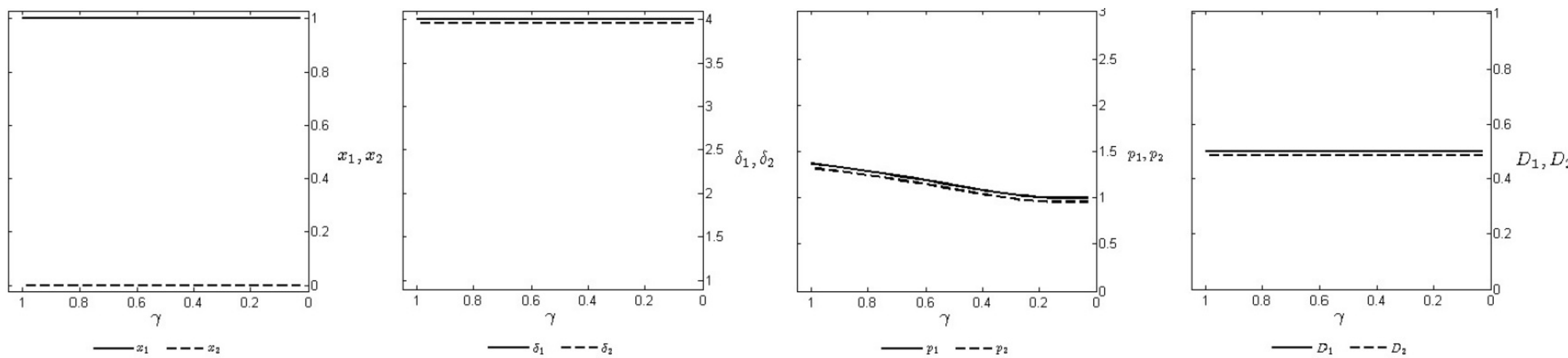
Proposition 3. *Under Assumptions 5 and 7, there exist two Nash equilibria in designs and qualities: (1) a min-max equilibrium, in which firms minimize design differentiation and maximize quality differentiation, given by:* $x_j = x_{-j} = 1/2$ *and* $\delta_j = 4$, $\delta_{-j} = 1$, *and (2) a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 4$.[17]

Proposition 3 implies that, in face of two equally-sized groups of consumers, one attentive and another inattentive, product attributes are instrumental in relaxing price competition, no matter the level of the cost of gathering and processing information of the high cost consumers. The reason is as follows. The group of attentive consumers rationally gathers and processes information that generates accurate signals about the attributes of the products in the market. As a result, the degree of uncertainty that rationally persists about those attributes, at the time those consumers make a purchase decision, is null. If this group of consumers is of a sizeable dimension (as in Assumption 7), the competing firms must engage in attribute differentiation strategies, in order to relax the otherwise fierce price competition (required to attract those attentive consumers). In particular, Proposition 3 establishes that two equilibrium strategies coexist, as depicted in Figure 4. A *min-max* equilibrium, in which firms maximize differentiation along the quality attribute dimension, $\delta_j = 4$ and $\delta_{-j} = 1$ (while maintaining no differentiation along the design dimension, $x_j = x_{-j} = 1/2$), and a *max-min* differentiation equilibrium, in which firms maximize differentiation along the design attribute dimension, $x_j = 1$ and $x_{-j} = 0$ (while maintaining no differentiation along the quality dimension, $\delta_j = \delta_{-j} = 4$). This establishes that the heterogeneity in information costs does not impact the robustness of the Nash equilibrium to small deviations in the unit

