



Exploring Risk Dynamics in Corporate Bonds: A Study of Effective and Macaulay Durations

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Dissertation written under the supervision of
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Dissertation submitted in partial fulfillment of requirements for the
MSc in Finance at the **Universidade Católica Portuguesa**

June 2023

Abstract

This dissertation examines the effect of firms' credit risk on interest rate risk using two real-world corporate bond portfolios in December 2022, one with low credit risk and the other with high credit risk. This is done by comparing the Macaulay duration with the effective duration resulting from the application of Leland and Toft (1996) structural credit risk model. It was found on average a Macaulay duration of 4.24 years versus an effective duration of 2.92 years for the high-risk portfolio, while the low-risk portfolio displayed a Macaulay duration of 9.04 years in contrast to an effective duration of 7.47 years. The difference between the Macaulay and effective durations highlights the limitations of traditional risk management techniques, such as immunization, which often do not take into account the relation between interest rates and credit risk.

Keywords: corporate bonds, interest rate risk, credit risk, structural models, Macaulay duration, duration, Leland and Toft (1996) model, bond price sensitivity, risk management.

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Resumo

Esta dissertação examina o efeito do risco de crédito das empresas sobre o risco da taxa de juro, utilizando dois portfólios de títulos corporativos reais em dezembro de 2022, um com baixo risco de crédito e outro com alto risco de crédito. Isto é conseguido comparando a duração de Macaulay com a duração efetiva resultante da aplicação do modelo de risco de crédito estrutural de Leland e Toft (1996). Os nossos resultados de pesquisa revelaram uma duração média de Macaulay de 4,24 anos contra uma duração efetiva de 2,92 anos para o portfólio de alto risco, enquanto o portfólio de baixo risco apresentou uma duração de Macaulay de 9,04 anos em contraste com uma duração efetiva de 7,47 anos. Esta diferença entre as durações de Macaulay e efetiva destaca as limitações das técnicas tradicionais de gestão de risco, como a imunização, que muitas vezes não levam em consideração a relação entre as taxas de juro e o risco de crédito.

Palavras-chave: títulos corporativos, risco da taxa de juro, risco de crédito, modelos estruturais, duração de Macaulay, duração, modelo de Leland e Toft (1996), sensibilidade do preço do título, gestão de risco.

Título: Exploração da Dinâmica do Risco em Títulos Corporativos: Um Estudo sobre Duração Efetiva e Duração de Macaulay

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Acknowledgments

Today I am completing a journey that has been both challenging and rewarding, made possible by the support, guidance, and contributions of many.

Foremost, I wish to extend a heartfelt gratitude to the city of Rio de Janeiro, my home throughout most of this journey. Its vibrant culture and entrancing beauty were both a captivating distraction and a profound source of inspiration, thereby contributing to the extended timeline of this dissertation. Yet, in retrospection, I wouldn't alter a single detail; the experience was inimitably enriching.

To my cherished circle of friends, who have been the soundboards for my frustrations, I offer my sincere thanks. A special mention is due here for my friend, Mariana. Her companionship and solidarity as we navigated simultaneous deadlines were invaluable. The shared experiences and sometimes struggles have fortified our bond, and I am very grateful for her presence.

At the core of this academic journey is my supervisor, Nuno Silva. I can say, unequivocally, that his attention, availability, and expertise have been instrumental in the completion of this dissertation. Had it not been for his continuous support, this journey might have extended well into 2025. His mentorship has been an invaluable asset, for which I am profoundly grateful.

Lastly, in the spirit of self-recognition, in the style of the artist Anitta, I would like to thank myself. This journey has been a testament to my resilience, dedication, and intellectual curiosity, and I am proud of the strides I've made.

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1. Introduction

The corporate bond market is a critical source of financing for businesses worldwide, providing trillions of dollars in debt issuance annually. It is an engine that fuels the growth and stability of various industries. This market has evolved over time, with various types of bonds available to investors. Given its importance, the measurement and management of risk associated with corporate bonds is a topic of significant interest and study in finance, attracting a mass of scholarly attention.

Interest rate risk is one of the primary risks associated with corporate bonds. It is a complex but crucial aspect of bond investment, driving significant shifts in market dynamics. When interest rates rise, bond prices usually decline because newly issued bonds offer higher yields, making existing bonds with lower yields less attractive to investors. The most commonly used measure of interest rate risk is duration, which captures the sensitivity of a bond's price to changes in interest rates. Academics have proposed different ways to compute duration. The simplest one is called Macaulay duration which measures the number of years it takes for the investor to recover the bond's price from its cash flows. This measure, while simplistic, provides a fundamental understanding of the dynamics of bond investment. Based on Macaulay duration, bonds with longer maturities generally have higher duration and are more sensitive to interest rate changes. While Macaulay duration takes only into account when each payment is made, other duration measures like effective duration, capture other factors such as call features, prepayment risk and credit risk. The latter refers to the risk that the issuer of a bond defaults on its debt obligations and is typically measured by the probability of default (PD), which is the likelihood that the issuer will be unable to meet its debt obligations, and the loss-given default (LGD), which is the amount of money that investors would lose if the issuer were to default.

One interesting question is whether interest rate risk and credit risk interact with each other. Whenever an increase in interest rates leads to a decrease (increase) in credit risk one might have that credit risk mitigates (amplifies) interest rate risk. In this case, effective duration may become far away from Macaulay duration. One way to examine this question is through the lens of a structural credit risk model, which incorporates both interest rate and credit risk into a single framework. The Leland and Toft (1996) model is a popular structural credit risk model that describes the dynamics of a firm's asset value and default probability over time. According to this model, higher interest rates lead to lower credit risk, which mitigates the

negative impact associated with the higher opportunity cost. As a result of this mechanism, bonds with high credit risk have a lower effective duration. This phenomenon is important to study because it has significant implications for the risk management of corporate bonds.

The goal of this dissertation is to illustrate this mechanism by applying the Leland and Toft (1996) model to two real corporate bond portfolios, one with low risk and the other with high risk, and comparing the Macaulay duration of both portfolios with the duration resulting from the Leland and Toft (1996) model in a certain moment of time. By analyzing these portfolios, we aim to demonstrate how the Leland and Toft (1996) model captures the relationship between interest rates and credit risk and how it can contribute to better risk management strategies for corporate bonds.

2. Literature Review

2.1. Interest Rates and Bond Prices: Empirical Evidence

The empirical relationship between risk-free interest rates and bond prices has been a central topic in finance research for decades. The theoretical expectation is that as risk-free interest rates rise, bond prices go down implying an increase in implicit bond yields. Intuitively, bond prices must go down because bond purchasers must be compensated accordingly to the new higher opportunity cost of capital. However, empirical evidence has suggested a more complex picture. In particular, some studies have found that the relationship between risk-free interest rates and bond prices may depend on firm-specific factors. This dissertation focus on one of these factors, notably, the firms' credit risk.

Chance (1990) provides empirical evidence on the relationship between risk-free interest rates, bond prices, and default risk. In this article, Chance highlights the importance of considering default risk when analyzing the sensitivity of bond prices to changes in interest rates, typically measured using duration-type measures. He argues that traditional measures of duration may not accurately reflect the sensitivity of these bonds to changes in interest rates due to the potential impact of default. The author presents empirical evidence that supports his argument, by examining the relationship between duration and the level of the risk-free interest rate, with a focus on zero coupon bonds. His work suggests that duration tends to increase with the level of the risk-free interest rate, which is consistent with the behaviour of investment-grade bonds. However, duration increases at a much lower rate for short-maturity bonds, whereas the duration of all bonds exhibits a tendency to asymptotically approach the maturity. By relating duration to the level of the risk-free interest rate, the results show that the duration of zero coupon bonds is influenced by the level of the interest rate, which in turn impacts the sensitivity of bond prices to interest rate changes. Additionally, one of the main conclusions of Chance (1990) is that high-yield bonds have durations lower than their maturities and, thus, are less sensitive to interest rate changes than investment-grade bonds.

Babbel et al. (1997) examine the impact of default risk on the effective duration of bonds. By using the yield on a comparable risk-free security, the authors are able to create a consistent basis for evaluating the effective duration of corporate bonds and for comparing the interest rate sensitivity of bonds with different credit ratings. This approach allows for greater

confidence in their duration measures, as it enables them to overcome the limitations of other models that may rely on different reference rates, and it ensures that the estimates of the impact of credit risk on effective duration are more accurate and comparable across different bond issuers and credit ratings. To illustrate the impact of credit risk on effective duration, Babbel et al. (1997) compare the effective duration of a BBB-rated corporate bond with a 10-year time to maturity to the effective duration of an AAA-rated corporate bond with a 10-year time to maturity. The authors estimate that the effective duration of the BBB-rated bond is around 8.1, while the effective duration of the AAA-rated bond is around 9.4. This suggests that credit risk shortens the effective duration of corporate bonds, as the lower-rated bond is less sensitive to changes in interest rates.

