Strategic Forward Contracting in Vertical Markets

by

Guilherme Fernandes Pedroso

Dissertation submitted in partial fulfillment of the requirements for the degree of

Master of Science in Economics

at the

Universidade Católica Portuguesa

Thesis Supervisor: Professor Duarte Brito

August 2018

[Page intentionally left blank]

Strategic Forward Contracting in Vertical Markets

Guilherme Fernandes Pedroso

Supervisor: Professor Duarte Brito

September 6, 2018

Abstract

There is a widespread belief that the introduction of forward contracts benefits firms if they compete in prices and harms them if they compete in quantities. The aim of this paper is analyze if this idea still holds in a vertical industry and, in order to do so, we build a model where forward contracts are introduced in a vertical industry where a manufacturer sells an input to two price setting retailers. We assume that firstly the input price is decided through bargaining. Then, the retailers decide on the amount of forward contracts they want to engage in and, finally, they set their prices (or quantities) in the spot market and the contracts are realized. We show that, under certain conditions, introducing a round of forward contracting might actually be detrimental to the retailers when they compete in prices and beneficial when competing in quantities.

Resumo

O consenso na literatura é de que a introdução de contratos futuros beneficia as empresas caso estas concorram em preços, e as prejudica caso concorram em quantidades. O objetivo desta tese é verificar se esta premissa se mantem numa industria vertical e, para esse efeito, um modelo foi criado onde os contratos futuros são introduzidos numa industria vertical caracterizada por um produtor que vende um input a dois retalhistas que competem em preços. Primeiro, assumimos que o preço do input é decidido através de uma negociação. Posteriormente, os retalhistas escolhem a quantidade de contratos futuros que querem assinar e, no final, decidem sobre os preços (ou quantidades) a praticar no mercado final e os contratos futuros são cumpridos. Nós mostramos que, em certos casos, introduzir uma ronda de contratos futuros pode ser prejudicial para os retalhistas caso compitam em preços e benéfico caso concorram em quantidades.

[Page intentionally left blank]

Acknowledgements

I would like to thank my advisor, professor Duarte Brito, for all his comments, insights and grateful suggestions. I would also like to thank Inês for all her advice and my family for all their support throughout this process.

[Page intentionally left blank]

Contents

List of Figures

1 Introduction

For several goods, there exists a spot market, where the goods are traded for immediate delivery, and a forward market, where contracts are exchanged to deliver the goods in the future at a pre-determined price. These forward markets are a frequent feature when exchanging commodities, like electricity. Here, spot trade is managed in the wholesale market, but sellers and buyers may also sign forward contracts for future delivery of electricity at a certain point in time for a previously agreed price.

Generally, forward contracts are linked with the desire of a group of agents to hedge against price risk.[1] Nevertheless, forward contracts can also be used strategically by firms to get an advantage over the competition. The study of this issue started with Allaz and Villa (1993), in which a two-period Cournot duopoly with homogeneous products model was built and firms could trade in both forward, with publicly observable contracts, and spot markets.[2] By committing to sell some units at a pre-determined price, a given producer has an incentive to produce more units. This happens because the price reduction that is expected to take place when output increases will not affect all the firm's sales, as some units have their price already established by the forward contract. This allows a firm to commit to be more aggressive and thus expand its best response function outwards, increasing its sales (and decreasing the rivals') in the new Nash equilibrium. If both firms introduce forward contracts, however, they both become more aggressive and will both become worse-off. Therefore, Allaz and Villa (1993) establish a Prisoner's Dilemma type of result where, even though engaging in forward contracting ends up being detrimental for both firms, they have an incentive to do so. Posterior research developed on this model. Bushnell (2007) extended the previous setting to accommodate a general number of oligopolists and increasing marginal costs, showing, implicitly, that the Prisoner's Dilemma is robust to these changes. On the other hand, Mahenc and Salanié (2004) built a model based on differentiated Bertrand competition and proved it yielded the opposite results when compared to Allaz and Villa (1993), i.e., firms would actually benefit from the introduction of the forward market, taking surplus away from the consumers by committing to buy: under price competition the Prisoner's Dilemma vanishes. This happens because firms buy a given percentage of their own output (i.e., take a long position), leading to higher prices than in the absence of the forward market. By agreeing to buy forward, firms have an incentive to increase prices (and thus reduce output) in the spot market as it makes the buying obligation at a pre-determined price less costly. These forward positions will expand both firms'

[1] Numerous studies have addressed this particular topic, namely Newbery (1984 and 1987).

[2] Hughes and Kao (1997) demonstrated the importance of public knowledge for the forward commitment.

upward sloping price best-response functions outwards, creating a competition softening effect that leads to higher equilibrium prices.

Several other papers have incorporated forward contracts into more dynamic environments. Marcus Aichele (2014) added a round of investment on top of a round of forward contracting, whereas Liski and Montero (2006) showed under which conditions forward contracting can facilitate tacit collusion in an infinite horizon model.

However, there is still a widespread belief that the introduction of forward contracts benefits firms if they compete in prices and harms them if they compete in quantities. The aim of this paper is analyze if this idea still holds in a vertical industry. In order to do so, we built a model where forward contracts are introduced in a vertical industry. In economics, a vertical relationship is one in which a product or service is supplied from one production activity to another. Vertical industries are usually characterized by upstream firms, such as manufacturers or wholesalers, and downstream firms such as retailers. Generally, the upstream firm sells an input to the downstream firms, who then transform it into the final product and sell it to final consumers.

In this setting, it may be the case that, even though the retailers compete in prices and therefore commit to a long position, like it is demonstrated by Mahenc and Salanié (2004), they can actually lose when engaging in forward contracting because of the response of the manufacturer. The upstream firm may change the input price (which in our model works like the marginal cost) as a result of the existence of downstream forward contracting and thus influence the retailers' outcome, leading to potential losses. On the other hand, if the retailers competed in quantities, the opposite result may apply i.e., the manufacturer could apply a discount to the downstream firms, leading to potential gains.

Our model with forward contracts is characterized by three stages. First, the upstream firm sells the input to two downstream firms, with the price being settled through bargaining, which occurs between the manufacturer and each retailer simultaneously. This methodology to set input prices is very similar to the one used in Dobson and Waterson (2007), who study the effects of resale price maintenance in a bilateral oligopoly framework. Then, the retailers face a round of forward contracting where they agree to buy/sell a given amount of output in the future at a pre-determined price. Finally, the downstream firms transform the input into output on a one-to-one ratio, forward contracts are realized and firms compete in prices in the spot market. We also address the case of quantity competition with differentiated products in an extension. To remove uncertainty/risk and only focus on the strategic effects of forward contracts, we assume, in line with Allaz and Villa (1993), Antelo and Bru (2002), Mahenc and

Salanié (2004), Bushnell (2007) and Marcus Aichele (2014), that agents have perfect foresight. We show that, under certain conditions and in specific market structures, introducing a round of forward contracting might actually be detrimental to the firms when they compete in prices and beneficial when competing in quantities, contrary to what happens when the vertical dimension of the industry is not considered. In particular, if the products are differentiated enough and the bargaining power is not too extreme (either to the retailers or to the manufacturer), there is a Prisoner's Dilemma type of result where both firms are worse off by engaging in forward contracting, despite the fact that firms compete in price. On the other hand, we also prove that if firms compete in quantities, the opposite result applies, implying that selling forward is not necessarily detrimental for the firms.

To the best of our knowledge only Antelo and Bru (2002) introduced a vertical industry where forward subcontracting between downstream firms is possible. Even though it also focuses in a vertical type of industry, the model substantially varies from the one presented in this paper. Contrary to ours, products are completely homogeneous, there is one more powerful firm downstream (and in an extension two) and a more efficient firm producing the input upstream and input prices are settled through a two-part tariff offered by the efficient upstream firm. Moreover, forward contracts are signed between the powerful downstream firm and a fringe of competitive retailers (horizontal subcontracting).

This thesis is organized as follows. Section 2 presents the price competition model and the corresponding equilibrium and Section 3 presents and interprets the main results, comparing them to other results in the literature. Section 4 presents the results for the quantity competition model. Finally, Section 5 summarizes all our findings. Appendix A presents all proofs and is followed by Appendix B, where a similar model was used, with the only difference being that the retailers compete in quantities and not in prices.

2 The model

We model the industry as a vertical oligopoly where an upstream firm, the *manufacturer* or firm m, sells a given final product to two downstream retailers, firms 1 and 2.[3] The two downstream firms sell differentiated products and compete on price. The marginal costs of the upstream firm are assumed to be constant and normalized to zero and the downstream firms have no costs other than the payments they make to the upstream firm. We assume that the upstream firm bargains simultaneously with the two downstream retailers over the

[3] Alternatively, one could think of firm m as the producer of an input that is sold to downstream firms that use it to produce the final goods on a one-to-one basis.

linear transfer price. To be more precise, we use the Nash Bargaining Solution to model the bargaining stage, in line with Dobson and Waterson (2007). During the bilateral and simultaneous negotiations, each pair manufacturer/retailer takes the transfer price of the other retailer as given. Moreover, the disagreement payoff will be the payoff each agent will get if the negotiation breaks down. In the case of a retailer, this payoff is zero, whereas in case of the manufacturer the payoff will be the profit he would get from only supplying the final product (or input) to the other downstream firm, which would in this case become a monopolist. The result drawn from this problem will represent a Nash equilibrium in Nash bargains, usually referred to as Nash-in-Nash.

The timing in our game is as follows:

1st) The two retailers and the manufacturer simultaneously bargain over the transfer prices.

2nd) The two retailers simultaneously make their decisions about the forward contracts.

3rd) The two retailers simultaneously set their retail prices.

As in Allaz and Villa (1993), we assume that the forward contracts are observable and binding. Moreover, in order to only focus on the strategic effects, we also assume that all agents have perfect foresight i. e., the price in the spot market will be the same as the price agreed in the forward contracts.

With respect to the demand for the final good we follow Singh and Vives (1984). Let the representative consumer's utility be given by:

$$U(q_1, q_2) = \frac{\alpha(q_1 + q_2)}{\beta(1-\gamma)} - \frac{(q_1)^2 + (q_2)^2}{2\beta(1-\gamma)(1+\gamma)} - \frac{\gamma q_1 q_2}{\beta(1-\gamma)(1+\gamma)}$$

where q_i represents the quantity sold by retailer i and α, β and $\gamma < 1$ are three positive parameters. Parameter γ represents the degree of substitution between the two goods, with a high γ meaning that the products are very close substitutes. In the limit case of $\gamma \rightarrow 1$ the representative consumer's utility depends only on the aggregate quantity $q_1 + q_2$, meaning that the products are perfect substitutes.

The demand for q_i is then given by:

$$q_i = \alpha - \beta p_i + \beta\gamma p_j$$

or

$$p_i = \frac{\alpha}{\beta(1-\gamma)} - \frac{q_i}{\beta(1-\gamma^2)} - \gamma\frac{q_j}{\beta(1-\gamma^2)}$$

Note that for equal prices, quantity demanded increases with α and γ and decreases with β.

In case retailer i becomes a monopolist, we have $q_j = 0$ and the corresponding inverse demand function is given by:

$$p_i = \frac{\alpha}{\beta(1-\gamma)} - \frac{q_i}{\beta(1-\gamma^2)} \Leftrightarrow q_i = \alpha(\gamma+1) - \beta\left(1-\gamma^2\right)p_i.$$

As for the representative consumer's surplus, CS, it is given by:

$$CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2 = \frac{q_1^2 + q_2^2 + 2\gamma q_1 q_2}{2\beta(1-\gamma)(\gamma+1)}$$

or, in a symmetric equilibrium, with $p_1 = p_2 = p$ and $q_1 = q_2 = q$:

$$CS = \frac{q^2}{\beta(1-\gamma)} = \frac{(\alpha - p\beta(1-\gamma))^2}{\beta(1-\gamma)}$$

As for welfare, W, given that all production costs are normalized to zero, it is merely given by the representative consumer's utility. In a symmetric equilibrium, it is given by:

$$W = q\frac{2\alpha - q}{\beta(1-\gamma)}$$

Welfare is obviously maximized at $q = \alpha$ or $p = 0$.

2.1 Model without forward contracts

We start by presenting the benchmark case in which there is no forward contracting. The game is as described above, except that the second stage is eliminated. As usual, we solve this game by backward induction, i.e., we start solving the model by the end, moving backwards until we reach the first stage, thus obtaining the Subgame Perfect Nash Equilibrium (SPNE).

2.1.1 Price competition stage

The profit function of retailer i is given by:

$$\Pi_i = (p_i - w_i)(\alpha - \beta p_i + \beta\gamma p_j)$$

where w_i is the input price for retailer i, determined through bargaining in the first stage. As we assume that there are no other costs of producing the good and that retailers transform the input into output in a one-to-one ratio, we can think of w_i as the retailers' marginal cost of producing good i. When the price competition stage is reached, we may have a duopoly (if both negotiations ended successfully) or a monopoly (if one negotiation broke down). The following Lemma presents the price equilibria for these two cases. The superscript D stands for duopoly, whereas the superscript M denotes the case of monopoly.

Lemma 1: *i) In case of duopoly, in equilibrium, retailer $i = 1, 2$ sets the following price and quantity:*

$$
\begin{aligned}
p_i^D(w_i, w_j) &= \frac{\alpha(\gamma+2) + \beta(2w_i + \gamma w_j)}{\beta(2-\gamma)(\gamma+2)}; \\
q_i^D(w_i, w_j) &= \frac{\alpha(\gamma+2) - \beta w_i(2-\gamma^2) + \beta\gamma w_j}{(2-\gamma)(\gamma+2)}
\end{aligned}
$$

and firms obtain the corresponding profit:

$$
\begin{aligned}
\Pi_i^D(w_i, w_j) &= \frac{(\alpha(\gamma+2) - \beta(2-\gamma^2) w_i + \beta\gamma w_j)^2}{(\gamma+2)^2(\gamma-2)^2 \beta} \\
\Pi_m^D(w_i, w_j) &= \frac{\alpha(w_i + w_j)(\gamma+2) - \beta(w_i + w_j)^2(2-\gamma^2) + 2\beta w_i w_j(\gamma+1)(2-\gamma)}{(\gamma+2)(2-\gamma)}
\end{aligned}
$$

ii) In case of monopoly, in equilibrium, retailer i sets the following price and quantity

$$
p_i^M(w_i) = \frac{1}{2}\frac{(\alpha + \beta w_i(1-\gamma))}{(1-\gamma)\beta}; \; q_i^M(w_i) = \frac{1}{2}(\gamma+1)(\alpha - \beta w_i(1-\gamma))
$$

and firms obtain the corresponding profit:

$$
\begin{aligned}
\Pi_i^M(w_i) &= \frac{1}{4}\frac{(\alpha - \beta w_i(1-\gamma))^2(\gamma+1)}{(1-\gamma)\beta} \\
\Pi_m^M(w_i) &= w_i \frac{1}{2}(\gamma+1)(\alpha - \beta w_i(1-\gamma))
\end{aligned}
$$

■

Under duopoly, retail prices increase with the two retailers' marginal costs, with each retailer's price being more sensitive to own cost than to rival's cost. Given the marginal costs, retail prices increase with α and γ and decrease with β.

In case of monopoly, a higher α increases the demand intercept, a higher β makes the demand more sensitive to price and a higher γ increases the demand intercept and makes it less sensitive to price. Thus, for a given marginal cost, retailer i's monopoly profit increases with α and γ and decreases with β. Having characterized the price equilibria as a function of the input prices, we now move to the bargaining stage.

2.1.2 Bargaining stage

In the first stage, transfer prices for the input are decided through bargaining between the manufacturer and each retailer simultaneously.

The transfer prices will be a result of two separate and simultaneous negotiations. The input price that results from the negotiation between retailer i and the manufacturer, is assumed to be:

$$w_i^{NF} = argmax_{w_i} \left[(\Pi_i^D(w_i, w_j) - 0)^\theta (\Pi_m^D(w_i, w_j) - \Pi_m^M(w_j))^{1-\theta} \right]$$

with $\Pi_m^D = w_i q_i + w_j q_j$ representing the manufacturer's profit under duopoly and $\Pi_m^M = w_j q_j$ representing the manufacturer's profit if negotiations with firm i break down and firm j will become a downstream monopolist. Parameter $\theta \in [0, 1]$ represents each retailer's relative degree of bargaining power in the negotiation (the higher the θ, the higher the power of the downstream firm in the negotiation), which may come from different time preferences or different attitudes towards risk (see Binmore et al., 1986). We assume that the two retailers have the same degree of bargaining power. The equilibrium is presented in the next proposition, where the superscript NF denotes the case of no forward contracting.

Proposition 1: *Without forward contracts, in equilibrium the input and output prices of retailer* $i = 1, 2$ *are, respectively*:

$$\begin{aligned} w_i^{NF} &= \frac{\alpha}{\beta} \frac{(1-\theta)(\gamma+2)}{(4-\gamma(\gamma+1)(2-2\theta\gamma+\theta\gamma^2))} \\ p_i^{NF} &= \frac{\alpha}{\beta} \frac{2\theta+\gamma+2\gamma^2+\theta\gamma-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-6}{(2-\gamma)(2\gamma+2\gamma^2-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-4)} \end{aligned}$$

and firms make the following equilibrium profits:

$$\begin{aligned} \Pi_i^{NF} &= \frac{\alpha^2}{\beta} \frac{(\gamma-2\theta+\theta\gamma+\gamma^2-\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-2)^2}{(\gamma-2)^2(2\gamma+2\gamma^2-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-4)^2} \\ \Pi_m^{NF} &= \frac{\alpha^2}{\beta} \frac{2(\gamma+2)(1-\theta)(\gamma-2\theta+\theta\gamma+\gamma^2-\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-2)}{(\gamma-2)(2\gamma+2\gamma^2-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-4)^2} \end{aligned}$$

■

The input prices can then be used to characterize the other variables in equilibrium. All the equilibrium values of the remaining variables (quantities, consumer surplus and welfare) are presented in the proof in the appendix.

