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Volatility-Managed Portfolios: Insights from 153 Factors in Global Markets

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Abstract

This Dissertation investigates the impact of volatility-managed portfolios across a set of 153 factors and 13 themes, segmented by Developed and Emerging markets. I demonstrate that dynamically adjusting portfolio risk in response to volatility enhances alpha, sharpe ratio, and mean-variance utility. This analysis demonstrates that volatility timing improves the performance of not only well-known factors like value, momentum, and profitability but also the vast majority of the 153 factors examined, across all countries and regions tested. This analysis aligns with the findings of Volatility-Managed Portfolios by Moreira and Muir (2017), challenging risk-based models and structural assumptions of time-varying expected returns. It shows how managed portfolios that take a cautious approach during high-volatility periods can have a real edge, providing practical insights for investors in both Developed and Emerging Markets who want to get the most out of factor-based strategies. However, as per the findings of Cederburg et al. (2020), the out-of-sample (OOS) results for these strategies are not as strong as one might expect based on the in-sample analysis.

Keywords: Volatility Management, Factor Investing, Sharpe Ratio, Alpha Generation, Mean-Variance Utility, Dynamic Portfolio Strategies, Emerging Markets, Developed Markets, Risk-Adjusted Returns.

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Resumo

Esta tese investiga o impacto de escalar portfolios num conjunto de 153 fatores e 13 temas com base na volatilidade passada, segmentados em mercados desenvolvidos e emergentes. Esta investigação documenta que ajustar dinamicamente o risco de um portfolio em resposta à volatilidade gera melhorias nos retornos anormais, razões de Sharpe e na utilidade de um investidor cuja função objetivo tem em consideração os dois primeiros momentos da distribuição: a média e a variância. Esta análise demonstra que escolher alocações de riqueza em função da volatilidade melhora fatores já bastante estudados como, Valor e Momento, assim como a grande maioria de outros fatores menos populares mas também incorporados nesta dissertação. A análise desta tese corrobora os resultados documentados por Moreira e Muir (2017) que coloca em questão os modelos de equilíbrio de risco e pressupostos estruturais da variação de retornos ao longo do tempo. É também demonstrado que escalar portfolios de acordo com a volatilidade traduz-se numa abordagem cautelosa em períodos de alta volatilidade o que por sua vez implica um ganho real.

Keywords: Volatilidade Passada, Razões de Sharpe, Melhorias nos retornos anormais, Utilidade, Mercados emergentes, Mercados desenvolvidos, Ganho real

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1 Introduction

Volatility management and factor investing have become key topics in finance, as investors look for ways to improve risk-adjusted returns. During the last few years factor investing has become a cornerstone of modern asset pricing, offering insights into the drivers of stock returns. Asset pricing originated from William F. (1964), Lintner (1965) and Mossin (1966) by creating the Capital Asset Pricing Model (CAPM), which introduced beta as a key measure of systematic risk, however, Black, Jensen, and Scholes (1972), identified that the relationship between beta and returns was inconsistent with the CAPM's predictions, creating debates on the validity of the model. Due to this fact, research started expanding the focus to additional variables beyond beta starting with Fama and French (1993) which introduced size and book-to-market factors, demonstrating that these characteristics significantly explain variations in stock returns. This started the shift from a single factor to multifactor models, which have since become established in asset pricing theory.

Today, academic research has uncovered hundreds of factors, Harvey, Liu, and Zhu (2016) highlight the expansion of these anomalies, emphasizing the importance of robust testing to identify economically meaningful factors. One of the anomalies that stood out is the volatility anomaly by Haugen and Heins (1975) that has earned significant attention for its counterintuitive finding that low-volatility stocks often outperform high-volatility ones. Moreira and Muir (2017) later introduced the concept of volatility-managed portfolios, showing that scaling portfolio risk based on past volatility can significantly boost Sharpe ratios and improve utility for investors in the U.S. market. My thesis builds upon their work, expanding the analysis to include 153 factors and 13 themes while examining Developed and Emerging Markets. seeking to provide new perspectives and address gaps identified in earlier research. Fleming et al. (2001) have emphasized the advantages of dynamic portfolio strategies by showing the economic benefits of using realized volatility for adjustments, while Campbell and Thompson (2008) demonstrated that timing expected returns can outperform simple historical averages. This thesis takes those ideas further by examining how volatility timing interacts with established multifactor models, including the Fama-French three-factor model (1996), the five-factor model (2015), and the Hou, Xue, and Zhang (2015) q-factor model, it also takes into consideration the findings of Barroso and Santa-Clara (2015), who highlighted the importance of risk timing in momentum strategies.

Going beyond the U.S. focused findings of Cederburg et al. (2020), this study looks at how volatility management performs across different regions showing variations in alpha and Sharpe ratio improvements, offering a more global perspective. An interesting observation is that the momentum theme consistently emerges as the strongest performer while investment theme occupies the opposite position across all regions which corroborates the U.S. specific results. Besides looking at different regions, this research goes deeper analyzing 13 themes, identifying not just the strongest but also the weakest performers, expecting that these findings will help investors improve their factor-based strategies for specific regional contexts. The analysis shows that while volatility management is effective, its impact varies by region since in the U.S., most factors produce positive alphas, with a significant portion being statistically significant while Developed Markets show similar trends but with slightly fewer positive alphas, suggesting possible regional constraints such as liquidity and maturity of the market. Emerging Markets show comparable alpha generation but with fewer statistically significant factors, reflecting higher variability. Sharpe ratio improvements follow a similar pattern, with the U.S. seeing moderate average improvements and Emerging Markets showing much higher gains, likely due to greater efficiency gains in less mature markets, by contrast, Developed Markets show a mean decline, which may reflect unique structural challenges to those regions.

Barroso and Detzel (2020) observed that volatility-managed portfolios tend to perform well only during periods of high liquidity and strong market sentiment, therefore this seems to limit their effectiveness in Developed Markets, where investors seem to demonstrate lower enthusiasm that often hurts performance, as demonstrated by Wang, Su and Duxbury (2021) where the research indicates that investor sentiment has a more immediate effect in emerging markets which are more prone to quicker shifts than developed markets where the effect is more enduring. Building on Moreira and Muir's (2017) framework, I also tested a crash indicator similar to theirs and confirmed that during crises exposure to factors decreases in the U.S. markets, furthermore, I extended this analysis to test if the same pattern holds for Developed and Emerging Markets, further validating the robustness of this behavior across regions which was not the case.

Beyond these established approaches, I conducted out-of-sample tests to examine the performance of efficient portfolios constructed using mean-variance optimization, which were then volatility-managed. The results revealed that volatility management on these out-of-sample portfolios performed significantly worse when compared to in-sample results and for the U.S. region I found small positive alphas, whereas Emerging Markets produced small and statisti-

cally insignificant alphas while Developed Markets showed negative insignificant alphas. These findings stand in contrast to the strong in-sample performance, highlighting a potential limitation in the framework proposed by Moreira and Muir (2017). The disparity between in-sample and out-of-sample results underscores the challenges of translating the theoretical benefits of volatility management into practical, real-world applications. This is particularly evident in the difficulties associated with accurately predicting factor returns and addressing market-specific constraints, as demonstrated by Cederburg et al. (2020).

This limitation applies specifically to Efficient portfolios, Markowitz (1952), prompting me to extend the analysis to out-of-sample testing for individual factors and other optimization methodologies. First, by isolating factors I aimed to provide a more nuanced understanding of how volatility management impacts individual out of sample performance and then by incorporating the Gao and Nardari (2018) approach I have tried to assess whether alternative methods of portfolio construction could better capture the benefits of volatility management. To further explore the out-of-sample impact, at a given point in time t , I computed the t-statistic of the intercept of regressing each managed factor on its original version using data until $t-1$. Then I analyzed for each, the Sharpe ratios from the scaled and plain versions from $t+1$ until the last sample date and evaluated the percentual increase in Sharpe ratio.

I analyzed this exercise for an estimation window of initially 10 years, but then gauged performance for wider windows of 11 to 40 years. For all windows I then extracted the median Sharpe ratio increase. Consistent with Cederburg et al. (2020) the variation in Sharpe ratio was either close to 0 or negative in the great majority of the windows. These results do not vary qualitatively when rolling estimation is employed. For both Developed and Emerging Markets, the estimation period was shortened to 2 years due to data constraints and in contrast to the U.S. region, where rolling and expanding methods yielded similar results, these alternative regions demonstrated better outcomes with the expanding method.

Finally, I implemented the Gao and Nardari (2018) optimization approach to assess whether alternative portfolio construction methods could better capture the benefits of volatility management. The results were inconclusive, as some universes, such as the Fama-French three-factor model with Momentum, Market Risk Premium (MRP), and HXZ with Momentum yielded positive Sharpe ratio changes, while others showed negative outcomes, one interesting factor that can be observable is that portfolios combined with momentum present larger gains from volatility

management implying that the factor is gaining from volatility management even out of sample. For the Emerging and Developed Markets, volatility management improves every single portfolio that was optimized using the Gao and Nardari approach. This consistency in improvement may be attributed to less data availability, resulting in fewer data points and influencing the observed results.

