Collusion in the Bertrand-Cournot Model: Analysing Strategic Behaviour and Market Outcomes

by

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Abstract

This dissertation examines the sustainability of collusion in a Cournot-Bertrand model involving three firms, where the strategic variable (price or quantity) is chosen before competition or collusion begins. Using the Singh and Vives (1984) demand model, the analysis considers both full and partial collusion, focusing on the role of a third firm that always competes independently. The critical discount factor is used to assess the ease of sustaining collusion under various scenarios. Results show that collusion is more stable when firms compete on quantities due to softer competition, while price-setting firms face stronger incentives to deviate. In mixed strategy scenarios, where some firms set prices and others set quantities, collusion becomes harder to maintain. The degree of product substitution also plays a crucial role, with higher substitution making collusion more fragile. For partial collusion, stability is greater when the third firm competes on price rather than quantity. The findings highlight the complexity of maintaining collusion in hybrid Cournot-Bertrand environments and suggest that coordination is easier when firms align their strategic variables. The dissertation also suggests potential extensions, including analyzing deviations by the third firm and assessing the impact on consumer welfare.

Keywords: Collusion,Bertrand,Cournot,Hybrid

Resumo

Esta dissertação examina a sustentabilidade do conluio num modelo de Cournot-Bertrand envolvendo três empresas, em que a variável estratégica (preço ou quantidade) é escolhida antes do início da concorrência ou do conluio. Utilizando o modelo de procura de Singh e Vives (1984), a análise considera tanto o conluio total como o parcial, centrando-se no papel de uma terceira empresa que compete sempre de forma independente. O fator crítico de desconto (d) é utilizado para avaliar a facilidade de manter a conluio nos vários cenários. Os resultados mostram que o conluio é mais estável quando as empresas competem em quantidades devido a uma concorrência mais suave, enquanto as empresas que fixam os preços enfrentam incentivos mais fortes para se desviarem. Em cenários de estratégia mista, em que algumas empresas fixam preços e outras fixam quantidades, o conluio torna-se mais difícil de manter. Relativamente ao conluio parcial, a estabilidade é maior quando a terceira empresa concorre em preço e não em quantidade. Os resultados evidenciam a complexidade da manutenção do conluio em ambientes mistos de Cournot-Bertrand e sugerem que a coordenação é mais fácil quando as empresas alinham as suas variáveis estratégicas. A dissertação também sugere potenciais extensões, incluindo a análise dos desvios da terceira empresa e a avaliação do impacto no bem-estar dos consumidores.

Palavras-Chave: Collusion,Bertrand,Cournot,Hybrid

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Contents

1 Introduction

Tacit collusion and the mode of coordination among firms in oligopolistic markets have been subjects of significant interest and debate in the field of industrial organization. Understanding how firms strategically behave and coordinate their actions is crucial for assessing market dynamics, competition policy, and consumer welfare. One key aspect of this research focuses on the choice between price and quantity coordination in tacit collusion scenarios.

In this context, Lambertini and Schultz (2003) contribute to the existing literature by examining whether firms engaged in oligopolistic competition tend to coordinate their actions through price or quantity adjustments. Tacit collusion refers to a situation where firms implicitly coordinate their behaviour without explicit agreements, aiming to achieve outcomes that resemble those of explicit collusion. By investigating the prevalent mode of coordination and its stability, this research provides insights into the strategic behaviour of firms and its implications for market outcomes.

The choice between price and quantity coordination holds substantial implications for market behavior. Price coordination, where firms collectively set prices, may lead to higher market prices and joint profits, while quantity coordination involves firms adjusting production levels to limit supply and secure larger market shares. Determining which mode of coordination is more prevalent and stable has significant implications for market efficiency, consumer welfare, and competition policy. Lambertini and Schultz's research aims to uncover the factors influencing this choice and its stability, by employing theoretical frameworks, the study seeks to identify the factors influencing the choice between price and quantity coordination and explore the implications for market dynamics.

In the context of strategic decision-making, the paper by Tremblay, Tremblay, and Isariyawongse (2011) explores how firms in oligopolistic markets decide on their competitive strategies (quantity vs. price competition) and the timing of their actions. Their model allows firms to choose whether to act as leaders or followers, integrating elements of both Cournot (quantity) and Bertrand (price) competition. This study highlights the interdependence of timing and type of competition, showing how these decisions influence market outcomes, such as prices, quantities, and profits.

The same authors in 2013 explore how advertising strategies influence market competition in the automobile industry, focusing on Honda and Scion. The study integrates traditional Cournot (quantity competition) and Bertrand (price competition) models with advertising that rotates demand curves. Honda, representing a Cournot-type firm, uses mass-market advertising to rotate demand counterclockwise, increasing quantity demanded at each price level.

In contrast, Scion, as a Bertrand-type firm, employs niche advertising that rotates demand clockwise, enhancing consumer willingness to pay. The findings demonstrate that Honda's strategy results in higher output levels, aligning with Cournot competition, while Scion's approach leads to higher prices, fitting the Bertrand model. This shows that firms with lower production costs benefit more from quantity competition and mass-market advertising, whereas firms with higher costs gain from price competition and targeted advertising. This study highlights the significant role of strategic advertising in shaping competitive behaviors and market outcomes.

Further expanding on these concepts, Tremblay and Tremblay (2011) investigate how product differentiation impacts market outcomes in a mixed duopoly with Cournot and Bertrand firms. They find that sufficient product differentiation allows both types of firms to coexist and achieve stable equilibrium. Without differentiation, the market dynamics shift, leading to a perfectly competitive outcome dominated by Cournot-type firms. This research illustrates the critical role of product differentiation in shaping competitive strategies and market equilibrium.

The previous articles focused on a game-theoretic framework involving two firms that made initial choices between quantity and price as their strategic variables. These choices led to three distinct subgames: Cournot competition when both firms chose quantities, Bertrand competition when both firms chose prices, and Cournot-Bertrand competition when each firm chose a different variable.

Expanding upon this line of inquiry, our model proposes incorporating a third firm into the framework to provide a more comprehensive analysis of strategic interactions in oligopolistic markets. This modification enables an examination of collusion scenarios in a trilateral setting, enhancing the complexity and realism of the analysis. The inclusion of additional firms also allows for the exploration of partial collusion dynamics, as seen in Escrihuela-Villar (2008) and Gabszewicz et al. (2017), particularly in cases of asymmetry among firms.

Moreover, the literature encompasses various studies elucidating different facets of oligopoly behavior. Escrihuela-Villar (2008) investigates partial coordination and mergers among quantity-setting firms, highlighting their strategic implications for market dynamics and competition outcomes. Gabszewicz et al. (2017) delve into the relationship between vertical differentiation and collusion, while Singh and Vives (1984) analyze price and quantity competition in differentiated duopolies, contributing theoretical insights into competition dynamics.

Additionally, Tremblay and Tremblay (2019) provide a comprehensive survey of oligopoly games and the Cournot-Bertrand model, synthesizing existing literature on strategic interactions among firms in quantity-setting environments. Wang and Wang (2021) examine the impact of managerial delegation on upstream collusion, Kovacic et al. (2018) explore serial collusion by multi-product firms, and Colombo (2015) investigate collusion dynamics in mixed oligopolies.

Together, these studies enrich our understanding of industrial organization and game theory, offering valuable insights into the strategic behavior of firms in oligopolistic markets, the determinants of collusion, and their implications for market outcomes and competition policy.

Our model focuses on studying the Cournot-Bertrand model with three firms, specifically examining a preliminary phase in which the firms choose their strategic variables before engaging in competition or collusion. After this stage, different subgames unfold which consist of six possible scenarios: i) All firms choose to compete in the Cournot strategy; ii) All firms choose to compete in the Bertrand strategy; iii) Two firms collude and choose the Bertrand strategy, while the third firm competes with the Cournot strategy; iv) Two firms collude and choose the Cournot strategy, while the third firm competes with the Bertrand strategy; v) The two colluding firms choose the Cournot-Bertrand strategy, while the third firm opts for the Cournot strategy; vi) The two colluding firms choose the Cournot-Bertrand strategy, while the third firm chooses the Bertrand strategy. The objective of this model is to determine which scenario exhibits the most stable (partial) collusion between the two firms. Additionally, if the third firm is removed from the analysis, the research aims to identify the most stable collusion scenario.