Even though it is not the aim of this paper to do so, it is still worth analyzing some effects in this model, especially how some variables are affected, in equilibrium, by the degree of bargaining power (θ) and the degree of product substitution (γ).

Regarding the equilibrium input price (w_i) the following corollary presents how it is affected by the model parameters.

Corollary 1. *Each symmetric equilibrium transfer price is (i) increasing in the degree of product substitution, γ, and (ii) decreasing in terms of the retailers' bargaining strength, θ. When the retailers have minimum bargaining strength (i.e., $\theta = 0$), the transfer prices are $\alpha/2\beta(1-\gamma)$. When the retailers have maximum bargaining strength (i.e., $\theta = 1$), the transfer prices are zero.* ■

If the degree of product substitution, γ, increases, the manufacturer's disagreement (monopoly) payoff increases relatively to its "agreement" profit level. This allows the upstream firm to raise the input price when bargaining with retailer i.

A higher retailer's bargaining strength, θ, puts more weight in maximizing the downstream firm profit function, which varies negatively with the transfer price, leading to lower input prices. If we only focus on maximizing the downstream firms' profit function, the optimal choice for the symmetric input price is obviously zero as the firms would be strictly worse off by choosing $w_i > 0$.

Regarding the retailers' prices (p_i), we can state the following:

Corollary 2. *In the symmetric equilibrium, the retail prices are (i) increasing in the degree of product substitution, γ, and (ii) decreasing in the retailer's bargaining strength, θ.* ■

When the downstream firms have more power in the negotiation, they are able to set lower input prices (w_i goes down), which act like lower marginal costs in our model. This, in turn, allows firms to set lower retail prices. On the contrary, when products become closer substitutes, the upstream firm has an incentive to increase the input prices, leading to higher marginal costs. This effect, on top of an increase in demand, causes the retailers to increase their prices.

2.2 Model with forward contracts

In this section, we solve the game including the stage of forward contracting inbetween the bargaining stage and the final, spot market, stage.

2.2.1 Price competition stage

The demand function is exactly identical to the one before. However, the profit function for the retailers has changed and is now given by:

$$\Pi_i = p_i(q_i - f_i) + p_i^f f_i - w_i q_i = (p_i - w_i)q_i + (p_i^f - p_i)f_i$$

where f_i is the forward position of firm i and p_i^f is the price agreed for these positions. If $f_i > 0$, the firm commits to sell quantity f_i of the product in the spot market at price p_i^f, whereas if $f_i < 0$ the firm commits to buy f_i at that same price from some agent to whom it will sell at price p_i. As in the previous model, we may have a duopoly if the negotiations in the first stage were successful or a monopoly if negotiations with one of the retailers broke down. The following Lemma presents the equilibrium retail prices in both cases, as a function of the other endogenous variables.

Lemma 2: *i) In case of duopoly, in equilibrium, retailer i sets the following price and sells the following quantity:*

$$\begin{aligned} p_i(w_i, w_j, f_i, f_j) &= \frac{\alpha(\gamma+2) + \beta(2w_i + \gamma w_j) - 2f_i - \gamma f_j}{\beta(2-\gamma)(\gamma+2)} \\ q_i(w_i, w_j, f_i, f_j) &= \frac{\alpha(\gamma+2) - (2-\gamma^2)(\beta w_i - f_i) + \gamma(\beta w_j - f_j)}{(2-\gamma)(\gamma+2)} \end{aligned}$$

ii) In case of monopoly, in equilibrium, retailer i sets the same price as in the model with no forward contracts. ■

In the case of duopoly, retail prices decrease with both f_i and f_j. When setting its retail price, retailer i faces the following familiar value versus volume trade-off: A lower spot price increases the volume of sales but the profit per unit sold is lower. A higher (positive) f_i means that a larger number of the units will be sold at the pre-determined forward price and thus, the lower spot price will impact a smaller number of units. This causes firm i's optimal retail price (given firm j's price) to drop. Naturally, the opposite happens for a negative f_i. Retail price is more sensitive to variations in the amount of forward contracts signed by the firm when compared to variations in the amount signed by the rival firm (as $2 > \gamma$), because the rival firm's forward contracts only affect the firm indirectly, through a rival's lower retail price.

In other words, from the set of FOC with respect to retail prices, we know that

$$\frac{\partial\left[(p_i - w_i)q_i + (p_i^f - p_i)f_i\right]}{\partial p_i} = 0 \Leftrightarrow \frac{\partial\left[(p_i - w_i)q_i\right]}{\partial p_i} = f_i$$

When setting the optimal retail price, retailer i considers two effects. The first one corresponds to the impact on retail profits of increasing the price: whatever is sold yields a higher profit per unit but the number of units sold is lower. In the absence of forward contracts, this term would be equal to zero at the retailer's optimal price. However, if the firm has committed to sell f_i units at a predetermined price, it will have to "buy" them at the higher spot price, thus having an additional loss. Therefore, the retailer will have an incentive not to price as high as in the absence of a positive f_i. On the contrary, if $f_i < 0$, it will price higher.

Moreover, retail prices are also increasing in the two input prices, w_i and w_j, which are the retailers' marginal cost of producing the final good. Thus, increasing the marginal cost gives an incentive for the firm to increase its price. Additionally, because the best responses are upward sloping, an increase in the rival's marginal cost increases its price, which also gives an incentive for the firm to increase its own price.

Finally, retail prices are increasing in α as it translates into a larger demand function, giving firms more room to increase prices.

2.2.2 Forward Contracting Stage

In the second stage, the forward contracting stage, the firms decide on the amount of forward contracts they want to engage in. Due to the perfect foresight assumption, $p_i = p_i^f$, which simplifies the retailers' profit functions. The profit of each retailer in this stage is thus:

$$\Pi_i(w_i, w_j, f_i, f_j) = (p_i(w_i, w_j, f_i, f_j) - w_i)q_i(w_i, w_j, f_i, f_j)$$

The equilibrium amount of forward contracts is presented in the next Lemma.

Lemma 3: *i) In case of duopoly, in equilibrium, retailer i sets the following amount of forward contracts:*

$$f_i(w_i, w_j) = \frac{\gamma^2 \left(\beta \left(4 - 3\gamma^2\right) w_i - \beta\gamma \left(2 - \gamma^2\right) w_j + \alpha \left(\gamma^2 - 2\gamma - 4\right)\right)}{\left(4 - \gamma^2 + 2\gamma\right)\left(4 - 2\gamma - \gamma^2\right)}$$

and firms obtain the corresponding profit:

$$\begin{aligned}
\Pi_i(w_i, w_j) &= \frac{2\left(4\beta w_i - 2\alpha\gamma - 4\alpha - 2\beta\gamma w_j + \alpha\gamma^2 - 3\beta\gamma^2 w_i + \beta\gamma^3 w_j\right)^2 \left(2 - \gamma^2\right)}{\left(\gamma^2 - 2\gamma - 4\right)^2 \left(2\gamma + \gamma^2 - 4\right)^2 \beta} \\
\Pi_m(w_i, w_j) &= \frac{\left(\alpha\left(2\gamma - \gamma^2 + 4\right)\left(w_i + w_j\right) - \beta\left(\left(4 - 3\gamma^2\right)\left(w_i^2 + w_j^2\right) - 2\gamma w_i w_j \left(2 - \gamma^2\right)\right)\right)\left(2 - \gamma^2\right)}{\left(2\gamma - \gamma^2 + 4\right)\left(4 - 2\gamma - \gamma^2\right)}
\end{aligned}$$

ii) In case of monopoly, in equilibrium, retailer i chooses not to engage in any forward *commitment, i.e.,* $f_i = 0$. ■

In case of duopoly, the amount of forward contracts retailer i will commit to varies positively with w_i and negatively with w_j. As seen above, forward contracts affect the retailers' profits only through the retail prices.

$$\frac{d\Pi_i}{df_i} = \frac{\partial\left[(p_i - w_i)q_i\right]}{\partial p_i}\frac{\partial p_i}{\partial f_i} + \frac{\partial\left[(p_i - w_i)q_i\right]}{\partial p_j}\frac{\partial p_j}{\partial f_i} = f_i\frac{\partial p_i}{\partial f_i} + (p_i - w_i)\beta\gamma\frac{\partial p_j}{\partial f_i}$$

with

$$\frac{\partial p_i}{\partial f_i} = \frac{-2}{\beta\left(2-\gamma\right)\left(\gamma+2\right)} < 0 \text{ and } \frac{\partial p_j}{\partial f_i} = \frac{-\gamma}{\beta\left(2-\gamma\right)\left(\gamma+2\right)} < 0.$$

The first term, which represents the impact of f_i on retailer i's profit if retailer j's price is kept constant, has the opposite sign of f_i (see FOC in the previous subsection). The second term, which represents the impact of f_i on retailer i's profit that results from changes in p_j (a strategic effect) is negative: more forward contracting by firm i makes firm j price more aggressively, leading to fewer sales by firm i. It is also the only term that depends on the two input prices. A higher w_i lowers the second term, whereas a higher w_j increases it. In fact, if w_i is higher the profit margin $p_i - w_i$ decreases (despite the fact that retail prices will also increase).[4] Therefore, the strategic effect becomes weaker as w_i increases and, as this effect impacts the retailer's profits negatively, this will lead to higher forward contracts.[5] A higher w_j leads to higher retail prices for both firms, and therefore a higher retail margin for retailer i. As such, an increase in w_j amplifies the negative effect and therefore should lead to less forward contracting by firm i.[6]

In case of a monopoly, the second effect does not exist and the retailer chooses not to commit forward as the biggest profit the monopolist can obtain is the monopolist profit. This goes in line with the results from Anderson and Sundaresan (1984).

As $\frac{\partial p_i}{\partial f_i}$ and $\frac{\partial p_j}{\partial f_i}$ have the same (negative) sign, it is clear that if retailer i is making a profit ($p_i - w_i > 0$) the first-order condition can only be verified with a negative f_i.

In fact, the corresponding firm i's equilibrium price and quantity is given by

$$\begin{aligned} p_i(w_i, w_j) &= \frac{-2\alpha\left(-2\gamma+\gamma^2-4\right)+\beta w_i\left(\gamma+2\right)\left(\gamma-2\right)\left(\gamma^2-2\right)-2\beta\gamma w_j\left(\gamma^2-2\right)}{\beta\left(-2\gamma+\gamma^2-4\right)\left(2\gamma+\gamma^2-4\right)} \\ q_i(w_i, w_j) &= \left(\gamma^2-2\right)\frac{\alpha\left(\gamma^2-2\gamma-4\right)+\beta w_i\left(4-3\gamma^2\right)-\beta\gamma w_j\left(2-\gamma^2\right)}{\left(2\gamma+\gamma^2-4\right)\left(-2\gamma+\gamma^2-4\right)} \end{aligned}$$

[4] This follows from $\frac{\partial p_i}{\partial w_i} = \frac{2}{(2-\gamma)(\gamma+2)} < 1$

[5] The amount of forward contracts is given by $|f_i|$. Thus, if $f_i < 0$ a higher f_i.means fewer forward contracts.

[6] Or a higher amount of forward contracting if $f_i < 0$.

As this quantity cannot be negative, we have $(\alpha(\gamma^2 - 2\gamma - 4) + \beta w_i (4 - 3\gamma^2) - \beta\gamma w_j (2 - \gamma^2)) \leq 0$ which implies that the equilibrium f_i is negative:

Corollary 3. *In a symmetric equilibrium, if $\gamma > 0$, firms always commit to buy, i.e., $f_i < 0$. If $\gamma = 0$, firms choose not to sign any forward contracts, i.e., $f_i = 0$.*

This result was expected since when firms compete in prices, they have an incentive to buy their own production back, as shown by Mahenc and Salanié (2004). Additionally, when the demand function faced by a retailer is completely independent from the price of the other retailer ($\gamma = 0$), each firm basically becomes a monopolist downstream and has no incentive to sign forward contracts, as showed by Anderson and Sundaresan (1984). As the bargaining stage precedes the stage in which forward contracts are defined, the transfer price is already set when firms choose how much forward contracts to sell/buy. Therefore, this result reproduces others in the literature.

Note also that

$$\begin{aligned} \frac{\partial^2 f_i}{\partial w_i \partial \gamma} &= \frac{64\beta\gamma(1-\gamma)(\gamma+1)(2-\gamma^2)}{(-2\gamma+\gamma^2-4)^2(2\gamma+\gamma^2-4)^2} > 0 \text{ and} \\ \frac{\partial^2 f_i}{\partial w_j \partial \gamma} &= \beta\gamma^2 \frac{104\gamma^2 - 34\gamma^4 + \gamma^6 - 96}{(-2\gamma+\gamma^2-4)^2(2\gamma+\gamma^2-4)^2} < 0. \end{aligned}$$

The effect of an increase in the transfer price in the amount of forward contracts is stronger the less differentiated the products are.

Moreover, $(4 - 3\gamma^2) > |\gamma(\gamma^2 - 2)| \Leftrightarrow (1-\gamma)(4 + 2\gamma - \gamma^2) > 0$: as expected, equilibrium forward contracts depend more on the own input price than on the rival's.

2.2.3 Bargaining stage

Finally, we must solve the first stage of the game, the bargaining problem. As in the previous model, the disagreement payoff for each retailer will be zero. In addition, the disagreement payoff for the manufacturer remains the same as, optimally, the other retailer (who becomes a monopolist) will afterwards choose $f_i = 0$.

Therefore, the optimal transfer prices will be derived from:

$$w_i^F = argmax_{w_i} \left[(\Pi_i^D(w_i, w_j) - 0)^\theta (\Pi_m^D(w_i, w_j) - \Pi_m^M(w_j))^{1-\theta} \right]$$

where the superscript F denotes the existence of the forward contracting stage. The next proposition presents the solution to this problem.

Proposition 2: *In equilibrium, the input prices and retail prices with forward contracts are:*

$$w_i^F = \frac{\alpha}{\beta}\frac{(1-\theta)(4-\gamma^2+2\gamma)(2-\gamma^2)}{(2(1-\gamma)(2-\gamma^2)(4+2\gamma-\gamma^2)+\theta\gamma^2(\gamma+1)(4-2\gamma-\gamma^2))}$$

$$p_i^F = \frac{\alpha}{\beta}\frac{\theta(\gamma(\gamma+2)(16\gamma-2\gamma^2-6\gamma^3+\gamma^4-4)-16)-\gamma(2-\gamma)(32\gamma+12\gamma^2-4\gamma^3-\gamma^4+4)+48}{(4-\gamma(\gamma+2))(16-2\gamma(2-\gamma)(6\gamma+\gamma^2-\gamma^3+2)+\theta\gamma^2(\gamma+1)(4-\gamma^2-2\gamma))}$$

and firms make the following equilibrium profits:

$$\Pi_i^F = \frac{2(2-\gamma^2)((\gamma-1)(\gamma^2-2)(-2\gamma+\gamma^2-4)+2\theta(2\gamma+3\gamma^2-3\gamma^3+\gamma^5-4))^2}{(2\gamma+\gamma^2-4)^2(2(\gamma-1)(\gamma^2-2)(-2\gamma+\gamma^2-4)+\theta\gamma^2(\gamma+1)(2\gamma+\gamma^2-4))^2}\frac{\alpha^2}{\beta}$$

$$\Pi_m^F = \frac{((\gamma-1)(-2\gamma+\gamma^2-4)(\gamma^2-2)+2\theta(2\gamma+3\gamma^2-3\gamma^3+\gamma^5-4))}{(2(\gamma-1)(-2\gamma+\gamma^2-4)(\gamma^2-2)+\theta\gamma^2(\gamma+1)(2\gamma+\gamma^2-4))^2(2\gamma+\gamma^2-4)} \times 2\left(\gamma^2-2\right)^2(\theta-1)\left(\gamma^2-2\gamma-4\right)\frac{\alpha^2}{\beta}$$

■

The next section discusses this result and presents the impact of forward contracting.[7]

3 Results

Regarding the input (or transfer) price, we can conclude the following:

Corollary 4. *In a symmetric equilibrium where $\gamma > 0$ and $\theta \in]0;1[$, introducing forward contracts causes the equilibrium transfer price to be higher than in the model without forward contracts.*[8] ■

It is easier to explain this result if one starts by assuming extreme cases such as $\theta = 0$, that is, the manufacturer has all the bargaining power in the negotiations and it will set the transfer price that maximizes its own profit or $\theta = 1$, that is, the retailers choose the transfer price that maximizes their individual profits.

Assume first that $\theta = 0$. As the manufacturer's costs are the same with or without forward contracting, the possibility of different transfer prices follows from the different demand functions that the monopolist manufacturer will consider when choosing w_i and w_j. The demand

[7] When comparing the two models (with and without forward contracting), we will always assume that $\gamma > 0$. If the products were completely different (i.e., $\gamma = 0$), firms would become monopolists and thus would choose not to engage in any forward contracts, making the two models equivalent.

