MALMQUIST INDEXES USING A GEOMETRIC DISTANCE FUNCTION (GDF)

Maria Conceição AS Portela
Universidade Católica Portuguesa, Rua Diogo Botelho, 1327, 4169-005 Porto, Portugal, csilva@porto.ucp.pt

Emmanuel Thanassoulis
Aston Business School, Aston Triangle, B4 7ET Birmingham, UK, e.thanassoulis@aston.ac.uk

ABSTRACT

Traditional approaches to calculate total factor productivity change through Malmquist indexes rely on distance functions. In this paper we show that the use of distance functions as a means to calculate total factor productivity change may introduce some bias in the analysis, and therefore we propose a procedure that calculates total factor productivity change through observed values only. Our total factor productivity change is then decomposed into efficiency change, technological change, and a residual effect. This decomposition makes use of a non-oriented measure in order to avoid problems associated with the traditional use of radial oriented measures, especially when variable returns to scale technologies are to be compared.

INTRODUCTION

Malmquist indexes using DEA effici-ency measures calculated in relation to a constant returns to scale (CRS) technology, are argued to be equivalent to a total factor productivity (TFP) index (see e.g. Färe et al. 1994, 1998). This is easily proved for single input-output technologies, but for multiple input-output technologies the calculated TFP Malmquist index has some problems. In this paper we refer in particular to problems arising from the fact that a reference technology is used relative to which technical efficiency is assessed. Therefore, we put forward a TFP measure that uses observed values only, and does not require any specifications about the technology. The proposed TFP measure is then decomposed into efficiency change, technolo-gical change, and a residual effect, which re-flects scale and allocative shifts.

In our TFP decomposition we try to resolve some problems in existing metho-dologies such as the approach of Färe et al. (1994) (FGNZ throughout) and the approach of Ray and Desli (1997) (RD throughout). Both approaches calculate the TFP Malmquist productivity index in the same way (through radial efficiency scores calculated in relation to a CRS technology), but they decompose it differently. In the FGNZ approach the techno-logical change component is calculated with reference to a CRS frontier, while in the RD approach it is calculated with reference to a VRS (variable returns to scale) frontier. The FGNZ approach has the disadvantage of not accounting for changes in the VRS technology, while the RD approach may result in some computational problems because some VRS-DEA models might be infeasible when assessments involve cross-period data (Bjurek, 1996).

Both the FGNZ and the RD appro-aches are based on radial efficiency measures, that may provide different results for some components of the Malmquist index depending on the model orientation. The use of non-oriented efficiency measures solves the pro-blem of sensitivity of the solution to the model's orientation, while at the same time solving the computational problems inherent in the RD approach.

Examples of non-oriented efficiency measures that have been used in this context are the directional distance function used by Chambers et al. (1996) and Chung et al. (1997), and the hyperbolic efficiency measure used by Zofio and Lovell (2001).

Another problem of the FGNZ and RD approaches to calculating Malmquist indexes is that they rely on radial measures and so they do not account for slacks. If slacks are important sources of inefficiency, then the resulting Malmquist indexes will be biased not fully reflecting performance. Some authors have addressed this problem and solved it through the use of non-radial efficiency measures (see for example, Grifell-Tatjé (1998) and Thrall (2000)).

The measure of efficiency that we use in this paper to decompose TFP is non-oriented and is able to account for all the sources of inefficiency, therefore avoiding the above mentioned problems. This measure was first proposed in Portela and Thanassoulis (2002) and is called Geometric Distance Function (GDF).

GEOMETRIC DISTANCE FUNCTION

Let the vector $\mathbf{x} = (x_1, \ldots, x_m) \in R^m$, correspond to inputs used to produce an output vector $\mathbf{y} = (y_1, \ldots, y_s) \in R^s$, in a technology involving *n* production units. Consider that, for each production unit, Pareto-efficient input and output levels are known and are equal to $(\mathbf{x}^*, \mathbf{y}^*) = ((x_1^*, \ldots, x_m^*), (y_1^*, \ldots, y_s^*))$. We denote these efficient levels of production by targets, as they correspond to projections of each observation on the Pareto-efficient frontier. We shall not detail at this point the procedure used to calculate such target points, but the reader may assume that they are calculated by any known DEA procedure.