The paper is organized as follows. Section 2 presents the model and the basic assumptions. Section 3 interprets the main results, comparing them to other results in the literature and with each other; Section 4 presents the conclusions and further extensions of the model.

2 The model

We model an industry with three firms that sell symmetrically differentiated products. Firms are assumed to make their decisions in two stages. However, this dissertation does not model the first-stage decision, it is taken as given. In the second stage, firms play an infinitely repeated game in which the strategic variable (either quantity or price) is chosen simultaneously.

To capture the dynamics of this industry, we integrate the Singh and Vives (1984) demand model into a Cournot-Bertrand framework for three firms. This involves using a demand curve formulation that considers consumer preferences, market size, and pricing strategies as outlined by Singh and Vives (1984).

In this model, the inverse demand functions are given by:

$$\begin{aligned} p_1 &= 1 - q_1 - \gamma q_2 - \gamma q_3 \\ p_2 &= 1 - q_2 - \gamma q_1 - \gamma q_3 \\ p_3 &= 1 - q_3 - \gamma q_2 - \gamma q_1 \end{aligned}$$

where p_i, represents the price of the product sold by firm i, q_i represents the quantity sold by firm i, and γ is a positive parameter (smaller than 1) that represents the degree of substitution between the three goods. A high γ indicates that the products are very close substitutes, implying that consumers view them as nearly identical alternatives whereas a γ equal to zero would mean each firm was the monopolist seller of an independent product. By representing the substitution effect with the same parameter γ, we assume that the degree of substitution is equal between all pairs of goods. As for the cost structure, we assume firms have constant marginal costs, normalized to zero. Each firm's profit π_i is then a function of its output q_i and price p_i:

$$\pi_i = p_i q_i$$

In the first stage, each firm decides whether to compete by setting prices (Bertrand competition) or by setting quantities (Cournot competition). This strategic choice significantly impacts the competitive landscape in the second stage.

In the second stage, firms may either compete or collude based on their initial strategic choices. In competitive scenarios, firms act independently to maximize their own profits, either by adjusting prices in the Bertrand model or quantities in the Cournot model. In collusive scenarios, firms cooperate to maximize joint profits. Collusion can be partial, where only some firms collude, or total, where all firms agree to collude. The sustainability of such collusion depends on several factors, including the degree of substitution γ and the discount factor δ assumed to be common to all firms, which reflects the firms' valuation of future profits.

The aim of this dissertation was to assess in which of the scenarios collusion is easier to sustain, in particular with the introduction of a third firm that always competes. As seen in previous papers, if companies have pre-agreed quantity or price contracts, and the goods are substitutes, as is our case, for reasons of calculation proven previously, it is a dominant strategy for the company to choose quantities (Singh and Vives, 1984) in terms of Nash equilibrium profits. When analyzing the discount factors in an industry with two firms, past research (Lambertini and Schultz (2003)) has concluded that if the goods are substitutes, then $\delta^Q < \delta^P$. This implies the following: If $\delta < \delta^P$, firms will set quantities in the normal phase; If

$\delta \in [\delta^Q, \delta^P]$, firms will be able to sustain collusion and obtain monopoly profits; If $\delta < \delta^Q$, firms will earn less than monopoly profits due to limited cooperation.

To be able to compare with previous work, we calculated the critical δ that allows collusion to be sustained for the 4 different scenarios of total collusion and 6 of partial collusion. We now investigate the sustainability of the collusive outcome in the infinitely repeated game.

2.1 Total Collusion

We begin by examining an industry characterized by complete collusion, as illustrated by Singh and Vives, 1984 and Lambertini and Schultz (2003). In this scenario, the three firms involved must choose between maintaining their collusive agreement or deviating from it, with specific punitive outcomes for deviation.

Given the choices in the first stage, where the firms decide their strategic variable, there are four alternative industry scenarios for the infinitely repeated game that follows:

	Firm 1	Firm 2	Firm 3
Scenario 1	P	P	P
Scenario 2	Q	Q	Q
Scenario 3	P	P	Q
Scenario 4	P	Q	Q

In scenario 1, all firms choose to be price setters; in the scenario 2, all firms decide to be quantities setters; in scenario 3, two firms decide to be price setters and the third quantity; and for scenario 4, two firms decide to be quantity setters and the third decide on prices.

There are multiple equilibria for the infinitely repeated game that unfolds after the first stage. We are interested in the situations in which the collusive prices or quantities are sustained in equilibrium. This happens when firms care sufficiently about the future, that is, when the discount factor is high.

Let θ_i represent the strategic variable, chosen on the first stage by firm i, which can be either q_i (quantity) or p_i (price). Let θ_i^P represent the Nash equilibrium value for θ_i in the one-shot version of the repeated game and let θ_i^C represent its value under collusion.

Using the standard trigger strategies, which can be defined as "firm i chooses $\theta_i = \theta_i^C$ in each period if no other outcome than $[\theta_1^C, \theta_2^C, \theta_3^C]$ was observed and $\theta_i = \theta_i^P$ otherwise", collusion is sustained by the trigger strategies if and only if :

$$\pi_i^C \frac{1}{1-\delta} \geq \pi_i^D + \pi_i^P \frac{\delta}{1-\delta} \Leftrightarrow \delta \geq \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^P}$$

where π_i^C, π_i^D and π_i^P represent, respectively, the collusion, deviation and punishment outcomes for firm i. The next sections present these payoffs.

2.1.1 Collusion payoffs

In the collusion stage, firms in an oligopoly cooperate to maximize their joint profits. Instead of competing aggressively by setting lower prices or increasing output, they agree (explicitly or tacitly) to set higher prices or limit output, resembling a monopolistic structure. This agreement allows firms to earn higher profits than they would under pure competition. The objective is to maximize total industry profit Π.

The total industry profit Π is the sum of the individual profits of each firm i, denoted as π_i

$$\Pi = \sum_{i=1}^{3} \pi_i$$

In the collusion stage, firms act as a single entity, akin to a monopoly, to decide on their strategic variables. Their goal is to maximize the combined profit:

$$\max \sum_{i=1}^{3} p_i q_i$$

To find the optimal strategy for each firm, we need to derive the first-order conditions (FOCs) for profit maximization. The first-order condition for maximizing total profit with respect to each strategic variable θ_i is given by:

$$\frac{\partial \Pi}{\partial \theta_i} = 0, i = 1, 2, 3$$

Lemma 1: *In case of total collusion, firm i' s profit is, in all scenarios,* $\pi_i^C = \frac{1}{4(2\gamma+1)}$. *The corresponding price for price setting firms is* $p_i = \frac{1}{2}$ *and the quantity for the quantity setting firms is* $q_i = \frac{1}{2(2\gamma+1)}$. ■

The payoffs are the same in any scenario because when colluding firms operate as a multi-product monopolist, being irrelevant whether the strategic variable is price or quantity.

2.1.2 Deviation payoff

When deviating, one firm may choose to break away from the collusive agreement to gain an immediate profit advantage. During this stage, the deviating firm aims to unilaterally maximize its own profit while the other firms continue to adhere to the collusive agreement.

We assume that the firm that deviates is firm i. The objective for firm i is to maximize its profit π_i independently:

$$\max \pi_i$$

given the strategic variables θ_j^C from other firms $j \neq i$.

The first-order condition for firm i's profit maximization is given by:

$$\frac{\partial \pi_i}{\partial \theta_i} = 0$$

Lemma 2: *Firm i' s profit when it unilaterally deviates, π_i^D, and the strategic variable when deviating are presented for each scenario in the following table:*

Firm i

Scenario 1 (p)	$p_i = \frac{1}{2(\gamma+1)}$, $i = 1,2,3$ $\pi_i^D = \frac{1}{4(1-\gamma)(2\gamma+1)(\gamma+1)}$, $i = 1,2,3$
Scenario 2 (p)	$q_i = \frac{\gamma+1}{2(2\gamma+1)}$, $i = 1,2,3$ $\pi_i^D = \frac{1}{4}\frac{(\gamma+1)^2}{(2\gamma+1)^2}$, $i = 1,2,3$
Scenario 3 (p)	$p_i = \frac{2+2\gamma-\gamma^2}{4(2\gamma+1)}$, $i = 1,2$ $\pi_i^D = \frac{\left(2+2\gamma-\gamma^2\right)^2}{16(2\gamma+1)^2(1-\gamma)(\gamma+1)}$, $i = 1,2$
Scenario 3 (q)	$q_3 = \frac{1}{2(2\gamma+1)(1-\gamma)}$ $\pi_3^D = \frac{1}{4(\gamma+1)(2\gamma+1)(1-\gamma)}$
Scenario 4 (p)	$p_1 = \frac{\gamma+1}{2(2\gamma+1)}$ $\pi_1^D = \frac{1}{4}\frac{(\gamma+1)^2}{(2\gamma+1)^2}$
Scenario 4 (q)	$q_i = \frac{2+2\gamma-\gamma^2}{2(1-\gamma)(2\gamma+1)(\gamma+1)}$, $i = 2,3$ $\pi_i^D = \frac{\left(2+2\gamma-\gamma^2\right)^2}{16(2\gamma+1)^2(1-\gamma)(\gamma+1)}$, $i = 2,3$

■

Under full collusion, all firms receive the same payoff. Therefore, when analyzing gains from deviation, we can focus solely on these deviation payoffs.

