Alternative approach to characteristics-based investments

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Abstract

Previous studies have shown significant return predictive ability of specific firm related variables, however, most of them only test one or a few variables at the same time. With the objective of exploring this shortcoming, this thesis analyzes the out-of-sample predictive ability of 12 firm characteristics to forecast returns. Forecasts are computed through cross-sectional Fama-Macbeth-style regressions. Moreover, we adopt 5 different combinations of characteristics in order to test the combined predictive ability of characteristics in each set. Additionally, we consider 4 estimation periods for the slopes derived from cross-sectional regressions. The main objective of this thesis is the development of a profitable firm-characteristics related strategy for small investors. Therefore, we take into consideration 3 important and real constraints faced by small investors: i) a limited value available to implement the strategy, ii) no possibility of short-selling and iii) the payment of transaction costs. We find high predictive ability in all combinations, and among all lengths of estimation periods under analysis, which leads the investment strategy to outperform the market throughout the sample, regardless of the combinations tested. However, the combination of all 12 characteristics and 1-year rolling slopes stands out, leading the investment strategy to yield an average monthly return of 2% net of transaction cost and an annualized Sharpe Ratio of 0.96 contrasting with 0.39 from the S&P500. The strategy transforms $20,000 invested in 1982 into $28,843,056 at the end of 2015.

Estudos anteriores têm mostrado uma capacidade relevante de previsão de retornos por parte de variáveis específicas da empresa, contudo, a maioria desses estudos analisa apenas uma ou um pequeno número de variáveis de cada vez. Com o objectivo de explorar essa imperfeição, esta tese analisa a capacidade de 12 características da empresa para prever retornos. As previsões são calculadas através de regressões do estilo Fama-Macbeth. Adotamos 5 combinações diferentes de características por forma a testar o poder combinado de previsão das características em cada conjunto. Adicionalmente, consideramos 4 períodos de previsão para os coeficientes achados nas regressões. O objectivo principal desta tese é o desenvolvimento de uma estratégia lucrativa, com base nas características da empresa, para pequenos investidores. Nesse sentido consideramos 3 reais e importantes limitações enfrentadas por pequenos investidores: i) valor limitado para implementação da estratégia, ii) a não possibilidade de realizar vendas a descoberto e iii) o pagamento de custos de transação. Encontramos uma elevada capacidade de previsão em todas as combinações e em todos os períodos de previsão analisados. A estratégia de investimento apresenta desempenhos superiores ao do mercado ao longo da amostra, independentemente da combinação testada. Contudo, a combinação das 12 características e coeficientes de 1 ano destaca-se, levando a estratégia de investimento a lucrar um retorno médio mensal de 2% líquidos de custos de transação e um Sharpe Ratio anualizado de 0.96, contrastando com 0.39 do S&P500. A estratégia transforma $20,000 investidos em 1982, em $28,843,056 no final de 2015.

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1. Introduction

Many financial players, such as investors, companies, analysts, among others, struggle to perform portfolio optimization on a daily basis. To accomplish this, they collect information about financial assets. However, part of that information is related to expectations about the future. Therefore, the accuracy of the data is an important issue given its impact on the optimization process. A slightly less accurate data can dramatically change the portfolio optimization output, and therefore investment strategies and their profits.

In this thesis we perform a portfolio optimization using characteristic-based expected returns. The main objective is the development of an investment strategy based on firm characteristics that would be economically feasible for small investors. For that purpose, we closely follow Lewellen´s (2014) analysis in order to find the basket of firm characteristics with higher predictive capability. To do so, we run cross-sectional Fama-Macbeth (FM) style regressions of stock returns on lagged characteristics. Afterwards, we apply in-sample average slopes out-of-sample, in order to forecast stock return over the subsequent month.

Given the importance and the influence of returns in markets, there has been a considerable dedication to develop models that forecast as accurately as possible the return of assets. Hence, this translates into a large pool of financial literature on the topic. Several researchers developed asset pricing models that are still very well-known and used in present days. An example of that is Sharpe (1964) where he presents the Capital Asset Pricing Model (CAPM), and Fama and French (1993) where they develop the Fama French 3 factor model. In addition, there are also other distinguished asset pricing models that result from previous research. Carhart (1997), for instance, develops the Carhart 4 factor model, which combines the Fama French 3 factor model with the factor so called Momentum, introduced by Jagadeesh and Titman (1993).

However, returns on their own are not a measure of the quality of a model, therefore a mean-variance analysis provides a trade-off between the average return and its variance. The variance of returns is a measure of the risk faced by investors. Often, in the literature, the comparison between returns and the risk faced is used in order to characterize the quality of the model used. As a result, risk is an important component to take into account when drawing an investment strategy. Merton (1987), Barberis and Huang (2001) and Ang, Hodrick, Xing and Zhang (2006) analyze the relationship between volatility and returns. They conclude that the market prices risk negatively. Thus, one can achieve higher returns through a skilled investment picking or over a higher exposure to risk. Hence, risk has an important role in the evaluation of models, allowing for the comparison between the level of return and the level of risk that each model delivers. Accordingly, the most efficient model is the one with the highest returns for a certain level of risk, or in other words, the one with the lowest level of risk for a certain level of return. Small investors are particularly sensitive to the level of risk faced given the inability to diversify their positions. Thus, the analysis of the risk faced by investors is especially important in our study. All in all, it determines whether it makes sense for small investors to implement a characteristic-based investment strategy.

Overall, there has been an important contribution of financial literature to the evolution of analysis techniques. Different assumptions made on models, the use of new mathematical tools, as well as more reliable and extended data are contributions that have led to infer different conclusions, according to previous analysis. An example of such is Simin (2008) that finds evidence of the existence of time-variance in risk-premiums, which reduces the accuracy of the expectations of CAPM and Fama French 3 factor model. Moreover, Goyal and Welch (2008) also show the differences between in-sample and out-of-sample (OOS) prediction capability of many asset pricing models which were suggested in previous academic literature. They conclude that those models have poor OOS prediction capability. Additionally, Fama and French (2015) prove that CAPM, Fama French 3 factor model and Carhart 4 factor model have drawbacks when incorporating patterns in returns, mainly in small stocks. Based on these findings, Fama and French (2015) suggest a new asset pricing model using five factors.

Parallel to this well-known discussion on these models, there has been an upgrowth in the analysis of firm characteristics to predict stock returns. Haugen and Baker (1996) test a pool of forty six observable characteristics of firms and conclude there is a significant prediction capability to forecast returns. Moreover, Fama and French (2008), and Lewellen (2014), when performing a cross-sectional analysis, find a high degree of firm characteristics in predicting stock returns.

Although there are several facts evidencing the existence of correlation between firm characteristics and stock returns, there is not much work on how investors can take advantage of this relationship in a profitable trading strategy. Lewellen (2014) covers this issue by applying slopes from FM style regressions with the aim of measuring the predictive ability of forecasts based on the slopes of firm characteristics. This analysis dictates whether the returns are due to chance or due to the quality of the model and, therefore, if firm characteristics can be applied in a true investment strategy. The author focuses his investigation on 15 firm characteristics while creating three portfolios with different combinations of characteristics. His results show a significant stock return prediction power. Additionally, it is shown that returns computed from the model coincide with the realized returns consistently over time. Furthermore, Lewellen's trading strategy reveals valuable risk-return trade-off with a Sharpe Ratio of approximately 0.70 for large stocks[1].

Bessembinder, Cooper and Zhang (2015) claim that factor-based models have limited success in predicting portfolio returns. They also state and prove that characteristic-based models are successful in predicting stock returns, supporting Lewellen's results. This strengthens our arguments that it is possible to take advantage of this predictive ability, so as to use it in an investment strategy. Additionally, Mclean and Pontiff (2016) validate the idea that sophisticated investors indeed learn with academic publications on how to improve their trading strategies. This highlights the practical implementation and relevance of the portfolio optimization topic.

