Finding Closest Targets in Non-Oriented DEA Models: The Case of Convex and Non-Convex Technologies

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Abstract

This paper draws attention for the fact that traditional Data Envelopment Analysis (DEA) models do not provide the closest possible targets (or peers) to inefficient units, and presents a procedure to obtain such targets.

It focuses on non-oriented efficiency measures (which assume that production units are able to control, and thus change, inputs and outputs simultaneously) both measured in relation to a Free Disposal Hull (FDH) technology and in relation to a convex technology. The approaches developed for finding close targets are applied to a sample of Portuguese bank branches.

JEL classification: C14, D24, G21

Introduction

Efficiency measurement in Data Envelopment Analysis (DEA) requires both the identification of a reference point on the boundary of the production possibility set (PPS) and the use of some measure of distance from that point to another being analysed. The two issues (identification of the boundary point and the distance measure used) are traditionally performed simultaneously. The basic DEA model as introduced by Farrell (1957) and later developed by Charnes et al. (1978), uses an oriented radial measure of efficiency, which identifies a point on the boundary with the same mix of inputs (input orientation) or outputs (output orientation) of that of the observed unit. The conservation of the mix in movements towards the boundary of the PPS is the characteristic that makes the resulting distance measure radial.

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In many practical situations, however, it is desirable to use measures of efficiency that are non-oriented and non-radial in character. Any measure of efficiency that does not assume equiproportional reductions of inputs or outputs is non-radial. The first non-radial measure of technical efficiency dates back to 1978 and is due to Färe and Lovell (1978). The interest of researchers in non-radial measures arises mainly from the fact that radial (or Farrell) efficiency measures do not correspond to the Pareto-Koopmans definition of technical efficiency.[1] This issue is known in the DEA literature as the indication or slacks problem, as the main characteristic of radial efficiency measures is that they ignore the possible existence of slacks associated with the projected points on the production frontier. This issue motivated a discussion (see e.g., Russell, 1985; Lovell and Schmidt, 1988; Kerstens and Vanden-Eeckaut, 1995) and although many authors still favour the use of radial measures (mainly because of its many useful properties), non-radial efficiency measures are increasing in popularity. A weakness of radial measures, is the perceived arbitrariness in imposing targets preserving the mix within inputs or within outputs, when the firm's very reason to change its input/output levels might often be the desire to change that mix (Chambers and Mitchell, 2001, p. 32).

The non-radial Färe-Lovell efficiency measure is oriented. That is, it aims at changing inputs or outputs but not both. To the authors' knowledge, the first (non-radial) non-oriented measures of efficiency were introduced in 1985. One of these, the hyperbolic measure of technical efficiency, is due to Färe et al. (1985) and the other, the additive model, is due to Charnes et al. (1985). Non-oriented measures are relevant in many practical situations. Take for example the banking context were the use of an intermediation approach (see Colwell and Davis, 1992, for details) specifies inputs in the form of costs and outputs in the form of revenues. Some of the costs and revenues are controllable, and so the obvious approach to follow is non-oriented, i.e., permitting at the same time reduction of inputs and increase of outputs (which in this case would translate in an increase in profits).[2]

As pointed out by one of the referees to this paper, the distinction between oriented and non-oriented measures of efficiency is mainly of theoretical interest only, because in practice the analyst needs to identify the variables which can be modified, and then efficiency is measured with reference to those variables. Non-radial oriented measures assume *a priori* that the variables to be modified are only on the input or on the output side, while in practice they are often on both sides. Non-oriented measures are, therefore, more general and more flexible in the sense that they allow for changes in all the factors.

One of the key practical outcomes in an efficiency assessment is the identification of targets. However, one of the drawbacks of the traditional non-oriented DEA models is that they either impose strong restrictions on the movements towards the efficient frontier, or they aim at maximising slacks. Both these facts contribute to finding targets and peers that are not the closest to the units being assessed. We will define measures of closeness later; suffice is to say at this point that the closer the targets to a unit, the less the change in its operations needed to reach its targets. If Pareto-efficiency can be achieved by imposing inefficient units less effort than that demanded by traditional efficiency measures, then it is at least of practical value to find the closest targets for each inefficient unit we can. Close targets in this sense are in line with the original spirit of DEA of showing each production unit in the best possible light. The idea of finding closest targets and peers has appeared in the literature both associated with oriented models (see e.g., Coelli, 1998; or Cherchye and Puyenbroeck, 2001a) and non-oriented models (see e.g., Frei and Harker, 1999; Golany et al., 1993). It is our intention to explore this issue for the most general case of non-oriented efficiency measures. In addition, we will restrict our analysis to technical efficiency. In this sense, we shall allow production units to move in all directions to improve their technical efficiency, as long as inputs are not increased and outputs are not decreased.

This paper analyses the issue of finding closest targets both in Free Disposal Hull (FDH) and in convex technologies. The next section introduces these production technologies and presents some existing non-oriented measures of efficiency. Section 2 discusses the concept of closeness of targets. Section 3 explains the approach developed to find closest targets for FDH technologies, while Section 4 presents the equivalent approach for convex technologies. In Section 5 these procedures are applied to a set of Portuguese bank branches, and Section 6 summarises the conclusions.

Non-Radial-Non-Oriented Measures of Efficiency

Consider a technology represented by $T = \{(\mathbf{x}, \mathbf{y}) \in R_+^{m+s} \mid \mathbf{x} \text{ can produce } \mathbf{y}\}$, where, for each unit $j(1, \dots, n)$, $\mathbf{x}_j = (x_{1j}, \dots, x_{mj}) \in R_+^m$ is an input vector producing an output vector $\mathbf{y_j} = (y_{1j}, \dots, y_{sj}) \in R_+^s$. This paper addresses the two production correspondences $T(\mathbf{x}, \mathbf{y})^{FDH}$ and $T(\mathbf{x}, \mathbf{y})^{BCC}$, which can both be specified by equation (1). When S equals $\{0, 1\}$, then T corresponds to an FDH technology ($T(\mathbf{x}, \mathbf{y})^{FDH}$) (Deprins et al., 1984), while when S equals $[0, +\infty[$, T corresponds to a BCC technology ($T(\mathbf{x}, \mathbf{y})^{BCC}$) (Banker et al., 1984).

$$T(\mathbf{x}, \mathbf{y}) = \left\{ (\mathbf{x}, \mathbf{y}) \in R_+^{m+s} \;\middle|\; \sum_{j=1}^{n} \lambda_j \mathbf{y_j} \geq \mathbf{y}, \sum_{j=1}^{n} \lambda_j \mathbf{x_j} \leq \mathbf{x}, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \in S, j = 1, \dots, n \right\} \quad (1)$$

$T(\mathbf{x}, \mathbf{y})^{FDH}$ assumes only free disposability of inputs and outputs being, therefore, a non-convex technology. $T(\mathbf{x}, \mathbf{y})^{BCC}$, on the other hand, is convex and assumes variable returns to scale (VRS). If the constraint normalizing the sum of lambdas was dropped in $T(\mathbf{x}, \mathbf{y})^{BCC}$ one would have a constant returns to scale (CRS) technology.