The profit from deviation depends only on the strategic behavior of the other firms. For example, the profit a price setter earns by deviating in Scenario 1 is identical to the profit a quantity setter earns by deviating in Scenario 3. In both cases, the other two firms are price setters, and the deviating firm acts as a monopolist over the residual demand when the others are adhering to collusive prices. Thus, it does not matter whether the firm sets prices or quantities in these scenarios. Similarly, the profit of a deviating firm in Scenario 2, where both rivals are quantity setters, is the same as the profit of the price setter in Scenario 4, as the other firms are setting quantities in both cases. Lastly, the profit from deviation for a price setter in Scenario 3 equals that of a quantity setter in Scenario 4, as the deviating firm competes against both a price setter and a quantity setter in each instance. From these comparisons, we conclude that it is more profitable to deviate when both rivals compete on price and less profitable when both rivals compete on output (with the hybrid case in between).[1]

2.1.3 Punishment payoff

The punishment is the reaction of the other firms to the deviation. The goal of the punishment phase is to deter future deviations by making the cost of deviating higher than the short-term gain. As mentioned in the trigger strategy definition, this corresponds to the Nash equilibrium of the one shot game, which is not the same in all scenarios. Each firm maximizes its own profit independently:

$$\max \pi_i$$

Firm i's FOC becomes

$$\frac{\partial \pi_i}{\partial \theta_i} = 0$$

Lemma 3: *Firm i' s profit in the Nash equilibrium of the one shot game, π_i^P, is presented for each scenario in the following table:*

	Firm 1	Firm 2	Firm 3
Scenario 1 (p,p,p)	$p_1 = \frac{1-\gamma}{2}$ $\pi_1^P = \frac{(1-\gamma)(\gamma+1)}{4(2\gamma+1)}$	$p_2 = \frac{1-\gamma}{2}$ $\pi_2^P = \frac{(1-\gamma)(\gamma+1)}{4(2\gamma+1)}$	$p_3 = \frac{1-\gamma}{2}$ $\pi_3^P = \frac{(1-\gamma)(\gamma+1)}{4(2\gamma+1)}$
Scenario 2 (q,q,q)	$q_1 = \frac{1}{2(\gamma+1)}$ $\pi_1^P = \frac{1}{4(\gamma+1)^2}$	$q_2 = \frac{1}{2(\gamma+1)}$ $\pi_2^P = \frac{1}{4(\gamma+1)^2}$	$q_3 = \frac{1}{2(\gamma+1)}$ $\pi_3^P = \frac{1}{4(\gamma+1)^2}$
Scenario 3 (p,p,q)	$p_1 = \frac{(3\gamma+2)(1-\gamma)}{2(3\gamma-\gamma^2+2)}$ $\pi_1^P = \frac{(1-\gamma)(3\gamma+2)^2}{4(\gamma+1)(-3\gamma+\gamma^2-2)^2}$	$p_2 = \frac{(3\gamma+2)(1-\gamma)}{2(3\gamma-\gamma^2+2)}$ $\pi_2^P = \frac{(1-\gamma)(3\gamma+2)^2}{4(\gamma+1)(-3\gamma+\gamma^2-2)^2}$	$q_3 = \frac{\gamma+2}{2(3\gamma-\gamma^2+2)}$ $\pi_3^P = \frac{(\gamma+2)^2(1-\gamma)(2\gamma+1)}{4(\gamma+1)(-3\gamma+\gamma^2-2)^2}$
Scenario 4 (p,q,q)	$p_1 = \frac{(\gamma+2)(1-\gamma)}{2(2+\gamma-2\gamma^2)}$ $\pi_1^P = \frac{(\gamma+2)^2(1-\gamma)^2}{4(2+\gamma-2\gamma^2)^2}$	$q_2 = \frac{2-\gamma}{2(2+\gamma-2\gamma^2)}$ $\pi_2^P = \frac{(2-\gamma)^2(1-\gamma)(\gamma+1)}{4(2+\gamma-2\gamma^2)^2}$	$q_3 = \frac{2-\gamma}{2(2+\gamma-2\gamma^2)}$ $\pi_3^P = \frac{(2-\gamma)^2(1-\gamma)(\gamma+1)}{4(2+\gamma-2\gamma^2)^2}$

■

From Lemma 3, we conclude the following:[2]

[1] The inequalities are presented in the Appendix.
[2] The inequalities are presented in the Appendix.

1) Price competition is less profitable than quantity (output) competition when firms use the same strategic variable.

2) Firms setting quantities achieve higher profits than those setting prices when strategies differ (regardless of how many choose quantities or prices)

3) The profit of a quantity setter increases with the number of quantity setters:

These conclusions imply that in a punishment phase following deviations from collusion, quantity setters are less negatively impacted than price setters. This is because quantity setters enjoy higher payoffs, and this advantage grows as more firms opt for quantity setting, making them more resilient to punishment.

2.1.4 Results

In our model, in scenarios 3 and 4, deviation can either occur for two reasons: we can observe the deviation of a company that competes à la Cournot or of one that competes à la Bertrand. The critical δ to be considered for the respective scenarios must be the maximum δ between the two deviations since collusion is maintained if and only if the 3 companies have incentives to sustain it. So the critical δ for scenarios 3 and 4 is as follows: $\max(\delta^Q, \delta^P)$.

The following table presents the critical discount factors:

	Critical Discount Factor
Scenario 1 (ppp)	$\frac{1}{2-\gamma^2}$
Scenario 2 (qqq)	$\frac{(\gamma+1)^2}{4\gamma+\gamma^2+2}$
Scenario 3 (ppq)	$\max(\frac{\left(-3\gamma+\gamma^2-2\right)^2}{4(3\gamma-2\gamma^2-\gamma^3+2)(\gamma+1)}, \frac{\left(-3\gamma+\gamma^2-2\right)^2(\gamma+2)^2}{(7\gamma+\gamma^2+4)(20\gamma-17\gamma^3+\gamma^4+8)}) = \frac{\left(-3\gamma+\gamma^2-2\right)^2}{4(3\gamma-2\gamma^2-\gamma^3+2)(\gamma+1)}$
Scenario 4 (pqq)	$\max(\frac{(\gamma+2)^2\left(-\gamma+2\gamma^2-2\right)^2}{(\gamma-2\gamma^2+4)(12\gamma-12\gamma^2-11\gamma^3+6\gamma^4+8)}, \frac{\left(-\gamma+2\gamma^2-2\right)^2}{4(3\gamma-2\gamma^2-2\gamma^3+2)}) = \frac{(\gamma+2)^2\left(-\gamma+2\gamma^2-2\right)^2}{(\gamma-2\gamma^2+4)(12\gamma-12\gamma^2-11\gamma^3+6\gamma^4+8)}$

The critical discount factor is larger in scenario 1 than in scenario 2, meaning that it is harder to sustain collusion when all firms are price setters than when they all are quantity setters:

$$\frac{1}{2-\gamma^2} - \frac{(\gamma+1)^2}{4\gamma+\gamma^2+2} = \frac{\gamma^3(\gamma+2)}{(2-\gamma^2)(4\gamma+\gamma^2+2)} > 0$$

As seen above, it is more profitable to deviate when rivals compete on price, but the punishment is also harsher in this case. The first effect dominates the second one. In scenarios 3 and 4, the highest critical discount factor is the one of the quantity setter, who is more likely to deviate. This follows from the fact that quantity setters have a higher punishment payoff.