In total, we use 12 characteristics out of Lewellen's 15 characteristics, as they are the most commonly studied in academic literature. Moreover, all of these characteristics are proven to be individually related with future returns, according to previous studies. We construct 5 models with different combinations and number of characteristics. Using rolling averages of past cross-sectional slopes to predict stock returns, we test 10, 5, 3 and 1-year rolling windows. These 4 estimation periods are combined with each model of characteristics, allowing to scrutinize the combination with higher return predictive capability. We find that shorter estimation windows are better than longer estimation periods on capturing new information, which increases the ability of the strategy to react accordingly and to predict future returns. Therefore, the shorter the rolling window, the higher the strategy performance.

Among all combinations under analysis, the investment strategy yields average monthly returns up to 2% net of transaction costs and Annualized Sharpe Ratios ranging between 0.39 and 0.96. Since the strategy is to be implemented by small investors, the initial monetary value available to implement the strategy is $20,000. At the end of the 33 years horizon in which the strategy is tested, the value available to implement it amazingly ranges between $783,814 and $28,843,056. More importantly, the strategy has always positive alphas, despite the combination of characteristics and length of estimation. This means that the strategy creates

[1] Lewellen (2014) defines Large Stocks as stocks bigger than NYSE median capitalization

value for investors. This goes against the market efficiency hypothesis which states that it is impossible for an active investment strategy to create value above the value delivered by a passive strategy in the Market portfolio.

2. Data

To perform our analysis, we combine the Center for Research in Security Prices (CRPS) with Compustat databases. CRPS provides the monthly market data from January 1958 to December 2015 while Compustat database supplies the quarterly accounting data from January 1961 to December 2015. From these two databases we access prices, number of shares outstanding, volume traded, monthly returns, book value, total assets, debt, quarterly sales and quarterly income, which are ultimately used to compute firm characteristics. With all characteristics computed, we implement FM style regressions of stock returns on lagged characteristics. Market data is assumed to be immediately available, while accounting data is assumed to be available 2 months after the end of the quarter[2]. In addition, the 3 Fama French factors as well as risk-free rate are obtained in the Kenneth R. French data library[3] whereas the T-Bill rates are collected from the Board of Governors of the Federal Reserve System[4].

Moreover, we test a monthly rebalancing with two objectives in mind. Firstly, to have relatively updated information. Secondly, to get a relatively low degree of rebalancing in positions and, consequently, lower transaction costs.

The analysis is performed using the following 12 characteristics, adapted from Lewellen (2014):

LogSize_{-1}	= Log of the market value of the equity at the end of the prior month. The market value is computed multiplying the price of stock by the number of shares outstanding.
LogB/M_{-1}	= Log of the book value divided by the market value at the end of the prior month. The book value of the equity is determined by the total common equity, adding deferred taxes and subtracting the preferred

[2] We consider accounting data to be available 2 months after the end of the quarter given the SEC rules for quarterly reports. Companies have between 40 and 45 days after the end of the quarter, depending on their Market Capitalization, to publish their report.

[3] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[4] http://www.federalreserve.gov/releases/h15/current/

	stock redeemable, or subtracting the par value if the redeemable value is not available.
$\text{Return}_{-2,-12}$	= Stock return between month -12 and month -2.
$\text{LogIssues}_{-1,-36}$	= Log growth in split-adjusted shares between month -36 and month -1.
ROA_{Yr-1}	= Income in the previous quarter divided by the average total assets in the previous fiscal year.
LogAG_{Yr-1}	= Log of the growth in total assets in the prior fiscal year.
$\text{DY}_{-1,-12}$	= Dividend per share over the prior 12 months divided by price at month -1.
$\text{LogReturn}_{-13,-36}$	= Log stock return from month -36 to month -13.
$\text{LogIssues}_{-1,-12}$	= Log growth in split-adjusted shares from month -12 to month -1.
$\text{Turnover}_{-1,-12}$	= Average monthly turnover from month -12 to month -1. Turnover is the ratio between the number of shares traded and the total number of existing shares.
Debt/Price_{-1}	= Current plus long-term debt divided by market value at month -1.
$\text{Sales}_{Q-1}/\text{Price}_{-1}$	= Sales in the prior quarter divided by market value at month -1.

The aforementioned 12 characteristics are tested to find the most statistically significant ones. Based on the ones with highest relevance, several combinations are developed with the purpose of finding the portfolio of combinations with the most accurate results. Therefore, we develop 5 different combinations. The first one includes all of the 12 characteristics, whereas the second considers 6 characteristics (LogSize_{-1}; LogB/M_{-1}; $\text{Return}_{-2,-12}$; $\text{LogIssues}_{-1,-36}$; ROA_{Yr-1} and LogAG_{Yr-1}), and the third examines only 3 characteristics (LogSize_{-1}; LogB/M_{-1} and $\text{Return}_{-2,-12}$). Additionally, we develop a combination with the 8 statistically significant characteristics at a 5% significance level, as well as a combination of the 9 characteristics stated in the literature as relevant to predict stock returns. It is important to notice that we exclude companies that, during the sample, disclosed negative sales and negative book value of the equity.

Table 1 presents the descriptive statistics for the 12 firm characteristics and for the monthly returns. The values presented are the time-series averages of the monthly cross-sectional mean, standard deviation and sample size for the different variables. In order to minimize the impact of extreme values, firm characteristics are winsorized at their 1^{st} and 99^{th} percentiles.

Table 1
Descriptive Statistics 1961-2015
The sample includes all common stocks in CRSP database from January 1961 to December 2015. CRSP supplies stock prices, returns, shares outstanding, dividends paid and turnover from its monthly data while book equity, sales, debt, total assets and earnings are supplied by the quarterly data of Compustat. Accounting data is assumed to be known 2 months after the end of the quarter, whereas market data is assumed to be immediately known. The following table provides the time-series averages of the cross-sectional mean (Avg), standard deviation (Std) and sample size (N) for the different variables used in the analysis. Return mean and standard deviation are stated in percentage.

	Full Sample		
	Avg	Std	N
Return (%)	1.20	14.16	3777
LogSize_{-1}	4.54	1.90	3782
LogB/M_{-1}	-0.40	0.72	3090
$\text{Return}_{-2,-12}$	0.12	0.45	3472
$\text{LogIssues}_{-1,-36}$	0.09	0.22	3026
$\text{ROA}_{\text{Yr-1}}$	0.01	0.03	2875
$\text{LogAG}_{\text{Yr-1}}$	0.10	0.20	2827
$\text{DY}_{-1,-12}$	0.03	0.08	2589
$\text{LogReturn}_{-13,-36}$	0.07	0.55	2896
$\text{LogIssues}_{-1,-12}$	0.03	0.10	3585
$\text{Turnover}_{-1,-12}$	0.08	0.08	3141
Debt/Price_{-1}	0.82	1.37	3069
$\text{Sales}_{\text{Yr-1}}/\text{Price}_{-1}$	0.66	0.87	3108

Table 1 shows that the average monthly cross-sectional mean return is 1.2% while its standard deviation is 14.16%. Moreover, the sample has a monthly average of 3777 companies with valid returns. Analyzing the firm characteristics, we realize that Size is, on average, the most volatile characteristic. This result is contrary to Lewellen's results where Sales/Price is the most volatile characteristic. When taking a closer look, we find a similar value for the volatility of Size. In contrast, the volatility of Sales/Price is less than half of the value the author finds. Regarding ROA, we obtain a much less volatile characteristic than Lewellen. However, these differences could be explained by the distinctive way we compute the characteristic Sales/Price and ROA. While he uses the annual reported sales and the annual reported income, we use the quarterly reported sales and the quarterly reported income of that specific quarter for Sales/Price and ROA, respectively.

A major part of these characteristics is highly persistent and stable on a monthly basis. This is caused by either a static or a slow change in level variables (such as B/M and Size), or flow variables computed only on a quarterly basis (like Sales and Earnings). Additionally, some of these characteristics are highly correlated with each other. This correlation is caused because they measure the same aspect of the company (such as long-term and short-term stock issues), and because they are related and impact each other (like stock issue and turnover). These issues lead to the existence of multicollinearity in regressions. Nonetheless, in this case, it does not raise barriers because the core of our analysis is the overall predictive power of the different combinations of characteristics, and not of each slope separately.