Each of the two production possibilities sets (PPS) above is bounded by a frontier, where target points are located. A number of measures exist to calculate the distance between observed points and target points. The ones in DEA are known as efficiency measures. Radial measures may find targets that, although lying on the frontier, are not on its Pareto-efficient subset. On the other hand, non-radial measures have the purpose of assuring that the identified targets lie on the Pareto-efficient subset of the frontier. Most of the studies that apply non-radial measures of efficiency use their oriented version (like the Färe-Lovell (see Färe and Lovell, 1978; Färe et al., 1985) or the Zieschang (1984) efficiency measures). Such studies can be found for example in Dervaux et al. (1998), Ruggiero and Bretschneider (1998), Kerstens and Vanden-Eeckaut (1995), De Borger and Kerstens (1996) or Cherchye and Puyenbroeck (2001b), both in the FDH context and in the context of convex frontiers.

The non-oriented DEA models in the literature share the common feature of maximising slacks. As a consequence, the targets these models identify are the furthest rather than the closest from each production unit being assessed. For some models, like the additive model of Charnes et al. (1985) or its variant the RAM (Range Adjusted Measure) as proposed by Cooper et al. (1999), this objective of slack maximisation is explicit in the objective function of the DEA models. See for example the objective function of the RAM model that is shown in (2), where slacks (normalised by the ranges) are being maximised. The traditional additive model simply maximises the sum of slacks, or alternatively, in one of its units invariant versions, it maximises slacks normalised by the observed input and output levels (see Charnes et al., 1985; Green et al., 1997).

$$\mathrm{RAM}_o = \min\left\{1 - \frac{1}{m+s}\left(\sum_{r=1}^{s}\left(\frac{s_{ro}}{R_r}\right) + \sum_{i=1}^{m}\left(\frac{e_{io}}{R_i}\right)\right)\right\},$$
$$\text{where } R_r = \max_j\{y_{rj}\} - \min_j\{y_{rj}\}, \quad R_i = \max_j\{x_{ij}\} - \min_j\{x_{ij}\} \qquad (2)$$

The model of Färe et al. (1985) defined in (3) in reference to T (which can be both $T(\mathbf{x}, \mathbf{y})^{FDH}$ or $T(\mathbf{x}, \mathbf{y})^{BCC}$) also maximises slacks, though this is not explicit in the objective function.

$$\mathrm{FGL}_o = \left\{\min \frac{\sum_{i=1}^{m} h_{io} + \sum_{r=1}^{s} 1/g_{ro}}{m+s} \;\middle|\; (h_{io}x_{io}, g_{ro}y_{ro}) \in T, g_{ro} \geq 1, 0 \leq h_{io} \leq 1\right\} \qquad (3)$$

This model is a generalisation of the hyperbolic measure of efficiency,[3] where inputs and outputs are allowed to change by different proportions. Using the relationship shown in (4) (see Cooper et al., 1999, for details)

$$h_i x_{io} = x_{io} - e_i \Leftrightarrow h_i = 1 - \frac{e_i}{x_{io}} \quad \text{and} \quad g_r y_{ro} = y_{ro} + s_r \Leftrightarrow g_r = 1 + \frac{s_r}{y_{ro}} \qquad (4)$$

it is possible to show that the objective function of (3) is equivalent to:

$$\frac{1}{m+s}\left(m - \sum_{i=1}^{m}\frac{e_i}{x_{io}} + \sum_{r=1}^{s}\frac{y_{ro}}{y_{ro}+s_r}\right) \approx 1 - \frac{1}{m+s}\left(\sum_{i=1}^{m}\frac{e_i}{x_{io}} + \sum_{r=1}^{s}\frac{s_r}{y_{ro}}\right) \qquad (5)$$

meaning that model (3) also maximises slacks.

The well known directional distance function introduced by Chambers et al. (1996, 1998) is also a non-oriented measure of efficiency that aims at maximising slacks. Indeed, it is defined as $\mathrm{Dir}_o = \{\max \beta_o \mid (x_{io} - \beta_o g_{x_i}, y_{ro} + \beta_o g_{y_r}) \in T\}$, where $\mathbf{g} = (-\mathbf{g_x}, \mathbf{g_y})$ is a directional vector chosen *a priori*. Dividing all inputs and outputs by the directional vector, reduces this measure to the maximisation of a normalised slack value. The directional model is, however, more restrictive than the measures referred to previously in the sense that it strongly limits the direction to be followed towards the production frontier. This means that an optimal solution to Dir_o will potentially result in targets that do not lie on the Pareto-efficient subset of the production frontier, as β cannot account for all the sources of inefficiency. Some references on the use of the above mentioned measures both in FDH and in convex technology settings can be found in De Borger and Kerstens (1996), Bardhan et al. (1996), or Cherchye et al. (2001).

The above mentioned measures will not be used in this paper for finding closest targets. Our objective is on the one hand to find an appropriate measure of efficiency and, on the other hand, to operationalise this measure so that closer targets can be found. For reasons that will be explained bellow, the above measures have some drawbacks in measuring efficiency in a non-oriented context.