The GDF as first defined in Portela and Thanassoulis (2002), assumes the form shown in (1), where θ_i represents the ratio between a target input and an observed input i (x_i^*/x_i) and β_r represents the ratio between a target output and an observed output r (y_r^*/y_r).

$$GDF = \frac{(\prod_i \theta_i)^{1/m}}{(\prod_r \beta_r)^{1/s}} \qquad (1)$$

The GDF is defined in (1) as the ratio between the geometric mean of inputs' contraction towards target levels, and the geometric mean of outputs' expansion towards target levels. As target levels can be calculated through any known procedure, the GDF is in fact a general measure that encompasses other existing efficiency measures in the literature.

When both inputs and outputs are allowed to change towards the efficient fron-tier, the GDF is a non-oriented measure that incorporates both input contraction and output expansion towards that frontier. It can also incorporate all the sources of inefficiency as long as target levels used in (1) are Pareto-efficient and so overcomes the problem of not reflecting slacks in the efficiency measure.

PROBLEMS WITH TRADITIONAL WAYS OF CALCULATING TFP

Consider the single input-output case where a measure of productivity change from period *t* to *t+1* is given by the ratio P_{t+1}/P_t, where $P_t = y_t/x_t$ in time period *t*. Using Figure 1, P_{t+1}/P_t corresponds to the distance between the rays that pass through a given observation in period *t* and *t+1*. The highest and leftmost of these rays in each time period is the CRS frontier of that period (associated with highest productivity).

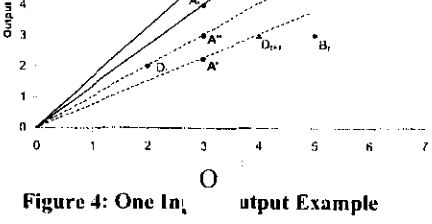

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Figure 4: One In utput Example

Obviously the distance between the rays that pass through, for example, points D_{t+1} and D_t in Figure 1 can be calculated with reference to any other ray. Taking as reference the ray through unit A_t, we find that the distance between the rays that pass through D_{t+1} and D_t is $OA'/OA'' = (OA'/OA_t)/(OA''/OA_t)$. Based on this fact, existing approaches propose the use of DEA efficiency measures defined in relation to CRS technologies to calculate productivity change indexes for the general case of multiple inputs/multiple outputs. Note, however, that productivity change is not dependent on efficiency or functional form of the efficient frontier as defined in DEA. The use of distance functions is just a means to operationalise the concept for the multiple input-output case. This approach relies, however, on efficiency being calculated in relation to a unique referent line or plane. This necessarily happens in the single input-output case as the ray presenting maximum productivity in each time period is unique. However, in the multiple input-output case, CRS technologies are defined by a cone that has multiple facets, and projections on this cone may happen on any of its facets.

To illustrate the above point, consider the example in Table 1, where 5 units produ-cing one output (y) from 2 inputs (x_1 and x_2) are considered. In this table we also show the growth in partial productivity between periods t and $t+1$ [given by the ratio $\Delta(y/x_i) = (y_{t+1}/x_{it+1}) / (y_t/x_{it})$]. Inspecting these growth ratios in Table 1, it is clear that units 1, 3 and 5 increased their productivity from t to $t+1$, while the productivity of unit 4 decreased in the same period. Note also, that unit 5 shows the highest productivity increase from t to $t+1$ since the partial productivity growth ratios are together the highest that can be found.

	Unit	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Period t	y	12	14	26	26	8
	x_1	5	16	16	17	12
	x_2	13	12	26	15	14
Period t+1	y	22	12	26	20	8
	x_1	8	12	8	15	6
	x_2	14	11	25	14	10
Growth	$\Delta(y/x_1)$	1.15	1.14	2	0.87	2
	$\Delta(y/x_2)$	1.70	0.94	1.08	0.82	1.4

Table 3: One Input Two Outputs Example

Using the approach of Färe et al., (1994) to calculate a Malmquist total factor productivity index (M_j) results in the values shown in Table 2. Note that according to this approach M_j is the geometric mean of M_j^t and M_j^{t+1}, where each value is calculated on the basis of an efficiency measure γ_{jt}^t indica-ting the radial efficiency of unit j as observed in period t and assessed in relation to the technology of period t (superscript), i.e.

$$M_j = \left(\frac{\gamma_{jt+1}^t}{\gamma_{jt}^t} \times \frac{\gamma_{jt+1}^{t+1}}{\gamma_{jt}^{t+1}} \right)^{1/2}.$$