[8] If $\theta = 1$, there is no change in the transfer price as retailers would always choose the minimum level possible, i.e., zero. As explained below, the input price also does not change for $\theta = 0$.

function that the monopolist faces from retailer i is given by:

$$\begin{aligned} q_i &= \alpha - \beta \frac{\alpha(\gamma+2) + \beta(2w_i + \gamma w_j) - 2f_i - \gamma f_j}{\beta(2-\gamma)(\gamma+2)} + \beta\gamma \frac{\alpha(\gamma+2) + \beta(2w_j + \gamma w_i) - 2f_j - \gamma f_i}{\beta(2-\gamma)(\gamma+2)} \\ &= \frac{\alpha(\gamma+2) - (2-\gamma^2)(\beta w_i - f_i) + \gamma(\beta w_j - f_j)}{(2-\gamma)(\gamma+2)} \end{aligned}$$

where f_i and f_j can either be zero (no forward contracting) or the negative valued equilibrium functions of w_i and w_j derived above.

With $\theta = 0$, when maximizing its profit $\Pi_m^D = w_i q_i + w_j q_j$ the manufacturer sets w_i such that:

$$\frac{d\Pi_m(w_i, w_j)}{dw_i} = q_i + w_i \frac{dq_i}{dw_i} + w_j \frac{dq_j}{dw_i} = 0$$

By increasing the transfer price for retailer i by +1 the manufacturer will: i) get an additional revenue of q_i from the infra-marginal units purchased by retailer i, ii) will sell fewer units to this retailer, losing $w_i \frac{\partial q_i}{\partial w_i}$, and iii) will sell more units to retailer j, getting an additional $w_j \frac{\partial q_j}{\partial w_i}$. We now need to see how these terms are affected by the existence of forward contracting.

The first term decreases when f_i is introduced but increases when f_j is introduced (as f_i and f_j are both negative). Due to forward contracts, the number of units sold for the same input price levels changes by:

$$\frac{f_i(2-\gamma^2) - \gamma f_j}{(2-\gamma)(\gamma+2)}$$

Note that for a given, common $f_i = f_j = f < 0$ the change in demand is negative and but it becomes closer to zero as γ increases.

It is likely that the first term, the direct effect, dominates,[9] meaning that forward contracts lower the demand faced by the manufacturer, an effect that, per se, would lead to lower transfer prices. As retailers commit to a long position, the manufacturer upstream faces a reduction in its demand, which all else constant would lead to a decrease in price.

The second term can be decomposed in

$$\frac{dq_i}{dw_i} = \underset{-}{\frac{\partial q_i}{\partial w_i}} + \underset{++}{\frac{\partial q_i}{\partial f_i}} \underset{++}{\frac{\partial f_i}{\partial w_i}} + \underset{-}{\frac{\partial q_i}{\partial f_j}} \underset{-}{\frac{\partial f_j}{\partial w_i}}$$

where the last two additive terms only exist under forward contracting (and the first term is the same, regardless of the existence of forward contracts). As these two terms are positive, they make the derivative less negative (a more rigid demand) which leads, all else constant, to

[9]When forward contracts are introduced, q_i changes by $\frac{(2-\gamma^2)}{(2-\gamma)(\gamma+2)}$ due to f_i and $\frac{-\gamma}{(2-\gamma)(\gamma+2)}$ due to f_j. Since $(2-\gamma^2) > \gamma$, the direct effect always dominates the indirect effect in a symmetric equilibrium.

higher prices: The manufacturer anticipates that a higher transfer price for firm i will increase its forward contracts (because it is negative this means fewer forward contracts). This makes firm i relatively more aggressive, leading to lower retail prices and a higher output. At the same time, a higher transfer price for firm i will decrease the rival's forward contracts, making them more negative. This makes firm j less aggressive, making it price higher, and therefore increasing the output of retailer i. Therefore, the demand from retailer i does not decrease as much after an input price increase as it would in the absence of forward contracts. This effect (a more rigid demand for each retailer) would lead, per se, to a higher price.

Finally, the last term can be decomposed in

$$\frac{dq_j}{dw_i} = \underset{+}{\frac{\partial q_j}{\partial w_i}} + \underset{-}{\frac{\partial q_j}{\partial f_i}} \underset{++}{\frac{\partial f_i}{\partial w_i}} + \underset{++}{\frac{\partial q_j}{\partial f_j}} \underset{-}{\frac{\partial f_j}{\partial w_i}}$$

Again, the first term is the same, regardless of the existence of forward contracts and the last two additive terms only exist under forward contracting. Increasing w_i increases the demand by the other retailer, but this effect is now smaller with forward contracts, which would also lead to a lower transfer price.

If $\theta = 0$, these three effects compensate each other and the equilibrium transfer prices are the same with or without forward contracts. In fact, making $\theta = 0$ we have that

$$w_i^{NF} = w_i^F = \frac{\alpha}{\beta} \frac{1}{2(1-\gamma)}.$$

What about if $\theta = 1$? In this case the optimal w would be the as low as possible, that is, zero. If each retailer could choose the transfer price to maximize own profit (given the other retailer's transfer price):

$$\Pi_i = (p_i - w_i)(\alpha - \beta p_i + \gamma \beta p_j)$$

with p_i and p_j functions of $w_i, w_j, f_i(w_i, w_j)$ and $f_j(w_i, w_j)$, retailer i would consider the following effects:

$$\frac{d\Pi_i}{dw_i} = -q_i + \frac{\partial \Pi_i}{\partial p_i}\frac{dp_i}{dw_i} + \frac{\partial \Pi_i}{\partial p_j}\frac{dp_j}{dw_i}$$

with

$$\begin{aligned} \frac{dp_i}{dw_i} &= \frac{\partial p_i}{\partial w_i} + \frac{\partial p_i}{\partial f_i}\frac{\partial f_i}{\partial w_i} + \frac{\partial p_i}{\partial f_j}\frac{\partial f_j}{\partial w_i} \\ \frac{dp_j}{dw_i} &= \frac{\partial p_j}{\partial w_i} + \frac{\partial p_j}{\partial f_i}\frac{\partial f_i}{\partial w_i} + \frac{\partial p_j}{\partial f_j}\frac{\partial f_j}{\partial w_i} \end{aligned}$$

As f_i will be chosen in the next stage to maximize the same function, the effects through f_i are zero:[10]

$$\frac{\partial \Pi_i}{\partial p_i}\frac{\partial p_i}{\partial f_i}\frac{\partial f_i}{\partial w_i}+\frac{\partial \Pi_i}{\partial p_j}\frac{\partial p_j}{\partial f_i}\frac{\partial f_i}{\partial w_i}=0$$

and what remains is:

$$\frac{d\Pi_i}{dw_i}=-q_i+\frac{\partial \Pi_i}{\partial p_i}\left(\frac{\partial p_i}{\partial w_i}+\frac{\partial p_i}{\partial f_j}\frac{\partial f_j}{\partial w_i}\right)+\frac{\partial \Pi_i}{\partial p_j}\left(\frac{\partial p_j}{\partial w_i}+\frac{\partial p_j}{\partial f_j}\frac{\partial f_j}{\partial w_i}\right)$$

When compared to what would happen with forward contracts, the difference is the term:

$$\left(\underset{-}{\frac{\partial \Pi_i}{\partial p_i}}\underset{-}{\frac{\partial p_i}{\partial f_j}}+\underset{+}{\frac{\partial \Pi_i}{\partial p_j}}\underset{--}{\frac{\partial p_j}{\partial f_j}}\right)\underset{-}{\frac{\partial f_j}{\partial w_i}}=\gamma\frac{\alpha\left(\gamma^2-2\gamma-4\right)+\beta w_i\left(4-3\gamma^2\right)-\beta\gamma w_j\left(2-\gamma^2\right)}{\beta\left(2\gamma+\gamma^2-4\right)\left(-2\gamma+\gamma^2-4\right)}\frac{\partial f_j}{\partial w_i}$$

which is positive (recall the condition above for positive quantities). This leads to a larger (that is less negative) $\frac{d\Pi_i}{dw_i}$, the relevant derivative in the retailer's perspective: by increasing w_i the rival will decrease forward contracts (i.e., make them more negative) which makes the rival less aggressive, thus increasing its price to the benefit of retailer i.

Note that the SOC for maximum is not verified.[11] So, we still have a corner solution: $w_i=0$. However, the derivative in the presence of forward contracts is not as negative as in the absence of forward contracts. When, as result of the bargaining process the two FOC are weighted to determine the optimal w, this increase in benefits to one of the bargaining parts of a larger w makes the equilibrium transfer price increase.

Summing up, the existence of forward contracts does not change the optimal transfer prices in the perspective of the manufacturer or of each retailer. These are, respectively, $w_i=\frac{\alpha}{\beta}\frac{1}{2(1-\gamma)}$ and $w_i=0$. However, it does change the respective derivatives. In fact, the FOC for the bargaining problem is given by:

$$(1-\theta)\Pi_i^D\frac{d\Pi_m}{dw_i}+\theta(\Pi_m^D-\Pi_m^M)\frac{d\Pi_i}{dw_i}$$

with the second term always negative. As we have seen before, the introduction of forward contracts makes the first term more positive and the second term less negative, resulting in an increase in w_i. However, the increase is greater for intermediate values of θ. When θ is too extreme (either close to 0 or 1), we only have one of the terms increasing, resulting in a smaller

[10] Note that we cannot state that $\frac{\partial \Pi_i}{\partial p_i}=0$ because in the last round of interaction, the profit function that will be maximized is $\Pi_i=(p_i-w_i)q_i+(p_i^f-p_i)f_i$ and not $\Pi_i=(p_i-w_i)q_i$ (in case of forward contracting).

[11] $4\left(2-\gamma^2\right)\left(4-3\gamma^2\right)\frac{\beta\left(4-3\gamma^2\right)}{(2\gamma+\gamma^2-4)^2(-2\gamma+\gamma^2-4)^2}>0$

increase in w_i.

As a result, the transfer price increases with the existence of forward contracts, because an increase in the transfer price is not as negative for each retailer as in the absence of such contracts.

We can also state the following concerning the input price:

Corollary 5. *In a symmetric equilibrium where $\gamma > 0$ and $\theta < 1$, the increase in the equilibrium transfer price due to the introduction of forward contracts is higher when (i) γ is high (ii) retailers' relative degree of bargaining strength is not too extreme.* ■

When γ increases, firms have an additional incentive to trade forward since the contracts become more profitable when competition is toughest, as stated by Mahenc and Salanié (2004). This means that, for the same value of w_i, we should expect a higher amount of forward contracts signed, increasing the two effects previously described.[12] Thus, for a given level of retailers' bargaining strength, the input price will increase more with the introduction of forward contracts if γ is high.

Part (ii) of this corollary was explained above. When $\theta = 0$ or $\theta = 1$, the equilibrium transfer price is the same with or without forward contracts. It is for intermediate values of bargaining power that the fact that an increase in transfer prices hurts retailers less with forward contracts is felt and results in a higher transfer price.

Moreover, we can also conclude the following:

Corollary 6. *In a symmetric equilibrium, forward contracts are (i) increasing (negatively) with the degree of substitution between the final products and (ii) increasing (negatively) with the retailers' bargaining strength.* ■

Even though firms have an incentive to buy less of their production back when the input price increases, the fact that forward contracts become more profitable as γ is higher makes retailers sign a greater amount of forward contracts. Therefore, we reach the same result of Mahenc and Salanié (2004) regarding the impact of the degree of substitution between the products on the amount of forward contracts signed, despite having another effect, the increase in w_i, working in the opposite direction. Also, a higher degree of bargaining strength for the downstream firms translates into a lower marginal costs (lower input prices), giving firms an additional incentive to buy more of their production back.

In addition, we can write the following regarding the differences in retail prices, consumer surplus and welfare:

[12] I.e., the changes in $\frac{d\Pi_m}{dw_i}$ and $\frac{d\Pi_i}{dw_i}$ when forward contracts are introduced.

Corollary 7. *In a symmetric equilibrium where $\gamma > 0$, introducing forward contracts leads to (i) an increase in the retail price (ii) a decrease in consumer surplus (iii) a decrease in welfare.* ■

The introduction of forward contracts in a vertical industry yields the same results regarding the quantity/price, consumer surplus and overall welfare as in Mahenc and Salanié (2004). In fact, firms commit to a long position, expanding their upward sloping best response functions, which on top of the increase in the input prices, increases the prices in the retail market. This action harms consumers and brings down overall welfare.

Finally, we look at the effect of introducing a round of forward contracting in the equilibrium profit. Regarding the upstream firm's profit, we can conclude the following:

Remark 1. *In a symmetric equilibrium where $\gamma > 0$, introducing forward contracts is beneficial for the manufacturer if (i) γ is large (ii) the degree of retailers' bargaining strength is intermediate.* ■

The difference in the manufacturer's profit between the two models is given by $\Pi_m^F - \Pi_m^{NF}$. Figure 1 presents the difference in the manufacturer's profit between the two models.

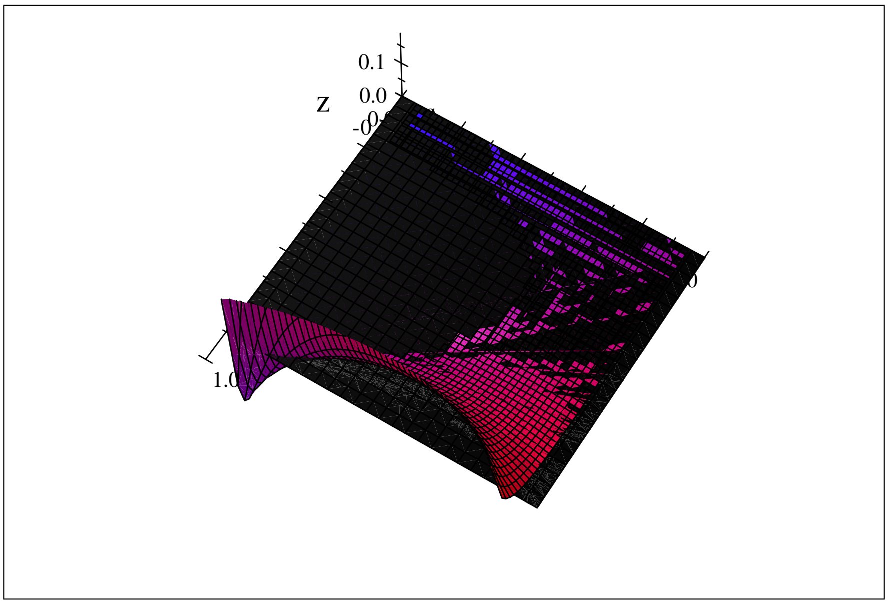

Figure 1: Difference in the manufacturer's profit with forward contracting and without forward contracting as a function of γ and θ.

which shows the manufacturer is more likely to win with the introduction of forward contracting if γ is high and/or θ is high.

Regarding the downstream firms' profit, we can state the following:

Remark 2. *In a symmetric equilibrium where $\gamma > 0$, introducing forward contracts* *is beneficial for the retailers if (i) γ is large (ii) the retailers' degree of bargaining power is extreme.* ■

The difference in retailer i's profit between the two models is given by $\Pi_i^F - \Pi_i^{NF}$. Figure 2 presents the difference in the retailers' profit between the two models.

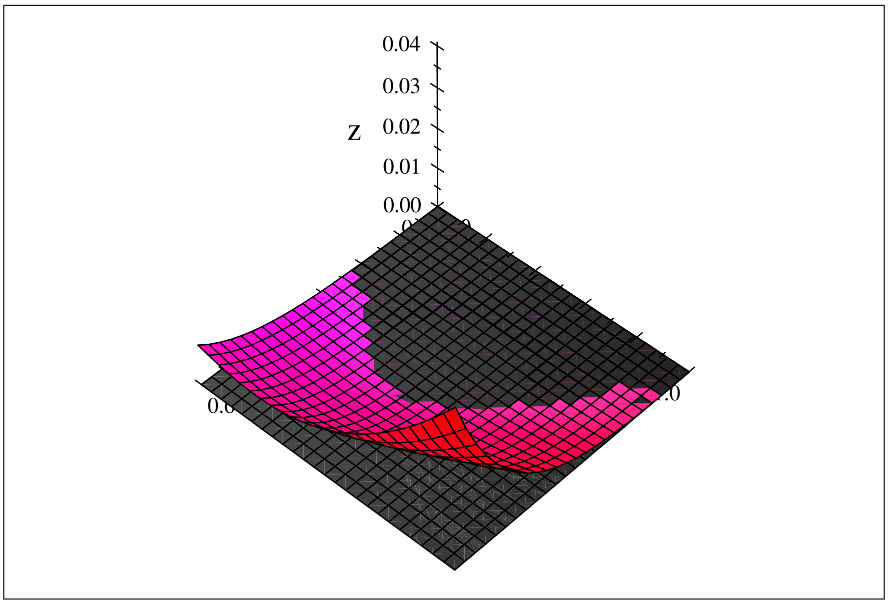

Figure 2: Difference in the retailer's profit with forward contracting and without forward contracting as a function of γ and θ.

Showing that the retailers are more likely to benefit from the introduction of forward contracts if γ is high and/or the value of θ is somewhat extreme.

Forward contracting has two effects on the retailers and manufacturer profit: for a given input price, it allows retailers to soften competition, which is profitable for them but leads to lower sales and **lower** profits for the manufacturer. However, for intermediate levels of bargaining power, forward contracting leads to higher input prices, which is not profitable for the retailers but it is profitable for the manufacturer. When the products are very differentiated, the retailers' gains from engaging in a forward commitment do not compensate the losses from an increase in the transfer price. As seen above, a very extreme degree of bargaining power (either in favor of the retailers or of the manufacturer), yields a lower increase in the input price.[13] Therefore, the retailers are more likely to find the forward commitment profitable in this case then when the retailers' degree of bargaining strength is intermediate and the increase in the input price is the highest**.** The opposite applies for the manufacturer.

