Predicting the probabilities of default for the banking sector in the United States

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Dissertation written under the supervision of Nuno Silva

Dissertation submitted in partial fulfilment of requirements for the MSc in Economics, at the Universidade Católica Portuguesa, 08.01.2020

Abstract

Title: Predicting the probabilities of default for the banking sector in the United States

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Key words: Structural credit risk models, Banking, Probability of Default

This dissertation implements a structural credit risk model to predict the probabilities of default for the eight largest retail banks in the United States. Similar to Goldstein, Ju, and Leland (2001), the model implemented in this dissertation relates the project/asset value with the firm capacity to generate earnings. Most papers in the literature consider that firms have fixed financial costs and that shareholders decide whenever to liquidate the firm based on the distance between some underlying earnings measure and these fixed costs. This assumption is not reasonable in the case of banks. As an alternative to incorporate shareholders strategic default decision, non-financial fixed costs are considered. These non-financial fixed costs are defined as the non-interest expenses and proxy the operational leverage of the bank. The analysed sample period contains 19 consecutive years, covering the dotcom crisis, the financial crisis and several minor crises in between. The model was calibrated by applying the iterative approach proposed by Vassalou and Xing (2004). The average computed probability of default for the whole sector ranged between 0.06% in 2006 and 5.80% during the financial crisis. These results were compared with the probabilities of default and the distances-to-default implied by Moody's and Standard & Poor's credit ratings for the period between 2006 and 2018. Though the probabilities of default show a low not significant correlation, the distances-to-default have a correlation of 53.27%, which was found to be significant at the usual significance levels.

Resumo

Título: Previsão das probabilidades de incumprimento para o sector bancário nos Estados Unidos

Autor: Lukas Weisel

Palavras-chave: Modelos de risco de crédito estrutural, Banca, Probabilidade de inadimplência

Esta tese implementa um modelo estruturado de risco de crédito para prever as probabilidades de falência dos oito maiores bancos comerciais nos Estados Unidos. Com base no trabalho de Goldstein, Ju and Leland (2001), o modelo implementado compara o valor do projeto/ativo com a capacidade da empresa de gerar receitas. A maioria da literatura considera que as empresas têm custos financeiros fixos e que os acionistas decidem liquidar a empresa sempre que se verificar uma determinada diferença entre as receitas e estes custos fixos. Esta premissa, no entanto, não é razoável no caso dos bancos. Como alternativa para a decisão estratégica dos acionistas de liquidar o banco, são também considerados custos fixos não-financeiros. Estes custos são definidos como despesas sem juros, representando um valor aproximado da alavancagem operacional do banco. O período analisado é composto por 19 anos consecutivos, cobrindo principalmente a crise da bolha da internet e a crise financeira de 2007/08. O modelo foi construído aplicando a abordagem sugerida por Vassalou e Xing (2004). A probabilidade média de falir calculada varia entre 0.06% em 2006 e 5.80% durante a crise financeira. Estes resultados foram comparados com as probabilidades de falência e distâncias para a falência calculadas pela Moody's e Standard & Poor's para o período entre 2006 e 2018. Apesar das probabilidades de falência demostrarem uma correlação baixa e não significante, as distâncias para a falência têm uma correlação de 53.27%, sendo significante para os usuais valores de significância.

Acknowledgements

First of all, I would like my supervisor Nuno Silva for his constant effort with any problems and the excellence of his teaching ability. Without any hesitation he provided a high-level support that enabled me to develop my knowledge in this area. Further I would like to thank the Católica Lisbon School of Business and Economics for the practical relevance of the materials taught throughout the whole programme.

Especially, I want to thank my family for their support over all these years to contribute perceiving this master. Without your support this wouldn't have been possible.

Finally, I would like to thank all the friends that I was able to get to know over the last one-and-a-half years. You were always supportive, made me feel like being in good company, and enhanced this whole experience.

Table of Contents

Index of Tables

Index of Figures

Index of Appendix

1. Introduction

Banks mainly generate earnings from three different sources. First, through the so-called interest rate margin. The main task of a retail bank is to collect deposits, for which they pay interest, and to grant credit to customers, to which they charge interest. This difference is expectedly positive. Second, banks charge commissions on their payment services or similar. Third, banks earn interest and dividends from investing in government or rated assets. The business model just described leaves banks exposed to a variety of different sources of risk. Three in particular, notably, credit, liquidity risk and market risk. Credit risk is defined as the risk that emerges from customers that are unable to repay their loan or contractual obligations towards the bank. Once this risk materialises it includes lost principal and interest, while increasing collection costs for the bank. Especially during the subprime crisis many banks suffered from significant losses through loans towards high-risk borrowers. These were caused by subprime mortgage borrowers. This can be measured by banks stating it as a "loss given default", which monitors the amount that got lost due to the default. The liquidity risk emerges from maturity transformation, which appears from the different maturity characteristics of deposits and loans. Short-term deposits are used to create long-term loans, which limit the banks financial resources in the case that depositors want to take their money out. Market risk emerges from the investing of some of the retail banks liquidity in bonds and other tradable securities, like government bonds or similar low-risk assets. Banks are measuring their market risk by using the value-at-risk (VaR) approach. It can be defined as the maximal value that a portfolio can suffer from and is known as one of the predominant risk management approaches used by banks (Berkowitz and O'Brien, 2002). Even though these are generally perceived as low-risk assets they increase their exposure to the market and create market risk. Operating risk can arise from employee errors, fraud or events that disrupt the production process. These can cause losses for the company. The economy is in demand of capital, which can cause banks to develop risk-taking incentives, which can be a problem in combination with the naturally low level of equity hold. The amount of capital held by a bank is measuring the potential amount a bank can absorb. Due to an increase in the return on equity that banks can achieve by using debt instead, the banks are holding only a low amount of capital. Debt is cheaper than equity, because the level of equity in a company can only be increased by retaining profits, decreasing dividend payments or buying shares back. Banks prefer not to use these options as they are not in line with shareholders wants or highly costly. Consequently, regulators aim to increase this low

level of capital to prevent bailouts paid by taxpayers as the capital is used to absorb losses when risks materialise.

Given that banks have a critical role in the economy and the risks that they face, they are regulated institutions. These regulations are intended to work as a framework to create a safer environment for companies and prevent financial crises. These regulations contain capital requirements that must be fulfilled by banks to be allowed to operate. In the last years, buffer requirements for cyclical risk were also added to the regulatory framework. This risk emerges from the cyclical peaks and downturns, which can have a significant effect on the business cycles of companies. Nevertheless, it is safe to say that these regulations are supposed to work as a financial cushion towards losses and prevent banks from overly taking risk into their books, but they don't guarantee that a bank cannot go bankrupt.

Several methodologies were proposed to measure the probability of banks going bankrupt. One of those methodologies are structural credit risk models. Structural credit risk models can be applied to overcome some of the previously defined shortcomings of traditional risk management metrics and models to predict the probabilities of default of financial institutions. Structural models use market data, which allow them to create forward-looking results. Consequently, this type of data enables the model to predict with a higher rate of precision and a more market realistic way.

Credit risks' close link towards default caught up the interest of many researchers and academics. For this reason, we are applying a structural model to analyse their exposure towards risk and analyse the likelihoods of default during times of crisis. In comparison to the previously mentioned approaches, structural models are very complex and difficult to successfully apply to financial institutions from an empirical perspective. Nevertheless, this dissertation aims towards the implementation of a structural model to predict the probabilities of default for the financial sector in the United States. In order to achieve this, a data set that contains 19 years of data from the eight largest retail banks in the United States is analysed. The model implemented in this dissertation is a modified version of the static Goldstein, Ju, Leland (2001) model, which is an EBIT-based structural model. This model finds application in the prediction process of the probabilities of default of these companies and therefore can retrieve information about their performance during the dotcom-crisis, the financial crisis of 2008, and an increase around 2014 – 2015 due to market changes. Throughout this time period, it was possible to retrieve information about the performance of the eight largest retail banks in the United States, which are an employer for many people and financial providers for a whole nation, that almost went bankrupt or that had to be bailed out. Finally, the probabilities retrieved from the GJL

(2001)-model were compared with the implied probabilities of default from the credit ratings of professional agencies. Further, the probabilities of default were used to compute the DD's and compare these with the rating implied ones by applying an InverseNormal function.

The structure of the dissertation is following the order of: Chapter 2 – Literature Review, Chapter 3 –Model, Chapter 4 – Data Inputs and Model calibration, Chapter 5 – Results, and finally in Chapter 6 – Conclusion.

2. Literature Review

2.1 Structural models

The application of structural credit risk models can be traced back to the pioneering papers of Black and Scholes (1973) and Merton (1974). In these papers corporate securities are seen as contingent claims on a firm's underlying assets. In the beginning, the pioneering paper of Black and Scholes (1973) first proposed an explicit formula to price call options on common stocks. Following joint work with Merton they also stated that in the hypothetical case of a company that issued a single pure discount bond and has a predetermined liquidation date, equity can be seen as a call option on the firm assets with strike equal to nominal liabilities. Thus, the Black-Scholes formulas could be used to price stocks. Therefore, this framework could be used to measure the value of equity. While the debt can be valued by applying put-call parity. This makes debt a covered call or something equivalent to a risk-free loan implied by a short put option. The put premium in this case can be understood as a compensation for the default risk. From this on it is possible to compute the probabilities of default with this framework. These are defined as the probability with which the company is unable to meet their outstanding financial obligations (Merton, 1974). The Black-Scholes formula used the market value of assets, the asset volatility, the risk-free rate, time-to-maturity, and the face value of debt as the strike price of the option (Black & Scholes 1973). This approach to measure credit risk became known as the Merton model. The usage of market data enables the prediction of forward-looking results and therefore are one of the main advantages of structural credit risk models.

Black and Cox (1976) improved on Merton's model by disbanding the assumption that companies can only default once outstanding debt matures. In their model the company defaults when the asset value passes a certain barrier the first time. They justify that default can occur at any point in time with the company not being able to meet its collateral agreements. The possibility of early default makes the model more realistic. They were the first ones to propose the "first passage time model". Everything else equal, the introduction of the possibility of early default leads to an increase in the chances of the firm defaulting. However, in contrast to Merton model where the asset value of a company can sink to any level before the debt is maturing, in Black-Cox debt holders' losses are capped by the recovered value at the barrier (Black & Cox, 1976).

A more complex approach was proposed by Geske (1977). In this paper, every time a payment is due shareholders have to decide whether or not do default. Shareholders exercise the option to stay when the asset values are at a significantly high level, which makes it possible to raise

new equity and continue with the payments to creditors. In contrast, they abandon the firm when the asset value is reaching a predefined low level where it is not possible to raise enough equity to meet the financial obligations to the creditors. The latter leads to the takeover of the company's assets by the creditors (Imerman, 2011). This approach was first proposed by Geske and further developed by Geske and Johnson (1984). This mechanism can further be compared to the one of compound options. These are defined as an option, which underlying asset is also an option. Therefore, these obtains can widely be applied through the possible combinations of call and put options. The advantage of this is that the bank got an exposure towards the underlying option now, but by hedging the exposure due to long-term costs of paying for the option now. Companies can therefore use these options while planning on costly projects and secure financial sources before going into the project. If they are unable to realize the project, they just lose their exposure towards the option, but without initiating costs of financing the unrealized project. Hence, financial institutions gain a sort of insurance policy from using compound options (Geske, 1979).

As this dissertation is about the computation of probabilities of default as important indicators in credit risk models it is essential to mention the paper about public firm expected default frequency (EDF), authored by Sun, Munves, and Hamilton (2012). Their paper follows the analytical calculation approach of default frequencies and their model belongs to the class of structural credit risk models. Their approach contains the distance-to-default (DD) which is of special interest for this dissertation. This approach first got proposed by Moody's KMV expected default frequency risk calculation model (Crosbie and Bohn, 2003). It represents a count of standard deviations that the asset value is away from financial distress, while assuming that there is a normal distribution. Hence, the distance-to-default can be seen as a market-based approach of corporate default risk, due to his inputs of the market value of the asset, the default point and the asset volatility. The measure captures macro-economic and firm-specific events in a closed-form solution, which can be seen as an advantage towards the classic Merton model (Crosbie and Bohn, 2003). They define the numerator as the difference between the logarithm of the asset value and the logarithm of the default point, which captures the firms' leverage. Meanwhile, the denominator discounts the leverage of the firm by asset volatility and therefore captures the denominator. Further, it can capture the overall risk level through the asset volatility. This formula can be used as a base to enhance the amount of credit risk parameters in a dissertation, which is using market information. The usage of market information allows forward-looking solutions towards default risk.

Brockman and Turtle (2003) proposed in their paper another approach using a barrier. In their paper, they proposed to use the down-and-out call (DOC) valuation model of Merton (1974).
The DOC framework works through exercising the call option once the barrier is hit and therefore terminates the company transferring the asset values from asset ownership to creditors (Brockman and Turtle, 2003). They find that this implication of a barrier leads to dominating results compared to the accounting-based approach of Altman Z-scores in almost all of their 7,787 firm-years tested.

In addition to the previously mentioned papers, there is a vast literature determined towards structural models that cover important corporate finance questions. These papers should be mentioned as well, due to their assumptions and findings. These can further be used to enhance structural models for the computation of default probabilities.

In the paper of Leland (1994), tax rates and bankruptcy costs are introduced, which allows the author to analyse the optimal capital question. In this model they assume that the shareholders set an endogenous threshold barrier at the lowest asset value where equity is still positive. The model assumes one class of debt with a fixed coupon and an infinite maturity. Once the asset value of the bank is reaching the barrier the shareholders make the strategical decision to default the company.

Goldstein, Ju, and Leland proposed a model that is using a cash flow-based approach to predict the optimal capital structure in a dynamic setting (i.e. they consider the possibility of issuing further debt in the future). They state that most models are limited by a static capital structure, while assuming a dynamic capital structure has significant effects on the amount of initial debt issued and the risk linked to these bond issuing companies. They assume that companies, in reality, adjust their debt levels with changing firm values and therefore tackle these previous assumptions (Goldstein, Ju, Leland, 2001).

The Leland- and the GJL-model are two examples of especially interesting corporate finance papers that can provide important insights to the measure of probabilities of default.

2.2 Structural models applications to the banking sector

Some papers using structural models focus on the banking sector.

Episcopos (2007) adapts the model proposed by Brockman and Turtle (2003) to the banking sector. This paper finds that their barrier option model is useful for bank regulators. Stockholders are left behind with a down-and-out call option, which leads to a decrease in their incentive to increase asset risk. As an increase in the regulatory barrier leads to a wealth transfer

from shareholders to in their case the federal deposit insurance corporation. This can be understood as a management tool for a deposit insurance premium schedule (Episcopos, 2007). The model assumes closure barriers to become trivial, while the down-and-in option works as a prompt corrective action (PCA). Thus, the barriers can be used as policy tools for banks and other financial institutions to meet capital requirements and restrictions from regulators (Episcopos, 2007).

Further, Flannery (2005) assumed in his work that asset values follow different stochastic processes. This could be taken into account for inputs in structural models to compute more realistic results. Assets following this assumption could increase the analysing power of banks capital structure works in times of financial crisis and how the valuation of contingent capital is affected by declines in the asset value, as more extreme events could be captured. This assumption could be applied to predict default probabilities through a structural model and retrieve information about the difference in their performance.

Following the previous assumption of asset values that follow different stochastic processes, Pennacchi (2010) stated the assumption that banks asset values follow a jump-diffusion process, while the bank issues short-term deposits, shareholder’s equity, and fixed- or floating-coupon contingent capital bonds (Pennacchi, 2010). The papers goal is to find evidence about the pricing mechanism of contingent capital in times of financial distress and how credit spreads are evolving at this time in the case of issuing banks. Further, they explore the risk-taking incentives of banks that issue several forms of contingent claims compared to banks that offer non-convertible subordinated debt. They find that banks that issue contingent capital are more likely to face moral hazard incentives to increase their asset's jump risk. In addition, they find that contingent capital is a feasible and cost-saving opportunity to mitigate financial distress when it is targeted to transform at an early stage (Pennacchi, 2010).

First of all, the implication of a jump-diffusion process, which permits sudden declines in asset values, led to contingent claims, that normally would have been risk-free or contain zero credit spreads, becoming positive. This provides more realistic results in times of crisis. They find that credit spreads of contingent claims tend to have an inverse relationship with the bank's capital, which can be explained through the closer range to conversion. Regarding the risk-taking incentive, they couldn't find evidence that a bank is issuing contingent capital to transfer value from contingent claims investors to banks shareholders because this incentive is given when issuing comparable subordinated debt as well (Penachhi, 2010). Even though earnings that follow a jump-diffusion process can monitor sudden declines in a better way, the

geometric-Brownian motion process, that is used by a lot of models, is also able to display declines realistically.

Due to the increasing connection of the global economy and the key role financial institutions take in this, it is important to predict future probabilities of default with a high level of precision. Even non-systemic crises can cause high costs for the public through bailouts of companies, that are defined to be systematic. Consequently, structural models can also be used to predict the likelihood with which banks are in need of a bail-out and the costs linked to these. Correia, Dubiel-Teleszynsky, and Población (2017) applied a Leland (1994)-style model, as it allows to model a more realistic approach and has the advantage of cash flows as an observable variable and the opportunity to model changes in the tax rate. Therefore, their model can indicate times with higher likelihoods of default and the costs that would occur out of it. These findings could help governments to work out regulations to increase their performance in a crisis. They find that the main costs of banking bailouts are caused by equity injections, deposit insurances and loan guarantees (Correia, Dubiel-Teleszynsky & Población, 2017).