One might thus consider that the larger the number of price setters, the harder it is to sustain collusion. However, as the following figure illustrates, that is not the case. The critical discount factor does not necessarilly decrease when one (or two) price setters are replaced by quantity setters. This happens because such replacement increases market asymmetry (by having price and quantity setters in a hybrid model) and this is a factor known to make collusion harder to sustain.

Another conclusion is that the type of industry that is more likely to sustain collusion depends crucially on γ the degree of product differentiation. With full collusion, the higher γ the harder it is to sustain collusion, regardless of the decision variables that firms select.

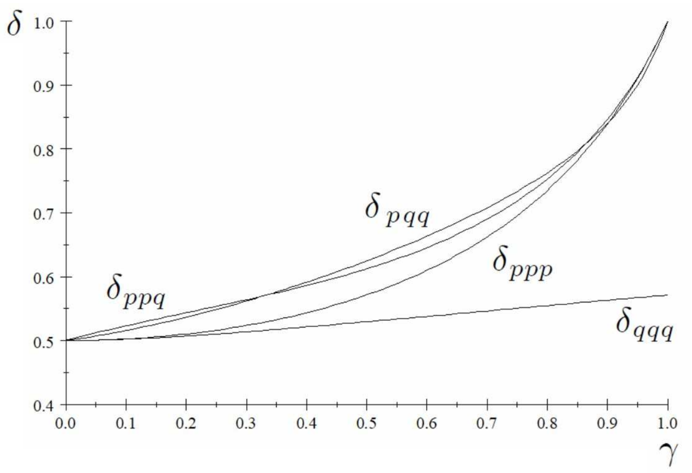

Figure 1: Crtical discount factors for total collusion

Collusion is harder to sustain in a hybrid case because the critical discount factor in scenario 3 is always larger than in scenario 1:

$$\frac{(-3\gamma+\gamma^2-2)^2}{4\,(3\gamma-2\gamma^2-\gamma^3+2)\,(\gamma+1)}-\frac{1}{2-\gamma^2}=\frac{1}{4}\frac{\gamma\,(4+10\gamma+4\gamma^2-\gamma^3)\,(1-\gamma)^2}{(\gamma+1)\,(2-\gamma^2)\,(2+3\gamma-2\gamma^2-\gamma^3)}>0$$

For $\gamma\in[0.330\,00, 0.864\,16]$, collusion is harder to sustain in scenario 4. Otherwise, it is harder to sustain in scenario 3.

Figure 1 provides valuable insights into how different strategic combinations of price-setting and quantity-setting influence the sustainability of collusion in a three-firm hybrid model. When a third firm is introduced, we have the same conclusions as if we had only two firms: the critical discount factor for price-setting firms, denoted δ^P, is higher than that for quantity-setting firms, δ^Q. This implies that price collusion (δ_{ppp}) becomes less sustainable than quantity collusion (δ_{qqq}) in this specific setting.

Collusion is easier to maintain when firms choose quantities as their strategic variable, as indicated by the δ_{qqq} curve, which remains the lowest across the entire graph. In Cournot competition, where firms compete based on output levels, competition is typically less aggressive because quantity adjustments are slower and less reactive to changes in demand compared to price adjustments. This leads to a softer competitive environment, where the temptation to deviate from a collusive agreement by producing more than the agreed-upon quantity is lower. Additionally, the punishment for deviating from collusion—reverting to non-collusive competition—is less severe than in price competition, making firms more likely to stick to the collusive agreement.

In contrast, the δ_{ppp} curve, representing price collusion, is higher than δ_{qqq}, suggesting that price collusion is harder to sustain. Price competition tends to be more aggressive. Even a small reduction in price by one firm can allow it to capture the entire market, making the incentive to deviate from collusion stronger. However, the punishment for deviation is also more severe, as prices drop significantly, leaving all firms with drastically reduced profits. As a result, collusion in price-setting environments requires a higher discount factor. Firms need to highly value future profits to maintain collusion in such environments. If they are not sufficiently patient, collusion is unlikely to be sustained due to the immediate and severe nature of price competition.

The mixed strategy scenarios (δ_{ppq} and δ_{pqq}) present additional complexities. When firms employ different strategic variables (some choosing price, others choosing quantity), collusion becomes more difficult to sustain. The δ_{ppq} scenario, where two firms set prices and one sets quantity, illustrates this challenge. The two price-setting firms face a much greater temptation to undercut their competitors, leading to greater instability. In this case, the price setters have stronger incentives to deviate from the collusive agreement, making it harder to maintain coordination. The differing incentives between firms using different strategies complicate collusion because the firms are effectively playing different games: price setters are operating in a Bertrand-like environment, while quantity setters are in a Cournot-like environment. The resulting misalignment of incentives makes collusion less stable.

In the δ_{pqq} scenario, where two firms set quantities and one sets price, the dynamics are somewhat different. The two quantity-setting firms create a stabilizing force, making collusion easier to maintain despite the presence of one price-setting firm. This suggests that quantity setters tend to stabilize collusion even in mixed strategy environments because their competitive behavior is less aggressive. In contrast, when more firms are setting prices, as in the δ_{ppq} scenario, collusion becomes more fragile due to the higher incentives for price setters to undercut.

As the parameter γ increases across all scenarios, the critical discount factor δ also rises, indicating that collusion becomes more difficult to sustain in more competitive or volatile markets. A rising γ might represent an increase in market competitiveness or demand sensitivity, which heightens the potential rewards from breaking a collusive agreement. In such environments, firms may be tempted to deviate from collusion to capture larger market shares or benefit from more intense competition. However, the punishment phase (returning to non-collusive competition) is also more severe, further destabilizing collusion. Firms need to place a higher value on future profits (i.e., have a higher discount factor) to maintain collusion in such competitive markets.

Punishment dynamics differ significantly between Cournot and Bertrand competition settings. In Cournot competition, firms collude by restricting their output to keep prices high. The temptation to deviate arises because a firm that marginally increases its output can capture a larger market share without causing a drastic reduction in the market price. This makes the gains from deviation relatively high and the immediate costs lower when compared to Bertrand competition.

However, in a Bertrand setting, especially when the γ is close to 1, even an infinitesimal price cut allows a firm to capture the entire market, thereby earning monopoly profits. This creates an extreme incentive to deviate, as the payoff from undercutting rivals is maximized. In contrast, Cournot deviations, while profitable, do not offer the same scale of immediate rewards, since an increase in output only shifts the market equilibrium and does not fully monopolize the market. Thus, the incentive to deviate in a Bertrand setting, where even a slight price reduction can result in monopoly profits, can be stronger than in Cournot, where the reward for deviation is more moderate due to the nature of quantity competition.

In scenarios where the degree of substitution between products is high, firms face increased competitive pressure, which makes collusion harder to maintain. In scenarios 1 and 2, where all firms use the same strategic variable (either all set prices or all set quantities), as the degree of substitution increases, the critical discount factor decreases slightly, implying that collusion becomes somewhat easier. However, in scenarios 3 and 4, where firms use different strategic variables, increasing the degree of substitution leads to a rise in the critical discount factor, making collusion more difficult. This indicates that a higher degree of substitution amplifies the temptation to deviate, as firms can gain more by capturing additional market share, but the punishment for deviation becomes more severe in a highly competitive environment.

In hybrid strategy scenarios, the interplay between firms using different strategic variables increases the complexity of collusion, as the misalignment of incentives makes it harder to maintain. Additionally, as the degree of substitution between firms' products increases, collusion becomes more fragile, particularly in highly competitive environments. Successful collusion requires a careful balance of these factors, along with effective monitoring and credible punishment mechanisms to deter firms from deviating from the collusive agreement.

2.2 Partial Collusion

Partial collusion refers to a situation where firms in an oligopoly collude to a certain extent but do not fully achieve the joint-profit-maximizing outcome. Instead, they settle on an intermediate level of output or pricing that is more profitable than pure competition but less profitable than full collusion. In our model we are considering that two of the three firms fully collude, and the third one competes by maximizing its own profit. For simplicity purposes, we are assuming that firm 1 and 2 are the ones who collude and firm 3 is the maverick firm that never considers other possibility than independent own profit maximization. So we will have the following scenarios:

	Firm 1	Firm 2	Firm 3
Scenario 1	P	P	P
Scenario 2	Q	Q	Q
Scenario 3	P	Q	P
Scenario 4	P	Q	Q
Scenario 5	P	P	Q
Scenario 6	Q	Q	P

In scenario 1, all firms are price setters; in the scenario 2, all firms are quantities setters; in scenario 3 and 5, two firms are price setters and the third sets quantity; in scenario 4 and 6, two firms are quantity setters and the third one sets prices.

