Using Option-Implied Information in Portfolio Selection and Risk Management

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Dissertation written under the supervision of Dr. Pedro Barroso

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All errors and omissions are solely my responsibility.

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Abstract

The objective of this dissertation is two-fold. The first objective is to examine whether one can use implied information (implied volatility and implied correlations) from the options market to improve the out-of-sample performance of an all-stock optimized portfolio. Portfolio performance is measured using three metrics, namely, returns, volatility, and the Sharpe Ratio. The second objective is to examine the risk metrics of the portfolios to analyze whether a portfolio created using option-implied information is better at predicting risk than one using a conventional sample covariance matrix. This is done by calculating the portfolios VaR using a variety of methodologies. Empirically, this dissertation finds that the use of option-implied volatility when estimating the covariance matrix was able to increase the Sharpe Ratio of both constrained and unconstrained portfolios. There was no improvement to performance when option-implied correlation was added to the optimization process, thus the primary mechanism for improving performance was the ability to predict asset volatility. The risk management aspect of the dissertation provides two interesting findings. It finds that the use of a covariance matrix using option implied information is better at estimating hit rates than the sample covariance matrix. Also, there is evidence that the use of option implied information in the portfolio selection process reduces tail risk.

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Abstrato

O objetivo desta dissertação é duplo. O primeiro objetivo é examinar se é possível usar informações implícitas (volatilidade implícita e correlações implícitas) do mercado de opções para melhorar o desempenho fora da amostra de uma carteira otimizada com todas as ações. O desempenho da carteira é medido usando três métricas, a saber, retornos, volatilidade e o Índice de Sharpe. O segundo objetivo é examinar as métricas de risco dos portfólios para analisar se um portfólio criado usando informações implícitas de opções é melhor na previsão de risco do que um usando uma matriz de covariância de amostra convencional. Isso é feito calculando o VaR das carteiras usando uma variedade de metodologias. Empiricamente, este artigo conclui que o uso da volatilidade implícita na opção ao estimar a matriz de covariância foi capaz de aumentar o índice de Sharpe de carteiras restritas e irrestritas. Não houve melhoria no desempenho quando a correlação implícita na opção foi adicionada ao processo de otimização, portanto, o mecanismo principal para melhorar o desempenho foi a capacidade de prever a volatilidade do ativo. O aspecto de gerenciamento de risco da dissertação fornece duas descobertas interessantes. Ele descobre que o uso de uma matriz de covariância usando informações implícitas de opções é melhor para estimar as taxas de acerto do que a matriz de covariância de amostra. Além disso, há evidências de que o uso de informações implícitas nas opções no processo de seleção da carteira reduz o risco de cauda.

Título: Usando Informações Implícitas nas Opções na Seleção de Portfólio e Gerenciamento de Risco

Nom: Mackenzie Mark Galvão Ferreira

Palavras-chave: média-variância, volatilidade implícita da opção, correlação implícita da opção, otimização da carteira, valor em risco (VaR), risco de cauda

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1. Introduction

In Markowitz's canonical paper, "Portfolio Selection", he states in his conclusion, "one suggestion as a tentative μ and σ is to use the observed μ and σ for some period on the past. I believe better methods, which take into account more information, can be found" (Markowitz, 1952). Many advancements have been made since 1952 in the field of portfolio selection, most of them relying on improving estimates based on historical data. A plausible method of improving portfolio selection could be to take into account more information by using implied information from the options market.

This dissertation will attempt to improve portfolio selection by using forward-looking data implied from the options market to improve the estimation of the covariance matrix instead of trying to improve the estimation of said matrix using historical data (Ledoit and Wolf (2003), Jorion, (1985)). The contribution of this work is two-fold. First, the analysis is focused on whether using option-implied information can help an investor successfully improve portfolio selection, where success is measured by out-of-sample performance. Second, a risk assessment is conducted of the portfolios created using the option-implied information.

This dissertation focuses on estimating two matrices crucial for portfolio selection: the volatility matrix (which is a zeros matrix with volatilities on the diagonal) and the correlation matrix. The portfolios weights will be obtained using a variety of close-end solutions to the mean-variance problem. Due to the complexity of estimating returns using option-implied data, this dissertation will simply assume the mean return across all assets for the sample period (Metron, 1980). From these calculations a total of four portfolios are constructed using different combinations of option-implied variables. These four portfolios are then compared to three benchmarks to test whether option-implied information can be used to improve out-of-sample performance.

This dissertation then presents the results of a risk assessment on the four portfolios versus the benchmarks. Here the VaR, hit rates, and cVaR of the portfolios are calculated. Due to evidence in the literature suggesting that returns usually exhibit leptokurtosis (Mandelbrot, (1963), (2004)), the VaR calculations do not assume a normal distribution (after a Jarque-Bera test is performed). Instead, the distribution that best fits the data, using a maximum likelihood criterion, is selected.

Since Black and Scholes presented their now famous model (Black and Scholes, 1973), it has been clear that options prices contain information about the ex-post realized moments of the underlying asset, the most common of these moments being volatility (Poon and Granger, 2005). This provides a justification for the choice of using implied volatility to estimate the volatility matrix[1]. The most comprehensive work on the use of options implied information as a tool in portfolio selection was done by DeMiguel, Plyakha, Uppal, and Vilkov (2013). In this paper, they conclude that the use of implied volatility in portfolio selection helps improve the volatility and Sharpe Ratios of a variety of portfolios. Implied correlations and other moments were less effective in improving results. This dissertation adds to the existing literature by focusing on the behaviour of the covariance matrix using option-implied data as well as assessing the risk of these constructed portfolios by using traditional methods of risk management.

The structure of the dissertation is as follows: Section 2 is an overview of the data used, both the options data and stock data. Section 3 explains the theoretical justification for option-implied data as well as the methodology for the calculation of implied volatility and implied correlation. Section 4 discusses the methodology used to create the portfolios (which consist of portfolios that use option-implied information and a series of benchmark portfolios) as well as theoretical background of the mean-variance problem. Section 5 presents the results. Section 6 provides a risk assessment of the portfolios, done primarily by calculating the VaR across a variety of methodologies. Section 7 concludes.

2. Data

This section lays out and describes the stock and options data that was used in the dissertation. All stock and options data were obtained using the Wharton Research Data Services (WRDS). Within WRDS, all of the stock data was obtained from the Centre for Research in Security Prices, more commonly referred to as CRSP and all of the options data was obtained from IvyDB, also known as OptionsMetrics. The sample period covered in this research are the 237 months starting in January 2000 and ending in September 2019.

[1] Note that any reference made to the volatility matrix is a zeros matrix with the stock volatilities on the diagonals.

The data on the monthly risk-free rate over the sample period was obtained from the Kenneth French Data Library.

2.1 Data on Stock Returns

In this dissertation, the all-stock portfolios created consist of the largest 100 companies listed on the NYSE, NASDAQ, or AMEX (as these are the exchanges covered in CRSP), for each month across the sample period. The largest 100 companies are given by the 100 companies with the largest market capitalizations. Market capitalization is defined as the close price of the stock per share on the last day of the month multiplied by the number of shares outstanding on that day. The total data set contains 336 companies across the 237 months of the sample period. Each month's portfolio allocation decisions are based on assigning a weight to each of the 100 stocks for the given month. Due to missing or inappropriate data the portfolios created using option-implied information were always made of less than 100 stocks. On average each portfolio contained 69 stocks.

2.2 Data on Stock Options

As aforementioned, this dissertation uses the OptionsMetrics database to retrieve the stock options data. OptionsMetrics data set contains all index and equity options listed in the US. The first step is to obtain all equity options for the 336 companies in the sample that were traded in the sample period. It must be stated that all of these options are American styled as that is the only option type available on NYSE, NASDAQ, and AMEX exchanges. Traditionally, the use of American options makes working with options more difficult as it gives the holder the right to exercise prior to expiration. This stipulation of American options contracts should not have a material impact on implied volatility calculations for short maturities (Anderson and Bondarenko, 2007)[2]. A series of filters is then applied to the data; the first being to isolate options traded on days that correspond to the third Friday of the *t*-th month and expire on the third Friday of the following month. As per CBOE regulation, US equity options expire on the third Friday of the month, thus the calculations will use options that have approximately one month (21 trading day) to expiration. This is the main reason that the portfolios will all have a one-month estimation period and a one-month rebalancing

[2] Anderson and Bondarenko define short maturities as 21 days to expiration which is the same as this dissertation.

period. Several further filters are put in place such as: eliminating all observations that were illiquid (where volume is used as a proxy for liquidity and illiquidity is defined as volume equals 0), eliminating any bids that were less than $0.125, and eliminating arbitrage opportunities by removing observations where the bid price was greater than the ask. These filters are common in the options literature (Faias and Santa-Clara, 2009). Other filters common in the literature that involve "the Greeks" were omitted as those are calculated from parametric models such as the Black-Scholes model. All the calculations for volatility in this dissertation seek risk-neutral, model free values (that are then risk-adjusted).

3. Option-Implied Information

This section explains the methodology and computation of the option-implied moments that were used for portfolio selection. Each subsection provides a brief theoretical justification for the use of option-implied moments as a method of forecasting the realized moments. The first subsection will justify detail the calculations used to obtain the implied volatilities and the second subsection will justify and explain the implied-correlations calculations.

3.1 Implied Volatilities Using Options

To start this sub-section, it will be justified that implied volatility is an appropriate way to predict realized volatility and further how one should go about calculating implied volatility. The initial hypothesis is that the volatility implied in an options price is the markets prediction as to the future volatility of the underlying over the remaining life of the option and therefore it should be a better predictor of future realized volatility than historical volatility. Evidence from Christensen and Hansen (2002) suggest that implied volatility is in fact an unbiased and efficient forecast of ex-post realized volatility. An equally important finding in this paper was that volatility is not autocorrelated, meaning that past realized volatility was not a significant variable in predicting future realized volatility once controlling for implied volatility. This research begins to build the argument of using implied volatility as a predictor of realized volatility, which is a necessary variable in problems of portfolio optimization.