2 Data

2.1 Raw data

The dataset used in this study is sourced from the Global Factor Data by Jensen, Kelly, and Pedersen (JKP) (2022). See the website jkpfactors.com, for more details about the data and its construction. For the U.S. region and Developed Markets, the dataset covers the period from January 2, 1926, to December 31, 2023, however, not all factors begin in 1926 due to limitations in early data availability, whereas for Emerging Markets, the data begins on January 2, 1986, and extends to December 31, 2023. As with the other regions, some factors in Emerging Markets have different starting dates, reflecting differences in when data became accessible. This dataset builds on the 153 factors analyzed by Jensen, Kelly, and Pedersen (2022) and is organized into 13 themes across 93 countries where themes include well-known categories such as value, momentum, and profitability, each representing distinct aspects of asset pricing anomalies, for this reason the data's depth make it particularly suited for a study that aims to uncover regional and thematic variations in volatility-managed portfolios as it is this case.

In addition to the dataset, I incorporated the NBER-based Recession Indicators for the United States from the Period following the Peak through the Trough to identify periods of economic downturns which I have used as a crash indicator in my analysis allowing for the assessment of portfolio performance during periods of heightened market stress.

2.2 Factor and Portfolio Construction

Factor portfolios are constructed following the methodology detailed by JKP, using country-specific tercile sorting, which means that stocks are ranked based on specific characteristics, such as book-to-market ratios or past returns, and sorted into terciles (top/middle/bottom third) using breakpoints derived from non-micro stocks (those above the NYSE 20th percentile by

market capitalization). Micro-cap stocks are then allocated across terciles based on the same breakpoints. Within each tercile, stocks are weighted by their market equity, ensuring balanced and tradable portfolios by minimizing the influence of tiny or mega-cap stocks.

The resulting factor return is defined as the excess return of a long-short zero-net-investment strategy, where the portfolio is long the high-tercile and short the low-tercile, where each factor requires a minimum of five stocks in both the long and short legs with weight defined based on value and at least 60 monthly observations to be included in the sample. For accounting factors, characteristics are updated using the most recent accounting data, starting four months after the fiscal period ends and factor clusters (themes) are constructed by equal-weighting the returns of factors within a specific category, following Jensen et al. (2022). This process ensures that the portfolios remain aligned with targeted exposures while maintaining practical tradability.

As seen in Table 2, the summary statistics of monthly returns for the market, size, and momentum factors across different regions align with findings from previous research. The mean returns, volatility levels, and Sharpe ratios follow expected patterns, with higher volatility in emerging markets and more stable returns in developed economies, the excess kurtosis and skewness values highlight the presence of fat-tailed distributions and asymmetry, consistent with prior studies on asset pricing.

Table 1. Theme Definitions

Theme	Definition
MOM (Momentum)	Stocks with strong past returns.
QUAL (Quality)	Financially strong and profitable firms.
MRP (Market Risk Premium)	Excess return over risk-free.
PG (Profit Growth)	Companies with increasing earnings growth.
LOWVOL (Low Volatility)	Stocks with stable price movements.
RMW (Robust Minus Weak)	Strong profitability minus weak firms.
SEAS (Seasonality)	Patterns based on calendar effects.
HML (High Minus Low)	Value stocks minus growth stocks.
DI (Debt Issuance)	Firms issuing new debt securities.
STR (Sentiment Trading)	Driven by investor sentiment shifts.
ACC (Accruals)	Non-cash earnings affecting profits.
INV (Investment)	Firms’ asset growth strategies.
LOWLEV (Low Leverage)	Companies with minimal financial debt.

Table 2. Summary statistics

The table presents the summary statistics of the monthly returns the market, size and value factors for each of the regions under analysis. Kitchen sink refers to the factor that combines all the factors and themes. The frequency of returns is monthly.

	United States				Developed Economies				Emerging Economies			
	MRP	Size	MOM	Kitchen Sink	MRP	Size	MOM	Kitchen Sink	MRP	Size	MOM	Kitchen Sink
Mean Annualized	8.14 (4.37)	2.52 (2.11)	5.84 (3.49)	1.96 (4.90)	7.41 (3.99)	2.18 (2.01)	5.10 (2.48)	5.50 (3.47)	1.68 (0.34)	5.08 (3.76)	4.98 (2.69)	3.84 (3.18)
Volatility Annualized	18.48	11.81	16.45	3.97	18.39	6.69	12.52	15.71	30.77	8.14	10.95	7.44
Sharpe ratio Annualized	0.44 (4.35)	0.21 (2.11)	0.35 (3.49)	0.49 (4.87)	0.40 (3.98)	0.33 (2.00)	0.41 (2.47)	0.35 (3.46)	0.05 (0.34)	0.62 (3.73)	0.45 (2.68)	0.52 (3.17)
Excess Kurtosis	7.55 (52.93)	40.17 (281.20)	17.85 (124.30)	41.61 (291.64)	7.57 (53.02)	9.20 (40.10)	4.99 (21.48)	15.22 (106.57)	39.73 (173.18)	7.67 (32.62)	17.99 (75.19)	33.06 (144.09)
Skewness	0.14 (1.99)	3.69 (51.69)	-2.16 (-30.02)	-3.51 (-49.15)	0.20 (2.86)	-0.58 (-5.08)	-0.80 (-6.91)	0.68 (9.58)	2.66 (23.21)	1.05 (8.97)	1.29 (10.78)	3.67 (32.02)
AR(1)	0.09 (3.26)	0.17 (5.71)	0.08 (2.68)	0.08 (2.84)	0.11 (3.64)	0.12 (2.57)	0.20 (4.22)	0.12 (4.23)	0.07 (1.49)	0.09 (1.85)	0.10 (2.08)	0.34 (7.28)
Minimum	-29.18	-12.49	-45.96	-12.94	-29.18	-14.34	-21.84	-29.18	-46.00	-8.35	-14.79	-13.17
Maximum	39.20	47.46	21.99	6.57	39.20	9.99	14.68	39.20	101.95	16.73	29.37	18.04
Alpha	- (-)	0.48 (0.43)	7.82 (4.83)	3.11 (10.16)	- (-)	2.30 (2.10)	6.34 (3.16)	0.33 (0.36)	- (-)	5.09 (3.78)	5.11 (2.77)	3.75 (3.19)
Beta	- (-)	0.25 (14.72)	-0.25 (-9.83)	-0.14 (-29.64)	- (-)	-0.02 (-0.90)	-0.20 (-5.42)	0.70 (48.55)	- (-)	0.03 (2.01)	-0.04 (-2.03)	0.06 (4.99)
Information Ratio	-	0.04	0.49	1.03	-	0.34	0.52	0.04	-	0.63	0.47	0.52
Observations	1, 179	1, 176	1, 164	1, 179	1, 176	456	444	1, 176	456	434	419	456
JB test	2, 806***	81, 745***	16, 352***	87, 468***	2, 818.94***	1, 634***	509***	11, 449***	30, 528***	1, 144***	5, 769***	21, 787***

3 Methodology

The construction of volatility-managed portfolios, as per Moreira and Muir (2017), involves scaling a portfolio using a constant that is calibrated such that the unconditional volatilities of the original and the volatility-managed factors are equal which ensures comparability between the two strategies.

$$f_{t+1}^{\sigma} = \frac{c}{\hat{\sigma}_t^2 f} f_{t+1} \quad (1)$$

Where f_t and f_{t+1}^{σ} are the original and managed factors monthly returns and c is the standard deviation normalizing constant. $\hat{\sigma}_t^2 f$ is the realized variance over the 22 trading days preceding the portfolio adjustment at the start of the month $t+1$ for factor f . The time series of the volatility is shown in Figure 1. The rolling window ensures that the variance estimate reflects the most recent market conditions. This statistic is computed as follows:

$$\hat{\sigma}_t^2 f = \sum_{d=1}^{22} (r_{s_{t+1}-d}^f - \bar{r}_t^f)^2 \quad (2)$$

