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Abstract

Recent years have witnessed an increased interest, by competition agencies, in assessing the competitive effects of partial acquisitions. We propose an empirical structural methodology to quantify the coordinated effects of such acquisitions on differentiated products industries, by evaluating the impact of such acquisitions on the minimum discount factors for which coordination can be sustained. The methodology can deal with settings involving all type of owners and ownership rights: owners that can be internal to the industry (rival firms) and external to the industry; and ownership rights that can involve financial interests and corporate control, can be direct and indirect, can be partial or full. We provide an empirical application of our proposed methodology to several acquisitions in the wet shaving industry that give rise to cross- and common-ownership structures. The results seem to suggest that the incentives of (i) the acquiring party's firm to coordinate are non-decreasing after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners), (ii) the acquired firm to coordinate are non-decreasing after acquisitions involving full or partial corporate control rights, but non-increasing after acquisitions involving full or partial financial rights, and (iii) the remaining firms in the industry to coordinate are non-increasing after an acquisition (again, independently of whether it involves full or partial financial or corporate control rights, by internal or external owners).

JEL Classification: D12, C54, L13, L41, L66

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1 Introduction

Full acquisitions complete and permanently eliminate competition among the firms involved in the transaction. This constitutes the basic element of a merger analysis. *Partial acquisitions*, in contrast, do not completely and permanently eliminate competition among firms. Nevertheless, they may present - and recent empirical work confirms this - significant competitive concerns (see, e.g., Azar, Schmalz and Tecu, 2016; Azar, Raina and Schmalz, 2016).[1] As a consequence, competition agencies have taken an increased interest in assessing the anti-competitive effects of partial acquisitions.

Following the long theoretical literature in industrial organization, agencies have typically focused on acquisitions settings involving owners that are *internal* to the industry (rival firms), which induce a *cross-ownership* structure. Some recent examples include the UK Competition Commission assessment of the BskyB's proposed acquisition of a 17.9% stake in ITV and the European Commission assessment of the News Corporation's proposed acquisition of an approximately 25% stake in Premiere.

However, the phenomenal growth of private equity investment in recent years has led agencies to focus also on acquisitions settings involving owners that are *external* to the industry,[2] but participate in more than one competitor firm, which induce a *common-ownership* structure. A recent example includes the FTC assessment of the Kinder Morgan buyout by (among others) private equity funds managed and controlled by the Carlyle Group and Riverstone Holdings LLC, which already held a significant partial ownership position in Magellan Midstream, a major competitor of Kinder Morgan.

Partial acquisitions induce *unilateral* and *coordinated effects* concerns. The assessment of the former has been recently studied by Brito, Ribeiro and Vasconcelos (2014) and Brito *et al.* (2017) who propose screening indicators (for phase I-type of investigations) and an empirical structural methodology (for phase II-type of investigations) to do so. This article focuses on the assessment of the latter.

The coordinated effects of partial acquisitions (as of mergers) flow from the repeated interaction among firms in the industry, an interaction that provides a structure in which an *agreed* coordinated outcome may be supported, not by explicit negotiation, but as a tacit non-cooperative equilibrium, under the credible threat that *deviations* from this coordinated arrangement would trigger *punishment* by rivals (e.g., a reversion to competitive behavior). In analyzing the coordinated effects of partial acquisitions, competition agencies need to evaluate whether a proposed acquisition *changes* the manner in which firms in the industry interact, increasing the strength, extent or likelihood of coordinated conduct. To do so, they need to evaluate the impact of the proposed acquisition on the three regimes of the tacit coordination model: *agreement*, *deviation*, and *punishment*. We propose an empirical methodology to quantitatively perform this evaluation in cases of *actual* and *hypothetical* partial *horizontal* acquisitions.

The proposed methodology considers a *structural setting* where oligopolistic firms interact repeatedly over *time* (Friedman, 1971) and across *markets* (Bernheim and Whinston, 1990). Firms are modelled as *asymmetric multi- and differentiated-product organizations* (Rothschild, 1999; Vasconcelos, 2005; Kuhn,

[1] Azar, Schmalz and Tecu (2016) examine the U.S. airline industry and find that the interlinks in the ownership of the airlines matters for how the airlines compete. Azar, Raina and Schmalz (2016) find the same relation in the U.S. banking industry.

[2] A key issue in the explanation of private equity growth over the past few years is the fact that private equity investment has been a crucial source of financing for many entrepreneurial ventures (Lerner, Leamon and Hardymon, 2012).

2004) in order to encompass real-world industry features, and are assumed to follow *grim-trigger strategies*, the most basic enforcement mechanism to sustain a coordinated arrangement (Friedman, 1971). This structural setting is used to simulate the counterfactual stream of operating profits of firms under the different regimes of the tacit coordination model (Davis, 2006; Davis and Huse, 2010), which, in turn, are used to evaluate quantitatively the likelihood of coordinated conduct *pre*- and *post*-acquisition.

To do so, we identify and distinguish acquisitions according to whether they involved *financial* or *corporate control* rights in the lines of O'Brien and Salop (2000), Brito, Ribeiro and Vasconcelos (2014), Brito, Cabral and Vasconcelos (2014), and Brito *et al.* (2017). The former refers to the right of the (partial) owner to receive the stream of profits generated by the firm from its operations and investments, while the latter refers to the right of the (partial) owner to influence the decisions that affect the firm. We need to identify and distinguish the two rights because partial horizontal acquisitions that do not result in effective control present competitive concerns distinct from those that involve effective control. When a party (internal or external to the industry) acquires a partial *financial* right in a firm, it acquires a share of its profits. As a consequence, such acquisition may impact the likelihood of coordinated conduct by reducing the incentive of the *acquiring party's firm* to deviate from the agreement *and* to punish (since, in both cases, it shares in the losses thereby inflicted on the acquired rival). On the other hand, when a party (internal or external to the industry) acquires a *corporate control* right in a firm, it acquires the ability to influence the competitive conduct of that firm. Such influence may impact the likelihood of tacit coordination by reducing the incentive of the *acquired firm* to punish the acquiring party's firm.

The proposed empirical structural methodology can cope with acquisition settings involving all types of owners and ownership rights: owners that can be internal to the industry (rival firms) and external to the industry; and ownership rights that can involve financial interests and corporate control, can be direct and indirect, can be partial or full.[3] Further, this structural approach to assess the coordinated effects of partial horizontal acquisitions has not been, to our knowledge, examined in any other academic study. Moreover, for competition policy issues, it may constitute a preferable approach compared to the current indirect approach focused on measures of market concentration and on informal analyses of the features of the market conducive to coordinated interaction.

We also provide an empirical application of the methodology to a variety of actual and hypothetical acquisitions in the wet shaving industry that give rise to cross- and common-ownership structures. The results seem to suggest that the incentives of (*i*) the *acquiring party's firm* to coordinate are non-decreasing after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners), (*ii*) the *acquired firm* to coordinate are non-decreasing after acquisitions involving full or partial corporate control rights, but non-increasing after acquisitions involving full or partial financial rights, and (*iii*) the *remaining firms in the industry* to coordinate are non-increasing after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners).

This article is organized as follows: Section 2 reviews the literature, Section 3 presents the empirical

[3] Following Flath (1991), an owner has an indirect partial ownership right in firm B if it holds a partial ownership right in firm A and, in turn, firm A holds a partial ownership right in firm B.

structural methodology used to evaluate the coordinated effects of partial horizontal acquisitions, Section 4 provides the above mentioned empirical application, and Section 5 concludes.

2 Literature Review

The relevant literature can be divided into two strands. The first strand of literature examines the theoretical impact of partial competitor ownership on the likelihood of a tacit coordinated agreement. The second strand of literature relates to the quantitative evaluation of the coordinated effects of mergers.

2.1 The Coordinated Effects of Partial Horizontal Acquisitions

The strand of the literature that examines the theoretical impact of partial competitor ownership on the likelihood of a tacit coordinated agreement began with Reynolds and Snapp (1986). They argue that, in markets where entry is difficult, a partial cross-ownership of *financial rights* and small joint ventures can facilitate tacit coordination among rivals, since such rights cause deviating firms to share (internalize) some of the cost imposed on rivals.

Malueg (1992) formally examines this argument in the context of an infinitely repeated *Cournot* homogeneous-product *symmetric duopoly* model with grim-trigger strategies in which each firm's single external owner holds an *identical* partial financial right in the rival. His analysis extends the literature by showing that a partial cross-ownership of financial rights has in fact *two* conflicting effects on the likelihood of a tacit coordinated agreement. On the one hand, they can facilitate tacit coordination by *reducing* the incentive of firms to deviate from the coordinated arrangement because cheaters internalize some of the cost imposed on rivals, as argued by Reynolds and Snapp (1986). On the other hand, they can hinder tacit coordination by *increasing* the incentive of all firms to deviate because a partial cross-ownership of financial rights softens market competition and induce, in case of defection from the agreement, a less severe punishment (e.g., a reversion to a more profitable Cournot-Nash equilibrium). Following the dynamic oligopoly theoretical literature, he measures the likelihood of a tacit coordinated agreement in terms of the set of discount factors for which tacit coordination can be sustained and finds that the net result of the two effects is, in general, ambiguous and depends critically on the assumed shape of the demand function, that can alter both quantitatively and qualitatively the impact of such cross-ownership of partial financial rights on the above mentioned set of discount factors.

Gilo, Moshe and Spiegel (2006) extend Malueg (1992)'s analysis to the context of an infinitely repeated *Bertrand* homogeneous-product *symmetric n-firm oligopoly* model in which firms *and* external owners may hold complex, *not necessarily identical*, partial financial rights in rivals, and follow grim-trigger strategies. In this framework, the static Bertrand-Nash equilibrium is not impacted by partial acquisitions, which allows the authors to focus the impact analysis on the first (positive) effect identified by Malueg (1992). This establishes that a partial cross- or common-ownership of financial rights *never* hinders tacit coordination, but *can* facilitate it by reducing the incentive of firms to deviate from the coordinated arrangement (because deviating firms internalize some of the cost imposed on rivals). They show that, under the above setting, a partial cross- or common-ownership of financial rights *does* facilitate tacit coordination if and only if a set of conditions is satisfied cumulatively. If either one of these conditions fails, the likelihood of a tacit coordinated agreement is not affected. Gilo, Spiegel and Temurshoev (2009) *relax the symmetry assumption* in Gilo, Moshe and Spiegel (2006) and generalize the set of conditions that must be satisfied cumulatively in order for a partial cross- or common-ownership of financial rights to facilitate tacit coordination.[4]

Finally, de Haas and Paha (2016) examine Reynolds and Snapp (1986)'s argument in the context of an infinitely repeated *symmetric duopoly* model that combines characteristics borrowed from Malueg (1992) and Gilo, Moshe and Spiegel (2006), so to establish a more comprehensive setting. In particular, they (*i*) consider *Cournot* or *Bertrand* homogeneous-product competition, (*ii*) allow firms *and* external owners to hold complex, *not necessarily identical*, partial financial rights in rivals, and (*iii*) follow grim-trigger strategies. Moreover, they extend the literature by (*iv*) allowing firms to engage in *Bertrand* differentiated-product competition and (*v*) introducing a competition agency, which may detect and sanction collusion. Under this comprehensive setting, they establish the following results. First, in the context of *Bertrand* homogeneous-product competition, a partial cross- or common-ownership of financial rights *never* hinders tacit coordination, in line with Gilo, Moshe and Spiegel (2006). Second, in contexts of *Cournot* homogeneous-product competition and *Bertrand* differentiated-product competition, a partial cross- or common-ownership of financial rights have an ambiguous impact on tacit coordination, in line with Malueg (1992). Moreover, it *can* hinders tacit coordination under a wider set of assumptions than was suggested by Malueg (1992), a conclusion that is particularly prevalent in the presence of the competition agency.

2.2 Quantifying the Coordinated Effects of Mergers

The second strand of literature relates to the quantitative evaluation of the coordinated effects of mergers, which began with Kovacic *et al.* (2007, 2009). They propose to measure the magnitude of coordinated effects by evaluating how a merger affects (*i*) the firms' *incentives* for *post*-merger tacit coordinated behavior and (*ii*) the *stability* of such behavior. The former is quantified by the raw difference between punishment (e.g., competitive) and coordination profits, denoted *incremental profits from coordination*, and the latter by the raw difference between deviation and coordination profits, denoted *incremental profits from deviations.* The proposed procedure involves three steps: (first) the selection of a competition model, (second) the calibration of the model to the relevant features of the *pre*-merger market, and (third) the use of the calibrated model to compute the profitability of coordination and deviation from that agreement. The approach assumes that the probability of coordination *increases* with the incremental profits from coordination and *decreases* with the incremental profits from deviations. The authors apply this procedure to several acquisitions by Hospital Corporation of America in the Chattanooga, Tennessee

[4] Gilo, Moshe and Spiegel (2006) show that an increase in the partial financial right of firm r in a rival s do facilitate coordination "if and only if (*i*) each firm in the industry holds a stake in at least one rival, (*ii*) the maverick firm in the industry (the firm with the strongest incentive to deviate from a collusive agreement) has a direct or an indirect stake in firm r, and (*iii*) firm s is not the industry maverick." Following Flath (1992) the maverick has an indirect stake in firm r if it holds a direct stake in firm t and, in turn, firm t holds a direct stake in firm r. Gilo, Spiegel and Temurshoev (2009) show that an increase in the partial financial right of firm r in a rival s do facilitate colusion "if and only if (*i*) the maverick firm in the industry has a direct or an indirect stake in firm r, and (*ii*) firm s is not the industry maverick."

area using a *Bertrand* differentiated-product model and allowing for the possibility of *post*-merger quality improvements among the merging firms, differential costs, and capacity constraints.

Davis (2006) and Sabatini (2006), working initially independently and then jointly in Davis and Sabatini (2011), extend Kovacic *et al.* (2007, 2009)'s procedure suggesting that the impact of a merger on the likelihood of a tacit coordinated agreement can only be properly captured by incorporating Kovacic *et al.* (2007, 2009)'s incremental profits, which are *static*, in a *dynamic* oligopoly model. The proposed procedure is closely related to that used to simulate the unilateral price effects of mergers in differentiated product markets and it involves three steps: (first) the estimation of the industry's demand system, (second) the use of the *pre*-merger data, jointly with the estimated demand and an appropriate assumption about the nature of *pre*-merger prices, to infer marginal costs, and (third) the simulation, using the inferred marginal costs, of the counterfactual stream of profits of firms under the different regimes of the tacit coordination model: *agreement*, *deviation*, and *punishment*. The authors provide *two alternatives* to quantify the coordinated effects of a merger. If the discount factors of the firms in the industry are known (inferred from internal documents or estimated from a rate of return model following the financial economics literature), the effects can be evaluated directly by examining how the merger impacts the incorporated constraints. Alternatively, and closely paralleling the dynamic oligopoly theoretical literature, the impact of a merger on the likelihood of a tacit coordinated agreement can be evaluated by examining how it affects the minimum discount factors that sustain that agreement. Davis and Huse (2010) provide the first application of this proposed methodology to Hewlett Packard and Compaq's merger in the network server industry. They account for multi-market contact, the presence of a competitive fringe and of a competition agency, and show that, *ceteris paribus*, the incentives to collude often fall as a result of a merger. Ivaldi and Lagos (2016) examine the robustness of Davis and Huse (2010) results for a broad range of consumer and firm characteristics. Their results suggest that mergers strengthen the incentives of the merging parties to coordinate and weaken the incentives of non-merging parties, with the former effect being stronger overall.

3 Empirical Structural Methodology

We propose an empirical structural methodology that attempts to link the above two strands of the literature.

3.1 The Setup

There are F multi-product firms in the industry, indexed by $f \in \Im \equiv \{1, \ldots, F\}$, which interact repeatedly over time and across markets. In each period $s \in \Psi \equiv \{1, \ldots, t, \ldots, \infty\}$ and market $m \in \Upsilon \equiv \{1, \ldots, M\}$ each firm f produces some subset, Γ_{fms}, of the J_s alternative products available in the period. There are also K owners, indexed by $k \in \Theta \equiv \{1, \ldots, F, \ldots, K\}$, who may include not just owners from the subset $\Theta \backslash \Im$ that are external to the industry (and can engage in common-ownership), but also owners from the subset $\Im$ that are internal to the industry (and can engage in cross-ownership).[5] Finally, there

[5] The set $\Theta \backslash \Im$ denotes the set Θ excluding the firms in the subset $\Im$.

is a proposed acquisition in period t. This implies that $s < t$ denotes a *pre*-acquisition period and $s \geq t$ denotes a *post*-acquisition period (assuming the acquisition is allowed by the competition agency).

As discussed above, the coordinated effects of partial acquisitions depend heavily on whether the ownership rights transacted in the acquisition are financial or corporate control rights. In order to capture the distinction between these two rights, we consider that the total stock of each firm f is composed of voting stock and non-voting (preferred) stock. Both give the holder the right to a share of the profits, but only the former gives the holder the right to vote for the Board or to participate in other decisions.

The degree of financial rights of owner k in firm f is represented by $0 \leq \phi_{kf} \leq 1$, with $\sum_{k\in\Theta} \phi_{kf} = 1$, which denotes the owner's holdings of total stock in the firm, regardless of whether it be voting or non-voting stock. The degree of corporate control rights of owner k in firm f is represented by $0 \leq \gamma_{kf} \leq 1$, with $\sum_{k\in\Theta} \gamma_{kf} = 1$, which denotes a measure of the owner's influence over the decision-making within the firm. This measure will, in general, be a function of the vector of holdings of voting stock of all owners in the firm.[6] Typically, the larger the owner's holdings of voting stock in the firm, the greater the degree of control over the decision making within the firm. However, the relationship is firm-specific and may not necessarily be linear. For instance, an owner holding 49% of voting stock in a firm may have no influence over the firm's decision-making if one other owner holds 51%. In contrast, an owner holding 10% of voting stock in a firm may effectively control the firm's decision-making if each of the remaining owners holds a trifling amount of voting stock. This implies that competition agencies, in order to apply our proposed empirical structural methodology, must beforehand evaluate the corporate-control structure of each firm (i.e., determine which holders of voting stock can actually influence the decision-making within the firm and in which degree) before and after the proposed acquisition.[7] We will discuss below two alternative approaches to do so from the vector of holdings of voting stock of all owners in the firm.

3.2 Cross-Ownership

We model acquisition settings involving the subset $\Im$ of firms that are internal to the industry in the lines of Ellerman (1991) and Brito *et al.* (2017), who note that a cross-ownership of financial and corporate control rights *changes the distribution of the corresponding rights* among external owners. In particular, it changes the distribution of ownership rights among external owners in a way that induces a common-ownership of rights among external owners, even if this common-ownership is - in the absence of cross-ownership - non-existent. In order to see why, note - for example - that an external owner with a sole *direct* ownership right in a firm (for example, firm A) has in fact an *ultimate* ownership right in two rival firms, firm A and rival firm B, if firm A has an ownership right in firm B.

Formally, we have that the ultimate ownership rights of external owner k in firm f, ϕ^u_{kf} and γ^u_{kf}, includes not just the direct ownership rights in the firm, ϕ_{kf} and γ_{kf}, but also the indirect ownership rights that may arise from having ultimate ownership rights in a rival $g \in \Im \backslash f$ if that rival holds, in turn,

[6] This makes clear that while an owner can hold a financial right in a firm without holding also a corporate control right, she can not hold a corporate control right in a firm without holding also a financial right.

[7] Financial and corporate control rights may also depend on the period s. We chose not make this dependence explicit in order to avoid having to introduce an additional subscript.

ownership rights in firm f. This implies that for all $k \in \Theta \backslash \Im$ and $f \in \Im$, we have:

$$\begin{aligned} \phi_{kf}^{u} &= \phi_{kf} + \sum\nolimits_{g \in \Im \backslash f} \phi_{kg}^{u} \phi_{gf} \\ \gamma_{kf}^{u} &= \gamma_{kf} + \sum\nolimits_{g \in \Im \backslash f} \gamma_{kg}^{u} \gamma_{gf}, \end{aligned} \tag{1}$$

where $\Im \backslash f$ denotes the set $\Im$ not including firm f.

Let $\mathbf{F}$ and $\mathbf{C}$ denote the $(K-F) \times F$ matrices capturing the direct financial and corporate control rights, respectively, of *external owners*, with typical elements ϕ_{kg} and γ_{kg} representing the corresponding direct ownership rights of external owner k in firm g. Let also $\mathbf{F}^{*}$ and $\mathbf{C}^{*}$ denote the $F \times F$ matrices capturing the direct financial and corporate control rights of *internal owners*, with zero diagonal elements, $\phi_{ff} = 0$ and $\gamma_{ff} = 0$, and off-diagonal elements, $0 \leq \phi_{fg} \leq 1$ and $0 \leq \gamma_{fg} \leq 1$ (if $f \neq g \in \Im$), representing the corresponding direct ownership rights of firm f in firm g. Brito *et al.* (2017) show that, under the assumption that external owners hold ownership rights in *at least one firm of the industry*, we can solve for the ultimate ownership rights of each external owner as a function of the direct ownership rights of all owners (internal and external), as follows:

$$\begin{aligned} \mathbf{F}^{u} &= \mathbf{F} \left(\mathbf{I}_{F} - \mathbf{F}^{*} \right)^{-1} \\ \mathbf{C}^{u} &= \mathbf{C} \left(\mathbf{I}_{F} - \mathbf{C}^{*} \right)^{-1}, \end{aligned} \tag{2}$$

where $\mathbf{I}_{F}$ denotes a $F \times F$ identity matrix while $\mathbf{F}^{u}$ and $\mathbf{C}^{u}$ denote the $(K-F) \times F$ matrices capturing the ultimate financial and corporate control rights, respectively, of external owners, with typical elements ϕ_{kg}^{u} and γ_{kg}^{u} representing the corresponding ultimate ownership rights of external owner k in firm g. Further, Brito *et al.* (2017) also show that the ultimate ownership rights of external owners established in matrices $\mathbf{F}^{u}$ and $\mathbf{C}^{u}$ are non-negative and sum up to one for any given firm f. This makes clear that a cross-ownership of ownership rights changes the distribution of those rights among external owners, as the *ultimate* ownership rights of an external owner in any given firm are not necessarily equal to her *direct* ownership rights in the firm, but the sum of all ownership rights (direct and ultimate) in the firm, is the same.

