(Welfare) Effects of Joint versus Independent Bargaining

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This thesis examines the impact of joint bargaining on market outcomes and welfare in comparison to individual negotiations. Using theoretical models rooted in game theory and industrial organization, it introduces the Nash-in-Nash bargaining framework into the Hotelling model for the first time. The analysis considers three distinct scenarios: individual firm price setting, collective consumer bargaining with producers, and collective bargaining on both sides of the market. These scenarios are further investigated under both symmetric and asymmetric cost structures. With respect to the bargaining setup, the findings reveal that negotiating jointly, rather than independently, can improve a party's outcome (surplus), a result that applies to both consumers and producers. Bargaining power primarily influences the distribution of welfare without affecting the total amount of welfare generated. However, the presence of cost asymmetry adds an efficiency dimension to the bargaining process, where greater consumer bargaining power can enhance total welfare by aligning prices and demand allocation more effectively with cost structures. The study contributes to the understanding of bargaining dynamics by offering policy recommendations and identifying pathways for further research, including for example the incorporation of heterogeneous consumer preferences and dynamic negotiation strategies.

Keywords: Nash(-in-Nash), Hotelling, Bargaining (Joint & Independent), Welfare

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Esta tese examina o impacto da negociação conjunta nos resultados do mercado e no bem-estar em comparação com as negociações individuais. Utilizando modelos teóricos enraizados na teoria dos jogos e na organização industrial, introduz pela primeira vez a estrutura de negociação Nash-in-Nash no modelo de Hotelling. A análise considera três cenários distintos: fixação de preços por empresas individuais, negociação colectiva dos consumidores com os produtores e negociação colectiva em ambos os lados do mercado. Estes cenários são investigados mais detalhadamente sob estruturas de custos simétricas e assimétricas. No que diz respeito à configuração da negociação, as conclusões revelam que a negociação conjunta, e não de forma independente, pode melhorar o resultado (excedente) de uma parte, um resultado que se aplica tanto aos consumidores como aos produtores. O poder negocial influencia principalmente a distribuição do bem-estar sem afetar a quantidade total de bem-estar gerado. Contudo, a presença de assimetria de custos acrescenta uma dimensão de eficiência ao processo de negociação, onde um maior poder de negociação do consumidor pode melhorar o bem-estar total, alinhando os preços e a alocação da procura de forma mais eficaz com as estruturas de custos. O estudo contribui para a compreensão da dinâmica de negociação, oferecendo recomendações políticas e identificando caminhos para futuras pesquisas, incluindo, por exemplo, a incorporação de preferências heterogéneas dos consumidores e estratégias de negociação dinâmicas.

Palavras-chave: Nash(-in-Nash), Hotelling, Negociação (conjunta e independente), Bem-estar

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List of Symbols

c	Marginal cost (used for symmetric costs)
c_A, c_B	Marginal costs of Firm A and Firm B (used for asymmetric costs)
δ	Marginal cost difference between firms ($c_A = c_B + \delta$)
t	Transportation cost
v	Reservation utility (maximum willingness to pay of a consumer)
θ	Bargaining power parameter ($\theta = 0$: Firms hold all bargaining power, $\theta = 1$: Consumers hold all bargaining power)
z	Integration variable
p_A^*, p_B^*	Equilibrium prices
p_{bench}^*	Equilibrium price in the symmetric cost model with independent firm pricing
p_{collC}^*	Equilibrium price in the symmetric cost model with consumers bargaining collectively with each of the firms independently
$p_{collBoth}^*$	Equilibrium price in the symmetric cost model with consumers and producers bargaining jointly
$x_{indifferent}$	Indifferent consumer
D_A, D_B	Demands of Firm A and Firm B
π_A, π_B	Profit functions of Firm A and Firm B
CS_A, CS_B	Consumer surplus from consumers purchasing from Firm A (Firm B)
CS	Total consumer surplus
CS_{bench}	Total consumer surplus in the symmetric cost model with independent firm pricing

CS_{collC}	Total consumer surplus in the symmetric cost model with consumers bargaining collectively with each of the firms independently
$CS_{collBoth}$	Total consumer surplus in the symmetric cost model with consumers and producers bargaining jointly
DA_A	Consumer's disagreement payoff for negotiations with Firm A
PS_A, PS_B	Producer surplus Firm A (Firm B)
PS	Total producer surplus
PS_{bench}	Total producer surplus in the symmetric cost model with independent firm pricing
PS_{collC}	Total producer surplus in the symmetric cost model with consumers bargaining collectively with each of the firms independently
$PS_{collBoth}$	Total producer surplus in the symmetric cost model with consumers and producers bargaining jointly
TW	Total Welfare

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Table of Contents

I. Foundations of the Study

A. Introduction

For an extended period, economists have engaged with the subject of welfare and its distribution, as well as the factors that exert an influence on it. While numerous factors influence overall welfare, one of the primary factors affecting welfare, particularly its distribution, is the bargaining structure of potential negotiations between consumers and firms, as well as the bargaining power of the parties involved. The objective of this master's thesis is to examine the impact of joint bargaining on welfare, in comparison to independent bargaining. Joint bargaining is a term used to describe a situation in which prices are negotiated between sellers (firms) and an organization acting on behalf of the buyers (consumers). This study examines the effects of buyer coalitions on price negotiations with firms, both in isolation and in collusion. The objective is to ascertain the welfare effects and distributional impacts of such practices. The topic is particularly pertinent, as it is highly applicable in real-life contexts, such as the healthcare sector. As “in many markets, prices are determined via bilateral negotiations rather than set by one of the sides or via an auction” (Gowrisankaran, Nevo and Town 2015, 172).

For example, in their 2003 study, Capps, Dranove, and Satterthwaite examine the market power of suppliers in option demand markets, with a particular focus on the healthcare sector. The authors demonstrate that intermediaries, such as managed care organizations, leverage their networks to secure more favorable terms for consumers, thereby emphasizing the significance of collective negotiation in enhancing welfare outcomes. Gowrisankaran, Nevo and Town (2015) build on this research, further analyzing the role of bargaining between managed care organizations (MCOs) and hospitals in shaping prices within the healthcare sector, highlighting the broader implications of bargaining dynamics in this industry. Their model elucidates the way joint negotiation affects price sensitivity and outcomes. For instance, the researchers demonstrate that hospital mergers frequently result in elevated prices due to alterations in bargaining leverage, thereby illustrating the substantial impact of negotiation structures on economic welfare. Lewis and Pflum (2016) even demonstrated this phenomenon in hospitals situated in disparate market contexts. The same bargaining dynamics can be observed in other sectors as well, such as digital platforms, including e-commerce marketplaces like Amazon. Amazon exemplifies joint bargaining as an intermediary by negotiating with suppliers “on behalf of customers,” leveraging its position to secure favorable terms that ultimately benefit consumers (Streitfeld and Eddy 2014). Further, Crawford and Yurukoglu's (2012) study offers a valuable framework for understanding the effects of bargaining dynamics on welfare. In their analysis of the cable television industry, intermediaries, such as cable providers, negotiate prices with channel owners on behalf of consumers. This is analogous to the concept of joint bargaining, whereby a coalition or intermediary represents a group to negotiate more favorable terms. The study illustrates how bargaining structures, such as bundling and renegotiation, have a considerable impact on consumer surplus and input costs, underscoring the importance of negotiation dynamics in determining welfare distribution.

Although this research is inspired by real-world applications such as healthcare markets, digital platforms, and the cable television industry, its objective is to derive more general theoretical insights that can be applied across diverse market structures. This thesis employs a two-pronged approach, combining Nash bargaining with spatial competition of the Hotelling model, to enhance our comprehension of coalition-based negotiations and their ramifications for economic welfare.

B. Hotelling model

In his 1929 article, "Stability in Competition", Hotelling introduces a spatial competition model to study the strategic behavior of firms. By incorporating the concept that the relative positioning of firms in relation to consumers influences competition, Hotelling addressed the limitations of earlier models, such as those proposed by Cournot, Bertrand, and Edgeworth, which often failed to incorporate realistic consumer behavior and market dynamics, as he emphasized “the gradualness in the shifting of customers from one merchant to another as their prices vary independently”, a factor “ignored in the examples worked out by Cournot, Amoroso, and Edgeworth” (Hotelling 1929, 44). As realistically, when one firm increases its price while competitors “keep theirs fixed, the diminution in volume of his sales will in general take place continuously rather than in the abrupt way” (Hotelling 1929, 41).

In his model, Hotelling presented a linear market, such as a street, where two firms compete for consumers who are uniformly distributed along the line. The transportation costs incurred by consumers are proportional to their distance from a given firm, which in turn influences their purchasing decisions. The firms set prices and locations independently with the objective of maximizing their profits. The total price paid by a consumer is the firm's price plus the transportation cost.

Although the model is foundational to the study of spatial competition, it is not without notable limitations. To cite but a few of its shortcomings, the model assumes a linear market, which serves to simplify the analysis of real-world markets while simultaneously introducing difficulties regarding the endpoints. Secondly, the model considers only homogeneous products that are differentiated by location, thereby ignoring other forms of product differentiation. Thirdly, the model assumes a fixed demand curve and a lack of price elasticity, which fails to account for scenarios where consumer purchases are dependent on price or utility. Fourth, the Principle of Minimum Differentiation (which was later shown to be invalid) predicts the clustering of firms at the center, a result that is often contradicted by real-world observations. Furthermore, the model treats transportation costs as the sole determinant of consumer choice, overlooking other factors that may influence consumer decisions, such as brand loyalty, quality, or advertising.

Since its initial presentation in 1929, Hotelling's model has constituted a prominent feature of economic literature, undergoing numerous extensions and applications. In the process, many of its shortcomings have been addressed and refined in subsequent research.

To extend the spatial competition, Salop (1979) introduces the circular market model, addressing the edge effects of Hotelling's linear market. D'Aspremont, Gabszewicz, and Thisse (1979) “show that the so-called Principle of Minimum Differentiation […] is invalid” (D'Aspremont, Gabszewicz, and Thisse 1979, 1145). Anderson, de Palma, and Thisse (1992) and Tirole (1988) further examine Hotelling's model by incorporating elastic demand, thereby addressing one of its limitations. Moreover, Chamberlin (1933) introduced the concept of monopolistic competition, building on Hotelling's insights to examine the impact of product differentiation on market structure and entry. There are numerous additional applications of the Hotelling model, including multi-dimensional and generalized models, specific applications, and empirical studies.

Despite some of its limitations, the Hotelling model offers a straightforward yet powerful framework for analyzing market dynamics, making it highly relevant to real-world applications. Its mathematical simplicity provides a solid foundation for incorporating advanced analytical tools, such as Nash bargaining, enabling deeper exploration of negotiation outcomes within competitive markets.

