Solving the Volatility Puzzle with Extreme Returns

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Abstract

Motivated by the research of Ang, Xing, and Zhang (2006, 2009), which documents a negative relation between idiosyncratic volatility (IVOL) and future stocks returns, this analysis uses extreme daily returns to undo this effect and solve the recent found idiosyncratic volatility puzzle. This thesis uses data from the U.S stock market for the period from July 1962 to May 2018 to compute portfolios sorted on extreme daily returns and to run Fama MacBeth multivariate analysis. Through this analysis this study finds that maximum daily returns (MAX) are able to reverse the negative effect of idiosyncratic volatility. It also shows that when using both maximum and minimum daily returns (MIN), the effect of idiosyncratic volatility disappears completely. Finally, this study shows evidence that the idiosyncratic volatility effect diminishes in more recent years, specifically from 1991 to May 2018. Concluding that for the second period, both MIN and MAX individually do a good work at canceling the IVOL puzzle.

Key Words: Idiosyncratic Volatility, Cross-section of stock returns, Lottery-like payoffs, Extreme Returns

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Resumo

Motivado pela investigação de Ang, Xing e Zhang (2006, 2009), que documenta uma relação negativa entre a volatilidade idiossincrática (IVOL) e retornos futuros de acções, esta análise utiliza retornos extremos diários para desfazer este efeito e solucionar o puzzle recentemente encontrado referente à volatilidade idiossincrática. Esta tese utiliza dados do mercado de ações dos EUA no período de Julho de 1962 a Maio de 2018 para construir carteiras de acções com base em retornos diários extremos e também executa análises Fama MacBeth com múltiplas variáveis. Através desta análise, este estudo considera que os retornos máximos diários (MAX) são capazes de reverter o efeito negativo da volatilidade idiossincrática. Também mostra que, ao usar os retornos diários máximo e mínimo (MIN), o efeito da volatilidade idiossincrática desaparece completamente. Finalmente, este estudo mostra evidências de que o efeito da volatilidade idiossincrática diminui nos anos mais recentes, especificamente de 1991 a Maio de 2018. Concluíndo que, para o segundo período, tanto o MIN quanto o MAX fazem um bom trabalho para cancelar o puzzle do IVOL.

Key Words: Volatilidade Idiosincrática, Cross-section do retorno de acções, Ativos tipo Lotarias, Retornos extremos

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1. Introduction

1.1.Research Problem Description

Motivated by the research of Ang, Xing, and Zhang (2006, 2009), which documents a negative relation between idiosyncratic volatility and future stocks returns, this analysis uses extreme daily returns to try to undo this effect and solve the recent found idiosyncratic volatility puzzle. Since this strange negative relation was found, several different authors have tried to explain it with different economic effects, based on different theories such as investors lottery-preferences, markets frictions and even corporate events.

Based on investor's preferences Bali, Cakici and Whitelaw (2011) study the relation between maximum daily returns to future expected returns and find a robust and negative relation which is in line with both lottery-preferences theory and the cumulative prospect theory modeled by Barberis and Huang (2008). This negative relation is explained by the fact that "investors may be willing to pay more for stocks that exhibit extreme positive returns and thus these stocks exhibit lower future returns" (Bali et al. 2011). At last their study examines how maximum daily returns contribute to solve the volatility puzzle, finding that maximum daily returns are able to reverse the negative sign of idiosyncratic volatility to subsequent future returns for equal-weighted portfolios. However, analysis for value-weighted portfolios still show a negative relation and therefore the existence of the puzzle.

Based on different literature such as skewness preference and the already mentioned cumulative prospect-theory, the effect of minimum daily return is expected to be the opposite of that of MAX. One would expect stocks with large minimum daily returns to exhibit a subsequent positive return. As discussed by Amaya et al. (2015) investors have a preference for positive skewed stock returns, which leads these stocks to command lower subsequent returns. On the contrary, investors should demand higher returns for holding stocks with negatively skewed returns. In addition, investors overweight small probabilities of large losses so stocks with larger minimum returns tend to be undervalued and for that should command positive subsequent returns.

This study examines the relation between extreme negative returns and future expected returns and most important, examines how good is MIN on solving the idiosyncratic volatility puzzle. After trying to understand how extreme returns work together to undo the idiosyncratic volatility effect a final analysis explores how IVOL and a set of models comprising IVOL, MAX and MIN behave across different time periods.

1.2. Objective and Hypothesis Development

This thesis contributes to the existing literature on the cross-section of expected stock returns and more specifically adding on to the idiosyncratic volatility puzzle research. It investigates the relation of extreme returns to future expected returns and how these extreme returns contribute to solve the volatility puzzle. The thesis tests if the results of previous studies such as Ang et al. (2006, 2009) and Bali et al. (2011) hold true for a different sample period by including the 2008 financial crisis and the post-crisis years. More importantly it studies if MIN, which is expected to have a similar effect of MAX at solving the volatility puzzle, shows statistically robust results on canceling this effect. At last the thesis develops and tests a jointly model of MAX and MIN that is able to cancel the IVOL puzzle for different time samples.

The thesis starts by studying how good are extreme returns on predicting future expected returns.

- Can extreme returns predict subsequent stock returns?

Hypothesis 1: Maximum and Minimum daily returns are able to predict the cross-section of future expected returns.

Secondly, this thesis studies the relation of extreme daily returns with idiosyncratic volatility.

- Can extreme returns cancel the idiosyncratic volatility puzzle?

Hypothesis 2: Minimum daily returns have a similar effect at cancelling out the idiosyncratic volatility puzzle as maximum daily returns.

- How does this relation behave across different time periods?

Do the previous models based on IVOL and extreme returns behave the same across different specifications?

Hypothesis 3: A model based on IVOL, MAX and MIN work differently for different sub-samples based on time.

1.3. Thesis Contribution

The contribution of this thesis extends to two main areas, first to the literature on the cross-section of expected stocks returns, specifically on the extensive literature on investors lottery preferences and lottery-like assets. Second, it adds to the recent literature on the idiosyncratic volatility puzzle.

The thesis extends the sample period used by Ang et al. (2006, 2009) and Bali et al. (2011) to include the 2008 financial crisis and the post-crisis years to study idiosyncratic volatility puzzle alone and next to study how can extreme positive returns reverse this unexpected effect. Furthermore, it adds to the literature an important analysis which is how do negative extreme returns work to cancel out the idiosyncratic volatility effect.

For the first analysis conducted on how extreme returns predict subsequent stock returns I start by testing this relation with univariate portfolio-level analysis and then I study the robustness of MAX and MIN for a set of control variables that have been commonly discussed to explain the cross-section of expected returns: size and book-to market as defined in Fama and French (1993), betas as discussed in Scholes and Williams (1977) and Dimson (1979), total skewness as in Boyer Mitton and Vorkink (2010), price as discussed in Campbell and Shiller (1988) and momentum as in Jegadeesh and Titman (1993).

The analysis shows that maximum daily returns do a good job at predicting the cross-section of future stock returns, where the High – Low portfolio produces a negative and statistically robust effect across value and equal-weighted portfolios. An investment strategy that explores the opposite strategy by buying stocks on the Low MAX portfolio, i.e. stocks with the lowest maximum daily return over the past one-month, and shorting stocks on the High MAX, i.e. stocks with the highest maximum daily return over the past one-month, produces an average monthly return of 0.79 percent with a Newey-West t-statistic of 2.86 with corresponding Carhart four-factor alpha (hereafter referred to as 4FF alpha) of 1.04 percent with a Newey-West t-statistic of 4.69.

As for minimum daily returns, for the equal-weighted portfolio the analysis shows a positive but not statistically robust effect of MIN on future expected returns. As shown in more detail further in the analysis the expected positive effect is only exhibited for the equal-weighted portfolio as the MIN effect is mostly driven by lottery-like assets. These are typically small, cheap and illiquid stocks. This effect is likely affected by large stocks, that might lead the results to be not significant and negative for the MIN value-weighted portfolio.

An interesting conclusion might be drawn from the different results for MAX and MIN. To explore the MAX mispricing an arbitrageur would have to short stocks with large maximum daily returns and buy stocks with low maximum returns. This means an investor would be shorting stocks that are small in size, i.e. with low market capitalization and highly illiquid. While this strategy would be too risky and costly, an investment strategy based on the MIN effect, would buy stocks with large minimum daily returns and short stocks with low minimum returns. This means buying the small, illiquid stocks. Arbitrageurs can more easily pursue an investment strategy that exploits the MIN mispricing when compared to a strategy based on the MAX. The analysis shows that while mispricing based on MAX might not be exploited because of the cost and risk, the mispricing based on MIN, is likely to be exploited and so its effect is being lost.

Addressing the second research question, I compute double sorted portfolios and Fama Macbeth regressions. For the two different models I find a statistically significant relation between idiosyncratic volatility and sub-sequent stock returns. I also find that MAX works well on reversing the effect of IVOL on future stock returns, with the average of time-series IVOL coefficient of 0.2674, with a Newey-West t-statistic of 4.17, when controlling for a set of variables, as listed before. As for MIN, individually it does not show to be enough to cancel or reverse the IVOL effect. However, when IVOL is modeled with MAX and MIN jointly, these two effects appear to be enough to cancel the idiosyncratic volatility effect. Where the average time-series IVOL coefficient is of 0.1185 with Newey-West t-statistic of 1.60 and more importantly after controlling for a set of variables commonly discussed to explain the cross-section of expected returns, exhibiting an average coefficient of -0.05 with a non-significant Newey-West t-statistic of -0.73.

For the last part of the analysis, regarding the third research question, the main conclusion is that data exhibits different behavior in IVOL from 1962 to 1990 when compared to the period starting in 1991 and lasting until the end of the sample period, May 2018. I conclude that for the second period, both MIN and MAX individually do a good work at canceling the IVOL puzzle, specifically for the value-weighted portfolios.

The remainder of the thesis is organized as follows. Section 2 presents a thorough analysis of previous research on idiosyncratic volatility. Section 3 is divided into 3 parts. Section 3.1 starts by describing the dataset used in the analysis. Next, section 3.2 confirms the results for MAX, and presents results for MIN. For both variables it is presented the univariate portfolio analysis, summary statistics, bivariate portfolio-level analysis and cross-sectional regressions. Section 3.3 focus on both the relationship between idiosyncratic volatility with MAX and MIN. As last section 4 explores the model specified with IVOL MAX and MIN for different specification related to time periods. Section 5 concludes.

2. Literature Review

In a market with poorly-diversified investors, where idiosyncratic volatility cannot be hedged away, stocks with high IVOL earn high returns to compensate investors for holding firm-specific risk. This was the result previously shown by Merton (1987). However, Ang et al. (2006, 2009) reach to a different conclusion. In these two papers the authors find that against previous literature, stocks that recently exhibited high idiosyncratic volatility tend to earn low future average returns. Their finding is commonly known as the idiosyncratic volatility puzzle. The authors measure idiosyncratic volatility with respect to the Fama and French (1993) factor model. They start by studying this relation for U.S. markets and later extend their research into international market, finding that around the world, across different markets, this effect is highly statistically significant. Ang et al. (2009) "conclude that the puzzle of low returns to high-idiosyncratic-volatility stocks have low returns is a global phenomenon". In their analysis, the authors also find that when controlling for a U.S idiosyncratic volatility factor, the alphas of portfolio strategies exploiting idiosyncratic volatility effect in international markets become insignificant. Concluding that "global idiosyncratic volatility effect is captured by a simple U.S. idiosyncratic volatility factor".

Since this puzzle has first emerged, several different authors have tried to solve it using different theories. Following I present a thorough analysis of the different approaches in a chronological order by year of publication.