3.3 Common-Ownership

Having established that a cross-ownership of rights among rival firms induces a common-ownership of rights by external owners in the firms involved, we now model the latter. We follow O'Brien and Salop (2000) and Brito *et al.* (2017) in arguing that a common-ownership of rights *may induce a conflict* of objectives among external owners and that the manager of the firm must weight the (eventual) conflicting objectives of the different external owners according to the corporate control structure of the firm, which determines the influence of each of those owners over the decision-making within the firm. In order to see why this is the case, note - for example - that an external owner of firm A who also holds financial rights in a rival firm B typically wants firm A to pursue a less aggressive strategy than the strategy desired by an external owner with no financial rights in firm B.

In order to model this potential conflict of objectives, we make two assumptions.

Assumption 1 *The objective function of external owners is captured by the present discounted value of the stream of returns from their overall ultimate financial rights holdings.*

Assumption 2 *The manager of a firm weights the potential conflict of objectives among external owners by maximizing a weighted sum of the returns of the firm's controlling external owners, where the weight associated to the returns of an owner in any given firm is given by the ultimate corporate control rights of the owner in the firm.*

Assumptions 1 and 2 imply the following mathematical formulation for the weight function of the manager of any firm f in period t, denoted ϖ_{ft}:

$$\varpi_{ft} = \sum\nolimits_{k \in \Theta} \gamma^{u}_{kf} \mathrm{PDR}_{kt}, \tag{3}$$

where PDR_{kt} denotes the present discounted value of the stream of returns of external owner k from her overall financial rights holdings (in period t's terms). Azar (2017) shows that this mathematical formulation can be microfounded through a probabilistic voting model in which two potential managers compete in Downsian lines for the owner's votes. Kamada and Kojima (2013)'s equivalence result establishes that the same mathematical formulation can be microfounded through a costly voting model in which owners vote if and only if the voting cost is smaller than the perceived gain from doing so.

Azar (2017) shows that if the two candidates maximize the expected vote share within the firm, the corporate control rights of the firm's external owners can be measured by their voting stock holdings. Alternatively, if the two candidates maximize the probability of winning the election, the corporate control rights of the firm's external owners can be measured by their Banzhaf (1965)'s power index.

We model the present discounted value of the stream of returns of external owner k from her overall financial rights holdings (in period t's terms) as follows:

$$\mathrm{PDR}_{kt} = \sum\nolimits_{s=t}^{\infty} \delta^{s-t} \sum\nolimits_{g \in \Im} \phi^{u}_{kg} \pi_{gs} \tag{4}$$

where $\delta \in (0, 1)$ denotes the common discount factor of external owners, capturing the the weight that they place in future returns as measured by the industry's cost of equity, and π_{gs} denotes the operating profit of firm g in period s.

We model the discount factor to be the same for all firms and all time periods, but both features are illustrative and presented for simplicity. They can be relaxed in line with Harrington (1989) and Friedman (1971), respectively. Further, we model the operating profits to account for a differentiated products industry and asymmetric multi-product firms in line with Rothschild (1999), Kuhn (2004) and Vasconcelos (2005), in order to encompass real-world industry features. As such, the operating profit of firm g in period s is defined over the set of different markets in which it is active on and over the set of different products it manufactures, as follows:

$$\pi_{gs} = \sum\nolimits_{m\in\Upsilon} \left\{ \sum\nolimits_{j\in\Gamma_{gms}} \left(p_{jms} - mc_{jms}\right) q_{jms}\left(\mathbf{p}_{gms}, \mathbf{p}_{-gms}\right) - C_{gms} \right\}. \tag{5}$$

p_{jms} and mc_{jms} denote the price and the (possibly asymmetric) marginal cost of product j in market m and period s, respectively. $q_{jms}\left(\mathbf{p}_{gms}, \mathbf{p}_{-gms}\right)$ denotes the quantity of product j in market m and period s, which is (by definition of market) a function of the vector of prices of the products available in the market: those produced by firm g, which we denote by $\mathbf{p}_{gms}$ and those produced by all other firms, which we denote by $\mathbf{p}_{-gms}$. Finally, C_{gms} denotes the fixed cost of production of firm g in market m and period s. This establishes that the operating profit of each firm g in any given period s is a function of the full price vector $\mathbf{p}_s = \left(\mathbf{p}_{gs}, \mathbf{p}_{-gs}\right)$ played in period s, where $\mathbf{p}_{gs}$ and $\mathbf{p}_{-gs}$ aggregate $\mathbf{p}_{gms}$ and $\mathbf{p}_{-gms}$, respectively, across the set of markets Υ as follows: $\mathbf{p}_{gs} = \left(\mathbf{p}_{g1s}, \ldots, \mathbf{p}_{gms}, \ldots, \mathbf{p}_{gMs}\right)'$ and $\mathbf{p}_{-gs} = \left(\mathbf{p}_{-g1s}, \ldots, \mathbf{p}_{-gms}, \ldots, \mathbf{p}_{-gMs}\right)'$. This assumes that we are ruling out dynamic effects, which could be important for either durable or storable products. However, this assumption is, again, merely illustrative and presented for simplicity. Let $\pi_{gs}\left(\mathbf{p}_s\right)$, for all g and s, express this mathematical dependence.

The above framework establishes that the weight function of the manager of any firm f in period t can be written as follows:

$$\begin{aligned} \varpi_{ft} &= \sum\nolimits_{k\in\Theta} \gamma^u_{kf} \sum\nolimits_{s=t}^{\infty} \delta^{s-t} \sum\nolimits_{g\in\Im} \phi^u_{kg} \pi_{gs}\left(\mathbf{p}_s\right) \\ &= \sum\nolimits_{s=t}^{\infty} \delta^{s-t} \sum\nolimits_{g\in\Im} l_{fg} \pi_{gs}\left(\mathbf{p}_s\right), \end{aligned} \tag{6}$$

where the weight $l_{fg} = \sum_{k\in\Theta} \gamma^u_{kf} \phi^u_{kg} \geq 0$ for any $f, g \in \Im$ denotes the typical element of the $F \times F$ matrix $\mathbf{L} = \left(\mathbf{C}^u\right)^\top \mathbf{F}^u$.[8] This weight captures the ultimate financial and corporate control rights that the external owners of firm f hold over firm g. Without loss of generality, we normalize the weight on the own-operating profit to be one by dividing the weight function of the manager of each firm f by l_{ff}. This implies that the manager of firm f in period t maximizes the following, entirely equivalent, weight function:

$$\varpi'_{ft} = \sum\nolimits_{s=t}^{\infty} \delta^{s-t} \sum\nolimits_{g\in\Im} w_{fg} \pi_{gs}\left(\mathbf{p}_s\right), \tag{7}$$

where $w_{fg} = l_{fg}/l_{ff} \geq 0$ for any $f, g \in \Im$ denotes the typical element of the $F \times F$ *normalized* weight matrix $\mathbf{W} = \mathit{diag}\left(\mathbf{L}\right)^{-1} \mathbf{L}$, and $\mathit{diag}\left(\mathbf{L}\right)$ is the $F \times F$ matrix formed by substituting zeros for all off-diagonal elements of $\mathbf{L}$.[9] This establishes that the weight function of the manager constitutes a real valued function of (i) the path of the vector of prices for the infinite sequence of time periods $s > t$: $\{\mathbf{p}_t, \mathbf{p}_{t+1}, \ldots, \mathbf{p}_s, \ldots\}$, and (ii) matrix $\mathbf{W}$. This weight function can cope with a multitude of general industry ownership structures, involving owners that can be internal (represented in matrices $\mathbf{F}^*$ and $\mathbf{C}^*$) and external (represented in matrices $\mathbf{F}$ and $\mathbf{C}$) to the industry; and ownership rights that can involve

[8] In order to see why the weights l_{fg} are non-negative, note that $\gamma^u_{kf} \geq 0$ and $\phi^u_{kg} \geq 0$ for all $k \in \Theta$, $k \notin \Im$, and all $f, g \in \Im$.

[9] In order to see why the weights w_{fg} are non-negative for any $f, g \in \Im$, note that - as discussed above - an owner can not hold a corporate control right in a firm without holding a financial right in that firm. This implies that $l_{ff} > 0$ and, in turn, that $w_{fg} \geq 0$ (since, as discussed above, $l_{fg} \geq 0$).

financial interests (represented in matrices $\mathbf{F}^*$ and $\mathbf{F}$) and corporate control (represented in matrices $\mathbf{C}^*$ and $\mathbf{C}$), can be direct and indirect, partial or full. Moreover, in structures in which *cross- and common-ownership rights are absent*, the above weight function reduces to the present discounted value of the firm's stream of operating profits. Appendix A.1 derives this result.

3.4 Competitive Setting

Having described the weight function of the manager of the firm, we now address the competitive setting in the industry. We make two *alternative* assumptions about this setting.

3.4.1 Non-Cooperative Behavior

We begin by assuming that the firms behave non-cooperatively.

Assumption 3a *The manager of every firm f competes in prices in each time period.*

In a setting, as discussed above, that rules out dynamic effects, the non-cooperative equilibrium price vector p_{ft}^{nc} of firm f in any period t is, under Assumption 3a, the solution to the following maximization problem:

$$\begin{aligned} \mathbf{p}_{ft}^{nc} &= \arg\max_{\mathbf{p}_{ft}} \sum\nolimits_{g\in\Im} w_{fg}\pi_{gs}\left(\mathbf{p}_s\right) \\ &= \arg\max_{\mathbf{p}_{ft}} \sum\nolimits_{g\in\Im} w_{fg}\left(\sum\nolimits_{m\in\Upsilon}\sum\nolimits_{j\in\Gamma_{gmt}}\left(p_{jmt}-mc_{jmt}\right)q_{jmt}\left(\mathbf{p}_{fmt},\mathbf{p}_{-fmt}\right)-C_{gmt}\right). \end{aligned}$$

Aksoy-Pierson, Allon and Federgruen (2010) established the conditions under which a non-cooperative equilibrium, in fact an unique equilibrium with positive prices, exists for the general multi-product price competition model with random coefficients multinomial logit demand functions (that we will consider below), see Theorem 6.1 therein. The unique non-cooperative equilibrium price p_{jms}^{nc} of any product j from firm f in market m and period t must then satisfy the following first-order condition:

$$q_{jmt}\left(\mathbf{p}_{fmt}^{nc},\mathbf{p}_{-fmt}^{nc}\right)+\sum\nolimits_{g\in\Im} w_{fg}\sum\nolimits_{r\in\Gamma_{gmt}}\left(p_{rmt}^{nc}-mc_{rmt}\right)\left(\partial q_{rmt}\left(\mathbf{p}_{fmt}^{nc},\mathbf{p}_{-fmt}^{nc}\right)/\partial p_{jmt}\right)=0, \quad (8)$$

where $\mathbf{p}_t^{nc}=\left(\mathbf{p}_{ft}^{nc},\mathbf{p}_{-ft}^{nc}\right)$ denotes the vector of the non-cooperative equilibrium industry prices in period t.

3.4.2 Coordinated Behavior

The repeated choice of period t's vector of the non-cooperative equilibrium prices in all subsequent periods $s>t$ is a sub-game perfect equilibrium. However, it is well known that there may exist more profitable strategies (Luce and Raiffa, 1957; Friedman, 1971). The interaction of the firms in the industry over time provides a formal structure that may support a coordinated outcome as a non-cooperative equilibrium, under the credible threat that deviations from this arrangement would trigger punishment by rivals.

When choosing strategy $\mathbf{p}_{ft}$, the manager of each firm f knows (and therefore can condition upon) the strategies chosen by the managers of every other firm in all previous periods. We make the following assumption about the price strategy adopted by firm managers to eventually support a coordinated outcome as a non-cooperative equilibrium.

Assumption 3b *The manager of each firm f adopts the following grim-trigger strategy σ_f^{grim}:*

$$\begin{aligned} \mathbf{p}_{ft} &= \mathbf{p}_{ft}^{c} \\ \mathbf{p}_{fs} &= \mathbf{p}_{fs}^{c} \text{ if } \mathbf{p}_{gl} = \mathbf{p}_{gl}^{c}, \quad \forall g, \ s > t, \ l = t, \ldots, s-1, \\ \mathbf{p}_{fs} &= \mathbf{p}_{fs}^{nc} \text{ otherwise} \end{aligned} \tag{9}$$

where $\mathbf{p}_{gl}^{c}$ denotes the coordinated equilibrium price vector of firm g in period l.

In this type of strategy, the manager of each firm *agrees* to set coordination prices in every period and trust the managers of each other firm to continue to do so indefinitely. Naturally, in face of this coordinated conduct, individual managers may be tempted to increase the returns of the firm's external owners for a period or so by *deviating* from the arrangement. However, should any single manager in any past period choose something different trust vanishes and triggers retaliation. Each manager (credibly) *punishes* the deviant manager by reverting permanently to a position in which no manager has any short-term temptation to deviate: under the above strategy, the non-cooperative equilibrium prices.

We acknowledge that the Nash reversion that characterizes the grim-trigger strategies established in Assumption 3*b*, while sub-game perfect, is not in general optimal. Abreu (1986, 1988) discusses more sophisticated forms of retaliation, *optimal punishments*, that support the *maximal* degree of coordination for arbitrary values of the discount factor of the external owners. These optimal punishments have a stick-and-carrot structure that, for example, may include temporary price wars: should any single manager in any past period deviate from the coordinated arrangement, firms revert to a war state in which managers set below non-cooperative equilibrium price levels for some period of time (stick) before reverting, if no manager deviates from the war state arrangement, to the coordinated arrangement again (carrot). Although the extension of the methodology to Abreu (1986, 1988)'s optimal punishments is a very interesting potential area for future research, in this article, we focus on developing an empirical methodology to quantify coordinated effects for the grim-trigger strategies' benchmark. We do so, first, because this type of strategies has the advantage of requiring simple calculations and of being easily understood by market participants. Second, because as pointed out by Harrington (1991):

> It is quite natural to think of a punishment strategy as being an industry norm with respect to firm conduct (...). Furthermore, once a norm is in place, firms may be hesitant to change it (...). Thus, even though the norm might not be the best in some sense (for example, it might not be a most severe punishment strategy), firms might choose to maintain it if it seems to work. In light of this interpretation of a punishment strategy, it seems plausible that the grim trigger strategy would be a commonly used norm. (page 1089)

We make the following assumption regarding our understanding of coordinated behavior, defined in Assumption 3b, in a setting of a cross- and common-ownership structure of financial and corporate control rights:

Assumption 4 *Under coordination, the manager of each firm f weights the stream of operating profit of every rival as its own.*

Assumption 4 implies that, under tacit coordination, the normalized weight $w_{fg} = 1$ for any $f, g \in \Im$ and, as a consequence, the manager of firm f in period t maximizes the present discounted value of the stream of industry profits:

$$\varpi''_{ft} = \sum_{s=t}^{\infty} \delta^{s-t} \sum_{g \in \Im} \pi_{gs} \left(\mathbf{p}_s \right). \tag{10}$$

The coordinated equilibrium price vector $\mathbf{p}^c_{ft}$ of every firm f in any period t, is therefore the solution to the following maximization problem:

$$\mathbf{p}^c_{ft} = \arg\max_{\mathbf{p}_{ft}} \sum_{g \in \Im} \left(\sum_{m \in \Upsilon} \sum_{j \in \Gamma_{gmt}} \left(p_{jmt} - mc_{jmt} \right) q_{jmt} \left(\mathbf{p}_{fmt}, \mathbf{p}_{-fmt} \right) - C_{gmt} \right). \tag{11}$$

The unique coordination price p^c_{jmt} of any product j from firm f in market m and period t must then satisfy the following first-order condition:

$$q_{jmt} \left(\mathbf{p}^c_{fmt}, \mathbf{p}^c_{-fmt} \right) + \sum_{g \in \Im} \sum_{r \in \Gamma_{gmt}} \left(p^c_{rmt} - mc_{rmt} \right) \left(\partial q_{rmt} \left(\mathbf{p}^c_{fmt}, \mathbf{p}^c_{-fmt} \right) / \partial p_{jmt} \right) = 0, \tag{12}$$

where $\mathbf{p}^c_t = \left(\mathbf{p}^c_{ft}, \mathbf{p}^c_{-ft} \right)$ denotes the vector of the coordinated equilibrium industry prices in period t. This yields that, under tacit coordination, each manager *fully* internalizes the effects of price changes on the stream of operating profits of all the firms in the industry.

The above coordinated equilibrium price vector is supported by the supergame grim strategy vector $\sigma^{grim} = \left(\sigma^{grim}_1, \ \ldots, \ \sigma^{grim}_f, \ \ldots, \ \sigma^{grim}_F \right)$ if, for every firm f and period t, the following *non-deviation* condition is satisfied:

$$\sum_{s=t}^{\infty} \delta^{s-t} \sum_{g \in \Im} w_{fg} \pi_{gs} \left(\mathbf{p}^c_s \right) > \sum_{g \in \Im} w_{fg} \pi_{gt} \left(\mathbf{p}^d_{gt}, \mathbf{p}^c_{-gt} \right) + \sum_{s=t+1}^{\infty} \delta^{s-t} \sum_{g \in \Im} w_{fg} \pi_{gs} \left(\mathbf{p}^{nc}_s \right) \tag{13}$$

i.e., if the value of the weight function of the manager of each firm f when playing the *agreed* coordination prices in the current and every single future period (the left term of the above inequality) exceeds the value of the weight function when *deviating* from the arrangement in the current period and revert to the *punishment*, non-cooperative equilibrium, prices in every future period (the right term of the above inequality).

In order to determine whether the supergame grim strategy vector σ^{grim} satisfies the set of non-deviation conditions (13) and, in fact, constitutes a sub-game perfect equilibrium, we must characterize $\pi_{gt} \left(\mathbf{p}^d_{gt}, \mathbf{p}^c_{-gt} \right)$ for all $g \in \Im$, i.e., the operating profit of every firm when *deviating* from the arrangement in the current period. To do so, we must determine the (sub-)vector of the deviation equilibrium prices $\mathbf{p}^d_{gt}$. The manager of every deviating firm g chooses the price vector $\mathbf{p}^d_{gt} \neq \mathbf{p}^c_{gt}$ that yields the maximum possible value for her weight function, given that all other managers are keeping to the arrangement by setting $\mathbf{p}^c_{-gt}$. This implies the deviation equilibrium price vector p^d_{gt} of firm g in period t is the solution to the following maximization problem:

$$\mathbf{p}^d_{gt} = \arg\max_{\mathbf{p}_{gt}} \sum\nolimits_{h\in\Im} w_{gh} \left(\sum\nolimits_{m\in\Upsilon} \sum\nolimits_{j\in\Gamma_{hmt}} \left(p_{jmt} - mc_{jmt}\right) q_{jmt}\left(\mathbf{p}_{hmt}, \mathbf{p}^c_{-hmt}\right) - C_{hmt} \right). \tag{14}$$

Following Bernheim and Whinston (1990), given that in any multimarket tacit coordination equilibrium, firms know that deviations will be punished in all markets (Abreu, 1988), if a firm decides to deviate, it will do so in every market. The unique deviation price p^d_{jmt} of any product j from deviating firm g in market m and period t must then satisfy the following first-order condition:

$$\begin{aligned} & q_{jmt}\left(\mathbf{p}^d_{gmt}, \mathbf{p}^c_{-gmt}\right) + \sum\nolimits_{r\in\Gamma_{gmt}} \left(p^d_{rmt} - mc_{rmt}\right) \left(\partial q_{rmt}\left(\mathbf{p}^d_{gmt}, \mathbf{p}^c_{-gmt}\right)/\partial p_{jmt}\right) + \\ & + \sum\nolimits_{h\in\Im, h\neq g} w_{gh} \sum\nolimits_{r\in\Gamma_{hmt}} \left(p^c_{rmt} - mc_{rmt}\right) \left(\partial q_{rmt}\left(\mathbf{p}^d_{gmt}, \mathbf{p}^c_{-gmt}\right)/\partial p_{jmt}\right) = 0. \end{aligned} \tag{15}$$

Having characterized $\pi_{gt}\left(\mathbf{p}^d_{gt}, \mathbf{p}^c_{-gt}\right)$ for all deviating firms g, we can now determine whether the supergame grim strategy vector σ^{grim} constitutes, in fact, a sub-game perfect equilibrium. However, beforehand, we must define a rule to evaluate *future* aggregate profits (under the three regimes of the tacit coordination model). We assume the following benchmark.