3. Analysis

In this section, we explain the methodology used while analyzing, discussing and comparing the obtained results. The main objective is to understand whether the assumptions made allow us to implement an investment strategy based on firm characteristics and low availability of cash.

3.1. FM style Regressions and estimated slopes

The regressive study is based on FM style regression to estimate stock returns from characteristics of firms. At the end of all of the calculations, it is possible to access how each factor defines stock returns, finding the premium related to the exposure to those factors.

Breaking down the process, the first FM style regression step consists of cross-sectional regressions. Stock returns are defined as the dependent variable while characteristics are the independent variables, therefore the regressions are run with the following approach:

$$R_{i,t+1} = \alpha_i + \beta_{F1,t} * F_{1,i,t} + \beta_{F2,t} * F_{2,i,t} + \ldots + \beta_{Fm,t} * F_{m,i,t} + \varepsilon_{i,t} \quad (1)$$

Where $R_{i,t+1}$ is the stock return at time t+1 of company i, α is the constant and $\beta_{F1,t}$ is the coefficient, at time *t*, relative to the first characteristic denoted as F_1. As mentioned before, there are up to 12 characteristics per portfolio and thus, m of F_m can go up to 12, depending on the portfolio under analysis. This process provides us with the β's which represents the exposure of returns to each factor at a certain month *t*.

Additionally, we aim to test the statistical significance of the characteristics elected to be used in the investment strategy. Table 2 presents the average slopes and t-statistics for each variable, for the different combinations of characteristics. It also provides the R^2 for each combination. The values presented are the result of 660 regressions from January 1961 to December 2015. Only companies with valid data to compute monthly returns and with all the characteristics in that specific combination are considered. Most of the results are coherent with previous findings in academic literature.

By analyzing the slopes presented in table 2, B/M and past 12-month Returns are significantly positive whereas Size is significantly negative, at a 95% confidence level across all combinations. Additionally, all characteristics maintain their sign constant over the different combinations, changing only in magnitude. Therefore, ROA, Dividend Yield and Sale/Price have positive and statistically significant slopes. On the other hand, Stock Issues from the past 36 months and the market leverage have negative and statistically significant slopes at a 5% significance level. Turnover and Stock Issues in the past 12 months likewise long-term monthly returns have negative but not statistically significant slopes, at a 5% significant level. However, the last two are statistically significant at a 10% level.

Asset Growth in the past year presents negative slopes across all combinations. Although Asset Growth has statistically significant slopes for the combinations of 6 and 9 firm characteristics, the slope in the combination of 12 characteristics is not significant, for a 95% confidence level.

In the combination of 8 firm characteristics, one may expect small changes in the level of slopes and t-statistics, when comparing with the 12 characteristics combination due to the removal of the non-significant variables. Nonetheless, the value of the slope of B/M almost doubles, also increasing the t-statistics. In contrast, the t-statistics of ROA drops to almost half of the value and a reduction in the slope is noted, as well.

Table 2
Fama-MacBeth regressions, 1961-2015
The following table provides the average slopes, the t-statistics (in parenthesis) and the R^2 from the Fama-MacBeth-style cross-sectional regressions. Monthly returns are regressed on prior month firm characteristics (as previously defined) for the combination of 3, 6, 8, 9 and 12 Firm Characteristics (Firm Charac.). The sample includes all common stocks available on CRSP with valid data to compute monthly returns. Market data such as stock prices, returns, shares outstanding, dividends, and turnover are supplied by the monthly file of CRSP while accounting data such as book equity, total assets, debt, sales and earnings come from the quarterly file of Compustat. All accounting data is assumed to be known 2 months after the end of the quarter, following SEC rules, whereas market data is assumed to be immediately known.

	Slopes				
	3 Firm Charac.	6 Firm Charac.	12 Firm Charac.	8 Firm Charac.	9 Firm Charac.
$LogSize_{-1}$	-0.16	-0.14	-0.09	-0.11	-0.14
	(-3.7)	(-3.82)	(-2.84)	(-3.18)	(-3.62)
$LogB/M_{-1}$	0.33	0.29	0.13	0.22	0.15
	(4.61)	(4.82)	(2.30)	(3.55)	(2.64)
$Return_{-2.-12}$	0.44	0.56	0.44	0.52	0.67
	(2.46)	(3.39)	(2.82)	(3.16)	(4.22)
$LogIssues_{-1.-36}$		-0.83	-0.47	-0.54	-0.52
		(-4.15)	(-3.65)	(-4.27)	(-4.02)
ROA_{Yr-1}		7.55	4.76	3.23	5.20
		(2.14)	(4.14)	(2.21)	(2.12)
$LogAG_{Yr-1}$		-0.58	-0.23		-0.44
		(-3.26)	(-1.75)		(-2.69)
$DY_{-1.-12}$			1.27	1.41	
			(5.32)	(5.6)	
$LogReturn_{-13.-36}$			-0.16		
			(-1.84)		
$LogIssues_{-1.-12}$			-0.55		-0.49
			(-1.67)		(-1.55)
$Turnover_{-1.-12}$			-0.13		-2.05
			(-0.07)		(-1.09)
$Debt/Price_{-1}$			-0.06	-0.08	
			(-2.54)	(-2.85)	
$Sales_{Yr-1}/Price_{-1}$			0.24	0.23	0.31
			(4.81)	(4.14)	(3.62)
R^2	0.002	0.002	0.003	0.003	0.003

In the combination of 9 characteristics, we withdraw the ones found in academic literature as non-significant from the 12 characteristics. Again, slight changes were expected in the level of slopes and t-statistics. Nevertheless, the t-statistics of past 12 month return almost doubles whereas it falls to almost half the value in ROA, while remaining statistically significant. The slope of the Asset Growth almost doubles while the t-statistics falls enough to become statistically significant, even at a 1% significance level. This drop goes in line with the academic literature, such as Cooper, Gulen, and Schill (2008) and Daniel and Titman (2006). Lastly, the coefficient of Turnover is more than 15 times bigger than in the combination of 12 characteristics. Its t-statistics becomes considerably more negative, but not enough to be significant, not even at a 20% significance level. This goes against the findings of Lee and Swaminathan (2000) who conclude that Turnover in the last 12 months is negatively and significantly related with future returns, especially on stocks that had a poor performance in the past 12 months.

Overall, we can highlight two differences in relation to the results in academic research. As previously stated, Lee and Swaminathan (2000) prove that Turnover has a negative and significant relationship with future returns. While negative, the slope of Turnover is not significant in none of the two combinations that consider Turnover. Black and Scholes (1974) suggest that it is not possible to prove an impact of Dividend Yield on returns. In the financial literature there are different results, however in some studies a low predictive power of dividends to forecast future returns is found. The combinations that consider the Dividend Yield present significantly positive slopes, even for a 99.9% confidence level, which supports the results of Christie (1990).

It is extremely relevant to note that interpreting the low values of the FM R^2 as informative would lead to erroneous conclusions about the predictive power of the variables analyzed. The FM R^2 provides information mainly on the power of variables to explain the contemporaneous volatility and not the predictive power of those characteristics. The relevant component when forecasting future stock returns is the predictive power of estimators, in this case firm characteristics.

3.2. Rolling averages slopes

Before estimating expected returns, we have to apply rolling averages over the FM cross-sectional slopes. Knowing all β’s, we consider 1, 3, 5 and 10-years rolling windows to compute rolling averages. Those rolling averages are used as slopes to compute the expected return. Therefore, it is pertinent to evaluate its behavior during the sample.

Following Lewellen's analysis and results, we first apply the methodology in the 10-year rolling window, which is the estimation window that provides higher performances in Lewellen's study. Figure 1 gives us a brief idea of the behavior of the rolling slopes throughout time. Those rolling slopes are the result of running FM style regressions over the combination of all 12 characteristics. We exhibit only 8 of those 12 characteristics divided over Panel A and Panel B for visual clarity.