Structural models can further be used towards the pricing of hybrid securities. These represent an important way for banks to raise capital in times of liquidity and/or market risk. The so-called reverse convertible debentures (RCD) could be analysed in future literature towards its effect to lower probabilities of default. This financial instrument would automatically convert into common equity once a market capital ratio of a bank reached a predetermined level. The current stock market price is used when the RCD converts, which hands over the full cost to shareholders, as a price of their risk-taking decision (Flannery, 2005). Structural models could therefore also be used to pursue the likelihood of a government bailout and therefore be in the interest of national supervisors. Further research could implement this kind of financial instrument towards a structural model and predict their efficiency to reduce the probabilities of default. In the case that good results, the government could create regulations towards banks to require them to hold a specific amount of these instruments like a capital requirement does for equity.

Additionally to the previous findings for hybrid securities, they seem to be a lower-cost alternative towards equity allowing for tax advantages (Barucci & Del Viva, 2013). Barucci and Del Viva apply an option pricing formula for contingent convertible-bonds (Coco-bonds). These provide financial services for banks, deleverage bank's balance sheets in times of high financial risk and reduce the expected bankruptcy costs while increasing the stability of the financial sector. Therefore, this financial instrument can reduce the likelihood of bankruptcy significantly in times of financial crisis. Overall they state that this instrument provides a lower

bankruptcy barrier and lower bankruptcy costs. Even though the instrument can do this for a company it comes at the cost of a higher yield spread of callable bonds, which is equivalent to an increase in the insurance premium (Barucci & Del Viva, 2013). Hence, several more instruments could be tested by structural models to predict if they can actively lower the probabilities of default for retail banks.

3. Model

The model used in this dissertation is largely based on the static version of Goldstein, Ju and Leland (2001) EBIT-based structural model. In this chapter, this model is explained in-depth and followed by a part about the calibrations of the model that were applied.

3.1 EBIT-based Model (Goldstein, Ju, Leland, 2001)

Goldstein, Ju and Leland (2001) assume that a company holds a project that generates a payout flow. The dynamics of this payout flow under measure P are given by[1]

Equation 1

$$\frac{d\delta}{\delta} = \mu_P dt + \sigma dz,$$

where μ_p and σ are constants. Here, μ_p, describes the drift of the project, while σ describes the volatility of the project returns. This equation is stated under the physical measure P. The physical measure P is true probability measure, which needs to be used to predict the probabilities of default later on. By discounting all expected future earnings flows at a certain constant discount rate, the project value can be obtained. Precisely, the value of the claim to the earnings flow is

Equation 2

$$V(t) = E_t^Q\left(\int_t^{\infty} ds \delta_s e^{-rs}\right)$$
$$= \frac{\delta_t}{r-\mu},$$

where $\mu = (\mu_P - \theta\sigma)$ is usually called the risk-neutral drift and r describes the risk-free rate. Both are assumed to be constant. θ is the market price of risk. The market price of risk is the compensation that investors require to invest in a certain project by a unit of risk. The risk premium is described by $\theta\sigma$. By applying Ito's lemma to the previously defined project value function, one can obtain the project value dynamics. This can be written under measure P (equation 3a) or under measure Q (equation 3b):[2]

Equation 3a

$$\frac{dV}{V} = \mu_P dt + \sigma dz^P$$

[1] A probability measure is a function that gives probabilities to states of the world.
[2] Measure Q is an alternative probability measure. Similar to measure P, measure Q gives probabilities to states of the word. However, measure Q gives higher probabilities to the bad states.

Equation 3b

$$\frac{dV}{V} = \mu dt + \sigma dz^Q.$$

It is clear from equation 1 and 3a that V and δ share the same dynamics under measure P (i.e. both follow a geometric Brownian motion with drift μ_P and volatilityσ). The same is true under measure Q.

Now, define the project payout ratio k as the ratio of the payout flow and the project value. In this case we have that:

Equation 4

$$k = \frac{\delta_t}{V} = r - \mu.$$

By rearranging equation 4, we can obtain the following:

Equation 5

$$\mu = r - k.$$

Substituting μ from equation 3b and rewriting the process, one obtains

Equation 6

$$\frac{dV}{V} = (r - k)dt + \sigma_t W_t^Q.$$

The value of any contingent claim on this asset can be found by solving the below partial differential equation (PDE):

Equation 7

$$(r - k)VF_v + \frac{\sigma^2}{2}V^2F_{VV} + F_t + P = rf,$$

where P in this case is defined as the payout flow specific to the contract we want to price. In the case of time independent securities, this PDE reduces to a second-order ordinary differential equation (ODE):

Equation 8

$$0 = (r - k)VF_v + \frac{\sigma^2}{2}V^2F_{VV} + F_t + P - rf,$$

We are interested in finding a pricing function (called F function in this case) that fulfils the condition given by equation 8 and the characteristics of the contract we want to price. The solution to equation 8 is generally found as the sum of the general solution to the homogenous equation, where the payout associated with the claim is equal to zero (i.e. P=0), and adding a particular solution. This particular solution depends on the contract we want to price. In our model, we will assume that the firm defaults whenever V falls to a certain level, hereafter denominated as V_B. This value is also called the default barrier and establishes an important condition to obtain the pricing of all securities we are interested.

The general solution to the above second-order ordinary differential equation is given by

Equation 9

$$F_{GS} = A_1 V^{-y} + A_2 V^{-x},$$

where

Equation 10

$$x = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]$$

and

Equation 11

$$y = \frac{1}{\sigma^2}\left[\left(\mu - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right].$$

In the case of a security that entitles its owner to all future payout flows, we call the F function as V_{Solv} and it can be shown that V is a particular solution. As a result, V_{Solv} can be shown to be given by.

Equation 12

$$V_{Solv} = V + A_1 V^{-y} + A_2 V^{-x}.$$

In equation 12, A_1 and A_2 are constants, which are determined by the boundary conditions specific to the contract that we want to price, which in this case is the value of all future payout flow until V_B is reached. The solution of x is positive, while the one for y is negative. This leads to an increase in the first term when V increases. For determining A_1 and A_2 notice that for $V \gg V_B$, the claim approaches the total firm value V. This implies that $A_1 = 0$. For the case of $V = V_B$, the value of the claim vanishes, as there is no value for the claimant left. This constraint determines that value of A_2. Thus,

Equation 13

$$V_{Solv} = V - V_B p_B(V),$$

where

Equation 14

$$p_B(V) = \left(\frac{V}{V_B}\right)^{-x}.$$

Following a similar rationale, the value of all future non-interest payments $V_{non-int}$ can be shown to be given by:

Equation 15

$$V_{non-int} = \frac{C}{r} - [1 - p_B(V)].$$

The equity value can be computed as the difference between these two claims:

Equation 16

$$E(V) = (1 - \tau_{eff})(V_{Solv} - V_{non-int}),$$

where τ_{eff} is the effective tax rate.

Until now it was assumed that V_B was some parameter in the model. In GJL (2001), however, the default level is chosen by the shareholder which acts strategically. It is the point at which defaulting is the preferred action over caring on the firm. Once the firm value passed this barrier, the remaining firm value is divided between debt, government and bankruptcy cost. Following GJL (2001), they state that there is an optimal default level V_B at which the equity value will be maximized while limited liability is given. They obtain the optimal default level V_B by invoking the smooth pasting condition[3]

Equation 17

$$\frac{\partial E}{\partial V}_{V=V_B} = 0.$$

Through solving

Equation 18

$$V_B^* = \lambda \frac{C^*}{r}$$

where

Equation 19

$$\lambda \equiv \left(\frac{x}{x+1}\right).$$

are used to solve this function.

For the final step of the dissertation it is necessary to define the expected return of the asset, μ_v. Following, this function follows

Equation 20

$$\mu_v = r + \theta * \sigma_v,$$

where

σ_v is the standard deviation of the asset, while r and θ are already defined above.

The functions stated above can be used as auxiliary functions to finally reach the goal of the prediction of the probabilities of default. The probabilities of default are computed based on the assets volatility and the expected growth rate of the project value.

[3] Can be understood as a boundary condition, that is used for American options. The options' value is maximized by following an exercise strategy, which makes option value and deltas continuous (Wilmott, 2006).

The formula used follows the form

Equation 21

$$P_{nd}(\sigma,\mu_v) = \phi\left(\frac{\left(\mu_v - k - \frac{\sigma^2}{2}\right)T - \ln\left(\frac{V_B}{V_1}\right)}{\sigma\sqrt{T}}\right) - e^{\left(\frac{2}{\sigma^2}\right)\left(\mu_v - k - \frac{\sigma^2}{2}\right)\ln\left(\frac{V_B}{V_1}\right)}\phi\left(\frac{\left(\mu_v - k - \frac{\sigma^2}{2}\right)T + \ln\left(\frac{V_B}{V_1}\right)}{\sigma\sqrt{T}}\right),$$

and needs to be subtracted by 1 in the end to find the real default probabilities.

3.2 Adapting GJL model to the banking sector

Goldstein, Ju, and Leland (2001) are defining their coupon payments to be constant interest payments, which is an unrealistic approach towards analysing banks debt. Nevertheless, the assumption of fixed costs is mandatory to analyse the companies and their contractual obligations to stay solvent. Due to this, the approach of Eisdorfer, Goyal, and Zhdanov (2019) is adapted to this dissertation. Following the constant interest payments are substituted by non-interest expenses. These allow monitoring the operational costs of the bank.

Consequently, the equation 15 is rewritten to implement the fixed cost claimant, V_{FC}, is following

Equation 22

$$V_{FC} = \frac{Non\text{-}interest}{r}\left[1 - p_B(V)\right],$$

where previously C is substituted by the non-interest expenses.

The implication of operational fixed costs into the GJL (2001)-model it is additionally necessary to rewrite equation 18, which is the default barrier,

Equation 23

$$V_B = \lambda \frac{Non\text{-}interest}{r}.$$

Through the implication of operational fixed costs, the nominator of the optimal default barrier is increasing, which is increasing the level of the barrier. Consequently, the operational costs increase the coupon payments and followingly the financial obligations that the company has to fulfil increase. This will have an increasing effect on the probabilities of default.

4. Data Inputs and Model Calibration

Table 1 – Banks selected

Company Name	Ticker
Citigroup Inc.	C
BB&T Corporation	BBT
Bank of America	BAC
PNC Financial Services Group. Inc.	PNC
Capital One Financial Corporation	COF
JPMorgan Chase & Co.	JPM
U.S. Bancorp	USB
Wells Fargo & Company	WFC

4.1 Calibration of the GJL (2001) -model

This section describes the implementation of the iterative approach, that was proposed by Vassalou and Xing (2004) for the Merton (1974) model, which can be used for the proposed GJL (2001)-model approach presented in Section 3.1. This iterative approach is used to compute the volatility of the asset and the payout ratio for each company, which are further going to be used for the default probability prediction. Further, these variables can be used to find the asset value of the company from historical equity prices.

4.1.1 Iterative Approach – Asset Value and Assert Return volatility

The iterative approach proposed by Vassalou and Xing (2004) follows five steps:

1. Defining a tolerance level of convergence for the volatility variable.
2. Defining an initial guess to start with. The historical equity return volatility was used to work as an initial guess for the equity return volatility in this case.
3. Create a time series of asset values by using the model value of equity and the observed equity.
4. Computing the standard deviation of the asset returns of these newly retrieved asset returns, which are further used.
5. Computing this process in a repeating manner with the newly computed volatility to converge towards the tolerance level. As the value of two consecutive iterations is below the tolerance level the iteration stops.

This part of the iterative approach provides us with the σ_v, which can be used further to predict the asset value of the desired company.

The iterative approach also got used to predict k. In the model k is defined as the project values payout ratio, which links the state variable (δ) to the asset value of the company.

Therefore, the iterative approach used in this dissertation to determine σ_v and k worked through the following steps:

1. Defining the same tolerance level of convergence as for the volatility variable.
2. Defining an initial guess of 20% for k.
3. Applying the iterative approach to the model equations in section *3.1 EBIT-based Model*.
4. Computing k through the previously defined ratio of the mean of EBT over the mean of the asset value vector.
5. Computing this process in a repeating manner until two consecutive estimates of k are falling under the previously set level of convergence.

This process provides the variables σ_v and k that are needed to further recover the asset value of companies in the data sample.

4.1.2 Computation of the market price of risk

The market risk premium used in this dissertation is a mandatory variable to compute the probabilities of default under the physical measure P.

The market risk premium got computed by rewriting the expected return formula under the CAPM model.

The CAPM model proposes that the expected return of on equity follows

Equation 24

$$\mu_e = rf + \beta_i(ER_m - rf),$$

where μ_e is the expected return on equity, rf the risk-free rate, β_i the beta coefficient, and $(ER_m - rf)$ the equity risk premium.

Following this, the expected return on equity can be rewritten as

Equation 25

$$\mu_e = rf + \beta * EQRP,$$

where EQRP is defined as the equity risk premium proposed by Damodaran.

On GJL (2001)-model the return on equity follows

Equation 26

$$\mu_e = rf + Market\ price\ of\ risk\ (\theta) * \sigma_e.$$

Followingly, the formula for the market price of risk (θ) emerges from setting the equations 25 and 26 equal and solving for the

Equation 27

$$Market\ price\ of\ risk\ (\theta) = \frac{\beta * EQRP}{\sigma_e}$$

for each bank.

4.2 Data Inputs

The application of this model to predict the probabilities of defaults requires the usage of a combination of accounting data and market data. Therefore, the data used was downloaded from multiple websites:

Table 2 – Inputs and their sources

Inputs	Source
Net Income before taxes	Compustat - Capital IQ
Market Value	Thomson Reuters
Non-interest expense	Compustat - Capital IQ
Beta coefficient	Thomson Reuters
Equity risk premium (EQRP)	Professionals Website
Interest rate (U.S. Treasury – 30-year bond)	Thomson Reuters

4.2.1 Immediate Variables

This section is providing information about the variables that got immediately obtained through accounting or market data. The accounting variables used in this dissertation are, notably, the net income before taxes, the non-interest expense. The market variables are the beta coefficient, the market value, the equity risk premium, the interest rate and the tax rates.

These variables are used in the iterative approach stated in Chapter 4.1.1 to find the estimated volatility, the asset value, the payout ratio k, and to define the fixed costs.

State variable (δ)

The state variable, δ, in this dissertation is defined as the sum of the earnings before taxes and the banks' operational fixed costs. The earnings before taxes used in this dissertation are proxied

by the net income before taxes. As further mentioned, the banks' operational fixed costs are proxied by using the non-interest expenses of the banks. The GJL (2001)-model implies an assumption that the dynamics of the state variable follow a geometric Brownian motion. This assumption implies that their log changes follow a normal distribution. To test this, we are applying a Shapiro-Wilk test in the program R. The significance level we are testing for is chosen for an alpha level of 0.05 and therefore a p-value of less than 0.05 signals a rejection of the null hypothesis, which is that the data is normally distributed. For a p-value higher than 0.05 we don't reject the null hypothesis, which does not mean that the data is not normally distributed (we just can't reject that hypothesis).

Table 3 – Shapiro-Wilk Test for Normality

Shapiro Wilk Test	p-value
Citigroup	0.0292
BB&T Corporation	0.0075
Bank of America	0.0614
PNC Financial Services Group. Inc.	0.1466
Capital One Financial Corporation	0.0000
JPMorgan Chase & Co.	0.3367
U.S. Bancorp	0.2069
Wells Fargo & Company	0.0000

The null hypothesis can be accepted for the Bank of America, Capital One, JPMorgan Chase, U.S. Bancorp that the dynamics of their state variables follow a normal distribution. Followingly, we reject the test for normal distribution in the cases of Citigroup, BB&T, PNC and Wells Fargo & Co and conclude therefore that their state variables are not normally distributed.

Also, the skewness of the companies' state variables was analysed. A negative skewness normally refers to a longer or "fatter" tail on the left side of the distribution, while a positive skewness refers to a longer or fatter tail on the right side. Financial institutions prefer a negative skewness, due to the small amount of very high outliers and the majority of smaller values. Therefore, the skewness retrieves information about the extreme values in comparison to one and the other tail. The skewness of the companies' state variable is presented in the following table:

Table 4 – Skewness of companies' state variable

C	BBT	BAC	COF	PNC	JPM	USB	WFC
-0.87	-0.01	-0.40	0.19	-0.02	-0.37	-0.50	-0.11

It is to see that the state variables of Citigroup, U.S. Bancorp and JPMorgan Chase have the largest negative skewness and therefore they are most likely to suffer from the largest decreases in values. Nevertheless, negative skewness is preferred due to the likelihood of frequent gains and only a few large losses.

Further, the kurtosis of the state variables was analysed as well to retrieve further information about the distribution of the latter. The information that can be observed are the extreme values in either tail. Means that a low kurtosis displays fewer extreme values, while a high kurtosis displays higher extreme values. The results are presented in the following table:

Table 5 – Kurtosis of the companies' state variable

C	BBT	BAC	COF	PNC	JPM	USB	WFC
5.20	1.94	2.05	1.60	1.21	1.50	2.18	1.22

Following these results, it is to say, that Citigroup, Bank of America, and BB&T are going to suffer the most extreme values in their state variables.

Equity value

In this dissertation, we assume to know the equity value from retrieving the market value of the companies in the sample. The data got retrieved from the Thomson Reuters Data stream platform.