Assumption 5 *Future consumer preferences, product characteristics, and firm marginal costs do not change over time.*

Assumption 5 implies that, in line with Ivaldi and Vagos (2016), the future operating profit of every firm f is time-independent $\pi_{fs}\left(\mathbf{p}_s\right) = \pi_f\left(\mathbf{p}\right)$ for any firm f and period $s = t, t+1, t+2, \ldots$. This rules out settings with, for example, future demand growth, future demand fluctuations (deterministic or not) and future innovative activity. However, this benchmark is merely illustrative. The proposed methodology is not constrained to it and remains valid under alternative settings.

In the above benchmark, the set of non-deviation conditions (13) that ensure that the supergame grim strategy vector σ^{grim} constitutes a sub-game perfect equilibrium can be written, for every firm f, as:

$$\frac{1}{1-\delta} \sum\nolimits_{g\in\Im} w_{fg}\pi_g\left(\mathbf{p}^c\right) > \sum\nolimits_{g\in\Im} w_{fg}\pi_g\left(\mathbf{p}^d_f, \mathbf{p}^c_{-f}\right) + \frac{\delta}{1-\delta} \sum\nolimits_{g\in\Im} w_{fg}\pi_g\left(\mathbf{p}^{nc}\right) \tag{16}$$

or

$$\sum\nolimits_{g\in\Im} w_{fg}\left(\pi_g\left(\mathbf{p}^d_f, \mathbf{p}^c_{-f}\right) - \pi_g\left(\mathbf{p}^c\right)\right) < \frac{\delta}{1-\delta} \sum\nolimits_{g\in\Im} w_{fg}\left(\pi_g\left(\mathbf{p}^c\right) - \pi_g\left(\mathbf{p}^{nc}\right)\right) \tag{17}$$

This inequality makes clear the supergame grim strategy vector σ^{grim} constitutes a sub-game perfect equilibrium if, for the manager of every firm f, the one-shot benefit from deviating the tacit coordinated agreement in a given period (the left term of the above inequality, which captures the weighted sum - across all firms in which the owners have, direct or indirectly, financial and/or corporate control rights on - of the operating profit difference between deviation and coordinated behavior) is more than compensated by the present discounted value of the benefit from maintaining coordination in all succeeding periods (the right term of the above inequality, which captures the present discounted value of the weighted sum of the operating profit difference between coordinated and non-cooperative behavior).

An alternative interpretation, paralleling the dynamic oligopoly theoretical literature, can be derived by solving the set of non-deviation conditions in terms of the *minimum discount factor* $\delta_f^{\min}$ of external owners that sustains equilibrium coordinated conduct by the manager of firm f:

$$\delta > \delta_f^{\min} = \frac{\sum_{g \in \Im} w_{fg} \left(\pi_g \left(\mathbf{p}_f^d, \mathbf{p}_{-f}^c \right) - \pi_g \left(\mathbf{p}^c \right) \right)}{\sum_{g \in \Im} w_{fg} \left(\pi_g \left(\mathbf{p}_f^d, \mathbf{p}_{-f}^c \right) - \pi_g \left(\mathbf{p}^{nc} \right) \right)}, \tag{18}$$

which implies that for equilibrium coordinated conduct to be sustained in the industry, external owners need to be sufficiently patient in the sense that the weight they place in future profits must exceed a *critical threshold*, given by:

$$\delta > \delta^{crt} = \max \left\{ \delta_1^{\min}, \ldots, \delta_f^{\min}, \ldots \delta_F^{\min} \right\}. \tag{19}$$

This critical threshold constitutes, in the lines of Malueg (1992), our proposed quantitatively measure of the likelihood of a non-cooperative coordinated arrangement.

3.5 Quantifying the Coordinated Effects of Partial Horizontal Acquisitions

In analyzing the coordinated effects of partial horizontal acquisitions, competition agencies need to evaluate whether a proposed acquisition *changes* the manner in which firms in the market interact, increasing the likelihood of coordinated conduct. Our methodology proposes to assess this by quantifying the impact of the acquisition on the critical threshold of the discount factor. To do so, competition agencies need to compute the *non-cooperative, coordination, and deviation* equilibrium prices, both *pre-* and *post-*acquisition. We propose to simulate these equilibrium prices by solving the set of first-order conditions (8), (12) and (15), respectively. In order to do so, we require information on the marginal cost of each relevant product, on the corresponding own- and cross-price effects, and on the elements of matrix $\mathbf{W}$ for both the *pre-* and *post-*acquisition setting. Our methodology proposes to infer this information assuming that the competition agency can obtain (i) *sales data* on prices and quantities for all the relevant products in the industry across the different markets for a collection of past time periods, (ii) *product data* containing observed characteristics of all the relevant products, and (iii) *ownership data* on the structure of voting and non-voting stock of the different firms, *pre-* and *post-*acquisition. The methodology involves several steps similar to Davis (2006) and Davis and Huse (2010), adapted for our setting.

Step 0: Model and Estimate Consumer Demand

Step 0 consists of using the *sales data* and the *product data* to estimate consumer demand and assess the degree of substitutability between the competing products. This step is instrumental in computing the *own- and cross-price effects* required for the *non-cooperative, coordination, and deviation* first-order conditions. We defer the description of step 0 to the next section when we will introduce the consumer demand model, a random coefficients multinomial logit demand function, in the context of our empirical application.

Step 1: Recover Unobserved Marginal Costs & Identify the Competitive Setting of Firms in the Industry

Step 1 uses the *sales data,* the *ownership data pre*-acquisition, and the *own- and cross-price effects* estimated in step 0 to recover unobserved marginal cost information. The procedure involves using the estimated price-effects, jointly with observed price and quantity data to back-out marginal cost information from the set of first-order conditions. However, to do so, we must, beforehand, identify the correct set of first-order conditions to use. In other words, we must identify the competitive setting of firms that gave rise to the observed data. Our methodology proposes to perform this identification by recovering marginal cost information using the sets of first-order conditions (8) and (12), derived under alternative Assumptions 3a and 3b and assume a *non-cooperative* and a *coordinated* competitive setting, respectively.

We do so for each market m in period t, the period of the proposed acquisition, as follows. Let $\mathbf{p}_{mt} = (\mathbf{p}_{fmt}, \mathbf{p}_{-fmt})'$ denote the vector of prices in market m and period t, and $\mathbf{\Omega}_{mt}$ denote a matrix with jr element given by $\Omega_{mt,rj} = -w_{fg}\partial q_{rmt}(\mathbf{p}_{mt})/\partial p_{jmt}$ for $r \in \Gamma_{gmt}$, $j \in \Gamma_{fmt}$. In vector notation, the first-order conditions for each market m in period t become:

$$\mathbf{q}_{mt}(\mathbf{p}_{mt}) - \mathbf{\Omega}_{mt}(\mathbf{p}_{mt})(\mathbf{p}_{mt} - \mathbf{mc}_{mt}) = 0, \tag{20}$$

where $\mathbf{q}_{mt}(\mathbf{p}_{mt})$ and $\mathbf{mc}_{mt}$ denote vectors of the quantities and marginal costs in market m and period t, respectively. We can then recover the vectors of price-cost margins and marginal costs, respectively, for each market m in period t, as follows:

$$\begin{aligned} \mathbf{p}_{mt} - \mathbf{mc}_{mt} &= \mathbf{\Omega}_{mt}(\mathbf{p}_{mt})^{-1}\mathbf{q}_{mt}(\mathbf{p}_{mt}) \\ \mathbf{mc}_{mt} &= \mathbf{p}_{mt} - \mathbf{\Omega}_{mt}(\mathbf{p}_{mt})^{-1}\mathbf{q}_{mt}(\mathbf{p}_{mt}). \end{aligned} \tag{21}$$

The only difference between the *non-cooperative* and *coordination* cases refers to the voting and non-voting stock used to derive matrix $\mathbf{W}$, which is subsequently used to compute matrix $\mathbf{\Omega}_{mt}$. If firms are behaving non-cooperatively, the typical element of matrix $\mathbf{W}$, given by w_{fg} for any $f, g \in \Im$, is computed using the voting and non-voting stock of the different firms *pre*-acquisition. If, however, firms are already coordinating, the typical element of matrix $\mathbf{W}$ is given by $w_{fg} = 1$ for any $f, g \in \Im$. We test the two cases in the lines of the empirical literature that attempts to evaluate the observed conduct of firms. We do so by comparing the recovered margin or cost information to a crude observed equivalent margin or cost measure. Recent examples that attempt to identify the competitive setting of firms from observed equilibrium prices includes Nevo (2001), Slade (2004), Salvo (2010) and Molnar, Violi and Zhou (2013). This enables us to determine the correct behavior of firms *pre*-acquisition and, consequently, the correct set of recovered marginal costs.

There are three important aspects about this empirical procedure to recover marginal costs. First, it relies on the ability to consistently estimate the price effects in step 0, an issue we address in the next section, when we introduce our demand model in the context of our empirical application. Second, it assumes constant marginal costs. However, this assumption can easily be relaxed and the procedure extended to deal with non-constant marginal costs. In this case, the set of first-order conditions differ slightly from the above and marginal costs can be recovered by estimating a marginal cost function using, for example, a method of moments approach. Third, it assumes solely two diametrically opposed competitive settings, *non-cooperative* and *coordinated*. However, this assumption can, again, easily be relaxed and the procedure extended to deal with intermediate competitive settings involving, for example, to *not all-inclusive* tacit coordination agreements in the lines of Nevo (1998) and Molnar, Violi and Zhou (2013). In this case, the elements of matrix $\mathbf{W}$ will combine elements of the two settings above.

Step 2: Compute the Critical Threshold *Pre*-Acquisition

Step 2 uses the *sales data,* the *ownership data pre-acquisition*, the *consumer demand model* and the *own- and cross-price effects* estimated in step 0, and, finally, the *behavior of firms* and the *unobserved marginal costs* identified in step 1, to simulate *counterfactual* equilibrium prices *pre*-acquisition. We do so for the three regimes of the tacit coordination model: *agreement*, *deviation*, and *punishment*.

If step 1 concludes that firms are behaving non-cooperatively, the observed prices and quantities in the *sales data* are already the *punishment* (*non-cooperative*) equilibrium ones and so do not need to be simulated. We have only to simulate *agreement*, (*coordination*) and *deviation* equilibrium prices. We do so by making use of the set of first-order conditions (12) and (15) for each market m in period t. If, however, step 1 concludes that firms are already coordinating, the observed prices and quantities in the *sales data* denote instead *coordination* equilibrium values. This implies that we have only to simulate *non-cooperative* and *deviation* equilibrium prices. We do so by making use of the set of first-order conditions (8) and (15), again, for each market m in period t. In either case, having computed the counterfactual equilibrium prices, we use the consumer demand model estimated in step 0 to derive the counterfactual equilibrium quantities.

Finally, we use the equilibrium prices and quantities, jointly with the unobserved marginal costs recovered in step 1, to compute the equilibrium operating profit of each firm in the industry. We do this for the three regimes of the tacit coordination model: *agreement*, *deviation*, and *punishment*. We then use those operating profits to compute the critical threshold for the discount factor that sustains σ^{grim} as a sub-game perfect equilibrium *pre*-acquisition.

Step 3: Compute the Critical Threshold *Post*-Acquisition

Step 3 uses the *sales data,* the *ownership data post-acquisition*, the *consumer demand model* and the *own- and cross-price effects* estimated in step 0, and, finally, the *behavior of firms* and the *unobserved marginal costs* identified in step 1, to simulate counterfactual equilibrium prices *post*-acquisition., i.e., the *counterfactual* equilibrium prices that would arise in period t if the competition agency allows the acquisition. We do so under the assumption that the proposed acquisition does not alter the competitive setting among firms nor the vector of marginal costs. This implies that we just have to simulate the *post*-acquisition counterfactual *punishment* (*non-cooperative*) and *deviation* equilibrium prices. The reason being that, under this assumption, the *agreement* (*coordination*) equilibrium prices *pre*- and *post*-acquisition coincide. In order to understand why, note that the set of first-order conditions in equation (12) does not depend on $\mathbf{W}$ since each firm internalizes the effects of price changes on the stream of operating profits of all firms in the industry.

We simulate the counterfactual *non-cooperative* and *deviation* equilibrium prices *post*-acquisition by making use of the set of first-order conditions (8) and (15) for each market m in period t. The only difference refers to the voting and non-voting stock used to derive matrix $\mathbf{W}$, which is subsequently used to compute matrix $\mathbf{\Omega}_{mt}$. In step 3, the typical element of matrix $\mathbf{W}$, given by w_{fg} for any $f, g \in \Im$, used to derive the set of first-order conditions (8) and (15) is computed using the voting and non-voting stock of the different firms *post*-acquisition.

Having computed the counterfactual equilibrium prices, we use the consumer demand model estimated in step 0 to derive the counterfactual equilibrium quantities. Finally, we use the equilibrium prices and quantities, jointly with the unobserved marginal costs recovered in step 1, to compute the equilibrium operating profit of each firm in the industry in each of the three regimes of the tacit coordination model. Again, we use these operating profits to compute the critical threshold for the discount factor that sustains σ^{grim} as a sub-game perfect equilibrium *post*-acquisition.

Finally, note that although we assume that the proposed acquisition does not alter the competitive setting among firms, the proposed methodology is not constrained to having the same assumption of firm behavior before and after the acquisition. If the proposed acquisition does alter the competitive setting among firms, the methodology idea remains valid, the only difference being that the equilibrium prices *post*-acquisition must solve the corresponding (new) set of first-order conditions. In such cases, the methodology requires the simulation not only of the *non-cooperative* and *deviation* equilibrium prices, but also of the *coordination* equilibrium prices, all according to the new behavioral setting. This will also be required, even in the absence of changes in the behavioral setting, if the partial acquisition incorporates eventual cost efficiencies that impact the marginal costs.

Having described the supply side of the model and the empirical structural methodology that can be used to quantify the impact on the likelihood of a non-cooperative coordinated arrangement that would result from a proposed acquisition, we move on to address the empirical illustration.

4 Empirical Application

In this section, we present an illustration of the structural methodology used to evaluate the coordinated effects of partial horizontal acquisitions. We apply our framework to several acquisitions in the wet shaving industry. On December 20, 1989, the Gillette Company, contracted to acquire the wet shaving businesses of Wilkinson Sword trademark outside of the (at the time) 12-nation European Community to Eemland Management Services BV (Wilkinson Sword's parent company) for \$72 million, which included the United States operations. It also contracted to acquire 22.9% of the non-voting stock of Eemland for about \$14 million. Gillette said that its reason for participating in Eemland was solely its wish to acquire various Wilkinson Sword trade marks and wet-shaving activities in certain countries outside the

European Community.

At the time, consumers in the United States annually purchased over $700 million of wet shaving razor blades at the retail level. Five firms supplied all but a nominal amount of these blades: Gillette Company, BIC Corporation, Warner-Lambert Company, Wilkinson Sword Inc., and American Safety Razor Company. On January 10, 1990, the Department of Justice (DoJ) instituted a civil proceeding against Gillette. The complaint alleged that the effect of the acquisition by Gillette may have been substantially to lessen competition in the sale of wet shaving razor blades in the United States. Shortly after the case was filed, Gillette voluntarily rescinded the acquisition of Eemland's wet shaving razor blade business in the United States. Gillette said it decided to settle the case to avoid the time and expense of a lengthy trial. However, Gillette still went through with the acquisition of 22.9% of the non-voting stock of Eemland and of all worldwide assets and businesses of Wilkinson Sword trademark from Eemland. Because Eemland kept the Wilkinson Sword's United States wet shaving razor blades business, Gillette had became one of the largest, if not the largest, owner in a competitor, giving rise to a financial cross-ownership structure. The DoJ (1990) allowed the acquisition under a condition of no agreement and communication between the two firms[10] However, even when the acquiring party cannot influence the conduct, agree or communicate with the target firm, the partial acquisition may still raise antitrust concerns about unilateral and coordinated effects. Brito, Ribeiro and Vasconcelos (2014) empirically examine the unilateral effects of this acquisition. In the present paper, we empirically examine its coordinated effects. In addition to the above *actual* operation, we also empirically examine five additional *hypothetical* acquisitions to illustrate the full variety of acquisitions that may be addressed by our empirical methodology.

The article proceeds by describing the data and performing some preliminary analysis. We then move on to describe the demand model, the estimation procedure and discuss the identifying assumptions. This is important since our proposed methodology relies on the ability to consistently estimate the price effects in step 0. We then present the demand estimation results that we use to compute the implied marginal costs and, finally, simulate the coordinated effects of the different acquisitions.

4.1 Data Description and Preliminary Analysis

We use scanner data collected from July 1994 to June 1996 from the Dominick's Finer Foods (DFF) chain in the Chicago metropolitan area. The dataset covers 29 different product categories at the store level. It includes weekly sales, prices and retail profit margins for each universal product code (UPC) and store of the chain. We supplemented the data with ZIP code (*i*) demographic information obtained from the Decennial Census 2000, and (*ii*) industry structure obtained from the Business Patterns 1998 databases.

In order to investigate the implications of the above acquisitions in the wet shaving industry, we focus on the grooming category. In particular, we focus on disposable razor products to avoid the complications that the tied-goods nature of demand poses for modeling in other razor products.

[10] "Gillette and Eemland shall not agree or communicate an effort to persuade the other to agree, directly or indirectly, regarding present or future prices or other terms or conditions of sale, volume of shipments, future production schedules, marketing plans, sales forecasts, or sales or proposed sales to specific customers (...)." (DoJ, 1990).

The sample covers 30 products from 6 brands in 81 stores (across 7 counties in the Chicago metropolitan area) for 104 weeks. Gillette is the dominant brand with an average share of 59.5% of the total number of razors sold in each store and week combination. DFF's private label is the second biggest-selling brand with an average share of 20.6%, followed by Shick, with an average share of 14.0% and BIC, with an average share of 5.6%. Personna and Wilkinson Sword have very residual average shares.

We define a product to be specific to a gender segment (men or women). This implies that, for example, Schick Slim Twin and Schick Slim Twin Women are classified as distinct products. Women products account for an average share of 17.3% of the total number of razors sold in every store and week. DFF stores carry an average of 13.2 different products in each store and week combination. However, in contrast with the substantial brand concentration, at the product level, there is slightly more fragmentation. Gillette Good News is the market leader with an average share of 14.2% of the weekly total number of razors sold in each store.

Each product is typically offered in several package sizes, with the top four sizes accounting for an average share of more than 99% of the weekly total number of razors sold in each store: 10 razors packages (41.5%), 5 razors packages (41.4%), 12 razors packages (11.3%) and 15 razors packages (5.2%). A product-package size combination defines an UPC. The sample covers 56 UPCs and DFF stores carry an average of 17.3 different UPCs in each store and week combination. Table 1 details the volume market shares for the top-6 brands, products and package sizes. Appendix B.1 describes in more detail the dataset and the different price discrimination features of the price variable that must be incorporated into the structural model and justify the aggregation of the original weekly data by *quarter*.

4.2 Step 0: Model and Estimate Consumer Demand

The supply-side of our empirical methodology, outlined in the previous section, relies on the ability to consistently estimate own- and cross-price effects in step 0. Here, we introduce the consumer's utility function and the assumptions of the demand side of the model. We model consumer demand using the multinomial random-coefficients logit model in the lines of McFadden and Train (2000), where consumers are assumed to purchase at most one unit of one of the products available in the market. We consider a differentiated products setting similar to Berry, Levinsohn and Pakes (1995, hereafter BLP). The estimation approach allows for consumer heterogeneity and controls for price endogeneity.

4.2.1 The Setup

In each market m (here defined as a store) and period s (here defined, as discussed above, as a quarter), there are I_{mt} consumers, indexed by i, each of which chooses among the subset of alternative products (here defined as an UPC) available. Let $j = 1, \ldots, J_{ms}$ index the inside UPC alternatives to the consumer in market m and period s. The no purchase choice (outside alternative) is indexed by $j = 0$.

4.2.2 Consumer Flow Utility

The consumer flow utility is expressed in terms of the indirect utility obtained from each of the available alternatives. We begin by specifying the indirect utility from choosing an inside alternative. The utility derived by consumer i from purchasing UPC j in market m and period s at retail price p^r_{jms} is assumed to be of the form:

$$\begin{aligned} u_{ijms} &= \bar{u}_{ijms}\left(p^r_{jms}, b_j, x_{jms}, w_{ms}, \xi_{jms}\right) + \varepsilon_{ijms} \\ &= \alpha_i p^r_{jms} + \varphi\left(b_j\right) + \beta_i x_{jms} + \tau_i w_{ms} + \xi_{jms} + \varepsilon_{ijms}, \end{aligned}$$

where b_j denotes the number of disposable razors included in UPC j (i.e., its package size), x_{jms} denotes a K_x-dimensional vector of observed characteristics of UPC j in market m and period s (observed by the consumer and the econometrician), w_{ms} denotes a K_w-dimensional vector of observed characteristics of the competitive environment of each market m (and potentially period s) to account for variations in the shopping alternatives that consumers have for making their purchases, and ξ_{jms} denotes the mean valuation for the unobserved characteristics of UPC j in market m and period s (observed by the consumer, but unobserved by the econometrician), which may potentially be correlated with price. Finally, ε_{ijms} is a random shock to consumer choice.

Prices of the different package sizes are, as discussed in Appendix B.1., typically nonlinear in size. $\varphi\left(b_j\right)$ denotes the component of the utility function associated to package size, which we assume to be non-linear in b_j. Following McManus (2007), a linear specification for both price and package size would be inappropriate. If the marginal utility from increasing size is constant, given that price schedules are typically concave in size, then (if the random shock is omitted from the model) all consumers with sufficiently high valuation to purchase a small size would prefer a larger size to the small one.