Collusion is sustainable if the pair of colluding firms have a discount factor that is above the critical level, defined as above.

2.2.1 Collusion payoffs

In this scenario, firms 1 and 2 cooperate to maximize their joint profits, while firm 3 acts independently to maximize its own profit. This creates a dynamic where firms 1 and 2 behave as a single entity, and firm 3 operates competitively within the same market.

Firms 1 and 2 aim to maximize their combined profit Π, which is the sum of their individual profits $\pi_1$and π_2:

$$\Pi = \sum_{i=1}^{2} \pi_i$$

Each firm's profit π_i is a function of its output q_i and the price p_i it sets. The combined objective can be written as:

$$\max \sum_{i=1}^{2} q_i p_i$$

To find the optimal strategies for firms 1 and 2, we derive the first-order conditions (FOCs) for joint profit maximization. Let θ_i represent the strategic variable for firm i, which can be either quantity q_i or price p_i. The FOCs for maximizing the joint profit Π are given by:

$$\frac{\partial \Pi}{\partial \theta_i} = 0$$

Firm 3, on the other hand, operates independently to maximize its own profit π_3:

$$\max p_3 q_3$$

To determine the optimal strategy for firm 3, we derive the first-order condition for its profit maximization. Let θ_3 represent the strategic variable for firm 3. The FOC for maximizing π_3 is:

$$\frac{\partial \pi_3}{\partial \theta_3} = 0$$

Lemma 4: *In case of partial collusion by firms 1 and 2, firm $i = 1, 2$ and 3's profit, π_i and π_3, as well as the relevant strategic variables are represented, for each scenario, in the* *following table:*

	Firm $i = 1, 2$	Firm 3
Scenario 1	$\pi_i^C = \frac{1}{4}\frac{(1-\gamma)(3\gamma+2)^2}{(2\gamma+1)(2+2\gamma-\gamma^2)^2}$	$\pi_3^C = \frac{(1-\gamma)(\gamma+1)^3}{(2\gamma+1)(2+2\gamma-\gamma^2)^2}$
(p, p, p)	$p_i = \frac{1}{2}\frac{(3\gamma+2)(1-\gamma)}{2+2\gamma-\gamma^2}$	$p_3 = \frac{(1-\gamma)(\gamma+1)}{2+2\gamma-\gamma^2}$
Scenario 2	$\pi_i^C = \frac{1}{4}\frac{(\gamma+1)(2-\gamma)^2}{(2+2\gamma-\gamma^2)^2}$	$\pi_3^C = \frac{1}{(2+2\gamma-\gamma^2)^2}$
(q, q, q)	$q_i = \frac{1}{2}\frac{2-\gamma}{2+2\gamma-\gamma^2}$	$q_3 = \frac{1}{2\gamma-\gamma^2+2}$
Scenario 3	$\pi_i^C = \frac{1}{4}\frac{(1-\gamma)(2\gamma+1)(\gamma+2)^2}{(2+4\gamma-\gamma^3)^2}$	$\pi_3^C = \frac{(1-\gamma)(\gamma+1)^3}{(2+4\gamma-\gamma^3)^2}$
(p, q, p)	$p_1 = \frac{1}{2}\frac{(1-\gamma)(2\gamma+1)(\gamma+2)}{2+4\gamma-\gamma^3}, q_2 = \frac{1}{2}\frac{\gamma+2}{2+4\gamma-\gamma^3}$	$p_3 = \frac{(1-\gamma)(\gamma+1)^2}{2+4\gamma-\gamma^3}$
Scenario 4	$\pi_i^C = \frac{1}{4}\frac{(\gamma+1)(\gamma+2)^2(1-\gamma)^2}{(2+2\gamma-2\gamma^2-\gamma^3)^2}$	$\pi_3^C = \frac{(1-\gamma)(\gamma+1)}{(2+2\gamma-2\gamma^2-\gamma^3)^2}$
(p, q, q)	$p_1 = \frac{1}{2}\frac{(1-\gamma)(\gamma+2)(\gamma+1)}{2+2\gamma-2\gamma^2-\gamma^3}, q_2 = \frac{1}{2}\frac{(1-\gamma)(\gamma+2)}{2+2\gamma-2\gamma^2-\gamma^3}$	$q_3 = \frac{1}{2+2\gamma-2\gamma^2-\gamma^3}$
Scenario 5	$\pi_i^C = \frac{1}{4}\frac{(3\gamma+2)^2(\gamma-1)^2}{(\gamma+1)(2+2\gamma-3\gamma^2)^2}$	$\pi_3^C = \frac{(1-\gamma)(2\gamma+1)}{(\gamma+1)(2+2\gamma-3\gamma^2)^2}$
(p, p, q)	$p_i = \frac{1}{2}\frac{(3\gamma+2)(1-\gamma)}{2+2\gamma-3\gamma^2}$	$q_3 = \frac{1}{2+2\gamma-3\gamma^2}$
Scenario 6	$\pi_i^C = \frac{1}{4}\frac{(1-\gamma)(2\gamma+1)(2-\gamma)^2}{(2+2\gamma-3\gamma^2)^2}$	$\pi_3^C = \frac{(1-\gamma)^2(\gamma+1)^2}{(2+2\gamma-3\gamma^2)^2}$
(q, q, p)	$q_i = \frac{1}{2}\frac{2-\gamma}{2+2\gamma-3\gamma^2}$	$p_3 = \frac{(1-\gamma)(\gamma+1)}{2+2\gamma-3\gamma^2}$

From Lemma 4, we conclude the following:[3]

Scenario 1 - All firms raise their prices, but those engaged in collusion tend to increase them more significantly than a non-colluding firm. The independent firm's pricing strategy inadvertently benefits the colluding firms, as it creates the illusion of competition while allowing the colluders to maintain higher profits.

Scenario 2 - When colluding firms reduce their output, the non-colluding firm increases its output, which negatively impacts the colluders. This independent firm's actions undermine the colluders' efforts to control supply and maintain higher prices.

Scenario 3 - When colluding firms raise prices and reduce output, the non-colluding firm also raises its price, which ends up benefiting the colluders. This independent firm's behavior reinforces the colluders' strategy by supporting higher market prices while maintaining the appearance of competition.

Scenario 4 - When colluding firms raise prices and reduce output, the non-colluding firm increases its output, which undermines the colluders' strategy. This independent firm's higher production weakens the colluders' control over supply and prices, ultimately harming their efforts to maintain elevated profits.

Scenario 5 - When colluding firms raise prices, the non-colluding firm increases its output, which disrupts the colluders' strategy. This independent firm's higher production weakens the

[3]The inequalities are presented in the Appendix.

colluders' ability to sustain higher prices, ultimately harming their profits.

Scenario 6 - When colluding firms reduce output, the non-colluding firm raises its price, which ultimately benefits the colluders. This independent firm's price increase aligns with the colluders' strategy, supporting higher market prices and reinforcing their ability to profit.

2.2.2 Deviation payoff

In this stage, either firm 1 or 2 deviate from the collusive strategies, taking into account that the other firms maintain what was decided in the collusion stage, and also firm 3. Firm i $(i = 1, 2)$ deviates:

$$\max \pi_i$$

given the strategic variables θ_j^* from other firms.