Unit	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
γ_{jt}^t	1	0.673	0.853	1	0.398
γ_{jt+1}^{t+1}	1	0.694	1	0.909	0.509
γ_{jt}^{t+1}	0.785	0.742	0.636	1.103	0.364
γ_{jt+1}^t	1.397	0.647	1.354	0.857	0.692
M_j^t	1.397	0.961	1.588	0.857	1.74
M_j^{t+1}	1.275	0.94	1.57	0.82	1.4
M_j	1.33	0.948	1.58	0.841	1.56

Table 4: Malmquist results for illustrative example

These results show some contradi-ction to what was expected from the partial productivity ratios, especially because unit 5 does not have the highest Malmquist index as one would expect. At the same time, while it is clear that unit 4 exhibited a productivity de-crease (and the Malmquist index correctly identifies this decrease), the index suggests a productivity decline for unit 2. Yet looking at Table 1 we would suggest unit 2 had on balance a productivity increase, because the growth on $\Delta(y/x_1)$ is higher than the decline in $\Delta(y/x_2)$.

The reasons for the above counter-intuitive behaviour of the Malmquist TFP index can be better explained trough Figure 2 where observations in $t+1$ are represented by dots and observations in t are represented by crosses.

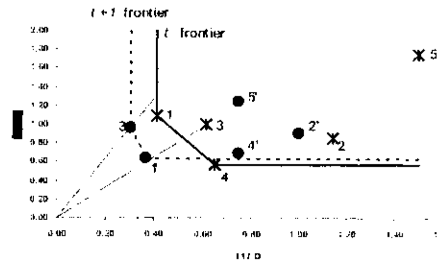

Figure 5: Two Inputs one output example

It is clear in Figure 2 that the hyperplane against which efficiency is mea-sured is not necessarily the same for an observation in t and $t+1$. For example, unit 3 as observed in t is projected on the hyperplane defined by units 1 and 4 in period t, but unit 3 observed in $t+1$ is projected on the hyperplane of the t frontier defined only by unit 1. In this facet slack in input 2 is not reflected in the efficiency measure. (Similar observations can be made about other units in Figure 2.) Note that the Malmquist index of units 2, 4, and 5 as evaluated in relation to the $t+1$ frontier (M_j^{t+1}) in Table 2 is exactly equal to the partial productivity change of input 2 $\Delta(y/x_2)$ in Table 1. This means that when productivity change for these units is evaluated in relation to the $t+1$ frontier one of the inputs (in this case input 1) is completely neglected in the analysis. Such a result is due to projections on the 'flat' part of the frontier of $t+1$ in Figure 2. that satisfy free disposability of input 1. This fact strengthens what was previously said about the importance of using efficiency measures that account for all sources of inefficiency.

In summary, the measurement of efficiency with reference to different facets of the same frontier for a given unit observed in two different time periods, causes biased results on Malmquist TFP indexes that are based on such efficiency measures. In the next section we propose a GDF based approach that attempts to solve the problems outlined.

MALMQUIST TYPE INDEXES BASED ON THE GDF

The GDF measure defined in (1) can be used to calculate a TFP index based on observed values only. This TFP measure can be decomposed into three components, namely efficiency change (EFCH), technological change (THCH), and a residual effect (RES), so that TFP = EFCH × THCH × RES. The way each one of the above terms is computed through the GDF is presented next.

Calculating TFP

The GDF as defined in (1) used efficiency measures computed with reference to observed and target input-output levels. We modify (1) as shown in (2) to measure TFP. The key change in going from (1) to (2) is that in (2) we use measures of the distance between input-output levels observed in t and $t+1$.

$$TFP(x_t, y_t, x_{t+1}, y_{t+1}) = \frac{\left(\prod_r \frac{y_{r_{t+1}}}{y_{r_t}}\right)^{1/s}}{\left(\prod_i \frac{x_{i_{t+1}}}{x_{i_t}}\right)^{1/m}} \qquad (2)$$

For the single input-output case it is easy to see that (2) corresponds to a TFP index. In the multiple input-output case the GDF is a ratio between a geometric mean of output growth and a geometric mean of input growth.

If we apply (2) to the units in Table 1 the total factor productivity values are those shown in Table 3.

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
TFP-GDF	1.3967	1.034	1.4999	0.84765	1.6733

Table 5: TFP Results for illustrative exa-mple based on the GDF

Productivity growth is identified for all units except unit 4 as expected. Note also that unit 5 offers now the highest productivity growth, exactly as one would expect from the partial productivity ratios calculated in Table 1. The GDF seems therefore a good alternative to calculate total factor productivity change, having the advantage of relying only on observed values and making, therefore, no assumptions about the technology.