Finally, we can derive the additional insight:

Corollary 8. *When the retailers have maximum bargaining strength (i.e., $\theta = 1$) and*

[13] See Corollary 5.

$\gamma > 0$, *introducing forward contracts is (i) always profitable for the retailers (ii) never profitable for the manufacturer.* ■

This is the expected result from introducing forward agreements in a differentiated Bertrand competition environment (the one found in Mahenc and Salanié (2004)). The manufacturer will make zero profits as, when $\theta = 1$, the retailers will set the input prices to zero in both models.

4 Extension

In addition to the pricing model, we have derived an identical model in Appendix B, with the only difference being that the downstream firms compete in quantities instead of prices. Therefore, as expected, most of the results obtained had the opposite sign. The retailers best response functions are now downward sloping, meaning that when one increases its quantity, the best response of the other is to decrease his. So, retail quantities will now increase with the level of a retailer's own forward contracts (as prices did), but decrease with the rival's forward contracts, which is due to the downward sloping best response functions. In fact, when the competitor increases its forward contracts, its best response function expands making it produce more, causing the price to drop and thus giving the other retailer an incentive to reduce its output. This is the same reason why, even though the relationship between own input price and the quantity produced is the same as in the pricing model, the impact of the rival's input price on the quantity produced by the retailer is now different. An increase in the rival's "marginal cost", leads to an inwards shift of the rival's best response function, causing its quantity to decrease and thus increasing the spot price, giving the retailer an incentive to increase its quantity. Nevertheless, as in the price competition case, the direct effect is always larger than the indirect effect, given the firms engage in the same amount of forward contracts.

This trend of opposite results continues when analyzing the effect of a change in the input prices on the amount of forward contracts signed. Contrary to the price competition model, the amount of forward contracts varies negatively with the firm's input cost and positively with the rival's input cost. Now, the impact on the retailer's profit from its own engagement in forward contracting that results from changes in the competitor's quantity (strategic effect) is positive: more forward contracting by one retailer causes the rival to lower its quantity, leading to a higher retail price. This effect is weaker when the retailer's input price increases as this will lead to a reduction in the quantity produced, diminishing the positive effect of introducing forward contracts (the higher retail price) and thus leading to a lower amount of forward contracts. Conversely, the strategic effect gets stronger when the rival's input price increases as it will cause the retailer to produce more, leading to more forward engagement.

Moreover, we reached the expected result regarding the amount of forward contracts in a quantity competition environment: the retailers will commit to sell, i.e., they will commit to a short position, following the conclusions from Allaz and Villa (1993). As before, if products are completely different, the retailers become monopolists and thus have no incentive to sign any forward commitment (as evidenced by Anderson and Sundaresan (1984)).

Regarding the input prices in a symmetric equilibrium, the results are the same for the very extreme cases where either the manufacturer or the retailers have all the bargaining power: introducing forward contracts does not change the optimal input price in the perspective of the party that has all the bargaining power. However, for intermediate levels of retailer's bargaining strength, the input price is always lower when forward contracts are introduced. The explanation relies on the same principles as before. We assume first that the manufacturer has all the bargaining power when choosing the input prices. In this case, increasing an input price will lead to the same three effects as in the price competition case: the manufacturer i) gets an additional revenue from the quantity sold from the infra-marginal units purchased by the retailer, ii) sells fewer units to this retailer, and iii) sells more units to the rival retailer. Nevertheless, introducing forward contracts now affects these terms in the opposite way. Firstly, the quantity demanded by a given retailer will increase, on one hand, if he signs forward contracts but it will also decrease, on the other hand, if the competitor signs forward contracts. As before, the direct effect dominates and thus, from this effect alone, one should see an increase in the transfer price when the forward contracts are introduced as it becomes more profitable to do so. Secondly, what forward contracts do is that they make the demand for the input more elastic. This means that, when they are introduced, increasing the input price for a given retailer will decrease his demand for the input more but also increase the demand from the rival retailer more. Thus, this first effect should lead to a reduction in the transfer price, while the second effect should lead to an increase. When the manufacturer has all the bargaining power, these three effects cancel each other and the optimal input price in the manufacturer's perspective does not change. However, for other values of retailer's bargaining strength the benefits of a lower transfer price are higher.

Now, if we assume that the retailers have all the bargaining power, they would want its "marginal cost" to be as low as possible, i.e., zero. Nevertheless, just like in the previous case, the magnitude of the derivative changes. In fact, when forward contracts are introduced, increasing the input price leads to a larger decline in the retailer's profit: after an increase in own input price, the rival would increase its forward contracts, becoming more aggressive, which brings the price down and hurts the retailer. Therefore, contrary to the first model, this favors a lower bargained input price.

Moreover, for the same reasons as in the pricing model, this decrease is larger for intermediate levels of bargaining strength. Additionally, when products become closer substitutes, firms have an additional incentive to trade forward as the gains from unilaterally signing forward contracts are higher (even though the firm will eventually lose in equilibrium due to the rival's response). This means that, for the same input price, we should expect a higher amount of forward contracts signed, increasing the two effects previously described. Therefore, for a given level of retailer's bargaining strength, the input price will decrease more with the introduction of forward contracts if the products downstream are closer substitutes.

Also, the amount of forward contracts is increasing in both the degree of substitution and retailer's degree of bargaining strength, just like in the first model.[14] Additionally, because firms commit to sell, there is an expansion in their downward slopping best responses outward, increasing aggregate quantity, which brings prices downs. In turn, this increases consumer surplus and overall welfare as argued by Allaz and Villa (1993).

Finally, regarding the changes in the manufacturer's and the retailers' profits, the quantity competition model yields opposite results when compared to the price competition model. As before, forward contracting has two effects on the retailers and manufacturer profit: for a given input price, retailers compete more fiercely as they commit to sell, bringing down their profit but increasing the profit for the manufacturer as the demand for the input increases. Nevertheless, for intermediate levels of bargaining power, forward contracting leads to lower input prices, which is profitable for the retailers but not profitable for the manufacturer. When the products are very differentiated, the retailers' gains from a lower input price compensate the losses from engaging in a forward commitment. A very extreme degree of bargaining power (either in favor of the retailers or of the manufacturer), yields a lower decrease in the input price. Therefore, the retailers are less likely to find the forward commitment profitable in this circumstance then when the retailers' degree of bargaining strength is intermediate and the decrease in the input price is higher. The opposite applies for the manufacturer.

[14] Even though an increase in the degree of substitution would cause the input price to increase, which would lead, everything else constant, to less forward contracts, the effect of the additional gains from unilaterally signing forward contracts is stronger and thus the retailers end up increasing their forward commitments.

5 Conclusion

With this paper we hope to have demystified the idea that when firms commit to a long position (i.e., commit to buy) they always win and that when they commit to a short position (i.e., commit to sell), they always loose. In fact, we have proved that, in a vertical industry, introducing forward contracts in a certainty environment might lead to losses when firms commit to buy or profits when firms commit to sell.

This is a rather interesting result as it highlights the danger of analyzing the impact of forward contracts through the change in firm's profits. According to previous results in the literature about Cournot competition and forward contracts, if the profit of the firms goes down when forward contracts are introduced, then it must be the case they compete in quantities and welfare goes up. However, in our case, we can have the opposite result. Conversely, we should not also assume that, just because firms are better off when a round of forward contracting is introduced, it must be the case that they compete in prices and welfare has gone down. Thus, the policies around forward contracts should not be decided based on the change in firms profits but rather if they decide to commit to buy or to commit to sell.

Moreover, the upstream firm should not be completely against or for the introduction of forward contracts. We have shown that it can benefit from its introduction, whether the downstream firms compete in quantities or prices. In fact, the manufacturer can profit even if the retailers also profit. For instance, in the quantity competition case, for a certain set of values, both the manufacturer and the retailers benefit from the engagement in forward contracts, showing that introducing forward contracts can benefit everyone. This contrasts with the results from Allaz and Villa (1993), where even though total welfare goes up, the retailers lose by committing to sell.

Nevertheless, our model makes many restrictive assumptions that could be developed on. Firstly, we only allow for two players downstream and we assume linear costs for all the players. One extension could be the introduction of various players downstream and increasing marginal costs, like in Bushnell (2007). Finally, we assume there is no uncertainty, which is obviously not realistic. This assumption is only used to focus exclusively on the strategic aspect of the forward contracts. One suggestion for future research could be the introduction of uncertainty about the spot price when the forward contracts are being negotiated. This, combined with risk-averse producers, would increase the incentive for the firms to sell forward in case of Cournot competition, as stated by Allaz and Villa (1993). However, if firms produced differentiated goods and competed in prices, the incentive of selling their output forward due to the introduction of uncertainty would be contrary to the usual competition-softening effect of taking a long position, as discussed by Mahenc and Salanié (2004).

References

[1] Aichele, M. (2014). Strategic Investment, Forward Markets and Competition. Tilburg University Working Paper.

[2] Allaz, B. and J-L Vila, 1993, "Cournot competition, forward markets and efficiency", Journal of Economic Theory, 59.

[3] Anderson, R., & Sundaresan, M. (1984) Futures Markets and Monopoly. In: Anderson, R. (ed.). The Industrial Organization of Futures Markets. Lexington Books, Lexinton Mass.

[4] Antelo, M. and Lluís Bru, 2002. "Forward contracts and competition," Spanish Economic Review, Springer;Spanish Economic Association, vol. 4(4), pages 281-300.

[5] Bushnell, J. (2007) "Oligopoly Equilibria in Electricity Contract Markets", Journal of Regulatory Economics 32 (3), 225-245.

[6] Dobson, P., Waterson, M., 2007, "The Competition Effects of Industry-wide Vertical Price Fixing in Bilateral Oligopoly", *International Journal of Industrial Organization* 25, 935–962.

[7] Hughes, J.S. and J.L. Kao (1997), "Strategic Forward Contracting and Observability," International Journal of Industrial Organization 16, 121-133.

[8] Liski, M., and J.-P. Montero (2006), Forward trading and collusion in oligopoly, Journal of Economic Theory 131, 212-230.

[9] Mahenc, P. & Salanie, F., 2004. "Softening competition through forward trading," Journal of Economic Theory, Elsevier, vol. 116(2), pages 282-293.

[10] Newbery, D. (1984) Manipulation of Futures Markets by a Dominant Producer. In: Anderson, R. (ed.). The Industrial Organization of Futures Markets. Lexington Books, Lexinton Mass.

[11] Newbery, D. (1987) Futures Markets, Hedging and Speculation. In: Eatwell et al (eds.). The New Palgrave: a Dictionary in Economics. The MacMillan Press Limited, London.

[12] Singh, N., and X. Vives, 1984, Price and Quantity Competition in a Differentiated Duopoly, *Rand Journal of Economics* 15, 546-554.

Appendix A - Proofs for the price competition models

Proof of Lemma 1:

(i) In case of duopoly, each downstream firms maximizes its own profit by choosing p_i. The first-order condition (FOC) is thus:

$$\frac{d}{dp_i}(\Pi_i) = \alpha - 2\beta p_i + \beta w_i + \beta\gamma p_j = 0$$

Solving this system of two equations yields the equilibrium price as a function of w_i and w_j:

$$p_i = \frac{(\alpha(\gamma+2) + 2\beta w_i + \beta\gamma w_j)}{(\gamma+2)(2-\gamma)\beta}$$

and firms obtain the corresponding profit:

$$\Pi_i = \frac{\left(2\alpha + \alpha\gamma - 2\beta w_i + \beta\gamma w_j + \beta\gamma^2 w_i\right)^2}{(\gamma+2)^2(\gamma-2)^2\beta}$$
$$\Pi_m = \frac{\alpha(w_i+w_j)(\gamma+2) - \beta(w_i+w_j)^2(2-\gamma^2) + 2\beta w_i w_j(\gamma+1)(2-\gamma)}{(\gamma+2)(2-\gamma)}$$

(ii) If the negotiations between the upstream firm and one of the downstream firms break down, the manufacturer only sells the input to the other retailer. Thus, retailer i becomes a monopolist in the second stage of the game, facing the following demand:

$$q_i^M = \alpha(\gamma+1) - \beta\left(1-\gamma^2\right)p_i$$

The retailer will maximize its own profit, Π_i^M, by choosing p_i. The first order condition (FOC) is thus:

$$\frac{d}{dp_i}(\Pi_i) = (\gamma+1)(\alpha - 2\beta p_i + \beta w_i + 2\beta\gamma p_i - \beta\gamma w_i) = 0$$

which yields the equilibrium price:

$$p_i^M = \frac{1}{2}\frac{\alpha + \beta w_i(1-\gamma)}{(1-\gamma)\beta}$$

and quantity:

$$q_i^M = \frac{1}{2}(\alpha - \beta w_i + \beta\gamma w_i)(\gamma+1)$$

This quantity will be the demand the manufacturer will face if the negotiations between the manufacturer and retailer j break down. Therefore, we can write the profit of the manufacturer in case of disagreement as:

$$\Pi_m^M = w_i q_i = \frac{1}{2}\left(\alpha - \beta w_i + \beta\gamma w_i\right)\left(\gamma+1\right) w_i$$

Proof of Proposition 1:

The transfer prices will be a result of two separate and simultaneous negotiations:

$$w_i = argmax_{w_i}\left[(\Pi_i^D - 0)^{\theta}(\Pi_m^D - \Pi_m^M)^{1-\theta}\right]$$

Solving the system of equations looking for the symmetric equilibrium ($w_i = w_j$), yields the equilibrium input prices:[15]

$$w_i = \frac{\alpha}{\beta}\frac{\left(1-\theta\right)\left(\gamma+2\right)}{\left(4-\gamma\left(\gamma+1\right)\left(2-2\theta\gamma+\theta\gamma^2\right)\right)}$$

and firms make the following equilibrium profits:

$$\begin{aligned}
\Pi_i &= \frac{\left(\gamma - 2\theta + \theta\gamma + \gamma^2 - \theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 2\right)^2 \alpha^2}{\left(2\gamma + 2\gamma^2 - 2\theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 4\right)^2 \left(\gamma - 2\right)^2 \beta} \\
\Pi_m &= \frac{2\left(-\gamma - \gamma^2 + 2 - \theta\left(\gamma - \gamma^2 - \gamma^3 + \gamma^4 - 2\right)\right)\left(1-\theta\right)\left(\gamma+2\right)\alpha^2}{\left(2\gamma + 2\gamma^2 - 2\theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 4\right)^2 \beta\left(2-\gamma\right)}
\end{aligned}$$

Also, in equilibrium, retail prices, quantities, consumer surplus and welfare will be given by:

$$\begin{aligned}
p_i &= \frac{\left(2\theta + \gamma + \theta\gamma + 2\gamma^2 - 2\theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 6\right)\alpha}{\left(2\theta\gamma^2 - 2\gamma^2 - 2\gamma + \theta\gamma^3 - \theta\gamma^4 + 4\right)\left(\gamma - 2\right)\beta} \\
q_i &= \frac{\left(\gamma - 2\theta + \theta\gamma + \gamma^2 - \theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 2\right)\alpha}{\left(2\theta\gamma^2 - 2\gamma^2 - 2\gamma + \theta\gamma^3 - \theta\gamma^4 + 4\right)\left(\gamma - 2\right)} \\
CS &= \frac{\alpha^2}{\beta}\frac{\left(\gamma - 2\theta + \theta\gamma + \gamma^2 - \theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 2\right)^2}{\left(2\gamma + 2\gamma^2 - 2\theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 4\right)^2\left(\gamma - 2\right)^2\left(1-\gamma\right)} \\
W &= \frac{\alpha^2}{\beta}\frac{\left(\theta\left(\gamma + 7\gamma^2 - \gamma^3 - 5\gamma^4 + 2\gamma^5 - 2\right) + \left(7 - 4\gamma\right)\left(1-\gamma\right)\left(\gamma+2\right)\right)}{\left(2\gamma + 2\gamma^2 - 2\theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 4\right)^2\left(2-\gamma\right)^2\left(1-\gamma\right)} \\
&\quad \times \left(\left(\gamma+2\right)\left(1-\gamma\right) + \theta\left(2 - \gamma + \gamma^2 + \gamma^3 - \gamma^4\right)\right)
\end{aligned}$$

Proof of Corollary 1:

[15] There is another possible solution for the system of equations where $w_i = \frac{\alpha}{(1-\gamma)\beta}$. However, this solution is not relevant for the discussion as it argues that the manufacturer chooses the maximum the consumers would be willing to pay for the final goods, causing the demand to be zero. Therefore, this solution will be dismissed.

Follows from

$$
\begin{aligned}
\frac{\partial w_i}{\partial \theta} &= -\frac{\alpha}{\beta}\frac{(\gamma+2)\left(2-\gamma^2\right)\left(2-\gamma+\gamma^2\right)}{\left(2\gamma+2\gamma^2-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-4\right)^2}<0 \\
\frac{\partial w_i}{\partial \gamma} &= \frac{\alpha}{\beta}(1-\theta)\frac{8(\gamma+1)(1-\theta\gamma)+\gamma^2\left(6\theta\gamma+3\theta\gamma^2+2\right)}{\left(2\gamma+2\gamma^2-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-4\right)^2}>0
\end{aligned}
$$

Also, substituting $\theta=1$ into $w_i=\frac{(\gamma+2)(\theta-1)\alpha}{(2\gamma+2\gamma^2-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-4)\beta}$ yields $w_i=0$ and substituting $\theta=0$ into $w_i=\frac{(\gamma+2)(\theta-1)\alpha}{(2\gamma+2\gamma^2-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-4)\beta}$ yields $w_i=\frac{\alpha}{2\beta(1-\gamma)}$.