The estimation approach allows for general parameter heterogeneity. α_i denotes consumer i's price sensitivity. β_i denotes the parameters representing consumer i's preference for the observed characteristics included in the vector x_{jms}, and τ_i denotes consumer i's valuation of shopping alternatives. In particular, we allow for observed and unobserved heterogeneity in price sensitivity, α_i:

$$\alpha_i = \alpha + \eta d_i + \gamma v_i,$$

where d_i is a vector of demographic variables and v_i is a vector of random-variables drawn from a normalized multivariate normal distribution that allows for unobserved heterogeneity. η is a vector of parameters that represent how price sensitivity varies with demographics, while γ is a scaling vector. We allow d_i to include the *age* of the consumer, as well as her *household size* and annual *household income.* For the remaining parameters, we set $\beta_i = \beta$ and $\tau_i = \tau$.

We now move on to specify the indirect utility from not purchasing. The utility derived by consumer

i from this outside option in market m and period s is assumed to be of the form:

$$\begin{aligned} u_{i0ms} &= \bar{u}_{i0ms}(\xi_{0ms}) + \varepsilon_{i0ms} \\ &= \xi_{0ms} + \eta_0 d_i + \gamma_0 v_i + \varepsilon_{i0ms}, \end{aligned}$$

where ξ_{0ms} and ε_{i0ms} denote the mean valuation of not purchasing and a random shock to consumer choice, respectively, in market m and period s. Because utility is ordinal, the preference relation is invariant to positive monotonic transformations. As a consequence, the model parameters are identifiable up to a scalar, which implies that a normalization is required. The standard practice is to normalize the mean valuation of this outside option, ξ_{0ms}, to zero.

Having described the indirect utility from the different alternatives available to the consumer, we now address her maximization problem: consumers are assumed to purchase one unit of the alternative that yields the highest utility. Because consumers are heterogeneous (d_i, v_i, ε_{ims}), the set of consumers that choose UPC j in market m and period s is given by:

$$A_{jms} = \left\{ (d_i, v_i, \varepsilon_{ims}) \,|\, u_{ijms} \geq u_{ilms} \forall l = 0, 1, \ldots, J_{ms} \right\},$$

where $\varepsilon_{im} = (\varepsilon_{i0ms}, \ldots, \varepsilon_{iJ_{mt}ms})^{\top}$. If we assume a zero probability of ties, the aggregate quantity of UPC j in market m and period s is given by the integral over the mass of consumers in region A_{jms} times the corresponding size of the market:

$$q_{jms} = \Lambda_{ms} \int_{A_{jms}} dP^*(d, v, \varepsilon_{ms}) = \Lambda_{ms} \int_{A_{jms}} dP^*_d(d)\, dP^*_v(v)\, dP^*_{\varepsilon}(\varepsilon_{ms}),$$

where Λ_{ms} denotes the size of market m in period s, and $P^*(d, v, \varepsilon_{ms})$ denotes the population distribution function of the consumer types (d_i, v_i, ε_{ims}). We assume d, v and ε_{ms} to be independent. The last equality is just a consequence of this assumption.

4.2.3 Estimation Procedure

Having described the consumer demand model, we address the estimation procedure. We estimate the parameters of the demand model assuming the empirical distribution of demographics for $P^*_d(d)$, independent normal distributions for $P^*_v(v)$, and a Type I extreme value distribution for $P^*_{\varepsilon}(\varepsilon_{ms})$. The latter assumption allows us to integrate the ε's analytically which implies that the ξ's, the mean valuation for the unobserved characteristics constitute the only source of sampling error. This gives an explicit structural interpretation to the error term and, thereby, circumvents the critique provided by Brown and Walker (1989), related to the addition of ad-hoc errors and their induced correlations. After integrating the ε's, the aggregate quantity of UPC j in market m and period s is given by:

$$q_{jms} = \Lambda_{ms} \int_{A_{jms}} \left[\frac{\exp(\bar{u}_{ijms})}{\sum_{k=0}^{J_{ms}} \exp(\bar{u}_{ikms})} \right] dP^*_d(d)\, dP^*_v(v).$$

Our estimation procedure follows the algorithm used by BLP and Nevo (2000). It involves searching for the parameters that equate *observed* and *predicted* aggregated quantities of each UPC j in each market m and period s.

4.2.4 Price Endogeneity and Identification

The pricing decision of firms takes into account *all* characteristics of a UPC. This introduces correlation between prices and UPC characteristics and, in particular, between prices and the mean valuation for the *unobserved* UPC characteristics across markets and time periods (that constitute the structural error term of the demand model). As a consequence, instrumental variable techniques are required for consistent estimation. Controlling for the (market- and time-invariant) mean valuation for the unobserved UPC characteristics and for UPC-invariant market/time deviations from that mean by using fixed effects decreases the requirements on the instruments, since the correlation between prices and the mean valuation for those specific unobserved UPC characteristics is fully accounted for and does not require an instrument. In order to understand why this is the case, note that we can model $\xi_{jms} = \xi_j + \xi_{ms} + \Delta\xi_{jms}$ and capture ξ_j and ξ_{ms} by UPC and market/time fixed effects, where ξ_j denotes the (market- and time-invariant) mean valuation for the unobserved characteristics of UPC j and ξ_{ms} denotes the UPC-invariant market/time deviations from that mean. However, it does not completely eliminate the need for instrumental variable techniques since UPC-specific market/time deviations from that mean $\Delta\xi_{jms}$ are still expected to be correlated with prices.

We now provide an informal discussion of identification. We have already noted that because utility is ordinal, the preference relation is invariant to positive monotonic transformations. As a consequence, the model parameters are identifiable up to a scalar, which implies that a normalization is required. Without loss of generality, and as discussed above, we normalize the mean utility of the outside option, ξ_{0ms}, to zero. Given this restriction, the identification of the remaining parameters is standard given a large enough sample. The fixed effects ξ_j and ξ_{ms} are identified from variation in market shares across the different UPC and markets/time periods, respectively. The taste parameters β and the parameters in $\varphi(b_j)$ are identified from variations in the observed UPC characteristics and package sizes. The mean value of the price coefficient, α, is identified from variation in prices. The competition environment coefficients, τ, are identified from variation in the number of grocery stores, convenience stores and pharmacies across ZIP codes. The parameters in vector η are identified from variation in demographics across ZIP codes and, finally, the parameters in vector γ are identified from variation in market shares due to unobserved factors.[11]

Because of price endogeneity, it will be appropriate to use instruments rather than the variation in the actual prices to empirically identify the model's parameters. We follow Davis and Huse (2010) in using three types of instruments for the price of UPC j in market m and period s. First, we use the median price of UPC j in period s across markets in other counties, in the lines of Hausman, Leonard and Zona (1994, hereafter HLZ). Second, we use the number of other own firm UPCs and the number of rival firms UPCs that are offered in that market and period, as well as the sum of package sizes of other own firm

[11] Note that η_0 and γ_0 are not identified separately from an intercept in $\bar{u}_{ijms}$ that varies with consumer characteristics.

UPCs and the sum of package sizes of rival firms UPCs that are offered in that market and period, in the lines of BLP. Third, we use the latter BLP-type instruments within the same gender segment, in the lines of Bresnahan, Stern and Trajtenberg (1997, hereafter BST): the number of other same segment and firm UPCs and the number of same segment rival firms UPCs that are offered in that market and period, as well as the sum of package sizes of other same segment and firm UPCs and the sum of package sizes of same segment rival firms UPCs that are offered in that market and period.

In order for an instrument to be valid, it needs to be simultaneously (i) correlated with the endogenous variable price p^r_{jms} and (ii) uncorrelated with the mean valuation variations in the unobserved UPC characteristics $\Delta\xi_{jms}$. The validity of the former condition can be tested by regressing the endogenous variable on the full set of instruments: the instruments excluded from the demand equation plus all the exogenous explanatory variables in the demand equations (the F-test of the joint significance of the excluded instruments constitutes a statistic commonly used for such test). The validity of the latter condition is more difficult to test and, although, if the demand equations are over-identified (the number of excluded instruments exceeds the number of included endogenous variables), the overidentifying restrictions may be tested via the J statistic of Hansen (1982), there are limits to the extent to which the uncorrelation condition in itself can be tested in an entirely convincingly way.

4.2.5 Consumer Demand Estimation Results

Table 2 presents the demand estimation results, with the different columns reporting distinct specifications that vary on both the covariates included, the estimation procedure and the type of price instruments. Specification (1) reports the results of an ordinary least squares standard multinomial logit model regression. This first specification includes price, demographic and competition variables as covariates. Further, it considers a quadratic functional form for $\varphi(b_j)$ and captures heterogeneity by interacting price with observable demographic characteristics. The coefficients on these different covariates are all of the expected sign but mostly statistically insignificant. The price coefficient is one example of the latter, suggesting that the average consumer is price insensitive. The interactions with household size and consumer age are also statistically insignificant suggesting that these observed demographics do not explain price sensitiveness. The interaction with household income is, however, highly significant indicating that households with higher income are less price sensitive. The coefficients on package size suggest that consumers value package size at a statistically significant decreasing rate. Finally, the coefficients on demographic and competition covariates are statistically insignificant. This indicates that the utility of purchasing (and not purchasing) is not explained by the observed demographics nor impacted by the number of nearby grocery, convenience stores and pharmacies.

The structural error term of specification (1) includes the full ξ_{jms} since the specification does not control for the mean valuation for the unobserved characteristics. In specification (2), we include UPC fixed effects in order to fully control for ξ_j.[12] This increases the absolute value of the price coefficient, which suggests that prices may be positively correlated with that mean, which will underestimate consumer price sensitivity if not accounted for. We interpret the effects on the price coefficient as evidence

[12] Moreover, this captures flexible non-linearities in $\varphi(b_j)$.

that controlling for ξ_j matters. The price coefficient suggests that the average consumer is, in fact, price sensitive. The interactions with household size and consumer age remain statistically insignificant indicating that these observed demographics do not explain price sensitiveness. Similarly, the interaction with household income remains highly significant suggesting that households with higher income are less price sensitive. While most demographic covariates remain statistically insignificant, the coefficient on age becomes statistically significant indicating that the utility of purchasing lowers with age. Finally, the coefficients on the competition covariates seem to suggest that the utility of not purchasing is higher with more nearby pharmacies in the area, while the number of nearby grocery and convenience stores remain not having a statistically significant impact.

Specification (2) controls for UPC fixed effects that capture the mean valuation for the unobserved UPC characteristics. However, it does not fully control for ξ_{jms}. The error term includes UPC-invariant and UPC-specific market/time deviations from that mean: ξ_{ms} and $\Delta\xi_{jms}$, respectively, both of which, as argued above, are taken into account in the pricing decision of firms, potentially introducing correlation with the price covariate. Specifications (3), (5) and (7) report the results of a generalized method of moments standard multinomial logit model regression that replicate specification (2) using each of the types of instruments described above to account for the correlation between prices and ξ_{ms} and $\Delta\xi_{jms}$. The effect on the price coefficient seems sensitive to the choice of instruments. Although the first stage F-test of the joint significance of the excluded instruments is statistically significant, the over-identification test is rejected, for all types of instruments, suggesting that the identifying assumptions are not valid.

In order to reduce the requirements on the instruments, we estimate specifications (4), (6) and (8) that include market- and time-fixed effects, ξ_m and ξ_s, respectively, that control for ξ_{ms}, UPC-invariant market and time deviations from the valuation mean. ξ_{ms} may be a function of unobserved demographics, and if those unobserved demographics are correlated with prices, ξ_{ms} will be correlated with prices. The inclusion of these fixed effects increases the absolute value of the price coefficient, which suggests that prices may be positively correlated with ξ_{ms}, which will underestimate consumer price sensitivity if not accounted for. We interpret the effects on the price coefficient as evidence that controlling for ξ_{ms} matters. The first stage F-test of the joint significance of the excluded instruments is, again, statistically significant for all types of instruments. However, controlling for the unobserved demographics via ξ_{ms} eliminates the omitted-variable bias and improves the over-identification test statistic. In the case of the BLP type instruments, the improvement is such that the instruments are no longer rejected, suggesting that the BLP identifying assumption is valid. We explored the sensitivity of our results to the inclusion of market/time fixed effects, ξ_{ms}, and they were found to be robust. In order to avoid increasing unnecessarily the dimensionality of our problem, we controlled for ξ_{ms} using market- and time-fixed effects, ξ_m and ξ_s, respectively.

Finally, specification (9) reports the results for the full multinomial random-coefficients logit model with BLP type instruments. The results suggest that the average consumer is price sensitive. The interaction with household income is, once again, statistically significant confirming that households with higher income are less price sensitive. The remaining interactions with household size and consumer age are statistically insignificant indicating that these observed demographics do not explain price sensitiveness. The standard deviation coefficients are also statistically insignificant, which suggests that most of the heterogeneity is due to demographics.

Table 3 reports a sample of the estimated median (across the 643 market-time period combinations) own- and cross-price elasticities computed according to the estimates from specification (9) in Table 2. The average (across the 56 UPCs) of the median of the estimates of the own-price elasticity is -8.9. While such elasticities may seem relatively high, when one takes into account the fact that there is a large number of UPCs typically produced by large multiproduct firms, the elasticities seem quite reasonable. If we were to look at own-price elasticities across products or brands, considering the cross-price elasticities of all the other UPCs that the company owns, the magnitudes would be lower. The average of the median of the estimates of the cross-price elasticity is 0.1. By a similar argument as above, while such elasticities may seem relatively low, if we were to look at cross-price elasticities across products or brands, the magnitudes would be higher.

4.3 Step 1: Recover Unobserved Marginal Costs & Identify the Competitive Setting of Firms in the Industry

We now move on to recover the unobserved marginal costs of all the relevant UPCs and identify the competitive setting of firms in the industry. In a typical competition policy case, we would perform such analysis *pre*-acquisition. However, the specificities of the data used in the demand estimation (step 0) require that we slightly adjust the methodology. Our data ranges from 1994 to 1996, a period that postdates Gillette's acquisition of 22.9% of non-voting stock in Eemland. As a consequence, we are required to recover marginal costs and identify the competitive setting of firms in the industry in a *post*-acquisition period (and perform counterfactuals about *prior facts)*. To do so, we focus on the data from 1994 so that we recover the marginal costs and identify the competitive setting of firms for the earliest *post*-acquisition period possible. Let $s > t$ denote this period.

We identify the competitive setting of firms that gave rise to 1994 data by recovering marginal cost information using two sets of first-order conditions: *non-cooperative* and *coordination*, (8) and (12), respectively. However, to do so, the specificities of the data used in the demand estimation (step 0) require that we, once more, slightly adjust the methodology. The reason being that, in our empirical application, we use retail data to infer manufacturer behavior. As a consequence, we must rewrite the operating profit of each manufacturer firm in terms of the retail price.

Let p^w_{jms} denote the manufacturer's price of UPC j in market m in period s and mc^w_{jms} denote the manufacturer's marginal cost of producing an additional pack of UPC j in market m and period s, and transporting it from the plant to the retailer store. Hence, the operating profit of each manufacturer firm f in period s, defined in terms of the manufacturer's price, is given by:

$$\pi_{fs}\left(\mathbf{p}^w_s, \mathbf{p}^r_s\right) = \sum_{m \in \Upsilon} \left\{ \sum_{j \in \Gamma_{fms}} \left(p^w_{jms} - mc^w_{jms}\right) q_{jms}\left(\mathbf{p}^r_{fms}, \mathbf{p}^r_{-fms}\right) - C_{fms} \right\}, \tag{22}$$

where $\mathbf{p}^w_s$, $\mathbf{p}^r_s$, $\mathbf{p}^r_{fms}$ and $\mathbf{p}^r_{-fms}$ have the same interpretation as before, but with reference to the manufacturer and retail's price, respectively.

Let also $mg^r_{jms} = p^r_{jms} - p^w_{jms} - mc^r_{jms}$ denote the margin of the retailer from selling a pack of UPC j in market m in period s, where p^r_{jms} is (as before) the corresponding retail price and mc^r_{jms} is the retailer's marginal cost of getting the additional pack to the store shelves and selling it. We can rearrange this margin in terms of the manufacturer's price ($p^w_{jms} = p^r_{jms} - mc^r_{jms} - mg^r_{jms}$) and use the result to rewrite the operating profit of each manufacturer firm f in period s in terms of the retail price as follows:

$$\pi_{fs}\left(\mathbf{p}^r_s\right) = \sum_{m\in\Upsilon}\left\{\sum_{j\in\Gamma_{fms}}\left(p^r_{jms} - mc^r_{jms} - mg^r_{jms} - mc^w_{jms}\right) q_{jms}\left(\mathbf{p}^r_{fms}, \mathbf{p}^r_{-fms}\right) - C_{fms}\right\}. \tag{23}$$

This implies that, in our application, the vector of marginal costs recovered, from either the set of first-order conditions (8) or (12), for each market m and period s, include three elements:

$$\left(\mathbf{mc}^w_{ms} + \mathbf{mc}^r_{ms} + \mathbf{mg}^r_{ms}\right) = \mathbf{p}^r_{ms} - \boldsymbol{\Omega}_{ms}\left(\mathbf{p}^r_{ms}\right)^{-1}\mathbf{G}\mathbf{q}_{ms}\left(\mathbf{p}^r_{ms}\right), \tag{24}$$

where $\mathbf{mc}^w_{ms}$, $\mathbf{mc}^r_{ms}$ and $\mathbf{mg}^r_{ms}$ denote the vectors of manufacturer marginal cost, retailer marginal cost, and retailer margin, respectively, in market m and period s. Although we do not model the interaction between manufacturers and retailers explicitly, this is consistent with a wide variety of models of manufacturer-retailer interaction, since it allows the retailer margin to be free floating over UPCs, markets and time periods.

4.3.1 Recover Unobserved Marginal Costs under Non-Cooperative Behavior

In order to recover the vector of marginal costs using the first set of first-order conditions, referent to the *non-cooperation* case, we must derive matrix $\boldsymbol{\Omega}_{ms}$ using a normalized weight matrix $\mathbf{W}$, with a typical element given by w_{fg} for any $f, g \in \Im$, computed using the *post*-acquisition 1994's voting and non-voting stock of the owners in the different manufacturing firms. We make use of two sources of information to determine data: proxy statements (schedule 14A) filled by firms with the Securities and Exchange Commission, and competition agencies decisions regarding operations involving any of the firms.

Two comments are in order at this point to compute the normalized weight matrix $\mathbf{W}$. First, we use the data on the voting and non-voting stock to compute the financial and voting rights of each owner, which is relatively straightforward. We must then infer the corporate control rights of each owner from the vector of voting rights within the firm. To do so, we must make an assumption regarding the measurement of corporate control rights. Second, public data is restricted to identify large external owners, whose rights (directly or together with affiliates) typically exceeds 5%. As a consequence, we must make an assumption regarding the financial and corporate control rights of the remaining minority external owners. As such, we make the following two assumptions:

Assumption 6 *The corporate control right each owner has over the decision-making within a firm is measured by the voting rights she holds in the firm.*

This assumption is, as shown by Azar (2017), microfounded through a probabilistic voting model in which two potential managers compete in Downsian lines for the owner's votes by maximizing the expected vote share within the firm. Further, it constitutes a natural benchmark and it is merely illustrative. Moreover, it has relatively innocuous implications in our application *post*-acquisition, since owners, as we will discuss below, do not have conflicting views on the best strategy to pursue. As a consequence, the recovered marginal costs are invariant to the control weight's distribution among owners.[13] Finally, the proposed methodology is not constrained to this assumption. As suggested by Azar (2017), we can, alternatively, microfound measuring corporate control by the Banzhaf (1965)'s power index.[14]

Assumption 7 *Minority external owners do not engage in common-ownership and coalesce.*

This assumption also constitutes a natural benchmark and it is merely illustrative. It implies that each firm's minority external owners agree on the best strategy to pursue, coalescing into a single fictitious concentrated external owner. As a consequence, the recovered marginal costs are invariant to the distribution of financial and corporate control rights among those owners. This constitutes the limiting case in which the degree of influence of a firm's minority external owners over the decision-making within the firm is assumed to be the maximum possible. Please see Leech (2002) and Levy (2011) for a description of alternative approaches.

Table 4 presents the financial and the corporate control rights (under Assumptions 6 and 7) of each owner, both internal and external, over the manufacturing firms in the industry in 1994. Let $\Im \equiv \{1, \ldots, 6\}$ denote the set of owners that are internal to the industry, each of which is indexed by j, and $\Theta \backslash \Im \equiv \{7, \ldots, 22\}$ denote the set of owners that are external to the industry, each of which is indexed by k.[15] Table 4, Panel A addresses the ownership rights of the *six* internal owners. It suggests that, in 1994, the firms in the industry did engage in cross-ownership. On March 22, 1993, Warner-Lambert acquired Wilkinson Sword for $142 million to Eemland, that had put the razor blade company up for sale the year before. The sale was prompted after the European Commission, in November, ordered Gillette to sell its stake in Eemland because of antitrust concerns. Table 4, Panel B addresses the ownership rights of the *sixteen* external owners (including the fictitious minority owners). It suggests that, in 1994, external owners did not engage in common-ownership at all. Having established the financial and the corporate control rights of each owner, we have all the information required to compute the elements w_{fg} for all $f, g \in \Im$ of the normalized weight matrix $\mathbf{W}$ in the non-cooperation case. As an illustration, Appendix

[13] Although, in our empirical application, *post*-acquisition, the recovered marginal costs are not affected by the distribution of corporate control rights among owners, in general this is not true. In cases of cross- or common-ownership, the owners of a firm will typically have conflicting views on the best strategy to pursue. Owers with interests in different firms may hence use their voting rights to influence the manager to pursue a less aggressive strategy than the strategy desired by the remaining owners. As a consequence, the question of how to measure corporate control rights is key and a careful evaluation of the true corporate control rights is essential. If, for instance, the assumed distribution of corporate control rights overestimates the true influence that cross- or common-owners have over the manager of a firm, the methodology will infer a less aggressive behavior towards rivals and consequently, for a given observed price level, overestimate marginal costs.