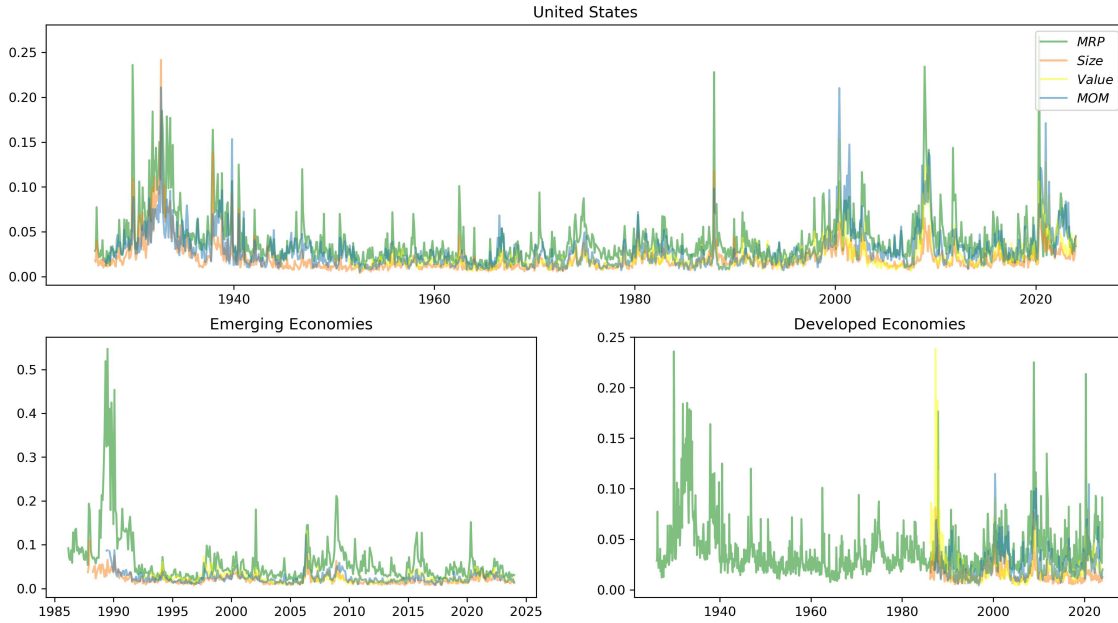
Where the $r_{s_{t+1}-d}^f$ is the daily return for a given factor (f) d days before the first trading day of month $t + 1$ and \bar{r}_t^f is the average daily return of f in month t . To compute the abnormal return of volatility management, we regressed each of the scaled factors on its original counterpart.

$$f_t^{\sigma} = \alpha^f + \beta^f f_t + \epsilon_t^f \quad (3)$$

As in Moreira and Muir (2017), $\alpha^f > 0$ translates into an increase in Sharpe ratio through a combination of the original factor (f_t) and the managed factor (f_t^{σ}) in relation to that of f_t . To ensure robust statistical inference, the covariance matrix of regression estimates was adjusted for potential autocorrelation and heteroskedasticity in the errors, this approach based on Newey and West (1987) which applies the Heteroskedasticity and Autocorrelation Consistent covariance estimator, correcting for both time-series dependencies (autocorrelation) and non-constant variance in the residuals (heteroskedasticity), by addressing these issues, the adjusted covariance matrix provides more reliable estimates of standard errors, increasing the validity of regression inferences.

Figure 1. Realized Volatility series

The figure describes realized volatility series for Momentum, Size, Value and the Market risk premium, computed by taking the square root of the output of (2) for the US and Developed and Emerging markets.



4 In Sample Scaling

Throughout this section we will extend the analysis in Moreira and Muir (2017) to a greater number of factors for both the United States and the Developed and Emerging markets.

4.1 US extended analysis

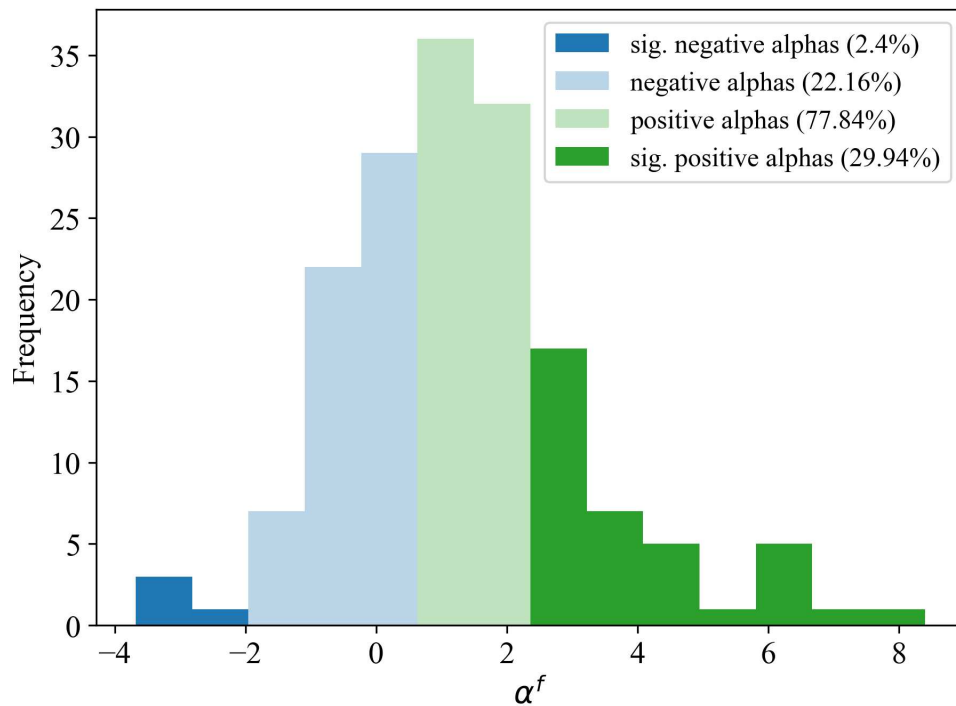
Table 2 presents the regression outputs of the top performing factors according to (3), for the United States. They were chosen based on the abnormal return of regressing the scaled strategy on its original version. Of these top-alpha factors, 50% relate to momentum strategies. If we also take into account seasonality and the price-to-high constructions, the former percentage improves to 80%. A key takeaway is that the highest alpha values are predominantly concentrated in the momentum theme and strategies that involve sorting the cross-section according to a prior returns characteristic. After controlling for the 3 Fama-French factors, the momentum strategy formed on 12 – 1 months, still delivers the highest risk adjusted returns and the abnormal returns of prior return strategies are still significant and drop only marginally. This aligns with the work of Moreira and Muir (2017), Barroso and Santa Clara (2015) and Cederburg et al. (2020), who highlight the exceptional responsiveness of momentum strategies to volatility management.

Given that the R^2 is lower than 0.4 for each of the factors we can say that the scaling is not fully explained by the original strategy. This points out that the stochastic behavior of a strategy, that is managed based on prior volatility, can be driven by other factors other than its non-scaled version.

For the 10 strategies in Table 3, most β^f estimates are statistically significant at the 1% level and positive, indicating that on average when the returns of the original factor increase, the scaled counter-part's profits also increase. However, since the estimated β^f s are lower than 1 for most strategies we can see that volatility managing reduces the exposure to the original factors. This is a natural sequence of the findings of Barroso and Santa Clara (2015) who report that momentum's risk is highly predictable and more prior risk, i.e., realized volatility before t , will lead to more standard deviation in t . Therefore, volatility managing momentum portfolios can lead to $\beta^f < 1$ of (3) precisely because of the portfolio dampening the exposure to a prior return factor when prior volatility $\hat{\sigma}_t^2 f$ is high (implying higher future volatility) ¹.

Figure 2. Alphas Across US factors

The figure shows an histogram of the annualized percentage α s of (3). The blue (green) colors stand for negative (positive) alphas. (Un)filled bins represent (in)significant alphas at the 5% level. Each of which is estimated with all available data for each factor.



¹Bear in mind that this is only valid for the momentum factors in Table 3, not necessarily to the other factors.

In the table above, we have only discussed the top 10 performing factors and themes in our sample. We would like to capture the effect of scaling across all the portfolios available in our sample. To this extent, in Figure 2 we show a histogram with all the abnormal returns from (3) for each of the long-short strategies. By examining the entire distribution of factor alphas, we gain a better understanding of how volatility management interacts with different strategies for which the presence of both significant positive and negative abnormal returns reinforces the need for a nuanced approach when applying volatility scaling. Each α was computed using the data available between the managed factor and its original version. From Figure 2 we can see that the majority of the factors (77.84%) have positive abnormal returns, of which 38.46% are significant, which further evidences that the majority of factors can benefit from scaling with previous variance and it is not just restricted to a handful of factors. On the other hand, (22.16%) of which (2.04%) are significant, which shows that certain strategies may be more sensitive to volatility scaling, potentially due to structural differences, factor definitions, or market conditions that diminish the effectiveness of the approach.

While previous research (Moreira and Muir (2017) and Cederburg et al. (2020)) has focused exclusively on the U.S., it remains unclear whether volatility management exhibits similar benefits across other economies. To address this, I extend the analysis to Developed and Emerging Markets, assessing whether the same patterns observed in the U.S. holds across regions with differing market characteristics.

Table 3. Volatility managed factors regressed on original factors

The Table shows regression outputs of the top 10 factors and themes, in US markets, in terms of the alpha, of regressing a volatility managed factor on itself, i.e. (3). The factor farthest to the left(right) has the highest(lowest) alpha. t-statistics are in parenthesis and corrected for heteroskedasticity and autocorrelation using the consistent estimator of Newey and West (1987) where the maximum autocorrelation order in the errors assumed was of 12. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The alphas and root mean squared errors are all annualized. The Alpha 3FF is the estimated intercept after controlling also for the 3 Fama French factors.

