Price Competition with Cost Heterogeneity and Prominence

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Dissertation submitted in partial fulfilment of requirements for the MSc in Economics, at the Universidade Católica Portuguesa, September 12, 2022.

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Abstract

This thesis aims to show how in a price comparison platform, where prominence is auctioned to the retailers with different marginal costs, the low-cost retailer will, counter-intuitively, under certain conditions, be incentivized to charge higher prices on average than its high-cost counterpart. This will happen because consumers will have to incur heterogeneous search costs when shopping in the platform, therefore some consumers will purchase from the prominent retailer while others will continue shopping and purchase from the cheapest firm. This implies that the prominent firm faces a more inelastic demand and, as such, is more strongly incentivized to charge higher prices. The low-cost firm can, for the same price, extract higher margins from consumers, therefore, if the proportion of consumers with high search costs is high enough, then the low-cost retailer will outbid the other firm in the auction for prominence. A prominent low-cost firm faces contradictory effects, the lower marginal cost incentivizes lower average prices yet the exclusive access to a segment of consumers with rigid demand incentivizes higher average prices. If that segment is sufficiently large the latter effect will dominate and the low-cost firm will charge a higher expected price than the high-cost firm.

Keywords: Game Theory, Pricing, Platforms, Prominence

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Resumo

É o objectivo desta tese explicar como, numa plataforma para comparar preços, onde proeminencia é leiolada a retalhistas com custos marginais diferentes, o retalhista com custos marginais mais baixos, contra-intuitivamente, em certas condições, vai cobrar preços mais altos em média do que os outros. Isto acontece porque os consumidores têm que incorrer custos de procura heterogéneos quando utilizam a plataforma, consequentemente alguns consumidores compram ao retalhista prominente enquanto que outros compram ao retalhista que pratica o preço mais baixo. Isto implica que a empresa prominente enfrenta uma procura menos elastica e que, assim sendo, é mais fortemente incentivada a praticar preços mais altos. A empresa com custos mais baixos consegue, para o mesmo preço, obter margens mais elevadas. Consequentemente, se a proporção de consumidores desinformados for suficientemente elevada, esta empresa está disposta a licitar mais do que as outras para obter prominencia. Uma empresa prominente com custos baixos está sujeita a efeitos contraditórios, os custos marginais inferiores incentivam preços médios mais baixos, no entanto acesso exclusivo aos consumidores desinformados incentiva preços médios mais altos. Se este segmento da procura for suficientemente grande o segundo efeito domina e a empresa com os custos mais baixos cobra um preço médio mais alto que uma empresa com custos mais altos.

Palavras Chave: Teoria de Jogos, Preços, Plataformas, Proeminência

Acknowledgements

First and foremost I would like to show my deep appreciation for the help and guidance I received from my supervisors, Professor Catarina Reis and Professor Samir Mamadehussene, whose extensive comments and suggestions were of paramount importance for the writing of this thesis.

I would also like to thank again Professor Samir Mamadehussene for giving me the original idea for the model and for introducing me to the literature of a field of economics I had never studied before, the research on platforms and prominence.

Finally I would like to thank my family and friends for the invaluable support given to me throughout this time.

Contents

1 Introduction

It is well established in the economic literature that when firms which operate at different marginal costs compete in a market, those with higher marginal costs charge higher prices (Reinganum 1978). This is true even in the case of Bertrand competition (Blume 2003). To understand why this happens we should look at the incentives faced by firms when setting prices. There are two direct effects in a firm's profit when increasing its price. The first one, the benefit, is that, if the firm increases the price, it will have a higher markup. The surplus that is obtained for each unit that is still sold will increase. The other effect, the drawback, is that, for a standard demand, by increasing the price total sales will decrease. The firm's marginal cost will affect these two effects differently, a firm with higher marginal costs will have a lower markup, hence the drawback from increasing prices will not be as costly, while the upside from increasing prices will not be affected by the marginal costs. This will incentivize high-cost firms to charge higher prices when compared to their low-cost counterparts. Notice that this will happen regardless of how competitive the market is, even in a monopoly a higher marginal cost will still lead to a higher price.

My thesis shows how, under specific circumstances, in a market with cost heterogeneity, the firms with lower marginal costs might end up charging the highest prices on average. This happens when we have a price-comparison platform, that consumers can use to search the various producers and compare their prices, such as Amazon or eBay. Such platforms sell prominence to firms, by, for example, making it so that their name appears on top of all the others. This impacts prices, because the firm-level demand of the prominent firm is different than that of the other non-prominent firms. I consider two groups of consumers, informed consumers, with low search costs, who will search the platform for the best deal and uninformed consumers, with high search costs, that buy from whatever firm is prominent. Because uninformed consumers always buy from the prominent firm, as long as the price is not higher than the maximum amount they are willing to pay for the good, the firm level demand of the prominent firm is less elastic than that of the non-prominent firm.

I show that, under certain market parameters, the low-cost firm is willing to bid more for prominence than any of the high-cost firms. And since having prominence makes the firm level demand faced by that firm less elastic, that firm charges higher prices than those of its high-cost competitors. This happens if the proportion of consumers with higher search costs is large enough. Since the low-cost firm can get a higher markup it has a stronger incentive to buy prominence. Meaning that the less price sensitive customers will be allocated to the low-cost firm, leading to it charging higher prices on average.

In the first stage of the game the two firms compete for a segment of demand, the uninformed consumers. Whoever becomes prominent is incentivized to charge higher prices, as that firm faces a less elastic demand. So, by becoming prominent, the firm is gaining exclusive access to the uninformed consumers, but is also partially abdicating the ability to sell to the informed consumers, since, by charging higher prices, it'll be less likely for that firm to have the lowest price in the market. The low-cost firm can extract a higher markup, so if there is a large number of uninformed consumers that firm is willing to bid more for prominence than the high-cost firm. This happens because the gain from becoming prominent is that the firm will be able to sell to a large number of consumers at a high price, forgoing selling to the small number of informed consumers. If the opposite is true, if there is a small number of uninformed consumers, then the benefit from prominence for the low-cost firm, which is the ability to sell to a small number of consumers at a high price, will not be large enough to compensate the cost of forgoing selling to the large number of informed consumers, so the low-cost firm is not as strongly incentivized to bid for prominence.

This also happens in other similar settings, whenever there is a market for prominence. When firms advertise in order to have consumers remember their brand or products instead of their competitor's, they are investing in prominence. They are using resources trying to make their firm-level demand less elastic. Because advertising opportunities are scarce, firms end up competing for prominence, just like they will in this model. There's only 15 minutes to advertise during the halftime of a football game, just like there's only a limited amount of spots to advertise on a web page. Low-cost firms, due to being able to extract larger surpluses, often have higher demand for advertising, leading to less elastic demands and higher prices for those firms when compared to high-cost firms. Another case when this might happen is when firms compete for the best shelves at a supermarket. Many consumers buy from the shelves at eye-level more often, without searching for alternatives on other shelves. Since there's a limited amount of space on those shelves, firms compete for placement. Again they are competing for prominence that makes their firm-level demand less elastic, incentivising prominent firms to increase prices and potentially leading low-cost firms, which might have higher demand for prominence due to their higher margins, to charge higher prices. This paper shows under what circumstances this happens.

2 Literature Review

2.1 Search Costs and Production Costs

Reinganum 1978 has shown that, in a market where prices are unknown to identical consumers who incur search costs and have elastic demands, cost heterogeneity will lead to price dispersion and the firms with higher marginal costs will charge higher prices. A lot of research has been made about markets where consumers have different search costs and how that can lead to price heterogeneity. Varian 1980 shows how, in a market where there are heterogeneous search costs (but consumers have unitary demands), it's profitable for firms to randomize prices in order to discriminate between informed and uninformed consumers. Shelegia 2012 builds on Varian's model by introducing cost heterogeneity for firms and shows how even a small asymmetry in cost can lead to a large price dispersion.

2.2 Prominence and platforms

It's clear that prominence can have a significant effect on demand, whether through advertising, branding, or how the firm is ranked in a price-comparison platform. Consumers will often sample the prominent firm first. Ho and Imai 2006 and Meredith and Salant 2013 show how the order by which candidates are listed in voting ballots impacts both voter share and who wins office. Consumers often use price-comparison platforms to choose a firm to buy from. The ranking of the various firms on the platform impacts the amount of clicks it receives, as shown by Ansari and Mela 2003, Ghose and Yang 2009, Baye et al. 2006, Agarwal, Bain, and Chamberlain 2012 and Glick, Millstein, and Orsillo 2014.

Being prominent has a negative impact in demand elasticity. Arbatskaya 2007 shows that firms that are ranked higher in the consumer search order charge higher prices.

This paper aims to show how low-cost firms have higher demand for prominence due to having higher markups. This impacts the price distribution by incentivizing the low-cost firms to charge higher prices.

3 Model Setup

Consider a homogeneous good and a unit consumer with inelastic demand over that good. Each consumer is willing to pay up to a certain price (v) for one unit of the good. Consider also a platform where firms can advertise their price to consumers, however one firm will be able to buy prominence in that platform, and consumers incur a search cost of finding the firm with the lowest price. For heterogeneous search costs there will be two types of consumers, informed consumers who search the platform and buy from the firm offering the lowest price ($1 - \theta$) and uninformed consumers who buy from the firm with prominence in the platform (θ). Consider also that, if two firms have the same lowest price in the market, the informed consumers will split equally between both firms.

Consider a sequential one-shot game with two stages. In the first stage there's an second price auction for prominence and in the second stage firms will compete simultaneously on prices. In that stage, the firm with prominence (j) will face the following demand

$$D_j(p_1, ..., p_N) = \begin{cases} \frac{1-\theta}{n} + \theta & \text{if } p_j \leq p_i \wedge p_j \leq v, \ \forall \ i \neq j \\ \theta & \text{if } p_j > p_i \wedge p_j \leq v, \ \text{for some } i \neq j \\ 0 & \text{if } p_j > v \end{cases}$$

Where N is the number of firms in the market and n is the number of firms charging the lowest price.

A firm without prominence (k) will face the following demand

$$D_k(p_1, ..., p_N) = \begin{cases} \frac{1-\theta}{n} & \text{if } p_k \leq p_i \wedge p_k \leq v, \ \forall \ i \neq k \\ 0 & \text{if } p_k > p_i \vee p_k > v, \ \text{for some } i \neq j \end{cases}$$

Notice that, for the sake of simplicity, we are considering that in the case where several different firms charge the same lowest price in the market, demand from the informed consumers is split equally among those firms. If instead we consider that demand is distributed randomly or one firm, specific or arbitrary, gets the entire demand the results will be the same.