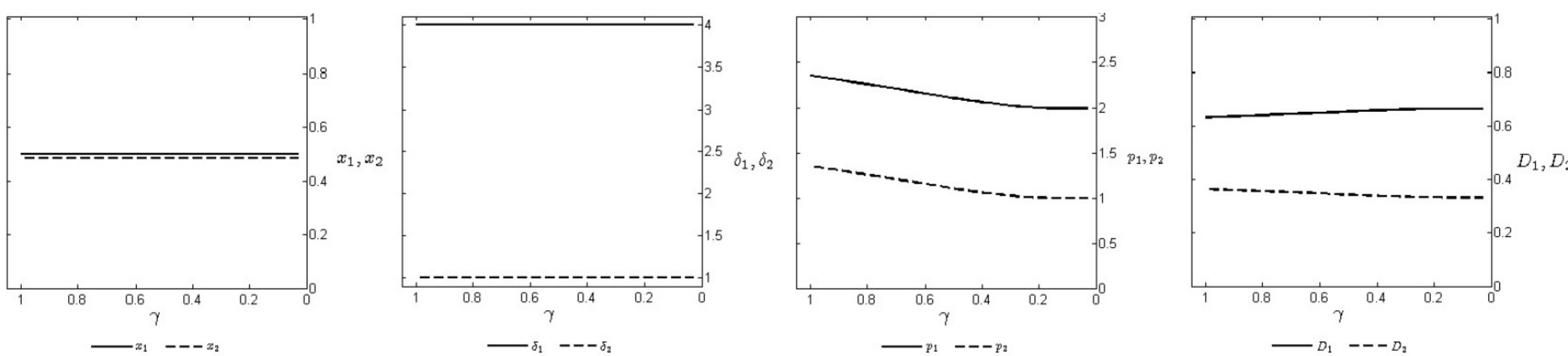
[17] Proposition 3 implies that Assumption 1 is consistent with a rational expectations assumption, since the two products are - for all possible values of the unit cost of gathering and processing information - exchangeable.

Figure 4: *Equilibria under Assumptions 5 and 7*

max-min equilibrium:



min-max equilibrium:



cost of information. Interestingly, firms are again not indifferent between the two equilibria. The latter yields, as discussed above, a symmetric outcome in terms of price, aggregate demand, and profit, whereas the former yields an asymmetric outcome in terms of price, aggregate demand, and profit, which favours the high-quality firm.

Quality Improvement Costs

We now examine the implications of Assumption 7 for a realization of the production technology costs in which there are costs associated with choosing different product qualities, i.e., under Assumption 6. The following result establishes the corresponding first stage Nash equilibrium in designs and qualities.

Proposition 4. *Under Assumptions 6 and 7, there exists a single Nash equilibrium in designs and qualities: a max-min equilibrium, in which firms maximize design differentiation and minimize quality differentiation, given by:* $x_j = 1$, $x_{-j} = 0$ *and* $\delta_j = \delta_{-j} = 1$.[18]

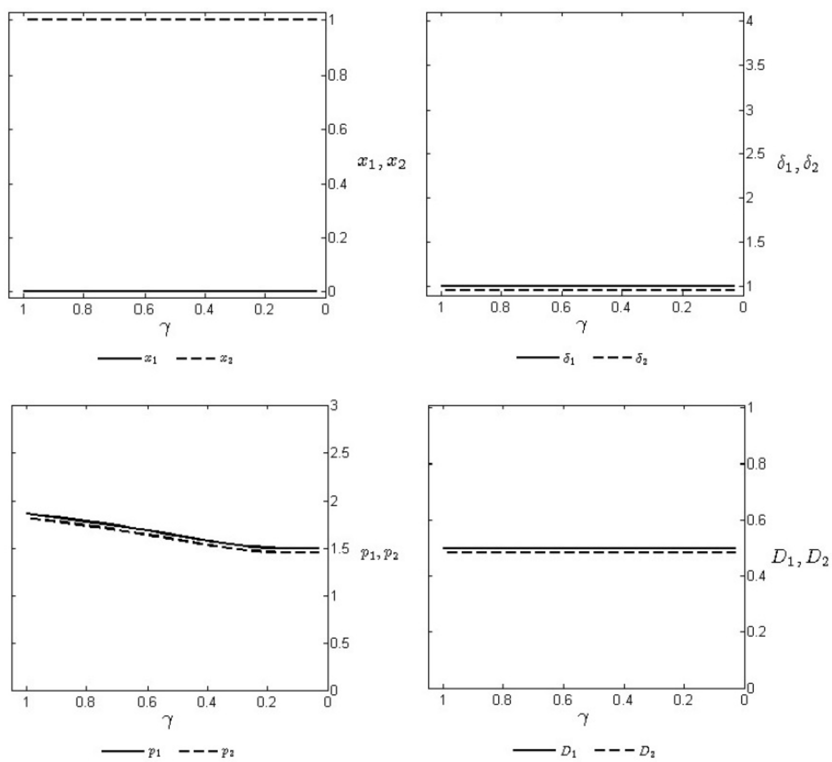
Proposition 4 implies, as in the costless quality case, that, in face of two equally-sized groups of consumers, one attentive and another inattentive, product attributes are instrumental in relaxing price competition, no matter the level of the cost of gathering and processing information of the inattentive consumers - for exactly the same reason as described above. However, in the costly quality case, in contrast with the costless quality case, a single *max-min* differentiation strategy exists in equilibrium, as depicted in Figure 5. The reason is as follows. Differentiation along one attribute dimension is, as demonstrated in the costless quality case, more than enough to relax price competition. This implies that differentiation strategies along the quality attribute dimension are substitutes of differentiation strategies along the design attribute dimension. Since the latter are now costly, firms in equilibrium pursue the former.

