UNIVERSIDADE CATÓLICA PORTUGUESA

Dynamic Rebalancing Strategies Applied to Factor Portfolios

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Resumo

O reequilíbrio de portefólios é uma ferramenta essencial para manter alocações de ativos alvo e otimizar perfis de risco e retorno. Esta tese compara o impacto de vários métodos de reequilíbrio baseados em regras tanto periódicas como de limites, no contexto de investimento por fatores. Particularmente, esta tese estuda os efeitos da frequência de reequilíbrio, do limiar de reequilíbrio e das proporções de ajuste parcial no desempenho.

Embora reequilibrar frequentemente melhore a exposição a fatores de retorno e consequentemente os retornos brutos, também aumenta os custos de transação, indicando a presença de um ponto ótimo de equilíbrio para o controlo de custos. Notavelmente, estratégias periódicas simples demonstram retornos comparáveis às mais complexas estratégias com base em limites, embora estas últimas mostrem maior consistência na obtenção de retornos mais altos. Consequentemente, a semelhança entre portefólios emerge como um preditor mais confiável de retornos em comparação com o período de reequilíbrio. A tese conclui que o reequilíbrio parcial, mas frequente, oferece desempenho superior quando alterações nos preços das ações aproximam um movimento browniano geométrico.

Palavras-chave: Reequilíbrio de portefólios, Investimento por fatores, Gestão de portefólios, Custos de transação

Abstract

Portfolio rebalancing serves as a critical mechanism for maintaining targeted asset allocations and optimizing risk-return trade-offs. This thesis compares the impact of various calendar- and threshold-based rebalancing strategies in the context of factor investing. Specifically, it scrutinizes the influence of rebalancing frequency, rebalancing threshold, and partial rebalancing proportion on performance.

While frequent rebalancing enhances factor exposure and gross returns, it simultaneously increases transaction costs, indicating the presence of an optimal trade-off point. Notably, simple calendar-based strategies demonstrate returns on par with their more intricate, threshold-based counterparts, although the latter exhibit greater consistency in achieving high returns. Consequently, portfolio similarity emerges as a more reliable predictor of returns compared to rebalancing period. Partial yet frequent rebalancing consistently delivers superior performance as stock price changes approximate a Geometric Brownian Motion with a drift component.

Keywords: Portfolio rebalancing, Factor investing, Portfolio management, Transaction costs

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List of Abbreviations

ADV - Average Daily Volume

APT - Arbitrage Pricing Theory

CAPM - Capital Asset Pricing Model

ES - Expected Shortfall

GBM - Geometric Brownian Motion

ICAPM - Intertemporal Capital Asset Pricing Model

IR - Information Ratio

ITG - Investment Technology Group, Inc.

MDD - Maximum Drawdown

MPT - Modern Portfolio Theory

NTR - No-trade Region

RRR - Return-to-risk Ratio

SAA - Strategic Asset Allocation

TAA - Tactical Asset Allocation

TE - Tracking Error

VCV - Variance-covariance

VaR - Value-at-risk

1 Introduction

Wise investment portfolios are purposefully designed to help investors achieve their desired risk and return objectives (Reilly & Brown 2011; Bodie et al. 2023). However, as time progresses and asset prices change, a portfolio's actual composition will drift away from the optimal allocation initially intended (Fabozzi & Markowitz 2011). Portfolio rebalancing describes the process of realigning a portfolio's actual composition with the optimal allocation, bringing the investment strategy back to its desired parameters (Perold & Sharpe 1995). The promise of active portfolio management to keeping strict risk profiles, achieving high return targets, or targeting and tracking certain characteristics makes regular portfolio rebalancing a crucial determination. Yet, deciding when and how to rebalance portfolios is a complex question and most investment managers pivot to rather simplistic ad-hoc rules, which are typically either calendar-based or tied to simple ratios or metrics.

Concomitantly, factor investing has become an increasingly critical investment strategy in the financial industry over the past few decades. This approach, based on the idea that certain characteristics or "factors" can explain differences in returns, allows investors to construct portfolios that emphasise assets with desirable traits (Fama & French 2015). Factors such as value, quality, momentum, and low volatility have been identified as drivers of returns, offering a more refined perspective than traditional asset-class-based investment strategies (Asness et al. 2019; Gupta et al. 2022). As empirical research continues to validate the robustness of these factors across different market conditions and geographies, factor investing is gaining wider acceptance. With the continued growth and development of technology and data analysis capabilities, the popularity of factor investing is set to further increase, allowing investors to take a more nuanced and effective approach to portfolio construction and risk management (Ang 2014).

Regarding portfolio rebalancing, the existing literature centers around the existence of a rebalancing premium (Booth & Fama 1992), in comparing different rebalancing strategies (Perold & Sharpe 1995), and, more recently, incorporating the impact of transaction costs (Leland 2000). However, the vast majority of past research has focused on portfolios with a reduced number of assets (usually 2 or 3) with limited practical applicability. To bridge this gap, this thesis considers an updated dataset of about 27 years (1995-2022), with a much more broad array of assets: the Axioma US equity universe consisting of over 1,000 stocks at any point in time representative of the largest US publicly-listed stocks.

Past research has explored two primary approaches to portfolio rebalancing: calendar-based and threshold-based strategies. Calendar-based methods involve rebalancing at fixed time intervals, such as daily, monthly, or annually (Zilbering et al. 2015). Threshold-based methods, on the other hand, trigger rebalancing when specific metrics, such as asset weight or tracking error, reach a predefined level (Donohue & Yip 2003). Recent studies have also introduced the notion of partial rebalancing, particularly when transaction costs are involved, suggesting that moving only partway towards the optimal portfolio may be more efficient (Olson 1998).

The existing literature has limited research on the interplay among various rebalancing variables such as partial rebalancing proportion and rebalancing threshold. Moreover, there is scope for exploring new rebalancing strategies that use advanced distance or similarity functions incorporating more data points. This thesis addresses these gaps by introducing more robust measures that include information on asset weights, factor scores, and correlations. It evaluates a range of distance functions that are both theoretically grounded and industry-relevant. The primary objective is to understand how different rebalancing variables interact with each other and affect portfolio performance, aiming to identify the most optimal settings for superior performance.

In the absence of rebalancing costs, an investor aims to align their portfolio as closely as possible to the optimal target, whether it be a Markowitz-efficient portfolio or a model influenced by long-term asset return drivers like value, quality, and momentum factors. However, optimal factor portfolios are dynamic, subject to changes in factor characteristics and return expectations over time (Gârleanu & Pedersen 2013). Crucially, transaction costs introduce frictions that may render constant and full rebalancing towards the optimal portfolio undesirable.

Past research has predominantly constrained its assumptions about transaction costs to either being proportional to turnover or to the square of turnover. This thesis contributes to bridging this gap in the literature by adopting a more comprehensive transaction cost model, informed by past academic literature and industry experience. This is achieved via the state-of-the-art parametric I-star model - a log-linear regression model that uses the transaction size, volatility, and trading volume as key inputs to estimate market impact costs and has proven to be extremely reliable (Kissel 2013).

The assumption of a robust transaction cost model allows this thesis to explore a further dynamic rebalancing approach that constantly weights off the expected benefit of trading against the costs associated with the implied transactions. Mathematically, trading towards the target makes sense if the added performance improvement exceeds expected transaction costs:

$$\Delta E(r) > E(Transaction\,Costs) \tag{1}$$

The challenge lies in efficiently estimating the pick-up in expected return from bringing the actual portfolio closer to the optimal. I derive this via reverse Markowitz (1952) optimisation by converting the holdings of the optimal portfolio into implied active returns (implied alphas).

Furthermore, very little has been published on rebalancing strategies in the context of factor investing. This study endeavors to fill this gap by examining such strategies in a multifactor model portfolio, which equally weights value, quality, and momentum factors. While the primary focus is on factor investing, the insights gleaned from this research are likely to have broader applicability beyond this specific context.

In the context of factor investing, optimal portfolios are built using stock attributes intended to mirror underlying factors such as book-to-market equity for value (Fama & French 1992), return-on-equity for quality (Novy-Marx 2013), and 12-1-month price momentum (Jegadeesh & Titman 1993). However, these attributes are constantly changing and are subject to error due to accounting mismeasurement, missing values, and random errors (Frazzini et al. 2015). Consequently, portfolios based on these characteristics are subject to noise and may not accurately reflect the theoretical factors. This thesis hypothesizes that less frequent rebalancing may reduce noise, leading to smoother factor exposure and improved risk-adjusted returns. Therefore, I examine the impact of varying rebalancing frequencies on the performance of a multifactor portfolio.

Recent research has highlighted the variability in factor returns over different time spans, indicating that the historical premiums for these factors are primarily driven by short periods of exceptionally high returns (Fama & French 2018*b*; Arnott et al. 2019). Khang & Picca (2021) propose that adaptive rebalancing - more frequent during periods of rapid factor exposure changes and less frequent otherwise - can optimise the capture of factor premiums while reducing rebalancing costs. This suggests that for factor investing, threshold-based rebalancing strategies may be more effective than calendar-based ones as they allow for better tracking of the optimal portfolio.

While frequent rebalancing enhances factor exposure and gross returns, it simultaneously increases transaction costs, indicating the presence of an optimal equilibrium point for turnover control. Notably, straightforward calendar-based strategies demonstrate returns on par with their more intricate, threshold-based counterparts, although the latter exhibit greater consistency in achieving high returns. Consequently, portfolio similarity emerges as a more reliable predictor of returns compared to rebalancing frequency. I find that partial, yet frequent, rebalancing consistently delivers superior performance.

2 Literature Review

2.1 Portfolio Theory

Portfolio theory provides a structured approach to investment and portfolio building. Its inception has led to the development of strategic asset allocation methodologies, justified by the significant influence target asset allocation has on portfolio performance (Brinson et al. 1986). Over the years, various models and factors have enriched this theory to better explain asset returns.

Modern Portfolio Theory (MPT), introduced by Markowitz (1952), was a groundbreaking development, offering a quantitative approach to asset selection and emphasizing risk aversion and diversification. The theory revolves around the principle of maximising expected return for a specific level of risk (or vice-versa) through diversification. Sharpe (1964), Treynor (1965), Lintner (1965) and Mossin (1966) improved upon the ideas of the MPT by introducing the Capital Asset Pricing Model (CAPM). The CAPM spotlights market beta as a systematic risk measure and proposes that the expected asset return relates to its inherent risk, emphasising that only systematic risk fetches higher returns. Merton (1973) expanded the CAPM, resulting in the Intertemporal Capital Asset Pricing Model (ICAPM). The ICAPM considers risks linked to changing investment opportunities, introducing hedging demand and acknowledging multiple systematic risk sources. Finally, the Arbitrage Pricing Theory (APT) by Ross (1976) stands out as a flexible multifactor model compared to the CAPM. It allows multiple macroeconomic factors to dictate expected asset returns and introduces arbitrage opportunities if deviations occur. Its flexibility is both its strength and limitation due to the unspecified factor selection.

2.2 Factor Investing

Following from the APT, factor investing is an investment approach that targets specific drivers of return, known as factors. The evolution of factor investing has unfolded through a series of pivotal studies over the years. It commenced with Banz (1981) shedding light on the outperformance of small-cap stocks, and Basu (1983) illustrating that lower price-to-earnings ratio stocks often yield higher returns. The traditional notion of market rationality was then questioned by research from Shiller (1981), Shiller (1984), and DeBondt & Thaler (1985), setting the stage for factor-based strategies. Fama & French (1992) added the size and value factors to the CAPM to account for additional sources of systematic risk. Momentum was brought to the forefront by Jegadeesh & Titman (1993), and further validated by Carhart (1997) who showed how adding it to the Fama & French 3-factor model enhanced performance. The predictive power of asset growth on future returns was highlighted by Cooper et al. (2008). Research by Novy-Marx (2013) stressed the predictive capacity of operating profitability, an assertion later corroborated by Hou et al. (2015) who proposed their q-model as a superior predictor based on profitability and asset growth. This led to the introduction of the Fama & French (2015) five-factor model, enhancing the scope for assessing expected returns by incorporating operating profitability and asset growth into the original market, size, and value factors. This has led to increased popularity of a quality factor associated with high profitability, low leverage, and robust governance. Asness et al. (2019) demonstrate the positive relationship between high-quality stocks and superior risk-adjusted returns. By systematically targeting and weighting these factors in a portfolio, factor investors aim to achieve superior risk-adjusted returns compared to traditional market-cap-weighted indices (Ang 2014).

The most extensively researched and documented factors in asset pricing are size, value, quality and momentum. The size factor suggests that smaller firms typically offer higher average returns than larger firms (Banz 1981; Fama & French 1992). However, the size premium has failed to materialize since its discovery and has been highly contested due to issues like data errors and insufficient adjustments for risk and liquidity (Alquist et al. 2018). Despite its contested stand-alone effectiveness, small-cap exposure has been shown to enhance the performance of other factors like value and momentum (Blitz & Hanauer 2020).

The value factor posits that stocks with low valuation ratios, such as book-to-market value or price-to-earnings, typically outperform those with high valuation ratios over time (Basu 1983). Various explanations have been proposed for this phenomenon. Fama & French (1992) argue that value stocks are inherently riskier, thereby offering higher expected returns as compensation. Zhang (2005) posits that value companies face higher operating leverage risks, justifying the premium. Behavioral explanations, such as those from Lakonishok et al. (1994), suggest that investors' over-extrapolation of past growth rates results in mispricing, making value stocks underpriced. Barberis & Shleifer (2003) propose that the value premium emerges due to investors' style-based categorization of stocks, which leads to mispricing and creates opportunities for value investing.

The quality factor suggests that high-quality stocks, characterized by strong balance sheets and stable earnings, outperform lower-quality counterparts, offering superior risk-adjusted returns. Novy-Marx (2013) found that profitability is a key indicator, with highly profitable firms consistently outperforming less profitable ones. Asness et al. (2019) argue that a multi-dimensional approach to quality, incorporating factors like profitability, earnings stability, and growth, yields a more robust quality factor. Behavioral explanations, such as those provided by Frazzini et al. (2018), propose that the quality premium is attributable to investor underreaction to a firm's fundamentals, leading to temporary mispricing that corrects over time, thereby generating superior returns.

The momentum factor posits that securities performing well recently are likely to continue doing so, while underperformers continue to lag. Behavioral theories suggest investor overreaction and underreaction to news as possible explanations. From a risk perspective, momentum is seen as capturing unexplained systematic risk. Empirical studies confirm the efficacy of momentum strategies across various markets and time frames. Jegadeesh & Titman (1993) documented significant excess returns from momentum strategies in the U.S., while Rouwenhorst (1998) observed similar results across 12 European markets. Grundy & Martin (2001) argue that momentum profitability persists even when accounting for other factors. Moskowitz et al. (2012) broaden the understanding of momentum by examining it from a time-series perspective. Asness et al. (2013) further corroborate the ubiquity of momentum by finding its presence across multiple asset classes. Daniel & Moskowitz (2006) study the downside of momentum, particularly during momentum crashes when past winners subsequently underperform.

2.3 Portfolio Rebalancing

The notion of an "optimal portfolio" goes back to Markowitz's (1952) MPT that posits that portfolios can be optimized for the highest return at a given level of risk or the lowest risk at a given level of return. However, the optimal portfolio is by no means a universal consensus as investment managers widely differ in their choices of investment strategies. Investment managers place significant emphasis on strategic asset allocation, as it profoundly impacts portfolio performance (Brinson et al. 1986). However, market volatility can disrupt the portfolio's initial risk-return configuration, necessitating periodic rebalancing (Perold & Sharpe 1995), creating the need for portfolio rebalancing to maintain the optimal risk-return profile of the portfolio. This rebalancing process involves readjusting the portfolio's asset composition by selling overweight assets and purchasing underweight ones to restore the targeted risk-return profile.

2.3.1 Benefits of Rebalancing

Bernstein (2001) succinctly highlights the benefits of portfolio rebalancing:

"First, it keeps your portfolio's risk within tolerable limits. Second, it generates a bit of excess return. And third, it will instill the discipline and mental toughness essential to investment success."

Risk Management

Regular portfolio rebalancing is essential for maintaining an investor's specific risk-return trade-off and achieving their investment objectives, as emphasized by Bernstein (2001). Failing to rebalance may cause the portfolio to deviate from its planned strategy, affecting the risk and return profiles. Periodic recalibration ensures enduring diversification and mitigates the risks of overexposure to particular assets or sectors. Rebalancing acts as a preventive measure against unintended risk elevation and strategy deviation, thereby sustaining the optimal risk-return balance (Buetow et al. 2002; Perold & Sharpe 1995).

Edesses (2017) challenges the conventional view by arguing that the higher standard deviation in buy-and-hold strategies often stems from greater upside potential, not downside risk. Consequently, buy-and-hold might offer better protection against worst-case scenarios, whereas rebalancing could lead to notable losses, especially during market downturns, by redirecting wealth into consistently underperforming assets.

Rebalancing Premium

The rebalancing premium refers to the extra return that an investor can earn by periodically rebalancing a diversified portfolio, as compared to an unmanaged one (Perold & Sharpe 1995; Bernstein 2001; Bouchey et al. 2012).

$$\text{Rebalancing Premium} = r_{rebalanced} - r_{buy\&hold} \tag{2}$$

$R_{rebalanced}$ - *rebalanced portfolio return*

$R_{buy\&hold}$ - *buy-and-hold portfolio return*

According to Perold & Sharpe (1995), the rebalancing premium emerges from a contrarian buying and selling pattern aligned with asset price oscillations and mean-reversion. Edesses (2017) asserts that, in a strategic asset allocation (SAA) context, rebalancing outperforms a buy-and-hold approach. Numerous studies have corroborated that rebalancing significantly contributes to alpha generation, even when controlling for known factors like size, value, and momentum (DeMiguel et al. 2009; Bouchey et al. 2012; Plyakha et al. 2012; Plyakha et al. 2014; Maeso & Martellini 2020).

Nevertheless, rebalancing may not always guarantee increased returns. Sometimes, drifting portfolios can outperform rebalanced ones, especially when a specific asset significantly excels (Perold & Sharpe 1995; Willenbrock 2011). Plaxco & Arnott (2002) observes that, while rebalanced portfolios demonstrate lower volatility, they can sometimes lag in returns. Wise (1996) and Edesses (2017) reveal rebalanced portfolios outperform only two-thirds of the time, with buy-and-hold significantly surpassing rebalancing in the remaining instances. Hilliard & Hilliard (2018) suggests buy-and-hold's outperformance is due to greater momentum factor exposure and finds no alpha in rebalancing.

Perold & Sharpe (1995), Bouchey et al. (2012), and Hallerbach (2014) find that rebalancing strategies benefit from the mean reversion of asset prices, allowing portfolios to capitalize on price fluctuations by selling high and buying low, thereby potentially improving returns. Conversely, in situations where asset returns exhibit significant trends, unmanaged portfolios may naturally lean toward the best-performing assets, enhancing their overall returns. Higher volatility increases the potential benefits of rebalancing by allowing for capitalization on mean-reversion tendencies in asset prices. Lower correlation among asset classes generally allows for counterbalancing effects that reduce portfolio volatility, potentially enhancing long-term returns through frequent rebalancing (Bouchey et al. 2012).

Behavioral Discipline

Bernstein (2001) and Beach & Rose (2005) underscore the role of rebalancing in fostering disciplined and systematic investing, particularly useful in volatile markets. Rebalancing helps mitigate emotionally driven decision-making often seen during market upheavals by consistently returning the portfolio to its target allocations. This approach not only maintains a suitable risk-return balance but also anchors investment decisions to a predetermined strategy, thereby potentially enhancing long-term profitability.

2.3.2 Rebalancing Costs

While continuous rebalancing could ideally maintain optimal asset allocation, transaction costs can substantially erode investment performance. To mitigate these costs, investors often reduce trading frequency and volume, thereby incurring a small liquidity premium (Constantinides 1986). In the context of a two-asset universe comprising equity and bonds, studies by Arnott & Lovell (1993), Tsai (2001), and Harjoto & Jones (2006) indicate that a rebalanced strategy, even accounting for transaction costs, tends to outperform the corresponding buy-and-hold strategy.

Research by Leland (2000), Donohue & Yip (2003), and Liu (2019) examines rebalancing in the presence of transaction costs, establishing the concept of a no-trade region (NTR) around the target asset allocations where Trading only occurs if asset proportions deviate outside this region. Dybvig & Pezzo (2020) expands on this by considering various types of transaction costs, such as bid-ask spread, brokerage fees, and price pressure, in a single-period mean-variance model, finding that the impact on returns is highly dependent on the trading cost function. Taxes and the time and labor spent to rebalance are also costs that should be considered (Tokat & Wicas 2007; Zilbering et al. 2015). Leland (2000) considers the impact of capital gains tax, showing that it significantly affects both the upper and lower limits of the NTR.

2.3.3 Rebalancing Strategies

In a **buy-and-hold** strategy, an investor sets an initial asset allocation and leaves it unchanged, allowing the portfolio to drift with market movements. While this approach can result in an accumulation of high-performing assets, Plaxco & Arnott (2002) argue that buy-and-hold is generally less profitable and harder to justify compared to rebalancing, even when one asset class significantly outperforms.

Calendar-based rebalancing adjusts portfolios at set timeframes like monthly, quarterly, or annually. While straightforward and easy to implement, this method can lead to either excessive rebalancing, incurring additional costs, or insufficient rebalancing, resulting in an undesirable drift from the target allocation. Nonetheless, its effectiveness and widespread usage in industry practice are supported by various studies, including those by Arnott & Lovell (1993), Zilbering et al. (2015), and Zhang et al. (2022).

Threshold-based rebalancing is an event-driven investment strategy that adjusts a portfolio when asset allocations deviate from target levels by a predefined criterion, such as tracking error or a fixed percentage of weights (Olson 1998; Chan & Ramkumar 2011). It closely aligns with the concept of a NTR, as elaborated by Leland (2000). This approach is responsive to market fluctuations but necessitates frequent monitoring. The strategy can be further refined by setting asset-specific tolerance bands, varying by characteristics like volatility, which adds complexity but enhances responsiveness to market conditions (McCalla 1997; Masters 2003; Donohue & Yip 2003; Chan & Ramkumar 2011).

Under **partial rebalancing**, asset weights are readjusted back to a point between the current and the optimal allocation. For example, if stocks rise from a 60% target to 70% of the portfolio, the investor might rebalance to 65% instead of back to 60%. Chan & Ramkumar (2011); Dybvig & Pezzo (2020) show this method helps minimise rebalancing costs. According to Zakamouline (2006), Pliska & Suzuki (2004), and Tokat & Wicas (2007), the optimal approach to partial rebalancing is contingent upon the transaction cost function.

Mixed rebalancing strategies integrate various rebalancing rules, such as calendarand threshold-based methods, to optimize portfolio management (Zhang et al. 2022). In such an approach, a portfolio may be adjusted at fixed time intervals, and also when asset allocations deviate by a specified percentage from target allocations. Buetow et al. (2002) state that by integrating frequent monitoring alongside preset threshold deviation intervals enhances the effectiveness of the overall rebalancing strategy. Chan & Ramkumar (2011) and Sullivan (2008) highlight key variables in tweaking rebalancing strategies, including adjusting the rebalancing threshold, varying threshold across assets, and setting a partial rebalancing proportion.

2.3.4 Rebalancing and Factor Investing

Factor investing aims for precise tracking of an optimal portfolio to attain desired exposure to specific return-driving factors. While frequent rebalancing would theoretically enhance factor exposure and subsequent returns, transaction costs limit the practicality of this approach. Consequently, portfolio managers generally opt for less frequent rebalancing intervals, such as monthly, when employing factor strategies.

The efficacy of factor investing has traditionally been influenced by a few instances of exceptional returns, which were aligned with significant changes in market leadership during market cycle transitions (Fama & French 2018*a*; Arnott et al. 2019). Recent research by Khang & Picca (2021) demonstrate that daily rebalancing particularly during these critical transitions could have substantially amplified factor premiums. This pattern persists even when the increased transaction costs resulting from higher turnover are taken into account. Therefore, the authors contend that strategic timing for more frequent rebalancing may be crucial for maximizing the potential of factor premiums. This underscores the potential advantages of a dynamic rebalancing strategy based on a suitable function that quantifies the difference between the actual and the optimal portfolio in the context of factor investing.

Studies by DeMiguel et al. (2009), Platen & Rendek (2010), Bouchey et al. (2012), Plyakha et al. (2012), and Maeso & Martellini (2020), enhance the optimistic view on the merits of portfolio rebalancing within factor investing by showing that rebalancing can yield performance gains, even when accounting for traditional factors such as market risk premium, size, value, and momentum.

Arnott et al. (2023) explore how effective rebalancing methods can enhance the performance of investment strategies like smart beta and factor strategies and illustrate how trade prioritisation for stocks with the most compelling signals can maximise factor premia and minimise trading costs. Masters (2003) explores the idea of taking advantage of asset correlations and differences in asset class volatilities and transaction costs to reduce tracking error through engaging in the most cost-effective rebalancing trades.

3 Data

This thesis studies a sample of 4,429 large- and mid-cap publicly-listed stocks from 1995-2022 (6,695 trading days). The daily number of stocks with complete data varies between 1,165 and 1,597, serving as the thesis' benchmark market portfolio. Daily data on returns, risk, factor exposure, and market cap is downloaded from the Axioma U.S. Equity database. Daily returns are calculated as simple returns from open to close of the trading day. This thesis employs a dataset of 591,232 unique ITG transaction cost estimates from Invesco's actual transactions from 2012-2022 to model transaction costs. The dataset includes variables like transaction size, type (buy/sell), stock volatility, and average daily volume (ADV). Daily volatility and ADV for the thesis' stock universe are calculated using a 21-day rolling window.

4 Methodology

This section details the methodologies for constructing factor exposures, factor portfolios, and variance-covariance matrices, along with procedures for estimating transaction costs and rebalancing strategies.

All computations were carried out using R and C++.[1]

4.1 Factor Exposures and Factor Portfolios

The process begins with the construction of long-short dollar-neutral factor portfolios for the most studied return drivers - value, quality, and momentum. I then create a model portfolio that equally weights the former three, as this configuration has been shown to yield the highest and most robust Sharpe ratios in academic models according to Fama & French (2018*a*).

The value factor in this thesis is constructed using book-to-market value as the valuation criterion. The quality factor amalgamates multiple profitability indicators, namely return-on-equity, return-on-assets, cash-flow-to-assets, cash-flow-to-income, sales-to-assets, and gross margin. The momentum factor adheres to its conventional definition, differentiating winners and losers based on returns from the past 12 months, excluding the most recent month.