Figure 1. 10-year rolling slopes, 1982 – 2015
The figure plots the 10-year rolling averages of the slopes from the FM style regressions. It is applied over the combination with all 12 characteristics. The horizontal axis indicates the last date in the rolling window. Panel A presents the results for Size, Book-to-Market (B/M), Return in the last 12 months (Return12) and 3 years stock issuance (Issues36). Panel B presents the rolling slopes for the Asset Growth (AG), Dividend Yield over the last 12 months (DY12), 1 year stock issuance (Issues12) and Turnover in the last 12 months (Turnover12). We consider all common stocks from CRSP with valid data to compute monthly returns. Market data comes from CRSP monthly file and it is assumed to be immediately known. Accounting data is supplied by Compustat quarterly file and it is assumed to be known 2 months after the end of the quarter, following SEC rules.

It is possible to note that most of the rolling slopes shrink towards zero. Rolling slopes are relatively stable, changing mainly in magnitude but not in sign. Therefore, they lie mostly in the same side of the horizontal axis over the period, while appearing less volatile at the end. Turnover and last 12 month returns seem to be the major exceptions. The former changes sign several times, but it seems more stable and closer to zero at the end of the sample. The latter presents a significant drop in 2008 turning into a negative value in 2009. This can be associated to the 2008 Financial Crises which represented a colossal negative impact on most momentum strategies. It is interesting to note that the rolling slopes do not seem to be significantly affected by relevant financial events such as the Black Monday on 19th October of 1987 and the energy crisis that started in 2003, when oil prices started to skyrocket. The only variable that drops in level due to these events is the last 12 month returns.

3.3. Expected and Observed Returns

We are now in a position to estimate future returns. In order to simulate the ability to implement the investment strategy at the end of each month, the expected returns of the following month are computed from the product between the rolling slopes and the end-of-the-month characteristics. Hence, at the beginning of each month, the strategy only requires past and known information to determine which positions to take in that month. The following equation is used:

$$\widehat{R_{i,t+1}} = \delta_0 + \delta_{F1,t} * F_{1,i,t} + \delta_{F2,t} * F_{2,i,t} + \cdots + \delta_{Fm,t} * F_{m,i,t} \tag{2}$$

Where $\widehat{R_{i,t+1}}$ is the expected return of the company *i* based on the model at time *t*+1. Once again, m reaches values up to 12, depending on the portfolio of characteristics under analysis. $\delta_{F1,t}$ is set as the rolling average of the β_{F1} of all companies between the month *t-1* and *t-x*. . Since the analysis considers 1, 3, 5 and 10 years of timespan for rolling windows, *x* takes the value of 12, 36, 60 and 120 months.

Our main objective is to develop a feasible and profitable investment strategy. Notice that the performance of the model increases the closer the expected return is to the realized return. Therefore, we can state that the performance of the model is directly dependent on the accuracy of the expected returns. Thus, to reach our main objective we have to correctly predict stock returns. Accordingly, the accuracy of expected returns may dramatically change positions and profits.

Table 3 compares the average expected return of the monthly portfolios, computed from 10-year rolling slopes, with the average observed returns. This comparison is done for all combination of characteristics under consideration. By looking at the standard deviations, we are able to detect whether the estimations reproduce the cross-sectional variation in the expected stock returns. This allow us to infer the quality of expected portfolio returns before applying the investment strategy. Despite the higher volatility (standard deviation of the expected returns from 6.08% to 7.55% comparing with 0.88% to 1.21%), the averages of the observed returns are close to the averages of expected returns.

Additionally, we can also observe that differences between average expected and observed returns are always inferior to 1 percentage point, ranging from 0.36 to 0.80 percentage points. More important than a low average difference is the fact that these differences are not statistically significant, neither at a 5% nor at a 10% significance level. The only exception is the combination with 3 characteristics whose difference is statistically significant, assuming a 95% confidence interval. Additionally, this combination yields the highest difference between expected and observed returns. Consequently, we can conclude a relatively high quality of 10-year rolling slopes to forecast returns.

Table 3
Expected and Observed Returns, 10-year rolling slopes
The table provides the time-series average (Avg) and standard deviation (Std) of the expected and observed monthly returns of portfolios constructed based on different combinations of characteristics. It also presents the differences between expected and observed average returns (Diff) and their t-statistics (t-stat). The expected returns are computed by the product between 10-year rolling slopes and the previous end-of-the-month characteristics. The observed returns consist in the evolution of prices relative to the previous month, not taking into account transaction costs. The sample includes all common stocks from CRSP with valid data to compute monthly returns. Market data comes from CRPS monthly file and it is assumed to be immediately known. Accounting data is supplied by Compustat quarterly file and it is assumed to be known 2 months after the end of the quarter, following SEC rules.

	Univariate properties (%)					
	Expected		Observed			
Model	Avg	Std	Avg	Std	Diff	t-stat
12 Firm Characteristics	2.32	0.88	1.95	6.57	0.36	1.11
6 Firm Characteristics	2.54	1.00	2.09	6.98	0.45	1.30
3 Firm Characteristics	2.63	1.21	1.84	7.35	0.80	2.15
8 Firm Characteristics	2.28	0.91	1.87	6.08	0.41	1.37
9 Firm Characteristics	2.61	0.87	2.18	7.55	0.42	1.13

Unexpectedly, the combination of 12 characteristics has the second most stable average expected and observed monthly returns. Given that 4 of the 12 characteristics seem not to be statistically significant, we may expect those 4 characteristics to increase noise in the estimation, and therefore increasing volatility.

Besides the quality of 10-year rolling slopes to forecast returns, we carry out the same comparison for the same sets of characteristics but with different length of rolling slopes. Table 4 compares the average expected returns and the average observed returns of monthly portfolios using 5, 3 and 1-year rolling slopes.

Although in Table 3 it is not possible to identify any kind of trend in terms of number of firm characteristics and average return or standard deviation of returns, a tendency can be noted in Table 4. Inside each set of characteristics, decreasing the length of the rolling window increases both expected and observed monthly average returns of portfolios. In general, the 1-year rolling window is the one which leads to higher volatility in expected and observed average returns. This makes sense given that the smaller the rolling window, the more affected it is by new information. Surprisingly, the 5-year rolling window is not always the length that leads to more stable returns, being surpassed twice by the 3-year rolling window (in the case of 12 and 3 characteristics). However, when comparing the results from Table 4 with the ones in Table 3, we notice that, in general, the 10-year length is the one with the lowest, yet more stable average expected and observed returns.

Once again, the averages of observed returns are smaller, less stable but similar to the averages of expected returns for all sets of characteristics and lengths of rolling slopes. The differences between expected and realized returns range between 0.1 and 0.69 percentage points. However, these differences are not statistically different from zero for a 95% confidence level. The exception is the combination between 9 characteristics and 1-year rolling windows whose difference is statistically significant. These results corroborate the previews results about the good quality of return estimations. Similarly to Lewellen (2014), it seems that cross-sectional FM style regressions are a solid way to predict future returns. Contrasting with the low predictability of time-series regressions as studied by Goyal and Welch (2008). Notwithstanding, there are evidences that the model overestimates expected returns.

Table 4
Expected and Observed Returns, Alternative rolling slopes
The table presents the time-series average (Avg) and standard deviation (Std) of the expected and observed monthly returns of portfolios constructed based on different combinations of characteristics. It also shows the differences between expected and observed average returns (Diff) and their t-statistics (t-stat). The expected returns are computed by the product between rolling slopes and the previous end-of-the-month characteristics. The table considers 5, 3 and 1-year rolling slopes for the different combinations of characteristics. The observed returns consist in the evolution of prices relative to the previous month, not taking into account transaction costs. The sample includes all common stocks from CRSP with valid data to compute monthly returns. Market data comes from CRPS monthly file and it is assumed to be immediately known. Accounting data is supplied by Compustat quarterly file and it is assumed to be known 2 months after the end of the quarter, following SEC rules.