Fooladi et al. (1997) argue that the fact that research has focused mostly on default-free bonds is an issue for portfolio managers, who hold significant amounts of bonds with default risk, including 'high yield' or junk bonds and often approximate duration by pretending that all bonds are default-free, something that produces significant errors in practice. To address this, they derive a general expression for the duration of bonds that are not default-free based on the Jonkhart (1979) formulation of the term structure of interest rates. The authors illustrate that the new duration measures differ substantially from Macaulay's duration and demonstrate the economic significance of ignoring default risk when employing immunization strategies for corporate bonds. Numerical results reveal significant differences between their new measures of duration and conventional Macaulay duration for bonds with default risk, further showing that their innovative method effectively balances interest rate risk and default risk in a majority of situations, both before and after the occurrence of events that impact bond prices. In one setting, they consider a three-year, zero-coupon bond with a face value of \$100,000 and a 10% chance of default each year. The risk-free interest rate is assumed to be 5% per year. The authors calculate a modified duration of 2.52 years using their new measure of duration, compared to a Macaulay duration of 3 years. This illustrates the significant differences that can arise between duration measures when default risk is not taken into account. In another setting, the authors consider a similar three-year bond with a 20% chance of default in the second year and no default risk in the first or third years. They calculate a modified duration of 2.3 years using their new measure of duration, compared to a Macaulay duration of 2.4 years. These results highlight the importance of considering default risk when calculating bond duration.

Ambastha et al. (2010) present an empirical study investigating the relationship between interest rate risk and credit risk, where they have conducted several studies on the empirical duration of investment-grade and high-yield bonds. In their research, the results show that empirical durations decline as credit ratings worsen, implying that interest rate sensitivity is lower for high-yield bonds compared to investment-grade bonds. The sensitivity to interest rates decreases as credit spreads widen, even across bonds with the same credit quality, so when comparing bonds within the same credit quality (e.g., within the investment-grade or high-yield category). However, there is a clear separation in interest rate sensitivity between investment-grade and high-yield bonds. Investment-grade bonds maintain their sensitivity to interest rates even when spreads widen, while high-yield bonds exhibit equity-like behaviour with zero or even negative empirical durations. These results are consistent with prior research on the topic, showing that investment-grade corporate bonds exhibit a higher average duration than high-yield corporate bonds, something that traditional duration measures like Macaulay duration do not capture.

2.2. Corporate Debt Valuation Models

Two primary approaches exist for valuing corporate debt, intensity-based models and structural models. In this section, I review some of the most important contributions of both types of models.

Intensity-based models are a class of models used to value corporate debt, which takes default as a random event whose instantaneous intensity or hazard rate (i.e. probability of occurring in a short time interval conditional on not occurring until that moment) typically changes over time. The primary aim of these models is to capture the dynamic nature of default risk without specifically linking it to a specific economic cause.

Jarrow and Turnbull (1995) introduced an important intensity-based model for valuing corporate debt. In their model the default intensity follows a mean-reverting process. The authors also extended their model to incorporate a term structure of interest rates, allowing for the estimation of the term structure of credit spreads. The Jarrow-Turnbull model has become a fundamental model in credit risk analysis and is widely used by practitioners and academics. The model has also been extended in various ways to account for other sources of risk, such as liquidity risk and systematic risk. While the Jarrow-Turnbull model has its limitations, such as

assuming a constant recovery rate, it remains a widely used and influential model in the field of credit risk analysis.

Duffie and Singleton (1999) developed an intensity-based model for pricing credit derivatives, which is a type of financial instrument used to manage credit risk. The model builds on the idea that the likelihood of default varies over time and can be captured by a stochastic process, or intensity, that evolves over time. The authors use a continuous-time framework to model the evolution of the intensity, which is linked to the credit spread of the underlying corporate bond. They also introduce a default barrier, which represents the threshold at which the issuer defaults, and use it to compute the probability of default and the value of the credit derivative. Duffie and Singleton's (1999) model is known for its simplicity and flexibility, and its ability to capture the dynamic nature of default risk. The model has also been extended to incorporate other factors that affect credit risk, such as interest rate and liquidity risk.

Jarrow and Protter (2004) present an information-based perspective to compare the performance of structural and reduced-form models for credit risk. They argued that reduced-form models, such as the intensity-based models, have the advantage of being able to incorporate all available information about credit risk, while structural models may not capture all relevant information. However, they also pointed out that structural models can provide valuable insights into the underlying economic factors driving default risk, which may not be readily available in reduced-form models. Jarrow and Protter's analysis suggests that both structural and reduced-form models have their own strengths and limitations, and the choice of model depends on the research question and available data.

Structural models, on the other hand, rely on the contingent claims pricing theory to estimate the value of corporate debt. These models of contingent claims pricing and credit risk started from the work of Fischer Black & Myron Scholes (1973) and Robert Merton (1974). In their seminal option pricing model, Black & Scholes (1973) suggested understanding the equity of a given company as a European call option on the company's assets with a strike price equal to its nominal debt. On the other hand, debt is suggested to be perceived as a risk-free bond less the value of a put option on the company's assets. The firm defaults whenever assets are worth less than nominal debt value at debt maturity. Thus, the probability of default corresponds to the probability of the put option ending up in the money. (Black & Scholes, 1973) (Merton, 1974)

Black and Cox (1976) performed some adaptations in the previously mentioned framework, in an attempt of adjusting the model's main critics. Black and Cox assumed a first passage time model when it was considered the possibility of the firm defaulting on its debt prior to debt maturity. The authors justify this with debt covenants. Opposite to the Black-Scholes-Merton model, company performance started to matter before debt maturity. This consideration leads to an increase in the probability of default. However, it also leads to higher recovery values. The authors conclude that covenants benefit debt value as long as there are no distress costs. By adding these new inclusions to the model, Black and Cox were also able to replicate more accurately the credit spreads observed in historical data. (Black & Cox, 1976)

In 1994, the Black and Cox model (1976) was extended by Leland. With the introduction of certain bankruptcy costs and corporate income taxes, Leland adjusted the model in order to determine the optimal capital structure that would be compatible with the trade-off theory of optimal capital structure. In this model, the implications of bankruptcy costs and taxes were studied for the firm optimal capital structure. In contrast to the main model in Black and Cox (1976), Leland (1994) assumes an endogenous barrier. It is endogenous because it is chosen by shareholders who decide on whether they want to keep paying the firm coupons on issued debt. Bankruptcy occurs when shareholders no longer find it worthwhile to keep the company running. Shareholders' decision on keeping the company alive depends on the model parameters. The lower the coupon rate and the higher the volatility, the interest rate, and the tax rate, the lower the barrier is, which implies that shareholders wait for longer to assess whether the company recovers or not. (Leland, 1994)

Longstaff and Schwartz (1995) introduced a more complex version of the structural credit risk model originally proposed by Black and Cox (1976). In contrast to earlier models that assumed a constant interest rate, Longstaff and Schwartz allowed for stochastic interest rates to vary over time. Additionally, they incorporated two stochastic risk factors, one related to the asset value and another related to risk-free interest rates, which allowed for a more accurate assessment of credit risk. This improvement enabled Longstaff and Schwartz to develop a model that can capture the relationship between default probability, interest rate fluctuations, and credit spreads in a single framework. Furthermore, the model's ability to account for the maturity of debt, a crucial factor affecting credit risk, represents another important advantage over previous models. (Longstaff & Schwartz, 1995)

Leland and Toft (1996) addressed the assumption of perpetual debt, which represented a further development in Leland's earlier works (Leland, 1994). In opposition to the original assumption, Leland and Toft (1996) consider that the firm must rollover its maturing debt. This is done in a way that does not affect the overall amount of debt in any specific moment. The rollover risk creates financial pressure to the firm. When rolling over under negative conditions, replacing debt is more costly, and consequently shareholders must incur additional costs. Everything else constant, this leads shareholders to leave the company earlier than in Leland. Furthermore, in Leland (1994) distress costs are irrelevant to determine the barrier once the coupon rate is set, while in Leland and Toft (1996) distress costs are still relevant due to the rollover. When comparing both models' results, Leland and Toft found their model more capable to replicate the historical default and credit spreads, in comparison to what Leland had achieved previously. (Leland & Toft, 1996)

In 2001, Goldstein, Ju and Leland made another breakthrough improvement to the existing literature on optimal capital structure. In Leland (1994) model, the company is perceived as a combination of assets, tax benefits, and distress costs (Leland, 1994). On the other hand, Goldstein, Ju, and Leland (2001) consider the company as a claim on a project continuously generating earnings. In this case, shareholders, debt holders, the government, and distress costs are perceived as claimants on this perpetual project. In Leland (1994) the asset value is understood as completely exogenous to the model, while Goldstein, Ju, and Leland (2001) relate the project value to the company's ability to generate earnings. Once again, the barrier is set by shareholders as the result of an optimal stopping time problem, with similarities to what occurs in Leland (1994).