Consider also two different types of firms, low-cost and high-cost firms, all with constant marginal costs and no fixed costs. We'll normalize the marginal cost of the low-cost types to zero and the marginal cost of the high-cost types shall be denominated by c. Let's assume $v > c$, otherwise the high-cost firms are not willing to supply a positive quantity of the good and there's no firm heterogeneity. The equilibrium prices will depend on the number of firms of each type that are in the market.

4 Equilibrium with two firms

We have two firms, both with different constant marginal costs and no fixed costs, a high-cost firm with marginal cost c and a low-cost firm with marginal cost normalized to zero.

The game proceeds as follows:

1. Both firms will bid in an auction for prominence;

2. Both firms set their price simultaneously.

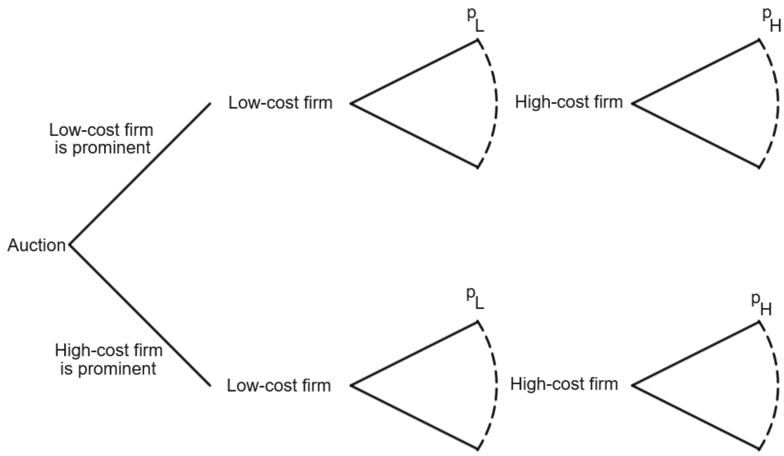


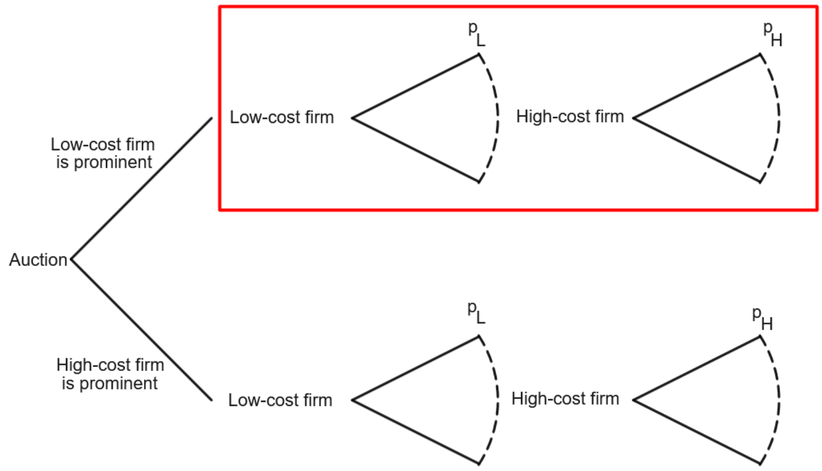
Figure 1: Decision tree

4.1 Second stage

In the second stage of the game there are two sub-games, one where the low-cost is prominent and another where the high-cost is prominent.

4.1.1 The low-cost firm has prominence

We shall begin by analyzing the sub-game where the low-cost firm is prominent.



Firm-level demands will be the following

$$D_L = \begin{cases} 1 & \text{if } p_L < p_H \wedge p_L \leq v \\ \frac{1-\theta}{2} + \theta & \text{if } p_L = p_H \wedge p_L \leq v \\ \theta & \text{if } p_L > p_H \wedge p_L \leq v \\ 0 & \text{if } p_L > v \end{cases}$$

$$D_H = \begin{cases} 1-\theta & \text{if } p_H < p_L \wedge p_H \leq v \\ \frac{1-\theta}{2} & \text{if } p_H = p_L \wedge p_H \leq v \\ 0 & \text{if } p_H > p_L \vee p_H > v \end{cases}$$

Given these demand functions we can derive the profit function for each firm,

$$\pi_L = \begin{cases} p_L & \text{if } p_L < p_H \wedge p_L \leq v \\ p_L \frac{1+\theta}{2} & \text{if } p_L = p_H \wedge p_L \leq v \\ p_L \theta & \text{if } p_L > p_H \wedge p_L \leq v \\ 0 & \text{if } p_L > v \end{cases}$$

$$\pi_H = \begin{cases} (p_H - c)(1-\theta) & \text{if } p_H < p_L \wedge p_H \leq v \\ (p_H - c)\frac{1-\theta}{2} & \text{if } p_H = p_L \wedge p_H \leq v \\ 0 & \text{if } p_H > p_L \vee p_H > v \end{cases}$$

Let's start by trying to find a pure strategy Nash equilibria.

If the low-cost firm were to charge any price $p_L \in (c, v]$ then the high-cost firm's best response would be to charge slightly below p_L and sell to $1-\theta$ consumers, if the low-cost firm were to charge $p_L \leq c$ then any price $p_H > p_L$ would be optimal for the high-cost firm, as they would always have zero profits and any $p_H \leq p_L$ would lead to losses. If the low-cost firm charges $p_L > v$ then the best response of the high-cost firm would be to charge $p_H = v$.

If $p_H \leq v$ then the low-cost firm's best response would either be to charge slightly below p_H and sell to all consumers or to charge v and sell to θ consumers, whatever is more profitable. The low-cost firm's profit if they charge v and sell only to the uninformed consumers is $v\theta$, the profit if they undercut the high-cost firm's price and sell to all consumers is p_H. If $p_H > v$ then the low-cost firm's best response is to charge v.

The following graph plots the best responses for both firms:

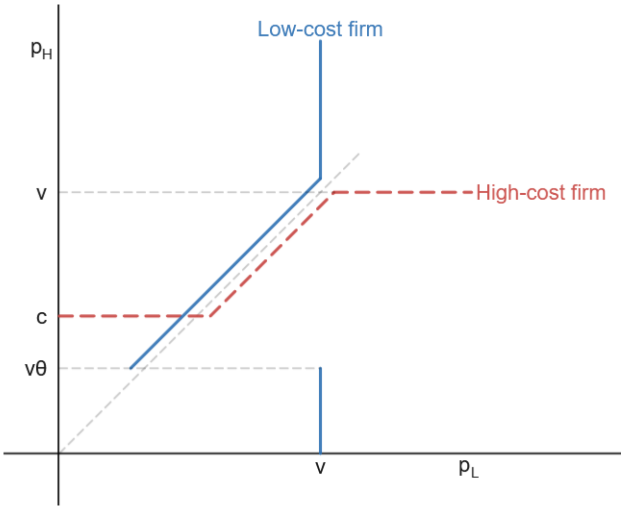


Figure 2: Best responses when $v\theta < c$

This is the case where $v\theta < c$. As we can see there is an intersection of the two best response curves at $(p_L, p_H) = (c - \epsilon, c)$. If the proportion of uninformed consumers, θ, is small (smaller than $\frac{c}{v}$) the incentive for the prominent low-cost firm to charge higher prices is not strong enough and we get the Bertrand result for asymmetric firms, a pure strategy Nash equilibrium where the low-cost meets the entire demand at a price lower but arbitrarily close to the marginal cost of the other firm, c.

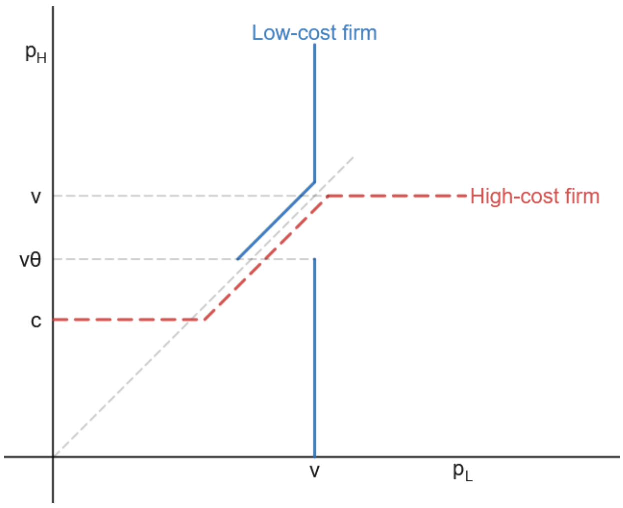


Figure 3: Best responses when $v\theta > c$

This is the case where $v\theta > c$. As we can see there is no intersection of the best response curves. Therefore, in this case, we have a mixed strategy Nash equilibrium where both firms will set a random price.

In order to find the mixed strategy NE we will have when $v\theta > c$, we need to know in what interval will firms mix the price.

Lemma 1. *No firm will price above the reservation price with positive probability,* $f_i(p) = 0 \; \forall \; p > v$

Proof. Looking at the demand function, we can see that $p > v$ is not optimal, since it would lead to no sales, and no profits, regardless if the firm has prominence or if it has the lowest price in the market. □

It's also clear that the high-cost firm won't be willing to price below it's marginal cost c, as it would either lead to no sales, and no profits, or $1-\theta$ sales, and negative profits. From there we can also conclude that the low-cost firm is also not willing to price below it's competitor's marginal cost, since charging below c would not increase demand, only decrease the price, leading to lower profits than what they could obtain by charging c. Further we will see that this constraint is not binding, and the lower bound of both firms price strategy is higher than c.