Amaya et al. (2015) using high frequency data, define a measure of past realized weekly skewness and find that after controlling for IVOL, high-skewness stocks earn low future returns, while stocks with low-skewed return distributions are compensated with higher subsequent returns. They conclude that their measure of "skewness provides a partial explanation to the IVOL puzzle". Chabi-Yo and Yang (2009) show that the negative relation of idiosyncratic volatility to stock returns is related to a stock's co-skewness with the market portfolio.

Fu (2009) shows the time-varying property of idiosyncratic volatilities, which invalidates Ang et al. (2006, 2009) to explain the relation between idiosyncratic risk and future expected return. Fu (2009) using the exponential GARCH model, estimates a measure of expected idiosyncratic volatility, finding “a significantly positive relation between the estimated conditional idiosyncratic volatilities and expected returns”. The author also shows how results presented by Ang et al. can be explained by return reversals of small stocks that exhibit high idiosyncratic volatility.

Bali and Cakici (2008) use a different approach to study idiosyncratic volatility and the cross-section of expected stock returns. They prove that different data treatments and model specifications have a big impact on finding the IVOL puzzle. This study examines the idiosyncratic volatility effect for different idiosyncratic volatility measures: for three different weighting schemes, value, equal-weighted and inverse-volatility-weighted; for two different samples one comprising NYSE, AMEX, and NASDAQ, and one comprising only stocks on NYSE; and finally, for different portfolio formation breaking points. Their main conclusion is that when testing the robustness of this anomaly across different specification this does not hold, concluding that “there is no robust, significant relation between idiosyncratic volatility and expected returns”.

On a very different setting Jiang, Xu and Yao (2009) study the relation of idiosyncratic volatility to corporate events. They find that the IVOL puzzle is not a result of market anomalies resulting from investors preferences, rather they find that “idiosyncratic volatility is inversely related to future earning shocks”. The authors further conclude that the power of idiosyncratic volatility to predict future returns is “induced by its information content about future earnings”. One important point in common to market frictions literature is that these authors find evidence that the idiosyncratic volatility puzzle is stronger among stocks usually traded by “less sophisticated investors”. As found by Han and Kumar (2009) this anomaly is concentrated in stocks dominated by retail investors. These investors are also more likely to be poorly-diversified and drive the mispricing of stocks according to their preferences.

Han and Lesmond (2011) model a microstructure effect on daily returns, enclosing the bid-ask spread. They find prove that this factor is able to eliminate the predictive power of IVOL to future returns. The authors also find a significant reduction in the relation of idiosyncratic volatility to expected future returns after exogenous shocks to liquidity.

In a paper from 2011, Bali and Cakici studding lotter-like assets and investors preference for lottery-like assets, revisit the idiosyncratic volatility puzzle. After studying the predictive power of extreme positive returns to future stock returns, the authors do a very thorough analysis on how maximum daily returns reverse the negative effect of IVOL to expected returns. They draw their results from a set of robustness tests after which they conclude that idiosyncratic volatility is only a proxy for extreme returns. When modeling both of these effects jointly, first in double-sorted portfolio and second in a Fama MacBeth regression analysis, they find a positive and statistically significant relation between IVOL and future returns for equal-weighted portfolios.

Chen and Petkova (2012) solve the puzzle by studying innovations in average volatility. The authors "decompose the aggregate market variance into an average correlation component and an average variance component". This study finds that only innovations to average variance as proxied by the average variance factor have a negative effect in the cross-section of expected stock returns. High idiosyncratic volatility portfolios relatively to the Fama-French (1993) model have positive exposures to innovations, resulting in lower expected returns.

In a more recent study, Hou, Loh (2016) command a very extensive study where they select the several different alternatives used before to solve the idiosyncratic volatility puzzle and study how each one individually and all jointly contribute to solve this anomaly. The authors develop a model based on Fama MacBeth cross-section regressions to "quantify the contribution of each explanation either by itself or when evaluated against competing explanations" Hou, Loh (2016). Among others, the variables analyzed are skewness, co-skewness, maximum daily return, short-term reversals, a measure of illiquidity as defined in Amihud (2002), the fraction of trading days with a zero return, bid-ask spread and the stock's exposure to the average variance component of the market variance and unexpected earnings.

Their main findings are, first that many of the existing explanations actually explain less than 10 percent of the puzzle. The second main finding is that explanation based on investor's lottery preferences, namely the maximum daily returns and market frictions, namely short-term reversals are the best alternatives at explaining the puzzle. And finally, as per their specifications, "all existing explanations account for 29–54% of the puzzle in individual stocks and 78–84% of the puzzle in idiosyncratic volatility-sorted portfolios."

After a thorough analysis of the different approaches, I decided to further study investors lottery preferences. Specifically, since most of papers that examine this preference despite the very comprehensive analysis fail to study how good are extreme negative returns on the prediction of expected future returns and how do this relate to the idiosyncratic volatility puzzle. As described before in this study one would expect the minimum daily returns to have an opposite effect to the of maximum daily returns. Specially if MAX does not proxy for idiosyncratic volatility as discussed by Bali et al. (2011), MIN should not just have the opposite effect but also cancel or reverse the negative effect as defined in the idiosyncratic volatility puzzle.

Evidence shows that investors have a favor lottery-like assets, these assets tend to be and behave like lotteries, they are typically cheap and hold a small probability of a large gain. As discussed by Thaler and Ziemba (1988) lotteries remain very popular despite the prevalence of negative returns. Later studies by Garret and Sobel in 1999 on Economic Letters show "that lottery players may be risk-averse but favor positive skewness of returns". Finally, Kumar studies this preference in the stock market. First the author shows that propensity for gambling is correlated to investment decisions and second the author shows that "individual investors prefer stocks with lottery features". Lottery like assets tend to underperform given investors preference for positive skewness, that causes these stocks to be overvalued. Contrary to this, stocks with low or negative skewness, representing small probabilities of very large losses, should be undervalued, leading to future positive returns Barberis and Huang (2008). Based on the different literature presented, the thesis studies the relation between extreme negative return to subsequent stock returns and the impact of this effect on the idiosyncratic volatility puzzle.

3. Data & Methodology

3.1. Data

The data used in this analysis, comes from CRSP and COMPUSTAT. This analysis uses both daily and monthly stock returns from the three major U.S stock exchanges, the New York Stock Exchange, the American Stock Exchange and Nasdaq for the period from July 1962 to May 2018. Data is treated to include only ordinary common shares, by selecting only stocks with share code 10 and 11. Next it is required at least 15 daily returns within each month for each stock. Daily data is used to calculate maximum and minimum daily stock returns for each firm for each month. It is also used to calculate monthly one-month idiosyncratic volatility, illiquidity and skewness over the past one-month. Monthly data is used to calculate market capitalization used as a proxy for firm size and intermediate-term momentum. COMPUSTAT is the second data set, this is used to source balance sheet data for the calculation of Book-to-Market ratio for all the firms for the period in analysis. A detailed description of the variables is presented in the Appendix.

To have a clearer view of the data set used I present the evolution of number of stocks per year, and the summary statistic for all the variables used in the analysis. The average number of stocks in the sample, across the whole period is 3,972. Figure 1 shows that the sample starts with 725 stocks in 1962, which is also the minimum number of stocks throughout the sample period, reaches a maximum of 6,645 stocks in 1997, year since when the number of stocks starts to decrease until the end of the period where it reaches 2,615 stocks.

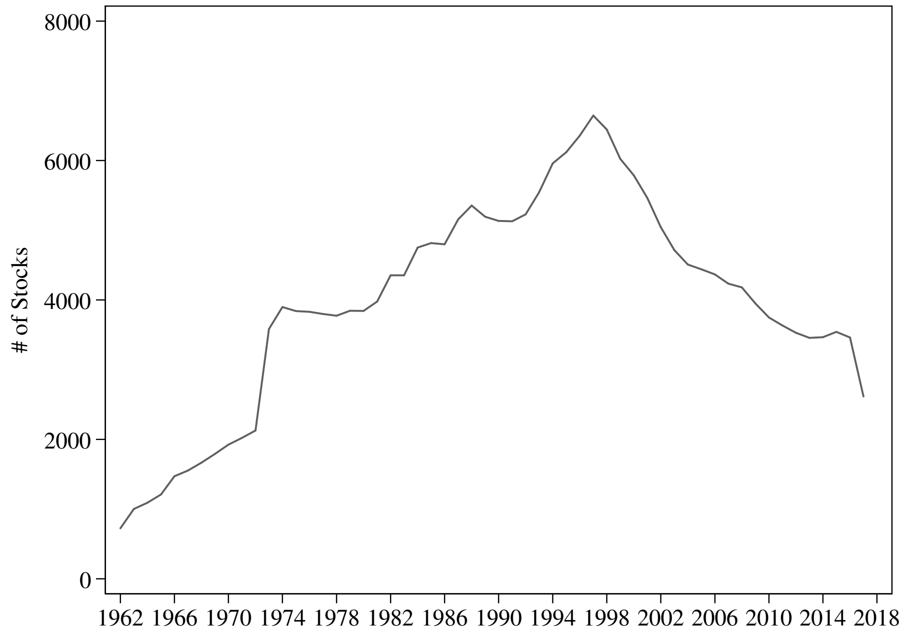

Figure 1. Number of Stocks per year.
This figure shows the evolution of the number stocks for the sample period used in this analysis, 1962 to 2018. Stocks considered are from the three major U.S. stocks exchanges – NYSE, AMEX & NASDAQ. Only ordinary common shares are considered.

Table 1 presents the summary statistics unconditional. Later in the thesis I will be presenting the summary statistic for each decile portfolio, based on sorting on extreme returns. After computing all variables as mentioned above, I work with monthly data. There are around 2,500,000 monthly observations in the data set. The data shows a lot of dispersion for each variable. The average maximum daily return for the sample is of 7.51 percent, while the average minimum return is of 6.11 percent. MAX shows to have more dispersion when compared to MIN. The average IVOL for the sample is 2.82.

	MAX (%)	MIN (%)	SIZE ($000,000)	Price ($)	Beta	BM ratio	ILIQ (10^4)	IVOL	MOM (%)	SKEW
Obs.	2 646 167	2 646 167	2 644 232	2 644 232	2 646 167	2 472 711	2 447 472	2 646 167	2 408 720	2 624 125
Mean	7.51	6.11	1 686	31.56	0.83	0.91	8.53	2.82	0.15	0.20
S.D	9.03	5.37	11 278	1290.80	2.52	1.16	574.37	2.65	0.70	0.94
Min	0.00	0.00	0	0.02	-118.64	-1.51	0.00	0.00	-1.00	-4.19
Max	1900.00	95.40	882 331	297600.00	255.31	10.62	853869.00	349.07	91.00	4.19

Table 1. Summary statistics.
This table presents summary statistics unconditional of sorting. It shows the number of observations (Obs.), the mean, the standard deviation (S.D.), minimum (Min) and maximum (Max) for all the variables used in this paper. Data used is the sample period from 1962 to 2018. Stocks considered are from the three major U.S. stocks exchanges – NYSE, AMEX & NASDAQ. Only ordinary common shares are considered.

To further examine the behavior of extreme daily return across the sample period, figures 2 and 3 graph these returns into box-plots for different periods across the sample. For simplicity the years were grouped into groups of 4. To understand the economic power it is important to notice that this represent daily returns. The median ranges between 3.7 percent in 1962 through 1965 and 7.8 percent in the period from 1998 until 2001. There is some increase in the median value for maximum daily returns in the period starting in 1990, extending until the period ending in 2001. After this, the maximum daily return median drops significantly. One other important analysis is the level of skewness. Throughout the sample period, MAX appears to be always positive skewed, this means, with higher probability of extreme positive returns. This effect is larger also from 1990 to 2001, with the box-plots presenting larger tails. At last it appears that the level of dispersion in the MAX increases from 1990 to 2001.