4 Managerial and Policy Implications

This section summarizes the managerial and policy implications of our results. We begin by addressing the managerial implications. We focus on three. First, Propositions 1 and 2 imply that managers of firms that face a single homogeneous group of inattentive consumers in their information costs *do not have an incentive to differentiate their products when information costs are high*, but should *increase differentiation as those information costs fall*, so to relax

[18]Proposition 4 implies that Assumption 1 is consistent with a rational expectations assumption, since the two products are - for all possible values of the unit cost of gathering and processing information - exchangeable.

Figure 5: *Equilibrium under Assumptions* 6 *and* 7



the otherwise increasing price competition. Independently of whether quality improvement is costly or not.

Implication 1 *When information costs are homogenous across consumers, firms have an incentive to respond to lower information costs by increasing differentiation.*

As a consequence, we may infer that when information costs are homogenous across consumers, information costs and product differentiation are *substitutes.* Further, since product differentiation countervails the negative impact on prices, we may also infer, as depicted in Figures 2 and 3, that equilibrium price levels *may increase* as the unit cost of gathering and processing information decreases. These implications are consistent with the standard search literature. See, for example, as discussed above, Kuksov (2004), Bar-Isaac et al. (2012), Larson (2013), and Fishman and Levy (2015).
Second, Propositions 3 and 4 imply that managers of firms that face two equally-sized and heterogeneous groups of consumers in terms of their information costs, one attentive (that can gather and process information at approximately no cost) and another inattentive (that must incur a cost to gather and process information), should *always differentiate their products as maximum as possible*, independently of whether quality improvement is costly or not.

Implication 2 *When information costs are heterogenous across consumers, the incentives of firms to respond to lower information costs by increasing differentiation only holds if the proportion of consumers with low information costs in the market is small.*

As a consequence, we may infer, as depicted in Figures 4 and 5, that equilibrium price levels *do not increase* (and, in fact, tend to decrease) as the unit cost of gathering and processing information of the inattentive consumers decreases if the proportion of consumers with low information costs (and consequently of attentive consumers) in the market is small. In order to see why note that, as this unit cost decreases, inattentive consumers rationally gather and process more information, which generates more precise signals about the attributes of the products in the market. As a consequence, the degree of uncertainty that rationally persists about product attributes at the time those consumers make a purchase decision, decreases, which in turn increases price competition (to attract not only the attentive consumers, but also the inattentive ones). This result seems to suggest that the incentives of firms, identified by the standard search literature, to respond to lower information costs by increasing differentiation *depend critically* on the heterogeneity of those costs across consumers. This implication is consistent with mixed empirical evidence on the impact of lower information costs on equilibrium prices (see, for example, Lynch and Ariely, 2000; Brown and Goolsbee, 2002).

Third, Propositions 1 to 4 imply that, in the two cases above, firms *do not need to differentiate their products along all attribute dimensions*. Differentiation along one attribute dimension is more than enough to relax price competition. In a costless quality setting, firms may, in equilibrium, differentiate along *any* attribute dimension, in a costly quality setting, firms should, in equilibrium, differentiate along the *least-costly* attribute dimension.