Efficiency Change and Technological Change Components

The GDF in (2) used to calculate TFP is not an efficiency measure as it does not account for distances between observed and target levels but between two points observed in different time periods. However, if the GDF is to be used to calculate the efficiency change and technological change components of TFP then assumptions regarding the technological specification of the frontier are required.

Consider a measure $GDF^t(x_t, y_t)$,

as calculated through (1), representing the efficiency measure of the vector (**x**, **y**) as observed in period t and projected against technology of period t (superscript). A Malmquist type index based on the GDF is given by (3).

$$MGDF = \left(\frac{GDF^t(x_{t+1}, y_{t+1})}{GDF^t(x_t, y_t)} \times \frac{GDF^{t+1}(x_{t+1}, y_{t+1})}{GDF^{t+1}(x_t, y_t)}\right)^{1/2} \qquad (3)$$

Similarly to other approaches, this index can be decomposed in efficiency change (EFCH) and technological change (THCH) as shown in (4), where MGDF = EFCH × THCH.

$$MGDF = \frac{GDF^{t+1}(x_{t+1}, y_{t+1})}{GDF^{t}(x_t, y_t)} \times \left(\frac{GDF^{t}(x_{t+1}, y_{t+1})}{GDF^{t+1}(x_{t+1}, y_{t+1})} \times \frac{GDF^{t}(x_t, y_t)}{GDF^{t+1}(x_t, y_t)} \right)^{1/2} \quad (4)$$

As the GDF is a general measure, the above decomposition is also general and enco-mpasses as special cases other decomposition approaches in the literature. Note, however, that MGDF is not necessarily equal to TFP as it is usually measured in the literature. We consider that MGDF is simply the product of efficiency change and technological change. TFP as calculated in the previous section includes these components but may also include another component as will become clearer later.

Assume now a single input-output case, where technical efficient projections of each observation are identified by the super-script $*t$ if the projection lies on the t frontier and by $*t+1$ if the projection lies on the $t+1$ frontier. In this case, each of the MGDF components assumes the form shown in (5).

$$EFCH = \frac{\dfrac{x_{t+1}^{*t+1} / x_{t+1}}{y_{t+1}^{*t+1} / y_{t+1}}}{\dfrac{x_t^{*t} / x_t}{y_t^{*t} / y_t}},$$

$$THCH = \left(\frac{\dfrac{x_{t+1}^{*t}}{y_{t+1}^{*t}}}{\dfrac{x_{t+1}^{*t+1}}{y_{t+1}^{*t+1}}} \times \frac{\dfrac{x_t^{*t}}{y_t^{*t}}}{\dfrac{x_t^{*t+1}}{y_t^{*t+1}}} \right)^{1/2} \quad (5)$$

The EFCH component in (5) is interpreted in the usual way, i.e. when it is higher than one the efficiency of observation in $t+1$ evaluated in relation to the $t+1$ frontier (measured for the single input output case as

$$\frac{x_{t+1}^{*t+1} / x_{t+1}}{y_{t+1}^{*t+1} / y_{t+1}})$$

is higher than the efficiency of observation in t evaluated in relation to the t frontier, and therefore there was an efficiency increase from t to $t+1$. (When EFCH is lower than one there was an efficiency decrease in moving from t to $t+1$). In the same way a THCH component higher than one means technological progress and a THCH compo-nent lower than one means technological re-gress. Note that technical change may be re-organised so that we have a product of input change (ICH) and output change (OCH). That is.

$$THCH = \left(\frac{x_{t+1}^{*t}}{x_{t+1}^{*t+1}} \times \frac{x_t^{*t}}{x_t^{*t+1}} \right)^{1/2} \times \left(\frac{y_{t+1}^{*t+1}}{y_{t+1}^{*t}} \times \frac{y_t^{*t+1}}{y_t^{*t}} \right)^{1/2}$$

An input change factor greater than 1 means that the frontier at t has higher inputs than the frontier at $t+1$. That is, there was an impro-vement (decrease) in inputs in moving from t to $t+1$. If the output change is higher than 1, it means that outputs in $t+1$ are on average higher than outputs in t, which also means an improvement in outputs in moving from t to $t+1$. So progress from t to $t+1$ happens when both input and output change are greater than 1. Obviously one may have movements in different directions and in this case the resulting technological progress or regress will depend on which factor dominates the other. Note that the input and output change compo-nents of technological change are closely related to the input and output scale bias defined in Färe et al. (1997a).