Regarding the MIN, figure 3 shows the minimum daily return, which is the original value multiplied by -1. Overall, MIN is relatively lower than MAX, with median values ranging from 3.05 percent in the period 1962 – 1965 to 6.73 percent in 1998 to 2001. Nevertheless, the behavior of MIN is similar to the one of MAX across time. On one hand, median values increase in the period from 1990 to 2001, on the other hand, the dispersion of minimum daily returns is also larger in this period.

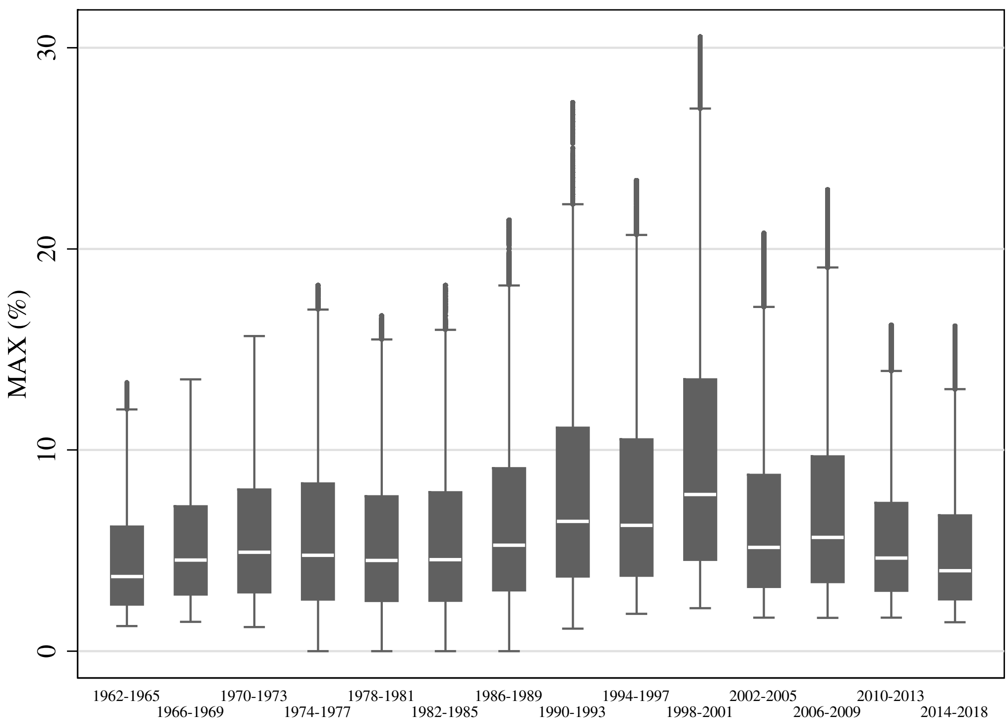

Figure 2. Box-plot for maximum daily return (MAX), across time.
This figure presents box-plots graphs for the maximum daily return. For simplicity years were grouped into periods of 4, for a total of 14 periods presented. The variables MAX is winsorized for each period at 5% and 95% level. Max is presented in percentage terms.

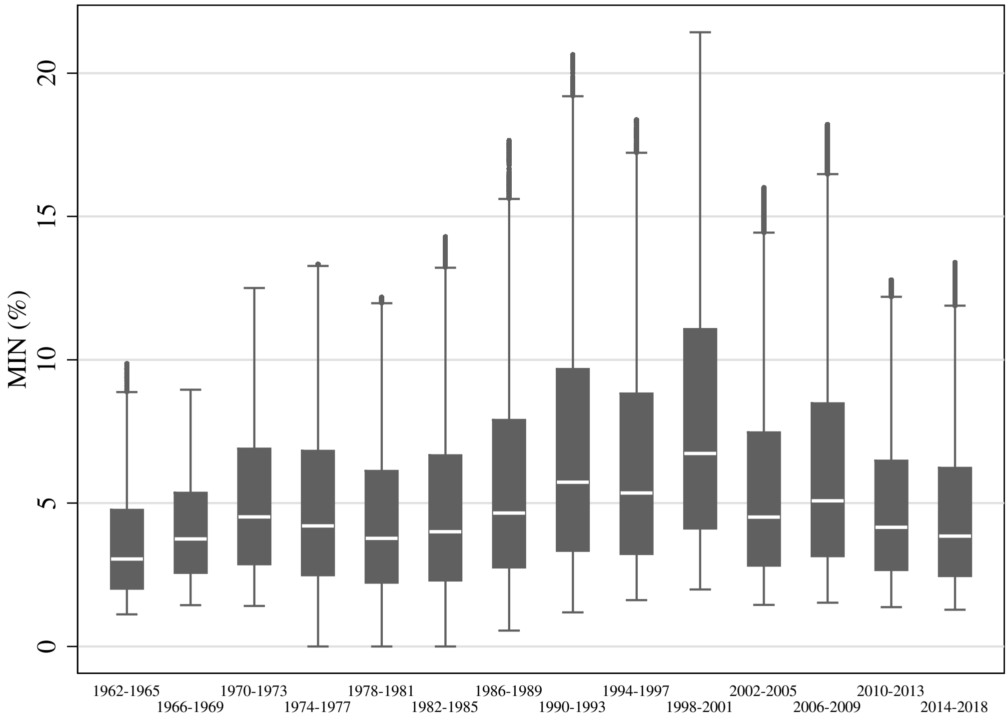

Figure 3. Box-plot for minimum daily return (MIN), across time.
This figure presents box-plots graphs for the maximum daily return. For simplicity years were grouped into periods of 4, for a total of 14 periods presented. The variables MIN is winsorized for each period at 5% and 95% level. Max is presented in percentage terms.

3.2.Extreme Returns and Future Expected Return

3.2.1. Univariate Portfolio Analysis

For this analysis, first, both variables MAX and MIN are computed. MAX is the maximum daily return within each month for each stock, while MIN is the symmetric of the minimum daily return within each month for each stock, i.e. it is the minimum daily return multiplied by -1. Every month I sort stocks into decile portfolios based on previous month MAX or MIN. Table 1 presents the equal and value-weighted average portfolio monthly returns for stocks sorted on MAX for the period from in July 1962 to May 2018. In addition, table 1 also presents 4FF alphas. The model uses a market factor defined as the excess to the U.S. one-month T-bill of the value-weighted return on the market portfolio as defined by Fama and French. SIZE, HML, MOM, represent the size, value and momentum factors as defined in Fama and French (1993) and Carhart (1997). The model for 4FF alphas is defined in equation (1), where $R_{i,t}$ is the return of decile portfolio i on month t.

Where rf_t is the risk-free rate for month t, Rm_t is the market return on month *t.* All alphas later presented are derived from the following model:

$$R_{i,t} - rf_t = \alpha_i + \beta_{1,i}(Rm_t - rf_t) + \beta_{2,i}SIZE_{,t} + \beta_{3,i}HML_t + \beta_{3,i}MOM_t + \varepsilon_{i,t} \quad (1)$$

Low MAX portfolio comprises the stocks with the lowest maximum daily returns, with an average maximum return of 1.26 percent while High MAX portfolio includes the stocks with the highest maximum daily returns, with an average maximum return of 24.83 percent.

Deciles	VW Portfolios		EW Portfolios		
	Average return	4FF Alpha	Average return	4FF Alpha	Average MAX
Low MAX	0.93	0.08	1.23	0.33	1.26
2	0.97	0.07	1.34	0.34	2.48
3	0.93	0.02	1.44	0.39	3.30
4	1.00	0.04	1.42	0.35	4.09
5	0.99	0.05	1.40	0.31	4.95
6	1.03	0.01	1.33	0.25	5.97
7	0.94	-0.07	1.25	0.18	7.26
8	0.82	-0.26	1.19	0.13	9.05
9	0.56	-0.48	1.00	-0.07	12.11
High MAX	0.14	-0.95	0.62	-0.40	24.83
High - Low	-0.79	-1.04	-0.62	-0.73	
t-stat	(-2.86)	(-4.69)	(-2.33)	(-3.26)	

Table 2. Average portfolio returns and alphas for univariate sorts on MAX

Every month, stocks are sorted on maximum daily return (MAX) over the past month to form 10 decile portfolios. The table reports both the value (VW) and equal-weighted (EW) average monthly returns, the 4FF alphas and the average maximum daily return of stocks in the portfolio formation month, all values in percentage terms. For the High – Low portfolio, Newey-West (1987) t-statistics are reported in parentheses.

The first important consideration about these results is that they show evidence for the rationale behind previous literature that defend investors preference for lottery-like stocks. Despite their underperformance compared to nonlottery stocks as studied by Kumar (2009), some investors perceive these stocks as lottery-like assets, that give a small probability of a very large gain. For instance, Markowitz (1952) presumes some investors might prefer to "take large chances of a small loss for a small chance of a large gain."

This relation can be understood both through average monthly returns and 4FF alphas. Focusing in value-weighted portfolios, while average MAX increases monotonically across decile 1 to 10, average monthly returns are very close for deciles 1 to 8 (0.93 to 0.82 percent), and decrease largely on the two last deciles, to 0.56 percent in decile 9 and more drastically in decile 10 where average monthly return decreases to 0.14 percent. Regarding 4FF alphas, the first 7 deciles also have similar alphas, which are all close to zero, while decile 8 through 10 present negative returns.

The second important consideration is that it confirms the results by Bali et al. (2011) who do this analysis for the period from July 1962 to December 2005 for the same set of companies. This analysis adds a very large portion of data to the study, specially including both the periods before and after 2008 financial crisis.

At last, it is important to notice the economically and statistically significant return for High – Low portfolios. Both average monthly returns and 4FF alphas, for both equal and value weighted portfolios are significant, with the value-weighted portfolio presenting most economically significant results, with an average monthly return of -0.79 percent, with a Newey-West (1987) t-statistic of -2.86, and a 4FF alpha of -1.04 percent with a Newey-West t-stat of -4.69.

	VW Portfolios		EW Portfolios		
Deciles	Average return	4FF Alpha	Average return	4FF Alpha	Average MIN
Low MIN	0.93	0.05	1.19	0.30	1.22
2	0.89	0.01	1.11	0.11	2.32
3	1.04	0.10	1.21	0.18	3.03
4	1.00	0.02	1.26	0.18	3.69
5	1.07	0.06	1.29	0.19	4.41
6	1.04	0.10	1.22	0.13	5.22
7	1.02	-0.02	1.18	0.09	6.23
8	0.77	-0.29	1.07	0.00	7.59
9	0.53	-0.54	1.07	0.00	9.81
High MIN	-0.02	-1.06	1.64	0.66	17.32
High - Low	-0.95	-1.11	0.45	0.36	
t-stat	(-3.49)	(-5.89)	(1.46)	(1.34)	

Table 3. Average portfolio returns and alphas for univariate sorts on MIN

Every month, stocks are sorted on minimum daily return (MIN) over the past month to form 10 decile portfolios. The table reports both the value (VW) and equal-weighted (EW) average monthly returns, the 4FF alphas and the average minimum daily return of stocks in the portfolio formation month, all values in percentage terms. For the High – Low portfolio, Newey-West (1987) t-statistics are reported in parentheses.

This analysis proposes a different approach to study the relation between idiosyncratic volatility and extreme returns. Instead of using MAX, it explores the relation between idiosyncratic volatility and minimum returns. As referred before, one should expect the effect of MIN to be symmetric to the one of MAX. As per Barberis and Huang (2008), according to the cumulative prospect theory, small probabilities of very large losses are overweighed by investors, which in turn causes these stocks to be undervalued.