The main difference between the barrier introduced by Leland (1994) and the one found in this model, is that in the latter the firm has a continuous pay-out that can be utilized to pay the coupons. While the firm remains solvent, earnings are divided between the shareholders, debt holders, and the government. If the firm incurs default, the project value after distress costs is shared between debt holders and the government. Furthermore, it is considered that the firm can issue additional debt later depending on the evolution of the project earnings. Consequently, it is optimal for the firm to issue considerably less debt in the early stages, which is usually aligned with what is observed in real life. This option is also responsible for increasing the shareholder interest in the project, causing them not to abandon the company and hold on for longer without declaring bankruptcy. As a result of both the lower initial debt and the increase

in shareholder interest, the value of the barrier is considerably lower than the one found in a static model with the same amount of coupon (Goldstein, Ju, & Leland, 2001).

2.3. Are Structural Models able to produce better Duration Estimates?

Acharya and Carpenter (2002) and Schaefer and Strebulaev (2008) are two influential articles that have analysed whether structural models are able to better measure interest rate risk in the case of bonds with credit risk.

Acharya and Carpenter (2002) propose a structural credit risk model that incorporates stochastic interest rates and endogenous bankruptcy risk, providing a more realistic framework for valuing and hedging corporate bonds compared to models that assume constant interest rates or exogenous default events. They argue that their model offers a better fit for corporate bond prices and empirical durations, particularly for low credit quality bonds, and that durations based on their endogenous barrier model are closer to empirical findings than those produced by exogenous barrier models.

The authors investigate the valuation of callable, defaultable corporate bonds under stochastic interest rates and firm value, considering the issuer's optimal call and default rules. By analyzing the interaction between call provisions and default risk, they provide analytical results on valuation, optimal call, and default rules, as well as numerical studies on hedging dynamics. This study sheds light on patterns in bond duration and sensitivity to firm value, showing that the endogenous bankruptcy model explains empirical patterns in the spread-rate relationship better than typical exogenous models. The authors numerically study the dynamics of hedging, using the results on exercise boundaries (optimal call and default rules) to explain patterns in bond duration and sensitivity to firm value. They also link bond duration to the slope coefficient in regressions of changes in corporate yield spreads on changes in interest rates, finding that the endogenous bankruptcy model offers a superior explanation of empirical patterns compared to exogenous bankruptcy models.

Acharya and Carpenter (2002) found that bond durations were influenced by the distance to the exercise boundary (call or default). For instance, bond durations tended to be lower when close to the exercise boundary and higher when far away from it. The numerical results also showed that the duration of callable defaultable bonds was hump-shaped as a function of firm value, while the duration of pure defaultable bonds increased with firm value.

This suggests that structural models with more realistic assumptions, like the one proposed in this study, may produce better duration estimates. These results highlight the importance of integrating both interest rate and bankruptcy risk into bond valuation models.

Schaefer and Strebulaev (2002) investigate the predictive ability of structural credit risk models, particularly the Merton model and its extensions, in estimating hedge ratios for corporate bonds against the equity of the underlying firm. The hedge ratio measures a bond's price sensitivity to fluctuations in the firm's asset value. The authors use a large sample of corporate bonds to demonstrate that key variables in structural models can explain a significant portion of returns on both investment-grade and high-yield bonds. Despite the perceived failure of structural models, the Merton model's equity hedge ratio predictions are not rejected for credit ratings between AAA and B. This challenges the notion that structural models are inadequate in capturing credit risk. They find that the key variables in structural models, which include the returns on the issuing firm's equity and riskless bonds, explain 43-66% of the returns on investment-grade bonds and about 35% for high-yield bonds.

Schaefer and Strebulaev (2002) contend that the Merton model captures the sensitivity of corporate debt value to changes in corporate asset value, a crucial aspect of credit risk. However, the model fails to explain the low interest rate sensitivity of corporate bonds. Including additional market-wide factors, such as changes in the yield spread on US Treasury bonds, S&P 500 index returns, VIX index changes, and Fama-French SMB and HML factors, does not alter the sensitivity of corporate bonds to risk-free rates and equity. These variables have a significant, unpredicted impact on corporate bond returns, which is not clearly related to credit exposure.

While structural models may not accurately predict bond prices, they provide reasonably precise hedge ratio estimates, implying that their poor performance may result from the influence of non-credit factors rather than an inability to capture corporate debt's credit exposure. The findings suggest that structural models are useful tools for understanding and managing credit risk in corporate bonds, but their accuracy may vary depending on the specific assumptions and parameters used.

3. Model

3.1. Model setup and debt securities valuation

Similar to Merton (1974), Black and Cox (1976), and Brennan and Schwartz (1978); Leland and Toft (1996) consider that the firm maintains productive assets with an unleveraged value V , which undergoes a continuous diffusion process exhibiting consistent proportional volatility σ :

$$\frac{dV}{V} = [\mu(V, t) - \delta]dt + \sigma dz \quad (1)$$

where $\mu(V, t)$ represents the total expected rate of return on asset value V ; δ represents the constant fraction of value paid out to security holders; and dz represents the increment of a standard Brownian motion. The process perpetuates indefinitely unless it occurs the declining of V to a default-triggering value V_B , which is assumed to remain constant. Afterwards, V_B will be endogenously determined. They assume the existence of a default-free asset, providing a continuous interest rate r .

The authors consider a bond issuance with a maturity of t periods from the present, continuously paying a fixed coupon flow $c(t)$ and featuring a principal $p(t)$, where $p(t)$ denotes the fraction of asset value V_B acquired by debt of maturity t if bankruptcy occurs. Through risk-neutral valuation and by designating $f(s; V, V_B)$ as the density of the initial passage time s to V_B from V when the drift rate amounts to $r - \delta$, they determine the value of debt with maturity t :

$$d(V; V_B, t) = \int_0^t e^{-rs} c(t) [1 - F(s; V, V_B)] ds + e^{-rt} p(t) [1 - F(t; V, V_B)] + \int_0^t e^{-rs} \rho(t) V_B f(s; V, V_B) ds \quad (2)$$

The first term in equation (2) is the discounted expected value of the coupon flow. This coupon flow is paid continuously with probability $[1 - F(s)]$, where s represents time and $F(s)$ represents the cumulative distribution function of the first passage time to bankruptcy. The second part is the expected discounted value of principal repayment. The third component is the expected discounted value of the asset fraction that will be allocated to debt with maturity t in the event of bankruptcy. They integrate the first term by parts yielding:

$$d(V; V_B, t) = \frac{c(t)}{r} + e^{-rt} \left[p(t) - \frac{c(t)}{r} \right] [1 - F(t)] + \left[\rho(t)V_B - \frac{c(t)}{r} \right] G(t) \quad (3)$$

$F(t)$ is obtained from Harrison (1990) and $G(t)$ is derived from Rubinstein and Reiner (1991):

$$F(t) = N[h_1(t)] + \left(\frac{V}{V_B} \right)^{-2a} N[h_2(t)] \quad (4)$$

$$G(t) = \left(\frac{V}{V_B} \right)^{-a+z} N[q_1(t)] + \left(\frac{V}{V_B} \right)^{-a-z} N[q_2(t)] \quad (5)$$

where

$$\begin{aligned} q_1(t) &= \frac{(-b - z\sigma^2 t)}{\sigma\sqrt{t}}; & q_2(t) &= \frac{(-b + z\sigma^2 t)}{\sigma\sqrt{t}}; \\ h_1(t) &= \frac{(-b - a\sigma^2 t)}{\sigma\sqrt{t}}; & h_2(t) &= \frac{(-b + a\sigma^2 t)}{\sigma\sqrt{t}}; \\ a &= \frac{\left(r - \delta - \left(\frac{\sigma^2}{2} \right) \right)}{\sigma^2}; & b &= \ln\left(\frac{V}{V_B} \right); & z &= \frac{[(a\sigma^2)^2 + 2r\sigma^2]^{1/2}}{\sigma^2} \end{aligned}$$

$N(\cdot)$ represents the cumulative standard normal distribution. They incorporate these expressions into equation (3), resulting in a closed-form solution for the value of risky debt when V_B remains constant. They define that $x = a + z$, noting that as $t \rightarrow \infty$:

$$d(V; V_B, t) \rightarrow \frac{c(\infty)}{r} \left[1 - \left(\frac{V}{V_B} \right)^{-x} \right] + \rho(\infty)V_B \left(\frac{V}{V_B} \right)^{-x} \quad (6)$$

Except for the value of x , this equation is identical to the one derived by Leland (1994a) for infinite-horizon risky debt when $\rho(\infty) = (1 - \alpha)$, where α represents the fraction of asset value lost in bankruptcy, and $c(\infty)$ represents the coupon paid by the bond with infinite maturity. Until this point, they assume that V_B is an exogenous constant, as in Longstaff and

Schwartz (1995), which is probably not optimal for arbitrary capital structures and leads to the next section.