Lemma 2. *There is no gap where the* $f(p) = 0$*.*

Proof. Let's prove by contradiction. Consider that there is a gap (p_1, p_2) where $f_i(p) = 0 \; \forall \; p_1 < p < p_2$ in the price strategy of one firm, i. It would never be profitable for the other firm to charge a price in that gap other than, perhaps, a price lower but arbitrarily close to p_2. Now consider a price, $\tilde{p}$, such that $p_1 < \tilde{p} < p_2$. Firm i would have higher expected profits charging $\tilde{p}$ than charging p_1 and therefore not be willing to play a mixed strategy. It follows that there must be no gaps for the mixed strategy Nash equilibrium to exist. □

Lemma 3. *The upper bound of the price distribution of the non prominent firm is never higher than the upper bound of the price distribution of the prominent firm,* $\overline{p}_{NP} \leq \overline{p}_P$ *(*$\overline{p}_P$ *and* $\overline{p}_{NP}$ *are the upper bounds of the price distributions of the prominent and non-prominent firms respectively)*

Proof. The non-prominent firm will only have positive sales if it sells to the informed consumers, which will only happen if it has the lowest price in the market. Therefore there's no incentive for that firm to charge any price above the upper bound of the price strategy of the prominent firm. □

Lemma 4. *The lower bound of the price distribution of both firms is the same,* $\underline{p}_H = \underline{p}_L$*, as long as* $\underline{p}_L \geq c$

Proof. If one firm were to charge a price $\tilde{p}$ such that $\tilde{p} < \underline{p}_i$, $\underline{p}_i$ being the lower-bond of the other firm's strategy, it would have higher profits by charging a price higher than $\tilde{p}$ and lower than $\underline{p}_i$ rather than $\tilde{p}$, as it would sell the same quantity with certainty but at a higher price. This is true for the low-cost firm always and for the high-cost firm as long as $p_H \geq c$. This proof also implies that there can't be a point mass at $\underline{p}$. □

Proposition 1. *The support of the distribution of the price charged by the prominent low-cost firm is the following,* $p_L \in [v\theta, v]$

Proof. Considering that the non-prominent high-cost firm will never price above the high-bound price of the prominent low-cost firm, any $\overline{p}_L < v$ would be sub-optimal for the low-cost firm, as charging $\overline{p}_L$ or v would lead to the same sales (since with certainty it will not have the lowest price in the market) and the latter would lead to higher profits.

The low-cost firm will have the following profit when charging v

$$\pi_L(v) = v\theta$$

This firm must be indifferent between v and any other price in it's price strategy. If it charges a price equal to the lower bound it will sell to the informed consumers with certainty, since both firms have the same lower bound and no point mass at that price. From here we can determine $\underline{p}$

$$\pi_L(\underline{p}) = \pi_L(v)$$
$$\underline{p} = v\theta$$

□

Lemma 5. *There are no point masses in the price strategy of the non-prominent firm.*

Proof. If the price strategy of the non-prominent firm was such that there was a positive probability of charging $\tilde{p}$, such that $\underline{p}_i \leq \tilde{p} \leq \overline{p}_i$, then the prominent firm would not be indifferent between charging $\tilde{p}$ and a price lower but arbitrarily close to $\tilde{p}$ and therefore not willing to play a mixed strategy. □

Lemma 6. *There is a point mass in the price strategy of the prominent firm at the upper bound.*

Proof. If the prominent low-cost firm's price strategy where to follow a non-degenerate distribution then the non-prominent high-cost firm would not be willing to charge a price equal to v or even any price arbitrarily close to v, since it would not be the lowest price in the market and therefore have no profits. If this firm charges a $p_H = v\theta$ it will sell to the informed consumers with certainty and have the following profits

$$\pi_H(v\theta) = (v\theta - c)(1 - \theta)$$

Since this mixed strategy Nash equilibrium exists when $v\theta > c$ it follows that the low-cost firm must have positive expected profits when playing a mixed strategy.

The Nash equilibrium only exists as long as there is a positive probability that the low-cost firm charges v. It must, therefore, be the case that the low-cost firm follows a degenerate distribution with a point mass at the reservation price. □

This also implies that $p_H \in [v\theta, v)$. If $\overline{p}_H$ was not arbitrarily close to v then the low-cost firm would not be willing to charge any price p such that $\overline{p}_H < p < v$. We know from Lemma 2 that there can't be any gaps.

Now we need to find the distribution of prices for each firm such that the other firm is willing to mix. Again, we know that the low-cost firm will be indifferent between any price such that $p \in [v\theta, v]$, so

$$\pi_L(v) = E[\pi_L(p)]\ \forall\, p \in [v\theta, v]$$

Charging v will lead to θ sales with certainty, since this firm would never have the lowest price in the market and therefore sell only to the uninformed consumers.

$$\pi_L(v) = v\theta$$

Charging any price p will either lead to θ sales, if it is the highest price, or to 1 sales, if it is the lowest price. The probability that the low-cost firm will have a lower price than that of the high-cost firm can be obtained from the price strategy of the high-cost firm. The cumulative distribution function, $F_H(p)$, gives us the probability that the price charged by that firm, p_H, will be less than or equal to a price, p. Therefore the probability that $p_L < p_H$, for a given p_L, is equal to $1 - F_H(p_L)$.

The profit of charging a price, p, such that $v\theta \leq p \leq v$ is

$$E[\pi_L(p)] = p(\theta + (1-\theta)(1 - F_H(p)))$$

It follows that, for the low-cost to be indifferent between any price, $p \in [\theta v, v]$,

$$F_H(p) = 1 - \frac{(v-p)\theta}{p(1-\theta)}$$

We also know that the high-cost firm will be indifferent between any price, p, such that $p \in [v\theta, v)$, so

$$\pi_H(v\theta) = E[\pi_H(p)]\ \forall\, p \in [v\theta, v)$$

If the high-cost firm charges $v\theta$ it will have $1-\theta$ sales, since it will be the lowest price in the market with certainty and therefore sell to the informed consumers.

$$\pi_H(v\theta) = (v\theta - c)(1 - \theta)$$

Charging a price p, such that $v\theta < p < v$, will either lead to $1 - \theta$ sales, if it is the lowest price in the market, or to no sales otherwise. Just like before, the probability that this firm has the lowest price in the market will come from the price strategy of the other firm. Let $F_L(p)$ be the cumulative distribution function of the low-cost firm's price. The probability that $p_H < p_L$, for a given p_H, is equal to $1 - F_L(p_H)$.

The profit of charging a price, p, such that $v\theta \leq p < v$ is

$$E[\pi_H(p)] = (p - c)(1 - \theta)(1 - F_L(p))$$

It follows that, for the high-cost to be indifferent between any price, $p \in [\theta v, v)$,

$$F_L(p) = 1 - \frac{v\theta - c}{p - c}$$

We know that the low-cost will charge v with a positive probability, and that it'll never charge above that value, so

$$P(p_L = v) = 1 - F_L(v) = \frac{v\theta - c}{v - c}$$

Below we can see the cumulative distribution functions for the prices of each firm ($F_L(p)$ in blue and $F_H(p)$ in dashed red). Notice that, as we have seen, the low-cost firm follows a degenerate cumulative distribution function. We can see the point mass probability at $p = v$ where the probability jumps to 100%.

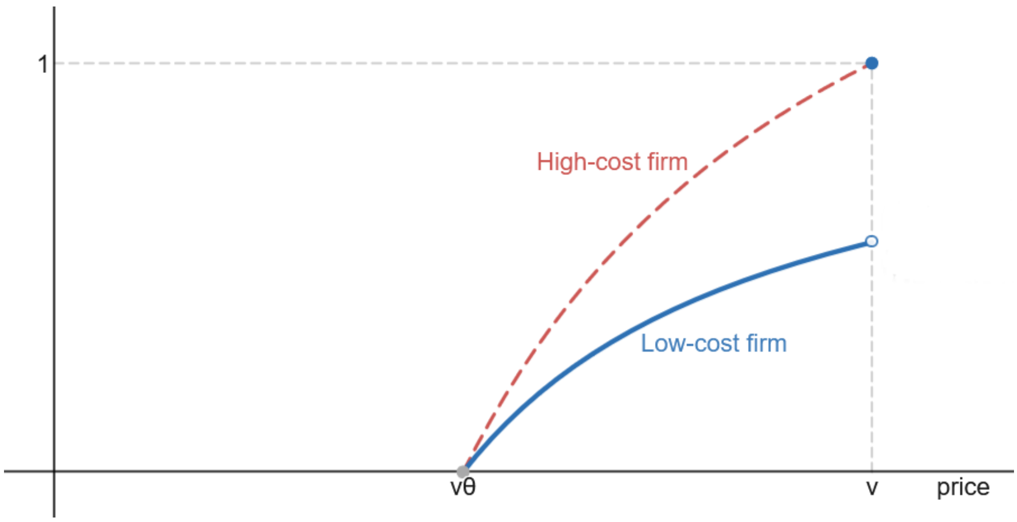


Figure 4: Equilibrium cumulative distributions

Below we can see the probability density functions for the prices of each firm ($f_L(p)$ in blue and $f_H(p)$ in dashed red).

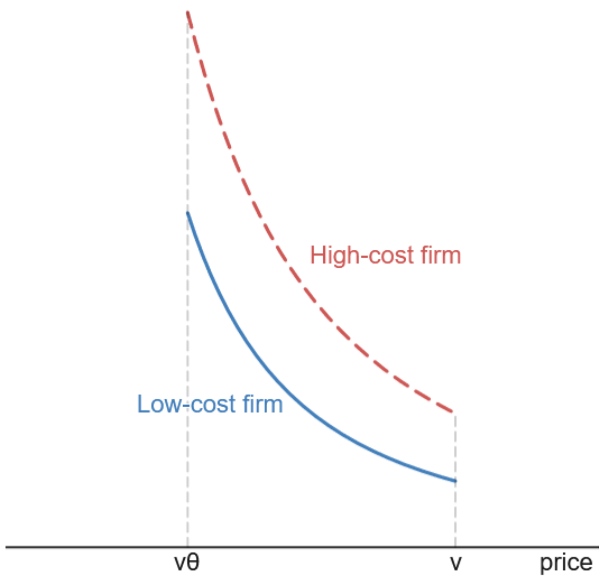
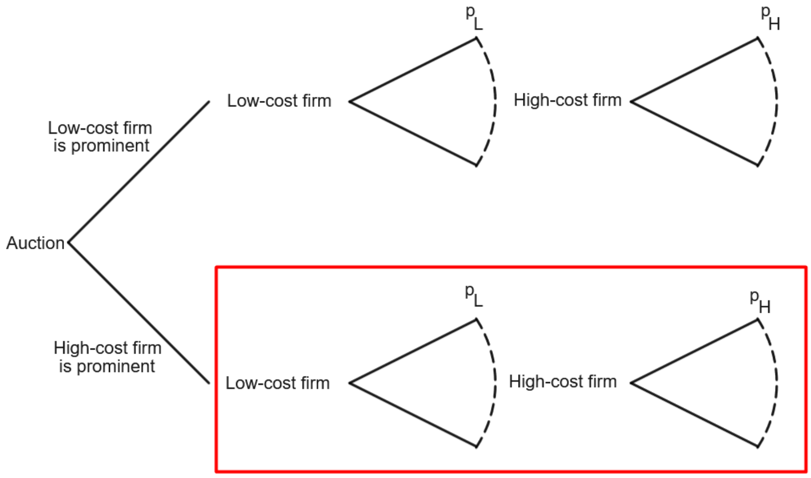


Figure 5: Equilibrium probability densities

As we can see the probability density is higher for lower prices, as charging higher prices decreases the probability of having the lowest price in the market. A similar result is found in Varian 1980, there the graph of the density is "U-shaped", *stores tend to charge extreme prices with higher probability than they charge intermediate prices*. The intuition was that firms either charge low prices to try to sell to the informed consumers or charge high prices to exploit the surplus of the uninformed consumers. Unlike Varian's model, where every firm had sales to uninformed consumers, here only one firm has access to that segment of the market, the low-cost firm. Remember that there is a point mass probability at $p = v$ and with that we get a "U-shaped" density, the low-cost firm either charges low prices to sell to the informed consumers or charges $p = v$ to fully extract the surplus of the uninformed consumers. This does not happen for the non-prominent high-cost firm. The high-cost firm can sell only to the informed consumers and only if it succeeds in undercutting it's competitor, so its probability density function is a strictly decreasing curve.