The definition of the minimum variable (MIN) is the actual return multiplied by -1, i.e. Stocks on decile 1 (Low MIN) are the stocks with higher minimum returns, and stocks on decile 10 (High MIN) are the stocks with the lowest minimum return, so the more extreme negative return stocks are in decile portfolio 10.

The equal-weighted portfolio returns shows the expected results. The average raw monthly return increases dramatically in decile portfolio 10. The average monthly return difference between High MIN and Low MIN is 0.45 percent with a Newey-West t-statistic of 1.46. However, the value-weighted portfolio returns have a strong negative and significant average monthly return and 4FF alpha. These results might be influenced by firms with large market capitalization, which leads to believe that the MIN effect seems to be limited to small-cap stocks.

3.2.2. Summary Statistics

Deciles	MAX	SIZE	Price	MIN	Beta	BM	ILIQ	IVOL	MOM	SKEW
Low MAX	1.26	735.93	22.34	2.24	0.28	0.80	0.29	0.81	8.32	-0.71
2	2.43	578.63	25.63	2.77	0.55	0.73	0.11	1.11	11.20	-0.14
3	3.20	409.17	22.95	3.30	0.70	0.71	0.12	1.37	10.60	-0.02
4	3.96	307.60	20.38	3.80	0.81	0.69	0.16	1.63	9.89	0.06
5	4.80	231.22	17.84	4.33	0.91	0.68	0.22	1.91	9.19	0.14
6	5.78	172.68	15.21	4.91	1.00	0.68	0.31	2.22	7.81	0.22
7	7.02	126.61	12.42	5.57	1.07	0.70	0.48	2.61	5.47	0.31
8	8.74	93.45	9.91	6.38	1.14	0.69	0.79	3.12	2.42	0.43
9	11.56	64.34	7.37	7.50	1.18	0.73	1.63	3.93	-2.69	0.62
High MAX	18.89	35.35	4.45	10.20	1.22	0.75	5.46	6.10	-14.00	1.07

Table 4. Median values of stock characteristics for decile portfolios sorted on MAX
Every month, stocks are sorted on maximum daily return (MAX) over the past month to form 10 decile portfolios. The table reports, for each decile, the averages across the months in the sample of the median values within each month for a several firm characteristics.

Table 4 presents, for each decile portfolio, the average across all months in the sample, of the median values for a set of stock characteristics. All characteristics but BM ratio show a pattern across different deciles. Values for MAX are similar to average MAX returns presented in table 1 for all deciles but decile 10. Here, the difference between average and median MAX return is of about 6 percentage points. This decile appears to have a right-skewed distribution, where the average is higher than the median, specially comparing to other deciles.

The table presents size, measured as price times the number of shares outstanding at the end of the month. As MAX increases, the stock's market capitalization decreases. This effect is more dramatical for deciles 8, 9 and 10. This indicates that smaller stocks dominate the high MAX portfolios.

Regarding the price, stocks on the high MAX portfolio tend to have lower prices. This is not surprising and appears to be in accordance with the lottery-like asset theory of cheap assets. One could argue that the results shown in table 1 hold only for micro-cap stocks with low prices. However as shown in table 1, the results prove to be significant also for value-weighted portfolios, however this might also be driven by other factors. As per the MIN, as MAX increases the MIN return becomes more negative. This means that stocks with higher maximum daily returns appear to have also lower minimum daily returns, or larger losses.

Betas are calculated by regressing, on each month, daily stock returns on the market daily excess returns. It appears that stocks with higher MAX return are more exposed to market risk. In table 1, the 4FF alpha should control for this effect and can potentially explain the larger difference in alphas compared to the average returns.

Illiquidity is measured following Amihud (2002), this is the average of daily ratio of return to trading volume in dollars, across each month for each stock. Illiquidity does not varies from decile 1 to 7; however, it increases considerably in decile 8, 9 and 10. Idiosyncratic volatility increases from decile 1 to 10. Stocks with higher MAX return, exhibit also lower minimum daily return and this can be a proxy for volatility as discussed by Bali et. Al (2011). The final characteristics reported are intermediate-term momentum (MOM) and skewness (SKEW). High MAX stocks have negative cumulative returns in the past 11 months and high positive skewness.

From the different characteristics analyzed, some would be associated with higher returns, although these stocks appear to have negative expected return. Stocks with higher market risk, that are more illiquidity and with higher volatility should compensate investors by exhibiting higher returns in the cross-section. However, this is not to the case for stocks with high MAX returns, as despite exhibiting high volatility, high illiquidity and high market risk, they still earn larger negative returns compared to stocks with lower maximum returns.

Deciles	MAX	SIZE	Price	MIN	Beta	BM	ILIQ	IVOL	MOM	SKEW
Low MIN	2.29	807.11	25.67	1.25	0.29	0.81	0.32	0.80	9.19	0.73
2	3.07	604.04	26.73	2.27	0.58	0.73	0.08	1.14	10.80	0.32
3	3.68	420.96	23.46	2.94	0.71	0.71	0.11	1.40	10.50	0.25
4	4.25	309.11	20.28	3.58	0.82	0.70	0.16	1.65	9.87	0.21
5	4.87	227.77	17.34	4.26	0.91	0.69	0.23	1.93	8.82	0.18
6	5.61	168.63	14.46	5.05	0.97	0.68	0.35	2.24	7.51	0.16
7	6.47	123.17	11.63	6.01	1.06	0.67	0.55	2.61	5.19	0.13
8	7.56	87.74	8.98	7.29	1.11	0.68	1.01	3.10	1.87	0.09
9	9.19	58.82	6.45	9.34	1.10	0.69	2.23	3.85	-2.99	0.02
High MIN	12.75	36.12	4.06	14.24	1.08	0.73	7.65	5.69	-12.50	-0.19

Table 5. Median values of stock characteristics for decile portfolios sorted on MIN

Every month, stocks are sorted on minimum daily return (MIN) over the past month to form 10 decile portfolios. The table reports, for each decile, the averages across the months in the sample of the median values within each month for several firm characteristics.

Table 5 presents the characteristics for decile portfolios formed based on sorting stocks on minimum daily returns. Portfolio High MIN holds the stocks with the lowest minimum returns. Minimum daily return (MIN) is defined as the original minimum return multiplied by -1. Opposite to the distribution of returns exhibited in portfolio High for MAX, which appeared to be positive skewed, portfolio 10 has an average return lower than median return, with a median return of -12.75 percent compared to an average return of -17.32 (table 3) in portfolio High MIN. This appears to have left-skewed distribution as it can be also proved by negative skewness of -0.19.

Regarding size, price, market beta, idiosyncratic volatility (IVOL) and illiquidity stocks with the lowest minimum daily return have the same characteristics of stocks with highest maximum daily return. Only these seem be even more illiquid. As per intermediate-term momentum this has the same pattern as for MAX. Intermediate-term momentum is negative. As shown by Jegadeesh and Titman (1993) stocks tend to follow a continuation pattern over a 12 months period. The momentum effect however is contrary to the positive effect of MIN and might cancel out the positive MIN effect. Thus, it is important to control for it, to make sure that MIN still commands any effect on the cross-section of expected returns.

When comparing MAX and MIN effect both rely on the mispricing exhibited on stocks with either extreme positive or extreme negative returns. As described in previous literature both of these effects seem to be relevant for stocks that are lottery-like assets. Like lotteries, these stocks are cheap and have small probabilities of extreme returns. While for MAX, high positive returns can lead investors to pay-more for these stocks, leading to lower future returns, for MIN, the existence of extreme low returns may lead investors to undervalue these stocks which in turn translates into higher future returns.

While to explore the MAX effect, an investor would have to short stocks on decile portfolio 10, which would be costly and risky, what might discourage arbitrageurs from pursuing strategies based on this effect, for the MIN effect, this would mean an investor buying stocks on decile portfolio 10, which are illiquid and cheap. Arbitrageurs can more easily pursuit a strategy based on the MIN rather than on MAX effect. This can be a partial explanation for why I find economical and statistically significant results for both equal and value-weighted portfolios for MAX, while the results are not robust for MIN.

3.2.3. Bivariate Portfolio Analysis

This section present results for portfolios sorted on MIN after controlling for a series of different variables. The table presents both monthly average returns and 4FF alphas as well as corresponding Newey-West t-statistic. To create these portfolios, stocks are first sorted based on each different variable, 10 deciles are created based on the given variable. Secondly, each decile is sorted into other 10 deciles based on the minimum daily return. In the end there are 100 (10x10) bins in total. For instance, for SIZE, stocks are first sorted on SIZE and 10 deciles are created, these have dispersion in SIZE. Within each of these 10 SIZE deciles, stocks are then sorted on minimum daily returns to produce 10 MIN deciles within each of the 10 SIZE deciles. Producing a total of 100 decile portfolios. Table 6 presents the average monthly return and 4FF alphas across the 10 deciles for each variable. For instance, for SIZE, table 6 presents for the Low MIN portfolio, the average monthly return across the 10 SIZE deciles for the stocks that on the MIN sorting fall under the decile portfolio 1. This bivariate analysis intends to create portfolios sorted on minimum daily return, that have dispersion in MIN but similar levels of each control variable.

Panel A: Average returns for value-weighted portfolios							
Deciles	SIZE	BM	BETA	MAX	SKEW	MOM	ILIQ
Low MIN	1.02	0.89	0.83	0.79	0.82	0.82	0.79
2	1.18	1.03	0.90	0.77	0.92	0.89	0.95
3	1.17	1.09	0.97	0.91	1.00	0.91	1.01
4	1.31	1.09	1.00	0.88	1.11	0.93	1.13
5	1.30	1.12	0.98	0.86	1.08	0.93	1.05
6	1.30	1.12	0.99	0.89	1.10	0.92	1.12
7	1.32	1.09	0.96	0.89	0.97	0.90	1.03
8	1.21	1.10	0.93	0.92	0.96	0.69	1.00
9	1.18	0.89	0.97	0.97	0.57	0.74	1.01
High MIN	1.02	0.52	0.57	0.60	0.33	0.50	0.83
High - Low	-0.01	-0.37	-0.26	-0.18	-0.49	-0.32	0.04
t-stat	(-0.07)	(-3.35)	(-2.48)	(-1.93)	(-3.55)	(-3.06)	(0.41)
4FF Alpha	-0.07	-0.54	-0.40	-0.27	-0.65	-0.58	-0.06
t-stat	(-0.83)	(-5.33)	(-4.03)	(-2.94)	(-5.25)	(-5.76)	(-0.68)

Panel B: Average returns for equal-weighted portfolios							
Deciles	SIZE	BM	BETA	MAX	SKEW	MOM	ILIQ
Low MIN	1.05	0.95	0.99	0.88	0.92	0.92	0.95
2	1.19	1.05	1.02	0.91	1.07	1.06	1.10
3	1.20	1.15	1.12	1.05	1.14	1.09	1.14
4	1.34	1.20	1.12	1.02	1.25	1.15	1.29
5	1.33	1.26	1.18	1.07	1.30	1.19	1.24
6	1.32	1.21	1.22	1.20	1.26	1.27	1.35
7	1.38	1.25	1.26	1.25	1.22	1.25	1.33
8	1.30	1.26	1.28	1.41	1.26	1.31	1.34
9	1.30	1.30	1.35	1.61	1.27	1.45	1.40
High MIN	1.23	1.89	1.95	2.15	1.84	1.87	1.43
High - Low	0.18	0.93	0.96	1.27	0.92	0.95	0.48
t-stat	(1.82)	(8.96)	(9.93)	(15.77)	(8.00)	(10.58)	(4.50)
4FF Alpha	0.11	0.85	0.90	1.22	0.81	0.68	0.42
t-stat	(1.25)	(8.77)	(9.47)	(15.06)	(7.77)	(8.19)	(4.17)

Table 6*. *Average portfolio returns and alphas for bivariate sorts on MIN and several controls

Every month, stocks are double sorted on a control variable and on minimum daily return (MIN) over the past month to form 10 decile portfolios with dispersion in MIN but similar levels of the control variable. This table reports average monthly returns across the ten control deciles. For the High – Low portfolio, Newey-West (1987) t-statistics are reported in parentheses.