This thesis constructs factor portfolios as 100% long and 100% short, with each stock's weight determined by its factor exposure. Factor exposure signifies the asset's return sensitivity to the underlying factors, quantifying how much of an asset's performance is attributable to those factors. Given a multifactor model with N factors,

[1]This thesis involves exceptionally high computational demands due to two main factors. Firstly, the portfolio rebalancing framework necessitates daily computations that depend on prior days' results, making vectorization of calculations unfeasible. Secondly, the study analyzes a staggering 173,061 distinct rebalancing strategies formed through various combinations of parameters (during the full research process this number was more than octuple). Despite considerable optimization efforts in R and the use of C++ functions, substantial computational resources across multiple systems were required to complete the calculations within the given timeframe.

the return r_t of an asset at time t can be represented as:

$$r_t = \alpha + \beta_1 f_{1,t} + \beta_2 f_{2,t} + \cdots + \beta_N f_{N,t} + \epsilon_t \tag{3}$$

α - *expected return independent of the factors*

β_i - *exposure of the asset to the i^{th} factor*

$f_{i,t}$ - *return of the i^{th} factor at time t*

ϵ_t - *stochastic component of idiosyncratic return*

The β values indicate an asset's sensitivity to respective factors; a β of 1 means a 1% factor return change results in a 1% asset return change, ceteris paribus.

The thesis takes a practitioner's approach, employing a detailed, daily-updated factor portfolio construction process that diverges from traditional academic frameworks as seen in seminal works such as Fama & French (1992), Carhart (1997), Fama & French (2015), and Hou et al. (2015).

Sourcing Raw Factor Exposures from Axioma

The thesis begins by obtaining daily raw factor exposures for momentum, quality, and value from Axioma. These raw factor exposures correspond to standardised measures of book-to-market value for value, a composite of return-on-equity, return-on-assets, cash-flow-to-assets, cash-flow-to-income, sales-to-assets, and gross margin for quality, and 12-1 past months' returns for momentum. Therefore, these raw factor exposures are subject to unwanted bias which I disentangle in further steps.

Industry-standardisation of Factor Exposures

The studies by Asness et al. (2000), Novy-Marx (2013), and Ehsani et al. (2023) emphasize the greater predictive power of firm-specific over industry-wide factor exposures.

Consequently, I industry-standardize factor exposures to eliminate potential industry-related risk and isolate the firm-specific component. I pool the factor exposures into industry groups and de-median within each industry.[2]

$$E^{(1)} = E^{(0)} - \text{industry median}^{(0)} \tag{4}$$

$E^{(1)}$ - *vector of factor exposures after industry-standardisation*

$E^{(0)}$ - *vector of factor exposures sourced directly from Axioma*

industry median$^{(0)}$ - *vector of industry median raw factor exposures*

Asset	ROE			De-medianed Value	Rank
1	5%	Industry 1	Subtract 7.5%	-2.5%	5
2	10%			2.5%	3
3	1%	Industry 2	Subtract 7%	-6%	7
4	13%			6%	1
5	20%	Industry 3	Subtract 15%	5%	2
6	15%			0%	4
7	10%			-5%	6

Table 1: Industry-standardisation and ranking process

[2]Numeric superscripts indicate the step in the factor exposure construction process.

Gaussian Standardisation of Factor Exposures

Next, I create a rank vector from the industry-neutralized exposures and map it to a standard normal $N(0,1)$ distribution to control for outliers and skewness. The following formula is applied to each element in the rank vector:

$$E_i^{(2)} = \begin{cases} \Phi\left(\frac{\text{rank}_i - \overline{\text{rank}}}{\sigma_{\text{rank}}}\right) & \text{if rank}_i < 0 \\ 1 - \Phi\left(\frac{\text{rank}_i - \overline{\text{rank}}}{\sigma_{\text{rank}}}\right) & \text{if rank}_i > 0 \\ 0.5 & \text{if rank}_i = 0 \end{cases} \tag{5}$$

$E_i^{(2)}$ - i^{th} *element in the factor exposures after Gaussian standardisation vector*

$\Phi(\cdot)$ - *cumulative distribution function of the standard normal distribution*

$rank_i$ - i^{th} *element of rank*

$\overline{rank}$ - *mean of rank*

σ_{rank} - *standard deviation of rank*

Finally, I re-subtract the industry median post-Gaussian standardisation to control for residual industry biases.

$$E^{(3)} = E^{(2)} - \text{industry median}^{(2)} \tag{6}$$

$E^{(3)}$ - *vector of factor exposures after re-subtracting the industry median*

$E^{(2)}$ - *vector of factor exposures after Gaussian standardisation*

$industry\ median^{(2)}$ - *vector of industry medians of* $E^{(2)}$

Beta- and Industry-neutralisation of Factor Exposures

Following Blitz (2023), I neutralise unintended systematic risks by regressing out market beta and industry effects from standardised factor exposures.

$$E^{(3)} = \psi_0 \cdot \beta_i + \psi_1 \cdot \text{ind}_1 + \psi_2 \cdot \text{ind}_2 + \cdots + \psi_N \cdot \text{ind}_N + \epsilon \tag{7}$$

β_i - *vector of market betas*

ind_i (for i=1 to N) - vectors of industry dummy variables

ψ_i (for i=0 to N) - vectors of coefficients

N - *total number of industries*

ϵ - *regression residual*

The regression residuals represent the portion of the standardised factor exposures that cannot be explained by the market beta and therefore constitute the market and industry-neutralised factor exposures (E^*).

$$E^* = \epsilon \tag{8}$$

I calculate new market betas tailored to this thesis' stock universe, using the standard formula (Sharpe 1964; Lintner 1965; Mossin 1966).

$$\beta_i = \frac{\text{Cov}(r_i, r_M)}{\text{Var}(r_M)} \tag{9}$$

β_i - *beta for stock i*

r_i - *time-series vector of stock i returns*

r_M - *time-series vector of market returns*

Factor Portfolio Construction from Factor Exposures

The final factor exposures are used to create dollar-neutral factor portfolios with 100% long and 100% short exposure. The weights, denoted by w, are linearly proportional to the value of the factor exposures, giving more weight to stocks with higher factor sensitivity.

$$w_i = \begin{cases} \frac{E_i}{\sum E_i > 0} & \text{if } E_i \geq 0 \\ -\frac{E_i}{\sum E_i < 0} & \text{if } E_i < 0 \end{cases} \tag{10}$$

w_i - *weight for stock i in the factor portfolio*

E_i - *factor exposure for stock i*

The final factor portfolios are therefore dollar-, beta-, and industry-neutral.

This factor construction process is conducted for value, quality, and momentum. A combined multifactor model portfolio is constructed by equal-weighting the former three.

$$\text{Model} = \frac{1}{3} \times \text{Value} + \frac{1}{3} \times \text{Quality} + \frac{1}{3} \times \text{Momentum} \tag{11}$$

4.2 Stock Variance-Covariance Matrix

Computational complexity, noisy data, and missing empirical data for certain stocks makes the direct calculation of variance-covariance (VCV) estimates from historical data a hopeless task. Instead, a more effective approach models factor returns according to fundamental factors like value, quality, momentum, and industry. These modelled factor returns are then employed to derive estimates for stock variances and covariances.

Following Litterman (2003), the stock-by-stock VCV matrix can be derived from stocks' factor exposures, the factor VCV matrix, and a diagonal matrix of idiosyncratic variances.

$$\Sigma = EFE' + \Delta \tag{12}$$

Σ - *stock-by-stock total VCV matrix*

E - *factor exposure matrix*[3]

F - *factor VCV matrix*

Δ - *idiosyncratic VCV matrix*

A stock VCV matrix is constructed for each day in the sample period.

Factor and Stock-specific Returns

I start by modelling equity returns with a simple linear model as proposed by Campbell et al. (1997):

$$r_t = E_{t-1} f_t + \epsilon_t \tag{13}$$

r_t - *return of a stock at time t*

E_{t-1} - *factor exposure matrix at time* $t-1$

f_t - *factor return at time t*

ϵ_t - *error term*

The ordinary least squares solution to this problem is well known.

$$\hat{f}_t = \left(E'_{t-1} E_{t-1}\right)^{-1} E'_{t-1} r_t \tag{14}$$

Yet, as one of the Gauss-Markov conditions states that residuals must be homoscedastic (which is rarely observed with stock return data), I correct for the heteroscedastic return data by scaling each stock's residual by the inverse of its residual variance, thereby estimating the factor returns by cross-sectional weighted least squares regression with

[3]Here E denotes a different factor exposure matrix from before. E is now an $N \times M$ matrix where N is the total number of stocks and M is the total number of factors.

the weighting matrix W. Specifically, the factor returns are estimated as:

$$\hat{f}_t = \left(E'_{t-1} W_t E_{t-1}\right)^{-1} E'_{t-1} W_t r_t \tag{15}$$

And stock specific returns are given by the regression residual, ϵ_t.

Factor Variance-Covariance Matrix

Once I have a time series vector of returns for each factor, I compute a factor VCV matrix (F) by applying the usual covariance formula to each combination of factors using a rolling window of 500-1,000 days.

$$F = \begin{pmatrix} \sigma_{1,1} & \cdots & \sigma_{1,j} \\ \vdots & \ddots & \vdots \\ \sigma_{i,1} & \cdots & \sigma_{i,j} \end{pmatrix} \tag{16}$$

$\sigma_{i,j}$ - *covariance between factor i and factor j*

Systematic Variance-Covariance Matrix

The factor VCV matrix (F) is multiplied with the factor exposure matrix (E) composed of a row for each stock and a column for each factor to formulate the systematic VCV matrix (Ω).

$$\Omega = EFE' \tag{17}$$

Idiosyncratic Variance-Covariance Matrix

The idiosyncratic VCV matrix is simply a diagonal matrix with the variances of the stock-specific returns obtained as the residuals of the regression in the first step. Each element of the diagonal vector of the idiosyncratic VCV matrix is filled out using the usual variance formula and a rolling window of 500-1,000 days.

$$\Delta = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_i \end{pmatrix} \tag{18}$$

Δ - *diagonal matrix with stock-specific variances*

σ_i - *variance of stock i*

Total Variance-Covariance Matrix

Finally, I add the specific VCV matrix (Δ) to the systematic VCV matrix (Ω) to get the total stock VCV matrix (Σ).

$$\Sigma = \Omega + \Delta \tag{19}$$

4.3 Implied Alphas

Implied alphas (or active return estimates) are essential for calculating expected return later in this thesis. As the factor portfolios represent the factor allocation that an investor would seek to implement in a completely unconstrained investment environment, I back out the implied alphas from the portfolio holdings by reverse Markowitz (1952) optimisation as also illustrated in Black & Litterman (1992) and Idzorek (2019).

$$L(w, \lambda) = w'\mu - \frac{1}{2}\lambda w'\Sigma w \tag{20}$$

$$\frac{\partial L}{\partial w} = \mu - \lambda w = 0 \tag{21}$$

$$w^* = \frac{1}{\lambda}\Sigma^{-1}\mu \tag{22}$$

Assuming $\lambda = 1$, we obtain:

$$\mu = \Sigma w^* \tag{23}$$

λ - *risk-aversion coefficient*

μ - *vector of implied alphas*

Σ - *stock-by-stock VCV matrix*

Hence, the obtained implied alphas (μ), represent the implied return estimates needed in an unconstrained mean-variance optimisation to obtain the same factor portfolios.

4.4 Transaction Costs

Inclusion of transaction costs is vital when assessing portfolio rebalancing strategies. Ignoring these costs could overestimate a strategy's profitability. Frequent rebalancing may appear favorable based on raw returns, but its net benefit may diminish or vanish once transaction costs are considered.

My choice for a transaction cost model is inspired by market microstructure literature on equities and internal Invesco estimates. For example, Corwin & Schultz (2012) report that the average bid–ask spread is 50 bps for large-cap U.S. companies, which implies 25 bps for a small enough one-way trade with no market impact. Khang & Picca (2021) assume 30 bps during normal times and 60 bps during periods of increased volatility as a realistic set of transaction costs for a factor investor who can demand liquidity of small enough quantities on the margin.

To allow for increased flexibility I employ the I-Star model - a market impact[4] model that estimates transaction costs according to variable stock- and transaction-specific characteristics (Kissel 2013).[5] The favored I-Star configuration utilises a power function

[4]Market impact, a vital consideration for large traders like institutional investors, refers to how buying or selling an asset influences its price against the trader, thereby affecting transaction costs.

[5]The model has evolved significantly to adapt to changes in the trading landscape, including algorithmic trading,

that integrates variables such as imbalance (size), volatility, liquidity, and daily trading patterns.

$$I^*_{\text{bps}} = a_1 \cdot \left(\frac{Q}{ADV}\right)^{a_2} \cdot \sigma^{a_3} \tag{24}$$

I^*_{bps} - *instantaneous impact of a particular transaction measured in bps*

Q - *transaction size*

ADV - *average daily volume for the stock being traded*

σ - *volatility of that stock*

a_1, a_2, a_3 - *parameters to be estimated*

The parameters a_1, a_2, and a_3 are estimated using transaction cost data through a process diligently outlined in Kissel (2013), which employs a log transformation of the I-Star model. This transformation facilitates linear regression and corrects for heteroscedasticity, allowing for parameter estimation via ordinary least squares regression.

$$\ln(I^*_{\text{bps}}) = \ln(a_1) + a_2 \cdot \ln\left(\frac{Q}{ADV}\right) + a_3 \cdot \ln(\sigma) \tag{25}$$

My estimated parameters are $a_1 = 0.0617$, $a_2 = 0.405$, and $a_3 = 0.601$. These values are consistent with the feasible parameter bounds delineated in Kissel (2013) of $0.01 \leq a_1 \leq 0.1$, $0.1 \leq a_2 \leq 1.0$, and $0.1 \leq a_3 \leq 1.0$.

The I-Star model's parameters, estimated from total transaction cost data, are considered to reflect comprehensive transaction costs. This is justified by Wyart et al. (2008), who demonstrate that bid-ask spreads are proportional to market impact costs, suggesting a similar pattern for total transaction costs. However, the I-Star model's estimated parameters are based on transaction cost data starting in 2012, although the thesis sample goes back to 1995. Given that transaction costs have decreased over time, as indicated by Diaz-Ruiz et al. (2020) and Hiraki & Skiadopoulos (2023), these estimates may be too low. To be conservative and consistent with the academic literature on proportional transaction costs and internal Invesco estimates, I augment the

dark pools, and regulatory shifts, among other factors.

I-Star model estimates with a 20bps premium.

$$\text{Transaction Cost}_{\text{bps}} = 20\text{bps} + I^*_{\text{bps}} \tag{26}$$

Transaction Cost$_{bps}$ - transaction costs in bps of turnover

Because the I-Star model relies on order size to estimate transaction costs, I assume a total size of $100 million for the model portfolio.

4.5 Rebalancing Strategies

The essence of this thesis focuses on implementing and assessing various rebalancing strategies. Each strategy follows a specific rule for when to rebalance, which occurs at day's end when criteria are met. Portfolios are adjusted to maintain 100% long and 100% short positions, aligning with the target portfolio.

4.5.1 Calendar-based Rebalancing

This thesis comprehensively evaluates calendar-based rebalancing by rebalancing every N days, where $N = 1, 2, 3, ..., 250$.[6]

4.5.2 Threshold-based Rebalancing

Threshold-based rebalancing strategies assume a certain function for the distance or similarity between the actual and optimal portfolio. A distance function increases as the difference between the actual and optimal portfolio increases, accounting for direction and magnitude of that difference. On the other hand, similarity functions increase and the difference between the actual and optimal portfolio decreases, accounting for

[6]The choice of maximum $N = 250$ corresponds to approximately one trading year.

direction only. Whenever the result of this function reaches a pre-defined threshold, the actual portfolio is rebalanced.

I explore a variety of distance and similarity functions to govern portfolio rebalancing, each evaluated across 201 distinct thresholds. These thresholds are informed by both economic logic and empirical data. Industry norms shape the lower boundaries for some metrics like correlation, while others like Euclidean and Chebyshev distance lack standardized limits. This study establishes a generous lower boundary for correlation at 0.1. The portfolio is then allowed to drift without rebalancing, and daily calculations for all similarity/distance functions are conducted. When the correlation metric reaches the set threshold of 0.1, the corresponding values for the alternative metrics are observed and utilized in the formulation of their respective limits.

Function	Type	Information	True Range	Imposed Range
Correlation	Similarity	Weights	-1 to +1	+0.1 to +1
Euclidean Distance	Distance	Weights	0 to +∞	0 to +0.1
Chebyshev Distance	Distance	Weights	0 to +∞	0 to +0.025
Transfer Coefficient	Similarity	Weights and Covariances	-1 to +1	+0.1 to +1
Tracking Error	Distance	Weights and Covariances	0 to +∞	0 to +0.02

Table 2: Summary of similarity and distance functions

Correlation

Correlation serves as a similarity metric in this study, quantifying the linear relationship between the weights of the actual and optimal portfolios.[7]

$$\rho_{w,w^*} = \frac{\text{Cov}(w, w^*)}{\sigma_w \sigma_{w^*}} \tag{27}$$

w - *vector of actual portfolio weights*

[7]Cosine similarity was initially considered as an additional metric. However, it was found to produce results nearly identical to those generated by correlation, due to its mathematical form: $CS = \cos(\theta) = \frac{w \cdot w^*}{\|w\| \|w^*\|}$.

w^* - *vector of optimal portfolio weights*

σ_w - *standard deviation of the actual weights vector*

σ_{w^*} - *standard deviation of the optimal weights vector*

Correlation rages between -1 and +1, with +1 signifying perfect positive similarity between the actual and optimal portfolio, and -1 indicating perfect negative similarity.

Euclidean Distance

Euclidean distance is a measure of the straight-line distance between two points in an N-dimensional space.

$$D_{\text{Euclidean}}(w, w^*) = \sqrt{\sum_{i=1}^{N}(w_i^* - w_i)^2} \tag{28}$$

N - *total number of assets*

w_i^* - *optimal weight of asset i*

w_i - *actual weight of asset i*

Euclidean distance is a metric that ranges from 0 to $+\infty$. Larger values denote greater divergence, while smaller values signify higher similarity.

Chebyshev Distance

Chebyshev distance measures the greatest distance between two vectors along any coordinate dimension, or simply the maximum distance along one axis.

$$D_{\text{Chebyshev}}(w, w^*) = \max_{i=1}^{n} |w_i - w_i^*| \tag{29}$$

Chebyshev distance ranges from 0 to $+\infty$. Higher values indicate greater divergence, while lower values denote increased similarity.

Transfer Coefficient

The transfer coefficient is a similarity measure defined as the cross-sectional correlation between forecasted active asset returns and actual active weights adjusted for risk (Grinold 1989; Grinold & Kahn 2000). It can also be more intuitively understood as the risk weighted correlation between the actual and optimal active weights (Clarke et al. 2006).

$$TC = \frac{w'\Sigma w^*}{\sqrt{w'\Sigma w}\sqrt{w^{*\prime}\Sigma w^*}} \tag{30}$$

Σ - *asset VCV matrix*

The transfer coefficient ranges from -1 to +1, serving as a gauge for the alignment between the actual and optimal portfolios. Values near +1 indicate high similarity, while values closer to -1 suggest divergence.

Tracking Error (ex-Ante)

Ex-ante tracking error quantifies the anticipated deviation of an actual portfolio from its optimal counterpart, employing portfolio weights and a variance-covariance matrix. It serves as a measure of active risk and is frequently utilized in portfolio management, particularly for portfolios aligned with specific benchmarks.

$$TE = \sqrt{(w - w^*)^\top \Sigma (w - w^*)} \tag{31}$$

Tracking error ranges from 0 to $+\infty$, serving as a measure of divergence between the actual and optimal portfolios. A larger tracking error indicates greater divergence, while a lower one signifies higher similarity.

4.5.3 Expected Return Pick-up vs Expected Transaction Costs

This thesis introduces and assesses a new rebalancing strategy that rebalances the portfolio whenever the expected return pick-up from rebalancing is greater than the expected transaction costs.

$$\Delta E(r) > E(\text{Transaction Costs}) \tag{32}$$

$$\Delta E(r) > \text{Turnover} \cdot \text{Transaction Costs}_{\text{bps}} \tag{33}$$

$\Delta E(r)$ - *expected return pick-up*

Expected return pick-up is calculated as the sum of the differences in implied alphas between the actual and optimal portfolio.

$$\Delta E(r) = \sum_{i=1}^{N} \mu_i \cdot (w_i^* - w_i) \tag{34}$$

μ_i - *implied alpha of stock i*

N - *total number of stocks*

w_i - *actual weight of stock i*

w_i^* - *optimal weight of stock i*

Turnover is computed as the sum of the absolute differences between the individual asset weights of the actual and optimal portfolio.

$$\text{Turnover} = \sum_{i=1}^{N} |w_i^* - w_i| \tag{35}$$

Transaction costs in bps of turnover terms are the variable estimated by the transaction cost model.

4.5.4 Partial Rebalancing

Partial rebalancing refers to the concept of rebalancing only a certain proportion of the way back to the optimal portfolio. According to Chan & Ramkumar (2011) and Dybvig & Pezzo (2020), this approach helps minimise rebalancing costs while still maintaining an allocation that is close to the optimal.

In this thesis, the proportion for partial rebalancing is varied from 0 to 1 in 201 equally-spaced intervals for all strategies, except those using the transfer coefficient and tracking error as the threshold function[8].[9]

4.6 Performance Metrics

I evaluate each rebalancing strategy using multiple performance metrics. These are:

1. Cumulative return
2. Standard deviation
3. Return-to-risk
4. Total turnover
5. Total transaction costs
6. Tracking Error (Ex-Post)
7. Value-at-Risk
8. Expected Shortfall
9. Maximum Drawdown

Each metric is calculated both gross and net of transaction costs, and all are annualized. The calculation methodologies are provided in the Appendix.

[8]Calculating daily covariance matrices for these two functions is extremely computationally intensive and time-prohibitive.

[9]This results in a total 173,061 combinations: 50,250 for calendar-based, 40,401 for each simple threshold-based, 603 for each covariance-involved threshold-based, and 201 for each expected-return strategies.

4.7 Rebalancing Period vs Portfolio Similarity

Two regressions are conducted to assess the efficacy of calendar- versus threshold-based rebalancing in increasing factor exposure and returns. Gross returns serve as the dependent variable, while the independent variable is either the period or a similarity/distance coefficient. The regression with the higher R^2 value will indicate which method is more powerful in explaining returns. If similarity/distance coefficients consistently yield higher R^2 values than the period, then threshold-based rebalancing is deemed superior for enhancing factor exposure and returns.

The regression utilizes data from the calendar-based rebalancing strategy, which is simulated with varying rebalancing periods and three specific proportion values (0.05, 0.5, and 1). Daily calculations for each similarity and distance measure are performed. These measures are then averaged over the entire sample period, yielding a cross-sectional dataset comprising periods, mean similarity/distance coefficients, and returns.

I utilise two regression models:

$$y = \alpha + x + \epsilon \tag{36}$$

$$y = \alpha + x + x^2 + x^3 + \epsilon \tag{37}$$

y - *vector of gross returns*

α - *intercept*

x - *vector of periods or similarity/distance coefficients*

ϵ - *error term*

Econometric procedures for assessing statistical significance and addressing heteroscedasticity and autocorrelation are detailed in the Appendix.

5 Empirical Analysis

This section discusses empirical results of calendar- and threshold-based rebalancing strategies, including a final strategy balancing expected return and transaction costs.

In order to streamline the presentation and improve readability, this section contains exclusively graphs for one similarity/distance function: correlation. Graphs pertaining to other functions are relegated to the Appendix for comprehensive review.

5.1 Calendar-based Rebalancing

The thesis finds a strong negative correlation between rebalancing period and returns, especially when the rebalancing proportion is 100%. This suggests that frequent, full rebalancing improves returns and factor exposure, discrediting the idea that less frequent rebalancing could reduce noise and increase factor returns. Moreover, in the absence of frequent rebalancing, increasing the rebalancing proportion emerges as an effective way to maintain portfolio similarity and returns.

When transaction costs are considered, the optimal rebalancing strategy changes. Frequent rebalancing, while maximizing gross returns, incurs significant transaction costs that erode net returns. Net returns thus exhibit a pattern of increase up to an optimal rebalancing period, after which they decay. Lower rebalancing proportions result in reduced turnover and transaction costs, shifting the net return optimization. Notably, rebalancing proportions below 1 yield higher net returns within the optimal period. A positive relationship exists between the rebalancing period and optimal proportion up to a specific point.

As rebalancing period increases, risk (as measured by standard deviation, VaR, ES, and MDD) decreases for higher proportions, attributed to reduced exposure to factor/systematic risk from less frequent rebalancing. For lower proportions, the standard deviation initially declines, then rises due to minimal overall rebalancing, resulting in a more concentrated, less diversified, riskier portfolio. Transaction costs amplify these patterns. The proportion that minimizes standard deviation grows with the rebalancing period.

The return-to-risk ratio largely corroborates findings from the examination of returns, both gross and net, but shows less stable correlations between rebalancing period and maximizing proportion, likely due to the inclusion of standard deviation as a risk metric.

The findings suggest an optimal rebalancing period of 0-40 days for return and 0-50 days for return-to-risk, varying with rebalancing proportion. For a proportion of 1, the optimal period is 15-40 days, consistent with common industry practice of monthly rebalancing, yielding a return of 0.9%-1.1% and a return-to-risk ratio around 0.4.

Tracking error rises concavely with longer rebalancing periods, is higher for portfolios with lower rebalancing proportions, and is minimally impacted by transaction costs.

Results show increasing volatility in all metrics as the rebalancing period extends, attributed to greater divergence in portfolio holdings due to infrequent rebalancing and portfolio drift. The degree of this divergence is influenced by the rebalancing proportion. If a portfolio is fully rebalanced, its composition varies more across different strategies than when adjusted by just 5%, leading to more consistent post-rebalancing portfolios in the latter case. Lower proportions therefore result in more homogeneous portfolios and reduced metric volatility.