In the multiple input-output case the above technological and efficiency change components are calculated as shown in (6), where again technological change is the product of input change and output change.

$$EFCH = \frac{\dfrac{(\prod_i \theta_{it+1}^{t+1})^{1/m}}{(\prod_r \beta_{rt+1}^{t+1})^{1/s}}}{\dfrac{(\prod_i \theta_{it}^{t})^{1/m}}{(\prod_r \beta_{rt}^{t})^{1/s}}},$$

$$THCH = \left(\left(\prod_i \frac{x_{it+1}^{*t}}{x_{it+1}^{*t+1}} \times \prod_i \frac{x_{it}^{*t}}{x_{it}^{*t+1}} \right)^{1/m} \right)^{1/2} \times \quad (6)$$

$$\left(\left(\prod_r \frac{y_{rt+1}^{*t+1}}{y_{rt+1}^{*t}} \times \prod_r \frac{y_{rt}^{*t+1}}{y_{rt}^{*t}} \right)^{1/s} \right)^{1/2}$$

The EFCH and THCH components in (6) include those existing in the literature, though being more general because they can handle situations where non-oriented models are used to calculate target levels. If both inputs and outputs change towards the technical efficient frontier, then the ratios considered in (6) account simultaneously for these changes. These ratios can be calculated both when targets lie on a CRS frontier or on a VRS frontier. We shall use, however, only the latter technological specification for reasons that will become clearer in the next section.

Residual Effect

The MGDF in (4) can alternatively be decomposed as shown in (7) where it equals the product of a TFP index as calculated through the GDF and a residual component that is scale related.

$$MGDF = \frac{\left(\prod_r \frac{y_{r_{t+1}}}{y_{r_t}}\right)^{1/s}}{\left(\prod_i \frac{x_{i_{t+1}}}{x_{i_t}}\right)^{1/m}} \times \left(\frac{\left(\prod_r \frac{y^{*t}_{r_t}}{y^{*t}_{r_{t+1}}}\right)^{1/s}}{\left(\prod_i \frac{x^{*t}_{i_t}}{x^{*t}_{i_{t+1}}}\right)^{1/m}} \times \frac{\left(\prod_r \frac{y^{*t+1}_{r_t}}{y^{*t+1}_{r_{t+1}}}\right)^{1/s}}{\left(\prod_i \frac{x^{*t+1}_{i_t}}{x^{*t+1}_{i_{t+1}}}\right)^{1/m}} \right)^{1/2} \quad (7)$$

Note that it is TFP that one wants to decompose, and therefore the above is better expressed as (8).

$$TFP = MGDF \times \left(\frac{\left(\prod_i \frac{x^{*t}_{i_t}}{x^{*t}_{i_{t+1}}}\right)^{1/m}}{\left(\prod_r \frac{y^{*t}_{r_t}}{y^{*t}_{r_{t+1}}}\right)^{1/s}} \times \frac{\left(\prod_i \frac{x^{*t+1}_{i_t}}{x^{*t+1}_{i_{t+1}}}\right)^{1/m}}{\left(\prod_r \frac{y^{*t+1}_{r_t}}{y^{*t+1}_{r_{t+1}}}\right)^{1/s}} \right)^{1/2} \quad (8)$$

To see that the square root in (8) is scale related, consider the single input-output case, where the above (8) reduces to (9).

$$TFP = MGDF \times \left(\frac{\frac{x^{*t}_t}{x^{*t}_{t+1}}}{\frac{y^{*t}_t}{y^{*t}_{t+1}}} \times \frac{\frac{x^{*t+1}_t}{x^{*t+1}_{t+1}}}{\frac{y^{*t+1}_t}{y^{*t+1}_{t+1}}} \right)^{1/2} \quad (9)$$

The second term of this decompo-sition compares changes between input and output targets along the t and the $t+1$ frontier. As it is arbitrary to measure these changes on the t or on the $t+1$ frontier the geometric mean between both is taken in (9). As all the points considered in the square root in (9) are efficient points, the movements between these points (on each frontier) can only reflect the exploitation of scale economies or changes in the mix of operations.

The TFP as calculated through the GDF approach decomposes, therefore, in MGDF (which includes a technological change and efficiency change components) and in a residual component (RES) that is scale related. Note that if input and output targets were calculated in relation to a CRS technology, then in (9) one would have TFP = MGDF as the residual component would equal 1 (the proof is omitted for sake of brevity). On the other hand, if target points are calculated in relation to a VRS technology, then it can be proven that the above decomposition in (9) is equivalent to the RD approach, where the residual component in (9) is equal to the RD scale effect. For the multiple input-output case, the RD and the GDF approaches yield diffe-rent TFP and RES components, but can yield the same efficiency change and technological change components when the same efficiency models are used in both approaches (to cal-culate efficiency scores in the RD model and target levels in the GDF model).