Implication 3 *If firms have an incentive to respond to lower information costs by increasing differentiation, they should do so along the least-costly attribute dimension.*

This implication makes clear that when the cost of an attribute dimension in a market changes, there can be radical shifts in product attributes and prices. It is consistent with, for example, the shift observed in the U.S. coffee market after the entry of Starbucks, which moved from a low-quality equilibrium to a high-quality equilibrium after a decrease in the cost of quality. Bordalo et al. (2016) explain this shift with the introduction, by Starbucks, of a technology which drastically reduced the cost of quality improvement. This cost reduction induces an increase in quality and, in turn, an increase in the attention that consumers assign to quality. We provide an alternative explanation. In a costless quality setting, firms may, in equilibrium, differentiate along any attribute dimension, while in a costly quality setting, firms should, in equilibrium, differentiate along the least-costly attribute dimension.
We now address the policy implications. We focus on one main implication. Lemma 1 implies that consumers may optimally select information strategies that do not fully eliminate their uncertainty, i.e., they *may choose to be rationally inattentive* when making a purchase decision. This creates market power for firms and relaxes price competition. The existing literature provided no basis for regulators to countervail this market power reducing firms' obfuscation of product information (and thus reducing consumer information costs). The reason being that, according to the existing literature, lower information costs could lead to higher equilibrium prices if firms respond to those lower costs by increasing product differentiation (so to decrease price competition). In contrast, Propositions 3 and 4 provide a basis for this intervation if the proportion of consumers with high information costs (and consequently of inattentive consumers) in the market is small since, in such case, equilibrium prices do not increase (and, in fact, tend to decrease) as the information costs of inattentive consumers decrease.

Implication 4 *When information costs are heterogenous across consumers, regulators should intervene to reduce obfuscation of product information if the proportion of consumers with high information costs in the market is small.*

5 Conclusion

We contribute to the literature of product positioning under consumer information frictions (Kuksov, 2004; Bar-Isaac et al., 2012; Larson, 2013; Fishman and Levy, 2015) in three aspects. First, we examine product positioning on a *multi-attribute* space. Second, we consider the money, time and effort required to gather and process information is *heterogeneous* across consumers. Third, we model the process of gathering and processing information according to the *rational inattention* framework. We show that, under this setting, the competition is inherently different from the existing literature. First, firms do have an incentive to respond to lower information costs by increasing differentiation but solely *along the least-costly attribute dimension* and *if the proportion of attentive consumers in the market is small.* This implies that accounting for multi-attributes and heterogeneous information costs is important. Second, the equilibrium outcomes derived from our theoretical framework are (unlike most standard search literature) robust to small deviations in information costs. This implies that modelling consumer limited attention is important.

We acknowledge the analysis is concentrated on very special cases. To our defense, most of the extant rational inattention literature that addresses firms and consumers' endogenous decisions in duopoly differentiated product settings has followed a similar approach, due to the framework's mathematical complexity. We hope, however, that our contribution can be seen as a stepping stone in the direction of a more complete and general analysis; and that our findings can provide practitioners and regulators with insights regarding equilibrium multi-attribute differentiation strategies when consumers have heterogeneous limited attention.

This article leaves many other issues yet to be explored. As discussed above, we believe that incorporating the following features constitute very interesting areas of future research: (*i*) considering a higher number of firms in the market, (*ii*) considering that consumers' income is not large enough for all consumers to find a product that generates a positive utility in equilibrium, (*iii*) relaxing the additive separability between income and prices in the conditional indirect utility function, (*iv*) including reputation issues that arise in multiple period settings, (*v*) accounting for more general consumers' prior beliefs, and (*vi*) considering more general functional forms of heterogeneity in consumers information costs.

Appendix

In this Appendix, we provide the proofs to the various lemmas stated in the main text.

Proof of Lemma 1. The probability that each consumer i purchases, in the second stage, product j, conditional on the realization $(\mathbf{x}, \delta, \mathbf{p})$ and the information strategy, $F(\mathbf{s}_i, \mathbf{x}, \delta, \mathbf{p})$, chosen in the first stage, is given by $\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=\int_{\mathbf{s}_i\in\Gamma_j}F(d\mathbf{s}_i|\mathbf{x},\delta,\mathbf{p})$, where Γ_j denotes the set of signals that lead to the choice of product j.