The function of any options price takes a variety of parameters into account such as volatility, time to maturity, moneyness, and prevailing interest rates. Hence, given the price of the option, it is possible to solve for its implied volatility. The problem that ensues with model-specific implied volatilities is that the measure is deeply impacted by the moneyness of the given option. Instead of using the traditional Black-Scholes model to calculate the implied volatility, a model free implied volatility (MFIV) measure will be used. This is a more appropriate way to commence the implied volatility calculation as it is a risk-free nonparametric value that contains information from the entire Black-Scholes volatility smile (Vaden, 2008). The MFIV calculation that this dissertation will replicate was originally presented in a theoretical form by Britten-Jones and Neuberger (2000). Using Eq. (1) one obtains a risk-neutral measure of implied volatility. Later in the section, the conversion of implied volatility from a risk-neutral metric into an appropriately risky one is explained. To compute the MFIV, use the practical approximation presented in Jiang and Tian (2005). The equation for the MFIV is given by the definite integral:

Eq. (1)
$$MFIV = \sqrt{2\int_{k_{min}}^{k_{max}} \frac{C(T,K) - \max\,(S_0 - K, 0)}{K^2}\, dk}$$

where C (T, K) is the price of a call with T days to maturity and a strike price of K, and S is the strike price at *t*=0. The equation is integrated from K_{min} to K_{max}, or alternatively from the call with the lowest liquid strike price to the call with the highest liquid strike price. Given that the equation is integrated across all strike prices (with liquid call options) it subsumes all the information in the volatility smile. Due to the fact that this equation is solved numerically using the trapezoidal rule across discrete values of K, there will be a minor discretization error. These discretization errors are "unlikely to have any impact on the calculation of model free implied volatility" according to Jiang and Tian (2005).

Stating that portfolio selection across a panel of risky assets is not a risk-neutral process is axiomatically true and therefore it would be inappropriate to use the MFIV as the implied volatility measure. Thus, the MFIV needs to be converted into an appropriate measure of implied volatility. The difference between the MFIV and the realized volatility is the volatility risk premium (Fassas and Papadamou, 2018). As presented in the work of Bollerslev, Gibson, and Zhou (2011) one can use the historical realized volatility to estimate the volatility risk premium. Assuming that the magnitude of the historical volatility risk premium (HVRP) is proportional to the realized volatility

(DeMiguel, Plyakha, Uppal, and Vilkov, 2013), the HVRP at time *t* can be estimated by taking the ratio of MFIV and realized volatility (RV) at time *t*.

Eq. (2)
$$HVRP_{i,t} = \frac{MFIV_{i,t}}{RV_{i,t}}$$

The HVRP is calculated on the third Friday of each month and takes into account data from the previous month (or approximately 21 trading days). Assuming that the HVRP for period $t + \Delta t$ can be approximated by the HVRP at time *t*, a predicted value for RV can be obtained by using:

Eq. (3)
$$\widehat{RV_{i,t+\Delta t}} = \frac{MFIV_{i,t+\Delta t}}{HVRP_{i,t}}$$

Note that Eq. (3) is simple a rearranged version of Eq. (2) where the HVRP lagged back one month.

3.2 Implied Correlations Using Options

This sub-section explores whether implied correlation is an appropriate way to predict the realized pairwise correlations and further how one should go about calculating implied correlations. A reasonable initial hypothesis would be that using implied information should allow for the calculation of more accurate estimates of the correlation matrix. There is evidence of this in other domains besides that of an all-stock portfolio. Walter and Lopez (2000) find that implied correlations are helpful in forecasting future realized correlation matrices, but the calculated implied correlations do not fully and appropriately incorporate all historical data.

The most appropriate method for the implied correlation calculation in an all-stock portfolio is the method presented by Buss and Vilkov (2012). Prior to the Buss-Vilkov method, there is little research about creating an implied correlation matrix for a portfolio of stocks. Hence, the Buss-Vilkov method is regarded as the most realistic model for calculating an implied correlation matrix (Numpacharoen and Numpacharoen, 2013).

Before going through the process for computing implied correlation, recall the equation for the variance of a portfolio composed of N individual stocks, $i = \{1, \ldots, N\}$:

Eq. (4) $$\sigma_{P,t}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{i,t}w_{j,t}\sigma_{i,t}\sigma_{j,t}\rho_{ij,t}$$

where $w_{i,t}$ and $w_{j,t}$ are the weight assigned[3] to the *i*-th and *j*-th asset, $\sigma_{p,t}$, $\sigma_{i,t}$, and $\sigma_{j,t}$ are the volatilities of the portfolio, *i*-th, and *j*-th asset, respectively in the *t*-th time period. $\rho_{ij,t}$ is the correlation coefficient between the *i*-th, and *j*-th asset in the *t*-th time period. The volatilities that were estimated in the previous section will now be used where $\widehat{\sigma_p}$, $\widehat{\sigma_i}$, and $\widehat{\sigma_j}$ are the implied volatilities of the portfolio, *i*-th asset, and *j*-th asset, respectively. Once these predicted values are substituted into the equation for portfolio volatility, there are N x (N-1) / 2 unknown correlations (as it is known that all values along the diagonal of the matrix are 1's). To compute these pairwise correlations, the assumptions made in this dissertation are the same as those in Buss and Vilkov (2012). The first assumption states that pairwise correlation coefficients can differ from each other but may not exceed 1 nor be less than -1. The second is the correlation matrix must be a symmetrical matrix with all non-negative eigenvalues. The Buss-Vilkov approach assumes that the implied pairwise correlation between two assets will differ from the historical correlation by a fixed proportion, ψ_t. This means that:

Eq. (5) $$\rho_{ij,t} - \widehat{\rho_{ij,t}} = \psi_t(1 - \rho_{ij,t})$$

where $\rho_{ij,t}$ is the actual pairwise correlation between the *i*-th, and *j*-th asset and $\widehat{\rho_{ij,t}}$ is the implied pairwise correlation between the *i*-th, and *j*-th asset. The above equation can be rearranged to obtain:

Eq. (6) $$\widehat{\rho_{ij,t}} = \rho_{ij,t} - \psi_t(1 - \rho_{ij,t})$$

Substituting in all of the implied volatilities for the historical and Eq. (6) into the original equation for portfolio variance the following equation is obtained:

[3] Note that the weights used in this calculation are based on the value-weights. These are the weights that were used in Buss and Vilkov (2012). The implied correlations are not recalculated for each portfolio as the weighting of the portfolios constructed in the following section should not have any impact on the estimated correlations as the composition of weights has no impact on the realized correlations.

Eq. (7) $$\hat{\sigma}_{p,t}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \hat{\sigma}_{i,t}\hat{\sigma}_{j,t}(\rho_{ij,t} - \psi_t(1-\rho_{ij,t}))$$

One can then derive, from the above equation, an explicit expression for the parameter ψ_t:

Eq. (8) $$\psi_t = -\frac{\hat{\sigma}_{P,t}^2 - \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \hat{\sigma}_{i,t}\hat{\sigma}_{j,t}\rho_{ij,t}}{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \hat{\sigma}_{i,t}\hat{\sigma}_{j,t}(1-\rho_{ij,t})}$$

It should be noted that this equation calculates the fixed proportion that the implied correlation deviates from the expected correlation. The parameter ψ_t is then substituted into Eq. (6) in order to obtain all the pairwise correlations that pertain to the N x (N-1) / 2 that need to be estimated. From this the implied correlation matrix is constructed, where all values on the diagonal are ones. Buss and Vilkov find that all assumptions are satisfied when $\psi_t \in$ (-1,0]. For the fixed proportion, ψ_t, the correlation risk premium is greater in magnitude for pairs of stocks with a low correlation coefficient. This is consistent with empirical evidence on the matter (Mueller, Stathopoulos and Vedolin, 2017).

4. Portfolio Construction

This section details the construction of the various portfolios used in the analysis. It also provides some theoretical background on the mean-variance problem and proposes the solution of the Markowitz portfolio (which becomes the minimum variance portfolio due to the assumptions made regarding estimating returns). In order to ensure robustness in the results, several portfolios are created which rely exclusively on historical data. Comparing portfolios constructed without option-implied data to portfolios constructed with option-implied data will prove whether option-implied data improves the out-of-sample performance of an optimized portfolio.

4.1 Naïve Diversification (1/N) Portfolio

The first portfolio created, which will serve as a benchmark to measure against the implied information optimized portfolios, is a naïve diversification portfolio, known as the 1/N portfolio. This portfolio allocates the same weight across all stocks. Specifically, that weight is given by the expression 1/N, where N is the number of stocks in the portfolio. There is a sufficient body of evidence (DeMiguel, Garlappi, and Uppal (2009) and Jacobs, Muller, and Weber (2014)) that shows that this strategy performs quite well against a variety of portfolio optimization strategies even though it does not require optimization itself. For example, the Sharpe Ratio of the 1/N portfolio used in this dissertation was approximately 27% greater than that of the S&P 100 (OEX) over the same time frame. DeMiguel, Garlappi, and Uppal show that out-of-sample no method of portfolio construction was able to consistently beat the naïve diversification strategy (finding that in order to have an appropriate sample size one would need 3000 observations for a 25-asset portfolio). Due to its simplicity and robustness, it is an appropriate benchmark against the proposed modification of optimization.