	MOM^σ	MOM_{6-1}^σ	MSP^σ	MOM_{9-1}^σ	$PRCH_{252d}^\sigma$	MOM_{Theme}^σ	$QUAL_{Theme}^\sigma$	SEA_{1Ylag}^σ	MOM_{12-7}^σ	$SEA_{2-5Ylag}^\sigma$
MOM	0.54*** (5.68)									
MOM_{6-1}		0.50*** (5.33)								
MSP			0.52*** (4.51)							
MOM_{9-1}				0.55*** (6.06)						
$PRCH_{252d}$					0.48*** (5.40)					
MOM_{Theme}						0.49*** (4.86)				
$QUAL_{Theme}$							0.38*** (3.00)			
SEA_{1Ylag}								0.56*** (6.34)		
MOM_{12-7}									0.55*** (5.21)	
$SEA_{2-5Ylag}$										0.63*** (6.16)
Alpha	8.39*** (5.51)	7.38*** (4.70)	6.66*** (4.62)	6.58*** (5.14)	6.57*** (3.80)	6.53*** (5.26)	5.87*** (4.35)	5.72*** (4.11)	4.79*** (3.56)	4.66*** (4.47)
Alpha 3FF	6.29*** (4.52)	6.10*** (4.16)	5.01*** (3.96)	4.87*** (4.05)	6.21*** (3.83)	5.13*** (4.38)	5.22*** (4.03)	5.03*** (3.79)	3.64*** (2.82)	4.20*** (4.39)
N	1,163	1,169	1,163	1,166	1,170	1,172	1,163	1,163	1,163	1,115
R^2	0.29	0.25	0.28	0.30	0.23	0.24	0.14	0.32	0.31	0.40
RMSE	48.00	46.50	41.70	46.10	52.80	38.80	38.80	46.00	40.20	26.50

4.2 Regional analysis

Tables 5 and 6 repeat the analysis in Table 3 for both Developed and Emerging Economies. As in the US, Momentum is pervasive in Developed economies with 3 Winner Minus Loser strategies placed on the top 10 factors sorted by alpha. Similarly, in Emerging markets, 40% of the top 10 highest abnormal return strategies are related to Momentum. Both regions also feature the price-to-high and seasonal factors reinforcing, once more, that prior return strategies benefit the most with scaling. Figure 3 shows that the positive alpha prominence also holds in Emerging and Developed economies with a proportion of 73.65% in respect to all the factors in the sample². In Developed countries a greater fraction of the factors is significantly positive than the Emerging markets, but this can potentially be attributed to the fact that more data is available to estimate each of the Developed factor alpha's decreasing the coefficient standard deviations, on average, boosting t-statistics and subsequently significance. These findings imply that volatility management is not only effective in the US, as demonstrated by Moreira and Muir (2017) and Cederburg et al.(2020), but also provides a significant edge for investors in both Developed and Emerging markets. By dynamically adjusting exposures, this approach enhances factor performance across regions, making it a valuable tool for investors seeking to improve risk-adjusted returns globally. The results reinforce the applicability of volatility management beyond the U.S., offering a robust framework for capitalizing on the advantages of factor-based strategies.

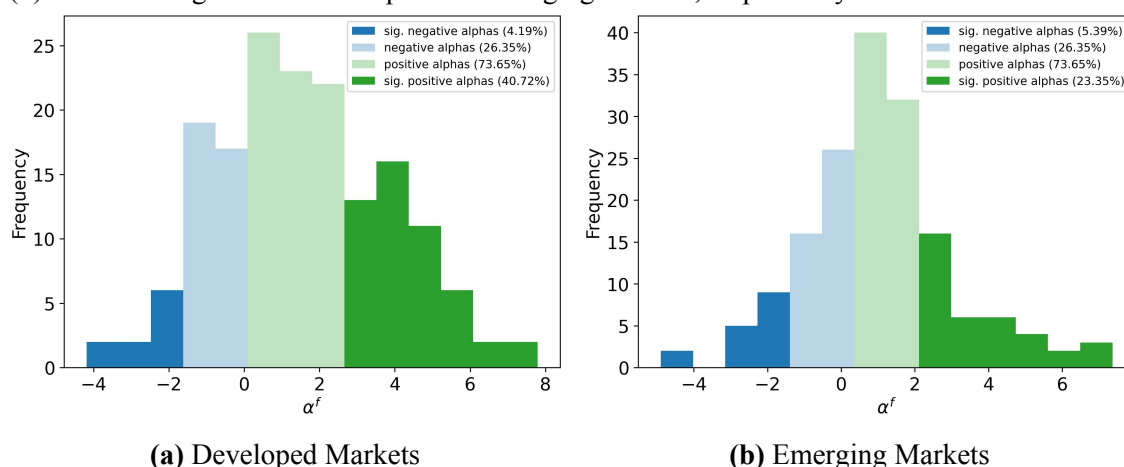
The analysis further examines the rankings of themes based on alpha across the three regions. Table 4 ranks the themes for each of the regions. Momentum, the Market factor and Profit Growth consistently rank among the top-performing themes for the U.S., Developed, and Emerging markets, highlighting their robust and reliable alpha generation under volatility management. Conversely, Low Leverage and Investment themes consistently rank in the bottom three across all regions, reinforcing their historical underperformance relative to other strategies. These findings underscore the dominance of momentum-based strategies and the strength of Profit Growth themes, while also identifying themes that may require further refinement or alternative approaches to deliver value for investors.

While the ranking analysis highlights which factors benefit the most from volatility management

²Even though this proportion is the same for both regions, only 104 match across the 2 regions (of the 123 out of 167 above-zero alpha factors).

Figure 3. Alphas for all the factors and themes across other regions

The figure shows the distribution of the intercept of (3) annualized and in percentage across the full 153 factors and remaining themes in other regions. The translucent colors refer to non-significant factors at the 5 % level, while the filled bars refer to factors statistically different than 0. (a) and (b) are the histograms of Developed and emerging markets, respectively.



under normal market conditions, an important question remains: how do these strategies perform in periods of heightened market stress?

Table 4. Rank of themes by Alpha

The Table shows the alphas of regressing (3) on each of the 10 themes below for US, Emerging and Developed markets, on Panels A., B. and C., respectively. The data used for this regression intercept varies from factor-to-factor, depending on data availability.

Rank	Panel A. US		Panel B. Emerging		Panel C. Developed	
	Factor	Alpha	Factor	Alpha	Factor	Alpha
1	MOM^σ	6.53	MOM^σ	5.73	MOM^σ	4.37
2	$QUAL^\sigma$	5.87	MRP^σ	5.55	MRP^σ	3.93
3	MRP^σ	4.39	PG^σ	5.06	PG^σ	2.77
4	PG^σ	4.09	$SEAS^\sigma$	4.06	HML^σ	1.97
5	$LOWVOL^\sigma$	2.28	$QUAL^\sigma$	3.55	RMW^σ	1.94
6	RMW^σ	1.72	HML^σ	1.88	$LOWVOL^\sigma$	1.77
7	$SEAS^\sigma$	1.70	RMW^σ	1.27	DI^σ	1.74
8	HML^σ	1.48	STR^σ	0.86	$QUAL^\sigma$	1.53
9	DI^σ	0.78	$LOWVOL^\sigma$	0.23	$SEAS^\sigma$	0.98
10	STR^σ	0.72	ACC^σ	0.11	ACC^σ	0.92
11	ACC^σ	0.60	DI^σ	0.05	$LOWLEV^\sigma$	-0.06
12	INV^σ	0.60	INV^σ	-0.38	STR^σ	-0.5
13	$LOWLEV^\sigma$	0.46	$LOWLEV^\sigma$	-4.9	INV^σ	-0.77

Table 5. Developed Economies: Volatility managed factors regressed on original factors

The Table shows regression outputs of the top 10 factors and themes, in Developed markets, in terms of the alpha, of regressing a volatility managed factor on itself, i.e. (3). The factor farthest to the left(right) has the highest(lowest) alpha. t-statistics are in parenthesis and corrected for heteroskedasticity and autocorrelation using the consistent estimator of Newey and West (1987) where the maximum autocorrelation order in the errors assumed was of 12. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The alphas and root mean squared errors are all annualized. The Alpha 3FF is the estimated intercept after controlling also for the 3 Fama French factors.