Matějka and McKay (2015) show (see Corollary 1 therein) that the collection of the conditional probabilities above for consumer i, $\mathcal{P}=\{\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\}_{j\in\{1,2\}}$, is induced by a solution to her decision problem if and only if it solves the following optimization problem.

$$\max_{\mathcal{P}=\{\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\}_{j\in\{1,2\}}}\sum_{j\in\{1,2\}}\int_{(\mathbf{x},\delta,\mathbf{p})}u_{ij}\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\,G(d\mathbf{x},d\delta,d\mathbf{p})-c(\mathcal{P},G(\mathbf{x},\delta,\mathbf{p});\gamma_i),\tag{16}$$

subject to:

$$\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\geq 0,\quad \forall j\in\{1,2\}\text{ and }\forall(\mathbf{x},\delta,\mathbf{p})\in\mathbb{R}^6\tag{17}$$

$$\sum_{j\in\{1,2\}}\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=1,\quad \forall(\mathbf{x},\delta,\mathbf{p})\in\mathbb{R}^6.\tag{18}$$

The cost of information (given in equation (3)), can be calculated from $\mathcal{P}$, as follows:

$$\begin{aligned}c(\mathcal{P},G(\mathbf{x},\delta,\mathbf{p});\gamma_i) &= \gamma_i\Bigg(-\sum_{j\in\{1,2\}}\Pr^0_{ij}\log\left(\Pr^0_{ij}\right) \qquad (19)\\ &+\int_{(\delta,\mathbf{x},\mathbf{p})}\left(\sum_{j\in\{1,2\}}\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\log\left(\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\right)\right)G(d\mathbf{x},d\delta,d\mathbf{p})\Bigg).\end{aligned}$$

where $\Pr^0_{ij}=\int_{(\mathbf{x},\delta,\mathbf{p})}\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\,G(\mathbf{x},\delta,\mathbf{p})$ denotes the unconditional probability (i.e. before engaging in any information gathering and processing strategy) that the consumer purchases product j, which is computed across the different realizations of $(\mathbf{x},\delta,\mathbf{p})$ according to the prior belief $G(\mathbf{x},\delta,\mathbf{p})$. Under Assumption 1, we have that $\Pr^0_{ij}=1/2$ for all i and j.

The Lagrangian of the problem above is:

$$\begin{aligned}\mathcal{L}(\mathcal{P}) &= \sum_{j\in\{1,2\}}\int_{(\mathbf{x},\delta,\mathbf{p})}u_{ij}\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\,G(d\mathbf{x},d\delta,d\mathbf{p})-\gamma_i\Bigg(-\sum_{j=1}^{2}\frac{1}{2}\log\left(\frac{1}{2}\right) \qquad (20)\\ &+\int_{(\mathbf{x},\delta,\mathbf{p})}\left(\sum_{j\in\{1,2\}}\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\log\left(\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\right)\right)G(d\mathbf{x},d\delta,d\mathbf{p})\Bigg)\\ &+\int_{(\mathbf{x},\delta,\mathbf{p})}\lambda_{ij}(\mathbf{x},\delta,\mathbf{p})\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\,G(d\mathbf{x},d\delta,d\mathbf{p})\\ &-\int_{(\mathbf{x},\delta,\mathbf{p})}\rho_i(\delta,\mathbf{x},\mathbf{p})\left(\sum_{j\in\{1,2\}}\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)-1\right)G(d\mathbf{x},d\delta,d\mathbf{p}),\end{aligned}$$

where $\lambda_{ij}(\mathbf{x},\delta,\mathbf{p})\geq 0$ denotes the Lagrange multipliers associated to restriction (17) and $\rho_i(\mathbf{x},\delta,\mathbf{p})$ denotes the Lagrange multipliers associated to restriction (18).

As $\Pr^0_{ij}=\frac{1}{2}>0$, then the first order conditions with respect to the conditional probabilities associated to the two products, $\Pr_{i1}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)$ and $\Pr_{i2}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)$, are given by:

$$u_{i1}+\lambda_{i1}(\mathbf{x},\delta,\mathbf{p})-\rho_i(\mathbf{x},\delta,\mathbf{p})+\gamma_i\left(\log\left(\Pr^0_{i1}\right)+1-\log\left(\Pr_{i1}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\right)-1\right) = 0 \quad (21)$$

$$u_{i2}+\lambda_{i2}(\mathbf{x},\delta,\mathbf{p})-\rho_i(\mathbf{x},\delta,\mathbf{p})+\gamma_i\left(\log\left(\Pr^0_{i2}\right)+1-\log\left(\Pr_{i2}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\right)-1\right) = 0 \quad (22)$$