4.2 Mean-Variance Problem

Any agent tasked with creating an optimized portfolio must make decisions regarding the asset allocation. Usually, this asset allocation decision is framed in terms of what proportion will be allocated to risky assets and what proportion will go to risk-free assets. Since the purpose of this dissertation is to measure the way in which implied information from the options market aids the selection of risky assets, the model presented here will not allow for allocation to the risk-free asset. Now that it has been stated that the investor is to allocate his/her wealth across risky assets, the question is reframed to: what is the maximum payout an investor can receive while maximizing their utility function?

Two things are known about the investor. They must pick a set of weights that are feasible (i.e., lies on the efficient frontier, *F*) and this combination of assets must yield the highest possible $\bar{u}$ (i.e., lies on the highest indifference curve). This corresponds to a solution of the problem (as presented by Bodi, Kane, and Marcus, 2013):

Eq. (9)
$$\max_{E[R_p]} E[R_p] - \frac{1}{2}\frac{\sigma_p^2}{A}$$

subject to:

$$\sigma_p = F(E[R_p])$$

where A is the risk tolerance of the investor, $E[R_p]$ is the expected return of the portfolio, σ_p^2 is the variance of the portfolio and the constraint simply ensures that the volatility of the portfolio lies on the efficient frontier.

To solve this problem, substitute the constraint into the objective function and take its first-order condition. Upon rearranging the equation in terms of the risk tolerance level the following equation is obtained:

Eq. (10)
$$A = \sigma_p \frac{d\sigma_p}{dE[R_p]}$$

This clearly demonstrates that less risk tolerant investors (those who have a smaller A value) invest less in the risky portfolio. This is the result of σ_p being a convex function of $E[R_p]$.

Due to issue of utility and tolerance for risk being outside the scope of this dissertation, the assumption is made that the investors tolerance for risk is sufficiently high that they will maximize their utility by solving for the optimal portfolio weights and allocating all of their wealth into that portfolio.

4.2.1 Markowitz Portfolio

The Markowitz portfolio (also commonly known as the tangency portfolio) is defined as the portfolio of risky assets in which the Sharpe Ratio is maximized. According to the two-fund separation theorem any investor can separate their asset allocation in two steps: First, they find the portfolio that maximizes the Sharpe Ratio. Second, they decide on the proportion, *p,* to invest in the portfolio and (1-*p*) is invested in the risk-free asset depending on the investors attitude towards risk (Kim and Boyd, 2008). For the sake of this dissertation a sufficiently high-risk tolerance will be assumed such that the investor puts all of their wealth in the Markowitz portfolio. The closed-form solution to the Markowitz portfolio is given by:

Eq. (11) $$w = \frac{\hat{\Sigma}^{-1}\,(\hat{\mu})e}{e^{\mathrm{T}}\hat{\Sigma}^{-1}\,(\hat{\mu})\,e}$$

subject to

$$w^{\mathrm{T}}\,e = 1$$

where $\hat{\Sigma}^{-1}$ is the inverse of the covariance matrix, w is a vector of weights, e is a vector of 1's, and $\hat{\mu}$ is a vector of adjusted returns (returns minus the risk-free rate). A constraint that the vector of weights must sum to unity is presented to ensure that an investor does not borrow or lend at the risk-free rate.

Due to the complex task of estimating returns, $\hat{\mu}$ will be calculated as the grand mean of all assets across the time frame (Merton (1980), Okhrin and Schmid (2006)). The decision to forgo the estimation of returns was not taken lightly but there are several problems with returns estimations. Several methods of returns predictability were considered including anomalies, implied information, traditional characteristics, such as the Fama-French 3 Factor model, and using historical sample data.

Although there appear to be anomalies in estimating returns using options data (look no further than internet forums such as r/wallstreetbets for some of the most outlandish claims for returns predictability using options data), anomalies tend to increase the predictability of returns only during certain time frames (Linnainamaa and Roberts, 2018). It is beyond the scope of this dissertation to test the predictability of anomalies and find across what sample periods they are effective. The second consideration was using factors such as model-free implied volatility, model-free implied skewness, and call-put implied volatility spreads. These are used by DeMiguel, Plyakha, Uppal, and Vilkov (2013) and they find that the predictability of returns decreases as the holding period increases. The longest holding period they had was a fortnight. This dissertation uses holding periods twice as long. Thus, this was not an appropriate method of estimating returns. The final two candidates would be to use more traditional methods such as the Fama-French model or to use a plug-in approach using historical data. Both of these have the same problem as they do not subsume any information from the options market. There is concern that using them would weaken the dissertations' claim about the ability to use option-implied information to improve portfolio selection by not appropriately isolating the correct cause for performance improvement.

In the absence of a reasonable solution to deal with returns estimations using option-implied information, another method of dealing with returns must be found. This dissertation settles on using Stein's shrinkage estimation as presented in Jorion (1985). In this dissertation, an "extreme shrinkage" (if standard shrinkage is the forcing of a value *toward* some term, extreme shrinkage is the shrinkage of a value *to* said term) is done to each asset mean until the mean of each asset is the grand mean across all assets in the sample. "Extreme shrinkage" is appropriate as there is evidence that there is no long-term difference in expected returns in the cross-section of stocks (Keloharju, Linnainmaa, and Nyberg, 2020). Reducing each assets' mean to the grand mean across all assets yields the global minimum variance portfolio. One of the large advantages of using this approach is that since the global minimum variance portfolio is only dependent on the estimated covariance matrix, it will more appropriately represent the quality of the covariance matrix estimator (Ledoit and Wolf, 2017). This is particularly important here as it displays most accurately the effects of the use of option-implied information in the portfolio optimization (as there will be no estimation errors in returns). The use of "extreme shrinkage" as an estimate of returns, and thus obtaining the global minimum variance portfolio, is quite common in the literature (Jorion (1985), Jagannathan and Ma (2003), DeMiguel, Garlappi, Nogales, and Uppal (2009), Ledoit and Wolf (2003), (2017)). These papers show that global minimum variance portfolios perform exceptionally well especially when compared to a tangency portfolio where nothing is done to reduce the estimation errors in returns.

4.2.2 Global Minimum Variance Portfolio

Minimum variance optimization is simply a closed-form solution to the aforementioned optimization problem in which the obtained portfolio has the lowest possible volatility along the efficient fronteir. Using matrix notation, the solution to the weights of the minimum variance portfolio is given by:

Eq. (12) $$w_{min} = \frac{\hat{\Sigma}^{-1}\, e}{e^{\mathrm{T}}\hat{\Sigma}^{-1}\, e}$$

subject to:

$$w^{\mathrm{T}}\, e = 1$$

where $\hat{\Sigma}^{-1}$is the inverse of the covariance matrix, w is a vector of weights, and e is a vector of 1's. The constraint applied to the weights presented is done to ensure that an investor allocates all of the money across risky assets. In other words, they cannot borrow at the risk-free rate to lever up ($w^{\mathrm{T}}\, e > 1$) and they cannot lend at the risk-free rate ($w^{\mathrm{T}}\, e > 1$). Two sets of optimization calculations are run: one in which the weights are constrained to the positive domain (this is also referred to as the no-short-sale constraint), the other in which the weights are free to be positive or negative. The weights obtained from this optimization process compose a portfolio that is not only on the efficient frontier but also one in which no other combination of weights can yield less volatility.

The estimated covariance matrix from Eq. (12) here can be decomposed into:

Eq. (13) $$\hat{\Sigma} = diag(\hat{\sigma})\, \hat{\Omega}\, diag(\hat{\sigma})$$

where $diag(\hat{\sigma})$ is the volatility matrix for a given timeframe. In the benchmark portfolios, these volatilities are estimated using historical data, in the other portfolios these volatilities are estimated using implied volatilities. The $\hat{\Omega}$ term is an estimated correlation matrix. To estimate the correlation matrix some portfolios are created using only historical data while others are created with option-implied data.

There are several methods in the literature that have been shown to improve out-of-sample performance of minimum variance portfolios, the most common among them is the constrained portfolio. Jagannathan and Ma (2003) argue that theoretically implementing a no-short-sale constraint should hurt portfolio performance but empirically they find the opposite is true. Thus, in the results section there will be a comparison of the performance of constrained and unconstrained portfolios.

The constrained weights cannot be found in a closed-from solution as in the case for the unconstrained circumstance. In order to obtain the constrained weights, the vector of weights from the unconstrained solution is taken and truncated at 0. This provides a new vector with no weights less than 0 but whose sum is not equal to 1. In order to resolve this, the weights are recalculated as a proportion of the sum of the vector, thus providing us a solution which satisfies the constraint that the weights must sum to unity.

4.2.2.1 Positive Definiteness of Covariance Matrix

While calculating the covariance matrices, in order to prevent instances in which the estimated portfolio variance yielded a negative value, a verification had to be done to ensure that the covariance matrix was positive definite. When necessary, to correct any covariance matrix that was not positive definite, the nearest positive definite matrix is used. This is done by first verifying that the covariance matrix for the i-th month cannot be decomposed by the Cholesky method. The Cholesky method is a decomposition of a Hermitian positive define matrix, thus if decomposition is not possible the matrix is not positive definite. When unable to compute this decomposition, the nearest-positive definite matrix in the Frobenius norm is found using a slightly altered methodology proposed by Highman (1988).

In order to avoid the negative portfolio variance problem, a matrix must be obtained that is positive definite but the Highman method finds the nearest positive semi definite (PSD) matrix. Presented here is the methodology to calculate the nearest PSD as per the Highman method. After this explanation the modification which forces the matrix from the positive semi-definite space to the positive definite space is presented.