	Univariate properties (%)					
	Expected		Observed			
Model	Avg	Std	Avg	Std	Diff	t-stat
12 Firm Characteristics						
5 years	2.42	1.21	2.03	6.43	0.39	1.22
3 years	2.52	1.25	2.22	6.19	0.30	1.00
1 year	3.21	2.12	2.70	6.35	0.50	1.65
6 Firm Characteristics						
5 years	2.50	1.36	2.40	7.13	0.10	0.28
3 years	2.58	1.32	2.46	7.37	0.12	0.33
1 year	3.37	2.40	2.72	7.87	0.65	1.71
3 Firm Characteristics						
5 years	2.55	1.54	2.11	7.56	0.43	1.13
3 years	2.57	1.45	2.26	7.56	0.32	0.84
1 year	3.16	2.45	2.67	7.93	0.49	1.26
8 Firm Characteristics						
5 years	2.45	1.29	1.83	6.01	0.62	2.05
3 years	2.47	1.28	2.06	6.01	0.41	1.37
1 year	3.01	2.01	2.43	6.19	0.58	1.93
9 Firm Characteristics						
5 years	2.61	1.22	2.44	7.53	0.17	0.46
3 years	2.75	1.35	2.46	7.55	0.28	0.76
1 year	3.56	2.48	2.87	7.32	0.69	1.97

3.4. Investment strategy implementation

After computing the expected return for all companies, they are ranked with the objective to compare and to understand which companies should be elected for the investment strategy.

Firms ranking highest, in other words the ones with the highest expected return for the following month, are the ones selected to invest in. As the main objective is to find a feasible investment strategy for small investors no short-position is taken. Short-selling enables an increase in returns, nonetheless it is not always available and, even when it is available, short-selling tends to be highly costly. Additionally, many investors are reluctant to have short-positions on their portfolios as, in fact, short-positions can lead to unlimited losses, thus increasing the risk of implementing the strategy.

DeMiguel, Garlappi and Uppal (2009) compare the performance of 14 models of optimal asset allocation, relative to the 1/N policy. It turns out that none of the 14 models is able to consistently outperform out-of-sample the 1/N portfolio. The naïve allocation delivers higher out-of-sample Sharpe ratio and certainty-equivalent return. These results are confirmed by Plyakha, Uppal and Vilkov (2012) who compare the performance of equal-weighted portfolio with the performance of value and price-weighted portfolios. They conclude that the equal-weighted portfolio delivers higher performances in terms of total mean return, 4 factor alpha, Sharpe ratio, and certainty-equivalent return, than both value and price-weighted portfolios.

Considering the results of DeMiguel, Garlappi and Uppal (2009) and Plyakha, Uppal and Vilkov (2012), the portfolio is equally-weighted among the selected stocks. The strategy starts with $20,000 invested in the 20 companies with the highest expected performance in the following month. For each selected company, we maximize the number of shares that it is possible to invest with the 1/20 of the value of the fund. If the maximum number of shares possible to buy does not totally cover the 1/20 of the fund, the remaining value is invested in the 1-month U.S. T-Bill. By doing so, we ensure the investment of the entire value available for the strategy. Above all, if the expected return of a selected company is negative, the value that would be invested in those stocks is fully invested in the 1-month U.S. T-Bill. This prevents us from investing in companies that are predicted to perform negatively during the following month as it would not be realistic to do so. This yields positive, yet low returns, decreasing the volatility of the portfolio.

The number of companies invested in changes with a rate of 1 company per $1,000 of variation in the value available to invest, up to a maximum of 300 companies. This brings diversification benefits and therefore less volatility, however it has a negative impact on the portfolio returns due to the increase in transaction costs. Following Plyakha, Uppal and Vilkov´s (2012) approach, we apply 50 basis points as transaction costs on selling orders. This assumption is extremely relevant given its negative impact on the amount available each month to implement the strategy.

It should be highlighted that we do not consider any capital gain/loss taxes given the nature of our rebalancing. U.S. tax authorities differentiate short and long-term capital gains/losses. A capital gain is considered long-term if the holding period of the asset is greater than 1 year. In that case, the tax rate only depends on the value of the capital gain/loss. Contrarily, the tax bracket applied in short-term capital gains depends on the investor's total income. Short-term capital gains/losses are joined with the investor's income to define the applicable tax bracket. Hence, the taxes to pay over strategy capital gains/losses depends not only on the size of the gain/loss but also on the investor's labor income.

Despite the number of characteristics and the length of the rolling slopes, the strategy is implemented in the beginning of March 1982, which is the date when there is information available to all combinations. Following Lewellen's results, we start by executing the strategy with 10-year rolling slopes for each combination of characteristics under analysis. Table 5 shows the time-series average return, standard deviation, minimum and maximum return and Annualized Sharpe Ratio of the realized monthly returns. We observe a clear drop in average returns when comparing them with Table 3. Average realized returns in Table 5 range from 1.20% to 1.60% while the average observed returns in Table 3 range from 1.84% to 2.18%. Such a drop can be explained by the introduction of transaction costs.

Interestingly, when using 10-year rolling slopes, the expected returns of the top selected companies are always positive. This happens even during financial crises when many of the selected stocks have negative return. Considering 120 observations in the rolling window, extreme events such as the beginning of a financial crisis are smoothed, limiting the strategy ability to react to those events. Thus, it is relevant to analyze the performance of the strategy with shorter rolling windows.

Table 5
Realized Performances, 10-year rolling slopes
The table shows the time-series average (Avg), standard deviation (Std), minimum return (Min. Return), maximum return (Max. Return) and the annualized Sharpe Ratio (Ann. Sharpe) of the realized monthly returns of investment strategy. The 10-year rolling slopes are considered for the different set of characteristics. Transaction costs are assumed to be 50 basis points for each sell order. The sample includes all common stocks from CRSP with valid data to compute monthly returns. Market data comes from CRPS monthly file and it is assumed to be immediately known. Accounting data is supplied by Compustat quarterly file and it is assumed to be known 2 months after the end of the quarter, following SEC rules. Kenneth French's Data Library supplies the risk-free rate.

	Univariate properties (%)				
Model	Avg	Std	Min. Return	Max. Return	Ann. Sharpe
12 Firm Characteristics	1.27	6.54	-28.54	38.96	0.50
6 Firm Characteristics	1.25	7.36	-24.49	36.18	0.43
3 Firm Characteristics	1.20	7.70	-30.97	36.12	0.39
8 Firm Characteristics	1.21	6.16	-29.75	39.08	0.49
9 Firm Characteristics	1.60	9.57	-27.91	115.51	0.46

Table 6 shares the features of Table 5 but for 5, 3 and 1-year rolling windows. It is relevant to note that, shorter lengths of rolling windows predict negative performances of top ranking stocks[5] more accurately, mainly during periods of recession. This corroborates the idea that shorter estimation windows have higher ability to react to new information, increasing strategy performance.

Surprisingly, there are situations in which shorter rolling windows provide more stable returns, while also presenting higher average returns. As an example, in the set with 12 characteristics, the 3-year rolling slopes provide higher and less volatile average returns than the 10 and 5-year slopes. Likewise, in the set of 9 characteristics, the 1-year rolling slopes provide the highest and the most stable average returns among the analyzed lengths of rolling windows.

When comparing the minimum and maximum average return for each portfolio, we note that the values of minimum average returns are similar among all portfolios, ranging from -23.28% to -31.70%. In contrast, the values of maximum average returns are widely dispersed ranging from 29.10% to 115.51%. The 1-year rolling slopes are the ones that always present the highest average return. Regardless, they never present neither the highest maximum average return nor the lowest minimum return.

[5] Ranked based on expected return of stock

Table 6
Realized Performances, 5, 3 and 1-year rolling slopes
The table display the time-series average (Avg), standard deviation (Std), minimum return (Min. Return), maximum return (Max. Return) and the annualized Sharpe Ratio (Ann. Sharpe) of the realized monthly returns of investment strategy for the different set of characteristics. The table shows the particular performance of the strategy when using 5, 3 and 1-year rolling slopes. Transaction costs are assumed to be 50 basis points for each sell order. The sample includes all common stocks from CRSP with valid data to compute monthly returns. Market data comes from CRPS monthly file and it is assumed to be immediately known. Accounting data is supplied by Compustat quarterly file and it is assumed to be known 2 months after the end of the quarter, following SEC rules. Kenneth French's Data Library supplies the risk-free rate.