Given that we follow Neven and Thisse (1987, 1990) in assuming that y_i is large enough for all consumers to find a product that generates a positive utility in equilibrium, we have that $u_{ij}>0$. As a consequence, the above set of first order conditions implies that $\Pr_{i1}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)>0$ and $\Pr_{i2}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)>0$. In order to see why whenever $\Pr^0_{ij}>0$, we must have $\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)>0$, suppose (without loss of generality) that $\Pr_{i1}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=0$, which implies $\log\left(\Pr_{i1}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)\right)=-\infty$, on a set of positive measure with respect to $G(\mathbf{x},\delta,\mathbf{p})$. Since we assume that $\Pr^0_{i1}>0$, we have $\log\left(\Pr^0_{i1}\right)>-\infty$. This implies, since $\lambda_{i1}(\mathbf{x},\delta,\mathbf{p})\geq 0$, that $\rho_i(\mathbf{x},\delta,\mathbf{p})=\infty$ on a set of positive measure to make the first order condition (21) hold. However, if $\rho_i(\mathbf{x},\delta,\mathbf{p})=\infty$, then, in order for the first order condition (22) to hold for all realizations $(\mathbf{x},\delta,\mathbf{p})$, we must have $\Pr_{i2}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=0$ or $\lambda_{i2}(\mathbf{x},\delta,\mathbf{p})=\infty$. But $\lambda_{i2}(\delta,\mathbf{x},\mathbf{p})>0$ will only be satisfied if $\Pr_{i2}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=0$, when restriction (17) is binding. This implies (without loss of generality) that if $\Pr_{i1}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)$, then $\Pr_{i2}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=0$. However, this is not possible, since then: $\sum_{j\in\{1,2\}}\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=0$, which contradicts restriction (18).
As a consequence, whenever $\Pr^0_{ij}=\frac{1}{2}>0$, we must have $\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)>0$. This implies that restriction (17) does not bind, and so we must have $\lambda_{ij}(\mathbf{x},\delta,\mathbf{p})=0$. Therefore, the first order condition for any product $j\in\{1,2\}$ can be rearranged to:

$$\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=\frac{1}{2}e^{(u_{ij}-\rho_i(\mathbf{x},\delta,\mathbf{p}))/\gamma_i}=\frac{\frac{1}{2}e^{u_{ij}/\gamma_i}}{e^{\rho_i(\mathbf{x},\delta,\mathbf{p})/\gamma_i}}. \quad (23)$$

If we substitute this result into restriction (18), we have that $e^{\rho_i(\mathbf{x},\delta,\mathbf{p})/\gamma_i}=\sum_{k\in\{1,2\}}\frac{1}{2}e^{u_{ik}/\gamma_i}$, which yields:

$$\Pr_{ij}(\mathbf{x},\delta,\mathbf{p};\gamma_i,\theta_i,v_i)=\frac{e^{u_{ij}/\gamma_i}}{\sum_{k\in\{1,2\}}\Pr^0_{ik}\,e^{u_{ik}/\gamma_i}}. \quad (24)$$

□

Proof of Lemma 2. The heart of the proof lies in establishing that, in this setting of single-products firms in which firm j and firm $-j$ set prices to maximize profits, the aggregate demand function and the cost function faced by each firm satisfy Mizuno (2003)'s five conditions for the existence of a unique (pure strategies) price equilibrium. Let $C_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)=mc(\delta_j)\,D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)$ denote the cost function faced by firm j. These five conditions are:

(i) $D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)$ is strictly positive and strictly decreasing in p_j on $\mathbb{R}^2$.

(ii) $D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)=D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}+\tau\mathbf{1}\right)$ for all τ, where $\mathbf{1}=(1,1)'$.

(iii) $D_j\left(\bar{\mathbf{x}},\bar{\delta},p^H_j,p^H_{-j}\right)D_j\left(\bar{\mathbf{x}},\bar{\delta},p^L_j,p^L_{-j}\right)\geq D_j\left(\bar{\mathbf{x}},\bar{\delta},p^H_j,p^L_{-j}\right)D_j\left(\bar{\mathbf{x}},\bar{\delta},p^L_j,p^H_{-j}\right)$ for $p^H_j>p^L_j$, and $p^H_{-j}>p^L_{-j}$.

(iv) $C_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)$ is convex in $D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)$.