Let this newly obtained matrix be called the nearest PSD matrix. To find the nearest PSD, let X be a real NxN matrix. First compute the symmetric part of X as defined by $Y = (X+X^T)/2$. Then find the eigen decomposition of Y, which is given by $Y = QDQ^T$, where D is a diagonal matrix of the eigenvalues and Q is a matrix of the eigenvectors. The diagonal matrix of eigenvalues is then augmented to satisfy the following piecewise:

$$D_i = \begin{cases} \lambda_i, & \text{if } \lambda_i \geq 0 \\ 0, & \text{if } \lambda_i < 0 \end{cases}$$

where λ_i is the eigenvalue of matrix X. Thus, the nearest PSD matrix of X is given by Z = QD_+Q^T. Although this provides the nearest-PSD matrix it will contain eigenvalues of 0 which means it will not be invertible, a necessary feature of the covariance matrix in order to solve for the weights of the minimum-variance portfolio. Thus, in lieu of zero the eigenvalues are set to a very small positive number (Jewbali, 2009); for the case of this dissertation $1e^{-10}$ is used. Now the nearest positive definite matrix of X is given by Z' = QD_+Q^T, where D_+ is a diagonal matrix that contain only positive values on the diagonal. Z' is then used as the new covariance matrix for the remainder of the calculations for the *i*-th month.

5. Performance

This section will be a discussion on the dissertations' findings regarding the ability to use forward-looking option-implied information to enhance the out-of-sample performance of an all-stock optimized portfolio. Section *5.1* will simply present the findings for the benchmark portfolios. Those are the portfolios whose weights were obtained using only historical data. Also present will be the results of naïve 1/N diversification portfolio. Section *5.2* will be broken down into two subsections. The first will deal with the results in which the volatilities were estimated using the options data and the estimation of the correlation matrix is based on historical data. The second subsection of *5.2* will present the portfolio's performance when both the estimates for volatility and correlation were obtained using implied information from the options data. This is done in order to study how the addition of each piece of implied information affects the results. Furthermore, the calculation of the implied correlation matrix is a function of the implied volatilities, thus it would be inappropriate to claim that a portfolio created using historical volatilities and implied correlation was created independent of implied volatility.

There will be 3 metrics to evaluate the out-of-sample performance of the various portfolios: the annualized portfolio returns (RET), annualized volatility (VOL), and the annualized Sharpe ratio (SR). It should be noted all calculations assume frictionless markets.

5.1 Performance of Portfolios Without Using Option-Implied Information

Tables 1 presents the results of the benchmark portfolios all of which contain no implied data from the options market. There is a total of three benchmark portfolios. The first of these is the naïve diversification 1/N portfolio. The following two portfolios are both calculated using the closed-form solution for the minimum-variance portfolio. The portfolios with the sub-scripts C and UC represent constrained and unconstrained weights, respectively.

Table I

Mean-Variance Portfolios without Option-Implied Information

Table I reports the performance of the 1/N portfolio and various benchmark portfolios. The nomenclature is as follows: BM represents a benchmark portfolio, GMV is a minimum variance portfolio, C is a portfolio with a no-short-sale constraint, and UC is a portfolio with unconstrained weights. All of these portfolios are created using historical data and none rely on option-implied information. In parenthesis, the *p-value* with respect to the 1/N portfolio. The null hypothesis is that the portfolio being evaluated is no better than the benchmark. Thus, a small *p-value* means the null is rejected.

Strategy	RET	VOL	SR
1/N	11.82%	19.05%	0.59
BM-GMV-C	8.28%	22.37%	0.36
	(0.999)	*(0.999)*	*(0.999)*
BM-GMV-UC	0.50%	31.56%	0.02
	(0.999)	*(0.999)*	*(0.999)*

In the case of each of these portfolios the estimation period was based on 1 month's observations of daily historical data. The rebalancing period was also monthly. Also reported in the table are three performance metrics aforementioned.

Below the RET and VOL metrics, the value in parenthesis is the *p-value* of a difference in means test that tests whether the returns of a given portfolio are statistically different than the 1/N portfolio. Note that this is a one-sided test where the null hypothesis is that the given portfolio obtained returns no better than the 1/N portfolio. Thus, an extremely small *p-value* leads us to reject the null. Although a large *p-value* has no bearing in this test, it can tell us whether a portfolio performed statistically significantly worse than the benchmark.

Below the SR metric, the value in parenthesis is the *p-value* of a test to see whether the Sharpe Ratio of a given portfolio is statistically different than the 1/N portfolio. Here a studentized bootstrap inferencing is conducted as presented in Ledoit and Wolf (2008). To generate the bootstrap data for each portfolio, resampling from the original pairs of returns and volatility is done with replacement. From the bootstrapped data the Sharpe Ratio is obtained. This process is then repeated 1000 times for each portfolio. Thus, there are 1000 bootstrapped Sharpe Ratios from each portfolio. The difference is taken between a given portfolios' Sharpe Ratio and its associated benchmark, for the case of Table I the associated benchmark is the 1/N portfolio.

Eq. (14) $$\Delta = SR_{benchmark} - SR_{evaluated\ porfolio}$$

The test, as originally presented in Ledoit and Wolf (2008), used has a null hypothesis, $H_0: \Delta = 0$. In this dissertation the slight alteration is made in order to make the test one tailed: $H_0: \Delta > 0$. This allows the test to be one tailed and see if the evaluated portfolio obtains a Sharpe Ratio that is no better than the benchmark. From this difference of Sharpe Ratios, a confidence interval is created with the nominal level of $1 - \alpha$. If this confidence interval does not contain a 0, the null can be rejected in favour of the alternative which states that the Sharpe Ratios are statistically different.

Note that in the tables which portray portfolio performance all *p-values* in the RET and VOL columns are created by the difference in means methodology and all *p-values* in the SR column are created using studentized bootstrapping.

From this table it can be seen that the minimum-variance portfolios did not reduce volatility when compared to the 1/N portfolio. Though it may seem counter-intuitive it is consistent with some of the existing literature (DeMiguel, Garlappi, Uppal, 2009). In *Optimal versus Naive Diversification* the authors state that "the 1/N strategy typically outperforms the sample-based mean-variance strategy if one were to make no adjustment at all for the presence of estimation error". This is consistent with the results as this dissertation makes no adjustments for the presence of estimation errors. This holds true across all three metrics across both optimized portfolios.

5.2 Performance of Portfolios Using Option-Implied Information

This subsection will discuss the results of portfolios using implied information. First the portfolio results with only implied volatility (where the estimation of the correlation matrix used historical data) then the portfolio results when both implied volatility and implied correlation were used in the optimization process.

5.2.1 Performance of Portfolio Using Only Implied Volatility

The literature is clear regarding the predictive power of implied volatility when calculated using the model-free methodology then adjusting for the volatility risk premium (DeMiguel, Plyakha, Uppal, and Vilkov, 2013). The equation for the covariance matrix is given by:

Eq. (13) $$\hat{\Sigma} = diag(\hat{\sigma})\ \hat{\Omega}\ diag(\hat{\sigma})$$

where $diag(\hat{\sigma})$ is the estimate for the volatility's matrix (in this subsection the volatilities matrix uses the implied volatilities calculated in section *3.1*) and $\hat{\Omega}$ is the historical correlation matrix as an estimate of the true correlation matrix. This covariance matrix that was used for the calculations in Table II. Table II presents the two portfolios calculated using implied volatilities to estimate the volatilities matrix and the historical correlations to estimate the correlation matrix.

Tables II and III contains two sets of *p-values* below the RET and VOL metrics: the first is a one-tailed test of the difference in means against the 1/N portfolio. The second is a one-tailed test of difference of means against the corresponding benchmark from Table I. There are also two sets of *p-values* below the SR metric. These are one-tailed tests to see if the SR obtained was no better than the benchmark. This was done using the bootstrapping method explained in Section *5.1*. Again, the first *p-value* tests the given portfolio against the 1/N portfolio and the second *p-value* tests the given portfolio against the appropriate benchmark.

Table II shows that both minimum variance portfolios perform very well compared to 1/N portfolio as well as their respective benchmarks with the most noticeable improvement occurring in the unconstrained minimum-variance portfolio. This portfolio was able to obtain a SR of 1.23. Both constrained and unconstrained minimum-variance portfolios saw a statistically significant increase in the Sharpe Ratio even though neither portfolio experienced the expected reduction in volatility (DeMiguel, Plyakha, Uppal, and Vilkov, 2013).

Even though there was an increase in the Sharpe Ratio of the constrained GMV portfolio, the increase was not sufficient to yield a higher Sharpe Ratio than the 1/N portfolio. Despite this, there is sufficient evidence to suggest that using option-implied volatility (as calculated with the model free method then adjusting for the volatility risk premium) has a significant improvement on the portfolio performance of a minimum-variance portfolio when compared with traditional optimization.

Table II

Mean-Variance Portfolios using Option-Implied Volatility

Table II reports the performance of the 1/N portfolio and various portfolios that use risk corrected model free implied volatility and historical correlations. The nomenclature is as follows: BM represents a benchmark portfolio, GMV is a minimum variance portfolio, C is a portfolio with a no-short-sale constraint, and UC is a portfolio with unconstrained weights. All of these portfolios are created using implied volatility to estimate the volatility matrix and historical correlation to estimate the correlation matrix. The first value in parenthesis is the *p-value* with respect to the 1/N portfolio. The second value in parenthesis is the *p-value* with respect to the portfolios' most appropriate benchmark from Table I. The null hypothesis is that the portfolio being evaluated is no better than the benchmark. Thus, a small *p-value* means the null is rejected.

Strategy	RET	VOL	SR
1/N	11.82%	19.05%	0.59
GMV-C	14.13%	21.99%	0.60
	(0.003)	*(0.999)*	*(0.396)*
	(0.000)	*(0.041)*	*(0.000)*
GMV-UC	33.57%	23.75%	1.23
	(0.000)	*(0.999)*	*(0.000)*
	(0.000)	*(0.000)*	*(0.000)*

5.2.2 Performance of Portfolio Using Implied Volatility and Implied Correlation

This section discusses the results obtained from using implied volatility and an option-implied correlation matrix in order to compute portfolio selection. These portfolios use the same computations for implied volatility as section *5.2.1* but also use a risk-corrected option-implied correlation matrix (as calculated in section *3.2*).