4.1.2 The high-cost firm has prominence

Now lets analyze the sub-game where the high-cost firm is prominent.



Firm-level demands will be the following

$$D_L = \begin{cases} 1-\theta & \text{if } p_L < p_H \wedge p_L \leq v \\ \frac{1-\theta}{2} & \text{if } p_L = p_H \wedge p_L \leq v \\ 0 & \text{if } p_L > p_H \vee p_L > v \end{cases}$$

$$D_H = \begin{cases} 1 & \text{if } p_H < p_L \wedge p_H \leq v \\ \frac{1-\theta}{2} + \theta & \text{if } p_H = p_L \wedge p_H \leq v \\ \theta & \text{if } p_H > p_L \wedge p_H \leq v \\ 0 & \text{if } p_H > v \end{cases}$$

Given these demand functions we can derive the profit function for each firm,

$$\pi_L = \begin{cases} p_L(1-\theta) & \text{if } p_L < p_H \wedge p_L \leq v \\ p_L\frac{1-\theta}{2} & \text{if } p_L = p_H \wedge p_L \leq v \\ 0 & \text{if } p_L > p_H \vee p_L > v \end{cases}$$

$$\pi_H = \begin{cases} p_H - c & \text{if } p_H < p_L \wedge p_H \leq v \\ (p_H - c)\left(\frac{1-\theta}{2} + \theta\right) & \text{if } p_H = p_L \wedge p_H \leq v \\ (p_H - c)\theta & \text{if } p_H > p_L \wedge p_H \leq v \\ 0 & \text{if } p_H > v \end{cases}$$

Let's start by trying to find a pure strategy Nash equilibria.

If the high-cost firm were to charge any price such that $0 < p_H \leq v$ then the low-cost firm's best response would be to charge slightly below p_H and sell to $1-\theta$ consumers. If the high-cost firm charges $p_H > v$ then the low-cost firm's best response is to charge v. If the high-cost firm charges 0 the low-cost firm is indifferent between any price.

If the low-cost firm were to charge $p_L > v$ then the best response of the high-cost firm would be to charge v. If the low-cost firm charges any price $c < p_L \leq v$ then the high-cost firm's best response is either to charge slightly below p_L and sell to all consumers, leading to a profit of $p_L - c$ or to charge v and sell only to the uninformed consumers for a profit of $(v - c)\theta$. By comparing profits we conclude that the first option is better if

$$p_L > v\theta + c(1 - \theta)$$

The following graph plots the best responses for both firms:

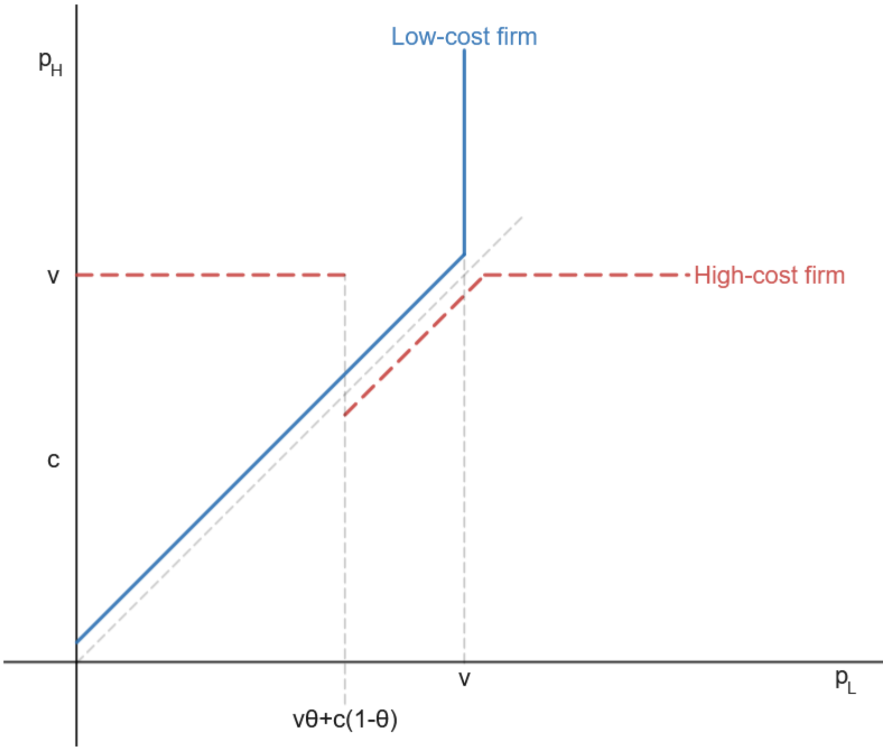


Figure 6: Best responses

Notice that, unlike the previous case, we don't have any pure strategy Nash equilibria. The intuition is simple, the non-prominent low-cost firm would always profit from undercutting it's competitor. The high-cost prominent firm will prefer to either undercut the low-cost firm, if it charges a high price, or charge the reservation price, if the low-cost firm charges a low price. They can't both undercut each other and if the high-cost firm where to charge a high price so would the low-cost firm. In the sub-game analyzed before it was possible, if c was large enough, for the prominent firm to price at the other firm's marginal cost. In this sub-game that is never possible because the prominent firm has a higher marginal cost than the non-prominent firm.

If we only have two firms and the high-cost one is prominent we don't have any pure strategy Nash equilibria, however we do have a mixed strategy NE.

Again, in order to find the mixed strategy NE, we need to know in what interval will firms mix the price. All lemmas evoked in the previous subgame still hold except for Lemma 6.

Lemma 1 still applies. Charging $p > v$ is never optimal, regardless of the circumstances, since it would lead to no sales, and no profits.

The prominent high-cost firm still won't be willing to price below it's marginal cost c, as it would necessarily lead to negative profits. From there we can also conclude that the low-cost firm is also not willing to price below it's competitor's marginal cost, since charging below c would not increase demand, only decrease the price, leading to lower profits than what they could obtain by charging c. Again, we will see further that this constraint is not binding, and the lower bound of both firms price strategy is higher than c.

Lemma 2 still applies, there will be no gaps in the price strategies of either firm.

Lemma 3 still applies, the non-prominent low-cost firm is not willing to price above the upper bound of the prominent high-cost firm's price strategy.

Lemma 4 still applies.

Proposition 2. *The support of the distribution of the price charged by the prominent high-cost firm is the following* $p_H \in [v\theta + c(1-\theta), v]$

Proof. The first part of this proof is similar to that of Proposition 1. Considering that the low-cost firm will never price above the upper-bound price of the high-cost firm (Lemma 3), any $\overline{p}_L < v$ would be sub-optimal for the low-cost firm, as charging $\overline{p}_L$ or v would lead to the same sales (since with certainty it will not have the lowest price in the market) and the latter would lead to higher profits.

The high-cost firm will have the following profit when charging v

$$\pi(v) = (v - c)\theta$$

If it charges a price equal to the lower bound it will sell to the all consumers with certainty, since both firms have the same lower bound and no point mass at that price.

$$\pi_H(\underline{p}) = \underline{p} - c$$

This firm must be indifferent between v and any other price in it's price strategy. From here we can determine $\underline{p}$

$$\pi_L(\underline{p}) = \pi_L(v)$$
$$\underline{p} = v\theta + c(1-\theta)$$

□

Lemma 5 still applies, therefore there are no point masses on the price strategy of the non-prominent firm.

Lemma 7. *There is a point mass in the price strategy of the prominent firm at the upper bound.*

Proof. This proof will be similar to the proof for Lemma 6. If the prominent high-cost firm's price strategy where to follow a non-degenerate distribution then the low-cost firm would not be willing to charge a price equal to v or even any price arbitrarily close to v, since it would not be the lowest price in the market and therefore have no profits. If this firm charges a $p_L = v\theta + c(1-\theta)$ it will sell to the informed consumers with certainty and have the following profits

$$\pi_H(v\theta + c(1-\theta)) = (v\theta + c(1-\theta)(1-\theta)$$

This is always positive, implying that the non-prominent low-cost firm must have positive expected profits when playing a mixed strategy.

The Nash equilibrium only exists as long as there is a positive probability that the high-cost firm charges v. It must, therefore, be the case that the high-cost firm follows a degenerate distribution with a point mass at the reservation price. □

This also implies that $p_L \in [v\theta + c(1-\theta), v)$. If $\overline{p}_H$ was not arbitrarily close to v then the high-cost firm would not be willing to charge any price p such that $\overline{p}_L < p < v$. We know from Lemma 2 that there can't be any gaps.