[14] Goppelsroeder, Schinkel and Tuinstra (2008) suggests we could alternatively measure corporate control by the Shapley-Shubik (1954)'s power index.

[15] Table 4 includes some external owners that, in 1994, post-acquisition, did not hold any stock in any of the manufacturer firms in the industry. They are included for modelization purposes because, as it will become apparent in the next section, they held stock, in 1989, pre-acquisition.

B.2 describes the step-by-step details of this computation, which yields:

$$\mathbf{W} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \end{bmatrix}.$$

This result implies that (i) the owners of BIC, Gillette, American Safety Razor and Private Labels agree (and give the appropriate incentives) that the pricing decisions of each (corresponding) manager should maximize the present discounted value of the firm's stream of own operating profits, (ii) the owners of Warner-Lambert and Wilkinson Sword agree (and give the appropriate incentives) that the pricing decisions of each (corresponding) manager should maximize the present discounted value of the stream of *joint* operating profits.

Having established the elements of the normalized weight matrix $\mathbf{W}$, we can then derive matrix $\mathbf{\Omega}_{ms}$ using the consumer demand model and the price effects estimated in step 0. Finally, we can couple $\mathbf{\Omega}_{ms}$ within the set of first-order conditions (8) to recover the marginal costs under non-cooperative behavior for each UPC and market in 1994. Table 5 presents these recovered costs (that include the three elements described above) for a selection of UPCs. Given that they vary by market and time period, we present the median of each selected UPC across the 162 market-time combinations. The overall median recovered marginal cost per UPC is \$2.235, which reflects a median margin of \$0.455 over the observed prices.

4.3.2 Recover Unobserved Marginal Costs under Coordinated Behavior

In order to recover the vector of marginal costs under the *coordination* case, the procedure is relatively simpler when compared to the one outlined above. The reason being that we must derive matrix $\mathbf{\Omega}_{ms}$ using a normalized weight matrix $\mathbf{W}$ with a typical element straightforwardly given by $w_{fg} = 1$ for any $f, g \in \Im$. We do so using, as before, the consumer demand model and the price effects estimated in step 0. Finally, we can couple $\mathbf{\Omega}_{ms}$ within the set of first-order conditions (12) to recover the marginal costs for each UPC and market in 1994. Table 5 presents the median of these recovered costs (that, again, include the three elements described above) for a selection of UPCs. The overall median recovered marginal cost per UPC is \$2.113, which reflects a median margin of \$0.577 over the observed price, slightly higher than the one established under non-cooperative behavior.

4.3.3 Identify the Competitive Setting of Firms in the Industry

In order to identify the competitive setting of firms in the industry, we follow the empirical literature that attempts to evaluate the observed conduct of firms. To do so, we compare the recovered cost information to a crude observed equivalent cost measure. The procedure is as follows.

First, we decompose (with the obvious exception of private labels) the *two* sets of recovered marginal costs (each of which include, as discussed above, three elements: $mc^r_{jms} + mg^r_{jms} + mc^w_{jms}$) into two parts: mc^w_{jms} and $(mc^r_{jms} + mg^r_{jms})$. We do so by using the gross retail margin $(p^r_{jms} - p^w_{jms})$, a variable not used in the demand side estimation for exactly this purpose, to capture $(mc^r_{jms} + mg^r_{jms})$, since $(p^r_{jms} - p^w_{jms}) = (mc^r_{jms} + mg^r_{jms})$. This allows us to straightforwardly retrieve mc^w_{jms}.

Second, we use the decomposed portion mc^w_{jms} to compute (for reasons that will become apparent in a moment) an industry operating profit (gross of fixed costs) to sales ratio, as follows:

$$salesratio^{ind}_s = \frac{\sum_{f\in\Im^*}\sum_{m\in\Upsilon}\sum_{j\in\Gamma_{fms}}\left(p^w_{jms} - mc^w_{jms}\right) q_{jms}\left(\mathbf{p}^r_{fms}, \mathbf{p}^r_{-fms}\right)}{\sum_{f\in\Im^*}\sum_{m\in\Upsilon}\sum_{j\in\Gamma_{fms}} p^w_{jms} q_{jms}\left(\mathbf{p}^r_{fms}, \mathbf{p}^r_{-fms}\right)}, \tag{25}$$

where $\Im^*$ denotes the set $\Im$ not including the firm manufacturing private labels (for which we can not perform the above decomposition).

Third, we compute 95 percent confidence intervals for this industry operating profit to sales ratio using the following bootstrap procedure: (*i*) we assume that the estimated demand parameters are the true means and that the estimated variance-covariance matrix of the demand parameters is the true variance-covariance matrix, (*ii*) we take 2,000 random draws of the demand parameters assuming a multivariate normal distribution, (*iii*) we compute the above ratio for each draw of the demand parameters, which we then use to construct a percentile bootstrap 95% confidence interval. Figure 1 displays the 95% confidence interval for the above ratio inferred from the *two* sets of recovered marginal costs, i.e., under *non-cooperative* and *coordinated* behavior, respectively. The former ranges from 20.0% to 38.4%, while the latter ranges from 28.2% to 99.8%.

Finally, we confront the 95 percent confidence intervals *inferred* from the two behaviourally assumptions to the *actual* industry ratio. To do so, we use crude accounting estimates. The 1994's 10-K regulatory filings of the manufacturers in the industry support an industry operating profit to sales ratio of 29.8%. This implies that we can not reject the null hypothesis that the firms behavior is *non-cooperative* neither that it is *coordinated*. As a consequence, we proceed the analysis considering the marginal costs recovered under both assumptions, which are presented in Table 5.

4.4 Step 2: Compute the Critical Threshold *Pre*-Acquisition

Having identified the competitive setting of firms and recovered the corresponding unobserved marginal costs for all the relevant UPCs, we proceed to step 2 in order to compute the critical threshold *pre*-acquisition. To do so, we first simulate the *counterfactual* equilibrium prices that would have arisen under the *financial* and *corporate control* rights of each owner, both internal and external, over the manufacturing firms in the industry *pre*-Gillette's 1989 initial offer. These rights are presented (under Assumptions 6 and 7) in Table 6. We use this information to compute the elements w_{fg} for all $f, g \in \Im$ of the normalized weight matrix $\mathbf{W}$ in 1989. Appendix B.3. presents the step-by-step details of this computation, which yields $\mathbf{W} = \mathbf{I}$. This implies that, in 1989, *pre*-acquisition, owners agree (and give the appropriate incentives) that each manager should maximize the present discounted value of the firm's stream of own operating profits. This result derives from the fact that cross- and common-ownership rights are absent (since Wilkinson Sword is, in the *pre*-acquisition period, independent of the remaining firms in the industry).

4.4.1 Counterfactual Equilibrium Prices

We simulate the counterfactual equilibrium prices under the assumption that the competitive setting among firms and the vector of marginal costs were not altered between 1989 and 1994. To do so, we make use of the 1989's normalized weight matrix **W** (which, as discussed above, is diagonal), the consumer demand model, the price effects estimated in step 0, and the two sets of unobserved marginal costs identified in step 1 - within the first-order conditions (8), (12) and (15), respectively, to derive the counterfactual equilibrium *pre*-acquisition prices for each UPC and market under the *three regimes of the tacit coordination model*.

Table 7 reports the median of the results (across all markets) for a sample of UPCs. We begin by addressing the *punishment* regime. If we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are behaving non-cooperatively, the results suggest that, under 1989's structure of financial and corporate control rights, in which Wilkinson Sword is independent of the remaining firms in the industry, the median *punishment* (*non-cooperative*) equilibrium price would have been \$2.690. This equilibrium price is exactly the same as the median price observed in 1994, despite the fact that, in the *pre*-acquisition structure, Warner-Lambert would not internalize the effects of price changes on the stream of operating profits of Wilkinson Sword (and vice-versa), since Wilkinson Sword scale within the industry is trifling.[16] If, however, we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are engaging in coordinated behavior, the results suggest that, under 1989's structure of financial and corporate control rights, the median *non-cooperative* equilibrium price would have been \$2.578. This equilibrium price is lower than the median price observed in 1994 (\$2.690) since non-cooperative prices do not internalize the effects of price changes on the stream of operating profits of rival firms.

We now address the *agreement* regime. If we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are behaving non-cooperatively, the results suggest that the 1989's median *agreement* (*coordinated*) equilibrium price would have been \$2.768, \$0.078 higher than the equilibrium price arising in a non-cooperative competitive setting. Moreover, an analysis by firm (non-tabulated) suggests that this price difference is larger for smaller firms, indicating that those are the ones that tend to benefit more from the full internalization induced by tacit coordination. If, however, we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are engaging in coordinated behavior, the results suggest that the 1989's median *coordinated* equilibrium price would have been \$2.690, which equals the median price observed in 1994 since coordinated equilibrium prices do not depend on **W**.

Finally, we address the *deviation* regime. The results suggest that, for both sets of unobserved

[16] The only median impact is on Wilkinson Sword UPCs. The median *non-cooperative* equilibrium prices for WS Colors 5r and WS Ultra Glide Twin 5r would be \$1.268 and \$1.668 (Table 7), respectively, both slightly lower than the 1994 observed prices of \$1.290 and \$1.690 (Table 5) since, in the *pre*-acquisition industry structure, Wilkinson Sword does not internalize the effects of price changes on Warner-Lambert UPCs.

marginal costs identified in step 1, the incentive to defect is non-negligible, with each deviant firm undercutting coordinated prices considerably, to a level close to that of the non-cooperative equilibrium. Moreover, an analysis by firm (non-tabulated) suggests that this price decrease is larger for smaller firms, since they typically enjoy a smaller portfolio effect.

Having computed the counterfactual equilibrium prices for the three regimes of the tacit coordination model, we then use them as an input to the consumer demand model estimated in step 0 so to derive the corresponding counterfactual equilibrium quantities. We then use the equilibrium prices and quantities, jointly with the unobserved marginal costs recovered in step 1, to compute each firm's counterfactual equilibrium operating profit for each market and time period. We perform this analysis, again, for each regime of the tacit coordination model. We then extrapolate the corresponding results for the US economy as a whole. In order to do so, we compute, for each firm and regime of the tacit coordination model, the average annual operating profit across the different markets (here a store-quarter combination) and multiply the corresponding result by the US economy annual potential market to obtain the counterfactual annual operating profit of each firm for the US economy as a whole. Finally, we use the latter to (*i*) examine the incentives from deviating and maintaining the tacit coordinated agreement, and (*ii*) compute the minimum discount factor, both, for each firm in the market. We will address each, in turn, below.

4.4.2 Non-Deviation Conditions

The annual benefit of each firm f in the market from deviating the tacit coordinated agreement is, since *pre*-acquisition $\mathbf{W} = \mathbf{I}$, captured by the difference between the firm's deviation and coordinated counterfactual equilibrium annual operating profit, i.e., $\pi_f \left(\mathbf{p}_f^d, \mathbf{p}_{-f}^c \right) - \pi_f \left(\mathbf{p}^c \right)$, while the annual benefit from maintaining coordination is captured by the difference between the firm's coordinated and non-cooperative counterfactual equilibrium annual operating profit, i.e., $\pi_f \left(\mathbf{p}_f^d, \mathbf{p}_{-f}^c \right) - \pi_f \left(\mathbf{p}^{nc} \right)$. The results, presented in Table 8, are consistent with the theoretical literature on the impact of firm asymmetry on the likelihood of coordination: Compte, Jenny and Rey (2002), Vasconcelos (2005), and Kuhn (2004).

First, in the absence of binding capacity constraints, smaller firms tend to be *maverick firms*, i.e., tend to have the greatest relative incentive to deviate, since they are typically the ones that can benefit the most from the demand effect induced by the disruption of the coordinating agreement. If we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are behaving non-cooperatively, the results suggest that, under 1989's structure of financial and corporate control rights, the benefit of smaller firms like American Safety Razor and Wilkinson Sword from deviating is 13.149% and 13.881% of their annual coordinated operating profit, respectively. If, however, we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are engaging in coordinated behavior, the results suggest a benefit of 25.631% and 30.165%, respectively.

Second, larger and smaller firms, both, tend to benefit the most from maintaining the coordinated arrangement. The latter because coordination allows them to enjoy a substantial price increase, as discussed above. The former because coordination allows them to benefit from a small price increase (when compared with the larger firms) over a large demand base. If we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are behaving non-cooperatively, the results suggest that, under 1989's structure of financial and corporate control rights, the benefit of a large (small) firm like Gillette (Wilkinson Sword) from maintaining the coordinated arrangement is around 4.754% (2.807%) of the annual coordinated operating profit. If, however, we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are engaging in coordinated behavior, the results suggest that the benefit of a large (small) firm like Gillette (Wilkinson Sword) from maintaining the coordinated arrangement is around 8.812% (8.541%) of the annual coordinated operating profit.

4.4.3 Minimum Discount Factor and Critical Threshold

The minimum discount factors that ensure that the supergame grim strategy vector σ^{grim} constitutes a sub-game perfect equilibrium are also presented in Table 8. If we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are behaving non-cooperatively, the minimum discount factors, under 1989's structure of financial and corporate control rights, range from 0.244 (Gillette) to 0.928 (American Safety Razor). If, however, we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are engaging in coordinated behavior, the minimum discount factors range from 0.271 (Gillette) to 0.830 (American Safety Razor). In both cases, the results imply that the critical threshold is given by American Safety Razor's minimum discount factor: 0.928 and 0.830, respectively. In order to assess the coordinated effects of partial horizontal acquisitions, we propose to evaluate their impact on these two sets of critical thresholds.

However, before doing so, and for completeness, we examine if *pre-acquisition*, i.e., under 1989's structure of financial and corporate control rights, the grim strategy vector σ^{grim} would have constituted, *in fact*, a sub-game perfect equilibrium. We perform this examination by comparing, for each firm, the above derived minimum discount factors to the *actual* discount factors. This requires us to infer the actual discount factor of each firm's owners in 1989. To do so, we compute the cost of equity of each firm using a CAPM asset pricing model, in the lines of Davis and Huse (2010).[17] The results, also presented in Table 8, confirm the validity of modelling the discount factor to be the same for all firms. Further, they suggest that, if we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are behaving non-cooperatively, the 1989's minimum discount factors are lower than the actual discount factors for all firms but one, American Safety Razor. This seems to indicate that, if firms in 1994 were behaving non-cooperatively, the supergame grim strategy vector σ^{grim} would not have constituted a sub-game perfect equilibrium *pre*-acquisition. Or, in other words, it seems to indicate that, if firms in 1994 were behaving non-cooperatively, they would also have been behaving non-cooperatively in 1989. If, however, we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are engaging in coordinated behavior, the 1989's minimum discount factors are lower than the actual discount factors, for all firms, with no exception. This seems to indicate that, if firms in 1994 were engaging in coordinated behavior, the supergame grim strategy vector σ^{grim} would have constituted a sub-game perfect equilibrium *pre*-acquisition. Or, in other words, it seems to indicate that, if firms in

[17] In order to derive the cost of equity, we (1) use the market yield on U.S. Treasury securities at 10-year constant maturity, quoted on investment basis, as the risk-free rate, (2) use Damodar implied equity risk premiums for the US market, calculated using the S&P 500, as the market risk premium, and finally, (3) use published historical betas from Thomson Reuteurs.

1994 were engaging in coordinated behavior, they would also have been engaging in coordinated behavior in 1989.

4.5 Step 3: Compute the Critical Threshold *Post*-Acquisition

Having identified the *pre*-acquisition critical threshold, we proceed to step 3 in order to empirically examine if such threshold is impacted by Gillette's acquisition of 22.9% of the non-voting stock of Eemland. However, in order to illustrate the applicability of our empirical methodology, in addition to the above actual acquisition, we also empirically examine, as a comparison, the impact of five other hypothetical acquisitions on critical thresholds. In total, we address the following set of actual and hypothetical acquisitions cases:

1. Gillette acquires 100% of the voting stock of Wilkinson Sword. This acquisition induces a merger, the extreme case of a *financial and a corporate control cross-ownership* structure, and is examined in order to illustrate that such acquisitions are nested in our empirical structural methodology.

2. Berkshire Hathaway, Gillette's largest external owner, acquires 100% of the voting stock of Wilkinson Sword. This acquisition induces a *financial and a corporate control common-ownership* structure and is examined in order to illustrate the differential impact of acquisitions involving 100% of the voting rights by internal and external owners.

3. Gillette acquires 22.9% of the voting stock of Wilkinson Sword. This acquisition induces a *financial and a corporate control cross-ownership* structure and is examined in order to illustrate the differential impact between full acquisitions of voting rights and partial acquisitions of voting rights by internal owners.

4. Berkshire Hathaway acquires 22.9% of the voting stock of Wilkinson Sword. This acquisition induces a *financial and a corporate control common-ownership* structure and is examined in order to illustrate the differential impact of partial acquisitions of voting rights by internal and external owners.

5. Gillette acquires 22.9% of the non-voting stock of Wilkinson Sword. This acquisition induces a *financial cross-ownership* structure and is examined in order to illustrate the differential impact between acquisitions of voting and non-voting rights by internal owners.

6. Berkshire Hathaway acquires 22.9% of the non-voting stock of Wilkinson Sword. This acquisition induces a *financial common-ownership* structure and is examined in order to illustrate the differential impact of partial acquisitions of nonvoting rights by internal and external owners.

The procedure is similar to the one outlined in step 2 and begins by simulating the *counterfactual* equilibrium prices that would arise *post*-acquisition. To do so, we must recompute the normalized weight matrix $\mathbf{W}$ for each of the above six acquisition cases, which we present in Appendix B.4. The results suggest the following. Acquisition case 1 would have induced the managers of Gillette and Wilkinson Sword to behave, effectively, as a single entity, maximizing the present discounted value of their joint stream of operating profits. Acquisition case 2 would have induced the managers of Gillette and Wilkinson Sword to maximize the present discounted value of a weighted average of the stream of operating profits of the two firms. The reason is as follows. One the one hand, Wilkinson Sword's manager would internalize Gillette's stream of operating profits because Berkshire Hathaway, Wilkinson Sword's single controlling owner, would hold financial rights in Gillette. On the other hand, Gillette's manager would internalize Wilkinson Sword's stream of operating profits because Berkshire Hathaway, one of Gillette's controlling owners, would hold financial rights in Wilkinson Sword. This suggests that *full acquisitions that give rise to a common-ownership structure of financial and corporate control rights align the interests of the firms involved in the acquisition in the same qualitative vein as a merger. The only difference is solely on the weight attributed to the involved rival firm's operations.*

Acquisition cases 3 and 4 would also have induced the managers of Gillette and Wilkinson Sword to maximize the present discounted value of a weighted average of the stream of operating profits of the two firms. The reason is as follows. One the one hand, Wilkinson Sword's manager would internalize Gillette's stream of operating profits because at least one of Wilkinson Sword's controlling owners, all Gillette's controlling external owners via Gillette's cross-ownership stake in case 3 and Berkshire Hathaway in case 4, would hold financial rights in Wilkinson Sword. On the other hand, Gillette's manager would internalize Wilkinson Sword's stream of operating profits because at least one of Gillette's controlling owners, all Gillette's controlling external owners via Gillette's cross-ownership stake in case 3 and Berkshire Hathaway in case 4, would hold financial rights in Wilkinson Sword. This suggests that *partial acquisitions that give rise to a cross- or common-ownership structure of financial and corporate control rights align the interests of the firms involved in the acquisition in the same qualitative vein as a merger. The only difference is, again, solely on the weight attributed to the involved rival firm's operations.*

Finally, acquisition cases 5 and 6 would have induced (for case 5, in fact, induced, since the acquisition actually occurred) the manager of Gillette to maximize the present discounted value of a weighted average of the stream of operating profits of the two firms. The reason is as follows. At least one of Gillette's controlling owners, all Gillette's controlling external owners via Gillette's cross-ownership stake in case 5 and Berkshire Hathaway in case 6, would hold financial rights in Wilkinson Sword. Note, in contrast with the previous acquisition cases, the manager of Wilkinson Sword would not internalize Gillette's stream of operating profits because none of Wilkinson Sword's controlling owners would hold financial rights in Gillette. This suggests that *partial acquisitions that give rise to a cross- or common-ownership structure of financial rights do not align the interests of the firms involved in the acquisition in the same qualitative vein as a merger.*

To sum up, the results suggest a stark contrast among the six acquisition cases above. Acquisition cases 1 to 4, by involving *corporate control*, change the pricing incentives of two firms, Gillette (the acquiring party, either the firm itself or its controlling external owners) and Wilkinson Sword (the acquired firm), while acquisition cases 5 to 6, by involving solely *financial rights*, change the pricing incentives of a single firm, Gillette (the acquiring party, either the firm itself or its controlling external owners). In addition to this stark contrast, the six acquisition cases above also differ on the weight attributed to the (involved) rival stream of operating profits, whose quantification is a key advantage of our proposed methodology.

Having established the new normalized weight matrix **W** for each of the above six acquisition cases, we can simulate the *counterfactual* equilibrium prices that would arise *post*-acquisition.