Table III follows the same layout as Table II in that it presents the performance metrics of three portfolios. The first portfolio is again the 1/N and the following two are the minimum variance portfolio solved using the closed-form solution (unconstrained) and truncated solution (constrained). Here again, there are two *p-values* in parenthesis below the RET, VOL and the SR metrics. The *p-values* in the RET and VOL columns are for a one-tailed test of the difference in means. The *p*-values in the SR column tests, using student bootstrap inferencing, whether the

Sharpe Ratio obtained from the optimization is no better than the Sharpe Ratio of the benchmark portfolio. The first *p-value* tests the given portfolio against the 1/N portfolio and the second tests the given portfolio against its corresponding benchmark in Table I.

Table III

Mean-Variance Portfolios using Option-Implied Volatilities and Correlation

Table III reports the performance of the 1/N portfolio and various portfolios that uses risk corrected model free implied volatility to estimate the volatilities matrix and option-implied correlations to estimate the correlation matrix. The nomenclature is as follows: GMV is a minimum variance portfolio, C is a portfolio with a no-short-sale constraint, and UC is a portfolio with unconstrained weights. The first value in parenthesis is the *p-value* with respect to the 1/N portfolio. The second value in parenthesis is the *p-value* with respect to the portfolios' most appropriate benchmark from Table I. The null hypothesis is that the portfolio being evaluated is no better than the benchmark. Thus, a small *p-value* means the null is rejected.

Strategy	RET	VOL	SR
1/N	11.82%	19.05%	0.59
GMV-C	7.73%	22.36%	0.33
	(0.999)	*(0.999)*	*(0.999)*
	(0.299)	*(0.489)*	*(0.999)*
GMV-UC	-6.51%	31.27%	-0.21
	(0.999)	*(0.999)*	*(0.999)*
	(0.999)	*(0.444)*	*(0.999)*

From Table III it can be seen that the addition of an implied correlation matrix to the optimization process does not lead to an improvement in out-of-sample performance by any metric used in this dissertation. Across both minimum-variance portfolios the introduction of the implied correlation matrix led to lower returns, higher volatility, and a lower Sharpe Ratio. Furthermore, the unconstrained portfolios obtained negative returns.

The conclusion can be drawn that using option-implied volatilities in tandem with an option-implied correlation matrix does not yield any improvement in portfolio performance as measured by returns, volatility, or Sharpe Ratio. In their analysis of the impact of various option-implied moments on portfolio selection, DeMiguel, Plyakha, Uppal, and Vilkov (2013) suggest that the poor performance is due to the fact that a covariance matrix based on an implied correlation matrix is “highly unstable over time”. Thus, the ensuing weights obtained with this matrix are highly variable and are very likely to obtain poor performance out-of-sample.

6. Portfolio Risk Assessment

Taking a risk measured approach to these portfolios requires an answer to one of the most common question from an investor: How much can the investor lose? The most reasonable solution to this question is to compute the Value-at-Risk (VaR). VaR is a measure of the largest possible loss within a given probability over a given timeframe. It allows investors to make a statement such as “*X*% of the time, we do not expect to lose more than *Y*% over the next T time span.” That is its main advantage; it summarizes risk into one easy to understand sentence. Over the past 25 years it has become one of the most commonly used risk management tools for financial institutions. One of the primary reasons for this is due to J.P. Morgan’s decision to make their RiskMetrics database and documentation freely available to everyone. The RiskMetrics documentation is highly dependent on VaR.

6.1 Calculating VaR

The formal definition of VaR is given by the probability of a loss greater than the VaR will occur must be less than or equal to the level of confidence. Mathematically, it is given by:

Eq. (15) $$P(L > VaR) \leq 1 - c$$

where c is the confidence level and L is the loss. There are three primary methods by which VaR is calculated: the delta-normal method, the historical method, and using a Monte Carlo simulation (RiskMetrics, 1996). Monte Carlo simulations are primarily used for portfolios whose instruments or payoffs are not lineally dependant on market variables (i.e., stock options), thus it is beyond the purview of this dissertation. The other two methods will be explored.

6.1.1 Delta-Normal VaR

The delta-normal method can be implemented when the relationship between the payout and the underlying is 1-to-1. The change in value of a stock is always given by a delta of one. The convexity of this payoff function (gamma) is always equal to 0 (RiskMetrics, 1996).

A portfolio's rate of return can be expressed as:

Eq. (16) $$R_{p,t+1} = w_t \, R_{t+1}$$

where w_t is a vector of weights and $R_{t+1}$is a vector of returns. Since the portfolios presented in this dissertation are well diversified, the initial hypothesis is made that the returns are normally distributed. Portfolios can be considered normally distributed if their elements are jointly normal variables. This normality assumption is supported by the central limit theorem which states that when independent random variables are added to a distribution their distribution approaches normal (as the number of randomly drawn value approaches infinity) even if they were not drawn from a normal distribution.

Recall that portfolio variance, using matrix notation, is given by:

Eq. (17) $$\sigma^2\left(R_{p,t+1}\right) = \; w_t \, \Sigma \, w_t$$

where w_t is a vector of weights, σ^2 is portfolio variance and Σ is an estimate of the covariance matrix. The portfolio VaR is then given by:

Eq. (18) $$VaR = \; \alpha_{CV} \sqrt{w_t \, \Sigma \, w_t}$$

where α_{CV} is the critical value that corresponds to the α confidence level of the continuous distribution. Traditionally, the estimated covariance matrix used in the VaR calculation is the sample covariance matrix of the *t*-th time frame, which in turn serves as an estimate for the covariance matrix in the $t + 1$ time frame.

Although the initial assumptions stated were that returns were normally distributed, there is reason to doubt that these assumptions hold in an empirical setting. In the literature, there is evidence that percent changes in many variables are not normally distributed as they exhibit leptokurtosis or "fat-tails" (Hull and White (1997), Mandelbrot (1963), (2004)).

In order to test the validity of these some of the assumptions made regarding the distribution, two tests will be done on each of the portfolios: a Ljung-Box Test and a Jarque-Bera test. The Ljung-Box tests if the sample data is autocorrelated with a lag of *k* periods. If they are autocorrelated, then the assumption that returns are independently distributed is too strong. The second is a Jarque-Bera test, which is a goodness-of- fit test that tests whether the kurtosis and skewness of the sample data match that of a normal distribution.

The Ljung-Box test is run twice per portfolio, once with a 1-month lag and again with a 12-month lag (1 year). All of these tests yield low scores and provide *p-values* very close to 1. For the results of this test please refer to Appendix A. The null hypothesis for this test is that the data is independently distributed. Since all the *p-values* are well above the critical value of 0.05, the null hypothesis is not rejected and there is evidence that the data is not autocorrelated at the 95% confidence level. This begins to build the argument for using the normal distribution to model the VaR calculations. The central limit theorem provides further evidence to this claim. A normal distribution is an appropriate distribution, if the sample size is large enough. The question is whether the data set used in this dissertation qualifies as a large enough sample size. Finding whether the sample size is large enough can be done by running a Jarque-Bera test. If the null hypothesis of excess kurtosis equals skewness equals 0 is rejected, then the sample size is not large enough to assume a normal distribution.

Table IV

Test for Normality using Jarque-Bera Test

Table IV reports the Jarque-Bera Statistic for each of the portfolios. This test will confirm or deny whether the returns follow a normal distribution. The Strategy column is divided into 3 sections: Historical - portfolios created using only historical data (the benchmark portfolios), Implied Vol. - portfolios created using implied volatilities and historical correlations, and IV & Implied Corr. - portfolios created using both implied volatilities and implied correlations. The null hypothesis of the test is a joint hypothesis stating: the returns of the portfolio exhibit no skewness and no excess kurtosis. Again, a small p-value means the null is rejected

Strategy	JB Test	p-value
Historical		
1/N	6609.98	0.000
BM-GMV-C	50477.60	0.000
BM-GMV-UC	39392.24	0.000
Implied Vol.		
GMV-C	248337.39	0.000
GMV-UC	470289.51	0.000
IV & Implied Corr.		
GMV-C	156.17	0.000
GMV-UC	36502.03	0.000

The Jarque-Bera test is used to see if there is evidence to support the assumption of normality. All these tests yield large scores that produce *p-values* of almost 0 in all cases. The null hypothesis is a joint hypothesis that skewness and excess kurtosis are equal to 0. Since the *p-values* are well below the critical values, the null is rejected as there is evidence that either the skewness or excess kurtosis (or both) does not equal 0.

This means that an appropriate distribution needs to be found in order to find the α to solve Eq. (19). Since no general hypothesis can be made regarding the distribution, several distribution candidates are chosen. The distribution candidates are primarily the more common continuous distributions, namely: normal, uniform, exponential, logistic, lognormal, gamma, laplace, f, and t. Each of these candidates is then fitted and the maximum likelihood estimation of parameters is obtained. The vector of parameters from the maximum likelihood estimation is then run though a log likelihood function where the best candidate will obtain the best value (Myung, 2002). Please refer to Appendix B for the results of these calculations. The candidate distribution with the lowest value (since the function is a negative log likelihood function the distribution with the smallest value is the most appropriate) is then selected. In the case of each of the portfolios, the t-distribution was the most appropriate candidate.