Proof of Corollary 2:

Follows from

$$
\frac{\partial p_i}{\partial \theta}=-\frac{\alpha}{\beta}\frac{(\gamma+2)\left(2-\gamma^2\right)\left(2-\gamma+\gamma^2\right)}{(2-\gamma)\left(2\gamma+2\gamma^2-2\theta\gamma^2-\theta\gamma^3+\theta\gamma^4-4\right)^2}<0
$$

As for the degree of product substitution we have

$$
\begin{aligned}
\frac{dp_i}{d\gamma} &= \frac{\partial p_i}{\partial \gamma}+\frac{\partial p_i}{\partial w_i}\frac{\partial w_i}{\partial \gamma}+\frac{\partial p_i}{\partial w_j}\frac{\partial w_j}{\partial \gamma} \\
&= \frac{\partial p_i}{\partial \gamma}+\frac{2}{(2-\gamma)(\gamma+2)}\frac{\partial w_i}{\partial \gamma}+\frac{\gamma}{(2-\gamma)(\gamma+2)}\frac{\partial w_j}{\partial \gamma}>0
\end{aligned}
$$

because $\frac{\partial p_i}{\partial \gamma}=\frac{\alpha(\gamma+2)^2+\beta\left(4w_j+4\gamma w_i+\gamma^2 w_j\right)}{\beta(2-\gamma)^2(\gamma+2)^2}>0$, $\frac{\partial w_i}{\partial \gamma}>0$ and $\frac{\partial w_j}{\partial \gamma}>0$.

Proof of Lemma 2:

(i) In case of duopoly, each downstream firms maximizes its own profit by choosing p_i. The first order condition (FOC) is thus:

$$
\frac{d}{dp_i}(\Pi_i)=\alpha-f_i-2\beta p_i+\beta w_i+\beta\gamma p_j=0
$$

Solving this system of two equations yields the equilibrium price as a function of w_i, w_j, f_i and f_j :

$$
p_i=\frac{\left(\alpha(\gamma+2)-2f_i-\gamma f_j+2\beta w_i+\beta\gamma w_j\right)}{(\gamma+2)(2-\gamma)\beta}
$$

and firms obtain the corresponding profit:

$$\begin{aligned}\Pi_i &= \frac{\left(2\alpha+\alpha\gamma-2\beta w_i+\beta\gamma w_j+\beta\gamma^2 w_i\right)^2}{(\gamma+2)^2(\gamma-2)^2\beta}+\frac{(2f_i+\gamma f_j)\left(-2\alpha(\gamma+2)+(2f_i+\gamma f_j)\right)}{(\gamma+2)^2(\gamma-2)^2\beta}+f_i p_i^f+\\ &+\frac{\left(2\gamma f_j\left(2-\gamma^2\right)-f_i\left(8-4\gamma^2+\gamma^4\right)\right)w_i-2\gamma\left(2f_i+\gamma f_j\right)w_j}{(\gamma+2)^2(\gamma-2)^2}\\ \Pi_m &= \frac{\alpha(w_i+w_j)(\gamma+2)-\beta(w_i+w_j)^2\left(2-\gamma^2\right)+2\beta w_i w_j(\gamma+1)(2-\gamma)}{(\gamma+2)(2-\gamma)}+\\ &+\frac{f_i\left(w_i\left(2-\gamma^2\right)-\gamma w_j\right)+f_j\left(w_j\left(2-\gamma^2\right)-\gamma w_i\right)}{(2-\gamma)(\gamma+2)}\end{aligned}$$

(ii) In case negotiations break down, retailer i becomes a monopolist facing the following demand[16]:

$$q_i^M=\left(\beta-\beta\gamma^2\right)\left(\frac{\alpha}{\beta(1-\gamma)}-p_i\right)$$

The retailer will maximize its own profit, Π_i^M, by choosing p_i. The first order condition (FOC) is thus:

$$\frac{d}{dp_i}(\Pi_i)=\alpha+\alpha\gamma-f_i-2\beta p_i+\beta w_i+2\beta\gamma^2 p_i-\beta\gamma^2 w_i=0$$

which yields the equilibrium price:

$$p_i^M=\frac{1}{2}\frac{\left(f_i-\alpha\gamma-\alpha-\beta w_i+\beta\gamma^2 w_i\right)}{(\gamma+1)(\gamma-1)\beta}$$

In the previous stage of the game, the retailer chooses the optimal amount of forward contracts to engage in. Due to the perfect foresight assumption (i.e., $p_i^f=p_i$), the monopolist will maximize a simplified profit function:

$$\Pi_i=(p_i-w_i)q_i$$

in order to f_i. The FOC will then be:

$$\frac{d}{df_i}(\Pi_i)=\frac{1}{2}(\gamma+1)^{-1}(\gamma-1)^{-1}\beta^{-1}f_i=0$$

which yields:

$$f_i=0$$

[16] This also serves as a proof for the second part of Lemma 3

Therefore, the price the monopolist will choose will be:

$$p_i^M = \frac{1}{2} \frac{(\beta\gamma w_i - \beta w_i - \alpha)}{(\gamma - 1)\beta}$$

and the corresponding quantity is:

$$q_i^M = \frac{1}{2}(\alpha - \beta w_i + \beta\gamma w_i)(\gamma + 1)$$

Therefore, we can write the profit of the manufacturer in case of disagreement as:

$$\Pi_m^M = w_i q_i = \frac{1}{2}(\alpha - \beta w_i + \beta\gamma w_i)(\gamma + 1) w_i$$

which is identical to the one found in the no forward contract case.

Proof of Lemma 3:

(i) Retailer i will maximize its profit by choosing f_i. The FOC will then be:

$$\frac{d}{df_i}(\Pi_i) = -\frac{(8f_i + 2\alpha\gamma^2 + \alpha\gamma^3 - 4\gamma^2 f_i - \gamma^3 f_j - 2\beta\gamma^2 w_i + \beta\gamma^3 w_j + \beta\gamma^4 w_i)}{(\gamma + 2)^2 (\gamma - 2)^2 \beta} = 0$$

Solving this system of two equations yields the equilibrium amount of forward output each firm will commit:

$$f_i = \frac{(4\beta w_i - 2\alpha\gamma - 4\alpha - 2\beta\gamma w_j + \alpha\gamma^2 - 3\beta\gamma^2 w_i + \beta\gamma^3 w_j)\gamma^2}{(2\gamma + \gamma^2 - 4)(\gamma^2 - 2\gamma - 4)}$$

and firms obtain the corresponding profit:

$$\begin{aligned} \Pi_i &= (-2)\frac{(\gamma^2 - 2)(4\beta w_i - 2\alpha\gamma - 4\alpha - 2\beta\gamma w_j + \alpha\gamma^2 - 3\beta\gamma^2 w_i + \beta\gamma^3 w_j)^2}{(\gamma^2 - 2\gamma - 4)^2 (2\gamma + \gamma^2 - 4)^2 \beta} \\ \Pi_m &= \frac{\left(\alpha(2\gamma - \gamma^2 + 4)(w_i + w_j) - \beta\left((4 - 3\gamma^2)\left(w_i^2 + w_j^2\right) - 2\gamma w_i w_j (2 - \gamma^2)\right)\right)(2 - \gamma^2)}{(2\gamma - \gamma^2 + 4)(4 - 2\gamma - \gamma^2)} \end{aligned}$$

Proof of Corollary 3:

See proof of Corollary 6.

Proof of Proposition 2:

The transfer prices will be a result of two separate and simultaneous negotiations:

$$w_i = argmax_{w_i} \left[(\Pi_i^D - 0)^\theta (\Pi_m^D - \Pi_m^M)^{1-\theta} \right]$$

Solving the system of equations looking for the symmetric equilibrium ($w_i = w_j$), yields the equilibrium input prices:[17]

$$w_i = \frac{\alpha}{\beta} \frac{(1-\theta)(4-\gamma^2+2\gamma)(2-\gamma^2)}{(2(1-\gamma)(2-\gamma^2)(4+2\gamma-\gamma^2)+\theta\gamma^2(\gamma+1)(4-2\gamma-\gamma^2))}$$

and firms make the following equilibrium profits:

$$\Pi_i = \frac{2(2-\gamma^2)((1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)+2\theta(2\gamma+3\gamma^2-3\gamma^3+\gamma^5-4))^2}{(2\gamma+\gamma^2-4)^2(2(1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)+\theta\gamma^2(\gamma+1)(2\gamma+\gamma^2-4))^2} \frac{\alpha^2}{\beta}$$

$$\Pi_m = \frac{2(\gamma^2-2)^2 \begin{pmatrix} (1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4) \\ +2\theta(2\gamma+3\gamma^2-3\gamma^3+\gamma^5-4) \end{pmatrix} (\theta-1)(\gamma^2-2\gamma-4)}{(2(1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)+\theta\gamma^2(\gamma+1)(2\gamma+\gamma^2-4))^2(2\gamma+\gamma^2-4)} \frac{\alpha^2}{\beta}$$

Also, in equilibrium, retail prices, quantities, forward contracts will be given by:

$$p_i = \frac{((2-\gamma^2)(6-4\gamma-\gamma^2)(2\gamma-\gamma^2+4)+\theta(\gamma(\gamma+2)(16\gamma-2\gamma^2-6\gamma^3+\gamma^4-4)-16))}{(2(1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)-\theta\gamma^2(\gamma+1)(4-2\gamma-\gamma^2))(2\gamma+\gamma^2-4)} \frac{\alpha}{\beta}$$

$$q_i = \frac{((1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)+2\theta(2\gamma+3\gamma^2-3\gamma^3+\gamma^5-4))(2-\gamma^2)}{(2(1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)-\theta\gamma^2(\gamma+1)(4-2\gamma-\gamma^2))(4-2\gamma-\gamma^2)}\alpha$$

$$f_i = \frac{(4\gamma-8\theta+4\theta\gamma+10\gamma^2-4\gamma^3-3\gamma^4+\gamma^5+6\theta\gamma^2-6\theta\gamma^3+2\theta\gamma^5-8)\gamma^2}{(2(1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)-\theta\gamma^2(\gamma+1)(4-2\gamma-\gamma^2))(2\gamma+\gamma^2-4)}\alpha$$

Finally, equilibrium consumer surplus and welfare amount to:

$$CS = \frac{((1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)+2\theta(2\gamma+3\gamma^2-3\gamma^3+\gamma^5-4))^2}{(2(1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)+\theta\gamma^2(\gamma+1)(2\gamma+\gamma^2-4))^2} \times \frac{(\gamma^2-2)^2}{(2\gamma+\gamma^2-4)^2(1-\gamma)} \times \frac{\alpha^2}{\beta}$$

$$W = \frac{((1-\gamma)(2-\gamma^2)(4+2\gamma-\gamma^2)+2\theta(4-2\gamma-3\gamma^2+3\gamma^3-\gamma^5))(2-\gamma^2)}{(2(1-\gamma)(2-\gamma^2)(-2\gamma+\gamma^2-4)+\theta\gamma^2(\gamma+1)(2\gamma+\gamma^2-4))^2} \times \frac{\begin{pmatrix} 2\theta(4\gamma+26\gamma^2-8\gamma^3-23\gamma^4+5\gamma^5+5\gamma^6-8)+ \\ (1-\gamma)(2-\gamma^2)(14-8\gamma-3\gamma^2)(4+2\gamma-\gamma^2) \end{pmatrix}}{(2\gamma+\gamma^2-4)^2(1-\gamma)} \times \frac{\alpha^2}{\beta}$$

[17] As before, there is another possible solution for the system of equations where $w_i = \frac{\alpha}{(1-\gamma)\beta}$. For the same reasons mentioned above, this solution will be dismissed.

Proof of Corollary 4:

The difference in the equilibrium input prices between the two models, $w_i^F - w_i^{NF}$ is, after dividing by $\frac{\alpha}{\beta}(1-\theta)$, given by:

$$\frac{\theta\gamma^2(\gamma+1)(8-12\gamma^2+3\gamma^3+4\gamma^4-\gamma^5)}{(4-2\gamma(\gamma+1)+\theta\gamma^2(\gamma+1)(2-\gamma))(16-2\gamma(2-\gamma)(6\gamma+\gamma^2-\gamma^3+2)+\theta\gamma^2(\gamma+1)(4-\gamma(\gamma+2)))}$$

We now show that both the numerator and the two factors in the denominator are positive.

i) As $\frac{\partial(8-12\gamma^2+3\gamma^3+4\gamma^4-\gamma^5)}{\partial\gamma} = \gamma(9\gamma+16\gamma^2-5\gamma^3-24) < 0$, the term $(8-12\gamma^2+3\gamma^3+4\gamma^4-\gamma^5)$ decreases with γ. Evaluating it at $\gamma = 1$ we obtain $-12*1+3*1+4*1-1+8 = 2 > 0$. So, the numerator is always positive.

ii) As $4-2\gamma(\gamma+1) > 0$ the first factor in the denominator is also positive.

iii) As for the second factor, a sufficient condition for it to be positive is that $16 - 2\gamma(2-\gamma)(6\gamma+\gamma^2-\gamma^3+2) > 0$. This is true because

$$\frac{\partial(16-2\gamma(2-\gamma)(6\gamma+\gamma^2-\gamma^3+2))}{\partial\gamma} = -2\left(4(1-\gamma)\left(6\gamma+3\gamma^2+1\right)+5\gamma^4\right) < 0$$

and, evaluated at $\gamma = 1$ it is equal to $16-2(2-1)(6+1-1+2) = 0$.

Proof of Corollary 5:

(i) The increase in the transfer price is maximized at:

$$\frac{\partial\left(w_i^F - w_i^{NF}\right)}{\partial\theta} = 0$$

from where one obtains

$$\theta^*(\gamma) = 2(1-\gamma)\frac{2(1-\gamma)(\gamma+2)(4+2\gamma-\gamma^2)-r(\gamma)}{\gamma^2(\gamma+1)(24\gamma+4\gamma^2-9\gamma^3+3\gamma^4-24)}$$

with

$$r(\gamma) = \sqrt{(\gamma+2)(2-\gamma+\gamma^2)(4-3\gamma^2)(4-2\gamma-\gamma^2+\gamma^3)(2\gamma-\gamma^2+4)}.$$

The other root is negative. Figure 3 represents θ^* as a function of γ. As can be seen we have $\theta^* < \frac{1}{2}$.

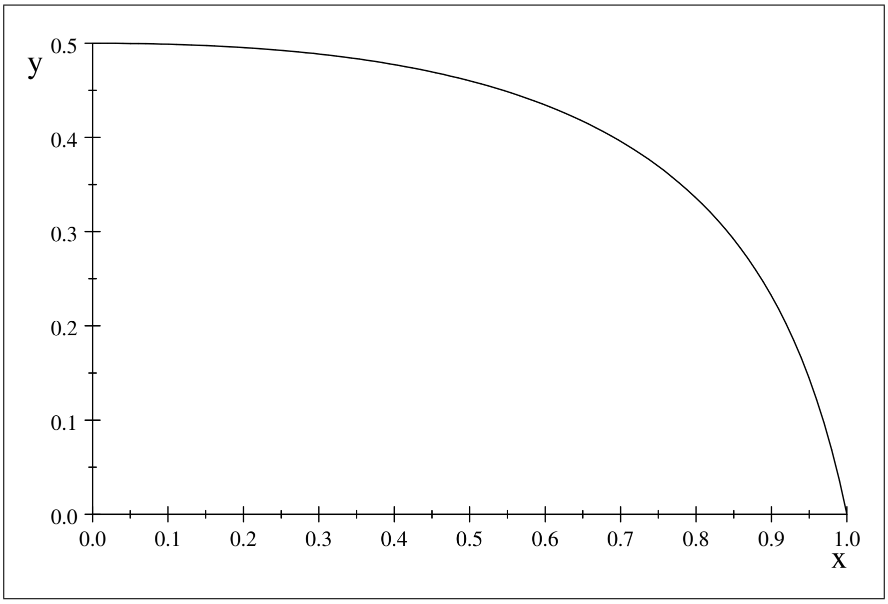

Figure 3: θ^* as a function of γ

Deriving $w_i^F - w_i^{NF}$ with respect to γ yields:

$$\frac{\theta\gamma \times \left[\begin{array}{c} \theta^2\gamma^4 \begin{pmatrix} 256\gamma^2 - 16\gamma^3 - 176\gamma^4 + 36\gamma^5 + \\ 42\gamma^6 - 5\gamma^7 - 8\gamma^8 + \gamma^9 - 128 \end{pmatrix} (\gamma+1)^2 + \\ 2\theta\gamma^3(\gamma-2) \begin{pmatrix} 80\gamma + 40\gamma^2 - 52\gamma^3 - 18\gamma^4 + \\ 11\gamma^5 + 28\gamma^6 - 4\gamma^7 - 6\gamma^8 + \gamma^9 - 96 \end{pmatrix} (\gamma+1)^2 + \\ 4(1-\gamma) \times \begin{pmatrix} 512\gamma - 512\gamma^2 - 1024\gamma^3 + 608\gamma^4 + 888\gamma^5 - 388\gamma^6 \\ -382\gamma^7 + 114\gamma^8 + 67\gamma^9 - 17\gamma^{10} - 3\gamma^{11} + \gamma^{12} + 256 \end{pmatrix} \end{array}\right]}{(2\gamma + 2\gamma^2 - 2\theta\gamma^2 - \theta\gamma^3 + \theta\gamma^4 - 4)^2 (8\gamma + 20\gamma^2 - 8\gamma^3 - 6\gamma^4 + 2\gamma^5 - 4\theta\gamma^2 - 2\theta\gamma^3 + 3\theta\gamma^4 + \theta\gamma^5 - 16)^2}$$

At $\theta = 0$ the derivative of the term in brackets in the numerator (which is a parabola in θ) is

$$2\gamma^3(2-\gamma)(\gamma+1)^2\left(96 - \gamma^9 + 6\gamma^8 + 4\gamma^7 - 28\gamma^6 - 11\gamma^5 + 18\gamma^4 + 52\gamma^3 - 40\gamma^2 - 80\gamma\right) > 0$$

At $\theta = 1$, the numerator is equal to:

$$\begin{pmatrix} 1024\gamma - 4096\gamma^2 - 1664\gamma^3 + 6656\gamma^4 + 224\gamma^5 - 5200\gamma^6 + 1064\gamma^7 + \\ 2176\gamma^8 - 802\gamma^9 - 586\gamma^{10} + 241\gamma^{11} + 128\gamma^{12} - 38\gamma^{13} - 18\gamma^{14} + 3\gamma^{15} + 1024 \end{pmatrix} > 0$$

At $\theta = 0$, the numerator is equal to:

$$4(1-\gamma)\left(\begin{array}{c} 512\gamma - 512\gamma^2 - 1024\gamma^3 + 608\gamma^4 + 888\gamma^5 - 388\gamma^6 + \\ -382\gamma^7 + 114\gamma^8 + 67\gamma^9 - 17\gamma^{10} - 3\gamma^{11} + \gamma^{12} + 256 \end{array}\right) > 0$$

So, it is always positive.