	MOM^σ	$PRCH_{252d}^\sigma$	$NOAAT^\sigma$	MOM_{9-1}^σ	DFA^σ	MOM_{12-7}^σ	$FCFME^\sigma$	$QROE^\sigma$	NEI^σ	SEA_{1Ylag}^σ
MOM	0.51*** (5.23)									
$PRCH_{252d}$		0.51*** (5.03)								
$NOAAT$			0.33** (2.04)							
MOM_{9-1}				0.50*** (4.83)						
DFA					0.38** (2.58)					
MOM_{12-7}						0.61*** (6.60)				
$FCFME$							0.41*** (3.07)			
$QROE$								0.46*** (2.81)		
NEI									0.49*** (3.51)	
SEA_{1Ylag}										0.53*** (4.38)
Alpha	7.79*** (3.22)	7.49*** (3.37)	6.63*** (3.15)	6.11*** (2.85)	5.81*** (3.77)	5.80*** (3.97)	5.76** (2.50)	5.70*** (2.73)	5.41*** (3.56)	5.26** (2.35)
Alpha 3FF	6.40*** (2.90)	6.49*** (3.36)	6.07*** (2.92)	5.12*** (2.66)	6.09*** (3.97)	5.44*** (3.17)	5.77** (2.34)	6.31*** (3.24)	6.12*** (4.17)	4.29** (2.04)
N	443	448	455	446	455	443	430	296	424	443
Rsquare	0.26	0.26	0.11	0.25	0.14	0.37	0.17	0.20	0.23	0.28
RMSE	37.34	40.41	39.09	36.70	34.49	26.11	38.14	37.43	31.78	37.04

Table 6. Emerging Markets: Volatility managed factors regressed on original factors

The Table shows regression outputs of the top 10 factors and themes, in Emerging markets, in terms of the alpha, of regressing a volatility managed factor on itself, i.e. (3). The factor farthest to the left(right) has the highest(lowest) alpha. t-statistics are in parenthesis and corrected for heteroskedasticity and autocorrelation using the consistent estimator of Newey and West (1987) where the maximum autocorrelation order in the errors assumed was of 12. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The alphas and root mean squared errors are all annualized. The Alpha 3FF is the estimated intercept after controlling also for the 3 Fama French factors.

	MOM^σ	MOM_{9-1}^σ	$PRCH_{252d}^\sigma$	MOM_{Theme}^σ	MOM_{6-1}^σ	MRP^σ	MSP^σ	F_{score}^σ	PG_{Theme}^σ	SEA_{1Ylag}^σ
<i>MOM</i>	0.76*** (6.68)									
<i>MOM</i> ₉₋₁		0.68*** (5.30)								
<i>PRCH</i> _{252d}			0.73*** (6.42)							
<i>MOM</i> _{Theme}				0.71*** (7.10)						
<i>MOM</i> ₆₋₁					0.73*** (6.07)					
<i>MRP</i>						0.43** (2.25)				
<i>MSP</i>							0.87*** (7.59)			
<i>F</i> _{score}								0.66*** (3.35)		
<i>PG</i> _{Theme}									0.48** (2.41)	
<i>SEA</i> _{1Ylag}										0.69*** (8.98)
Alpha	7.36*** (4.48)	7.24*** (4.40)	6.89*** (5.39)	5.73*** (5.22)	5.73*** (3.85)	5.55 (0.99)	5.53*** (5.16)	5.19*** (4.47)	5.06*** (4.46)	4.56*** (3.54)
Alpha 3FF	7.29*** (4.70)	7.47*** (4.60)	7.07*** (5.90)	5.88*** (5.55)	5.94*** (4.10)	2.74 (0.46)	5.64*** (5.08)	5.21*** (4.49)	5.19*** (4.69)	4.40*** (3.52)
N	414	418	421	427	421	455	414	337	420	416
R^2	0.45	0.44	0.53	0.49	0.49	0.19	0.54	0.37	0.22	0.47
RMSE	28.20	27.17	26.54	20.45	27.10	96.28	23.12	24.45	17.20	26.00

4.2.1 Recession Analysis

To address this, I examine factor exposures during recessions, using NBER indicators to identify economic downturns and assess whether volatility management adjusts exposure dynamically. To capture this impact, I included these recession dummies in a regression framework. I thus regressed each managed factor on the following regressors:

$$f_t^\sigma = \delta_0^\sigma + \delta_1^\sigma f_t + \delta_2^\sigma f_t \cdot 1_{rec} + \eta_t \quad (4)$$

where the δ_0 term stands for the abnormal return after accounting for the variables on the right-hand-side of (4), δ_1^σ is the exposure of the managed factor's return to f_t in non-recession periods and δ_2^σ stands for the increment in the exposure in recessive periods. The results reveal that the majority of δ_2^σ s became negative across all regions, indicating that the volatility-managed strategy reallocates capital in a way that, per dollar invested, reduces exposure to the long-short strategy during periods of heightened market stress. These findings validate the adaptability of volatility-managed portfolios, which dynamically lower exposures during economic uncertainty, aligning with the conclusions of Moreira and Muir (2017). They further emphasize the role of volatility management in mitigating downside risks and enhancing portfolio resilience during economic stress.

However, certain factors displayed distinctive behaviors that diverge from this general trend. In Developed markets (Table 8), the (NOAAT) stand out by maintaining or even increasing their exposure during recessions. These factors are rooted in company fundamentals, reflecting financial health and operational efficiency, which provide a stable foundation and remain relatively unaffected by short-term market volatility. The Change in Financial Assets factor (DFA) also exhibited stable or positive beta changes, signaling investor confidence in firms with sound financial management during economic downturns. In Emerging markets (Table 9), where factors are predominantly driven by prior returns, the F-score factor stands out as the only one with increased exposure during recessions, highlighting its robustness and its alignment with metrics of financial stability and operational quality. In the US, all factors followed the common trend of declining exposure during recessions, consistent with previous research, which suggests that factor dynamics in the US market are more sensitive to economic downturns, with the scaled strategy reducing exposure across all the factors in Table 7, i.e., $\delta_2^\sigma < 0$.

The contrasting behaviors of factors can largely be attributed to their nature. Factors driven by prior returns, such as momentum and price-to-high strategies, are more sensitive to market sentiment and volatility, which are amplified during crises. These factors depend on trends and short-term performance patterns that often weaken under heightened market stress, resulting in a reduction in exposure due to the dynamic risk rebalancing of volatility management. Conversely, fundamentally driven factors like NOAAT, DFA and F-score reflect intrinsic financial and operational stability. Their resilience in maintaining or increasing exposure during crisis under volatility management highlights their defensive characteristics and their potential value for investors seeking stability during crises.

While previous research, including Moreira and Muir (2017), has demonstrated the benefits of volatility management in the U.S. market, its effectiveness within optimized portfolios across Developed and Emerging Markets remains largely unexplored. Given that investors typically construct diversified portfolios rather than relying on single factors, I extend the analysis to evaluate whether volatility management enhances risk-adjusted returns in a Markowitz mean-variance optimized portfolio beyond the U.S. This broader framework allows for a more comprehensive understanding of the global applicability of volatility management.

Table 7. USA Markets: Recessions

The Table shows regression outputs of the top 10 factors and themes, in US, in terms of the alpha, according to (4). t-statistics are in parenthesis and corrected for heteroskedasticity and autocorrelation using the consistent estimator of Newey and West (1987) where the maximum autocorrelation order in the errors assumed was of 12. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The alphas and root mean squared errors are all annualized. The Alpha 3FF is the estimated intercept after controlling also for the 3 Fama French factors.

	MOM^σ	MOM_{6-1}^σ	MSP^σ	MOM_{9-1}^σ	$PRCH_{252d}^\sigma$	MOM_{Theme}^σ	$QUAL_{Theme}^\sigma$	SEA_{1Ylag}^σ	MOM_{12-7}^σ
MOM	0.69*** (6.80)								
$MOM \cdot 1_{rec}$	-0.36*** (-2.76)								
MOM		0.58*** (5.01)							
$MOM \cdot 1_{rec}$		-0.22 (-1.54)							
MSP			0.76*** (7.52)						
$MSP \cdot 1_{rec}$			-0.49*** (-3.88)						
MOM				0.66*** (5.98)					
$MOM \cdot 1_{rec}$				-0.30** (-1.99)					
$PRCH$					0.59*** (5.34)				
$PRCH \cdot 1_{rec}$					-0.30** (-2.44)				
MOM						0.66*** (5.83)			
$MOM \cdot 1_{rec}$						-0.38*** (-2.69)			
$QUAL$							0.63*** (3.73)		
$QUAL \cdot 1_{rec}$							-0.48*** (-2.92)		
SEA								0.68*** (5.82)	
$SEA \cdot 1_{rec}$								-0.31** (-2.09)	
Alpha	7.88*** (5.59)	7.26*** (4.70)	6.16*** (4.78)	6.13*** (5.12)	6.79*** (3.95)	6.18*** (5.23)	5.97*** (4.71)	5.23*** (3.90)	4.56*** (3.46)
Alpha 3FF	6.35*** (4.52)	6.16*** (4.20)	5.12*** (4.18)	4.84*** (4.07)	6.65*** (4.06)	5.15*** (4.46)	5.40*** (4.27)	4.96*** (3.76)	3.92*** (2.98)
R^2	0.32	0.26	0.34	0.32	0.25	0.27	0.20	0.34	0.33
N	1,163	1,169	1,163	1,166	1,170	1,172	1,163	1,163	1,163
RMSE	46.97	46.23	39.99	45.51	52.19	37.96	37.49	45.28	39.45

Table 8. Developed Markets: Recessions

The Table shows regression outputs of the top 10 factors and themes, in Developed Markets, in terms of the alpha, according to (4). t-statistics are in parenthesis and corrected for heteroskedasticity and autocorrelation using the consistent estimator of Newey and West (1987) where the maximum autocorrelation order in the errors assumed was of 12. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The alphas and root mean squared errors are all annualized. The Alpha 3FF is the estimated intercept after controlling also for the 3 Fama French factors.