3.2. Contingent claims value with endogenous default barrier and a stationary debt structure

In the specific case of perpetual debt studied by Leland (1994), coupon flows are constant leading to a constant endogenous default barrier. Leland and Toft (1996) consider an alternative case where the firm finances itself with finite maturity debt that still leads to a constant coupon outflow. In their case, the firm continuously issues a constant principal amount of new debt with T years maturity, which, upon maturity, is repaid at face value if the firm remains solvent. This new bond principal is issued at a rate $p = (P/T)$ per year, where P represents the total principal value of all outstanding bonds. As long as the firm stays solvent, the total outstanding debt principal at any time s will be P . Debt securities follow a uniform distribution of principal across maturities within the interval $(s, s + T)$, where $s = 0$ designates the current time. Bonds with a principal p yield a constant coupon rate $c = (C/T)$ per year, which implies that the aggregate coupon paid by all outstanding bonds amounts to C per year. Consequently, total debt service payments remain time-independent, equating to $(C + P/T)$ per year.

Denote as $D(V; V_B, T)$ the total value of debt when debt of maturity T is issued. As referred in the previous section, whenever bankruptcy occurs, $(1 - \alpha)V_B$ is distributed among bondholders. This implies that the sum of all fractional claims $\rho(t)$ for debt of all outstanding maturities equates to $(1 - \alpha)$ (one can write $\rho = (1 - \alpha)$) and thus $\rho(t) = \rho/T$ per year for all t .

The value of all outstanding bonds is given by the sum of all current debt securities with maturity from 0 (the oldest debt security issued) to T (the most recent debt security issued):

$$\begin{aligned}
 D(V; V_B, T) &= \int_{t=0}^T d(V; V_B, t) dt \\
 &= \frac{C}{r} + \left(P - \frac{C}{r} \right) \left(\frac{1 - e^{-rT}}{rT} - I(T) \right) + \left((1 - \alpha)V_B - \frac{C}{r} \right) J(T)
 \end{aligned} \tag{7}$$

where

$$I(T) = \frac{1}{rT} (G(T) - e^{-rT} F(T))$$

$$J(T) = \frac{1}{z\sigma\sqrt{T}} \left(- \left(\frac{V}{V_B} \right)^{-a+z} N[q_1(T)] q_1(T) + \left(\frac{V}{V_B} \right)^{-a-z} N[q_2(T)] q_2(T) \right)$$

Thereafter, they state that the total market value of the company, denoted by v , corresponds to the sum of the asset value, plus the value of the tax benefits minus the value of the bankruptcy costs. In this case, tax benefits accumulate at a rate of τC per year, provided that $V > V_B$, where τ represents the corporate tax rate. In contrast, distress costs accrue at a rate of αV_B . They follow Leland (1994a, equation (12)) and determine that the total firm value can be expressed as:

$$v(V; V_B) = V + \frac{\tau C}{r} \left[1 - \left(\frac{V}{V_B} \right)^{-x} \right] - \alpha V_B \left(\frac{V}{V_B} \right)^{-x} \quad (8)$$

And value of equity as:

$$E(V; V_B, T) = v(V; V_B) - D(V; V_B, T) \quad (9)$$

The authors invoke the condition of smooth-pasting, which has the characteristic of optimizing, in relation to V_B , the valuation of both equity and the firm as a whole, while subject to the constraint of equity's limited liability $E(V) \geq 0$ for all $V \geq V_B$. Through this condition, the authors determine the equilibrium bankruptcy-triggering asset value V_B :

$$V_B = \frac{\left(\frac{C}{r} \right) \left(\frac{A}{rT} - B \right) - \frac{AP}{rT} - \frac{\tau C x}{r}}{1 + \alpha x - (1 - \alpha) B} \quad (10)$$

where

$$A = 2ae^{-rT}N(a\sigma\sqrt{T}) - 2zN(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}}n(z\sigma\sqrt{T}) + \frac{2e^{-rT}}{\sigma\sqrt{T}}n(a\sigma\sqrt{T}) + (z - a)$$

$$B = -\left(2z + \frac{2}{z\sigma^2T}\right)N(z\sigma\sqrt{T}) - \frac{2}{\sigma\sqrt{T}}n(z\sigma\sqrt{T}) + (z - a) + \frac{1}{z\sigma^2T}$$

In these expressions $n(\cdot)$ represents the standard normal density function. When T approaches infinity, it can be demonstrated that V_B converges to $(1 - \tau)(Cx/r)/(1 + x)$, as presented in Leland (1994a). They observe that V_B depends on the maturity chosen of debt, for any given values of bond principal P and coupon rate C , on the contrary of other models consistent with flow-based bankruptcy or presenting a positive net worth covenant, which usually imply that V_B is independent of debt maturity and present a relevant difference to their model regarding the effect of maturity on debt value.

3.3. Base case parameters

In order to facilitate comparisons, the authors consider a "base case" environment where they attribute the following parameters:

Notation	Variable	Base case value
V	Asset value	100
r	Default-free interest rate	7.5%
δ	Payout ratio	7%
σ	Asset volatility	20%
α	Bankruptcy cost fraction	50%
τ	Corporate tax rate	35%

Finally, they assume that the coupon is established in a way that newly issued debt sells at par value ($d = p$). This implies that ($c = C/T$) is determined as the smallest solution to the given equation:

$$d(V; c, p)|_{V=100} = p \tag{11}$$

4. Data

4.1. Data Set

The companies considered for the data set of this dissertation were retrieved from two portfolios chosen from the website of iShares, which is a family of exchange-traded funds (ETFs) managed by BlackRock, one of the world's largest asset managers. These ETFs are composed of investments in a variety of assets like stocks, bonds, and commodities from different regions and sectors. The first portfolio chosen was a high yield corporate bond ETF with higher interest rates (yields), lower credit rating, and a higher risk of default compared to other bonds. This portfolio is denominated as high yield portfolio (HYP). The second portfolio selected was an investment grade corporate bond ETF with bonds issued by companies with a high credit rating, indicating a lower risk of default. This portfolio is denominated as investment grade portfolio (IGP). The timespan considered focuses on the 5-year period from 2018 to 2022.

The list of the constituents of each portfolio and their respective accounting data were retrieved from Refinitiv Eikon Datastream. The initial population of companies were 385 for the high-risk portfolio and 315 companies for the low-risk one. Financial companies and companies from the insurance sector were excluded, as financial services companies typically present a balance sheet structure and a risk profile very different from the one observed in other sectors. After these first adjustments in terms of data restrictions, the list of constituents of the high yield portfolio resulted in 253 companies and the list of constituents for the investment grade portfolio resulted in a total of 299.

In the second part of treating the data, companies that did not present relevant accounting values in all periods under analysis were also excluded. With this final restriction, 31 companies from the high yield portfolio were eliminated, resulting in 222 final companies; while 20 companies were excluded from the investment grade portfolio, leading to a total of 279 final companies.

Regarding the final samples of companies for both portfolios, it is important to analyse them in certain relevant metrics.

4.2. Data Analysis

Regarding the final samples of companies for both portfolios, it is important to study them in certain relevant metrics. Therefore, an analysis of the distribution was conducted by sector, country, dimension (in terms of equity market value), leverage ratio (the metric approached was Debt-To-Equity), and finally credit rating (in order to assess the credit risk implied to the company).

High Yield Portfolio

Table 1 presents a detailed breakdown of the high yield portfolio sample, organized by sector and region in December 2022.

Panel A: HYP Companies by Sector	Number of Companies	Percentage
Basic Industry	12	5,41%
Brokerage/Asset Managers/Exchanges	2	0,90%
Capital Goods	23	10,36%
Communications	25	11,26%
Consumer Cyclical	48	21,62%
Consumer Non-Cyclical	35	15,77%
Electric	9	4,05%
Energy	30	13,51%
Industrial Other	1	0,45%
Reits	11	4,95%
Technology	19	8,56%
Transportation	7	3,15%
Total	222	100,00%
Panel B: HYP Companies by Country	Number of Companies	Percentage
Australia	1	0,45%
Belgium	1	0,45%
Canada	12	5,41%
Israel	1	0,45%
Italy	1	0,45%
Macau	1	0,45%
United Kingdom	2	0,90%
United States	203	91,44%
Total	222	100,00%

Table 1 - HYP Sample Description – December 2022

Subsequently, in Figures 1 and 2 it is possible to observe the distribution of the companies in terms of dimension and leverage ratio. As mentioned previously, the metrics chosen to analyse these topics were equity market value and debt-to-equity. Regarding

dimension, the value of equity is concerned to the last month of the timeseries. In terms of leverage, it was observed the 5-year average debt-to-equity ratio for the timespan considered in this study. All values were retrieved from Refinitiv Eikon Datastream.