Table 6, panel A, presents the results for value-weighted portfolios while panel B has the results for the value-weighted portfolios. As MIN effect is driven by small stocks and value-weighted portfolios cancel its effect, this analysis will be performed focusing on equal-weighted portfolio. When referring to value-weighted panel A results, it will be specified.

When controlling for firm size, defined as the natural logarithm of end of month price times end of month number of shares outstanding, the average value-weighted monthly return difference between high and low MIN portfolios becomes very close to zero, although still negative, but with no statistical significance at any level. When comparing to the results in panel B, for equal-weighted portfolio, the average monthly return is still positive and statistically significant at 5 percent level. This is in line with the results presented in the univariate analysis. Also, as shown in table 5, stocks with high MIN are typically small and micro-cap stocks. When controlling for size the MIN effect is enhanced, and in the value weighted portfolio, the negative effect for the MIN which is opposite to the expected positive relation between MIN and future returns loses significance and becomes close to zero.

When controlling for book-to-market ratio, the effect of MIN is preserved for both the value and equal weighted portfolio. Specifically, for Panel B, both average monthly returns and 4FF alphas are better in both economical and statistical terms, when compared to the univariate portfolio analysis with an average monthly return difference between decile 10 and 1 of 0.93 percent and a 4FF alpha of 0.85 percent, with Newey-West t-statistic of 8.96 and 8.77 respectively. In respect to market BETA, defined as normal CAPM. The effects of MIN are also better in this analysis and similar to the results for book-to-market.

The other three important effects to control for are MAX, Skewness, and intermediate-term momentum. MAX has been used in previous literature to study the relationship to future returns and its impact of idiosyncratic volatility puzzle first studied by Ang et al. (2006). Bali et al. (2011) discussed how MIN should have the symmetric effect of MAX. Here maximum daily return is controlled for and the MIN effect is still preserved and very high with an average monthly return for the High – Low portfolio of 1.27 percent and t-statistic of 15.77, and a 4FF alpha of 1.22 percent with a Newey-West t-statistic of 15.06.

Skewness has been discussed to drive both MIN and MAX effect. Amaya et al. (2015) study the relation between realized skewness and future returns. Their study concludes that "investors accept low returns ang high volatility because they are attracted to high positive skewness". This preference for positive skewness could explain the positive effect found between MIN and future returns as investors should demand a compensation for holding stocks with negative skewness. And like so, MIN could be proxying for skewness. The results presented do not show evidence that the extreme negative returns effect is subsumed by the level of skewness, with the average monthly returns for the High – Low portfolio of 0.92 percent for a Newey-West t-statistic of 8.00 and an 4FF alpha of 0.81 percent for a t-statistic of 7.77. As for intermediate-term momentum (MOM) and illiquidity, results are still economically significant and present very high statistical significance. The same analysis is computed for portfolios sorted on MAX using the same set of variables as controls. For simplicity results are presented in the appendix. Regarding maximum daily returns the majority of the results are statistically significant. It is important to focus that opposite to MIN that keeps its effect, the MAX High-Low portfolio when controlling for skewness is not statistically significant for the equal-weighted portfolio. Showing evidence that the effect of MAX might be proxying for positive skewness.

To further investigate the relation between extreme negative returns and future stocks return, a cross-sectional analysis is undertaken. Both the univariate and bivariate portfolio analysis present a big limitation which is to control for multiple effects simultaneously. To tackle this issue, I examine the cross-sectional relation between MIN and expected return using Fama and MacBeth (1973) regressions. First a set of variables including MIN and MAX are analyzed independently. Secondly, MIN and MAX are each one analyzed while controlling for different effects. The final model, following equation (2) includes both MIN, MAX and the all the control variables. Each month *t* the following cross-sectional regression is estimated:

$$R_{i,t+1} = \gamma_{0,t} + \gamma_{1,t} MIN_{i,t} + \gamma_{2,t} MAX_{i,t} + \gamma_{3,t} BETA_{i,t} + \gamma_{4,t} SIZE_{i,t} + \gamma_{5,,t} BM_{i,t} + \gamma_{6,,t} MOM_{i,t} + \gamma_{8,t} ILIQ_{i,t} + \varepsilon_{i,t+1} \quad (2)$$

where $R_{i,t+1}$ is the monthly return of each *i*th stock for month *t*+1. The cross-section regressions are run on MIN, MAX, BETA, SIZE, BM, MOM and ILIQ. Where MOM is the intermediate-term momentum calculated over the 11 months ending 2 months before the return in t +1. Table 7 has the time-series average of the coefficients for each variable from July 1962 to May 2018. It is of particular interest to analyze how MAX and MIN behave independently and jointly.

MAX	MIN	BETA	SIZE	BM	MOM	ILIQ	SKEW
-0.0275							
(-2.56)							
	0.0605						
	(3.14)						
		-0.0252					
		(-0.84)					
			-0.1961				
			(-4.87)				
				0.2097			
				(5.93)			
					0.3531		
					(2.05)		
						0.0354	
						(2.95)	
							-0.1360
							(-4.12)
		0.0030	-0.1564	0.1110	0.4513	0.0341	-0.2333
		(0.11)	(-4.06)	(3.20)	(2.88)	(3.06)	(-8.04)
-0.0671		0.0371	-0.1961	0.0975	0.3852	0.0402	-0.0576
(-6.27)		(1.56)	(-5.68)	(2.97)	(2.53)	(3.61)	(-1.85)
	0.0058	-0.0046	-0.1349	0.1080	0.5173	0.0321	-0.2248
	(0.37)	(-0.21)	(-4.25)	(3.40)	(3.53)	(2.86)	(-5.76)
-0.0664	0.1094						
(-8.49)	(6.88)						
-0.1542	0.1566	0.0122	-0.1457	0.1068	0.4475	0.0395	0.4249
(-9.46)	(7.01)	(0.56)	(-4.65)	(3.37)	(3.02)	(3.54)	(8.20)

Table 7. Cross-sectional regressions of stock returns on MAX, MIN and firm characteristics.
Every month, excess stock returns in month $t+1$ are regressed on a subset of predictor variables in month t, including MIN and MAX, producing a time-series of coefficients for each explanatory variable. Reported in this table are the averages of these coefficients. Newey-West (1987) t-statistics are presented in parentheses. In the appendix, a definition for each variable presented is found.

First, table 7 presents univariate regressions, where MAX, and MIN are analyzed individually. For this, the time series average of MAX coefficients is -0.0275, this is negative and statistically significant, with Newey-West t-statistic of -2.56. Confirming the results presented before, this shows a significant negative relation between MAX and expected future returns. As for MIN, which is the coefficient of interest, the average of time-series coefficients is of 0.0605 with a Newey-West t-statistic of 3.14. Here, the MIN effect is also in line with the results for equal-weighted portfolio analysis, proving a positive and highly significant relation between the minimum daily observation and next month expected return.

For the other individual control variables, coefficients are also as expected. Against expectation BETA has a negative coefficient, although this is not significant and changes sign when used in the multivariate analysis, although none of the studied relations having any statistical significance. As for SIZE, book-to-market (BM), momentum and illiquidity as defined in Amihud (2002), these appear to have significant coefficients across different model specifications.

It is of interest to analyze in detail the model where MAX and MIN are modeled together. The average coefficient for both variables becomes larger with higher economic significance and also more statistical significance that when modeled separately. MAX has a negative time-series average coefficient of -0.0664 with Newey-West t-statistic of -8.49 while MIN has an average coefficient of 0.1566 with a Newey-West t-statistic of 7.01.

3.3.Idiosyncratic Volatility and Extreme Returns

3.3.1. Finding the Idiosyncratic Volatility Puzzle

In this section I examine the relation between extreme returns and idiosyncratic volatility, hereafter referred to as IVOL. First, it is important to find evidence that the idiosyncratic volatility puzzle exists. To prove this, I first do a portfolio level analysis of stocks sorted on the previous one-month IVOL. The second part of this section explores in more detail how the IVOL effect behaves when controlling for extreme returns. I explore this in two ways, first by creating double sorted portfolios of stocks, where I create 10 IVOL decile portfolios, controlling for MIN and MAX, individually. Second, I present Fama MacBeth cross-section regressions for a set of different models, to examine MIN and MAX together and their relationship with IVOL.

Table 8 presents the average monthly return and 4FF alpha, along with corresponding Newey-West t-statistics for both Value- Weighted (VW) and Equal-Weighted (EW) portfolios.

Deciles	VW Portfolios Average return	4FF Alpha	EW Portfolios Average return	4FF Alpha	Average IVOL
Low IVOL	0.96	0.10	1.11	0.21	0.64
2	0.95	0.03	1.21	0.19	1.09
3	0.99	0.04	1.31	0.26	1.38
4	1.04	0.07	1.37	0.29	1.68
5	1.03	0.03	1.38	0.30	2.00
6	1.08	0.06	1.35	0.26	2.38
7	0.83	-0.16	1.25	0.16	2.84
8	0.60	-0.46	1.15	0.10	3.46
9	0.34	-0.68	1.05	0.02	4.47
High IVOL	-0.17	-1.18	1.03	0.03	8.07
High - Low	-1.13	-1.28	-0.09	-0.18	
t-stat	(-3.69)	(-5.47)	(-0.28)	(-0.66)	

Table 8. Average portfolio returns and alphas for univariate sorts on IVOL
Every month, stocks are sorted on idiosyncratic volatility (IVOL) over the past month to form 10 decile portfolios. The table presents both the value (VW) and equal-weighted (EW) average monthly returns, the 4FF alphas and the average IVOL of stocks in the portfolio formation month, all values in percentage terms. For the High – Low portfolio, Newey-West (1987) adjusted t-statistics are reported in parentheses.

Stocks exhibit different levels of IVOL, and data shows clear evidence of extreme IVOL values for a set of stocks, these are the stocks in both Low IVOL and High IVOL portfolios. Looking at the average IVOL in each decile portfolio, the variation across decile 1 to 9 is relatively small ranging from 1.09 to 4.47, however in decile portfolio 1(Low IVOL), idiosyncratic volatility drops to 0.64. Also, in decile portfolio 10 (High IVOL) the average idiosyncratic volatility increases dramatically to 8.07, almost the double of decile portfolio 9.

The important aspect to study is if this dispersion in IVOL has any impact in future expected returns, and if so, as per the idiosyncratic volatility puzzle, is this relation negative. i.e., do stocks with high idiosyncratic volatility have low future returns?

The results in the table show evidence to support the idiosyncratic volatility puzzle. All High – Low portfolio monthly returns and 4FF alphas are negative. While results for value-weighted portfolios are highly significant, with an average monthly return for High – Low portfolio of -1.13 percent with a Newey-West t-statistic of -3.69 and a 4FF alpha of -1.28 percent with a Newey-West t-statistics of -5.47. The results for equal-weighted portfolio, although both average return and 4FF alpha in High – Low being negative, none is significant at any level. The idiosyncratic volatility effect appears to be only relevant for value-weighted portfolios as for equal-weighted portfolios, there is no indication of an idiosyncratic volatility effect. This result is robust to evidence found by Bali and Cakici (2008) and Bali et al. (2011) but not robust to evidence found by Ang et al. (2006).