Figure 1 – Quarterly Equity Values (in millions $)

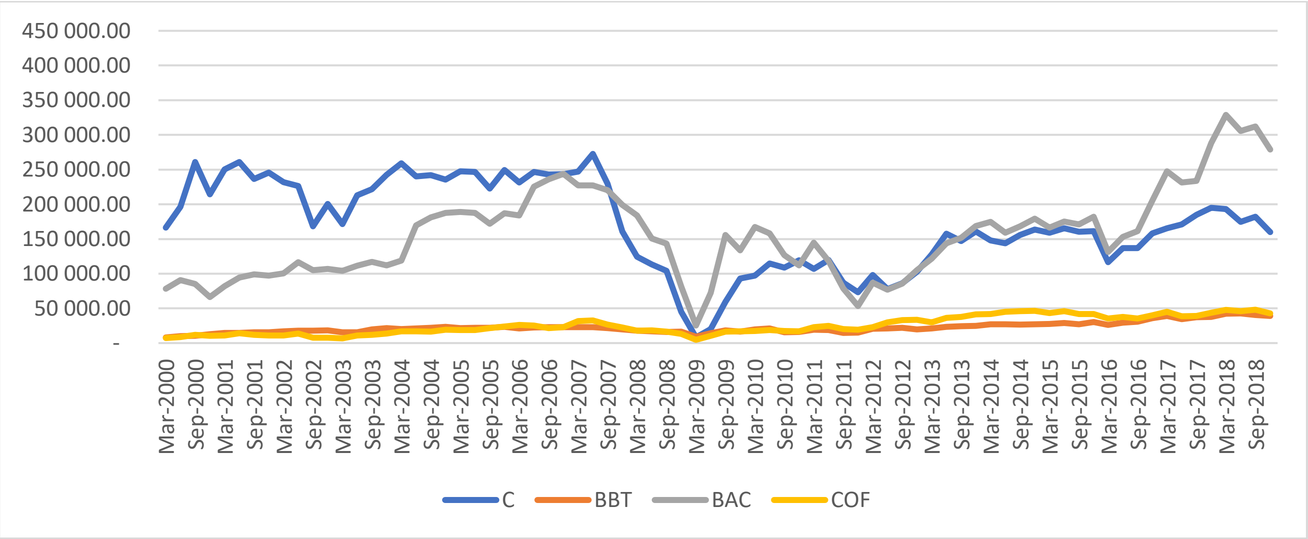
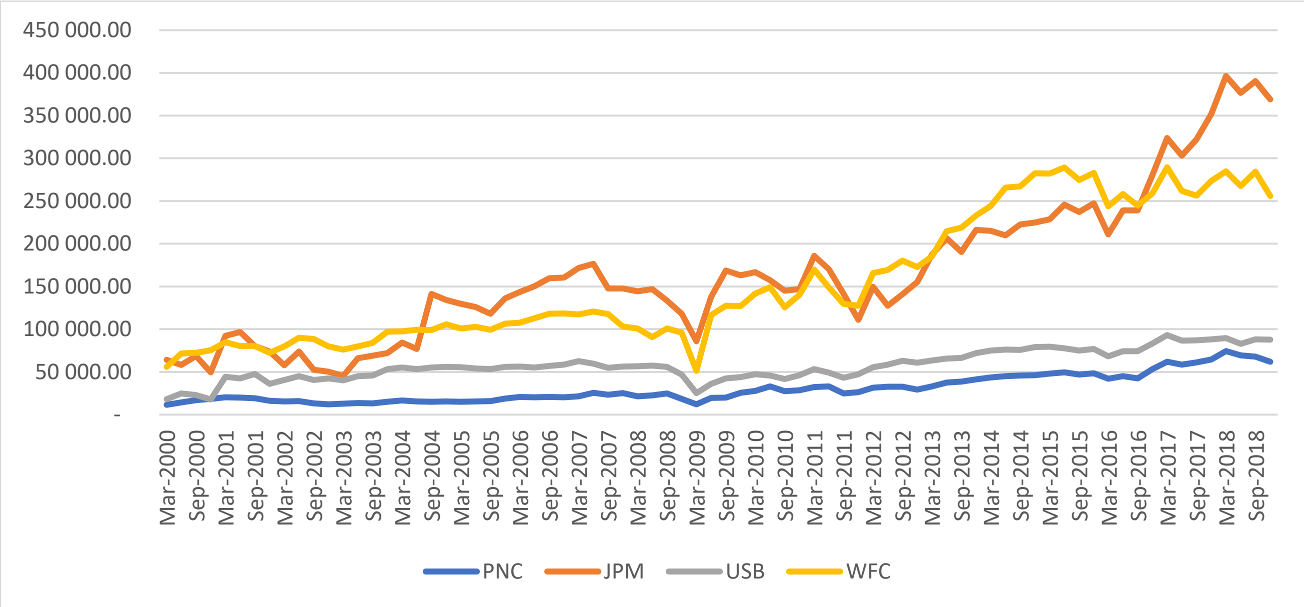


Figure 2 – Quarterly Equity Values (in millions $)

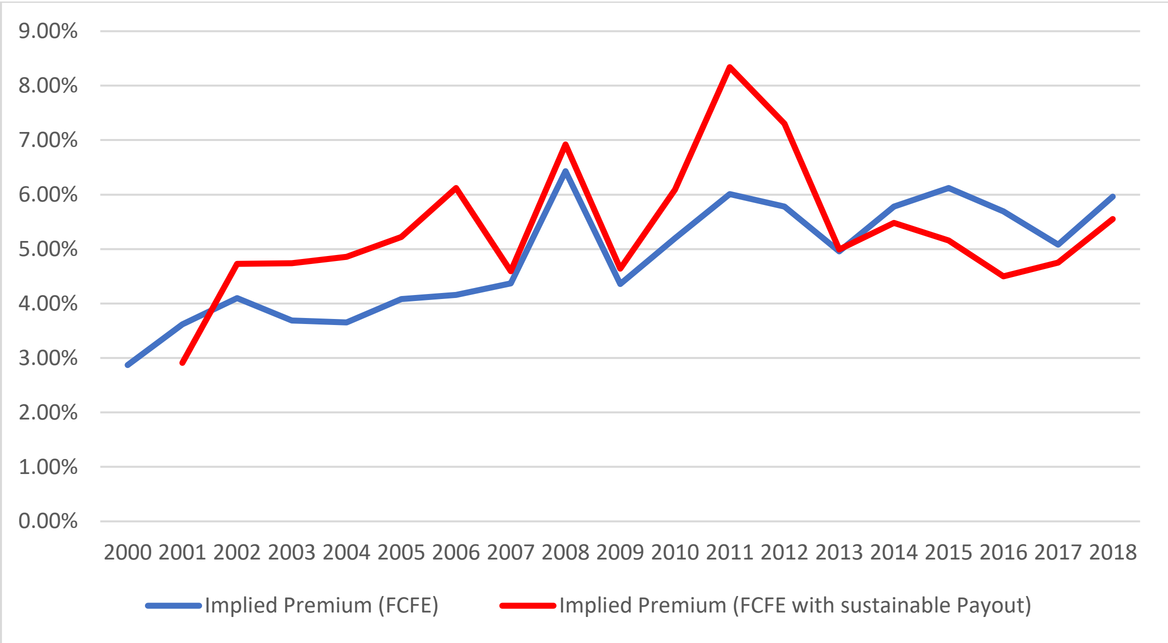


The market value is defined as the share price multiplied by the number of ordinary shares issued. Therefore, it is to see that all financial institutions perceived major declines within their equity values throughout the financial crisis of 2007 – 2008. The largest losses in their market capitalization were observed for Citigroup, Bank of America, and JPMorgan Chase. Even though, these declines are a signal that shareholders "flee" to quality investments and can indicate future solvency risk (Gray and Jobst, 2010).

<u>Equity Risk Premium (EQRP)</u>

The equity risk premium mentioned previously got retrieved from the website of Professor Damodaran. He started collecting, cleaning and publishing individual company data on his website since the late 1990s to make it available for public use. One of the estimates that is going to be applied in this dissertation is the Free Cash Flow to Equity (FCFE) method. This valuation method is following two slightly different approaches. Their difference can be found within the sustainability of the payout, which can affect the growth rate and their overall levels over time. Due to a lack of data for the year 2000 data is retrieved for this case in a different way. For these missing years, the equity risk premium is assumed to be the implied premium derived from FCFE, for the rest the implied premium from FCFE with sustainable payout is taken into consideration. Applying this data resolved that the implied premium from FCFE contains lower rates for the missing years, which decrease the prediction of the default probabilities.

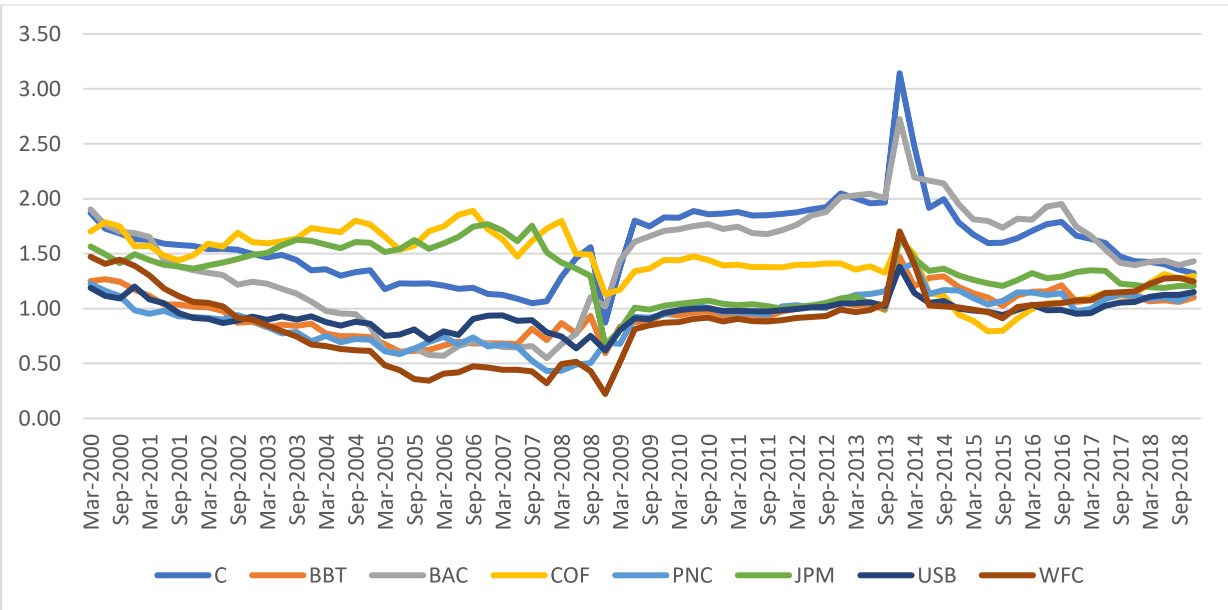
Figure 3 – Historical Free Cashflow to Equity comparison



<u>Historical Betas (β)</u>

To compute the market price of risk, it is necessary to retrieve the historical betas of the companies of the sample. These beta coefficients are a measure of systemic risk. Therefore, they enable us to look at time periods of increased volatility and exposure to the market. The quarterly beta coefficients of the companies are presented in the following figure:

Figure 4 – Historical Beta Coefficients β



Substantially it is to say that the historical beta coefficients vary in their value, but they are showing a similar behaviour over the time periods. It can be seen that the beta coefficients tend

to decrease while the crisis took place. After the crisis, they were increasing again to a pre-crisis level.
Analysing this behaviour, the betas that were taken into consideration here are computed on the basis of a long duration of time, which enables them to return more robust solutions. Further, the fact of the long duration could cause a variation in the results, that is not wanted and therefore we are assuming a constant, average beta for every bank. The constant value gets justified under the assumption that the banks did not change their business model drastically through the time period analysed.

Table 6 – Mean beta coefficient β

C	BBT	BAC	PNC	COF	JPM	USB	WFC
1.60	1.43	1.33	0.89	0.96	0.92	1.43	0.96

Tax Rates

As in the paper published by Goldstein, Ju, and Leland (2001), we assume a constant tax rate equal to 20% over the whole time. Hence, we assume a $Taxrate_{Dividend}$ of 20% and $Taxrate_{Corporate}$ of 20%. This assumption was considered throughout the whole paper and led to the computation of an effective tax rate of

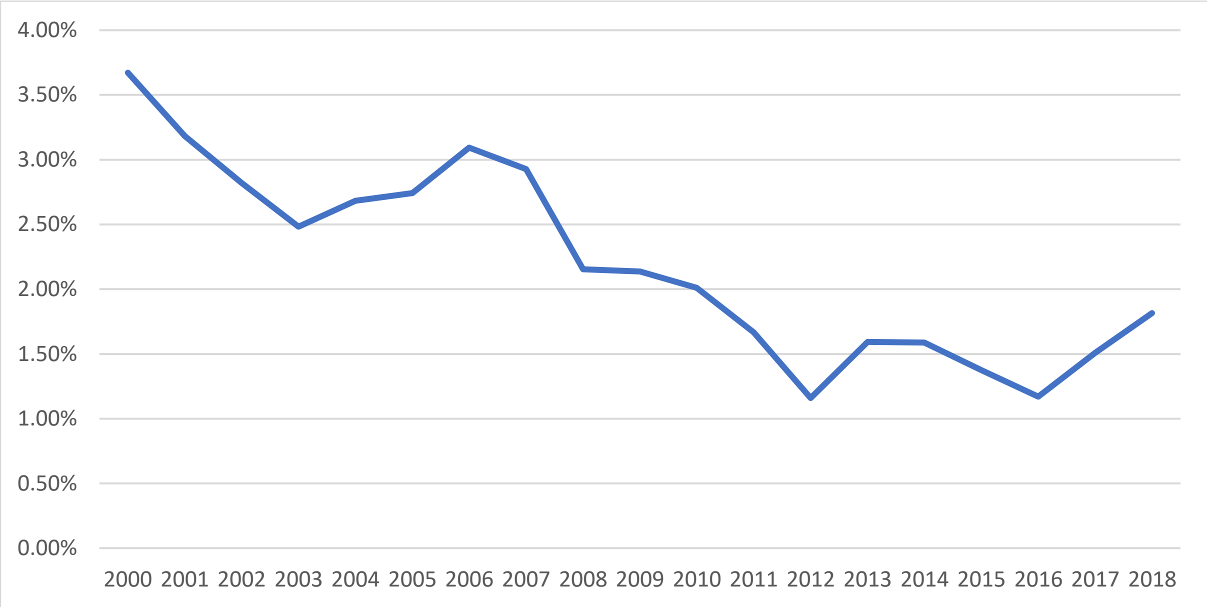
$$(1 - Taxrate_{Effective}) = (1 - Taxrate_{Dividend}) * (1 - Taxrate_{Corporate}),$$

which leads up to an effective tax rate of 36% for interest payments to investors.

Interest rate (RF)

The interest rate used in this dissertation is the 30-year U.S. treasury bond bid rate. This rate is decreasing over time. As we want to measure the effect of taxes on the whole company process it is necessary to reduce the interest rate by the effective tax rate, which enables us to retrieve following interest rate over the time span.

Figure 5 – Interest rate



<u>Historical equity volatility σ_e</u>

The volatility of the equity, σ_e, got empirically computed with the program R. Therefore, the standard deviation of the difference of the log changes of the equity was taken into consideration. Further, the sample size got cleaned from outliers by setting upper and lower limits of three standard deviations as boundary conditions. To annualize the results, due to the usage of weekly data, got annualized by multiplying them by the square root of 52.

Table 7 – Annualized equity volatility σ_e

C	BBT	BAC	PNC	COF	JPM	USB	WFC
35.27%	25.92%	34.58%	26.78%	38.35%	31.23%	26.20%	25.83%

This table provides information about the financial institutions with the highest and lowest volatile equities. COF and C show high volatility of more than 35%, while WFC and BBT possess the lowest volatility of around 26%. Regarding the regular definition of volatility, the companies with the highest volatilities are meant to be the ones with the highest risk related to the size of changes in their assets.

<u>Expected rate of return for equity μ_e</u>

Further, the CAPM parameters were used to compute the expected rate of return of the equity of the financial institutions. These were calculated as mentioned in section 4.2.2. Computation

of market price risk. Further, the expected rate of return for equity is used as an auxiliary function to compute the market price of risk.

Table 8 – Average expected rate of return on equity μ_e

C	BBT	BAC	COF	PNC	JPM	USB	WFC
11.98%	8.50%	11.03%	11.05%	8.32%	10.43%	8.52%	8.04%

The CAPM parameters of interest rate and the equity risk premium proposed by Damodaran are the same for each bank, due to the same geographic location and at the same time. Therefore, the only difference in this computation is the beta coefficient. Due to these similarities in the inputs the pure effect of risk that an investment is adding to the portfolio can be observed in the different average expected returns.