To find the price distributions we use the same method as before. Lets start by determining the cumulative distribution function of the low-cost firm. If the high-cost firm charges v

it will sell only to the uninformed consumers and have the following profits

$$\pi_H(v) = (v - c)\theta$$

If it charges some price, p, such that $v\theta + c(1-\theta) \leq p < v$ it will either sell to all consumers or to only the uninformed, depending on whether it charges the lowest price in the market. Let $F_L(p)$ be the cumulative distribution function of the low-cost firm. The probability that the price p, charged by the high-cost firm, is lower than p_L is $1 - F_L(p)$. The expected profits of this firm will be the following

$$E[\pi_H(p)] = (p - c)(\theta + (1 - \theta)(1 - F_L(p)))$$

In order to be willing to play a mixed strategy the high-cost firm must be indifferent between charging v or any p such that $v\theta + c(1 - \theta) \leq p < v$, therefore

$$\pi_H(v) = E[\pi_H(p)]$$

From here we can determine the price strategy for the low-cost firm

$$F_L(p) = 1 - \frac{(v - p)\theta}{(p - c)(1 - \theta)} \; \forall \, p \in [v\theta + c(1 - \theta), v]$$

Now lets find the price strategy of the high-cost firm. If the low-cost firm charges $v\theta+c(1-\theta)$ it will sell only to the informed consumers with certainty, since the high-cost firm will not price below $v\theta + c(1 - \theta)$ and doesn't have a point mass there either.

$$\pi_L(v\theta + c(1 - \theta)) = (v\theta + c(1 - \theta))(1 - \theta)$$

If the high-cost charges a price p, such that $v\theta + c(1 - \theta) < p < v$ it will either sell to the informed consumers or have no sales, depending on whether it charges the lowest price in the market. If $F_H(p)$ is the cumulative distribution function of the high-cost firm, then the probability that $p_L < p_H$ is $1 - F_H(p)$.

$$E[\pi_L(p)] = p(1 - \theta)(1 - F_H(p))$$

The low-cost firm will have to be indifferent between charging any price p, such that $v\theta + c(1 - \theta) \leq p < v$, therefore

$$\pi_L(v\theta + c(1 - \theta)) = E[\pi_L(p)]$$

It follows that

$$F_H(p) = 1 - \frac{v\theta + c(1 - \theta)}{p}$$

Just like the sub-game analyzed before, the price strategy of the prominent firm includes a point mass at v

$$P(p_H = v) = 1 - F_H(v) = \theta + \frac{c(1-\theta)}{v}$$

Below we can see the cumulative distribution functions for the prices of each firm ($F_L(p)$ in blue and $F_H(p)$ in dashed red). Notice how again we can see the point mass probability at $p = v$ in the cumulative distribution function of the prominent high-cost firm.

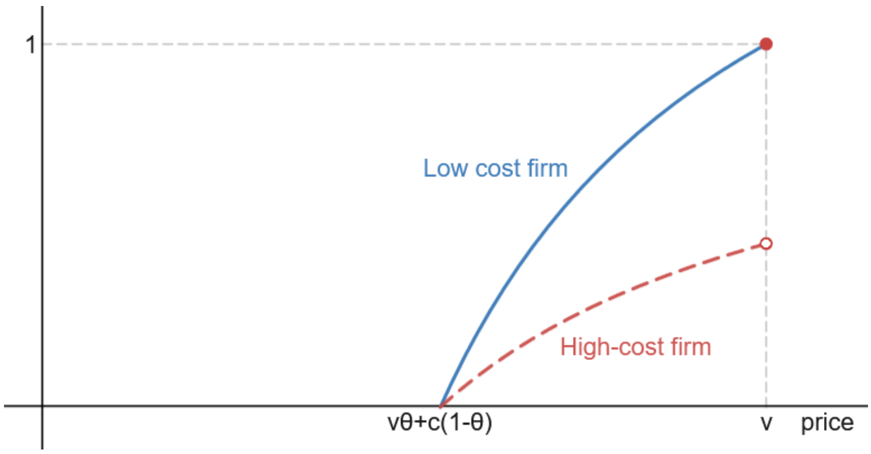


Figure 7: Equilibrium cumulative distributions

Below we can see the probability density functions for the prices of each firm ($f_L(p)$ in blue and $f_H(p)$ in dashed red).

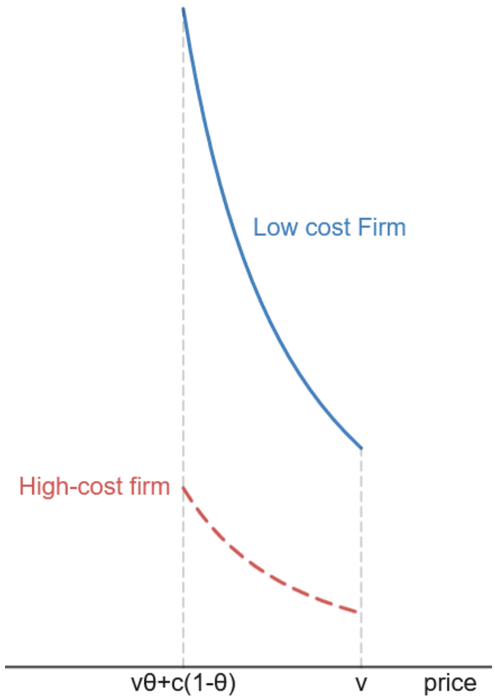


Figure 8: Equilibrium probability densities

Notice how, similar to the other sub-game, the probability density is higher for lower prices, as charging higher prices decreases the probability of having the lowest price in the market. This time the high-cost firm is prominent and it has a point mass probability at $p = v$. As a result we get a "U-shaped" density, the prominent high-cost firm either charges low prices to sell to the informed consumers or charges $p = v$ to fully extract the surplus of the uninformed consumers. The non-prominent low-cost firm can sell only to the informed consumers and only if it succeeds in undercutting it's competitor, so, just like the non-prominent high-cost firm in the other sub-game, its probability density function is a strictly decreasing curve.

4.2 First stage

To know what will happen in the first stage we need to know how much each firm would benefit from prominence, that's how much they would be willing to pay for prominence (W_L and W_H). For that we must compare expected profits in both cases for each firm.

Let's start with the case where, $v\theta > c$, notice that this is the condition required for the existence of a Nash equilibrium of mixed strategies in the case where the low-cost firm becomes prominent.

As we have seen before (see Proposition 1), when charging the reservation price v the prominent low-cost firm will sell only to the uninformed consumers, this allows us to determine the expected profits of this firm.

$$E[\pi_L^{\text{Prominence}}] = v\theta$$

If the low cost firm is not prominent we can derive it's expected profits by multiplying the lower bound of the price distribution by the proportion of informed consumers (see Proposition 2).

$$E[\pi_L^{\text{No prominence}}] = [(v - c)\theta + c](1 - \theta)$$

We can determine the maximum amount this firm is willing to bid for prominence in the first stage of the game by computing the difference between the two cases.

$$W_L = E[\pi_L^{\text{Prominence}}] - E[\pi_L^{\text{No prominence}}]$$
$$W_L = 2c\theta - c + v\theta^2 - c\theta^2$$

We can employ the same method to determine how much the high-cost firm is willing to bid for prominence. We know that if this firm charges the reservation price v it will only sell to the uninformed consumers at a profit of $v - c$ per sale (see Proposition 2).

$$E[\pi_H^{\text{Prominence}}] = (v - c)\theta$$

We can determine the expected profits if this firm is not prominent by multiplying the lower bound of the price distribution by the proportion of uninformed consumers.

$$E[\pi_H^{\text{No prominence}}] = (v\theta - c)(1 - \theta)$$

Finally we can compute the difference to know how much this firm is willing to bid for prominence.

$$W_H = E[\pi_H^{\text{Prominence}}] - E[\pi_H^{\text{No prominence}}]$$
$$W_H = c + v\theta^2 - 2c\theta$$

Whichever firm is willing to pay more will get prominence, so if $W_L > W_H$, which happens if $\theta > 2 - \sqrt{2}$, the low-cost will have prominence in the second stage of the game.

Now let's consider the case where $v\theta < c$. If the low-cost is prominent we will have a pure strategy Nash equilibrium, the standard Bertrand equilibrium for firms with different marginal costs. If the high-cost firm is prominent then we only have a mixed strategy Nash equilibrium.

The prominent low-cost firm will sell to all consumers at the marginal cost of it's competitor.

$$\pi_L^{\text{Prominence}} = c$$

If instead this firm is not prominent the expected profit is given by the same expression as before.

$$E[\pi_L^{\text{No prominence}}] = [(v-c)\theta + c](1-\theta)$$

Again, the maximum amount this firm is willing to bid to become prominent is given by the difference.

$$W_L = E[\pi_L^{\text{Prominence}}] - E[\pi_L^{\text{No prominence}}]$$
$$W_L = v\theta^2 - c\theta^2 - v\theta + 2c\theta$$

The expected profits of the prominent high-cost firm are given by the same expression as before

$$E[\pi_H^{\text{Prominence}}] = (v-c)\theta$$

However if this firm is not prominent then it will not have any sales

$$E[\pi_H^{\text{No prominence}}] = 0$$

The maximum amount that the high-cost firm is willing to pay in the first stage of the game in order to become prominent is given by the difference in profits

$$W_H = E[\pi_H^{\text{Prominence}}] - E[\pi_H^{\text{No prominence}}]$$
$$W_H = v\theta - c\theta$$

Again the low-cost firm will be willing to outbid high-cost for large enough values of θ, if $\theta > 2 - \frac{c}{v-c}$.

Notice that if $\frac{c}{v} < 2 - \sqrt{2}$ then $2 - \frac{c}{v-c} > 2 - \sqrt{2}$, meaning that $\theta > 2 - \frac{c}{v-c}$ implies $\theta > 2 - \sqrt{2}$. If $\frac{c}{v} > 2 - \sqrt{2}$ then $2 - \sqrt{2} > 2 - \frac{c}{v-c}$, meaning that $\theta > 2 - \sqrt{2}$ implies $\theta > 2 - \frac{c}{v-c}$.

So if $\theta > 2 - \sqrt{2} \vee \theta > 2 - \frac{c}{v-c}$ the low-cost will have prominence.

In the two following figures we can see the differences in the willingness to pay for prominence for each firm (the low-cost firm in blue and the high-cost firm in dashed red). Figure 9 graphs the case when $\frac{c}{v} < 2 - \sqrt{2}$, therefore the low-cost firm will outbid the high-cost firm for prominence if $\theta > 2 - \sqrt{2}$.