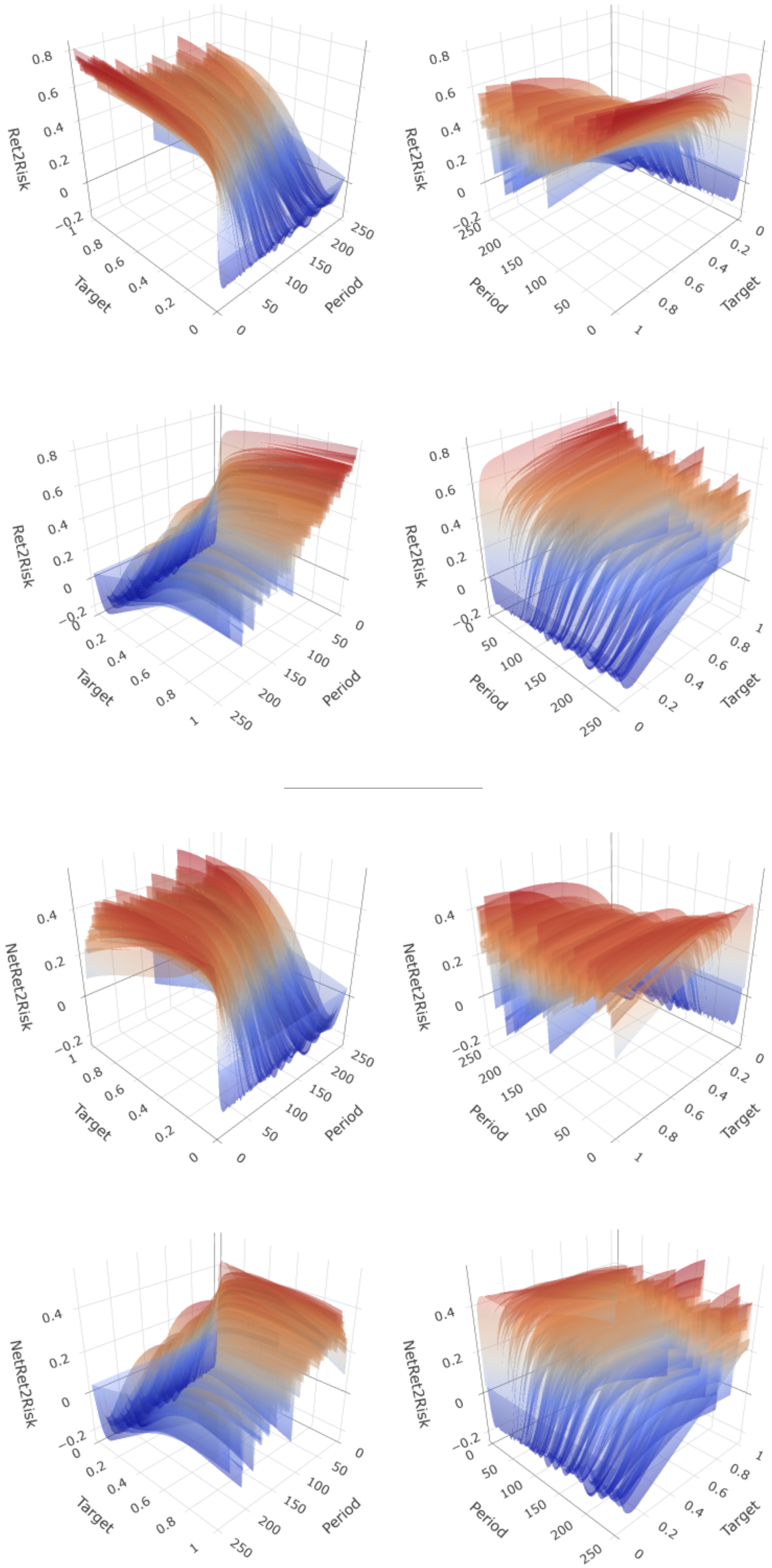

Figure 1: Return 3D Plots: Calendar-based Rebalancing

The 3D graphs display the relationship between return (gross and net), rebalancing period and partial rebalancing proportion. The top 4 graphs pertain to gross return, while the bottom 4 show return net of transaction costs. Red and blue indicate high and low metric values, respectively.

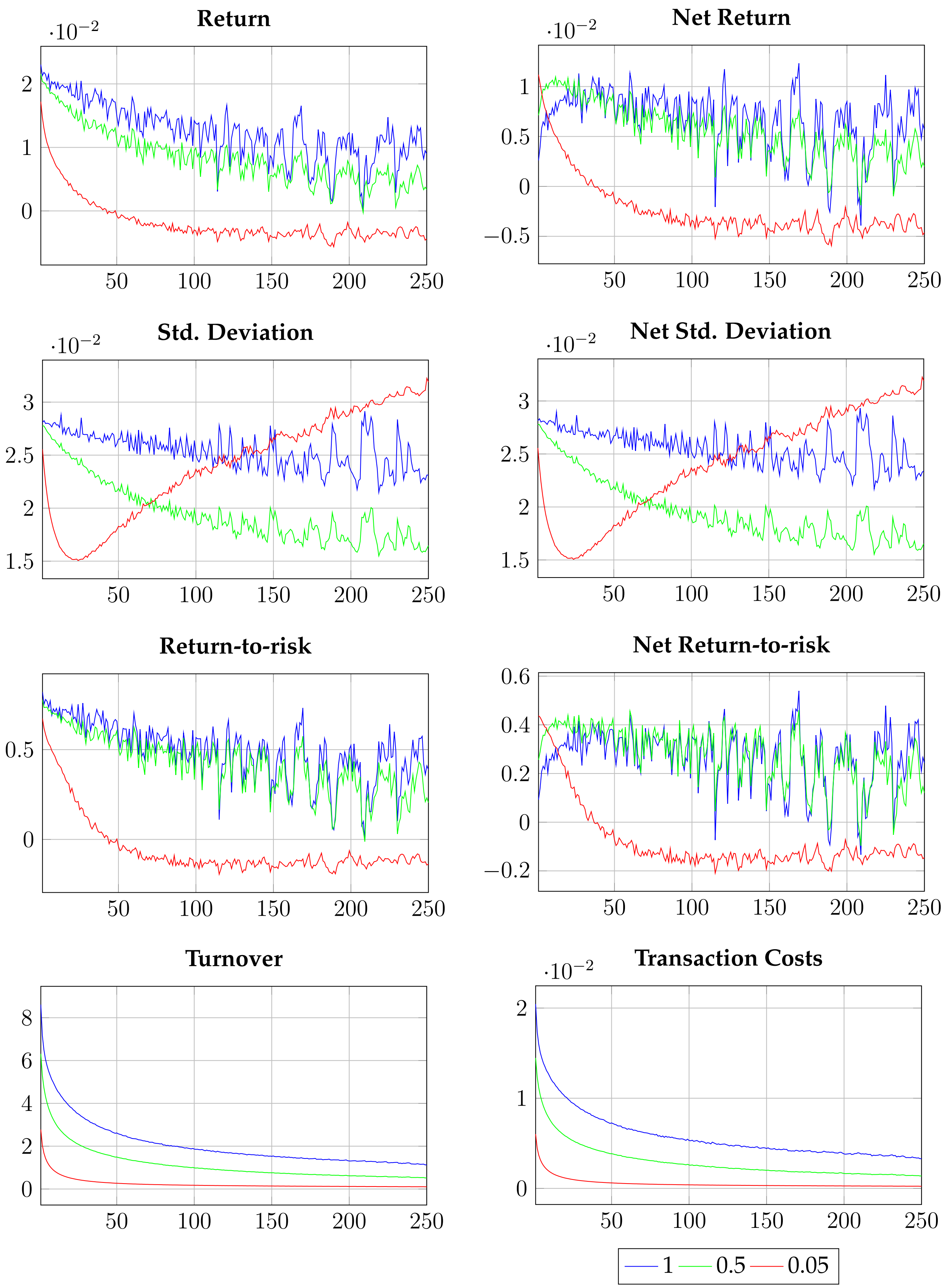

Figure 2: Rebalancing Performance Metrics: Calendar-based (Pt. 1)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. Three lines in each graph represent partial rebalancing proportions of 1, 0.5, and 0.05. A proportion of 0 is omitted as its performance is constant.

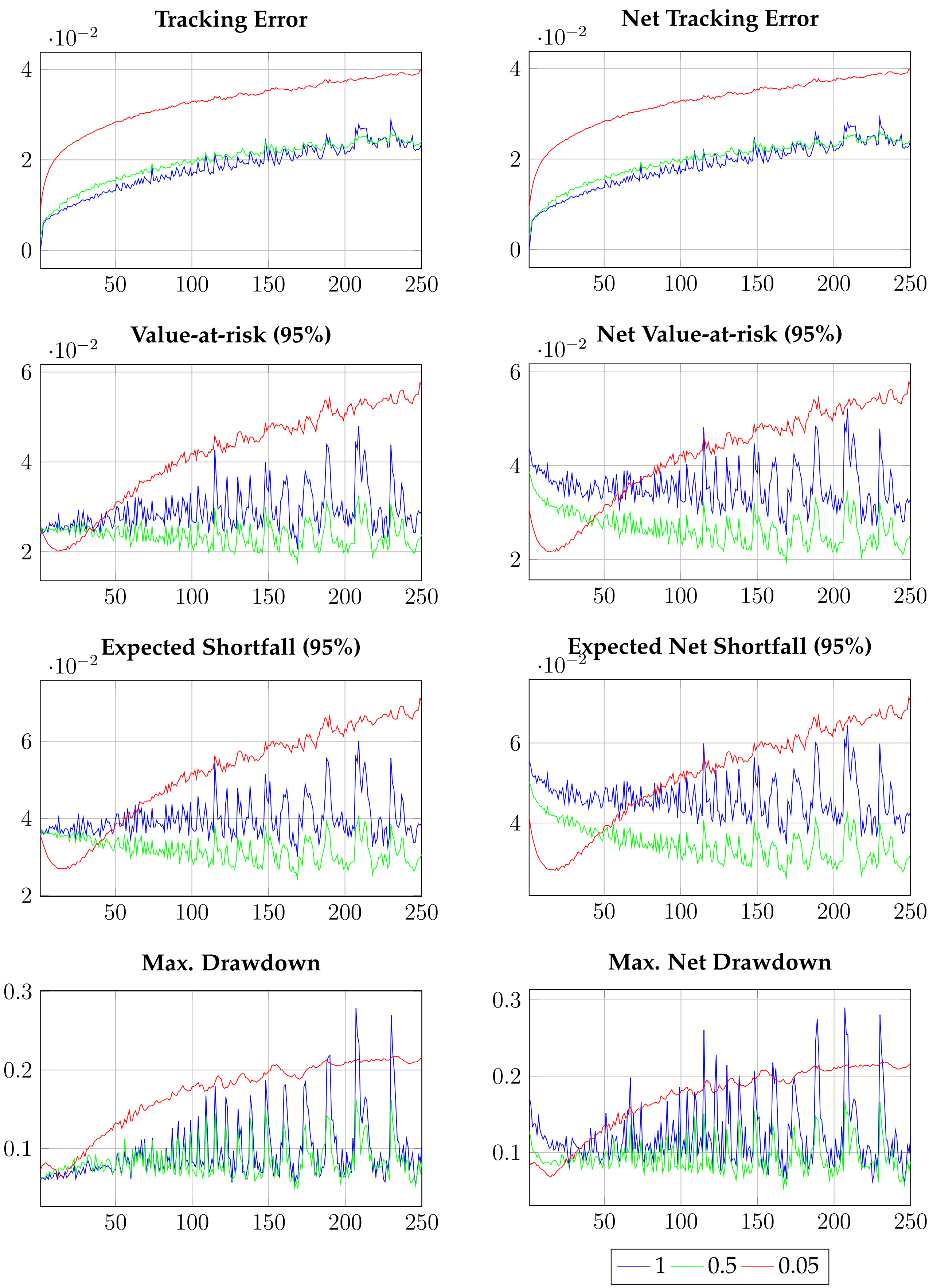

Figure 3: Rebalancing Performance Metrics: Calendar-based (Pt. 2)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. Three lines in each graph represent partial rebalancing proportions of 1, 0.5, and 0.05.

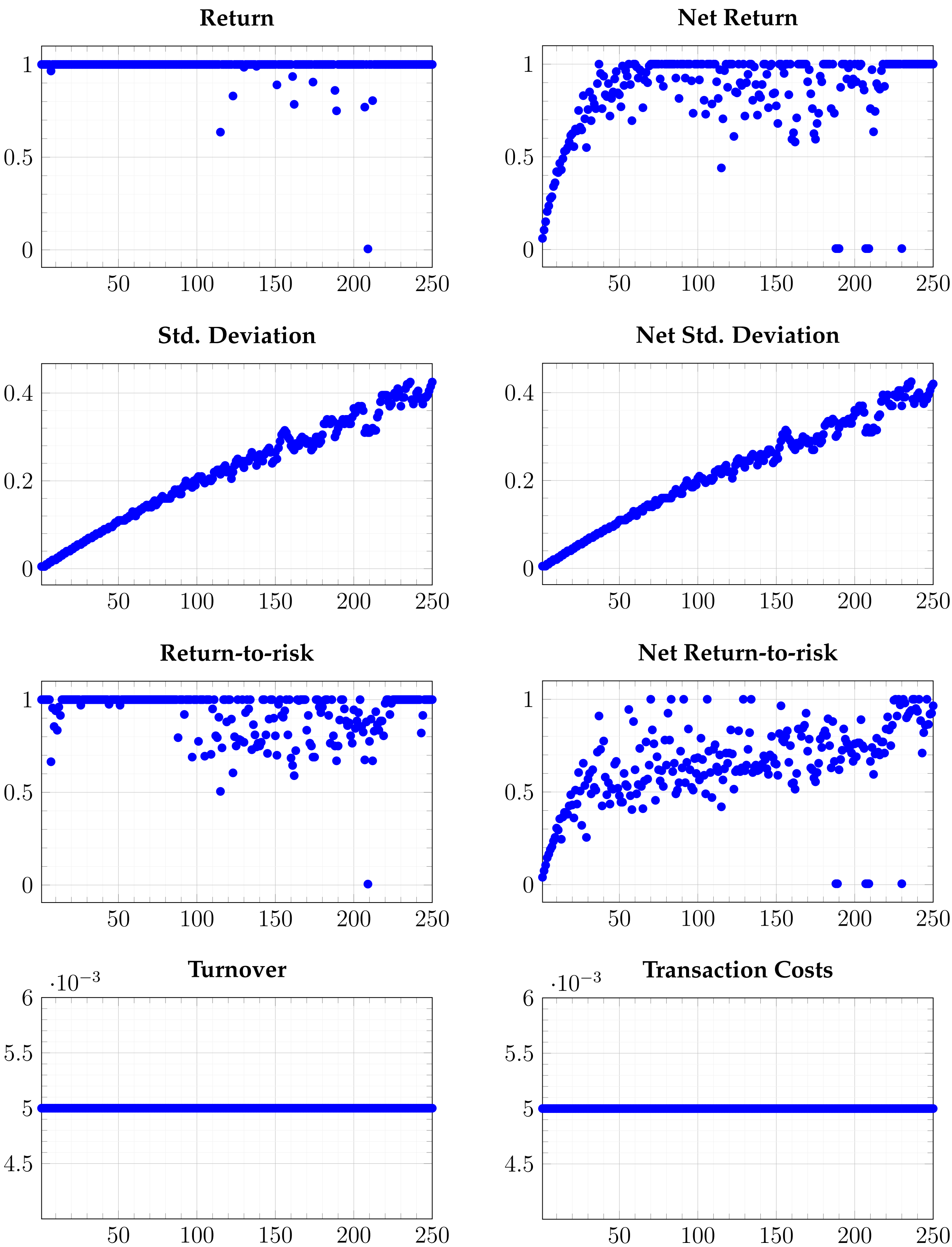

Figure 4: Optimal Proportion and Period Relationship (Pt. 1)

The graphs show how rebalancing period and optimal proportion values relate. The x-axis represents the period, and the y-axis shows the proportion optimizing a given metric, such as maximizing return or minimizing standard deviation.

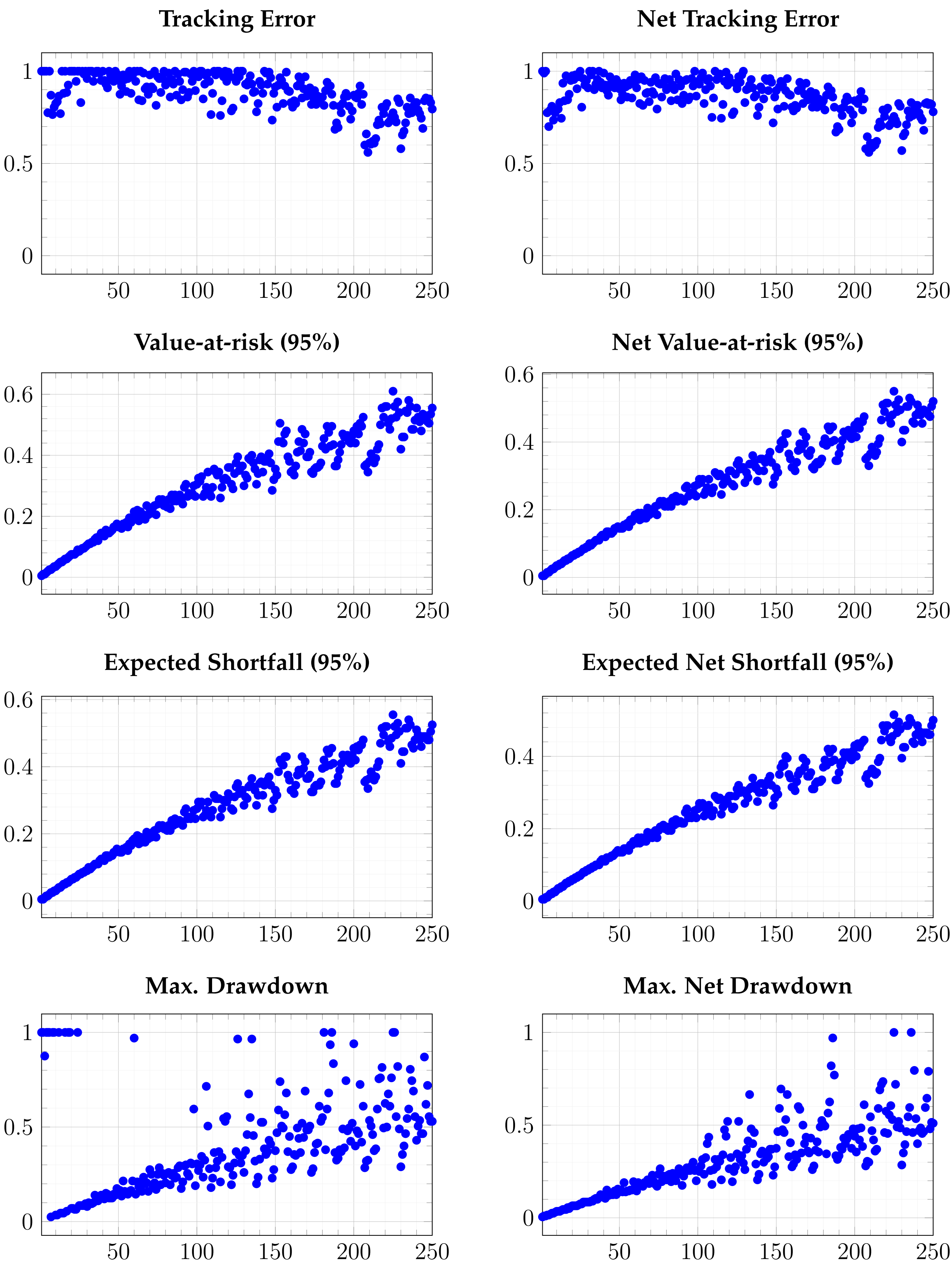

Figure 5: Optimal Proportion and Period Relationship (Pt. 2)

The graphs show how rebalancing period and optimal proportion values relate. The x-axis represents the period, and the y-axis shows the proportion optimizing a given metric, such as maximizing return or minimizing standard deviation.

5.2 Threshold-based rebalancing

For ease of reference I use the standardised term 'distance threshold' that quantifies the allowable drift from an optimal portfolio configuration. While distance functions utilize this threshold directly, similarity functions operate inversely. Therefore, an increase in distance threshold is synonymous with a decrease in similarity threshold.

Threshold-based findings largely mirror calendar-based results.

There is a strong negative, close to linear relationship between distance thresholds and gross returns across various distance functions. Correlation and tracking error show the most linear relationships, followed by transfer coefficient, Euclidean distance, and finally Chebyshev distance. The degree of linearity is influenced by the rebalancing proportion: higher proportions yield linear relationships, while lower proportions lead to convex curves. This supports the idea that higher portfolio similarity improves factor exposure and returns. The link between the optimal rebalancing proportion and distance threshold is generally weak but increasing. This is likely because less frequent rebalancing requires larger rebalancing proportions to maintain portfolio optimality. However, increasing the distance threshold makes the portfolio more vulnerable to performance metric volatility, contributing to the observed fragility in this relationship.

When transaction costs are considered, the optimal distance threshold values increase. Lower distance thresholds may boost factor exposure and gross returns through frequent rebalancing, but these gains are offset by the increased portfolio turnover and transaction costs. In cases with high rebalancing proportions, net returns increase with the distance threshold until they reach an optimal zone, after which they decline. When rebalancing proportions are low, optimal net returns are attained at minimal distance thresholds. Similar to calendar-based rebalancing, higher net returns are generated when rebalancing proportions are below 1. Additionally, there is a positive correlation between the distance threshold and the rebalancing proportion that maximizes net returns, up to a certain point.

The average coefficient serves as a measure of portfolio similarity. Its graphs show

how effectively the threshold value translates into portfolio similarity. The relationship between this coefficient and threshold value is generally linear, with the direction dependent on whether a similarity or distance function is employed. Specifically, a higher distance threshold corresponds to decreased portfolio similarity.

Frequent rebalancing coupled with a reduced rebalancing proportion engenders a notable improvement in returns. Despite the reduction in rebalancing proportion, portfolio similarity remains relatively stable, as indicated by the average coefficient metric. This is consistent with studies by Chan & Ramkumar (2011) and Dybvig & Pezzo (2020). I hypothesize that the observed effect could be partly due to stock price changes following a Geometric Brownian Motion (GBM) model with a drift component. To elaborate, I mathematically define stock price changes as:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{38}$$

S_t - *stock price at time t*

μ - *expected return*

σ - *volatility*

dW_t - *Brownian motion/standard Wiener process*

If stock prices obey a GBM with a constant drift, then stock returns follow the process:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t \tag{39}$$

$$r_t = \mu dt + \sigma dW_t \tag{40}$$

r_t - *return over an infinitesimal time period dt*

Consequently, portfolio weight changes must also follow a GBM with constant drift:

$$d\omega_t = \mu\omega_t dt + \sigma\omega_t dW_t \tag{41}$$

ω_t - *stock portfolio weight at time t*

Therefore, full-scale portfolio rebalancing jointly corrects for both constant and stochastic factors affecting weight changes. Utilizing a reduced rebalancing proportion primarily corrects for the constant drift while allowing stochastic components to naturally mean-revert to zero, thereby reducing transaction costs without sacrificing portfolio similarity.

For all functions except transfer coefficient, all risk metrics (standard deviation, VaR, ES, and MDD) exhibit a common trend: an initial decline followed by an uptick as the distance threshold increases. This is due to increased factor risk at lower thresholds being offset by risks from portfolio concentration and reduced diversification. The observed patterns manifest differently across distance functions; they are more gradual and curvilinear for correlation, and more abrupt for Euclidean distance, Chebyshev distance, and tracking error. The pattern for transfer coefficient is very irregular. The pattern becomes more pronounced when transaction costs are considered. The proportion that minimizes risk increases with period, supporting the previous conclusions.

The return-to-risk ratio largely confirms findings from return analysis but shows less stable correlations between rebalancing period and maximizing proportion, especially compared to calendar-based methods.

The sequence from more linear to more concave tracking error among similarity/distance functions can be delineated as follows: correlation, tracking error, Euclidean distance, Chebyshev distance, and transfer coefficient. Contrary to observations from calendar-based rebalancing, tracking error shows marginal sensitivity to variations in rebalancing proportion and is largely indifferent to the inclusion of transaction costs.

Similar to the calendar-based strategy, threshold-based rebalancing approaches manifest a growing volatility in performance metrics as the distance threshold increases.

All threshold-based rebalancing strategies yield similar annualized returns (0.9%-1.1%) and return-to-risk ratios (~0.4) in optimal zones, aligning with outcomes from calendar-based strategies. However, threshold-based methods show less "slicing" patterns and volatility in plots, suggesting more consistent realization of targeted returns or return-to-risk for portfolio managers using this approach.

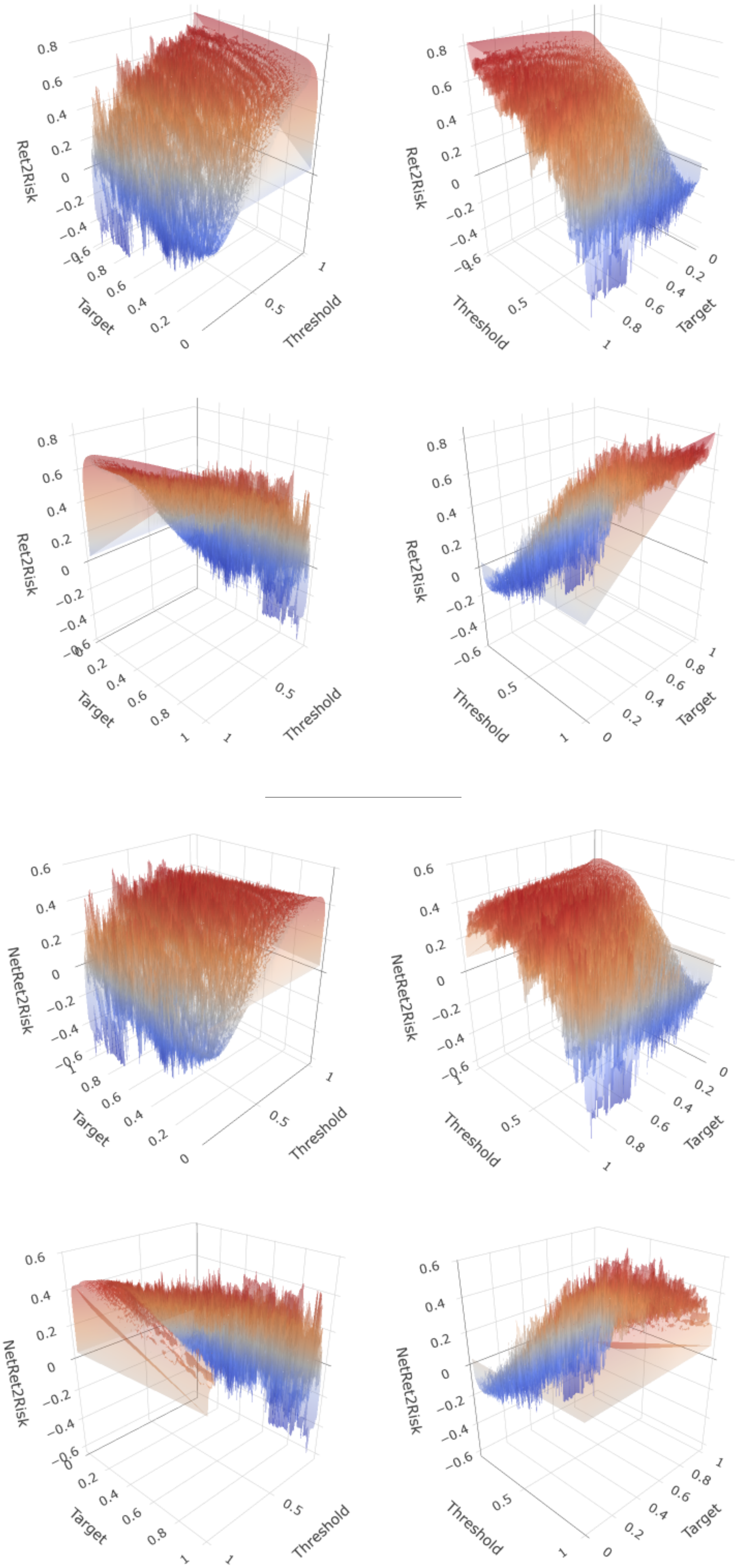

Figure 6: Return 3D Plots: Calendar-based Rebalancing

The 3D graphs display the relationship between return (gross and net), rebalancing period and partial rebalancing proportion. The top 4 graphs pertain to gross return, while the bottom 4 show return net of transaction costs. Red and blue indicate high and low metric values, respectively.