3.3.2. Idiosyncratic Volatility Puzzle and Extreme Returns

To further examine the idiosyncratic puzzle and the relation between extreme returns and IVOL, bivariate sorted portfolios are constructed. In panel A of table 9, average returns and alphas are reported for the double sorted portfolios. To create these, I first sort stocks on IVOL and then within each of the 10 IVOL deciles, I sort stocks on either MAX or MIN. This approach allows to create portfolios sorted on MAX or MIN, controlling for IVOL. The table presents average monthly returns across the 10 IVOL control deciles. This creates 10 decile portfolios with different levels of MAX/MIN but similar levels of IVOL. In this table the figures of interest are the High – Low return and the 4FF alphas. In regard to MAX, the maximum daily return appears to preserve its effect even after controlling for IVOL, monthly returns and 4FF alphas have high statistical significance. As per MIN, in the equal-weighted portfolio, both the average monthly return and the 4FF alpha for High – Low portfolio, increase both the economical and statistical significance when compared to univariate portfolio results presented in table 3. As for value-weighted portfolios, in the univariate portfolio analysis the results for High – Low portfolio had high economical and high statistical significance, but when controlling for IVOL, MIN loses all statistical power. This is interesting as the negative effect of minimum daily returns to sub-sequent stock returns was a result that went against the expectations, as discussed before this could possibly be driven for other effect other than the actual MIN. And it appears this is a possible explanation given the loss of statistical power in the bivariate portfolios.

In panel B I reverse the analysis. Here stocks are first sorted on MAX or on MIN. Secondly, within each of the 10 MAX/MIN decile portfolios created, stocks are sorted into 10 deciles based on IVOL. Reported in the table are the average monthly returns across the 10 MAX/MIN portfolios. This allows to create 10 decile portfolios with different levels of IVOL, but similar levels of either MAX or MIN.

For maximum daily returns (MAX), the value-weighted analysis shows that the IVOL effect becomes less economical and statistically significant when compared to the univariate portfolio. While in the univariate IVOL portfolios, the High – Low average monthly return is -1.13 percent with a t-statistic of -3.69, now this is -0.20 percent with a t-statistic of -2.11. The effect is similar for the 4FF alpha that decreases from -1.28 percent in the univariate portfolio analysis to -0.25 when controlling for MAX. As for equal-weighted portfolios, an interesting effect appears. When controlling for MAX, equal-weighted Portfolios exhibit a positive and highly significant relation between IVOL and future expected returns. Showing evidence that the IVOL puzzle disappears.

In regards to IVOL portfolios controlling for minimum daily return (MIN). In the value-weighted portfolio similarly to MAX, the effect decreases when compared to the univariate IVOL analysis. Interestingly that for equal-weighted portfolios, now that I control for MIN, idiosyncratic volatility has a larger negative impact that is highly significant with an average monthly return for High – Low portfolio of -0.49 percent with a t-statistic of -4.92.

From this table several conclusions can be drawn. First it shows a potential solution for the idiosyncratic volatility puzzle with extreme high returns represented by MAX. Second, it seems that the minimum daily return has an opposing impact for value and equal-weighted portfolios. While it decreases the effect of IVOL for value-weighted portfolios, the effect becomes larger in negative terms for the equal-weighted, more importantly it turns the High – Low average monthly return and 4FF alpha into a statistically significant coefficient. In conclusion, the results found in data prove that MIN have an opposite behavior to MAX in respect to expected stock return, however MIN has not shown to have the same impact on idiosyncratic volatility as MAX.

Panel A: Portfolios sorted on MAX or MIN controlling for IVOL				
	MAX		MIN	
Deciles	VW	EW	VW	EW
Low MAX/MIN	1.05	1.83	0.78	0.83
2	0.99	1.46	0.77	0.91
3	0.93	1.44	0.71	1.01
4	0.88	1.30	0.80	1.13
5	0.87	1.19	0.86	1.21
6	0.70	1.11	0.79	1.26
7	0.81	1.12	0.88	1.34
8	0.75	1.07	0.92	1.52
9	0.77	0.98	0.91	1.61
High MAX/MIN	0.67	0.99	0.77	1.72
High - Low	-0.39	-0.84	-0.01	0.90
t-stat	(-4.55)	(-13.63)	(-0.11)	(13.86)
Four-factor Alpha	-0.45	-0.97	-0.08	0.82
t-stat	(-4.82)	(-14.71)	(-0.87)	(11.89)

Panel A: Portfolios sorted on IVOL controlling for MAX or MIN				
	MAX		MIN	
Deciles	VW	EW	VW	EW
Low MAX/MIN	0.88	0.96	0.90	1.21
2	0.86	1.02	1.03	1.37
3	0.00	1.06	1.01	1.45
4	0.91	1.09	0.95	1.44
5	0.85	1.09	0.90	1.32
6	0.82	1.14	0.86	1.30
7	0.81	1.24	0.79	1.29
8	0.84	1.34	0.80	1.24
9	0.76	1.48	0.55	1.13
High MAX/MIN	0.68	2.07	0.42	0.79
High - Low	-0.20	1.11	-0.49	-0.42
t-stat	(-2.11)	(13.40)	(-4.92)	(-5.53)
Four-factor Alpha	-0.25	1.13	-0.61	-0.49
t-stat	(-2.60)	(13.42)	(-6.10)	(-6.75)

Table 9. Average portfolio returns and alphas for bivariate sorts on MIN, MAX and IVOL
Every month decile portfolios are formed by double sorting stocks on a IVOL and then on either the minimum daily return (MIN) or maximum daily returns (MAX) over the past month. Similarly, each month decile portfolios are formed by double sorting stocks on either the minimum daily return (MIN) or maximum daily returns (MAX) and secondly by sorting these in IVOL. Average monthly returns across the ten control deciles are presented. For the High – Low portfolio Newey-West (1987) adjusted t-statistics are reported in parentheses.

To examine the relation between extreme returns and IVOL more closely, the following analysis computes Fama-MacBeth regressions. It studies the cross-sectional relation between extreme returns, MAX and MIN, and idiosyncratic volatility, allowing to control for a set of different control variables, namely firm Beta (BETA), the size (SIZE), book-to-market (BM), intermediate-term momentum (MOM), illiquidity (ILIQ) and skewness (SKEW). Table 10 presents several different models to study each of the effect individually and jointly.

MAX	MIN	IVOL	BETA	SIZE	BM	MOM	ILIQ	SKEW
-0.0275								
(-2.56)								
	0.0605							
	(3.14)							
		-0.0168						
		(-0.39)						
-0.1246		0.3616						
(-10.52)		(5.27)						
	0.1366	-0.2510						
	(10.16)	(-6.27)						
-0.0664	0.1094							
(-8.49)	(6.88)							
-0.0919	0.0807	0.1185						
(-7.20)	(5.76)	(1.60)						
-0.1428		0.2674	0.0419	-0.1552	0.0994	0.4500	0.0360	0.1214
(-8.63)		(4.17)	(1.74)	(-5.21)	(3.17)	(3.05)	(3.18)	(3.56)
	0.1578	-0.4285	-0.0186	-0.1706	0.1021	0.4400	0.0412	0.1053
	(7.36)	(-8.18)	(-0.85)	(-5.56)	(3.23)	(2.97)	(3.68)	(2.95)
-0.1477	0.1649	-0.0461	0.0074	-0.1476	0.1039	0.4500	0.0394	0.4340
(-8.01)	(7.33)	(-0.73)	(0.34)	(-5.03)	(3.31)	(3.05)	(3.46)	(8.23)

Table 10. Cross-sectional regressions of stock returns on IVOL and firm characteristics.
Every month, excess stock returns in month *t+1* are regressed on a subset of predictor variables in month *t*, including MIN and MAX and IVOL, producing a time-series of coefficients for each explanatory variable. Reported in this table are the averages of these coefficients. Newey-West (1987) t-statistics are presented in parentheses. In the appendix, a definition for each variable presented is found.

The three first models, study the cross-sectional relation between each variable of interest, MAX, MIN and IVOL and future expected returns. As expected from previous analysis presented in table 2, the maximum daily return, has a negative and statistically significant relation to expected returns. The minimum daily return also proves the results found in the analysis presented before, with an average coefficient of 0.065 with a Newey-West t-statistic of 3.14. Finally, idiosyncratic volatility has a negative average coefficient but that is not significant. This result might be reflecting the relation of IVOL to expected returns in equal-weighted portfolios, which proved also to be negative but not significant.

As for the analysis where MAX or MIN are included jointly with IVOL, MAX one more time, proves to reverse the idiosyncratic volatility puzzle, with IVOL having an average coefficient of 0.3616 with a Newey-West t-statistic of 5.27. For MIN, when added to the regression it increases the statistical significance of IVOL coefficient to -6.27.

I also analyze MAX and MIN together, and both effects seem to explain expected returns in opposing ways. While MAX has an average coefficient of -0.0664 with a t-statistic of -8.49, MIN has an average coefficient of 0.1094 with a t-statistic of 6.88. So, MAX and MIN appear to have a different impact in IVOL. When I model MAX, MIN and IVOL together, MAX and MIN remain statistically significant, whereas IVOL although still positive loses its statistical power.

Finally, I conduct the same analysis but controlling for a set of variables that literature has proved to impact future expected returns. When IVOL is modeled with the set of controls, its average coefficient remains negative and becomes statistically significant. When I add MAX to this model, the average coefficient for IVOL again reverses its sign, becoming positive with an average coefficient of 0.2674 and t-statistic of 4.17 when controlling for a set of variables. This, together with previous results seem to solve the IVOL volatility puzzle reported by Ang et al. (2006, 2009). When MIN is added to IVOL plus controls model, its sign is still negative and significant, with an average coefficient of -0.4285 and a t-statistic of -8.18. The final model represented in equation (3), where both MAX and MIN, IVOL and the set of controls is included, IVOL seems to lose its statistical power, whereas MAX and MIN hold highly significant t-statistics and average coefficient of -0.1477 for MAX and 0.1649 for MIN. With these two variables I am able to eliminate the effect of idiosyncratic volatility. This final model is defined as in equation (3).

$$R_{i,t+1} = \gamma_{0,t} + \gamma_{1,t}MAX_{i,t} + \gamma_{2,t}MIN_{i,t} + \gamma_{3,t}IVOL_{i,t} + \gamma_{4,t}BETA_{i,t} + \gamma_{5,t}SIZE_{i,t} + \gamma_{6,t}BM_{i,t} + \gamma_{7,t}MOM_{i,t} + \gamma_{8,t}ILIQ_{i,t} + \gamma_{9,t}SKEW_{i,t} + \varepsilon_{i,t+1} \quad (3)$$

4. Further Analysis

The last analysis explores the relation between extreme returns and idiosyncratic volatility in two different periods. I conduct an analysis for two different time periods, the goal is to understand if the idiosyncratic volatility puzzle was more important or driven in a specific time period or if the idiosyncratic volatility puzzle holds true throughout the whole period in analysis.

4.1. Idiosyncratic Volatility and Extreme Returns for different time periods

To conduct the analysis for different time periods, I first compute the cumulative returns from the High – Low (H-L) portfolio of stocks sorted on IVOL. Figure 5 shows the evolution of cumulative returns for the whole period for the value-weighted and equal-weighted portfolios. From the analysis for the whole sample period, value-weighted portfolio presents a statistically robust negative relation of IVOL and expected future returns, however from figure 5, this effect appears to be more pronounced in the period from 1962 to 1990. After 1990 the effect appears to vanish. For the equal-weighted portfolio, where the unconditional analysis did not exhibit a statistically robust negative relation of IVOL to expected future returns, the graph depicts a more volatile relation until 1990, with periods of positive relation such as 1964 until the end of 1968 and from 1975 to the end of 1980 and periods where there was in fact a negative effect, depicting an idiosyncratic volatility puzzle such as the period from 1969 until the end of 1974 and the period starting in 1981 until 1990. After this period the graph depicts a still negative but more stable relation of IVOL to future expected returns.