Proof of Corollary 3 and 6:

The amount of forward contracts in equilibrium is

$$f_i = \alpha\gamma^2 \frac{2\theta(2\gamma + 3\gamma^2 - 3\gamma^3 + \gamma^5 - 4) + \gamma(\gamma - 2)(-6\gamma - \gamma^2 + \gamma^3 - 2) - 8}{(4 - 2\gamma - \gamma^2)(16 - 2\gamma(2-\gamma)(6\gamma + \gamma^2 - \gamma^3 + 2) + \theta\gamma^2(\gamma + 1)(4 - \gamma(\gamma + 2)))}$$

In the proof of Corollary 4 we established that the denominator is positive. The numerator is always negative because it is linear in θ and:

i) For $\theta = 0$ we have

$$\begin{aligned} \gamma(\gamma - 2)\left(-6\gamma - \gamma^2 + \gamma^3 - 2\right) - 8 &= \\ -\gamma^4(1-\gamma) - 2(1-\gamma)(\gamma + 1)\left(4 - 2\gamma - \gamma^2\right) &< 0 \end{aligned}$$

which is always true.

ii) For $\theta = 1$ we have

$$\begin{aligned} 2\left(2\gamma + 3\gamma^2 - 4 - 3\gamma^3 + \gamma^5\right) + \gamma(\gamma - 2)\left(-6\gamma - \gamma^2 + \gamma^3 - 2\right) - 8 &= \\ -\left(4 - 3\gamma^2\right)\left(4 - \gamma(\gamma + 2) + \gamma^3\right) &< 0 \end{aligned}$$

which is also always true.

As for the impact of θ on the amount of forward contracting:

$$\frac{\partial f}{\partial \theta} = \frac{-\alpha\gamma^2(1-\gamma)(4 + 2\gamma - \gamma^2)(4 - \gamma(\gamma + 2) + \gamma^3)(4 - 3\gamma^2)(2 - \gamma^2)}{(4 - \gamma(\gamma + 2))(2\gamma(\gamma - 2)(-6\gamma - \gamma^2 + \gamma^3 - 2) + \theta\gamma^2(\gamma + 1)(2\gamma + \gamma^2 - 4) - 16)^2} < 0$$

Finally, the impact of γ is given by:

$$\frac{\partial f}{\partial \gamma} = \frac{\left[\begin{array}{c} \theta^2\gamma^3(2\gamma + \gamma^2 - 4)(12\gamma + 30\gamma^2 - 2\gamma^3 - 26\gamma^4 - 4\gamma^5 + 5\gamma^6 - 8) + \\ \theta\left(\begin{array}{c} 2\gamma^{11} - 40\gamma^{10} + 25\gamma^9 + 318\gamma^8 - 278\gamma^7 - 800\gamma^6 + \\ 960\gamma^5 + 512\gamma^4 - 1344\gamma^3 + 512\gamma^2 + 640\gamma - 512 \end{array}\right) \\ -2(4-\gamma)(\gamma - 1)^2(\gamma^2 - 2)^2(-2\gamma + \gamma^2 - 4)^2 \end{array}\right]}{\frac{(2\gamma + \gamma^2 - 4)^2(8\gamma + 20\gamma^2 - 8\gamma^3 - 6\gamma^4 + 2\gamma^5 - 4\theta\gamma^2 - 2\theta\gamma^3 + 3\theta\gamma^4 + \theta\gamma^5 - 16)^2}{2\alpha\gamma}}$$

The sign depends only on the sign of the numerator, which is a parabola in θ such that:
At $\theta = 0$ the numerator takes value

$$-2(4-\gamma)(\gamma-1)^2\left(\gamma^2-2\right)^2\left(-2\gamma+\gamma^2-4\right)^2 < 0$$

At $\theta = 1$ the numerator takes value

$$-(4-\gamma)\left(3\gamma^2-4\right)^2\left(-2\gamma-\gamma^2+\gamma^3+4\right)^2 < 0$$

The derivative in θ at $\theta = 0$ is

$$\left(2\gamma^{11}-40\gamma^{10}+25\gamma^9+318\gamma^8-278\gamma^7-800\gamma^6+960\gamma^5+512\gamma^4-1344\gamma^3+512\gamma^2+640\gamma-512\right)$$

which is always negative.

Proof of Corollary 7:

(i) The difference in the equilibrium retail prices (divided by $\frac{\alpha}{\beta}$) between the two models is given by:

$$p_i^F - p_i^{NF} = \gamma^2 \frac{-g_1(\gamma)\theta^2 + g_2(\gamma)\theta + g_3(\gamma)}{(2-\gamma)(4-\gamma(\gamma+2))\, g_4(\gamma) g_5(\gamma)}$$

with

$$\begin{aligned}
g_1(\gamma) &:= (1-\gamma)(\gamma+2)(\gamma+1)\left(2\left(2(1-\gamma)(2-\gamma)(3\gamma+2)+\gamma^6\right)+7\gamma^4(1-\gamma)\right) > 0 \\
g_2(\gamma) &:= \left(-16\gamma-88\gamma^2+24\gamma^3+26\gamma^4-11\gamma^5+7\gamma^6-3\gamma^8+\gamma^9+64\right) \\
g_3(\gamma) &:= 2(\gamma+2)\left(2-\gamma^2\right)\left(4+2\gamma-\gamma^2\right)(1-\gamma)^2 > 0 \\
g_4(\gamma) &:= \left(4-2\gamma(\gamma+1)+\theta\gamma^2(\gamma+1)(2-\gamma)\right) > 0 \\
g_5(\gamma) &:= \left(16-2\gamma(2-\gamma)\left(6\gamma+\gamma^2-\gamma^3+2\right)+\theta\gamma^2(\gamma+1)\left(4-2\gamma-\gamma^2\right)\right) > 0
\end{aligned}$$

The denominator is clearly positive. The numerator is an inverted parabola in θ so, the minimum is either at $\theta = 0$ or $\theta = 1$. At $\theta = 0$:

$$g_3(\gamma) = 2(\gamma+2)\left(2-\gamma^2\right)\left(4+2\gamma-\gamma^2\right)(1-\gamma)^2 > 0$$

At $\theta = 1$:

$$-g_1(\gamma)+g_2(\gamma)+g_3(\gamma) = \left(2-\gamma+\gamma^2\right)\left(2-\gamma^2\right)\left(4-3\gamma^2\right)\left(4-\gamma(\gamma+2)+\gamma^3\right) > 0$$

Therefore the retail price always increases with forward contracts.

(ii) and (iii): The results for consumer surplus and welfare follow from the fact that both decrease with retail price in a symmetric equilibrium.

Proof of Corollary 8:

(i) When the retailers have maximum bargaining strength (i.e., $\theta = 1$), the difference between the retailers' profit in the two models is given by:

$$\Pi_i^F - \Pi_i^{NF} = \frac{(4-3\gamma)\gamma^3}{(2\gamma+\gamma^2-4)^2(\gamma-2)^2}\frac{\alpha^2}{\beta}$$

which is always positive as α, β and γ are all positive parameters below one.

(ii) When the retailers have maximum bargaining strength (i.e., $\theta = 1$), the difference between the manufacturer's profit in the two models is given by:

$$\Pi_m^F - \Pi_m^{NF} = 0$$

since in both cases the input prices are set to zero.

Appendix B - Quantity competition

In this appendix we derive the solution for an identical model as the one before, with the only difference being the fact that the retailers compete in quantities (Cournot with differentiated products) instead of prices. All the other assumptions are kept the same. However, we will not derive as many propositions and corollaries as before since our main goal from introducing this model is to focus on the variation of the firms' profits. The timing in this alternative game is as follows:

1st) The two retailers and the manufacturer simultaneously bargain over the transfer prices.

2nd) The two retailers simultaneously make their decisions about the forward contracts.

3rd) The two retailers simultaneously set their quantities.

As before, we solve the game by backward induction, looking for the SPNE. We assume the same utility function as in the price competition model. Therefore, the demand will be given by:

$$p_i = \frac{\alpha}{\beta(1-\gamma)} - \frac{q_i}{\beta - \beta\gamma^2} - \gamma\frac{q_j}{\beta - \beta\gamma^2}$$

In case retailer i becomes a monopolist, the corresponding inverse demand function is given by:

$$p_i = \frac{\alpha}{\beta(1-\gamma)} - \frac{q_i}{\beta - \beta\gamma^2}$$

where q_i represents the quantity sold by retailer i and α, β and $\gamma < 1$ are three positive parameters with the same meaning as before. Consumer surplus and welfare have the expressions presented in the main text.

Model without forward contracts

As in the price model, when there is no forward contracting, the second round of the game is eliminated.

Quantity competition stage

The profit function of retailer i is given by:

$$\Pi_i = (p_i - w_i)q_i$$

with w_i having the same meaning as in the previous model. When the quantity competition stage is reached, we may have a duopoly (if both negotiations ended successfully) or a monopoly (if one negotiation broke down). The equilibrium is as follows:

i) In case of duopoly, each downstream firms maximizes its own profit by choosing q_i. The first order condition (FOC) is thus:

$$\frac{d}{dq_i}(\Pi_i) = \frac{\alpha + \alpha\gamma - 2q_i - \gamma q_j - \beta w_i + \beta\gamma^2 w_i}{(\gamma+1)(1-\gamma)\beta} = 0$$

Solving this system of two equations yields the equilibrium quantities as a function of w_i and w_j:

$$q_i = \frac{(\alpha(2-\gamma) - \beta(1-\gamma)(2w_i - \gamma w_j))(\gamma+1)}{(\gamma+2)(2-\gamma)}$$

and firms obtain the corresponding profit:

$$\Pi_i^D = \frac{(\alpha(2-\gamma) - \beta(1-\gamma)(2w_i - \gamma w_j))^2(\gamma+1)}{(1-\gamma)\beta(\gamma+2)^2(\gamma-2)^2}$$

ii) In case of monopoly, in equilibrium, retailer i sets exactly the same price/quantity as in the price competition model.

Bargaining stage

The first stage will work in the exact same way as before. Thus, transfer prices will be a result of two separate and simultaneous negotiations:

$$w_i = argmax_{w_i}\left[(\Pi_i^D - 0)^\theta(\Pi_m^D - \Pi_m^M)^{1-\theta}\right]$$

with $\Pi_m^D = w_i q_i + w_j q_j$ and $\theta \in [0, 1]$.

Solving the system of equations looking for the symmetric equilibrium ($w_i = w_j$), yields the equilibrium input prices[18]:

$$w_i = \frac{1}{2}\frac{1-\theta}{1-\gamma}\frac{\alpha}{\beta}$$

and firms make the following equilibrium profits:

$$\begin{aligned}\Pi_i &= \frac{1}{4}\frac{(\theta+1)^2(\gamma+1)}{(1-\gamma)(\gamma+2)^2}\frac{\alpha^2}{\beta}\\ \Pi_m &= \frac{1}{2}\frac{(\theta+1)(1-\theta)(\gamma+1)}{(1-\gamma)(\gamma+2)}\frac{\alpha^2}{\beta}\end{aligned}$$

In addition, in equilibrium, retail prices, quantities, consumer surplus and welfare will be given

[18] As in the previous model, there is another possible solution for the system of equations where $w_i = \frac{\alpha}{(1-\gamma)\beta}$. For the same reasons mentioned before, this solution will be dismissed.

by:

$$
\begin{aligned}
p_i &= \frac{1}{2}\frac{(3+\gamma-\theta(\gamma+1))}{(1-\gamma)(\gamma+2)}\frac{\alpha}{\beta} \\
q_i &= \frac{1}{2}\frac{(\theta+1)(\gamma+1)}{(\gamma+2)}\alpha
\end{aligned}
$$

$$
\begin{aligned}
CS &= \frac{1}{4}\frac{(\theta+1)^2(\gamma+1)^2}{(1-\gamma)(\gamma+2)^2}\frac{\alpha^2}{\beta} \\
W &= \frac{1}{4}(\theta+1)(\gamma+1)\frac{7+3\gamma-\theta(\gamma+1)}{(1-\gamma)(\gamma+2)^2}\frac{\alpha^2}{\beta}
\end{aligned}
$$

Regarding the equilibrium input prices (w_i), we can derive the same conclusions as in the price competition model:

Corollary 9. *Each symmetric equilibrium transfer price is (i) increasing in the degree of substitution of the retailers (ii) decreasing in terms of retailer's bargaining strength. When the retailers have maximum bargaining strength (i.e., $\theta = 1$), the transfer prices are zero.*

Proof:
(i) $\frac{d}{d\gamma}(w_i) = \left(-\frac{1}{2}\right)\frac{(\theta-1)\alpha}{(\gamma-1)^2\beta} > 0$
(ii) $\frac{d}{d\theta}(w_i) = \frac{1}{2}\frac{\alpha}{(\gamma-1)\beta} < 0$ ■

Finally, we can state the following regarding the equilibrium retail quantities (q_i):

Corollary 10. *In the symmetric equilibrium, the retail quantities are (i) increasing in γ (ii) increasing in the retailer's bargaining strength.*

Proof:
(i) $\frac{d}{d\gamma}(q_i) = \frac{1}{2}\frac{(\theta+1)\alpha}{(\gamma+2)^2} > 0$
(ii) $\frac{d}{d\theta}(q_i) = \frac{1}{2}\frac{(\gamma+1)\alpha}{(\gamma+2)} > 0$ ■

As in the pricing model, when the downstream firms have more power in the negotiation, they are able to set lower input prices, which allows firms to set a higher quantity. Conversely, when γ increases, the upstream firm has an incentive to increase the input prices, leading to higher marginal costs. This effect alone should lead to a decrease in quantity. However, an increase in γ leads to an increase in demand, giving the retailers an incentive to produce more. This effect is stronger than the increase in w_i and thus q_i increases.

Model with forward contracts

In this section, we solve the game including the stage of forward contracting inbetween the bargaining stage and the final, spot, stage.

Quantity competition stage

Due to the introduction of forward contracts, the profit function for the retailers is now given by:

$$\Pi_i = p_i(q_i - f_i) + p_i^f f_i - w_i q_i = (p_i - w_i)q_i + (p_i^f - p_i)f_i$$

where f_i and p_i^f still have the same meaning as in the price competition model. Moreover, we may have a duopoly if the negotiations in the first stage were successful or a monopoly if negotiations broke down.

(i) In case of duopoly, each downstream firms maximizes its own profit by choosing q_i. The first order condition (FOC) is thus:

$$\frac{d}{dq_i}(\Pi_i) = \frac{(\alpha + \alpha\gamma + f_i - 2q_i - \gamma q_j - \beta w_i + \beta\gamma^2 w_i)}{(\gamma+1)(1-\gamma)\beta} = 0$$

Solving this system of two equations yields the equilibrium quantity and price as a function of w_i, w_j, f_i and f_j:

$$\begin{aligned} q_i(w_i, w_j, f_i, f_j) &= \frac{\alpha(\gamma+1)(2-\gamma) + 2f_i - \gamma f_j - \beta(1-\gamma)(\gamma+1)(2w_i - \gamma w_j)}{4-\gamma^2} \\ p_i(w_i, w_j, f_i, f_j) &= \frac{\alpha(\gamma+1)(2-\gamma) - f_i(2-\gamma^2) - \gamma f_j + \beta(1-\gamma)(\gamma+1)((2-\gamma^2)w_i + \gamma w_j)}{\beta(1-\gamma)(2-\gamma)(\gamma+2)(\gamma+1)} \end{aligned}$$

ii) In case of monopoly, in equilibrium, retailer i sets the same quantity as in the model with no forward contracts.