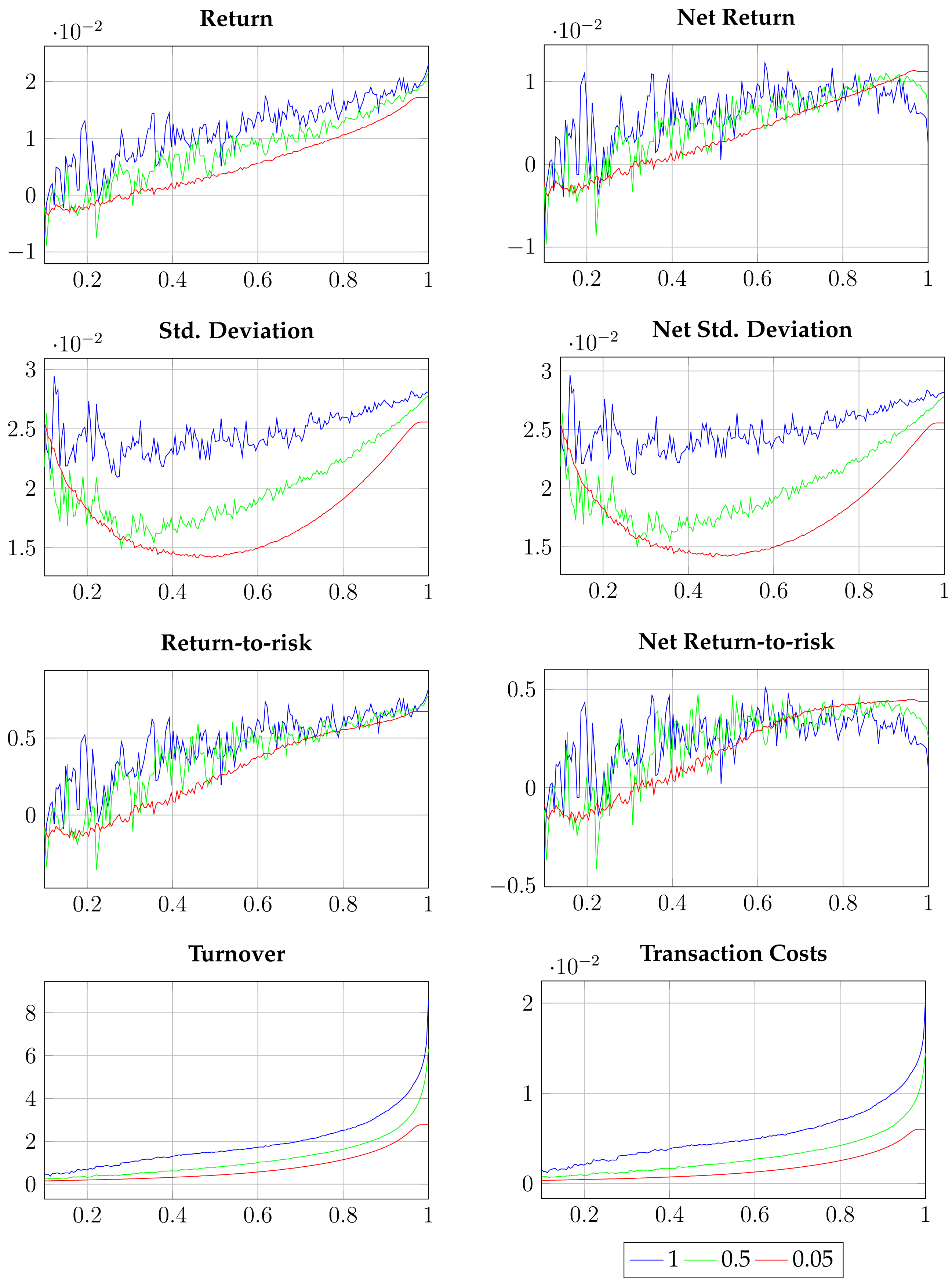

Figure 7: Rebalancing Performance Metrics: Correlation (Pt. 1)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. Three lines in each graph represent partial rebalancing proportions of 1, 0.5, and 0.05.

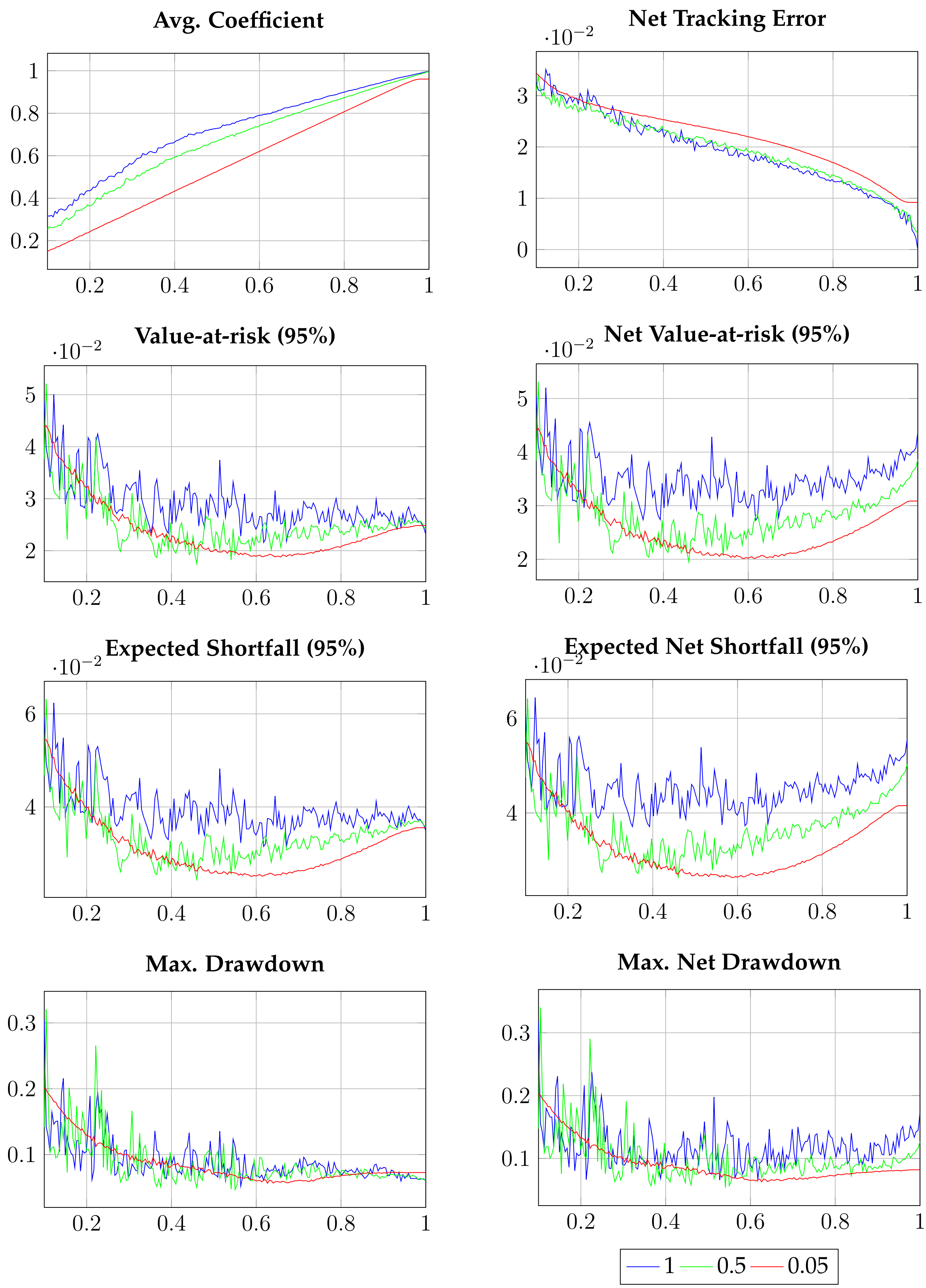

Figure 8: Rebalancing Performance Metrics: Correlation (Pt. 2)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. Three lines in each graph represent partial rebalancing proportions of 1, 0.5, and 0.05.

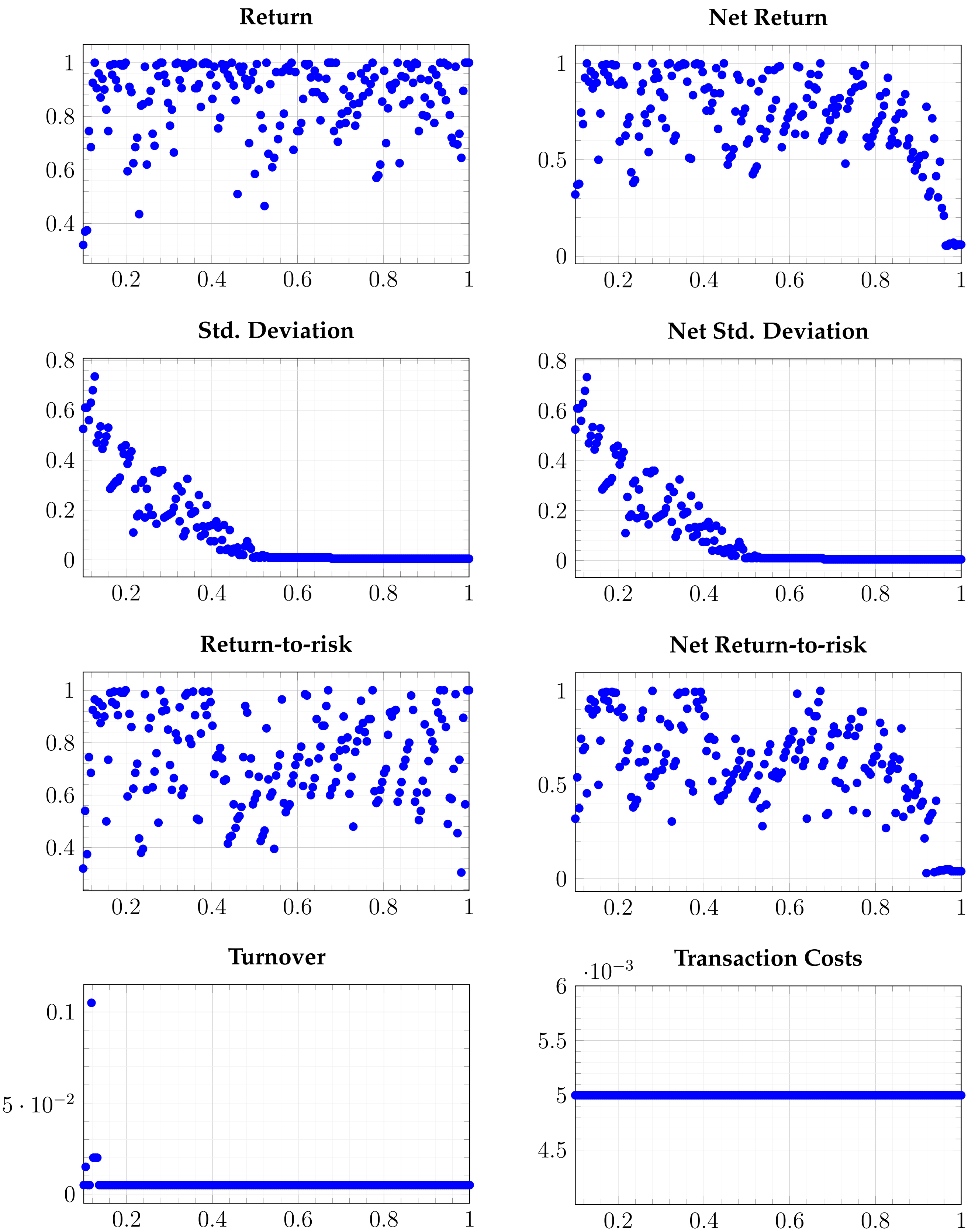

Figure 9: Optimal Proportion and Correlation Relationship (Pt. 1)

The graphs show how rebalancing period and optimal proportion values relate. The x-axis represents the period, and the y-axis shows the proportion optimizing a given metric, such as maximizing return or minimizing standard deviation.

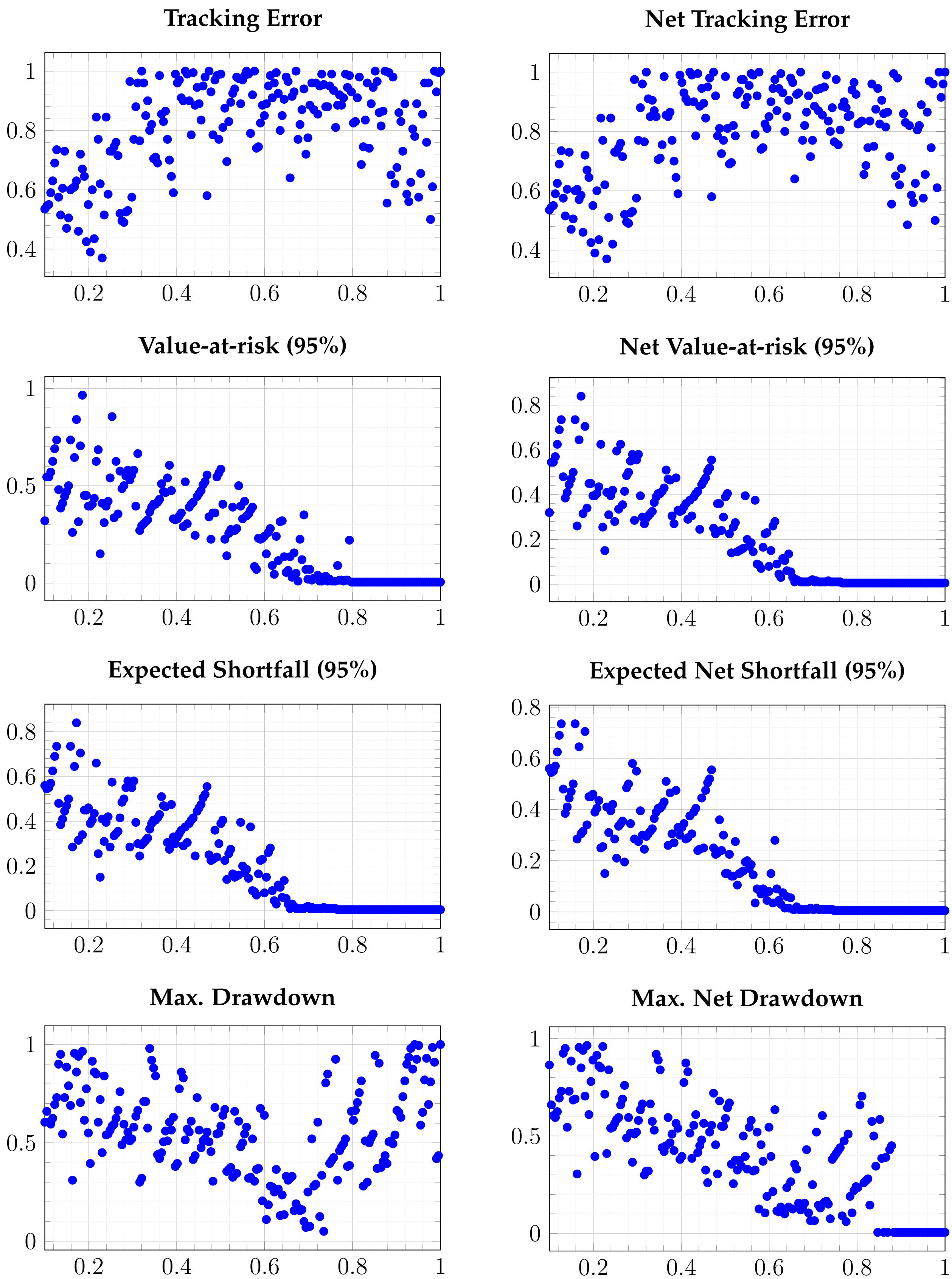

Figure 10: Optimal Proportion and Correlation Relationship (Pt. 2)

The graphs show how rebalancing period and optimal proportion values relate. The x-axis represents the period, and the y-axis shows the proportion optimizing a given metric, such as maximizing return or minimizing standard deviation.

5.3 Expected Return Pick-up vs Expected Transaction Costs

This strategy aims to rebalance the portfolio when expected return gains outweigh expected transaction costs. Expected returns, calculated as daily implied alphas, secure gains before the next rebalancing event, assuming accurate return expectations.

In the simulation, no rebalancing occurs because the expected daily return pick-up is too low to ever justify the transaction costs. Consequently, the return graphs effectively display a buy-and-hold strategy.

The following graphs show cumulative gross and net returns for this strategy at three partial rebalancing levels (0.05, 0.5, and 1) and a daily rebalanced portfolio for comparison. The x-axis indicates cumulative returns, while the y-axis shows time in years.

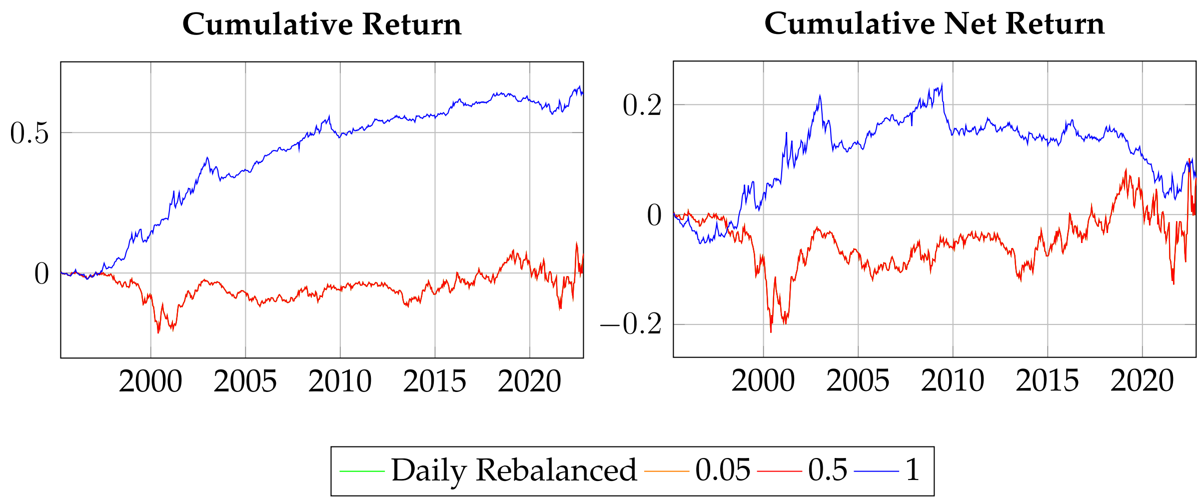

Figure 11: 1-day $\Delta E(r)$ vs $E(TC)$: Cumulative Gross and Net Return

To further scrutinise the strategy's viability, I adjust the initial assumptions to include a 22-day compounding period for the 1-day expected return pick-up, assuming constant implied alphas over this period.

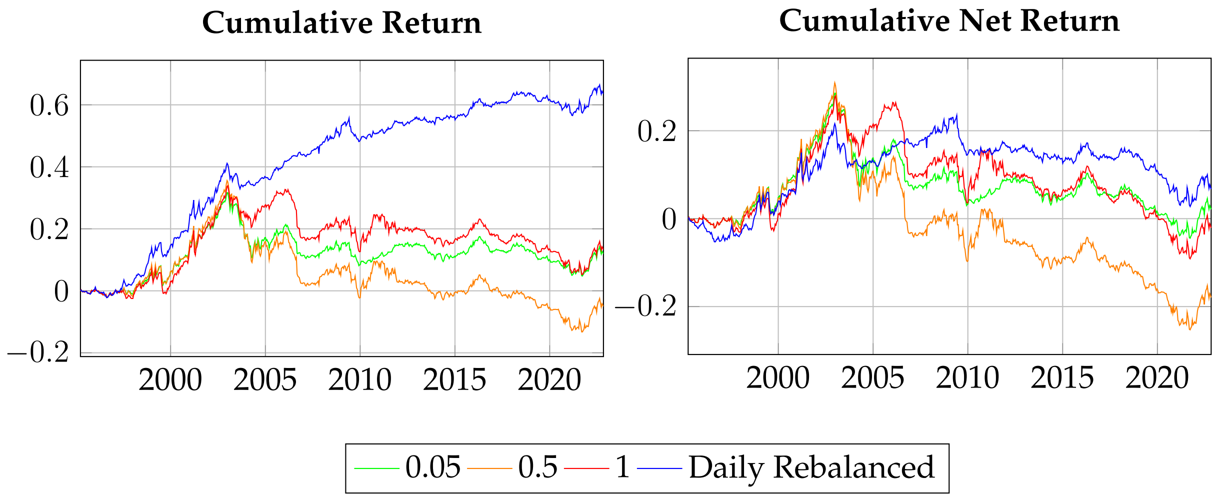

Figure 12: 22-day $\Delta E(r)$ vs $E(TC)$: Cumulative Gross and Net Return

Even under more lenient assumptions, the strategy still underperforms, yielding both gross and net returns inferior to a daily rebalanced portfolio. Consequently, the strategy is deemed ineffective for portfolio rebalancing.

5.4 Rebalancing Period vs Portfolio Similarity

The thesis employs linear and cubic regressions to analyze gross return, considering rebalancing period or similarity/distance coefficients as independent variables for rebalancing proportions of 0.05, 0.5, and 1. Comprehensive summary statistics are included in the Appendix.

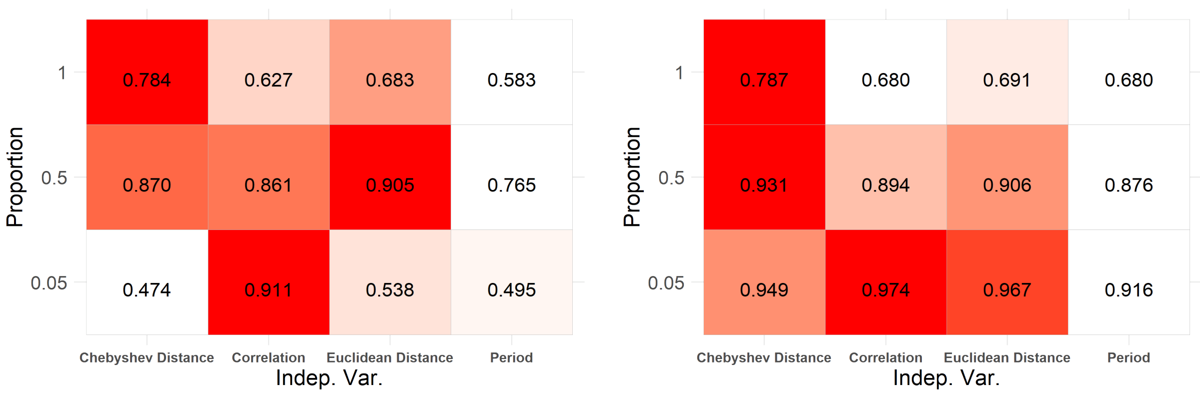

Figure 13: Heatmap of Relative R^2: Linear Model vs Cubic Model

The R^2 values[10] reveal that portfolio similarity, not rebalancing period, better explains variance in gross returns. Therefore, returns are more closely tied to portfolio similarity than to rebalancing period or frequency. These finding are consistent with the graphs from previous sections: calendar-based strategies show more "slicing" and volatility than threshold-based strategies. The latter generally follows a smoother trajectory and contains regions with notably less volatility, highlighting its relative stability. Therefore, rebalancing strategies that emphasize maintaining a high portfolio similarity are more consistent than conventional calendar-based strategies predominantly employed by portfolio managers.

5.5 Periods of Elevated Rebalancing Frequency

This section examines if specific time frames favor frequent rebalancing by collecting data on daily rebalancing occurrences for each threshold-based strategy, across various thresholds and proportions.

The analysis identifies periods of increased rebalancing activity during times of financial upheaval, namely the Dot-Com bubble, the 2008-2009 financial crisis, and the COVID-19 pandemic. The degree of increased rebalancing varies by distance function; it is most noticeable with Euclidean distance and least with correlation. The transfer coefficient and tracking error strategies show more pronounced patterns, possibly due to the limited data tied to a single rebalancing proportion of 1.[11]

The results indicate that the concentration of rebalancing activities during financial crises may depend on the chosen rebalancing proportion. This opens avenues for further research on the relationship between rebalancing thresholds and proportions, and their effect on temporal clustering patterns.

[10] The heatmap numbers represent absolute R^2 values for specific strategies, while the colors are scaled according to relative R^2 values across all functions for each proportion.

[11] The dataset is more limited for transfer coefficient and tracking error strategies due to fewer simulated combinations.

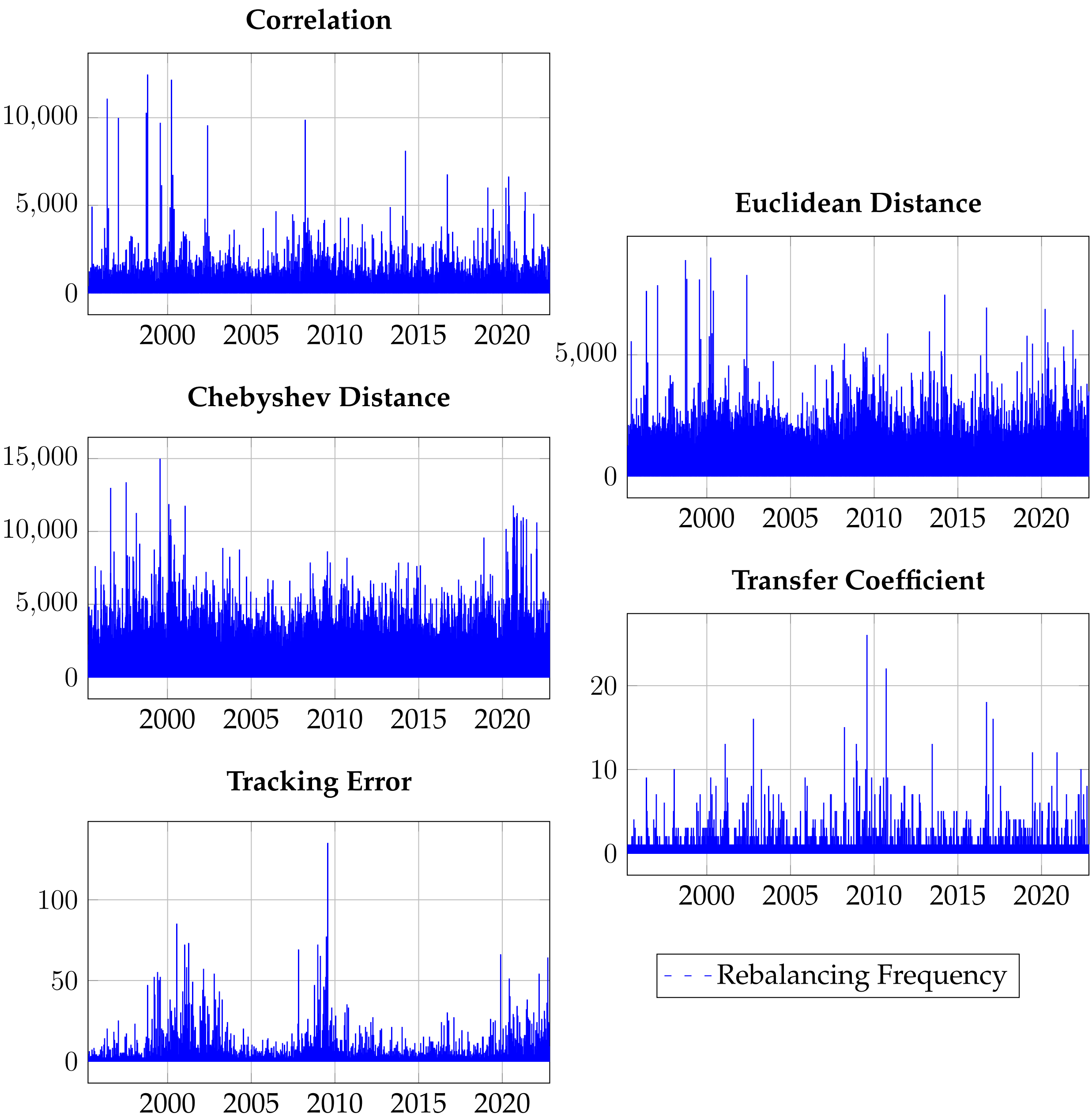

Figure 14: Daily Rebalancing Frequency

6 Conclusion

The thesis elucidates the intricate relationship between rebalancing strategies, returns, and risk in factor investing. It highlights the added complexity introduced by varying rebalancing proportions, emphasizing the necessity for customized rebalancing approaches.