An appropriate measure of efficiency in a non-oriented context should be capable of incorporating all the sources of inefficiency, while at the same time retaining the meaning of radial efficiency measures. The directional and hyperbolic measures do not satisfy the first requirement, while the RAM, additive model, and FGL model do not satisfy the second requirement. Before showing why this is so, we will present a measure that satisfies both requirements. This is the measure developed by Brockett et al. (1997), which will be referred to as BRWZ throughout this paper. This measure was originally developed to be used *a posteriori,* that is, after targets have been found, but it can also be used directly in any DEA model. The BRWZ efficiency measure is shown in (6).

$$\text{BRWZ}_o = \frac{1}{m}\left(\sum_{i=1}^{m}\frac{x_{io}-e_i^*}{x_{io}}\right)\times\frac{1}{s}\left(\sum_{r=1}^{s}\frac{y_{ro}}{y_{ro}+s_r^*}\right)$$
$$\Leftrightarrow \text{BRWZ}_o = \frac{\sum_{i=1}^{m} h_{io}\times\sum_{r=1}^{s} 1/g_{ro}}{m\times s} \qquad (6)$$

The first expression in (6) assumes that all inefficiencies are captured by additive slack values (e_i^* and s_r^*, where the star means an optimal value of the input and output slacks as resulting from the solution of some DEA model which projects on the Pareto-efficient boundary). The equivalent second expression (see relationships in (4)) for the BRWZ measure in (6) makes it possible to show its similarity to oriented measures under certain circumstances.[4] The multiplicative version of the BRWZ measure is similar to the FGL model in (3), but instead of adding the factors on the numerator and denominator it multiplies them. The multiplication of these factors makes the BRWZ measure closer to oriented efficiency measures. To illustrate this fact, Figure 1 presents two units in a single input/output space.

Unit A is inefficient and it can be projected on the CRS efficient boundary in three different ways. The input oriented efficiency measure (IO) of unit A is 45% which obviously equals its output oriented efficiency measure (OO). Let us assume that the non-oriented (NO) movement of unit A leads to point B. This means that the inputs of unit A should be contracted by 0.6 and outputs expanded by 1.33. With these values the BRWZ efficiency measure equals $0.6\times\frac{1}{1.33}=45\%$.[5] The FGL efficiency measure equals $\frac{0.6+\frac{1}{1.33}}{2}=67.5\%$, and the RAM measure equals $1-0.5(\frac{2}{2}+\frac{1}{1})=0\%$ if we assume that our sample consists only of A and B, or $1-0.5(\frac{2}{2.75}+\frac{1}{3.67})=50\%$ if we assume that our sample consists of A, B, A' and A''. If the target NO point was point A'' then one would expect non-oriented measures to coincide with the output oriented measure as the projection is the same. This coincidence only happens for the BRWZ efficiency measure which would still be 45%. The FGL efficiency measure would equal $\frac{1+0.45}{2}=72.5\%$, and the RAM would equal $1-0.5(0/2.75+3.67/3.67)=50\%$ or it would be negative if our sample consists only of units A and B.

This simple example shows that the BRWZ measure is indeed closer to the meaning of radial measures as it encompasses as special cases the Farrell radial input and output

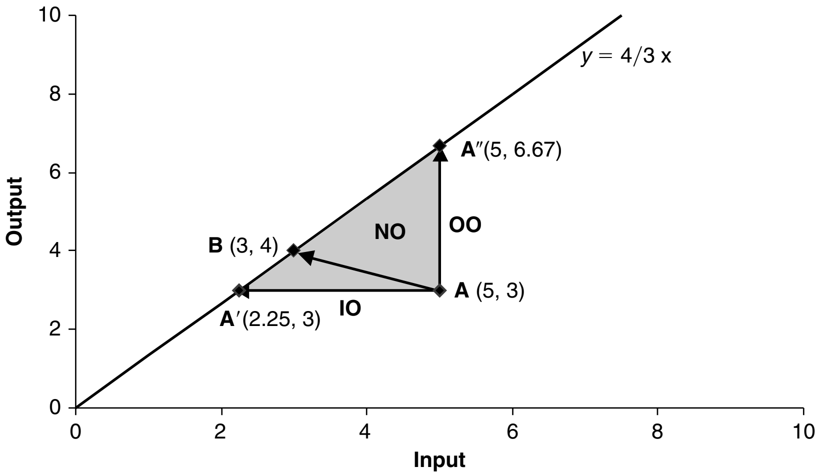

Figure 1. Single input output example.

oriented measures. For example, assuming that all inputs change equiproportionaly (and so each $h_i = \theta$) and that outputs are not allowed to change (and so each $g_r = 1$), the BRWZ measure reduces to θ, which coincides with the Farrell measure of input efficiency. In addition the BRWZ measure is units invariant which is a considerable advantage. The RAM, as Cooper et al. (2001) note, was defined in a VRS technology and should not be applied when CRS prevail. This constitutes an important limitation of this measure. Apart from this, the RAM measure has also the disadvantage of being very sensitive to the composition of the sample as we show above. The inclusion of one unit with an unusually small or large amount of one input (or output) could greatly change the results for many units (see also Steinmann and Zweifel, 2001).

One possible disadvantage of the BRWZ efficiency measure is that it weights equally all ratios of target to observed input or output level. Yet not all ratios reflecting short falls from target input-output levels may represent equal 'worth.' This is especially true in contexts where input and output prices are known and shares of inputs and outputs are substantially different between units. While we acknowledge this shortcoming of our measure of distance we do note that it is cast here in the framework of reflecting distance from a technically efficient boundary rather than from some value (cost or revenue) frontier. Distances from value frontiers and associated concepts of allocative efficiency are important but not being addressed in this paper.

Closer Targets and Efficiency

The objective of finding closest targets implies the definition of closeness. In general, one says that unit B is closer to A than to C, if in order to move from A to B, the changes required in inputs and outputs are smaller than the changes required in order to move from

A to C. Such changes can be expressed, for example, in terms of ratios of inputs and output levels at the two different points concerned. Thus the larger the ratios x^*/x and y/y^*, where the star denotes a target point, the closer the target (x^*, y^*) will be to the unit at (x, y). Obviously in a non-oriented space with multiple inputs and outputs one needs to choose a form of aggregating the above ratios. In our case, the BRWZ efficiency measure was chosen for this aggregation. Thus, the closer the target point to an observed point the higher the BRWZ efficiency as a measure of the distance between the two points.

The closeness between two points can also be measured using an L_p metric. Such metrics are not expressed in ratio form but in difference form. Therefore they have the disadvantage of not being units invariant. The L_p distance between two points (A and B) is given by $[\sum_{i=1}^{n} |A_i - B_i|^p]^{1/p}$. If A is an observed point and B is a target point on the Pareto-efficient frontier, then L_1 is simply the sum of slacks, as yielded by the additive DEA model. Most of the traditional efficiency measures can be related to L_p metrics as shown by Briec (1998).