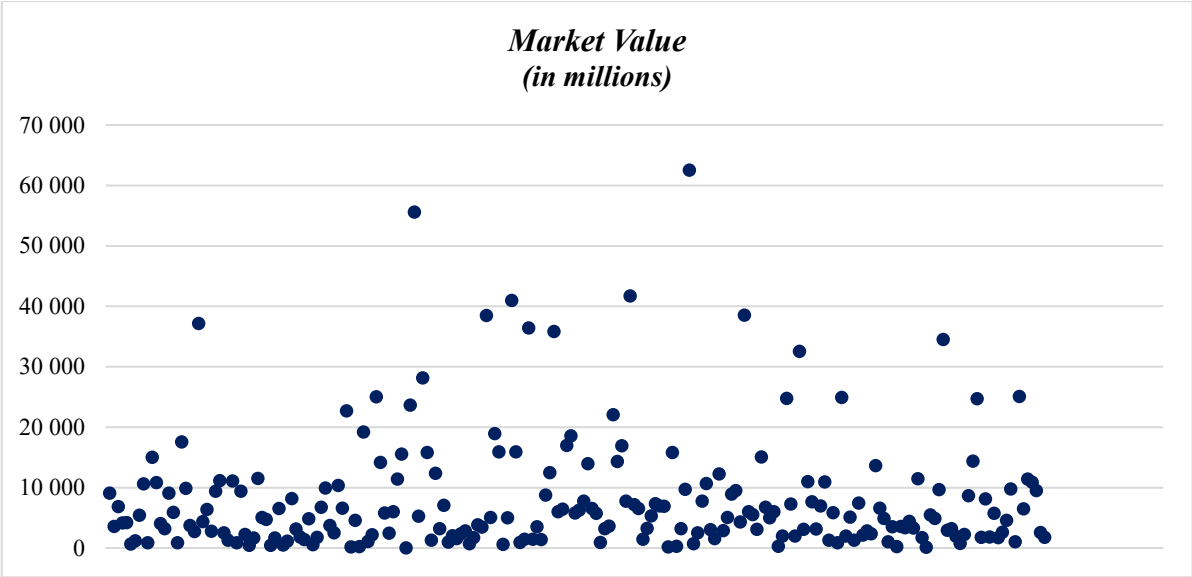


Figure 1 - HYP Market Value illustration – December 2022

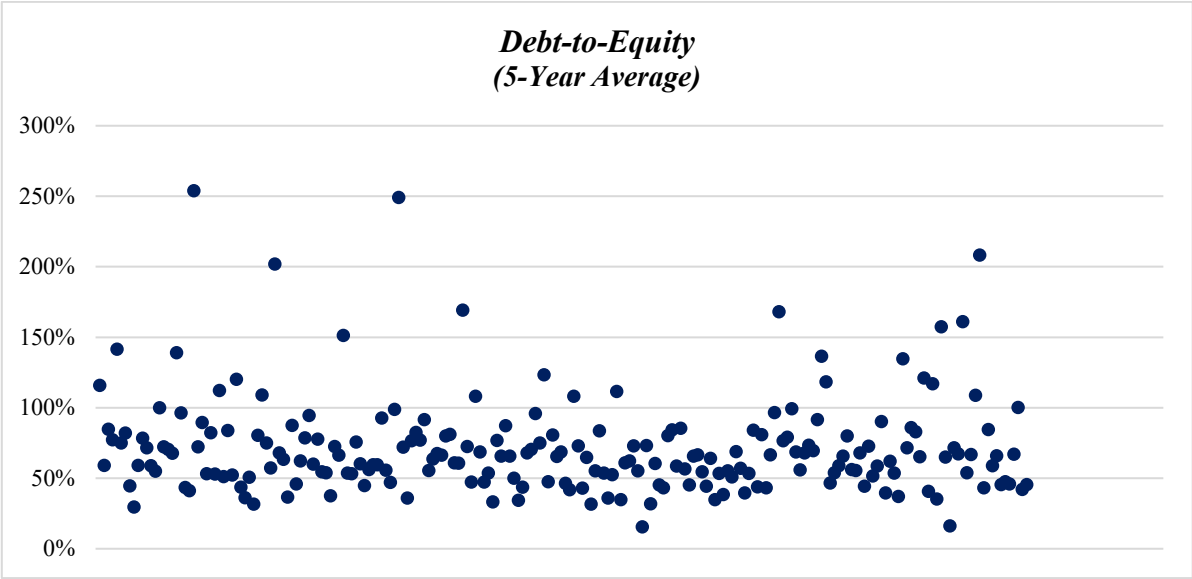


Figure 2 - HYP Debt-to-Equity illustration – December 2022

In the last part of this data analysis, the goal is to illustrate our portfolio regarding its maturity (in Figure 3) and credit quality (in Figure 4). The following values were retrieved from iShares website and despite being allusive to the original portfolio before data adjustments, they are still representative for our sample. The website states that credit quality ratings on underlying securities of the fund are received from S&P, Moody’s and Fitch and converted to the equivalent S&P major rating category. In terms of maturity, the breakdown

below is based on Weighted Average Life, which is the average length of time to the repayment of principal for the securities in the fund.

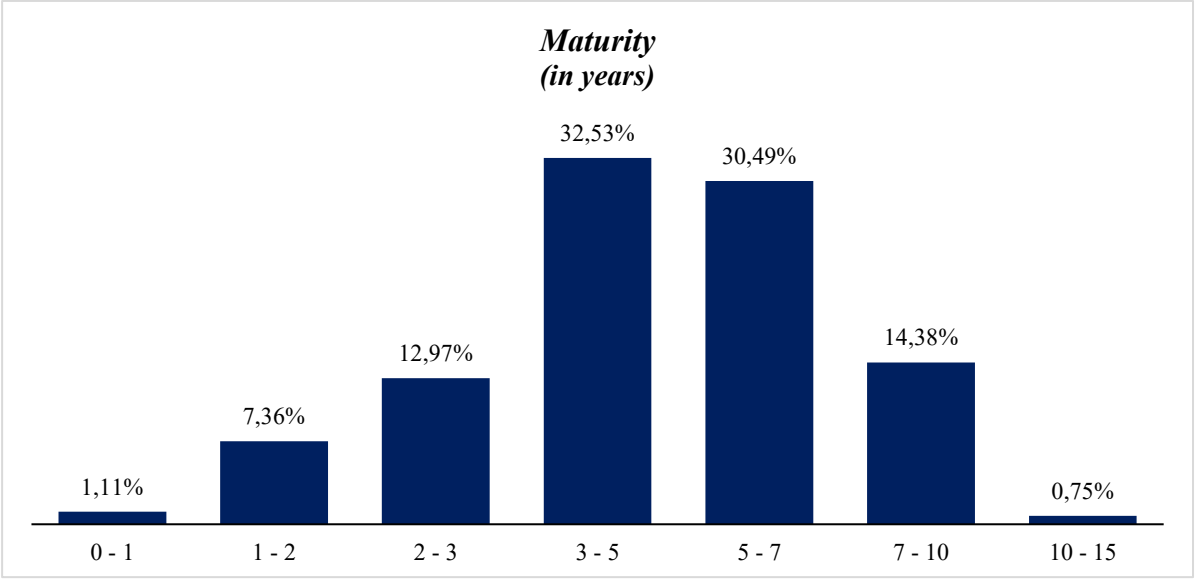


Figure 3 - HYP Maturity levels by % of the Fund – December 2022¹

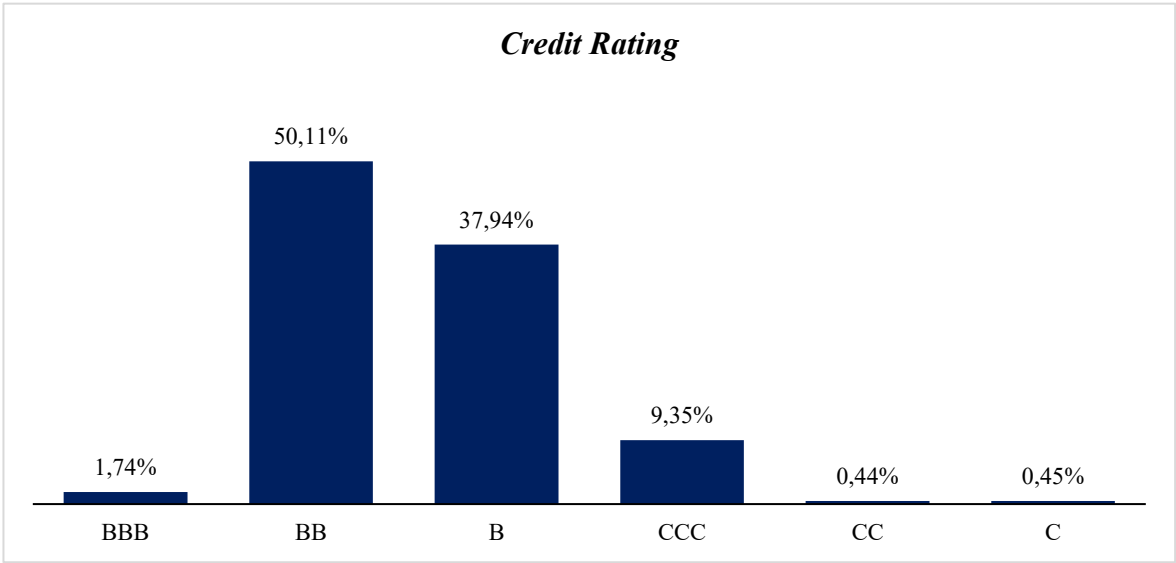


Figure 4 - HYP Credit Quality levels by % of the Fund – December 2022¹

¹ Numbers might not add up to 100% due to rounding.