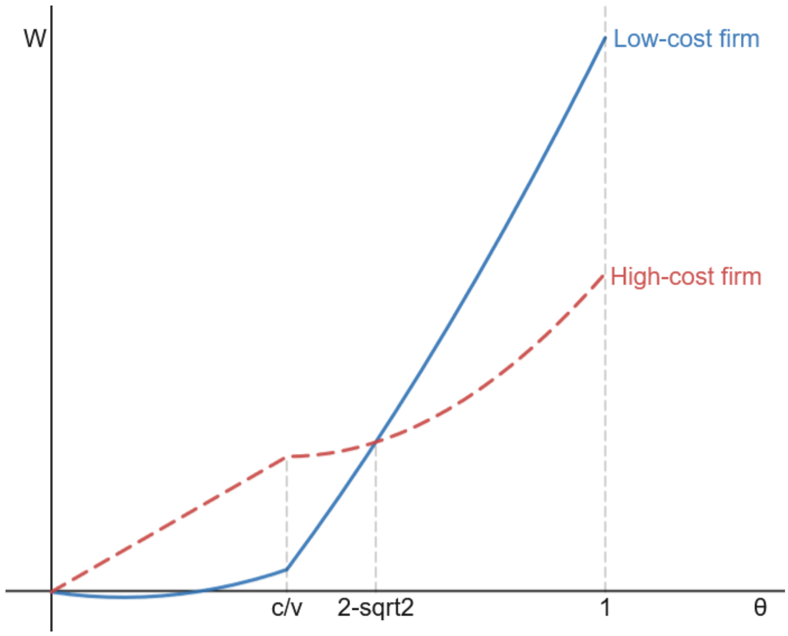


Figure 9: Willingness to pay for prominence when $\frac{c}{v} < 2 - \sqrt{2}$

The following graph, Figure 10, graphs the case when $\frac{c}{v} > 2-\sqrt{2}$, therefore the low-cost firm will outbid the high-cost firm for prominence if $\theta > 2 - \frac{c}{v-c}$. Notice that $2 - \frac{c}{v-c}$ can be negative, for a high enough c and a low enough v. In that case , when $\frac{c}{v} > \frac{2}{3}$, the low-cost firm will outbid the high-cost firm regardless of how many uninformed consumers there are (see Figure 11).

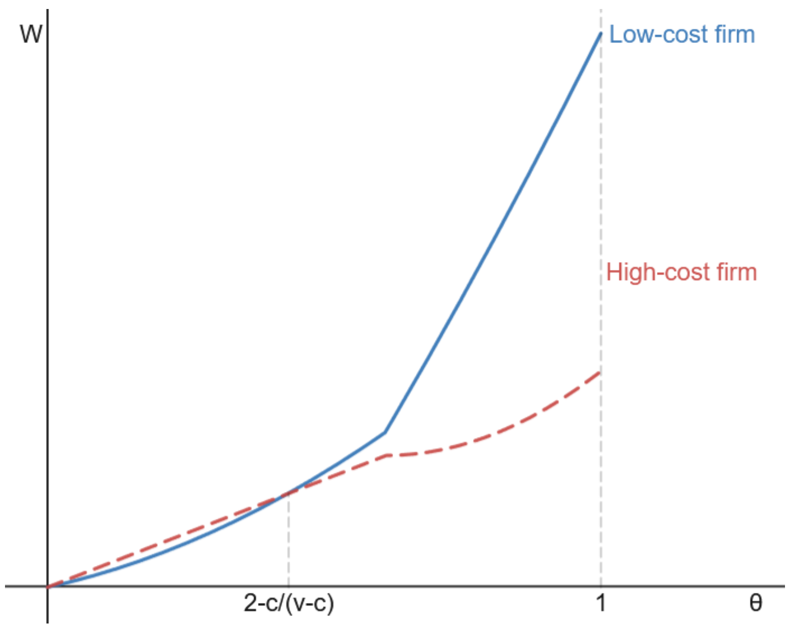


Figure 10: Willingness to pay for prominence when $2 - \sqrt{2} < \frac{c}{v} < \frac{2}{3}$

The higher c is, relative to v, the more likely a given θ will be large enough for the low-cost firm to outbid the high-cost firm in the auction for prominence.

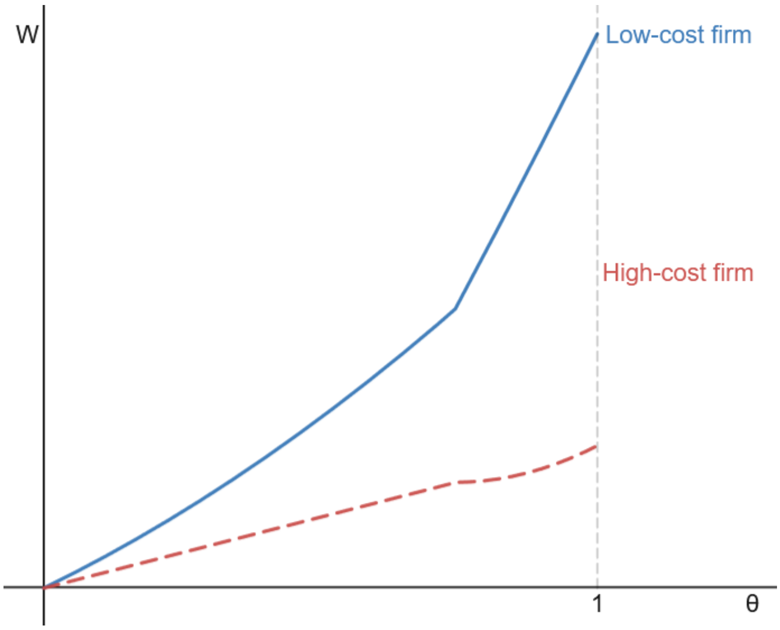


Figure 11: Willingness to pay for prominence when $\frac{c}{v} > \frac{2}{3}$

4.3 Comparing prices

When will the low-cost firm charge a higher expected price than high-cost? If the low-cost is the prominent firm and $v\theta > c$, then the expected prices will be the following

$$
\begin{aligned}
E[p_L] &= \int_{v\theta}^{v} p f_L(p) dp + \frac{c\theta - c}{v - c} v = \\
&= (v\theta - c)\left[\ln \frac{v-c}{v\theta - c} - \frac{c}{v-c} + \frac{c}{v\theta - c}\right] + \frac{v\theta - c}{v - c} v \\
E[p_H] &= \int_{v\theta}^{v} p f_H(p) dp = \\
&= \frac{v\theta}{1-\theta} \ln \frac{1}{\theta}
\end{aligned}
$$

If the low-cost is prominent but instead $v\theta < c$ then, as we have seen before, we will have a standard Bertrand equilibrium with cost heterogeneity, where only the low-cost firm will sell, at a price lower but arbitrarily close to the marginal cost of the high-cost firm.

If the high-cost is the prominent firm, the expected prices will be

$$
\begin{aligned}
E[p_L] &= \int_{v\theta+c(1-\theta)}^{v} p f_L(p) dp = \\
&= \frac{(v-c)\theta}{1-\theta} \ln \frac{1}{\theta} + c \\
E[p_H] &= \int_{v\theta+c(1-\theta)}^{v} p f_H(p) dp + v\theta + c(1-\theta) = \\
&= [v\theta + c(1-\theta)] \ln \frac{v}{v\theta + c(1-\theta)} + v\theta + c(1-\theta)
\end{aligned}
$$

In the following graphs we can see the expected prices of both firms, let's consider first the case is when $\frac{c}{v} > 2 - \sqrt{2}$

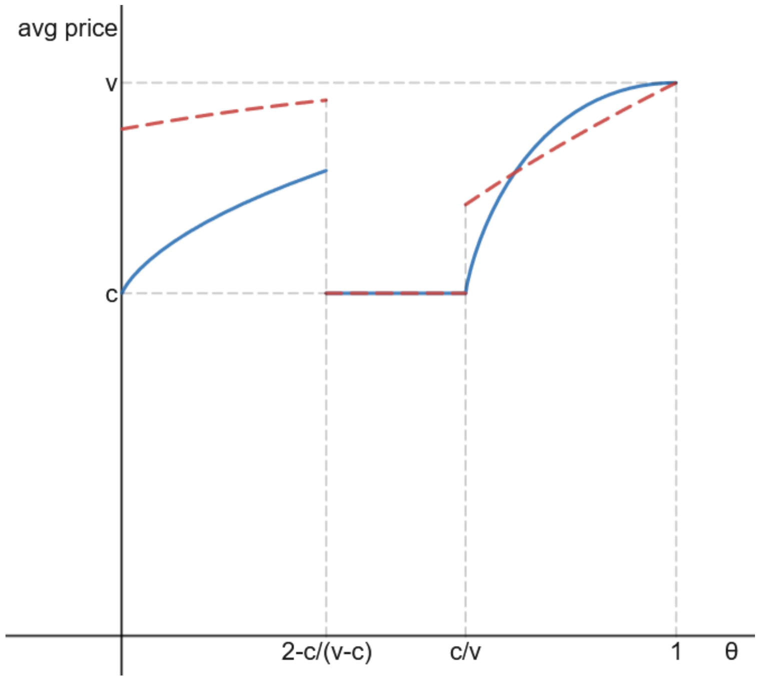


Figure 12: Average prices if $2 - \sqrt{2} < \frac{c}{v} < \frac{2}{3}$

Notice how we can see that for a small proportion of uninformed consumers ($\theta < 2 - \frac{c}{v-c}$) the high-cost firm will become prominent and charge a higher average price than the low-cost firm. Both effects contribute positively for a higher average price, having a higher marginal cost and being prominent. Notice that this section might not exist, if $\frac{c}{v} > \frac{2}{3}$, see Figure 13 for an example.

If the proportion of uninformed consumers is higher than $2 - \sqrt{2}$ but still lower than $\frac{c}{v}$ then the low cost firm will be willing to outbid it's competitor in the first stage of the game and, because $v\theta < c$, there will be a pure strategy Nash equilibrium where the price will be c.

Notice that this section might not exist, if $\frac{c}{v} < 2 - \sqrt{2}$, see Figures 14 and 15 for an example.