Firm i's FOC becomes

$$\frac{\partial \pi_i}{\partial \theta_i} = 0$$

Lemma 5: *Firm i's profit, π_i^D, and its relevant strategic variable when it unilaterally deviates are presented, for each scenario, in the following table:*

Firm $i = 1, 2$

Scenario 1 (p)	$p_i = \frac{1}{4}\frac{(1-\gamma)(\gamma+2)(3\gamma+2)}{(\gamma+1)(2+2\gamma-\gamma^2)}$ $\pi_i^D = \frac{1}{16}\frac{(1-\gamma)(\gamma+2)^2(3\gamma+2)^2}{(\gamma+1)(2\gamma+1)(2+2\gamma-\gamma^2)^2}$
Scenario 2 (q)	$q_i = \frac{1}{4}\frac{(2-\gamma)(\gamma+2)}{2+2\gamma-\gamma^2}$ $\pi_i^D = \frac{1}{16}\frac{(2-\gamma)^2(\gamma+2)^2}{(2+2\gamma-\gamma^2)^2}$
Scenario 3 (p)	$p_1 = \frac{1}{4}\frac{(1-\gamma)(3\gamma+2)(\gamma+2)}{2+4\gamma-\gamma^3}$ $\pi_1^D = \frac{1}{16}\frac{(1-\gamma)(\gamma+2)^2(3\gamma+2)^2}{(\gamma+1)(2+4\gamma-\gamma^3)^2}$
Scenario 3 (q)	$q_2 = \frac{1}{4}\frac{(\gamma+2)^2}{(2+4\gamma-\gamma^3)}$ $\pi_2^D = \frac{1}{16}\frac{(1-\gamma)(2\gamma+1)(\gamma+2)^4}{(\gamma+1)(2+4\gamma-\gamma^3)^2}$
Scenario 4 (p)	$p_1 = \frac{1}{4}\frac{(1-\gamma)(\gamma+2)^2}{2+2\gamma-2\gamma^2-\gamma^3}$ $\pi_1^D = \frac{1}{16}\frac{(1-\gamma)^2(\gamma+2)^4}{(2+2\gamma-2\gamma^2-\gamma^3)^2}$
Scenario 4 (q)	$q_2 = \frac{1}{4}\frac{(2-\gamma)(\gamma+2)}{2+2\gamma-2\gamma^2-\gamma^3}$ $\pi_2^D = \frac{1}{16}\frac{(1-\gamma)(\gamma+1)(2-\gamma)^2(\gamma+2)^2}{(2+2\gamma-2\gamma^2-\gamma^3)^2}$
Scenario 5 (p)	$p_i = \frac{1}{4}\frac{(1-\gamma)(2-\gamma)(3\gamma+2)}{2+2\gamma-3\gamma^2}$ $\pi_i^D = \frac{1}{16}\frac{(1-\gamma)(2-\gamma)^2(3\gamma+2)^2}{(\gamma+1)(2+2\gamma-3\gamma^2)^2}$
Scenario 6 (q)	$q_i = \frac{1}{4}\frac{(2-\gamma)(3\gamma+2)}{(\gamma+1)(2+2\gamma-3\gamma^2)}$ $\pi_i^D = \frac{1}{16}\frac{(1-\gamma)(2-\gamma)^2(3\gamma+2)^2}{(\gamma+1)(2+2\gamma-3\gamma^2)^2}$

In scenarios 3 and 4 the colluding firms have different strategic variables and therefore there are two different types of deviation: by the price setting firm or by the output setting firm.

2.2.3 Punishment payoff

As in the case of full collusion, firms maximize their own profit independently. The equilibrium profits and strategic variable choices are the same as in the case of full collusion because the punishment payoff corresponds to the Nash equilibrium of the one-shot game which is the same regardless of the number of colluding firms.

2.2.4 Results

In our model, in scenario 3 and 4, deviation can either occur for two reasons: we can observe the deviation of a company that competes à la Cournot or one that competes à la Bertrand. So the critical δ to be considered for the respective scenarios must be the maximum δ between the two deviations, as we previously observed in the case of full collusion.

	Critical Discount Factor
Scenario 1 $[(pp)p]$	$\frac{(3\gamma+2)^2}{(2\gamma+1)(16\gamma+5\gamma^2-2\gamma^3+8)}$
Scenario 2 $[(qq)q]$	$\frac{(\gamma-2)^2(\gamma+1)^2}{(1-\gamma)(8+8\gamma-3\gamma^2-\gamma^3)}$
Scenario 3 $[(pq)p]$	$\max\left\{\frac{\left(-3\gamma+\gamma^2-2\right)^2}{(\gamma+1)(16\gamma+\gamma^2-3\gamma^3+8)}, \frac{(\gamma+2)^2\left(-3\gamma+\gamma^2-2\right)^2}{(\gamma+1)(16\gamma+\gamma^2-3\gamma^3+8)(3\gamma+2)^2}\right\} = \frac{\left(-3\gamma+\gamma^2-2\right)^2}{(\gamma+1)(16\gamma+\gamma^2-3\gamma^3+8)}$
Scenario 4 $[(pq)q]$	$\max\left\{\frac{\left(-\gamma+2\gamma^2-2\right)^2(\gamma+2)^2}{(8\gamma-7\gamma^2-4\gamma^3+8)(\gamma-2)^2}, \frac{\left(-\gamma+2\gamma^2-2\right)^2}{8\gamma-7\gamma^2-4\gamma^3+8}\right\} = \frac{\left(-\gamma+2\gamma^2-2\right)^2(\gamma+2)^2}{(8\gamma-7\gamma^2-4\gamma^3+8)(\gamma-2)^2}$
Scenario 5 $[(pp)q]$	$\frac{\left(2+3\gamma-\gamma^2\right)^2}{(\gamma+1)(8\gamma-11\gamma^2+\gamma^3+8)}$
Scenario 6 $[(qq)p]$	$\frac{\left(2+\gamma-2\gamma^2\right)^2}{16\gamma-3\gamma^2-12\gamma^3+8}$

As before, in scenarios 3 and 4, when colluding firms are asymmetric, the quantity setter is more likely to deviate.

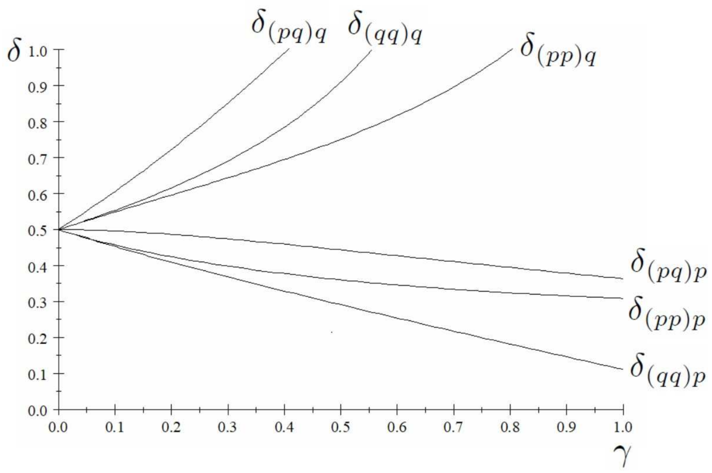

Figure 2: Critical discount factors for partial collusion

It is possible to rank the scenarios in terms of how likely it is that collusion is sustained. The ranking from the hardest to sustain collusion to the easiest to sustain collusion is: Scenario 4, 2, 5, 3, 1, 6.

The three cases in which collusion is harder to sustain are scenarios, 4, 2 and 5. All these have in common that the maverick firm competes by setting output. The scenarios in which collusion is easier to sustain (scenarios 3, 1 and 6) are those in which the maverick firm competes on price.

When we now analyze only the collusion between 2 of the 3 companies in the market, the results obtained are closer to the results of previous papers (Lambertini and Schultz (2003)), but with the same divergences as the total collusion results.

Firstly, when we look at scenarios 2 ($\delta_{(qq)q}$) and 6 ($\delta_{(qq)p}$), in which both colluding companies compete à la Cornout, the stability results are quite different. We found that when the third company also competes on quantities, we have exactly the same results as before, a very high delta, but when the third company competes on prices, the delta is the lowest, showing higher stability.

In this scenario, two of the firms are colluding by coordinating the quantities they produce, while the third firm competes by setting prices. The colluding firms, by agreeing on the quantities to produce, effectively control partially the supply in the market . This coordination allows them to restrict partially the supply and maintain higher market prices. The market becomes more predictable for the colluding firms because their joint output decisions significantly influence the market price, reducing the competitive impact of the third firm's pricing strategy.

The third firm, which competes by setting prices, finds its pricing decisions constrained by the quantities supplied by the colluding firms. Even if this third firm tries to undercut prices, the limited supply orchestrated by the colluding firms ensures that the overall market price remains relatively high. The power of the colluding firms in controlling supply diminishes the price-setting firm's ability to dominate the market through price competition alone.

Moreover, the punishment mechanism for any deviation from the collusive agreement is robust in this setup. If one of the colluding firms decides to produce more than agreed, the increased supply directly impacts the market price, leading to reduced profits for both colluding firms. This immediate negative consequence of deviation helps sustain the collusion, as both firms understand that any breach of the agreement will quickly harm their profits.