Investment Grade Portfolio

Table 2 presents the same detailed breakdown but this time for the high yield portfolio sample, organized by sector as well as region.

Panel C: IGP Companies by Sector	Number of Companies	Percentage
Basic Industry	18	6,45%
Brokerage/Asset Managers/Exchanges	6	2,15%
Capital Goods	26	9,32%
Communications	19	6,81%
Consumer Cyclical	30	10,75%
Consumer Non-Cyclical	59	21,15%
Electric	20	7,17%
Energy	33	11,83%
Industrial Other	1	0,36%
Natural Gas	2	0,72%
Reits	12	4,30%
Technology	44	15,77%
Transportation	8	2,87%
Utility Other	1	0,36%
Total	279	100,00%
Panel D: IGP Companies by Country	Number of Companies	Percentage
Australia	1	0,36%
Belgium	1	0,36%
Brazil	1	0,36%
Canada	12	4,30%
France	1	0,36%
Germany	1	0,36%
Luxembourg	1	0,36%
Macau	1	0,36%
Netherlands	2	0,72%
Spain	1	0,36%
Switzerland	1	0,36%
United Kingdom	6	2,15%
United States	250	89,61%
Total	279	100,00%

Table 2 - IGP Sample Description - December 2022

Once again, in Figures 5 and 6 it is possible to observe the distribution of the companies in terms of dimension and leverage ratio with the same metrics and conditions previously mentioned for the first portfolio.

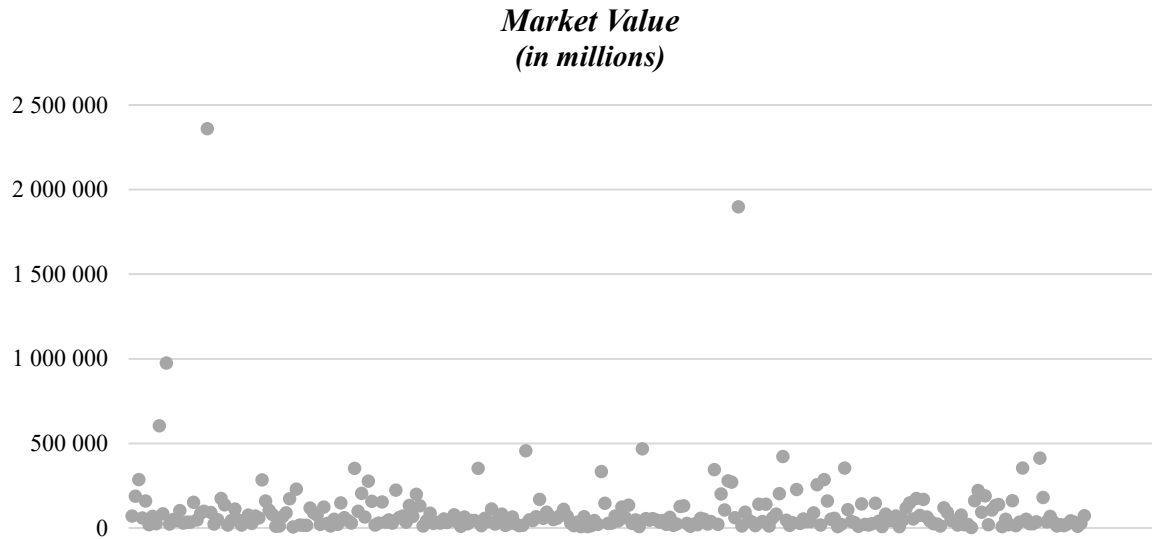


Figure 5 - IGP Market Value illustration – December 2022

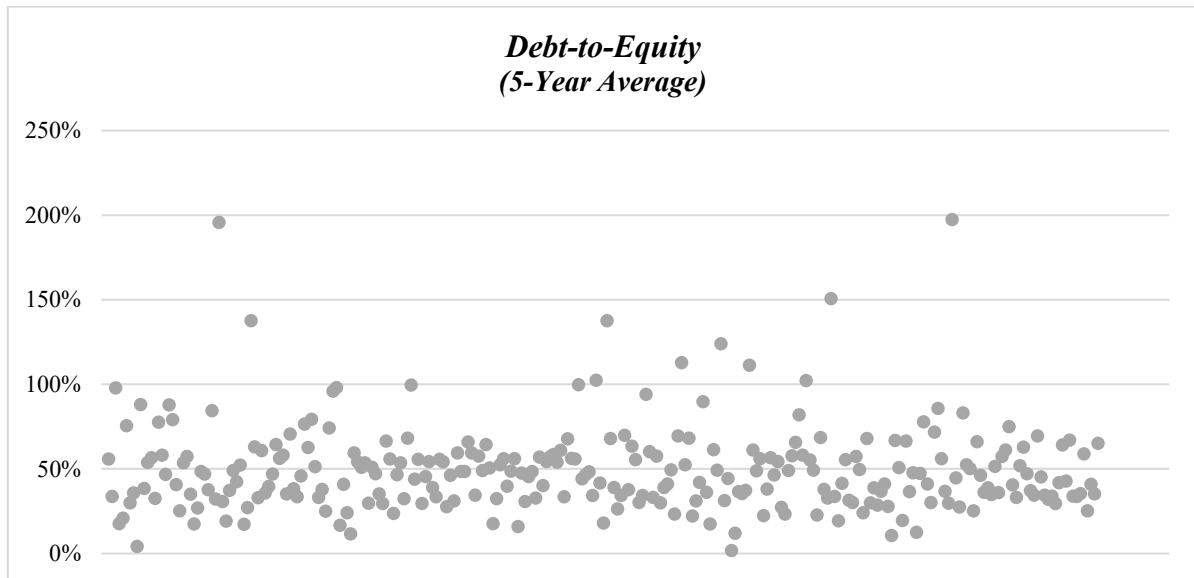


Figure 6 - IGP Debt-to-Equity illustration – December 2022

In the last part of the data analysis for the investment grade portfolio, this portfolio is also illustrated in terms of maturity (in Figure 7) and credit quality (in Figure 8). The process performed was repeated as before and every observation stated previously for the high yield portfolio applies to this case as well.