C. Bargaining

The concept of bargaining underwent a profound transformation at the hands of John F. Nash, as evidenced by his seminal paper, "The Bargaining Problem," published in 1950. Nash developed an axiomatic framework for resolving a two-person bargaining problem, wherein two rational agents engage in negotiations with the objective of dividing a surplus in a manner that is mutually beneficial to both parties. “The Nash bargaining problem is a function of the value to each party from agreement relative to the value without agreement, and hence depends on the objective functions of the parties” (Gowrisankaran, Nevo & Town 2015, 173). The Nash bargaining solution is the one that maximizes the weighted product of the parties' gains from bargaining. It has been demonstrated that this bargaining solution is the only one that satisfies four fundamental axioms. The solution is Pareto-efficient, invariant to equivalent payoff representations, independent of irrelevant alternatives, and, in the case of equal weights, symmetric. This makes it an ideal tool for the simpler negotiation analysis presented in this thesis.

Building on this initial framework, Nash extended his analysis to cooperative game theory in his 1953 paper, "Two-Person Cooperative Games." Collectively, these two papers established the mathematical and theoretical foundations for bargaining theory. They formalized how two parties with potentially conflicting interests can collaborate to divide a surplus efficiently and fairly, serving as the cornerstone of modern bargaining theory.

Following Nash's foundational work (1950, 1953), Rubinstein's (1982) alternating-offers model demonstrated how the Nash solution emerges dynamically when time preferences are included. Binmore, Rubinstein, and Wolinsky (1986) further bridged static and dynamic bargaining models, showing how equilibrium outcomes converge to the Nash solution given “the parties' impatience and the parties' fear of breakdown of the negotiations” (Binmore, Rubinstein, and Wolinsky 1986, 187). Collectively, these contributions established a definitive connection between strategic bargaining processes and Nash's axiomatic framework, paving the way for subsequent developments such as Nash-in-Nash bargaining.

This thesis will also analyze the more complex case where consumers bargain collectively with two firms at the same time, i.e., address scenarios where multiple, interdependent negotiations occur simultaneously. In this case “interdependence means that the consequences to a firm of taking an action depend not just on that firm's action, but also on what actions its competitors take” (Moorthy 1985, 262). To derive a solution, it will be necessary to incorporate the concept of Nash-in-Nash bargaining, initially introduced by Horn and Wolinsky (1988) in their seminal paper, "Bilateral Monopolies and Incentives for Merger." They investigated a scenario in which upstream suppliers negotiated simultaneously with downstream firms in bilateral oligopolies, emphasizing how the result of each negotiation affected the disagreement payoffs in the others. As they explain, “we combine a bargaining model with a duopoly model to examine how input prices and profits are affected by the structures of the upstream and downstream industries, by the demand relations among the final products, and by the nature of bargaining between suppliers and firms” (Horn and Wolinsky 1988, 408). Their “bargaining solution is a set of transfer prices between ‘upstream' and ‘downstream' firms in which the price negotiated between any pair of firms is the Nash bargaining solution (Nash 1950) for that pair given that all other pairs reach agreement” (Collard-Wexler, Gowrisankaran, and Lee 2019, 165). By integrating elements of bargaining theory with a standard duopoly framework, their approach becomes highly relevant and insightful for the context of this thesis. However, whereas their model focuses on how the structure of upstream and downstream industries affects input prices, profits, and the incentives for mergers, this thesis will address the implications of horizontal bargaining, focusing on interactions between consumers and producers and their impact on welfare and equilibrium. To this end, it will integrate the technique of Nash(-in-Nash) bargaining in the Hotelling model.

Although Horn and Wolinsky's work did not explicitly formalize Nash-in-Nash bargaining, it established the foundation for analyzing interdependent bargaining dynamics by focusing on the structural interdependencies between multiple bilateral negotiations. The formalization and explicit naming of Nash-in-Nash bargaining occurred subsequently, with the publication of "Nash-in-Nash Bargaining - A Microfoundation for Applied Work " by Collard-Wexler, Gowrisankaran, and Lee (2019) . They named it Nash-in-Nash solution, as the “solution can be cast as a ‘Nash equilibrium in Nash bargains'—that is, separate bilateral Nash bargaining problems within a Nash equilibrium to a game played among all pairs of firms” (Collard-Wexler, Gowrisankaran, and Lee 2019, 165). By extending “Rubinstein (1982) alternating offers game to bilateral oligopoly” and proving that “a Nash-in-Nash limit equilibrium exists”, they “provide support for the Nash-in-Nash solution as a credible bargaining framework […|” (Collard-Wexler, Gowrisankaran, and Lee 2019, 192).

Extensions of Nash bargaining have investigated its applicability in complex market structures and multi-party negotiation settings. In their 2007 study, Dobson and Waterson examine the bargaining outcomes in vertical oligopolies with differentiated products, emphasizing how bargaining power imbalances influence market outcomes and welfare in bilateral negotiations. In a further development of Nash bargaining theory, Escrihuela-Villar, Ferrarese, Iozzi and Tolvanen (2022) introduce the concept of size effects and demonstrate how an increase in bargaining power may, counterintuitively, result in a reduction in a party's payoff. These insights provide a foundation for understanding interdependent negotiations and the distribution of surplus in spatial and collaborative bargaining contexts, which are central to this thesis.

In their 1999 paper, "The Role of Firm Size in Bilateral Bargaining", Chipty and Snyder demonstrate the applicability of Nash bargaining to empirical contexts. The study, entitled "A Study of the Cable Television Industry," examines the impact of firm size on bargaining outcomes within the context of a bilateral monopoly framework. The authors employ the U.S. cable television industry as a case study to investigate the influence of the relative size of buyers (cable operators) and sellers (broadcasters) on input prices and profitability. They utilize a bargaining model based on Nash's solution, incorporating factors such as bargaining power asymmetries and outside options.

In sum, these contributions establish Nash-in-Nash bargaining as a powerful tool for understanding interdependent negotiations, as "the Nash-in-Nash solution provides easily computable payments for complicated environments with interdependencies" (Collard-Wexler, Gowrisankaran, and Lee 2019, 165).

D. Research Approach

The choice to combine Nash-in-Nash bargaining with the Hotelling model is based on how well each framework handles the complexities of negotiation and market competition. The Nash-in-Nash approach is well-suited to the analysis of collective bargaining scenarios, wherein the outcomes of one negotiation can influence those of another. This is due to its ability to capture the interdependencies across multiple bilateral negotiations. The Hotelling model, with its focus on spatial competition and consumer behavior, provides a complementary perspective by incorporating how market positioning and price sensitivity shape firm strategies and consumer welfare. Together, these frameworks provide a robust analytical tool for investigating the interplay between bargaining structures, market dynamics, and welfare distribution, integrating theoretical insights with real-world applications.

This thesis employs the Nash(-in-Nash) bargaining framework to calculate equilibrium prices for horizontal joint bargaining situations within the Hotelling model, introducing two novel scenarios. At the outset, it was uncertain whether this approach would yield meaningful results. However, equilibrium prices are successfully derived for two distinct joint bargaining scenarios. The first considers consumers collectively bargaining with two firms that act independently, and the second examines a setting where both consumers and firms engage in collective bargaining. For comparison, a benchmark scenario is included in which firms unilaterally set prices without negotiation—an extensively studied and well-established model.

The mathematical derivation of equilibrium prices is conducted for two cases: (1) symmetric costs and (2) asymmetric costs. These prices form the basis for analyzing welfare outcomes in joint bargaining situations, shedding light on the interplay between bargaining power, cost structures, and market dynamics. This thesis makes a meaningful contribution to the Hotelling model by introducing two newly calculated scenarios with explicitly derived equilibrium prices, addressing cases that have not been previously formalized and extending the framework in a novel direction.

II. Symmetric cost model

A. Benchmark model

This study employs a simplified version of the Hotelling model as a foundational case for analyzing price-setting by firms in the absence of bargaining. In this benchmark scenario, each firm independently sets prices (p_A and p_B) to maximize profits, and consumers choose based on these posted prices. The fundamental tenets of this model are consistent with the classical Hotelling setup. The total number of consumers is normalized to 1. Consumers are distributed uniformly along a linear market, from zero to one, and transportation costs (t) are proportional to the distance between consumers and firms. Additionally, products are homogeneous, aside from their spatial differentiation. The two firms are identical in all respects except for their respective positions: firm A is located at position zero, while firm B is located at position 1. The consumer's reservation utility is given by (v) which denotes the maximum willingness to pay or the baseline utility that a consumer derives from purchasing a product (excluding transportation costs and price). The value of v is assumed to be sufficiently high to ensure a covered market, meaning all consumers participate and purchase from the available options. The minimum reservation utility inequality will be outlined for each model and case, as needed.

To derive the demand function and consequently the firm's profit function, we need to derive the indifferent consumer which is defined as the individual whose net utility from purchasing from either firm is equal. This point (x) defines the market share boundary between the two competing firms, as it marks the point at which consumers are equally satisfied with either option:

$$v - p_A - t \cdot x = v - p_B - t \cdot (1 - x)\,;$$

$$x_{indifferent} = \frac{p_B + t - p_A}{2t}\,.$$

In accordance with the value x of the indifferent consumer, the demand for the two firms is derived as follows:

$$D_A = \frac{p_B + t - p_A}{2t}\,;$$

$$D_B = 1 - \frac{p_B + t - p_A}{2t}\,.$$

The profit function of the firms can be expressed as follows:

$$\pi_A = \frac{(p_A - c)(p_B + t - p_A)}{2t} \, ;$$

$$\pi_B = (p_B - c)\left(1 - \frac{p_B + t - p_A}{2t}\right) .$$

Each firm maximizes its profit with regard to its own price:

$$\frac{\partial \pi_A}{\partial p_A} = 0 \xrightarrow{isolate\ for\ p_A} p_A = \frac{p_B}{2} + \frac{t}{2} + \frac{c}{2} \, ;$$

$$\frac{\partial \pi_B}{\partial p_B} = 0 \xrightarrow{isolate\ for\ p_A} p_B = \frac{p_A}{2} + \frac{t}{2} + \frac{c}{2} \, .$$

Solving this system of equations delivers the price that both firms will choose:

$$p_A^* = p_B^* = t + c \, .$$

The results of the benchmark model reveal that both firms set the same equilibrium price, $p_A^* = p_B^* = t + c$. This outcome is driven by the interplay of transportation costs (t) and the marginal production costs (c), which together determine the optimal pricing strategy for each firm. Since the firms are symmetrically positioned along the linear market and consumers are uniformly distributed, they share the market equally when prices are identical.