4.5.1 Counterfactual Equilibrium Prices

We simulate the counterfactual equilibrium prices under the assumption that the competitive setting among firms and the vector of marginal costs are not altered *post*-acquisition. As a consequence, we just have to simulate counterfactual equilibrium prices for two regimes of the tacit coordination model: *punishment* and *deviation*. The reason being that, under the above assumption, the *agreement* (*coordinated*) equilibrium prices *pre*- and *post*-acquisition coincide. We perform the simulation by making use of the new normalized weight matrix **W** for each of the above six acquisition cases, the consumer demand model, the price effects estimated in step 0, and finally the two sets of unobserved marginal costs identified in step 1, which we coupled within the first-order conditions (8) and (15), respectively, for each UPC and market. The results of this simulation are reported in Table 9, which presents the median percentage change in the counterfactual *punishment* (*non-cooperative*) and *deviation* equilibrium prices, for each of the above six acquisition cases, relative to the corresponding *pre*-acquisition levels for a sample of UPCs across all markets.

In acquisition cases 1 to 4, the manager of Gillette internalizes Wilkinson Sword's stream of operating profits and vice-versa. The results suggest that this internalization induces an increase in the non-cooperative and deviation equilibrium prices of the two firms: between 0.000% to 0.058% for Gillette and between 0.226% to 26.298% for Wilkinson Sword, depending on the focus (i.e., on the acquisition case under analysis, on the UPC under consideration, on the assumption about the firms 1994 behavior, and on whether the emphasis is on non-cooperative or deviation equilibrium prices). This increase is higher for Wilkinson Sword since it internalizes a higher number of UPCs than Gillette. Further, the results also suggest that prices are not strong strategic complements, since the remaining firms, BIC, American Safety Razor, Private Labels and Warner-Lambert, maintain prices unchanged or increase them very slightly, up to 0.002%, depending on the focus. Finally, the results are consistent with the DoJ's decision of instituting a civil proceeding against Gillette's initial proposed acquisition of 100% of the voting stock of Wilkinson Sword, since although the price impact on the industry as a whole is diminute, the impact on Wilkinson Sword UPCs is substantial.

In acquisition cases 5 to 6, the manager of Gillette internalizes Wilkinson Sword's stream of operating profits, but not vice-versa, since the acquisitions do not involve corporate control. The results suggest, again, that this internalization induces an increase in Gillette's non-cooperative and deviation equilibrium prices: between 0.000% to 0.013%, depending on the focus. Further, the results also suggest that prices are, again, not strong strategic complements, since the remaining firms, BIC, American Safety Razor, Private Labels, Warner-Lambert and Wilkinson Sword, maintain prices unchanged. Finally, the results are consistent with the DoJ's decision of allowing Gillette's 22.9% nonvoting rights acquisition, since the price impact is extremely low for all UPCs, including those from Wilkinson Sword.

Having computed the counterfactual *non-cooperative* and *deviation* equilibrium prices for each of the above six acquisition cases, we use them as an input to the consumer demand model estimated in step 0 so to derive the corresponding counterfactual equilibrium *non-cooperative* and *deviation* quantities. We then use these equilibrium prices and quantities, jointly with the unobserved marginal costs recovered in step 1, to compute each firm's equilibrium operating profit for each market under each of the above six acquisitions. We perform this analysis, again, for the above two regimes of the tacit coordination model: *punishment* and *deviation*. We then extrapolate the corresponding results for the US economy as a whole, again, for each of the two regimes of the tacit coordination model and for each of the six acquisition cases. Finally, we use the latter to (i) examine the incentives from deviating and maintaining the tacit coordinated agreement, and (ii) compute the minimum discount factor, both, for each firm in the market. We will address each, in turn, below.

4.5.2 Non-Deviation Conditions

We use the *post*-acquisition counterfactual *non-cooperative* and *deviation* equilibrium annual operating profit of each firm in the market, jointly with the corresponding counterfactual *coordinated* equilibrium annual operating profit previously derived, to compute the annual benefit of deviating from the tacit coordinated agreement and the annual benefit from maintaining coordination, again, for each firm in the market and for each of the six acquisition cases. Since *post*-acquisition $\mathbf{W} \neq \mathbf{I}$, the former is captured by the difference between the weighted sum of the deviation and the coordinated counterfactual equilibrium annual operating profits across all firms in which the owners have, direct or indirectly, financial and/or corporate control rights on, i.e., $\sum_{g\in\Im} w_{fg}\left(\pi_g\left(\mathbf{p}_f^d, \mathbf{p}_{-f}^c\right) - \pi_g\left(\mathbf{p}^c\right)\right)$, while the latter is captured by the difference between the weighted sum of the coordinated and the non-cooperative counterfactual equilibrium annual operating profits across, again, all firms in which the owners have, direct or indirectly, financial and/or corporate control rights on, i.e., $\sum_{g\in\Im} w_{fg}\left(\pi_g\left(\mathbf{p}_f^d, \mathbf{p}_{-f}^c\right) - \pi_g\left(\mathbf{p}^{nc}\right)\right)$.

The impact of acquisitions on the annual benefit of deviating from the tacit coordinated agreement and the annual benefit from maintaining coordination is, in general, ambiguous and depends critically, both quantitatively and qualitatively, on the estimated substitution patterns of consumers among products and on the type of acquisition. This illustrates the importance of an empirical structural methodology to quantitatively evaluate them in a differentiated products setting.

The results of this analysis for the six acquisition cases described above are reported in Table 10. We begin by addressing the results relative to the benefit of deviating from the tacit coordinated agreement. They suggest that (i) the incentives of the *acquiring party's firm* to deviate decrease after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners), (ii) the incentives of the *acquired firm* to deviate decrease after acquisitions involving full or partial corporate control rights, by internal or external owners, but are unaffected after acquisitions involving full or partial financial rights, by internal or external owners, and (iii) the incentives of the *remaining firms in the industry* to deviate are unaffected after an acquisition (independently of whether it involves, again, full or partial financial or corporate control rights, by internal or external owners).

We now address the results relative to the benefit of maintaining the tacit coordinated agreement.

They suggest that (*i*) the incentives of the *acquiring party's firm* to maintain coordination decrease after acquisitions involving full or partial corporate control rights, by internal or external owners, but increase after acquisitions involving full or partial financial rights, (*ii*) the incentives of the *acquired firm* to maintain coordination increase after acquisitions involving full or partial corporate control rights, by internal or external owners, but decrease after acquisitions involving full or partial financial rights, and (*iii*) the incentives of the *remaining firms in the industry* to maintain coordination decrease after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners).

4.5.3 Minimum Discount Factor and Critical Threshold

We also use the *post*-acquisition counterfactual *non-cooperative* and *deviation* equilibrium annual operating profit of each firm in the market, jointly with the corresponding counterfactual *coordinated* equilibrium annual operating profit previously derived, to compute the minimum discount factors that ensure that the supergame grim strategy vector σ^{grim} constitutes a sub-game perfect equilibrium. The results are presented in Table 11 and seem to suggest that the minimum discount factor of (*i*) the *acquiring party's firm* is non-increasing after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners), (*ii*) the *acquired firm* is non-increasing after acquisitions involving full or partial corporate control rights, but non-decreasing after acquisitions involving full or partial financial rights, and (*iii*) the *remaining firms in the industry* is non-decreasing after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners). These results are consistent with Davis and Huse (2010) and Ivaldi and Lagos (2016). Further they extend their scope by distinguishing that the ownership right involved in the acquisition matters for the incentives of the *acquired* firm to coordinate.

Further, the results also imply that the critical threshold, in all acquisition cases and the two sets of unobserved marginal costs identified in step 1, is given by American Safety Razor's minimum discount factor. This suggests, since American Safety Razor is not, direct or indirectly, involved in any of the acquisition cases, that the likelihood of coordinated conduct would have not increased after any those acquisitions (since the minimum discount factor for an firm not involved in an acquisition is non-decreasing).

Finally, and for completeness, we examine if *post-acquisition*, the grim strategy vector σ^{grim} would have constituted, *in fact*, a sub-game perfect equilibrium. We perform this examination by comparing, for each firm, the above derived minimum discount factors to the actual discount factors. The results, also presented in Table 11, suggest that, if we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are behaving non-cooperatively, the *post*-acquisition's minimum discount factors are lower than the actual discount factors for all firms but one, American Safety Razor. This seems to indicate that, if firms in 1994 were behaving non-cooperatively, the supergame grim strategy vector σ^{grim} would not have constituted a sub-game perfect equilibrium *post*-acquisition. If, however, we use the set of unobserved marginal costs derived under the assumption that firms in 1994 are engaging in coordinated behavior, the *post*-acquisition's minimum discount factors are lower than the actual discount factors, for all firms, with no exception. This seems to indicate that, if firms in 1994 were engaging in coordinated behavior, the supergame grim strategy vector σ^{grim} would have constituted a sub-game perfect equilibrium *post*-acquisition.

5 Conclusions

This article proposes an empirical structural methodology to quantitatively evaluate the coordinated effects of actual and hypothetical partial horizontal acquisitions. The proposed methodology can cope with acquisition settings involving all types of owners and ownership rights: owners that can be internal to the industry and external to the industry; and ownership rights that can involve financial interests and corporate control, can be direct and indirect, can be partial or full.

To do so, we consider a structural setting where asymmetric multi- and differentiated-product firms interact repeatedly over time and across markets, and are assumed to follow the most basic enforcement mechanism, grim-trigger strategies, in order to sustain a coordinated arrangement. This structural setting is used to simulate firms' counterfactual outcomes under the different regimes of the tacit coordination model and evaluate quantitatively the likelihood of coordinated conduct *pre*- and *post*-acquisition by examining the minimum discount factors for which tacit coordination can be sustained. This structural approach to partial horizontal acquisitions may constitute a preferable approach compared to the current indirect approach focused on measures of market concentration and on informal analyses of the features of the market conducive to coordinated interaction.

We also provide an empirical application of the methodology to a variety of actual and hypothetical acquisitions in the wet shaving industry that give rise to cross- and common-ownership structures. The results seem to suggest that the incentives of (i) the *acquiring party's firm* to coordinate are non-decreasing after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners), (ii) the *acquired firm* are non-decreasing after acquisitions involving full or partial corporate control rights, but non-increasing after acquisitions involving full or partial financial rights, and (iii) the *remaining firms in the industry* are non-increasing after an acquisition (independently of whether it involves full or partial financial or corporate control rights, by internal or external owners).

This article leaves many issues yet to be explored. Extensions of this methodology to Abreu (1986, 1988)'s optimal punishments, to not all-inclusive tacit coordination agreements, and to partial *vertical* acquisitions constitute very interesting potential areas for future research.

Appendix A. Empirical Structural Methodology

A.1. Weight Function of the Manager

In structures in which cross- and common-ownership rights are absent, our proposed weight function reduces to the present discounted value of the firm's stream of operating profits. In order to see why, note that, in those cases, we have that $\mathbf{F}^u = \mathbf{F}$ and $\mathbf{C}^u = \mathbf{C}$ (since in the absence of cross-ownership $\mathbf{F}^*$ and $\mathbf{C}^*$ constitute null matrices). This implies that the weight function of the manager of firm f is given by:

$$\begin{aligned} \varpi_{ft} &= \sum\nolimits_{s=t}^{\infty} \delta^{s-t} \sum\nolimits_{k\in\Theta} \gamma_{kf}\phi_{kf}\pi_{fs}(\mathbf{p}_s) \\ &= \sum\nolimits_{s=t}^{\infty} \delta^{s-t} \left(\sum\nolimits_{k\in\Theta} \gamma_{kf}\phi_{kf}\right) \pi_{fs}(\mathbf{p}_s). \end{aligned} \tag{26}$$

In turn, this implies that the weight matrix $\mathbf{L}$ is a diagonal matrix and that the normalized weight matrix $\mathbf{W} = \mathbf{I}$. As consequence, we have that:

$$\varpi'_{ft} = \sum\nolimits_{s=t}^{\infty} \delta^{s-t} \pi_{fs}(\mathbf{p}_s). \tag{27}$$

In other words, with no partial ownership rights of any kind, our proposed weight function yields that the manager of each firm should maximize the present discounted value of firm's stream of own operating profits.

Appendix B. Empirical Application

B.1. Data Description and Preliminary Analysis

An important question is obviously whether the dataset is representative of the whole population buying disposable razor products. For purposes of Gillette's acquisition of 22.9% of the non-voting stock of Eemland, the DoJ (1990) characterized the industry as follows:

> Gillette accounts for 50% of all razor blade units (...). The next closest competitor is BIC with 20%, followed by Warner-Lambert with 14%, Wilkinson with 3%, and American Safety Razor with less than 1% of unit sales. (page 9)

Because this industry characterization does not account for private labels, we must be cautious in a straightforward comparison with our dataset. However, it does suggest that our data is reasonably representative, although slightly overrepresenting Gillette and underrepresenting BIC and Wilkinson Sword.

We now move on to describe the dataset in more detail. Table B1, Panel A presents summary purchase statistics at the UPC level. Although there is evidence of substantial heterogeneity across stores and weeks, the median store in the sample sells 2 packages of 5 men razors per UPC per week at a price of \$3.10 per package, generating 38.9% gross retail margin. This margin is computed with reference to the average acquisition cost of the items in inventory, an issue we address in more detail in the main text. Table B1, Panel B presents summary statistics at the store level. $17,539$ households visit and purchase something in the median store per week. The potential market size in a given time period is defined in terms of the number of purchases of razor packages and assumed to be proportional to the weekly number of household visits of each store. The proportionality factor is assumed to be the percentage of households buying razor products times the probability of a purchase in any given visit. According to the IRI Builders Suite (Bronnenberg, Kruger and Mela, 2008) 28.5% of US households purchase razor blades in a year, with an average purchase cycle of 106 days. Furthermore, according to Food & Beverage Marketing (Degeratu, Rangaswamy and Wu, 2000), US households visit regular grocery stores about 7.9 times per month on the average. This translates into a median potential market size of 181.7 package purchases per store and week, a potential market that a median of 7 grocery stores, 3 convenience stores and 5 pharmacies compete for each week. We explored the sensitivity of our results to the proportionality factor assumption and all the main conclusions were found to be robust. Finally, Table B1, Panel C presents summary demographic statistics of each store surrounding area (same

ZIP code). The median consumer is 40-year-old within an household consisting of two members and an annual income of $57,457.

Having described the main data summary statistics, we now examine in more detail the price variable. Temporary price promotions are important marketing tools in the pricing strategy of many nondurable goods and disposable razors are no exception, as the high price variance and the (occasional) negative gross retail margin reported in Table B1, Panel A suggest. Prices in the sample display a classic high-low pattern: products have a *regular level* that remains constant for long periods of time with occasional temporary reductions. High-low pricing allows firms to discriminate between (*i*) informed and uninformed consumers; (*ii*) consumers with different inventory holding costs; and (*iii*) price-sensitive switchers and store-loyal consumers. While the classic high-low pattern is easy to spot, regular price levels are hard to define because they may change over time. We define a temporary price promotion in the lines of Dossche, Heylen and Van den Poel (2010): as any sequence of prices that is below at least 95 percent of the most left and the most right adjacent prices. Table B2 characterizes DFF's temporary price promotions. Following the typical pattern of setting regular price levels that remain constant for long periods of time, the median prices set by this supermarket chain across all UPCs, stores and weeks are non-promoted. Occasional temporary reductions account for only 11.5% of all price observations and, although there is evidence of substantial heterogeneity, consist of a median 20.8% discount every 4 weeks.

In an environment characterized by temporary price discounts, it is important to examine how consumers respond to price cuts. As Hendel and Nevo (2006a) show, demand estimation based on temporary price reductions may mismeasure the long-run responsiveness to prices. This is of fundamental importance in a setting like ours that relies on the ability to consistently estimate own- and cross-price elasticities. The first two columns in Table B3 addresses this issue by comparing, per package size, the percentage of weeks that a UPC was on promotion and the percentage of razors sold during those weeks. The results suggest that consumers do respond to temporary price discounts: the percentage of quantity sold on promotion is larger than the percentage of weeks that the promoted price is available. This is consistent with the hypothesis that consumers respond to temporary price cuts by accelerating (anticipating) purchases and hold inventories for future consumption (i.e. stockpile). The main alternative explanation that consumers simply increase their consumption in response to a price reduction is less valid in the wet shaving setting. In order to avoid mismeasuring the long-run responsiveness to prices due to temporary price reductions, we aggregate the data quarterly.

Having characterized the price discrimination induced by temporary price promotions, we now address a second form of discrimination: discrimination induced by price nonlinearity in package size. Nonlinear pricing can be used by oligopolistic firms as a screening mechanism to price discriminate between types of consumers that hold private information about their tastes by nudging consumers to self-select (according to their tastes) into a given price-package size combination. Disposable razors are once again no exception. Prices in the sample display a non-linear schedule in package size, which is reported in Table B3. The last column of the table presents the quantity discount associated with the biggest-selling package sizes. In a context where not all products are sold in all package sizes and all DFF's stores, we analyzed the nonlinearity in package size in the lines of Hendel and Nevo (2006b), using a regression of the price per 5 razors on size dummy variables, controlling for temporary price promotions as well as product and store fixed effects. The quantity discount of each package size is then computed as the ratio of the coefficient on the corresponding size dummy variable to the constant. The results show that prices do exhibit quantity discounting. As a consequence, price nonlinearity constitutes a feature of the market that must be incorporated into the structural model.

B.2. Recover Unobserved Marginal Costs under Non-Cooperative Behavior

Having established, in Table 4, the structure of financial and corporate control rights of the six manufacturing firms in the industry, we have all the required information to construct the four matrices that are instrumental in computing the normalized weight matrix $\mathbf{W}$: matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, which capture eventual cross-ownership among internal owners, and matrices $\mathbf{F}$ and $\mathbf{C}$, which capture eventual common-ownership from external owners. We address first the former. Matrices $\mathbf{F}^*$ and $\mathbf{C}^*$ denote, in our application, (6×6) matrices. The diagonal elements are, by definition, zero. The off-diagonal elements, ϕ_{fg} and γ_{fg}, represent the cross-ownership financial and corporate control rights of firm f on firm g, respectively, for all $f \neq g \in \Im \equiv \{1, \ldots, 6\}$. In both cases, the rows and columns are ordered from $f = 1$ to $f = 6$. Given that in 1994 firm 5, Warner-Lamber, owns 100% of the voting rights in firm 6, Wilkinson Sword, and this constitutes the only cross-ownership stake in the industry, we have that (i) $\phi_{fg} = 0$ and $\gamma_{fg} = 0$ for all $f \in \{1, \ldots, 4\}$ and $g \in \Im$ (and vice-versa), (ii) $\phi_{5,6} = 1$ and $\gamma_{5,6} = 1$, and (iii) $\phi_{6,5} = 0$ and $\gamma_{6,5} = 0$. This implies the following 1994's $\mathbf{F}^*$ and $\mathbf{C}^*$ matrices:

$$\mathbf{F}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \mathbf{C}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

We now address the latter. Matrices $\mathbf{F}$ and $\mathbf{C}$ denote, in our application, (16×6) matrices. The typical element is given by ϕ_{kf} and γ_{kf}, respectively, for all $f \in \Im$ and all $k \in \Theta \backslash \Im$. The rows are ordered from $k = 7$ to $k = 22$, while the columns are ordered from $f = 1$ to $f = 6$. For instance, external owner Berkshire Hathaway, indexed as $k = 9$, has financial and corporate control rights on Gillette, indexed as $f = 2$, of 10.9% and 10.7%, respectively. As a consequence, we have that $\phi_{9,2} = 0.109$ and $\gamma_{9,2} = 0.107$. Formally, the 1994's $\mathbf{F}$ and $\mathbf{C}$ matrices are given by:

$$\mathbf{F} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1090 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8910 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0516 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9484 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1070 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8930 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0516 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9484 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

Having constructed matrices $\mathbf{F}^*$, $\mathbf{C}^*$, $\mathbf{F}$ and $\mathbf{C}$, we have all the necessary information to compute the 1994's matrices $\mathbf{F}^u$ and $\mathbf{C}^u$, which capture the ultimate ownership rights, as described in section 3.2. In our application, this computation yields:

$$\mathbf{F}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1090 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8910 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0516 & 0.0516 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9484 & 0.9484 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \mathbf{C}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1070 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8930 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0516 & 0.0516 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9484 & 0.9484 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

Finally, we can use $\mathbf{F}^u$ and $\mathbf{C}^u$ to compute the 1994's weight matrices $\mathbf{L}$ and $\mathbf{W}$, as described in section 3.4. This computation yields:

$$\mathbf{L} = \begin{bmatrix} 0.6535 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8073 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.2762 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9022 & 0.9022 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.9022 & 0.9022 \end{bmatrix} \quad \mathbf{W} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \end{bmatrix}.$$