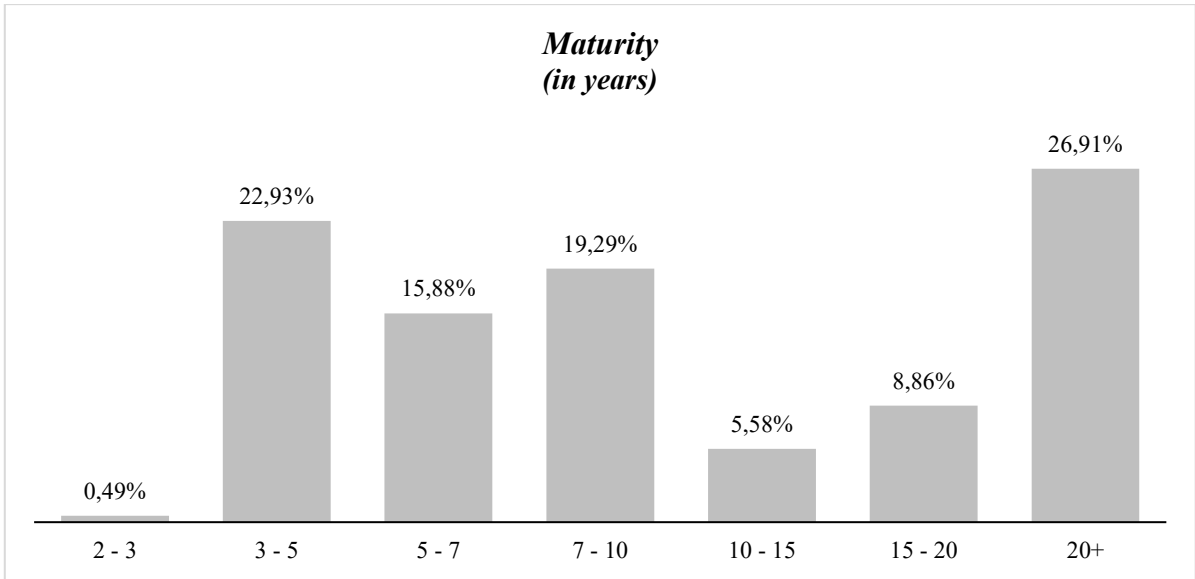


Figure 7 - IGP Maturity levels by % of the Fund – December 2022²

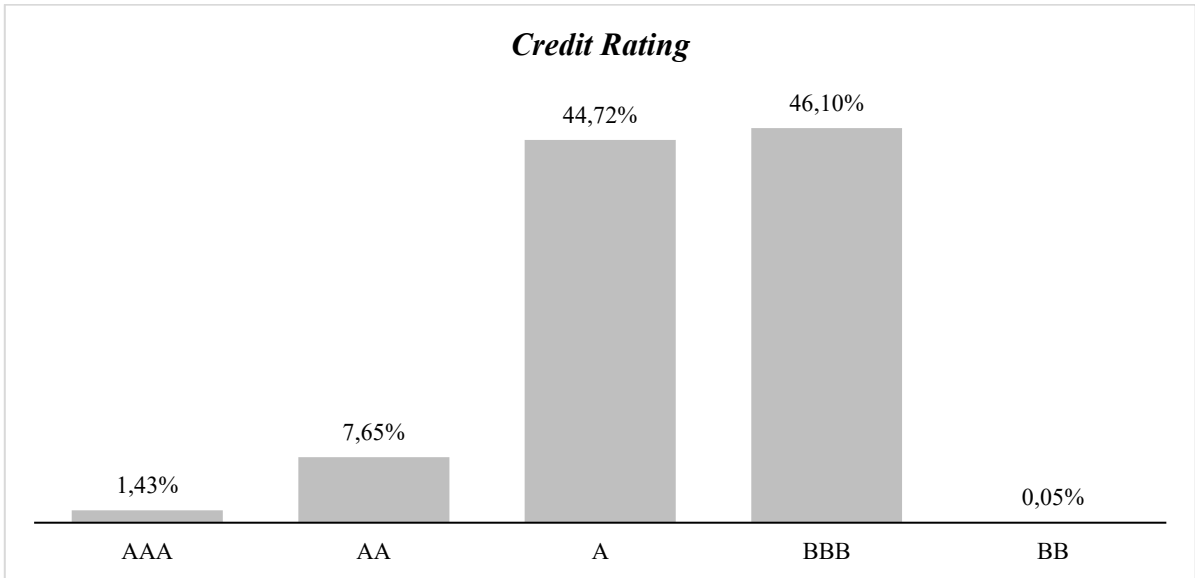


Figure 8 - IGP Credit Quality levels by % of the Fund – December 2022²

² Numbers might not add up to 100% due to rounding.

5. Calibration Strategy

This section presents how the several model inputs were calibrated to implement the Leland and Toft (1996) model. Section 5.1 describes the model inputs that were obtained through accounting data and assumptions stated in the original model. In section 5.2, the iterative strategy used to calibrate the asset value (corresponding to the value of V) and the asset volatility (represented by σ) is described.

5.1. Model Inputs

Payout Ratio

The calculation of the payout ratio was approached by the following steps:

1. Set a value of δ for each month, using the monthly values of equity, debt, dividends paid and interest expense in the following equation:

$$\delta = \frac{\textit{Dividends Paid} + \textit{Interest Expense}}{\textit{Equity} + \textit{Debt}} \quad (12)$$

2. For each company, compute the average of the calculated monthly payout ratio for the entire time series.

Interest Expense

The total coupon paid by all outstanding bonds C for each company was obtained from Refinitiv Eikon Datastream by retrieving the values of Interest Expense on Debt for every month in our 5-year time series.

Principal Value

The principal value of all outstanding bonds P was also obtained from Refinitiv Eikon Datastream by retrieving the monthly values of Total Debt for each company.

Risk-free Interest Rate

To facilitate comparisons with the Leland and Toft (1996) study, this dissertation also assumes the same 'base case' value for variable r , which is 0.075.

Bankruptcy cost fraction

Once again, the condition of the “base case” environment is assumed following the original study of Leland and Toft (1996). Consequently, the value of α is assumed to be 0.5.

Corporate Tax Rate

The same assumptions were made as the two previous parameters, setting τ at 0.35.

Maturity

In alignment with our reference article, T was employed as 5 years as the input for the model, considering it yields an average maturity of 3 years, a value that appears to be reasonable.

5.2. Iterative Approach

To calculate the asset value and the asset volatility, a similar approach to the one proposed by Vassalou and Xing (2004) was followed:

1. Defining a level of convergence: similar to Vassalou and Xing (2004), the level of convergence set was 0.0001.
2. Setting an initial value for V and for σ . The initial estimate of V was set as the sum of debt value (corresponding to the nominal value of total debt in each month) and equity value (corresponding to the market value of equity in each month). The initial estimate of σ was set at 20%.
3. For each month, the values of assets were estimated, by solving the model equity valuation formula (i.e., equation (9)) with the observed market value of equity in each month.
4. Computing a new guess for σ by:
 - a. Calculating the log returns based on asset prices for each month.
 - b. Determining the annualized standard deviation of the log returns.
5. Repetition of steps 3 to 5 until the estimated σ from two consecutive iterations finally converge.

6. Results

6.1. Macaulay Duration vs Effective Duration for each bond

This section compares the Macaulay and effective duration of every bond present in the two portfolios presented in the previous sections. The Macaulay duration offers a measure of the percent change in bond price due to a uniform shift in default-free interest rates (bonds with no default risk). Assuming the bond, with maturity T , is sold at par, and the yield is represented by R (where $R = c/p$), the Macaulay Duration of the bond is given by:

$$\text{Macaulay Duration} = \frac{1 - e^{-RT}}{R} \quad (13)$$

Macaulay duration does not consider the impact of changes in the risk-free rate on the probability of default. This leads to a critical question: How sensitive are “effective” measures of duration to the presence of default risk? By an "effective" measure of duration, this term is referring to a formulation that precisely predicts the percentage change in the value of a (risky) bond in response to an adjustment in the default-free interest rate.

Using Leland and Toft (1996) theoretical framework and our initial case parameters, we initially calculated the percentage shift in the value of a newly issued bond with maturity T , responding to a 1 percent increase in the default-free interest rate r (from 7.5% to 8.5%). This measure gave us an alternative duration estimate termed as the "effective duration." However, to refine this methodology and account for potential bias in interest rate changes, we have modified our approach to take into consideration both an increase and decrease of 1 percentage point (pp) in the default-free interest rate. We first calculate the impact of a 1pp rise in the yield (from 7.5% to 8.5%) and then evaluate the impact of a 1pp drop (from 7.5% to 6.5%). By averaging the absolute values of the impact in these two scenarios, we obtain a more balanced measure of the effective duration, which is less sensitive to the direction of interest rate changes.

After these computations were made, the effective duration as a function of the Macaulay duration was plotted in Figures 9 and 10, for our two portfolios' bonds in December 2022. Plotting the relationship between the effective duration and Macaulay duration for every bond present in our two portfolios, the following illustrations were achieved:

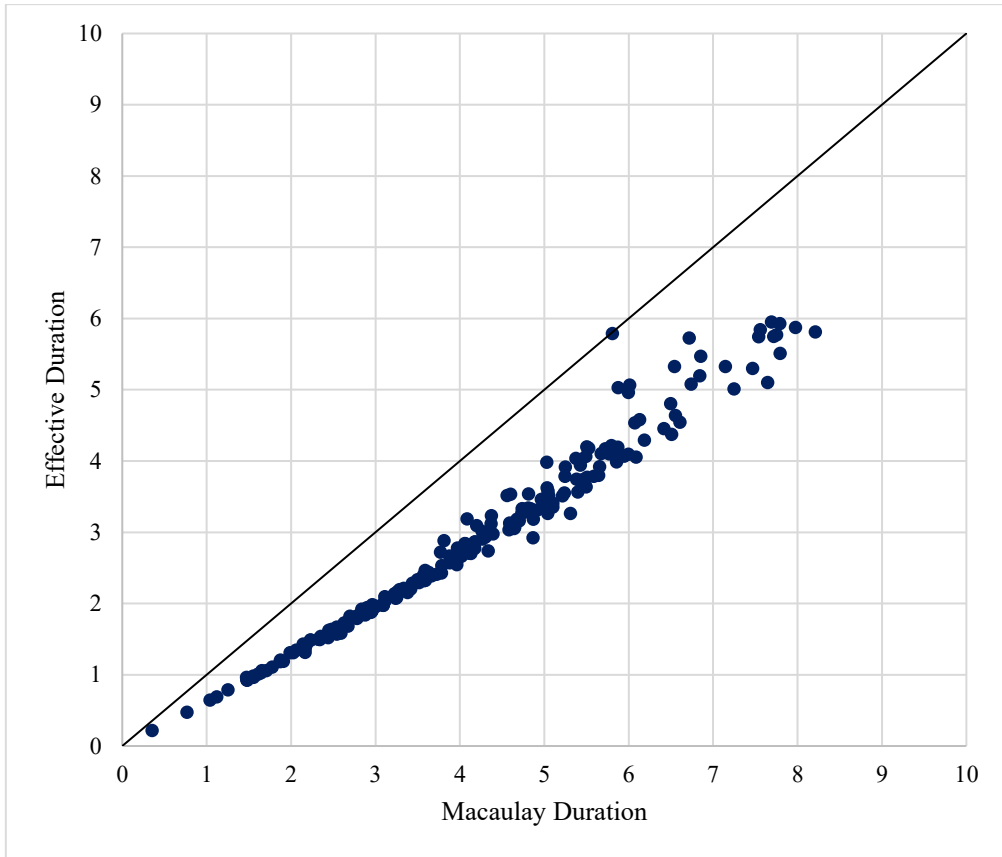


Figure 9 - HYP Effective Duration as a function of Macaulay Duration – December 2022