The inclusion of transportation costs reflects the cost consumers face based on their distance from a firm. Higher transportation costs (t) reduce the price sensitivity of consumers, as their decision to buy is influenced more by location than by small price differences. This allows firms to incorporate (t) into their pricing without losing demand. Additionally, the marginal cost of production (c) forms the baseline price, ensuring that each firm covers its costs.

Given the symmetrical setup of the model, the indifferent consumer is located at the midpoint of the market, at 0.5. Using the equilibrium prices, consumer surplus is calculated by combining the surplus of consumers purchasing from Firm A (CS_A), covering the range from 0 to 0.5, and those purchasing from Firm B (CS_B), spanning from 0.5 to 1:

$$CS = CS_A + CS_B = \int_0^{0.5} v - p_A^* - t \cdot z \, dz + \int_{0.5}^{1} v - p_B^* - t \cdot (1 - z) \, dz$$

$$= v - c - 1.25t \, .$$

Following this logic, producer surplus is obtained by summing the profits of Firm A and Firm B:

$$PS = PS_A + PS_B = \left(p_A^* - c\right) \cdot \left(\frac{1}{2}\right) + \left(p_B^* - c\right) \cdot \left(1 - \frac{1}{2}\right)$$

$$= t\,.$$

Consequently, total welfare can be expressed as:

$$TW = CS + PS = v - c - 0.25t\,.$$

As observed, total welfare is determined by the minimum reservation utility, marginal costs, and transportation costs. A detailed analysis of consumer surplus, producer surplus, and total welfare is provided in the subsequent chapters.

B. Model with joint bargaining on consumer's side

Consider now the model with collective bargaining, where consumers are represented by a single bargaining entity that negotiates prices with each firm on their behalf. This entity helps establish the prices that will be offered to consumers, who can then freely choose their preferred supplier. This structural change fundamentally alters the dynamics of price-setting and the distribution of welfare.

To derive the prices in this setup, it is necessary to maximize the Nash product with respect to the price. The Nash product for the bargaining between firm A and the entity bargaining on behalf of the consumers can be expressed as follows:

$$(CS_A + CS_B - DA_A)^{\theta} \cdot (PS_A - 0)^{1-\theta}\,.$$

Since both firms are symmetrical, it is sufficient to perform the calculations for one negotiation—in this case, the bargaining between the consumer coalition and Firm A. The Nash product incorporates the surplus of both parties in the event of a successful agreement: the producer surplus for Firm A (PS_A) and the total consumer surplus, which includes consumers purchasing from both Firm A and Firm B ($CS_A + CS_B$). Additionally, the Nash product accounts for the disagreement payoffs, representing the fallback utilities for each party if the negotiation fails. In this framework, it is assumed that if the negotiation fails, no consumers purchase from Firm A, and all consumers instead purchase Firm B's product. Consequently, the disagreement payoff for Firm A is zero. A distinctive feature of this Nash product is the incorporation of the parameter Θ, which serves to determine the bargaining power of the two parties involved. Specifically, Θ in (0,1), where (Θ = 0) indicates that firms hold all the bargaining power, while (Θ = 1) signifies that consumers have full bargaining power.

As in the initial model, the expression for the indifferent consumer is derived to calculate the consumer surpluses:

$$x_{indifferent} = \frac{p_B + t - p_A}{2t} .$$

To calculate the consumer surplus derived from the portion of consumers purchasing from Firm A (CS_A), we compute the integral over the range of consumers who choose Firm A, which spans from 0 to $x_{indifferent}$:

$$CS_A = \int_0^{x_{indifferent}} (v - p_A - t \cdot z)\, dz$$

$$= -\frac{(p_B + t - p_A)(p_B + t + 3p_A - 4v)}{8t} .$$

In the same manner, we calculate the consumer surplus for the portion of consumers purchasing from Firm B (CS_B) by integrating over the range of consumers choosing Firm B, which spans from $x_{indifferent}$ to 1:

$$CS_B = \int_{x_{indifferent}}^{1} v - p_B - t \cdot (1 - z)\, dz$$

$$= -\frac{(t - p_B + p_A)(p_A + 3p_B + t - 4v)}{8t} .$$

In the event of an unsuccessful negotiation, it is assumed that all consumers would purchase from Firm B. This would result in the following disagreement payoff (DA_A):

$$DA_A = \int_0^1 v - p_B - t \cdot (1 - z)\, dz$$

$$= -\frac{t}{2} - p_B + v .$$

Regarding Firm A, the producer surplus (PS_A) can be calculated as follows:

$$PS_A = (p_A - c) \cdot x_{indifferent}$$

$$= \frac{(p_A - c)(p_B + t - p_A)}{2t}.$$

To determine the equilibrium prices, it is necessary to maximize the Nash product capturing the joint surplus of both parties. This necessitates the balancing of the producer surplus for the firm and the total consumer surplus, while accounting for the disagreement payoffs.

$$\frac{\partial (CS_A + CS_B - DA_A)^{\theta} \cdot (PS_A - 0)^{1-\theta}}{\partial p_A} = 0$$

$$\rightarrow p_A = \frac{1}{2}\Theta c - \frac{1}{2}\Theta p_B - \frac{1}{2}\Theta t + \frac{1}{2}c + \frac{1}{2}p_B + \frac{1}{2}t$$

Given the symmetrical configuration of our model, the price maximizing the Nash product of the bargaining between Firm B and the entity representing consumers is as follows:

$$\rightarrow p_B = \frac{1}{2}\Theta c - \frac{1}{2}\Theta p_A - \frac{1}{2}\Theta t + \frac{1}{2}c + \frac{1}{2}p_A + \frac{1}{2}t.$$

The solution to the system of equations yields the equilibrium prices, which are as follows:

$$p_A^* = p_B^* = \frac{\Theta c - \Theta t + c + t}{1 + \Theta} = c + \frac{1 - \Theta}{1 + \Theta}t.$$

To ensure that all consumers purchase the product at the equilibrium prices, it is necessary to verify that the consumer who is worst off, namely the indifferent consumer at x = 0.5, still derives a non-negative net utility. As this consumer is located furthest from both firms, their utility serves as the critical threshold for market participation:

$$v - p_A^* - t \cdot \left(\frac{1}{2}\right) \geq 0.$$

Rearranging this inequality yields a restriction on the minimum reservation utility v, which ensures that even the worst-off consumer is willing to buy the product:

$$\rightarrow \frac{\Theta c - \Theta t + c + t}{1 + \Theta} + \frac{t}{2} \leq v.$$

This condition establishes the lower bound for v, ensuring that the market remains fully covered, and all consumers choose to participate despite their transportation costs and distance from the firms.

A similar check is required for the scenario in which negotiations fail, as this introduces a new condition to ensure that all consumers still derive a non-negative net utility. This case imposes an even stricter requirement on the minimum reservation utility v:

$$v - p_A^* - t \geq 0\,;$$

$$\rightarrow \frac{\Theta c - \Theta t + c + t}{1 + \Theta} + t \leq v\,.$$

The equilibrium prices are determined by a weighted combination of the marginal cost (c), the transportation cost (t) and the bargaining power parameter (Θ). It can be interpreted as a weighted average of the consumer-optimal price (c) and the firm's equilibrium price (c + t), with the latter incorporating the full transportation cost. This demonstrates how bargaining power determines the extent to which the price aligns with the preferences of either consumers or producers.

The derivative $\frac{6pA^*}{6c} = 1 > 0$, shows that the equilibrium price p_A^* or p_B^* increases proportionally to the change in marginal cost (c). This is expected because (c) directly affects the firm's production costs, which are reflected in the price. If marginal cost increase by 1 unit, the price also increases by 1 unit, regardless of (Θ).

The result $\frac{6pA^*}{6t} = \frac{1-\Theta}{1+\Theta} \geq 0$, represents the change in price with respect to transportation cost (t). The change is contingent upon the relative bargaining power of the parties involved. If firms hold all the bargaining power, i.e. Θ=0, the price fully reflects changes in transportation cost. With higher transportation costs consumers are less likely to switch to competitors, making demand less elastic. When t is higher, consumers are less responsive to price differences, allowing firms to charge higher prices.

If consumers hold all the bargaining power, i.e. Θ=1, transportation cost changes have no effect on prices, as consumers prevent firms from factoring t into the prices during bargaining. For $0 < \Theta < 1$, the rate of change of prices lies between 0 and 1, depending on the relative bargaining power. The closer Θ is to 1 (stronger consumer bargaining power), the smaller the fraction of transportation costs is incorporated into the prices. In conclusion, the price of a product will increase in line with the costs of transportation, but the rate of increase will be reduced as consumers gain greater bargaining power. This is consistent with the hypothesis that more powerful consumer coalitions can negotiate more favorable terms.

As shown by $\frac{\partial p_A^*}{\partial \Theta} = -\frac{2t}{(1+\Theta)^2} \leq 0$, as Θ increases (meaning consumers gain more bargaining power), the equilibrium prices decrease, reflecting that stronger consumer coalitions can negotiate lower prices from firms. The magnitude of the effect is contingent upon the transportation cost (t). The negative impact of the parameter Θ on prices is amplified when transportation costs are high. If these were zero, price competition between the two firms would be so aggressive that, regardless of consumers' bargaining power, price would equal marginal cost. The squared term $(1 + \Theta)^2$ in the denominator indicates that as Θ increases, the impact of further increases in consumer bargaining power is reduced. In other words, the marginal effect of bargaining power on prices is reduced as consumers already hold substantial power.

Moreover, in the absence of consumer coalition bargaining power ($\Theta = 0$), the equilibrium prices observed are the same as in the benchmark model ($p_A^* = p_B^* = t + c$). This outcome is consistent with our expectations. When consumers engage in independent bargaining with firms that hold significant market power, they typically lack the ability to influence price outcomes through negotiation.

With equilibrium prices p_A^* and p_B^* established, the subsequent step is to analyze the implications for the distribution of welfare among consumers and firms. Given the symmetry of the setup, the indifferent consumer remains at 0.5, and firms A and B share the market equally. Accordingly, the total consumer surplus (CS) can be calculated as follows:

$$CS = \int_0^{0.5} v - p_A^* - t \cdot z dz + \int_{0.5}^{1} v - p_B^* - t \cdot (1 - z)\, dz$$

$$= \frac{(0.75t - c + v)\Theta - 1.25t - c + v}{1 + \Theta}.$$

Subsequently, the total producer surplus (PS) is defined as follows:

$$PS = \left(p_A^* - c\right) \cdot \left(\frac{1}{2}\right) + \left(p_B^* - c\right) \cdot \left(1 - \frac{1}{2}\right)$$

$$= \frac{t(1 - \Theta)}{1 + \Theta}.$$

The consumer surplus (CS) and producer surplus (PS) serve to illustrate the influence of bargaining power (θ) on the distribution of welfare. As the bargaining power of consumers increases, consumer surplus rises because they can negotiate more favorable prices ($\frac{\partial CS}{\partial \Theta} =$

$\frac{2t}{(1+\Theta)^2} \geq 0$). Conversely, the producer surplus is diminished as firms forfeit surplus due to lower negotiated prices ($\frac{6PS}{6\Theta} = -\frac{2t}{(1+\Theta)^2} \leq 0$). This redistribution illustrates how the formation of more powerful consumer coalitions results in a reallocation of welfare from firms to consumers.