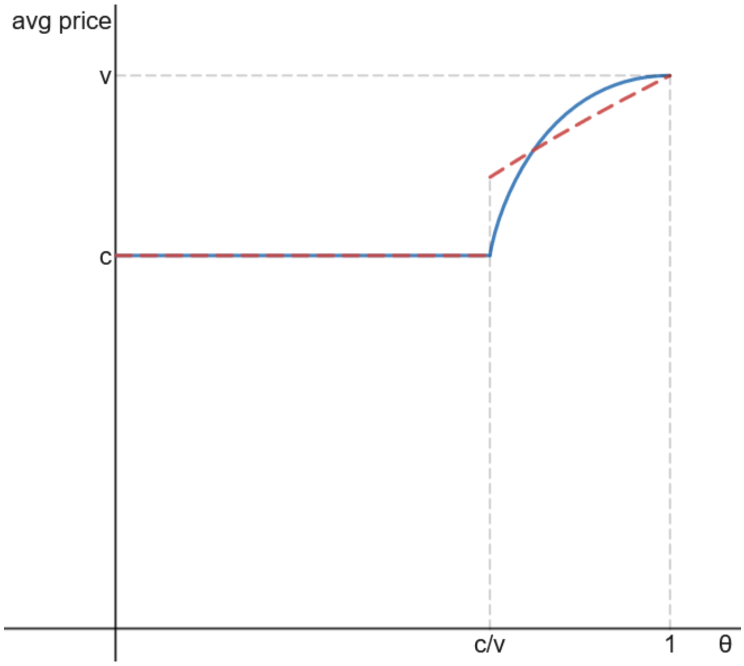


Figure 13: Average prices if $\frac{c}{v} > \frac{2}{3}$

If the proportion of uninformed consumers is large ($\theta > \frac{c}{v}$) the low-cost firm will become prominent and be willing to play a mixed strategy. Now we have effects going in the opposite direction. Having a lower marginal cost, the low-cost firm is more strongly incentivized to charge a lower price than the high cost firm, however, being prominent, this firm is incentivized to charge a higher price. We can see in this example how for a θ larger, but still close to $\frac{c}{v}$ the average price or the low-cost firm is still lower than the average price of the high-cost firm, here the cost effect dominates the prominence effect. However for a sufficiently large proportion of uninformed consumers the low-cost firm will charge a higher average price than the high-cost firm, here the prominence effect dominates the cost effect. This will happen if the following condition is verified.

$$\frac{\theta - C}{\theta} \ln \frac{1 - C}{\theta - C} + 1 > \frac{1}{1 - \theta} \ln \frac{1}{\theta}\,, \textit{ where } C \equiv \frac{c}{v}$$

This inequation can always be verified for a high enough level of θ, see the Appendix for proof.

Now let's consider a different case, where $\frac{c}{v} > 2 - \sqrt{2}$

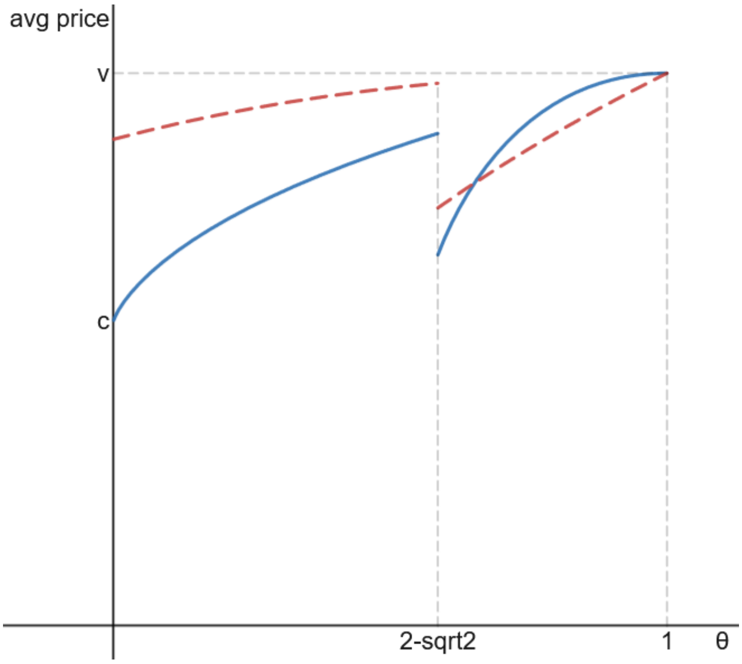


Figure 14: Average prices if $0.4774 < \frac{c}{v} < 2 - \sqrt{2}$

This case is similar to the previous one. As we can see, for a small proportion of uninformed consumers ($\theta < 2 - \sqrt{2}$) the high-cost firm will become prominent and charge a higher average price than the low-cost firm. Again the intuition is that having a higher marginal cost and being prominent both incentivize higher prices.

Here there is no case where we can have the pure strategy Nash equilibrium since it would require both $\theta < \frac{c}{v}$ and the low-cost firm being prominent, which only happens if $\theta > 2 - \frac{c}{v-c}$. Since this is the case where $\frac{c}{v} > 2 - \sqrt{2}$ that is not possible.

If the proportion of uninformed consumers is large ($\theta > 2 - \sqrt{2}$) then the low-cost firm will outbid the high-cost firm in the first stage of the game and become prominent. Just like the previous case, a prominent low-cost firm might still charge a lower average price if the proportion of consumers is larger than, but closer to $2 - \frac{c}{v-c}$. Again, here we have the cost effect dominating (notice that this case might not exist, see next graph for an example). If instead we have a sufficiently large θ then the prominence effect will dominate and the low-cost firm will charge a higher average price than the high-cost firm.

Notice that if $\frac{c}{v} < 0.4774$ then the low-cost firm being prominent is enough to result in that firm charging a higher average price, see Figure 15. We still require $\theta > 2 - \sqrt{2}$ for that firm to be prominent, the difference here is that there is no interval of θ where the non-prominent firm charges a higher average price.

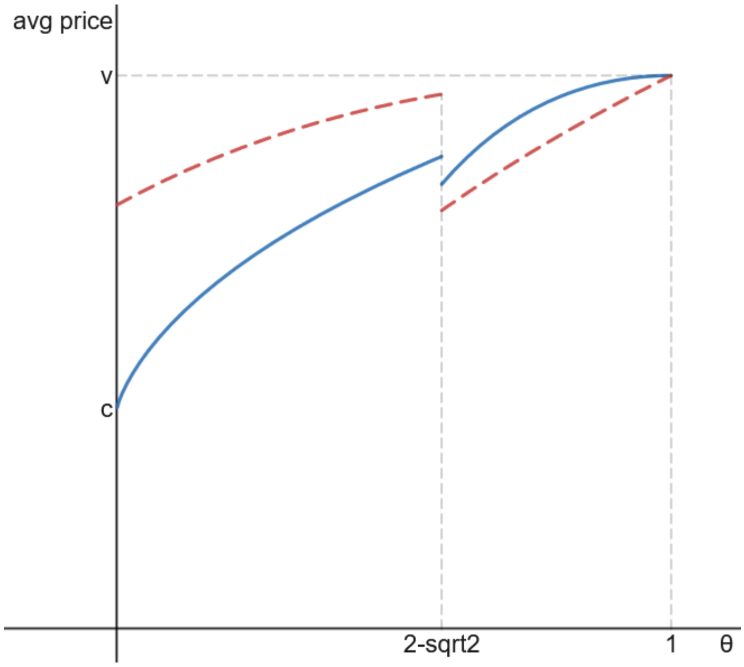


Figure 15: Average prices if $\frac{c}{v} < 0.4774$

5 Equilibrium with many firms

Let's now consider the case when there is one low-cost firm, with zero marginal costs, and any number of high-cost firms larger than one, each with the same marginal cost c.

5.1 Second stage

In the second stage of the game the equilibrium will depend on which firm has prominence.

5.1.1 The low-cost firm has prominence

In this case there's a pure strategy Nash equilibrium. If $v\theta > c$, the low-cost firm will charge v and sell to the uninformed consumers, for a profit of $v\theta$, and the high-cost firms will all charge c, split the demand of the informed consumers among each other and have no profits. Notice that it's not possible to sustain a mixed strategy Nash equilibrium because the high-cost firms are playing the standard Bertrand game with symmetric firms and as a result the only options available to the prominent high-cost firm are to charge the reservation price v to the uninformed consumers or to undercut all the high-cost firms, since $v\theta > c$ the latter strategy is more profitable.

If instead $v\theta < c$, then we will get the Bertrand equilibrium with asymmetric firms, where the low-cost firm will meet the entire demand at a price lower but arbitrarily close to the marginal cost of the other firms, for a profit of c, while the other firms have no sales and no profits. This is the case where the number of uninformed consumers, θ, is not large enough to incetivize the low-cost firm to charge a high price and extract the full surplus of the informed consumers.

In this case competition drives the price charged to the informed consumers down to the marginal cost of the high-cost firms, as there are many non-prominent firms competing for those consumers. If the proportion of uninformed consumers is large enough, the prominent low-cost firm is incentivized to forgo the rest of the market in order to fully extract the surplus from those consumers, if not then the low-cost will undercut all the high-cost firms *à la Bertrand*.

5.1.2 A high-cost firm has prominence

If one of the high-cost firms has prominence then that firm will charge v and sell only to the informed consumer for a profit of $(v-c)\theta$. The other firms will play the asymmetric Bertrand game and the low-cost firm will meet all the demand of the informed consumers by selling at a price lower but arbitrarily close to the marginal cost of the other firms, for a profit of $c(1-\theta)$. All the other high-cost firms will have no sales and no profits. Notice that the prominent high-cost firm is never willing to undercut the low-cost firm as that would lead to negative profits.

5.2 First stage

In the first stage which firm wins the auction will depend, again, on how much each firm benefits from having prominence. Let's consider first the case where $v\theta > c$.

If the low-cost firm becomes prominent it will charge a price equal to the reservation price v and sell only to the uninformed consumers.

$$\pi_L^{\text{Prominence}} = v\theta$$

If that firm does not become prominent then it will charge a price equal to the marginal cost of the high-cost firms and sell to all informed consumers.

$$\pi_L^{\text{No prominence}} = c(1-\theta)$$

We can determine the maximum amount that this firm is willing to bid for prominence by computing the difference in profits in both cases.

$$W_L = v\theta - c(1-\theta)$$

If a high-cost firm becomes prominent it will charge a price equal to the reservation price and sell only to the uninformed consumers.

$$\pi_H^{\text{Prominence}} = (v - c)\theta$$

All other non-prominent high-cost firms will either charge a price equal or above their marginal cost and have no sales.

$$\pi_H^{\text{No prominence}} = 0$$

As a result, in the first stage of the game, a high-cost firm is willing to pay up to all their potential profit to become prominent.

$$W_H = (v - c)\theta$$

The low-cost will become prominent when $W_L > W_H$, this happens when $\theta > \frac{1}{2}$. Just like in the version of the model with only two firms, for a high enough θ the low-cost firm will outbid any of the high-cost firms in the auction for prominence.