In order to confirm this assumption, a Kolmogorov-Smirnov test is run. A Kolmogorov-Smirnov test quantifies the distance between the empirical distribution function and in this case the cumulative distribution function of a reference distribution, in this case the t-distribution. The null hypothesis of this test states that, given that the reference assumption is the t-distribution, the sample data is drawn from the t-distribution. Table V displays the results of the Kolmogorov-Smirnov test statistic as well as the corresponding *p-values*. It is clear that in all portfolios the *p-value* is well above the critical value for rejection. Thus, the null cannot be rejected, and the remainder of the calculations will be done under the assumption that the sample data follows a t-distribution.

Table V

Test for Goodness of Fit using Kolmogorov-Smirnov Test

Table V reports the Kolmogorov-Smirnov Statistic for each of the portfolios. This test is a goodness of fit test that confirms or denies whether the returns of the given portfolio returns could be drawn from the reference distribution, which in this case is the t-distribution. The Strategy column is divided into 3 sections: Historical - portfolios created using only historical data (the benchmark portfolios), Implied Vol. - portfolios created using implied volatilities and historical correlations, and IV & Implied Corr. - portfolios created using both implied volatilities and implied correlations. Again, a small *p-value* means the null is rejected.

Strategy	KS Statistic	*p-value*
Historical		
1/N	0.043727	0.755
BM-GMV-C	0.053389	0.500
BM-GMV-UC	0.032054	0.967
Implied Vol.		
GMV-C	0.048046	0.648
GMV-UC	0.034719	0.937
IV & Implied Corr.		
GMV-C	0.047918	0.652
GMV-UC	0.042868	0.774

Returning now to Eq. (19), a suitable value for α can be easily found. It is important to note that in all VaR calculations in Table VI use the sample covariance matrix to calculate estimates of volatility. Since each month a new covariance matrix is used to estimate VaR, the time-series average VaR is presented as a proxy for the VaR over the sample period. Table VI presents the results of the VaR calculations at the 95% VaR (and thus α is 5%). In the tables, there is a variable called Hit Rate. This value represents the number of times the VaR threshold was hit in the 237-month sample. It is important to keep in mind that if the sample distribution was exactly a t-distribution the VaR threshold would be hit h number of times, where $h = \alpha * 237$ (where 237 is the number of months in the sample period). So, the 10%, 5%, and 1% VaR thresholds should be hit 24 times, 12 times, and 2 times, respectively. Any deviation from that is due to the inaccuracy of the assumption of the distribution of returns being a t-distribution, even though it is the known distribution that most resembles the distribution of returns. In this dissertation, only the 95% VaR and cVaR of each portfolio will be presented. See Appendix C for the 90% and 99% VaR for each portfolio as well as the cVaR at each of those levels[4]. Occasionally in cVaR calculations no value was obtained as there were no instances where the VaR threshold was hit. When this is the case, the tables present n/a in the cVaR column.

Table VI

95% VaR of Each Portfolio

Table VI reports the mean 95% VaR, the Hit Rates and cVaR of each of the portfolios. The expectation is that the portfolios will experience losses greater than the value in the VaR column 5% of the time. Hit Rate is the number of times the losses in a month were equal to or exceeded the VaR. cVaR is the expected loss given that losses are greater than the 95% VaR. The Strategy column is divided into 3 sections: Historical - portfolios created using only historical data (the benchmark portfolios), Implied Vol. - portfolios created using implied volatilities and historical correlations, and IV & Implied Corr. - portfolios created using both implied volatilities and implied correlations.

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-1.84%	54	-5.24%
BM-GMV-C	-1.39%	68	-4.68%
BM-GMV-UC	-26.23%	56	-58.07%
Implied Vol.			
GMV-C	-2.01%	58	-5.02%
GMV-UC	-9.30%	37	-6.74%
IV & Implied Corr.			
GMV-C	-1.99%	61	-5.09%
GMV-UC	-15.47%	40	-20.09%

It is clear from Table VI that there is a discrepancy between the number of times the portfolio should be experiencing losses greater than VaR and the number of times the portfolio actually experiences losses greater than VaR. This is not inconsistent with what is expected from the literature, as aforementioned. For more information on the ability of the delta normal VaR to predict the actual VaR an analysis of and comparison with the historical VaR will be conducted later in this dissertation.

[4] Calculated along with the VaR and presented in all tables with VaR calculations, will be the cVaR. cVaR, also referred to as expected shortfall, is the conditional value-at-risk and it seeks to quantify the loss given the VaR threshold is crossed. If VaR is concerned with "*X*% of the time, we do not expect to lose more than *Y*% over the next *T* time span", then cVaR is concerned with "If our losses exceed *Y*%, we expect to lose *Z*% over time span *T*" where $Z>Y$.

6.1.2 VaR Under Different Covariance Matrices

This section calculates VaR in a slightly unorthodox way: by using the covariance matrices that were constructed using implied information. Here, the weights obtained from a given portfolio are applied to different estimates of the covariance matrix to first produce a volatility estimate. The volatility estimate is then multiplied by the α (refer to Eq. (18)) and will thus result in a value for VaR. This is a robustness test done to see whether the use of a covariance matrix constructed using option-implied data produces better VaR estimates than the VaR estimates calculated using the sample covariance matrix. See Figure 1 for a flow chart using the weights of the naïve diversification strategy.

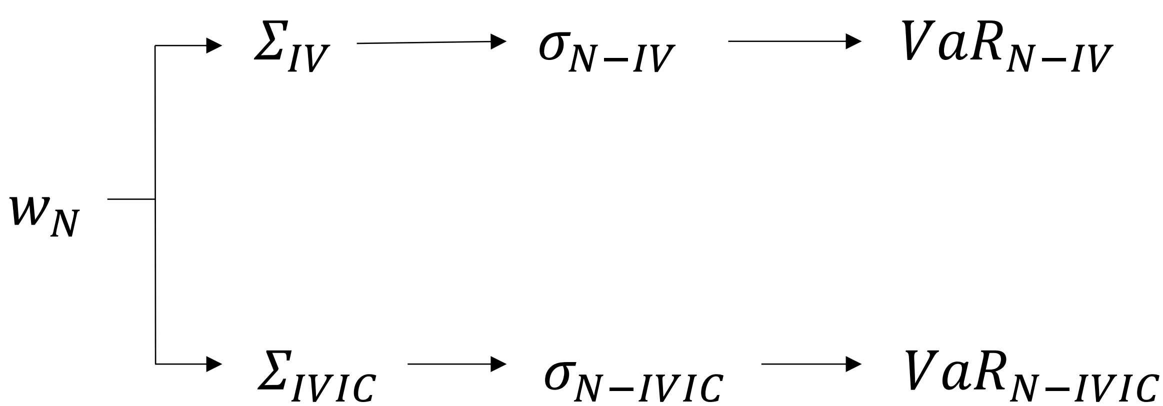

Figure 1. This is a visual representation of the way the estimate of volatility is calculated using the weights of a given portfolio and the different covariance matrices constructed with option-implied data. Figure 1 shows the process specifically for the weights of the naïve diversification strategy multiplied by the covariance matrix (IV) for one volatility estimate and covariance matrix (IVIC) for another. IV is the covariance matrix which used implied volatility and historical correlation and IVIC is the covariance matrix which used implied volatility and implied correlation. All weights obtained from the optimization calculations were used and substituted for w_N.

In Figure 1, which is an illustrative example of the VaR calculation process, w_N is the weights vector from the naïve portfolio, Σ_{IV} is the estimated covariance matrix obtained by using the implied volatilities historical correlation, and Σ_{IVIC} is the estimated covariance matrix obtained using the implied volatilities and implied correlations. These matrices are then applied to Eq. (17) which gives an estimate of volatility for each covariance matrix. This process is repeated using all constrained and unconstrained weights that were calculated in subsections *4.1* and *4.2*. After all these new estimates of volatility are calculated they are substituted into Eq. (18) and a new estimate of VaR can be obtained. This means that, when including VaR estimates using the sample covariance matrix, in total 21 VaR estimates are obtained for each month in the sample period (7 portfolios, thus 7 weight vectors and 3 estimates for the covariance matrix: sample, implied volatility (IV), and implied volatility and implied correlation (IVIC), which are used on each set of weights). For full VaR results of calculations using covariance matrices with option-implied information refer to Appendix E, F, and G.

Table VII presents the hit rates of these new VaR estimates and compares it to the VaR estimates that used the sample covariance matrix.

Table VII

Hit Rates of 95% VaR of Each Portfolio

Table VII reports the hit rates of VaR calculated using different covariance matrices. The first column is the hit rate at the 95% VaR using the sample covariance matrix. The next two columns use the weights from the given strategy and the covariance matrix using implied volatility (IV), then the covariance matrix using implied volatility and implied correlation (IVIC). The Strategy column is divided into 3 sections: Historical - portfolios created using only historical data (the benchmark portfolios), Implied Vol. - portfolios created using implied volatilities and historical correlations, and IV & Implied Corr. - portfolios created using both implied volatilities and implied correlations.

	Covariance Matrices		
Strategy	Sample	IV	IVIC
Historical			
1/N	54	4	1
BM-GMV-C	68	8	3
BM-GMV-UC	56	13	4
Implied Vol.			
GMV-C	58	46	18
GMV-UC	37	33	24
IV & Implied Corr.			
GMV-C	61	37	35
GMV-UC	40	35	28

Table VII shows the hit rates for all portfolios when using different estimates of the covariance matrix. As previously mentioned, and consistent with expectations, hit rates calculated using the sample covariance matrix greatly underestimate the tail risk. For a given 95% VaR, the hit rate should be approximately 12; yet in some portfolios, hit rates 5-fold larger were obtained. In the case of every portfolio, the use of option-implied information to construct the covariance matrix helped reduce the hit rate. The low hit rates obtained from the covariance matrices using implied data cannot always be considered an improvement in estimation. In some cases, the decrease in hit rates is due to the high instance of volatility estimates that once multiplied by the appropriate $\alpha_{C.V}$ yielded a VaR less than -1, a value which is not a possible return in an unlevered portfolio. Thus, for these months it would have been impossible for the VaR to be hit even if the portfolio value went to 0.