In this environment, a colluding firm may have a stronger incentive to deviate from the agreement by increasing its output to gain a larger share of the market, which can disrupt the collusive arrangement. The temptation to increase individual profits by unilaterally expanding production is higher because the immediate market price impact is less directly apparent and more diffused across the combined output of all firms.

Coordinating quantities among two firms while accounting for the independent actions of the third firm adds complexity and instability to the collusion. The third firm's independent quantity decisions introduce variability and unpredictability to the market price, making it harder for the colluding firms to maintain their agreed quantities without external interference.

Regarding the other scenarios, the conclusions and interpretations of the results are exactly the same as the previous ones: greater collusion stability with fewer companies competing on quantity.

Concerning the relationship between the discount factor and the degree of substitution, we found a difference from the total collusion scenarios. We observe that when the third firm, the non-colluding firms, compete on prices, the δ decrease as γ increases. When γ increases, meaning that products become more substitutable, customers are more sensitive to price differences. In this scenario, if the non-colluding firm competes aggressively on price, it can capture a significant portion of the market because customers find it easy to switch to the cheaper product. Also, if they try to maintain higher prices or restrict output as part of their collusive strategy, the third firm can easily undercut them, leading to a large loss in market share for the colluding firms. As a result, the colluding firms have weaker market power and lower profits from sustaining the collusive agreement.

3 Conclusion

This dissertation aimed to evaluate the ease of sustaining collusion with the introduction of a third company in markets where goods are substitutes. Extending the analysis to both full and partial collusion scenarios, it investigated the roles of strategic variables like pricing and quantity in collusion stability.

The results show significant changes in collusion dynamics when a third company is introduced. In scenarios where all companies employ the same strategic variable (either all on price or all on quantity), collusion is more readily maintained. However, when mixed strategies are used (some firms focus on price, others on quantity), the complexity and difficulty of sustaining collusion increase.

In the Bertrand model, where firms compete on prices, collusion is more stable due to the immediate and severe consequences of price deviations, the transparency of pricing strategies, and the deterrent effect of potential price wars. Conversely, in the Cournot model, where firms compete on quantities, collusion is harder to maintain due to stronger incentives for firms to deviate by increasing output and the delayed detection of such deviations.

The hybrid Bertrand-Cournot model adds further complexity, as firms must align both pricing and quantity strategies to maintain collusion. The interplay between the degree of substitution and the discount factor critically influences the feasibility of collusion. High substitution increases the temptation to deviate, while a high discount factor encourages long-term cooperation. Effective collusion in this model requires robust monitoring, credible punishment mechanisms, and strong communication among firms.

For partial collusion scenarios, where only two out of three firms collude, the results align more closely with previous studies but also highlight the same divergences found in full collusion scenarios. Specifically, when the third firm competes on prices, collusion stability is higher due to controlled supply by colluding firms. In contrast, when all firms independently compete on quantities, maintaining collusion becomes more challenging due to stronger incentives to deviate and weaker punishment mechanisms.

For possible extensions and analysis of this model, I would first recommend solving the first stage of the model, where firms make their decisions with respect to their strategic variable. And secondly, to see what impact the introduction of the third company would have on consumer welfare, which would bring advantages for policy adequacy in terms of competitive arrangements.

In conclusion, the Bertrand model inherently supports more stable collusive arrangements among a small number of firms due to the immediate and severe consequences of price deviations and the transparency of pricing strategies. On the other hand, the Cournot model's quantity competition provides stronger incentives for firms to deviate and complicates monitoring and enforcement of collusive agreements. Therefore, while both models present challenges for sustaining collusion, the nature of price competition in the Bertrand model creates a more conducive environment for collusive stability among three firms.

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4 Appendix

From Lemma 2, we conclude the following:

It is more profitable to deviate when both rivals compete on price and less profitable when both rivals compete on output (with the hybrid case in between).

$$\frac{1}{4(1-\gamma)(2\gamma+1)(\gamma+1)} - \frac{(2+2\gamma-\gamma^2)^2}{16(2\gamma+1)^2(1-\gamma)(\gamma+1)} = \frac{1}{16}\frac{(4-\gamma)\gamma^3}{(1-\gamma)(\gamma+1)(2\gamma+1)^2} > 0$$

$$\frac{(2+2\gamma-\gamma^2)^2}{16(2\gamma+1)^2(1-\gamma)(\gamma+1)} - \frac{1}{4}\frac{(\gamma+1)^2}{(2\gamma+1)^2} = \frac{1}{16}\frac{(5\gamma+4)\gamma^3}{(1-\gamma)(\gamma+1)(2\gamma+1)^2} > 0$$

From Lemma 3, we conclude the following:

1) Price competition is less profitable than quantity (output) competition when firms use the same strategic variable:

$$\frac{(1-\gamma)(\gamma+1)}{4(2\gamma+1)} - \frac{1}{4(\gamma+1)^2} = -\frac{1}{4}\frac{(\gamma+2)\gamma^3}{(2\gamma+1)(\gamma+1)^2} < 0$$

2) Firms setting quantities achieve higher profits than those setting prices when strategies differ (regardless of how many choose quantities or prices)

$$\frac{(\gamma+2)^2(1-\gamma)(2\gamma+1)}{4(\gamma+1)(-3\gamma+\gamma^2-2)^2} - \frac{(1-\gamma)(3\gamma+2)^2}{4(\gamma+1)(-3\gamma+\gamma^2-2)^2} = \frac{1}{2}\frac{(1-\gamma)\gamma^3}{(\gamma+1)(2+3\gamma-\gamma^2)^2} > 0$$

$$\frac{(2-\gamma)^2(1-\gamma)(\gamma+1)}{4(2+\gamma-2\gamma^2)^2} - \frac{(\gamma+2)^2(1-\gamma)^2}{4(2+\gamma-2\gamma^2)^2} = \frac{1}{2}\frac{(1-\gamma)\gamma^3}{(2+\gamma-2\gamma^2)^2} > 0$$

3) The profit of a quantity setter increases with the number of quantity setters:

$$\frac{1}{4(\gamma+1)^2} - \frac{(2-\gamma)^2(1-\gamma)(\gamma+1)}{4(2+\gamma-2\gamma^2)^2} = \frac{1}{4}\gamma^3\frac{2-2\gamma^2+\gamma^3}{(\gamma+1)^2(-\gamma+2\gamma^2-2)^2} > 0$$

$$\frac{(2-\gamma)^2(1-\gamma)(\gamma+1)}{4(2+\gamma-2\gamma^2)^2} - \frac{(\gamma+2)^2(1-\gamma)(2\gamma+1)}{4(\gamma+1)(-3\gamma+\gamma^2-2)^2} =$$

$$\frac{1}{4}\gamma^3(1-\gamma)\frac{32\gamma+26\gamma^2-14\gamma^3-16\gamma^4+\gamma^5+8}{(\gamma+1)(-3\gamma+\gamma^2-2)^2(-\gamma+2\gamma^2-2)^2} > 0$$

From Lemma 4, we conclude the following:

Scenario 1 - When all firms raise their prices, those engaged in collusion tend to increase them more significantly than a non-colluding firm.

$$\frac{1}{2}\frac{(3\gamma+2)(1-\gamma)}{2+2\gamma-\gamma^2}-\frac{1-\gamma}{2} = \frac{1}{2}\frac{\gamma(\gamma+1)(1-\gamma)}{2+2\gamma-\gamma^2}>0$$
$$\frac{(1-\gamma)(\gamma+1)}{2+2\gamma-\gamma^2}-\frac{1-\gamma}{2} = \frac{1}{2}\frac{\gamma^2(1-\gamma)}{2+2\gamma-\gamma^2}>0$$

Scenario 2 - When colluding firms reduce their output, the non-colluding firm increases its output, which negatively impacts the colluders.

$$\frac{1}{2}\frac{2-\gamma}{2+2\gamma-\gamma^2}-\frac{1}{2(\gamma+1)} = -\frac{1}{2}\frac{\gamma}{(\gamma+1)(2+2\gamma-\gamma^2)}<0$$
$$\frac{1}{2\gamma-\gamma^2+2}-\frac{1}{2(\gamma+1)} = \frac{1}{2}\frac{\gamma^2}{(\gamma+1)(2+2\gamma-\gamma^2)}>0$$