In the case of duopoly, the retail quantity increases with f_i but decreases with f_j. By increasing f_i, firm i is either committing to sell more of its output in the future (if $f_i > 0$) or committing to buy less of its own production in the spot market (if $f_i < 0$). Either way, this gives an incentive for the firm to increase its output q_i as it reduces the price of good, making the provision of f_i less costly. In reality, we can think of an increase in the amount of forward contracts as a parallel expansion outwards of firm i best response function. Conversely, when f_j increases, firm j best response function expands making it produce more, causing the price to drop and thus giving firm i an incentive to reduce its equilibrium output. As in the price competition case, the effect of f_i is always larger ($2 > \gamma$), since the effect from the competitor is only indirect.

The set of FOC with respect to retail quantities is:

$$\frac{\partial\left[(p_i - w_i)q_i + (p_i^f - p_i)f_i\right]}{\partial q_i} = 0 \Leftrightarrow p_i - w_i + \frac{\partial p_i}{\partial q_i}q_i - \frac{\partial p_i}{\partial q_i}f_i = 0$$

When setting the optimal quantity, retailer i has to take into consideration three effects. Firstly, he will realize the unit margin $p_i - w_i$ on the additional sales. Secondly, the price of infra-marginal sales decreases, with an impact on profit of $\frac{\partial p_i}{\partial q_i}q_i$. Finally, the third effect comes from the introduction of forward contracts. This effect is positive if the firm commits to sell ($f_i > 0$) or negative if the firm commits to buy ($f_i < 0$). As in the previous model, if the firm has committed to sell f_i units at a predetermined price, it will have to "buy" them at the higher spot price, thus having a loss. Therefore, the retailer will have an incentive to increase the quantity produced in order to lower the spot price.

Additionally, retail quantities are increasing in α as it translates into a larger demand function, giving firms an incentive to sell more. Finally, retail quantities are decreasing in w_i and increasing in w_j, which are the retailers' marginal cost of producing the final good. Increasing the marginal cost of the own retailer causes its best response function to shift inwards, reducing the quantity produced. On the other hand, an increase in the rival's marginal cost, leads to an inwards shift of the rival's best response function, causing its quantity to decrease and thus increasing the spot price, giving the retailer an incentive to increase its quantity.

Forward contracting stage

In this stage, firms choose the amount of forward contracts. As in the price competition model, we assume agents have perfect foresight. The profit of each retailer in this stage is thus:

$$\Pi_i = (p_i - w_i)q_i$$

which will be a function of f_i, f_j, w_i and w_j. Once more, this stage may involve a duopoly or a monopoly.

(i) In case of a duopoly, each retailer i will maximize its profit by choosing f_i. The FOC will then be:

$$\frac{d}{df_i}(\Pi_i) = \frac{8f_i - 2\alpha\gamma^2 - \alpha\gamma^3 + \alpha\gamma^4 - 4\gamma^2 f_i + \gamma^3 f_j + 2\beta\gamma^2 w_i - \beta\gamma^3 w_j - 2\beta\gamma^4 w_i + \beta\gamma^5 w_j}{(\gamma+2)^2(\gamma-2)^2(\gamma+1)(\gamma-1)\beta} = 0$$

Solving this system of two equations yields the equilibrium amount of forward output each firm will commit to:

$$f_i = \gamma^2 (\gamma + 1) \frac{\alpha (4 - 2\gamma - \gamma^2) - \beta (1 - \gamma) (w_i (2 - \gamma) (\gamma + 2) - 2\gamma w_j)}{(4 + 2\gamma - \gamma^2) (4 - 2\gamma - \gamma^2)}$$

and firms obtain the corresponding profit:

$$\Pi_i = \frac{2 (\gamma + 1) (2 - \gamma^2) (\alpha (4 - 2\gamma - \gamma^2) - \beta (1 - \gamma) (w_i (2 - \gamma) (\gamma + 2) - 2\gamma w_j))^2}{\beta (1 - \gamma) (-2\gamma + \gamma^2 - 4)^2 (2\gamma + \gamma^2 - 4)^2}$$

$$\Pi_m = 2 (\gamma + 1) \frac{\alpha (4 - 2\gamma - \gamma^2) (w_i + w_j) + \beta (1 - \gamma) \left(4\gamma w_i w_j - (2 - \gamma) (\gamma + 2) \left(w_i^2 + w_j^2\right)\right)}{(-2\gamma + \gamma^2 - 4) (2\gamma + \gamma^2 - 4)}$$

(ii) In case of a monopoly, retailer i chooses not to engage in any forward commitment, i.e., $f_i = 0$. So, the optimal quantity and corresponding profit are identical to those in the previous models.

In case of duopoly, the amount of forward contracts retailer i will commit to varies negatively with w_i and positively with w_j. The FOC is given by:

$$\frac{d\Pi_i}{df_i} = \frac{\partial [(p_i - w_i)q_i]}{\partial q_i} \frac{\partial q_i}{\partial f_i} + \frac{\partial [(p_i - w_i)q_i]}{\partial q_j} \frac{\partial q_j}{\partial f_i} = f_i \frac{\partial p_i}{\partial q_i} \frac{\partial q_i}{\partial f_i} + (-\frac{\gamma}{\beta - \beta\gamma^2}) q_i \frac{\partial q_j}{\partial f_i} = 0$$

The first term, which represents the impact of f_i on retailer i's profit if retailer j's quantity is kept constant has the opposite sign of f_i. The second term, which represents the impact of f_i on retailer i's profit that results from changes in q_j (a strategic effect) is positive. A higher w_i lowers the second term whereas a higher w_j increases it. As we saw before, a higher w_i leads to a reduction in the quantity produced q_i. Thus, the strategic effect becomes weaker and, as this effect impacts the firm's profit positively, it will lead to less forward contracts. On the other hand, an increase in w_j causes q_i to increase (as previously shown). This, in turn, amplifies the strategic effect leading to an increase in the amount of forward contracts.

Also, we know the last term of the FOC is positive because $-\frac{\gamma}{\beta - \beta\gamma^2} < 0$, $q_i > 0$ and $\frac{\partial q_j}{\partial f_i} < 0$ (as it is shown before). Therefore, in order for this condition to hold, it must be the case that the first term is negative. This only happens if $f_i > 0$ since $\frac{\partial p_i}{\partial q_i} < 0$ and $\frac{\partial q_i}{\partial f_i} > 0$. Therefore, we can state the following:

Corollary 11. *In a symmetric equilibrium, if $\gamma > 0$, firms always commit to sell, i.e., $f_i > 0$. If $\gamma = 0$, firms choose not to sign any forward contracts, i.e., $f_i = 0$.* ■

This is the expected result when firms compete in quantities and are allowed to sign forward contracts, following the conclusions from Allaz and Villa (1993). If $\gamma = 0$, the retailers become monopolists and thus have no incentive to sign any forward commitment (as evidenced by

Anderson and Sundaresan (1984)).

Also, as in the previous model, the amount of forward contracts f_i is more sensitive to changes in w_i compared to changes in w_j since $|(\gamma+2)(\gamma-2)| > 2\gamma$.[19]

In case of a monopoly, we reach exactly the same solution as in the price competition model. The second effect does not exist and the retailer chooses not to commit forward as the biggest profit the monopolist can obtain is the monopolist profit.

Bargaining stage

The optimal transfer prices will be given by the solution to the following problem:

$$w_i = argmax_{w_i}\left[\Pi_i^D - 0)^{\theta}(\Pi_m^D - \Pi_m^M)^{1-\theta}\right]$$

with $\Pi_m^D = w_i q_i + w_j q_j$ and $\theta \in [0,1]$.

Solving the system of equations looking for the symmetric equilibrium ($w_i = w_j$), yields the equilibrium input prices:[20]

$$w_i = \frac{2(1-\theta)(4-2\gamma-\gamma^2)}{(1-\gamma)(16-8\gamma-4\gamma^2+\theta\gamma^2(2\gamma-\gamma^2+4))}\frac{\alpha}{\beta}$$

and firms make the following equilibrium profits:

$$\Pi_i = \frac{2(2-\gamma^2)(4\gamma-8\theta+4\theta\gamma+2\gamma^2-2\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-8)^2(\gamma+1)}{(1-\gamma)(8\gamma+4\gamma^2-4\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-16)^2(\gamma^2-2\gamma-4)^2}\frac{\alpha^2}{\beta}$$

$$\Pi_m = \frac{8(4\gamma-8\theta+4\theta\gamma+2\gamma^2-2\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-8)(\gamma+1)(4-2\gamma-\gamma^2)(1-\theta)}{(\gamma^2-2\gamma-4)(8\gamma+4\gamma^2-4\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-16)^2(1-\gamma)}\frac{\alpha^2}{\beta}$$

In addition, in equilibrium, retail prices, quantities, forward contracts will be given by:

$$p_i = \frac{(4(2\gamma+\gamma^2-4)(-\gamma+\gamma^2-3)+\theta(-8\gamma+20\gamma^2+8\gamma^3-6\gamma^4-2\gamma^5+\gamma^6-16))}{(4\theta\gamma^2-4\gamma^2-8\gamma+2\theta\gamma^3-\theta\gamma^4+16)(4-\gamma^2+2\gamma)(1-\gamma)}\frac{\alpha}{\beta}$$

$$q_i = \frac{2(4\gamma-8\theta+4\theta\gamma+2\gamma^2-2\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-8)(\gamma+1)}{(8\gamma+4\gamma^2-4\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-16)(4-\gamma^2+2\gamma)}\alpha$$

$$f_i = \frac{(4\gamma-8\theta+4\theta\gamma+2\gamma^2-2\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-8)(\gamma+1)\gamma^2}{(4\theta\gamma^2-4\gamma^2-8\gamma+2\theta\gamma^3-\theta\gamma^4+16)(\gamma^2-2\gamma-4)}\alpha$$

[19]Note that $\frac{d}{dw_i}(f_i) = \frac{(\gamma+1)(\gamma+2)(\gamma-2)(\gamma-1)\gamma^2\beta}{(4-\gamma^2-2\gamma)(\gamma^2-2\gamma-4)}$ and $\frac{d}{dw_j}(f_i) = (-2)\frac{(\gamma+1)(\gamma-1)\beta\gamma^3}{(2\gamma+\gamma^2-4)(\gamma^2-2\gamma-4)}$.

[20]As in the previous model, there is another possible solution for the system of equations where $w_i = \frac{\alpha}{(1-\gamma)\beta}$. For the same reasons mentioned before, this solution will be dismissed.

Finally, consumer surplus and welfare are:

$$
\begin{aligned}
CS &= \frac{\left(2(\gamma+1)\frac{-8\theta+4\gamma+2\gamma^2+4\theta\gamma-2\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-8}{(-2\gamma+\gamma^2-4)(8\gamma+4\gamma^2-4\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-16)}\right)^2}{(1-\gamma)}\frac{\alpha^2}{\beta} \\
W &= \frac{2(\gamma+1)(4\gamma+2\gamma^2+\theta(4\gamma-2\gamma^2-2\gamma^3+\gamma^4-8)-8)}{(4+2\gamma-\gamma^2)(8\gamma+4\gamma^2-4\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-16)(1-\gamma)}\times \\
&\left(2+\frac{2(\gamma+1)(4\gamma+2\gamma^2+\theta(4\gamma-2\gamma^2-2\gamma^3+\gamma^4-8)-8)}{(-2\gamma+\gamma^2-4)(8\gamma+4\gamma^2-16+\theta\gamma^2(-2\gamma+\gamma^2-4))}\right)\frac{\alpha^2}{\beta}
\end{aligned}
$$

Results

Regarding the input (or transfer) price, we can conclude the following:

Corollary 12. *In a symmetric equilibrium where $\gamma > 0$ and $\theta \in]0;1[$, introducing forward contracts causes the equilibrium transfer price to always be lower than in the model without forward contracts.*[21] ■

Proof: The difference in the equilibrium input prices between the two models is given by:

$$w_i^F - w_i^{NF} = -\frac{1}{2}\frac{(1-\theta)\theta(4-\gamma^2+2\gamma)\gamma^2}{(4(4-2\gamma-\gamma^2)+\theta\gamma^2(4-\gamma^2+2\gamma))(1-\gamma)}\frac{\alpha}{\beta}$$

which is negative. ■

As in the price competition model, we will explain this result by assuming extreme cases, such as $\theta = 0$ and $\theta = 1$.

The demand function that the monopolist upstream firm faces from retailer i is given by:

$$q_i = \frac{\alpha(\gamma+1)(2-\gamma)+2f_i-\gamma f_j-\beta(1-\gamma)(\gamma+1)(2w_i-\gamma w_j)}{4-\gamma^2}$$

where f_i and f_j can either be zero (no forward contracting) or the equilibrium functions of w_i and w_j derived above.

With $\theta = 0$, when maximizing its profit $\Pi_m^D = w_i q_i + w_j q_j$ the manufacturer sets w_i such that:

$$\frac{d\Pi_m(w_i,w_j)}{dw_i} = q_i + w_i\frac{dq_i}{dw_i} + w_j\frac{dq_j}{dw_i} = 0$$

The effects when w_i increases are the same as under price competition. Nevertheless, they will now change differently when forward contracts are introduced.

Contrary to the first model, the first term increases when f_i is introduced but decreases

[21] If $\theta = 1$, there is no change in the transfer price as retailers would always choose the minimum level possible, i.e., zero.

when f_j is introduced, since f_i and f_j are both positive. Due to forward contracts, the number of units sold for the same transfer price levels changes by:

$$\frac{2f_i - \gamma f_j}{(2-\gamma)(\gamma+2)}$$

Note that for a given, common $f_i = f_j = f$ the change in demand is $\frac{1}{\gamma+2} > 0$ and becomes closer to zero as γ increases.

In the first term, the direct effect dominates,[22] meaning that forward contracts increase the demand faced by the manufacturer, an effect that, per se, would lead to an increase in the transfer prices. As retailers commit to a short position, the upstream manufacturer sees an increase in the demand for the input, which all else constant would lead to an increase in price.

The second term, as before, can be decomposed in:

$$\frac{dq_i}{dw_i} = \underset{-}{\frac{\partial q_i}{\partial w_i}} + \underset{++}{\frac{\partial q_i}{\partial f_i}} \underset{-}{\frac{\partial f_i}{\partial w_i}} + \underset{-}{\frac{\partial q_i}{\partial f_j}} \underset{+}{\frac{\partial f_j}{\partial w_i}}$$

where the last two additive terms only exist under forward contracting. As these two terms are negative, they make the derivative more negative (a more elastic demand) which leads, all else constant, to lower prices: the manufacturer anticipates that a higher transfer price for firm i will decrease its forward contracts. This makes firm i relatively less aggressive, setting a lower output. Also, a higher transfer price for firm i will increase the rival's forward contracts. This makes firm j more aggressive, setting a higher output, and therefore lowering the output of retailer i. Therefore, the demand from retailer i will decrease more after a transfer price increase when forward contracts are introduced. This effect would lead, *ceteris paribus*, to a lower input price.

Finally, the last term can be decomposed in

$$\frac{dq_j}{dw_i} = \underset{+}{\frac{\partial q_j}{\partial w_i}} + \underset{-}{\frac{\partial q_j}{\partial f_i}} \underset{-}{\frac{\partial f_i}{\partial w_i}} + \underset{++}{\frac{\partial q_j}{\partial f_j}} \underset{+}{\frac{\partial f_j}{\partial w_i}}$$

Increasing w_i increases the demand by the other retailer, and with forward contracts this effect is bigger. Therefore, everything else constant, we should expect a higher transfer price.

As in the price competition model, when $\theta = 0$, the three effects compensate each other and the equilibrium transfer prices do not change with the introduction of forward contracts.

[22] As $f_i > 0$ and $f_j > 0$, when forward contracts are intoduced, q_i changes by $\frac{2}{(2-\gamma)(\gamma+2)}$ due to f_i and $\frac{-\gamma}{(2-\gamma)(\gamma+2)}$ due to f_j. Since $2 > \gamma$, the direct effect always dominates the indirect effect in a symmetric equilibrium.

Making $\theta = 0$ we have that:

$$2\frac{(\theta-1)(2\gamma+\gamma^2-4)\alpha}{(8\gamma+4\gamma^2-4\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-16)(\gamma-1)\beta} = \frac{1}{2}\frac{(\theta-1)\alpha}{(\gamma-1)\beta} = \frac{\alpha}{\beta}\frac{1}{2(1-\gamma)}.$$

If $\theta = 1$, the optimal w would be, as before, the lowest possible, that is, zero. If each retailer could choose the transfer price to maximize own profit (given the other retailer's transfer price):

$$\Pi_i = (\frac{\alpha}{\beta(1-\gamma)} - \frac{q_i}{\beta-\beta\gamma^2} - \gamma\frac{q_j}{\beta-\beta\gamma^2} - w_i)(q_i)$$

with q_i and q_j functions of $w_i, w_j, f_i(w_i, w_j)$ and $f_j(w_i, w_j)$, it would consider the following effects:

$$\frac{d\Pi_i}{dw_i} = -q_i + \frac{\partial \Pi_i}{\partial q_i}\frac{dq_i}{dw_i} + \frac{\partial \Pi_i}{\partial q_j}\frac{dq_j}{dw_i}$$

with

$$\begin{aligned}\frac{dq_i}{dw_i} &= \frac{\partial q_i}{\partial w_i} + \frac{\partial q_i}{\partial f_i}\frac{\partial f_i}{\partial w_i} + \frac{\partial q_i}{\partial f_j}\frac{\partial f_j}{\partial w_i} \\ \frac{dq_j}{dw_i} &= \frac{\partial p_j}{\partial w_i} + \frac{\partial q_j}{\partial f_i}\frac{\partial f_i}{\partial w_i} + \frac{\partial q_j}{\partial f_j}\frac{\partial f_j}{\partial w_i}\end{aligned}$$

As f_i will be chosen in the next stage to maximize the same function, the effects through f_i are zero:

$$\frac{\partial \Pi_i}{\partial q_i}\frac{\partial q_i}{\partial f_i}\frac{\partial f_i}{\partial w_i} + \frac{\partial \Pi_i}{\partial q_j}\frac{\partial q_j}{\partial f_i}\frac{\partial f_i}{\partial w_i} = 0$$

and what remains is:

$$\frac{d\Pi_i}{dw_i} = -q_i + \frac{\partial \Pi_i}{\partial q_i}\left(\frac{\partial q_i}{\partial w_i} + \frac{\partial q_i}{\partial f_j}\frac{\partial f_j}{\partial w_i}\right) + \frac{\partial \Pi_i}{\partial q_j}\left(\frac{\partial q_j}{\partial w_i} + \frac{\partial q_j}{\partial f_j}\frac{\partial f_j}{\partial w_i}\right)$$

When compared to what would happen with forward contracts, the difference is the term:

$$\left(\underset{+}{\frac{\partial \Pi_i}{\partial q_i}}\underset{-}{\frac{\partial q_i}{\partial f_j}} + \underset{-}{\frac{\partial \Pi_i}{\partial q_j}}\underset{+}{\frac{\partial q_j}{\partial f_j}}\right)\underset{+}{\frac{\partial f_j}{\partial w_i}}$$

which is negative. Thus, contrary to the first model, this leads to a lower optimal w_i in the retailer's perspective: by increasing w_i the rival would increase its forward contracts, becoming more aggressive, bringing the price down and hurting retailer i.