Interestingly, the total welfare (TW), defined as the sum of consumer and producer surplus, is independent of the bargaining power parameter (Θ):

$$TW = CS + PS = v - c - 0.25t\,.$$

This is because any increase in consumer surplus is precisely balanced by a corresponding reduction in producer surplus. Consequently, while bargaining power exerts an influence on the distribution of welfare, it does not affect the overall efficiency of the market. These finding highlights that the function of bargaining power is to establish equity (the distribution of welfare) rather than to ensure efficiency (the total welfare generated). This is a direct consequence of the assumption that demand is rigid, that the market is covered and that firms are symmetric. Prices merely divide total welfare, without affecting total output or transportation costs,

Transportation costs (t) play a significant role in determining total welfare. The negative term $-\ 0.25t$ in the total welfare equation indicates that higher transportation costs reduce overall welfare, as these costs represent a form of inefficiency or deadweight loss. Transportation costs act as a friction in the market, preventing a more efficient allocation of goods and reducing the utility that could otherwise be generated. If transportation costs were zero, consumers could choose products purely based on price and quality, maximizing their utility without geographical constraints. In an alternative interpretation, the transportation cost can be interpreted as a disutility cost that consumers face when purchasing a version of the product that is not the one, they prefer the most. No transportation costs would simply mean that all consumers would be purchasing their preferred version of the product, with positive consequences on welfare. This highlights the importance of minimizing transportation costs to enhance market efficiency. On the other hand, total welfare increases with v, the reservation utility of consumers, showing that higher perceived value of the product improves overall market outcomes.

In conclusion, while the equilibrium prices already provided insights into the effect of bargaining power, the welfare analysis enriches the understanding by showing how bargaining redistributes welfare without affecting total efficiency. Furthermore, the analysis underscores the significance of transportation costs and consumer valuation in influencing total welfare. These findings indicate that while bargaining primarily affects equity, reducing transportation costs directly improves overall market efficiency.

C. Model with joint bargaining on both sides

Now consider a scenario where both consumers and firms engage in collective bargaining. This fundamentally changes the structure of the Nash product. For consumers, the total consumer surplus in the case of a successful negotiation remains unchanged, i.e. $(CS_A + CS_B)$. However, because consumers are now bargaining collectively with both firms rather than negotiating with them independently, the breakdown of negotiations results in a significant change: consumers no longer have the option to switch to the alternative firm. Instead, they are left unable to purchase any product, resulting in a disagreement payoff of zero.

On the firm side, the optimization shifts from maximizing the producer surplus of an individual firm, i.e. (PS_A), to maximizing the total producer surplus of both firms collectively, i.e. $(PS_A + PS_B)$. Like consumers, if negotiations fail, the firms collectively face a disagreement payoff of zero.

With x still representing the indifferent consumer, as derived earlier, the Nash product in this scenario is adjusted to reflect the collective bargaining framework. It is structured as follows:

$$\text{Nash}-\text{product} = (CS_A + CS_B - 0)^{\Theta} \cdot (PS_A + PS_B - 0)^{1-\Theta}$$

$$= \left(\int_0^{x_{indifferent}} v - p_A - t \cdot z \, dz + \int_{x_{indifferent}}^{1} v - p_B - t \cdot (1 - z) \, dz\right)^{\Theta}$$

$$\cdot \left((p_A - c) \cdot x_{indifferent} + (p_B - c) \cdot (1 - x_{indifferent})\right)^{1-\Theta}.$$

Solving the system of first-order conditions $\frac{\delta Nash-product}{\delta p_A} = 0$ and $\frac{\delta Nash-product}{\delta p_B} = 0$, the equilibrium prices are derived as follows:

$$p_A^* = p_B^* = \Theta c + \frac{1}{4}\Theta t - \Theta v - \frac{1}{4}t + v = \Theta c + (1 - \Theta)\left(v - \frac{1}{4}t\right).$$

To ensure that all consumers purchase the product at the equilibrium prices, it is necessary to verify that the consumer who is worst off, namely the indifferent consumer at x = 0.5, still derives a non-negative net utility. As this consumer is located furthest from both firms, their utility serves as the critical threshold for market participation:

$$v - p_A^* - t \cdot \left(\frac{1}{2}\right) \geq 0 \,.$$

Rearranging this inequality yields a restriction on the minimum reservation utility v, which ensures that even the worst-off consumer is willing to buy the product:

$$v \geq c + \frac{1}{4} \cdot t + \frac{1}{4} \cdot \frac{t}{\Theta} \,.$$

The equilibrium prices are determined by a weighted combination of the marginal cost (c), the transportation cost (t), the consumer's reservation utility (v) and the bargaining power parameter (Θ). The price represents a balance between the consumer-optimal price (c) and the firm-optimal price ($v - \frac{1}{4}t$), which corresponds to the consumer's willingness to pay adjusted for the average transportation cost. The bargaining power parameter Θ determines the outcome of the price, deciding whether it aligns more closely with the firm-optimal price ($v - \frac{1}{4}t$), the consumer-optimal price (c), or a weighted mix of the two.

The result $\frac{\delta p_A^*}{\delta c} = \Theta \geq 0$ indicates that marginal production costs (c) continue to have a positive influence on prices, but no longer in a proportional manner as seen in the case of independent firm pricing. The factor Θ acts as a weight, showing that the extent to which c affects prices depends on the relative bargaining power of the parties.

In the case of consumers having full bargaining power (Θ=1), prices increase fully and proportionally with marginal costs c. Consumers, as the dominant party in the negotiation, ensure that the price reflects only the true production cost. They push back against firms adding any extra margin unrelated to c. Effectively, the price becomes cost-plus, with firms setting prices just high enough to cover production costs without adding additional surplus for themselves.

With firms having full bargaining power (Θ=0), prices are completely insensitive to changes in marginal costs. When firms dominate the negotiation, they collectively set prices based on the maximum willingness to pay (reservation utility, v) rather than marginal costs. Firms are no longer constrained by c, as they can extract surplus from consumers without necessarily justifying their pricing based on cost. This shields prices from fluctuations in c, allowing firms to prioritize profit maximization over cost-based pricing.

Prices are no longer directly tied to c because the Nash product balances both collective consumer surplus and producer surplus, with the influence of c weighted by Θ.

The derivative $\frac{6pA^*}{6t} = \frac{\theta}{4} - \frac{1}{4} \leq 0$ shows, when producers dominate negotiations (Θ=0), they strategically lower prices as transportation costs increase. This counterintuitive behavior reflects producers' efforts to ensure market coverage, as higher transportation costs could discourage consumers from purchasing. To maintain demand, producers absorb some of the burden by lowering prices slightly.

When consumers dominate (Θ=1), they ensure that prices are unaffected by transportation costs. This effectively prevents producers from using the reduced demand elasticity caused by transportation costs to increase prices, maintaining stable prices regardless of t.

For intermediate bargaining power, prices respond moderately to transportation costs. As Θ increases, the influence of t diminishes, reflecting the growing ability of consumers to negotiate prices that do not factor in transportation inefficiencies.

The marginal impact of the consumer's reservation utility is given by $\frac{6pA^*}{6v} = 1 - \Theta \geq 0$. With total firm bargaining dominance (Θ=0), prices increase proportionally with v as firms take full advantage of consumers' willingness to pay. They set prices that align with the reservation utility (v), capturing as much consumer surplus as possible.

When consumers dominate (Θ=1), the derivative becomes zero, meaning v has no impact on prices. Consumers have full bargaining power and negotiate prices independently of their reservation utility. Prices are determined by costs rather than willingness to pay, preventing firms from extracting additional surplus based on v.

The result $\frac{6pA^*}{6\Theta} = c + \frac{t}{4} - v$ is ≤ 0 as long as the minimum reservation inequality, defined earlier, holds. As expected, as bargaining power shifts toward consumers (Θ increases), prices decrease.

Following a similar approach to the earlier model, we derive the total consumer surplus (CS) and producer surplus (PS) as follows:

$$CS = \int_{0}^{0.5} v - p_A^* - t \cdot z dz + \int_{0.5}^{1} v - p_B^* - t \cdot (1 - z)\, dz$$

$$= -\Theta c - \frac{1}{4}\Theta t + \Theta v\,;$$

$$PS = \left(p_A^* - c\right) \cdot \left(\frac{1}{2}\right) + \left(p_B^* - c\right) \cdot \left(1 - \frac{1}{2}\right)$$

$$= \Theta c + \frac{1}{4}\Theta t - \Theta v - c - \frac{1}{4}t + v\,.$$

In this model, when firms bargain collectively and hold full bargaining power, they can extract the entire surplus, leaving consumers with no surplus at all. Similarly, when consumers bargain collectively and hold full bargaining power, they are able to capture the entire surplus, leaving the producers with no surplus at all.

Total welfare (TW) therefore is:

$$TW = CS + PS = v - c - 0.25t\,.$$

The welfare analysis shows that bargaining power redistributes welfare between consumers and producers but does not affect total efficiency. Transportation costs reduce overall welfare as a market inefficiency, while higher reservation utility (v) improves market outcomes. Reducing transportation costs directly enhances efficiency, highlighting their significance in welfare outcomes.

D. Comparative Analysis

To compare the equilibrium prices of the three models, it is necessary to analyze how the different bargaining structures—independent pricing, consumer collective bargaining, and joint bargaining by both sides—affect the role of costs (c), transportation costs (t), reservation utility (v), and bargaining power (θ) in determining the equilibrium prices:

$$\text{Benchmark model: } p_{bench}^* = t + c\,;$$

$$\text{Model with joint bargaining on consumer's side: } p_{collC}^* = c + \frac{1 - \Theta}{1 + \Theta}t\,;$$

$$\text{Model with joint bargaining on both sides: } p_{collBoth}^* = \Theta c + (1 - \Theta)\left(v - \frac{1}{4}t\right).$$

To structure this analysis, three special cases will be examined: firstly, when consumers hold all the bargaining power (Θ=1); secondly, when firms hold all the bargaining power (Θ=0); and finally, when bargaining power is equally distributed (Θ=0.5).