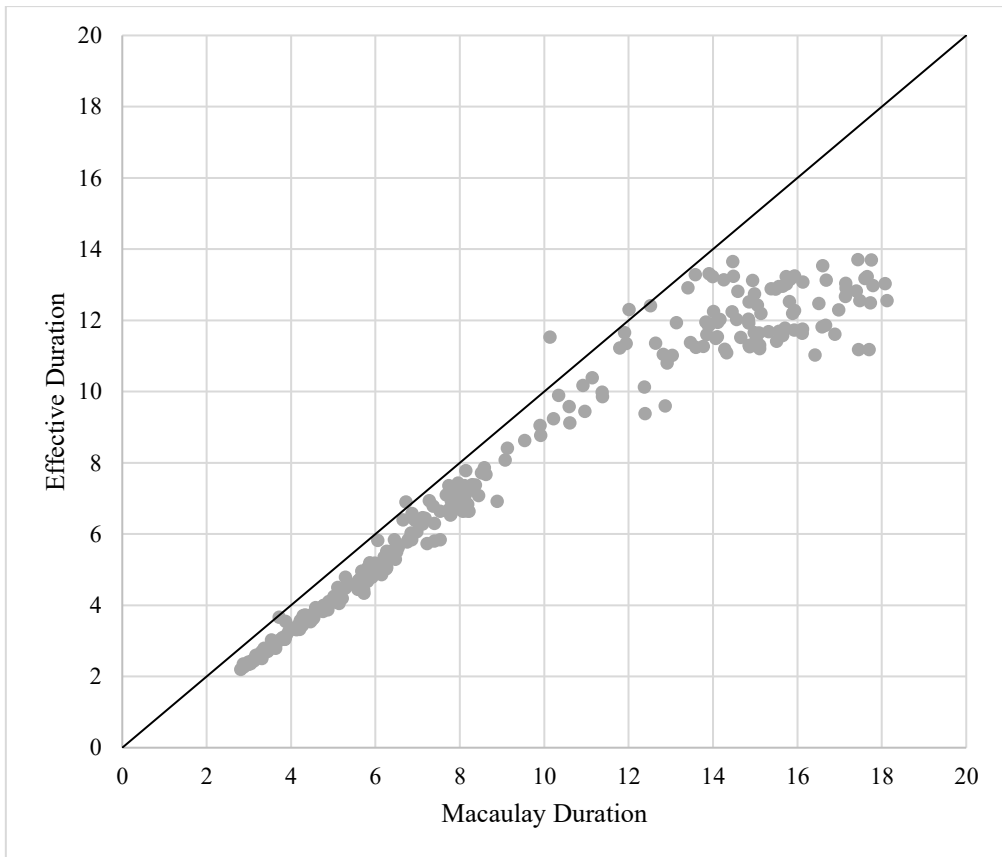


Figure 10 - IGP Effective Duration as a function of Macaulay Duration – December 2022

Observing Figure 9 and 10, it is noteworthy that there is considerable variation or dispersion in the effective duration for bonds with the same Macaulay duration. This can be accounted for by the differing characteristics of the bonds in our two portfolios. Even though two bonds may have the same Macaulay duration, their effective durations may differ due to variations in the credit risk of the issuers, coupons, payment schedules, and other bond-specific factors. The effective duration, which we have adjusted for both increases and decreases in interest rates, reflects these variations more accurately than the Macaulay duration, thus causing the observed dispersion.

Furthermore, dispersion tends to increase with larger Macaulay durations. This tendency can be explained by the higher sensitivity of long-maturity bonds to changes in the default-free interest rate. For longer maturities, changes in the risk-free rate may have a significant impact in the case of bonds with higher credit risk and small impact in the case of bonds with low credit risk. Consequently, the effective duration diverges more significantly from the Macaulay duration for these bonds, causing the observed trend towards greater dispersion.

Such large differences between effective and Macaulay duration highlight a critical issue in bond portfolio management. Traditional immunization techniques and other risk management methods often depend heavily on duration-matching approaches. Yet, our findings suggest that these methods may fall short when applied to corporate bonds, especially those with higher credit risk, as they fail to account for the nuanced impact of default risk on bond price sensitivity to interest rate changes.

6.2. Macaulay Duration vs Effective Duration for the total portfolios

In the previous section of this results discussion, we focused on the individual assessment of each bond within our two portfolios for a specific moment. We compared the Macaulay and effective durations of each bond, taking into account factors like credit risk and yield curve slope. Our approach aimed to provide a comprehensive understanding of how both interest rate changes and default risk influence bond value.

In this section, we follow the same reasoning, but we shift our attention from the individual bonds to the portfolios as a whole. To achieve this, we calculate the average Macaulay and effective durations for the entire portfolios in December 2022. This calculation takes into account the proportional exposure of each bond in the portfolio. This means that we weight the duration of each bond by its representation in the portfolio.

It is important to note that we have assumed an equal weight for all companies within the portfolio. In other words, we have assumed the same level of exposure to each company. This assumption simplifies the computation, as it essentially corresponds to calculating a simple arithmetic mean of the durations of all the bonds present in the portfolio. This approach provides an aggregate measure of the duration of the entire portfolio, which will allow us to better understand the overall interest rate risk and default risk of our portfolios.

Ultimately, our main findings were on average a Macaulay duration of 4.24 years versus an effective duration of 2.92 years for the high-risk portfolio, while the low-risk portfolio displayed a Macaulay duration of 9.04 years in contrast to an effective duration of 7.47 years.

7. Conclusion

This dissertation aimed to investigate the interaction between interest rate risk and credit risk in corporate bonds by comparing the Macaulay duration and effective duration of two corporate bond portfolios using the Leland and Toft (1996) structural credit risk model at a specific point in time - December 2022. The journey towards this research discovery has had the purpose of shedding light on the intricate balance between different types of risk in the bond market. The critical finding of this research is that the effective duration, a measure which incorporates the effect of interest rate changes on credit risk, diverges significantly in a way that is lower than the Macaulay duration, which does not account for this effect. This effect is even more pronounced for bonds with higher credit risk. This discovery is not only academically intriguing but also practically significant for professionals in the field of finance.

As previously stated in the introduction of this study, whenever an increase in interest rates leads to a decrease (increase) in credit risk it is plausible to consider that credit risk mitigates (amplifies) interest rate risk. And in this case, effective duration may actually become far away from Macaulay duration. According to Leland and Toft (1996), higher interest rates lead to lower credit risk, which mitigates the negative impact associated with the higher opportunity cost. As a result of this mechanism, bonds with high credit risk have a lower effective duration.

Significantly, the research findings align with those observed in the Leland and Toft study, providing further evidence for the relevance of their model in understanding and managing the risk in corporate bonds.

These findings underscore the limitations of traditional risk management techniques like immunization, which rely heavily on duration-matching approaches. Such strategies, while useful in some contexts, may prove inadequate when managing corporate bonds, particularly those with higher default risk. By not accounting for the effect of interest rate changes on credit risk, these approaches may have a wrong assessment of the portfolio interest rate risk.

While this dissertation provides compelling insights into the relationship of credit risk and interest rate risk in corporate bonds, it is crucial to highlight some limitations inherent to the methodology. In order to facilitate comparisons with the Leland and Toft (1996) model, several of their base case parameters used. However, it is important to emphasize that some of

these parameters, in particular the risk-free rate, could have been recalibrated to reflect the values observed during the years used to calibrate the model.

Despite these limitations, ultimately, our findings lead us to believe that the Leland and Toft (1996) model, by incorporating both interest rate and credit risk, offers a more sophisticated tool for analyzing and managing the risk of corporate bonds. This model allows us to account for the interplay between interest rate changes and default risk, providing a more accurate measure of the total risk associated with a bond or a bond portfolio at this specific moment in time. The significance of this result can be perceived as a milestone towards developing more robust financial strategies.

In conclusion, understanding the interaction between Macaulay and effective durations is critical in the field of corporate bond management. By applying the Leland and Toft (1996) model to real-world portfolios, this dissertation has shed light on the crucial role that credit risk plays alongside interest rate risk. These insights pave the way towards more robust risk management strategies for corporate bonds, driving our understanding of corporate bond management further.

8. References

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