The frequency of rebalancing exhibits a strong positive correlation with gross returns. Therefore, frequent rebalancing does not serve to mitigate noise emanating from the factor construction process. Rather, it amplifies factor exposure, thereby augmenting factor returns and associated risks. However, the impact of transaction costs is non-negligible and diminishes the merits of frequent rebalancing. An optimal balance exists between enhancing factor returns through frequent rebalancing and managing turnover.

For calendar-based rebalancing strategies, the optimal rebalancing frequency aligns closely with industry practice, falling within a window of 15 to 40 days. Rebalancing at intervals that adhere to this time frame yields performance metrics, specifically return and return-to-risk ratios, comparable to those achieved by threshold-based strategies. Notably, all threshold-based strategies demonstrate uniform performance when operating within their individually optimal parameter ranges for similarity or distance thresholds. Optimal returns range between 0.9%-1.1% and the optimal return-to-risk ratio fluctuates around 0.4.

While calendar-based rebalancing may subject investors to more uncertain performance, threshold-based strategies predicated on similarity or distance measures offer smoother and more reliable outcomes. Portfolio similarity statistically outperforms rebalancing period as a predictor of gross returns. Thus, practitioners are advised to employ threshold-based strategies that focus on portfolio similarity.

The optimal rebalancing strategy is to maintain high portfolio similarity through frequent, threshold-based rebalancing, ideally coupled with minimal rebalancing proportions to manage transaction costs effectively. This is consistent with studies by Chan & Ramkumar (2011) and Dybvig & Pezzo (2020). By frequently rebalancing small proportions, the strategy corrects portfolio weights for the costant drift component of portfolio weight changes, while avoiding unnecessary trades induced by the stochastic element that mean-reverts to zero over time.

Temporal analysis reveals a heightened frequency of rebalancing during periods of financial distress, corroborating findings from Khang & Picca (2021). This thesis suggest that the clustering of rebalancing events during periods of financial instability may be contingent upon the assumed rebalancing proportion value. Consequently, a more comprehensive investigation into the interplay between rebalancing thresholds, proportions, and their impact on temporal clustering patterns could offer valuable insights.

The findings of this thesis strongly suggest that threshold-based strategies exhibit more stable performance compared to calendar-based approaches of portfolio rebalancing. However, determining the optimal similarity or distance function to maximize consistency or performance remains inconclusive and a promising area for further study.

Another avenue for future research lies in the comprehensive modelling of the interrelationships among variables such as rebalancing thresholds, partial rebalancing proportions, frequency of rebalancing, and performance metrics like return and risk. Preliminary observations suggest that complex yet strong relationships exist among these variables, that are amenable to robust mathematical modelling.

Extending the scope of this study to incorporate other factor- and non-factor-based portfolios, as well as different geographical markets, could also yield important results.

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Appendices

A Derivation of Volatility Drag and Diversification Return

As per Bouchey et al. (2012).

In alignment with the theories prevalent in option-pricing and capital growth literature, it is posited that returns adhere to a pattern defined by a geometric Brownian motion process:

$$\frac{dS}{S} = \mu \cdot dt + \sigma \cdot dz \tag{42}$$

S - *asset price*

μ - *expected return*

dt - *time increment*

σ - *volatility*

dz - *random normal variable* $N(0,1)$

Using Itô's Lemma, it can be shown that the above stochastic differential equation has the solution:

$$dG = \left(\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right) dt + \frac{dG}{dS}\sigma S dz \tag{43}$$

Assuming prices follow a lognormal process, represented as $G = \ln(S)$, we get:

$$\frac{\partial G}{\partial S} = \frac{1}{S}, \ \frac{\partial^2 G}{\partial S^2} = -\frac{1}{S^2}, \ \frac{\partial G}{\partial t} = 0 \tag{44}$$

Substituting this back into Itô's formula gives:

$$dG = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dz \tag{45}$$

Consequently, the continuously compounded return, represented by $dG = dln\left(S\right) = \frac{dS}{S}$ becomes a geometric Brownian motion process with the following drift parameter:

$$g = \mu - \frac{\sigma^2}{2} \tag{46}$$

For a fixed-weight portfolio, the long-term growth from the above equation transforms into:

$$g_p = \sum_{i=1}^{N} w_i \mu_i - \frac{1}{2} \sum_{i,j=1}^{N,M} w_i \sigma_{ij} w_j \tag{47}$$

g_p - *continuously compounded portfolio return*

w_i - *weight of asset i in the portfolio*

w_j - *weight of asset j in the portfolio*

μ_i - *expected return of asset i*

σ_{ij} - *return covariance of assets i and j*

To emphasize the impact of diversification on portfolio return, they solve the equation (46) for μ and substitute it into equation (47) to get:

$$g_p = \sum_{i=1}^{N} w_i \left(g_i + \frac{\sigma_i^2}{2} \right) - \frac{1}{2} \sum_{i,j=1}^{N,M} w_i \sigma_{ij} w_j \tag{48}$$

This can be further rearranged to:

$$g_p = \sum_{i=1}^{N} w_i g_i + d \tag{49}$$

where

$$d = \frac{1}{2} \sum_{i=1}^{N} w_i \sigma_i^2 - \frac{1}{2} \sum_{i,j=1}^{N,M} w_i \sigma_{ij} w_j \tag{50}$$

Equation (49) expresses the portfolio growth rate as a sum of the individual asset growth rates plus a premium d, derived from diversification and rebalancing. This premium is positive when correlations are less than one, indicating a positive impact of rebalancing to fixed weights. The first term of d represents the weighted sum of the component asset variances, and the second is the portfolio variance. The balance between an increase in asset volatility, which bolsters growth potential from rebalancing, and portfolio variance, which decreases growth, largely hinges on the correlation among assets.

B Performance Metrics

I compute several performance metrics to evaluate each rebalancing strategy. Each relevant metric is computed both in gross terms and net of transaction costs. All metrics are annualised. To calculate metrics net of transaction costs I simply subtract the transaction costs on each day from the return on each day.

Cumulative Return

The return of a particular rebalancing strategy is defined differently according to whether we are rebalancing a long-short or a long-only factor portfolio.

The simplest case is that of the long-only portfolio, where the cumulative return of a rebalancing strategy is the compound return of the portfolio throughout the whole sample period. For a long-only portfolio, the cumulative return is computed as:

$$r = \prod_{t=1}^{T}(1 + r_t) \tag{51}$$

OR

$$r = \frac{p_T - p_0}{p_0} \tag{52}$$

T - *total number of time periods (in this case, days)*

r_t - *return at time t*

p_T - *is the asset price at time* T

p_0 - *is the asset price at time* 0

For long-short portfolios, the return of a single time period (e.g., day) is the return of the long leg of the portfolio minus the return of the short leg of the portfolio. However, in this kind of portfolio, the proceeds from shorting the short leg are used to fund the long leg. This means the initial invested capital, or the price of the initial portfolio nets out to zero. Therefore, because there is no initial invested capital, returns cannot be computed as a proportion of such an amount. Any return on a net initial investment of zero would be mathematically undefined. Therefore, it also does not make sense to compound returns.

Instead, returns on long-short portfolios are defined on a period-by-period basis either in monetary terms or relative to the original size (before weights drift) of either the long leg or the short leg of the portfolio. Because I will take the latter approach and define these returns as a percentage of a single long-short leg, it is important to reiterate that these returns are conceptually different from those of a long-only portfolio, as they are not returns on investment. As such, the returns of a long-short portfolio are comparable to returns of other long-short portfolios, but not necessarily with the returns of long-only portfolios. The returns of a long-short portfolio can be summed over time to compute the total return as:

$$r = \sum_{t=1}^{T} r_t \tag{53}$$

Standard Deviation

This is computed as an ex-post measure based on the realized population of return values:

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T}(r_t - \bar{r})^2}{T}} \tag{54}$$

$\bar{r}$ - *mean return over T time periods*

Information ratio

The Information Ratio (IR) is a financial metric used to assess the risk-adjusted return of a financial portfolio relative to a benchmark. It is calculated by dividing the portfolio's active return (the difference between the portfolio's return and the benchmark's return) by the tracking error, which measures the portfolio's deviation from the benchmark. It provides a way to assess the efficiency or effectiveness of an investment strategy in generating active return returns relative to the amount of active risk taken. A higher IR indicates that a portfolio has outperformed its benchmark with a given level of risk, while a lower IR suggests underperformance. Mathematically, the IR can be expressed as:

$$\text{IR} = \frac{R_a}{\sigma_a} \tag{55}$$

IR - *information ratio*

R_a - *active return*

σ_a - *active risk*

Return-to-risk

The return-to-risk ratio (RRR) is a financial metric that measures the relationship between the potential return and the level of risk associated with an investment or portfolio. It provides a way to assess the efficiency or effectiveness of an investment strategy in generating returns relative to the amount of risk taken. I compute this metric as the ratio of return to standard deviation.

$$RRR = \frac{r}{\sigma} \tag{56}$$

r - *portfolio return*

σ - *standard deviation of portfolio return*

Because factor strategies can be considered active strategies that take on additional risk to generate increased returns, the RRR can be considered akin to the information ration (IR). The IR is essentially the same as the RRR, but applied to active returns and active risk.

Total Turnover

Total turnover is a measure of the trading activity of a portfolio. It is the amount traded to get from the actual portfolio to the optimal portfolio on the act of rebalancing.

$$\text{Turnover} = |w^* - w| \tag{57}$$

$$\text{Total Turnover} = \sum_{i=1}^{T} \text{Turnover}_t \tag{58}$$

w^* - *optimal portfolio nominal weights*

w - *actual portfolio nominal weights*

$Turnover_t$ - *turnover incurred at time t*

Total Transaction Costs

Total transaction costs represent the sum of the time series of transaction costs computed by the transaction cost models employed multiplied by turnover.

$$\text{Transaction Costs} = \text{Turnover} \cdot \text{Transaction Costs}_{bps} \tag{59}$$

$$\text{Total Transaction Costs} = \sum_{t=1}^{T} \text{Transaction Costs}_t \tag{60}$$

$Transaction\ Costs_{bps}$ - *transaction costs in bps of turnover terms*

$Transaction\ Costs_t$ - *transaction costs incurred at time t*

Tracking Error (Ex-Post)

Tracking error (TE) is a measure of how closely a portfolio tracks or replicates the performance of its benchmark. In our case this is the actual and optimal portfolio respectively. Therefore, tracking error quantifies the variability or deviation in returns between the actual and the optimal portfolio over a specific time frame. Intuitively it can be understood as a measure of extra risk taken by not sticking to the optimal allocation or benchmark. A low tracking error suggests that the portfolio closely follows the optimal, while a higher tracking error indicates greater divergence from the optimal. Ex-post tracking error is computed from realized return values. In this case I assume a time frame of 250 days for tracking error calculation.

$$\text{TE} = \sqrt{\frac{1}{250}\sum_{t=1}^{T}\Big((r_t - r_t^*) - \overline{(r - r^*)}\Big)^2} \tag{61}$$

r_t - *actual portfolio return at time t*

r_t^* - *optimal portfolio return at time t*

r - *time series vector of actual portfolio return*

r^* - *time series vector of optimal portfolio return*

Value-at-Risk

Value at Risk (VaR) is a statistical measure used to quantify the level of risk within an investment portfolio over a specific time frame. It provides an estimate of the potential loss in value of a portfolio for a given confidence interval and time horizon. Intuitively, VaR answers the question: "Given a set confidence level (e.g., 95% or 99%), how much could we potentially lose over a given time period (e.g., a day or month)?" For a given confidence level and time frame, the VaR is defined as:

$$\text{VaR} = \mu - Z \cdot \sigma \tag{62}$$

μ - *mean return*

Z - *Z-score corresponding to the desired confidence level*

σ - *standard deviation of return*

Expected Shortfall

Expected Shortfall (ES), also known as Conditional Value at Risk (CVaR), is a risk metric that quantifies the expected value of losses beyond a specified VaR threshold. While VaR provides a threshold for worst-case losses at a certain confidence level, ES calculates the average of all losses exceeding that threshold within the specified confidence level. For a given time frame, the ES is defined as:

$$\text{ES} = \mu - \frac{f(Z)}{1-c} \cdot \sigma \tag{63}$$

μ - *mean return*

$f(Z)$ - *density function of the standard normal distribution evaluated at Z*

c - *confidence level*

σ - *standard deviation of return*

Maximum Drawdown

Maximum Drawdown (MDD) is a risk metric used to measure the largest single drop from peak to trough in the value of a portfolio before a new peak is achieved. It is an indicator of downside risk over a specified time frame. Mathematically, MDD is defined as:

$$\text{MDD} = \max\left(\frac{P_{\text{peak}} - P_{\text{trough}}}{P_{\text{peak}}}\right) \tag{64}$$

P_{peak} - *highest portfolio value up to a given time*

P_{trough} - *lowest portfolio value after P_{peak} and before the portfolio achieves a new peak*

C Regression Analysis Process

Evaluating the significance of coefficients and the regression

To test for the significance of individual coefficients I use a Student (1908) t-test which produces a test statistic that follows a student's t-distribution with $n - (k + 1)$ degrees of freedom.

$$H_0 : \beta_i = 0,$$
$$H_a : \beta_i \neq 0,$$

$$t = \frac{\hat{\beta}_i - r}{\sqrt{V[\hat{\beta}_i]}} \sim t_{(T-(k+1))} \tag{65}$$

β_i - *coefficient*

r - *restriction*

n - *sample size*

k - *number of explanatory variables*

To verify the overall significance of regressions I use an F-test which produces a test statistic that follows an F-distribution. When there is only one explanatory variable, the null hypothesis is the same as above and the F-test is the same as a t-test.

$$F = \frac{\frac{R^2}{k}}{\frac{1-R^2}{T-(k+1)}} \sim F_{(k,T-(k+1))} \tag{66}$$

To ensure the full ideal conditions are satisfied and the t and F statistics are valid, I test for heteroscedasticity and autocorrelation.

Testing for heteroscedasticity

I start with the null hypothesis that the error variances are all equal, against the alternative that they vary across observations.

$$H_0 : V[\epsilon_i] = \sigma^2,$$
$$H_a : V[\epsilon_i] = \sigma_i^2,$$

By estimating an auxiliary regression and obtaining its R^2, I compute a Breusch & Pagan (1979) LM test statistic that follows a chi-squared distribution with k degrees of freedom.

$$e_t^2 = \gamma_0 + \gamma_1 z_{i1} + \cdots + \gamma_k z_{ik} + \xi_i \rightarrow R^2_{\text{aux}}, \tag{67}$$

$$\text{LM} = n \times R^2_{\text{aux}} \sim \chi^2_k. \tag{68}$$

e - *residual*

γ_i - *coefficient*

z_{ik} - *explanatory variable*

ξ_i - *error term*

n - *sample size*

If the test statistic of either test is larger than the respective critical value assumed (p-value is lower than the respective significance level assumed) I reject the null hypothesis that the error variances are all equal.

Testing for autocorrelation

To test for serial correlation of the errors I employ a Breusch-Godfrey (1978; 1978) test of order p. I start with the null hypothesis that consecutive error terms are not correlated, against the alternative that autocorrelation is present.

$$H_0 : \text{Corr}(\epsilon_t, \epsilon_{t-1}) = \rho = 0, \tag{69}$$

$$H_a : H_0 \text{ does not hold} \tag{70}$$

I follow the suggestion of Newey & West (1994) and define the truncation lag p as:

$$p = \left\lfloor 4 \left(\frac{T}{100} \right)^{\frac{2}{9}} \right\rfloor \tag{71}$$

By estimating an auxiliary regression and obtaining its R^2, I compute a Breusch-Godfrey LM test statistic that follows a chi-squared distribution with p degrees of freedom.

$$e_t = \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \cdots + \gamma_p e_{t-p} + x_t' \delta + \xi_t \rightarrow R^2_{\text{aux}}, \tag{72}$$

$$LM_{\text{BG}} = n \times R^2_{\text{aux}} \sim \chi^2_p. \tag{73}$$

If the test statistic is larger than the critical value assumed (p-value is lower than the significance level assumed) I reject the null hypothesis that the errors are uncorrelated with lags up to order p.

Correcting for heteroscedasticity and autocorrelation

If heteroscedasticity or autocorrelation is detected within a given regression, I compute Newey & West (1987) heteroscedasticity- and autocorrelation-robust standard errors, that provide more reliable t and F statistics.

$$\hat{V}_{\mathrm{NW}}[\hat{\beta}] = T(X'X)^{-1}S^*(X'X)^{-1}, \tag{74}$$

$$S^* = \frac{1}{T}\sum_{t=1}^{T} e_t^2 x_t x_t' + \frac{1}{T}\sum_{l=1}^{p}\sum_{t=l+1}^{T} w_l e_t e_{t-l}(x_t x_{t-l}' + x_{t-l} x_t'), \tag{75}$$

$$w_l = 1 - \frac{l}{p+1}. \tag{76}$$

D Supplementary Results

This section contains supplementary sets of tables and graphs containing relevant results.

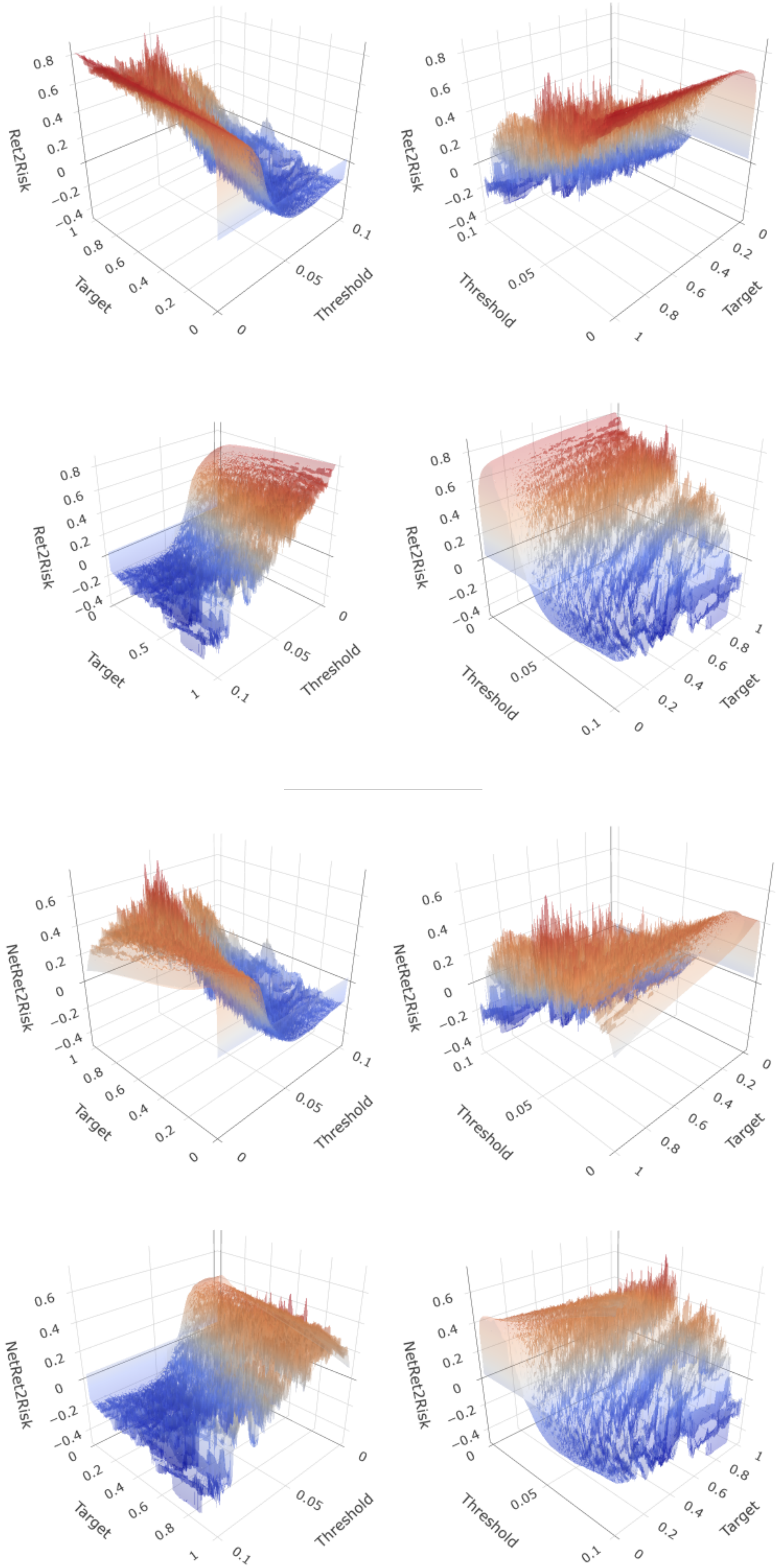

Figure 15: Return 3D Plots: Calendar-based Rebalancing

The 3D graphs display the relationship between return (gross and net), rebalancing period and partial rebalancing proportion. The top 4 graphs pertain to gross return, while the bottom 4 show return net of transaction costs. Red and blue indicate high and low metric values, respectively.

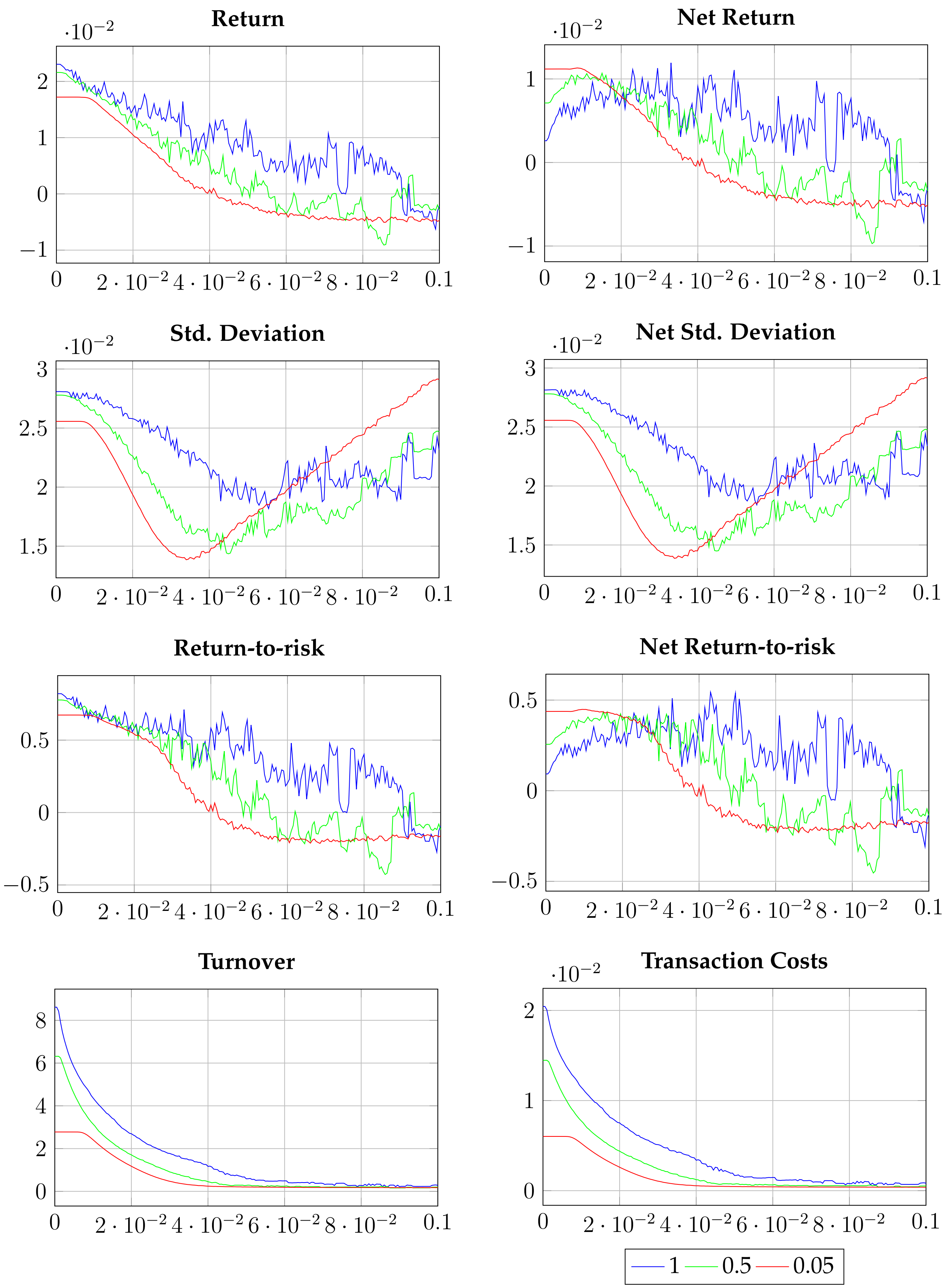

Figure 16: Rebalancing Performance Metrics: Euclidean Distance (Pt. 1)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. Three lines in each graph represent partial rebalancing proportions of 1, 0.5, and 0.05.