<u>Market price of risk θ</u>

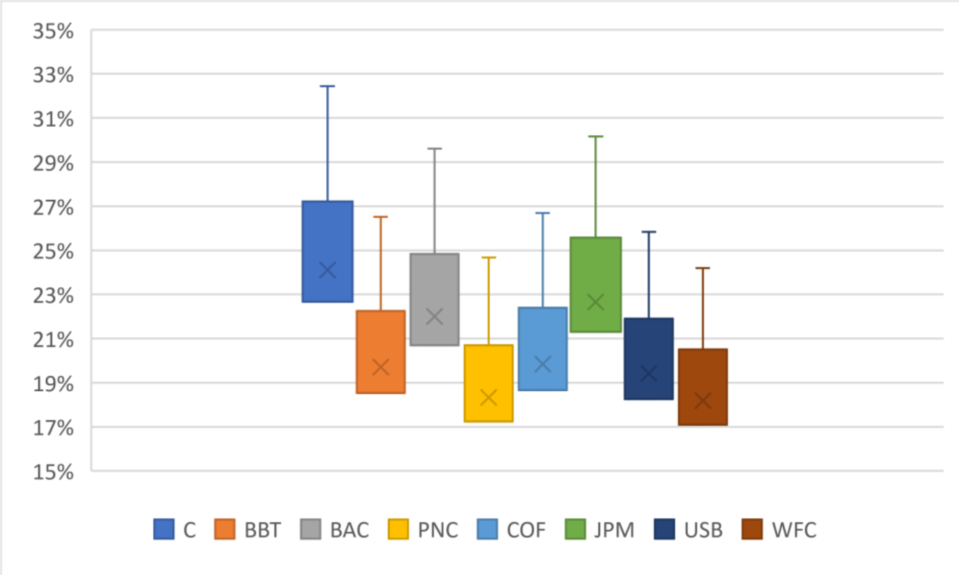
Further, we analysed the market price of risk by using equation 27. First of all, we computed the mean market price of risk.

Table 9 – Mean market price of risk θ

C	BBT	BAC	PNC	COF	JPM	USB	WFC
24.14%	19.47%	21.87%	18.14%	19.79%	22.32%	19.29%	17.71%

The result of the market price of risk retrieved from the companies' data was taken into consideration by computing the mean of it. That was done because the dataset only contains companies from the same sector, which includes similarity in the behaviour of the market price of risk in times of crisis. Therefore, it can also be seen as a risk-to-reward ratio for a market portfolio. The companies with the highest mean market price of risk are Citigroup, Bank of America, JPMorgan Chase, and BB&T.
Further, the market price of risk got analysed by displaying the results in a boxplot graph form. This provided further evidence about outliers, minimal and maximal values, and the mean of the market price of risk. These results can be observed in the Figures 6.

Figure 6 – Market price of risk boxplot



As shown in Table 9, the means of the market price of risk differ a lot over the sample. Overall, we can conclude that the banks market price of risk tends to be affected a lot by the time series of beta. The difference in the shapes of the boxplots can be explained by differences in the historical equity volatility as well. The significant differences between the equity volatilities cause the variation in the shape of the boxplots. Further, we are analysing the time series results of the market risk premium and observe it for in- and decreases.

Figure 7 – Panel A (Market price of risk)

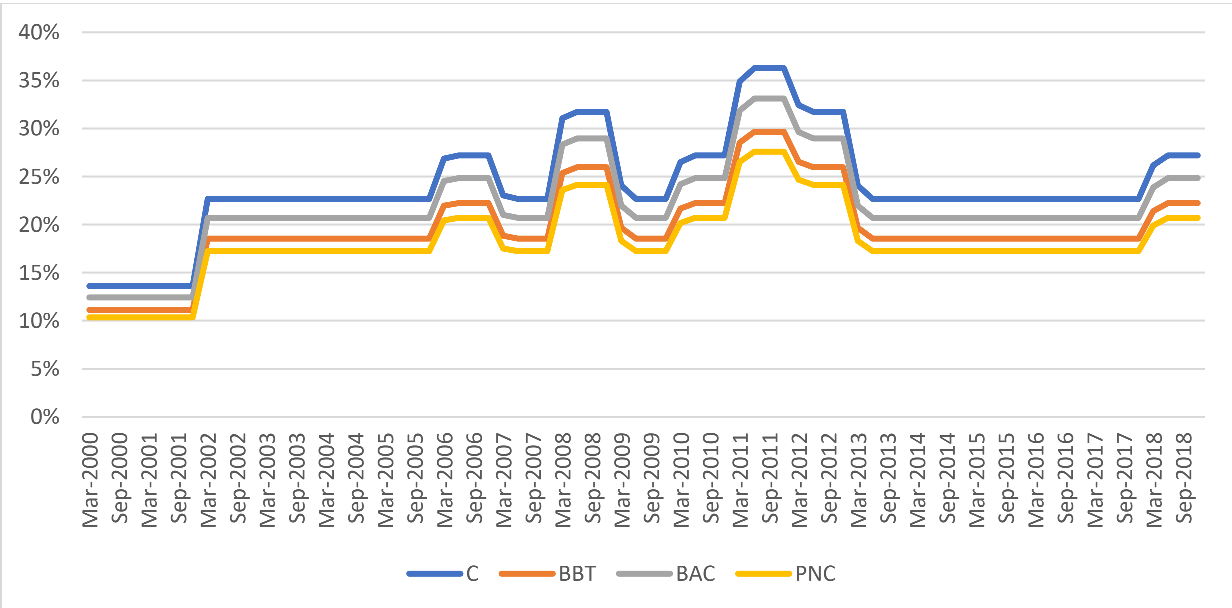


Figure 8 – Panel B (Market price of risk)

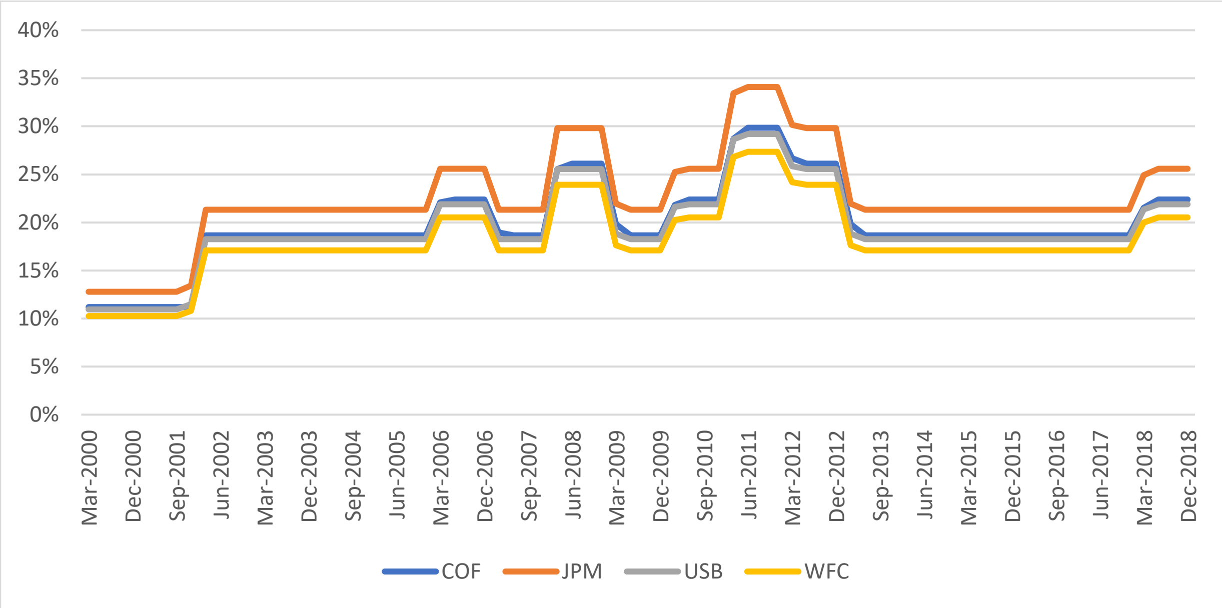


Figure 7 and 8 provide information about the dynamics of the market price of risk for the corresponding banks. It can be observed that the market price of risk for all banks are sinking in times of crisis and after these occur, they tend to increase again. These changes are mainly caused by the time series changes of the equity risk premium as they move similarly. As stated above, the previous variation through the time series of the betas was cleaned by using an average beta.

4.2.2 Iteratively obtained variables

This section is determined to the variables that got obtained from the iterative approach mentioned in chapter 4.1.1. These variables are the annualized volatility of the project σ_v and the payout ratio *k*.

Annualized volatility of the project σ_v

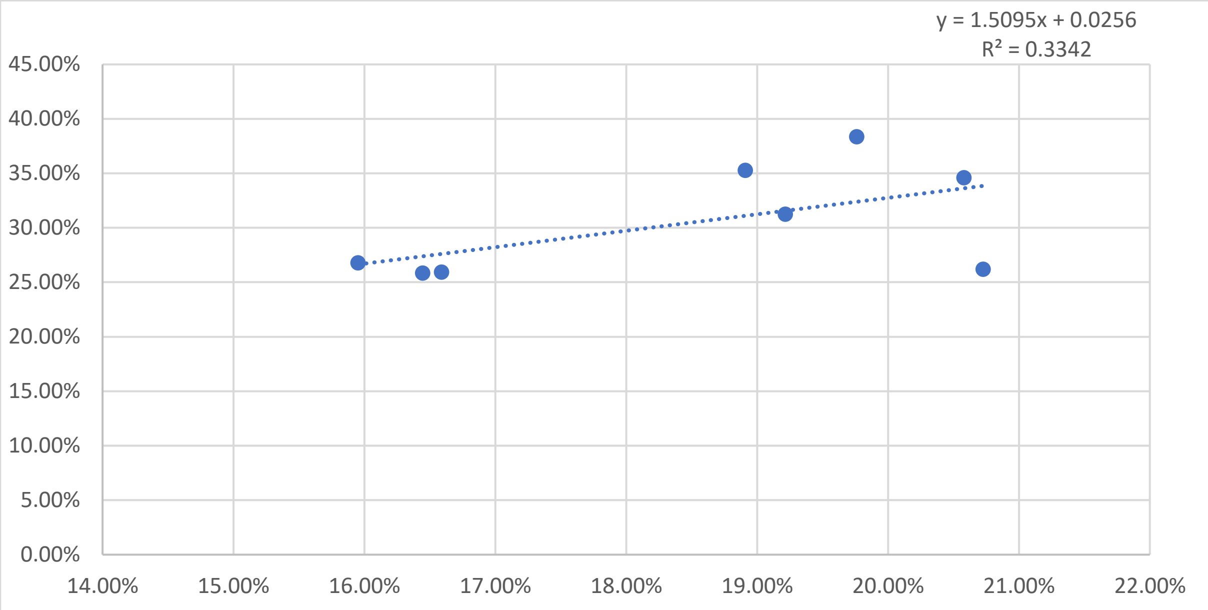
Table 10 – Projects volatility σ_v

C	BBT	BAC	PNC	COF	JPM	USB	WFC
20.50%	17.97%	22.20%	17.49%	21.43%	20.91%	22.61%	18.04%

In comparison to the historical equity volatility, σ_e, it is to say that the projects volatility, σ_v, is significantly lower in all cases, which can be explained through the naturally higher volatility of equity, when its defined as the market capitalization. As the market capitalization suffered

from significant decreases the higher rates can be explained through this. Another reason could be that the project volatility got modelled by implying the operational leverage.

Further there was a positive correlation between the volatility of the equity and the asset volatility observed.



<u>Payout ratio *k*</u>

Table 11 – Payout ratio for each company

C	BBT	BAC	PNC	COF	JPM	USB	WFC
3.00%	2.68%	3.15%	2.74%	3.32%	3.12%	2.44%	2.79%

The payout ratios *k* found within the financial sector of the United States contains similar values over the whole data sample. Even though there have been banks that have been hit by the financial crisis their payout ratios overall are similar to the ones that were hit less. In GJL (2001)'s paper, they state that there is a positive correlation between the level of C and the value for the payout ratio *k*, which tends to be not the case. Typically, a lower level of *k* can be found in companies that are reinvesting their earnings into growing business, research, or further developments. Overall, a positive and stable payout ratio is important for companies to indicate shareholders that the company is knowing about their risk and reducing it. Further, the reduction of default risk, through good risk management can lead to higher future payout ratios, which are then priced by the market by stock acquisitions, which was found by Charitou and Lambertides (2011).

5. Results

This chapter has two sections. The first section discusses the values obtained for the distance-to-default and the probability of default and explores the roots behind these results. The second section compares the obtained results with the ones implied by the credit ratings given by two of the most important rating agencies: Moody's and S&P.

5.1 Credit Risk Indicators

This section presents the credit risk metrics that were computed in this dissertation. These metrics were the default barrier-to-asset ratio, the drift of the process, the distance-to-default (DD) and the probabilities of default.

5.1.1 Default barrier-to-asset ratio

The barrier-to-asset ratio tells how far the bank is from default at a given moment in time. As such, it can be interpreted as a measure of leverage. The barrier-to-asset ratio is always lower than 1. The closer this ratio is to 1 the closer the company is to default. Differently from the distance-to-default, the barrier-to-asset ratio does not adjust for risk. Figure 9 and 10 show the barrier-to-asset ratio for the eight banks considered in this dissertation. The ratios range from 14.55% to 71.38%. For most of the banks the maximum value was obtained between Q1 and Q3 of 2009, while three banks obtained values above 50%. These three were Citigroup (71.38%), the Bank of America (51.37%), and Capital One (51.01%). The yearly results are presented in Appendix 5.

Figure 9 – Default barrier-to-asset ratio

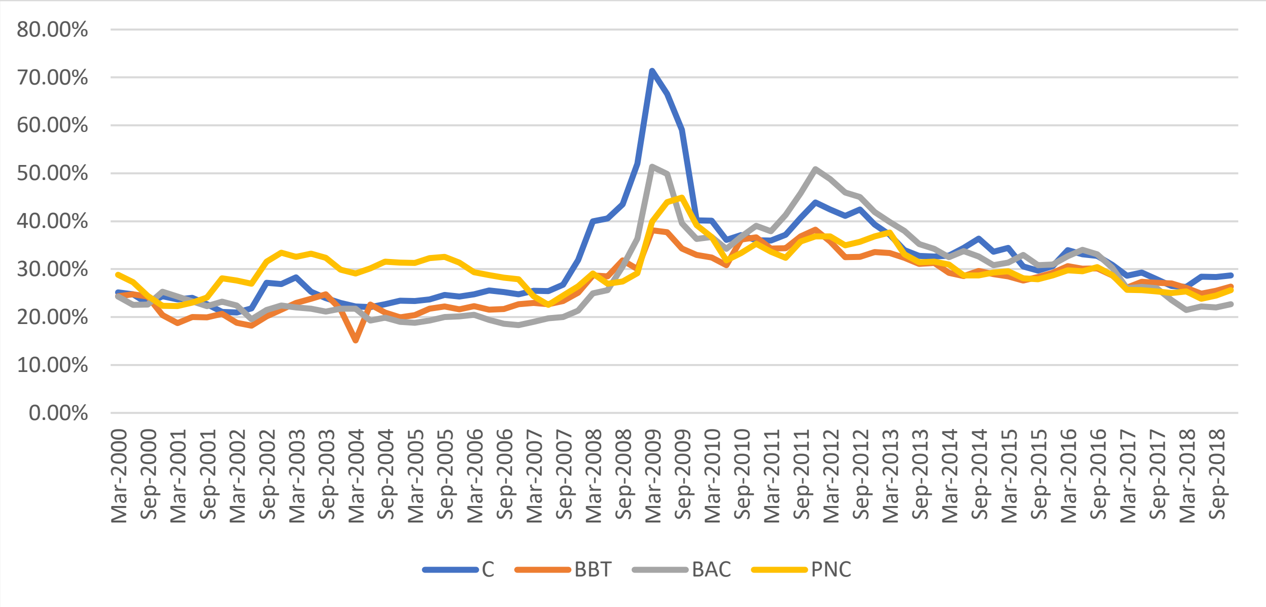
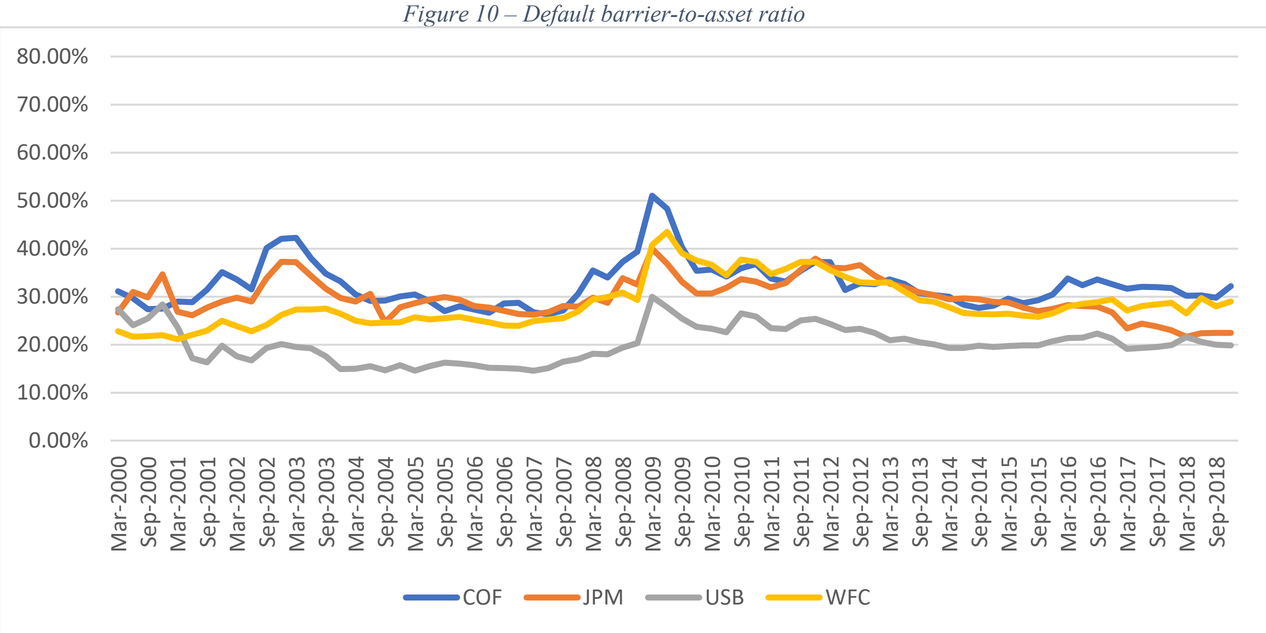


Figure 10 – Default barrier-to-asset ratio



5.1.2 Drift of the process μ_p

The drift of the process tells us the expected growth rate of the project value. It is defined as the difference between the expected return from the project μ_v and the payout ratio k. Figure 11 and 12 show the drift of the process. The drift ranges from 1.82% to 7.83%, while the average drift is 4.72%. The most banks obtained their maximum value in Q2 2011. Meanwhile the minimum value for the most banks were obtained in Q3 2016. As the market price of risk, the payout ratio and the project volatility are constant variables in this computation, its dynamics are linked to the dynamics of the interest rate. It is to say that the average drift is positive, which

implies that the banks are expected to deleverage over time. The yearly drifts of the process are presented in Appendix 7.