(v) $D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)$ is increasing in p_{-j} on $\mathbb{R}^2$.

Condition (i) consists of two parts. The first part of condition (i) requires aggregate demand to be strictly positive for every price vector on $\mathbb{R}^2$. From equation (8) it is straightforward to show that this condition is satisfied in our model. For every price vector on $\mathbb{R}^2$, $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,\theta_i,v_i\right)>0$ for every consumer i and product j, and consequently $D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)>0$ for every product j. The second part of condition (i) establishes the standard law of demand. In our model, note that from (8), we have:

$$\begin{aligned}
\frac{\partial D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)}{\partial p_j} &= \int_{\gamma_i}\int_0^1\int_0^1 \frac{\partial \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,,\theta_i\right)}{\partial p_j} P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right) \qquad (25)\\
&= -\int_{\gamma_i}\frac{1}{\gamma_i}\left(\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)\left(1-\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)\right)P_\gamma\left(d\gamma_i\right)\\
&= -\int_{\gamma_i}\frac{1}{\gamma_i}\left(\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)\right.\\
&\qquad\left.-\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)^2 P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)\right)P_\gamma\left(d\gamma_i\right).
\end{aligned}$$

Since $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)>0$ for all i and j, we have $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)<1$ for all i and j, because $\Pr_{i1}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)+\Pr_{i2}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)=1$. This result implies that the integrand $\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)>\Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)^2$, and therefore, using the inequality rule for definite integrals, we must have:

$$\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right) \quad > \quad \int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p};\gamma_i,v_i,\theta_i\right)^2 P_v\left(dv_i\right)P_\theta\left(d\theta_i\right). \qquad (26)$$

Because $\gamma_i>0$, this establishes that the second part of the condition is also satisfied, since we have that $\partial D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)/\partial p_j<0$ for every product j.

Condition (ii) requires aggregate demand for a product to depend only on price differences, which is also satisfied by our aggregate demand function for every product j:

$$\begin{aligned}
D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}+\tau\mathbf{1}\right) &= \int_{\gamma_i}\int_0^1\int_0^1 \Pr_{ij}\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}+\tau\mathbf{1};\gamma_i,v_i,\theta_i\right)P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right) \qquad (27)\\
&= \int_{\gamma_i}\int_0^1\int_0^1\left(\frac{e^{\left(-p_j-\tau-(v_i-\bar{x}_j)^2+\theta_i\bar{\delta}_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}}e^{\left(-p_k-\tau-(v_i-\bar{x}_k)^2+\theta_i\bar{\delta}_k\right)/\gamma_i}}\right)P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)\\
&= \int_{\gamma_i}\int_0^1\int_0^1\left(\frac{e^{(-\tau/\gamma_i)}e^{\left(-p_j-(v_i-\bar{x}_j)^2+\theta_i\bar{\delta}_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}}e^{(-\tau/\gamma_i)}e^{\left(-p_k-(v_i-\bar{x}_k)^2+\theta_i\bar{\delta}_k\right)/\gamma_i}}\right)P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)\\
&= \int_{\gamma_i}\int_0^1\int_0^1\left(\frac{e^{\left(-p_j-(v_i-\bar{x}_j)^2+\theta_i\bar{\delta}_j\right)/\gamma_i}}{\sum_{k\in\{1,2\}}e^{\left(-p_k-(v_i-\bar{x}_k)^2+\theta_i\bar{\delta}_k\right)/\gamma_i}}\right)P_\gamma\left(d\gamma_i\right)P_v\left(dv_i\right)P_\theta\left(d\theta_i\right)\\
&= D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right).
\end{aligned}$$

Condition (iii) requires the aggregate demand function $D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)$ of every product j to be totally positive of order 2 in prices. In order to show that this condition is, in fact, satisfied, it suffices to show that the population distribution function $P\left(\gamma_i,v_i,\theta_i\right)=P_\gamma\left(\gamma_i\right)P_v\left(v_i\right)P_\theta\left(\theta_i\right)$ is log concave. As Mizuno (2003) shows, if $P\left(\gamma_i,v_i,\theta_i\right)$ is log concave, $D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)$ is log concave by the Prekópa–Borel theorem for every product j. Furthermore, since, under condition (ii), aggregate demand for a product depends only on price differences, we can always rewrite the aggregate demand function as $D_j\left(\bar{\mathbf{x}},\bar{\delta},\mathbf{p}\right)=g_j\left(\bar{\mathbf{x}},\bar{\delta},p_j-p_k\right)$ for every products