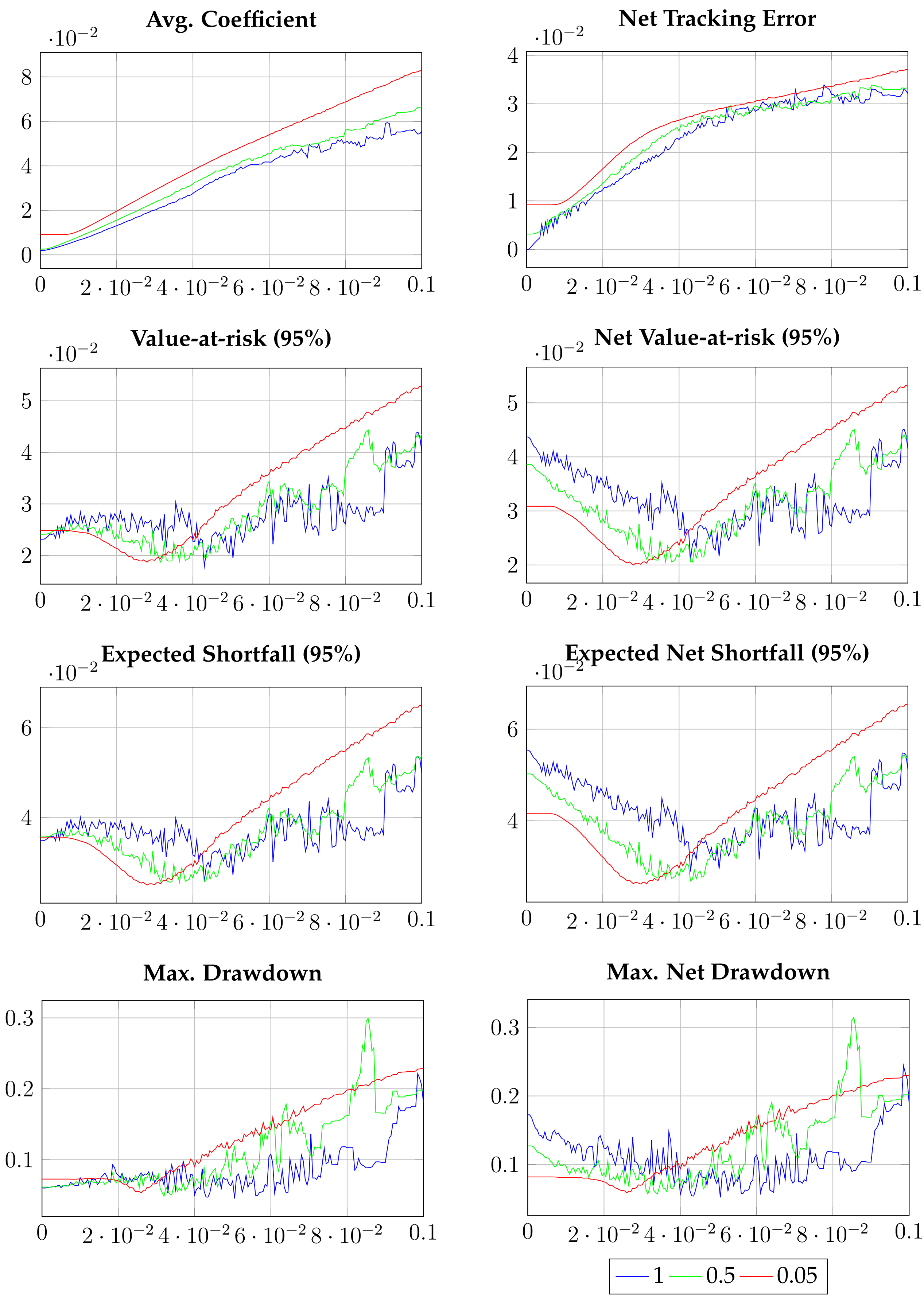

Figure 17: Rebalancing Performance Metrics: Euclidean Distance (Pt. 2)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. Three lines in each graph represent partial rebalancing proportions of 1, 0.5, and 0.05.

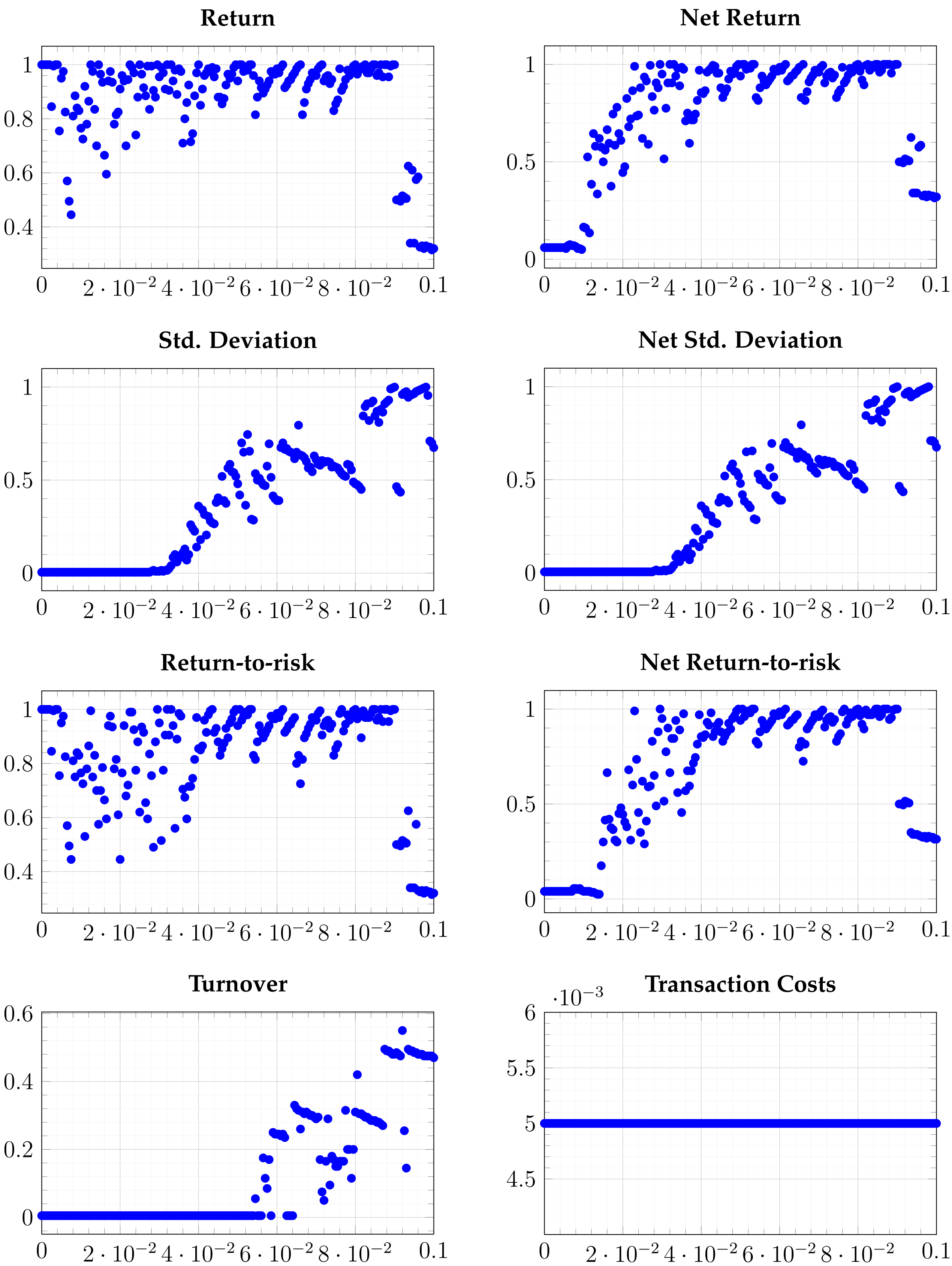

Figure 18: Optimal Proportion and Euclidean Distance Relationship (Pt. 1)

The graphs show how rebalancing period and optimal proportion values relate. The x-axis represents the period, and the y-axis shows the proportion optimizing a given metric, such as maximizing return or minimizing standard deviation.

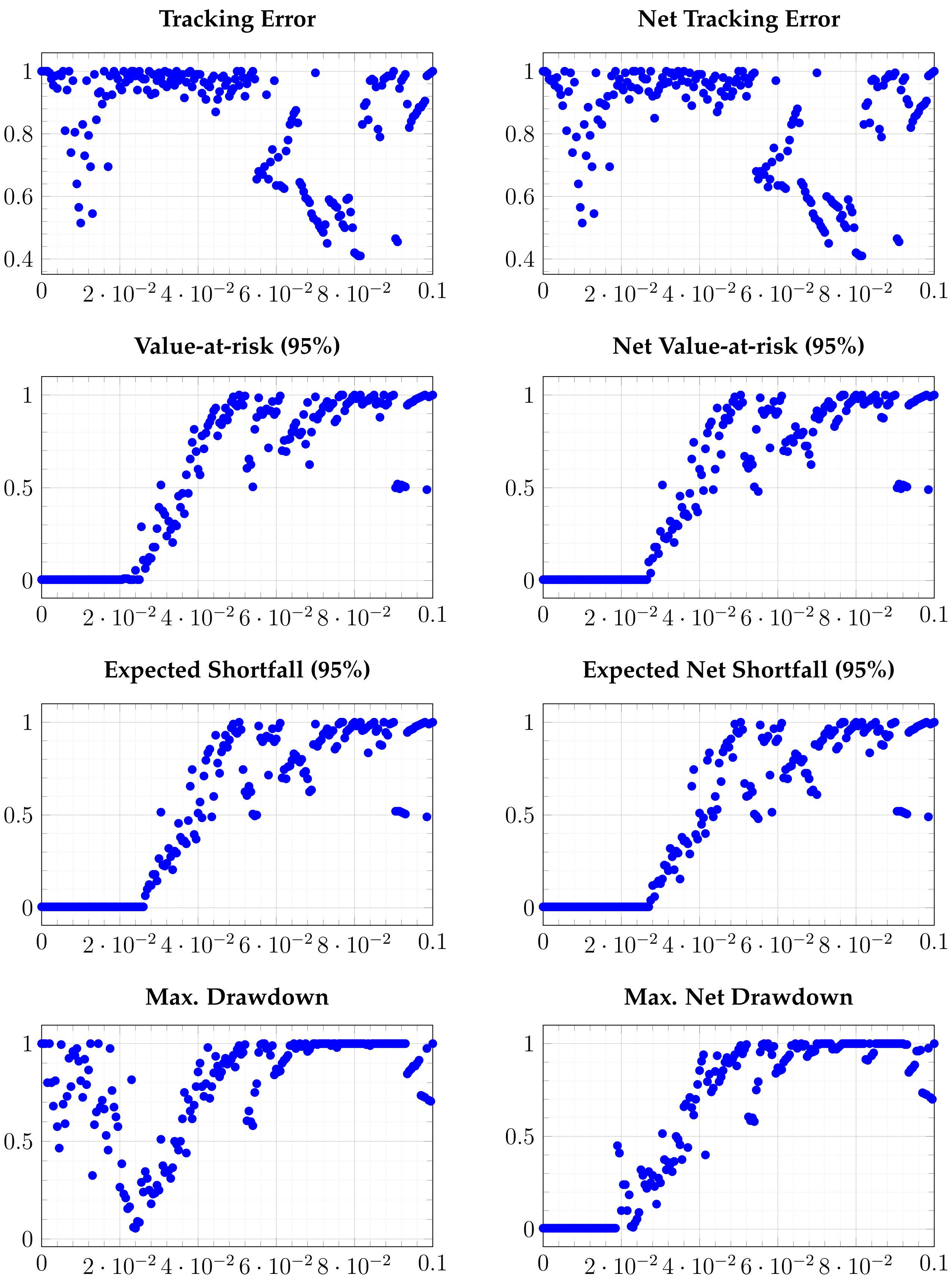

Figure 19: Optimal Proportion and Euclidean Distance Relationship (Pt. 2)

The graphs show how rebalancing period and optimal proportion values relate. The x-axis represents the period, and the y-axis shows the proportion optimizing a given metric, such as maximizing return or minimizing standard deviation.

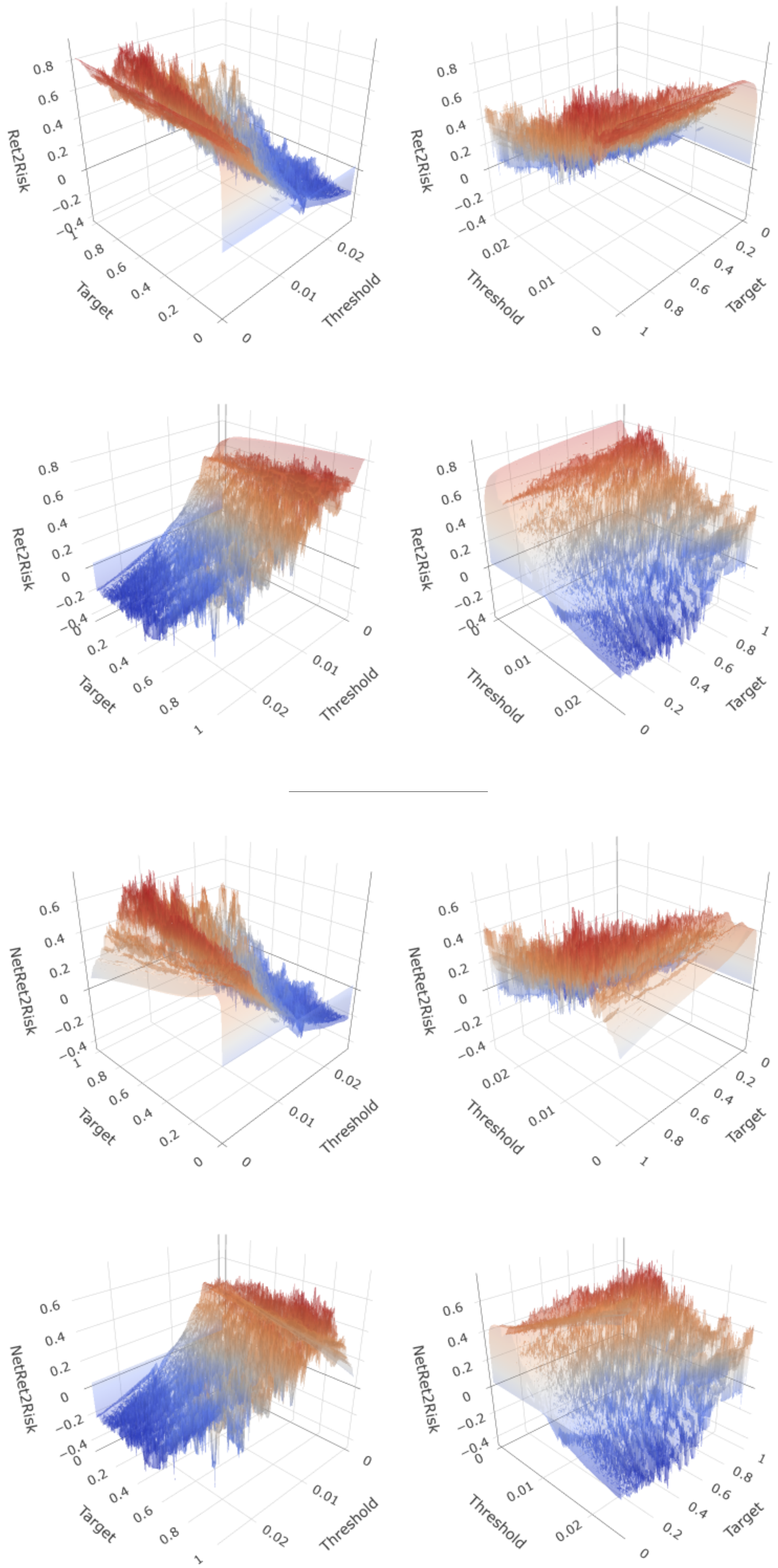

Figure 20: Return 3D Plots: Calendar-based Rebalancing

The 3D graphs display the relationship between return (gross and net), rebalancing period and partial rebalancing proportion. The top 4 graphs pertain to gross return, while the bottom 4 show return net of transaction costs. Red and blue indicate high and low metric values, respectively.

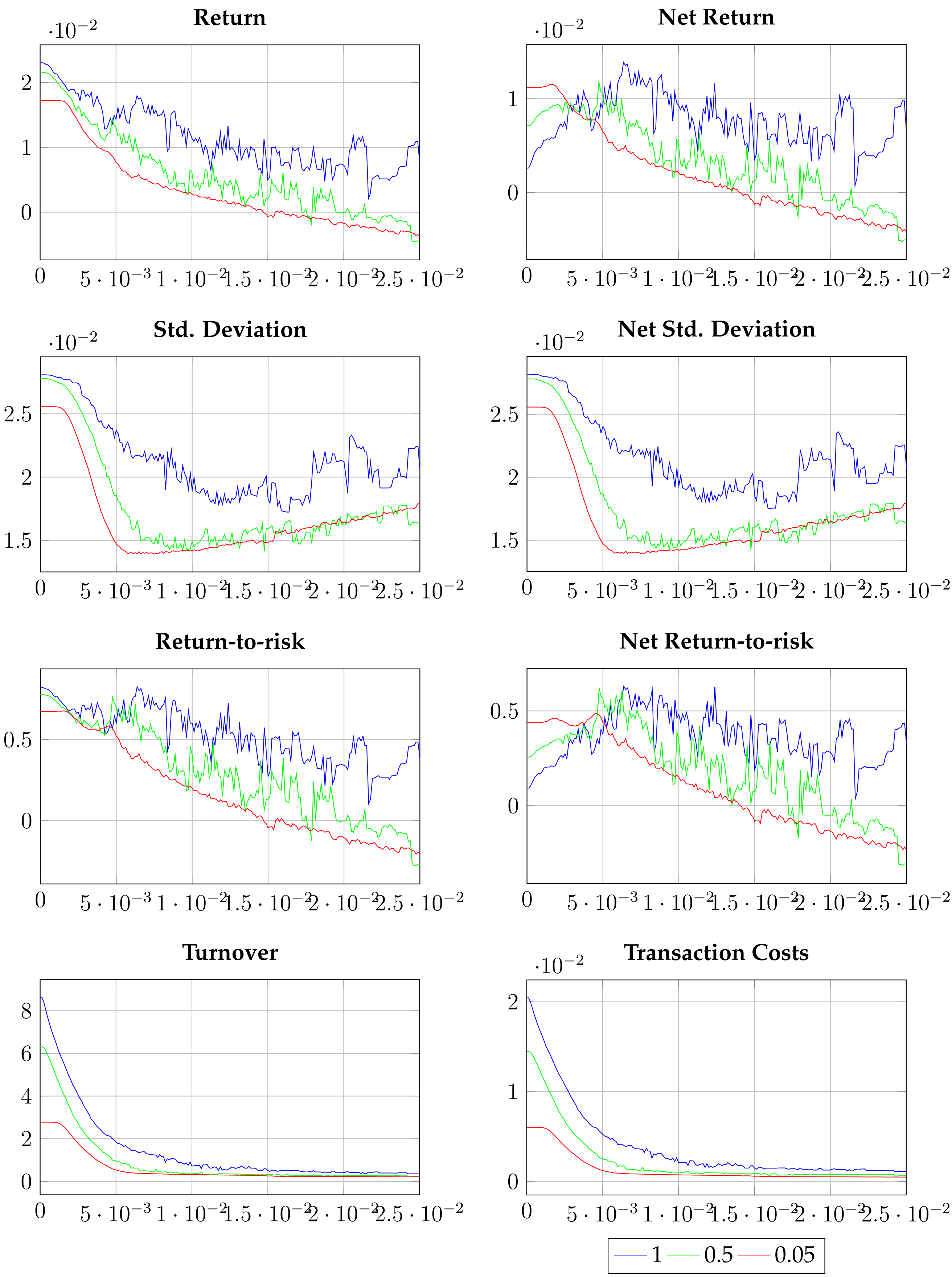

Figure 21: Rebalancing Performance Metrics: Chebyshev Distance (Pt. 1)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. Three lines in each graph represent partial rebalancing proportions of 1, 0.5, and 0.05.

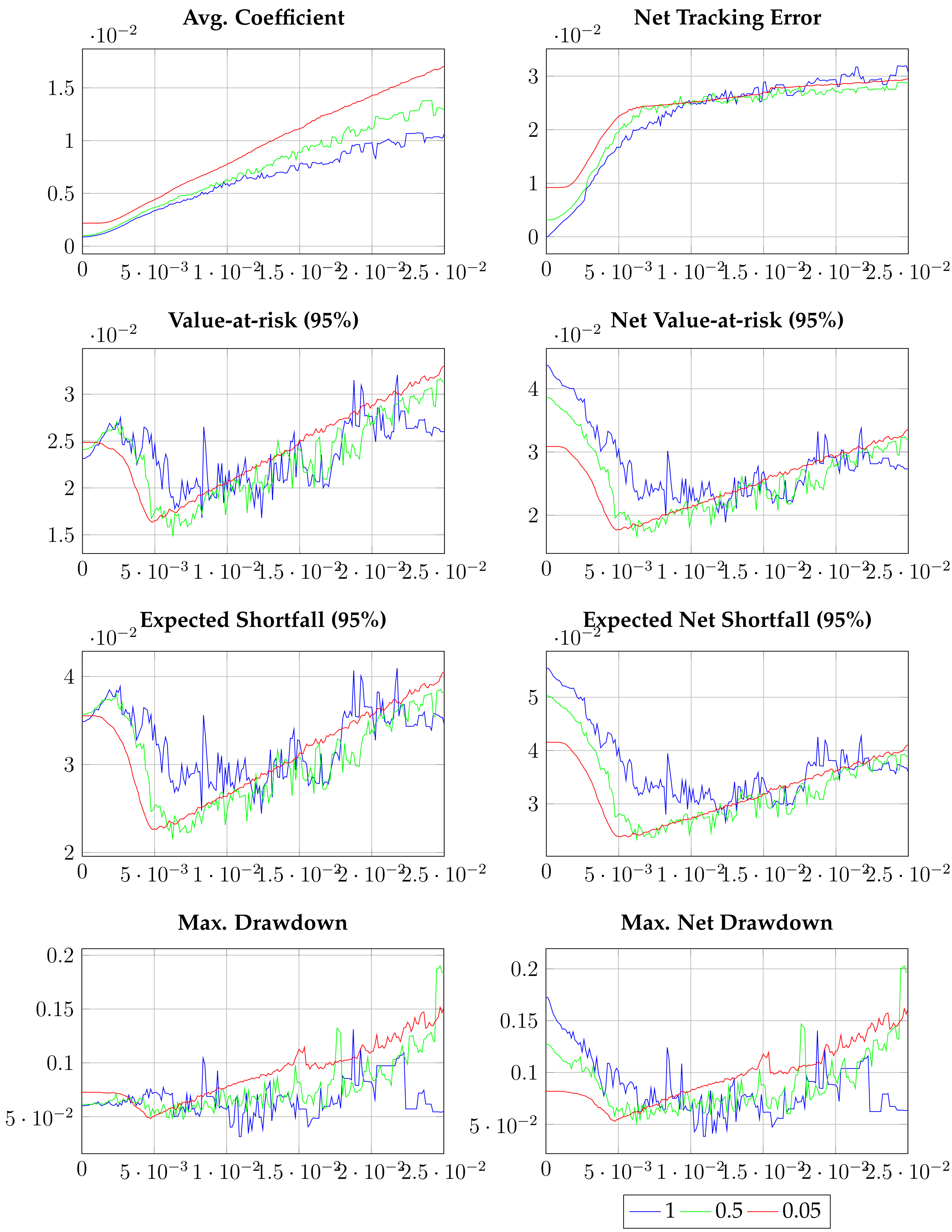

Figure 22: Rebalancing Performance Metrics: Chebyshev Distance (Pt. 2)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. Three lines in each graph represent partial rebalancing proportions of 1, 0.5, and 0.05.

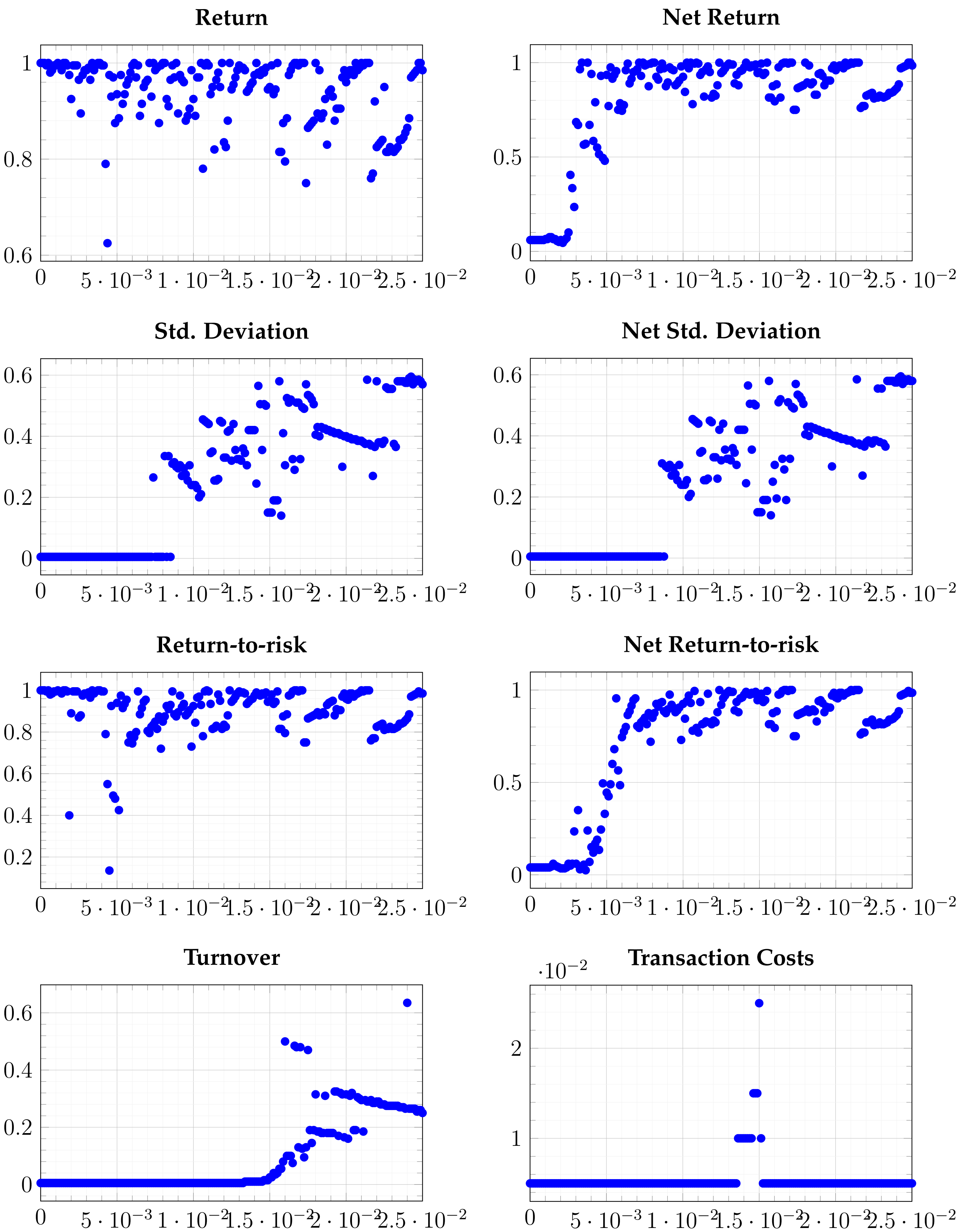

Figure 23: Optimal Proportion and Chebyshev Distance Relationship (Pt. 1)

The graphs show how rebalancing period and optimal proportion values relate. The x-axis represents the period, and the y-axis shows the proportion optimizing a given metric, such as maximizing return or minimizing standard deviation.