In the first case (Θ=1), the prices from the model with joint bargaining on the consumer side and the model with joint bargaining on both sides simplify to $(p_{collC}^{*} = c)$ and $(p_{collBoth}^{*} = c)$. Regardless of the bargaining structure of firms, consumers collectively negotiate the lowest price possible, leading to prices equal to the marginal cost c, ensuring that firms do not incur losses.

Next, for the case of (Θ=0), the prices from the model with joint bargaining on the consumer side and the model with joint bargaining on both sides simplify to $(p_{collC}^{*} = c + t)$ and $(p_{collBoth}^{*} = v - \frac{1}{4}t)$. For the case of joint bargaining on the consumer's side, when firms hold all the bargaining power (Θ = 0), they can effectively set prices to maximize their profits. This results in equilibrium prices identical to those in the benchmark model, where firms unilaterally determine prices without any negotiation constraints. If both sides bargain jointly, firms fully capitalize on consumers' reservation utility (v) while partially adjusting for transportation costs (t), demonstrating their dominance in extracting surplus.

When bargaining power is equally distributed (Θ=0.5), the outcomes of the two models reflect a balance between consumer and producer influence. In the consumer-side model $(p_{collC}^{*} = c + \frac{1}{3}t)$, the price is primarily cost-based, with transportation costs (t) playing a reduced role due to the balanced negotiation dynamics. In the both-sides model $(p_{collBoth}^{*} = \frac{1}{2}v + \frac{1}{2}c - \frac{1}{8}t)$, the price incorporates both the reservation utility (v) and production costs (c), while reflecting a partial adjustment for transportation inefficiencies $(-\frac{1}{8}t)$. Equally distributed bargaining power ensures a more balanced distribution of surplus between consumers and producers, demonstrating that shared bargaining power results in more evenly distributed outcomes compared to scenarios of unilateral dominance.

By comparing prices across the different bargaining structures, we can observe their impact on equilibrium prices and, consequently, the distribution of welfare. First, the equilibrium price p_{collC}^{*} is consistently lower than the equilibrium price in the benchmark model $(p_{collC}^{*} < p_{bench}^{*})$, except for the extreme case where firms have full bargaining power $(p_{collC}^{*} = p_{bench}^{*} \text{ for } \Theta = 0)$. This highlights how an entity negotiating on behalf of consumers strengthens their position and increases their surplus. Similarly, when comparing $p_{collC}{}^{*}$ with $p_{collBoth}{}^{*}$, we find that prices are consistently higher when firms bargain collectively rather than independently ($p_{collC}{}^{*} < p_{collBoth}{}^{*}$), except for full consumer bargaining power, where they are equal ($p_{collC}{}^{*} = p_{collBoth}{}^{*}$ for $\Theta = 1$). This outcome, which holds when v is sufficiently high as implied by the minimum thresholds, underscores how collective bargaining strengthens firms by enhancing their negotiating position. Concluding, we can remark that the bargaining structure plays a crucial role in influencing prices and illustrates how a party can secure advantages through joint bargaining compared to independently negotiating.

Moving forward to analyze welfare, it has already been established that total welfare remains the same across all three models, regardless of the parameters, including bargaining power. However, a closer examination of the distribution of welfare between consumer surplus (CS) and producer surplus (PS) provides deeper insights into how bargaining dynamics shape the allocation of benefits:

$$\text{Benchmark model: } \begin{array}{c} CS_{bench} = v - c - 1.25t \\ PS_{bench} = t \end{array};$$

Model with joint bargaining on consumer's side:

$$\begin{array}{c} CS_{collC} = \dfrac{(0.75t - c + v)\Theta - 1.25t - c + v}{1 + \Theta} \\ PS_{collC} = -\dfrac{t(-1 + \Theta)}{1 + \Theta} \end{array};$$

$$\text{Model with joint bargaining on both sides: } \begin{array}{c} CS_{collBoth} = -\Theta c - \frac{1}{4}\Theta t + \Theta v \\ PS_{collBoth} = \Theta c + \frac{1}{4}\Theta t - \Theta v - c - \frac{1}{4}t + v \end{array}.$$

In the models with joint bargaining, an increase in Θ leads to greater consumer surplus, as stronger bargaining power enables consumers to negotiate more favorable terms. In the model with joint bargaining on both sides, the extreme cases ($\Theta = 0$) or ($\Theta = 1$) lead to either consumers capturing the whole surplus ($\Theta = 1$) or firms extracting it fully ($\Theta = 0$). Interestingly, when only consumers bargain collectively, even in the extreme case with firms having full bargaining power, consumers still have some surplus left because both firms are competing and setting prices to maximize own profits. This highlights how the bargaining structure itself can influence the distribution of surplus, with joint bargaining on both sides allowing for more extreme outcomes, while consumer-side joint bargaining imposes limitations that prevent firms from fully dominating.

III. Asymmetric cost model

In this chapter, the analysis is extended to a scenario where firms face different marginal costs. While the previous models assumed symmetric marginal costs between firms (c), this new framework incorporates cost asymmetries, allowing for a more realistic exploration of price-setting and welfare distribution. Firm A will incur a marginal cost of (c_A), while Firm B will incur a marginal cost of (c_B). The same three bargaining structures—independent firm pricing, collective bargaining on the consumer side, and joint bargaining on both sides—are revisited under this assumption. The goal is to understand how differences in marginal costs influence equilibrium prices, welfare outcomes, and the balance of power in bargaining scenarios.

A. Benchmark model

The steps and calculations follow the same approach as outlined in the previous chapters. The formula for the indifferent consumer ($x_{indifferent}$) and the resulting consumer demands (D_A and D_B) remain unchanged. However, the profit maximization process for the firms differs, as each firm now optimizes its profit based on its individual marginal cost (c_A and c_B) rather than a uniform marginal cost (c). This adjustment results in the following profit functions:

$$\Pi_A = (p_A - c_A) \cdot D_A = \frac{(p_A - c_A)(p_B + t - p_A)}{2t};$$

$$\Pi_B = (p_B - c_B) \cdot D_B = (p_B - c_B)\left(1 - \frac{p_B + t - p_A}{2t}\right).$$

Solving the system of first-order conditions yields the following equilibrium prices:

$$p_A^* = t + \frac{2c_A}{3} + \frac{c_B}{3} \text{ and } p_B^* = t + \frac{c_A}{3} + \frac{2c_B}{3}.$$

The results for the model with differentiated marginal costs reveal that the equilibrium prices, (p_A^* and p_B^*), depend not only on the transportation cost (t) but also on the marginal costs of both firms (c_A and c_B). Each firm's price reflects a weighted contribution from its own marginal cost and the marginal cost of its competitor. This outcome highlights the interdependence of pricing strategies in markets where firms have different cost structures.

The inclusion of transportation costs (t) still plays a key role in determining equilibrium prices by influencing consumer sensitivity to price differences between firms. Additionally, the interaction of the firms' marginal costs ensures that the pricing strategy reflects both the firm's cost efficiency and the competitive dynamics of the market. When ($c_A = c_B$), the equilibrium prices simplify to the benchmark result, ($p_A^* = p_B^* = t + c$), demonstrating how symmetry in marginal costs leads to uniform pricing outcomes. In contrast, when costs differ, each firm's price adjusts to account for both its own cost structure and the influence of its competitor's costs. This adjustment reflects the competitive nature of price-setting under cost asymmetry.

B. Model with joint bargaining on consumer's side

Now consider the model with joint bargaining on the consumer's side, but with differentiated marginal costs for the firms. As previously noted, in this setup, consumers are represented by a unified bargaining entity that negotiates prices individually with each firm on their behalf. Despite the introduction of differentiated costs, the Nash product to be maximized with respect to prices retains its original structure. The Nash product for the bargaining between firm A and the entity bargaining on behalf of the consumers can still be expressed as follows:

$$(CS_A + CS_B - DA_A)^{\Theta} \cdot (PS_A - 0)^{1-\Theta} .$$

Only the definition of producer surplus (PS_A) is adjusted to account for firm A's individual marginal cost (c_A):

$$PS_A = (p_A - c_A) \cdot x_{indifferent} = \frac{(p_A - c_A)(p_B + t - p_A)}{2t}.$$

Solving the first-order condition for Firm A with respect to p_A (and vice versa for Firm B with respect to p_B) results in the following system of equations:

$$p_A = \frac{1}{2}\Theta c_A - \frac{1}{2}\Theta p_B - \frac{1}{2}\Theta t + \frac{1}{2}c_A + \frac{1}{2}p_B + \frac{1}{2}t ;$$

$$p_B = \frac{1}{2}\Theta c_B - \frac{1}{2}\Theta p_A - \frac{1}{2}\Theta t + \frac{1}{2}c_B + \frac{1}{2}p_A + \frac{1}{2}t .$$

Solving this system of equations yields the following equilibrium prices:

$$p_A^* = \frac{c_B\Theta^2 - t\Theta^2 - 2\Theta c_A + 4\Theta t - 2c_A - c_B - 3t}{\Theta^2 - 2\Theta - 3}$$

$$= \frac{(\Theta^2 - 1)c_B}{\Theta^2 - 2\Theta - 3} + \frac{(-2\Theta - 2)c_A}{\Theta^2 - 2\Theta - 3} + \frac{-\Theta^2 t + 4\Theta t - 3t}{\Theta^2 - 2\Theta - 3} ;$$

$$p_B^* = \frac{c_A\Theta^2 - t\Theta^2 - 2\Theta c_B + 4\Theta t - c_A - 2c_B - 3t}{\Theta^2 - 2\Theta - 3}$$

$$= \frac{(-2\Theta - 2)c_B}{\Theta^2 - 2\Theta - 3} + \frac{(\Theta^2 - 1)c_A}{\Theta^2 - 2\Theta - 3} + \frac{-\Theta^2 t + 4\Theta t - 3t}{\Theta^2 - 2\Theta - 3} .$$

For easier interpretation, the costs can be redefined in terms of their difference. Specifically, c_A can be expressed as (c_B + ð), where delta (ð) represents the marginal cost difference between the two firms. The equilibrium prices can then be expressed as:

$$p_A^* = \frac{\Theta^2 c_B - \Theta^2 t - 2\Theta(c_B + \delta) + 4\Theta t - 3c_B - 2\delta - 3t}{\Theta^2 - 2\Theta - 3}$$

$$= c_B + \frac{(-2\Theta - 2)\delta}{\Theta^2 - 2\Theta - 3} + \frac{-\Theta^2 t + 4\Theta t - 3t}{\Theta^2 - 2\Theta - 3}$$

$$= c_B - \left(\frac{2}{\Theta - 3}\right) \eth + \left(\frac{-\Theta + 1}{\Theta + 1}\right) t \; ;$$

$$p_B^* = \frac{\Theta^2(c_B + \delta) - \Theta^2 t - 2\Theta c_B + 4\Theta t - 3c_B - \delta - 3t}{\Theta^2 - 2\Theta - 3}$$

$$= c_B + \frac{(\Theta^2 - 1)\delta}{\Theta^2 - 2\Theta - 3} + \frac{-\Theta^2 t + 4\Theta t - 3t}{\Theta^2 - 2\Theta - 3}$$

$$= c_B + \left(\frac{\Theta - 1}{\Theta - 3}\right) \eth + \left(\frac{-\Theta + 1}{\Theta + 1}\right) t \; .$$

To ensure the market is fully covered again, a minimum inequality constraint on v is necessary. The most restrictive one is the following:

$$v - p_A^* - t \geq 0 \; ;$$

$$\frac{(-\Theta^2 + 4\Theta - 3)t}{\Theta^2 - 2\Theta - 3} + \frac{(-2\Theta - 2)\delta}{\Theta^2 - 2\Theta - 3} + c_B + t \leq v \; .$$

The equilibrium prices are influenced by firm B's marginal cost (c_B), the marginal cost difference between firms (ð) and the transportation costs (t).