From these findings the conclusion is drawn that not only can the use of option-implied data improve the portfolio selection process, but it can decrease the hit rate across all portfolios using the delta-normal method. It is important to note that while most of these VaR hit rates are much higher than the expected 12 (due to the VaR underestimating the amount of tail risk), they are an improvement when compared to using the sample covariance matrix, especially in the portfolios created using implied information.

6.1.3 Historical VaR

The historical simulation approach to calculating VaR is much more straight forward as it is nonparametric and makes no assumptions regarding the distribution of the data (Jorion, 2001). In this method, there is no need to calculate the portfolios mean returns or standard deviation. Instead, this approach uses the actual percentiles from the data to obtain the VaR measures. The reason that there are no assumptions regarding the distribution is because the actual historical distribution is used. The assumption regarding serial independence is also relaxed (Hendricks, 1996).

The calculation of this is quite simple. For the 237-month observation period the 95% VaR will be equal to the 12^{th} largest loss as 12 months represents approximately 5% of the total sample period. Table VIII presents the historical VaR and cVaR of all the portfolios at the 95% VaR levels. Please refer to Appendix D for the calculations of the 90% and 99% VaR.

Table VIII

95% VaR of Each Portfolio using Historical Method

Table VIII reports the 95% VaR based on the historical method. These VaR calculations are non-parametric and assume the future returns will follow the exact same distribution as past returns. Presented along with the VaR is the cVaR. The hit rates are omitted as they are implicitly obvious. The Strategy column is divided into 3 sections: Historical – portfolios created using only historical data (the benchmark portfolios), Implied Vol. – portfolios created using implied volatilities and historical correlations, and IV & Implied Corr. – portfolios created using both implied volatilities and implied correlations.

Strategy	95% VaR	95% cVaR
Historical		
1/N	-8.40%	-10.53%
BM-GMV-C	-8.78%	-10.67%
BM-GMV-UC	-100.00%	-100.00%
Implied Vol.		
GMV-C	-8.58%	-10.30%
GMV-UC	-8.05%	-11.38%
IV & Implied Corr.		
GMV-C	-8.01%	-10.58%
GMV-UC	-12.27%	-24.49%

This calculation yields expected results. When compared with Table VI it is clear that the 95% VaR using the historical method yields a higher loss than the time-series average VaR calculated using the delta-normal method with the sample covariance matrix. This statement is true for all portfolios save the implied volatility unconstrained portfolio. This presents evidence that the returns distribution is leptokurtic as the literature suggests. It furthermore shows how weak the sample covariance matrix is at predicting VaR. Although the use of option implied information in the covariance matrices does not fully capture tail risk, it does a better job than the sample covariance matrix.

The second key observation from Table VIII is that the cVaR of the portfolios constructed using implied information yield less intense losses than their appropriate benchmarks. This suggests that the use of option-implied information in portfolio selection may decrease the tail risk of the portfolio, especially in the case of unconstrained GMV portfolios. The ability of the option-implied portfolios to decrease tail-risk is likely due to one of two reasons. The first one is that the use of American options may bias the calculations as the market had priced in the risk of early execution.

This is unlikely the case as Anderson and Bondarenko (2007) claim that the use of American options should not have a material effect on the implied volatility calculations when short maturities are used. The second plausible solution is that options prices in general are designed to include many extreme scenarios, some of which do not happen, thus exaggerating tail risk. A portfolio selected using option-implied information will compensate for the high tail risks by reducing the weights in the assets with the highest implied volatility, creating a portfolio with narrower tails than would be expected from traditional portfolio optimization.

The main criticism and drawback of the historical method is that only one sample path is used, the historical one. Here, the implicit assumption is that the past is the best predictor of the near future. This means that if a historical window omits important events such as recession or periods of high volatility, the VaR will not capture their probability (for example, had the historical period of the portfolios been from 2010-2019 it would have 10 years of data but no recessions). Given that the sample period used in this dissertation captures two bear markets (the tech bubble and the Great Recession), the VaR calculated from this method could be reasonably reliable.

6.2 Statistical Significance of Hit Rates

It is difficult to say whether implied information actually helps the estimates of VaR given that the calculation is done month to month. Hence, for any given month an estimate of VaR can be calculated using the covariance matrix but the true VaR for an individual month cannot be calculated. Thus, it is much more appropriate to deal with hit rates. This dissertation has made the first step in demonstrating that the delta normal method of calculating VaR, especially when using the sample covariance matrix, underestimates the historical VaR in portfolios constructed both using historical data and option-implied information. In order to test whether this is true, the statistical significance of the hit rates must be found and used as a proxy for the accuracy of the VaR calculations. With this a definite statement can be made whether the overestimate by the delta normal VaR is significant.

To do this, a binomial test is conducted. This can be done because the issue of hit rates can be seen as a problem that takes a binomial distribution, where a successful trial is defined by a month in which losses surpass the VaR. Thus, hit rate, h, is a binomial random variable where $h \sim B(n,p)$

where n is the number of trails (in the case of this dissertation, the number for months in the sample data) and p is the success rate. First to form the proportion use:

Eq. (19)
$$\pi = \frac{h}{n}$$

When doing the test, for the purpose of simplicity, the binomial distribution can be assumed to be approximately normal if

Eq. (20)
$$n\pi(1-\pi) \geq 10$$

This inequality did not hold; thus, the calculation needs to be made using the binomial distribution[5]. The binomial test has a null hypothesis given by $H_0\colon \pi = \pi_0$ where π is the proportion from the sample and π_0 is the expected proportion.

Since the objective in this dissertation is to see whether the observed proportion is significantly more than the expected proportion, the null hypothesis is modified to $H_0\colon \pi \leq \pi_0$. The probability of finding a given value, in this case *h*, is given by:

Eq. (21)
$$\Pr(X = h) = \binom{n}{h} p^h (1-p)^{n-h}$$

where h is the number of times the VaR was hit in a given portfolio, n is the sample size, $\binom{n}{h}$ (*n* choose k) is the binomial coefficient, and p is the probability of hitting the VaR (for 95% VaR this value is 0.05). If $h < n\pi_0$, the cumulative probability is found with $\Pr(P \leq h)$. If $h \geq n\pi_0$, the cumulative probability is found with $\Pr(P \geq h)$. The *p-value* of this test is then given by this value.

Table IX displays the results from the test of statistical significance of the hit rates, where p is the probability of hitting the VaR is 5% and h is the actual hit rate of a given portfolio. This table reenforces the strength of the statement made in the previous subsection regarding the ability of using option-implied information in portfolio selection as a way to accurately model VaR.

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[5] Although in some portfolios this inequality held, it would be inconsistent to do some of the calculations assuming a normal distribution while others used the binomial distribution.

Although there is an improvement in the hit rate, it is not sufficient to reduce the observed hit rate to the theoretical expectation. With these results the definite statement can be made that the VaR calculated using the delta-normal method with the sample covariance matrix will always underestimate risk when compared to the historical method.

Table IX

Statistical Significance of Hit Rates

Table IX reports the statistical significance of the hit rates across all the portfolios at the 95% level. This is a test on the statistical significance of the hit rate and whether the obtained hit rate is less than the theoretical value (in this case given by 0.05*237≈12). When the p-value is sufficiently small we reject the null. Each p-value is calculated by taking the h value from the corresponding strategy and covariance matrix in Table VII. Since the table refers to the 95% VaR the appropriate p is 5%. The Strategy column is divided into 3 sections: Historical - portfolios created using only historical data (the benchmark portfolios), Implied Vol. - portfolios created using implied volatilities and historical correlations, and IV & Implied Corr. - portfolios created using both implied volatilities and implied correlations.

	Covariance Matrices		
Strategy	Sample	IV	IVIC
Historical			
1/N	0.999	0.007	0.000
BM-GMV-C	0.999	0.158	0.002
BM-GMV-UC	0.999	0.700	0.070
Implied Vol.			
GMV-C	0.999	0.999	0.970
GMV-UC	0.999	0.999	0.999
IV & Implied Corr.			
GMV-C	0.999	0.999	0.999
GMV-UC	0.999	0.999	0.999

A second test of statistical significance is also done. Here the goal is to measure whether the addition of more implied information to the covariance matrix helps statistically significantly decrease the hit rate. For this set of calculations, the h is the number of times the VaR was hit in a given portfolio and p is the probability of obtaining the empirically measured hit rate using the reference covariance matrix.

Table X shows clearly that the addition of implied information to the covariance matrix helps statistically significantly reduce the hit rate. Although this is not the case for each individual portfolio, as some portfolios do not have a statistically significant decrease at the 10% level, the general trend is clear. When using the IVIC covariance matrix, the estimate of VaR yielded hit rates statistically significantly less than when using the sample covariance matrix at the 5% confidence level across all portfolios. It is again important to mention that the decrease in hit rates cannot always be seen as an improvement in VaR calculations.

Table X

Statistical Significance of Hit Rate Improvement

Table X reports the p-values of hit rates compared to a reference hit rate. For these calculations the binomial equation is used where p is the expected probability of hitting the VaR (which was calculated using a reference covariance matrix) and h is given by the hit rate of the portfolio using one of the implied covariance matrices (i.e column Sample-IV tests the improvement in hit rates of a given portfolio using the IV covariance matrix against the hit rate of that portfolio using the sample covariance matrix). The Strategy column is divided into 3 sections: Historical - portfolios created using only historical data (the benchmark portfolios), Implied Vol. - portfolios created using implied volatilities and historical correlations, and IV & Implied Corr. - portfolios created using both implied volatilities and implied correlations.