Now let's consider the case where $v\theta < c$. Now the low-cost firm will always charge a price equal to the marginal cost of the high-cost firms, regardless of whether this firm is prominent or not. Being prominent is still beneficial, as the firm will be able to sell to more consumers at the same price. If the firm is prominent it sells to all consumer if not then it only sells to the informed consumers.

$$\pi_L^{\text{Prominence}} = c$$
$$\pi_L^{\text{No prominence}} = c(1 - \theta)$$

Again we can determine how much this firm is willing to pay for prominence in the first stage of the game by computing the difference.

$$W_L = c\theta$$

Just like in the previous case, where $v\theta > c$, a high-cost firm can only make positive profits if it becomes prominent. If they are prominent they can charge the reservation price and sell to the uninformed consumers, otherwise they will compete *à lá Bertrand* with all the other high-cost firms and have no sales.

$$\pi_H^{\text{Prominence}} = (v - c)\theta$$
$$\pi_H^{\text{No prominence}} = 0$$

Again, the maximum amount this firm is willing to pay in the auction is the difference in profits between the two cases which corresponds to their entire profits if they become prominent.

$$W_H = (v - c)\theta$$

The low cost firm will become prominent when $W_L > W_H$, which happens when $\frac{v}{c} < 2$. This means that if $v > 2c$ the low-cost firm will be prominent regardless of how many consumers are informed.

We can see that case, $v > 2c$, with the willingness to pay for both firms plotted in blue and red for the low-cost and the high-cost firms respectively.

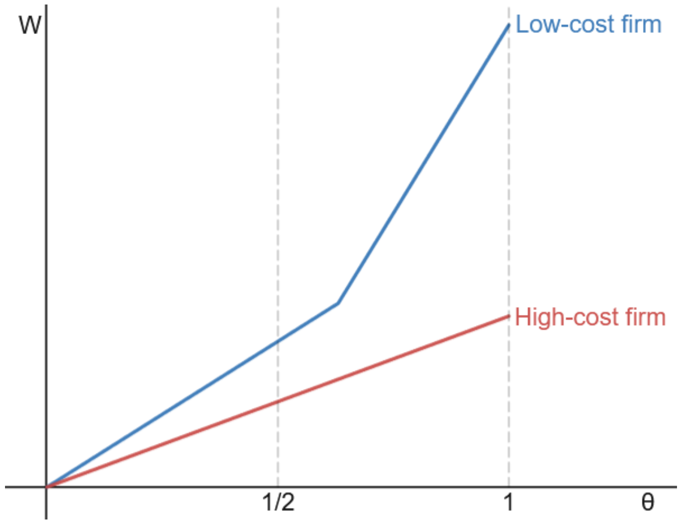


Figure 16: Willingness to pay for prominence when $v > 2c$

Notice that if the relative difference between the reservation price and the marginal cost of the high-cost firms is low enough, if the marginal cost represents at least 50% of the reservation price, the low-cost will always outbid the high-cost firms in the auction for prominence.

Now let's consider the other case, $v < 2c$. The low-cost will still be prominent as long as the proportion of informed consumers is sufficiently high, $\theta > \frac{1}{2}$.

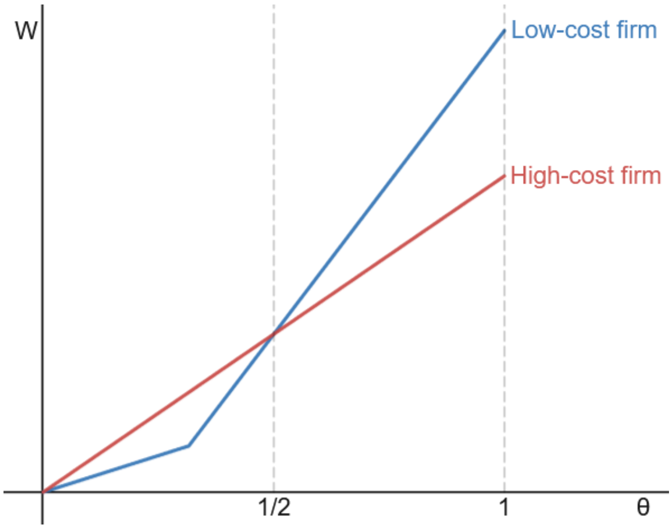


Figure 17: Willingness to pay for prominence when $v < 2c$

5.3 Comparing prices

In this version of the model there are no mixed strategy Nash equilibria. As a result of that comparing prices is not only easier but we can compare the actual prices, before we could only compare the average prices.

Notice that a prominent high-cost firm will always charge the reservation price, v, and a non-prominent low-cost firm will always charge a price equal to the marginal cost of the other firms, c. A prominent low-cost firm might charge either c or v depending on whether the proportion of uninformed consumers is large enough or not. The result is similar to the one derived in the version of the model with only two firms, a sufficiently high proportion of uninformed consumers incentivizes the low-cost firm to both outbid the high-cost firms in the first stage of the game and charge a higher price in the second stage. Just as before merely becoming prominent might not be enough to incentivize the low-cost firm to charge a higher price, the proportion of uninformed needed to result in the low-cost firm charging a higher price than the high-cost firm is the following

$$\theta > \max(\frac{1}{2}, \frac{c}{v})$$

6 Conclusion

We have seen how prominence affects price competition. The auction in the first stage is simply a market mechanism employed by the platform to allocate prominence among retailers and extract some of the surplus, however it ends up affecting the price competition between retailers that happens in the second stage of the game.

The low-cost firm charging a higher price than it's competitors is not a new result. If firms are pricing according to mixed strategies there will be a positive probability that the low-cost firm charges a higher price. However we still expect that the average price of the low-cost firm will be lower than that of the high-cost firm. In fact, as we have discussed before, in similar models we observe that a small difference in marginal costs can lead to a large difference in average prices (Shelegia 2012).

My thesis shows how the effect that results from making one firm prominent can eventually lead to the low-cost firm, which we would expect to see charging a lower average price, instead charge a higher average price than their competitor. Notice however, that this is not merely the result of adding a new exogenous effect to the model, one that counters the effect of price heterogeneity by incentivizing the low-cost firm to charge a higher average price. In this model prominence is endogenous, it's a scarce resource, supplied by the platform, that firms compete for. The relevant aspect of the model is that, under certain conditions, the low-cost firm will become prominent as a result of being the low-cost firm.

The prominent firm is incentivized to charge higher prices because it has exclusive access to the uninformed consumers, a segment of the demand that has a rigid demand function, they will buy one unit of the good from the prominent firm as long as the price does not exceed the reservation price v, charging a price lower than that only decreases the revenues from this group of consumers. The non-prominent firm merely competes with the prominent firm for the rest of the demand, the informed consumers. These consumers also have a rigid demand function, however, because both firms compete for this segment of the market, it can be profitable to charge a lower price than the reservation price, since the lower the price the higher the probability of undercutting the other firm.

The relevant question is, why does the low-cost firm become prominent? The intuition is the following. The low-cost firm can have a higher markup selling to the uninformed consumers than the high-cost firm, as the monopoly price will be the same, v, but the marginal costs will be different. So if there's a large number of uninformed consumers that segment of the market will be more profitable for the low-cost firm than for the high-cost and hence they'll be willing to bid more for prominence than the high-cost firm. Conversely, for a lower number of uninformed consumers, the prominent firm is still incentivized to charge higher prices, however forgoing the rest of the market is very costly if most consumers are informed, and for that segment of the market, again, the markup for the low-cost firm will be higher than that of the high-cost firm, so it would be more costly to forgo those consumers for the few uninformed consumers for the low-cost firm than for the high-cost firm.

In a sense, the low-cost firm, being more efficient, will compare the size of these two segments of the market. If there are a lot of uninformed consumers it'll buy prominence in the first stage of the game and sell at a high average price. If there are few uninformed consumers it won't buy prominence, allowing the high-cost firm to become prominent and charge higher prices, leaving the large number of informed consumers for the low-cost firm.

Notice that one firm being prominent can be beneficial for the other firm too, we have even seen that the willingness to pay for prominence of either firm can be negative for certain parameters, meaning that that firm would even be willing to pay so that the other firm becomes prominent instead. This happens because prices are strategic complements and one firm being incentivized to increase prices creates a strategic effect that incentivizes the other firm to increase prices too. This explains the case where the high-cost firm becomes prominent, which happens if we have a low proportion of uninformed consumers. In this case if the low-cost becomes prominent it will be more strongly incentivized to charge higher prices than the high-cost firm which results in a lower probability of selling to the informed consumers. If instead the high-cost firm is "allowed" to become prominent it will be more strongly incentivized to charge higher prices than the low-cost firm which results in a higher probability that the non-prominent low-cost firm sells to the informed consumers.

Finally notice that this result holds even if we have more than two firms. For a sufficiently high proportion of consumers in the market that are uninformed the low-cost firm will become prominent and charge a higher price than that of their high-cost competitors.

Appendix

Here is the proof that

$$\frac{\theta - C}{\theta} \ln \frac{1 - C}{\theta - C} + 1 > \frac{1}{1 - \theta} \ln \frac{1}{\theta}$$

can always be verified for a θ large enough (close enough to 1).

First, consider that the left side of the inequation takes the value of 1 when evaluated at $\theta = 1$. The right side is not defined at $\theta = 1$ but we can se that it aproaches 1 by computing the limit.

$$\lim_{\theta \to 1^-} \frac{1}{1 - \theta} \ln \frac{1}{\theta} = 1$$

Next we compute the derivatives for both sides with respect to θ.

$$\frac{\partial\left(\frac{\theta-C}{\theta}\ln\frac{1-C}{\theta-C}+1\right)}{\partial\theta}=\frac{C\ln\frac{1-C}{\theta-C}-\theta}{\theta^2}$$
$$\frac{\partial\left(\frac{1}{1-\theta}\ln\frac{1}{\theta}\right)}{\partial\theta}=-\frac{\theta\ln\theta-\theta+1}{(\theta-1)^2\theta}$$

When evaluated at $\theta = 1$ the derivative of the left side takes the value of -1. The derivative of the right side is not defined at $\theta = 1$ but we can see it approaches $-\frac{1}{2}$ by computing the limit.

$$\lim_{\theta\to 1^-} -\frac{\theta\ln\theta-\theta+1}{(\theta-1)^2\theta}=-\frac{1}{2}$$

This means that, while the two curves "meet" (they don't intercept because of the discontinuity) at $\theta = 1$, the left side curve is steeper, it has a slope of -1, while the right side curve is flatter, it has a slope of $-\frac{1}{2}$. When the curves meet at $\theta = 1$ the left side curve is coming from above and the right side curve from below. This implies that there is always an interval of values for θ, close to 1, where the inequality is verified.

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