We can illustrate concepts of closeness between points using a single input/output example as shown in Figure 2. Unit F is FDH and BCC inefficient. In the FDH case unit F is dominated by units B and C. Unit C is closer to F than is unit B. This can be seen in Table 1 where the BRWZ measure and some metric distances between points F and C, and F and B are presented. Clearly point B is the point that maximises the sum of slacks (see L_1 metric),

Table 1. Distance of F from points C, B, and (5, 5.33).

Point	BRWZ	L_1	L_2	L_∞
B	45%	3	$\sqrt{5}$	2
C	60%	2	$\sqrt{2.5}$	1.5
(5, 5.33)	56.25%	2.33	$\sqrt{5.44}$	2.33

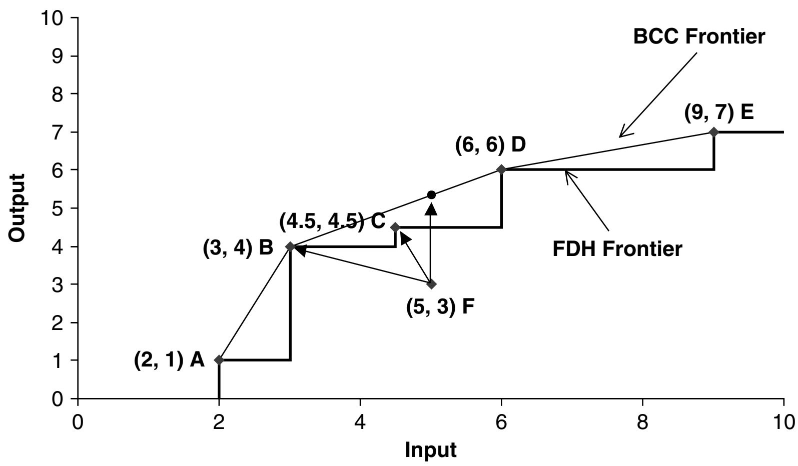

Figure 2. FDH frontier for a one input/output example.

meaning that the non-oriented models mentioned previously—additive, RAM, and FGL—identify point B as the target of unit F rather than point C. This happens both for the case of FDH and convex technology. In the convex context the closest point in terms of the BRWZ measure is point (5, 5.33)—a convex combination between points B and D. Table 1 shows that this point is closer to F than the target point B in terms of the BRWZ measure and in terms of the L_1 norm. As far as the other norms are concerned point B seems closer to point F than point (5, 5.33).

We favour the BRWZ measure because it is units invariant—a characteristic that is important when units of measurement are subjective. The example also shows that traditional DEA models do not necessarily provide targets that are as close as might be possible to the inefficient unit being assessed. As noted earlier, DEA models need to assure that units are projected on the Pareto-efficient frontier and for that purpose maximize slacks. This means that a single stage procedure using the BRWZ incorporated into a DEA structure (see model (7)) would not necessarily identify, in the same way as other DEA models, the closest targets.

$$\mathrm{BRWZ}_o = \min \left\{ \frac{\sum_{i=1}^{m} h_{io} \times \sum_{r=1}^{s} 1/g_{ro}}{m \times s} \;\middle|\; (h_{io}x_{io}, g_{ro}y_{ro}) \in T \right\} \tag{7}$$

Model (7) aims at minimising the BRWZ, because this is the only way to assure simultaneously that efficiency is measured and targets lying in the Pareto-efficient frontier are identified. However, these targets are not the closest, as the BRWZ efficiency resulting from (7) is not the maximum but the minimum (in the above example the solution of (7) identifies unit B as the target of unit F, making the BRWZ measure 45%). To find closest targets one needs to use multi-stage procedures so that we can maximise the objective function in (7), while at the same time assuring projection on the efficient frontier. The next two sections will provide means to achieve this, both in FDH technologies and in convex technologies.

Calculating Closer Targets in FDH Technologies

The interest in finding closer targets in relation to an FDH technology is twofold. First, the targets resulting from efficiency measurement in such a technology correspond to observable units, which might be desirable in some circumstances (for example when inputs and outputs are integer, or when it is likely that the production unit will be more comfortable comparing itself with a real unit rather then with a virtual one—farmers could be such a case). This characteristic makes FDH suitable for benchmarking purposes. Secondly, the non-convex nature of the FDH efficient frontier usually results in higher slack values than those obtained in convex technologies when the direction towards the frontier is restricted in some sense. As noted by De Borger and Kerstens (1996, p. 46) “empirical studies confirm that the amount of unmeasured technical efficiency or slacks is pervasive in FDH.” This is a sign that the use of efficiency measures that capture all the sources of inefficiency is potentially more important in FDH than in convex technologies.

The approach developed in this paper takes advantage of the fact that in FDH targets correspond to a single observed unit (peer), which simplifies their identification and the calculation of efficiency. Calculating efficiency requires first the knowledge of the set of dominating units for each dominated unit, and then the selection of the one (the closest) that should be used as the peer.

Our approach follows three steps (note that these steps are usually followed in practical applications of the FDH approach, although not necessarily using the same techniques nor the same criteria for finding peers):

Step I. Determine the set of non-dominated units (100% FDH efficient);

Step II. Determine a peer unit for each dominated unit;

Step III. Calculate the efficiency score.

Step I classifies all the units into one of two sets: *ND* or D. *ND* is the set of non-dominated (or dominating) units (units in relation to which no other unit exists presenting lower or equal inputs and higher or equal outputs) and D is the set of the remaining units, called dominated. Although this operation can be performed for each unit by comparing it with all the other units or with the current non-dominated set, such implementations become inefficient as the number of units grows. Techniques to handle dominant free sets are also relevant for multiple objective combinatorial optimization, where the state of the art implementations use structures like quad trees for drastically reducing the computational effort spent in such operations (Borges, 2000; Habenicht, 1982). In a quad tree representation of a dominant free set, each node represents a non-dominated unit and can have up to $(2^{s+m} - 2)$ branches, which are themselves also quad trees. Each one of those branches corresponds to a particular combination of inputs and outputs in order to guarantee that all units in a branch are dominated by the parent node only in exactly the same outputs and inputs. The discriminatory power of these structures, together with additional bounding techniques, makes them very efficient for handling domination relations and calculating L_∞ metrics. Our implementation uses an algorithm presented in Borges (2000), together with other well known quad tree algorithms to discriminate non-dominated units. These will also be used in step II, to find the units in *ND* that dominate the unit being assessed. Since these aspects are beyond the focus of the present paper, whose experiments could just as well have been performed using enumeration, we will not elaborate on them here, for the sake of conciseness.