	Univariate properties (%)				
Model	Avg	Std	Min. Return	Max. Return	Ann. Sharpe
12 Firm Characteristics					
5 years	1,35	6,23	-26,63	31,06	0.57
3 years	1,43	5,87	-24,68	31,18	0.65
1 year	2,00	6,02	-26,01	30,31	0.96
6 Firm Characteristics					
5 years	1,57	7,55	-24,02	59,11	0.57
3 years	1,57	7,68	-29,19	66,02	0.56
1 year	1,83	7,97	-25,52	54,94	0.65
3 Firm Characteristics					
5 years	1,60	8,65	-28,21	88,37	0.51
3 years	1,58	7,75	-31,70	70,28	0.56
1 year	1,88	7,42	-25,76	48,04	0.72
8 Firm Characteristics					
5 years	1,20	5,99	-26,24	34,44	0.51
3 years	1,32	5,60	-27,83	29,10	0.61
1 year	1,79	5,79	-24,80	30,84	0.87
9 Firm Characteristics					
5 years	1,66	8,00	-23,28	53,66	0.57
3 years	1,59	7,86	-28,73	61,37	0.55
1 year	1,91	7,33	-26,71	47,71	0.75

Given that one of our main questions is whether we can draw an efficient risk-return investment strategy, we should analyze the Sharpe Ratios of each combination of characteristics and length of rolling slopes. When comparing Table 5 with Table 6, a general increase in Sharpe Ratio is noticeable, alongside a decrease in the length of estimation windows. Hence, the 10-year rolling slopes are always the ones with the lowest Sharpe Ratio, while the 1-year slopes present the highest Sharpe Ratios. In accordance, we can conclude that increases in risk (volatility) delivered by 1-year slopes are compensated by higher increases in average returns. While Sharpe Ratios of 1-year slopes range from 0.65 to 0.96, the Sharpe Ratios of all the other slopes range from 0.39 to 0.65. In other words, the highest Sharpe Ratio among the 10, 5 and 3-year rolling slopes is equal to the lowest Sharpe Ratio of the 1-year slopes. These results contradict Lewellen's study, whose results conclude that the 10-year rolling slopes deliver the highest Sharpe Ratios.

3.5. Measurement and comparison of performance quality

In this subsection we analyze the performance quality comparing the behavior of our portfolios with the S&P500, as it is one of the most followed equity indexes. It is considered, by many investors, as the best representation of U.S. stock market and the best at signaling of U.S. business cycle. The S&P500 is a good proxy, not only for a diversified portfolio, but also for a passive strategy. This proxy is important for our analysis because it allows us to study the impact of the lack of diversification at the beginning of our strategy.

Academics show that the best way to invest in equities is through diversified portfolios, however it is too expensive to buy multiple individual shares. Given that in the U.S. small investors are not allowed to invest in hedge funds (except if they have a net worth of more than $1 million), they have to invest in either active investment strategies or passively in ETF's. So, comparing the strategy performance with the S&P500, we can understand if our active strategy is able to deliver a higher value to investors than a passive strategy.

In order to test the quality of our model, we experiment with a model in which stocks are randomly picked. We run 1,000 simulations of the implementation of the Random Picking model, in which the companies invested in each month are arbitrarily selected. Table 7 compares the performance of all combinations of firm characteristics and rolling windows length under analysis, the S&P500 and the 1,000 simulations of the Random Picking strategy.

Table 7
Performance comparison
The table shows the time-series average (Avg), standard deviation (Std) and annualized Sharpe Ratio (Ann. Sharpe) of the realized monthly returns for all sets of characteristics and length of rolling windows and for the S&P500. Additionally, we run 1,000 simulations in which stocks are randomly picked (Random Picking). For the Random Picking strategy, the values presented are the mean of the time-series average, standard deviation and annualized Sharpe Ratio of the realized monthly returns in each simulation. The sample includes all common stocks from CRSP with valid data to compute monthly returns. Market data comes from CRPS monthly file and it is assumed to be immediately known. Accounting data is supplied by Compustat quarterly file and it is assumed to be known 2 months after the end of the quarter, following SEC rules. Kenneth French's Data Library supplies the risk-free rate.

	Univariate properties (%)		
Model	Avg	Std	Ann. Sharpe
12 Firm Characteristics			
10 years	1.27	6.54	0.50
5 years	1.35	6.23	0.57
3 years	1.43	5.87	0.65
1 year	2.00	6.02	0.96
6 Firm Characteristics			
10 years	1.25	7.36	0.43
5 years	1.57	7.55	0.57
3 years	1.57	7.68	0.56
1 year	1.83	7.97	0.65
3 Firm Characteristics			
10 years	1.20	7.70	0.39
5 years	1.60	8.65	0.51
3 years	1.58	7.75	0.56
1 year	1.88	7.42	0.72
8 Firm Characteristics			
10 years	1.21	6.16	0.49
5 years	1.20	5.99	0.51
3 years	1.32	5.60	0.61
1 year	1.79	5.79	0.87
9 Firm Characteristics			
10 years	1.60	9.57	0.46
5 years	1.66	8.00	0.57
3 years	1.59	7.86	0.55
1 year	1.91	7.33	0.75
S&P500	0.82	4.34	0.39
Random Picking	0.39	10.77	-0.01

It is possible to see that, despite delivering more stable average monthly returns, the S&P500 has a substantially lower average monthly return than our strategy. Additionally, the relationship between average excess return and volatility of S&P500 is low, with an annualized Sharpe Ratio of 0.39 (from the several combinations, only the 3 firm characteristics with a 10-year rolling window presents a Sharpe Ratio as low). Therefore, we can conclude that our model (and several strategies) has a higher risk-return efficiency than a passive strategy.

When looking at the Random Picking model, it is possible to observe the significantly low mean average monthly return (the lowest of all with 0.39%) and the very high mean standard deviation (the highest of all with 10.77%). The 1,000 simulations have a mean Sharpe Ratio of -0.01, meaning that it is inefficient. It is relevant to say that in several simulations, the strategy runs out of money before the end of the sample. As a result, these values prove that the good results obtained by all combinations cannot be just due to chance.

It is highly relevant to note that the analyzed strategies are active ones, and therefore, they have shifting mean returns, thus increasing the standard deviation of returns. As a consequence, this makes the strategies appear riskier than what they really are when using the standard deviation of returns as a measurement of risk. Consequently, Sharpe Ratio underestimates the performance of the strategies.

Lastly, we scrutinize the value created through the alpha and the monetary value of each set at the end of the sample. Table 8 provides information regarding the alpha from both CAPM and Fama French 3 factor model and respective t-statistics. Moreover, Table 8 shows the monetary value of each combination of characteristics and length of rolling slopes to invest in at the end of December of 2015.

In general, the values of the alpha from CAPM and from Fama French 3 factor model, presented in Table 8, are similar for the same strategy, however there is a pattern in the value of the alphas. Alphas increase with the decrease in the length of rolling slopes. All strategies have positive alphas, meaning that all strategies, indeed, create value for investors. However, when analyzing the statistical significance of the positive alphas, we realize that not all are significantly greater than zero. The Fama French 3 factor presents a higher number of significant alphas, in which all 3 and 1-year rolling slopes have significant alphas, for a confidence level of 98%. The 5-year rolling slopes present significant Fama French alphas in the combinations of 6 and 9 firm characteristics, at a 5% significance level. Lastly, the 10-year rolling windows never present statistically significant alphas. This seems to contradict Lewellen's results in which 10-year rolling averages lead to better performances.

Table 8
Quality of Performances
The table presents the alpha from CAPM and from Fama French 3 factor model (Alpha CAPM and Alpha 3FF respectively) as well as the t-statistics (t-stat) of both alphas. Alphas are presented in percentage terms. It is also presented the monetary value of strategy in December of 2015 (End Value) for the different set of characteristics and length of rolling windows. All strategies start with $20,000 in March 1982. Transaction costs are assumed to be 50 basis points for each sell order. The sample includes all common stocks from CRSP with valid data to compute monthly returns. Market data comes from CRPS monthly file and it is assumed to be immediately known. Accounting data is supplied by Compustat quarterly file and it is assumed to be known 2 months after the end of the quarter, following SEC rules. Kenneth French's Data Library supplies the risk-free rate and the 3 Fama French factors.