	MOM^σ	$PRCH_{252d}^\sigma$	$NOAAT^\sigma$	MOM_{9-1}^σ	DFA^σ	MOM_{12-7}^σ	$FCFME^\sigma$	$QROE^\sigma$
<i>MOM</i>	0.60*** (5.28)							
<i>MOM</i> · 1 _{rec}	-0.35** (-2.13)							
<i>PRCH</i>		0.57*** (5.52)						
<i>PRCH</i> · 1 _{rec}		-0.24 (-1.17)						
<i>NOAAT</i>			0.32** (2.03)					
<i>NOAAT</i> · 1 _{rec}			0.67*** (3.05)					
<i>MOM</i>				0.55*** (5.02)				
<i>MOM</i> · 1 _{rec}				-0.23 (-0.99)				
<i>DFA</i>					0.37*** (2.60)			
<i>DFA</i> · 1 _{rec}					0.57** (2.01)			
<i>MOM</i>						0.63*** (6.52)		
<i>MOM</i> · 1 _{rec}						-0.10 (-0.36)		
<i>FCFME</i>							0.42*** (2.82)	
<i>FCFME</i> · 1 _{rec}							-0.07 (-0.33)	
<i>QROE</i>								0.48** (2.59)
<i>QROE</i> · 1 _{rec}								-0.09 (-0.41)
Alpha	7.36*** (3.26)	7.29*** (3.39)	6.70*** (3.20)	5.84*** (2.82)	5.65*** (3.60)	5.80*** (3.95)	5.81** (2.50)	5.72*** (2.75)
Alpha 3FF	6.26*** (2.98)	6.43*** (3.44)	6.18*** (2.99)	4.99*** (2.66)	5.88*** (3.76)	5.47*** (3.30)	5.84** (2.34)	6.46*** (3.36)
R^2	0.28	0.27	0.12	0.26	0.15	0.38	0.17	0.21
N	443	448	455	446	455	443	430	296
RMSE	36.82	40.15	38.92	36.53	34.41	26.11	38.17	37.45

Table 9. Emerging Markets: Recessions

The Table shows regression outputs of the top 10 factors and themes, in Emerging Markets, in terms of the alpha, according to (4). t-statistics are in parenthesis and corrected for heteroskedasticity and autocorrelation using the consistent estimator of Newey and West (1987) where the maximum autocorrelation order in the errors assumed was of 12. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. The alphas and root mean squared errors are all annualized. The Alpha 3FF is the estimated intercept after controlling also for the 3 Fama French factors.

	MOM^σ	MOM_{9-1}^σ	$PRCH_{252d}^\sigma$	MOM_{Theme}^σ	MOM_{6-1}^σ	MRP^σ	MSP^σ	F_{score}^σ
MOM	0.86*** (7.09)							
$MOM \cdot 1_{rec}$	-0.51*** (-3.45)							
MOM		0.74*** (4.63)						
$MOM \cdot 1_{rec}$		-0.33* (-1.85)						
$PRCH$			0.81*** (5.92)					
$PRCH \cdot 1_{rec}$			-0.38** (-2.42)					
MOM				0.75*** (6.38)				
$MOM \cot 1_{rec}$				-0.31** (-2.14)				
MOM					0.76*** (5.43)			
$MOM \cdot 1_{rec}$					-0.27* (-1.77)			
MRP						0.44** (2.04)		
$MRP \cdot 1_{rec}$						-0.10 (-0.41)		
MSP							0.92*** (7.15)	
$MSP \cdot 1_{rec}$							-0.38** (-2.12)	
F_{score}								0.65*** (3.24)
$F_{score} \cdot 1_{rec}$								0.43* (1.72)
Alpha	6.62*** (4.89)	6.54*** (4.05)	6.15*** (5.98)	5.35*** (5.15)	5.36*** (3.64)	5.33 (0.95)	5.31*** (5.26)	5.04*** (4.21)
Alpha 3FF	6.72*** (4.97)	6.83*** (4.16)	6.46*** (6.15)	5.56*** (5.34)	5.62*** (3.87)	1.66 (0.29)	5.57*** (5.25)	4.97*** (4.10)
R^2	0.48	0.46	0.55	0.50	0.49	0.19	0.55	0.38
N	414	418	421	427	421	455	414	337
RMSE	27.44	26.87	25.90	20.24	26.96	96.33	22.81	24.39

5 Efficient portfolios

The construction of efficient portfolios in this study starts with the implementation of in-sample Markowitz (1952) optimization, this process aims to construct portfolios that maximize returns for a given level of risk, based on the mean-variance utility approach which uses historical return data to determine asset weights that achieve the highest possible Sharpe ratio.

The optimality condition to set the optimal weights $w = (w_1, \dots, w_N)'$ for a set of N factors is:

$$w = \frac{\Sigma^{-1}\mu}{\iota'(\Sigma^{-1}\mu)} \quad (5)$$

Where Σ and μ are the covariance matrix and expect return vector of the N factors, respectively, and ι is a vector of ones of dimension N .

The optimization is implemented across different market universes, progressively increasing the number of factors included. It initially focuses solely on the Market Risk Premium (MRP), followed by the inclusion of all factors within the Fama and French (1993) Three-Factor model. Subsequently, the optimization extends to the Fama and French (2015) Five-Factor model and, finally, incorporates the q-factor model proposed by Hou, Xue, and Zhang (2015), as outlined in the methodology of Moreira and Muir (2017).

Following the construction of in-sample efficient portfolios, volatility management is applied to these portfolios, building on the framework introduced by Moreira and Muir (2017), the in-sample optimized portfolios are scaled to align conditional portfolio volatility with a predetermined target in a manner consistent with (1). Table 10 displays the main descriptive statistics of regressing a volatility-managed efficient portfolio on its original version across the US, Developed and Emerging markets in Panels 1, 2 and 3, respectively.

Notably, across all combinations and regions the α s are positive, so for each efficient portfolio there is another portfolio that combines it with its scaled counter-part that is able to improve upon the original Sharpe ratio. On the other hand, Sharpe ratios across different universes provide key insights into the effectiveness of volatility management, particularly in portfolios that include the momentum factor, as shown in Table 10.

In the U.S. market, the application of volatility management produces improvements in risk-adjusted returns, particularly for portfolios incorporating momentum-based strategies. The FF3-

MOM and MRP portfolios show increases in Sharpe ratios compared to their unmanaged counterparts. Even though HXZ and FF5 have a lower Sharpe ratio when scaled, even with momentum, the gap between the original portfolio and its volatility managed version becomes shorter with momentum (FF5-MOM and HXZ-MOM) reinforcing the sensitivity of momentum to volatility scaling. Furthermore, R^2 values across all specifications are lower than 50%, indicating that efficient portfolios only explain a portion of the volatility managed factors. The strategies that combine Momentum

In the developed markets, the benefits of volatility management are similar to the US particularly pronounced for portfolios that incorporate the momentum factor which aligns with the findings of Moreira and Muir (2017) and Cederburg et al.(2020), who highlighted the unique cyclical-ity and risk-return dynamics of momentum strategies, making them well-suited for volatility scaling.

Interestingly, the same pattern is not as evident in the emerging markets where volatility-managed portfolios still show overall improvements in Sharpe ratios, however, the magnitude of these improvements for momentum-included portfolios is less consistent and less pronounced. This suggests that the dynamics driving the success of volatility management in developed markets such as greater market efficiency and liquidity may not fully translate to the more variable and less mature market environments of emerging regions. It also raises the possibility that momentum factors in emerging markets may exhibit different behavioral characteristics, potentially due to structural or macroeconomic constraints.

Although in-sample results indicate that volatility-managed portfolios consistently outperform their unmanaged counterparts, a crucial test of robustness is whether these benefits persist out-of-sample. To evaluate the practical applicability of volatility management, I conduct out-of-sample tests using a 60-month rolling window, ensuring that portfolio weights are based on past information rather than future data.

6 Out of Sample analysis

In the previous sections I described the advantages of managing a portfolio with its prior volatility, but c in (1) was fit on all the data. Therefore, these benefits may not be representative of the true gains that one would make if the data used to estimate c in a given month t were a

Table 10. In sample efficiency portfolios

The Table shows the descriptive statistics of Markowitz portfolios with and without scaling in sample, for US, Developed and Emerging markets, in Panels 1.,2. and 3., respectively. The alphas, root mean squared errors, appraisal and Sharpe ratios are annualized. The alphas and Appraisal ratios are constructed with the estimates of (3).