Again, the SOC for maximum is not verified,[23] leading to a corner solution: $w_i = 0$. Now, the derivative is more negative than in the absence of forward contracts. When, as result of the bargaining process the two FOC are weighted to determine the optimal w, this increase in benefits to one of the bargaining parts of a lower w makes the equilibrium transfer price decrease.

Summing up, the existence of forward contracts does not change the optimal transfer prices in the perspective of the manufacturer or of each retailer. These are, as in the price competition model, $w_i = \frac{\alpha}{\beta}\frac{1}{2(1-\gamma)}$ and $w_i = 0$. Nevertheless, it does change the respective derivatives. The FOC for the bargaining problem is given by:

$$(1-\theta)\Pi_i^D \frac{d\Pi_m}{dw_i} + \theta(\Pi_m^D - \Pi_m^M)\frac{d\Pi_i}{dw_i}$$

with the second term always negative. The introduction of forward contracts lowers the first term and makes the second term more negative, resulting in a decrease in w_i. In this case, the decrease is greater for intermediate values of θ. When θ is too extreme (either close to 0 or 1), we only have one of the terms decreasing, resulting in a smaller decrease in w_i.

Therefore, we can conclude that the transfer price decreases with the existence of forward contracts, because now an increase in the transfer price affects each retailer more negatively when compared to when there was no forward contracts.

We can also state the following concerning the input price:

Corollary 13. *In a symmetric equilibrium where $\gamma > 0$ and $\theta < 1$, the decrease in the equilibrium transfer price due to the introduction of forward contracts is higher when (i) γ is high (ii) retailers' relative degree of bargaining strength is not too extreme.* ■

Proof:

(i) Deriving $w_i^F - w_i^{NF}$ with respect to γ yields:

$$\frac{d(w_i^F - w_i^{NF})}{d\gamma} = -\frac{128 - 4\gamma^2\left(-16\gamma + 2\gamma^2 + \gamma^3 + 40\right) + \theta\gamma^3\left(-2\gamma + \gamma^2 - 4\right)^2}{2\left(8\gamma + 4\gamma^2 - 4\theta\gamma^2 - 2\theta\gamma^3 + \theta\gamma^4 - 16\right)^2(1-\gamma)^2}(1-\theta)\,\theta\gamma\frac{\alpha}{\beta}$$

The denominator is clearly positive. The numerator is also positive because

$$\max_{\gamma\in[0,1]} 4\gamma^2\left(-16\gamma + 2\gamma^2 + \gamma^3 + 40\right) = 108 < 128$$

Therefore, we can conclude the sign of the derivative is negative, meaning the difference

[23] $4\frac{(\gamma+2)^2(\gamma-2)^2\left(\gamma^2-2\right)(\gamma+1)(\gamma-1)\beta}{(\gamma^2-2\gamma-4)^2(2\gamma+\gamma^2-4)^2} > 0$

gets more negative as γ increases.

(ii) Deriving $w_i^F - w_i^{NF}$ in order to θ yields:

$$\frac{d}{d\theta}\left(w_i^F - w_i^{NF}\right) = -\frac{\left(4\left(2\gamma+\gamma^2-4\right)(2\theta-1)+\theta^2\gamma^2\left(-2\gamma+\gamma^2-4\right)\right)\left(4-\gamma^2+2\gamma\right)\gamma^2}{2\left(8\gamma+4\gamma^2-4\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-16\right)^2(1-\gamma)}\frac{\alpha}{\beta}$$

The sign of this derivative depends on the sign of $\left(4\left(2\gamma+\gamma^2-4\right)(2\theta-1)+\theta^2\gamma^2\left(-2\gamma+\gamma^2-4\right)\right)$ which is an inverted parabola in θ which takes value $4(4-2\gamma-\gamma^2) > 0$ at $\theta = 0$ and $-(2-\gamma)(\gamma+2)(4-2\gamma+\gamma^2) < 0$ at $\theta = 1$. ■

Part (ii) of this corollary was explained above. When $\theta = 0$ or $\theta = 1$, the equilibrium transfer price is the same with or without forward contracts. It is for intermediate values of bargaining power that the fact that an increase in transfer prices hurts retailers more with forward contracts is felt and results in a lower transfer price. When γ increases, firms have an additional incentive to trade forward as the gains from unilaterally signing forward contracts are higher (even though the firm will eventually loose due to the rival's response). This means that, for the same value of w_i, we should expect a higher amount of forward contracts signed, increasing the two effects previously described. Therefore, for a given level of retailer's bargaining strength, the input price will decrease more with the introduction of forward contracts if γ is high.

Moreover, we can also conclude the following:

Corollary 14. *In a symmetric equilibrium, forward contracts are (i) increasing with γ (ii) increasing with the retailers' bargaining strength.* ■

Proof:

(i) Deriving f_i in order to γ yields:

$$\frac{d}{d\gamma}(f_i) = \frac{\gamma\alpha\left(a\theta^2+b\theta+c\right)}{\left(-2\gamma+\gamma^2-4\right)^2\left(8\gamma+4\gamma^2-4\theta\gamma^2-2\theta\gamma^3+\theta\gamma^4-16\right)^2}$$

The sign depends of this derivative is equal to the sign of $a\theta^2 + b\theta + c$ with:

$$a = \gamma^3\left(2\gamma-\gamma^2+4\right)\left(8\gamma+60\gamma^2+20\gamma^3-12\gamma^4-6\gamma^5+\gamma^6-16\right)$$
$$b = 10\gamma^8(2-\gamma)+228\gamma^7+32\gamma^3(5\gamma+4)(8-7\gamma)$$
$$+256\gamma(1-\gamma)\left(1+\gamma+\gamma^2+\gamma^3+\gamma^4\right)+512\gamma(1-\gamma)+1024 > 0$$
$$c = 8\left(8+14\gamma+4\gamma^2-\gamma^3\right)\left(2\gamma+\gamma^2-4\right)^2 > 0$$

At $\theta = 0$ we have that $a\theta^2 + b\theta + c = c > 0$. Moreover, the derivative of $a\theta^2 + b\theta + c$ with respect to θ, $2a\theta + b$ is, at $\theta = 0$, equal to $b > 0$. Finally, at $\theta = 1$ we have that

$a\theta^2 + b\theta + c = (14\gamma + 4\gamma^2 - \gamma^3 + 8)(\gamma - 2)^2 (\gamma + 2)^2 (-2\gamma + \gamma^2 + 4)^2 > 0$. Thus, for any $\theta \in [0, 1]$, $a\theta^2 + b\theta + c$ is positive.

(ii) Deriving f_i in order to θ yields:

$$\frac{d}{d\theta}(f_i) = \frac{2\gamma^2 (2 - \gamma)(\gamma + 2)(\gamma + 1)(4 - 2\gamma - \gamma^2)(4 - 2\gamma + \gamma^2)}{(4 + 2\gamma - \gamma^2)(8\gamma + 4\gamma^2 + \theta\gamma^2 (-2\gamma + \gamma^2 - 4) - 16)^2}\alpha$$

which is positive. ■

Part (i) is partially explained above. However, in addition to that effect, a higher γ causes the input price to increase, which would lead, all else constant, to lower forward contracts.[24] Nevertheless, the first effect dominates and firms end up engaging in more forward contracts. Moreover, as in the price competition model, a higher degree of bargaining strength for the downstream firms translates into lower marginal costs (lower input prices), which for the same reasons as mentioned above, leads to an increase in the amount of forward contracts.

Additionally, we can write the following regarding the introduction of forward contracts in our vertical arrangement[25]:

Proposition 4. *In a symmetric equilibrium where $\gamma > 0$, introducing forward contracts leads to (i) a decrease in the retail price (ii) an increase in consumer surplus (iii) an increase in welfare.* ■

Proof:

(i) The difference in the equilibrium retail quantities between the two models is given by:

$$q_i^F - q_i^{NF} = -\alpha\gamma^2 (\gamma + 1)\frac{\theta^2 (-2\gamma + \gamma^2 - 4)^2 + \theta(\gamma^4 - 8\gamma - 32) + 4(2\gamma + \gamma^2 - 4)}{2(\gamma + 2)(4 + 2\gamma - \gamma^2)(16 - 8\gamma - 4\gamma^2 + \theta\gamma^2 (4 + 2\gamma - \gamma^2))}$$

where q_i^F is the retail quantity in the model with forward contracts and q_i^{NF} is the retail quantity in the model without forward contracts. The numerator is a U-shaped parabola that takes value $4(2\gamma + \gamma^2 - 4) < 0$ when $\theta = 0$ and $-2(2 - \gamma)(\gamma + 2)(4 - 2\gamma + \gamma^2) < 0$ when $\theta = 1$. Thus, it is always negative, meaning that $q_i^F - q_i^{NF} > 0$. (ii) and (iii) follow directly from the fact that CS increases with quantity, as well as welfare (provided that $q < \alpha$, which is the case). ■

These results are the exact opposite from those of the price competition model. As firms

[24] Remember that, even though f_i decreases with w_i and increases with w_j, the direct effect is always bigger than the strategic effect, as proven before.

[25] As before, when comparing the two models, we will always assume that $\gamma > 0$. If the products were completely different (i.e., $\gamma = 0$), firms would choose not to engage in any forward contracts, making the two models equivalent and thus not worth comparing.

commit to sell, they expand their downward slopping best responses outward, increasing aggregate quantity, which brings prices downs. In turn, this increases consumer surplus and overall welfare as argued by Allaz and Villa (1993).

In what concerns the upstream firm's profit, we can conclude the following:

Remark 3. *In a symmetric equilibrium where $\gamma > 0$, introducing forward contracts is beneficial for the manufacturer if (i) products are differentiated enough (ii) the degree of retailers' bargaining strength is low enough.* ■

The difference in the manufacturer's profit between the two models is presented inFigure 5, which shows that the manufacturer is more likely to win with the introduction of forward contracting if γ is low and/or θ is low.

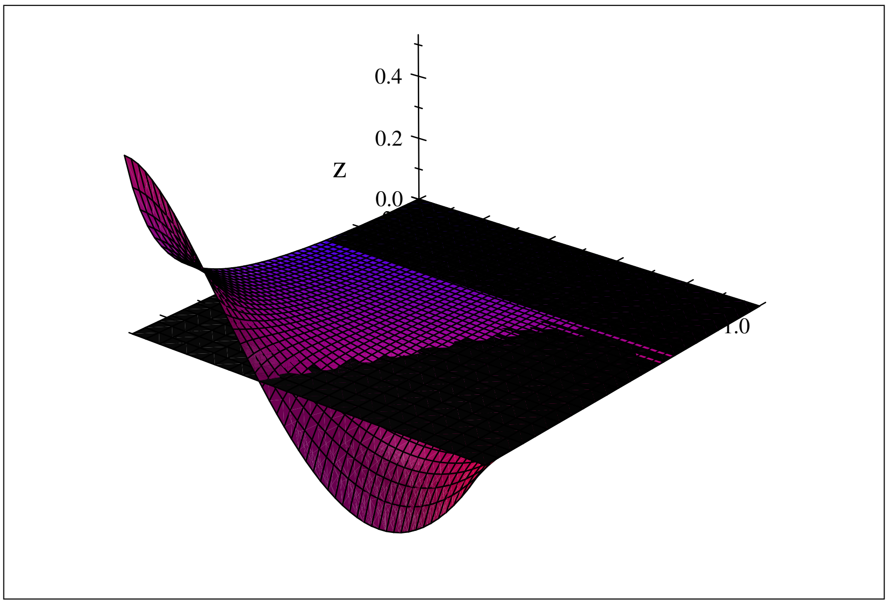

Figure 4: Difference in the manufacturer's profit with forward contracting and without forward contracting as a function of γ and θ (quantity competition).

Finally, regarding the retailers' profit we can conclude the following:

Remark 4. *In a symmetric equilibrium where $\gamma > 0$, introducing forward contracts is beneficial for the retailers if (i) products are differentiated enough (ii) the retailers' degree of bargaining power is not too extreme.* ■

The difference in the retailers' profit between the two models is presented in Figure 5 which shows that the retailers are more likely to benefit from the introduction of forward contracts if γ is low and/or the value of θ is intermediate.

As in the pricing model, forward contracting has two effects on the retailers and manufacturer profit: for a given input price, retailers compete more fiercely as they commit to sell, bringing down their profit but increasing the profit for the manufacturer as the demand for the

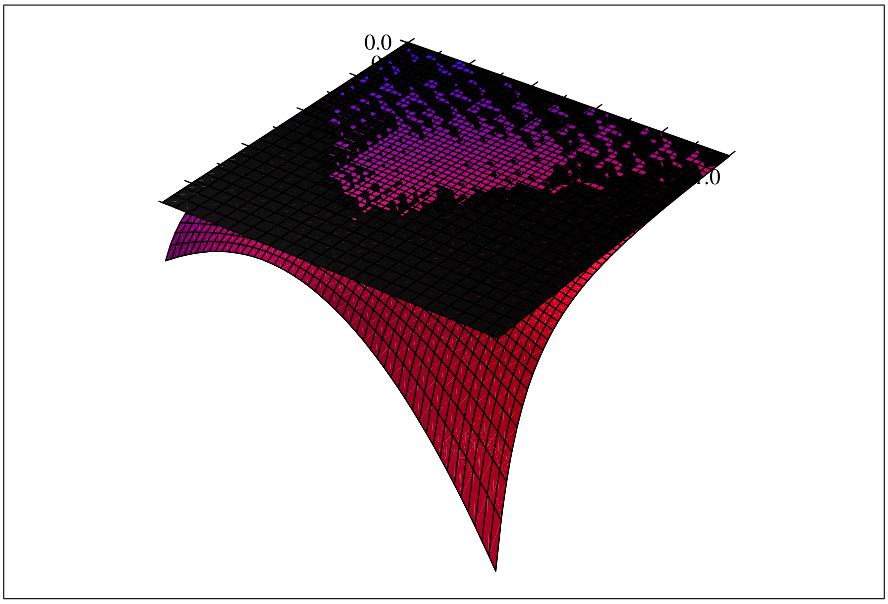

Figure 5: Difference in the retailer's profit with forward contracting and without forward contracting as a function of γ and θ (quantity competition).

input increases. Nevertheless, for intermediate levels of bargaining power, forward contracting leads to lower input prices, which is profitable for the retailers but not profitable for the manufacturer. When the products are very differentiated, the retailers' gains from a lower transfer price compensate the losses from engaging in a forward commitment. As seen above, a very extreme degree of bargaining power (either in favor of the retailers or of the manufacturer), yields a lower decrease in the input price.[26] Therefore, the retailers are less likely to find the forward commitment profitable in this case then when the retailers' degree of bargaining strength is intermediate and the decrease in the input price is the highest. The opposite applies for the manufacturer.

Finally, we can derive the additional insight:

Corollary 14. *When the retailers have maximum bargaining strength (i.e., $\theta = 1$) and $\gamma > 0$, introducing forward contracts is (i) never profitable for the retailers (ii) never profitable for the manufacturer.* ■

Proof:

(i) When the retailers have maximum bargaining strength (i.e., $\theta = 1$), the difference between the retailers' profit in the two models is given by:

$$\Pi_i^F - \Pi_i^{NF} = -\frac{(\gamma+1)(3\gamma+4)\gamma^3\alpha^2}{(\gamma^2-2\gamma-4)^2(\gamma+2)^2(1-\gamma)\beta}$$

which is always negative as α, β and γ are all positive parameters below one, leading the

[26] See Corollary 13.

numerator to be positive but the denominator to be negative.

(ii) When the retailers have maximum bargaining strength (i.e., $\theta = 1$), the difference between the manufacturer's profit in the two models is given by $\Pi_m^F - \Pi_m^{NF} = 0$ since, in both cases, the input prices are set to zero. ■

This is the expected result from introducing forward agreements in a Cournot competition environment (the one found in Allaz and Villa (1993)). The manufacturer will make zero profits as, when $\theta = 1$, the retailers will set the input prices to zero in both models.