As in the model with symmetric costs, examining p_B^* reveals that the minimum price is determined by the marginal cost. Like the previous models, transportation costs (t) influence equilibrium prices by affecting consumer sensitivity to price differences. However, what is new in this model is the impact of marginal cost differences on equilibrium prices.

If there is no difference in marginal costs ($c_A = c_B$), both firms will set the same price (as ð=0). However, if Firm B has lower marginal costs, i.e., ($c_A > c_B$, ð > 0), and assuming full firm bargaining power (Θ=0), Firm B's price increases by one-third of the marginal cost difference, while Firm A's price increases by two-thirds of the difference. This allows Firm B to capitalize on its cost advantage by raising its price by $(\frac{1}{3})$ð over its marginal cost c_B, whereas Firm A, due to its marginal cost disadvantage, struggles to fully cover its costs based on the first two components of the price equation ($c_A > c_B + (\frac{2}{3})$ ð, firm A's profit is still positive because of t). This outcome applies only in the extreme case where firms have full bargaining power.

In contrast, when consumers hold full bargaining power (Θ=1), Firm B's price is entirely unaffected by the marginal cost difference, while Firm A's price is linearly dependent on it. Considering the scenario where Firm B has lower marginal costs ($c_A > c_B$), (Θ=1), transportation costs no longer influence prices. As a result, Firm B's price equals its marginal cost (c_B). Meanwhile, Firm A's price is (c_B + ð), which equals c_A, meaning both firms' prices align precisely with their respective marginal costs.

A greater marginal cost difference enables the firm with lower marginal costs to set a price above its marginal cost (not taking into regard t), while the firm with higher marginal costs struggles to fully cover its own (although it still earns a profit ≥ 0 due to the incorporation of transportation costs, t). However, this effect diminishes as consumers gain more bargaining power and completely disappears when consumers hold full bargaining power.

From the derivatives $\frac{\partial}{\partial \delta}(p_A^*) = -\frac{2}{\Theta-3} > 0$ and $\frac{\partial}{\partial \delta}(p_B^*) = \frac{\Theta-1}{\Theta-3} \geq 0$, equilibrium prices clearly increase with larger cost asymmetries.

Using the equilibrium prices, the indifferent consumer can be identified and is determined by the following formula:

$$x_{indifferent} = \frac{1}{2} + \frac{(\Theta + 1)\delta}{2(\Theta - 3)t}.$$

As in the previous models, when there is no difference in marginal costs (ð = 0), the market is evenly divided between the firms, and the indifferent consumer is located at the midpoint.

With different marginal costs, it can no longer be guaranteed that the indifferent consumer lies within the interval [0,1]. Therefore, it is necessary to derive a condition on the parameter relationships to ensure that the indifferent consumer falls within this range. In essence, it is necessary to make sure that:

$$0 < x_{indifferent} < 1 \; ;$$

$$0 < \frac{1}{2} + \frac{(\Theta + 1)\delta}{2(\Theta - 3)t} < 1 \; ;$$

$$\frac{-(3 - \Theta)}{\Theta + 1} < \frac{\delta}{t} < \frac{(3 - \Theta)}{\Theta + 1} \, .$$

With the indifferent consumer determined, the welfare analysis can begin by calculating the total consumer surplus:

$$CS = v - c_B + \left(\frac{3\Theta - 5}{4\Theta + 4}\right) t - \left(\frac{1}{2}\right) ð + \left(\frac{(\Theta + 1)^2}{4(\Theta - 3)^2 t}\right) ð^2 \, .$$

As anticipated, consumer surplus increases proportionally with the reservation utility and decreases proportionally with marginal cost. Taking the derivatives with respect to transportation costs, it becomes immediately apparent that an increase in transportation costs results in a reduction in consumer surplus:

$$\frac{\&CS}{\&t} = \left(\frac{3\Theta - 5}{4\Theta + 4}\right) - \left(\frac{(\Theta + 1)^3}{4\Theta((\Theta + 1) - 3)^2}\right)\left(\frac{ð^2}{t^2}\right) < 0 \, .$$

Our analysis of equilibrium prices further reveals that consumer surplus decreases as cost asymmetries increase, since higher cost asymmetries lead to an increase in both equilibrium prices.

With the producer surplus given by the following expression, it becomes evident that producer surplus increases as cost differences become larger:

$$PS = \left(\frac{2\Theta - 2}{\Theta - 3}\right) ð \, .$$

When firms hold full bargaining power, the marginal effect of an increase in ð on producer surplus is $\frac{2}{3}$, but this effect gradually diminishes to zero as consumer bargaining power increases. Parameter ð represents a firm's cost efficiency relative to its competitor. When firms have strong bargaining power, they can leverage this efficiency to command higher prices and secure greater profits.

The total welfare is given by the following expression:

$$\mathrm{TW} = v - c_B - \left(\frac{1}{2}\right) ð - \left(\frac{1}{4}\right) t + \left(\frac{-3\Theta^2 + 2\Theta + 5}{4(\Theta - 3)^2 t}\right) ð^2 .$$

Welfare increases proportionally with reservation utility and decreases proportionally with marginal cost. Differentiating with respect to transportation costs reveals a clear negative impact of transportation costs on total welfare as well:

$$\frac{\&\mathrm{TW}}{\&t} = -\frac{1}{4} + \frac{(3\Theta^2 - 2\Theta - 5)}{4(\Theta - 3)^2} \frac{ð^2}{t^2} < 0 .$$

As demonstrated, an increase in the marginal cost difference, (ð), results in a decrease in consumer surplus and an increase in producer surplus. To assess which effect dominates and how welfare changes, we calculate the derivative with respect to ð:

$$\frac{\&\mathrm{TW}}{\&ð} = \frac{\frac{ð}{t}(\Theta + 1)(5 - 3\Theta) - (\Theta - 3)^2}{2(\Theta - 3)^2} .$$

Since the denominator is positive, the sign of the derivative is determined entirely by the numerator. The numerator, and thus the derivative, is positive when the following condition is satisfied:

$$\frac{ð}{t}(\Theta + 1)(5 - 3\Theta) - (\Theta - 3)^2 > 0 ;$$

$$\frac{ð}{t} > \frac{(\Theta - 3)^2}{(\Theta + 1)(5 - 3\Theta)} .$$

This is possible, as demonstrated by incorporating the condition that the indifferent consumer lies within the range of zero to one:

$$\frac{-(3 - \Theta)}{\Theta + 1} < \frac{(\Theta - 3)^2}{(\Theta + 1)(5 - 3\Theta)} < \frac{(3 - \Theta)}{\Theta + 1} .$$

In conclusion, greater marginal cost differences can potentially increase welfare. If Firm A is inefficient, meaning it has higher marginal costs, further inefficiency produces two effects. First, the higher production costs for Firm A negatively impact welfare. However, these increased costs and the resulting higher price for Firm A lead to a shift in sales toward Firm B, the more efficient producer. This positive effect outweighs the negative impact, ultimately resulting in a net increase in welfare.

Interestingly, in this setup with consumers bargaining collectively, greater consumer bargaining power results in an increase in total welfare, as demonstrated below:

$$\frac{\&TW}{\&\Theta} = \frac{4(\Theta - 1)\delta^2}{t(\Theta - 3)^3} \geq 0 \,.$$

When cost asymmetry exists between firms ($\delta > 0$), bargaining power starts to influence total welfare because it affects the efficiency of production, transportation costs and the distribution of surplus in the market. In scenarios where firms have different marginal costs, directing more demand toward the lower-cost firm improves overall efficiency by reducing production costs. Greater consumer bargaining power ($\Theta \rightarrow 1$) facilitates this outcome by pushing prices closer to marginal costs, which encourages consumers to allocate their purchases more effectively to the more efficient producer. This alignment between prices and production costs enhances market efficiency and contributes to higher total welfare.

In contrast, when firms hold more bargaining power ($\Theta \rightarrow 0$), they can set prices that may not fully reflect their cost efficiencies. This results in a less optimal allocation of demand, where the higher-cost firm may capture a larger share of the market than is economically efficient. Consequently, the overall production costs in the market increase, reducing total welfare.

In cases where marginal costs are symmetric ($\delta = 0$), bargaining power has no impact on total welfare. Since both firms have identical production costs, the distribution of demand between them does not affect efficiency. Bargaining power in such cases only redistributes welfare between consumers and producers without influencing the total amount of welfare generated.

Therefore, the presence of cost asymmetry introduces an efficiency dimension to the bargaining process. Greater consumer bargaining power not only redistributes welfare in their favor but also improves total welfare by ensuring that prices and demand allocation align more closely with cost efficiencies. In this scenario, total welfare would be maximized when consumers have full bargaining power. Without cost asymmetry, bargaining power merely shifts surplus between consumers and producers without affecting overall efficiency or welfare.