	MKT	FF3	FF3-MOM	FF5	FF5-MOM	HXZ	HXZ-MOM
Panel 1. US markets							
α	4.39	2.17	1.99	0.81	1.29	0.77	1.33
N	1,175	874	874	874	874	874	874
R^2	0.37	0.45	0.44	0.44	0.36	0.47	0.37
RMSE	50.94	28.95	14.45	7.99	7.69	10.37	9.56
Original Sharpe	0.44	0.55	0.86	0.95	1.16	0.81	1.02
Vol-Managed Sharpe	0.50	0.56	0.92	0.89	1.15	0.73	1.00
Appraisal Ratio	0.30	0.26	0.48	0.35	0.58	0.26	0.48
Panel 2. Developed markets							
α	3.94	0.45	2.56	1.17	1.20	0.99	1.53
N	1,175	455	443	455	443	455	443
R^2	0.39	0.43	0.40	0.28	0.38	0.30	0.38
RMSE	49.77	19.99	11.74	6.91	5.46	9.67	8.62
Original Sharpe	0.40	0.57	1.06	0.98	1.38	0.82	1.15
Vol-Managed Sharpe	0.47	0.43	1.25	1.01	1.47	0.74	1.19
Appraisal Ratio	0.27	0.08	0.76	0.59	0.76	0.35	0.61
Panel 3. Emerging markets							
α	6.42	1.15	1.34	1.33	1.07	1.99	1.34
N	455	379	379	379	379	379	379
R^2	0.21	0.53	0.44	0.41	0.46	0.49	0.49
RMSE	94.85	13.49	8.72	4.93	4.34	6.90	6.11
Original Sharpe	0.05	0.64	1.24	1.48	1.91	0.97	1.48
Vol-Managed Sharpe	0.23	0.67	1.23	1.66	1.93	1.38	1.57
Appraisal Ratio	0.23	0.30	0.53	0.93	0.85	1.00	0.76

set of months before or equal to t , which is the most realistic case. As such, out-of-sample performance will be addressed in this section.

We construct the volatility managed portfolios by at each month t setting the weight on a given portfolio based on its realized variance at month t $\hat{\sigma}_t^2 f$, evaluated according to (2), and a dynamic constant c_t , such that:

$$f_{t+1}^{\sigma, OOS} = \frac{c_t}{\hat{\sigma}_t^2 f} f_{t+1} \quad (6)$$

where c_t set to match the variance of $f_{t+1}^{\sigma, OOS}$ with f_{t+1} over the past 24 months before $t + 1$, ensuring no hindsight to compute the returns in $t + 1$.

In Table 11 I show the proportions, in percentage of positive and positively significant alphas, in Panels A and B, respectively, per region in and out-of-sample for all the 153 factors and themes in the sample. In Panel A it is perceptible that across the 3 regions the proportion of strategies

out-of-sample that deliver positive α are almost the same as in-sample. However, the percentage of strategies which have significant positive alphas gets majorly affected for the US, Developed and Emerging Markets, which points towards a deterioration performance.

Table 11. Alpha distributions in and out of sample

The Table shows the proportion of positive (Panel A) and positive significant (Panel B) alphas across the 153 factors and remaining themes, for the 3 regions, computed according to (3), where the volatility managed factor is constructed according to (6).

	Panel A. Positive			Panel B. Positive and significant		
	US	Developed	Emerging	US	Developed	Emerging
OOS	72.46	59.28	67.66	16.77	15.57	14.37
IS	77.84	73.65	73.65	29.94	40.72	23.35

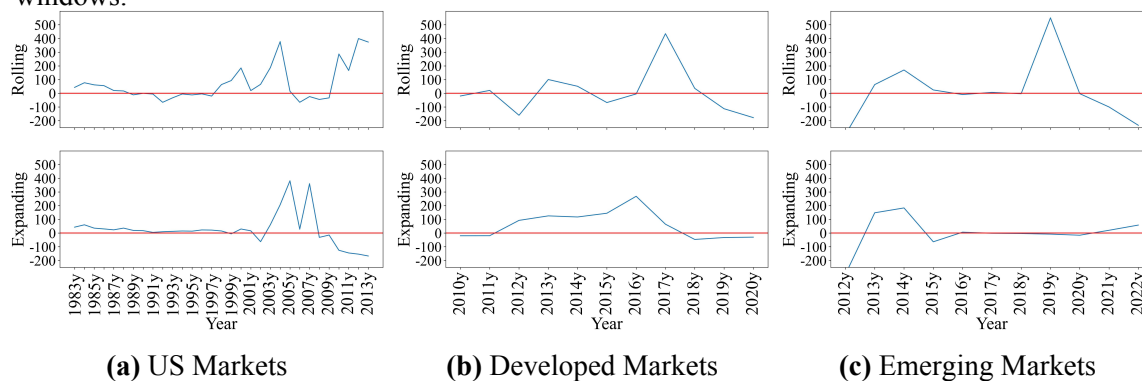
Even if the persistence of positive alphas OOS highlights the robustness of volatility management, it is not clear which of the strategies in Table 11 benefit the most with this. An investor generally would have a difficult time betting on all the 153 factors and themes and hope that a proportion of those strategies deliver a positive alpha. Most appropriate course of action would be choosing a handful of strategies to volatility manage based on past performance. I thus devised a method which is based on an investing principle that avoids too much rebalancing, where every year the investor looks back and picks all the factors which have a statistically positive alpha, volatility manages each of them and then equal weights these scaled factors into a portfolio.

One cannot ascertain how far into the past an investor should look at to determine which factors have positive alpha. To this end, Figure 4 exhibits the results of using rolling and expanding windows to estimate significant alphas, in the top and bottom rows, respectively. Each point in a given subplot means the percent increase in Sharpe ratio, measured with all months after the corresponding year, of the 1/N combination when all constituents are volatility scaled, according to (6), relative to the Sharpe ratio of the simple equal weighted combination. In this figure, what immediately catches the eye is the scale of the vertical axis. There are periods where managing is very useful such as 2003 in US in which scaling can deliver up to 400% increases in Sharpe ratio, but there are periods such as 2008 where using the expanding window method is very harmful for US investors where drops in Sharpe ratio go up to almost 200%. This variability also holds for Developed and Emerging Economies.

Even with these sporadic sizable positive increases in mean variance gains when we scale each

Figure 4. OOS equal weighted portfolios with yearly rebalancing

For each region, at each year, we compute the robust t-statistics for the alphas of all factors and then form a portfolio with those that exceed 1.96, i.e., significant at the 5% level. To form the portfolio we use either a 24 month rolling window or an expanding window, starting at 24 months. Then using all the data after the end of the year we pinpoint the Sharpe ratios for equal weighted combinations where all elements are scaled, according to (6), and unscaled. For each decade we then report the increase in Sharpe ratio with volatility managing. Panels (a), (b) and (c) stand for US, Developed and Emerging Markets and the top and bottom rows refer to rolling and expanding windows.



of the elements in a portfolio, there is an indisputable truth that for the great majority of the time these gains approach 0 for each of the regions and can induce investors into huge losses, even before factoring in transaction costs. These results are in line with Cederburg et al. (2020) who show that, in an out-of-sample set up, original strategies earn higher Sharpe ratios and certainty equivalent than the ones managed on prior volatility.

While the results indicate that volatility management can, at times, deliver improvements in Sharpe ratios, the variability in outcomes highlights that the effectiveness of scaling is highly dependent on market conditions. This reinforces the notion that blindly applying volatility scaling to all factors with a significant abnormal return may not be the optimal approach for investors.

7 Robustness Analysis

The method presented in Section 6 suggests that a more structured methodology is required to enhance the robustness of volatility-managed portfolios out-of-sample. One potential avenue is mean-variance optimization, as proposed by Markowitz, which allows for an explicit risk-adjusted allocation rather than equal weighting. In Table 12 we show the OOS Sharpe ratio of a dynamic combination that each month sets weights to maximize the Sharpe ratio and then its volatility managed version.

The constraints we impose assume that initially we are 100% invested in a riskless asset and then add to our portfolio a set of long-short strategies. Since these are self-financing and we are already fully invested in a risk-free security, the weights add up to 1 independent of the weights we assign to these zero investment portfolios. Therefore, realistically, the only relevant restriction would be that all the short positions in our portfolio do not represent more than 150% or 100% of wealth, otherwise it would be too risky for the long counterpart.

Under these limitations at each month t we set weights such that the absolute values of the weights are below 1.5 or 1. That way we ensure that the value of the short position will never exceed the above conditions. For instance if we have a portfolio with N assets the constraint will be:

$$\sum_{i=1}^N |w_t^i| \leq 1. \quad (7)$$

where w_t^i is the weight set in asset i at time t for $t + 1$ if we require a 100% constraint. We optimized a typical Mean variance Utility function where the risk aversion parameter was set to 10, as in [Faias and Santa Clara \(2017\)](#).

Then we volatility managed these portfolios by computing their realized variance with the 22 daily returns preceding t (using the weights set in t) and then applying (6). However, to prevent too extreme positions, we constrained this weight to take the value of 1 if the value given by (6) exceeds it.

The results in Table 12 highlight the impact of volatility management on mean-variance preferences across different factor models and market regions. Overall, the findings suggest that volatility scaling can enhance risk-adjusted returns, though its effectiveness varies depending on the specific model and market segment. In the US, managed portfolios generally show improved mean-variance preferences, particularly for models incorporating momentum, such as FF5-MOM and HXZ-MOM. However, the benefits are not uniform across all specifications, as some models, like FF3, exhibit only modest gains.