Step II finds a peer unit for each inefficient or dominated unit. In order to find this unit, we consider a subset of *ND*, named K, consisting of the units that dominate the unit being assessed. For each inefficient unit, its closest peer is determined through the BRWZ efficiency criterion, that is, calculating (8) for every unit $k \in K$.

$$\text{Peer of unit } o = \max_k \left(\frac{\sum_{i=1}^{m} x_{ik}/x_{io}}{m} \times \frac{\sum_{r=1}^{s} y_{ro}/y_{rk}}{s} \right) \tag{8}$$

Step III generates the efficiency of the unit being assessed in reference to the peer unit identified in the previous step. The measure of efficiency is given directly from the value obtained in (8). Therefore, steps II and III take place simultaneously.

The Case of Convex Frontiers

Extending the above procedure to convex boundaries is not straightforward because in this case target points may not correspond to observed units but to convex combinations of efficient units. This means that an enumeration oriented procedure which calculates the BRWZ measure for a set of potential target points can no longer be applied. The analogous approach to follow in the case of convex frontiers is to use model (7) but with a modified objective function so that the BRWZ is maximised instead of minimised.

Golany et al. (1993) and Frei and Harker (1999) also used DEA models for the case of convex frontiers where the objective function was the minimisation of slacks rather than their maximisation. In the first case, the authors minimised L_1 and L_2 norms in the non-oriented space, and in the second case the authors used L_2 norms for finding the closest targets for each unit, also in the non-oriented space. A concern we share with these authors, is that when distances as the above are being minimised (instead of maximised as is usual in DEA models) it is important to assure that the projected point lies on the Pareto-efficient boundary. For this purpose it is necessary to identify the efficient facets, or at least to have some knowledge about which units belong to which facet. Both, Golany et al. (1993) and Frei and Harker (1999) used the multiplier form of the additive model for the purpose of identifying these facets in the spirit of Ali and Seiford (1993, p. 130) (see also Huang et al., 1997; and Yu et al., 1996 who used similar procedures for determining efficient facets). The problem with the use of the additive model is that it does not assure the complete characterisation of the efficient facets. Take the example in Ali and Seiford (1993, p. 122), where solving the multiplicative additive model for units 1, 4 and 7 (belonging to the same facet) results in 2 hyperplanes: one spanning through units 1 and 4, and the other through units 4 and 7. The full dimension efficient facet (1, 4, 7) is not identified (see also Olesen and Petersen, 1996, who discuss full dimension and non-full dimension efficient facets). This is a direct result of the existence of multiple optimal solutions, which poses a problem in the identification of all efficient facets. Our approach departs from that of Golany et al. (1993) and Frei and Harker (1999) in the procedure used for identifying efficient facets, and also in the distance measure used. The latter is the units invariant BRWZ measure.

In order to identify efficient facets we recommend the use of the procedure proposed by Olesen and Petersen (2001): QHull (see Barber et al., 1996). QHull is a freely available software that is designed to identify all full dimension efficient facets (FDEF) in a DEA model. Each facet is identified in terms of the convex hull of the Pareto-efficient DMUs whose input-output levels span the facet. The procedure also identifies a supporting hyperplane equation for each facet (for details on the principles behind QHull see Olesen and Petersen, 2001, pp. 27–30). The procedure can also be modified to identify non-full dimensional efficient facets (private correspondence with Olesen). This involves the use of an augmented data set using artificial DMUs in addition to those observed. The details are beyond the scope of this paper.

Our procedure for finding the closest targets in convex technologies consists of three steps:

Step I. Determine the set of Pareto-efficient units (E) by solving the additive model;

Step II. Determine all Pareto-efficient facets (F_k) using QHull;

Step III. For each F_k, $k = 1, \ldots, K$ solve model (9) to find the closest targets for inefficient unit o.

$$\max \left\{ \text{BRWZ}_o = \frac{\sum_{i=1}^{m} h_{io} \times \sum_{r=1}^{s} 1/g_{ro}}{m \times s} \;\middle|\; \sum_{j \in F_k} \lambda_j y_{rj} = g_{ro} y_{ro}, \sum_{j \in F_k} \lambda_j x_{ij} = h_{io} x_{io}, \right.$$
$$\left. \sum_{j \in F_k} \lambda_j = 1, \lambda_j \geq 0, g_{ro} \geq 1, 0 \leq h_{io} \leq 1 \right\} \tag{9}$$

In order to assure projection of the efficient frontier only points on F_k are considered as potential projections of unit o in (9). The final BRWZ efficiency measure of unit o is the maximum value found for the measure after model (9) is solved for all K facets. Step III is repeated for each inefficient unit for which we wish to identify the closest targets. We can also formulate (9) as a single mixed integer linear program that should be solved only once for DMU_o in respect of all facets K, but in the interest of brevity we omit this formulation.

The foregoing procedure has been developed in a VRS context. It is, however, equally applicable to a CRS context. Model (9) will change in that the convexity constraint will be dropped. Further, the Pareto-efficient units and the efficient facets will change as we move from a VRS to a CRS technology.

Model (9) is non-linear and is not easily linearised. Nevertheless, there are several solvers that can handle non-linear models, whose constraints are linear. We used GAMS and its non-linear programming solver (CONOPT). Nevertheless, for computational convenience a model minimising the sum of normalised slacks, such as that in (10), could be used instead.

$$\min \left\{ \sum_{r=1}^{s} \gamma_{ro} + \sum_{i=1}^{m} \beta_{io} \;\middle|\; \sum_{j \in F_k} \lambda_j y_{rj} = y_{ro} + \gamma_{ro} y_{ro}, \sum_{j \in F_k} \lambda_j x_{ij} = x_{io} - \beta_{io} x_{io}, \right.$$
$$\left. \sum_{j \in F_k}^{n} \lambda_j = 1, \lambda_j \geq 0 \right\} \tag{10}$$

Such a model, although not equivalent to model (9), would likely result in similar targets as a normalised L_1 norm is being minimised. Note that this model is a generalisation of the directional distance function, that assumes a directional vector equal to the observed input and output vectors, and different expansion and contraction factors associated with each input and output. There is one important difference between (10) and directional distance function or additive DEA models. The contraction of inputs and expansion of outputs is minimised rather than maximised in (10) and this is only made possible by the constraints that ensure the projection point to lie on an efficient facet. The model in (10) is also similar to the preference model introduced by Thanassoulis and Dyson (1992). If there are any preferences for moving towards the frontier these can be incorporated in the model in (10) as detailed in Thanassoulis and Dyson (1992).