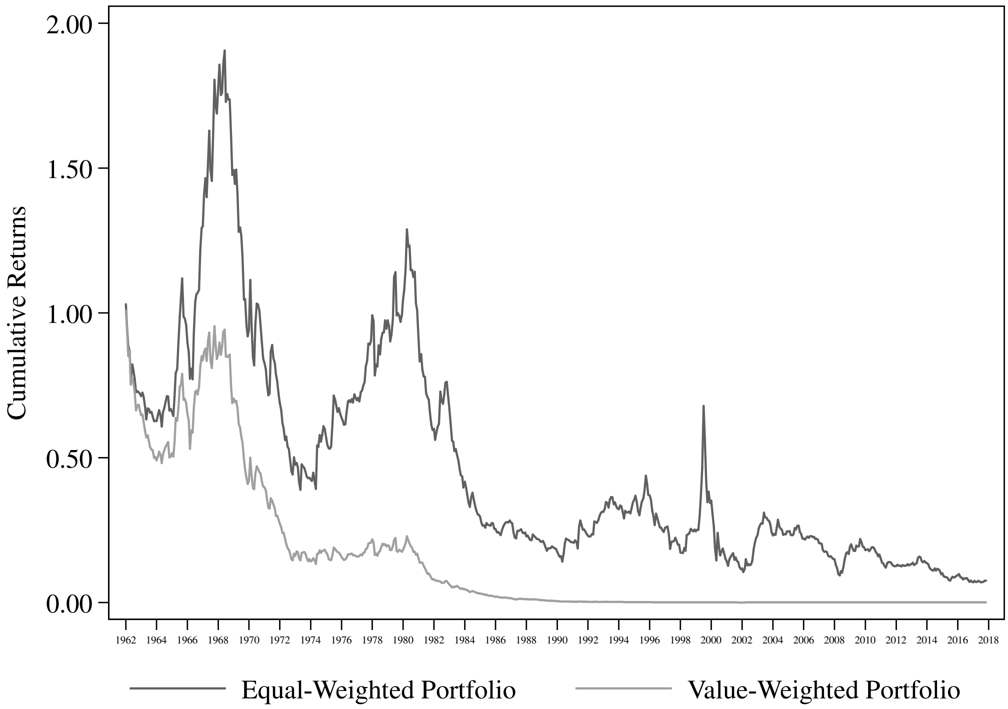

Figure 4. Cumulative returns for High – Low (H-L) sorted on IVOL.
Value and Equal-weighted portfolios sorted on Idiosyncratic volatility. The time period presented is from July 1962 to May 2018.

To understand the behavior of IVOL when controlling for extreme returns, Figure 6, 7 and 8 present the cumulative returns of the High – Low portfolio (H-L) of decile portfolios sorted on IVOL controlling for minimum daily returns and maximum daily.

From Graph 3, where the IVOL decile portfolios are controlled for MIN, it appears that there still persists an Idiosyncratic volatility puzzle for the period starting in 1962 that goes until the end of 1990. The effect is equal to both value and equal-weighted portfolios. After this period, the negative relation of idiosyncratic volatility to future expected returns seems to disappear and become close to zero, as indicated by the stable cumulative returns for both portfolios from 1990 to the end of the period in 2018.

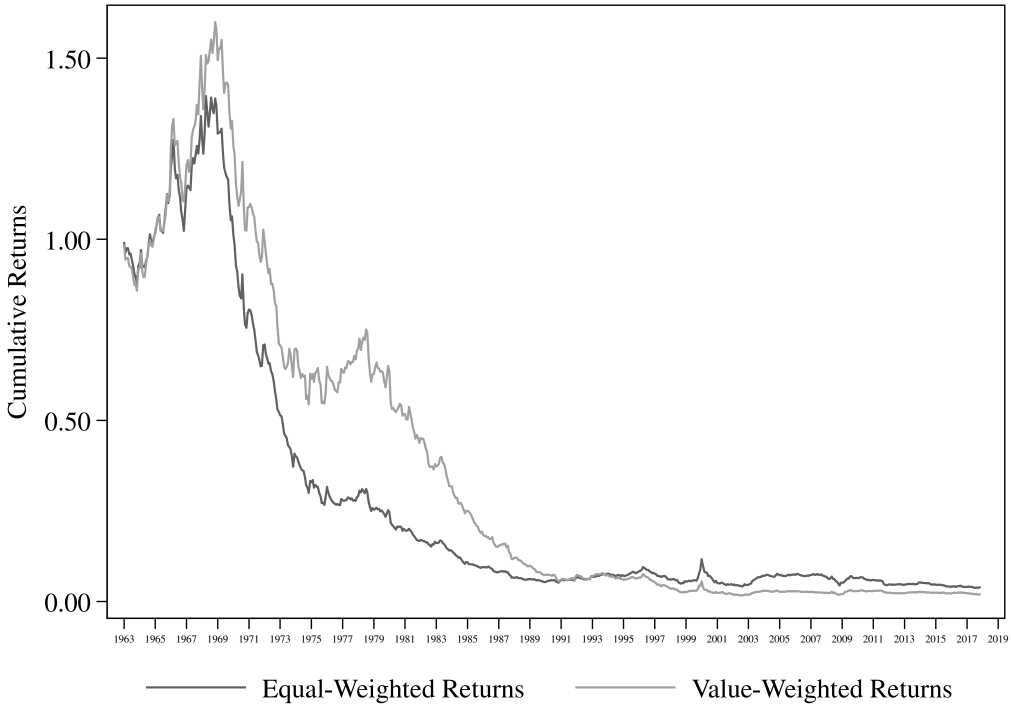

Figure 5. Cumulative returns for High – Low (H-L) sorted on IVOL controlling for MIN.
Value and Equal-weighted portfolios sorted on Idiosyncratic volatility and controlled for MIN. The time period presented is from July 1962 to May 2018.

Finally, I examine the cumulative returns for a High – Low (H-L) portfolio for a portfolio of stocks sorted on IVOL controlling for the maximum daily returns (MAX). From the table 9, this strategy for the whole period, has shown the existence of an IVOL puzzle for value-weighted portfolios, with a negative and statistically robust relation of IVOL to future expected returns. The two figures below depict the cumulative returns for value and equal-weighted portfolios. For the value-weighted in figure 7 and for the equal weighted portfolio in figure 8. It appears that for both weighting schemes, the IVOL effect also differs in the two periods mentioned. In figure 4 for a value-weighted portfolio, the relation of IVOL appears to be negative until the end of 1990, and relatively stable and close to zero from this year onwards, with the cumulative returns not changing throughout the last period, starting in 1991. In figure 5, the relation appears to be always positive but more prominent in the period starting in 1991.

From the analysis of the graphs, I split the sample period into two sub-samples. The first sub-sample starting in July 1962 and going until December 1990, and the second sub-sample starting in January 1991 lasting until the end of the period, May 2018. Table 11 presents the average monthly returns and 4FF alphas for the High – Low (H-L) portfolio, for the two weighting schemes, value and equal weighted.

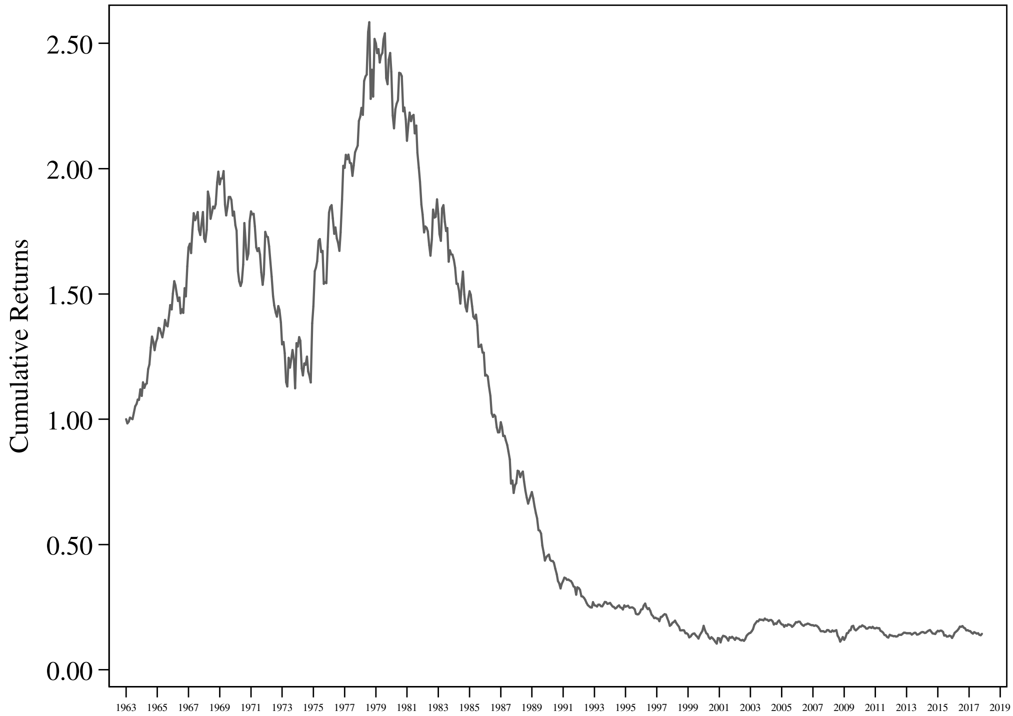

Figure 6. Cumulative returns for High – Low (H-L). sorted on IVOL controlling for MAX – Valuel-Weighted Portfolio. Value-weighted portfolios sorted on Idiosyncratic volatility and controlled for MAX. The time period presented is from July 1962 to May 2018.

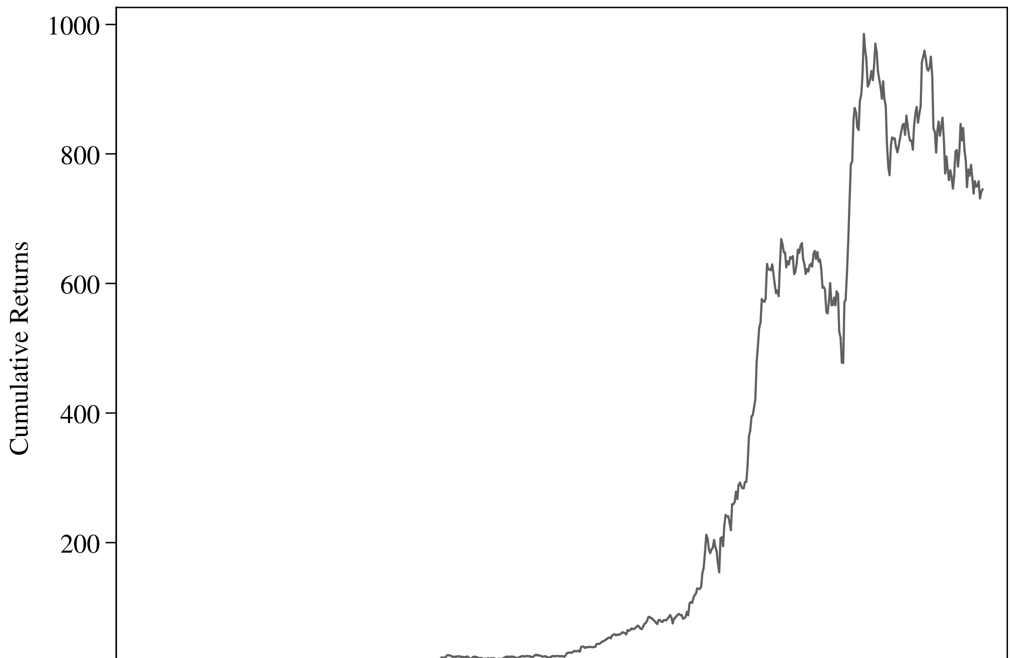

Figure 7. Cumulative returns for High – Low (H-L) sorted on IVOL controlling for MAX – Equal-Weighted Portfolio. Equal-weighted portfolios sorted on Idiosyncratic volatility and controlled for MAX. The time period presented is from July 1962 to May 2018.