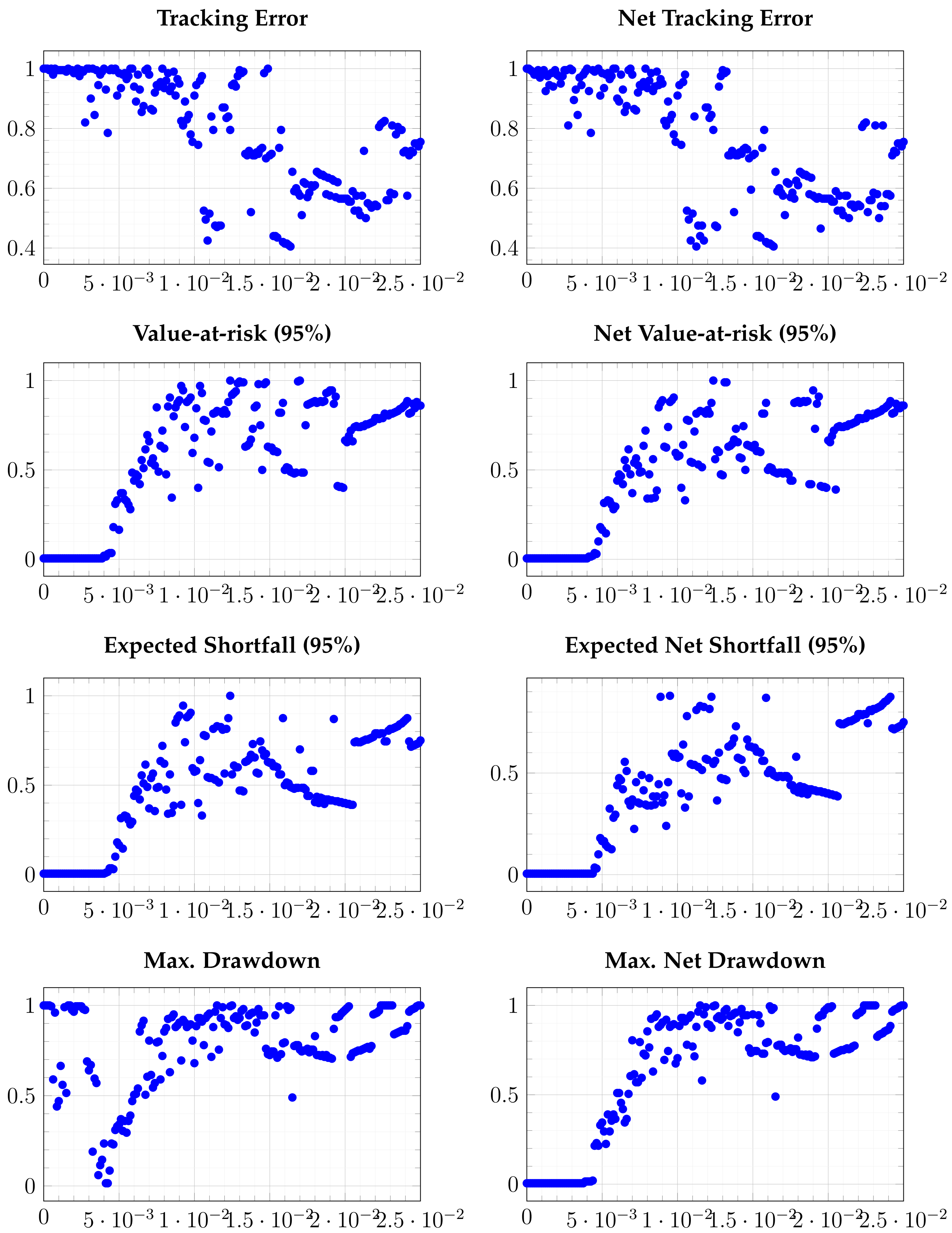

Figure 24: Optimal Proportion and Chebyshev Distance Relationship (Pt. 2)

The graphs show how rebalancing period and optimal proportion values relate. The x-axis represents the period, and the y-axis shows the proportion optimizing a given metric, such as maximizing return or minimizing standard deviation.

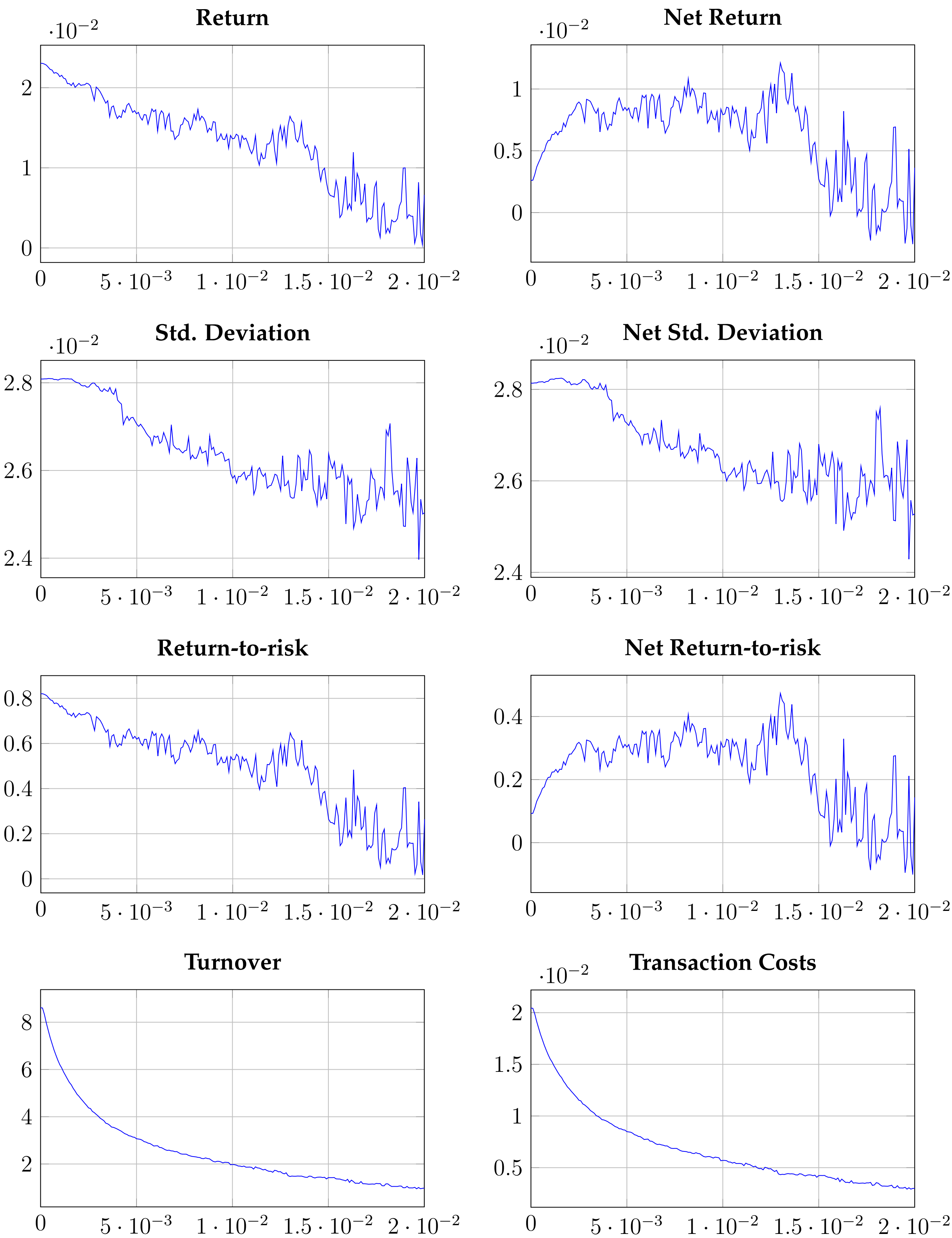

Figure 25: Rebalancing Performance Metrics: Transfer Coefficient (Pt. 1)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. The partial rebalancing proportion presented is 1.

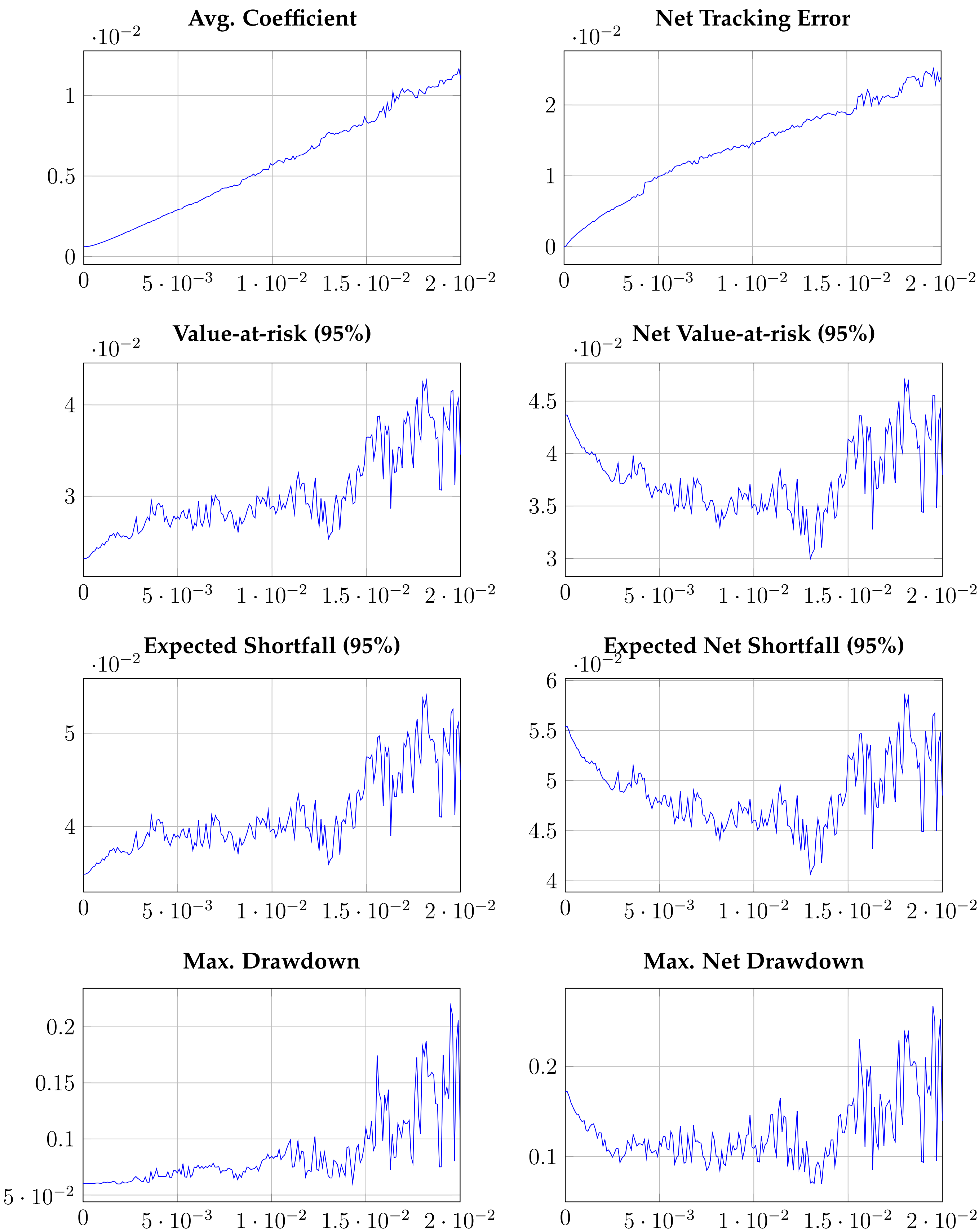

Figure 26: Rebalancing Performance Metrics: Transfer Coefficient (Pt. 2)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. The partial rebalancing proportion presented is 1.

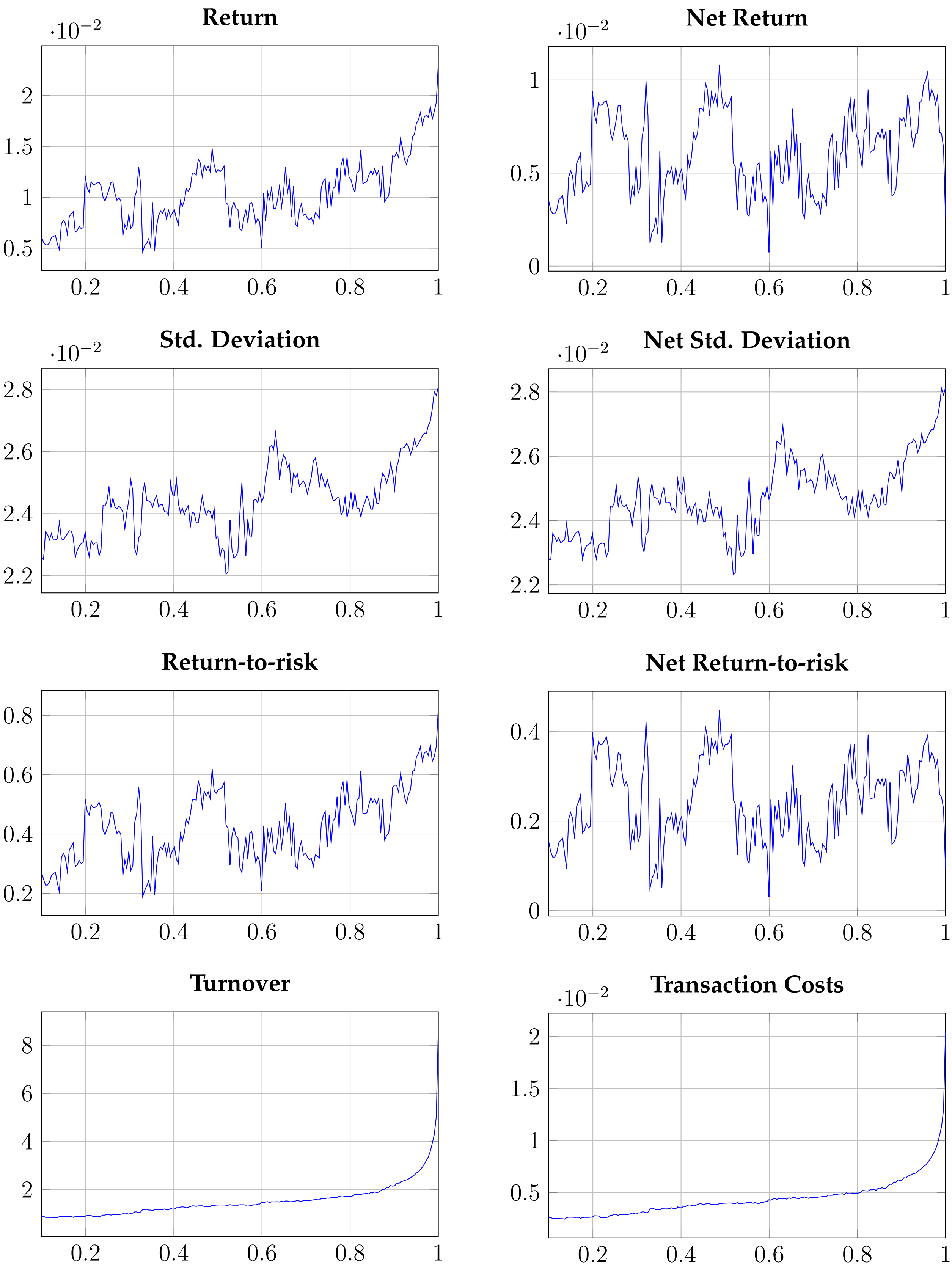

Figure 27: Rebalancing Performance Metrics: Tracking Error (Pt. 1)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. The partial rebalancing proportion presented is 1.

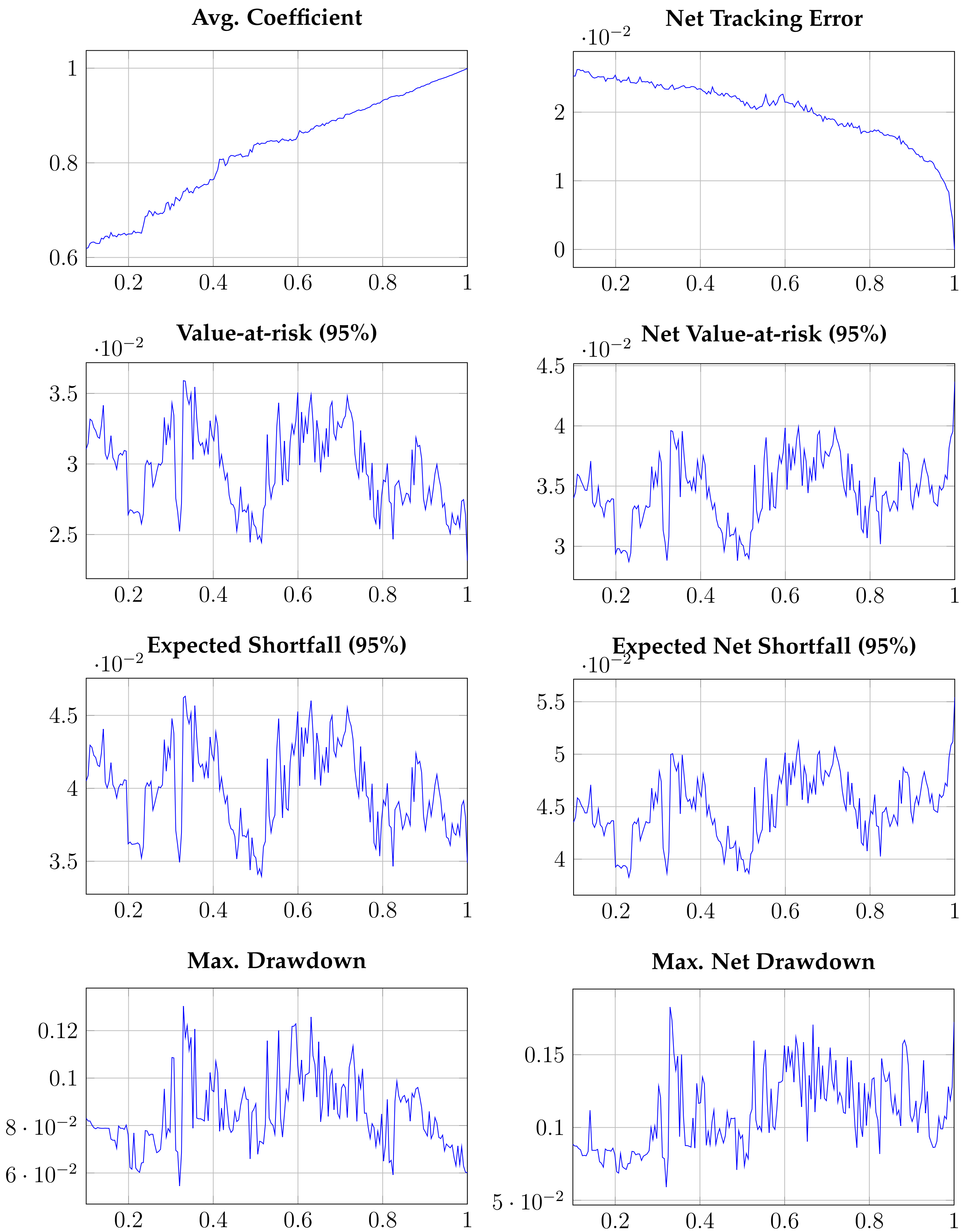

Figure 28: Rebalancing Performance Metrics: Tracking Error (Pt. 2)

Graph titles specify the y-axis metric, while the x-axis shows rebalancing period. The partial rebalancing proportion presented is 1.

Period	Target	Return	StdDev	Ret2Risk	Turnover	VaR95	CVaR95	MaxDrawd	TrackErr	NumRebal
1	0.05	0.0172	0.0256	0.673	2.7771	0.0248	0.0355	0.0727	0.0091	6953
1	0.25	0.0205	0.0275	0.7428	5.0587	0.0248	0.0363	0.0631	0.005	6953
1	0.5	0.0216	0.0278	0.777	6.3173	0.0241	0.0357	0.0614	0.003	6953
1	0.75	0.0224	0.0279	0.8013	7.3766	0.0236	0.0352	0.0607	0.0014	6953
1	1	0.023	0.0281	0.8206	8.6142	0.0232	0.0349	0.0601	0	6953
5	0.05	0.0107	0.0202	0.5322	1.2777	0.0224	0.0309	0.0811	0.0162	1390
5	0.25	0.0175	0.0258	0.6784	2.9109	0.0249	0.0357	0.0731	0.0089	1390
5	0.5	0.0193	0.027	0.7148	4.0314	0.0251	0.0364	0.0675	0.0072	1390
5	0.75	0.0201	0.0276	0.7264	4.8999	0.0254	0.0369	0.0656	0.0069	1390
5	1	0.0206	0.028	0.7351	5.7638	0.0255	0.0371	0.0642	0.007	1390
21	0.05	0.0038	0.0152	0.2497	0.4975	0.0211	0.0275	0.0772	0.0234	331
21	0.25	0.0121	0.0215	0.5612	1.4993	0.0233	0.0323	0.0773	0.0149	331
21	0.5	0.0154	0.0247	0.6243	2.2797	0.0252	0.0355	0.0746	0.0116	331
21	0.75	0.0171	0.0261	0.6547	2.9876	0.0259	0.0368	0.0725	0.0102	331
21	1	0.0181	0.027	0.6714	3.7809	0.0263	0.0376	0.0715	0.0097	331
63	0.05	-0.0021	0.0196	-0.1054	0.2296	0.0343	0.0425	0.1481	0.0296	110
63	0.25	0.0052	0.017	0.304	0.764	0.0228	0.0299	0.08	0.0205	110
63	0.5	0.0085	0.0212	0.3986	1.3069	0.0264	0.0353	0.0953	0.0167	110
63	0.75	0.0109	0.0246	0.4424	1.797	0.0295	0.0398	0.0967	0.0151	110
63	1	0.0129	0.0274	0.4719	2.3307	0.0321	0.0435	0.1003	0.0153	110
250	0.05	-0.0044	0.0319	-0.1374	0.1034	0.0568	0.0702	0.2163	0.0396	27
250	0.25	-0.0015	0.0185	-0.0803	0.285	0.032	0.0397	0.1338	0.0282	27
250	0.5	0.0034	0.0165	0.2055	0.5218	0.0237	0.0306	0.0822	0.0241	27
250	0.75	0.0066	0.0191	0.3439	0.7952	0.0248	0.0328	0.0881	0.023	27
250	1	0.009	0.0231	0.389	1.1329	0.029	0.0387	0.1017	0.0236	27

Table 3: Rebalancing Performance Metrics: Calendar-based

Period	Target	NetReturn	NetStdDev	NetRet2Risk	TrCosts	NetVaR95	CNetVaR95	MaxNetDrawd	NetTrackErr	AvgTCost
1	0.05	0.0112	0.0256	0.4375	0.006	0.0309	0.0416	0.0819	0.0092	0.0022
1	0.25	0.0091	0.0275	0.3307	0.0113	0.0362	0.0477	0.105	0.0051	0.0022
1	0.5	0.0071	0.0278	0.2561	0.0145	0.0386	0.0502	0.1271	0.0032	0.0023
1	0.75	0.0052	0.028	0.1853	0.0172	0.0408	0.0525	0.1466	0.0015	0.0023
1	1	0.0026	0.0281	0.0925	0.0204	0.0437	0.0554	0.1725	0	0.0024
5	0.05	0.0079	0.0202	0.3919	0.0028	0.0253	0.0337	0.0858	0.0163	0.0022
5	0.25	0.0107	0.0258	0.4149	0.0068	0.0318	0.0426	0.0844	0.0091	0.0023
5	0.5	0.0097	0.0271	0.3564	0.0097	0.0349	0.0462	0.0992	0.0074	0.0024
5	0.75	0.0081	0.0277	0.291	0.012	0.0375	0.0491	0.1163	0.0073	0.0025
5	1	0.0061	0.0281	0.218	0.0144	0.0401	0.0518	0.1372	0.0074	0.0025
21	0.05	0.0027	0.0152	0.176	0.0011	0.0223	0.0286	0.0793	0.0235	0.0022
21	0.25	0.0085	0.0215	0.393	0.0036	0.027	0.036	0.0833	0.015	0.0024
21	0.5	0.0097	0.0248	0.3916	0.0057	0.031	0.0414	0.0839	0.0118	0.0025
21	0.75	0.0094	0.0262	0.357	0.0077	0.0338	0.0447	0.0958	0.0106	0.0026
21	1	0.0081	0.0272	0.2965	0.0101	0.0367	0.048	0.1125	0.0103	0.0027
63	0.05	-0.0026	0.0196	-0.132	0.0005	0.0348	0.043	0.1509	0.0297	0.0023
63	0.25	0.0033	0.017	0.1929	0.0019	0.0247	0.0318	0.0836	0.0206	0.0025
63	0.5	0.005	0.0213	0.2373	0.0034	0.0299	0.0388	0.1041	0.0168	0.0026
63	0.75	0.006	0.0246	0.2427	0.0049	0.0346	0.0449	0.1278	0.0154	0.0027
63	1	0.0063	0.0275	0.2305	0.0066	0.0389	0.0504	0.1424	0.0157	0.0028
250	0.05	-0.0046	0.0319	-0.1448	0.0002	0.0571	0.0704	0.2173	0.0396	0.0023
250	0.25	-0.0022	0.0185	-0.1186	0.0007	0.0327	0.0405	0.1386	0.0283	0.0025
250	0.5	0.002	0.0165	0.1207	0.0014	0.0252	0.0321	0.0881	0.0242	0.0027
250	0.75	0.0043	0.0192	0.2259	0.0022	0.0273	0.0353	0.1	0.0232	0.0028
250	1	0.0057	0.0234	0.2431	0.0033	0.0328	0.0426	0.1197	0.0239	0.0029

Table 4: Rebalancing Net Performance Metrics: Calendar-based

The tables above display the results of calendar-based rebalancing for the model portfolio. On the left-hand-side of each graph is the rebalancing period and partial rebalancing proportion assumed for each strategy while the metrics are on the right. The top table presents gross performance metrics, while the bottom table includes transaction costs. It's worth noting that a proportion of 0 is not depicted because its performance remains unchanged, irrespective of the rebalancing period.