Figure 11 – Drift of the process μ_p

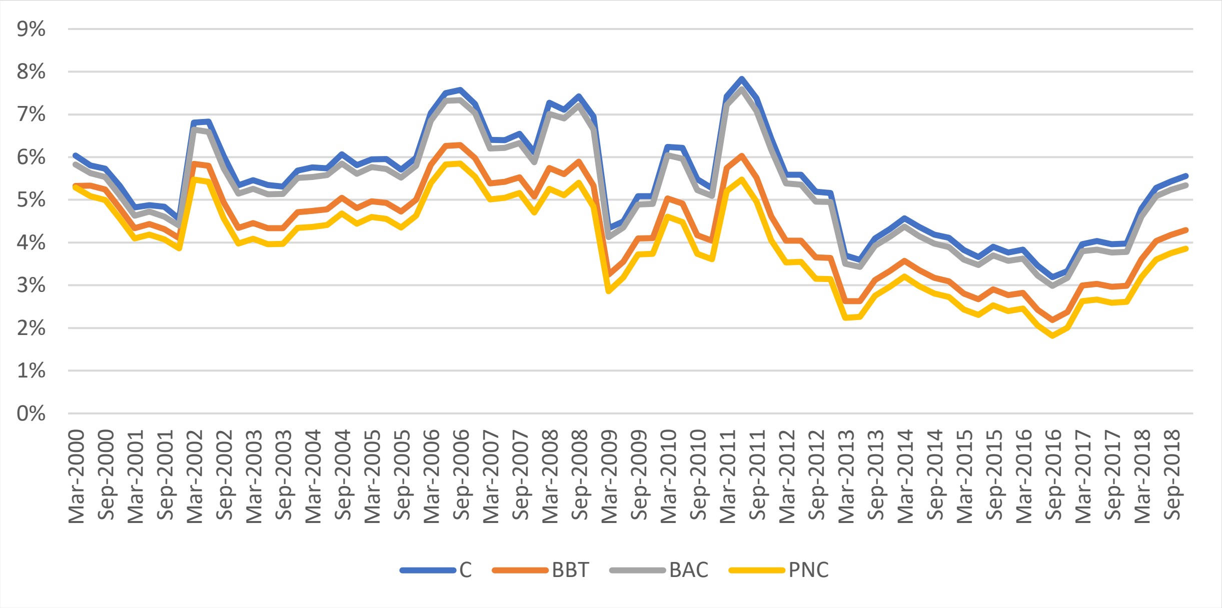
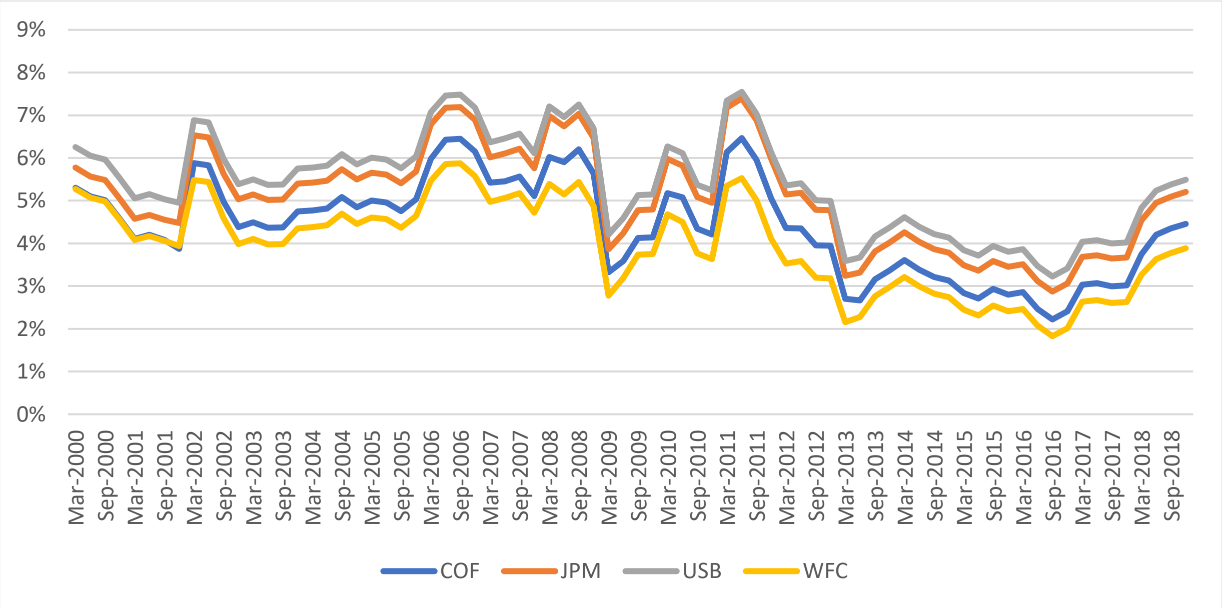


Figure 12 – Drift of the process μ_p



5.1.3 Distance-to-Default (5-year)

Figures 13 and 14 present the distance-to-default for the 8 banks considered. On average the distance-to-default is 3.16. The bank with the lowest average value is Citigroup (0.98) and the one with the highest average value is BB&T (2.62). The yearly distance-to-default values for all companies are presented in Appendix 6.

Figure 13 – Quarterly DD of companies

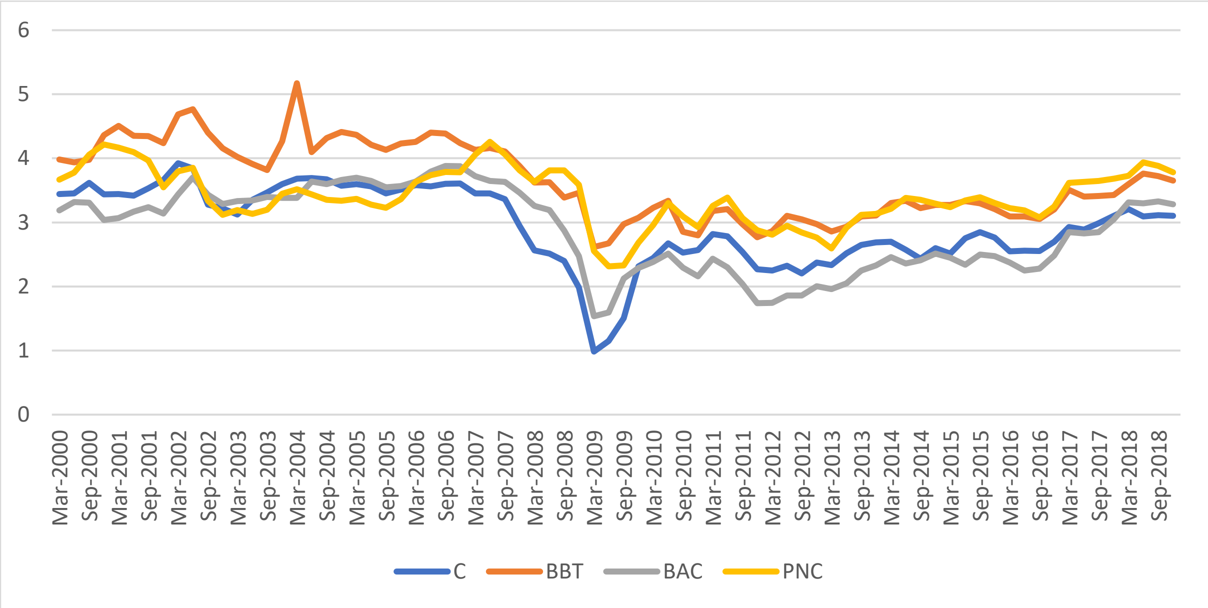
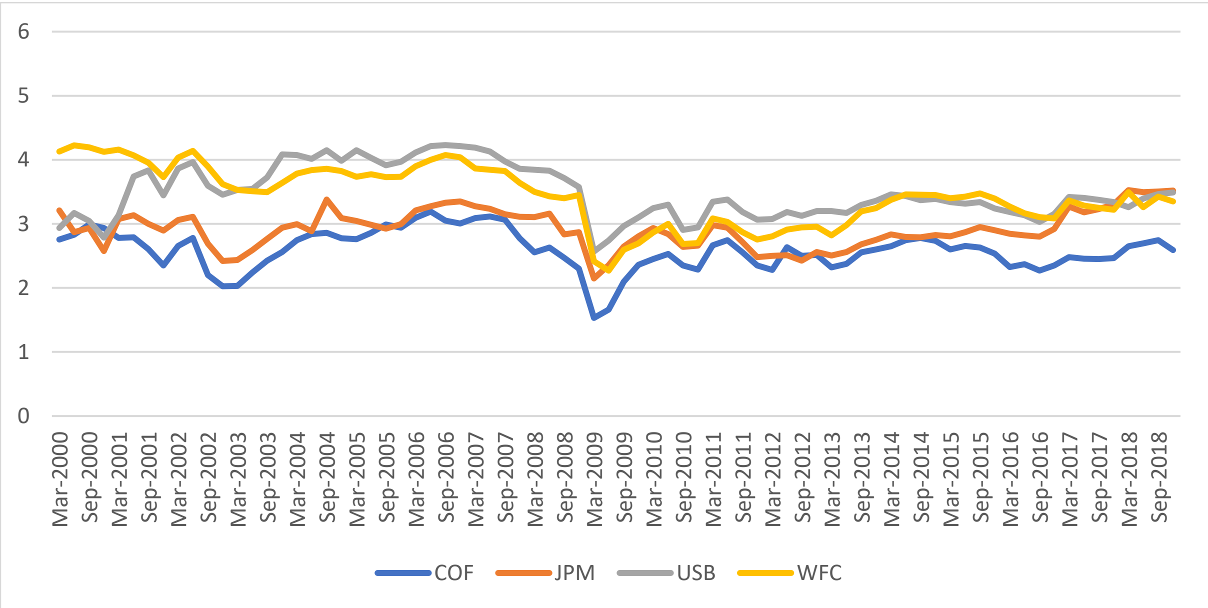


Figure 14 – Quarterly DD of companies



The smallest distance-to-default was found for Citigroup, in March 2009 with a value of 0.98. Further, the Bank of America was analysed to have the second-lowest value with 1.53 on the same date. The third lowest value of 1.54 for the data sample got retrieved for Capital One Financial Corp. on the 13th of March 2009. The distance-to-default for all banks are highly correlated with an average correlation value of 85.69%. This high level of correlation results from the high correlation observed in stock prices. Further, this could result from "similar" asset volatilities retrieved from the iterative approach. Through their similar values in a range from 17.49% to 22.61%, the difference in the distance-to-default's most likely occurs through different default points, the asset values, and the drifts. Overall, the distance-to-default captures

information about the distance-of-the assets from the barrier, which monitors leverage. Further, information regarding the risk through the volatility of the assets, and the expected change in leverage through the drift.

5.1.4 Probabilities of default (5-years)

The probabilities of default that got retrieved from applying the model on the banking sector are the core element of this dissertation. Further, the probabilities were computed as 5-year probabilities of default. All calculations were done on weekly frequency. This was important to better estimating the model parameters. For simplicity, we decided however to present quarterly graphs, while mentioning peaks in a weekly frame. In addition, yearly results are presented in Appendix 4. Figure 15 and Figure 16 present the probabilities of default in a quarterly timeframe.

Figure 15 – Quarterly probabilities of default

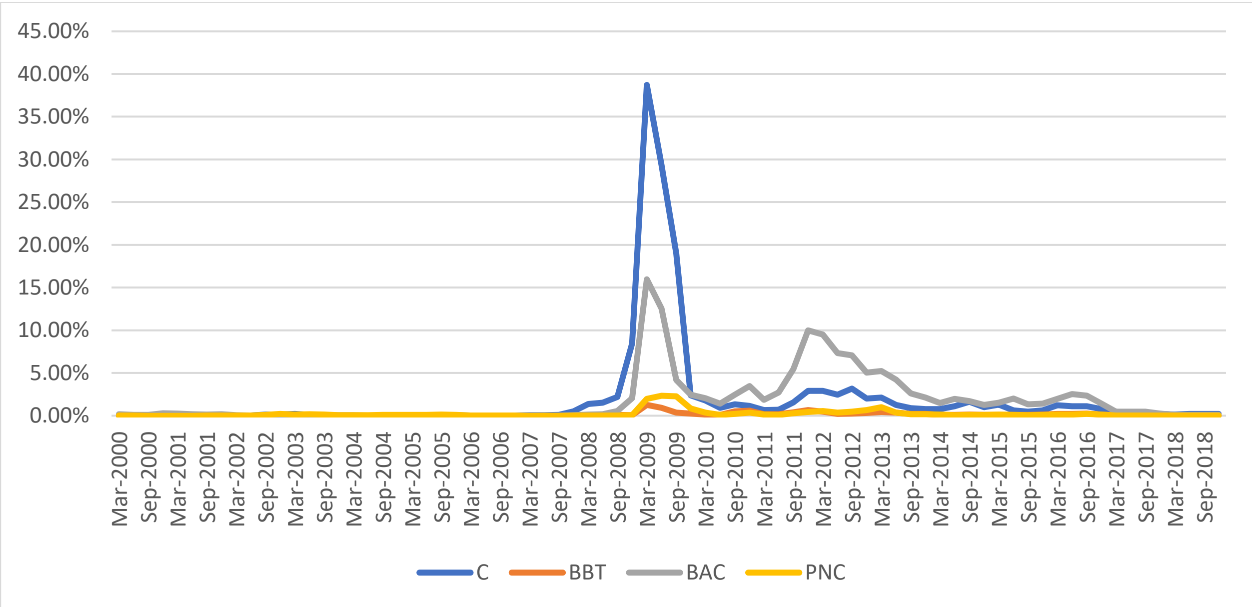


Figure 16 – Quarterly probabilities of default

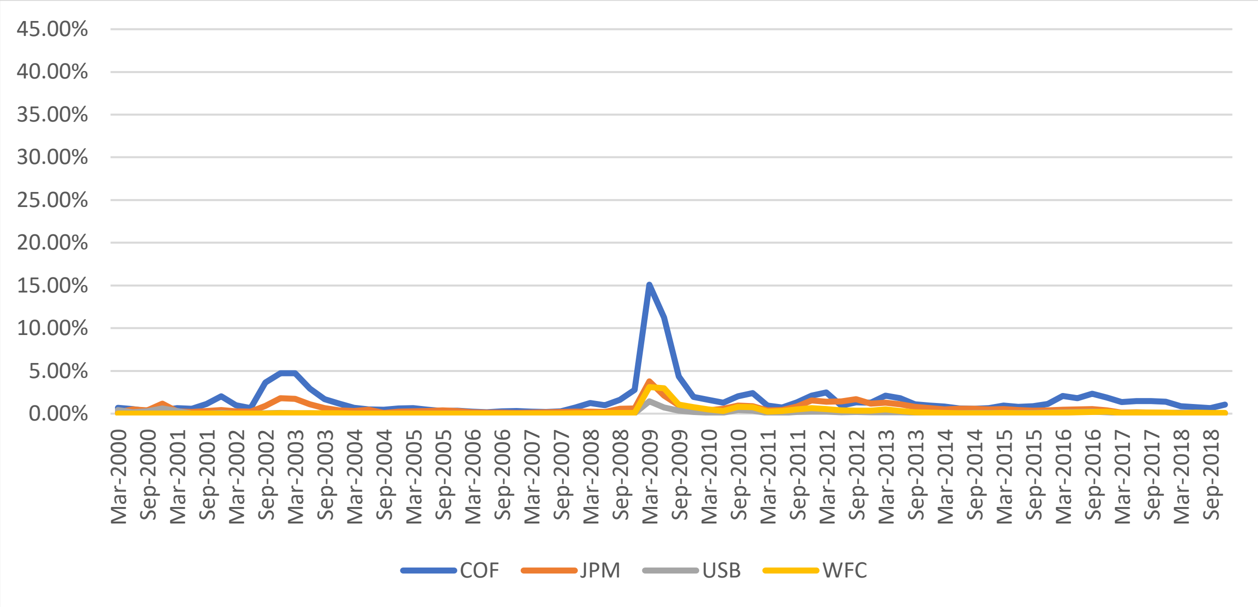


Figure 15 and 16 enable us to observe the general movements of the probabilities of default, even though extreme outliers are not observable. These extreme outliers are still observed throughout the calculations and consequently mentioned in the text. Before analysing the different impacts of the crisis, it is to say that most of the time the probabilities of default for the banks were close to zero. The average probability of default ranges from 0.06% in 2006 to 5.80% in 2009.