Scenario 3 - When colluding firms raise prices and reduce output, the non-colluding firm also raises its price, which ends up benefiting the colluders.

$$\frac{1}{2}\frac{(1-\gamma)(2\gamma+1)(\gamma+2)}{2+4\gamma-\gamma^3}-\frac{(3\gamma+2)(1-\gamma)}{2(3\gamma-\gamma^2+2)} = \frac{1}{2}\gamma(1-\gamma)\frac{5\gamma+3\gamma^2+\gamma^3+2}{(2+3\gamma-\gamma^2)(2+4\gamma-\gamma^3)}>0$$
$$\frac{1}{2}\frac{\gamma+2}{2+4\gamma-\gamma^3}-\frac{\gamma+2}{2(3\gamma-\gamma^2+2)} = -\frac{1}{2}\gamma(\gamma+2)\frac{1+\gamma-\gamma^2}{(2+3\gamma-\gamma^2)(2+4\gamma-\gamma^3)}<0$$
$$\frac{(1-\gamma)(\gamma+1)^2}{2+4\gamma-\gamma^3}-\frac{(3\gamma+2)(1-\gamma)}{2(3\gamma-\gamma^2+2)} = \frac{1}{2}\gamma^2(1-\gamma)\frac{4\gamma+\gamma^2+2}{(2+3\gamma-\gamma^2)(2+4\gamma-\gamma^3)}>0$$

Scenario 4 - When colluding firms raise prices and reduce output, the non-colluding firm increases its output, which undermines the colluders' strategy.

$$\frac{1}{2}\frac{(1-\gamma)(\gamma+2)(\gamma+1)}{2+2\gamma-2\gamma^2-\gamma^3}-\frac{(\gamma+2)(1-\gamma)}{2(2+\gamma-2\gamma^2)} = \frac{1}{2}\gamma(\gamma+2)(1-\gamma)\frac{1+\gamma-\gamma^2}{(2+\gamma-2\gamma^2)(2+2\gamma-2\gamma^2-\gamma^3)}>0$$
$$\frac{1}{2}\frac{(1-\gamma)(\gamma+2)}{2+2\gamma-2\gamma^2-\gamma^3}-\frac{2-\gamma}{2(2+\gamma-2\gamma^2)} = -\frac{1}{2}\gamma\frac{2+\gamma-\gamma^2-\gamma^3}{(2+\gamma-2\gamma^2)(2+2\gamma-2\gamma^2-\gamma^3)}<0$$
$$\frac{1}{2+2\gamma-2\gamma^2-\gamma^3}-\frac{2-\gamma}{2(2+\gamma-2\gamma^2)} = \frac{1}{2}\gamma^2\frac{2-\gamma^2}{(2+\gamma-2\gamma^2)(2+2\gamma-2\gamma^2-\gamma^3)}>0$$

Scenario 5 - When colluding firms raise prices, the non-colluding firm increases its output, which disrupts the colluders' strategy.

$$\frac{1}{2}\frac{(3\gamma+2)(1-\gamma)}{2+2\gamma-3\gamma^2}-\frac{(3\gamma+2)(1-\gamma)}{2(3\gamma-\gamma^2+2)} = \frac{1}{2}\gamma(1-\gamma)(2\gamma+1)\frac{3\gamma+2}{(2+3\gamma-\gamma^2)(2+2\gamma-3\gamma^2)}>0$$
$$\frac{1}{2+2\gamma-3\gamma^2}-\frac{\gamma+2}{2(3\gamma-\gamma^2+2)} = \frac{1}{2}\gamma^2\frac{3\gamma+2}{(2+3\gamma-\gamma^2)(2+2\gamma-3\gamma^2)}>0$$

Scenario 6 - When colluding firms reduce output, the non-colluding firm raises its price, which ultimately benefits the colluders.

$$\frac{1}{2}\frac{2-\gamma}{2+2\gamma-3\gamma^2}-\frac{2-\gamma}{2\left(2+\gamma-2\gamma^2\right)} = -\frac{1}{2}\gamma\left(1-\gamma\right)\frac{2-\gamma}{\left(2+\gamma-2\gamma^2\right)\left(2+2\gamma-3\gamma^2\right)}<0$$

$$\frac{\left(1-\gamma\right)\left(\gamma+1\right)}{2+2\gamma-3\gamma^2}-\frac{\left(\gamma+2\right)\left(1-\gamma\right)}{2\left(2+\gamma-2\gamma^2\right)} = \frac{1}{2}\gamma^2\left(1-\gamma\right)\frac{2-\gamma}{\left(2+\gamma-2\gamma^2\right)\left(2+2\gamma-3\gamma^2\right)}>0$$

The ranking of the critical discount factors in the case of partial collusion follows from:

$$\frac{\left(-\gamma+2\gamma^2-2\right)^2\left(\gamma+2\right)^2}{\left(8\gamma-7\gamma^2-4\gamma^3+8\right)\left(\gamma-2\right)^2}-\frac{\left(\gamma-2\right)^2\left(\gamma+1\right)^2}{\left(1-\gamma\right)\left(8+8\gamma-3\gamma^2-\gamma^3\right)}$$

$$=\frac{4\gamma\left(4-11\gamma^2+7\gamma^4-\gamma^6\right)\left(8+4\gamma-6\gamma^2-\gamma^3\right)}{\left(1-\gamma\right)\left(8+8\gamma-7\gamma^2-4\gamma^3\right)\left(8+8\gamma-3\gamma^2-\gamma^3\right)\left(2-\gamma\right)^2}$$

This expression may be positive or negative but for critical values below 1 it is always positive.

$$\frac{\left(\gamma-2\right)^2\left(\gamma+1\right)^2}{\left(1-\gamma\right)\left(8+8\gamma-3\gamma^2-\gamma^3\right)}-\frac{\left(2+3\gamma-\gamma^2\right)^2}{\left(\gamma+1\right)\left(8\gamma-11\gamma^2+\gamma^3+8\right)}$$

$$=\frac{8\gamma^2\left(4+7\gamma-4\gamma^2-4\gamma^3+4\gamma^4-\gamma^5\right)}{\left(1-\gamma\right)\left(\gamma+1\right)\left(8+8\gamma-11\gamma^2+\gamma^3\right)\left(8+8\gamma-3\gamma^2-\gamma^3\right)}>0$$

$$\frac{\left(2+3\gamma-\gamma^2\right)^2}{\left(\gamma+1\right)\left(8\gamma-11\gamma^2+\gamma^3+8\right)}-\frac{\left(-3\gamma+\gamma^2-2\right)^2}{\left(\gamma+1\right)\left(16\gamma+\gamma^2-3\gamma^3+8\right)}$$

$$=\frac{4\gamma\left(2+3\gamma-\gamma^2\right)^3}{\left(\gamma+1\right)\left(8\gamma-11\gamma^2+\gamma^3+8\right)\left(16\gamma+\gamma^2-3\gamma^3+8\right)}>0$$

$$\frac{\left(-3\gamma+\gamma^2-2\right)^2}{\left(\gamma+1\right)\left(16\gamma+\gamma^2-3\gamma^3+8\right)}-\frac{\left(3\gamma+2\right)^2}{\left(2\gamma+1\right)\left(16\gamma+5\gamma^2-2\gamma^3+8\right)}$$

$$=\frac{4\gamma\left(1+\gamma-\gamma^2\right)\left(28\gamma+24\gamma^2-5\gamma^3-7\gamma^4+\gamma^5+8\right)}{\left(2\gamma+1\right)\left(\gamma+1\right)\left(8+16\gamma+\gamma^2-3\gamma^3\right)\left(8+16\gamma+5\gamma^2-2\gamma^3\right)}>0$$

$$\frac{\left(3\gamma+2\right)^2}{\left(2\gamma+1\right)\left(16\gamma+5\gamma^2-2\gamma^3+8\right)}-\frac{\left(2+\gamma-2\gamma^2\right)^2}{16\gamma-3\gamma^2-12\gamma^3+8}$$

$$=\frac{8\gamma^2\left(\gamma+1\right)\left(13\gamma+8\gamma^2-10\gamma^3-8\gamma^4+2\gamma^5+4\right)}{\left(2\gamma+1\right)\left(8+16\gamma-3\gamma^2-12\gamma^3\right)\left(8+16\gamma+5\gamma^2-2\gamma^3\right)}>0$$