Illustrative Application to Bank Branches

This section applies the above procedures to a sample of 24 Portuguese bank branches which are located in mid-sized cities (as classified by the bank) in the northern region of Portugal. We use an intermediation approach of banking activities as this requires in principle non-oriented models. In this sense on the input side cost related variables are used (staff costs and other operating costs), and on the output side revenue related variables are used (value of current accounts, value of credit, and interest revenues).[6] We assume that all inputs and outputs are discretionary. The data correspond to the month of July 2001 and values are expressed in thousands of Euros. Our input-output set here is only illustrative. Table A1 in Appendix A contains the data used, as well as some descriptive statistics. The units that were identified as efficient both under FDH and under a convex VRS technology are also identified in Appendix A (see Table A1). Here we will only detail on the results of some inefficient units.

For the FDH case, the application of the additive units invariant model, the RAM model, and the FGL model result in the same peers for inefficient units in all the cases. This is illustrated in Table 2 which shows the BRWZ measure calculated *a posteriori* in relation to the targets identified by these models. It also shows the BRWZ efficiency measure obtained under our closer target (CT) FDH procedure. The BRWZ measure has the same value under all the procedures for identifying targets, except in two cases. The reason for this is simple: unit B10 dominates most of the units in the sample and most of them are solely dominated by this unit. As the set of potential referents consists of a single unit there is not much for the alternative procedures to choose. Only in two cases is there a genuine choice of targets to be made: the case of inefficient units B19 and B22. The first unit is dominated by B10 and also by B20, and the second unit is dominated by B10, B26, B50 and B52. The application of our CT procedure clearly identifies closer targets to units B19 and B22 (B20 and B52, respectively) as testifies a higher value of the CT BRWZ efficiency score in

Table 2. Results from additive-FDH, RAM-FDH, FGL-FDH and CT procedure.

Unit	Peer Unit	BRWZ Efficiency	BRWZ CT Efficiency
Unit B3	B10	67.02%	67.02%
Unit B5	B10	77.26%	77.26%
Unit B9	B16	64.70%	64.70%
Unit B13	B10	74.85%	74.85%
Unit B15	B10	53.57%	53.57%
Unit B19	B10	68.15%	81.30% (B20)
Unit B21	B10	71.87%	71.87%
Unit B22	B10	52.76%	78.00% (B52)
Unit B59	B10	74.00%	74.00%

Table 3. Comparison between models based on L_p metrics.

Unit	Additive, RAM, and FGL				CT FDH Procedure			
	Peer	L_1	L_2	L_∞	Peer	L_1	L_2	L_∞
Unit B19	Unit B10	4044.94	2920.62	2468.72	Unit B20	800.05	602.98	554.51
Unit B22	Unit B10	6514.76	5367.91	5213.83	Unit B52	1355.58	1004.59	900.77

Table 2. These higher efficiency scores also correspond to lower metric distances as can be seen in Table 3.

The above example shows that easier-to-achieve targets can be provided to some bank branches, showing them in a better light. If we relied on the traditional models to establish targets we would advise branch B22 to reduce (in thousands of Euros) its staff costs by 6.06 and its other operating costs by 2.96, while at the same time increasing the value of deposit accounts by 1276.8, the value of credit by 5213.83, and its interest revenues by 15.12. Such targets are more demanding than the alternative, which also renders B22 efficient, and corresponds to decreasing staff costs only by 2.02 and operating costs by 3.77, while increasing the value of current accounts by 444.73, the value of credit by 900.77, and interest revenues by only 4.29. Only for the case of other operating costs is the target more demanding in this second case, a fact that is more than compensated for by the much less demanding targets in the remaining variables.

In the convex VRS technology case, the application of the CT procedure to the bank branches example results (in its first step) in a set of efficient units that is shown in the last column of Table A1 in Appendix A. After obtaining the set of efficient units QHull was used to identify the set of efficient facets. These are: $F_1 = \{B10, B16, B20, B29, B50\}$; $F_2 = \{B20, B27, B29, B50, B57\}$; $F_3 = \{B10, B20, B27, B29, B50\}$; $F_4 = \{B10, B27, B56, B57\}$; $F_5 = \{B10, B11, B16, B29\}$; $F_6 = \{B10, B11, B26, B29\}$; $F_7 = \{B10, B26, B27, B29\}$, where the first three facets are full dimensional and the last four are not. In the third step, model (9) was applied to each inefficient unit in relation to each efficient facet. The facet chosen for projection in each case was the one maximising the objective function of model (9). Note that in some cases projection on some facets might be infeasible, but at least one facet shall result in a feasible solution.

The detailed results of applying the additive units invariant model, the RAM model and our CT convex procedure are shown in Appendix B.[7] The results in terms of the various BRWZ efficiency measures show that *BRWZ CT procedure* $\geq$ *BRWZ Additive* $\geq$ *BRWZ RAM*. We will sidestep the discussion of the relationship between the additive and RAM measures since is not the aim of this paper to analyse it. Concerning the results from our model, they confirm that it shows each inefficient unit in a much better light than the other two models not only in terms of the BRWZ measure but also in terms of L_p metric measures. Take for example units B15 and B59 shown in Table 4.

Results for these units show closer targets identified by the CT convex procedure than those identified by the additive model (the same being true for the RAM model). This fact is expressed in higher BRWZ efficiency scores and smaller L_p metrics, as illustrated for

Table 4. Distance to targets for inefficient units for the VRS case.

	B15			B59		
	Observed	Targets Additive	Targets CT Convex	Observed	Targets Additive	Targets CT Convex
x_1	11.717	11.717	11.487	13.338	13.338	12.606
x_2	29.314	24.726	16.122	24.820	24.820	19.030
y_1	4070.630	5682.936	4070.630	4354.301	6073.258	4475.281
y_2	6418.995	14409.226	6418.995	10889.840	14368.013	10889.840
y_3	40.328	69.268	45.086	57.033	74.865	57.033
L_1		9636.066	18.181		5214.962	127.502
L_2		8151.330	14.027		3879.796	121.121
L_∞		7990.231	13.193		3478.173	120.980
BRWZ		53.58%	73.83%		74.56%	84.82%

the two cases above (this fact can however be generalised to the entire sample of units). Interestingly the additive model tends to identify most of the inefficiencies associated with outputs, while the CT procedure for convex technologies identifies most of the inefficiencies associated with inputs. For the additive model the average BRWZ-input efficiency is 98.27% and the average BRWZ-output efficiency is 73.36%, while the corresponding values for the CT procedure are 90.72% and 92.03%, respectively (the RAM model also identifies most of the inefficiencies associated with outputs but to a lesser extent than the additive model: BRWZ-input efficiency is 93.67% and BRWZ-output efficiency is 75.89%). This clearly indicates that our procedure and the additive model identify different directions for improvement of inefficient units. The choice of the model to use should not, thus, be taken lightly.