j and $k \neq j$, which by the duality between log concave functions and totally positive of order 2 functions, establishes that $g_j\left(\bar{\mathbf{x}}, \bar{\delta}, p_j - p_k\right)$ and hence $D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right)$ is totally positive of order 2 in p_j and p_k. It remains to be shown that $P\left(\gamma_i, v_i, \theta_i\right) = P_\gamma\left(\gamma_i\right) P_v\left(v_i\right) P_\theta\left(\theta_i\right)$ is, in fact, log concave. The proposition establishes that $P_\gamma\left(\gamma_i\right)$ is a log concave function. Further, in our model, $P_v\left(v_i\right)$ and $P_\theta\left(\theta_i\right)$ are assumed to denote a uniform distribution. Since uniform distributions are log concave, and the product of log concave functions, is log concave, condition (iii) is, in fact, satisfied.

Condition (iv) requires the cost function to be convex in demand, which is satisfied in our model since we have that $\partial^2 C_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right) / \partial D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right)^2 = 0$.

Finally, condition (v) requires that any two product are gross substitutes, which again is satisfied in our model since $\partial D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right) / \partial p_{-j} > 0$. In order to see why, note that from (8), we have:

$$
\begin{aligned}
\frac{\partial D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right)}{\partial p_{-j}} &= \int_{\gamma_i} \int_0^1 \int_0^1 \frac{\partial \operatorname{Pr}_{ij}\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right)}{\partial p_{-j}} P_\gamma\left(d\gamma_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right) \qquad (28) \\
&= \int_{\gamma_i} \frac{1}{\gamma_i} \left(\int_0^1 \int_0^1 \operatorname{Pr}_{ij}\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right) \operatorname{Pr}_{i-j}\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)\right) P_\gamma\left(d\gamma_i\right) \\
&= \int_{\gamma_i} \frac{1}{\gamma_i} \left(\int_0^1 \int_0^1 \operatorname{Pr}_{ij}\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right) \left(1 - \operatorname{Pr}_{ij}\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right)\right) P_v\left(dv_i\right) P_\theta\left(d\theta_i\right)\right) P_\gamma\left(d\gamma_i\right),
\end{aligned}
$$

where the last equality is just a consequence of the fact that $\operatorname{Pr}_{ij}\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right) + \operatorname{Pr}_{i-j}\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right) = 1$, for every consumer i. This result establishes that our model implies $\partial D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right) / \partial p_{-j} = -\partial D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right) / \partial p_j$, which, using condition (i), ensures that condition (v) is, in fact, satisfied.

□

Proof of Lemma 3. Note that $\Pi_j\left(\bar{\mathbf{x}}, \bar{\delta}, p_j, p_{-j}\right) < \Pi_j\left(\bar{\mathbf{x}}, \bar{\delta}, mc_j, p_{-j}\right)$ for any $p_j < mc_j = mc\left(\delta_j\right)$, so that $p_j^* \geq mc_j, \forall j = 1, 2$. Furthermore, note also that $\partial \Pi_j\left(\bar{\mathbf{x}}, \bar{\delta}, mc_j, p_{-j}^*\right) / \partial p_j = D_j\left(\bar{\mathbf{x}}, \bar{\delta}, mc_j, p_{-j}^*\right) > 0$ since $D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right)$ is strictly positive for any price vector on $\mathbb{R}^2$.[19] Finally, note that since $D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right)$ is strictly decreasing in p_j, there must be some $p_j \in \left(mc_j, \infty\right)$ for which $\partial \Pi_j\left(\bar{\mathbf{x}}, \bar{\delta}, p_j, p_{-j}^*\right) / \partial p_j < 0$, so that $p_j^* < \infty, \forall j = 1, 2$.

□

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[19] As discussed in the proof of Lemma 2, from equation (8) it is straightforward to show that for every price vector on $\mathbb{R}^2$, $\operatorname{Pr}_{ij}\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}; \gamma_i, v_i, \theta_i\right) > 0$ for every consumer i and product j. This implies that $D_j\left(\bar{\mathbf{x}}, \bar{\delta}, \mathbf{p}\right) > 0$ for every product j.

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