At the beginning of Figure 15 and 16, around Q3 2000 and Q1 2001, there was an increase of the probabilities of default for the Capital One Finance and JPMorgan Chase. In the beginning, JPMorgan's default probability increased to around 1.2%. Capital One Finance's rate increased in Q3 2001 to around 2%. This increase can be explained through the burst of the so-called "dotcom" bubble. This bubble was created by speculative investors that overvalued tech companies, which was leading to strongly increasing equity markets in the previous years. As the bubble was at its peak, big tech companies placed sell orders on their stocks, which caused the bubble to explode. Therefore, the effect on especially these three banks could be caused through their financial involvement in holding their stocks or credits towards these companies. Further, it is to observe, that the default probability of Capital One Finance increased over the time frame of 2002 to 2003 and peaked at a value of 4.74%. This could have been caused by Capital One's acquisition of PeopleFirst Finance LLC in late 2001. Additionally, economists claimed that the United States suffered under a recession throughout the years of 2000 to 2002, which could as well explain the increases. JPMorgan Chase suffered from an increasing rate

that peaked at 1.74% again. The reasons for this increase could be dedicated to the recession or some late effects of J.P. Morgan & Co's merger with Chase Manhattan Bank.

Of special interest are the developments during the financial crisis of 2007 – 2008. First of all, it is to say that the default probabilities of all retail banks increased significantly in comparison to the previous months. The whole sector peaked with a value of 5.80% in 2009.

Although the whole sector suffered from significant increases some companies suffered especially from the crisis. The largest increase in their default probabilities was measured for Citigroup with a peak of 38.72% in Q1 of 2009. At the same time, the default probability of the Bank of America increased to 15.97% and Capital One Finance's peaked at 15.10%. These companies suffered the most from the financial crisis and therefore their probabilities of default were on the highest levels. In the case of Citigroup, JPMorgan Chase, and the Bank of America we can try to explain that on a macro-economic level that these companies were actively participating in the secondary and securitized mortgage market that was leading towards the financial crisis. While these securitized mortgages were traded the balance sheets of the banks gained strength and they were enabled to trade even more of these, which could explain the difference in the capital losses. As this bubble collapsed it materialized into losses that destroyed the capital of the banks and the global economy.

Additionally, it is to observe that after the financial crisis the probabilities of default nearly decreased to the pre-crisis level, but then increased again. The probabilities of default for the Bank of America raised to 10.01%, Capital Finance One and JPM increased to around 2.11%, while Citigroup increased to 2.89%. This increase was caused by a combination of the downgrading of the American credit rating from AAA to AA+ by Standard & Poor's on the 6th of August 2011, the European sovereign debt crisis in Spain and Italy, and international discussions about France's credit rating. These left the stock markets very volatile and caused the S&P 500 to decline by 6.2% on the 8th of August 2011, as an immediate reaction of the market[4].

Finally, there was an increase in the default probability to around 2.3% at the end of 2016 and the beginning of 2017 observable for Capital One Finance. This increase can be explained through a slumping oil price, the Brexit vote and the acquisition of Paribus by the company in October 2016[5].

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[4] Information retrieved from: https://web.archive.org/web/20120323002407/http://heraldnews.suntimes.com/business/6946196-420/dow-dropped-634-monday-in-worst-one-day-drop-since-december-2008.html

[5] Information retrieved from: https://www.americanbanker.com/news/capital-one-adds-to-its-growing-list-of-fintech-deals

5.2 Comparison to Rating Agencies

5.2.1 Implied probability of default from rating agencies

Finally, it is important to analyse the reasonability of the results. Accordingly, the credit ratings given by Standard & Poor's and Moody's were obtained and converted into probabilities of default using an historical transition matrix. The yearly results for the rating agencies are shown in Appendix 8 and 9. The lack of accessible data only enabled us to download the historical credit ratings for the time period of 2006 to 2018. These ratings were downloaded from the Thomson Reuters Eikon database and the corresponding annual reports. Figure 17 presents a time series for the probability of default using the proposed model and the referred historical transition matrix in conjunction with credit ratings. The probabilities of default used were average annual probabilities of default. The credit ratings were chosen according to the latest credit rating that was published in one year. On average the model leads to a probability of default only slightly above the one implied by credit ratings. The average default probability of 1.31% returned from the model for the corresponding time period, the implied default probability was on average 1.07%, in the case of Standard & Poor's, and 0.82%, in the case of Moody's. Looking by year, my results suggest that the model is overestimating the probabilities of default in times of crisis (2008-2009 and 2011-2013) and underestimating them in periods of economic growth (2006-2007 and 2014-2018).

Figure 17 – Credit Rating implied probabilities of default and model comparison

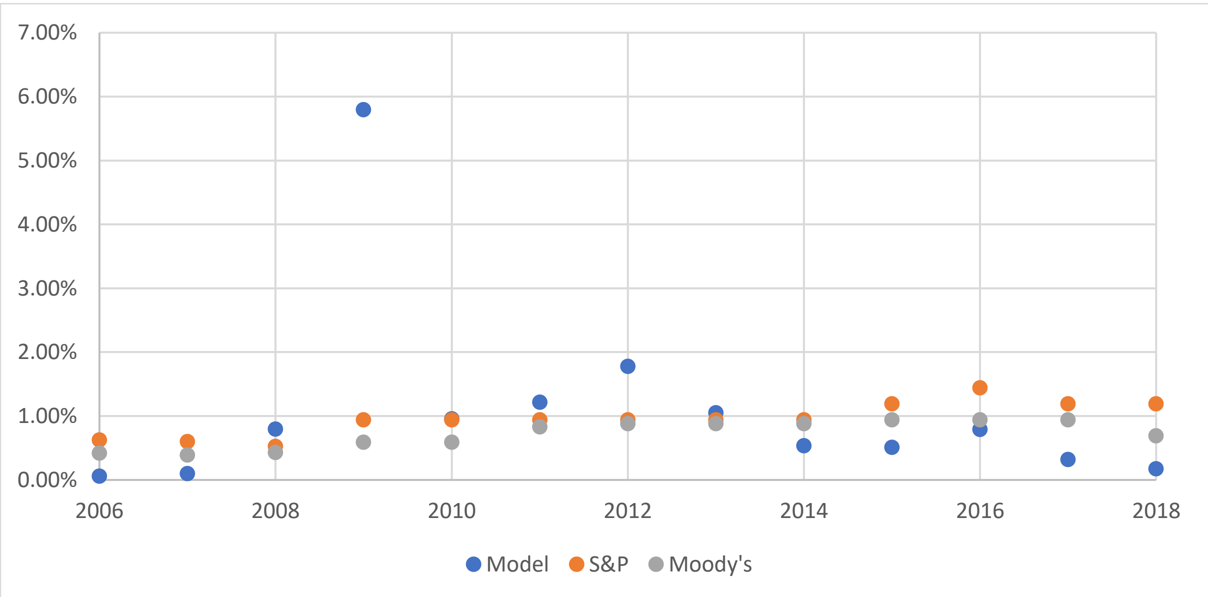


Figure 18 presents a scatter plot of the probabilities of default implied by the model and by the credit ratings. For credit ratings, the average between the probabilities of default implied

by Moody's and Standard & Poor's is presented. Figure 18 also presents the output from fitting a linear regression model to the data points.

Figure 18 – Scatter plot of model and implied rating probabilities of default

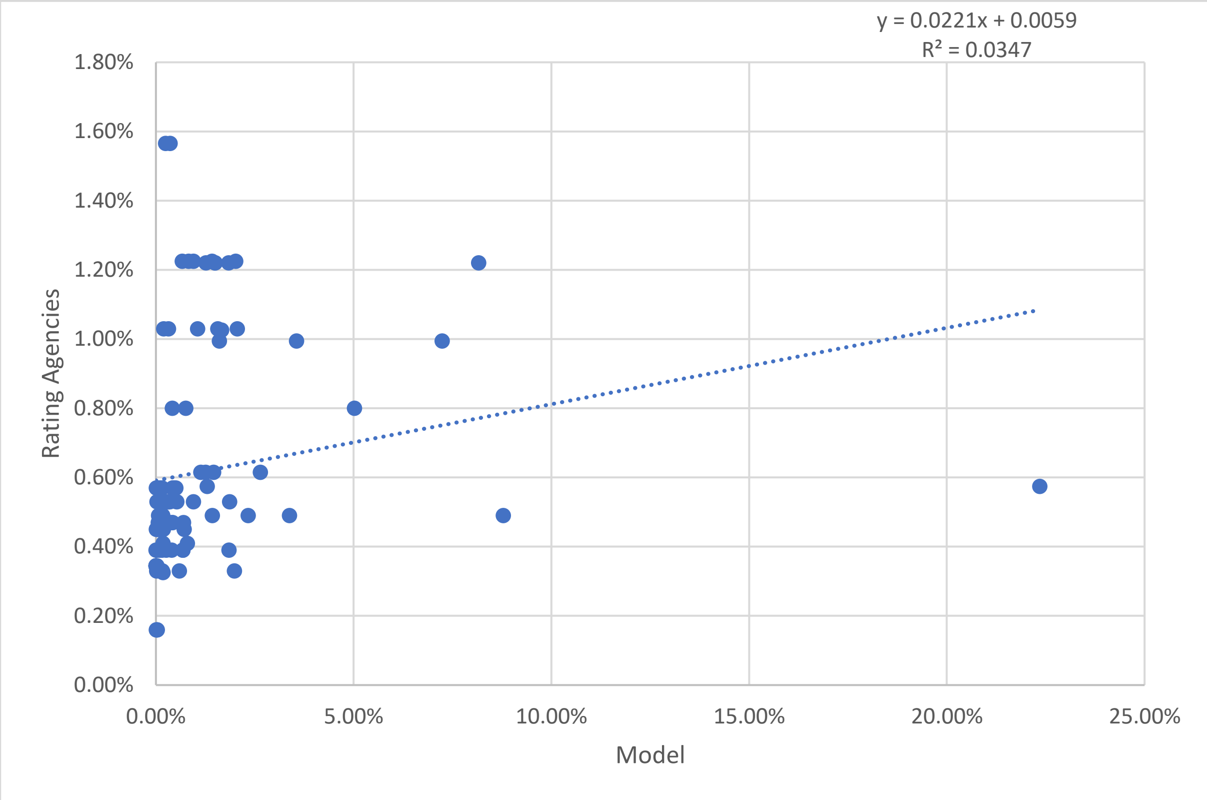


Figure 18 leads us to the conclusion that our model and the implied probabilities from the rating agencies share a slightly positive relation. The measured correlation between the results is 18.63%. Further, the results were tested using a "linear model" function in R, which retrieved a p-value of 0.0634. Thus, the coefficient is not significant at the usual 5% significance level, but it is significant at the less conservative 10% significance level.

5.2.2 Comparison to the inverse normal cumulative DD's

As the comparison between the probabilities of default implied by the credit ratings and the model probabilities of default returned only a weak positive correlation, we decided to test the relation between the symmetric of the inverse normal function of the probabilities of default. This measure is often called the distance-to-default. The excel formula "Norm.Inv" was used on the model PD's and the implied PD's from the credit ratings. This was done because the distance-to-default is generally more stable, meaning that this measure does decrease as much as the probability of default increases in times of crisis neither increases as much as the probability of default decreases in good times. Figure 19 presents a time series for the distance-

to-default implied by the model and by the credit ratings. The average implied distance-to-default ranges from 1.83 in 2009 to 3.52 in 2006. Overall, it is to say that the average value obtained is 2.72. Meanwhile the average value obtained from the implied ratings of Moody's was 2.55 and 2.53 for Standard & Poor's. Even though it is to state that the rating implied DD's don't go as low as the ones from the model in 2009. Before this and after 2014 the ones of the model seem to underestimate in comparison to the rating agencies.

Figure 19 – Correlation between model and agencies implied results

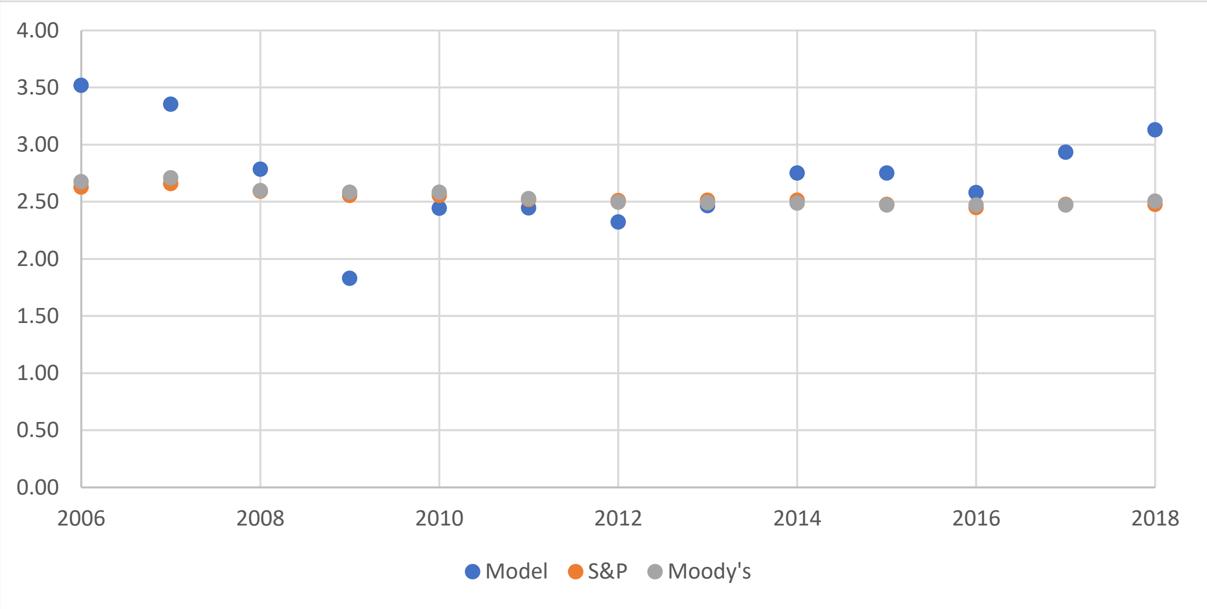
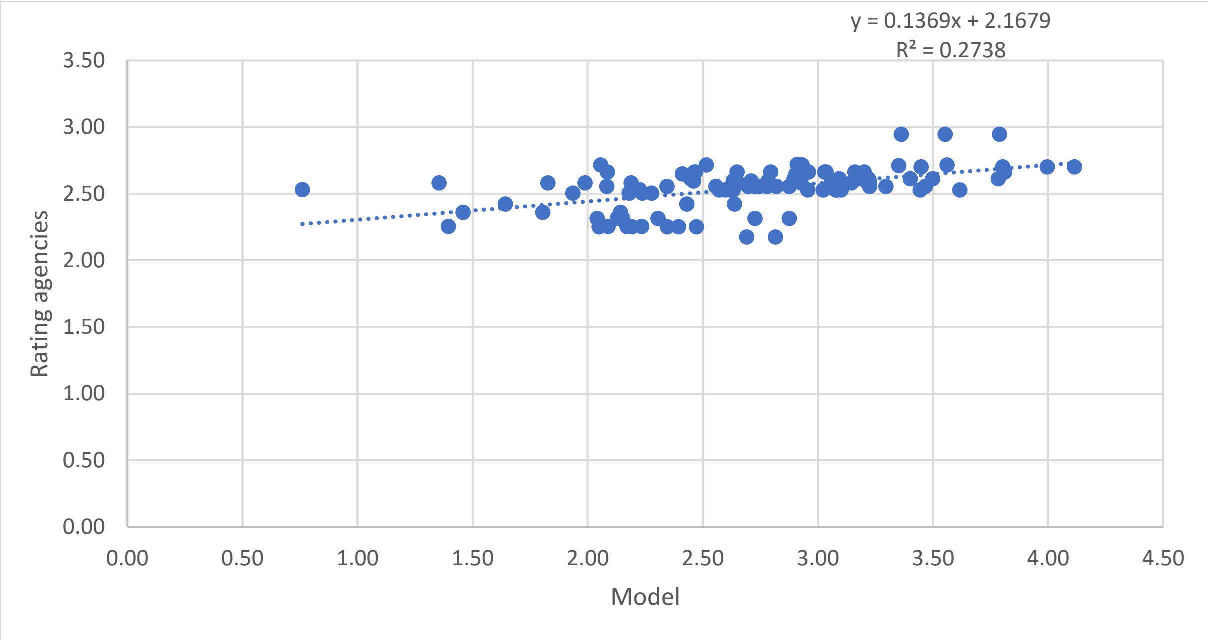


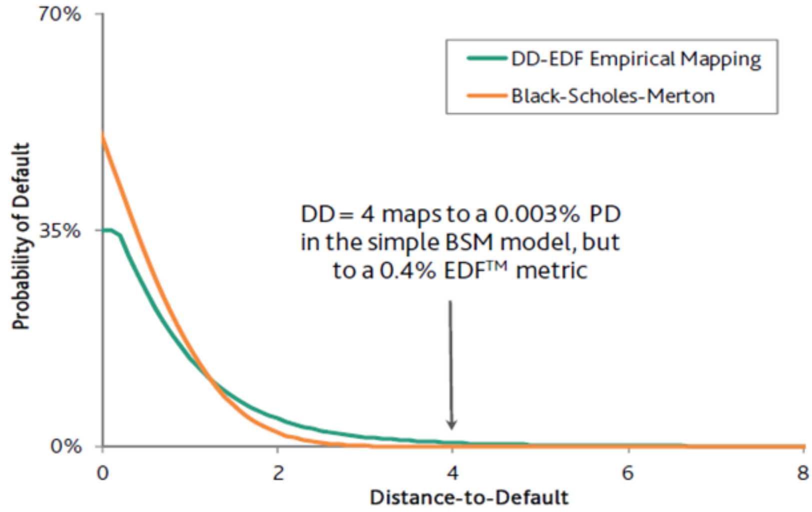
Figure 20 is similar to Figure 18, but this time the distance-to-default is presented. The results from the application on the model PD's are presented in Appendix 10. This time there is a positive correlation of 52.32% and a test of the results using the "linear model" function in R returns that the results are highly significant (p-value: 0.0001, significant for all the usual significance levels). The results of the rating agencies and the model can be described as broadly in line.