Table 5: Rebalancing Performance Metrics: Correlation

Threshold	Target	Return	StdDev	Ret2Risk	Turnover	VaR95	CVaR95	MaxDrawd	TrackErr	NumRebal
0.1	0.05	-0.0018	0.0255	-0.0715	0.1512	0.0438	0.0544	0.2007	0.0343	45
0.1	0.25	-0.0032	0.0227	-0.139	0.166	0.0405	0.05	0.1737	0.0321	11
0.1	0.5	-0.0018	0.0218	-0.0812	0.2423	0.0376	0.0467	0.1417	0.0308	8
0.1	0.75	-0.0079	0.0274	-0.2886	0.3482	0.053	0.0644	0.3079	0.0356	8
0.1	1	-0.0076	0.025	-0.3022	0.48	0.0488	0.0592	0.3052	0.0325	8
0.325	0.05	0.0008	0.0153	0.0555	0.2651	0.0243	0.0306	0.0919	0.0265	132
0.325	0.25	0.0016	0.0149	0.1074	0.3454	0.023	0.0292	0.0924	0.0251	35
0.325	0.5	0.0003	0.0178	0.0141	0.546	0.0291	0.0366	0.1323	0.0245	28
0.325	0.75	0.0059	0.0186	0.3189	0.7386	0.0246	0.0324	0.0728	0.0232	24
0.325	1	0.0068	0.0257	0.2638	1.1262	0.0355	0.0462	0.1266	0.0253	27
0.55	0.05	0.0043	0.0146	0.293	0.4872	0.0197	0.0258	0.0684	0.0229	320
0.55	0.25	0.0056	0.0162	0.3438	0.6527	0.0211	0.0278	0.0627	0.0214	86
0.55	0.5	0.01	0.0179	0.5618	0.89	0.0193	0.0268	0.047	0.0195	58
0.55	0.75	0.0092	0.0215	0.4296	1.2217	0.0261	0.0351	0.0918	0.019	52
0.55	1	0.0103	0.025	0.4131	1.6	0.0308	0.0412	0.1127	0.0191	50
0.775	0.05	0.0099	0.0184	0.5389	1.0395	0.0203	0.028	0.0662	0.0176	978
0.775	0.25	0.0109	0.0199	0.547	1.2455	0.0219	0.0302	0.0689	0.0161	236
0.775	0.5	0.0121	0.0221	0.545	1.5275	0.0243	0.0336	0.0691	0.0148	144
0.775	0.75	0.0119	0.0239	0.4958	1.883	0.0275	0.0375	0.0778	0.0137	119
0.775	1	0.0183	0.0259	0.7073	2.3704	0.0243	0.0351	0.0712	0.0134	111
1	0.05	0.0172	0.0256	0.673	2.7771	0.0248	0.0355	0.0727	0.0091	6953
1	0.25	0.0205	0.0275	0.7428	5.0587	0.0248	0.0363	0.0631	0.005	6953
1	0.5	0.0216	0.0278	0.777	6.3173	0.0241	0.0357	0.0614	0.003	6953
1	0.75	0.0224	0.0279	0.8013	7.3766	0.0236	0.0352	0.0607	0.0014	6953
1	1	0.023	0.0281	0.8206	8.6142	0.0232	0.0349	0.0601	0	6953

Table 6: Rebalancing Net Performance Metrics: Correlation

Threshold	Target	NetReturn	NetStdDev	NetRet2Risk	TrCosts	NetVaR95	CNetVaR95	MaxNetDrawd	NetTrackErr	AvgTCost
0.1	0.05	-0.0022	0.0255	-0.085	0.0003	0.0441	0.0548	0.2025	0.0343	0.0023
0.1	0.25	-0.0036	0.0227	-0.1578	0.0004	0.041	0.0505	0.1764	0.0321	0.0026
0.1	0.5	-0.0024	0.0218	-0.112	0.0007	0.0384	0.0475	0.1473	0.0309	0.0028
0.1	0.75	-0.0089	0.0279	-0.3204	0.001	0.0548	0.0664	0.3272	0.036	0.0029
0.1	1	-0.0091	0.0252	-0.3592	0.0015	0.0505	0.0611	0.3177	0.0327	0.0031
0.325	0.05	0.0002	0.0153	0.0161	0.0006	0.0249	0.0313	0.0941	0.0266	0.0023
0.325	0.25	0.0007	0.015	0.049	0.0009	0.0239	0.0301	0.0962	0.0252	0.0025
0.325	0.5	-0.0013	0.018	-0.0697	0.0015	0.0308	0.0383	0.1571	0.0247	0.0028
0.325	0.75	0.0038	0.0187	0.2041	0.0021	0.027	0.0348	0.081	0.0234	0.0028
0.325	1	0.0034	0.0261	0.1291	0.0034	0.0396	0.0505	0.1366	0.0258	0.003
0.55	0.05	0.0032	0.0146	0.2176	0.0011	0.0208	0.0269	0.0725	0.0229	0.0023
0.55	0.25	0.0039	0.0162	0.2432	0.0016	0.0227	0.0295	0.0685	0.0215	0.0025
0.55	0.5	0.0077	0.0179	0.4275	0.0024	0.0218	0.0293	0.0567	0.0197	0.0027
0.55	0.75	0.0058	0.0217	0.2681	0.0034	0.0298	0.0389	0.1089	0.0193	0.0028
0.55	1	0.0057	0.0253	0.2237	0.0047	0.036	0.0466	0.1464	0.0197	0.0029
0.775	0.05	0.0076	0.0184	0.413	0.0023	0.0226	0.0303	0.0707	0.0177	0.0022
0.775	0.25	0.0079	0.0199	0.3941	0.003	0.0249	0.0333	0.0829	0.0162	0.0024
0.775	0.5	0.0081	0.0222	0.3645	0.004	0.0284	0.0377	0.0932	0.015	0.0026
0.775	0.75	0.0067	0.024	0.2806	0.0051	0.0328	0.0428	0.1228	0.014	0.0027
0.775	1	0.0116	0.0262	0.445	0.0067	0.0314	0.0423	0.0998	0.0139	0.0028
1	0.05	0.0112	0.0256	0.4375	0.006	0.0309	0.0416	0.0819	0.0092	0.0022
1	0.25	0.0091	0.0275	0.3307	0.0113	0.0362	0.0477	0.105	0.0051	0.0022
1	0.5	0.0071	0.0278	0.2561	0.0145	0.0386	0.0502	0.1271	0.0032	0.0023
1	0.75	0.0052	0.028	0.1853	0.0172	0.0408	0.0525	0.1466	0.0015	0.0023
1	1	0.0026	0.0281	0.0925	0.0204	0.0437	0.0554	0.1725	0	0.0024

The tables above display the results of threshold-based rebalancing using correlation as the similarity measure, for the model portfolio. On the left-hand-side of each graph is the rebalancing threshold and partial rebalancing proportion assumed for each strategy while the metrics are on the right. The top table presents gross performance metrics, while the bottom table includes transaction costs.

Threshold	Target	Return	StdDev	Ret2Risk	Turnover	VaR95	CVaR95	MaxDrawd	TrackErr	NumRebal
0	0.05	0.0172	0.0256	0.673	2.7771	0.0248	0.0355	0.0727	0.0091	6953
0	0.25	0.0205	0.0275	0.7428	5.0587	0.0248	0.0363	0.0631	0.005	6953
0	0.5	0.0216	0.0278	0.777	6.3173	0.0241	0.0357	0.0614	0.003	6953
0	0.75	0.0224	0.0279	0.8013	7.3766	0.0236	0.0352	0.0607	0.0014	6953
0	1	0.023	0.0281	0.8206	8.6142	0.0232	0.0349	0.0601	0	6953
0.025	0.05	0.0077	0.0164	0.4683	0.7562	0.0193	0.0261	0.0553	0.0202	601
0.025	0.25	0.0084	0.0183	0.4623	0.9654	0.0216	0.0292	0.0685	0.0182	156
0.025	0.5	0.0103	0.0202	0.5114	1.2867	0.0229	0.0313	0.0733	0.017	107
0.025	0.75	0.0111	0.0234	0.4733	1.6502	0.0274	0.0371	0.0892	0.0154	93
0.025	1	0.0147	0.0249	0.5881	2.1586	0.0263	0.0367	0.0727	0.0143	93
0.05	0.05	-0.0019	0.0174	-0.1068	0.205	0.0305	0.0378	0.1225	0.0288	86
0.05	0.25	-0.0009	0.0163	-0.0549	0.2345	0.0277	0.0345	0.1091	0.0277	20
0.05	0.5	0.0006	0.0169	0.0384	0.2772	0.0271	0.0342	0.1086	0.0273	11
0.05	0.75	0.0041	0.0166	0.2454	0.3749	0.0233	0.0302	0.0861	0.0265	10
0.05	1	0.0083	0.0195	0.424	0.5869	0.0238	0.032	0.0701	0.0252	12
0.075	0.05	-0.0046	0.0234	-0.1984	0.1794	0.0432	0.0529	0.1853	0.0328	58
0.075	0.25	-0.0065	0.0221	-0.2932	0.1912	0.0429	0.0522	0.2248	0.0322	12
0.075	0.5	-0.0044	0.0173	-0.2517	0.2032	0.0328	0.0401	0.1505	0.0303	6
0.075	0.75	0.0038	0.0211	0.1795	0.25	0.031	0.0398	0.1185	0.0316	5
0.075	1	0.0001	0.0201	0.0071	0.3811	0.0329	0.0413	0.0961	0.0301	6
0.1	0.05	-0.0047	0.0291	-0.162	0.1732	0.0526	0.0648	0.2284	0.0369	47
0.1	0.25	-0.0052	0.0259	-0.2001	0.1733	0.0478	0.0586	0.2182	0.0344	9
0.1	0.5	-0.0029	0.0247	-0.1168	0.1982	0.0435	0.0538	0.1981	0.0331	5
0.1	0.75	-0.0098	0.0242	-0.4065	0.2229	0.0497	0.0598	0.3056	0.0338	4
0.1	1	-0.0021	0.0232	-0.0927	0.2802	0.0403	0.0499	0.1808	0.0319	4

Table 7: Rebalancing Performance Metrics: Euclidean Distance

Threshold	Target	NetReturn	NetStdDev	NetRet2Risk	TrCosts	NetVaR95	CNetVaR95	MaxNetDrawd	NetTrackErr	AvgTCost
0	0.05	0.0112	0.0256	0.4375	0.006	0.0309	0.0416	0.0819	0.0092	0.0022
0	0.25	0.0091	0.0275	0.3307	0.0113	0.0362	0.0477	0.105	0.0051	0.0022
0	0.5	0.0071	0.0278	0.2561	0.0145	0.0386	0.0502	0.1271	0.0032	0.0023
0	0.75	0.0052	0.028	0.1853	0.0172	0.0408	0.0525	0.1466	0.0015	0.0023
0	1	0.0026	0.0281	0.0925	0.0204	0.0437	0.0554	0.1725	0	0.0024
0.025	0.05	0.006	0.0164	0.3649	0.0017	0.021	0.0278	0.0621	0.0203	0.0022
0.025	0.25	0.0061	0.0183	0.3319	0.0024	0.024	0.0317	0.073	0.0183	0.0025
0.025	0.5	0.007	0.0202	0.3441	0.0034	0.0263	0.0348	0.0922	0.0172	0.0026
0.025	0.75	0.0066	0.0236	0.2782	0.0045	0.0322	0.042	0.1203	0.0157	0.0027
0.025	1	0.0085	0.0251	0.3391	0.0061	0.0328	0.0433	0.1086	0.0146	0.0028
0.05	0.05	-0.0023	0.0174	-0.1329	0.0005	0.031	0.0383	0.1261	0.0289	0.0022
0.05	0.25	-0.0015	0.0163	-0.09	0.0006	0.0283	0.0352	0.1133	0.0278	0.0025
0.05	0.5	-0.0001	0.017	-0.005	0.0007	0.028	0.0351	0.1131	0.0274	0.0026
0.05	0.75	0.0031	0.0167	0.183	0.001	0.0245	0.0315	0.0919	0.0266	0.0027
0.05	1	0.0066	0.0197	0.3346	0.0017	0.0258	0.034	0.0794	0.0254	0.0029
0.075	0.05	-0.005	0.0234	-0.2152	0.0004	0.0436	0.0534	0.1881	0.0329	0.0022
0.075	0.25	-0.007	0.0222	-0.3134	0.0005	0.0435	0.0528	0.2369	0.0323	0.0025
0.075	0.5	-0.0049	0.0174	-0.2814	0.0005	0.0336	0.0408	0.1567	0.0304	0.0027
0.075	0.75	0.0031	0.0212	0.1468	0.0007	0.0318	0.0407	0.1235	0.0317	0.0027
0.075	1	-0.001	0.0204	-0.0476	0.0011	0.0345	0.043	0.1047	0.0303	0.0029
0.1	0.05	-0.0051	0.0292	-0.175	0.0004	0.0531	0.0652	0.2301	0.037	0.0022
0.1	0.25	-0.0056	0.026	-0.2162	0.0004	0.0483	0.0592	0.2211	0.0346	0.0025
0.1	0.5	-0.0034	0.0248	-0.1374	0.0005	0.0441	0.0545	0.2017	0.0332	0.0026
0.1	0.75	-0.0105	0.0244	-0.4295	0.0006	0.0506	0.0608	0.3221	0.034	0.0029
0.1	1	-0.003	0.0234	-0.1272	0.0008	0.0414	0.0512	0.1889	0.0321	0.003

Table 8: Rebalancing Net Performance Metrics: Euclidean Distance

The tables above display the results of threshold-based rebalancing using Euclidean distance as the distance measure, for the model portfolio. On the left-hand-side of each graph is the rebalancing threshold and partial rebalancing proportion assumed for each strategy while the metrics are on the right. The top table presents gross performance metrics, while the bottom table includes transaction costs.

Threshold	Target	Return	StdDev	Ret2Risk	Turnover	VaR95	CVaR95	MaxDrawd	TrackErr	NumRebal
0	0.05	0.0172	0.0256	0.673	2.7771	0.0248	0.0355	0.0727	0.0091	6953
0	0.25	0.0205	0.0275	0.7428	5.0587	0.0248	0.0363	0.0631	0.005	6953
0	0.5	0.0216	0.0278	0.777	6.3173	0.0241	0.0357	0.0614	0.003	6953
0	0.75	0.0224	0.0279	0.8013	7.3766	0.0236	0.0352	0.0607	0.0014	6953
0	1	0.023	0.0281	0.8206	8.6142	0.0232	0.0349	0.0601	0	6953
0.00625	0.05	0.0056	0.014	0.3958	0.383	0.0175	0.0234	0.0584	0.0242	270
0.00625	0.25	0.0068	0.0144	0.4711	0.4636	0.0169	0.0229	0.0515	0.0234	63
0.00625	0.5	0.0115	0.016	0.7183	0.6166	0.0148	0.0215	0.0487	0.0225	40
0.00625	0.75	0.0147	0.0188	0.7821	1.0534	0.0162	0.024	0.0499	0.02	47
0.00625	1	0.0167	0.0217	0.7667	1.4464	0.0191	0.0282	0.0717	0.0195	50
0.0125	0.05	0.0013	0.0146	0.0886	0.2954	0.0227	0.0288	0.089	0.0258	174
0.0125	0.25	0.0015	0.0144	0.1048	0.3044	0.0222	0.0282	0.0839	0.0253	33
0.0125	0.5	0.0049	0.0149	0.3286	0.3456	0.0196	0.0258	0.0572	0.0248	17
0.0125	0.75	0.0059	0.0163	0.3613	0.4246	0.0209	0.0277	0.0694	0.0249	13
0.0125	1	0.0096	0.0179	0.5369	0.6055	0.0199	0.0274	0.038	0.025	13
0.01875	0.05	-0.0012	0.0164	-0.0717	0.2354	0.0282	0.035	0.1052	0.0283	115
0.01875	0.25	0.0001	0.0158	0.0057	0.2436	0.026	0.0326	0.0979	0.0278	22
0.01875	0.5	0.0025	0.0163	0.1536	0.3028	0.0244	0.0312	0.0789	0.0265	13
0.01875	0.75	0.004	0.0171	0.2359	0.3015	0.0241	0.0313	0.1006	0.0286	7
0.01875	1	0.0047	0.022	0.215	0.4476	0.0315	0.0407	0.131	0.0294	8
0.025	0.05	-0.0037	0.0179	-0.2053	0.217	0.033	0.0405	0.1509	0.0294	94
0.025	0.25	-0.0004	0.0174	-0.0239	0.2094	0.029	0.0362	0.1012	0.0291	16
0.025	0.5	-0.0044	0.0164	-0.2705	0.2382	0.0313	0.0382	0.1829	0.0286	9
0.025	0.75	0.004	0.0168	0.2388	0.3031	0.0237	0.0307	0.0697	0.0269	7
0.025	1	0.0078	0.0206	0.3814	0.3556	0.026	0.0346	0.0549	0.0305	6

Table 9: Rebalancing Performance Metrics: Chebyshev Distance

Threshold	Target	NetReturn	NetStdDev	NetRet2Risk	TrCosts	NetVaR95	CNetVaR95	MaxNetDrawd	NetTrackErr	AvgTCost
0	0.05	0.0112	0.0256	0.4375	0.006	0.0309	0.0416	0.0819	0.0092	0.0022
0	0.25	0.0091	0.0275	0.3307	0.0113	0.0362	0.0477	0.105	0.0051	0.0022
0	0.5	0.0071	0.0278	0.2561	0.0145	0.0386	0.0502	0.1271	0.0032	0.0023
0	0.75	0.0052	0.028	0.1853	0.0172	0.0408	0.0525	0.1466	0.0015	0.0023
0	1	0.0026	0.0281	0.0925	0.0204	0.0437	0.0554	0.1725	0	0.0024
0.00625	0.05	0.0047	0.014	0.3339	0.0009	0.0184	0.0243	0.062	0.0242	0.0023
0.00625	0.25	0.0056	0.0144	0.3886	0.0012	0.0181	0.0242	0.0562	0.0235	0.0025
0.00625	0.5	0.0098	0.0161	0.6095	0.0017	0.0167	0.0234	0.0515	0.0226	0.0027
0.00625	0.75	0.0117	0.019	0.6175	0.0029	0.0195	0.0275	0.06	0.0203	0.0028
0.00625	1	0.0125	0.0221	0.566	0.0041	0.0239	0.0331	0.0866	0.0199	0.0029
0.0125	0.05	0.0006	0.0146	0.043	0.0007	0.0235	0.0296	0.0939	0.0259	0.0022
0.0125	0.25	0.0007	0.0145	0.0516	0.0008	0.023	0.0291	0.0892	0.0254	0.0025
0.0125	0.5	0.004	0.015	0.2644	0.0009	0.0206	0.0269	0.0612	0.0249	0.0027
0.0125	0.75	0.0047	0.0165	0.2847	0.0012	0.0224	0.0293	0.0736	0.0251	0.0028
0.0125	1	0.0079	0.0181	0.4341	0.0017	0.022	0.0296	0.0443	0.0252	0.0029
0.01875	0.05	-0.0017	0.0164	-0.1035	0.0005	0.0287	0.0356	0.1098	0.0284	0.0022
0.01875	0.25	-0.0005	0.0159	-0.0318	0.0006	0.0267	0.0333	0.0993	0.0279	0.0024
0.01875	0.5	0.0017	0.0164	0.103	0.0008	0.0253	0.0322	0.0846	0.0266	0.0027
0.01875	0.75	0.0032	0.0173	0.1824	0.0009	0.0253	0.0325	0.1051	0.0287	0.003
0.01875	1	0.0034	0.0223	0.1517	0.0014	0.0332	0.0425	0.1404	0.0296	0.003
0.025	0.05	-0.0042	0.0179	-0.232	0.0005	0.0336	0.0411	0.161	0.0295	0.0022
0.025	0.25	-0.0009	0.0174	-0.0535	0.0005	0.0296	0.0368	0.1027	0.0292	0.0025
0.025	0.5	-0.0051	0.0164	-0.3078	0.0006	0.0321	0.0389	0.1954	0.0287	0.0026
0.025	0.75	0.0032	0.0169	0.1864	0.0009	0.0247	0.0318	0.0768	0.027	0.0028
0.025	1	0.0068	0.0207	0.3264	0.0011	0.0273	0.036	0.0633	0.0307	0.0031

Table 10: Rebalancing Net Performance Metrics: Chebyshev Distance

The tables above display the results of threshold-based rebalancing using Chebyshev distance as the distance measure, for the model portfolio. On the left-hand-side of each graph is the rebalancing threshold and partial rebalancing proportion assumed for each strategy while the metrics are on the right. The top table presents gross performance metrics, while the bottom table includes transaction costs.

Threshold	Target	Return	StdDev	Ret2Risk	Turnover	VaR95	CVaR95	MaxDrawd	TrackErr	NumRebal
0	1	0.023	0.0281	0.8206	8.6142	0.0232	0.0349	0.0601	0	6953
0.005	1	0.0167	0.0271	0.6189	3.0679	0.0278	0.0391	0.0697	0.0093	262
0.01	1	0.0136	0.0258	0.5276	1.9781	0.0288	0.0396	0.0847	0.0143	94
0.015	1	0.007	0.0264	0.2636	1.4236	0.0364	0.0475	0.1098	0.0181	47
0.02	1	0.0066	0.025	0.2646	0.9826	0.0346	0.045	0.1096	0.0237	25

Table 11: Rebalancing Performance Metrics: Transfer Coefficient

Threshold	Target	NetReturn	NetStdDev	NetRet2Risk	TrCosts	NetVaR95	CNetVaR95	MaxNetDrawd	NetTrackErr	AvgTCost
0	1	0.0026	0.0281	0.0925	0.0204	0.0437	0.0554	0.1725	0	0.0024
0.005	1	0.0083	0.0273	0.3029	0.0085	0.0366	0.048	0.1163	0.01	0.0028
0.01	1	0.0079	0.0262	0.3025	0.0057	0.0351	0.0461	0.1093	0.0147	0.0029
0.015	1	0.0027	0.0268	0.1016	0.0042	0.0414	0.0526	0.1572	0.0187	0.003
0.02	1	0.0036	0.0253	0.1436	0.003	0.038	0.0485	0.1397	0.0239	0.003

Table 12: Rebalancing Net Performance Metrics: Transfer Coefficient

The tables above display the results of threshold-based rebalancing using the transfer coefficient as the similarity measure, for the model portfolio. On the left-hand-side of each graph is the rebalancing threshold and partial rebalancing proportion assumed for each strategy while the metrics are on the right. The top table presents gross performance metrics, while the bottom table includes transaction costs.

Threshold	Target	Return	StdDev	Ret2Risk	Turnover	VaR95	CVaR95	MaxDrawd	TrackErr	NumRebal
0.1	1	0.0061	0.0226	0.2699	0.897	0.0311	0.0405	0.0831	0.0251	20
0.325	1	0.0112	0.0233	0.4799	1.0784	0.0272	0.0369	0.0683	0.023	26
0.55	1	0.0069	0.024	0.2867	1.3761	0.0326	0.0427	0.0966	0.021	37
0.775	1	0.0103	0.0245	0.4182	1.7255	0.0301	0.0403	0.0808	0.0175	60
1	1	0.023	0.0281	0.8206	8.6142	0.0232	0.0349	0.0601	0	6953

Table 13: Rebalancing Performance Metrics: Tracking Error

Threshold	Target	NetReturn	NetStdDev	NetRet2Risk	TrCosts	NetVaR95	CNetVaR95	MaxNetDrawd	NetTrackErr	AvgTCost
0.1	1	0.0035	0.0228	0.1536	0.0026	0.034	0.0435	0.0884	0.0253	0.0029
0.325	1	0.0081	0.0236	0.3419	0.0031	0.0308	0.0407	0.0802	0.0233	0.0029
0.55	1	0.0028	0.0244	0.115	0.0041	0.0373	0.0475	0.1406	0.0215	0.003
0.775	1	0.0053	0.0248	0.2128	0.005	0.0355	0.0458	0.131	0.018	0.0029
1	1	0.0026	0.0281	0.0925	0.0204	0.0437	0.0554	0.1725	0	0.0024

Table 14: Rebalancing Net Performance Metrics: Tracking Error

The tables above display the results of threshold-based rebalancing using tracking error as the similarity measure, for the model portfolio. On the left-hand-side of each graph is the rebalancing threshold and partial rebalancing proportion assumed for each strategy while the metrics are on the right. The top table presents gross performance metrics, while the bottom table includes transaction costs.

Proportion	Indep. Var.	R^2	F-test	alpha	x	p-val (alpha)	p-val (x)	BP	BG
0.05	Period	0.49524	0	0.00261	-0.00003	0.00001	0	0	0
0.05	Euclidean Distance	0.53768	0	0.00564	-0.13127	0	0	0	0
0.05	Chebyshev Distance	0.47422	0	0.00323	-0.24214	0	0	0	0
0.05	Correlation	0.91069	0	-0.00678	0.01883	0	0	0	0
0.5	Period	0.76471	0	0.01511	-0.00005	0	0	0.17202	0
0.5	Euclidean Distance	0.90532	0	0.02303	-0.62887	0	0	0	0
0.5	Chebyshev Distance	0.87012	0	0.02081	-2.43553	0	0	0.15599	0
0.5	Correlation	0.86114	0	-0.01379	0.03114	0	0	0.35211	0
1	Period	0.58321	0	0.01837	-0.00005	0	0	0	0
1	Euclidean Distance	0.68336	0	0.02275	-0.49601	0	0	0	0
1	Chebyshev Distance	0.78387	0	0.02418	-2.46656	0	0	0	0
1	Correlation	0.62653	0	-0.01127	0.03035	0.00012	0	0	0

Table 15: Linear Regression Summary Statistics

Proportion	Indep. Var.	R^2	F-test	alpha	x	x^2	x^3	p-val (alpha)	p-val (x)	p-val (x^2)	p-val (x^3)	BP	BG
0.05	Period	0.91616	0	0.01028	-0.00029	0	0	0	0	0	0	0	0
0.05	Euclidean Distance	0.96715	0	0.03224	-1.54662	21.79366	-100.54456	0	0	0	0	0	0
0.05	Chebyshev Distance	0.94916	0	0.01832	-2.74247	108.90744	-1379.54606	0	0	0	0	0	0
0.05	Correlation	0.97434	0	-0.0035	-0.00637	0.04009	-0.01308	0	0.08502	0	0.00469	0.00001	0
0.5	Period	0.87575	0	0.0195	-0.00019	0	0	0	0	0.00012	0.01517	0.00003	0
0.5	Euclidean Distance	0.9058	0	0.02411	-0.86791	14.5298	-261.77376	0	0.00008	0.31921	0.36518	0.00001	0
0.5	Chebyshev Distance	0.93138	0	0.02967	-6.90799	613.14413	-21003.31873	0	0	0.00006	0.0275	0.0033	0
0.5	Correlation	0.89363	0	-0.07781	0.32781	-0.44488	0.21561	0.01233	0.00802	0.00574	0.00175	0.00001	0
1	Period	0.67995	0	0.02093	-0.0001	0	0	0	0.002	0.8778	0.54961	0	0
1	Euclidean Distance	0.69057	0	0.02198	-0.15217	-28.99999	639.9398	0	0.65155	0.24808	0.22503	0	0
1	Chebyshev Distance	0.78698	0	0.02691	-4.26512	342.26982	-19397.60574	0	0.00448	0.35725	0.4732	0.00001	0
1	Correlation	0.68037	0	0.12087	-0.41066	0.4723	-0.16151	0.31506	0.3691	0.40762	0.48981	0	0

Table 16: Cubic Regression Summary Statistics

The tables above report summary statistics for two regressions. The first regression is a simple linear regression with gross return as the dependent variable and either period or threshold as the independent variabe. The second regression adds to the first regression a square and cubic term of the previously sole independent variable. The tables report the rebalancing proportion, the function used, the R^2, the F-test p-value, the coefficients for the alpha (intercept) and the independent variables, the p-values for those coefficients, the Breusch-Pagan p-value, and the Breusch-Godfrey p-value. Results are reported for partial rebalancing proportions of 0.05, 0.5, and 1.