As a final note on this example one can observe that BRWZ efficiency scores are higher for the convex than for the FDH case, for units that are inefficient under both technologies. The typical result in pure radial models is precisely the reverse because, as it is well known, FDH closely envelops the data and thus provides higher efficiency measures. In our example the closer envelopment resulted in more efficient units for the FDH case but not in higher BRWZ efficiency scores for inefficient units. Note that the range of targets in FDH is limited to observed units, while in the convex case this range is greatly expanded through convex combinations of Pareto-efficient units. This means that we can actually find closer targets in the convex case than in the FDH case, when we are not restricted to move in any direction and when the measure of efficiency used captures all the sources of inefficiency (that is, when it restricts targets to lie on the Pareto-efficient subset of the frontier).

Conclusion

The analysis of non-oriented measures of efficiency and their use to identify the closest targets for inefficient units was performed both considering FDH technologies and convex technologies. The chosen criterion of closeness is based on the maximisation of the BRWZ efficiency measure, which has the advantage over other efficiency measures of capturing all the sources of inefficiency and retaining a meaning that is close to that associated with oriented efficiency measures. In order to use this measure multi-stage procedures are required both in the FDH and in the convex case to find the closest targets. As our analysis restricts targets to lie on the Pareto-efficient subset of the production frontier, the multi-stage FDH procedure starts by choosing potential target units and then it takes the one maximising the BRWZ efficiency measure as the adopted target and peer. In the convex case the aim is also the maximisation of the BRWZ efficiency measure, which results in a non linear programming model, that requires knowledge on the efficient facets of the PPS. The application of our procedure to a real bank branch example shows that it provides closer and easier-to-achieve targets in both, the FDH and convex, cases.

Appendix A

Table A1. Bank branches data.

Unit	Staff Costs	Other Operating Costs	Current Accounts	Credit	Interest Revenue	FDH Eff.	VRS Eff.
B3	16.819	24.471	4892.629	10238.760	52.234		
B5	11.243	23.558	4777.107	8756.227	52.449		
B9	18.441	35.090	6450.385	12479.115	64.644		
B10	10.106	23.104	5223.611	12572.231	61.332	100%	100%
B11	15.129	32.781	7666.449	10221.426	67.682	100%	100%
B13	12.979	23.658	4991.984	10194.377	48.583		
B15	11.717	29.314	4070.630	6418.995	40.328		
B16	18.306	31.359	7561.477	21922.138	101.725	100%	100%
B17	16.505	31.574	6322.393	17323.595	81.404	100%	
B19	12.211	24.411	3663.067	10103.516	49.062		
B20	11.981	17.857	3899.831	10658.024	51.052	100%	100%
B21	12.689	25.489	4797.797	10281.063	48.822		
B22	16.166	26.062	3946.813	7358.401	46.214		
B26	12.041	19.688	5524.905	7393.716	48.912	100%	100%
B27	10.021	16.780	3394.509	8269.236	39.565	100%	100%
B29	12.739	18.505	5635.758	6667.397	63.048	100%	100%
B50	12.505	17.508	4745.698	9603.156	48.199	100%	100%
B51	15.178	21.418	5758.861	6007.936	64.210	100%	
B52	14.146	22.291	4391.541	8259.170	50.503	100%	
B53	12.959	20.117	5372.053	7323.490	64.076	100%	
B56	9.073	19.259	2888.434	8694.691	39.974	100%	100%
B57	9.747	13.004	2107.062	5012.420	24.202	100%	100%
B58	10.639	22.566	3344.774	10293.887	43.311	100%	
B59	13.338	24.820	4354.301	10889.840	57.033		
Average	13.195	23.529	4824.253	9872.617	54.523		
Max	18.441	35.090	7666.449	21922.138	101.725		
Min	9.073	13.004	2107.062	5012.420	24.202		
Stdev	2.646	5.494	1356.082	3640.361	15.456		

Appendix B

Table B1. Results from Additive Units Invariant Model (values are rounded).

Unit	Peers	Slacks (e_1, e_2, s_1, s_2, s_3)	BRWZ Efficiency
B3	B16(0.503), B50(0.497)	(1.39, 0, 1268.6, 5557.1, 22.87)	68.30%
B5	B10(0.80), B16(0.113), B50(0.086)	(0, 0, 670.53, 4620.72, 12.33)	78.04%
B9	B16	(0.135, 3.73, 1111.09, 9443.02, 37.08)	64.70%
B13	B10(0.431), B16(0.247), B29(0.323)	(0, 0, 941.5, 2780.4, 23.3)	76.77%
B15	B10(0.804), B16(0.196)	(0, 4.6, 1612.31, 7990.23, 28.94)	53.58%
B17	B10(0.22), B16(0.78)	(0, 2.03, 725.72, 2545.43, 11.45)	85.36%
B19	B10(0.67), B16(0.22), B29(0.11)	(0, 0, 2122.7, 3870.92, 21.38)	68.42%
B21	B10(0.66), B16(0.31), B29(0.03)	(0, 0, 1152.3, 4970.8, 24.9)	71.42%
B22	B16(0.62), B50(0.38)	(0.078, 0, 2537.8, 9852.6, 35.04)	53.37%
B51	B16(0.26), B29(0.32), B50(0.42)	(1.09, 0, 0, 5859.3, 2.6)	79.29%
B52	B10(0.077), B16(0.314), B50(0.61)	(0, 0, 1276.16, 5444.8, 15.5)	71.41%
B53	B10(0.078), B16(0.11), B20(0.25), B29(0.55), B50(0.006)	(0, 0, 0, 2521.204, 0)	91.46%
B58	B10(0.83), B16(0.02), B29(0.15)	(0, 0, 1980.7, 1564.91, 18.98)	73.05%
B59	B10(0.462), B16(0.326), B29(0.212)	(0, 0, 1718.96, 3478.2, 17.84)	74.56%

Table B2. Results from RAM Model (values are rounded).