	Covariance Matrices		
Strategy	Sample - IV	Sample - IVIC	IV -IVIC
Historical			
1/N	0.000	0.000	0.090
BM-GMV-C	0.000	0.000	0.040
BM-GMV-UC	0.000	0.000	0.003
Implied Vol.			
GMV-C	0.039	0.000	0.000
GMV-UC	0.270	0.010	0.051
IV & Implied Corr.			
GMV-C	0.000	0.000	0.402
GMV-UC	0.220	0.020	0.115

7. Conclusion

The weights of an optimized portfolio rely on estimates for volatilities, correlations, covariance matrices, which are simply the products of the diagonal volatilities' matrix and the correlation matrix, and expected stock returns. This dissertation analyzed how the use of implied information from the options market can be used to improve the out-of-sample performance of an all-stock portfolio by improving the estimates of the volatilities and correlations. Portfolio performance was measured along three metrics, namely: annualized returns, annualized volatility, and annualized

Sharpe Ratio. In this dissertation, a total of four portfolios are constructed using option-implied information. These portfolios obtain their weights using the minimum variance portfolio optimization. Constrained and unconstrained weights are obtained for the purpose of comparison. There are also several benchmark portfolios including a naïve diversification strategy, 1/N. After the presentation of the results, there is an attempt to quantify the risk of the constructed portfolios. This is done using a variety of VaR methodologies and analyzing hit rates.

The principal findings of this dissertation are that, by using options-implied volatilities, an investor is able to improve the performance of minimum-variance portfolios. The option-implied correlation matrix was much less helpful in improving portfolio performance. Pervious literature has suggested this is due to the resulting covariance matrix being highly unstable over time. Based on this empirical analysis it is clear that information extracted from the options market can be useful in improving the portfolio performance out-of-sample.

The benefits of using options-implied data are even more stark when analyzing risk metrics such as VaR, hit rates, and cVaR. These benefits can be deconstructed into two observations. The first is that the use of covariance matrices constructed using implied data in VaR calculations decreased the instance in hit rates. The second is that the use of option implied information in the portfolio selection process reduces the tail risk of the portfolio. This was evident in the fact that the historical VaR calculations yielded larger losses in traditionally optimized portfolios than in those constructed using option implied information. The same observation is true when analyzing the cVaR using the historical method.

Further research would need to verify that the use of option-implied data reduces the tail risk of optimized portfolios. Also, further analysis must be done regarding the claims a more accurate estimation of VaR can be obtained when using option-implied information to formulate the covariance matrix. This verification would involve a statistical analysis of the properties of said matrices to ensure the improvement in hit rates is a result of improved volatility estimates and not extreme volatility estimates which make the VaR is impossible to hit. Other further research should focus on the historical method of calculating VaR. The largest default of this method is it assumes that the distribution of previous results is the best predictor of the distribution in the near future. This dissertation does nothing to prove or disprove this assumption.

Appendix

Appendix A

Test for Autocorrelation using Ljug-Box Test with *k*-Period Lag

Strategy	LB Stat (*k*=1)	*p-value*	LB Stat (*k*=12)	*p-value*
Historical				
1/N	0.000029	0.995	4.840829	0.963
BM-GMV-C	0.001308	0.971	2.630068	0.997
BM-GMV-UC	0.000927	0.975	3.973316	0.983
Implied Vol.				
GMV-C	0.000021	0.996	0.734838	0.999
GMV-UC	0.000000	0.999	0.190937	0.999
IV & Implied Corr.				
GMV-C	0.029342	0.863	5.121024	0.953
GMV-UC	0.011711	0.913	5.347892	0.945

Appendix B

Results from Negative Log Likelihood Calculation Using Parameter Estimates from Maximum Likelihood Estimates

	Candidate Distributions								
Strategy	norm	uniform	expon	logistic	lognorm	gamma	laplace	f	t
Historical									
1/N	-351.614	-101.14	-190.745	-388.798	-366.233	-161.274	-398.136	-52.189	-399.756
BM-GMV-C	-313.614	-23.378	-185.313	-387.884	-360.945	88.9653	-401.175	-64.3906	-408.443
BM-GMV-UC	-231.965	77.7458	237.099	-314.756	-231.648	-208.708	-325.006	394.997	-341.176
Implied Vol.									
GMV-C	-199.92	111.149	-218.604	-356.236	-342.927	301.866	-371.421	-115.488	-397.443
GMV-UC	49.7704	366.644	-156.224	-233.134	-278.395	341.996	-261.441	-110.511	-334.877
IV & Implied Corr.									
GMV-C	-386.351	-211.736	-210.675	-398.947	-386.754	-386.584	-401.068	-133.624	-401.531
GMV-UC	-234.281	34.2013	236.665	-300.538	-234.009	311.365	-310.19	341.607	-318.404

Appendix C

90% VaR of Each Portfolio Using Sample Covariance Matrix

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-1.43%	60	-4.97%
BM-GMV-C	-1.08%	72	-4.55%
BM-GMV-UC	-21.83%	58	-58.36%
Implied Vol.			
GMV-C	-1.57%	66	-4.81%
GMV-UC	-7.38%	46	-6.37%
IV & Implied Corr.			
GMV-C	-1.55%	68	-4.73%
GMV-UC	-9.33%	46	-10.54%

Appendix C

99% VaR of Each Portfolio Using Sample Covariance Matrix

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-2.60%	46	-5.79%
BM-GMV-C	-1.97%	54	-5.47%
BM-GMV-UC	-33.36%	52	-60.81%
Implied Vol.			
GMV-C	-2.86%	46	-5.74%
GMV-UC	-12.20%	31	-7.02%
IV & Implied Corr.			
GMV-C	-2.82%	49	-5.44%
GMV-UC	-14.83%	29	-24.35%

Appendix D

90% VaR of Each Portfolio using Historical Method

Strategy	VaR	cVaR
Historical		
1/N	-5.36%	-8.54%
BM-GMV-C	-5.64%	-8.62%
BM-GMV-UC	-82.94%	-97.50%
Implied Vol.		
GMV-C	-5.14%	-8.38%
GMV-UC	-6.24%	-9.16%
IV & Implied Corr.		
GMV-C	-5.23%	-8.33%
GMV-UC	-7.91%	-17.20%

Appendix D

99% VaR of Each Portfolio using Historical Method

Strategy	VaR	cVaR
Historical		
1/N	-15.05%	-15.28%
BM-GMV-C	-13.78%	-14.97%
BM-GMV-UC	-100.00%	-100.00%
Implied Vol.		
GMV-C	-12.47%	-12.99%
GMV-UC	-14.74%	-15.66%
IV & Implied Corr.		
GMV-C	-12.57%	-13.54%
GMV-UC	-23.09%	-61.55%

Appendix E

90% VaR of Each Portfolio Using IV Covariance Matrix

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-12.98%	9	-7.45%
BM-GMV-C	-12.19%	12	-8.32%
BM-GMV-UC	-71.31%	20	-81.96%
Implied Vol.			
GMV-C	-2.60%	54	-4.64%
GMV-UC	-14.12%	37	-5.35%
IV & Implied Corr.			
GMV-C	-2.84%	49	-4.89%
GMV-UC	-18.60%	38	-10.09%

Appendix E

95% VaR of Each Portfolio Using IV Covariance Matrix

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-15.59%	4	-7.85%
BM-GMV-C	-14.99%	8	-7.76%
BM-GMV-UC	-77.80%	13	-84.30%
Implied Vol.			
GMV-C	-3.35%	46	-4.74%
GMV-UC	-16.79%	33	-5.53%
IV & Implied Corr.			
GMV-C	-3.65%	37	-4.77%
GMV-UC	-21.78%	35	-20.37%

Appendix E

99% VaR of Each Portfolio Using IV Covariance Matrix

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-20.53%	2	-8.06%
BM-GMV-C	-19.98%	3	-8.49%
BM-GMV-UC	-85.90%	7	-79.77%
Implied Vol.			
GMV-C	-4.75%	35	-5.41%
GMV-UC	-21.12%	27	-5.52%
IV & Implied Corr.			
GMV-C	-5.18%	31	-4.61%
GMV-UC	-27.33%	29	-23.25%

Appendix F

90% VaR of Each Portfolio Using IVIC Covariance Matrix

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-16.31%	1	-10.72%
BM-MV-C	-15.36%	3	-8.49%
BM-MV-UC	-82.09%	9	-77.32%
Implied Vol.			
MV-C	-18.09%	21	-4.85%
MV-UC	-18.09%	30	-5.51%
IV & Implied Corr.			
MV-C	-3.98%	33	-4.69%
MV-UC	-23.74%	31	-10.41%

Appendix F

95% VaR of Each Portfolio Using IVIC Covariance Matrix

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-19.87%	1	-10.72%
BM-MV-C	-19.02%	3	-8.49%
BM-MV-UC	-88.03%	4	-92.16%
Implied Vol.			
MV-C	-21.47%	18	-4.95%
MV-UC	-21.47%	24	-5.67%
IV & Implied Corr.			
MV-C	-5.12%	26	-4.66%
MV-UC	-27.89%	28	-23.98%

Appendix F

99% VaR of Each Portfolio Using IVIC Covariance Matrix

Strategy	Mean VaR	Hit Rate	cVaR
Historical			
1/N	-26.57%	0	n/a
BM-MV-C	-25.53%	1	-10.24%
BM-MV-UC	-94.54%	2	-84.32%
Implied Vol.			
MV-C	-26.78%	16	-4.88%
MV-UC	-26.78%	24	-5.67%
IV & Implied Corr.			
MV-C	-7.26%	18	-4.50%
MV-UC	-34.35%	24	-26.14%

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