C. Model with joint bargaining on both sides

Now, consider the scenario where both consumers and firms engage in collective bargaining, while firms face asymmetrical marginal costs. It is assumed that both Firm A's and Firm B's products are sold in the market. The Nash product remains largely unchanged, with the only difference being that the uniform marginal cost is replaced by each firm's individual marginal cost:

$$\text{Nash} - \text{product} = (CS_A + CS_B - 0)^{\theta} \cdot (PS_A + PS_B - 0)^{1-\theta}$$

$$= \left(\int_0^{x_{indifferent}} v - p_A - t \cdot z \, dz + \int_{x_{indifferent}}^{1} v - p_B - t \cdot (1 - z) \, dz\right)^{\theta} \cdot \left((p_A - c_A) \cdot x_{indifferent} + (p_B - c_B) \cdot (1 - x_{indifferent})\right)^{1-\theta} .$$

Solving the system of first-order conditions and using again ($c_A = c_B + ð$) delivers the following equilibrium prices:

$$p_A^* = \frac{(-1 + \Theta)t^2 + ((4c_B + 2\delta - 4v)\Theta + 4v + 2\delta)t - \delta^2(-1 + \Theta)}{4t}$$

$$= \Theta c_B + (1 - \Theta)\left(v - \frac{1}{4}t\right) + \left(\frac{\Theta + 1}{2}\right)ð + \frac{(1 - \Theta)}{4}\frac{ð^2}{t} ;$$

$$p_B^* = \frac{(-1 + \Theta)t^2 + ((4c_B + 2\delta - 4v)\Theta + 4v - 2\delta)t - \delta^2(-1 + \Theta)}{4t}$$

$$= \Theta c_B + (1 - \Theta)\left(v - \frac{1}{4}t\right) + \left(\frac{\Theta - 1}{2}\right)ð + \frac{(1 - \Theta)}{4}\frac{ð^2}{t} .$$

In this case, the minimum reservation utility required to ensure a fully covered market is:

$$v - p_A^* - t \cdot x_{indifferent} \geq 0 .$$

For ($\Theta > 0$) the minimum reservation utility is given by:

$$\frac{4\Theta c_B t - \Theta\,\delta^2 + 2\Theta\delta t + \Theta t^2 + \delta^2 + t^2}{4\Theta t} \leq v .$$

With the equilibrium prices determined, an additional condition must be met to ensure that the indifferent consumer falls within the interval from zero to one. Since this does not occur automatically in this case due to cost asymmetry, the following condition must be satisfied as well:

$$0 < x_{indifferent} = \frac{t — δ}{2t} < 1 ;$$

$$—1 < \frac{δ}{t} < 1 .$$

The equilibrium prices are influenced by firm B's marginal cost (c_B), the marginal cost difference between firms (ð), the transportation costs (t), the bargaining power parameter (Θ) and the reservation utility (v). As in the model with symmetric costs, examining p_B^* reveals that the minimum price is determined by the marginal cost. Transportation costs (t) influence equilibrium prices by affecting consumer sensitivity to price differences. An increase in reservation utility leads to higher equilibrium prices, with the rise being proportional for (Θ = 0).

The impact of the cost asymmetries is more difficult to grasp. For instance, the derivatives with respect to the cost difference ð can be expressed as:

$$\frac{\&p_A^*}{\&ð} = \frac{(t — δ)Θ + t + δ}{2t} ;$$

$$\frac{\&p_B^*}{\&ð} = \frac{(—1 + Θ)(t — δ)}{2t} .$$

Given the condition for the indifferent consumer to lie within the interval [0,1], it follows that transportation costs must dominate, i.e., t > ð. Under this condition, the derivative of p_A^* with respect to δ is positive, while the derivative of p_B^* is negative. This indicates that as ð increases, meaning Firm A has a higher marginal cost than Firm B, Firm A's price rises while Firm B's price falls.

This behavior reflects the interplay of cost asymmetry, transportation costs, and bargaining power. For Firm A, the higher marginal cost difference (ð) forces an upward adjustment in its price to account for its cost disadvantage, which is partially passed on to consumers. For Firm B, the cost advantage leads to a downward adjustment in its price, driven not by competitive pressure but by the joint bargaining process, which balances both firms' costs and consumers' interests.

As consumer bargaining power (Θ) increases, the magnitude of these price adjustments diminishes. With stronger consumer influence, equilibrium prices align more closely with marginal costs, reducing the sensitivity of both p_A^* *and* p_B^* to changes in ð.

Thus, a higher marginal cost difference (ð > 0) amplifies the price disparity between Firm A and Firm B under weaker consumer bargaining power but becomes less influential as consumers gain more control in the negotiation.

The broader impact of bargaining power can be analyzed by examining the derivative with respect to Θ, which is identical for both equilibrium prices:

$$\frac{\partial p_A^*}{\partial \Theta} = \frac{\partial p_B^*}{\partial \Theta} = c_B - \frac{\delta^2}{4t} + \frac{\delta}{2} + \frac{t}{4} - v .$$

By applying the minimum reservation utility condition, this expression simplifies to:

$$\frac{\partial p_A^*}{\partial \Theta} = \frac{\partial p_B^*}{\partial \Theta} = \frac{-\delta^2 - t^2}{4\Theta t} \leq 0 .$$

Again, in this bargaining setup with asymmetrical costs, higher consumer bargaining power ($\Theta \to 1$) consistently leads to lower equilibrium prices, as consumers effectively negotiate terms that align prices with cost efficiencies, diminishing firms' ability to incorporate transportation costs or extract surplus from cost differences.

The consumer surplus and producer surplus can be calculated as:

$$CS = \Theta v - \Theta c_B - \frac{\Theta t}{4} - \frac{\Theta \delta}{2} + \frac{\delta^2 \Theta}{4t} ;$$

$$PS = \left(1 - \Theta\right) v + \left(-1 + \Theta\right) c_B + \left(\frac{\Theta - 1}{4}\right) t + \left(\frac{\Theta - 1}{2}\right) \delta + \frac{(1 - \Theta)\delta^2}{4t} .$$

In addition to the effects already examined, we take the derivatives with respect to ð to further analyze its impact on the surpluses:

$$\frac{\partial CS}{\partial \delta} = \frac{\Theta(-t + \delta)}{2t} \leq 0 ;$$

$$\frac{\partial PS}{\partial \delta} = -\frac{(-1 + \Theta)(-t + \delta)}{2t} \leq 0 .$$

In conclusion, with total welfare given by: $TW = v - c_B - \frac{t}{4} - \frac{\delta}{2} + \frac{\delta^2}{4t}$, both consumer and producer surplus decline with increasing cost asymmetries, resulting in a decrease in total welfare. From the price analysis, it is evident that the bargaining power parameter Θ redistributes welfare: higher consumer bargaining power increases consumer surplus, while higher firm bargaining power reduces it. Conversely, producer surplus decreases with stronger consumer power or increases with greater firm power. However, the increase in one is exactly offset by the decrease in the other, meaning that total welfare remains unchanged regardless of the distribution of bargaining power.

D. Comparative analysis

By incorporating differentiated costs, we observe how the interplay between marginal cost differences (δ), transportation costs (t), and bargaining power (Θ) reshapes the market outcomes in each model.

Under independent pricing, where firms set prices unilaterally to maximize their profits, the equilibrium prices directly reflect both transportation costs and the individual marginal costs of the firms. Each firm's price becomes a weighted average of its own costs and the competitor's costs, moderated by transportation costs. Here, cost asymmetry introduces an inherent imbalance: the firm with lower marginal costs (c_B) gains a competitive advantage, while the higher-cost firm (c_A) struggles to maintain profitability. This dynamic highlights how cost differences affect the competitive landscape, even in the absence of bargaining.

A comparison of the joint bargaining models reveals a striking difference in how marginal cost differences impact equilibrium prices, depending on the bargaining structure. When firms bargain independently, an increase in marginal cost differences leads to an increase in both negotiated equilibrium prices. In contrast, when firms bargain collectively, the price of the higher-cost firm increases, while the price of the lower-cost firm decreases. This highlights the influence of different bargaining structures on pricing outcomes. Examining welfare, we have demonstrated that greater cost asymmetries result in higher welfare when only consumers bargain jointly, but lower welfare when both sides negotiate jointly.

As observed in the scenario where only consumers bargain jointly, marginal cost differences introduce an efficiency dimension to the bargaining process. In this setup, bargaining power not only redistributes welfare but also enhances total welfare by aligning prices and demand allocation more closely with cost efficiencies. This alignment ensures that resources are directed toward the more cost-efficient firm, improving overall market efficiency. However, this efficiency gain does not occur when firms bargain collectively. In the collective bargaining scenario, cost asymmetries instead create a welfare disadvantage, as higher cost differences reduce total welfare.

A comparison of total welfare in the joint bargaining scenarios shows that, in the absence of marginal cost differences ($\delta = 0$), welfare is identical in both cases: collective bargaining on the consumer side and joint bargaining on both sides ($TW = v - c_B - \frac{t}{4}$). Considering the presence of cost asymmetries ($\delta > 0$) and analyzing the total welfare formulas:

$$TW_{CollC} = v - c_B - \frac{t}{4} - \frac{ð}{2} + \frac{\delta^2}{4t} \cdot \left(\frac{-3\Theta^2 + 2\Theta + 5}{(\Theta - 3)^2} \right),$$

$$TW_{CollBoth} = v - c_B - \frac{t}{4} - \frac{ð}{2} + \frac{\delta^2}{4t},$$

it becomes evident that the difference in welfare between the two bargaining scenarios is determined by the term: $\left(\frac{-3\Theta^2+2\Theta+5}{(\Theta-3)^2}\right)$. For Θ in [0,1), this term is always less than one, indicating that welfare is higher in the scenario where both parties negotiate jointly. However, in the extreme case where consumers hold all bargaining power ($\Theta = 1$), as shown earlier, total welfare is maximized in the scenario where only consumers bargain jointly. In this case, the term $\left(\frac{-3\Theta^2+2\Theta+5}{(\Theta-3)^2}\right)$ equals one, meaning that welfare is identical in both scenarios. When the reservation utility is sufficiently large, as determined by the minimum reservation constraint, consumer surplus is higher in the scenario where only consumers bargain collectively, while producer surplus is higher when firms negotiate jointly too.

IV. Final results and remarks

Having demonstrated in the previous chapters that the integration of Nash-in-Nash bargaining into the Hotelling model is both feasible and effective, and after thoroughly analyzing the resulting calculations and insights, it is important to highlight the key findings derived from these models.

After analyzing the model with symmetric costs across its three scenarios—independent pricing, consumer collective bargaining, and joint bargaining by both sides—several consistent insights emerge regarding the factors that influence equilibrium prices and total welfare.

In all scenarios, the minimum price is anchored by the marginal cost (c), as firms must maintain profitability. This baseline reflects the cost structure of the market, ensuring that firms cover production costs. Transportation costs (t) also play a crucial role in determining equilibrium prices, as they affect consumer price sensitivity. Higher transportation costs reduce consumers' price sensitivity because purchasing decisions are increasingly influenced by location rather than small price differences. This enables firms to incorporate transportation costs into their prices without losing demand, particularly in the independent pricing scenario, where firms have full control over pricing strategies.