Figure 20 – Correlation of rating agencies and model DD's



But why do I obtain a significant relation between the distance-to-default and not the probabilities of default? We are trying to explain this fact by using figure 21 that was taken from the paper of Ferry, Hughes, and Ding (2012). It presents the relation between the distance-to-default computing from Merton's model and the probability of default evaluating that distance-to-default on the Normal distribution and an empirical distribution. It is clear that using the Normal distribution leads to an underestimation of the probability of default for high values of the distance-to-default and an overestimation for low values. The first is often attributed to the risk of sudden jumps which are not accounted in Merton's model. The second may result from many things. One possibility is the lack of mean reversion. In the case of banks there is another reason however that may justify this, which is the presence of government implicit guarantees. There is a vast literature stating that structural models using equity calibrated data tend to ignore these implicit guarantees (Gray and Jobst, 2010). These guarantees benefit the bank's debt holders, while they leave the equity values nearly unchanged. Structural models using the CDS spreads are better able to capture the expected losses suffered by banks debtholders because they take into account the government guarantees.

Figure 21 – Merton output and DD's comparison



6. Conclusion

The goal of this dissertation was the implementation of a structural credit risk model to the eight largest retail banks in the United States. The model chosen for this purpose was inspired by the papers of Goldstein, Ju and Leland (2001) and Eisdorfer, Goyal and Zhdanov (2019). As mentioned, these models were adapted to the case of a retail bank and further calibrated.

The model was applied for the period between 2000 and 2018. The chosen time period covers the effects of the dotcom bubble crisis, the financial crisis of 2008 – 2009, and the European sovereign debt crisis on the probabilities of default. The default probabilities varied over the time from 0.06% on average in 2006 to 5.80% on average in 2009. The probabilities of default peaked in the year 2009 with a value of 5.80% for the whole sector.

The results obtained were compared with the ones implied by the credit ratings given by Standard & Poor's and Moody's. Based on this comparison, one can conclude that, on average, the model overestimates slightly banks' probabilities of default. Interestingly, it was found that the model tends to significantly overestimate the probability of default in crisis times and underestimate it in normal times. The time series dynamics were also compared. The implied probabilities of default were found to have a small positive correlation with the probabilities of default that arise from the application of the model. This correlation is not significant at the 5% significance level, though. Next, correlations were computed using the distance-to-distress instead of the probability of default. In this case, a stronger and clearly significant positive correlation was found. Finally, this dissertation discusses why the relation between the distance-to-default is found be stronger than the relation between the probabilities of default. In addition to non-Normal returns, it is argued that structural models calibrated based on equity data tend to overestimate banks probabilities of default in crisis times because they do not take into account government implicit guarantees. These were shown to be very relevant throughout the financial crisis.

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Appendix

Appendix 1 – Non-interest expense (in $ million)

Date	C	BBT	BAC	COF	PNC	JPM	USB	WFC
2000	11,850.00	481.81	4,482.00	757.05	773.00	5,217.00	1,340.17	2,890.67
2001	11,767.35	480.37	4,679.50	951.51	787.50	5,456.50	1,375.68	3,168.50
2002	9,878.75	539.58	4,731.75	1,134.78	857.00	4,832.00	1,414.93	3,546.00
2003	10,884.50	735.62	4,892.75	1,144.57	857.50	5,377.75	1,409.74	4,167.75
2004	10,584.25	603.51	6,155.25	1,239.17	911.00	6,386.00	1,371.33	4,275.50
2005	11,564.00	765.75	6,924.75	1,421.03	1,037.00	8,898.50	1,438.50	4,768.50
2006	12,638.00	845.42	8,301.00	1,665.87	1,141.00	9,492.67	1,518.50	5,021.25
2007	14,291.00	904.25	8,923.50	1,873.18	1,013.00	10,124.92	1,623.25	5,587.00
2008	15,343.25	956.75	10,055.50	1,838.71	1,120.50	10,673.00	1,790.75	5,622.00
2009	15,085.25	1,095.25	14,695.25	1,815.38	1,988.42	12,576.50	1,967.00	10,507.75
2010	12,060.50	1,362.25	16,261.50	1,918.25	2,140.25	12,939.00	2,281.50	12,484.25
2011	12,795.50	1,400.25	17,726.50	2,161.25	2,181.50	14,478.50	2,328.50	12,527.50
2012	12,575.50	1,471.25	16,843.25	2,688.75	2,561.75	14,702.25	2,581.00	12,498.25
2013	12,712.25	1,455.50	16,399.00	3,020.50	2,541.00	14,865.50	2,335.25	12,415.00
2014	13,396.25	1,438.50	14,982.00	2,969.25	2,380.00	14,721.50	2,512.00	12,118.75
2015	11,896.50	1,418.25	14,380.00	3,126.25	2,401.50	14,180.25	2,616.75	12,512.50
2016	10,714.00	1,610.75	13,485.50	3,251.00	2,357.75	14,025.75	2,758.75	12,959.00
2017	10,494.00	1,689.75	13,539.00	3,443.50	2,444.50	14,512.75	2,869.75	13,735.75
2018	10,619.00	1,727.75	13,287.25	3,546.50	2,695.00	15,660.00	3,176.00	14,003.75

Appendix 2 – Earnings before taxes (in millions $)

Date	C	BBT	BAC	COF	PNC	JPM	USB	WFC
2000	5,493.25	289.05	3,084.50	202.85	468.75	2,541.00	1,170.38	1,637.75
2001	5,449.20	389.85	2,855.50	258.86	166.25	1,291.75	1,003.50	1,369.75
2002	4,998.75	457.49	3,279.75	383.52	474.67	1,453.25	1,382.37	2,213.50
2003	6,642.25	522.93	3,965.25	456.70	400.00	2,547.00	1,427.78	2,410.00
2004	7,491.50	581.97	5,474.00	590.02	436.25	2,798.00	1,592.75	2,691.75

2005	7,358.25	614.75	6,223.00	708.93	491.00	3,809.33	1,643.00	2,933.00
2006	7,409.75	622.75	8,194.50	1,027.91	484.75	4,853.00	1,724.00	3,186.25
2007	5,767.75	647.75	6,746.50	1,032.71	573.50	6,428.50	1,634.25	3,842.25
2008	471.25	780.50	5,390.75	431.36	572.67	5,151.25	1,024.75	2,784.75
2009	-1,978.00	800.25	10,195.25	393.25	1,290.33	7,645.75	688.75	4,752.50
2010	3,297.25	259.50	3,874.25	1,102.50	1,025.25	8,064.75	1,050.00	4,750.25
2011	3,656.00	411.00	1,998.00	1,181.75	1,056.50	7,912.25	1,580.00	5,914.00
2012	1,956.25	715.00	1,443.00	1,266.25	984.00	7,985.50	1,931.50	7,108.50
2013	4,874.25	792.50	5,560.75	1,699.00	1,422.00	9,253.50	1,941.00	8,157.25
2014	3,675.25	823.75	5,813.75	1,711.00	1,403.50	8,386.00	1,998.75	8,478.75
2015	6,206.50	834.75	5,817.50	1,549.25	1,376.75	8,289.50	2,007.50	8,410.25
2016	5,369.25	917.50	6,419.75	1,453.75	1,313.25	8,554.75	2,026.25	8,030.00
2017	5,773.00	958.25	7,496.75	1,456.50	1,372.50	8,810.25	1,879.25	7,769.25
2018	5,861.25	1,051.50	8,895.00	1,898.00	1,607.00	10,209.00	2,169.50	7,134.50

Appendix 3 – State variable (in millions $)

Date	C	BBT	BAC	COF	PNC	JPM	USB	WFC
2000	69,849.0	3,014.1	30,421.0	3,905.1	4,978.0	31,557.0	4,681.5	18,438.0
2001	66,762.2	3,589.7	30,826.0	5,093.4	4,023.0	26,162.0	4,014.0	18,391.0
2002	60,786.0	4,076.2	31,427.0	6,009.7	4,334.7	25,283.0	5,529.5	23,566.0
2003	69,678.0	4,661.4	36,016.0	6,683.5	5,076.0	31,844.0	5,711.1	26,830.0
2004	72,717.0	5,218.2	47,920.0	7,682.3	5,480.0	40,553.0	6,371.0	28,307.0
2005	75,713.0	5,634.0	53,161.0	8,547.3	6,253.0	41,816.3	6,572.0	30,622.0
2006	82,877.0	5,989.0	67,570.0	9,092.7	6,382.0	48,523.0	6,896.0	33,500.0
2007	82,280.0	6,206.0	64,100.0	11,947.6	6,590.0	67,208.0	6,537.0	37,993.0
2008	70,615.0	6,945.0	62,157.0	8,975.4	5,559.7	63,673.0	4,099.0	33,759.0
2009	41,525.0	7,912.0	104,773.0	8,785.0	12,025.3	81,618.0	2,755.0	68,030.0
2010	61,407.0	6,639.0	81,785.0	12,264.0	12,714.0	86,055.0	4,200.0	69,342.0
2011	66,580.0	7,430.0	78,844.0	13,919.0	13,331.0	89,660.0	6,320.0	73,032.0
2012	58,670.0	8,620.0	73,565.0	16,387.0	14,518.0	91,658.0	7,726.0	78,839.0
2013	68,815.0	8,961.0	85,386.0	18,931.0	15,369.0	96,381.0	7,764.0	81,471.0
2014	70,391.0	8,979.0	81,972.0	18,749.0	15,102.0	91,973.0	7,995.0	82,980.0
2015	69,246.0	9,183.0	79,804.0	18,877.0	14,970.0	89,202.0	8,030.0	83,690.0
2016	63,126.0	10,221.0	80,104.0	19,042.0	14,729.0	90,307.0	8,105.0	84,541.0
2017	65,272.0	10,770.0	83,956.0	19,686.0	15,888.0	94,745.0	7,517.0	85,989.0

2018	65,500.0	10,992.0	88,579.0	22,220.0	16,724.0	103,984.0	8,678.0	84,696.0

Appendix 4 – Yearly 5-year probabilities of default (PD)

Date	C	BBT	BAC	PNC	COF	JPM	USB	WFC	Average
2000	0.06%	0.01%	0.17%	0.02%	0.49%	0.55%	0.37%	0.00%	0.21%
2001	0.05%	0.00%	0.18%	0.02%	1.09%	0.31%	0.10%	0.01%	0.22%
2002	0.08%	0.00%	0.07%	0.09%	2.49%	0.81%	0.03%	0.01%	0.45%
2003	0.10%	0.01%	0.08%	0.14%	2.63%	0.96%	0.03%	0.04%	0.50%
2004	0.03%	0.00%	0.04%	0.08%	0.56%	0.27%	0.01%	0.01%	0.13%
2005	0.05%	0.00%	0.03%	0.11%	0.44%	0.32%	0.01%	0.02%	0.12%
2006	0.04%	0.00%	0.02%	0.02%	0.24%	0.12%	0.00%	0.01%	0.06%
2007	0.18%	0.01%	0.04%	0.01%	0.36%	0.17%	0.01%	0.02%	0.10%
2008	3.38%	0.06%	0.72%	0.03%	1.66%	0.40%	0.03%	0.08%	0.80%
2009	22.35%	0.70%	8.79%	1.86%	8.16%	1.85%	0.68%	1.98%	5.80%
2010	1.30%	0.34%	2.34%	0.27%	1.84%	0.69%	0.26%	0.59%	0.95%
2011	1.46%	0.35%	5.02%	0.24%	1.27%	0.80%	0.15%	0.42%	1.21%
2012	2.64%	0.32%	7.24%	0.51%	1.50%	1.43%	0.18%	0.41%	1.78%
2013	1.26%	0.30%	3.56%	0.44%	1.49%	0.95%	0.12%	0.27%	1.05%
2014	1.13%	0.11%	1.61%	0.10%	0.67%	0.53%	0.07%	0.06%	0.54%
2015	0.75%	0.11%	1.56%	0.10%	0.95%	0.42%	0.10%	0.06%	0.51%
2016	1.06%	0.20%	2.06%	0.16%	2.02%	0.47%	0.19%	0.17%	0.79%
2017	0.32%	0.06%	0.42%	0.03%	1.42%	0.12%	0.08%	0.11%	0.32%
2018	0.20%	0.03%	0.10%	0.01%	0.83%	0.05%	0.08%	0.08%	0.17%

Appendix 5 – Barrier-to-asset ratio

Date	C	BBT	BAC	PNC	COF	JPM	USB	WFC	**Average**
2000	27.95%	27.47%	26.97%	30.08%	32.30%	34.12%	30.10%	26.15%	29.39%
2001	26.06%	23.25%	26.17%	28.14%	34.05%	30.58%	22.35%	26.37%	27.12%
2002	27.22%	22.82%	24.15%	33.43%	39.52%	35.41%	21.37%	27.64%	28.95%
2003	27.83%	26.10%	24.07%	35.04%	39.45%	35.83%	20.39%	30.11%	29.85%
2004	25.40%	22.57%	22.45%	33.83%	32.34%	30.81%	17.80%	27.86%	26.63%
2005	26.86%	24.56%	22.06%	35.19%	31.31%	32.16%	18.22%	28.82%	27.40%
2006	28.23%	25.44%	21.89%	32.25%	30.72%	30.38%	18.04%	27.96%	26.86%
2007	30.50%	26.90%	22.70%	28.05%	30.54%	30.27%	18.58%	29.15%	27.09%
2008	46.41%	32.53%	31.80%	31.12%	38.91%	33.80%	21.55%	32.79%	33.61%
2009	60.97%	38.13%	46.33%	44.35%	45.84%	37.49%	29.33%	42.58%	43.13%
2010	39.80%	36.53%	38.94%	36.97%	37.94%	34.74%	27.21%	39.09%	36.40%
2011	41.75%	38.25%	46.04%	37.17%	37.09%	36.88%	26.78%	38.71%	37.83%
2012	43.41%	35.54%	47.32%	38.17%	35.50%	37.82%	25.41%	35.96%	37.39%
2013	36.33%	34.10%	38.84%	35.66%	33.90%	33.63%	22.87%	32.77%	33.51%
2014	36.61%	31.33%	34.57%	31.82%	30.68%	31.62%	21.74%	29.12%	30.94%
2015	33.51%	30.46%	33.56%	30.75%	31.53%	29.84%	22.13%	28.34%	30.02%

2016	34.82%	31.86%	34.52%	31.72%	35.10%	29.76%	23.67%	30.77%	31.53%
2017	30.28%	29.05%	27.49%	27.66%	33.96%	25.71%	21.61%	30.29%	28.26%
2018	30.26%	27.97%	24.15%	27.23%	32.79%	24.40%	22.84%	30.72%	27.55%
Average	34.43%	29.73%	31.26%	33.09%	34.92%	32.38%	22.74%	31.33%	
Volatility	8.99%	5.13%	8.76%	4.32%	4.02%	3.69%	3.64%	4.61%	

Appendix 6 – Distance-to-default yearly values (DD)

Date	C	BBT	BAC	PNC	COF	JPM	USB	WFC	Average
2000	3.49	4.06	3.21	3.93	2.88	2.90	2.98	4.17	3.45
2001	3.51	4.36	3.15	3.94	2.63	3.03	3.54	3.98	3.52
2002	3.57	4.50	3.46	3.53	2.42	2.82	3.72	3.92	3.49
2003	3.39	4.00	3.36	3.24	2.31	2.68	3.72	3.54	3.28
2004	3.66	4.50	3.57	3.41	2.81	3.09	4.06	3.83	3.61
2005	3.53	4.24	3.61	3.31	2.89	2.99	4.01	3.74	3.54
2006	3.59	4.32	3.80	3.73	3.09	3.29	4.19	4.00	3.75
2007	3.30	4.07	3.62	4.05	3.01	3.19	4.04	3.79	3.63
2008	2.37	3.53	2.95	3.71	2.49	2.99	3.74	3.45	3.15
2009	1.49	2.84	1.89	2.47	1.91	2.49	2.84	2.49	2.30
2010	2.55	3.05	2.34	3.07	2.41	2.77	3.10	2.81	2.76
2011	2.60	3.03	2.13	3.15	2.58	2.78	3.24	2.94	2.81
2012	2.29	3.00	1.87	2.84	2.48	2.50	3.15	2.90	2.63
2013	2.55	3.00	2.15	2.94	2.46	2.62	3.26	3.06	2.75
2014	2.58	3.28	2.43	3.31	2.73	2.81	3.41	3.43	3.00
2015	2.72	3.28	2.44	3.32	2.61	2.88	3.31	3.42	3.00
2016	2.59	3.11	2.35	3.18	2.33	2.85	3.13	3.16	2.84
2017	2.98	3.44	2.89	3.64	2.46	3.25	3.39	3.28	3.17
2018	3.13	3.68	3.31	3.83	2.67	3.51	3.40	3.38	3.36
Average	2.94	3.65	2.87	3.40	2.59	2.92	3.49	3.44	
Volatility	58.95%	58.21%	64.28%	41.64%	27.78%	26.86%	39.50%	46.10%	