Interpreting the RD scale change factor is not easy as testifies Lovell (2001) and Ray (2001), since it is not a straightforward ratio of scale efficiency in two different periods (as happens in the FGNZ approach). However, it is not clear that the scale related component of productivity change should re-flect changes in scale efficiency. For example, Lovell (2001) points out that the scale compo-nent of productivity change should reflect the influence of scale economies on productivity change rather than changes in scale efficiency. The author further points out that this contri-bution of scale economies to productivity change is provided by the scale component of the RD approach, whereas the contribution of the scale efficiency change of the FGNZ approach to explain scale economies is unclear. Being our residual component related with the RD approach its interpretation in terms of scale is not easy, especially for technologies using multiple inputs/outputs.

In order to shed some light on the interpretation of our residual component, note that in the single input-output case the square root in (9) can be alternatively written as:

$$\left(\frac{\dfrac{y_{t+1}^{*t}}{x_{t+1}^{*t}}}{\dfrac{y_t^{*t}}{x_t^{*t}}} \times \frac{\dfrac{y_{t+1}^{*t+1}}{x_{t+1}^{*t+1}}}{\dfrac{y_t^{*t+1}}{x_t^{*t+1}}} \right)^{1/2} .$$

Considering the produc-tion frontier at period t and assuming that $x^{*t}_{t+1} > x^{*t}_{t}$, we have that if $[y^{*t}_{t+1} / x^{*t}_{t+1}] / [y^{*t}_{t} / x^{*t}_{t}]$ is greater than 1 then the production function exhibits increasing returns to scale. If this ratio is equal to 1 we have constant returns to scale, and if it is lower than 1 we have decreasing returns to scale (see Diewert and Nakamura, 2003 who also put forward this interpretation). Each term of the above geometric mean can therefore be interpreted as containing informa-tion on the returns to scale properties of the production frontier.

Though attractive this interpretation it may have some problems. Note, for example, that the above interpretation implies $x^{*t}_{t+1} > x^{*t}_{t}$. If $x^{*t}_{t+1} < x^{*t}_{t}$, then a ratio of two input-output coefficients lower than 1 would indicate incre-asing returns while a value higher than 1 would indicate decreasing returns. Therefore the interpretation of values higher or lower than 1 is conditional to the relationship be-tween input levels at the two points being compared. Another difficulty relates to the RES component being an aggregate measure of returns to scale on both the t and $t+1$ frontiers. While a component on each frontier can be interpreted in the way suggested by Diewert and Nakamura (2003) a geometric mean of RTS on the t and $t+1$ frontier seems to be lacking an easy interpretation. For the multiple input-output case difficulties are even greater because movements along each produ-ction frontier may reflect, apart from scale effects, also mix effects. This means that in this case the interpretation of this factor becomes even more complicated. It is not our aim in this paper to deepen the analysis on the scale change component of the GDF measure. We interpret this factor simply as a residual effect that accounts for differences between TFP and a Malmquist index calculated in relation to a VRS technology. When the residual effect is greater than 1 it means that TFP change benefits from a positive influence through the exploitation of scale economies, whereas when it is lower than 1 this influence on TFP is negative.

CONCLUSION

This paper draws attention for some limitations of current approaches to calculating Malmquist indexes, and attempts to solve them through the use of a geometric distance func-tion (GDF) approach. The GDF is used here with two purposes. (i) To calculate a total factor productivity measure based on observed values only, and (ii) to calculate measures of technical efficiency that are non-oriented and account for all the sources of inefficiency. The latter use of the GDF solves the problem of infeasibility of some DEA models when VRS technologies are used, and resolves the ambi-guity resulting from the use of oriented models that yield conflicting information depending on efficiency measures being input or output oriented. The former use of the GDF to calcu-late TFP is consistent with the single input-output case, where it is widely accepted that a ratio of productivity at two different points in time reflects productivity change. Such ratios are based on observed values only, and do not require any assumptions regarding the form of the production frontier. In the multiple input-output case the most usual procedure to calcu-late TFP change is through Malmquist indexes that use efficiency measures calculated in re-lation to CRS technologies. We show through an example that this procedure may yield biased results due to changes in the reference hyperplane where units are projected.

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