In bargaining situations, the distribution of bargaining power (Θ) becomes the key determinant of price levels. When consumers have greater bargaining power, equilibrium prices decrease and align more closely with the consumer-optimal price, which only covers production costs (c). Conversely, when firms dominate the negotiation, prices approach the firm-optimal price, which incorporates transportation costs (t) and reservation utility (v) to maximize producer surplus. Thus, bargaining power determines the degree to which prices reflect the preferences of either consumers or producers.

Regarding welfare, the total amount of welfare remains consistent across all bargaining scenarios in the symmetric cost model. Total welfare, defined as the sum of consumer and producer surplus, increases with higher reservation utility (v) and decreases with higher marginal costs (c) and transportation costs (t). Transportation costs introduce inefficiencies, acting as a drag on welfare by limiting efficient allocation and consumer utility. Reducing transportation costs enhances market efficiency by allowing consumers to base purchasing decisions on price and quality rather than geographical constraints. On the other hand, higher reservation utility improves total welfare by reflecting greater perceived value for consumers, leading to better overall market outcomes.

While the distribution of welfare is sensitive to bargaining power, the total welfare itself is unaffected by it. This is because any increase in consumer surplus is exactly offset by a corresponding decrease in producer surplus, and vice versa. Bargaining power, therefore, influences equity—how welfare is distributed between consumers and producers—without affecting the overall market efficiency.

An important insight is that the bargaining structure itself significantly shapes the distribution of surplus. Specifically, the analysis reveals that when consumers bargain collectively, the equilibrium price is consistently lower than the benchmark price (except in the extreme case where firms have full bargaining power where the prices are equal). This underscores how collective consumer bargaining strengthens their position and increases their surplus. Conversely, when firms also negotiate collectively, prices are consistently higher compared to independent firm bargaining (except when consumers have full bargaining power, where the prices remain equal). This outcome highlights how collective bargaining strengthens the negotiating position of firms and emphasizes the crucial role of bargaining structures in influencing prices and surplus distribution.

In the model with joint bargaining on both sides, the extreme cases of bargaining power ($\Theta = 0$ or $\Theta = 1$) lead to one party capturing the entire surplus. When firms hold all the bargaining power ($\Theta = 0$), they fully extract the surplus, leaving consumers with none. Conversely, when consumers have full bargaining power ($\Theta = 1$), they capture the entire surplus, leaving producers with no share. This reflects the potential for highly unequal welfare distribution when both parties negotiate collectively.

When only consumers bargain collectively, even in the extreme case with firms having full bargaining power, consumers still have some surplus left because both firms are competing and setting prices to maximize own profits.

The introduction of cost asymmetries provides further valuable additional insights into the dynamics of bargaining and welfare. As demonstrated, the bargaining structure—whether firms negotiate independently or collectively with the entity representing consumers—significantly influences how cost asymmetries affect equilibrium prices. Cost asymmetries can positively or negatively impact total welfare, as shown for the cases of consumer collective bargaining and collective bargaining on both sides. When comparing welfare levels under cost asymmetries ($\eth > 0$), we observe that welfare is generally higher in the scenario where both sides negotiate jointly, except for the case where consumers have full bargaining power, in which welfare remains equal.

Perhaps the most notable insight arises from the model where consumers bargain collectively with each firm independently. Unlike the model with symmetric costs, where bargaining power merely redistributes welfare, the presence of cost asymmetries reveals that bargaining power can actively influence the total amount of welfare generated. Specifically, greater consumer bargaining power ensures that prices align more closely with marginal costs, directing demand more efficiently toward the lower-cost producer. This efficiency gain highlights how bargaining power, combined with cost asymmetries, can enhance overall market outcomes, challenging the initial impression that it solely affects welfare distribution.

Comparing the results of this thesis with empirical work, we observe significant similarities and alignment. Capps, Dranove and Satterthwaite (2003) were able to show for option demand markets where intermediaries, i.e. managed care organizations, identify and assemble networks of suppliers, i.e. hospitals and physicians, for consumers, that there is a “strong, precisely estimated positive relationship between WTP [willingness-to-pay] and profits […]” (Capps, Dranove and Satterthwaite 2003, 753) . In their reduced form bargaining model, where “each MCO negotiates bilaterally with each hospital”, “the WTP associated with a supplier […]” serves as a measure of market power of the supplier (Capps, Dranove and Satterthwaite 2003, 738). Similarly, the findings of this thesis reveal comparable insights. For instance, in the symmetric cost model, it is explicitly shown that as producer bargaining power increases, producer surplus—representing their profits—also rises. This alignment underscores the empirical applicability of the proposed model in this thesis, at least for markets of this nature.

In a similar vein, Gowrisankaran, Nevo, and Town (2015), employing the Nash-in-Nash bargaining framework, reinforce the notion that “a party to negotiations will earn more beneficial terms of trade by improving its bargaining leverage” (Gowrisankaran, Nevo, and Town 2015, 172) , aligning with the key findings of this thesis. Lewis and Pflum (2017) provide valuable insights by highlighting “the existence of important cross-market dependencies that can allow some hospitals to strengthen their market power by affiliating with out-of-market systems” (Lewis and Pflum 2017, 40). This finding suggests a compelling research direction that could offer intriguing extensions and applications for the approach taken in this thesis.

However, these insights are not confined to the hospital market. Crawford and Yurukoglu (2012), in their research on multichannel television markets, also highlight the critical role of bargaining power in shaping negotiated prices and welfare outcomes, specifically within the context of distributor-channel negotiations. This thesis results for horizontal bargaining coincide with “the conventional wisdom – that larger buyers pay lower prices to sellers” in the vertical case, as larger buyers can be assumed to have more bargaining power (Chipty and Snyder 1999, 337) . Interestingly, Chipty and Snyder reveals a notable exception as “merger [in certain cases] worsens rather than enhances buyers' bargaining position” (Chipty and Snyder 1999, 338). Future research could extend the models in this thesis to examine how market structure changes, such as mergers or acquisitions, influence bargaining dynamics, potentially reversing the expected effects of firm size on bargaining leverage.

This thesis offers valuable theoretical insights into the dynamics of collective bargaining and its welfare implications. Nevertheless, several limitations should be considered. While the models account for both symmetric and asymmetric cost structures, they rely on simplifying assumptions such as the use of a linear Hotelling framework. This may not fully capture the complexities of real-world markets, where consumer preferences and transportation costs might follow non-linear patterns. Additionally, the Nash bargaining model assumes fixed and constant bargaining parameters (e.g., Θ), which may not adequately reflect the variability and evolution of bargaining dynamics in practice, where these parameters could shift across different negotiations or market conditions. Disagreement payoffs are treated as static, assuming fixed fallback options for both consumers and producers. In reality, these payoffs can evolve dynamically and depend on strategic behavior or changing market conditions. The models also simplify the range of fallback options available to negotiating parties, which may underestimate the leverage derived from alternative opportunities. While the analysis of consumer collaboration provides meaningful insights, it does not account for practical challenges such as coordination costs, which could significantly impact the feasibility and effectiveness of forming consumer coalitions. Furthermore, the models assume a fixed set of market participants, overlooking the potential impact of firm or consumer entry and exit on bargaining dynamics.

These limitations, while inherent to theoretical modeling, open up numerous opportunities for further research to refine and expand the framework developed in this thesis, providing a more nuanced understanding of bargaining dynamics in real-world settings. For instance, during the development of this thesis, I explored the possibility of incorporating switching costs into the models, where consumers experience a disutility when changing from one firm to another.

Despite extensive use of mathematical software, solving these models for equilibrium prices proved infeasible within the scope of this work. However, with more time and resources, this remains a promising direction for future research.

Other potential extensions include assuming alternative or more complex disagreement payoffs. For example, one could model a scenario where firms have non-zero disagreement payoffs, reflecting the possibility of selling in alternative markets if negotiations fail. Another avenue for exploration is the inclusion of heterogeneous consumer preferences, where consumers differ in their valuations and transportation costs. Such an extension could provide a more realistic depiction of market dynamics and offer insights into how consumer diversity impacts bargaining outcomes and welfare.

Additionally, dynamic bargaining models could be developed to capture how bargaining power, disagreement payoffs, and strategies evolve over time. Such models would allow for the analysis of long-term bargaining dynamics and their implications for welfare distribution. On a more personal note, given my strong interest in behavioral economics, I am particularly intrigued by the potential to incorporate biases and irrationalities into the model. Understanding how these behavioral factors influence bargaining strategies and outcomes could offer a richer perspective on the complexities of real-world negotiations.

Building on the insights presented in this thesis, several policy recommendations emerge as practical applications of the findings, particularly when considering the social dimension of economics and its implications for equity, fairness, and societal impact.

Regulators could facilitate the formation of consumer coalitions, particularly in markets where individual consumers face significant power imbalances against large firms. Providing institutional or financial support for such coalitions can help empower consumers to negotiate better terms, ultimately leading to fairer welfare distribution and potentially lower prices. Transportation costs represent a significant inefficiency in market outcomes, as shown in this thesis. Policies aimed at reducing these costs, such as subsidies for logistics or investments in infrastructure, could enhance overall market efficiency and increase total welfare. In markets with cost asymmetries among firms, regulators should monitor and potentially intervene to ensure that firms with higher costs do not exploit collective bargaining structures to disproportionately affect prices and welfare. Policies that promote competitive pricing strategies could mitigate welfare losses due to cost asymmetries. Given the potential for mergers or acquisitions to alter bargaining dynamics, as discussed in the context of empirical studies, antitrust authorities should carefully evaluate how such market structure changes affect bargaining leverage. Policies should aim to prevent mergers that could worsen buyer or consumer bargaining positions, ensuring that market efficiency and welfare are preserved. Establishing transparency in negotiation processes could reduce inefficiencies and promote equitable outcomes. For example, disclosure requirements for pricing structures and bargaining agreements could improve accountability and prevent exploitation of weaker parties in negotiations.

Finally, this thesis has highlighted the critical role of joint bargaining dynamics and their significant impact on prices and welfare. A deep understanding of these dynamics is essential for designing targeted economic policies that not only enhance total welfare but also promote fairness and equity over the long term.

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VI. Appendix

The mathematical calculations for this thesis were performed using the software Maple and were thoroughly verified by the supervising professor using an alternative mathematical program. All essential formulas are included in the main text. In cases where a solution is presented directly without displaying the intermediate steps, it follows a similar approach to previously calculated models. Consequently, the intermediate equations have been omitted as they are deemed already familiar.