Model	Alpha CAPM	t-stat	Alpha 3FF	t-stat	End Value
12 Firm Characteristics					
10 years	0.20	0.87	0.11	0.66	$1,302,781
5 years	0.33	1.47	0.31	0.17	$1,970,987
3 years	0.49	2.20	0.57	3.35	$3,029,128
1 year	1.05	4.58	1.06	6.08	$28,843,056
6 Firm Characteristics					
10 years	0.22	0.74	0.24	0.98	$1,078,203
5 years	0.53	1.74	0.54	2.02	$3,773,128
3 years	0.59	1.82	0.71	2.51	$3,766,823
1 year	0.88	2.52	0.93	3.15	$9,587,996
3 Firm Characteristics					
10 years	0.11	0.38	0.10	0.40	$783,814
5 years	0.55	1.49	0.56	1.62	$3,316,738
3 years	0.63	1.87	0.74	2.44	$3,719,378
1 year	0.87	2.85	0.94	3.56	$12,122,719
8 Firm Characteristics					
10 years	0.17	0.79	0.07	0.43	$1,104,037
5 years	0.20	0.93	0.15	0.94	$1,152,551
3 years	0.40	1.88	0.46	2.78	$2,000,071
1 year	0.85	3.90	0.86	5.17	$12,882,861
9 Firm Characteristics					
10 years	0.52	1.24	0.45	1.14	$2,464,718
5 years	0.56	1.76	0.55	2.02	$4,727,733
3 years	0.57	1.74	0.72	2.57	$3,711,302
1 year	0.96	3.09	1.01	4.11	$15,458,393

In summary, the 1-year rolling slopes are the ones that create the greatest value to investors regardless of the number of characteristics and alpha considered. The combination of the 12 characteristics with the 1-year rolling slopes is the one which creates the highest value to investors, closely followed by the combination of 9 characteristics and 1-year rolling slopes. It is worth recalling that this set is composed of the 9 characteristics out of the 12 under analysis, stated by previous studies as having the ability to forecast future returns.

If we invested $20,000 in the beginning of March of 1982 in the S&P500, we would have $372,679 in December of 2015. Thus, when looking at the monetary value of each strategy in December of 2015, we clearly identify a superior total profitability of the analyzed strategies, especially using 1-year rolling slopes.

We have to highlight the value at the end of the sample period in the combination of all 12 characteristics and the 1-year rolling slopes, which is almost the double of the second highest value ($28.8M against $15.5M of the set with 9 characteristics and 1-year rolling slopes). Once again, it seems that, although 4 of the 12 characteristics are not statistically significant, they deliver positive value to the model. Otherwise, the models with 8 characteristics, in which we take out the 4 non-statistically significant, should perform at least as well as the models with 12 firm characteristics. However, the analysis verifies the positive value delivered by the 4 non-statistically significant characteristics.

3.6. Performance through time

Considering the previous results, this subsection focus on the analysis of the best performing combination (12 characteristics and 1-year rolling slopes). To simplify, we name our model with the 12 characteristics and the 1-year rolling slopes as the investment strategy. We analyze its performance over the sample and the impact of implementing the strategy in different time periods.

Figure 2 shows the cumulative excess returns of both the S&P500 and the investment strategy adjusted to a risk level of 5%. The investment strategy always presents a cumulative risk adjusted excess return higher than the S&P500. We can observe a similar drop in the cumulative returns in the end of 1987 of both the strategy and the S&P500, which can be explained by the crash in U.S. stock market in the renowned Black Monday. Although their similar behavior in this crash, they do not behave in the same way in the most recent financial events. While the S&P500 presents a drop in the sequence of the burst of the dot-com bubble, the investment strategy sees its cumulative returns increased. During the 2008 financial crisis, the S&P500 has a sharp loss in its cumulative returns, whereas the investment strategy is able to maintain its cumulative returns steady. On the one hand, there is evidence of a counter-cyclical ability of the investment strategy in bearish markets, on the other hand, there is a cyclical ability in bullish markets.

Figure 2. Cumulative risk adjusted returns, 1982-2015
The figure plots the cumulative excess returns adjusted to a 5% risk level for the S&P500 and for the investment strategy between March 1982 and December 2015. The investment strategy combines the 12 characteristics with the 1-year rolling slopes. The sample includes all common stocks from CRSP with valid data to compute monthly returns. Market data comes from CRPS monthly file and it is assumed to be immediately known. Accounting data is supplied by Compustat quarterly file and it is assumed to be known 2 months after the end of the quarter, following SEC rules. Kenneth French's Data Library supplies the risk-free rate.

With an overall view of the behavior of the cumulative risk adjusted excess returns along the entire sample, it is relevant to study the performance of the strategy throughout the sample. Moreover, we want to understand the impact in performance when implementing the strategy in different time periods. Table 9 shows the investment strategy performance as well as the performance of S&P500 over the sample. The S&P500 is our proxy for a passive strategy given all its attributes. The following 4 implementation periods are considered: March 1982, January 1991, January 2001, and January 2011. The time span is divided into four almost identical periods, and it is also ensured that they start right after negative, yet important, financial events. Moreover, the initial amount available to implement the strategy grows with inflation, which translates into a value of $20,000 in March 1982, $28,467.92 in January 1991, $37,021.28 in January 2001 and $46,633.89 in January 2011.

Table 9
Performance through time
The table exhibits the time-series average return (Avg), standard deviations (Std) and annualized Sharpe Ratio (Ann. Sharpe) of the realized monthly returns for both the strategy with 12 firm characteristics and 1-year rolling slopes and the S&P500. It also shows the number of different stocks (No Stocks) invested at the beginning of each period. The sample is divided into 4 periods, in which the performance of the investment strategy is compared with the S&P500. The strategy performance is simulated for different implementation dates. The table also provides the monetary value available in the strategy for the different implementation dates (End Value) and the monetary value if invested in the passive strategy (S&P500 End Value). The value invested at the beginning of each implementation date grows with the inflation which represents $20,000 in March 1982, $28,467.92 in January 1991, $37,021.28 in January 2001 and $46,633.89 in January 2011.

	Implementation date				
	03-1982	01-1991	01-2001	01-2011	S&P500
1982-1990					
Avg	2,03				1,15
Std	5,76				4,84
A. SR	0,84				0,37
No Stocks	20				500
1991-2000					
Avg	2,01	2,38			1,25
Std	6,97	11,65			3,85
A. SR	0,81	0,59			0,78
No Stocks	151	28			500
2001-2010					
Avg	2,27	2,24	2,35		-0,04
Std	5,78	5,85	6,62		4,61
A. SR	1,24	1,21	1,13		-0,18
No Stocks	300	300	37		500
2011-2015					
Avg	1,28	1,35	1,36	1,56	0,86
Std	5,02	5,12	5,13	5,46	3,37
A. SR	1,07	1,11	1,10	0,99	0,88
No Stocks	300	300	300	46	500
End Value	$28,843,056	$6,678,533	$996,430	$98,879	
S&P500 End Value	$372,679	$174,070	$56,001	$73,862	

Notwithstanding the implementation date, the strategy always yields higher, but also more volatile average monthly returns than the S&P500 (benchmark). Nevertheless, the strategy presents higher annualized Sharpe Ratios than the benchmark. This means that the investment strategy has a higher efficiency between risk and return above the risk-free rate than the passive strategy.

The passive strategy presents a higher Sharpe Ratio than our strategy only between 1991 and 2000, when the active strategy is implemented in January 1991. This can be justified by the lack of diversification benefits at the beginning of the implementation of the active strategy, which does not occur in the passive strategy. The passive strategy is an investment in the stock index with the 500 U.S. companies with the highest market capitalization, therefore it is a largely diversified position. On the contrary, the active strategy starts by investing in just 20 to 47 stocks, increasing with the money available to implement it. Therefore, a long position in the active strategy is not diversified at all until the monetary value of the strategy increases. Only after that, does the model allow for the possibility of investing in more companies. As explained before, the Sharpe Ratio underestimates the active strategies risk-return performances, notwithstanding, it allows comparing the performances of passive and active strategies.