For the portfolio sorted in IVOL, it appears to persist a negative relation of IVOL to expected returns, especially when examining the 4FF alpha that is negative and robust for both value and equal-weighted portfolios. This means, although it appeared that there was no idiosyncratic volatility puzzle in equal-weighted portfolios as found by Bali and Cakici (2008), this does hold true in the periods until 1990, where the puzzle persists for the two different weighting schemes.

From the analysis of IVOL portfolios sorted on MAX, data shows that MAX is enough to undo the IVOL puzzle from 1991 to 2018 for both value and weighted portfolios, but not for value-weighted portfolios for the period that starts in 1962 and goes until 1990. As for portfolios sorted on IVOL controlling for MIN, this effect similarly to MAX, appears to be able to undo the IVOL puzzle for the second sub-sample. One can argue that for recent periods of time, both MAX and MIN have a similar effect in IVOL, although it appears to be stronger for MAX from the analysis of equal weighted portfolios.

Sorting	Sub Sample	VW Portfolios		EW Portfolios	
		Average return	4FF Alpha	Average return	4FF Alpha
IVOL	1962 - 1990	-1.50	-2.07	-0.32	-0.82
		(-4.43)	(-9.32)	(-0.92)	(-3.54)
	1991 - 2018	-0.74	-0.97	0.16	0.11
		(-1.44)	(-2.84)	(0.31)	(0.26)
IVOL controlling for MAX	1962 - 1990	-0.24	-0.50	1.07	0.80
		(-1.93)	(-4.18)	(9.67)	(8.10)
	1991 - 2018	-0.17	-0.16	1.16	1.31
		(-1.13)	(-1.09)	(9.36)	(10.43)
IVOL controlling for MIN	1962 - 1990	-0.78	-1.20	-0.81	-1.13
		(-6.55)	(-10.39)	(-8.70)	(-12.69)
	1991 - 2018	-0.18	-0.24	-0.01	0.03
		(-1.14)	(-1.53)	(-0.10)	(0.24)

Table 11. Average returns and alphas for bivariate sorts on MIN, MAX and IVOL for different time sub-samples
The table reports both the value (VW) and equal-weighted (EW) average return and the 4FF alpha for High – Low (H-L) portfolio and the corresponding t-statistics. Average monthly returns as well as 4FF alphas are given in percentage terms. Newey-West (1987) t-statistics are reported in parentheses. The results are computed for two sub-samples, first for the period from July 1962 to the end of 1990, and second for the period starting in January 1991 and ending in May 2018.

5. Conclusion and Future Research

This thesis studies the relation between extreme returns and future stock returns. More precisely it addresses the relation between extreme negative daily returns and the return in the sub-sequent month. More importantly, it studies how these impacts the idiosyncratic volatility puzzle. In a final analysis it examines the behavior of IVOL across time.

While extreme returns can be proxied by either maximum or minimum daily returns, previous research have focused mostly on the relation of positive extreme returns to future expected returns and further used this effect to solve the idiosyncratic volatility puzzle. This study focus on studying both maximum and minimum daily returns in an attempt to compare the two effects and provide a robust analysis for the relation between extreme returns and IVOL. Where MIN and MAX are expected to have an opposite impact in the cross-section of returns, but a similar effect on the IVOL puzzle.

The analysis shows that maximum daily returns do a good job at predicting the cross-section of future stocks returns, where the High – Low portfolio produces a negative and statistically robust effect across value and equal-weighted portfolios. As for the idiosyncratic volatility puzzle, a negative relation between idiosyncratic volatility and sub-sequent stock returns is found in value-weighted portfolios. Throughout this analysis, maximum proved to work better then minimum daily return to cancel out the negative effect of idiosyncratic volatility. More importantly, MAX was able to reverse the negative impact of IVOL on future stock returns.

However, IVOL appears to have a distinctive behavior for different specifications. Not only it shows a distinctive behavior in different periods of time as it also exhibits a different behavior for different weighting schemes. When the analysis was conducted for a sub-sample, comprising only more recent data, specifically, for the period starting in January 1991 and going until May 2018, both MIN and MAX proved to be capable of canceling out the puzzle.

While different models were examined controlling for a set of variables, further analysis could be done, addressing the relation of extreme returns to other characteristics related to market imperfections. Interestingly, in regards to skewness of daily returns, the effect of MIN was not subsumed by it, however the effect of MAX disappeared, showing evidence that this effect might be proxying a skewness effect.

The idiosyncratic volatility puzzle seems difficult to solve and despite many different alternatives shown in previous research it appears that there is still not a common agreement for what might be driving this effect. Based on previous literature it would be important to further investigate this topic. Based on market imperfections theory, further research can investigate the relation between skewness and future expected returns, and more importantly, how does this effect work on solving this anomaly. Driving from this analysis one can investigate how does this effect relate to extreme returns.

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Appendix

Appendix 1: List of Variables

Maximum Daily Return (MAX): is the maximum daily return for stock *i* within each month t, where $R_{i,d}$ is the return on stock I on day d, and D_t is the total number of trading days within month *t.*

$$MAX_{i,t} = \max(R_{i,d}), d = 1, \ldots, D_t \quad (1)$$

Minimum Daily Return (MIN): is the minimum daily return for stock *i* within each month t, where $R_{i,d}$ is the return on stock I on day d, and D_t is the total number of trading days within month *t.*

$$MIN_{i,t} = \min(R_{i,d}), d = 1, \ldots, D_t \quad (2)$$

BETA: following Scholes and Williams (1977) and Dimson (1979) I compute stocks sensitivity to the market and compute beta as the sum of the lag, current and lead of the market sensitivity. Where $R_{i,d}$ is the return on stock *i* on day *d,* $Rm_{,d}$ is the market return on day d and rf_d is the risk-free rate on day *d.* The final Beta is the represented in equation (4), as the sum of the three beta coefficients.

$$R_{i,d} - rf_d = \alpha_i + \beta_{1,i}(Rm_{,d-1} - rf_{d-1}) + \beta_{2,i}(Rm_{,d} - rf_d) + \beta_{3,i}(Rm_{,d+1} - rf_{d+1}) + \varepsilon_{i,d} \quad (3)$$

$$\hat{\beta}_i = \hat{\beta}_{1,i} + \hat{\beta}_{2,i} + \hat{\beta}_{3,i} \quad (4)$$

Idiosyncratic Volatility (IVOL): this is estimated as the standard deviation of the residuals calculated over one-month for each individual stock. Where the return-generating process follows equation (5). The final IVOL is calculated, where $IVOL_{i,t}$ is the standard deviation of daily residuals of stock *i* in month *t.*

$$R_{i,d} - rf_d = \alpha_i + \beta_i(Rm_d - rf_d) + \varepsilon_{i,d} \quad (5)$$

SIZE: Firm size is defined each month as the natural logarithm of the market capitalization. Where *Shrout* is the number of shares outstanding for firm *i* at the end of month *t,* and *prc* is the price of stock *i* at the end of month *t.*

$$SIZE_{i,t} = \ln(Shrout_{i,t}\ x\ \mathrm{abs}(prc_{i,t})) \quad (6)$$

Book-to-Market (BM): Following Fama and French (1992), book-to-market in month *t* is calculated as the market value of equity at the end of December of the previous year, divided by the book value of common equity plus balance-sheet deferred taxes for the firm's latest fiscal year ending in the prior calendar year.

Intermediate-term momentum (MOM): As in Jegadeesh and Titman (1993), the momentum factor for each stock i in month t is defined as the cumulative return over the previous 11 months starting two month before the holding month.

$$MOM_{i,t} = \prod_{-2}^{-12}(1 + R_i) \quad (7)$$

Illiquidity (ILIQ): Following Amihud (2002), illiquidity is measured for each stock i in month t as the monthly average ratio between daily returns $(R_{i,d})$ and daily dollar trading volume $(Vol_{i,d})$.

$$ILIQ_{i,t} = \frac{1}{D_t}\sum_{1}^{D_t}\frac{R_{i,d}}{Vol_{i,d}} \quad (8)$$

Skewness (SKEW): The skewness of stock i for month t is computed using daily returns within each month, where μ is the mean return of stock i on month t, *and* S is the standard deviation.

$$SKEW_{i,t} = \frac{1}{D_t}\sum_{1}^{D_t}\frac{R_{i,d} - \mu_{i,t}}{s_i} \quad (9)$$

Appendix 2

Panel A: Average returns for value-weighted portfolios							
Deciles	BETA	BM	SIZE	MIN	SKEW	MOM	ILIQ
Low MAX	1.03	0.97	0.99	1.00	0.99	1.10	0.95
2	0.91	0.97	0.97	0.97	1.04	0.97	0.98
3	0.90	0.95	0.93	0.99	0.91	0.94	0.93
4	0.91	0.98	1.00	0.94	0.99	1.01	1.00
5	1.00	0.94	1.02	0.91	0.91	0.91	0.98
6	0.97	0.86	0.88	0.98	0.86	0.83	0.91
7	0.84	0.83	0.89	0.86	0.83	0.86	0.91
8	1.03	0.88	0.87	0.80	0.69	0.88	0.91
9	0.74	0.66	0.83	0.68	0.53	0.74	0.80
High MAX	0.40	0.21	0.68	0.89	0.27	0.65	0.71
High - Low	-0.60	-0.73	-0.29	-0.09	-0.70	-0.42	-0.23
t-stat	(-2.69)	(-2.83)	(-1.33)	(-0.63)	(-2.22)	(-1.95)	(-1.01)
4FF	-0.72	-0.90	-0.45	-0.28	-0.91	-0.72	-0.44
t-stat	(-4.55)	(-4.79)	(-2.66)	(-2.21)	(-3.99)	(-4.34)	(-2.49)

Panel B: Average returns for equal-weighted portfolios							
Deciles	BETA	BM	SIZE	MIN	SKEW	MOM	ILIQ
Low MAX	1.31	1.26	1.39	1.54	1.19	1.41	1.32
2	1.30	1.35	1.55	1.51	1.25	1.41	1.43
3	1.31	1.39	1.52	1.50	1.28	1.32	1.47
4	1.30	1.38	1.49	1.46	1.31	1.31	1.41
5	1.35	1.34	1.46	1.32	1.33	1.24	1.44
6	1.26	1.28	1.35	1.22	1.27	1.21	1.32
7	1.29	1.24	1.26	1.22	1.25	1.27	1.29
8	1.24	1.25	1.09	1.06	1.17	1.13	1.18
9	1.16	1.12	0.89	0.95	1.14	1.19	0.96
High MAX	1.00	0.95	0.51	0.69	1.39	1.01	0.70
High - Low	-0.29	-0.29	-0.87	-0.83	0.22	-0.37	-0.60
t-stat	(-1.37)	(-1.16)	(-3.89)	(-6.05)	(0.75)	(-1.93)	(-2.51)
4FF	-0.39	-0.39	-0.97	-0.97	0.06	-0.70	-0.72
t-stat	(-2.04)	(-1.85)	(-5.69)	(-8.72)	(0.26)	(-5.15)	(-3.68)

Average portfolio returns and alphas for bivariate sorts on MAX and several controls

Every month, stocks are double sorted on a control variable and on maximum daily return (MAX) over the past month to form 10 decile portfolios with dispersion in MAX but similar levels of the control variable. This table reports average monthly returns across the ten control deciles. For the High – Low portfolio, Newey-West (1987) adjusted t-statistics are reported in parentheses.