B.3. Compute the Critical Threshold *Pre*-Acquisition

Pre-Gillette's 1989 initial offer, the financial and corporate control rights are the industry is slightly different. Among other differences, Wilkinson Sword is fully held by Eemland's external owners and, thus, independent of the remaining firms in the industry. Comparing with the 1994 ownership structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{C}$. Let $\mathring{\mathbf{F}}^*$, $\mathring{\mathbf{C}}^*$, $\mathring{\mathbf{F}}$ and $\mathring{\mathbf{C}}$ denote the corresponding matrices *pre*-acquisition, which according to the financial and corporate control rights of the industry, *pre*-Gillette's 1989 initial offer, are given by:

$$\mathring{\mathbf{F}}^{*} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \mathring{\mathbf{C}}^{*} = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix},$$

and

$$\mathring{\mathbf{F}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \mathring{\mathbf{C}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Having constructed matrices $\mathring{\mathbf{F}}^{*}$, $\mathring{\mathbf{C}}^{*}$, $\mathring{\mathbf{F}}$ and $\mathring{\mathbf{C}}$, we have all the necessary information to compute the corresponding matrices $\mathring{\mathbf{F}}^u$ and $\mathring{\mathbf{C}}^u$. This computation yields:

$$\mathring{\mathbf{F}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \quad \mathring{\mathbf{C}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Finally, we can use $\mathring{\mathbf{F}}^u$ and $\mathring{\mathbf{C}}^u$ to compute the corresponding weight matrices $\mathring{\mathbf{L}}$ and $\mathring{\mathbf{W}}$. This computation yields:

$$\mathring{\mathbf{L}} = \begin{bmatrix} 0.6535 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7075 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.2762 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \quad \mathring{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

B.4. Compute the Critical Threshold *Post*-Acquisition

Acquisition Case 1. Gillette Acquires 100% of the Voting Stock of Wilkinson Sword

The (hypothetical) acquisition of 100% of the voting stock of Wilkinson Sword by Gillette gives rise to a cross-ownership structure in the industry. Comparing with the *pre*-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{C}$. Let $\tilde{\mathbf{F}}^*$, $\tilde{\mathbf{C}}^*$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ denote the corresponding matrices *post*-acquisition, which are then given by:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \tilde{\mathbf{C}}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix},$$

and

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \tilde{\mathbf{C}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{F}}^*$, $\tilde{\mathbf{C}}^*$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$, we have all the necessary information to compute the corresponding matrices $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$. This computation yields:

$$\tilde{\mathbf{F}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.8320 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \tilde{\mathbf{C}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0600 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.8320 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

Finally, we can use $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$ to compute the corresponding weight matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} = \begin{bmatrix} 0.6535 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7075 & 0.0000 & 0.0000 & 0.0000 & 0.7075 \\ 0.0000 & 0.0000 & 0.2762 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.7075 & 0.0000 & 0.0000 & 0.0000 & 0.7075 \end{bmatrix} \quad \tilde{\mathbf{W}} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Acquisition Case 2. Berkshire Hathaway Acquires 100% of the Voting Stock of Wilkinson Sword

The (hypothetical) acquisition of 100% of the voting stock of Wilkinson Sword by Berkshire Hathaway gives rise to a common-ownership structure in the industry. Comparing with the *pre*-acquisition structure, this implies changes to matrices $\mathbf{F}$ and $\mathbf{C}$. Let $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ denote the corresponding matrices *post*-acquisition, which are then given by:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \tilde{\mathbf{C}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

Having constructed *post*-acquisition matrices $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$, we have (jointly with *pre*-acquisition matrices $\mathring{\mathbf{F}}^*$ and $\mathring{\mathbf{C}}^*$) all the necessary information to compute the corresponding matrices $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$. This computation yields:

$$\tilde{\mathbf{F}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \tilde{\mathbf{C}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

Finally, we can use $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$ to compute the corresponding weight matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} \approx \begin{bmatrix} 0.6535 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7075 & 0.0000 & 0.0000 & 0.0000 & 0.1080 \\ 0.0000 & 0.0000 & 0.2762 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \quad \tilde{\mathbf{W}} \approx \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1527 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.108 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Acquisition Case 3. Gillette Acquires 22.9% of the Voting Stock of Wilkinson Sword

The (hypothetical) acquisition of 22.9% of the voting stock of Wilkinson Sword by Gillette gives rise to a cross-ownership structure in the industry. Comparing with the *pre*-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{C}^*$, as well as to matrices $\mathbf{F}$ and $\mathbf{C}$. Let $\tilde{\mathbf{F}}^*$, $\tilde{\mathbf{C}}^*$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ denote the corresponding matrices *post*-acquisition, which are then given by:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \quad \tilde{\mathbf{C}}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix},$$

and

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix} \quad \tilde{\mathbf{C}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{F}}^*$, $\tilde{\mathbf{C}}^*$, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$, we have all the necessary information to compute the corresponding matrices $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$. This computation yields:

$$\tilde{\mathbf{F}}^u \approx \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0247 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0137 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.1905 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix} \quad \tilde{\mathbf{C}}^u \approx \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0247 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0137 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.1905 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix}.$$

Finally, we can use $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$ to compute the corresponding weight matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} \approx \begin{bmatrix} 0.6535 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7075 & 0.0000 & 0.0000 & 0.0000 & 0.1620 \\ 0.0000 & 0.0000 & 0.2762 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.1620 & 0.0000 & 0.0000 & 0.0000 & 0.6315 \end{bmatrix} \quad \tilde{\mathbf{W}} \approx \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.2565 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Acquisition Case 4. Berkshire Hathaway Acquires 22.9% of the Voting Stock of Wilkinson Sword

The (hypothetical) acquisition of 22.9% of the voting stock of Wilkinson Sword by Berkshire Hathaway gives rise to a common-ownership structure in the industry. Comparing with the *pre*-acquisition structure, this implies changes to matrices $\mathbf{F}$ and $\mathbf{C}$. Let $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$ denote the corresponding matrices *post*-acquisition, which are then given by:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix} \quad \tilde{\mathbf{C}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix}.$$

Having constructed *post*-acquisition matrices $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{C}}$, we have (jointly with *pre*-acquisition matrices $\mathring{\mathbf{F}}^*$ and $\mathring{\mathbf{C}}^*$) all the necessary information to compute the corresponding matrices $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$. This computation yields:

$$\tilde{\mathbf{F}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix} \quad \tilde{\mathbf{C}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix}.$$

Finally, we can use $\tilde{\mathbf{F}}^u$ and $\tilde{\mathbf{C}}^u$ to compute the corresponding weight matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} \approx \begin{bmatrix} 0.6535 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7075 & 0.0000 & 0.0000 & 0.0000 & 0.0247 \\ 0.0000 & 0.0000 & 0.2762 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0247 & 0.0000 & 0.0000 & 0.0000 & 0.6469 \end{bmatrix} \quad \tilde{\mathbf{W}} \approx \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0350 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0382 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Acquisition Case 5. Gillette Acquires 22.9% of the Non-Voting Stock of Wilkinson Sword

The (hypothetical) acquisition of 22.9% of the non-voting stock of Wilkinson Sword by Gillette gives rise to a cross-ownership structure in the industry. Comparing with the *pre*-acquisition structure, this implies changes to matrices $\mathbf{F}^*$ and $\mathbf{F}$. Let $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{F}}$ denote the corresponding matrices *post*-acquisition, which are then given by:

$$\tilde{\mathbf{F}}^* = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix},$$

and

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix}.$$

Having constructed matrices $\tilde{\mathbf{F}}^*$ and $\tilde{\mathbf{F}}$, we have all the necessary information to compute the corresponding matrix $\tilde{\mathbf{F}}^u$. This computation yields:

$$\tilde{\mathbf{F}}^u \approx \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.0247 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0137 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.1905 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix}.$$

Finally, we can use the *post*-acquisition matrix $\tilde{\mathbf{F}}^u$ and the *pre*-acquisition matrix $\mathring{\mathbf{C}}^u$ to compute the corresponding weight matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} \approx \begin{bmatrix} 0.6535 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7075 & 0.0000 & 0.0000 & 0.0000 & 0.1620 \\ 0.0000 & 0.0000 & 0.2762 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix} \quad \tilde{\mathbf{W}} \approx \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

Acquisition Case 6. Berkshire Hathaway Acquires 22.9% of the Non-Voting Stock of Wilkinson Sword

The (hypothetical) acquisition of 22.9% of the non-voting stock of Wilkinson Sword by Berkshire Hathaway gives rise to a common-ownership structure in the industry. Comparing with the *pre*-acquisition structure, this implies changes to matrix $\mathbf{F}$. Let $\tilde{\mathbf{F}}$ denote the corresponding matrix *post*-acquisition, which is then given by:

$$\tilde{\mathbf{F}} = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix}.$$

Having constructed *post*-acquisition matrix $\tilde{\mathbf{F}}$, we have (jointly with *pre*-acquisition matrix $\mathring{\mathbf{F}}^*$) all the necessary information to compute the corresponding matrix $\tilde{\mathbf{F}}^u$. This computation yields:

$$\tilde{\mathbf{F}}^u = \begin{bmatrix} 0.7770 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.2230 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1080 & 0.0000 & 0.0000 & 0.0000 & 0.2290 \\ 0.0000 & 0.0600 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.8320 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1440 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.1240 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0780 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0700 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0610 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0510 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4720 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix}.$$

Finally, we can use *post*-acquisition matrix $\tilde{\mathbf{F}}^u$ and *pre*-acquisition matrix $\mathring{\mathbf{C}}^u$ to compute the corresponding weight matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{W}}$. This computation yields:

$$\tilde{\mathbf{L}} \approx \begin{bmatrix} 0.6535 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.7075 & 0.0000 & 0.0000 & 0.0000 & 0.0247 \\ 0.0000 & 0.0000 & 0.2762 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7710 \end{bmatrix} \quad \tilde{\mathbf{W}} \approx \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0350 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}.$$

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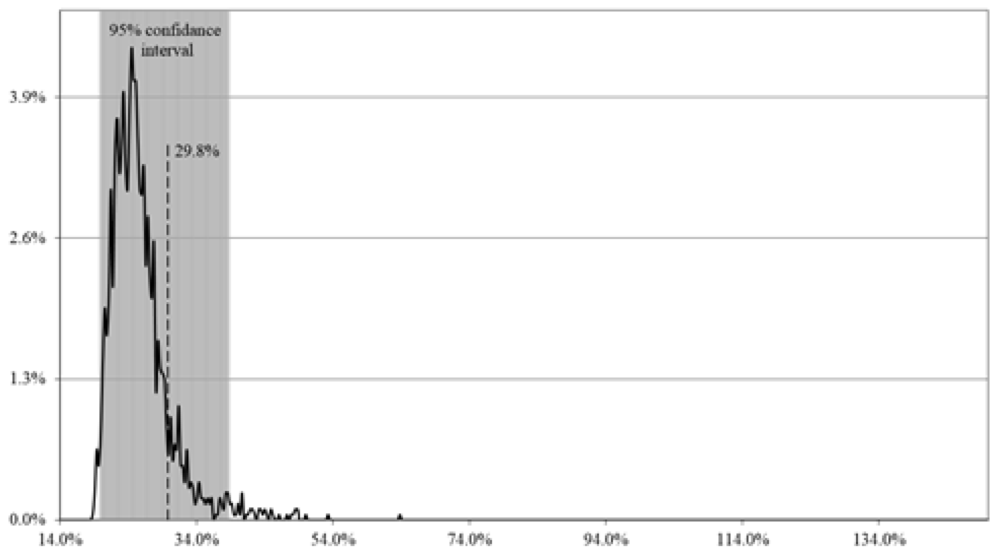
FIGURE 1

Non-Cooperative Industry Operating Profit to Sales Ratio



Coordinated Industry Operating Profit to Sales Ratio

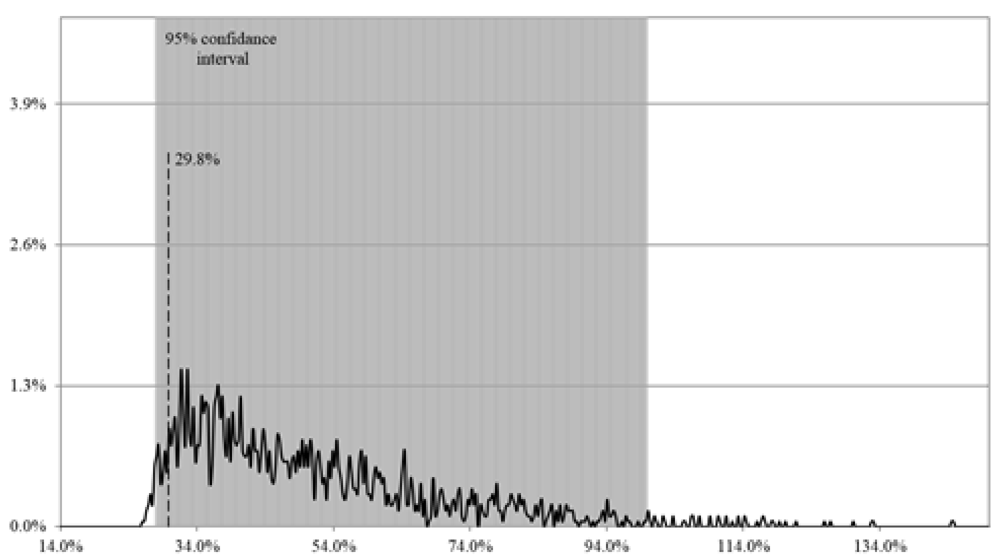


Table 1

*Volume Market Shares (%)**

	Mean	Median	Std	Min	Max
Panel A: Brand Level					
1. Gillette	59.538	61.538	14.737	0.000	95.037
2. Private Label	20.562	18.634	10.837	0.000	100.000
3. Schick	14.043	12.753	8.832	0.000	66.154
4. BIC	5.551	0.000	14.392	0.000	93.776
5. Personna	0.275	0.000	0.770	0.000	11.990
6. Wilkinson Sword	0.032	0.000	0.314	0.000	9.284
Panel B: Product Level					
1. G Good News	14.210	12.975	8.387	0.000	74.850
2. G Good News Plus	11.173	10.504	6.535	0.000	52.941
3. G Daisy Plus	9.553	8.467	6.767	0.000	45.455
4. WL Schick Slim Twin	8.832	7.634	6.988	0.000	56.893
5. G Good News Pivot Plus	6.959	6.094	5.313	0.000	48.980
6. G Good News Microtrac	6.891	6.061	5.552	0.000	54.545
Panel C: Package Size Level					
1. 10 Razors	41.482	41.667	13.978	0.000	97.162
2. 5 Razors	41.438	40.650	13.348	2.080	100.000
3. 12 Razors	11.328	10.480	7.384	0.000	56.376
4. 15 Razors	5.247	0.000	10.677	0.000	71.942
5. 3 Razors	0.378	0.000	0.886	0.000	12.060
6. 2 Razors	0.121	0.000	0.556	0.000	11.538

* The statistics presented are computed across the 8,346 store-week combinations. Volume market share denotes the percentage of the number of razors sold by brand, product and package size in the total number of razors sold in each market. B: BIC, G: Gillette, ASR: American Safety Razor, PL: Private Label, WL: Warner-Lambert, WS: Wilkinson Sword.

Table 2

*Demand Estimation Results**

	Logit OLS		Logit HLZ		Logit BLP		Logit BST		RC Logit BLP
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Standard Price Parameters									
Price	-0.074	-0.837	-0.921	-1.054	-0.442	-2.442	-0.627	-2.301	-2.516
	(0.364)	(0.216)	(0.188)	(0.124)	(0.487)	(0.239)	(0.510)	(0.232)	(0.352)
Price × HH Size	-0.021	-0.053	-0.074	-0.074	-0.256	-0.187	-0.216	-0.189	
	(0.160)	(0.100)	(0.083)	(0.055)	(0.220)	(0.083)	(0.233)	(0.088)	
Price × Age	0.016	0.007	0.009	-0.016	-0.083	-0.250	-0.020	-0.135	
	(0.261)	(0.135)	(0.115)	(0.076)	(0.292)	(0.135)	(0.314)	(0.140)	
Price × HH Income	0.113	0.123	0.159	0.144	0.154	0.181	0.082	0.161	
	(0.072)	(0.043)	(0.038)	(0.021)	(0.085)	(0.036)	(0.094)	(0.041)	
Standard Product Characteristics Parameters									
Package Size	0.029								
	(0.025)								
Package Size^2	-0.005								
	(0.001)								
Standard Demographic Parameters									
HH Size	-0.213	-0.141	-0.020		0.482		0.253		
	(0.591)	(0.290)	(0.240)		(0.633)		(0.665)		
Age	-0.460	-0.620	-0.557		-0.299		-0.576		
	(0.892)	(0.355)	(0.325)		(0.829)		(0.881)		
HH Income	-0.082	-0.112	-0.171		-0.274		-0.065		
	(0.268)	(0.125)	(0.106)		(0.256)		(0.283)		

Table 2

Extended

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Standard Competition Parameters									
Nearby Grocery Str.	0.704	0.646	0.383		0.429		0.731		
	(0.844)	(0.571)	(0.406)		(0.493)		(0.482)		
Nearby Conven. Str.	1.403	0.733	2.017		0.886		0.151		
	(1.479)	(0.953)	(0.737)		(0.849)		(0.849)		
Nearby Pharmacies	-1.313	-1.343	-1.556		-1.452		-1.568		
	(1.359)	(0.696)	(0.635)		(0.671)		(0.661)		
Random Coefficients: Standard Deviations									
Constant									0.030
									(2.501)
Price									0.047
.									(0.330)
Random Coefficients: Demographic Interactions									
Price × HH Size									-0.189
									(0.190)
Price × Age									-0.223
									(0.222)
Price × HH Income									0.131
									(0.068)
Control Parameters	—	j–	j–	jmt	j–	jmt	j–	jmt	jmt
No. End. Var./Instr.	4/0	4/0	4/28	4/28	4/16	4/16	4/16	4/16	6/16
R^2/Hansen J Statistic	0.03	0.55	146.39	159.15	150.59	18.044^+	154.82	105.54	17.398^+

* Based on 17,745 store-quarter-UPC observations. Standard errors clustered by store-brand in parentheses. HH denotes household. Nearby Grocery Str. and Nearby Conven. Str. denote the number of nearby grocery and convenience stores, respectively. No. End. Var./Instr. denote the number of endogenous variables and the number of instruments, respectively. Specification (1) includes a constant term. j, m and t denote UPC, market (store) and time (quarter) dummy variables. + denotes that the J statistic of Hansen is statistically significant at a 5 percent significance level.

TABLE 3

*Median Own- and Cross-Price Elasticities**

UPC	4	7	9	10	12	13	14	15	16
1. BIC Lady Shaver 10r	0.045	0.275	0.009	0.031	0.105	0.045	0.059	0.004	0.006
2. BIC Metal Shaver 5r	0.036	0.327	0.009	0.033	0.105	0.050	0.149	0.004	0.006
3. BIC Pastel Lady Shaver 5r	0.031	0.301	0.011	0.032	0.105	0.051	0.156	0.004	0.006
4. BIC Shaver 10r	-6.439	0.256	0.010	0.028	0.106	0.046	0.145	0.004	0.006
5. G Daisy Slim 5r	0.024	0.294	0.011	0.051	0.111	0.072	0.228	0.004	0.006
6. G Good News 3r	0.036	0.325	0.009	0.033	0.114	0.051	0.146	0.004	0.006
7. G Good News 10r	0.032	-12.877	0.010	0.032	0.109	0.051	0.165	0.004	0.006
8. G Good News Microtrac 5r	0.031	0.346	0.009	0.035	0.117	0.052	0.181	0.004	0.006
9. G Good News Pivot Plus 10r	0.022	0.387	-12.761	0.038	0.111	0.054	0.205	0.004	0.006
10. ASR Personna Flicker 5r	0.032	0.313	0.009	-10.221	0.113	0.053	0.187	0.004	0.006
11. PL Single Blade 5r	0.030	0.246	0.008	0.028	0.107	0.047	0.135	0.004	0.006
12. PL Twin Blade 5r	0.030	0.258	0.008	0.029	-4.538	0.047	0.140	0.004	0.006
13. WL Schick Slim Twin 5r	0.027	0.323	0.009	0.034	0.111	-7.277	0.157	0.004	0.006
14. WL Schick Slim Twin 10r	0.031	0.305	0.010	0.031	0.110	0.050	-10.901	0.004	0.006
15. WS Colors 5r	0.026	0.324	0.011	0.036	0.108	0.056	0.202	-3.650	0.007
16. WS Ultra Glide Twin 5r	0.023	0.336	0.011	0.033	0.110	0.058	0.205	0.004	-4.769

* Figures denote the median price elastiticities over the 643 store-quarter combinations. The elasticity in row i and column j represents the percentage change in market share of product i with a 1% change in price of product j. B: BIC, G: Gillette, ASR: American Safety Razor, PL: Private Label, WL: Warner-Lambert, WS: Wilkinson Sword. 3r, 5r and 10r denote package sizes of 3, 5 and 10 razors, respectively.