The period between 2001 and 2010 is when the active strategy has a higher performance, comparing with the S&P500. This period was significantly and negatively affected by the collapse of both the dot-com bubble and the 2008 financial crisis. These events affected the U.S. stock market so negatively that the S&P500 yielded a negative average monthly return during this period. Nevertheless, the active strategy was able to yield considerably high positive returns and remarkable Sharpe Ratios between 1.13 and 1.24. Interestingly, it was in this period that the active strategies implemented in March 1982, in January 1991 and in January 2001 provide the highest Sharpe Ratio. Reinforcing, in some sense, the idea of a counter-cyclical ability of the strategy during times of financial distress.

Overall, the differences in the monetary value of the active strategy and the passive strategy are extremely high across all time periods. The period with the lowest difference between the monetary value of both strategies is the one starting in January 2011, with a difference of $25,017, having both strategies started with $46,633.89. Even though this is the smallest difference, this means that, in a 5 year period, the active strategy had a total return 53.65 percentage points higher than the total return of the passive strategy. When looking at the highest difference, we notice that, when implementing both strategies in March 1982, our strategy surpasses the benchmark by $28,470,377, representing a total return 1,423.52 percentage points higher.

Figure 3 shows the distribution of average monthly returns for an investment horizon of 20 years. The first implementation date is March 1982 and the last is January of 1996, thus investing until December 2015. In total, figure 3 shows 167 implementation dates with 1 month intervals, which allows us to study the impact of different implementation dates in our strategy more intensively. Consequently, it is possible to identify a consistent superior average monthly return of the investment strategy using 12 characteristics and 1-year rolling slopes. The maximum average monthly return of the S&P500 is 1.07%, while the minimum average monthly return of the investment strategy is 1.7%.

Figure 3. Return distribution for an investment horizon of 20 years

The figure shows the distribution of average monthly returns for different implementation dates with an investment horizon of 20 years. The first implementation date is March 1982 which invests until February 2002. It is considered an interval of 1 month between implementation dates. The last implementation date is January 1996 which invests until December 2015. The x-axis presents bins of average monthly returns (values in percentage). The y-axis identifies the number of times an investment strategy had an average monthly return inside each bin. Histogram Investment Strategy is related with the investment strategy with 12 characteristics and 1-year rolling slopes. Histogram S&P500 is related with the S&P500 index.

Despite the fact that our strategy presents higher returns, the most important factor to consider is the risk-reward efficiency. The results show that investing in the S&P500 during 20 years presents a minimum annualized Sharpe Ratio of 0.13, a maximum of 0.46 and an average of 0.33 among the several implementation dates considered. On the other hand, the investment strategy presents a minimum annualized Sharpe Ratio of 0.41, a maximum of 0.69 and an average of 0.55. These results point towards the fact that a higher return from the strategy is not just related with higher exposure to risks, but a higher risk-return efficiency. Additionally, the superior performance of the strategy is not due to investing on a specific date. Among the 167 implementation dates, investing in the S&P500 never yields a higher (or even equal) Sharpe Ratio than the one yielded by the investment strategy. All in all, it seems that the investment strategy yields a superior performance, being preferable to a passive position in the S&P500.

4. Conclusion

Prediction of returns is a crucial topic in Finance, not only due to its several purposes, such as asset pricing, portfolio maximization and development of investment strategies, but also because of its impact on the economy. These are several reasons that make it such a studied and discussed topic in academic literature.

The main objective of this thesis is to find a group of firm characteristics, which combined with past cross-sectional slopes, allows investors to predict future stock returns in real-time. Ultimately, the aspiration is to be able to draw an economically feasible investment strategy to be implemented by small investors. For that purpose, we develop a dynamic model which requires an initial capital of $20,000.

Based on our results, we can state that cross-sectional regressions allow predicting, with a high degree of accuracy, future stock returns. Contrarily to Lewellen (2014), we find that the 1-year rolling window is the estimation length that yields higher average returns and Sharpe Ratios, while the 10-year rolling slopes are the ones that deliver the lowest ones. Although our results demonstrate that some characteristics are not statistically significant at a 5% significance level, it seems that all the 12 firm characteristics add value to the model. Therefore, the combination of the 12 characteristics with 1-year rolling slopes is the one presenting the best performance.

All combinations under analysis surpass the S&P500 in terms of average monthly return, Sharpe Ratio and profit over the initial amount invested. The several combinations yield average monthly returns ranging from 1.21% to 2%, annualized Sharpe Ratios as low as 0.39 and as high as 0.96. In terms of value created to investors (CAPM alpha), it ranges from 0.11% to 1.05%. In spite of the picked combination, it beats the passive strategy of investing in the S&P500, creating value to investors (positive alphas), which is evidence against the market efficiency hypothesis. Overall, the combination with the best performance presents the highest values in all measures indicated above, delivering a total profit of $28,823,056 over the entire sample.

Despite the performances obtained, there are some limitations in our analysis. Firstly, all accounting variables are obtained in quarterly reports. The lack of a higher measurement frequency of these variables leads us to consider constant values over the entire quarter, although they change on a daily-basis. As proved, these variables are important in the implementation of the investment strategy, thus, a higher frequency measurement may dramatically change positions and returns. Moreover, quarterly reports are not required to be audited, which raises uncertainty about the truthfulness of the reported values.

Secondly, the accounting data is assumed to be known 2 months after the end of the quarter, following SEC rules. However, these rules were only imposed in September 2002, meaning that it does not cover the entire timespan considered. Therefore, the 2 months assumption may be imperfectly used in a substantial part of our sample (January 1961 until August 2002). Additionally, companies may not take the maximum time allowed to publish their quarterly reports. In other words, the information may be available prior to what we consider, affecting negatively the predictive power of our model.

Moreover, the conclusions drawn are only true for the sample used and timespan analyzed. The application of the same methodology in a different time period and/or different markets (others than the U.S. stock market) may present distinctive results. Thus, the conclusions of this thesis should not be extrapolated out of our sample.

Furthermore, transaction costs are set in line with academic literature. As explained before, transaction costs have a relevant negative impact on returns, limiting the evolution of our strategy. Investors attempting to implement our strategy must pay attention to the value of transaction costs that they would have to bear. Any other value than the one considered in the analysis will yield different results which may lead to contradictory conclusions. In addition, we do not consider capital gain taxes. Given the short-term nature of the capital gains/losses in our strategy, the applicable tax rate varies across investors. The capital gains/losses are combined with investor's revenues, determining the tax rate that the investor will have to pay. The tax rate goes from 10% to 39.6%. Investing in the strategy could lead the investor to pay a higher tax rate over his labor income, due to capital gains.

Additionally, we suggest some alternative analysis that could be incorporated in the strategy presented. A different source of accounting data with higher measurement frequency can be considered to access the impact of the higher frequency in the predictive ability of the model to forecast future returns. Alternatively, a lower rebalancing frequency could be tested (such as bimonthly or quarterly rebalancing), which would allow to reduce transaction costs. A lower rebalancing frequency also reduces the impact of assuming accounting data to be stable over the entire quarter. Lewellen (2014) tests alternative rebalancing frequencies, however the alternatives he tests are too low in frequency (semester and annual rebalancing).

With the objective of reducing transaction costs, several thresholds regarding differences in expected returns could be investigated. As a result, investors would be advised on whether they should change investment positions. This would lead investors to reduce transactions and to change positions only whenever the additional return compensates the costs. Future studies can also examine the impact in the strategy performance of starting to invest with a different monetary value, as well as with different initial and maximum numbers of companies invested in. Additionally, alternative rates at which the number of companies increases can be tested. This rate affects the evolution of the diversification benefit delivered to the strategy.

Our last suggestion is to test the behavior of the strategy to alternative firm characteristics and estimation windows. As a result, this would affect the predictive capability of the model, and therefore its performance. However, it is important to take into account the difficulty of investors in accessing the required data.

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