Unit	Peers	Slacks (e_1, e_2, s_1, s_2, s_3)	BRWZ Efficiency
B3	B10(0.83), B16(0.17)	(5.36, 0, 718.04, 3881.43, 15.79)	66.28%
B5	B10(0.945), B16(0.055)	(0.69, 0, 575.05, 4330.11, 11.11)	77.14%
B9	B16	(0.13, 3.73, 1111.1, 9443.02, 37.08)	64.70%
B13	B10(0.933), B16(0.067)	(2.32, 0, 388.43, 3004.95, 15.5)	74.62%
B15	B10(0.804), B16(0.196)	(0, 4.6, 1612.31, 7990.23, 28.94)	53.58%
B17	B10(0.22), B16(0.78)	(0, 2.03, 725.72, 2545.43, 11.45)	85.36%
B19	B10(0.842), B16(0.158)	(0.81, 0, 1930.65, 3948.88, 18.66)	67.63%
B21	B10(0.71), B16(0.29)	(0.22, 0, 1101.04, 4991.63, 24.18)	71.23%
B22	B10(0.642), B16(0.358)	(3.12, 0, 2114.5, 8563.9, 29.6)	51.89%
B51	B10(0.285), B16(0.125), B29(0.59)	(2.5, 0, 0, 4243.7, 3.18)	77.68%
B52	B10(0.823), B29(0.177)	(3.6, 0, 904.9, 3269.15, 11.13)	68.87%
B53	B10(0.25), B16(0.04), B29(0.72)	(0.66, 0, 234.62, 1365.72, 0)	91.00%
B58	B10(0.883), B29(0.117)	(0.225, 0, 1927.1, 1586.7, 18.22)	72.71%
B59	B10(0.79), B16(0.21)	(1.53, 0, 1355.25, 3625.82, 12.7)	73.24%

Table B3. Results from CT procedure (values are rounded).

Unit	Facet/Peers	$(h_1, h_2, g_1, g_2, g_3)$	BRWZ Efficiency
B3	F_6/B10(0.41), B11(0.26), B26(0.33)	(0.72, 1, 1.22, 1, 1.13)	77.50%
B5	F_7/B10(0.25), B26(0.36), B27(0.21), B29(0.17)	(1, 0.84, 1.05, 1, 1)	90.42%
B9	F_5/B11(0.81), B16(0.19)	(0.85, 0.93, 1.185, 1, 1.15)	80.52%
B13	F_1/B16(0.06), B29(0.07), B50(0.86)	(0.994, 0.78, 1, 1, 1.085)	86.42%
B15	F_2/B29(0.45), B50(0.14), B57(0.41)	(0.98, 0.55, 1, 1, 1.118)	73.83%
B17	F_5/B10(0.1), B11(0.32), B16(0.59)	(1, 0.98, 1.165, 1, 1.07)	92.27%
B19	F_2/B20(0.9), B29(0.02), B57(0.08)	(0.967, 0.715, 1.0337, 1, 1)	83.22%
B21	F_6/B10(0.37), B11(0.35), B26(0.28)	(0.98, 1, 1.28, 1, 1.23)	85.45%
B22	F_6/B11(0.49), B26(0.51)	(0.84, 1, 1.664, 1.2, 1.26)	68.49%
B51	F_5/B11(0.19), B16(0.01), B29(0.8)	(0.87, 1, 1.05, 1.24, 1)	86.03%
B52	F_6/B10(0.07), B11(0.18), B26(0.75)	(0.88, 1, 1.34, 1, 1.053)	84.52%
B53	F_5/B10(0.03), B11(0.09), B16(0.02), B29(0.86)	(1, 1, 1.086, 1.015, 1)	96.87%
B58	F_4/B10(0.41), B56(0.59)	(0.89, 0.924, 1.15, 1, 1.126)	83.45%
B59	F_1/B16(0.08), B20(0.76), B29(0.16)	(0.945, 0.767, 1.0278, 1, 1)	84.82%

Notes

1. Pareto-Koopmans, technical efficiency is attained when an increase in any output (or a decrease in any input) requires a decrease in at least another output (or an increase in at least another input) (see e.g., Lovell, 1993).
2. Profit analysis has been recently advocated in the context of measuring efficiency in banking (see Berger et al., 1993), a field where cost oriented efficiency analysis has been the dominating approach.
3. Assuming that each h_{io} is constant and equal to θ, and that each g_{ro} is also constant and equal to $\frac{1}{\theta}$ reduces (3) to the hyperbolic efficiency measure.
4. The BRWZ measure is very similar to the slack based measure (SBM) of Tone (see Tone, 1993; and Tone, 2001). The SBM equals $(\frac{1}{s}\sum_{r=1}^{s}(\frac{y_{ro}+s_r}{y_{ro}}))^{-1} \times (\frac{1}{m}\sum_{i=1}^{m}(\frac{x_{io}-e_i}{x_{io}}))$, which is equivalent to $\sum_{i=1}^{m} h_i/m \times s/\sum_{r=1}^{s} g_r$, when the slacks are replaced by multiplying factors. We prefer the BRWZ to the SBM because the former uses an arithmetic mean of the input efficiency h_i and an arithmetic mean of the output efficiency $1/g_r$. The SBM uses an harmonic mean of the output efficiency whose rationale is not easy to understand.
5. It is easy to demonstrate that under constant returns to scale the BRWZ measure will provide the same efficiency measure (45%) for all non-oriented efficient targets in the line between A' and A'' in Figure 1. Replacing a point in the line segment between A' and A'' by (x, y), and knowing that the line passing through point A′ and A″ is $y = \frac{y_{A''}}{x_{A''}}x = \frac{y_{A'}}{x_{A'}}x$, then we have $\text{BRWZ}_A = \frac{x}{x_A} \times \frac{y_A}{y} = \frac{x}{x_A} \times \frac{y_A}{\frac{y_{A''}}{x_{A''}}x} = \frac{y_A}{x_A} \times \frac{x_{A''}}{y_{A''}} = \frac{y_A}{y_{A''}}$ because the input at point A and A'' is equal.
6. The interest revenues are net of interest costs and this is the reason why these are not considered on the input side. The bank could not provide interest costs and revenues disaggregated.
7. All results reported concern the use of model (9). Model (10) was also used and results are equal except for 4 units (B5, B9, B22, B53). In all the cases except B22 the facet of projection was the same both using model (9) and model (10). Obviously the BRWZ is maximum when model (9) is used.

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