In Developed Markets, the results are more mixed. While managed portfolios tend to perform better in multi-factor models such as FF5 and HXZ, the market factor (MKT) experiences a deterioration in performance when volatility managed. This suggests that simple market exposure in developed economies does not benefit from volatility scaling in the same way as factor-based strategies. In contrast, Emerging Markets consistently show strong improvements when port-

Table 12. Mean-Variance preferences optimization

The table below shows the annualized Sharpe ratios when using mean-variance preferences to maximize utility subject to the constraint imposed in (7) when the caps on shorting are 100% and 150% of wealth, with and without scaling. The set of factors on the columns refer to long-short strategies' weights we are optimizing. Panels A, B, C reflect the results of US, Developed and Emerging Markets, respectively.

Constraint	MKT		FF3		FF3-MOM		FF5		FF5-MOM		HXZ		HXZ-MOM	
	150	100	150	100	150	100	150	100	150	100	150	100	150	100
Panel A. US Markets														
Original	0.42	0.41	0.39	0.42	0.53	0.49	0.49	0.47	0.62	0.53	0.46	0.46	0.60	0.52
Managed	0.30	0.32	0.44	0.47	0.65	0.60	0.55	0.53	0.70	0.61	0.58	0.55	0.71	0.63
Panel B. Developed Markets														
Original	0.04	0.06	0.30	0.28	0.56	0.44	0.40	0.38	0.49	0.38	0.50	0.44	0.52	0.41
Managed	-0.06	-0.06	0.38	0.40	0.79	0.69	0.62	0.65	0.77	0.70	0.78	0.77	0.78	0.72
Panel C. Emerging Markets														
Original	0.04	0.05	0.23	0.19	0.48	0.40	0.36	0.31	0.42	0.31	0.18	0.13	0.37	0.27
Managed	0.18	0.19	0.29	0.31	0.72	0.67	0.50	0.46	0.60	0.55	0.32	0.33	0.54	0.52

folios are volatility managed. The gains are particularly pronounced in models that incorporate additional risk factors, suggesting that volatility scaling may be more effective in markets with higher inefficiencies and greater levels of volatility.

The table also presents results under two different constraint levels, 150 and 100, but the differences between them are relatively small. This indicates that the benefits of volatility management remain stable across different portfolio constraints. Additionally, the best-performing managed strategies tend to be those that include momentum factors, reinforcing the idea that momentum-based strategies benefit significantly from volatility scaling.

Taken together, these results suggest that while volatility management can improve mean-variance preferences, its effectiveness depends on the market environment and the specific factor model. The findings also align with prior research indicating that momentum-based strategies (Barroso and Santa Clara (2015), Cederburg (2020) and Daniel and Moskowitz (2016)). However, in certain cases, such as simple market exposure in Developed Markets, volatility management can even be detrimental, highlighting the need for careful consideration when applying these techniques.

To show that the benefit of scaling does not only come from scaling mean-variance efficient portfolios, but is also present when we account for higher-order preferences, we are going to

consider a problem that sets weights by maximizing the fourth order expansion of the power Utility function and where the risk aversion parameter was set to 10, as in Gau and Nardari (2018). Table 13 is, thus, constructed in the same way as 12, but the objective Utility function to initially set up the weights is that of Gau and Nardari (2018).

By integrating both Markowitz optimization and the Gao and Nardari (2018) approach in an OOS setting with volatility-managed strategies, we can evaluate whether different approaches mitigates the limitations observed in simple equal-weighted scaling strategies.

Table 13. Higher order preferences optimization

The table below shows the annualized Sharpe ratios when using preferences consistent with those of Gao and Nardari (2018) to maximize utility subject to the constraint imposed in (7) when the caps on shorting are 100% and 150% of wealth, with and without scaling. The set of factors on the columns refer to long-short strategies' weights we are optimizing. Panels A, B, C reflect the results of US, Developed and Emerging Markets, respectively.

	MKT		FF3		FF3-MOM		FF5		FF5-MOM		HXZ		HXZ-MOM	
Constraint	150	100	150	100	150	100	150	100	150	100	150	100	150	100
Panel A. US Markets														
Original	0.45	0.35	0.41	0.21	0.26	0.23	0.33	0.33	0.28	0.22	0.34	0.34	0.26	0.19
Managed	0.59	0.46	0.45	0.27	0.31	0.27	0.42	0.44	0.34	0.32	0.35	0.36	0.29	0.28
Panel B. Developed Markets														
Original	0.42	0.30	0.34	0.07	0.23	0.33	0.21	0.21	0.13	0.26	0.24	0.21	0.20	0.24
Managed	0.54	0.36	0.67	0.31	0.46	0.46	0.46	0.49	0.37	0.42	0.58	0.59	0.46	0.46
Panel C. Emerging Markets														
Original	0.20	0.16	0.16	0.05	0.16	0.10	0.15	0.11	0.11	0.07	0.08	0.07	0.04	0.08
Managed	0.54	0.36	0.23	-0.07	0.27	0.19	0.08	0.15	0.21	0.17	0.05	0.11	0.13	0.19

When comparing the results of Table 13 with those from Table 12 , it becomes evident that higher-order optimization methods, such as those proposed by Gao and Nardari (2018), provide a more nuanced approach to portfolio construction relative to traditional mean-variance optimization.

In the US market, the managed portfolios under higher-order optimization consistently outperform their original versions across most factor models, with notable improvements in Sharpe ratios. This suggests that incorporating dynamic adjustments to efficient portfolios enhances risk-adjusted returns, particularly in a sophisticated and competitive market like the US. The magnitude of improvement is generally greater than what was observed under mean-variance optimization, indicating that simple variance-based scaling might not be the most efficient method

for optimizing returns.

In Developed Markets, the results are even more pronounced. The managed portfolios exhibit significant increases in performance compared to their original counterparts, with some cases showing nearly double the returns (e.g., FF3-MOM). Unlike the mixed results seen under mean-variance optimization, higher-order optimization seems to provide more consistent benefits across different strategies. This highlights the importance of incorporating non-traditional risk measures when dealing with markets that may not exhibit the same level of efficiency as the US.

The Emerging Markets segment presents a more complex picture. While there are some cases where managed portfolios under higher-order optimization outperform (e.g., FF5-MOM and HXZ-MOM), the gains are generally less substantial than those observed in US and Developed Markets. Additionally, there are instances where managed portfolios actually underperform their original versions, suggesting that volatility scaling combined with higher-order preferences may not always yield superior results in less developed and more volatile markets. The differences in performance relative to traditional mean-variance optimization further reinforce the idea that emerging markets may require a tailored approach that accounts for structural inefficiencies and market frictions.

Overall, the comparison between mean-variance optimization and higher-order optimization methods underscores the importance of using more advanced frameworks when constructing portfolios, especially in environments where volatility management alone does not consistently yield positive results. The improvements seen in the US and Developed Markets suggest that methods like those of Gao and Nardari (2018) can add value by incorporating a broader set of risk considerations beyond variance.

8 Conclusion

This study extends the existing literature on volatility-managed portfolios by demonstrating their effectiveness across a broad set of factors in U.S., Developed, and Emerging Markets. While prior research, such as Moreira and Muir (2017) and Cederburg et al. (2020), has largely focused on U.S. factor portfolios, my findings confirm that volatility management generates positive abnormal returns globally, reinforcing its role as a valuable tool for investors seeking to enhance

risk-adjusted performance.

The analysis highlights that momentum strategies benefit the most from volatility scaling, with consistent improvements across all regions.

Beyond factor-level analysis, this study also evaluates the performance of volatility-managed efficient portfolios. While in-sample tests confirm improvements in Sharpe ratios across various Markowitz-optimized portfolios, out-of-sample (OOS) results reveal a more nuanced picture. Although volatility management still generates positive and significant alphas in OOS settings, the magnitude of these improvements is smaller compared to in-sample estimates. This aligns with the findings of Cederburg et al. (2020), who caution that volatility scaling does not universally improve risk-adjusted returns. The rolling-window strategy for selecting positive and significant factors further reinforces the idea that while volatility management can be effective, its benefits fluctuate across market conditions.

To assess the robustness of these findings, I implemented different optimization techniques, including Markowitz (1952) and the Gao and Nardari (2018) approach, to construct volatility-managed efficient portfolios. The results indicate that while volatility management enhances Sharpe ratios in some cases, its effectiveness varies depending on factor selection, portfolio constraints, and market structure. Developed Markets show consistent improvements when incorporating momentum factors, whereas Emerging Markets exhibit less stable benefits, likely due to liquidity constraints and structural inefficiencies.

The broader implications of these results suggest that volatility management should not be applied indiscriminately across all factors. Instead, investors should selectively integrate this approach based on factor characteristics, market conditions, and investment objectives. Future research could refine factor selection methodologies, explore alternative weighting schemes, and investigate whether dynamic adjustments to volatility-scaling parameters can further enhance performance. Additionally, further exploration into how macroeconomic conditions influence the effectiveness of volatility management may provide deeper insights into its role as a risk-adjustment tool.

Overall, this thesis contributes to the growing body of research on factor investing by demonstrating that volatility management extends beyond U.S. markets and remains relevant across different regions and portfolio structures. While the strategy provides a meaningful edge in many contexts, its implementation requires careful consideration, as its benefits vary depending

on factor type, market efficiency, and economic conditions.

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