TABLE 4

*1994 Financial and Corporate Control**

		BIC		G		ASR		PL		WL		WS	
f/k		F	CC	F	CC	F	CC	F	CC	F	CC	F	CC
Panel A: Internal Owners													
01	BIC Corporation	–	–	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
02	The Gillette Company	0.00	0.00	–	–	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
03	American Safety Razor Company	0.00	0.00	0.00	0.00	–	–	0.00	0.00	0.00	0.00	0.00	0.00
04	Private Label Companies	0.00	0.00	0.00	0.00	0.00	0.00	–	–	0.00	0.00	0.00	0.00
05	Warner-Lambert Company	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	–	–	100.00	100.00
06	Wilkinson Sword, Inc.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	–	–
Panel B: External Owners													
07	Bruno Bich	77.70	77.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
08	BIC Minority Owners	22.30	22.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
09	Berkshire Hathaway, Inc.	0.00	0.00	10.90	10.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	State Street Bank and Trust Co.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	G Minority Owners	0.00	0.00	89.10	89.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	Equitable (a)	0.00	0.00	0.00	0.00	14.40	14.40	0.00	0.00	0.00	0.00	0.00	0.00
13	Allsop Venture Partners III, LP	0.00	0.00	0.00	0.00	12.40	12.40	0.00	0.00	0.00	0.00	0.00	0.00
14	Goldman Sachs Group, LP	0.00	0.00	0.00	0.00	7.80	7.80	0.00	0.00	0.00	0.00	0.00	0.00
15	Scudder Stevens and Clarck	0.00	0.00	0.00	0.00	7.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00
16	Leucadia-Mezzanine (b)	0.00	0.00	0.00	0.00	6.10	6.10	0.00	0.00	0.00	0.00	0.00	0.00
17	Grantham Mayo Van Otter	0.00	0.00	0.00	0.00	5.10	5.10	0.00	0.00	0.00	0.00	0.00	0.00
18	ASR Minority Owners	0.00	0.00	0.00	0.00	47.20	47.20	0.00	0.00	0.00	0.00	0.00	0.00
19	PL Minority Owners	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	0.00	0.00	0.00	0.00
20	The Capital Group, Inc.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.16	5.16	0.00	0.00
21	WL Minority Owners	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	94.84	94.84	0.00	0.00
22	Eemland M. S. BV Minority Owners	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

* Figures are in percentage points. Adapted from Schedule 14A (proxy statement) information and competition agencies decisions. F denotes each (row) owner's holdings of total stock on (column) firm. CC denotes, under Assumption 6, each (row) owner's holdings of voting stock on (column) firm. BIC, G, ASR, PL, WL, and WS denote BIC Corporation, The Gillette Company, American Safety Razor Company, Private Label companies, Warner-Lambert Company, and Wilkinson Sword, Inc., respectively. (a) Equitable denotes the cumulative ownership of Equitable Capital Partners, LP, Equitable Deal Flow Fund, LP, Equitable Capital Partners (Retirement Fund), LP, and The Equitable Life Assurance Society of the United States. (b) Leucadia-Mezzanine denotes the cumulative ownership of Leucadia Investors, Inc. and Mezzanine Capital and Income Trust 2001 PLC.

TABLE 5

*Median Recovered Marginal Costs**

UPC	1994 Price	1994 Marginal Cost	
		Non-Cooperative	Coordinated
1. BIC Lady Shaver 10r	2.300	1.928	1.640
2. BIC Metal Shaver 5r	2.090	1.723	1.447
3. BIC Pastel Lady Shaver 5r	1.990	1.624	1.409
4. BIC Shaver 10r	2.390	2.022	1.836
5. G Daisy Slim 5r	2.223	1.773	1.533
6. G Good News 3r	1.990	1.517	1.364
7. G Good News 10r	4.781	4.316	4.203
8. G Good News Microtrac 5r	2.672	2.189	2.034
9. G Good News Pivot Plus 10r	4.390	3.894	3.760
10. ASR Personna Flicker 5r	3.990	3.627	3.412
11. PL Single Blade 5r	1.072	0.663	0.509
12. PL Twin Blade 5r	1.180	0.783	0.641
13. WL Schick Slim Twin 5r	2.058	1.681	1.473
14. WL Schick Slim Twin 10r	3.750	3.362	3.202
15. WS Colors 5r	1.290	0.915	0.663
16. WS Ultra Glide Twin 5r	1.690	1.314	1.071
Overall Median	2.690	2.235	2.113

* Figures denote the median price, the median recovered marginal cost under non-cooperative behavior, and the median recovered marginal costs under coordinated behavior across 162 store-quarter combinations. B: BIC, G: Gillette, ASR: American Safety Razor, PL: Private Label, WL: Warner-Lambert, WS: Wilkinson Sword. 3r, 5r and 10r denote package sizes of 3, 5 and 10 razors, respectively.

TABLE 6

*1989 Financial and Corporate Control**

f/k		BIC		G		ASR		PL		WL		WS	
		F	CC	F	CC	F	CC	F	CC	F	CC	F	CC
Panel A: Internal Owners													
01	BIC Corporation	–	–	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
02	The Gillette Company	0.00	0.00	–	–	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
03	American Safety Razor Company	0.00	0.00	0.00	0.00	–	–	0.00	0.00	0.00	0.00	0.00	0.00
04	Private Label Companies	0.00	0.00	0.00	0.00	0.00	0.00	–	–	0.00	0.00	0.00	0.00
05	Warner-Lambert Company	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	–	–	0.00	0.00
06	Wilkinson Sword, Inc.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	–	–
Panel B: External Owners													
07	Bruno Bich	77.70	77.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
08	BIC Minority Owners	22.30	22.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
09	Berkshire Hathaway, Inc.	0.00	0.00	10.80	10.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	State Street Bank and Trust Co.	0.00	0.00	6.00	6.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	G Minority Owners	0.00	0.00	83.20	83.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	Equitable (a)	0.00	0.00	0.00	0.00	14.40	14.40	0.00	0.00	0.00	0.00	0.00	0.00
13	Allsop Venture Partners III, LP	0.00	0.00	0.00	0.00	12.40	12.40	0.00	0.00	0.00	0.00	0.00	0.00
14	Goldman Sachs Group, LP	0.00	0.00	0.00	0.00	7.80	7.80	0.00	0.00	0.00	0.00	0.00	0.00
15	Scudder Stevens and Clarck	0.00	0.00	0.00	0.00	7.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00
16	Leucadia-Mezzanine (b)	0.00	0.00	0.00	0.00	6.10	6.10	0.00	0.00	0.00	0.00	0.00	0.00
17	Grantham Mayo Van Otter	0.00	0.00	0.00	0.00	5.10	5.10	0.00	0.00	0.00	0.00	0.00	0.00
18	ASR Minority Owners	0.00	0.00	0.00	0.00	47.20	47.20	0.00	0.00	0.00	0.00	0.00	0.00
19	PL Minority Owners	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	0.00	0.00	0.00	0.00
20	The Capital Group, Inc.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
21	WL Minority Owners	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00	0.00	0.00
22	Eemland M. S. BV Minority Owners	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00	100.00

* Figures are in percentage points. Adapted from Schedule 14A (proxy statement) information and competition agencies decisions. F denotes each (row) owner's holdings of total stock on (column) firm. CC denotes, under Assumption 6, each (row) owner's holdings of voting stock on (column) firm. BIC, G, ASR, PL, WL, and WS denote BIC Corporation, The Gillette Company, American Safety Razor Company, Private Label companies, Warner-Lambert Company, and Wilkinson Sword, Inc., respectively. (a) Equitable denotes the cumulative ownership of Equitable Capital Partners, LP, Equitable Deal Flow Fund, LP, Equitable Capital Partners (Retirement Fund), LP, and The Equitable Life Assurance Society of the United States. (b) Leucadia-Mezzanine denotes the cumulative ownership of Leucadia Investors, Inc. and Mezzanine Capital and Income Trust 2001 PLC.

TABLE 7

*Pre-Acquisition Counterfactual Prices**

	Non-Cooperative 1994 Marginal Costs			Coordinated 1994 Marginal Costs		
UPC	Coordinated	Deviation	Non-Cooperative	Coordinated	Deviation	Non-Cooperative
1. BIC Lady Shaver 10r	2.480	2.302	2.300	2.300	1.997	1.995
2. BIC Metal Shaver 5r	2.283	2.091	2.090	2.090	1.831	1.822
3. BIC Pastel Lady Shaver 5r	2.162	1.992	1.990	1.990	1.781	1.778
4. BIC Shaver 10r	2.516	2.391	2.390	2.390	2.208	2.205
5. G Daisy Slim 5r	2.297	2.226	2.223	2.223	2.042	2.011
6. G Good News 3r	2.075	2.001	1.990	1.990	1.871	1.855
7. G Good News 10r	4.850	4.784	4.781	4.781	4.681	4.667
8. G Good News Microtrac 5r	2.769	2.691	2.672	2.672	2.565	2.545
9. G Good News Pivot Plus 10r	4.509	4.410	4.390	4.390	4.290	4.271
10. ASR Personna Flicker 5r	4.158	3.990	3.990	3.990	3.774	3.773
11. PL Single Blade 5r	1.158	1.076	1.072	1.072	0.925	0.917
12. PL Twin Blade 5r	1.280	1.185	1.180	1.180	1.055	1.051
13. WL Schick Slim Twin 5r	2.257	2.063	2.057	2.058	1.865	1.859
14. WL Schick Slim Twin 10r	3.924	3.752	3.750	3.750	3.600	3.599
15. WS Colors 5r	1.466	1.268	1.268	1.290	1.022	1.021
16. WS Ultra Glide Twin 5r	1.862	1.668	1.668	1.690	1.431	1.430
Overall Median	2.768	–	2.690	2.690	–	2.578

* Figures are the median pre-acquisition counterfactual equilibrium prices across 162 store-quarter combinations, computed using the marginal costs recovered under non-cooperative and coordinated behavior. B: BIC, G: Gillette, ASR: American Safety Razor, PL: Private Label, WL: Warner-Lambert, WS: Wilkinson Sword. 3r, 5r and 10r denote package sizes of 3, 5 and 10 razors, respectively.

Table 8

*Pre-Acquisition Counterfactual Benefit from Deviation and Coordination and Minimum Discount Factors**

	Non-Cooperative 1994 Marginal Costs			Coordinated 1994 Marginal Costs			
firm	benefit from deviation	benefit from coordination	minimum discount factor	benefit from deviation	benefit from coordination	minimum discount factor	actual discount factor
BIC	8.242	1.915	0.811	15.656	5.317	0.746	0.904
G	1.537	4.754	0.244	3.275	8.812	0.271	0.906
ASR	13.149	1.016	0.928	25.631	5.263	0.830	0.903
PL	4.101	2.627	0.610	7.348	6.038	0.549	0.899
WL	7.503	1.531	0.831	13.278	4.744	0.737	0.898
WS	13.881	2.807	0.832	30.165	8.541	0.779	0.903

* The "benefit from deviation" denotes the difference between the deviation and the coordinated counterfactual equilibrium operating profit as a percentage of the coordinated operating profit. The "benefit from coordination" represents the difference between the coordinated and the non-cooperative operating profit as a percentage of the coordinated operating profit. Both figures are computed using the marginal costs recovered under non-cooperative and coordinated behavior. BIC, G, ASR, PL, WL, and WS denote BIC Corporation, The Gillette Company, American Safety Razor Company, Private Label companies, Warner-Lambert Company, and Wilkinson Sword, Inc., respectively.

Table 9

*Acquisition's Percentage Impact on Counterfactual Non-Cooperative and Deviation Prices**

	Non-Cooperative 1994 Marginal Costs						Coordinated 1994 Marginal Sosts					
	WS acquired by...						WS acquired by...					
	G	BH	G	BH	G	BH	G	BH	G	BH	G	BH
	100%	100%	22.9%	22.9%	22.9%	22.9%	100%	100%	22.9%	22.9%	22.9%	22.9%
	voting	voting	voting	voting	non	non	voting	voting	voting	voting	non	non
UPC					voting	voting					voting	voting
Panel A: Non-Cooperative Prices												
1. BIC Lady Shaver 10r	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
2. BIC Metal Shaver 5r	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
3. BIC Pastel Lady Shaver 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000
4. BIC Shaver 10r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5. G Daisy Slim 5r	0.023	0.004	0.006	0.001	0.005	0.001	0.037	0.006	0.009	0.001	0.008	0.001
6. G Good News 3r	0.036	0.006	0.009	0.001	0.008	0.001	0.058	0.009	0.015	0.002	0.013	0.002
7. G Good News 10r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8. G Good News Microtrac 5r	0.024	0.004	0.006	0.001	0.005	0.001	0.039	0.006	0.010	0.001	0.008	0.001
9. G Good News Pivot Plus 10r	0.014	0.002	0.003	0.000	0.003	0.000	0.021	0.003	0.005	0.001	0.004	0.001
10. ASR Personna Flicker 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11. PL Single Blade 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12. PL Twin Blade 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13. WL Schick Slim Twin 5r	0.001	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000
14. WL Schick Slim Twin 10r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15. WS Colors 5r	9.117	0.977	2.324	0.346	0.000	0.000	13.410	1.438	3.420	0.509	0.000	0.000
16. WS Ultra Glide Twin 5r	6.618	0.710	1.689	0.251	0.000	0.000	9.053	0.972	2.311	0.344	0.000	0.000
Overall Median	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 9

Extended

	Non-Cooperative 1994 Marginal Costs						Coordinated 1994 Marginal Costs					
	WS acquired by...						WS acquired by...					
UPC	G 100% voting	BH 100% voting	G 22.9% voting	BH 22.9% voting	G 22.9% non voting	BH 22.9% non voting	G 100% voting	BH 100% voting	G 22.9% voting	BH 22.9% voting	G 22.9% non voting	BH 22.9% non voting
Panel B: Deviation Prices												
1. BIC Lady Shaver 10r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2. BIC Metal Shaver 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3. BIC Pastel Lady Shaver 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4. BIC Shaver 10r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5. G Daisy Slim 5r	0.019	0.002	0.004	0.001	0.004	0.001	0.028	0.003	0.005	0.001	0.005	0.001
6. G Good News 3r	0.028	0.003	0.005	0.001	0.005	0.001	0.043	0.005	0.008	0.001	0.007	0.001
7. G Good News 10r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8. G Good News Microtrac 5r	0.019	0.002	0.004	0.000	0.003	0.000	0.029	0.003	0.005	0.001	0.005	0.001
9. G Good News Pivot Plus 10r	0.012	0.001	0.002	0.000	0.002	0.000	0.016	0.002	0.003	0.000	0.003	0.000
10. ASR Personna Flicker 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11. PL Single Blade 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12. PL Twin Blade 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13. WL Schick Slim Twin 5r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14. WL Schick Slim Twin 10r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15. WS Colors 5r	15.585	0.853	2.028	0.302	0.000	0.000	26.298	1.205	2.867	0.426	0.000	0.000
16. WS Ultra Glide Twin 5r	11.660	0.639	1.519	0.226	0.000	0.000	18.103	0.807	1.920	0.285	0.000	0.000

* Figures are the median post-acquisition percentage change (with respect to the pre-acquisition corresponding values) across 162 store-quarter combinations, computed using the marginal costs recovered under non-cooperative and coordinated behavior. B: BIC, G: Gillette, ASR: American Safety Razor, PL: Private Label, WL: Warner-Lambert, WS: Wilkinson Sword, BH: Berkshire Hathaway. 3r, 5r and 10r denote package sizes of 3, 5 and 10 razors, respectively.

TABLE 10

*Acquisition's Percentage Impact on the Counterfactual Benefit from Deviation and Coordination**

	Non-Cooperative 1994 Marginal Costs						Coordinated 1994 Marginal Costs					
	WS acquired by...						WS acquired by...					
	G	BH	G	BH	G	BH	G	BH	G	BH	G	BH
	100%	100%	22.9%	22.9%	22.9%	22.9%	100%	100%	22.9%	22.9%	22.9%	22.9%
	voting	voting	voting	voting	non	non	voting	voting	voting	voting	non	non
firm					voting	voting					voting	voting
Panel A: Benefit from Deviation												
BIC	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
G	-1.211	-0.186	-0.279	-0.043	-0.279	-0.043	-1.497	-0.230	-0.345	-0.053	-0.345	-0.053
ASR	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
PL	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
WL	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
WS	-86.154	-13.537	-30.341	-4.922	0.000	0.000	-84.518	-12.896	-28.994	-4.683	0.000	0.000
Panel B: Benefit from Coordination												
BIC	-3.262	-0.455	-0.867	-0.133	-0.346	-0.053	-2.515	-0.358	-0.687	-0.106	-0.258	-0.039
G	-0.153	-0.048	-0.130	-0.021	0.041	0.006	0.044	-0.019	-0.090	-0.015	0.092	0.014
ASR	-8.189	-1.152	-2.181	-0.336	-0.893	-0.136	-3.120	-0.448	-0.854	-0.132	-0.330	-0.050
PL	-1.714	-0.239	-0.456	-0.070	-0.177	-0.027	-1.573	-0.224	-0.431	-0.067	-0.157	-0.024
WL	-3.544	-0.496	-0.943	-0.145	-0.377	-0.058	-2.390	-0.341	-0.653	-0.101	-0.246	-0.038
WS	51,306.021	5,557.495	13,194.398	1,967.975	-0.737	-0.113	23,030.781	2,495.485	5,924.398	883.730	-0.453	-0.069

* Figures are the post-acquisition percentage change (with respect to the pre-acquisition corresponding values), computed using the marginal costs recovered under non-cooperative and coordinated behavior. BIC, G, ASR, PL, WL, and WS denote BIC Corporation, The Gillette Company, American Safety Razor Company, Private Label companies, Warner-Lambert Company, and Wilkinson Sword, Inc., respectively.

TABLE 11

*Post-Acquisition Counterfactual Minimum Discount Factors**

	Non-Cooperative 1994 Marginal Costs							Coordinated 1994 Marginal Costs							
		WS acquired by...							WS acquired by...						
	pre-	G	BH	G	BH	G	BH	pre-	G	BH	G	BH	G	BH	
	acqui-	100%	100%	22.9%	22.9%	22.9%	22.9%	acqui-	100%	100%	22.9%	22.9%	22.9%	22.9%	actual
	sition	voting	voting	voting	voting	non	non	sition	voting	voting	voting	voting	non	non	discount
firm						voting	voting						voting	voting	factor
BIC	0.811	0.817	0.812	0.813	0.812	0.812	0.812	0.746	0.751	0.747	0.748	0.747	0.747	0.747	0.904
G	0.244	0.242	0.244	0.244	0.244	0.244	0.244	0.271	0.268	0.271	0.270	0.271	0.270	0.271	0.906
ASR	0.928	0.934	0.929	0.930	0.928	0.929	0.928	0.830	0.834	0.830	0.831	0.830	0.830	0.830	0.903
PL	0.610	0.614	0.610	0.611	0.610	0.610	0.610	0.549	0.553	0.550	0.550	0.549	0.549	0.549	0.899
WL	0.831	0.836	0.831	0.832	0.831	0.831	0.831	0.737	0.741	0.737	0.738	0.737	0.737	0.737	0.898
WS	0.832	0.001	0.070	0.025	0.185	0.833	0.832	0.779	0.002	0.106	0.040	0.533	0.780	0.779	0.903

* B: BIC, G: Gillette, ASR: American Safety Razor, PL: Private Label, WL: Warner-Lambert, WS: Wilkinson Sword, BH: Berkshire Hathaway. 3r, 5r and 10r denote package sizes of 3, 5 and 10 razors, respectively.

TABLE B1

*Summary Statistics**

	Mean	Median	Std	Min	Max
Panel A: UPC Level					
Quantity (number of packages)	3.297	2.000	3.951	1.000	308.000
Price ($)	3.272	3.090	1.393	0.460	6.390
Gross Retail Margin (%)	41.108	38.890	15.889	-97.570	74.910
Package Size (number of razors)	7.377	5.000	3.052	1.000	20.000
Women Segment	0.209	0.000	0.407	0.000	1.000
Panel B: Store Level					
Number of Household Visits (000's)	17.481	17.539	4.675	1.686	30.640
Potential Market (number of packages)	181.079	181.684	48.431	17.465	317.395
Number of Grocery Stores	9.765	7.000	8.784	1.000	46.000
Number of Convenience Stores	4.296	3.000	3.404	0.000	16.000
Number of Pharmacies	5.556	5.000	3.637	0.000	14.000
Panel C: Demographic Level					
Age	41.537	40.221	19.228	10.000	79.000
Household Size	2.660	2.000	1.552	1.000	9.000
Household Income ($ 000's)	79.544	57.457	87.337	0.002	599.999

* Panel A statistics are based on 144,325 store-week-UPC observations. Gross Retail Margins denotes the margin in percent that DFF makes on the dollar for each item sold. Panel B Number of Household Visits and Potential Market statistics are based on 8,346 store-week combinations. Panel B competition statistics are based on 81 store observations. Panel C statistics are based on 2,000 simulated consumers for each of the 8,346 store-week combinations under analysis.

Table B2

*Temporary Price Promotions Characterization**

UPC Level	Mean	Median	Std	Min	Max
Promotion	0.115	0.000	0.319	0.000	1.000
Promotion Discount (%)	22.864	20.761	12.113	5.010	74.874
Duration from Last Promotion (weeks)	11.833	4.000	17.823	1.000	94.000

* Promotion statistics are based on 137,808 store-week-upc observations (since our temporary price promotion definition makes use of the first and last observation of the sequence of prices of each UPC in a given supermarket). Promotion Discount and Duration from Last Promotion statistics are conditional on a promotion and therefore are based on the corresponding 15,869 store-week-upc observations.

Table B3

*Temporary Price Promotions and Quantity Discount**

Package Size	Weeks on Promotion (%)	Quantity Sold on Promotion (%)	Quantity Discount (%)
5 Razors	11.427	19.027	–
10 Razors	11.967	23.959	29.635
12 Razors	11.755	15.489	52.555
15 Razors	6.199	7.875	61.278

* Weeks on Promotion and Quantity Sold on Promotion denote, conditional on package size, the percentage of weeks a promotion was offered and the percentage of number of packages sold on promotion, respectively. Figures are computed across all stores, weeks and UPCs. Quantity discount computed as the ratio of each dummy variable coefficient to the constant, from a regression of the price per 5 razors on size dummy variables, controlling for temporary price promotions as well as product and store fixed effects.