Appendix 7 – Yearly drift of the project μ_p

Date	C	BBT	BAC	PNC	COF	JPM	USB	WFC	Average
2000	5.72%	5.18%	5.52%	4.98%	5.00%	5.46%	5.95%	4.97%	5.35%
2001	4.77%	4.30%	4.59%	4.05%	4.06%	4.57%	5.05%	4.06%	4.43%
2002	6.25%	5.23%	6.03%	4.86%	5.27%	5.92%	6.27%	4.87%	5.59%
2003	5.45%	4.46%	5.26%	4.09%	4.49%	5.15%	5.50%	4.10%	4.81%
2004	5.84%	4.85%	5.65%	4.48%	4.88%	5.53%	5.88%	4.49%	5.20%
2005	5.90%	4.90%	5.70%	4.53%	4.94%	5.59%	5.94%	4.55%	5.26%
2006	7.34%	6.09%	7.13%	5.65%	6.25%	7.01%	7.30%	5.69%	6.56%
2007	6.37%	5.35%	6.16%	4.98%	5.39%	6.03%	6.38%	4.98%	5.70%
2008	7.19%	5.65%	6.94%	5.15%	5.94%	6.81%	7.03%	5.21%	6.24%

2009	4.75%	3.75%	4.57%	3.38%	3.79%	4.42%	4.77%	3.36%	4.10%
2010	5.80%	4.54%	5.58%	4.11%	4.70%	5.46%	5.75%	4.15%	5.01%
2011	7.27%	5.48%	7.02%	4.92%	5.90%	6.86%	7.01%	5.00%	6.18%
2012	5.39%	3.85%	5.16%	3.35%	4.15%	4.97%	5.19%	3.38%	4.43%
2013	3.93%	2.93%	3.75%	2.55%	2.97%	3.60%	3.95%	2.54%	3.28%
2014	4.31%	3.30%	4.10%	2.93%	3.33%	3.99%	4.34%	2.94%	3.65%
2015	3.79%	2.79%	3.59%	2.42%	2.82%	3.47%	3.83%	2.43%	3.14%
2016	3.45%	2.45%	3.25%	2.08%	2.49%	3.14%	3.49%	2.10%	2.81%
2017	3.99%	3.00%	3.79%	2.62%	3.03%	3.68%	4.03%	2.64%	3.35%
2018	5.27%	4.03%	5.07%	3.60%	4.19%	4.95%	5.23%	3.64%	4.50%
Average	5.41%	4.32%	5.20%	3.93%	4.40%	5.08%	5.42%	3.95%	4.72%

Appendix 8 – Implied probabilities of default (S&P)[6]

Date	C	BBT	BAC	PNC	COF	JPM	USB	WFC	**Average**
2006	0.32%	0.33%	0.33%	0.45%	2.11%	0.45%	0.36%	0.16%	0.62%
2007	0.32%	0.45%	0.16%	0.45%	2.11%	0.33%	0.36%	0.16%	0.60%
2008	0.49%	0.45%	0.45%	0.45%	1.03%	0.45%	0.36%	0.33%	0.53%
2009	0.49%	0.49%	0.49%	0.49%	1.42%	0.45%	0.45%	0.33%	0.61%
2010	0.49%	0.49%	0.49%	0.49%	1.42%	0.45%	0.45%	0.33%	0.61%
2011	0.57%	0.57%	0.57%	0.49%	1.42%	0.49%	0.45%	0.45%	0.65%
2012	0.57%	0.57%	0.57%	0.57%	1.42%	0.49%	0.49%	0.45%	0.67%
2013	0.57%	0.57%	0.57%	0.57%	1.42%	0.49%	0.45%	0.45%	0.66%
2014	0.57%	0.57%	0.57%	0.57%	1.42%	0.49%	0.45%	0.45%	0.66%
2015	0.57%	0.57%	1.03%	0.57%	1.42%	0.57%	0.45%	0.49%	0.74%
2016	1.03%	0.57%	1.03%	0.57%	1.42%	0.57%	0.45%	0.49%	0.81%
2017	1.03%	0.57%	0.57%	0.57%	1.42%	0.57%	0.45%	0.49%	0.74%
2018	1.03%	0.57%	0.57%	0.57%	1.42%	0.57%	0.45%	0.49%	0.74%

Appendix 9 – Implied probabilities of default (Moody's)[7]

Date	C	BBT	BAC	PNC	COF	JPM	USB	WFC	Average
2006	3.35	4.12	3.56	3.50	2.82	3.04	4.00	3.79	3.52
2007	2.91	3.81	3.36	3.78	2.69	2.93	3.80	3.55	3.36
2008	1.83	3.22	2.45	3.40	2.13	2.65	3.45	3.16	2.79
2009	0.76	2.46	1.35	2.08	1.39	2.09	2.47	2.06	1.83
2010	2.23	2.71	1.99	2.78	2.09	2.46	2.80	2.52	2.45
2011	2.18	2.70	1.64	2.82	2.24	2.41	2.96	2.63	2.45
2012	1.94	2.73	1.46	2.57	2.17	2.19	2.91	2.65	2.33

[6] Data collected from the *S&P Global Fixed Income Research 2018 annual global corporate report default study and rating transition report*

[7] Data collected from the *S&P Global Fixed Income Research 2018 annual global corporate report default study and rating transition report*

2013	2.24	2.75	1.80	2.62	2.17	2.34	3.03	2.78	2.47
2014	2.28	3.06	2.14	3.08	2.47	2.56	3.20	3.22	2.75
2015	2.43	3.05	2.15	3.10	2.35	2.63	3.09	3.22	2.75
2016	2.31	2.88	2.04	2.96	2.05	2.60	2.90	2.93	2.58
2017	2.73	3.22	2.64	3.45	2.19	3.02	3.17	3.06	2.94
2018	2.88	3.47	3.08	3.62	2.40	3.30	3.17	3.15	3.13
Average	2.31	3.09	2.28	3.06	2.24	2.63	3.15	2.98	2.72

Appendix 10 - Inverse normal function on model PD's (yearly results)

Date	C	BBT	BAC	PNC	COF	JPM	USB	WFC	Average
2006	3.35	4.12	3.56	3.50	2.82	3.04	4.00	3.79	3.52
2007	2.91	3.81	3.36	3.78	2.69	2.93	3.80	3.55	3.36
2008	1.83	3.22	2.45	3.40	2.13	2.65	3.45	3.16	2.79
2009	0.76	2.46	1.35	2.08	1.39	2.09	2.47	2.06	1.83
2010	2.23	2.71	1.99	2.78	2.09	2.46	2.80	2.52	2.45
2011	2.18	2.70	1.64	2.82	2.24	2.41	2.96	2.63	2.45
2012	1.94	2.73	1.46	2.57	2.17	2.19	2.91	2.65	2.33
2013	2.24	2.75	1.80	2.62	2.17	2.34	3.03	2.78	2.47
2014	2.28	3.06	2.14	3.08	2.47	2.56	3.20	3.22	2.75
2015	2.43	3.05	2.15	3.10	2.35	2.63	3.09	3.22	2.75
2016	2.31	2.88	2.04	2.96	2.05	2.60	2.90	2.93	2.58
2017	2.73	3.22	2.64	3.45	2.19	3.02	3.17	3.06	2.94
2018	2.88	3.47	3.08	3.62	2.40	3.30	3.17	3.15	3.13
Average	2.31	3.09	2.28	3.06	2.24	2.63	3.15	2.98	2.72

Appendix 11 – Model code in R

```
#GJL model functions
x_function <-function(rf, k, sig_a) {
  miu <- rf-k
  a <- sig_a^2/2
  b <- 2*rf*sig_a^2
  c <- (miu-a)^2
  d <- miu-a
  e <- d+sqrt(c+b)
  x <- e/sig_a^2
  return(x)
}
```

```
y_function <- function(rf, k, sig_a) {
  miu <- rf-k
  a <- sig_a^2/2
  b <- 2*rf*sig_a^2
  c <- (miu-a)^2
  d <- miu-a
  e <- d-sqrt(c+b)
  y <- e/sig_a^2
  return (y)
}

#Default Barrier Function
#found by invoking smooth pasting condition
v_b_function <- function( rf, k, sig_a, C) {
  l_d <- x_function(rf=rf, k=k, sig_a=sig_a) / (x_function(rf=rf, k=k, sig_a=sig_a)+1)
  V_b <- l_d*C*(1/rf)
  return (V_b)
}
#v_b_function(rf,k,sig_a,C=Non)

p_b_function <- function( v_a, rf, k, sig_a, C){
  R<- v_a/v_b_function(rf=rf, k=k, sig_a=sig_a, C=C)
  p_b <- R^(-1*x_function(rf=rf, k=k, sig_a=sig_a))
  return ((R>1)*p_b+(R<=1)*10^10)
  return (p_b)
}
#p_b_function(v_a,rf,k,sig_a,C)


v_non_function <- function( v_a, rf, k, sig_a, C ){
  v_non <- (1-p_b_function( v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C ))*C/rf
  return(v_int)
}
```

```
v_solv_function <- function(v_a, rf, k, sig_a, C){
  v_solv <- v_a - v_b_function( rf=rf, k=k, sig_a=sig_a, C=C )*p_b_function( v_a=v_a, rf=rf,
k=k, sig_a=sig_a, C=C)
  return(v_solv)
}

e_function <- function( v_a, rf, k, sig_a, C, TaxCorp, TaxDiv){
  Tx_eff <- (1-TaxCorp)*(1-TaxDiv)
  return(Tx_eff*(v_solv_function( v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C)-v_non_function (
v_a=v_a, rf=rf, k=k, sig_a=sig_a, C=C )))
}
#data<-WFC
#data<-JPM
#data<-BBT
#data<-PNC
#data<-USB
#data<-BAC
#data<-WFC
xpto<-as.matrix(data)

Time   <- xpto[,0]
date    <- as.numeric(xpto[,1])
EBT     <- as.numeric(xpto[,2])
Equity  <- as.numeric(xpto[,3])
Non     <- as.numeric(xpto[,4])
RF      <- as.numeric(xpto[,6])*0.01


FindV <- function(x, k, sig_a, TimeM) {
  ModelEquity<-e_function(v_a=x, rf=RF[TimeM], k=k, sig_a, C=Non[TimeM],
TaxCorp=0.2, TaxDiv=0.2)
  #print(ModelEquity)
  return(Equity[TimeM]-(ModelEquity>0 & ModelEquity<x)*ModelEquity)
```

```
}


#Finds the project value that matches equity value
Vfunction <- function(k, sig_a){
BBsolve(par=v_b_function(rf=RF,k=k,sig_a=sig_a,C=Non)+100000,fn=FindV,k=k,sig_a=si
g_a,1: 988)$par
}
#Vfunction(k,sig_a)

#Finds sig_a1
FindAssetVol <- function (k, Start_sig_a) {
  Error <- 10^10
  while (Error > 0.00001){
    RecoveryAssetVec_1 <- Vfunction(k, sig_a=Start_sig_a)
    log_ret <- diff(log(RecoveryAssetVec_1),lag=1)
    sig_a1 <-  sd(log_ret)*sqrt(52)
    Error<- abs(Start_sig_a-sig_a1)
    Start_sig_a <- sig_a1
  }
  return(sig_a1)
}

# K as average of  (EBIT+Non)/AssetVector
Findk<- function(Start_k, sig_a){
  Error <- 10^10
  for (i in 1: 988)
    while (Error>0.00001){
      RecoveryAssetVec_1 <- Vfunction(k=Start_k, sig_a)
      k_a1 <- (sum(EBT[1:988])+sum(Non[1:988]))/sum(RecoveryAssetVec_1)
      Error<- abs(Start_k-k_a1)
      Start_k <- k_a1
    }
  return(k_a1)
```

```
  return(RecoveryAssetVec_1)
}

FindEstimates<- function(Start_k, Start_sig){
  Error <- 10^10
  for (i in 1:992)
    while (Error>0.00001){
      sig_a1 <-FindAssetVol(k= Start_k , Start_sig_a= Start_sig )
      RecoveryAssetVec_1 <- Vfunction(k= Start_k,sig_a= Start_sig )
      k_a1 <- (sum(EBT[1:988])+sum(Non[1:988]))/sum(RecoveryAssetVec_1)
      Error<- abs(Start_k-k_a1)
      Start_k <- k_a1
    }
  return(sig_a1)

}

FindEstimates2<- function(Start_k, Start_sig){
  Error <- 10^10
  for (i in 1:992)
    while (Error>0.00001){
      sig_a1 <-FindAssetVol(k= Start_k , Start_sig_a= Start_sig )
      RecoveryAssetVec_1 <- Vfunction(k= Start_k,sig_a= Start_sig )
      k_a1 <- (sum(EBT[1:998])+sum(Non[1:998]))/sum(RecoveryAssetVec_1)
      Error<- abs(Start_k-k_a1)
      Start_k <- k_a1
    }
  return(k_a1)
}

#run the iterative approach at once and safe the values in sig_a & k
Sig_K <-FindEstimates(Start_k=0.05,Start_sig=0.2)
sig_a <- Sig_K[1]
```

```
Sig_K1 <-FindEstimates2(Start_k=0.05,Start_sig=0.2)
k    <- Sig_K1[1]

# Define Market price risk
EQRP <- as.numeric(xpto[,7])
beta <- mean(as.numeric(xpto[,8]))
miu_e<- RF+beta*(EQRP)

#Computing standard deviation of Equity and cleaning from outliers
sig_e1      <- sd(diff(log(Equity[1: 988])))
all_outliers<- diff(log(Equity[1: 988]))
limits <- 3*sig_e1
all_outliers<- all_outliers[!(all_outliers> limits)]
all_outliers<- all_outliers[!(all_outliers< -limits)]
b           <- boxplot(all_outliers)
sig_e1      <-sd(all_outliers)*sqrt(52)

Mk_Rsk <- function(EQRP,beta,sig_e1){
  Mk_Rsk    <- (beta*EQRP)/sig_e1
  return(Mk_Rsk)
}
Mk_Rsk <- Mk_Rsk(EQRP,beta,sig_e1)

# Sigma as standard deviation of log returns
Findmiu_a <- function(beta, EQRP, sig_a,Mk_Rsk){
  miu_a     <- RF+Mk_Rsk*sig_a
  return(miu_a)
}
miu_a <- Findmiu_a(beta, EQRP, sig_a,Mk_Rsk)

#Find miu_d => drift of the project/process
miu_d_function <- function(beta,EQRP,sig_e1,sig_a,k,Mk_Rsk){
  miu_d     <- RF+Mk_Rsk*sig_a-k
  return(miu_d)
```

```
}
miu_d <- miu_d_function(beta, EQRP, sig_e1,sig_a,k,Mk_Rsk)

#Probability of default function

RecoveryAssetVec_1 <- Vfunction(k,sig_a)
Barrier<- 1: 988
for(i in 1: 988){
  Barrier[i]<-v_b_function(rf=RF[i], k, sig_a, C=Non[i])
}
V_b_Ratio <- Barrier / RecoveryAssetVec_1
max(Barrier/RecoveryAssetVec_1)

#Gives distance to distress at each moment in time
D2D<-function(k, sig_a, miu_a, years, Time){
  Delta_t <- 1/52
  TimeT   <- 52*years
  a <- (miu_a - k - (sig_a^2/2))
  b <- TimeT*Delta_t
  c <- log(RecoveryAssetVec_1[Time]/v_b_function(rf=RF[Time], k=k, sig_a=sig_a,
C=Non[Time]))
  d <- sig_a*sqrt(b)
  e<- (c+a*b)/d
}
#Gives Probability of V being below V_B at time T (ignores first passage time)
AuxProbability<-function(k, sig_a, miu_a, years, Time){
  pnorm(-D2D(k, sig_a, miu_a, years, Time))
}
#Gives the probability of defaulting in "years"-years at time "Time"
PDfunc <- function(k, sig_a, miu_a, years, Time)
{
  Delta_t <- 1/52
  TimeT   <- 52*years
  a <- (miu_a - k - (sig_a^2/2))
```

```
  b <- TimeT*Delta_t
  c <- log(v_b_function(rf=RF[Time], k=k, sig_a=sig_a,
C=Non[Time])/RecoveryAssetVec_1[Time])
  d <- sig_a*sqrt(b)
  e <- pnorm(((a*b)-c)/d)
  f <- exp((2/sig_a^2)*a*c)*pnorm(((a*b)+c)/d)
  g <- e-f
  return(1-g)
}

PD_Series <- 1: 988
D2D_Series <- 1: 988
PD_Series_aux1 <- 1: 988

#Computes output
for(i in 1:988){
  D2D_Series[i]<-D2D(k, sig_a, miu_a=miu_a[i], years=5,Time=i)
  PD_Series_aux1[i]<-AuxProbability(k, sig_a, miu_a[i], years=5,Time=i)
  PD_Series[i] <- PDfunc( k, sig_a, miu_a[i], years=5, Time =i )
}

#Output
Output <- do.call(rbind.data.frame, Map('c',V_b_Ratio, PD_Series, PD_Series_aux1,
D2D_Series, RecoveryAssetVec_1, sig_a, k, sig_e1,miu_a,miu_d,miu_e,Mk_Rsk))
write.table(Output, file = "new_WFC2.csv",  sep=";", dec = ".")
```